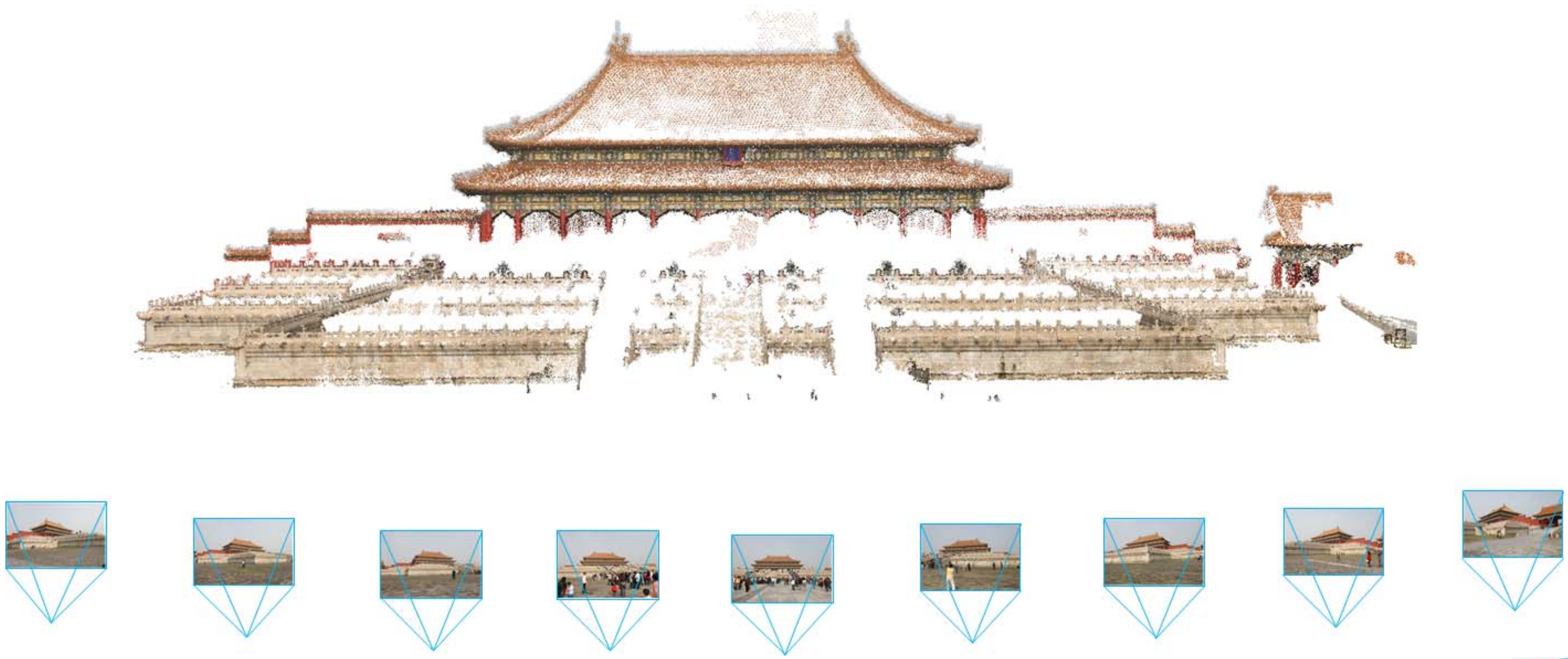
11. Structure-from-Motion



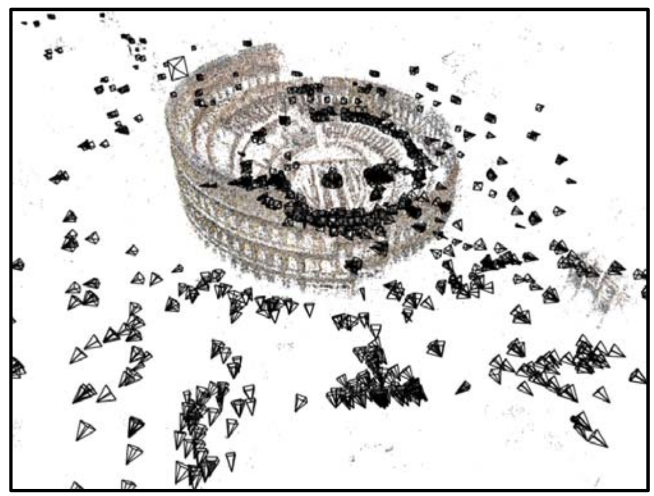
Outline

- Bundle Adjustment
- Rotation Parameterization
- Initializing BA

structure from motion的目标：通过无序图片序列，重建出这些图片中的物体的三维模型，并且求得拍摄每张图片的相机的位置与朝向。

Structure-from-Motion

- Given many images, how can we
  a) figure out where they were all taken from?
  b) build a 3D model of the scene?



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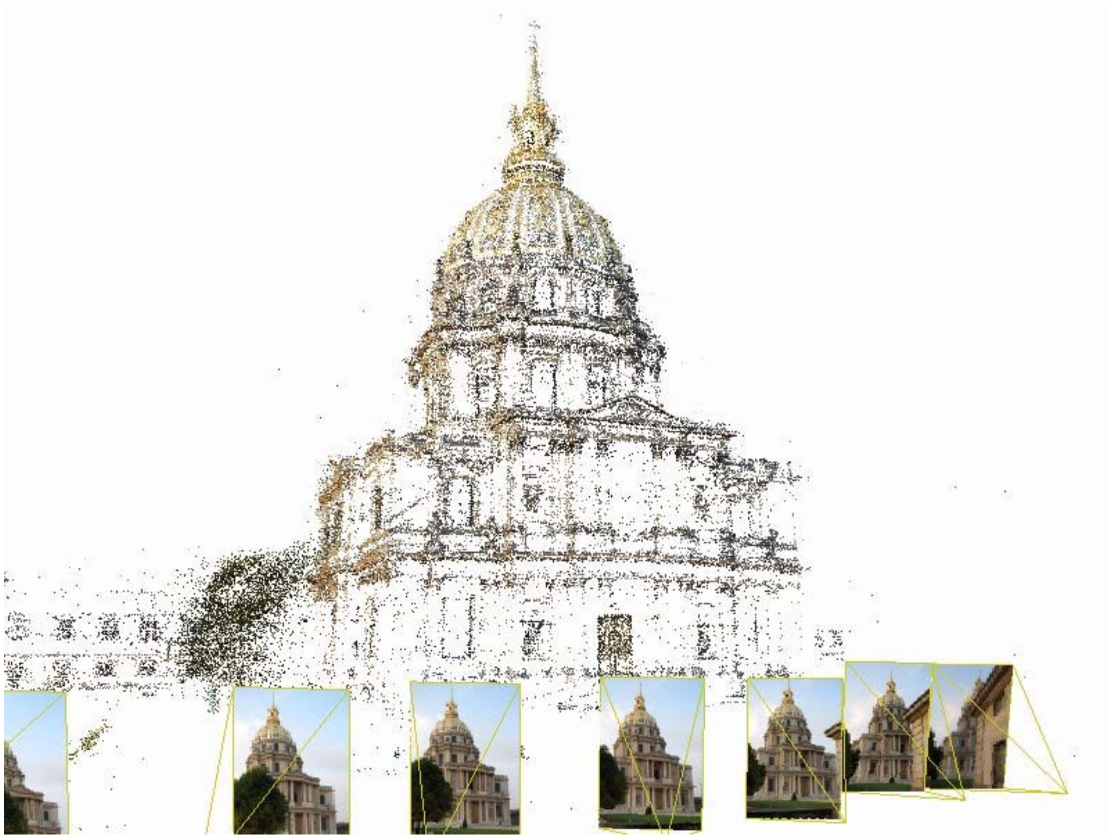
Structure-from-Motion

- Structure = 3D Point Cloud of the Scene
- Motion = Camera Location and Orientation
- SFM = Get the Point Cloud from Moving Cameras

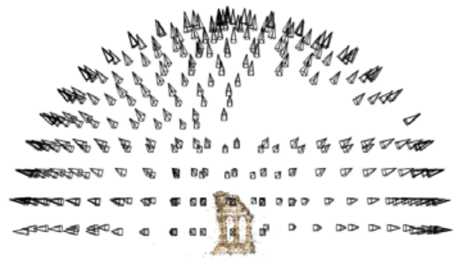
structure指的是场景的3D点云
motion指的是相机中心的坐标与朝向
SfM：从移动的相机中获得3D点云

