QML Assignment 1 - Mathematical Model E

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The notation used for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
I	Set of the different suppliers	$i \in I$
K	Set for the months	$k \in K$
J	Set for the produced types of steel	$j \in J$
Parameters		
Cr_i	percentage of chromium from supplier i	[%]
Ni_i	percentage of nickel from supplier i	[%]
Cu_i	percentage of copper from supplier i	[%]
max_i	maximum supply per i	[kg]
c_i	cost per kilogram for supplier i	[€/ kg]
d_{ik}	demand for produced steel type j in month k	[kg]
h_j	holding cost per kilogram for steel type j per month	[€/ month]
pNi_j	percentage of Nickel necessary for steel type j	[%]
pCr	percentage of chromium needed for all produced steel types	[%]
p_{max}	maximum weight producible in each month	[kg]
Cu_{max}	percentage of copper allowed	[%]
$c_{e,f}$	fixed cost for using electrolysis	[€]
$c_{e,kg}$	cost for using electrolysis per kilogram copper removed	[€/ kg]
Variables		
x_{ik}	weight of steel production type i in month k	[kg]
y_{jk}	produced steel of type j in month k	[kg]
z_{jk}	storage of steel type i in month k	[kg]
e_k	binary indicator for use of electrolysis in month k	Binary

The mathematical formulation then follows as:

$$\min \quad \sum_{i\in I}\sum_{k\in K}\sum_{j\in J}(c_i x_{ik} + z_{jk}h_j + e_k(c_{e,f} + c_{e,kg}x_{ik}Cu_i)) \qquad \forall i \in I, j \in J, k \in K \tag{1}$$

Subject to:

$$x_{ik} \leq x_{max} \qquad \forall i \in I, k \in K \tag{2}$$

$$\sum_{j\in J} y_{jk} \leq p_{max} \qquad \forall i \in I, k \in K \tag{3}$$

$$\begin{cases} y_{jk} - z_{jk} = d_{ik} & \text{if } k = 0 \\ y_{jk} + z_{j,k-1} - z_{jk} = d_{ik} & \text{if } k > 0 \end{cases} \qquad \forall j \in J, k \in K \tag{4}$$

$$\sum_{i\in I} x_{ik} = \sum_{j\in J} y_{jk} - \sum_{i\in I}(x_{ik}Cu_i)e_k \qquad \forall i \in I, j \in J, k \in K \tag{5}$$

$$\sum_{i\in I}(Ni_i x_{ik}) = \sum_{j\in J}(pNi_j y_{jk}) \qquad \forall i \in I, j \in J, k \in K \tag{6}$$

$$\sum_{i\in I}(Cr_i x_{ik}) = \sum_{j\in J} y_{jk}pCr \qquad \forall i \in I, j \in J, k \in K \tag{7}$$

$$\sum_{i\in I}(x_{ik}Cu_i)(1-e_k) \leq Cu_{max}\sum_{j\in J} y_{jk} \qquad \forall i \in I, j \in J, k \in K \tag{8}$$

The objective function (1) is written as a minimization of the costs of buying materials and the holding costs for storage from one month to another. Additionally, if electrolysis is used, all bought copper is removed which costs money too which should be minimized.

Constraint 1 (2) ensures that not more materials are bought than available per supplier. Constraint 2 (3) limits the total goods produced to be less than the 100 kg per month. Constraint 3 (4) is made up of two parts and is about meeting the demand. In the first month there is no storage yet, thus the storage is left out. In the every month there on out, the storage of the previous month is taken into account for the meeting of the demand. Constraint 4 (5) limits the acquired materials to the production in every month. In case electrolysis is used, the removed copper is subtracted from the produced material. Constraints 5 (6) and 6 (7) make sure the percentages of nickel and chromium are adequate. Constraint 7 checks if the percentage of copper is below the threshold in the produced material. In case electrolysis is used, the left side of this equation is set to 0 as all copper is removed.

Optimal allowable percentage of copper

With experimentation the optimal percentage of copper which can be allowed by the producer without any loss can be found. In case of a maximum percentage of 3% copper in all final steel, no electrolysis is needed and the total costs are 9646.78 euro. Any lower percentage as upper boundary will increase the costs, as electrolysis is expensive. When the allowable percentage is set to 2.9%, the costs already increase with 13.87 euro. When 0% copper is allowed, the total costs become 10896.86 euro, which is considerably more expensive than with a 3% copper allowance.

When an intermediate copper upper boundary is used, the electrolysis is used part of the time. This will not change the storage costs as the electrolysis is done over the produced goods. The produced goods are stored, so there is no profit to be gained by doing the electrolysis and storing the material.