Quantum Teleportation general concept

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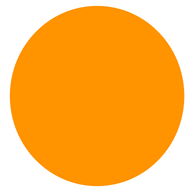
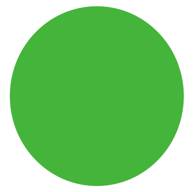
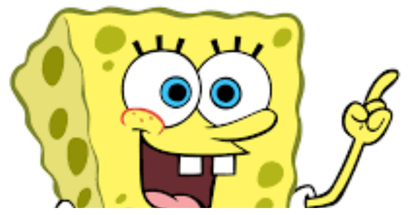
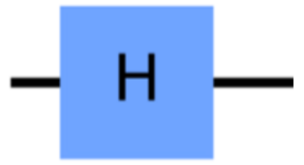
Before getting started:

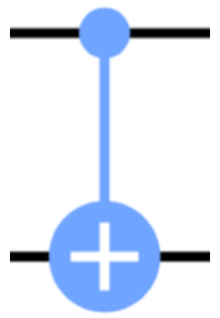
 $|q_1\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 

 $|q_2\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 

 $|q_0\rangle = \alpha |0\rangle + \beta |1\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$\text{Hadamard } H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$



$\text{Controlled Not } CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\text{Basis for CNOT}: \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$

Bell States:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|11\rangle + |00\rangle)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|11\rangle - |00\rangle)$$

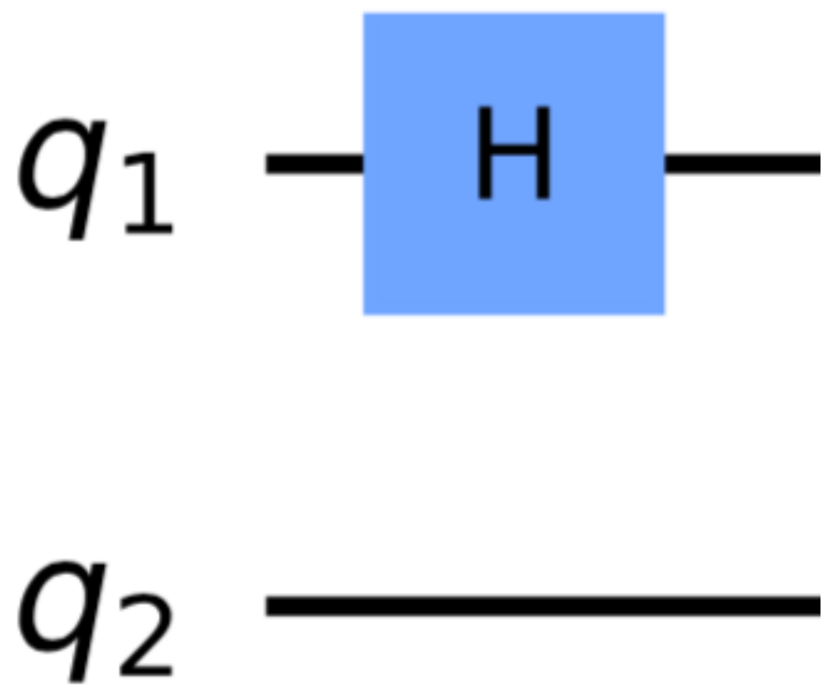
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|10\rangle - |01\rangle)$$

Step 1 + Step 2: To generate an entangled state

Step 1

Applying a Hadamard operation on q1.

$$q_1 : H\left|0\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\left|1\right\rangle + \left|0\right\rangle\right)$$

