UNIVERSITY OF VERONA

Department of Computer Science

TECHNICAL REPORT

Industrial Plants - Project 3

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Curriculum Computer Engineering for Robotics and Smart Industry

Verona 2024

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1. Project Objectives

The objective of the project is to perform a kinematic analysis of a multi-loop planar linkage. Figure 1 provides a visualization of the linkage, the crank AB rotates in the clockwise direction with a constant speed of 20 rad/s. The connecting rod BD then transfers the motion to the link CE, which has one of its ends (C) connected to the ground link through a revolute joint. In addition, the connecting rod EF transfers the motion to the slider in F, whose kinematic axis is along the y axis of the absolute reference frame. The lengths of the links are: AB = 160 cm, BD = 210 cm, CD = 190 cm, CE = 520 cm and EF = 630 cm.

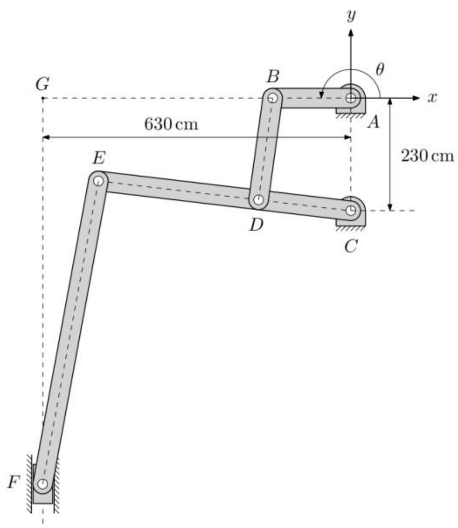


Figure 1. *Visualization of the linkage.*

In the following sections, an analytical solution for the kinematic analysis of the linkage will be first discussed. Following that, a simulation of the linkage using Simscape Multibody™ will be presented. The correctness of the results was validated by cross-checking between the analytical solution and the simulation outcomes. **Note**: all the numerical calculations was done by the Matlab script *project3_main_HuiShi.m*.

2. Calculate the number of degrees of freedom N of the linkage by applying the Grübler criterion

Grübler criterion:

$$N = 3(m-1) - 2c_1 - c_2\,, \tag{1}$$

where m is the number of links, c1 is the number of single-degree-of-freedom joints and c2 is the number of two-degree-of-freedom joints. This linkage is composed of 6 links (the ground link, link AB, link BD, link CE, link EF and the slider in F), 6 revolute joints and 1 prismatic joint. The obtained number of degrees of freedom N of the linkage is:

$$N = 3(6-1) - 2 \cdot 7 - 0 = 15 - 14 = 1 \tag{2}$$

As a result, the linkage can be operated with a single input motion. Once the position of one link is specified, the positions of the other links are automatically determined.

3. Decompose the linkage into a base mechanism and a number of dyads

The linkage can be decomposed into a base mechanism, a RRR dyad and a RRP dyad, as illustrated in Figure 2.

