4. Etxeechallenge

1.-

$I\,\psi(x) = \psi(-x)$ eta $\hat{I}^2 = \mathbb{1}$ autobalioa

$\xi(x)$ badaukagu $\Rightarrow$ $\hat{I}^2\,\xi(x) = \hat{I}\,\overbrace{\hat{I}\,\xi}^{\lambda\xi} = \lambda\,\hat{I}\,\xi = \lambda^2\,\xi(x)$

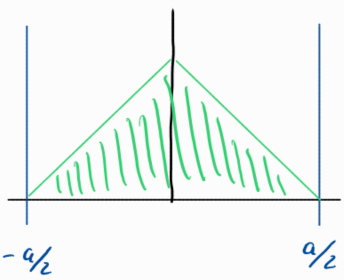
$\hookrightarrow \hat{I}^2\,\xi(x) = \hat{I}\,\xi(-x) = \xi(x)$ $\Bigg\}\Rightarrow$

$$\Rightarrow \lambda^2 = \frac{\xi(x)}{\xi(x)} = 1 \iff \boxed{\lambda = \pm 1} \longleftarrow \text{Autobalioak}$$

Hermitiarra $\iff$ $(\hat{I}\psi, \varphi) = (\psi, \hat{I}\varphi) \Rightarrow \int \psi^*(-x)\cdot\varphi(x)\,dx = \int \psi^*(x)\varphi(-x)\,dx$

$\psi, \varphi \in \mathbb{R}$ bakarrik $\Rightarrow$ $\hat{I}$ ez da Hermitiarra.

2.-



$$\psi(x,0) = \begin{cases} x + a/2\,; & -a/2 \le x \le 0 \\ -x + a/2\,; & 0 \le x \le a/2 \end{cases}$$

$$2\sqrt{\frac{6}{a^3\pi}}\cdot\left[\frac{1-\cos\left(\frac{ak}{2}\right)}{k^2}\right]$$

$$A(k) = \frac{4\sqrt{3}}{a\sqrt{a}}\left[\frac{1-\cos\left(\frac{ak}{2}\right)}{\sqrt{2\pi}\;k^2}\right] \Rightarrow |A(k)|^2 = \frac{16\cdot 3}{a^3\cdot 2\pi}\;\frac{\left(1-\cos\left(\frac{ak}{2}\right)\right)^2}{k^4} =$$

$$= \frac{24}{a^3\pi}\;\frac{\left(1-\cos\left(\frac{ak}{2}\right)\right)^2}{k^4} \Rightarrow \langle p^2\rangle = \int_{-\infty}^{\infty} \hbar^2\cdot\frac{24}{a^3\pi}\;\frac{\left(1-\cos\left(\frac{ak}{2}\right)\right)^2}{k^2}\,dk = \frac{12\,\hbar^2}{a^2}$$

Hau da $\langle E\rangle = \dfrac{\langle p^2\rangle}{2m} = \dfrac{6\,\hbar^2}{m a^2}$ //

5. Etxechallenge

Aurreko sistemaren t aldiunean energia $\frac{81\hbar^2\pi^2}{2ma^2}$ balioa neurtzeko probabilitatea.

osin infinituaren autobalioak.

Dakigu $\Psi(x,t) = \sum_n C_n \cdot \varphi_n \cdot e^{-i\frac{E_n}{\hbar}t}$

$$E = \frac{81\hbar^2\pi^2}{2ma^2} = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \iff \boxed{n=9}$$

$$C_9 = (\varphi_9, \Psi(x,0)) = \left[\int_{-a/2}^{0} \sqrt{\frac{2}{a}} \sin\left[\frac{9\pi}{a}(x+a/2)\right](x+a/2)\,dx + \right.$$

$$\left. + \int_0^{a/2} \sqrt{\frac{2}{a}} \sin\left[\frac{9\pi}{a}(x-a/2)\right](-x+a/2)\,dx\right]\sqrt{\frac{12}{a^3}} = -\frac{2\sqrt{2}}{81\left(\frac{1}{a}\right)^{3/2}\pi^2}\sqrt{\frac{12}{a^3}}$$

$$\Rightarrow \boxed{P_9 = |C_9|^2 = \frac{32}{2187\cdot\pi^4}}$$

6. etxe challenge

1. Galdera:

$$\frac{\hat{x}^2\hat{p} + \hat{p}\hat{x}^2}{2} = \frac{-i\hbar}{2}\left\{x^2\partial_x + 2x + x^2\partial_x\right\} = -i\hbar\left(x^2\partial_x + x\right) = x\overbrace{\left\{x(-i\hbar\partial_x) - i\hbar\right\}}^{\hat{p}\hat{x}} =$$

$$= \hat{x}\hat{p}\hat{x} \;//$$

2. Galdera

$\hat{I}$ hurrengoa egiten du $\hat{I}\,\psi(x) = \psi(-x)\,\hat{I}$ eta energia $\hat{T}$ eragilea, multzo osoa osatzen dute?

Multzo osoa izateko $[\hat{I}, \hat{T}] = 0$ izan behar da.

Lehenengo: dakigu $\hat{T} = \frac{\hat{p}^2}{2m} = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ dela

Eta $[\hat{I}, \hat{T}] = 0$ bada orduan $\hat{I}[\hat{T}\,\psi(x)] = \hat{T}[\hat{I}\,\psi(x)]$ izan beharko luke, hau da:

$$\left.\begin{array}{l} \hat{I}\left[\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)\right] = \frac{\hbar^2}{2m}\,\hat{I}\,\psi''(x) = \frac{\hbar^2}{2m}\,\psi''(-x) \\ \hat{T}\left[\hat{I}\,\psi(x)\right] = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(-x) = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\psi'(-x) = \frac{\hbar^2}{2m}\,\psi''(-x) \end{array}\right\} \Rightarrow$$

$\Rightarrow$ Berdinak direnez $[\hat{I}, \hat{T}] = 0$ da eta esan dezakegu $\{\hat{I}, \hat{T}\}$ multzo osoa dela.

3. Galdera.

m-partikulari $F = mg$ indarra m osoa baikia $\Psi(x,t)$ egoeran:

$t = 0$ denboran $\langle y_0 \rangle_\Psi = h$ eta $\langle v_y \rangle_\Psi = 0$

$t = \sqrt{h/g}$ bada $\langle y \rangle_\Psi = ?$

Ehrenfest-en teoremak direla eta:

$$\frac{d\langle p \rangle}{dt} = \langle F \rangle = -mg \Leftrightarrow \langle p \rangle = -mgt \Rightarrow \langle v_y \rangle = -gt$$

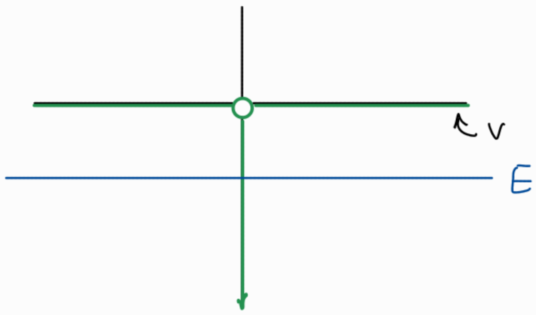
$$\Rightarrow \frac{d\langle y \rangle}{dt} = \frac{\langle p \rangle}{m} \Rightarrow \langle y \rangle = \langle y_0 \rangle - \tfrac{1}{2}gt^2 \Rightarrow \langle y(\sqrt{h/g}) \rangle = h - \frac{h}{2} = \frac{h}{2}$$

7. Etxeechallenge

1. Galdera

$V = -\alpha\, \delta(x)$ $\quad \alpha > 0$ $\quad$ V honek egoera lotuak izan ahal ditu?

Potentzialaren grafikoa egiten badugu:

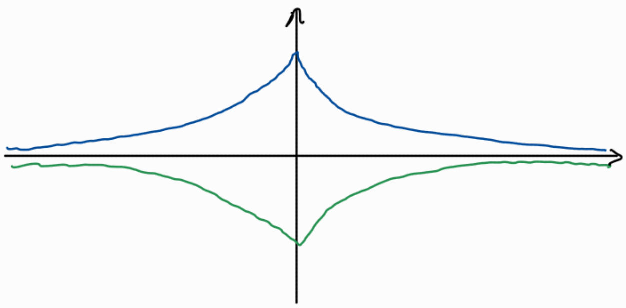


$$d(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

Posible da E hori? $\frac{\partial^2 \Psi}{\partial x^2} = \frac{2m}{\hbar}(V-E)\Psi$ -ren zeinua aztertuko dugu

$$(V-E) \begin{cases} > 0 & x = 0 \\ < 0 & x \neq 0 \end{cases} \qquad \Psi \begin{cases} \Psi < 0 & \cup \; x<0, \;\; \cap \; x>0 \\ \Psi > 0 & \cap \; x<0, \;\; \cup \; x>0 \end{cases}$$

Eta inflexio puntu bakarra izango duenez ez ditu oszilaziorik izango eta piko bat izango du $x=0$ puntuan:



Eta bien 2. deribatua $A\delta(x)$ bat izango da. Gainera biak simetrikoak izango dira.