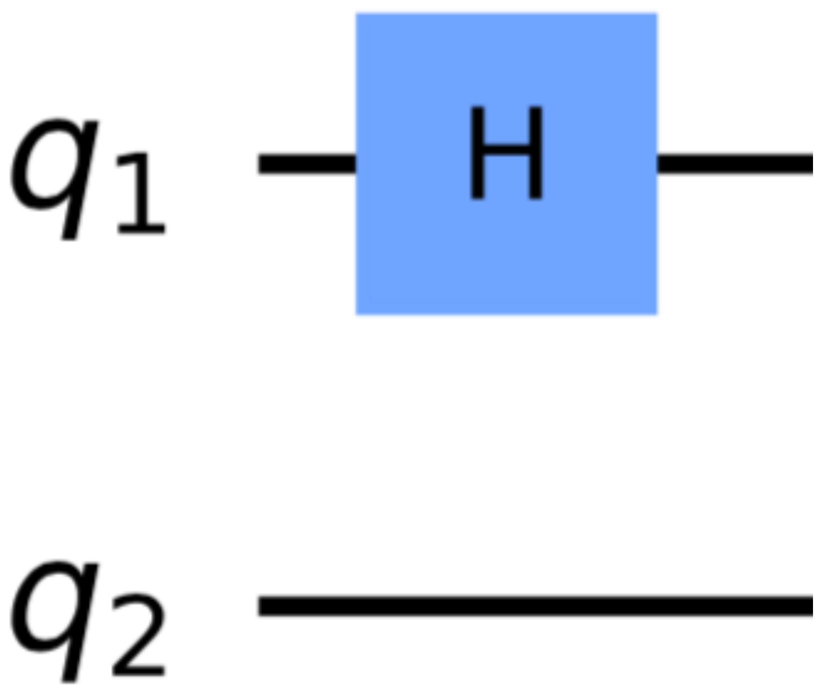


Step 1 + Step 2: To generate an entangled state

Step 1

Applying a Hadamard operation on q1.

$$q_1 : H\,|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

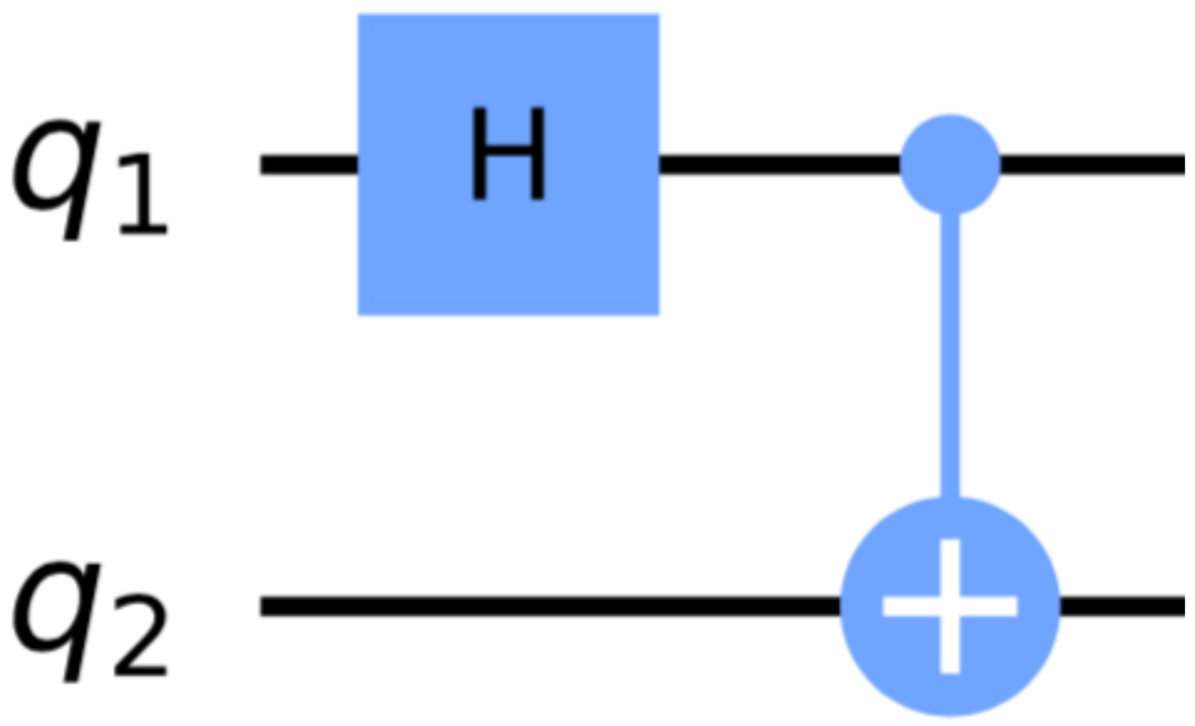


Step 2.

$$|q_1\rangle \otimes |q_2\rangle = \frac{1}{\sqrt{2}}\left[(|1\rangle + |0\rangle) \otimes |0\rangle\right] = \frac{1}{\sqrt{2}}\left(\boxed{|\overset{q1}{1}\overset{q2}{0}\rangle} + |\overset{q1}{0}\overset{q2}{0}\rangle\right)$$

$$|q\rangle = CNOT_{12}\,|q_1\rangle \otimes |q_2\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{array}{l} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \end{array}\begin{array}{l} \leftarrow |00\rangle \\ \leftarrow |01\rangle \\ \leftarrow |10\rangle \\ \leftarrow |11\rangle \end{array}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(|00\rangle + \boxed{|11\rangle}\right)$$: Bell pair



