Gamma I

Per poder ser GLM s'ha complir

$$\log\left(f\left(y|\theta,\Phi\right)\right) = \frac{y\theta - b\left(\theta\right)}{\Phi} + c\left(y,\Phi\right)$$

Gamma II

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$f_{Y_1}\left(y|\alpha,\beta\right)=\frac{\beta^\alpha}{\Gamma(\alpha)}y^{\alpha-1}e^{-\beta y}=e^{-\beta y+(\alpha-1)log(y)+\log\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)}$$

$$\log\left(f_{Y_1}\left(y|\alpha,\beta\right)\right)=$$

$$\overbrace{-\beta y}+\left(\alpha-1\right)log\left(y\right)+\alpha\log\left(\beta\right)-\log\left(\Gamma\left(\alpha\right)\right)$$

$$c\left(y,\Phi\right)=\left(\alpha-1\right)log\left(y\right)-\log\left(\Gamma\left(\alpha\right)\right)+c_1\left(y\right)+c_2\left(\Phi\right)$$

$$\Longrightarrow a\left(\Phi\right)=\alpha$$

Gamma III

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$\log\left(f_{Y_1}\left(y|\Phi,\beta\right)\right)=$$

$$\frac{\overbrace{-\beta\Phi}\,y+\Phi a(\Phi)\log(\beta)}{\Phi}+\left(a\left(\Phi\right)-1\right)log\left(y\right)-\log\left(\Gamma\left(a\left(\Phi\right)\right)\right)$$

$$\Longrightarrow\theta=-\beta\Phi\Rightarrow\beta=-\frac{\theta}{\Phi}$$

Gamma IV

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$\log\left(f_{Y_1}\left(y|\theta,\Phi\right)\right)=$$

$$\frac{\theta y+\Phi a(\Phi)\log\left(-\frac{\theta}{\Phi}\right)}{\Phi}+\left(a\left(\Phi\right)-1\right)log\left(y\right)-\log\left(\Gamma\left(a\left(\Phi\right)\right)\right)$$

$$\log\left(f_{Y_1}\left(y|\theta,\Phi\right)\right)=\frac{\theta y-\overbrace{\left(-\Phi a\left(\Phi\right)\log\left(-\theta\right)\right)}}{\Phi}-a\left(\Phi\right)\log\left(\Phi\right)+$$

$$\left(a\left(\Phi\right)-1\right)log\left(y\right)-\log\left(\Gamma\left(a\left(\Phi\right)\right)\right)$$

$$b\left(\theta\right)=-\Phi a\left(\Phi\right)\log\left(-\theta\right)\Longrightarrow a\left(\Phi\right)=\Phi^{-1}$$

Gamma V

$\log\left(f\left(y|\theta,\Phi\right)\right) = \frac{y\theta - b(\theta)}{\Phi} + c\left(y,\Phi\right)$

$$\log\left(f_{Y_1}\left(y|\theta,\Phi\right)\right) = \frac{\theta y - \overbrace{\left(-\log\left(-\theta\right)\right)}}{\Phi} - \Phi^{-1}\log\left(\Phi\right) +$$

$$\left(\Phi^{-1} - 1\right) log\left(y\right) - \log\left(\Gamma\left(\Phi^{-1}\right)\right)$$

$$\implies \begin{aligned} b\left(\theta\right) &= -\log\left(-\theta\right) \\ c\left(y,\Phi\right) &= \Phi^{-1}\log\left(y\Phi^{-1}\right) - \log\left(y\Gamma\left(\Phi^{-1}\right)\right) \end{aligned}$$

Gamma VI

Link canònic i funció de variància

$$\mu = \mathbb{E}\left(Y|\theta,\Phi\right) = b'\left(\theta\right) = \left(-\log\left(-\theta\right)\right)' = \frac{-1}{\theta}$$

$$\Longrightarrow link_{canonic}\left(\mu\right) = \frac{1}{\mu} = -\theta$$

$$Var\left(Y|\theta,\Phi\right) = \Phi b''\left(\theta\right) = \frac{\Phi}{\theta^2} = \Phi\mu^2$$

$$\Longrightarrow V\left(\mu\right) = \mu^2$$

Binomial I

Par a poder ser GLM s'ha de complir

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b\left(\theta\right)}{\Phi}+c\left(y,\Phi\right)$$