2. Galdera

$\Psi(x,t)$ ondorengo oinarri ortonormalean dugu $\{\phi_1, \phi_2, \phi_3\}$ eta

$$\Psi(x,t) = \sum_{n=1}^{3} C_n(t)\, \phi_n$$

$$\hat{H} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$ bada eta $\Psi(x,0) = [i\phi_1 - i\phi_2 + i\phi_3]$

Zein da edozein t aldiunean izango dugun $\Delta \hat{H}$?

$$(\Psi(x,0), \Psi(x,0)) = [-i\phi_1 + i\phi_2 - i\phi_3][i\phi_1 - i\phi_2 + i\phi_3] = 3 \Rightarrow$$

$$\Rightarrow \Psi(x,0) = \frac{1}{\sqrt{3}}[i\phi_1 - i\phi_2 + i\phi_3] \rightarrow$$ edozein t aldiunerako:

$$\Psi(x,t) = \frac{1}{\sqrt{3}}\left[i e^{i\frac{\varepsilon_1}{\hbar}t}\phi_1 - i e^{i\frac{\varepsilon_2}{\hbar}t}\phi_2 + i e^{i\frac{\varepsilon_3}{\hbar}t}\phi_3\right] \Rightarrow$$

$$\Rightarrow \left\{ c_1 = \frac{i e^{i\frac{\varepsilon_1}{\hbar}}}{\sqrt{3}}, \quad c_2 = \frac{-i e^{i\frac{\varepsilon_2}{\hbar}t}}{\sqrt{3}}, \quad c_3 = \frac{i e^{i\frac{\varepsilon_3}{\hbar}t}}{\sqrt{3}} \right\}$$

Beraz batazbestekoak berealakoak dira ($e^{i\alpha}$-k joango dira konjokatuak direla eta, ez du t-ren menpekotasunik edukiko emaitzak)

$$\langle \hat{H} \rangle_\Psi = \frac{1}{3}\begin{pmatrix} -i & i & -i \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} i \\ -i \\ i \end{pmatrix} = \frac{1}{3}(1+1+3) = \frac{5}{3}$$

$$\langle \hat{H}^2 \rangle_\Psi = \frac{1}{3}\begin{pmatrix} -i & i & -i \end{pmatrix}\underbrace{\begin{pmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 9 \end{pmatrix}}_{\hat{H}^2}\begin{pmatrix} i \\ -i \\ i \end{pmatrix} = \frac{1}{3}(1+1+9) = \frac{11}{3}$$

Hau da $$\Delta\hat{H} = \sqrt{\langle \hat{H}^2\rangle_\Psi - \langle \hat{H}\rangle_\Psi^2} = \sqrt{\frac{11}{3} - \frac{25}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

8. Etxeechallenge.

1. Galdera.

$\Psi(x,t)$ daukagu eta $\{\phi_1, \phi_2, \phi_3\}$ oinarri ortonormala

$$\Psi(x,t) = \sum_{n=1}^{3} c_n(t)\phi_n$$

Biak oinarri berean.

$$\hat{H} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$ eta $$\hat{A} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{pmatrix}$$ eta $\Psi(x,t=0) = i[\phi_1 - \phi_2 + \phi_3]$

Energia neurtzean 1 balioa neurtu dugu jarraian A neurtzean zein izango da 3 neurtzeko probabilitatea eta honen denboraren menpekotasuna?

$\hat{H}$ eta $\hat{A}$ daudenez oinarri berdinean daude orduan, A-ren autobalioak eta autofuntzioak aterako ditugu.

- $\hat{A}$-ren autobalioak:

$$\det(\hat{A} - a\,\mathbb{1}) = 0 \iff \begin{vmatrix} 5-a & 0 & 0 \\ 0 & 2-a & i \\ 0 & -i & 2-a \end{vmatrix} = 0 \Rightarrow \begin{matrix} a_3 = 1 \\ a_2 = 3 \\ a_1 = 5 \end{matrix}$$

eta autofuntzio bakoitza $\begin{cases} \alpha_1 = (0,-i,1) \Rightarrow \alpha_1 = \frac{1}{\sqrt{2}}(0,-i,1) = \frac{1}{\sqrt{2}}(-i\phi_2 + \phi_3) \\ \alpha_3 = (0,i,1) \Rightarrow \alpha_3 = \frac{1}{\sqrt{2}}(i\phi_2 + \phi_3) \\ \alpha_5 = (1,0,0) \Rightarrow \alpha_5 = \phi_1 \end{cases}$

orduan T matrizea:

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \Rightarrow T^{-1} = T^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Neurtu ezin dugunez orduan gure egoera izango da ψ_3 egoera gogoratzen badugu egoera hau $\psi_3 = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2)$ izango da. eta dakigunez:

$$\psi_3' = T^{-1}\psi_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ i/2 \\ -i/2 \end{pmatrix} \Rightarrow \psi_3' = \frac{-1}{\sqrt{2}}\alpha_5 + \frac{i}{2}\alpha_3 - \frac{i}{2}\alpha_1$$

$(\psi_3', \psi_3') = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 \longrightarrow$ normalizatuta $\Rightarrow \boxed{P(3) = \left|\frac{i}{2}\right|^2 = \frac{1}{4}}$ ez da t-ren menpe

2. Galdera

Partikula baten egoera $\psi(x,t)$ $\{\phi_1, \phi_2, \phi_3\}$ oinarri ortonormal $\psi(x,t) = \sum_{n=1}^{3} c_n(t)\,\phi_n$ $\hat{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ $\hat{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{pmatrix}$ $\hat{A}$ eta $\hat{B}$ trukakorrak eta multzo osoa?

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = \begin{pmatrix} 2 & 2 & i \\ 1 & 4 & i \\ 0 & -3i & 6 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 0 \\ 2 & 4 & 3i \\ -i & -2i & 6 \end{pmatrix} \neq 0 \Rightarrow$$ ez dira ez trukakorrak ez multzo osoa.

3. Galdera

$\{\phi_1, \phi_2, \phi_3\}$ eta $\hat{A} = \begin{pmatrix} 0 & 0 & a_0 \\ 0 & a_0 & 0 \\ a_0 & 0 & 0 \end{pmatrix}$ $\hat{B} = \begin{pmatrix} 2b_0 & -ib_0 & -ib_0 \\ ib_0 & 2b_0 & b_0 \\ ib_0 & b_0 & b_0 \end{pmatrix}$ $a_0, b_0 \in \mathbb{R}$

Craiki ψ hurrengo baldintzetan:

1. ψ egoeran $\hat{A}$ neurtzen badugu a_0 neurtu ziurtasun osos
2. ψ egoeran $\hat{B}$ neurtu b_0 neurtzeko probabilitate nulua.

A-ren balio eta bektoreak

$$\begin{vmatrix} -\lambda & 0 & a_0 \\ 0 & a_0-\lambda & 0 \\ a_0 & 0 & -\lambda \end{vmatrix} = 0 \iff \lambda^2(a_0-\lambda) - a_0^2(a_0-\lambda) = 0 \quad \begin{array}{l} \nearrow \lambda = -a_0 \\ \searrow \lambda = a_0 \quad m=2 \end{array} \Rightarrow$$

$$\Rightarrow \begin{cases} v_{-a} = (-1,0,1) \\ v_a = (1,0,1) \\ v_a' = (0,1,0) \end{cases} \longrightarrow \beta_a = \{v_{-a}, v_a, v_a'\} \text{ ortonormala}$$

B-rekin gauza berdina.

$$\begin{array}{l} \lambda = (2+\sqrt{3})b_0 \\ \lambda = b_0 \\ \lambda = (2-\sqrt{3})b_0 \end{array} \longrightarrow \begin{cases} v_+ = \left(\frac{-i}{2}(1+\sqrt{3}), \frac{1}{2}(1+\sqrt{3}), 1\right) \\ v_b = (i, 1, 0) \\ v_- = \left(\frac{i}{2}(-1+\sqrt{3}), \frac{1}{2}(1-\sqrt{3}), 1\right) \end{cases}$$

Esta claro que:

$\psi = a\,v_a + b\,v_a' = a\phi_1 + b\phi_2 + a\phi_3$ y ahora sabemos las convinaciones con B:

0 para que $P(b_0)=0$

$$a = a'\left(\frac{-i}{2}\right)(1+\sqrt{3}) + \cancel{b'}\,i + c'\frac{i}{2}(-1+\sqrt{3}) \quad (1)$$

$$b = \frac{a'}{2}(1+\sqrt{3}) + \cancel{b'} + \frac{c'}{2}(1-\sqrt{3}) \quad (2)$$

$$a = a' + c' \quad (3)$$

$$(1) \longrightarrow a = -\frac{i}{2}(a'+c') + i\frac{\sqrt{3}}{2}(c'-a') \overset{\times i}{\Rightarrow} a i = \frac{1}{2}(a'+c') + \frac{\sqrt{3}}{2}(a'-c')$$

$$(2) \longrightarrow b = \frac{1}{2}(a'+c') + \frac{\sqrt{3}}{2}(a'-c')$$

$$\Rightarrow \boxed{a i = b}$$

$$a i (-a i)$$

Hau de gure $\Psi = a\phi_1 + a i \phi_2 + a\phi_3$ Normalizatzeu baldintza:

$$(\Psi, \Psi) = a^2 + a^2 + a^2 = 1 \Leftrightarrow a = 1/\sqrt{3} \Rightarrow \boxed{\Psi = \frac{1}{\sqrt{3}}(\phi_1 + i\phi_2 + \phi_3)}$$

9. Etxeechallenge

2. Galdera

Oinarri ortonormala $\{\phi_1, \phi_2, \phi_3\}$, A eta B oinarri horretan

$$\hat{A} = \begin{pmatrix} 2 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 2 \end{pmatrix} \qquad \hat{B} = \frac{1}{2}\begin{pmatrix} 3 & -i\sqrt{2} & i \\ i\sqrt{2} & 2 & \sqrt{2} \\ -i & \sqrt{2} & 3 \end{pmatrix}$$

, zein egoeratan neur genezake $\hat{A}$ eta $\hat{B}$ zehaztasun osos?

Lehenengo trukakorrak diren aztertuko dugu. (izan behar dira trukakorrak biak oinarri berean baitaude).

$$\hat{A}: \begin{cases} a = 3 \;, \; m=1 \\ a = 1 \;, \; m=2 \end{cases} \longrightarrow \begin{cases} \psi_3^A = \frac{1}{\sqrt{2}}(i,0,1) = \frac{1}{\sqrt{2}}(i\phi_1 + \phi_3) \\ \psi_1^A = \frac{1}{\sqrt{2}}(-i,0,1) = \frac{1}{\sqrt{2}}(-i\phi_1 + \phi_3) \\ \psi_1'^A = (0,1,0) \end{cases}$$

$\longleftarrow$ Bastante estupido sa cando este...

$$\hat{B}: \begin{cases} b = 2 \;, \; m=2 \\ b = 0 \;, \; m=1 \end{cases} \longrightarrow \begin{cases} \psi_2^B = \frac{1}{\sqrt{2}}(i,0,1) = \psi_3^A \\ \psi_2'^B = \frac{1}{\sqrt{3}}(-i\sqrt{2},1,0) = \frac{1}{\sqrt{3}}(-i\sqrt{2}\phi_1 + \phi_2) \\ \psi_0^B = \frac{1}{\sqrt{4}}(-i,-\sqrt{2},1) = \frac{1}{2}(-i\phi_1 - \sqrt{2}\phi_2 + \phi_3) \end{cases}$$

Trukakorrak dira beraz multso osoa osatu dezakete, eta ikusten dugunez egingo dute endekatuta dauden balioak neurtzean bereiztu ahal dugulako zein den neurketa.

ψ_1^A eta $\psi_1'^A$ endekatuak daudenez B-ren oinarri bat osatu ahal dugu haien konbinazio lineal bateRin. eta hau:

$$\psi_0^B = \frac{1}{2}(-i\phi_1 - \sqrt{2}\phi_2 + \phi_3) = a\psi_1^A + b\psi_1'^A = \frac{a}{\sqrt{2}}(-i\phi_1 + \phi_3) + b\phi_2 \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \frac{-ia}{\sqrt{2}} = \frac{-i}{\sqrt{4}} \Leftrightarrow a = \frac{\sqrt{2}}{\sqrt{4}} = 1/\sqrt{2} \\ b = -\frac{\sqrt{2}}{\sqrt{4}} \Rightarrow b = -1/\sqrt{2} \end{array} \right\} \Rightarrow \boxed{\psi_1 \equiv \psi_0^B = \frac{1}{\sqrt{2}}\left(\psi_1^A - \psi_1'^A\right)}$$

ψ_1 autofuntzioa → A behagarriaren 1 balioa / → B behagarriaren 0 balioa $\overset{\text{Not}}{\equiv}$ (1, 0) autobalioa emango dugu.

Berdina egiten badugu A-rekin ψ_2^B eta $\psi_2'^B$-ren konbinazio lineal bat lortuko dugu ψ_3^A bektorea izateko. Orduan

$$\psi_3^A = a\,\psi_2^B + b\,\psi_2'^B = \psi_2^B = \psi_2$$

ψ_2 autofuntzioaren autobalioa (3, 2)

ψ_3 ateratzeko ortogonaltasuna erabiliko dugu: $\rightarrow \psi_3 = a\phi_1 + b\phi_2 + c\phi_3$

$$(\psi_1, \psi_3) = 0 \Leftrightarrow \frac{i}{\sqrt{4}}a - \frac{b}{\sqrt{2}} + \frac{c}{\sqrt{4}} = 0 \Rightarrow \frac{2ia}{\sqrt{4}} = \frac{b}{\sqrt{2}} \Leftrightarrow \boxed{b = \sqrt{2}\,i a}$$

$$(\psi_2, \psi_3) = 0 \Leftrightarrow \frac{-ia}{\sqrt{2}} + 0 \cdot b + \frac{c}{\sqrt{2}} = 0 \Rightarrow \boxed{c = ia}$$

Hau da $\psi_3 = -i\phi_1 + \sqrt{2}\,\phi_2 + \phi_3 \xrightarrow{\text{Normalizatuz}} \boxed{\psi_3 = \frac{1}{2}\left(-i\phi_1 + \sqrt{2}\,\phi_2 + \phi_3\right)}$

$$\psi_3 = \frac{\sqrt{2}}{2}\left(\psi_1^A + \psi_1'^A\right) = \frac{1}{\sqrt{2}}\psi_2^B + \sqrt{\frac{3}{2}}\,\psi_2'^B$$

$\rightarrow$ ψ_3-ren autobalioa (1, 2) izango da

Gure oinarria izango da

$$\left\{ \frac{1}{2}\left(-i\phi_1 - \sqrt{2}\,\phi_2 + \phi_3\right), \frac{1}{\sqrt{2}}\left(i\phi_1 + \phi_3\right), \frac{1}{2}\left(-i\phi_1 + \sqrt{2}\,\phi_2 + \phi_3\right) \right\}$$ izango da.

Berridatziz erantzunekin bat etortzeko:

$$\left\{ \underbrace{\frac{1}{2}(-i\phi_1+\phi_3)-\frac{1}{\sqrt{2}}\phi_2}_{\downarrow\cdot i},\ \underbrace{\frac{1}{\sqrt{2}}(i\phi_1+\phi_3)}_{\downarrow\cdot(-i)},\ \underbrace{\frac{1}{2}(-i\phi_1+\phi_3)+\frac{1}{\sqrt{2}}\phi_2}_{\downarrow\cdot i} \right\}$$

$$\left\{ \frac{1}{2}(\phi_1+i\phi_3)-\frac{i}{\sqrt{2}}\phi_2,\ \frac{1}{2}(\phi_1+i\phi_3)+\frac{i}{\sqrt{2}}\phi_2,\ \frac{1}{\sqrt{2}}(\phi_1-i\phi_3) \right\}$$

(a)

1. Galdera.

Atomo baten momentu angeluarraren modulua konstantea da eta bere balioa $\sqrt{2}\hbar$. $\hat{L}_z$ eta autofuntzioak $\{\phi_{-\hbar}, \phi_0, \phi_\hbar\}$ diren Hamiltondearen adierazpena: $\hat{H}=\hbar\omega_0\begin{pmatrix}2&0&1\\0&1&0\\1&0&2\end{pmatrix}$

Elektroiaren momentu angeluarraren OX osagaia L_x neurtzean baliorik handiena neurtu dugu. Goiko oinarrian $\hat{L}_x=\hbar/\sqrt{2}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$ $\hat{L}_x$ berriro neurtzean noiz lortuko dugu balio berbera.