Step 3: Expand the state of |q0, q1, q2>

$$|q_0\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|q_0, q_1, q_2\rangle = |q_0\rangle \otimes |q_1, q_2\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle) \otimes \tfrac{1}{\sqrt{2}}\,(|11\rangle + |00\rangle)$$

$$= \tfrac{1}{\sqrt{2}}\,(\alpha\,|011\rangle + \alpha\,|000\rangle + \beta\,|111\rangle + \beta\,|100\rangle)$$

$$= \tfrac{\alpha}{\sqrt{2}}\,(|01\rangle \otimes |1\rangle) + \tfrac{\alpha}{\sqrt{2}}\,(|00\rangle \otimes |0\rangle) + \tfrac{\beta}{\sqrt{2}}\,(|11\rangle \otimes |1\rangle) + \tfrac{\beta}{\sqrt{2}}\,(|10\rangle \otimes |0\rangle)$$

Step 3: Expand the state of |q0, q1, q2>

Replace $|01\rangle$, $|00\rangle$, $|10\rangle$, and $|11\rangle$ with the Bell states:

$$|01\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle)$$

$$|00\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle)$$

$$|11\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$$

$$|q_0\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix}$$

$$|q_0, q_1, q_2\rangle = |q_0\rangle \otimes |q_1, q_2\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|011\rangle + \alpha|000\rangle + \beta|111\rangle + \beta|100\rangle)$$

$$= \frac{\alpha}{\sqrt{2}}(|01\rangle \otimes |1\rangle) + \frac{\alpha}{\sqrt{2}}(|00\rangle \otimes |0\rangle) + \frac{\beta}{\sqrt{2}}(|11\rangle \otimes |1\rangle) + \frac{\beta}{\sqrt{2}}(|10\rangle \otimes |0\rangle)$$

$$= \left\{\left[\frac{\alpha}{2}(|\Phi^+\rangle - |\Phi^-\rangle) \otimes |1\rangle + \frac{\alpha}{2}(|\Psi^+\rangle - |\Psi^-\rangle) \otimes |0\rangle + \frac{\beta}{2}(|\Psi^+\rangle + |\Psi^-\rangle) \otimes |1\rangle + \frac{\beta}{2}(|\Phi^+\rangle + |\Phi^-\rangle \otimes |0\rangle)\right]\right\}$$

$$= \left[|\Phi^+\rangle \otimes \boxed{\left(\frac{\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right)} + |\Psi^+\rangle \otimes \boxed{\left(\frac{\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right)} + |\Phi^-\rangle \otimes \boxed{\left(\frac{-\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right)} + |\Psi^-\rangle \otimes \boxed{\left(\frac{-\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right)}\right]$$

|q2> can be written in a similar form to |q0>

Step 4: Apply a CNOT gate and a H gate

$$|q_0, q_1, q_2\rangle = \left[|\Phi^+\rangle \otimes \left(\frac{\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right) + |\Psi^+\rangle \otimes \left(\frac{\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right) + |\Phi^-\rangle \otimes \left(\frac{-\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right) + |\Psi^-\rangle \otimes \left(\frac{-\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right)\right]$$