Also Doable from Videos

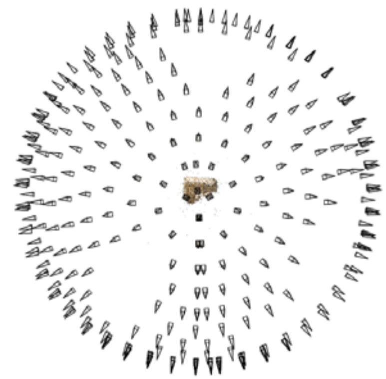
视频其实也就是图片序列，一样适用于SfM



Formulation



Reconstruction (side)



(top)

- Input: images with points in correspondence $p_{ij} = (u_{ij}, v_{ij})$
- Output
  - structure: 3D location X_i for each point p_i
  - motion: camera parameters R_j, t_j possibly K_j
- Objective function: minimize *reprojection error*

输入：目标上的每个像素在所有图片上的对应坐标。p(ij)代表的是目标上的第i个点在第j张图片上的坐标。有很多个p(ij)
输出：目标上第i个点的三维坐标；相机朝向，中心位置，可能还有相机内参矩阵。

为了从输入得到输出，手段就是minimize reprojection error。

π1表示的是第一个相机的投影矩阵，X1是三维点，p11是X1经过π1的投影得到的像素点（或者说第一个三维点在第一张图片上的投影）

X1经过投影得到的像素位置与特征提取找到的点之间存在误差，所有三维点在所有图片中的投影点与检测点之间都存在误差，将所有的误差进行求和，就得到了g（R，T，X），这就是最终的reprojection error，它与所有的旋转矩阵，转移矩阵，三维点有关。

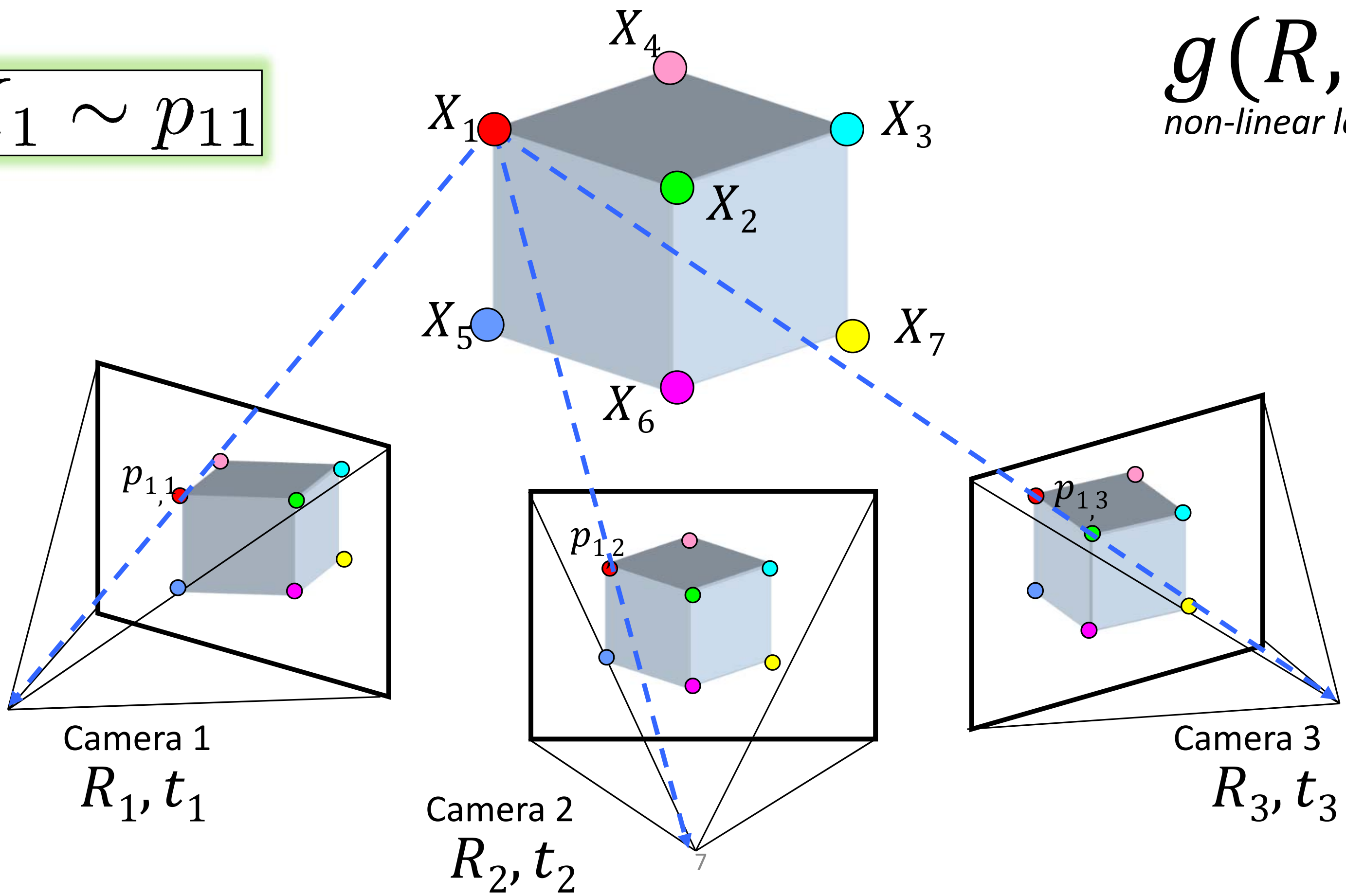
Formulation

$$\boxed{\Pi_1 X_1 \sim p_{11}}$$

minimize

$$g(R, T, X)$$

non-linear least squares



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三维点经过投影矩阵，得到的预测的像素坐标。可以理解为model prediction

由feature detection得到的像素坐标，相当于Obeservation。

Formulation

- Minimize sum of squared reprojection errors:

m个三维点，n张图片

$$g(\mathbf{X},\mathbf{R},\mathbf{T}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \underbrace{w_{ij}}_{\substack{\textit{indicator variable}: \\ \text{is point } i \text{ visible in image } j\text{ ?}}} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i,\mathbf{R}_j,\mathbf{t}_j)}_{\substack{\textit{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\textit{observed} \\ \text{image location}}} \right\|^2$$

第i个点在第j张图片中可见就取1，否则取0

- Minimizing this function is called *bundle adjustment*
  - Optimized using non-linear least squares, e.g. Levenberg-Marquardt

bundle指的是穿过同一个三维点的光束

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Problem size

- What are the variables?
  - Cameras and points
- How many variables per camera? 6
- How many variables per point? 3

- An example with moderate size

 466 input photos
 + > 100,000 3D points
 = very large optimization problem

Questions?

Bundle Adjustment

省略了考虑是否
可见的那个变量

- The objective function:

$$g(\mathbf{X},\mathbf{R},\mathbf{T}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left\|\mathbf{P}(\mathbf{x}_i,\mathbf{R}_j,\mathbf{t}_j) - \begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}\right\|^2$$

$$= \sum_{ij} e_{ij}^2(X_i, R_i, t_i, K_i)$$

将上述整理为一个函数，其表示的是点i个三维点在第j张图片上的投影误差。将所有关于i，j的误差平方求和，就是重投影误差。

  - $e_{ij} = P\left(X_i, R_j, t_j\right) - p_{ij}$ is the 'reprojection error' of X_i in the jth image
- The parameters: $\boldsymbol{X} \in \mathbb{R}^{3m}$, $\boldsymbol{R} \in \mathbb{R}^{3n}$, $\boldsymbol{T} \in \mathbb{R}^{3n}$

m个三维点，则X的维度是3m
n张图片，则R，T的维度是3n

  - Typically, $m \gg n$ (why?)
- The optimization method: Levenberg-Marquardt algorithm

对于待优化的函数，首先在初始值处进行泰勒展开，将二次项以上的都丢弃。

J表示的是函数f在p点处的梯度

Gauss-Newton Method Revisit

- Steps: 先将函数展开，丢弃掉2次及以上的项，将函数线性化
- 1. linearize the objective function (nearby an initial solution P_0)

$$f(P_0 + \Delta) \approx f(P_0) + J\Delta \qquad J = \frac{\partial f}{\partial P}$$

- 2. minimize the linearized objective function

$$\Delta = \arg\min \| f(P_0) + J\Delta \|^2$$

求出使得一阶展开最小的△

$$\Rightarrow J^T J\Delta = J^T f(P_0)$$

为什么要满足这个约束呢？

- 3. solve the linear system to update the initial solution

$$P_{i+1} = P_i + \Delta$$

- 4. iterate 1-3 until converge

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Linearize the re-projection error