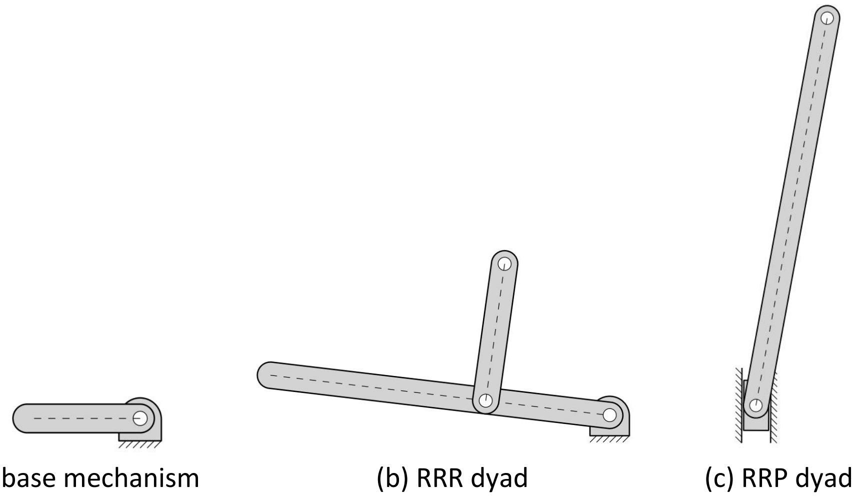


Figure 2. *Decomposition into Assur groups.*

4. Define the position-difference vectors used to solve the kinematic analysis of the linkage

Figure 3 demonstrates the defined position-difference vectors: $\overline{z}_1 = \overline{B-A}$, $\overline{z}_2 = \overline{D-B}$, $\overline{z}_3 = \overline{C-D}$, $\overline{z}_4 = \overline{A-C}$, $\overline{z}_5 = \overline{G-A}$, $\overline{z}_6 = \overline{F-G}$, $\overline{z}_7 = \overline{E-F}$, $\overline{z}_8 = \overline{C-E}$.

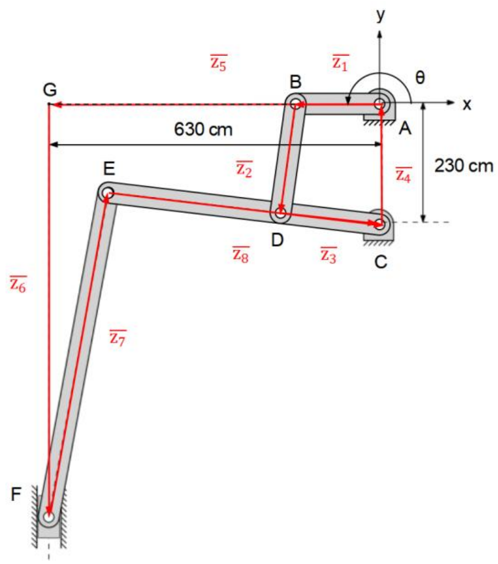


Figure 3. *Definition of position-difference vectors.*

From the closed polygons formed by the position-difference vectors through successive links and joints, two linearly independent loop-closure equations were obtained:

$$\overline{z}_1 + \overline{z}_2 + \overline{z}_3 + \overline{z}_4 = \overline{0} \tag{3}$$
$$\overline{z}_5 + \overline{z}_6 + \overline{z}_7 + \overline{z}_8 + \overline{z}_4 = \overline{0} \tag{4}$$

5. Solve the complete kinematic analysis of the linkage (position, velocity and acceleration analysis) when the angle θ is equal to 180°

5.1 Position analysis

5.1.1 Position analysis of the first loop-closure equation

Figure 4 shows the first kinematic loop defined by Equation (3). Equation (3) can be projected along the two principal axes as:

$$\begin{cases} z_1 \cos(\varphi_1) + z_2 \cos(\varphi_2) + z_3 \cos(\varphi_3) + z_4 \cos(\varphi_4) = 0 \\ z_1 \sin(\varphi_1) + z_2 \sin(\varphi_2) + z_3 \sin(\varphi_3) + z_4 \sin(\varphi_4) = 0 \end{cases} \tag{5}$$

From the given conditions, the magnitude of vector $\overline{z}_1$, $\overline{z}_2$, $\overline{z}_3$, and $\overline{z}_4$ are known as 1.6 m, 2.1 m, 1.9 m, and 2.3 m respectively. The direction of vector $\overline{z}_1$ is the generalized coordinate and is given by θ = 180° (π rad). The direction of vector $\overline{z}_2$ and $\overline{z}_3$ are unknown. The direction of $\overline{z}_4$ is $\pi/2$ as illustrated in Figure 4.