Lehenengo $\hat{L}_x$-ren autobalioak eta autofuntzioak egingo ditugu.

$$\hat{L}_x \begin{cases} l_\hbar = \hbar \\ l_{-\hbar} = -\hbar \\ l_0 = 0 \end{cases} \longrightarrow \begin{cases} \psi_\hbar^L = \frac{1}{\sqrt{4}}(1,\sqrt{2},1) = \frac{1}{\sqrt{4}}(\phi_{-\hbar}+\sqrt{2}\phi_0+\phi_\hbar) \\ \psi_{-\hbar}^L = \frac{1}{\sqrt{4}}(1,-\sqrt{2},1) \\ \psi_0^L = \frac{1}{\sqrt{2}}(-1,0,1) \end{cases}$$

Hau da neurtzean lortu dugun balioa $l_\hbar$ denez, lumengoa dugu:

$$\Psi_L(x,0)=\frac{1}{2}(\phi_{-\hbar}+\sqrt{2}\phi_0+\phi_\hbar)=\frac{1}{2}(\phi_{-\hbar}+\phi_\hbar)+\frac{1}{\sqrt{2}}\phi_0$$

Orain jakin nahi dugu Hamiltondarren autofuntzioen menpean aurreko uhin-funtzioaren adierazpena. Hau lortzeko:

$$\hat{H} \rightarrow \begin{cases} E = \hbar\omega_0 \\ E = \hbar\omega_0 \\ E = 3\hbar\omega_0 \end{cases} \longrightarrow \begin{cases} \phi_1 = \frac{1}{\sqrt{2}}(-1,0,1) = \frac{1}{\sqrt{2}}(-\phi_\hbar + \phi_{-\hbar}) \\ \phi_1' = (0,1,0) = \phi_0 \\ \phi_3 = \frac{1}{\sqrt{2}}(1,0,1) = \frac{1}{\sqrt{2}}(\phi_\hbar + \phi_{-\hbar}) \end{cases}$$

Or duzu: Ψ_H

$$\Psi_H = \frac{a}{\sqrt{2}}(-\phi_\hbar + \phi_{-\hbar}) + b\,\phi_0 + \frac{c}{\sqrt{2}}(\phi_\hbar + \phi_{-\hbar}) = \frac{1}{\sqrt{4}}(\phi_\hbar + \phi_{-\hbar}) + \frac{1}{\sqrt{2}}\phi_0$$

$$\Rightarrow \begin{cases} \frac{-a}{\sqrt{2}} + \frac{c}{\sqrt{2}} = \frac{1}{\sqrt{4}} \Rightarrow c - a = \frac{1}{\sqrt{2}} \Leftrightarrow c = \frac{1}{\sqrt{2}} + a \Rightarrow c = \frac{1}{\sqrt{2}} \\ b = +1/\sqrt{2} \\ \frac{a}{\sqrt{2}} + \frac{c}{\sqrt{2}} = \frac{1}{\sqrt{4}} \Rightarrow \frac{1}{\sqrt{2}} + a + a = \frac{1}{\sqrt{2}} \Rightarrow a = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \Psi_H(x,0) = \frac{1}{\sqrt{2}}(\phi_3 + \phi_1') \Rightarrow \Psi_H(x,t) = \frac{1}{\sqrt{2}}\left(\phi_3\, e^{-i3\omega_0 t} + \phi_1'\, e^{-i\omega_0 t}\right) \Rightarrow$$

$$\Rightarrow \Psi_L(x,t) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(\phi_\hbar + \phi_{-\hbar})\, e^{-i3\omega_0 t} + \phi_0\, e^{-i\omega_0 t}\right) =$$

$$= \frac{1}{2}\left((\phi_\hbar + \phi_{-\hbar})\, e^{-i3\omega_0 t} + \sqrt{2}\,\phi_0\, e^{-i\omega_0 t}\right)$$

eta $\Psi_L(x,t) = \Psi_L(x,t=0) \Leftrightarrow$

$$\frac{1}{2}\left((\phi_\hbar + \phi_{-\hbar})\, e^{-i3\omega_0 t} + \sqrt{2}\,\phi_0\, e^{-i\omega_0 t}\right) = \frac{1}{2}\left(\phi_\hbar + \phi_{-\hbar} + \sqrt{2}\,\phi_0\right) \Leftrightarrow$$

$$\begin{cases} e^{-i3\omega_0 t} = 1 \Rightarrow \cos 3\omega_0 t - i \sin 3\omega_0 t = 1 \\ e^{-i\omega_0 t} = 1 \Rightarrow \cos\omega_0 t - i\sin\omega_0 t = 1 \end{cases} \Rightarrow$$

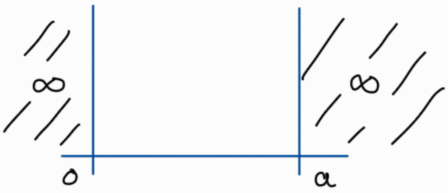
$$\Rightarrow \cos 3\omega_0 t - \cos\omega_0 t - i(\sin 3\omega_0 t - \sin\omega_0 t) = 0$$

Hau da $\begin{cases} \cos 3\omega_0 t - \cos\omega_0 t = 0 \rightarrow t = \frac{n\pi}{\omega_0}, \frac{\pi n}{\omega_0} - \frac{\pi}{2\omega_0} \\ \sin 3\omega_0 t - \sin\omega_0 t = 0 \rightarrow t = \frac{n\pi}{\omega_0}, \frac{\pi n}{\omega_0} - \frac{3\pi}{4\omega_0}, \frac{\pi n}{\omega_0} - \frac{\pi}{4\omega_0} \end{cases}$

Hau da biak aldi berean $\iff t = \frac{n\pi}{\omega_0}$ $\forall n \in \mathbb{N}$ bada

(a)

3. Galdera.



$$\Psi(x,0) = \begin{cases} x(a-x), & 0 \leq x \leq a \\ 0, & - \end{cases}$$

Edozein aldiunetan zein da energia 0 eta $\frac{3\hbar^2\pi^2}{ma^2}$ neurtzeko probabilitatea.

Lehenengo normalizatu behar dugu. Orduan: $\int_0^a |A|^2 (x(a-x))^2 dx =$

$= |A|^2 \frac{a^5}{30} \stackrel{!}{=} 1 \iff |A|^2 = \sqrt{\frac{30}{a^5}}^2$

Hau da $\Psi(x,0) = \sqrt{\frac{30}{a^5}}^2 x(a-x)$

Dakigu zein potentzial osin infinituaren oinarria eta $P(E_n) = |C_n|^2$ izango dela eta: atera nahi dugun energia $E = \frac{3\hbar^2\pi^2}{ma^2}$ jakin dezakegu zein den n: $\frac{n^2\hbar^2\pi^2}{2ma^2} = \frac{3\hbar^2\pi^2}{ma^2} \iff$

$\iff n^2 = 6$, hau da bakarrik 2 energia egongo dira 0 eta $n^2 = 6$ tartean E_1 eta $E_2 \Rightarrow P\left(0 < E < \frac{3\hbar^2\pi^2}{ma^2}\right) = |C_1|^2 + |C_2|^2$

orduan $|C_n|$ak ateratzeko nahiko erreza da.

Dakigu $\Psi(x,t) = \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$

$$\Psi(x,0) = \sum_{n=1}^{\infty} C_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) =$$

$$\int_0^a C_n \sin\left(\frac{n\pi}{a}x\right) \sin\left(\frac{m\pi}{a}x\right) \frac{2}{a}\, dx = \sqrt{\frac{60}{a^6}} \int_0^a x(a-x) \sin\left(\frac{m\pi}{a}x\right) dx.$$

$$\Longleftrightarrow \quad C_n = \frac{\sqrt{60}}{a^3} \int_0^a x(a-x) \sin\left(\frac{n\pi}{a}x\right) dx$$

Hau da

- $C_1 = \frac{\sqrt{60}}{a^3} \int_0^a x(a-x) \sin\left(\frac{\pi}{a}x\right) dx = \frac{8\sqrt{15}}{\pi^3}$
- $C_2 = \frac{\sqrt{60}}{a^3} \int_0^a x(a-x) \sin\left(\frac{2\pi}{a}x\right) dx = 0$

$$\Rightarrow P\left(0 < E < \frac{3\hbar^2\pi^2}{ma^2}\right) = |C_1|^2 = \left|\frac{8\sqrt{15}}{\pi^3}\right|^2 = \frac{960}{\pi^6} \quad // \qquad (d)$$

10. Etxeechallenge

1. Galdera

$\hat{x}$-ren adierazpen matriziala a zabalerako p.o.i-ko oinarrian.

$$\psi_n = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} & 0 \leq x \leq a \\ 0 & \text{besteetan.} \end{cases}$$

orduan

$$(\hat{x})_{mn} = (\psi_n, \hat{x}\psi_m) = \int_0^a \frac{2}{a} \sin\frac{n\pi x}{a} \cdot x \sin\frac{m\pi x}{a}\, dx$$

$n = m$ bada:

$$(\hat{x})_{nn} = -\frac{a\left[-1 - 2n^2\pi^2 + \cos(2n\pi) + 2n\pi \sin(2n\pi)\right]}{4n^2\pi^2} = \frac{a\, 2n^2\pi^2}{4n^2\pi^2} = a\,\frac{1}{2}$$