- Error function: $f(P) = g(\boldsymbol{X}, \boldsymbol{R}, \boldsymbol{T}) = \sum_{ij} e_{ij}^2(\boldsymbol{X}, \boldsymbol{R}, \boldsymbol{T})$
  - $e_{ij} = P(X_i, R_j, t_j) - p_{ij}$

- Linearize it by Taylor expansion:

$$e_{ij}(P) = e_{ij}(P_0) + J_{ij}\Delta$$

$J_{ij} \in \mathbb{R}^{\underline{2\times(3m+6n)}}$ is the Jacobian matrix, $\Delta \in \mathbb{R}^{3m+6n}$

雅克比函数描述的是二维平面的欧氏距离，所以它是两行的。然后要对所有点与矩阵求距离，所以是2×（3m+6n）

- The sparse structure of J_{ij}:

$$J_{ij}(\boldsymbol{X}, \boldsymbol{R}, \boldsymbol{T}) = \left(0, \cdots, \frac{\partial e_{ij}}{\partial X_i}, \cdots, \frac{\partial e_{ij}}{\partial R_j}, \frac{\partial e_{ij}}{\partial T_j}, \cdots, 0\right)$$

Jij是一个稀疏的矩阵，因为Jij只和第i个三维点，第j个相机的旋转矩阵和平移矩阵有关。对别的无关的地方求导，结果都是0

Linearize the re-projection error

- The linearized objective function:

$$f(P) = \sum_{ij} \left(e_{ij}(P_0) + J_{ij}\Delta \right)^2 \approx \boldsymbol{c} + 2\boldsymbol{b}^T\Delta + \Delta^T \boldsymbol{H}\Delta$$

with

$$\boldsymbol{b}^T = \sum_{ij} e_{ij}^T \mathrm{J_{ij}} \qquad \boldsymbol{H} = \sum_{ij} J_{ij}^T J_{ij} \in \mathbb{R}^{(3m+6n)\times(3m+6n)}$$

这个H十分稀疏

This is huge!

$\boldsymbol{H}$ is the Hessian matrix.

目标是要最小化关于△的二次函数，求解△。
做法是设置f对△求偏导的结果为0，经过简单的推导，能够得到下面这个式子
用过解线性方程可以将△求出，更新到P上去

- Set the partial derivative to zero:

$$\boldsymbol{H}\Delta = -\boldsymbol{b}$$

- Solving this linear system for improved results:

$$P \leftarrow P + \Delta$$

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Gauss-Newton Algorithm

- Repeat until convergence:
- 1. Compute the terms of linear systems:

$$\boldsymbol{b}^T = \sum_{ij} e_{ij}^T \mathrm{J}_{\mathrm{ij}} \qquad \boldsymbol{H} = \sum_{ij} J_{ij}^T J_{ij} \in \mathbb{R}^{(3m+6n)\times(3m+6n)}$$

- 2. Solve the linear systems by

$$\boldsymbol{H}\Delta = -\boldsymbol{b}$$

- 3. Update the previous results by:

$$P \leftarrow P + \Delta$$

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The Hessian

- The Hessian H is
  - Positive semi-definite
  - Symmetric
  - Sparse
- This allows efficient solution
  - Detailed late

Levenberg-Marquardt Algorithm

- Observations:
  - Gauss-Newton method typically converges very quickly
  - Sometimes diverges when initial solution is far off
  - Gradient descent (with line search) never diverges

- **How can we combine the advantages of both minimization methods?**

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Levenberg-Marquardt Algorithm

- Idea: Add a damping factor

$$(H + \lambda I)\Delta = -\mathrm{b}$$

- The effect of this damping factor:
  - Small λ, the same as Gaussian-Newton
  - Large λ, the same as gradient descendant
- Algorithm:
  - If error decrease, accept Δ and reduce λ
  - If error increase, reject Δ and increase λ

- Update the previous results by:

$$P \leftarrow P + \Delta$$

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Various Open Source Solvers

- PBA [Wu et al. 2011]
- Ceres [Google, 2012]
- G2O [Kuemmerle et al., 2011]
- SBA [Lourakis and Argyros, 2009]
- iSAM [Kaess et al., 2008]

Questions?

eij是第i个三维点在第j张图片上的投影误差，是一个2*1的向量，上面是x方向的误差，下面是y方向的误差？

将与三维点相关的放前面，与相机参数相关的放后面。两个红方块表示eij与第i个三维点有关，与第j个相机的参数有关（这里相机参数同事考虑了第j个相机的旋转矩阵和平移矩阵）

Structure of $\boldsymbol{b}$ and $\boldsymbol{H}$

$$\boldsymbol{b}^T = \sum_{ij} e_{ij}^T \mathrm{J_{ij}} \qquad \boldsymbol{H} = \sum_{ij} J_{ij}^T J_{ij}$$

- Remember J_{ij}'s sparse structure

$$J_{ij}(\boldsymbol{X}, \boldsymbol{R}, \boldsymbol{T}) = \left(0, \cdots, \frac{\partial e_{ij}}{\partial X_i}, \cdots, \frac{\partial e_{ij}}{\partial R_j}, \frac{\partial e_{ij}}{\partial T_i}, \cdots, 0\right)$$

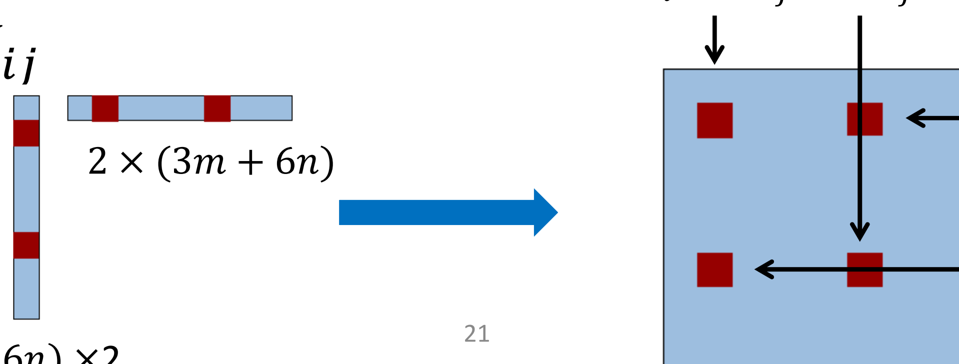
- So $b_{ij} = e_{ij}^T J_{ij}$

None zero only at X_i and R_j (and T_j)

1×2 $\quad 2 \times (3m + 6n)$

None zero at the diagonal at X_i and R_j (and T_j)

$$H_{ij} = J_{ij}^T J_{ij}$$



$2 \times (3m + 6n)$

$(3m + 6n) \times 2$

None zero at a few off diagonal elements

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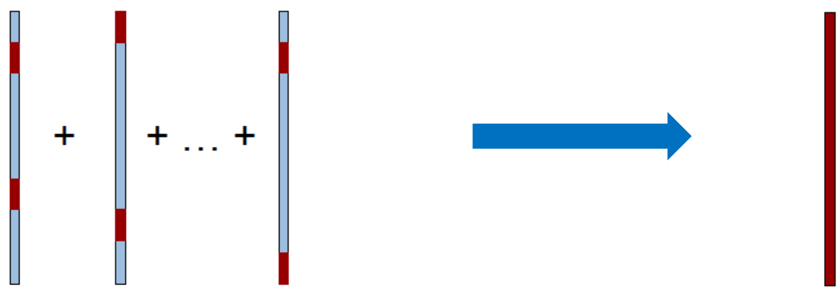
Structure of $\boldsymbol{b}$ and $\boldsymbol{H}$

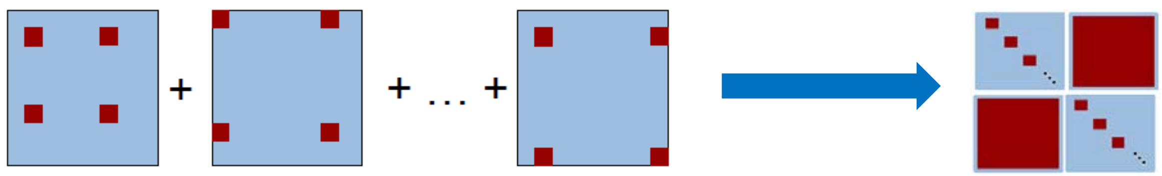
$$\boldsymbol{b}^T = \sum_{ij} b_{ij}$$



$\boldsymbol{b}$ is a dense vector

$$\boldsymbol{H} = \sum_{ij} J_{ij}^T J_{ij}$$

$\boldsymbol{H}$ has a special structure, if we order the parameters appropriately



