Kungliga Tekniska Högskolan

DD2424

Deep Learning in Data Science

Assignment 1

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1 Introduction

In this assignment, we will train and test a one layer network with multiple outputs to classify images from the CIFAR-10 datasets. To do so, we will train the network using a mini-batch gradient descent algorithm applied to a cost function that computes the cross-entropy loss of the classifier applied to the labelled training data and an L_2 regularization term on the weight matrix.
The training set is given by the file **data_batch_1.mat**, the validation by **data_batch_2.mat** and the test set by **test_batch.mat**.
This assignment will be solved using MATLAB.

2 Method

First, we compute the network function that will evaluate the classifier and returns a matrix **P** of probability. This is done thanks to equation 1 and 2.

$$\mathbf{s} = W\mathbf{x} + \mathbf{b} \tag{1}$$

$$SOFTMAX(\mathbf{s}) = \frac{exp(\mathbf{s})}{\mathbf{1}^T exp(\mathbf{s})} \tag{2}$$

The parameters **W** and **b** of our classifier are what we have to learn by exploiting labelled training data. We will use the same method presented in the lectures, that is to minimize the cross-entropy plus a regularization term. The cost function is given by:

$$J(D, \lambda, W, b) = \frac{1}{|D|} \sum_{(\mathbf{x},y)\in\mathcal{D}} l_{cross}(\mathbf{x}, y, W, \mathbf{b}) + \lambda \sum_{i,j} W_{i,j}^2 \tag{3}$$

where,

$$l_{cross}(\mathbf{x}, y, W, \mathbf{b}) = -log(p_y) \tag{4}$$

As said in the introduction, the learning algorithm is implemented with a mini-batch gradient descent algorithm that begin wit a sensible initialization of the parameters W and **b** and we update them with the following equations.

$$W^{(t+1)} = W^{(t)} - \eta \frac{\partial J(\mathcal{B}^{(t+1)}, \lambda, W, \mathbf{b})}{\partial W} \tag{5}$$

$$\mathbf{b}^{(t+1)} = \mathbf{b}^{(t)} - \eta \frac{\partial J(\mathcal{B}^{(t+1)}, \lambda, W, \mathbf{b})}{\partial \mathbf{b}} \tag{6}$$

Finally, we compute the gradients following the equations in the last slide of the lecture 3.
We set,

$$\mathbf{G}_{batch} = -(\mathbf{Y}_{batch} - \mathbf{P}_{batch}) \tag{7}$$

The gradient w.r.t W is given by:

$$\frac{\partial J}{\partial W} = \frac{1}{n_b}\mathbf{G}_{batch}\mathbf{X}_{batch}^T + 2\lambda W \tag{8}$$

and the gradient w.r.t **b** is given by:

$$\frac{\partial J}{\partial \mathbf{b}} = \frac{1}{n_b}\mathbf{G}_{batch}\mathbf{1}^T \tag{9}$$

I have successfully implemented the function $ComputeGradient$. To check whether my analytical gradient was correct or not I checked that the maximum absolute difference between numerical gradient (function provided) and my analytical gradient was less than **1e-6**. (see code)

3 Results

In this section, the results from different experiments will be exposed. The algorithm implemented has been tested under the following parameters:

- λ is the regularization parameter
- n_{epoch} is the number of time we go through all the dataset (epoch).
- n_{batch} is the size of each batch
- η is the learning rate

Setting 1 : $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .1$

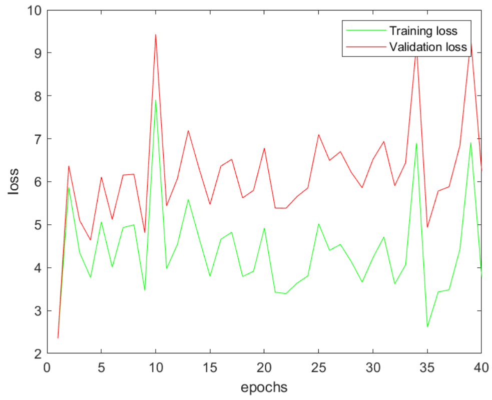


Figure 1: The graph of the training and validation loss computed after every epoch. The network was trained with the following parameter settings:$\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .1$

