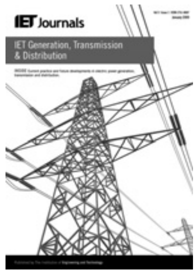
Published in IET Generation, Transmission & Distribution
Received on 16th June 2009
Revised on 30th September 2009
doi: 10.1049/iet-gtd.2009.0317



ISSN 1751-8687

Method for determining the maximum allowable penetration level of distributed generation without steady-state voltage violations

H.M. Ayres[1] *W. Freitas*[1] *M.C. De Almeida*[2] *L.C.P. Da Silva*[1]
[1]*Department of Electrical Energy Systems, University of Campinas, UNICAMP/FEEC/DSEE, Av. Albert Einstein, 400, 13083-852, Campinas, SP, Brazil*
[2]*Electrical Energy Department, DEE/CT/UFRN, CEP, 59072-970, Natal, RN, Brazil*
E-mail: walmir@dsee.fee.unicamp.br

Abstract: One of the main factors that may limit the penetration level of distributed generation (DG) in typical distribution systems is the steady-state voltage rise. The maximum amount of active power supplied by distributed generators into each system bus without causing voltage violations can be determined by using repetitive power flow studies. However, this task is laborious and usually time-consuming, since different loading level and generation operation modes have to be evaluated. Therefore this article presents a method that, based on only one power flow solution and one matrix operation, can directly determine the maximum power that can be injected by distributed generators into each system bus without leading to steady-state voltage violations. This method is based on the determination of voltage sensitivities from a linearised power system model. In addition, this article proposes a numerical index to quantify the responsibility of each generator for the voltage level rise in a multi-DG system. Based on this index, utility managers can decide which generators, and in which degree, should be penalised by the voltage rise or rewarded by not depreciating the voltage profile. The method is applied to a 70-bus distribution network. The results are compared with those obtained by repetitive power flow solutions in order to validate the proposed method.

1 Introduction

Currently, the usage of distributed generation (DG) is considerably increasing [1–15]. Supported by current political policies and global environmental issues, DG is expected to play an important role in electrical power systems in the future. Defined as generators installed directly in distribution systems or customer sites, DG technologies include combustion turbines, gas turbines, photovoltaic arrays, fuel cells and wind turbines [3, 4]. Although the insertion of DG plants into the distribution systems may benefit utilities, customers and the environment, DG may also cause operation and safety problems [3, 6, 14–16]. Characteristics such as the size, location and operation mode of the distributed generators are decisive in determining the impacts of DG on a distribution network. One of the most important technical problems concerning the installation of distributed generators in distribution networks is the steady-state voltage rise [3, 17]. Indeed, recently, this subject has called considerable attention of the technological community and many works have been devoted to deal with this important subject [18–26]. Utility engineers cannot easily determine the maximum active power that distributed generators can inject into the system without causing steady-state voltage violations as well as identify the generators responsible for the voltage rise in a multi-DG system. In order to obtain a generalised picture, successive power flow studies based on, for example, a try-and-error approach must be carried out. After running the power flow studies, the results must be post-processed in order to obtain useful information. Moreover, the analysis should be repeated for different load levels.

The problem of optimal siting and sizing of DG has been investigated by a number of works [5–13]. In some works [5, 6], the problem has been approached by using metaheuristic-based methods (or other non-classical methods), which demand the solution of several power flows. In other works [7–13], AC optimal power flow, or other classical techniques, has also been successfully applied to analyse this problem. The purpose of the method presented in this paper is not to replace any optimisation (classical or not)-based approach or even to be a complete method for DG analysis. The idea is to present a very simple approach based on the Jacobian sensitivities to direct estimation of the maximum amount of active power that distributed generators can inject into each system bus without causing voltage limit violation. This method is based on only one matrix operation and one power flow solution and assuming no substantial changes in the system structure. It can also be useful, for example, for optimum short-term operation planning or optimum allocation and sizing of distributed generators based on optimisation techniques. The linearised method is based on the determination of voltage sensitivities with relation to active and reactive power injections. The impact of the location, generation level and generator operation mode on the voltage profile can be estimated by using the proposed method. The comparison of the results obtained by the proposed method with those provided by repetitive power flow solutions demonstrates that the method has a very good accuracy.

In addition, a numerical index is proposed to determine the responsibility of each generator for the voltage rise in a multi-DG system. By using this index, the contribution of each generator on the nodal voltage variations can be calculated. As a result, the responsible generators can be penalised (or, sometimes, rewarded) by the voltage rising to an appropriate degree. This numerical index can be used in a constrained optimisation process in which additional voltage control, as load tap changer adjustment and shunt compensation switching, can be considered to maximise the production of DGs whereas nodal voltages are kept within their limits.

This paper is organised as follows. Section 2 describes the usage of the voltage sensitivities related to active and reactive power injections to estimate the system voltage profile in the presence of DG. Section 3 presents the proposed method for estimating the maximum power that distributed generators can inject into each system bus without causing voltage violations. Extensive validation studies are presented in Section 4 by using repetitive power flows. The proposed formulation cannot explicitly handle the presence of voltage regulation devices; this limitation is discussed in Section 5 taking into consideration step-type voltage regulators and shunt compensators. An appropriate index to determine the responsibility of each generator for the voltage level rise, in a multi-DG system, is presented in Section 6. A brief analysis of the computational requirements of the proposed method is presented in Section 7. Section 8 summarises the conclusions of the paper. Additional validation tests are presented in Appendix A. The usage of repetitive power flows to determine the maximum penetration level per bus is explained in Appendix B.

2 Voltage sensitivities determination

The proposed method for estimating the maximum allowable power injection into each system bus, which is presented in the next section, is based on the voltage sensitivities related to active and reactive power injections. Such sensitivities are obtained from the load flow Jacobian matrix, which can be determined from the linearised power system model for a given base case ($\boldsymbol{V}^0$, θ^0) as

$$\begin{bmatrix} \Delta \boldsymbol{P} \\ \Delta \boldsymbol{Q} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{J}_{P\theta} & \boldsymbol{J}_{PV} \\ \boldsymbol{J}_{Q\theta} & \boldsymbol{J}_{QV} \end{bmatrix}}_{\boldsymbol{J}} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta \boldsymbol{V} \end{bmatrix} \tag{1}$$

where Δ denotes small variations in the variables. The elements of the Jacobian matrix ($\boldsymbol{J}$) represent the sensitivities among the power injection variations ($\Delta \boldsymbol{P}$, $\Delta \boldsymbol{Q}$) and voltage variations ($\Delta \boldsymbol{V}$, $\Delta \boldsymbol{\theta}$) at the buses of the system. In this work, matrices and vectors are represented by boldface and italic characters, whereas matrix and vector elements are represented by only italic characters.

2.1 V-P sensitivity

Supposing that $\Delta \boldsymbol{Q} = 0$ and $\boldsymbol{J}_{Q\theta}^{-1}$ is non-singular, (1) can be rewritten as

$$\Delta \boldsymbol{P} = (\boldsymbol{J}_{PV} - \boldsymbol{J}_{P\theta}\boldsymbol{J}_{Q\theta}^{-1}\boldsymbol{J}_{QV})\Delta \boldsymbol{V} = \boldsymbol{J}_{RPV}\Delta \boldsymbol{V} \tag{2}$$

and

$$\Delta \boldsymbol{V} = \boldsymbol{J}_{RPV}^{-1}\Delta \boldsymbol{P} \tag{3}$$

where $\boldsymbol{J}_{RPV}$ is a reduced Jacobian matrix, which gives the voltage magnitude variations due to active power injection variations. The inverse of $\boldsymbol{J}_{Q\theta}$ matrix is feasible only if all buses are modelled as PQ buses, guaranteeing that $\boldsymbol{J}_{Q\theta}$ is a square matrix. This situation normally occurs in distribution systems, where the slack bus is the only bus that keeps a fixed voltage magnitude. In addition, DG plants are usually modelled as PQ buses since they do not contribute to the voltage control of the system [3].

The matrix $\boldsymbol{J}_{\mathrm{PV}}$ can be used directly in order to indicate which buses of the system will be more or less affected by the installation of a DG unit. However, matrix $\boldsymbol{J}_{\mathrm{PV}}$ by itself does not give sufficient information about the sensitivities

doi: 10.1049/iet-gtd.2009.0317

because the other matrices $J_{P\theta}$, $J_{Q\theta}$ and J_{QV} are neglected. On the other hand, matrix J_{RPV} is obtained without any approximation with respect to the characteristics of the system, since the relationships among variables V, θ, P and Q are preserved.