Structure of $\boldsymbol{H}$

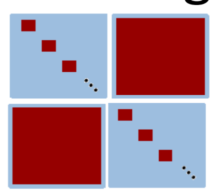
- Characteristic structure

$$\begin{pmatrix} J_C^T J_C & J_C^T J_P \\ J_P^T J_C & J_P^T J_P \end{pmatrix} \begin{pmatrix} \Delta_C \\ \Delta_P \end{pmatrix} = \begin{pmatrix} -b_C \\ -b_P \end{pmatrix}$$

Or

$$\begin{pmatrix} H_{CC} & H_{CP} \\ H_{PC} & H_{PP} \end{pmatrix} \begin{pmatrix} \Delta_C \\ \Delta_P \end{pmatrix} = \begin{pmatrix} -b_C \\ -b_P \end{pmatrix}$$

- Both H_{CC} and H_{PP} are block diagonal



$$\begin{pmatrix} \Delta_C \\ \Delta_P \end{pmatrix} = \begin{pmatrix} -b_C \\ -b_P \end{pmatrix}$$

- This can be solved using the Schur Complement

Schur Complement

- Given linear system

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

- If D is invertible, then by Gauss elimination,

$$(A - BD^{-1}C)x = a - BD^{-1}b$$
$$y = D^{-1}(b - Cx)$$

- This reduces computation complexity,
  i.e. from inverting a $(3m + 6n) \times (3m + 6n)$ matrix to inverting a $3m \times 3m$ and a $6n \times 6n$ matrix, each is block-diagonal

因为hessian的稀疏性，所以可以进行计算加速。可以从对大矩阵求逆转化成对小矩阵求逆

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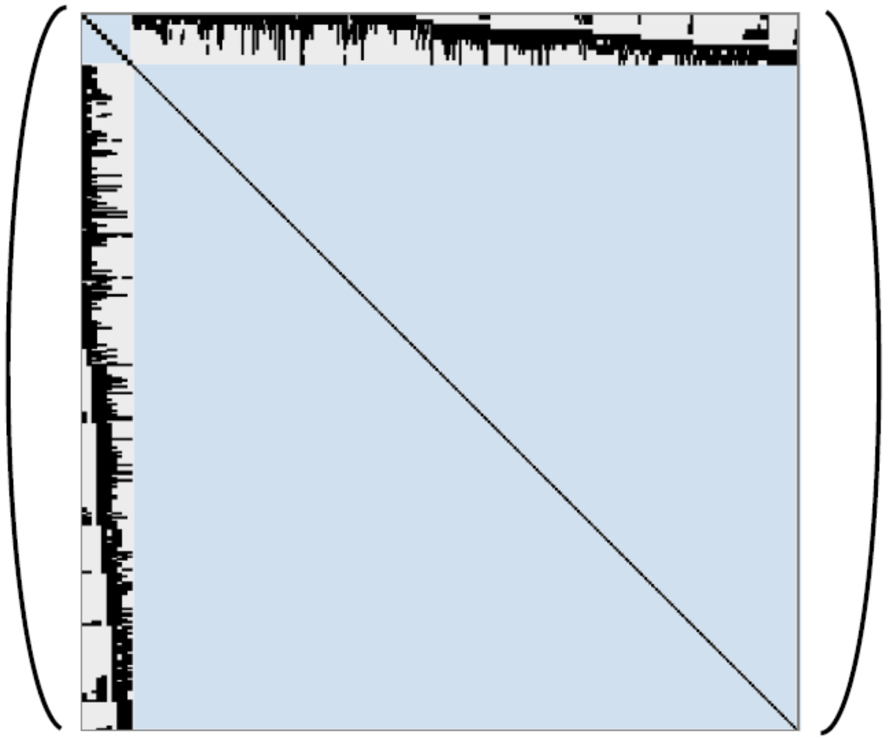
Example Hessian

$H =$ 

Questions?

Outline

- Bundle Adjustment
- Rotation Parameterization
- Initializing BA

Parameterizing Rotation Matrix

旋转矩阵是正交矩阵

- One last problem
  - Recall $J_{ij}(\boldsymbol{X},\boldsymbol{R},\boldsymbol{T}) = (0,\cdots,\frac{\partial e_{ij}}{\partial X_i},\cdots,\frac{\partial e_{ij}}{\partial R_j},\frac{\partial e_{ij}}{\partial T_i},\cdots,0)$
  - How do we parameterize $\boldsymbol{R}$?
- A rotation matrix is a 3x3 orthogonal matrix

$$\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

  - Also called the special orientation group SO(3)
- 9 parameters with 3 DoF!
  - The computed result might not be a rotation matrix, i.e. $R^T R \neq 1$

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用9个参数描述旋转矩阵的方法是不可取的，因为旋转矩阵只有3个自由度

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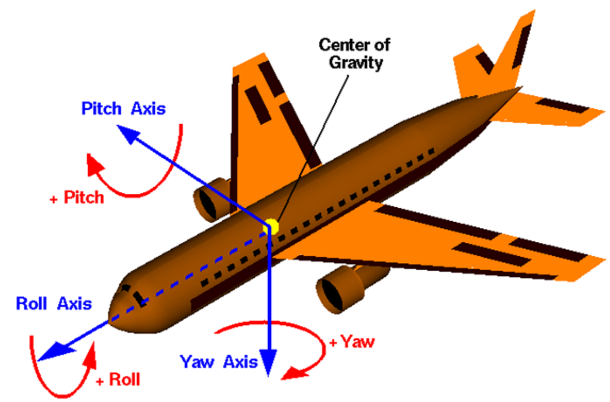
Representing $\boldsymbol{R}$ by 3 Angles

- Roll ϕ , Pitch θ, Yaw ψ
  - is very common in aerial navigation
- Conversion to 3x3 rotation matrix:



$$
\begin{aligned}
\mathbf{R} &= \mathbf{R}_Z(\psi)\mathbf{R}_Y(\theta)\mathbf{R}_X(\phi) \\
&= \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix} \\
&= \begin{pmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix}
\end{aligned}
$$

用欧拉角来描述R，理论上是可行的，但是很麻烦，因为欧拉角不能直接进行矩阵乘法

欧拉角和绕什么轴旋转的顺序有关。

Representing $\boldsymbol{R}$ by 3 Angles

- Advantage:
  - Minimal representation (3 parameters)
  - Easy interpretation
- Disadvantages:
  - Many “alternative” Euler representations exist (XYZ, ZXZ, ZYX, …)
  - Difficult to concatenate
  - Singularities (gimbal lock)
    - E.g. when $\theta = 90^o$, ϕ, ψ cannot be differentiated (2 DoFs combine to 1)

$$\boldsymbol{R} = \begin{bmatrix} 0 & 0 & -1 \\ \sin(\psi - \phi) & \cos(\psi - \phi) & 0 \\ \cos(\psi - \phi) & -\sin(\psi - \phi) & 0 \end{bmatrix}$$

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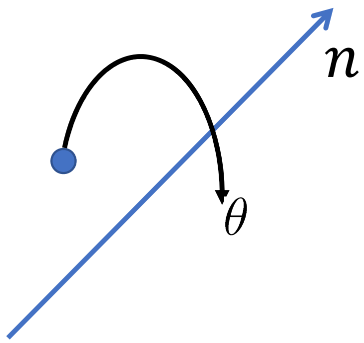
这种描述方法是周期性的，也不好优化，存在一些问题

可能采用

Axis-Angle Representation

- Represent rotation by
  - rotation axis n and angle θ
- 4 parameters (θ, n)
- 3 parameters $\theta \cdot n$
  - length is rotation angle
- Disadvantage:
  - Not a unique representation
  - Difficult to concatenate
  - Slow conversion



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Axis-Angle Representation

- Rodriguez' formula

$$\mathbf{R}(\hat{\mathbf{n}}, \theta) = \mathbf{I} + \sin\theta[\hat{\mathbf{n}}]_\times + (1 - \cos\theta)[\hat{\mathbf{n}}]_\times^2$$

- Inverse

$$\theta = \cos^{-1}\left(\frac{\mathrm{trace}(\mathbf{R}) - 1}{2}\right), \hat{\mathbf{n}} = \frac{1}{2\sin\theta}\begin{pmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{pmatrix}$$