$n \neq m$ bada.

$$(\hat{x})_{nm} = \frac{2a}{(m^2-n^2)^2\pi^2}\left[-2mn + \sin\cancel{[m\pi]}\,(\quad) + m\cos(m\pi)\left(2n\cos(n\pi) + \cancel{(n^2-m^2)\pi\sin(n\pi)}\right)\right]$$

$$= \frac{2a}{(m^2-n^2)^2\pi^2}\left(-2mn + 2mn\cos(m\pi)\cos(n\pi)\right) = \frac{4a}{\pi^2}\cdot\frac{mn}{(m^2-n^2)^2}\left[\cos(m\pi)\cos(n\pi) - 1\right]$$

Dakigu $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \Rightarrow \cos((n\pm m)\pi) = \cos(m\pi)\cos(n\pi) \pm 0$

Hau da: $(\hat{x})_{nm} = \frac{4a}{\pi^2}\frac{nm}{(m^2-n^2)^2}\left[\cos((n\pm m)\pi) - 1\right]$

Hau da $(\hat{x})_{nm} = \frac{4a}{\pi^2}\frac{nm}{(m^2-n^2)^2}\begin{cases} 0 & n\pm m \text{ bikoiti} \\ -2 & n\pm m \text{ bakoiti} \end{cases}$

Beraz

$$\boxed{(\hat{x})_{nm} = a\begin{cases} 1/2 & ;\ n=m \\ 0 & ;\ n\neq m \text{ eta } n\pm m \text{ bikoiti} \\ \dfrac{-8nm}{\pi^2(m^2-n^2)^2} & ;\ n\neq m \text{ eta } n\pm m \text{ bakoiti} \end{cases}}$$

2. Galdera.

m masadun partikula L zabalera duen p.o.i.

$$\Psi(x, t=0) = \begin{cases} x(L-x) & ;\ 0\leq x\leq L \\ 0 & ;\ \text{besteetan.} \end{cases} \qquad \langle \hat{H}\rangle_{\Psi(x,t)}\ ?$$

Lehenengo Normalizatzen dugu: $\Psi(x,t=0) = \sqrt{\frac{30}{L^5}}\,x(L-x)$

orduan:

$$\langle \hat{H} \rangle_\psi = (\psi, \hat{H}\psi) = \left(\psi, \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi\right) =$$

$$= \int_0^L \frac{30}{L^5}\left[x(L-x)\left(-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}(L-2x)\right)\right] dx = \cancel{+}\frac{30\hbar^2}{\cancel{2}m L^5}\int_0^L x(L-x)\,\cancel{(-2)}\,dx =$$

$$= \frac{30\hbar^2}{mL^5}\left[\frac{Lx^2}{2} - \frac{x^3}{3}\right]_0^L = \frac{30\hbar^2}{mL^2}\cdot\left[\frac{1}{2}-\frac{1}{3}\right] = \frac{30\hbar^2}{mL^2}\,\frac{1}{6} = \frac{5\hbar^2}{mL^2} \Rightarrow$$

$$\Rightarrow \boxed{\langle \hat{H} \rangle_\psi = \frac{5\hbar^2}{mL^2}} \rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \Rightarrow \boxed{n \approx 1}$$

bat dator aurreko taechallenge-ren emaitzarekin

3. galdera.

m masa duen partikula L zabalera duen p.o.i dago. neurtutako energia $E = \frac{\hbar^2\pi^2}{2mL^2}$ zein izango da lor dezakegun momentuaren baliorik probableena?

$$E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} = \frac{\hbar^2\pi^2}{2mL^2} \Leftrightarrow n=1 \Rightarrow$$ dakigu neurtu ondoren izango

dugun uhin-funtzioa : $\psi(x, t=0) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$ izango dela.

Beraz: $\langle \hat{p} \rangle = (\psi, \hat{p}\psi) = \frac{2}{L}\int_0^L (-i\hbar)\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx = 0$ izango da

gainera uhin funtzioa erreala denez dakigu $\langle \hat{p} \rangle = 0$ izango zela.

11. Etxeko challenge.

1. Galdera

m masa duen partikula a zabalerako p.o.i $[0,a]$

$$\Psi(x,t=0) = \left(1+\cos\frac{\pi x}{a}\right)\sin\frac{\pi x}{a}$$

Edozein t aldiunean masa "$\leftarrow$" ezkerreko aldean aurkitzeko probabilitatea.

Gure uhin funtzioa: $\Psi(x,t=0) = \sin\frac{\pi x}{a} + \underbrace{\cos\frac{\pi x}{a}\sin\frac{\pi x}{a}}_{\frac{1}{2}\sin\frac{2\pi x}{a}} \Rightarrow$

$$\Rightarrow \Psi(x,t=0) = \sin\frac{\pi x}{a} + \frac{1}{2}\sin\frac{2\pi x}{a} \overset{\text{Normalizatu}}{\Rightarrow} |A| = 2\sqrt{\frac{2}{5a}}$$

$$\Rightarrow \Psi(x,t=0) = \sqrt{\frac{8}{5a}}\sin\frac{\pi x}{a} + \sqrt{\frac{2}{5a}}\sin\frac{2\pi x}{a}$$

Ψ t-ren funtzioan jartzea erreza da:

$$\Psi(x,t) = \sqrt{\frac{8}{5a}}\sin\frac{\pi x}{a}\, e^{-i\frac{\varepsilon_1}{\hbar}t} + \sqrt{\frac{2}{5a}}\sin\frac{2\pi x}{a}\, e^{-i\frac{\varepsilon_2}{\hbar}t}$$

Hau da

$$|\Psi|^2 = \frac{8}{5a}\sin^2\frac{\pi x}{a} + \frac{2}{5a}\sin^2\frac{2\pi x}{a} + \frac{4}{5a}\cdot 2\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\varepsilon_2-\varepsilon_1}{\hbar}t\right)$$

orduan

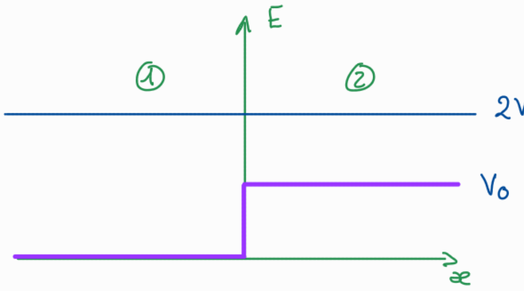
$$P(0\leq x\leq \tfrac{a}{2};t) = \int_0^{a/2}|\Psi|^2\,dx = \underbrace{\frac{2}{5}+\frac{1}{10}}_{1/2} + \frac{16}{15\pi}\cos\left(\frac{\varepsilon_2-\varepsilon_1}{\hbar}t\right)$$

$$\varepsilon_2-\varepsilon_1 = \frac{\hbar^2\pi^2}{2ma^2}(4-1) = \frac{3\hbar^2\pi^2}{2ma^2} \Rightarrow \boxed{P(0\leq x\leq a/2;t) = \frac{1}{2} + \frac{16}{15\pi}\cos\left(\frac{3\hbar\pi^2}{2ma^2}t\right)}$$

(c)

2. Galdera.

$E = 2V_0$ energia eta m masa dun partikula batek $V_0 > 0$ potentzial-jausia-ren kontra jotzen du. Zein da probabilitatea partikula isla tzeko. (elkarrekintzan)



Bi alde ditugu beraz gure uhin funtzioa bi osagaiz osatuta dago:

$$\Psi = \begin{cases} Ae^{ik_1x} + Be^{-ik_1x} & ; \quad k_1 = \dfrac{\sqrt{2mE}}{\hbar} \\ Ce^{ik_2x} + De^{-ik_2x} & ; \quad k_2 = \dfrac{\sqrt{2m(E-V_0)}}{\hbar} \end{cases}$$

Partikula ezkerretik datorrela suposatuz, $\boxed{D=0}$ dela esan dezakegu.