Equation (3) can be used to estimate the impact of multi-DG systems by representing ΔP as a diagonal matrix, with one entry of active power injection for each generator. In this case, each column of ΔV will represent the impact of one generator on the system voltage profile. The full voltage profile would be the summation of the several columns of ΔV. The disadvantage of (3), however, is that only unity power factor generators can be considered. This drawback can be solved by using the V-Q sensitivity, as explained in the next subsection.

2.2 V-Q sensitivity

Analogously to the V-P sensitivities, V-Q sensitivities can be determined by assuming $\Delta P = 0$ in (1), resulting in

$$\Delta \boldsymbol{Q} = (\boldsymbol{J}_{QV} - \boldsymbol{J}_{Q\theta}\boldsymbol{J}_{P\theta}^{-1}\boldsymbol{J}_{PV})\Delta \boldsymbol{V} = \boldsymbol{J}_{RQV}\Delta \boldsymbol{V} \tag{4}$$

and

$$\Delta \boldsymbol{V} = \boldsymbol{J}_{RQV}^{-1}\Delta \boldsymbol{Q} \tag{5}$$

where J_{RQV} is a reduced Jacobian matrix, which states the voltage magnitude variations with relation to the reactive power injection variations.

Equation (5) allows the estimation of the impact of generators with different power factors on the system voltage profile. Again, ΔQ can be organised as a diagonal matrix, whose elements would represent the absorption or injection of reactive power of each individual generator.

2.3 Application of V-P and V-Q sensitivities simultaneously

The information obtained from these two sensitivity matrices (J_{RPV} and J_{RQV}) permits the estimation of the voltage variations due to the installation of one or a group of generators with any desired power factor. By considering P_{DG} as a diagonal matrix whose elements represent the active power injection of each generator and Q_{DG} as a diagonal matrix whose elements represent the reactive power injection/absorption into the network, the voltage profile variation due to these additional generators can be estimated, respectively, as

$$\Delta \boldsymbol{V}_{(P_{\mathrm{DG}})} = \boldsymbol{J}_{RPV}^{-1}\boldsymbol{P}_{\mathrm{DG}} \quad \text{and} \quad \Delta \boldsymbol{V}_{(Q_{\mathrm{DG}})} = \boldsymbol{J}_{RQV}^{-1}\boldsymbol{Q}_{\mathrm{DG}} \tag{6}$$

where $\Delta V_{(P_{\mathrm{DG}})}$ and $\Delta V_{(Q_{\mathrm{DG}})}$ are, respectively, matrices that reflect the voltage profile deviation due to the new active and reactive power injections of the generators, assuming their installation at any bus of the system with respect to the base case. If just one generator is considered, matrices ΔV will provide just one non-null column. On the other hand, if a multi-DG case is considered, matrices ΔV will provide a non-null column for each generator, which should be summed in order to build up the new system voltage profile.

Therefore the estimated voltage profile after the installation of one or a group of new generators can be analytically expressed by

$$\boldsymbol{V} = \boldsymbol{V}^0 + \Delta \boldsymbol{V}_{(P_{\mathrm{DG}})} + \Delta \boldsymbol{V}_{(Q_{\mathrm{DG}})} \tag{7}$$

where V^0 is the voltage profile for the base case. The reactive power impact on the voltage profile can be negative or positive, depending on the generator power factor. The capacitive power factor leads to a voltage rise (positive signal) and inductive power factor to a voltage drop (negative signal).

Equation (7) allows one to estimate the voltage profile when the generator is installed at every possible bus of the system, with any lead or lag power factor and with any specified generation level. The simultaneous usage of the V-P and V-Q sensitivities to determine the voltage profile variation due to the installation of distributed generators is validated in Appendix A. In the following section, these sensitivities are used in the proposed method for estimating the maximum allowable power injection of distributed generators.

3 Determination of the maximum allowable power

The determination of the maximum power that a generator can inject into a system without causing steady-state voltage violations is difficult. Usually, this task is carried out based on successive power flow solutions, which have to be redone for different load levels. However, the maximum allowable active power injection can be directly determined by manipulating (7), which describes the effect of the installation of new distributed generators on the steady-state voltage profile. Assuming that the reactive power of a generator k can be expressed as a function of its power factor pf (i.e. $Q_{\mathrm{DG}k} = P_{\mathrm{DG}k}\ \tan[\cos^{-1}(\mathrm{pf}_k)]$), the impact of a DG unit k on the nodal voltage of bus m, can be expressed by

$$V_m = V_m^0 + J_{PQ_{mk}} P_{\mathrm{DG}_k} \tag{8}$$

where $J_{PQ_{mk}} = J_{RPV_{(m,k)}}^{-1} + J_{RQV_{(m,k)}}^{-1} \tan[\cos^{-1}(\mathrm{pf}_k)]$ is the equivalent active and reactive sensitivity, with the reactive sensitivity weighed by the generator power factor. From (8), the maximum active power that a DG unit with a given power factor installed at bus k can export to the

doi: 10.1049/iet-gtd.2009.0317

system without violating the superior voltage limit of bus m is

$$P^{m}_{DG_k} = \frac{\Delta V_m}{J_{PQ_{mk}}} \tag{9}$$

where $\Delta V_m = V^{\max} - V^0_m$ and $V^{\max}$ is the superior voltage limit considered for the system, typically 5%.

One DG unit can affect all the nodal voltages of the system; thus, the maximum amount of active power that can be injected at bus k will be different if the violation is considered for the different buses of the system. Consequently, the maximum power that can be injected at bus $k(P^{\max}_{\mathrm{DG}_k})$ is the minimum amount of power necessary to violate at least one nodal voltage of the system, and it can be expressed by

$$P^{\max}_{\mathrm{DG}_k} = \min\{P^1_{\mathrm{DG}_k}\ P^2_{\mathrm{DG}_k} \cdots P^n_{\mathrm{DG}_k}\} \tag{10}$$

where n is the number of buses of the network. A generalised form of (10) can be written as

$$\boldsymbol{P}^{\max}_{\mathrm{DG}} = \min(J^*_{PQ} \cdot \Delta V) = \min\left(\begin{bmatrix} J^*_{PQ11} & \cdots & J^*_{PQ1n} \\ \vdots & \ddots & \vdots \\ J^*_{PQn1} & \cdots & J^*_{PQnn} \end{bmatrix}_{n\times n} \times \begin{bmatrix} \Delta V_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Delta V_n \end{bmatrix}_{n\times n}\right) \tag{11}$$

$$\boldsymbol{P}^{\max}_{\mathrm{DG}} = \min\left(\begin{bmatrix} P^1_{\mathrm{DG}_1}\ P^2_{\mathrm{DG}_1} \cdots P^n_{\mathrm{DG}_1} \\ \vdots \\ P^1_{\mathrm{DG}_n}\ P^2_{\mathrm{DG}_n} \cdots P^n_{\mathrm{DG}_n} \end{bmatrix}_{n\times n}\right) = \begin{bmatrix} \min\{P^1_{\mathrm{DG}_1}\ P^2_{\mathrm{DG}_1} \cdots P^n_{\mathrm{DG}_1}\} \\ \vdots \\ \min\{P^1_{\mathrm{DG}_n}\ P^2_{\mathrm{DG}_n} \cdots P^n_{\mathrm{DG}_n}\} \end{bmatrix}_{n\times 1} = \begin{bmatrix} P^{\max}_{\mathrm{DG}_1} \\ \vdots \\ P^{\max}_{\mathrm{DG}_n} \end{bmatrix}_{n\times 1} \tag{12}$$

where $\boldsymbol{J}^*_{PQ}$ is a n dimension matrix whose elements are defined as $J^*_{PQ_{km}} = 1/J_{PQ_{mk}}$, $\boldsymbol{\Delta V}$ is a n dimension diagonal matrix formed by the elements $(\Delta V_1 = V^{\max} - V_1, \ldots, \Delta V_n = V^{\max} - V_n)$, and the operator min is defined as a function that returns the minimum element of each row of a matrix, that is, it is a line-by-line operation. Thus, each element of $\boldsymbol{P}^{\max}_{\mathrm{DG}}$ gives the maximum allowable power that the generator installed at the associated bus can inject without violating the voltage magnitude of any bus. Equation (12) considers the installation of a single generator on one-by-one basis and with a pre-specified power factor. In addition, only one matrix operation is necessary to simultaneously determine the maximum allowable power that can be injected into all system buses.