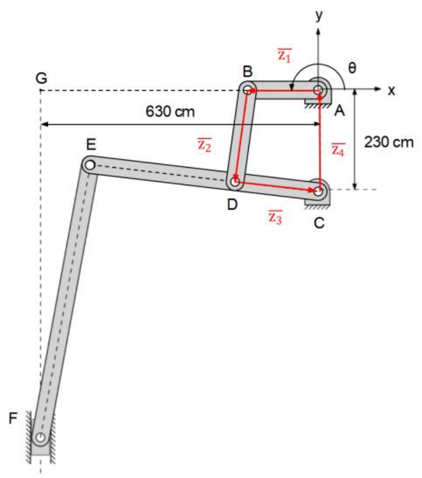


Figure 4. *First kinematic loop defined by equation (3).*

To solve Equation (5), the analytical solution of an RRR dyad was used. First an additional vector $\overline{z}_9$ connecting point C and point B is defined, as shown in Figure 5. Then the angles γ_{19}, γ_{29}, γ_{39} and γ_{49} are defined as the angles between $\overline{z}_1$ and $\overline{z}_9$, $\overline{z}_2$ and $\overline{z}_9$, $\overline{z}_3$ and $\overline{z}_9$, $\overline{z}_4$ and $\overline{z}_9$ respectively.

As the absolute reference frame (x, y) is defined in A, the coordinates of point B and C can be computed from the given conditions:

$$(x_B, y_B) = (-1.6\,\mathrm{m}, 0\,\mathrm{m}), \qquad (x_C, y_C) = (0\,\mathrm{m}, -2.3\,\mathrm{m}) \tag{6}$$

The magnitude of $\bar{z}_9$ was then computed from these coordinates:

$$z_9 = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = 2.802\,\mathrm{m} \tag{7}$$

Utilizing the following relation:

$$z_4^2 = z_1^2 + z_9^2 - 2z_1z_9\cos(\gamma_{19}) \tag{8}$$
$$z_3^2 = z_2^2 + z_9^2 - 2z_2z_9\cos(\gamma_{29}) \tag{9}$$

The angles γ_{19}, γ_{29} can be obtained as:

$$\gamma_{19} = \arccos\left(\frac{z_1^2 + z_9^2 - z_4^2}{2z_1z_9}\right) = 0.963\,rad \tag{10}$$
$$\gamma_{29} = \arccos\left(\frac{z_2^2 + z_9^2 - z_3^2}{2z_2z_9}\right) = 0.745\,rad \tag{11}$$

As φ_1 is known $(\theta = \pi)$, the direction of vector $\bar{z}_2$ can be calculated as:

$$\varphi_2 = \varphi_1 + (\pi - \gamma_{19} - \gamma_{29}) = 4.575\,rad \tag{12}$$

Exploiting the following relation:

$$z_2^2 = z_3^2 + z_9^2 - 2z_3z_9\cos(\gamma_{39}) \tag{13}$$

The angle γ_{39} is then obtained:

$$\gamma_{39} = \arccos\left(\frac{z_3^2 + z_9^2 - z_2^2}{2z_3z_9}\right) = 0.847\,rad \tag{14}$$

Since $\bar{z}_4$ is orthogonal to $\bar{z}_1$, γ_{49} is then calculated as:

$$\gamma_{49} = \pi/2 - \gamma_{19} = 0.608\,rad \tag{15}$$

Finally, the direction of vector $\bar{z}_3$ is obtained:

$$\varphi_3 = 2\pi - (\pi/2 - \gamma_{49} - \gamma_{39}) = 6.167\,rad \tag{16}$$

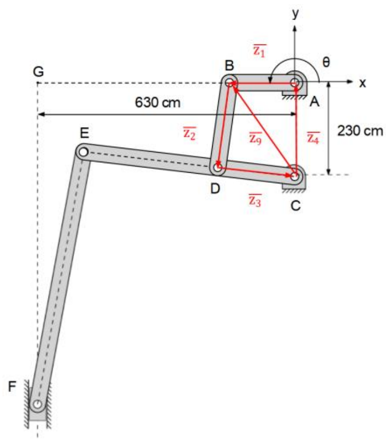


Figure 5. *Definition of the additional vector* $\overline{z}_9$ *.*

Summarization of the position analysis of the first loop-closure equation:

$$z_1 = 1.6\text{ m},\quad \varphi_1 = \pi\text{ rad}$$
$$z_2 = 2.1\text{ m},\quad \varphi_2 = 4.575\text{ rad}$$
$$z_3 = 1.9\text{ m},\quad \varphi_3 = 6.167\text{ rad}$$
$$z_4 = 2.3\text{ m},\quad \varphi_4 = \pi/\,2\text{ rad}$$