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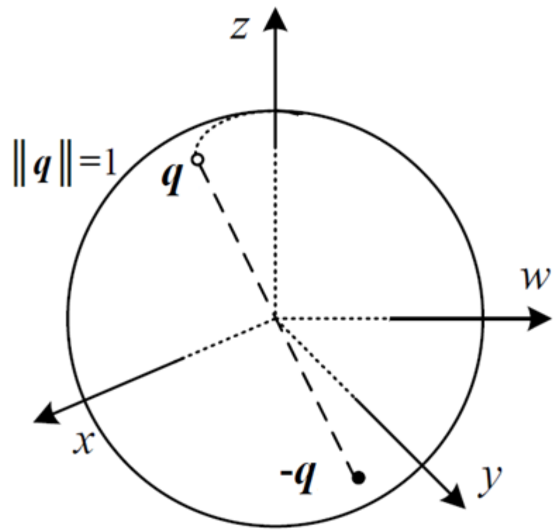
可能采用

Quaternions

- Quaternion $\mathbf{q} = (q_1, q_2, q_3, q_4)$
- It is an extension of complex numbers
  - $q_1 + q_2\boldsymbol{i} + q_3\boldsymbol{j} + q_4\boldsymbol{k}$
  - $\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = -1$
- Unit quaternions have $||\mathbf{q}|| = 1$
- Relation to angle-axis representation
  - $\mathbf{q} = (r, \boldsymbol{v}) = \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\,\boldsymbol{n}\right)$
- $\boldsymbol{q}$ and $-\boldsymbol{q}$ represent the same rotation



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Quaternions

- Advantage:
  multiplication, inversion and rotations are very efficient
- Concatenation

$$(r_1, \mathbf{v}_1)(r_2, \mathbf{v}_2) = (r_1 r_2 - \mathbf{v}_1 \cdot \mathbf{v}_2, r_1 \mathbf{v}_2 + r_2 \mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2)$$

- Inverse (=flip signs of real or imaginary part)

$$(r, \mathbf{v})^{-1} = (r, \mathbf{v})^{\star} \equiv (-r, \mathbf{v}) \equiv (r, -\mathbf{v})$$

- Rotate 3D vector $\mathbf{p} \in \mathbb{R}^3$ using a quaternion:

$$(r, \mathbf{v})(0, \mathbf{p})(r, \mathbf{v})^{\star}$$

Desired Rotation Parameterization

- No over-parameterization (avoid using 3×3 matrix)
  - Using 3 parameters to represent a rotation matrix
- No degeneracy (avoid using Euler angles)
  - Degeneracy: a subspace of the parameter space corresponds to a single rotation matrix
- The optimization algorithm can change parameters freely
  - The result is always a valid rotation matrix
- Still a somewhat unsolved problem

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Rotation Parameterization in BA

- During the LM optimization:
  - Compute the terms $\boldsymbol{H}, \boldsymbol{b}$, wrt the parameters to be optimized (e.g. quaternions)
  - Solve the linear systems by
$$(\boldsymbol{H} + \lambda \boldsymbol{I})\Delta = -\boldsymbol{b}$$
  - Update the previous results by:
$$P \leftarrow P + \Delta$$
- A quaternion has 4 parameters $\mathbf{q} = (q_1, q_2, q_3, q_4)$, we can:
  - Use 4 independent parameters and enforce $||\mathbf{q}|| = 1$ at each step;
  - Enforce the constraint $||\mathbf{q}|| = 1$, e.g. by Lagrange multiplier;
  - Focus on Δ (a small update), and parameterize it by 3 parameters (e.g. the last three elements of a quaternion, or a axis-angle representation).

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Questions?

Outline

- Bundle Adjustment
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Initializing the Bundle Adjustment

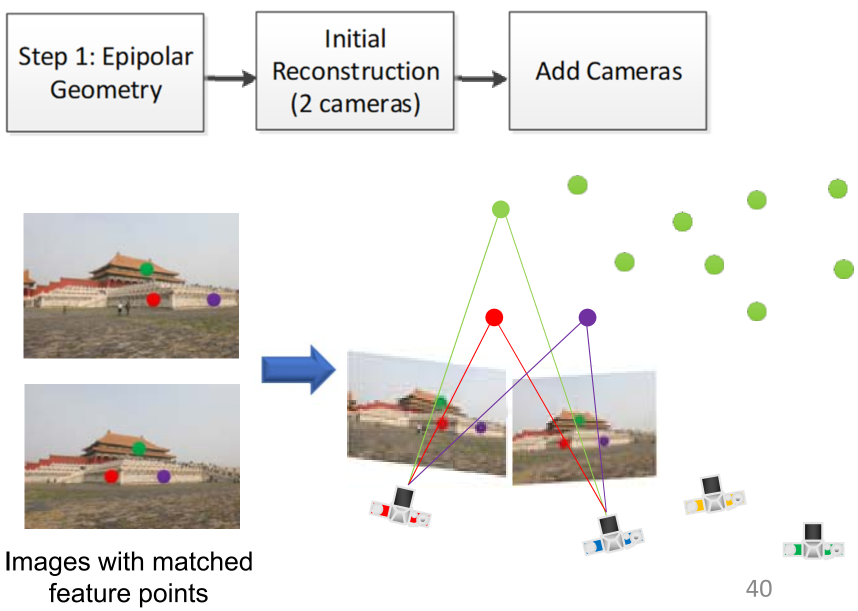
- Levenberg-Marquardt algorithm requires good initial guess for:
  - 3D points X_i
  - camera parameters R_j, t_j, K_j

需要初始化的参数是X，R，T；K有时候能得到

- How do we initialize?
- Two typical solutions:
  - Incremental Structure-from-Motion
  - Global Structure-from-Motion

Incremental Structure-from-Motion

1. Solve a two-view reconstruction (essential matrix, decomposition, triangulation)
2. Add cameras by resection with 3D-2D correspondences (resection, PnP)
   Might triangulate more points from the newly added cameras (resection)

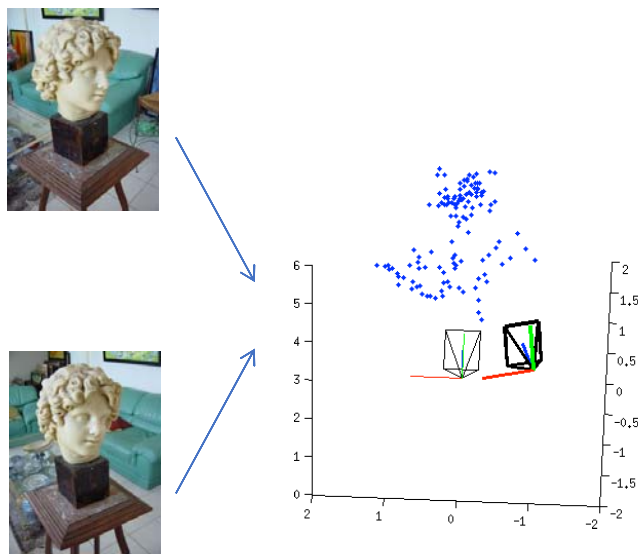


首先通过两张图片求出E或F，然后SVD求得相机内参矩阵，接着通过内参矩阵和图片上的对应点，triangulate（pnp）出对应的三维点。

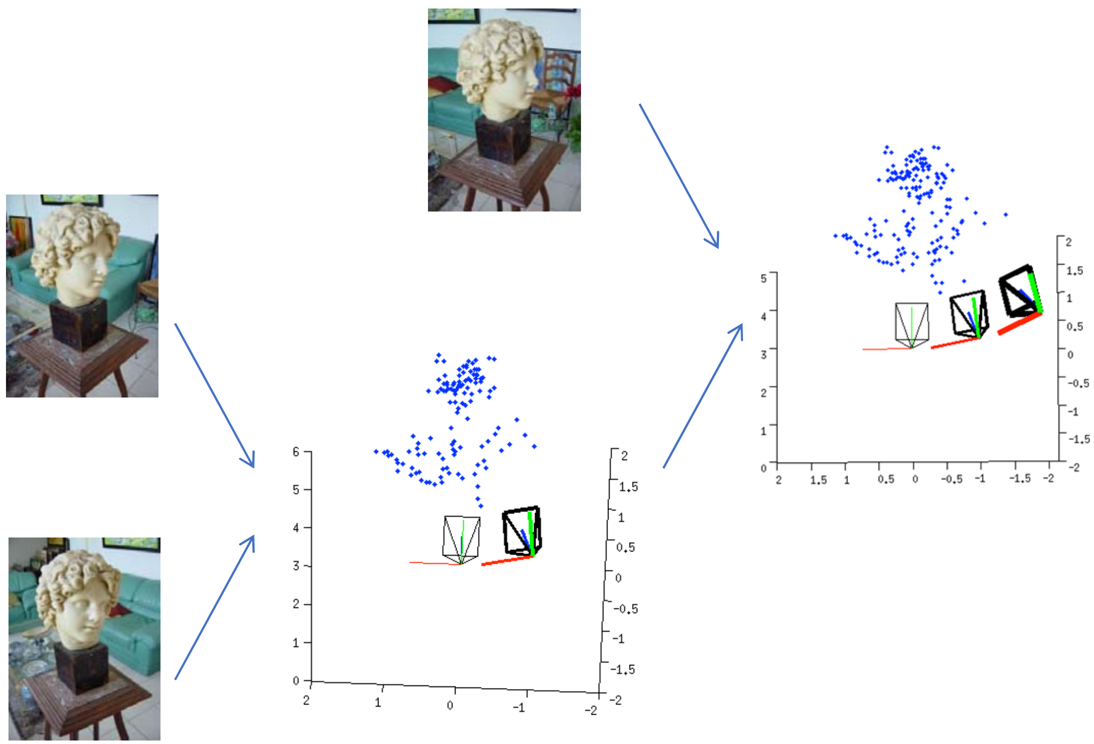
加入第三张图片，如果第三个相机内参未知，则先通过resection求出相机内参，在用pnp方法（如果内参已知则直接用pnp方法）重建出新的三维点（前两张图片中没有的，但存在于第三张图片和前两张图片中）