4 Validation studies for different scenarios

In this section, in order to verify the accuracy of the proposed method, its results are compared with the results provided by repetitive power flow solutions, which are taken as reference values. The usage of repetitive power flows to determine the maximum allowable active power injection per bus is explained in Appendix B. The single-line diagram of 70-bus test system used in this paper is shown in Fig. 1. The system data were obtained from [27]. The bus numeration was modified to facilitate the graphical visualisation of the maximum allowable active power injection per bus; however, the original numeration, as used in [27], is shown in parentheses in Fig. 1. Before installing a DG unit, the distribution network operator typically assesses the worst operating scenarios, which are usually related to the maximum and minimum network demand, in order to ensure that the network voltages will not be adversely affected due to the presence of the generators [17]. In the following examples, the maximum network active and reactive power demands are 3802.2 kW and 2694.6 kVAr, respectively, as described in [27], and the minimum active and reactive power demand was assumed equal to 20% of the maximum demand. The substation tap was adjusted at 1.04 pu to maintain the voltage magnitude of every bus within the allowable range (0.95–1.05 pu) for minimum and maximum network demand without the presence of distributed generators. In the sequence, the results obtained by the proposed method and by repetitive power flows for maximum and minimum demand and different values of DG power factor as well as scenarios with multi-DGs are analysed. Since the maximum demand is 3802.2 kW, in the following analyses, the maximum allowable generation per bus was limited to 4000 kW; however, higher values also can be analysed.

4.1 Maximum demand

By using the generalised expression (12), the maximum active power injection per bus of the test system is depicted in Fig. 2*a* for the case with maximum demand. In this example, the maximum generation was determined by considering that any generator to be installed into the network would operate with unity power factor. In this figure, the horizontal axis shows the system buses, and the vertical axis shows the amount of power in kW that one DG unit can inject into the respective bus without causing superior voltage limit violation in any bus. As an example, this graphical can be interpreted as follows: the maximum active power that one generator installed at bus 20 can inject into the system without causing voltage violations is 1190.4 kW. From this figure, one can see that the maximum allowable generation is very low for some buses, sometimes lower than 500 kW, since the installation of generators in these buses extremely affects the system voltage profile. On the other hand, the choice of some buses allows the installation of the highest capacity

IET Gener. Transm. Distrib., 2010, Vol. 4, Iss. 4, pp. 495–508
doi: 10.1049/iet-gtd.2009.0317

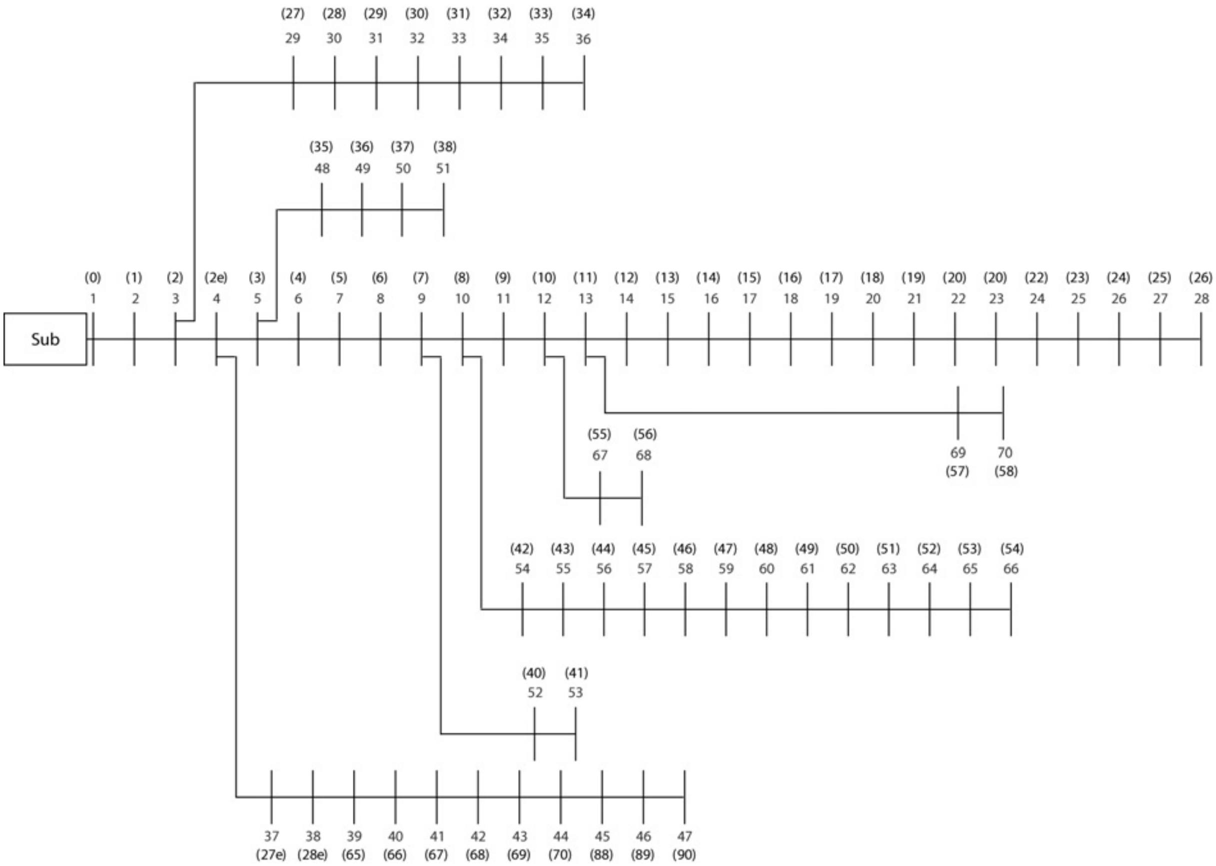


Figure 1 *Single-line diagram of the test system*

considered for the studies (4000 kW). These buses are electrically close to the main substation and have very low voltage sensitivities due to the constant voltage kept on the main substation. The results provided by the repetitive power flow method, where a step of 1.0 kW of power injection was used to determine the maximum allowable power injection, are compared with those obtained by the analytical method. This comparison reveals that the usage of (12) leads to very accurate results. For this example, the average error is only 3.1%, which is calculated by

$$A_{\text{error}} = \sum_{i=1}^{n} \left(\frac{\| P_{\text{DG}_i}^{\text{max}_{\text{sensitivities}}} - P_{\text{DG}_i}^{\text{max}_{\text{power flow}}} \|}{P_{\text{DG}_i}^{\text{max}_{\text{power flow}}}} \right) \times \frac{100}{n} \qquad (13)$$

where n is the number of buses; $P_{\text{DG}_i}^{\text{max}_{\text{sensitivities}}}$ is the maximum allowable generation of bus i determined by (12) and $P_{\text{DG}_i}^{\text{max}_{\text{power flows}}}$ is maximum allowable generation of bus i determined by repetitive power flows.

4.2 Minimum demand

Since minimum load condition is more restrictive to the installation of generators [17], special attention has to be given to this scenario. Thus, Fig. 2*b* shows the maximum active power that a generator can export without voltage violations for the minimum demand and unity power factor operation. The comparison of Figs. 2*a* and *b* reveals that the amount of active power for the scenario of minimum demand is lower than that for the maximum demand. In fact, it can be verified for the same generator at bus 20, which for this minimum demand scenario, it exports 447 kW without voltage violations. The comparison of the results obtained by using repetitive power flow solutions and by (12) shows that the proposed method has good performance. The average error between the methodologies is only 1.3% for this case.

4.3 Different generator operating modes

The operating mode of the generators also has to be evaluated since this technical aspect may produce different results in the analysis. In order to verify this aspect, Fig. 3*a* shows how the maximum active power varies according to the generator power factor calculated by using (12). In this analysis, the maximum demand scenario is assessed. The generator power factor is adjusted as unity, 0.9 inductive and 0.9 capacitive. The capacitive operation mode restricts the power that can be injected regarding the voltage limits. On the other hand, the inductive operation mode permits the distributed generator to inject a higher amount of power into the system. The comparison among the results obtained by the proposed method and by repetitive power flow is shown in Fig. 3*b*. In this case, the average errors for unity, inductive and capacitive power factors are 3.1%, 4.5% and 3.0%, respectively. Due to the simplicity of the