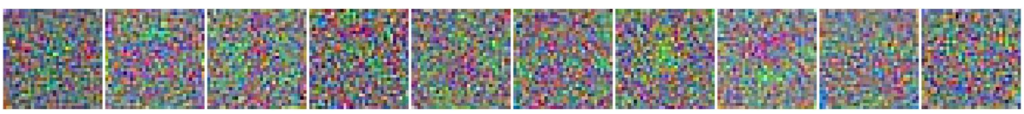


Figure 2: Weight matrix for the following parameter settings: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .1$

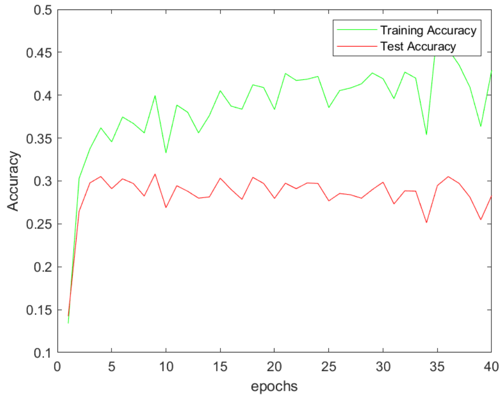


Figure 3: Accuracy of the network for the following parameter settings: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .1$. Final Accuracy for test set : 29.35%

Setting 2: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

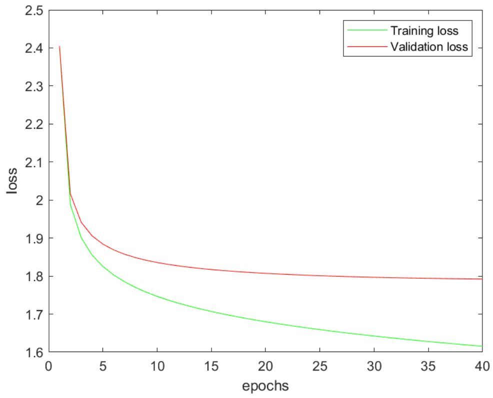


Figure 4: The graph of the training and validation loss computed after every epoch. The network was trained with the following parameter settings: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$.

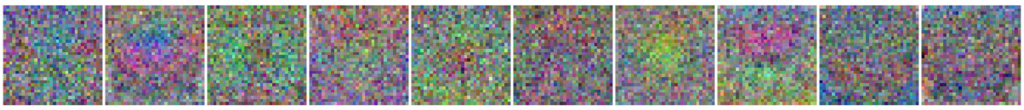


Figure 5: Weight matrix for the following parameter settings: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

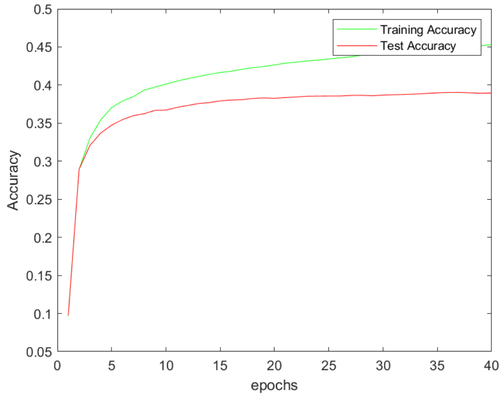


Figure 6: Accuracy of the network for the following parameter settings: $\lambda = 0$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$. Final Accuracy for test set : 38.97%

Setting 3: $\lambda = 0.1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

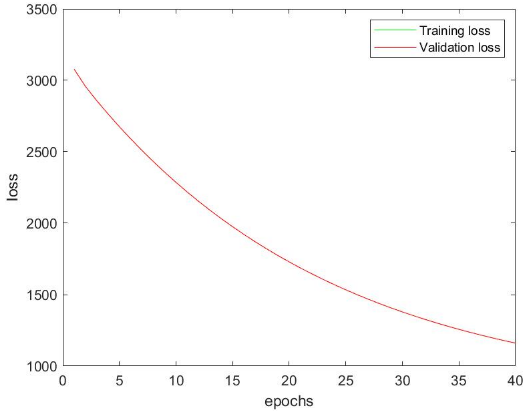


Figure 7: The graph of the training and validation loss computed after every epoch. The network was trained with the following parameter settings: $\lambda = 0.1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