以此类推。

two-view reconstruction

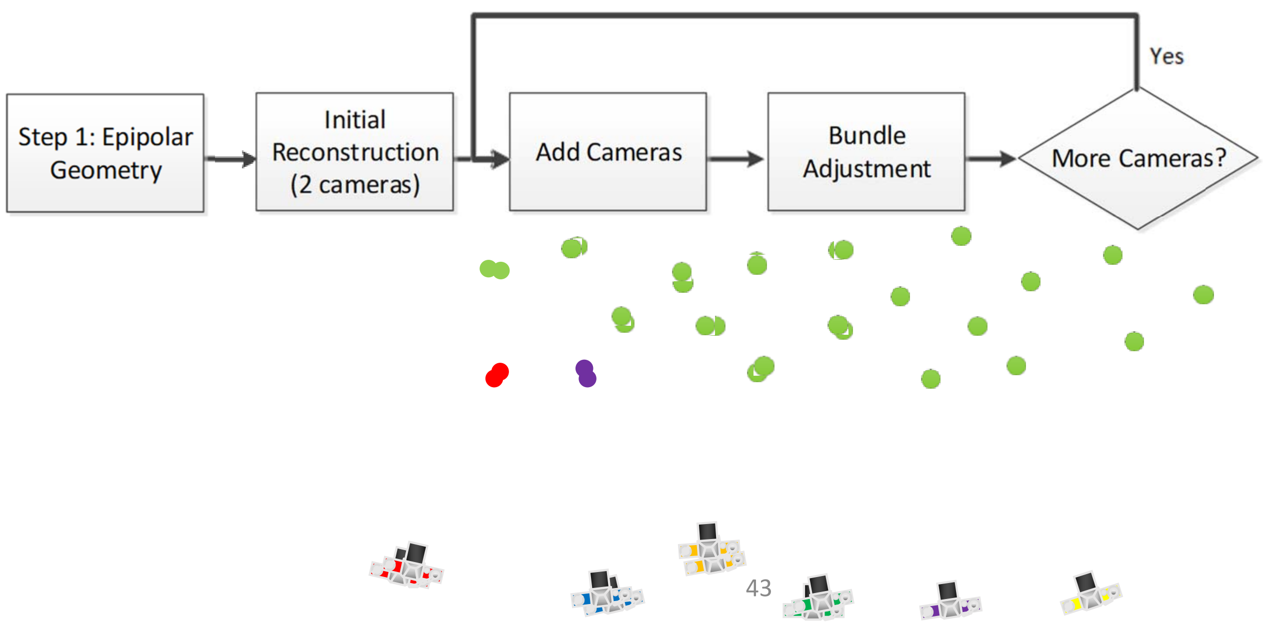


incrementally add the third view



Incremental Structure-from-Motion

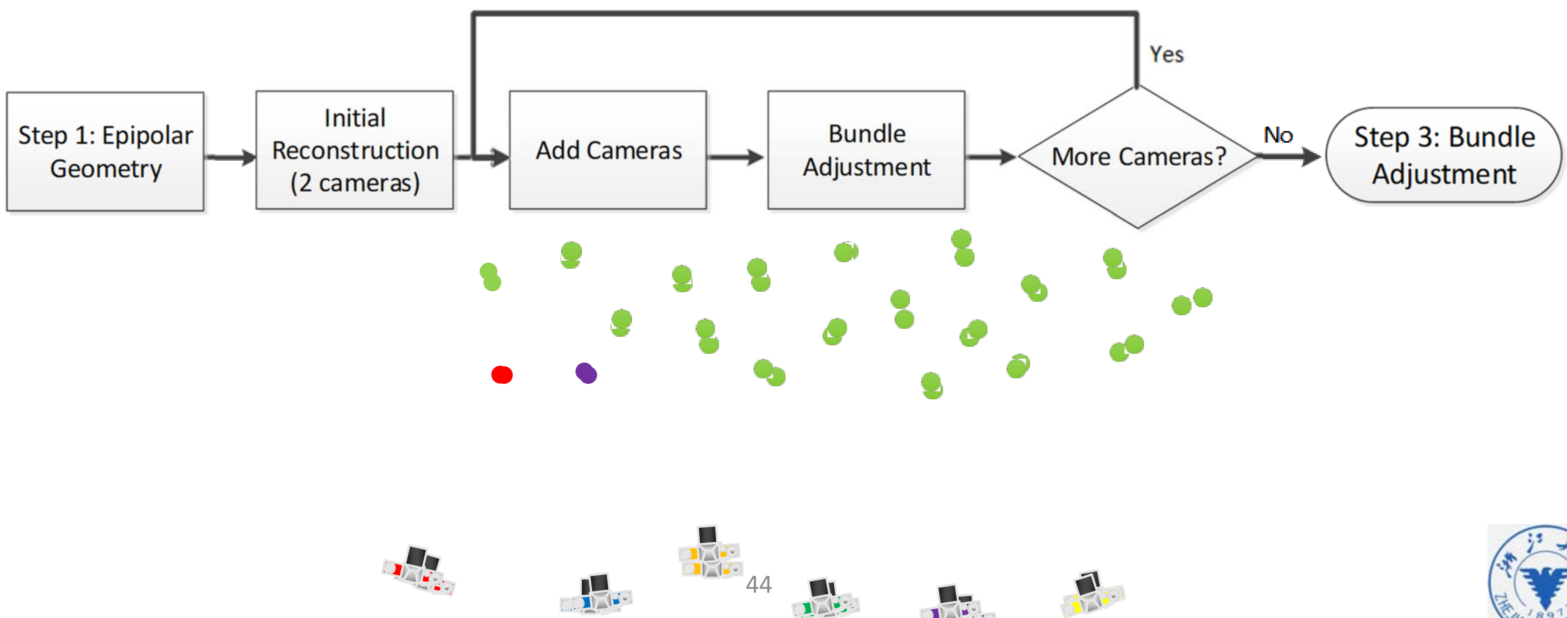
1. Solve a two-view reconstruction (essential matrix, decomposition, triangulation)
2. Add cameras by resection with 3D-2D correspondences (resection, PnP)
   Might triangulate more points from the newly added cameras (resection)
3. Repeat step-2 (with intermediate BA to reduce error accumulation)



应用resectioning的前提是三维点的未知是绝对精确的，但是pnp出来的三维点并不是绝对精确的，所以增量式sfm会造成误差累加。那么就会要在增加了一定数量的图片后做一次BA，来缩小误差。然后再加图片，再BA。在所有图片都添加完以后，再做一次整体的BA。

Incremental Structure-from-Motion

1. Solve a two-view reconstruction (essential matrix, decomposition,triangulation)
2. Add cameras by resection with 3D-2D correspondences (resection, PnP)
   Might triangulate more points from the newly added cameras (resection)
3. Repeat step-2 (with intermediate BA to reduce error accumulation)



Other Issues

如何选择初始的两张图？
如何选择下一张图？

- Which two images to begin with?
  - Maybe two images with high quality essential matrix
- Which is the next image to add (next-best-view)?
  - Maybe the one with most correspondences to existing 3D map

- Different answers to these questions lead to different result.

