QML Assignment 1 - Mathematical Model

25-10-2024

The notation used for this mathematical formulation is provided in Table 1.

Table 1: Notation

Sets and indices		
I	Set of the different suppliers	$i \in I$
K	Set for the months	$k \in K$
J	Set for the produced types of steel	$j \in J$
Parameters		
Cr_i	percentage of chromium from supplier i	[%]
Ni_i	percentage of nickel from supplier i	[%]
Cu_i	percentage of copper from supplier i	[%]
max_i	maximum supply per i	[kg]
c_i	cost per kilogram for supplier i	[€/ kg]
d_{ik}	demand for produced steel type j in month k	[kg]
h_j	holding cost per kilogram for steel type j per month	[€/ month]
pNi_j	percentage of Nickel necessary for steel type j	[%]
pCr	percentage of chromium needed for all produced steel types	[%]
p_{max}	maximum weight producible in each month	[kg]
Variables		
x_{ik}	weight of steel production type i in month k	[kg]
y_{jk}	produced steel of type j in month k	[kg]
z_{jk}	storage of steel type i in month k	[kg]

The mathematical formulation then follows as:

$$\min \quad \sum_{i \in I} \sum_{k \in K} \sum_{j \in J} (c_i x_{ik} + z_{jk} h_j) \qquad \forall i \in I, j \in J, k \in K \tag{1}$$

Subject to:

$$x_{ik} \leq x_{max} \qquad \forall i \in I, k \in K \tag{2}$$

$$\sum_{j \in J} y_{jk} \leq p_{max} \qquad \forall i \in I, k \in K \tag{3}$$

$$\begin{cases} y_{jk} - z_{jk} = d_{ik} & \text{if } k = 0 \\ y_{jk} + z_{j,k-1} - z_{jk} = d_{ik} & \text{if } k > 0 \end{cases} \qquad \forall j \in J, k \in K \tag{4}$$

$$\sum_{i \in I} x_{ik} = \sum_{j \in J} y_{jk} \qquad \forall i \in I, j \in J, k \in K \tag{5}$$

$$\sum_{i \in I} Ni_i x_{ik} = \sum_{j \in J} pNi_j y_{jk} \qquad \forall i \in I, j \in J, k \in K \tag{6}$$

$$\sum_{i \in I} Cr_i x_{ik} = \sum_{j \in J} y_{jk} pCr \qquad \forall i \in I, j \in J, k \in K \tag{7}$$

The objective function (1) is written as a minimization of the costs of buying materials and the holding costs for storage from one month to another. Constraint 1 (2) ensures that not more materials are bought than available per supplier. Constraint 2 (3) limits the total goods produced to be less than the 100 kg per month. Constraint 3 (4) is made up of two parts and is about meeting the demand. In the first month there is no storage yet, thus the storage is left out. In the every month there on out, the storage of the previous month is taken into account for the meeting of the demand. Constraint 4 (5) limits the acquired materials to the production in every month. Constraints 5 (6) and 6 (7) make sure the percentages of nickel and chromium are adequate.