$$H_0\left(CNOT_{01}\left(|q_0, q_1, \boxed{q_2\rangle}\right)\right)$$

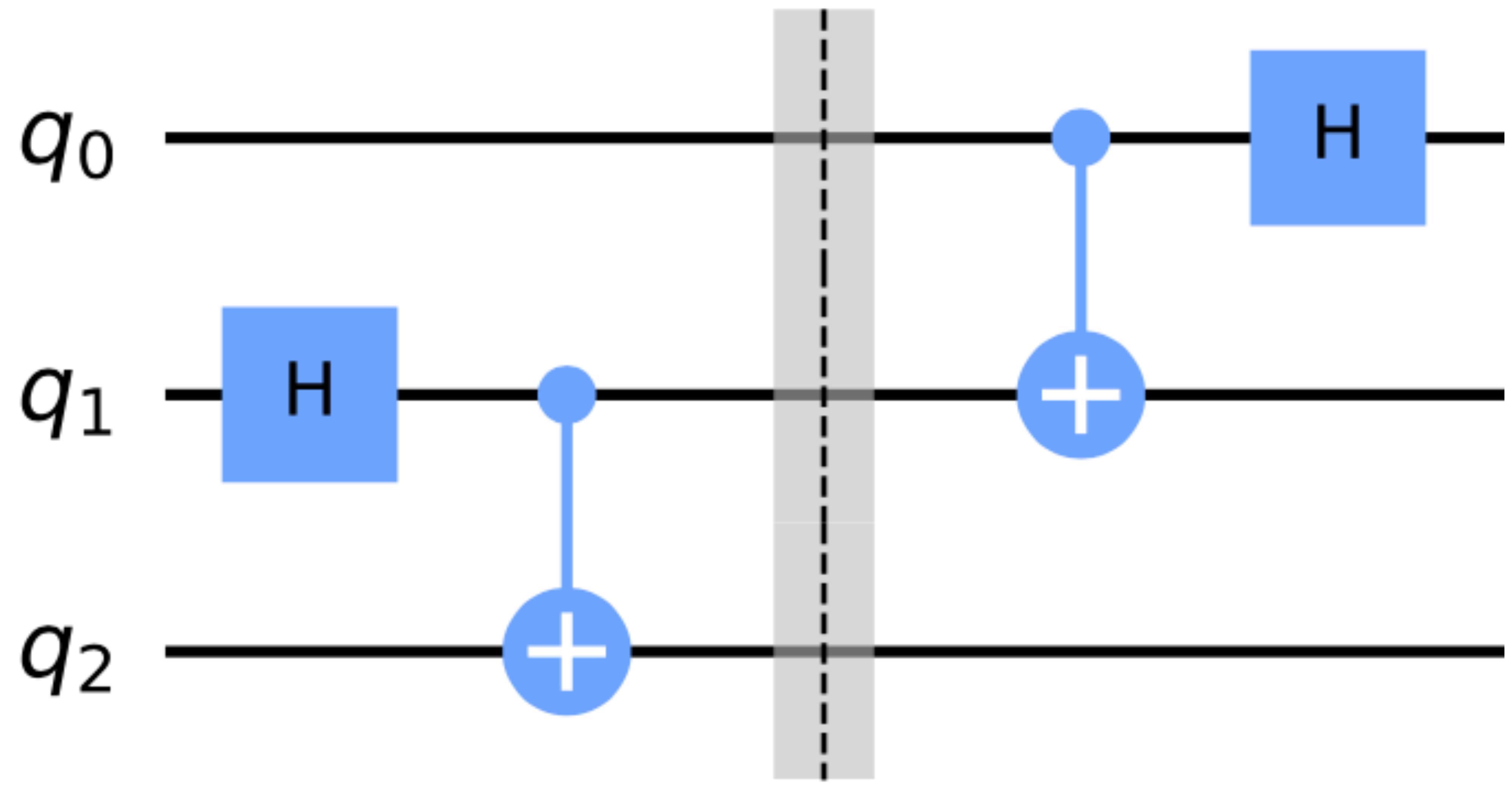
$$= \left[|01\rangle \otimes \boxed{\left(\frac{\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right)} + |00\rangle \otimes \boxed{\left(\frac{\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right)} + |11\rangle \otimes \boxed{\left(\frac{\alpha}{2}|1\rangle - \frac{\beta}{2}|0\rangle\right)} + |10\rangle \otimes \boxed{\left(\frac{\alpha}{2}|0\rangle - \frac{\beta}{2}|1\rangle\right)}\right]$$

$$H_0\left(CNOT_{01}\left(|\Phi^+\rangle\right)\right) = H_0\left(\frac{1}{\sqrt{2}}\left(|11\rangle + |01\rangle\right)\right) = \frac{1}{2}\left[\left(|0\rangle - |1\rangle\right) \otimes |1\rangle + \left(|0\rangle + |1\rangle\right) \otimes |1\rangle\right] = |01\rangle$$