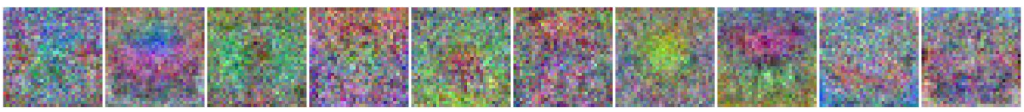


Figure 8: Weight matrix for the following parameter settings: $\lambda = 0.1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

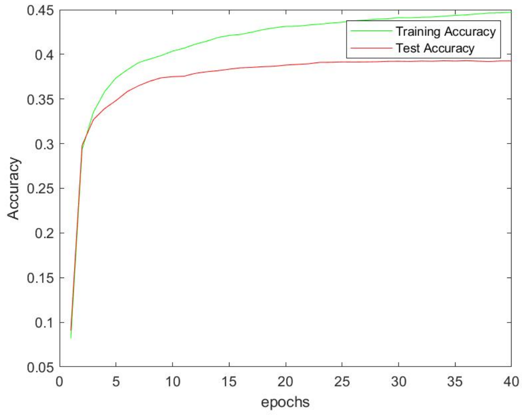


Figure 9: Accuracy of the network for the following parameter settings: $\lambda = 0.1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$. Final Accuracy for test set : 39.25%

Setting 4: $\lambda = 1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

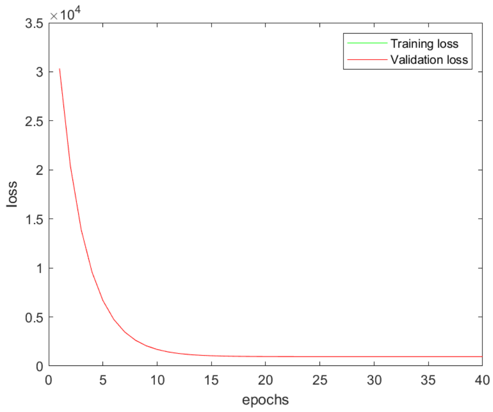


Figure 10: The graph of the training and validation loss computed after every epoch. The network was trained with the following parameter settings: $\lambda = 1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

Figure 11: Weight matrix for the following parameter settings: $\lambda = 1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$

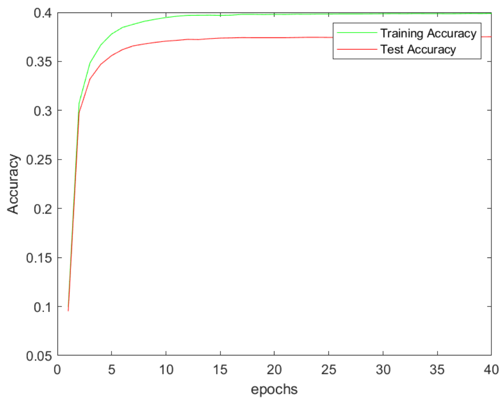


Figure 12: Accuracy of the network for the following parameter settings: $\lambda = 1$ $n_{epoch} = 40$ $n_{batch} = 100$ $\eta = .001$. Final Accuracy for test set : 37.52%

4 Conclusion

As we can see in the Fig.1 and Fig.4, having a high learning rate make the network not stable and it would not be recommended to use it, especially because the accuracy on the test set is pretty low. By reducing this parameter, it is shown that it will make the convergence slower but the update will be smoother.
Similarly we can compare the two last settings to analyze the influence of the regularization parameter on the network. Increasing the regularization parameter make the gap between the training loss and the validation lower but the values are higher. However the transition is sharper as increases and if it is too high the accuracy decreases as well.
For now, the best model found would be the second one since there aren't over fitting and the accuracy is good compared to the other.

5 Code

```
clear;
clc;
close all;
```

```
addpath Datasets\cifar-10-batches-mat\

[X_train,Y_train,y_train] = LoadBatch('data_batch_1.mat');
[X_val, Y_val, y_val] = LoadBatch('data_batch_2.mat');
[X_test, Y_test, y_test] = LoadBatch('test_batch.mat');

n = size(X_train,2);
d = size(X_train, 1);
K = size(Y_train,1);

lambda = 0;
n_epochs = 40;
n_batch = 100;
eta = .001;

[W, b] = GetParams(d, K);
```

```
%Check whether the gradient is correct or not
batch_size = 50;
eps =1e-10;

[ngrad_b, ngrad_W] = ComputeGradsNumSlow(X_train(:, 1 : batch_size), ...
    Y_train(:, 1 : batch_size), W, b, lambda, 1e-6);
P = EvaluateClassifier(X_train(:, 1 : batch_size),  W, b);
[grad_W, grad_b] = ComputeGradients(X_train(:, 1 : batch_size), ...
    Y_train(:, 1 : batch_size), P,  W, lambda);
%check gradients
gradcheck_b = max(abs(ngrad_b - grad_b));
gradcheck_W = max(abs(ngrad_W - grad_W));
if gradcheck_b <= 1e-6
    fprintf("Correct grad_b");
else
    fprintf("Incorrect grad_b");
end
```

```
Correct grad_b
```