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Drawbacks of Incremental SfM

- Poor run-time efficiency
  - Repetitively solving the nonlinear bundle adjustment (though locally)
  - Most of the computation time is spent on bundle adjustment

需要重复解决BA问题，这浪费了大部分的时间。

- Inferior results
  - Some cameras are fixed when solving the others
  - It is desirable to solve all cameras simultaneously

增量式SfM的思想有点类似与贪心式算法，它每一步都是聚焦于当前，而不是从全局的角度来考虑。所以大概率得不到一个最优解。

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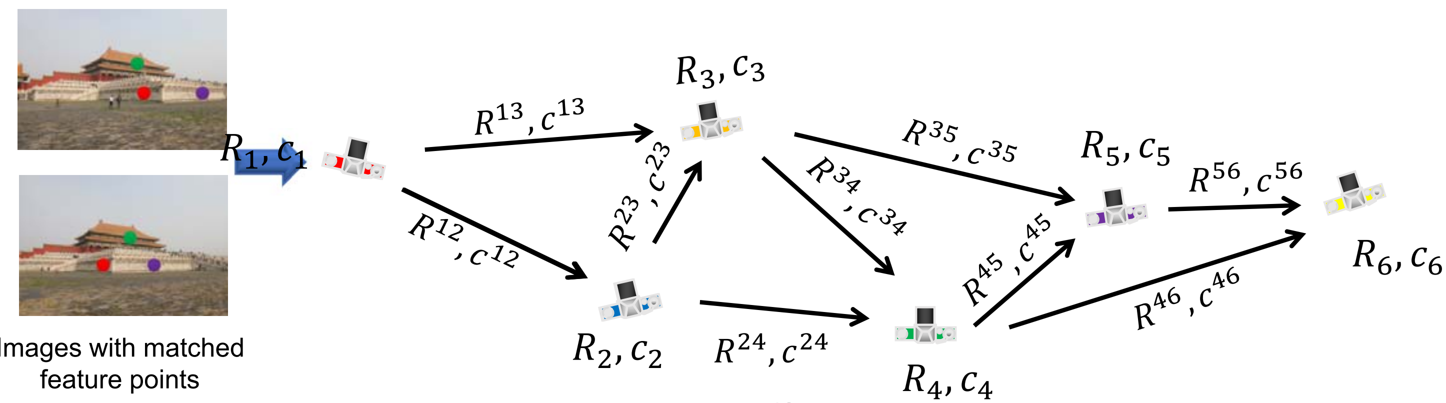
Questions?

Global Structure-from-Motion

- Solve all pairwise camera motion (essential matrices, decomposition)
- Register all cameras simultaneously from input pairwise motions



Images with matched feature points

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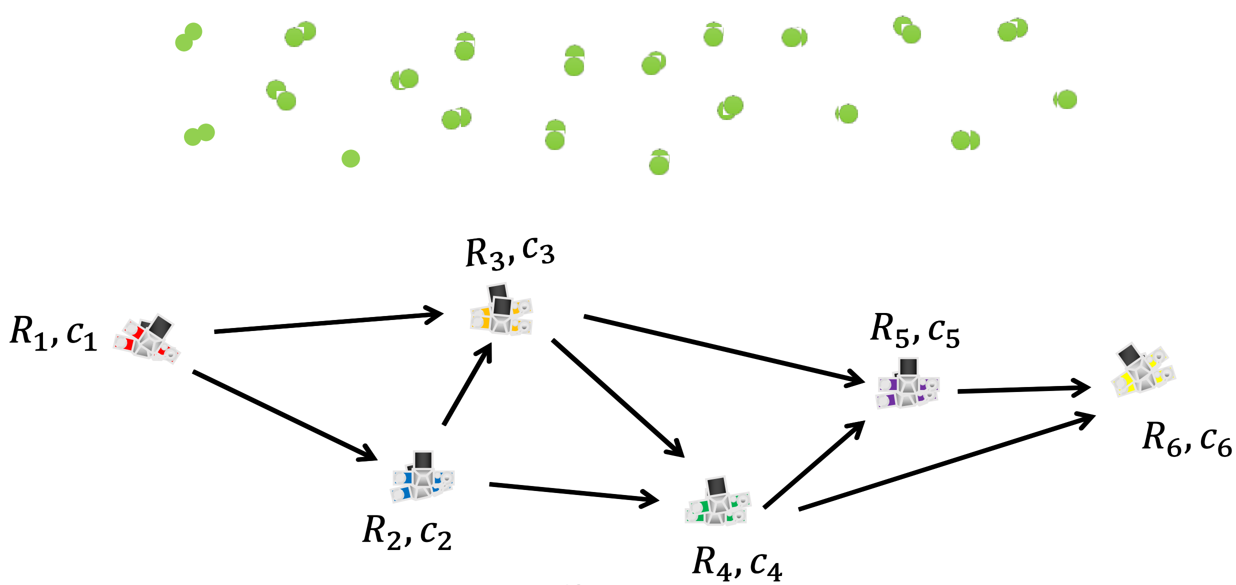
假设有n张图片，计算Cn2个image pair的相机内参，外参。
将这n张图片转换到同一个相机坐标系下。
进行一次BA

Global Structure-from-Motion

- Solve all pairwise camera motion (essential matrices, decomposition)
- Register all cameras simultaneously from input pairwise motions
- Bundle adjustment only once



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Rotation Averaging

Rij表示两个相机的相对旋转，可以从E中decompose出来。Ri& Rj表示的是两个相机在世界坐标系下的真是朝向，还是未知的

- Known relative rotation between two cameras

$$R_j = R^{ij} R_i$$

- Solving for R_i, R_j from all pairwise constraints
- In quaternion representation, $R_i = (r_i^1, r_i^2, r_i^3, r_i^4)$, therefore

$$\begin{pmatrix} r_j^1 \\ r_j^2 \\ r_j^3 \\ r_j^4 \end{pmatrix} = \begin{pmatrix} r_{ij}^1 & -r_{ij}^2 & -r_{ij}^3 & -r_{ij}^4 \\ r_{ij}^2 & r_{ij}^1 & -r_{ij}^4 & r_{ij}^3 \\ r_{ij}^3 & r_{ij}^4 & r_{ij}^1 & -r_{ij}^2 \\ r_{ij}^4 & -r_{ij}^3 & r_{ij}^2 & r_{ij}^1 \end{pmatrix} \begin{pmatrix} r_i^1 \\ r_i^2 \\ r_i^3 \\ r_i^4 \end{pmatrix}$$

$$\boldsymbol{r}_j = \mathcal{R}^{ij} \boldsymbol{r}_i$$

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通过quaternion描述旋转矩阵，进而Rj=Rij·Ri可以表示为下面那个矩阵乘法，也就是一个线性方程组，然后通过两幅图中的多对对应点求出Ri和Rj的四元数表示。

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Rotation Averaging

- Obtain a linear equations of $\boldsymbol{r}_i, \boldsymbol{r}_j$ for a pair (i, j)

$$\begin{bmatrix}\mathcal{R}^{ij} & -I\end{bmatrix}\begin{pmatrix}\boldsymbol{r}_i \\ \boldsymbol{r}_j\end{pmatrix} = 0$$

- Stack all equations, solve all $\boldsymbol{r}_i$ linearly
  - Ignore the unit quaternion constraint, i.e. $||\boldsymbol{r}_i|| = 1$
  - Normalize the result quaternions afterwards

旋转矩阵的模应该为1，索要要进行处理

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Rotation Averaging

- Similar linear solution from matrix representation
- From $R_j = R^{ij} R_i$, $R_i = \left[r_i^1, r_i^2, r_i^3\right]$, $R_j = [r_j^1, r_j^2, r_j^3]$, obtain 3 equations

$$r_j^k = R^{ij} r_i^k \qquad k = 1, 2, 3$$

- Similarly,

$$\begin{bmatrix} R^{ij} & -I \end{bmatrix} \begin{pmatrix} r_i^k \\ r_j^k \end{pmatrix} = 0 \qquad k = 1, 2, 3$$

- Stack all equations, solve all r_i^k linearly
  - Ignore the orthogonal matrix constraint, i.e. $R_i^T R_i = I$
  - Normalize the result matrix afterwards

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Rotation Averaging

- Rotation averaging is still an open problem
- Most recent methods apply nonlinear optimization after the linear initialization

Robust Relative Rotation Averaging

Avishek Chatterjee and Venu Madhav Govindu

[PAMI 2017]