$$H_0\left(CNOT_{01}\left(|\Psi^+\rangle\right)\right) = |00\rangle$$

$$H_0\left(CNOT_{01}\left(|\Phi^-\rangle\right)\right) = -|11\rangle$$

$$H_0\left(CNOT_{01}\left(|\Psi^-\rangle\right)\right) = -|10\rangle$$



Step 5: Alice performs the measurement on |q0, q1>

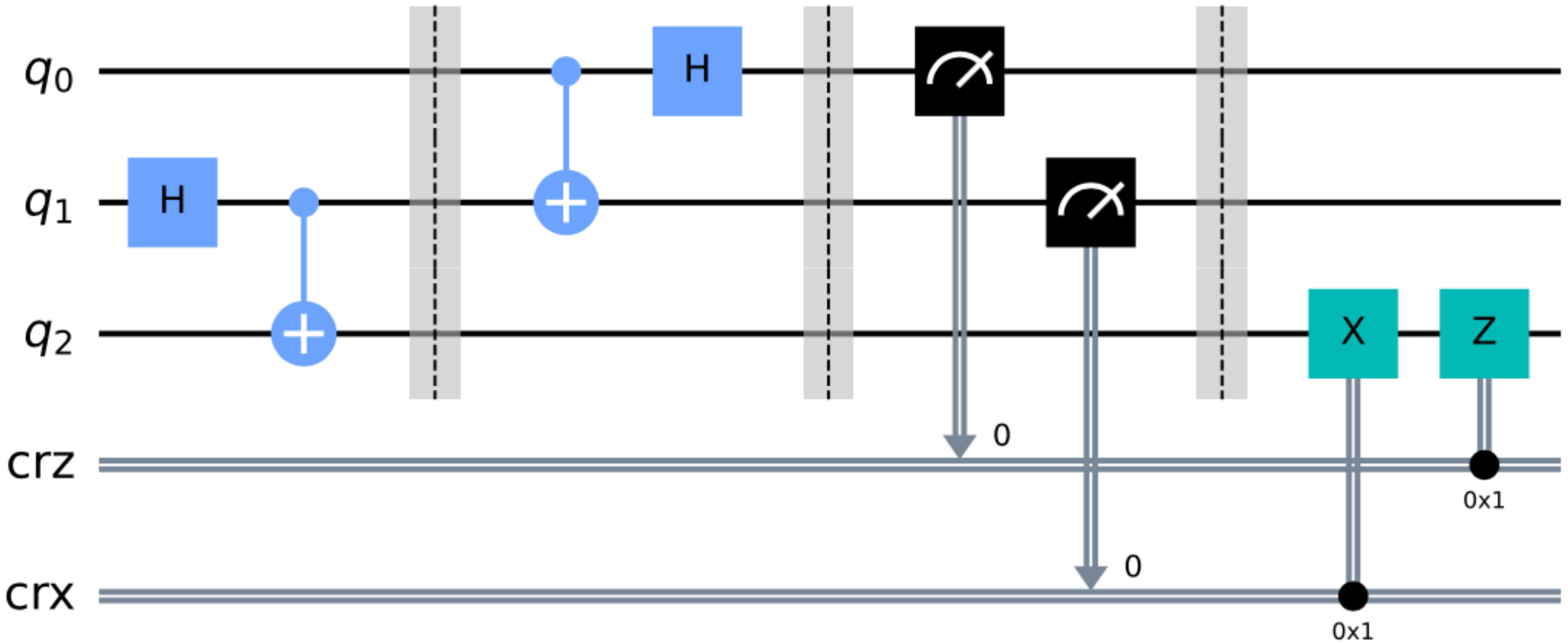
After applying the H gate and CNOT gate, |q0, q1, q2> is now:

$$|01\rangle \otimes \left(\frac{\alpha}{2}|1\rangle + \frac{\beta}{2}|0\rangle\right) + |00\rangle \otimes \left(\frac{\alpha}{2}|0\rangle + \frac{\beta}{2}|1\rangle\right) + |11\rangle \otimes \left(\frac{\alpha}{2}|1\rangle - \frac{\beta}{2}|0\rangle\right) + |10\rangle \otimes \left(\frac{\alpha}{2}|0\rangle - \frac{\beta}{2}|1\rangle\right)$$

After the measurement on |q0, q1>, the state will be collapsed into one of the following states

|q0, q1> |q2>

$$\begin{aligned}
& \boxed{|00\rangle} \otimes \boxed{(\alpha|0\rangle + \beta|1\rangle)} \xrightarrow{\text{Do nothing}} |q_2\rangle = \alpha|0\rangle + \beta|1\rangle \\
\text{or } & |01\rangle \otimes (\alpha|1\rangle + \beta|0\rangle) \xrightarrow{\text{Apply X gate}} |q_2\rangle = \alpha|0\rangle + \beta|1\rangle \\
\text{or } & |11\rangle \otimes (\alpha|1\rangle - \beta|0\rangle) \xrightarrow{\text{Apply XZ gate}} |q_2\rangle = \alpha|0\rangle + \beta|1\rangle \\
\text{or } & |10\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) \xrightarrow{\text{Apply Z gate}} |q_2\rangle = \alpha|0\rangle + \beta|1\rangle
\end{aligned}$$



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