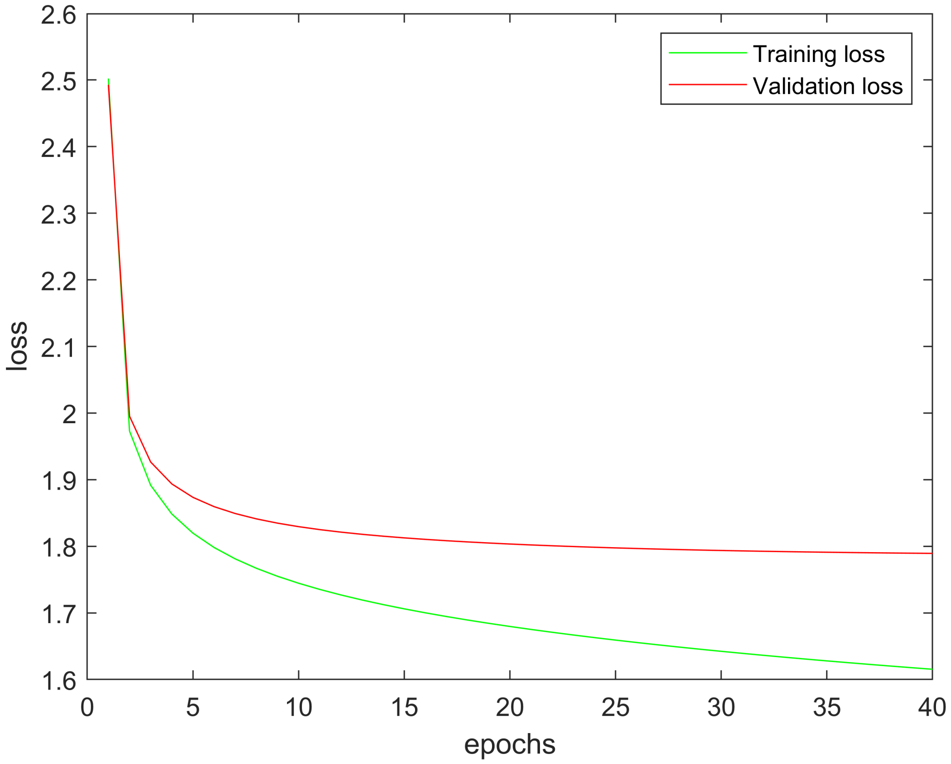
```
if gradcheck_W <= 1e-6
    fprintf("Correct grad_W");
else
    fprintf("Incorrect grad_W");
end
```

```
Correct grad_W
```

```
% %Preprocess raw input data on training set
mean_Xtr = mean(X_train, 2);
std_Xtr = std(X_train, 0 ,2);
```

```
%
% % mean_Xva = mean(X_val, 2);
% % std_Xva = std(X_val, 0 ,2);
% %
% % mean_Xte = mean(X_test, 2);
% % std_Xte = std(X_test, 0 ,2);
 %Normalize training validation and test data

X_train = X_train- repmat(mean_Xtr, [1, n]);
X_train = X_train./ repmat(std_Xtr, [1, n]);

X_val = X_val- repmat(mean_Xtr, [1, n]);
X_val = X_val./ repmat(std_Xtr, [1, n]);

X_test = X_test- repmat(mean_Xtr, [1, n]);
X_test = X_test./ repmat(std_Xtr, [1, n]);

% %eta_step = (eta-final_eta)/n_epochs;

cost_train = zeros(n_epochs, 1);
cost_eval = zeros(n_epochs, 1);

acc_test = zeros(n_epochs,1);
acc_train = zeros(n_epochs,1);

for i =1:n_epochs

    %DEBUGGING
    %Compute and store cost and accuracy for each epoch on all training set
    cost_train(i) = ComputeCost(X_train, Y_train, W, b, lambda);
    cost_eval(i) = ComputeCost(X_val, Y_val, W, b, lambda);

    acc_test(i) = ComputeAccuracy(X_test,y_test,W,b);
    acc_train(i) = ComputeAccuracy(X_train,y_train,W,b);

    for j = 1:n/n_batch
        j_start = (j-1)*n_batch + 1;
        j_end = j*n_batch;
        inds = j_start:j_end;
        Xbatch = X_train(:, j_start:j_end);
        Ybatch = Y_train(:, j_start:j_end);

        %Mini batch of X and Y
        [W, b] = MiniBatchGD(Xbatch,Ybatch,W,b,lambda,eta);
    end
end

%Plot cost scores
figure()
plot(1 : n_epochs, cost_train, 'g')
hold on
plot(1 : n_epochs, cost_eval, 'r')
hold off
xlabel('epochs');
```

```
ylabel('loss');
legend('Training loss', 'Validation loss');
```