Jarraitasuna erabiliz:

- $\Psi_1(0) = \Psi_2(0) \Rightarrow A + B = C$
- $\Psi_1'(0) = \Psi_2'(0) = ik_1(A-B) = ik_2 C \Rightarrow \dfrac{k_1}{k_2}A - \dfrac{k_1}{k_2}B = A + B \Leftrightarrow$

$$\Leftrightarrow \left(\frac{k_1}{k_2} - 1\right)A = B\left(\frac{k_1}{k_2} + 1\right) \Rightarrow (k_1 - k_2)A = (k_1 + k_2)B \Rightarrow \boxed{B = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)A}$$

$$C = A + B = A\left(\frac{k_1 + k_2 + k_1 - k_2}{k_1 + k_2}\right) = \boxed{\frac{2k_1 A}{k_1 + k_2} = C}$$

Hau jakinda isla tzeko probabilitatea:

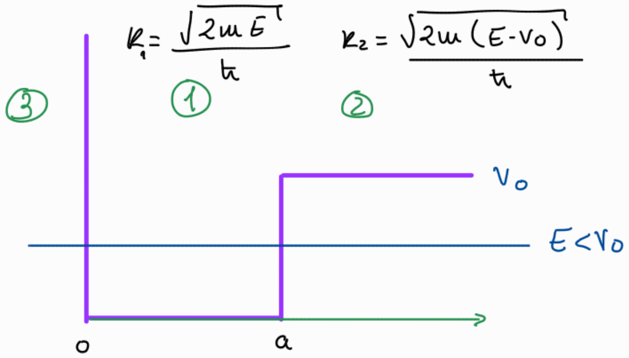
$$R = \frac{|B|^2 \frac{\hbar k_1}{m}}{|A|^2 \frac{\hbar k_1}{m}} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{(\sqrt{2V_0} - \sqrt{V_0})^2}{(\sqrt{2V_0} + \sqrt{V_0})^2} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2} + 1)^2} = 0'029 \Rightarrow \boxed{R_{\%} = \%2'94}$$

(e)

3. Galdera

m masadun partikula hu mengo osinean dago. $V(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < a \\ V_0 (>0) & x \geq a \end{cases}$

Zein da V_0-k bete behar duen erlazioa $E < V_0$ egoerak ez egoteko.



Ψ 3 osagaiz osatuta dago:

$$\Psi(x) = \begin{cases} 0 & ; \ x \leq 0 \\ Ae^{ik_1 x} + Be^{-ik_1 x} & ; \ 0 < x < a \\ Ce^{k_2 x} + De^{-k_2 x} & ; \ x \geq a \end{cases}$$

Lehenengo muga baldintzen bitartez:

- $\psi_1(0) = 0 \iff \boxed{B = -A} \Rightarrow \psi_1 = A \sin(k_1 x)$

- $\psi_2(x \to \infty) = 0 \iff \boxed{C = 0}$

Jarraitasunarekin:

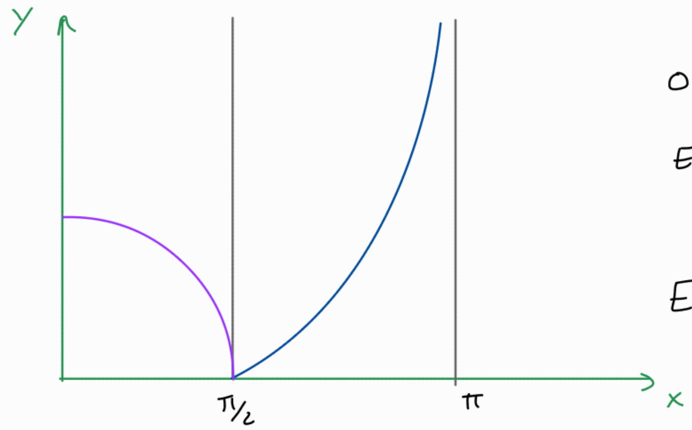
- $\psi_1(a) = \psi_2(a) \Rightarrow A\sin(k_1 x) = C e^{-k_2 a}$
- $\psi_1'(a) = \psi_2'(a) \Rightarrow Ak_1 \cos(k_1 x) = -k_2 C e^{-k_2 a}$

(·a) bi aldeetan

$\Rightarrow -\cot g(k_1 a)\, k_1 a = k_2 a$

Hau grafikoki ebasteko: $X = k_1 a$; $Y = k_2 a \Rightarrow$ $Y = -X\cot g(X)$

$X^2 + Y^2 = R^2$



Orduan $R = \pi/2$ denean $Y = 0$ izango da eta honen ondorioz $k_2 a = 0 \iff \boxed{E = V_0}$

Eta $\frac{2mV_0 a^2}{\hbar^2} = \frac{\pi^2}{4} \iff \boxed{V_0 = \frac{\hbar^2 \pi^2}{8ma^2}}$

Hau da $V_0 < \frac{\hbar^2\pi^2}{8ma^2}$ izan behar da goera loturik ez goteko.

(c)

12. Etxeechallenge

1. Galdera.

m masa duen partikula $V(x) = \frac{1}{2}Kx^2$ potentzialean.

$$\Psi(x, t=0) = \begin{cases} 1 & , \ x \in [0, d] \\ 0 & , \ \text{besteetan} \end{cases}$$

$d=\sqrt{\frac{2\hbar}{m\omega}}$, $\omega=\sqrt{\frac{k}{m}}$. Energia neurtzean zein da $E=\frac{1}{2}\hbar\omega$ neurtzeko probabilitatea.

Lehenengo uhin funtzioa normalizatu behar dugu:

$$(\Psi,\Psi)\overset{?}{=}1 \iff |A|^2\int_{-\infty}^{\infty}|\Psi|^2dx=|A|^2 d=1 \iff \boxed{|A|=d^{-1/2}}$$

Hau da, $\Psi(x,t=0)=\begin{cases}1/\sqrt{d} & , x\in[0,d]\\ 0 & , \text{besteetan}\end{cases}$

$E=\frac{1}{2}\hbar\omega$ izango da $n=0$ balioarentzako C_0-a jakin nahi dugu:

Lehenengo: $\psi_0=A\cdot e^{-\frac{m\omega}{2\hbar}x^2}$ non $A=\sqrt[4]{\frac{m\omega}{\pi\hbar}}$

Orduan $C_0=(\Psi,\psi_0)=\int_0^d\frac{1}{\sqrt{d}}\cdot\sqrt[4]{\frac{m\omega}{\pi\hbar}}\cdot e^{-\frac{m\omega}{2\hbar}x^2}dx=$

$$=\left(\frac{m\omega\pi}{\hbar}\right)^{1/4}\cdot\sqrt{\frac{\hbar}{2dm\omega}}\cdot\text{erf}\left(d\sqrt{\frac{m\omega}{2\hbar}}\right)=$$

$$=\left(\frac{m\omega\pi}{\hbar}\right)^{1/4}\left(\sqrt{\frac{m\omega}{2\hbar}}\;\frac{\hbar}{m\omega}\right)^{1/2}\text{erf}(1)=$$

$$=\left(\frac{\cancel{m\omega}\,\pi}{\cancel{\hbar}}\;\frac{\cancel{m\omega}}{2\cancel{\hbar}}\;\frac{\cancel{\hbar^2}}{\cancel{m^2\omega^2}}\right)^{1/4}\text{erf}(1)=\left(\frac{\pi}{2}\right)^{1/4}\cdot\text{erf}(1)=\frac{(2\pi)^{1/4}}{2}\text{erf}(1)$$

orduan $\boxed{P(E=\frac{1}{2}\hbar\omega)=\left|\frac{(2\pi)^{1/4}}{2}\text{erf}(1)\right|^2=0'445017\approx\%\,44'5}$

2. Galdera

m masa duen partikula $V(x)=\frac{1}{2}kx^2$ potentzialaren eraginpean dago. Hasierako egoera $\Psi(x,0)=\sum_{n=0}^{\infty}C_n\psi_n$ non ψ_n harmonikoaren autofuntzioak. $\langle x\rangle_{t=0}=x_0$ bada $\langle x\rangle_t$?

Dakigu: $\langle x \rangle_{t=0} = x_0$ beraz Ehrenfesten teorema erabiliko dugu:

$$\frac{d\langle \hat{x}\rangle}{dt} = \frac{\langle \hat{p}\rangle}{m} \Rightarrow \frac{d^2\langle \hat{x}\rangle}{dt^2} = \frac{1}{m}\frac{d\langle \hat{p}\rangle}{dt} = -\frac{1}{m}\left\langle \frac{\partial V}{\partial x}\right\rangle = -\frac{1}{m}\cdot K\langle \hat{x}\rangle \Rightarrow$$

$$\Rightarrow \langle \ddot{\hat{x}}\rangle + \frac{K}{m}\langle \hat{x}\rangle = 0 \Leftrightarrow \boxed{\langle \hat{x}\rangle(t) = A\cos(\omega t) + B\sin(\omega t) \quad \text{non} \quad \omega = \sqrt{\frac{K}{m}}}$$

Dakigu: $\langle \hat{x}\rangle(0) = x_0$ dela $\rightarrow$ $A\cos(\omega\cdot 0) + B\sin(\omega\cdot 0) = x_0 \Rightarrow \boxed{A = x_0}$

Gainera: $\langle \hat{p}\rangle_{t=0} = (\psi, \hat{p}\,\psi) = \underbrace{-i\sqrt{\frac{\hbar\omega m}{2}}}_{\alpha}(\psi, (\hat{a}-\hat{a}^+)\,\psi) =$

$$= -i\alpha\left\{\underbrace{\left(\sum c_n\psi_n, \sum c_n\hat{a}\psi_n\right)}_{0} - \underbrace{\left(\sum c_n\psi_n, \sum c_n\hat{a}^+\psi_n\right)}_{0}\right\} = 0$$

Berriz Ehrenfesten teorema erabiliz

$$\left.\frac{d\langle \hat{x}\rangle}{dt}\right|_{t=0} = \left.\frac{\langle \hat{p}\rangle}{m}\right|_{t=0} \Rightarrow (-x_0\sin(\omega 0) + B\cos(\omega 0))\,\omega = 0 \Leftrightarrow \boxed{B=0}$$

Hau da:

$$\boxed{\langle x\rangle(t) = x_0\cos(\omega t)}$$

3. Galdera.

m masadun partikula $V(x) = \frac{1}{2}Kx^2$. $\hat{a}\psi_\alpha = \alpha\psi_\alpha$ $\alpha\in\mathbb{C}$ izanda
$\langle \hat{p}^2\rangle_{\psi_\alpha}$?