5.1.2 Position analysis of the second loop-closure equation

Figure 6 provides the second kinematic loop defined by Equation (4). Equation (4) can be projected along the two principal axes as:

$$\begin{cases} z_5\cos(\varphi_5) + z_6\cos(\varphi_6) + z_7\cos(\varphi_7) + z_8\cos(\varphi_8) + z_4\cos(\varphi_4) = 0 \\ z_5\sin(\varphi_5) + z_6\sin(\varphi_6) + z_7\sin(\varphi_7) + z_8\sin(\varphi_8) + z_4\sin(\varphi_4) = 0 \end{cases} \tag{17}$$

From the given conditions, the magnitude of vector $\overline{z}_5$, $\overline{z}_7$, and $\overline{z}_8$ are known as 6.3 m, 6.3 m, and 5.2 m respectively. The magnitude of vector $\overline{z}_6$ depends on the value assumed by the prismatic joint in F. The direction of vector $\overline{z}_5$ and $\overline{z}_6$ are known as $\pi\, rad$ and $(3/2)\pi\, rad$ respectively. The direction of vector $\overline{z}_7$ depends on the value assumed by the joint variables. The direction φ_8 of $\overline{z}_8$ is the same as φ_3 (6.167 rad). Thus, the unknowns of the second loop-closure equation are magnitude of vector $\overline{z}_6$ and the direction of $\overline{z}_7$.

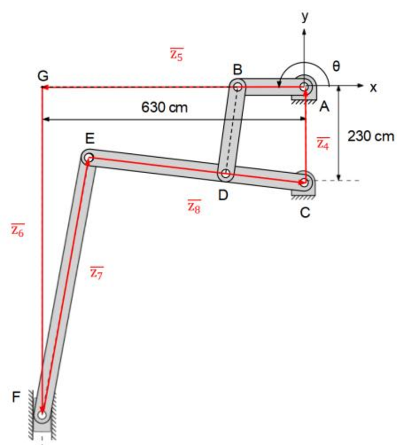


Figure 6. *Second kinematic loop defined by Equation (4).*

To calculate the unknowns, the known values can be exploited. As $\varphi_6 = (3/2)\pi$ rad , it can be known that:

$$\cos(\varphi_6) = 0, \quad \sin(\varphi_6) = -1 \tag{18}$$

Substitude Euqation (18) into Equation (17), we obtain:

$$\begin{cases} z_5\cos(\varphi_5) + z_7\cos(\varphi_7) + z_8\cos(\varphi_8) + z_4\cos(\varphi_4) = 0 \\ z_5\sin(\varphi_5) - z_6 + z_7\sin(\varphi_7) + z_8\sin(\varphi_8) + z_4\sin(\varphi_4) = 0 \end{cases} \tag{19}$$

Finally, Equation (19) can be solved as:

$$\varphi_7 = \arccos\left(\frac{-z_5\cos(\varphi_5) - z_8\cos(\varphi_8) - z_4\cos(\varphi_4)}{z_7}\right) = 1.39\,rad \tag{20}$$

$$z_6 = z_5\sin(\varphi_5) + z_7\sin(\varphi_7) + z_8\sin(\varphi_8) + z_4\sin(\varphi_4) = 7.896\,m \tag{21}$$

Summarization of the position analysis of the second loop-closure equation:

$$\begin{aligned} z_5 &= 6.3\text{ m}, & \varphi_5 &= \pi\text{ rad} \\ z_6 &= 7.896\,m, & \varphi_6 &= (3/2)\pi\text{ rad} \\ z_7 &= 6.3\text{ m}, & \varphi_7 &= 1.39\text{ rad} \\ z_8 &= 5.2\text{ m}, & \varphi_8 &= 6.167\text{ rad} \end{aligned}$$

5.2 Velocity analysis

5.2.1 Velocity analysis of the first kinematic loop

As the magnitude of vector $\overline{z}_1$, $\overline{z}_2$, $\overline{z}_3$, and $\overline{z}_4$ are known and constant, the relative velocity of these vectors is always null. From the given condition we know that the crank AB rotates in the clockwise direction with a constant speed of 20 rad/s, therefore, the angular velocity of $\overline{z}_1$ is 20 rad/s. The direction of $\overline{z}_4$ is known and constant, resulting in its null angular velocity. The direction of vectors $\overline{z}_2$ and $\overline{z}_3$ depends on the values assumed by the revolute joints, thus, the angular velocity of these vectors is different from zero.