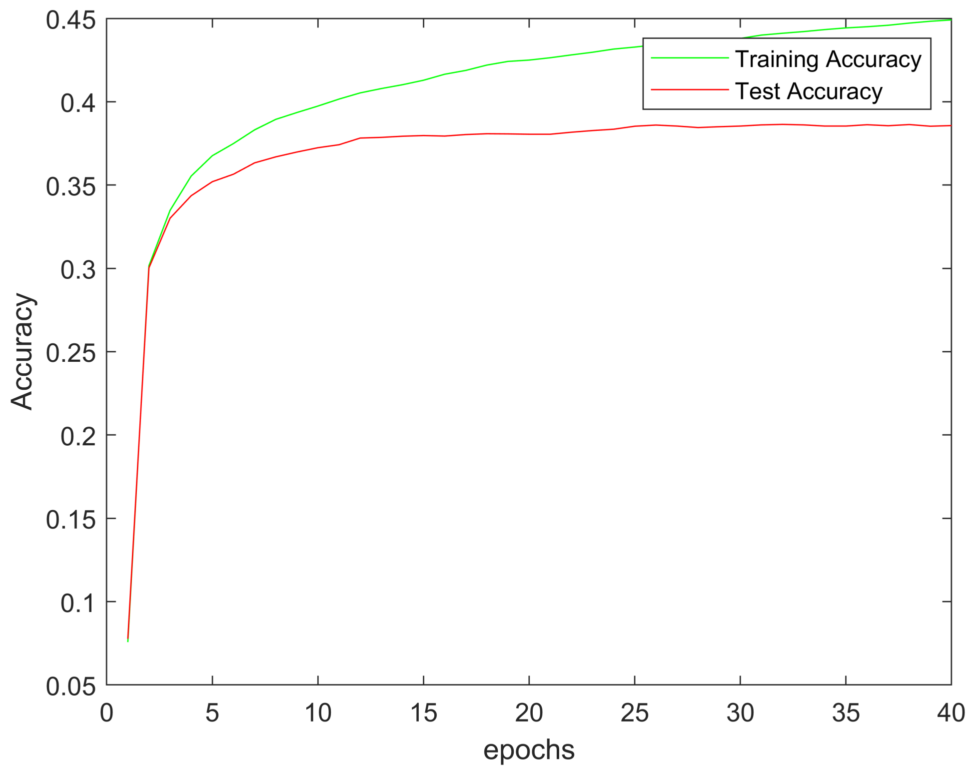

```
% Accuracy of the network
acc_train_f = ComputeAccuracy(X_train, y_train, W, b);
disp(['Training Accuracy:' num2str(acc_train_f*100) '%'])
```

```
Training Accuracy:44.97%
```