Binomial II

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$f_{Y_2}\left(y\right)=\binom{n}{y}\pi^{y}\left(1-\pi\right)^{n-y}\ ,\ n\in\mathbb{N}^{+}fix,\ y\in\{0,1,\ldots n\}\ ,\pi\in(0,1)$$

$$f_{Y_2}\left(y|\pi\right)=e^{\log(\pi)y+(n-y)\log(1-\pi)+\log\binom{n}{y}}$$

$$\log\left(f\right)=\overbrace{\left(\log\left(\pi\right)-\log\left(1-\pi\right)\right)y}+n\log\left(1-\pi\right)+\log\binom{n}{y}$$

$$\Longrightarrow a\left(\Phi\right)=constant=a$$

Binomial III

$$\log\left(f\left(y|\theta,\Phi\right)\right) = \frac{y\theta - b(\theta)}{\Phi} + c\left(y,\Phi\right)$$

$$\log\left(f_{Y_2}\left(y|\pi\right)\right) = \frac{\overbrace{\log\left(\frac{\pi}{1-\pi}\right)\Phi}\, y + \Phi n \log(1-\pi)}{\Phi} + \log\binom{n}{y}$$

$$\Longrightarrow \theta = \log\left(\frac{\pi}{1-\pi}\right)\Phi \Rightarrow \pi = \frac{e^{\frac{\theta}{\Phi}}}{1+e^{\frac{\theta}{\Phi}}}$$

Binomial IV

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$\log\left(f_{Y_2}\left(y|\theta\right)\right)=\frac{\theta y-\overbrace{\Phi n\log\left(1+e^{\frac{\theta}{\Phi}}\right)}}{\Phi}+\log\binom{n}{y}$$

$$\implies \Phi=1$$

Binomial V

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$\log\left(f_{Y_2}\left(y|\theta\right)\right)=\theta y-\overbrace{n\log\left(1+e^{\theta}\right)}+\log\binom{n}{y}$$

$$\Longrightarrow\quad\begin{aligned} b\left(\theta\right)&=n\log\left(1+e^{\theta}\right)\\ c\left(y,\Phi\right)&=\log\binom{n}{y}\end{aligned}$$

Link canònic i funcion de variància

$$\mu = n\pi = \mathbb{E}\left(Y|\theta\right) = b'\left(\theta\right) = \left(n\log\left(1+e^{\theta}\right)\right)' = \frac{ne^{\theta}}{1+e^{\theta}}$$

$$\Longrightarrow link_{canonic}\left(\mu\right) = \log\left(\frac{\mu}{n-\mu}\right) = \log\left(\frac{\pi}{1-\pi}\right) = \theta$$

$$Var\left(Y|\theta\right) = b''\left(\theta\right) = \frac{ne^{\theta}}{\left(1+e^{\theta}\right)^{2}} = \mu\left(1-\frac{\mu}{n}\right) = n\pi\left(1-\pi\right)$$

$$\Longrightarrow V\left(\mu\right) = \mu\left(1-\frac{\mu}{n}\right) = n\pi\left(1-\pi\right)$$

Binomial negativa I

Par a poder ser GLM s'ha de complir

$$\log\left(f\left(y|\theta,\Phi\right)\right) = \frac{y\theta - b\left(\theta\right)}{\Phi} + c\left(y,\Phi\right)$$

Binomial negativa II

$$\log\left(f\left(y|\theta,\Phi\right)\right)=\frac{y\theta-b(\theta)}{\Phi}+c\left(y,\Phi\right)$$

$$f_{Y_3}\left(y\right)=\frac{\Gamma(y+\rho)}{y!\Gamma(\rho)}\pi^y\left(1-\pi\right)^\rho,\ \ y\in\mathbb{N},\ \ \rho>0\ ,\pi\in(0,1)$$

$$f_{Y_3}\left(y|\pi,\rho\right)=\frac{\Gamma(y+\rho)}{y!\Gamma(\rho)}\pi^y\left(1-\pi\right)^\rho$$

$$\log\left(f_{Y_3}\left(y|\pi,\rho\right)\right)=$$

$$\overbrace{\log\left(\pi\right)y}+\rho\log\left(1-\pi\right)-\log\left(\Gamma\left(\rho\right)\right)+\log\left(\Gamma\left(y+\rho\right)\right)-\log\left(y!\right)$$

$$c\left(y,\Phi\right)=\log\left(\Gamma\left(y+\rho\right)\right)-\log\left(y!\right)+c_1\left(y\right)+c_2\left(\Phi\right)\Longrightarrow$$

$$a\left(\Phi\right)=\rho$$