Dakigunez $\hat{p} = -i\sqrt{\frac{\hbar\omega m}{2}}(\hat{a}-\hat{a}^+) \Rightarrow \hat{p}^2 = -\frac{\hbar\omega m}{2}(\hat{a}-\hat{a}^+)^2$

Orduan

$$\langle \hat{p}^2\rangle_{\psi_\alpha} = -\frac{\hbar\omega m}{2}(\psi_\alpha, (\hat{a}-\hat{a}^+)^2\psi_\alpha) = -\frac{\hbar\omega m}{2}(\psi_\alpha, (\hat{a}^2 - \hat{a}\hat{a}^+ - \hat{a}^+\hat{a} + \hat{a}^{+2})\psi_\alpha)$$

$$= -\frac{\hbar\omega m}{2}\left\{\underbrace{(\psi_\alpha, \hat{a}^2\psi_\alpha)}_{\alpha^2} - (\psi_\alpha, \hat{a}\hat{a}^+\psi_\alpha) - (\psi_\alpha, \hat{a}^+\hat{a}\,\psi_\alpha) + \underbrace{(\psi_\alpha, \hat{a}^{+2}\psi_\alpha)}_{(\alpha^*)^2}\right\} =$$

$$\bullet\ [\hat{a}, \hat{a}^{+}] = \hat{a}\hat{a}^{+} - \hat{a}^{+}\hat{a} = 1 \iff \hat{a}\hat{a}^{+} = 1 + \hat{a}^{+}\hat{a}$$

Ordnen: $\langle \hat{p}^2 \rangle_{\psi_\alpha} = -\frac{\hbar\omega m}{2}\left\{ \alpha^2 + \alpha^{*2} - \left(\psi_\alpha, (1 + 2\hat{a}^{+}\hat{a})\,\psi_\alpha\right)\right\} =$

$$= -\frac{\hbar\omega m}{2}\left\{ \alpha^2 + \alpha^{*2} - 1 + 2\left(\psi_\alpha, \hat{a}^{+}\hat{a}\,\psi_\alpha\right)\right\} =$$

$$= -\frac{\hbar\omega m}{2}\left\{ \alpha^2 + \alpha^{*2} - 2\alpha^*\alpha - 1 \right\} = \frac{\hbar\omega m}{2}\left\{ -(\alpha - \alpha^*)^2 + 1 \right\} =$$

$$= \frac{\hbar\omega m}{2}\left\{ 4\,\mathfrak{Im}^2(\alpha) + 1 \right\} \Rightarrow$$

$$\Rightarrow \boxed{\langle \hat{p}^2 \rangle_{\psi_\alpha} = 2\hbar\omega m\left[\mathfrak{Im}^2(\alpha) + \frac{1}{4} \right]}$$

13. Etxeechallenge

1. Galdera.

m masadun partikula bat 1D-ko horrengo $\hat{H}$-n dago:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \gamma \hat{x}^4$$

Dakigu $\hat{H}$ honi dagokion ψ_0, ε_0 eta ψ_1, ε_1

Kontziderazten badugu 3D-ko $\hat{H}$

$$\hat{H} = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + \gamma(x^4 + y^4 + z^4)$$

Aurrekoa jakinda zein izango da 1. egoera kitzikatuari dagokion energia, eta honen endakapena.

$$\hat{H}\Psi(\vec{r}) = \varepsilon\,\psi(\vec{r}) \Leftrightarrow \hat{H}_x \psi_{n_1}(x) + \hat{H}_y \psi_{n_2}(y) + \hat{H}_z \psi_{n_3}(z) = \varepsilon_{n_1}\psi_{n_1} + \varepsilon_{n_2}\psi_{n_2} + \varepsilon_{n_3}\psi_{n_3}$$

Orduan oinarrizko egoera $n_1 = n_2 = n_3 = 0$ denean izango da non $\varepsilon_{N=0} = 3\varepsilon_0$ non $N = n_1 + n_2 + n_3$ eta $g = 1$

1. egoera kitzikatua izango da $(n_1, n_2, n_3) = (1,0,0) = (0,1,0) = (0,0,1)$

orduan $\boxed{\varepsilon_{N=1} = 2\varepsilon_0 + \varepsilon_1 \quad g = 3 \quad \text{izango dira.}}$

2. Galdera.

e karga duen m masadun partikula 3D-ko osziladore harmoniko anisotropo baten eragin pean dago. Gainera, ε_0 modulua duen eremu elektriko baten eraginpean dago $\vec{u} = \hat{i} + \hat{j}$ noranzkoarekin. Zein izango da $\langle \vec{r} \rangle_t$? Oinarrizko egoeran.

Dakigunez

$$V(\vec{r}) = \frac{1}{2} K_1 x^2 + \frac{1}{2} K_2 y^2 + \frac{1}{2} K_3 z^2$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 \checkmark$$

$$F = eE_0 \left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} \right) \Rightarrow V_{el} = -\int E\,dl \Rightarrow V_{el} = -\frac{eE_0}{\sqrt{2}}(x+y)$$

Oinarrizko egoeran: $H_0(x) = 1$ beraz:

- $\psi_0(x) = A \exp\left[-\frac{m\omega_1}{2\hbar}\left(x - \frac{eE_0}{\sqrt{2}\,k_1}\right)^2\right]$

- $\psi_0(y) = B \exp\left[-\frac{m\omega_2}{2\hbar}\left(y - \frac{eE_0}{\sqrt{2}\,k_2}\right)^2\right]$

- $\psi_0(z) = C \exp\left[-\frac{m\omega_3}{2\hbar} z^2\right]$

Beraz Gaussiarrak direnez hirurak dakigu:

$$\left.\begin{aligned} \langle x \rangle_t &= \frac{eE_0}{\sqrt{2}K_1} \\ \langle y \rangle_t &= \frac{eE_0}{\sqrt{2}K_2} \\ \langle z \rangle_t &= 0 \end{aligned}\right\} \longrightarrow \boxed{\langle \vec{r} \rangle_t = \frac{eE_0}{\sqrt{2}}\left(\frac{\hat{i}}{K_2} + \frac{\hat{j}}{K_2}\right)}$$

3. Galdera:

$\vec{a}$ eta $\vec{b}$ bi norabideetako bektoreak eta $L_a = \vec{a}\cdot\vec{L}$ eta $L_b = \vec{b}\cdot\vec{L}$
$[\vec{a}\cdot\vec{L}, \vec{b}\cdot\vec{L}]$ zenbat izango da?

$$\boxed{[\vec{a}\cdot\vec{L}, \vec{b}\cdot\vec{L}] = i\hbar\,(\vec{a}\times\vec{b})\cdot\vec{L}}$$

14. Etxe challenge.

1. Galdera.

Hasierako egoera: $\Psi(\tilde{r}, t=0) = f(r)\, Y_e^m(\theta, \varphi)$

$l \neq 0$ finkatuta daudenez ezin ditugu aldi berean neurtu $\hat{L}_x$ eta $\hat{L}_y$ neurtu. Esan zein izan behar den m-ren balioa $(\Delta\hat{L}_x)^2 + (\Delta\hat{L}_y)^2$ baliorik txikiena izateko.