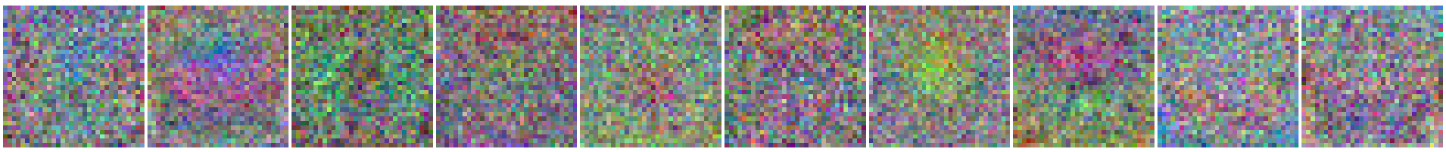
```
acc_test_f = ComputeAccuracy(X_test, y_test, W, b);
disp(['Test Accuracy:' num2str(acc_test_f*100) '%'])
```

```
Test Accuracy:38.52%
```

```
figure()
plot(1 : n_epochs, acc_train, 'g')
hold on
plot(1 : n_epochs, acc_test, 'r')
hold off
xlabel('epochs');
ylabel('Accuracy');
legend('Training Accuracy', 'Test Accuracy');
```




```
%Visualize the weight matrix W as an image and see what class template the
%network has learnt
K = 10;
for i = 1 : K
    im = reshape(W(i, :), 32, 32, 3);
    s_im{i} = (im - min(im(:))) / (max(im(:)) - min(im(:)));
    s_im{i} = permute(s_im{i}, [2, 1, 3]);
end
figure()
montage(s_im, 'size', [1, K]);
```



```
function [X,Y, y] = LoadBatch(file)
    %LOADBATCH reads in the data from a CIFAR-10 batch file
    %and returns the image and label data in separate files.
    A = load(file);
    X = double(A.data')/255;

    %label vector of size n
    y = A.labels';
    y = y + uint8(ones(1,length(y))); %Simplifying indexing
```

```
    %Create the image label matrix Y of size Kxn where K = #of labels.
    Y= zeros(10, length(y));
    for  i= 1:length(Y)
        Y(y(i),i) = 1;
    end
end

function P = EvaluateClassifier(X, W, b)
    %EVALUATECLASSIFIER that evaluates the network function, i.e. equations
    %(1, 2), on multiple images and returns the results.

    %Each column of X corresponds to an image and it has size dxn
    %W and b are the parameters of the network
    %Each column of P contains the probability for each label for the image
    %in the corresponding column of X. P has size Kxn

    s = W*X + b;
    P = softmax(s);

end

function p = softmax(s)
    %Compute the softmax of S, return a probability vector
    p = exp(s)./sum(exp(s));
end

function [W,b] = GetParams(d , K)
    %GETPARAMS returns the initial parameters of the model (random guess))
    W = 0.01*randn(K, d); %0.01 is the standard deviation
    b = 0.01*randn(K, 1);
end

function J = ComputeCost(X,Y,W,b, lambda)
    %COMPUTECOST  computes the cost function given by equation
    %(5) for a set of images
    P = EvaluateClassifier(X,W,b);

    J= sum(diag(-log(Y'*P))/size(X,2) + lambda*sumsqr(W));
end

function acc = ComputeAccuracy(X, y, W, b)
    %COMPUTEACCURACY computes the accuracy of the network’s predictions given
    %by equation (4) on a set of data.
    P = EvaluateClassifier(X,W,b);
    [~, pred] = max(P);
    acc = sum(pred == y)/length(P);
end

function [grad_W, grad_b] = ComputeGradients(X, Y, P, W, lambda)
%COMPUTEGRADIENTS evaluates, for a mini-batch, the gradients of the cost
%function w.r.t. W and b, that is equations (10, 11).
%X has size dxn
%Y has size Kxn
```

```
%P contains the probability for each label for the image in the
%corresponding column, has size Kxn

%grad_W is the gradient matrix of the cost J relative to W, has size
%Kxd
%grad_b is the gradient vector of the cost J relative to b, has size
%Kx1
    grad_W = zeros(size(W));
    grad_b = zeros(size(W, 1), 1);

    for i = 1 : size(X, 2)
        P_i = P(:, i);
        Y_i = Y(:, i);
        X_i = X(:, i);

        g = -(Y_i-P_i)';
        grad_b = grad_b + g';
        grad_W = grad_W + g'*X_i';

    end
    % divide grad by the number of entries in D
    grad_b = grad_b/size(X, 2);
    grad_W = grad_W/size(X, 2) + 2*lambda*W ;
end

function [Wstar, bstar] = MiniBatchGD(X, Y, W, b, lambda, eta)
    P = EvaluateClassifier(X, W, b);
    [grad_W, grad_b] = ComputeGradients(X, Y, P, W, lambda);

    %Update parameters
    Wstar = W-eta*grad_W;
    bstar = b-eta*grad_b;
end
```