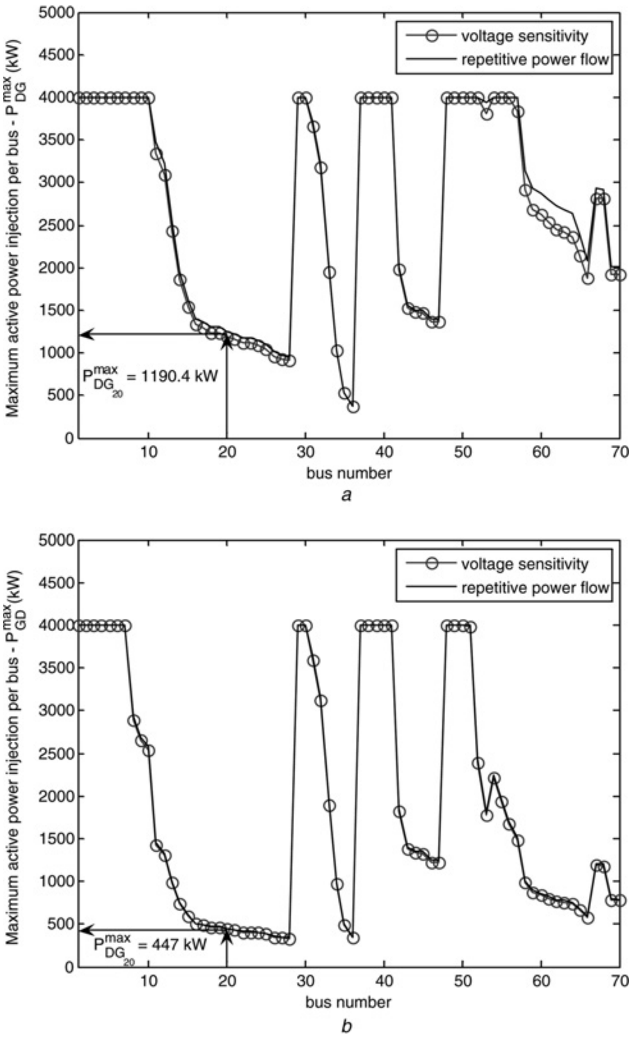


Figure 2 *Maximum active power injection per bus*

a Maximum demand and unity power factor
b Minimum demand and unity power factor

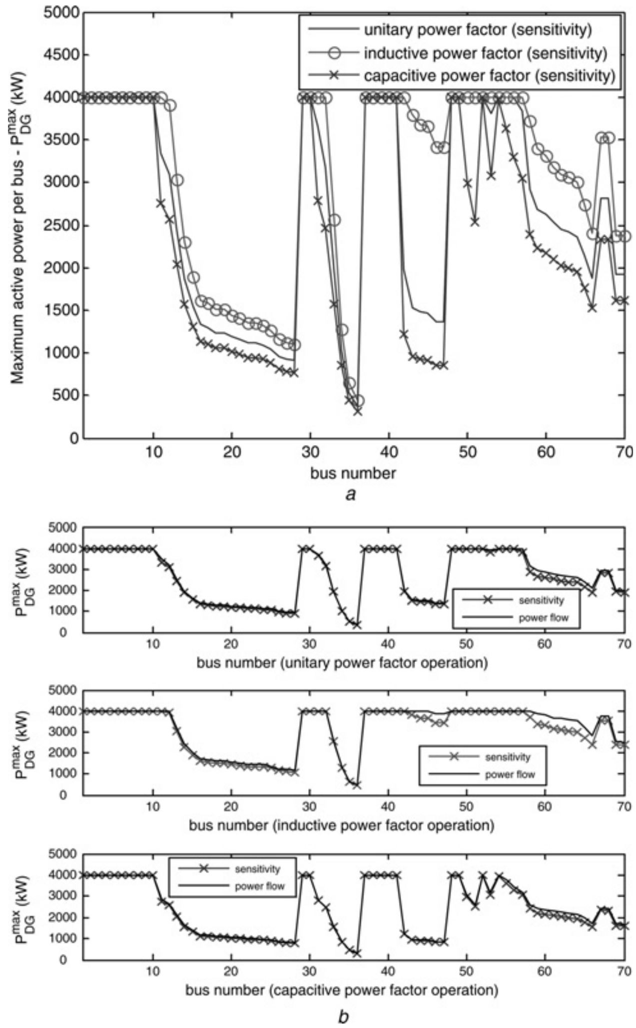


Figure 3 *Maximum active power injection for maximum demand and different power factors*

a Maximum power per bus determined by the proposed method
b Validation of the proposed method for different values of power factors

proposed method, many aspects of the problem can be quickly assessed and interpreted.

4.4 Evaluating multi-DG cases

In some circumstances, DG units are already connected to the distribution network, and some independent power producer requires the installation of a new generator into the system. The maximum allowable active power of the new generator considering the DG units already installed in the network should be determined. Therefore in this subsection, an assessment of the method in the presence of multi-DGs is presented. The first case analysed considers the presence of two generators previously connected at buses 20 and 45, which inject 500 and 1000 kW, respectively, operating at unity power factor.

Fig. 4*a* illustrates the method application for the maximum demand scenario. With two DG units already connected to the system, the allowable amount of new active power injection regarding the voltage limit is considerably decreased. Any combination of location, capacity and operating mode for generators can be directly evaluated to quickly find out the maximum allowable active power. The comparison of the results obtained by the proposed method and by repetitive power flow can be seen in this figure. For this case, the average error is 2.5%.

The second case analysed is related to the minimum demand, which is shown in Fig. 4*b*. It is assumed that three generators have already been installed at buses 10, 43 and 55 of the test network, and these DG units inject 400, 1000 and 800 kW, respectively. In addition, generators at buses 10 and 55 operate at 0.9 inductive power factor and generator at bus 43 operates with unity power factor. Moreover, it is considered that the new generator to be installed operates at 0.9 inductive power factor. Although the usage of generators consuming reactive power permits a major amount of active power to be exported to the system, the three DG units previously connected restrict this

IET Gener. Transm. Distrib., 2010, Vol. 4, Iss. 4, pp. 495–508
doi: 10.1049/iet-gtd.2009.0317

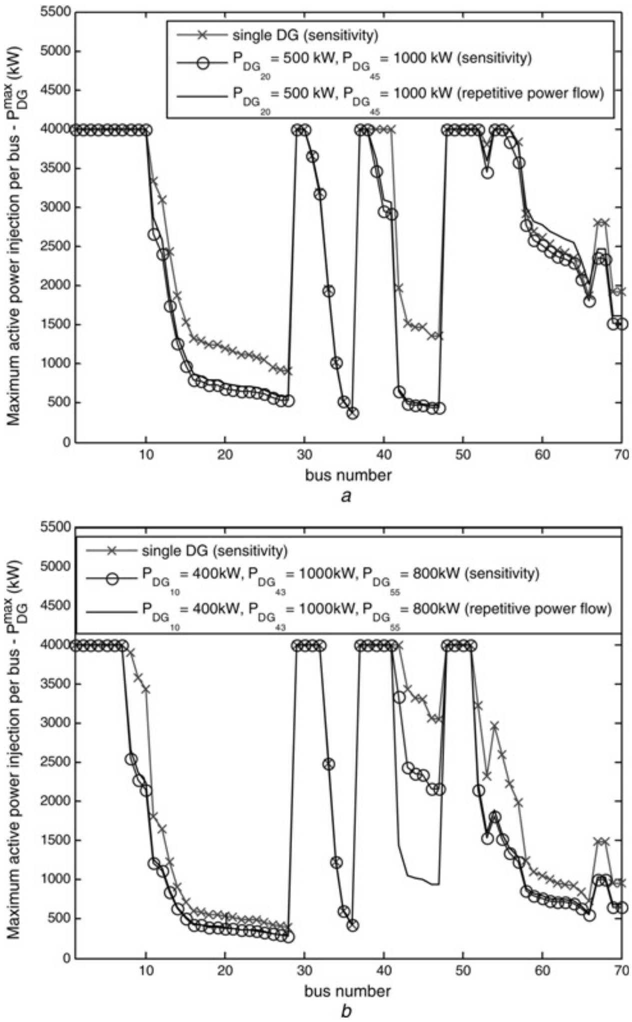


Figure 4 *Multi-DG cases*

a Maximum power per bus for maximum demand and unity power factor
b Maximum power per bus for minimum demand and inductive power factor

benefit. The maximum active power obtained by repetitive power flow is also plotted in Fig. 4*b* and the average error for this case is 1.5%.

5 Method application in the presence of voltage regulator devices

In order to maintain the nodal voltages within permissible limits, distribution power utilities usually install voltage regulator devices in their networks. Nowadays, the main devices used to voltage control are [28–30]: (a) step-type voltage regulators, which are basically autotransformers with several taps in the winding, or (b) shunt compensation devices, which can be switched capacitors, synchronous condensers or power electronic-based devices (e.g. SVC – static var compensator and DSTATCOM – distribution static synchronous compensator). The proposed method in this paper, as developed in Section 3, cannot explicitly handle these devices in its formulation. However, as will be seen in this section, the maximum permissible generation can be estimated by using the proposed method even in the presence of these voltage regulators since appropriate assumptions are made when calculating (12). The basic idea is that, for maximum penetration of DG, the voltage regulator devices will operate in their limits in order to reduce the nodal voltage, permitting more active power injection. This will be discussed by using two cases as examples. In the first case, lines 1–2 and 10–54 are replaced by two step-type voltage regulators, which have the capability of correcting the voltage by $\pm 10\%$ in 32 steps, as is usual in these devices [28]. In the second case, two ± 1500 and ± 2500 kVA DSTATCOMs are installed at buses 21 and 64, respectively. The models used to represent the step-type voltage regulator and the DSTATCOM can be found in [31, 32], respectively. These devices were controlled in order to keep constant terminal voltage. Both the results presented were obtained to minimum demand scenario since this is the most restrictive situation; however, similar results were found to maximum demand scenario.