To summarize:

$$\begin{aligned} \dot{z}_1 &= 0, & \dot{\varphi}_1 &= 20\text{ rad/s} \\ \dot{z}_2 &= 0, & \dot{\varphi}_2 &\neq 0 \\ \dot{z}_3 &= 0, & \dot{\varphi}_3 &\neq 0 \\ \dot{z}_4 &= 0, & \dot{\varphi}_4 &= 0 \end{aligned} \tag{22}$$

The first velocity loop equation is obtained by differentiating Equation (5) with respect to time:

$$\begin{cases} -z_1\sin(\varphi_1)\,\dot{\varphi}_1 - z_2\sin(\varphi_2)\,\dot{\varphi}_2 - z_3\sin(\varphi_3)\,\dot{\varphi}_3 = 0 \\ z_1\cos(\varphi_1)\,\dot{\varphi}_1 + z_2\cos(\varphi_2)\,\dot{\varphi}_2 + z_3\cos(\varphi_3)\,\dot{\varphi}_3 = 0 \end{cases} \tag{23}$$

The unknowns of Equation (23) are $\dot{\varphi}_2$ and $\dot{\varphi}_3$. We can then write Equation (23) in matrix form:

$$\underbrace{\begin{bmatrix} -\,z_2 \sin(\varphi_2) & -\,z_3 \sin(\varphi_3) \\ z_2 \cos(\varphi_2) & z_3 \cos(\varphi_3) \end{bmatrix}}_{[J]} \underbrace{\begin{Bmatrix} \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{Bmatrix}}_{\{\dot{x}\}} = -\underbrace{\begin{bmatrix} -\,z_1 \sin(\varphi_1) \\ z_1 \cos(\varphi_1) \end{bmatrix}}_{[A]} \underbrace{\{\dot{\varphi}_1\}}_{\{\dot{q}\}} \tag{24}$$

Utilizing matrix inversion, we can compute the unknowns:

$$\{\dot{x}\} =-\, [J]^{-1}[A]\{\dot{q}\} \tag{25}$$

The numerical computation was done in Matlab and the results are:

$$\begin{Bmatrix} \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{Bmatrix} = \begin{Bmatrix} 1.763 \text{ rad/s} \\ -16.688 \text{ rad/s} \end{Bmatrix} \tag{26}$$

Note that Equation (25) can be rewritten as:

$$\begin{Bmatrix} \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{Bmatrix} = \begin{bmatrix} K_{\varphi_2,\varphi_1} \\ K_{\varphi_3,\varphi_1} \end{bmatrix} \{\dot{\varphi}_1\}, \tag{27}$$

where K_{φ_2,φ_1} and K_{φ_3,φ_1} are the first order kinematic coefficients which relate the unknown velocities to the velocity of the generalized coordinate.

Summarization of the velocity analysis of the first kinematic loop:

$$\begin{aligned} \dot{z}_1 &= 0 \text{ m/s}, & \dot{\varphi}_1 &= 20 \text{ rad/s} \\ \dot{z}_2 &= 0 \text{ m/s}, & \dot{\varphi}_2 &= 1.763 \text{ rad/s} \\ \dot{z}_3 &= 0 \text{ m/s}, & \dot{\varphi}_3 &= -\,16.688 \text{ rad/s} \\ \dot{z}_4 &= 0 \text{ m/s}, & \dot{\varphi}_4 &= 0 \text{ rad/s} \end{aligned}$$

5.2.2 Velocity analysis of the second kinematic loop

As the magnitude of vector $\overline{z}_5$, $\overline{z}_7$, and $\overline{z}_8$ are known and constant, the relative velocity of these vectors is always null. The magnitude of vector $\overline{z}_6$ depends on the value assumed by the prismatic joint in F, thus, its relative velocity is different from zero. The direction of $\overline{z}_5$ and $\overline{z}_6$ are known and constant, resulting in their null angular velocities. The direction of vector $\overline{z}_7$ depends on the value assumed by the joint variables, therefore its angular velocity is different from zero. The direction φ_8 of $\overline{z}_8$ is the same as φ_3, resulting in their angular velocities are the same (-16.688 rad/s).