Lehenengo kalkulatu behar dugu ΔL_x eta ΔL_y:

$$\langle \hat{L}_x \rangle_\Psi = \left(Y_e^m, \left(\frac{\hat{L}_+ + \hat{L}_-}{2}\right) Y_e^m \right) = \frac{1}{2}\left\{ \left(Y_e^m, A_e^m Y_e^{m+1} \right) + \left(Y_e^m, B_e^m Y_e^{m-1} \right) \right\} = 0$$

$$\langle \hat{L}_y \rangle_\Psi = \frac{1}{2i}\left(Y_e^m, (\hat{L}_+ - \hat{L}_-) Y_e^m \right) = 0 \qquad \hat{L}_+\hat{L}_- - \hat{L}_-\hat{L}_+ = 2\hbar \hat{L}_z$$

$$\langle \hat{L}_x^2 \rangle_\Psi = \frac{1}{4}\left(Y_e^m, (\hat{L}_+^2 + \hat{L}_-\hat{L}_+ + \hat{L}_+\hat{L}_- + \hat{L}_-^2) Y_e^m \right)$$

$$= \frac{1}{4}\left(Y_e^m, (2\hat{L}_-\hat{L}_+ + 2\hbar\hat{L}_z) Y_e^m \right) =$$

$$= \frac{1}{2}\left(\left(\hat{L}_+ Y_e^m, \hat{L}_+ Y_e^m \right) + \hbar^2 m \right) = \frac{\hbar^2}{2}\left(l(l+1) - m(m+\cancel{1}) + \cancel{m} \right) =$$

$$= \frac{\hbar^2}{2}\left(l(l+1) - m^2 \right)$$

$$\langle \hat{L}_y \rangle_\Psi = \frac{1}{4}\left(Y_e^m, (\hat{L}_-\hat{L}_+ + \hat{L}_+\hat{L}_-) Y_e^m \right) = \frac{\hbar^2}{2}\left(l(l+1) - m^2 \right)$$

Beraz:

$$(\Delta\hat{L}_x)^2 = \frac{\hbar^2}{2}\left(l(l+1) - m^2 \right) \quad \text{eta} \quad (\Delta\hat{L}_y)^2 = \frac{\hbar^2}{2}\left(l(l+1) - m^2 \right)$$

Orduan $(\Delta\hat{L}_x)^2 + (\Delta\hat{L}_y)^2 = \hbar^2\left(l(l+1) - m^2 \right) \leftarrow$ minimoa $m = \pm l$ bada

orduan $\boxed{m = \pm l \quad (\Delta L_y)^2 + (\Delta L_x)^2 = l\hbar^2}$

2. Galdera.

$\Psi(\vec{r}, t=0) = (ar^3 + br^5)\, x^2\, e^{-\alpha r^6}$

L_z^8 aplikatuz gero bi balioetatik probableena zein izango da?

$x^2 = r^2 \sin^2\theta \cos^2\varphi \Rightarrow \Psi(\vec{r}, 0) = f(r,\theta)\left(e^{i\varphi} + e^{-i\varphi}\right)^2 =$

$= f(r,\theta)\left(e^{2i\varphi} + 2e^0 + e^{-2i\varphi}\right) \rightarrow$ Normalizatuz

$\Psi(\vec{r}, 0) = f(r,\theta)\dfrac{1}{\sqrt{6}}\left(e^{2i\varphi} + 2e^0 + e^{-2i\varphi}\right) =$

Beraz daukagu l_z-ren balioak:

$l_z = \hbar m \Rightarrow l_z^8 = (\hbar m)^8$ → $l_z = 0$; → $l_z^8 = 256\hbar^8$

Beraz $P(l_z^8 = 0) = \dfrac{4}{6} = \dfrac{2}{3}$

$P(l_z^8 = 256\hbar^8) = \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{2}{6} = \dfrac{1}{3}$

$\Rightarrow$ $\boxed{l_z^8 = 256\hbar^8 \text{ izango da } 1/3\text{-ko probabilitatearekin}}$

Etxechallenge 15.

1. Galdera.

$$\Psi(\vec{r},0) = e^{-\alpha r^2} r(\cos\theta \sin\theta \cos\varphi + r) \quad \text{non } \alpha > 0$$

$$\hat{u} = \frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} \quad \text{eta} \quad \hat{L}_u = \hat{L}\cdot\hat{u} = \frac{\hat{L}_x + \hat{L}_y}{\sqrt{2}} \quad \text{zein izango da } \langle \hat{L}_u \rangle$$

Lehenengo $\Psi(\vec{r},0)$ harmoniko esferikoetara:

$$\Psi(\vec{r},0) = e^{-\alpha r^2}\cdot r\left(\frac{e^{i\varphi}+e^{-i\varphi}}{2}\sin\theta\cos\theta + 1\right) = e^{-\alpha r^2} r\left(\sqrt{\frac{2\pi}{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \sqrt{4\pi}\, Y_0^0\right) =$$

$$= e^{-\alpha r^2} r\sqrt{2\pi}\left(\frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \sqrt{2}\, Y_0^0\right)$$

Normalizatzen badugu: $\Psi(\vec{r},0) = F(r)\sqrt{\frac{32}{15}}\left(\frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \sqrt{2}\, Y_0^0\right)$

orduan:

$$\langle \hat{L}_u \rangle = \langle \Psi | \hat{L}_u | \Psi \rangle = F(r)\frac{32}{15}\left\langle \frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \sqrt{2}\, Y_0^0 \right| \frac{\hat{L}_x + \hat{L}_y}{\sqrt{2}} \left| \frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \sqrt{2}\, Y_0^0 \right\rangle$$

$$= F(r)\frac{32}{15}\left[\left\langle \frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \frac{1}{\sqrt{2}} Y_0^0 \right| \hat{L}_x \left| \frac{1}{\sqrt{30}}\left(Y_2^{-1} - Y_2^{1}\right) + Y_0^0 \right\rangle + \right.$$

$$\left. + \underbrace{\left\langle \frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \frac{1}{\sqrt{2}} Y_0^0 \right| \hat{L}_y \left| \frac{1}{\sqrt{30}}\left(Y_2^{-1} - Y_2^{1}\right) + Y_0^0 \right\rangle}_{\text{Va a dar } \in \mathbb{C} \text{ así que fuera.}} \right] =$$

$$= F(r)\frac{32}{15}\left\langle \frac{1}{\sqrt{15}}\left(Y_2^{-1} - Y_2^{1}\right) + \frac{1}{\sqrt{2}} Y_0^0 \right| \frac{\hat{L}_+ + \hat{L}_-}{2} \left| \frac{1}{\sqrt{30}}\left(Y_2^{-1} - Y_2^{1}\right) + Y_0^0 \right\rangle \Rightarrow$$

$$\Rightarrow \boxed{\langle \hat{L}_u \rangle = 0}$$

2. Galdera

$\psi(\vec{r},0) = r^3 e^{-\alpha r^5} (x+y+2z) \qquad \alpha > 0$

L_x^2 neurtzean bi magnitude lortzen ditugu. Zein da txikiarena lortzeko probabilitatea.

- $$(x+y+2z) = r\left[(\cos\varphi + \sin\varphi)\sin\theta + 2\cos\theta \right] =$$

$$= r\left[\left(e^{i\varphi} + e^{-i\varphi} + \frac{e^{i\varphi} - e^{-i\varphi}}{i}\right)\frac{\sin\theta}{2} + 2\cos\theta \right] =$$

$$= r\left[(1-i)\, e^{i\varphi}\frac{\sin\theta}{2} + (1+i)\, e^{-i\varphi}\frac{\sin\theta}{2} + 2\cos\theta \right] =$$

$$= r\left[-(1-i)\sqrt{\frac{2\pi}{3}}\, Y_1^1 + (1+i)\sqrt{\frac{2\pi}{3}}\, Y_1^{-1} + 2\sqrt{\frac{4\pi}{3}}\, Y_1^0 \right] =$$

$$= r\sqrt{\frac{2\pi}{3}}\left[(1+i) Y_1^{-1} - (1-i) Y_1^1 + \sqrt{8}\, Y_1^0 \right]$$

Hau da gure uhin funtzioa:

$$\psi(\vec{r},0) = F(r)\left[\frac{(1+i)}{2\sqrt{3}} Y_1^{-1} + \frac{(1-i)}{2\sqrt{3}} Y_1^1 + \frac{2}{\sqrt{6}} Y_0^1 \right]$$

$$\hat{L}_x = \frac{\hbar}{\sqrt{2}} \begin{array}{cc} \begin{array}{ccc} Y_1^{-1} & Y_1^0 & Y_1^1 \end{array} & \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \begin{array}{c} Y_1^1 \\ Y_0^0 \\ Y_1^{-1} \end{array} \end{array} \implies \hat{L}_x^2 = \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \implies$$

$$\implies \begin{array}{l} \phi^{\hbar^2} = 1/\sqrt{2}\,(1,0,1) \\ \phi^{\hbar^2} = (0,1,0) \\ \phi^0 = 1/\sqrt{2}\,(-1,0,1) \end{array} \implies \psi(\vec{r},0) = F(r)\left[i\, 1/\sqrt{6}\, \phi^{\hbar^2} + 2/\sqrt{6}\, \phi^{\hbar^2} + 1/\sqrt{6}\, \phi^0 \right]$$

$$\implies P(0) = \left| 1/\sqrt{6} \right|^2 = 1/6$$