5.1 Step-type voltage regulator

Fig. 5*a* shows the maximum generation per bus by using repetitive power flows and the proposed method for the case with and without step-type voltage regulators installed in branches 1–2 and 10–54. From this figure, one can see that the usage of voltage regulators permits one to increase the maximum generation in some buses. It was verified that the final tap positions at the solution points determined by the repetitive power flow approach, when calculating the maximum generation per bus, are always at the extreme tap positions that lead to the maximum voltage reduction. Thus, in order to use (12) to determine the maximum generation in the presence of step-type voltage regulators, one needs to replace the branches with under load tap changer transformers by fixed off-nominal tap transformers with the tap positions fixed in the limit that lead to the maximum voltage reduction. This was performed in Fig. 5*a*. Comparing the results obtained by using sensitivities and repetitive power flows, it is possible to see that, based on the above assumption, the proposed method has a good estimation performance even in the presence of step-type voltage regulators. In this case, the average error is 3.2%. Similar performances were obtained to different demand scenarios and voltage regulators locations.

5.2 DSTATCOM

Fig. 5*b* shows the maximum generation per bus by using repetitive power flow and the proposed method for the case with and without two DSTATCOMs installed at buses 21 and 64 with capabilities of ± 1500 and ± 2500 kVA, respectively. From this figure, it is possible to see that the installation of DSTATCOMs permits one to increase the maximum generation in some buses. It was verified that

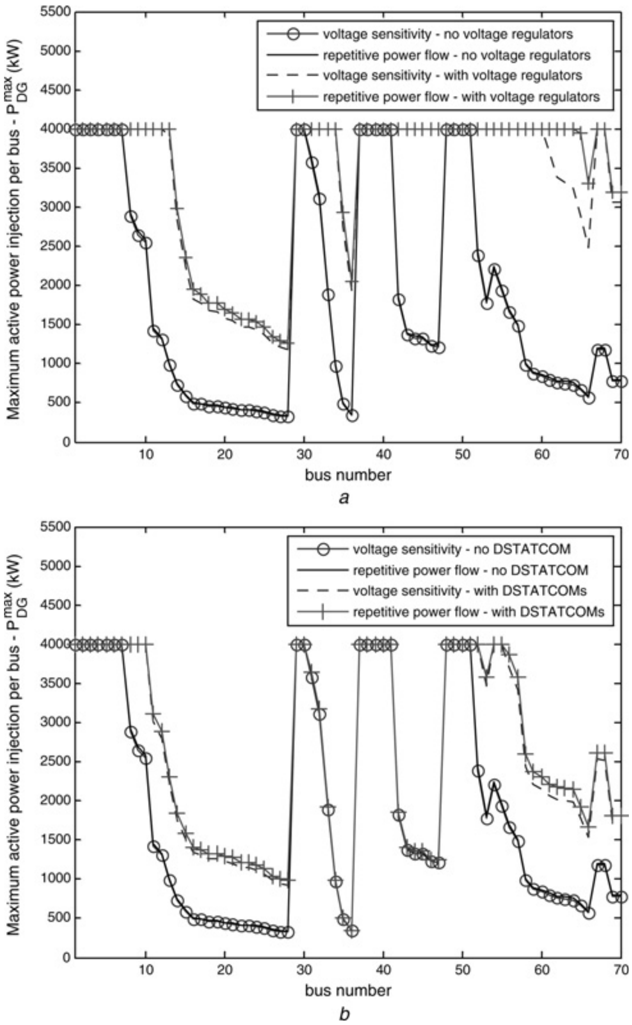


Figure 5 *Maximum power per bus in the presence of voltage regulators devices*

a Step-type voltage regulators in branches 1–2 and 10–54
b DSTATCOM at buses 21 and 64

the reactive power consumed by DSTATCOMs at the solution points determined by repetitive power flows, when calculating the maximum generation per bus, are always in the limit in order to cause the maximum voltage reduction. Thus, in the case of shunt voltage regulators, in order to use (12) to estimate the maximum generation per bus by using the proposed method, one just needs to consider a nodal consume of reactive power equal to the maximum capability of the shunt compensators at the respective buses. This was performed in Fig. 5*b*. In this case, the average error is 2.8%. Similar performances were obtained to different demand scenarios and shunt compensator locations. It is worth mentioning that similar results were found in the case of SVCs and synchronous condensers.

To sum up, although the proposed method cannot explicitly handle the usage of the main types of voltage regulator devices, it can still estimate the maximum allowable generation if appropriate assumptions are made to calculate the sensitivities.

6 Responsibility voltage factor

As previously seen, one of the aspects that can limit the number of distributed generators connected to distribution systems is the steady-state voltage rise [2, 17]. Thus, in a distribution network with two or more DG units, the contribution of each generator to the voltage rise must be identified. Therefore a numeric index to quantify the responsibility of each generator for the voltage level rise in a multi-DG system is proposed in this section.

According to (8), the impact of multi-DG units on a specific bus m of the system can be calculated by

$$V_m = V_m^0 + \sum_{i=1}^{n} J_{PQ_{mi}} P_{\mathrm{DG}_i} \tag{14}$$

If no DG unit is connected on a specific bus i, then P_{DG_i} is null. Therefore the total voltage deviation of bus m caused by all DG units is

$$\Delta V_m = V_m - V_m^0 = \sum_{i=1}^{n} J_{PQ_{mi}} P_{\mathrm{DG}_i} \tag{15}$$

From (15), the percentage impact of each generator with respect to an eventual voltage violation at bus m can be determined. In this work, this index is defined as the responsibility voltage factor or simply the responsibility factor (RF). Therefore the RF of a DG unit installed at bus k on the voltage magnitude of bus m can be expressed as

$$\boldsymbol{RF}_m^k = \frac{J_{PQ_{mk}} P_{\mathrm{DG}_k}}{\Delta V_m} 100\% \tag{16}$$

In a generalised form, the RFs can be computed by

$$\boldsymbol{RF} = \begin{bmatrix} \dfrac{J_{PQ_{11}} P_{\mathrm{DG}_1}}{\Delta V_1} & \cdots & \dfrac{J_{PQ1n} P_{\mathrm{DG}_n}}{\Delta V_1} \\ \vdots & \ddots & \vdots \\ \dfrac{J_{PQn1} P_{\mathrm{DG}_1}}{\Delta V_n} & \cdots & \dfrac{J_{PQnn} P_{\mathrm{DG}_n}}{\Delta V_n} \end{bmatrix} \times 100\% \tag{17}$$

Finally, a matrix equation to compute the RFs can be expressed by

$$\boldsymbol{RF} = \Delta \boldsymbol{V}^{-1} \boldsymbol{J}_{PQ} \cdot \boldsymbol{P}_{\mathrm{DG}} \times 100\% \tag{18}$$

where $\Delta\boldsymbol{V}$ and $\boldsymbol{P}_{\mathrm{DG}}$ are diagonal matrices formed by the elements $(\Delta V_1, \Delta V_2, \ldots, \Delta V_n)$ and $(P_{\mathrm{DG}_1}, P_{\mathrm{DG}_2}, \ldots, P_{\mathrm{DG}_n})$, respectively, and $\boldsymbol{RF}$ is a n dimension square matrix whose summation of any row is equal to 100%. Buses with no generators have null RFs.

The responsibility voltage factors can be used, for instance, on optimisation problems that aim to maximise the penetration level of DG. In this case, the algorithms can manage the compromise between the installed capacity of each generator and its RF with respect to critical voltages

doi: 10.1049/iet-gtd.2009.0317

(i.e. buses with eventual violations of the superior voltage limit).

6.1 Application of responsibility voltage factor

In this section, an example of how the concept of responsibility voltage factors can be applied to increase the allowable penetration level of distributed generators is presented. The voltage profile for minimum demand shown in Fig. 6*a* is used for this study. Two DG units operating at unity power factor are installed in the network. One delivers 400 kW at bus 18, whereas the power delivered by the other unit, located at bus 67, is varied from 0 to 250 kW through steps of 50 kW. If the generator at bus 67 injects more than 200 kW, the voltage at bus 18 (and its vicinity) violates the superior limit. Assuming a fixed power injection of 400 kW at bus 18, the maximum power injection at bus 67, by using (12), is 198 kW. Thus, the maximum allowable penetration level of distributed generators, in this case, is 598 kW ($P_{DG_{Total}} = P_{DG_{18}} + P_{DG_{67}}$).