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Translation Averaging

- Known relative rotation between two cameras

$$c_i - c_j = R_j^T t^{ij}$$

- Solving for c_i, c_j from all pairwise constraints
- Direct Linear Transform:

$$R_j^T t^{ij} \times (c_i - c_j) = 0$$

  - Problem 1: minimizing an algebraic error, faraway pairs are weighted more
  - Problem 2: cannot work on linear camera motion (i.e. all $(c_i - c_j)$ are colinear)

Essential matrices can only determine camera centers in a 'parallel rigid graph'

Robust Camera Location Estimation by Convex Programming

Onur Özyeşil[1] and Amit Singer[1,2]
[1]Program in Applied and Computational Mathematics, Princeton University
[2]Department of Mathematics, Princeton University
Princeton, NJ 08544-1000, USA
{oozyesil,amits}@math.princeton.edu

[CVPR 2015]

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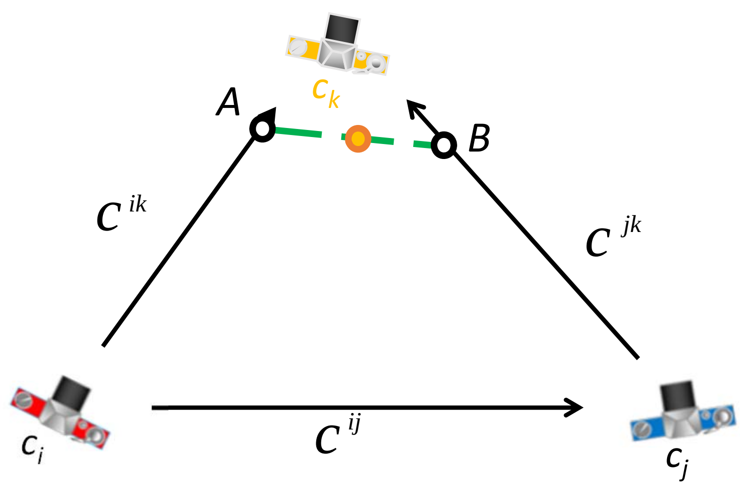
Translation Averaging

A novel linear equation for three cameras from the 'mid-point' algorithm

$$c_k = \frac{1}{2}\left[\boxed{c_i + M_1(c_j - c_i)} + \boxed{c_j + M_2(c_i - c_j)}\right]$$

Similar linear equations for c_i and c_j



M_1, M_2 are both known matrices, computed from scene points.

$$\boxed{A = c_i + M_1(c_j - c_i)}$$

$$\boxed{B = c_j + M_2(c_i - c_j)}$$

AB: the mutual perpendicular line

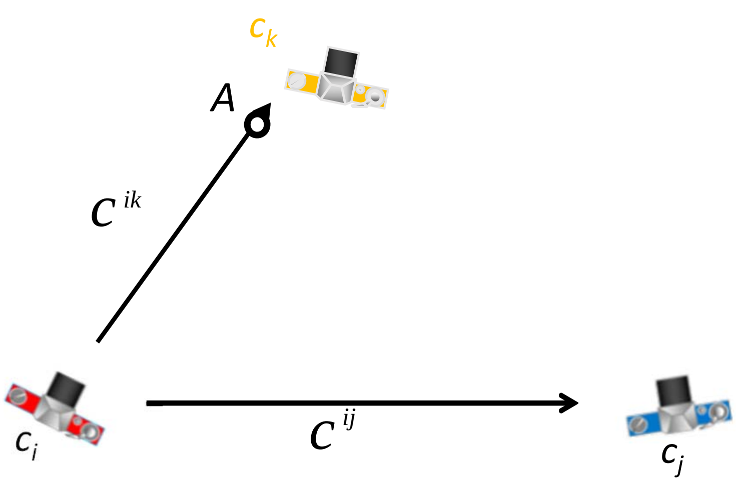
c_k: the middle point of AB

Translation Averaging

Geometric meaning of M_1

$$c_k = \frac{1}{2}\left[\boxed{c_i + M_1\left(c_j - c_i\right)} + \boxed{c_j + M_2\left(c_i - c_j\right)}\right]$$

1. rotate to match the orientation
2. shrink/grow to match the length

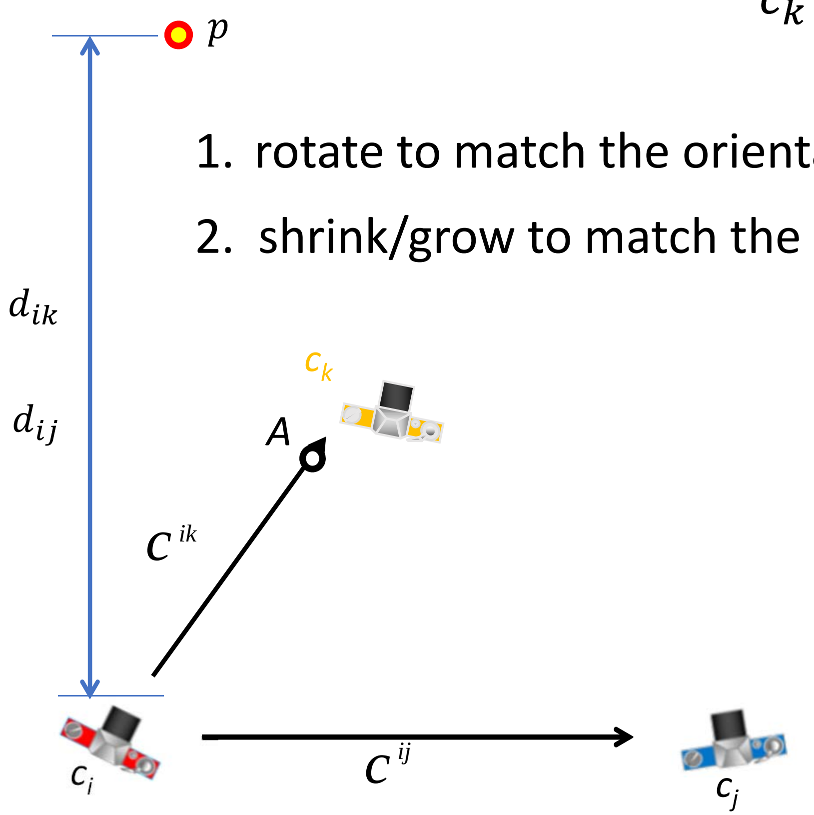


$$\boxed{A = c_i + M_1(c_j - c_i)}$$

Translation Averaging

Geometric meaning of M_1

$$c_k = \frac{1}{2}\left[\boxed{c_i + M_1(c_j - c_i)} + \boxed{c_j + M_2(c_i - c_j)}\right]$$



1. rotate_to match the orientation ➔ Known from essential matices
2. shrink/grow to match the length ➔ Known from a scene point

$$\frac{|c_i - c_j|}{|c_i - c_k|} = \frac{d_{ik}}{d_{ij}}$$

The ratio of a scene point's depths

d_{ik} is p's depth when reconstructed by the pair (i, k).

d_{ij} is p's depth when reconstructed by the pair (i, j).

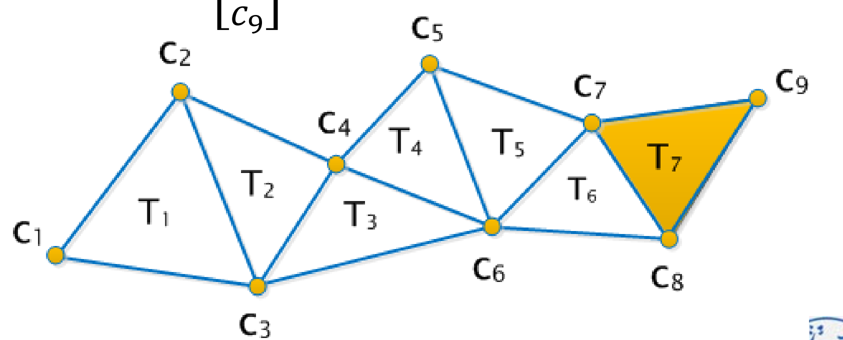
Translation Averaging

1. Collect equations from all triangles in the pose graph.

$$\boxed{AX = 0} \qquad X = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{bmatrix}$$

The pose graph:
each camera is a vertex,
connect two cameras if their
relative motion is known.

2. Solve all equations



$$A_1(c_1, c_2, c_3)^T = 0$$

cameras can be non-coplanar.

Translation Averaging

• More details in the papers

A Global Linear Method for Camera Pose Registration

Nianjuan Jiang[1,*] Zhaopeng Cui[2,*] Ping Tan[2]
[1] Advanced Digital Sciences Center, Singapore [2] National University of Singapore

[ICCV 2013]

Questions?