The responsibility voltage factors of the DG units installed at buses 18 and 67 are shown in Fig. 6*b*. DG unit 18 (DG_{18}) is the unit mainly responsible for the system voltage rise. Even when the DG unit 67 (DG_{67}) injects 250 kW, about 80% of the voltage deviation at bus 18 (the critical bus) is caused by DG_{18}. Thus, DG_{18} is the most responsible for the eventual voltage violations, and thereby, it restricts the amount of power that can be injected at bus 67 by the other DG unit. As well, DG_{18} reduces the possible economic benefits of DG_{67} by restricting its allowable power. DG_{18} can also limit the maximum penetration level of DG into the whole system.

Fig. 7*a* shows how the penetration level of DG($P_{DG_{Total}}$) varies with the capacity of the generators installed at buses

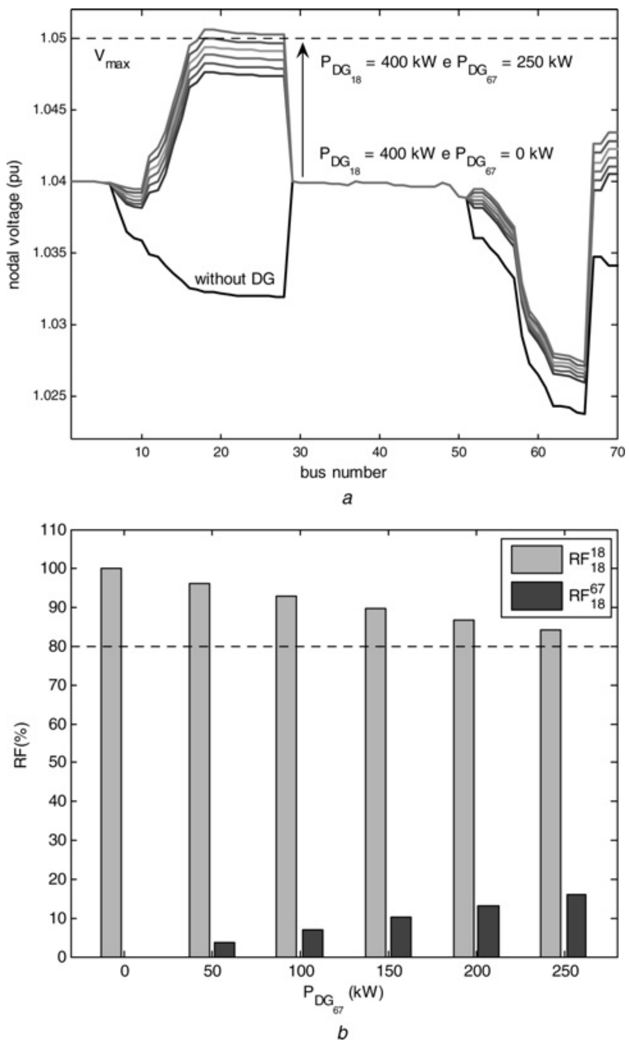


Figure 6 *Application of responsibility voltage factor*

a Voltage profile with DG unit at buses 18 and 67
b Responsibility voltage factor of bus 18

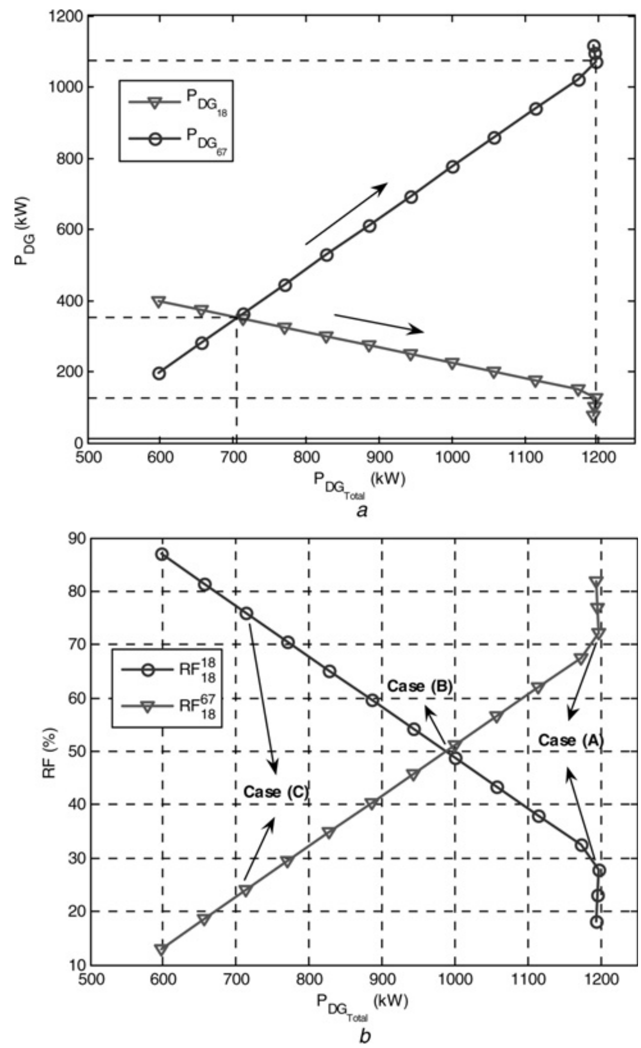


Figure 7 *DG penetration level and responsibility factor of bus 18 as function of DG_{18} and DG_{67}*

a DG penetration level
b Responsibility factor of bus 18

Table 1 Three different assumptions to establish a fair criterion

	RF_{18}^{18}, %	RF_{18}^{67}, %	$P_{DG_{18}}$, kW	$P_{DG_{67}}$, kW	$P_{DG_{total}}$, kW
Case a	27.8	72.2	125	1073	1198
Case b	50.0	50.0	230	759	989
Case c	76.7	23.3	353	353	706

18 and 67. In order to obtain this graphic, DG_{18} is varied from 400 to 75 kW through steps of −25 kW, and for each value of DG_{18}, the maximum power injection of DG_{67} that does not violate the voltage limits is calculated by using (12). The penetration level of DG can be significantly increased through the re-scheduling of power injections from buses 18 to 67. The maximum penetration level of DG (1198 kW) is obtained for $P_{DG_{18}} = 125$ kW and $P_{DG_{67}} = 1073$ kw. In addition, Fig. 7*a* reveals that DG has a maximum penetration level in this case, since $P_{DG_{Total}}$ starts decreasing around 1200 kW for any new re-scheduling.

However, with a maximum penetration level, the generation of DG_{18} is severely restricted, maybe unfairly. This problem raises the question of how to increase the penetration level of DG without unfairly restricting the generation of any DG unit. The concept of responsibility voltage factors can be used to find a compromise between the power injection of each DG unit and its contribution to the increase of the system voltage profile. Fig. 7*b* presents the behaviour of the responsibility voltage factors for the same test presented in Fig. 7*a*. As DG_{18} decreases, RF_{18}^{18} decreases and RF_{18}^{67} increases. Thus, three possible criteria to define the most adequate capacity of the two generators are the following:

6.1.1 Case (A) – maximum penetration level:

The maximum penetration level is 1198 kW. The generation levels of DG_{18} and DG_{67} are 125 and 1073 kW, respectively. In this case, the responsibility factors for these generators are $RF_{18}^{18} = 27.8\%$ and $RF_{18}^{67} = 72.2\%$, as indicated in Fig. 7*b* and Table 1. As discussed before, although the maximum penetration level is reached by scheduling the generation of both units, the generation level of DG_{18} is much restricted compared to DG_{67} installed on the system.

6.1.2 Case (B) – equal contribution of the DG units to the voltage rise:

The responsibility factors for the DG units are $RF_{18}^{18} = RF_{18}^{67} = 50\%$. The penetration level in this case is 989 kW, where the capacities of DG_{18} and DG_{67} are 230 and 759 kW, respectively. However, attributing equal responsibility for the voltage rise to both DG units can still be a severe measure for DG_{18}, since its sensitivity coefficient $J_{PQ_{(18,18)}}$ is almost twice that of $J_{PQ_{(18,67)}}$ related to the generator installed at bus 67.

6.1.3 Case (C) – identical power generation for both DG units:

The power injection by each generator is 353 kW, and therefore, the penetration level of DG is 706 kW, as can be also verified by Fig. 7*a* and Table 1. For this case, the responsibility voltage factors are $RF_{18}^{18} = 76.7\%$ and $RF_{18}^{67} = 23.3\%$. Nevertheless, this criterion may discourage the installation of DG units in locations in which the power injection would strongly impact the network voltage profile.

7 Analysis of computational requirements

In the proposed method, based on only one power flow solution and one matrix operation of (12), the maximum power that can be injected at any bus of the system without violations of nodal voltage limits can be determined. In the classical approach, for computing the maximum power injection, repetitive power flow solutions are necessary. Although, to reduce the number of power flow solutions, a binary search, which consists of searching for the limit value in an interval whose width is divided by two at each iteration, can be used, the number of power flows in real networks is still high. For example, with a given tolerance of 50 kW and an interval search of 0–4000 kW, the number nr of power flow runs per bus will be such that $2^{nr} = 4000/50$, which gives $nr = 7$ runs per bus and, therefore, in the real systems with thousands of buses, the number of power flows is very high. Even for the small test system, the total number of power flow runs is $7 \times 70 = 490$ runs. Therefore in this example, the proposed method provides a considerable computational gain, since to run one power flow solution is less time-consuming than to perform 490 power flow solutions. Although performing 490 power flow solutions is not considered to be a restrictive computational requirement for any modern personal computer, with an optimisation problem involving multi-DGs and several discrete control devices and load levels, where thousands of situations are assessed, the number of power flows considerably increases, and the proposed methodology may become a useful solution.

8 Conclusions

In this paper, an analytical methodology based on voltage sensitivity was proposed to directly estimate the maximum

doi: 10.1049/iet-gtd.2009.0317

allowable power that distributed generators can inject into a system without causing steady-state voltage violations and assuming no substantial changes in the system structure. Based on only one power flow solution and one matrix operation given by (12), the impact of adding new generators at every bus of the network, with any capacity and any lead or lag power factor can be estimated. The results obtained by the proposed method were compared with those provided by repetitive power flow solutions. The accuracy of the proposed method was shown to be adequate despite its simplicity, mainly for typical distribution systems, where the network is radial and the X/R relation is low. It is important to emphasise that, although the proposed method has presented a good performance, this is an approximated approach due to the linearisation of the system model. Consequently, it should be used as a first approximation and, then, the selected results should be confirmed and refined by using a complete power flow program.

A new responsibility voltage factor was also proposed and discussed in order to evaluate the contribution of each DG unit to the voltage rise in multi-DG systems. This factor can be used by utility managers to decide which generators, and in which degree, should be penalised by the voltage rise or rewarded by not depreciating the voltage profile. This factor can also be used, for example, in optimisation problems. In this case, based on this factor, different strategies can be traced to multi-DG system, as for example:

- to maximise the penetration level;
- to share the responsibility of the voltage rise equally among the DGs;
- to permit all DGs to produce the same amount of power.

To finalise, it is important to call the attention that other technical aspects have to be analysed when determining the maximum allowable generation such as protection systems, active power losses, stability issues, which cannot be addressed by the proposed method. Thus, this method should be used only during a preliminary investigation stage. In addition, when different means of voltage regulator should also be detailing investigated power flow studies should be used due to the limitation of the proposed method to explicitly handle these types of devices.

9 Acknowledgment

This work was supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazilian research agencies.

10 References

[1] CIGRÉ Working Group 37.23: 'Impact of increasing contribution of dispersed generation of the power systems'. Technical Report, CIGRÉ, 1999

[2] CIRED Working Group 4: 'Dispersed generation'. Technical Report, CIRED, 1999

[3] JENKINS N., ALLAN R., CROSSLEY P., KIRSCHEN D., STRBAC G.: 'Embedded generation' (Institute of Electrical Engineering, London, UK, 2000, 1st edn.)

[4] INVERNIZZI A., BUCHHOLZ B., STUBBE M., JENKINS N., DOWD B., CERAOLO M.: 'Distribution systems and dispersed generation: a new focus for CIGRE', *Electra*, 2004, **213**, pp. 17–21

[5] CELLI G., GHIANI E., MOCCI S., PILO F.: 'A multiobjective evolutionary algorithm for the sizing and siting of distributed generation', *IEEE Trans. Power Syst.*, 2005, **20**, (2), pp. 750–757

[6] OCHOA L.F., PADILHA-FELTRIN A., HARRISON G.P.: 'Evaluating distributed generation impacts with a multiobjective index', *IEEE Trans. Power Del.*, 2006, **21**, (3), pp. 1452–1458

[7] HARRISON G.P., WALLACE A.R.: 'Optimal power flow evaluation of distribution network capacity for the connection of distributed generation', *IEE Proc. Gener. Transm. Distrib.*, 2005, **152**, (1), pp. 115–122

[8] VOVOS P.N., HARRISON G.P., WALLACE A.R., BIALEK J.W.: 'Optimal power flow as a tool for fault level-constrained network capacity analysis', *IEEE Trans. Power Syst.*, 2005, **20**, (2), pp. 734–741

[9] DENT C.J., OCHOA L.F., HARRISON G.P., BIALEK J.W.: 'Efficient secure AC OPF for distributed generation uptake maximisation'. Proc. 2008 16th Power Systems Computation Conf. (PSCC'08), p. 7

[10] SIANO P., OCHOA L.F., HARRISON G.P., PICCOLO A.: 'Assessing the strategic benefits of distributed generation ownership for DNOs', *IET Gener. Transm. Distrib.*, 2009, **3**, (3), pp. 225–236

[11] JABR R.A., PAL B.C.: 'Ordinal optimisation approach for locating and sizing of distributed generation', *IET Gener. Transm. Distrib.*, 2009, **3**, (8), pp. 713–723

[12] GÖZEL T., HOCAOGLU M.H.: 'An analytical method for the sizing and siting of distributed generators in radial systems', *Electr. Power Syst. Res.*, 2009, **79**, pp. 912–918

[13] WANG C., NEHRIR M.H.: 'Analytical approaches for optimal placement of distributed generation sources in power

systems', *IEEE Trans. Power Syst.*, 2004, **19**, (4), pp. 2068–2076

[14] QUEZADA V.H.M., ABBAD J.R., ROMAN T.G.S.: 'Assessment of energy distribution losses for increasing penetration of distributed generation', *IEEE Trans. Power Syst.*, 2006, **21**, (2), pp. 533–540

[15] DUGAN R.C., MCDRMOTT T.E.: 'Distributed generation', *IEEE Ind. Appl. Mag.*, 2002, **8**, (2), pp. 19–25

[16] CONTI S., RAITI S., TINA G.: 'Small-scale embedded generation effect on voltage profile: an analytical method', *IEE Proc. Gener. Transm. Distrib.*, 2003, **150**, (1), pp. 78–86

[17] MASTERS C.L.: 'Voltage rise: The big issue when connecting embedded generation to long 11 kV overhead lines', *Power Eng. J.*, 2002, **16**, (1), pp. 5–12

[18] HIRD C.M., LEITE H., JENKINS N., LI H.: 'Network voltage controller for distributed generation', *IEE Proc. Gener. Transm. Distrib.*, 2004, **151**, (2), pp. 150–156

[19] BOLLEN M.H.J., SANNINO A.: 'Voltage control with inverter-based distributed generation', *IEEE Trans. Power Del.*, 2005, **20**, (1), pp. 519–520

[20] VILLACCI D., BONTEMPI G., VACCARO A.: 'An adaptive local learning-based methodology for voltage regulation in distribution networks with dispersed generation', *IEEE Trans. Power Syst.*, 2006, **21**, (3), pp. 1131–1140

[21] FREITAS W., VIEIRA J.C.M., MORELATO A., DA SILVA L.C.P., DA COSTA V.F., LEMOS F.A.B.: 'Comparative analysis between synchronous and induction machines for distributed generation applications', *IEEE Trans. Power Syst.*, 2006, **21**, (1), pp. 301–311

[22] VOVOS P.N., KIPRAKIS A.E., WALLACE A.R., HARRISON G.P.: 'Centralized and distributed voltage control: impact on distributed generation penetration', *IEEE Trans. Power Syst.*, 2007, **22**, (1), pp. 476–483

[23] VIAWAN F.A., KARLSSON D.: 'Combined local and remote voltage and reactive power control in the presence of induction machine distributed generation', *IEEE Trans. Power Syst.*, 2007, **22**, (4), pp. 2003–2012

[24] SENJYU T., MIYAZATO Y., YONA A., URASAKI N., FUNABASHI T.: 'Optimal distribution voltage control and coordination with distributed generation', *IEEE Trans. Power Del.*, 2008, **23**, (2), pp. 1236–1242

[25] VIAWAN F.A., KARLSSON D.: 'Voltage and reactive power control in systems with synchronous machine-based distributed generation', *IEEE Trans. Power Del.*, 2008, **23**, (2), pp. 1079–1087

[26] HISCOCK N., HAZEL T.G., HISCOCK J.: 'Voltage regulation at sites with distributed generation', *IEEE Trans. Ind. Appl.*, 2008, **44**, (2), pp. 445–454

[27] BARAN M.E., WU F.F.: 'Optimal capacitor placement on radial distribution systems', *IEEE Trans. Power Del.*, 1989, **4**, (1), pp. 725–734

[28] GÖNEN T.: 'Electric power distribution system engineering' (CRC Press, 2008, 2nd edn.)

[29] HINGORANI N.G.: 'Introducing custom power', *IEEE Spectr.*, 1995, **31**, (6), pp. 41–48

[30] ACHA E., AGELIDIS V.G., ANAYA-LARA O., MILLER T.J.: 'Power electronic control in electrical systems' (Newnes, 2002, 1st edn.)

[31] KERSTING W.H.: 'Distribution system modeling and analysis' (CRC Press, 2002, 2nd edn.)

[32] FREITAS W., MORELATO A., XU W., SATO F.: 'Impacts of AC generators and DSTATCOM devices on the dynamic performance of distribution systems', *IEEE Trans. Power Del.*, 2005, **20**, (2), pp. 1493–1501

11 Appendix A

11.1 Validation and application of V-P and V-Q sensitivities

The usage of *V-P* and *V-Q* sensitivities to determine the impact of a new generator on the system voltage profile by using (7) is explained in this appendix. In the following studies, three 1 MVA generators are added to the test system on buses 9, 25 and 66. The generator at bus 9 operates at 0.95 inductive power factor, the generator bus 25 at unity power factor and the generator installed at bus 66 at 0.95 capacitive power factor. Fig. 8*a* presents the voltage profile for the base case and the estimated voltage profile with the installation of one, two and three generators, respectively, by using (7). The first generator, installed at bus 25, causes a significant voltage rise even when operating at unity power factor. The buses on the main feeder are the most impacted, whereas the remaining buses suffer smaller voltage rise. The second generator, installed at bus 9, has little impact on the system voltage profile, due to its small distance from the main substation and to its inductive power factor operating mode. However, the installation of the second generator leads to voltage violations at the vicinity of bus 25, due to the combined effect of the two generators. The third generator, installed at bus 66, drastically affects the whole voltage profile, since it injects simultaneously active and reactive power into the

doi: 10.1049/iet-gtd.2009.0317

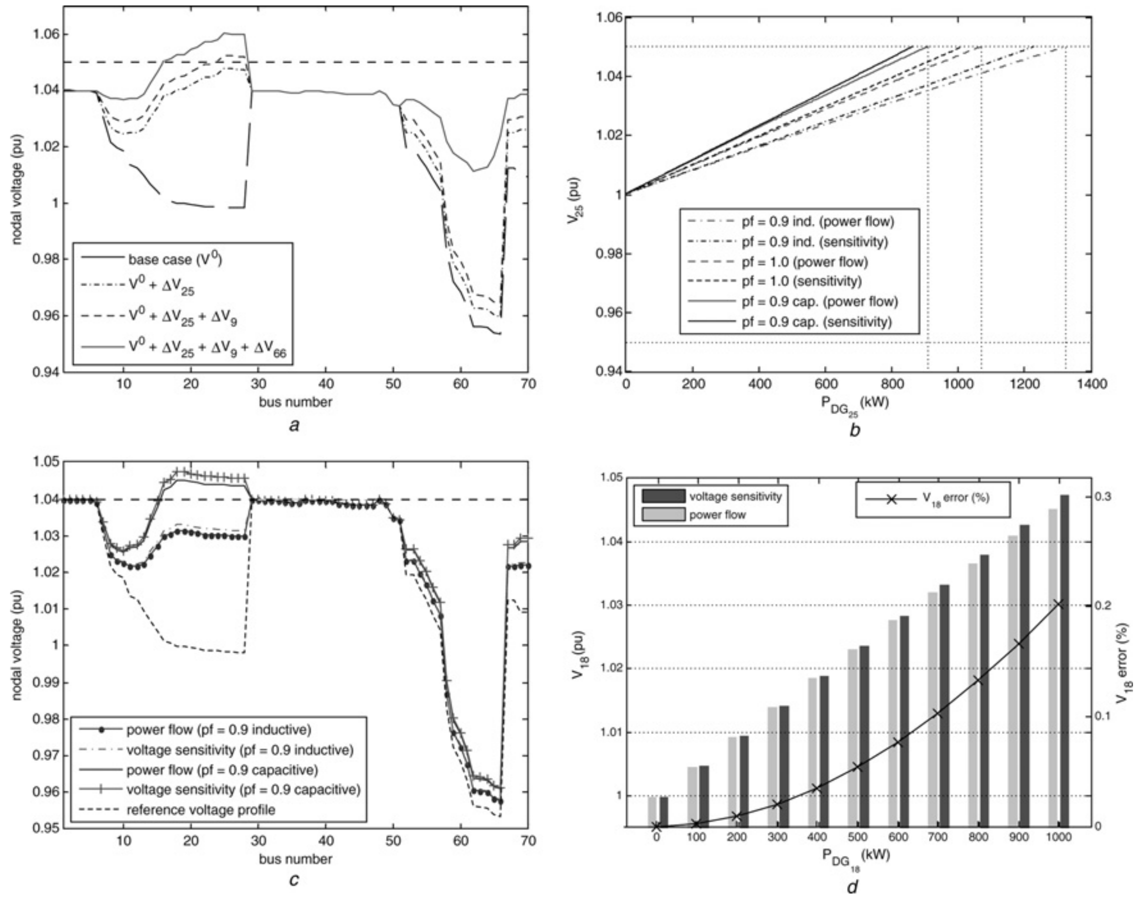


Figure 8 *Different validation tests for the proposed methodology*

a Voltage profile with DG units at buses 9, 25 and 66
b Maximum allowable active power of bus 25
c Voltage profile for different power factors for the DG unit
d Assessment of voltage sensitivity

system. The combined impact of the three generators leads to severe voltage violations at the end of the main feeder (buses 16–28).

To justify why the linearised equations (7) and (12) can be used, one DG unit is considered to have been installed at bus 25 of the test network. Fig. 8*b* illustrates the voltage behaviour of bus 25 when the active power injection of the generator is incremented by 1 kW-steps until the superior voltage limit is reached. For each step, a load flow is solved, and then the graphic is plotted. The analysis considers three different power factor operations (pf = 0.9 inductive, pf = 1.0 and pf = 0.9 capacitive) and the maximum demand scenario. The linear behaviour of the nodal voltage can be observed. Indeed, the slopes of these lines are approximately equal to the sensitivity coefficients of J_{PQ}, which are also shown in Fig. 8*b*. These results explain why the maximum capacity of the generators can be estimated by using sensitivities, as proposed in (12), which are linearised equations. The behaviour of distribution systems within narrow voltage variation is close to linear. This figure reveals that, as expected, the capacitive operating mode presents a higher sensitivity coefficient than that of the other operating modes. Therefore the amount of active power injected for this operating mode is the lowest for the three analysed cases.

In order to further validate the estimation of the system voltage profile with the addition of new generators by using (7), a DG unit of 1000 kW was installed at bus 18. The maximum demand scenario is considered, and two different values of power factor are analysed. The base case voltage profile (without any DG unit) and the new voltage profile after the addition of the generator are illustrated in Fig. 8*c*, which reveals that the results provided by the sensitivity-based methodology are very close to those obtained by using successive load flow solutions. The capacitive operating mode case presents the largest voltage variation. Consequently, the major error is related to this case. Fig. 8*d* presents the evolution of the error between the estimated and computed voltage magnitude of bus 18 as a function of the active power injected by the generator. Although the error increases when the power injection is incremented, the maximum error is within an acceptable range for a linear method (lower that 0.2%). In this case, 1000 kW of power generation at bus 18 corresponds to 25% of the total active power demand of the system ($\simeq$4000 kW).

12 Appendix B: maximum penetration by repetitive power flow

The procedure used to determine the maximum allowable penetration by repetitive power flow is summarised as follows:

Step 1: Define: active power step in kW (*step*); generator power factor ($\cos(\phi)$), and maximum power to be investigated (P_{limit});

Step 2: Run power flow for the base case;

Step 3: Do $k = 0$ and $\mathbf{P}_{\text{DG}}^{\max} = 0$;

Step 4: Do $k = k + 1$ (index for bus under investigation);

Step 5: $k > n$ (where n is the number of buses); Yes → finish process; No → go to Step 6.

Step 6: Do: $P_{\text{gen}}(k) = P_{\text{gen}}(k) + step$ (increase the active power of generator k); $Q_{\text{gen}}(k) = P_{\text{gen}}(k) \times \tan(a\cos(\cos(\phi)))$ (vary reactive power of generator k);

Step 7: Run power flow;

Step 8: Any nodal voltage > 1.05 or $P_{\text{gen}}(k) > P_{\text{limit}}$ Yes → Do $\boldsymbol{P}_{\text{DG}}^{\max}(k) = P_{\text{gen}}(k) - step$; and return to Step 4; No → Go to Step 6

IET Gener. Transm. Distrib., 2010, Vol. 4, Iss. 4, pp. 495–508

doi: 10.1049/iet-gtd.2009.0317