To summarize:

$$\begin{aligned} \dot{z}_5 &= 0, & \dot{\varphi}_5 &= 0 \\ \dot{z}_6 &\neq 0, & \dot{\varphi}_6 &= 0 \\ \dot{z}_7 &= 0, & \dot{\varphi}_7 &\neq 0 \\ \dot{z}_8 &= 0, & \dot{\varphi}_8 &=-\, 16.688 \text{ rad/s} \end{aligned} \tag{28}$$

The second velocity loop equation is obtained by differentiating Equation (17) with respect to time:

$$\begin{cases} \dot{z}_6 \cos(\varphi_6) - z_7 \sin(\varphi_7)\,\dot{\varphi}_7 - z_8 \sin(\varphi_8)\,\dot{\varphi}_8 = 0 \\ \dot{z}_6 \sin(\varphi_6) + z_7 \cos(\varphi_7)\,\dot{\varphi}_7 + z_8 \cos(\varphi_8)\,\dot{\varphi}_8 = 0 \end{cases} \tag{29}$$

The unknowns of Equation (29) are $\dot{z}_6$ and $\dot{\varphi}_7$. We can then write Equation (29) in matrix form:

$$\underbrace{\begin{bmatrix} \cos(\varphi_6) & -z_7 \sin(\varphi_7) \\ \sin(\varphi_6) & z_7 \cos(\varphi_7) \end{bmatrix}}_{[J]} \underbrace{\begin{Bmatrix} \dot{z}_6 \\ \dot{\varphi}_7 \end{Bmatrix}}_{\{\dot{x}\}} = -\underbrace{\begin{bmatrix} -z_8 \sin(\varphi_8) \\ z_8 \cos(\varphi_8) \end{bmatrix}}_{[A]} \underbrace{\{\dot{\varphi}_8\}}_{\{\dot{q}\}} \tag{30}$$

Utilizing matrix inversion, we can compute the unknowns:

$$\{\dot{x}\} =- [J]^{-1}[A]\{\dot{q}\} \tag{31}$$

The numerical computation was done in Matlab and the results are:

$$\begin{Bmatrix} \dot{z}_6 \\ \dot{\varphi}_7 \end{Bmatrix} = \begin{Bmatrix} -88.031 \text{ m/s} \\ -1.62 \text{ rad/s} \end{Bmatrix} \tag{32}$$

Note that Equation (31) can be rewritten in terms of first-order kinematic coefficients:

$$\begin{Bmatrix} \dot{z}_6 \\ \dot{\varphi}_7 \end{Bmatrix} = \begin{bmatrix} K_{z_6,\varphi_8} \\ K_{\varphi_7,\varphi_8} \end{bmatrix} \{\dot{\varphi}_8\} \tag{33}$$

Summarization of the velocity analysis of the second kinematic loop:

$$\begin{aligned} &\dot{z}_5 = 0 \text{ m/s}, \quad \dot{\varphi}_5 = 0 \text{ rad/s} \\ &\dot{z}_6 = -88.031 \text{ m/s}, \quad \dot{\varphi}_6 = 0 \text{ rad/s} \\ &\dot{z}_7 = 0 \text{ m/s}, \quad \dot{\varphi}_7 = -1.62 \text{ rad/s} \\ &\dot{z}_8 = 0 \text{ m/s}, \quad \dot{\varphi}_8 = -16.688 \text{ rad/s} \end{aligned}$$

5.3 Acceleration analysis

5.3.1 Acceleration analysis of the first kinematic loop

From the observations and analysis previously made on the magnitude and direction of the position-difference vectors, it can be known:

$$\begin{aligned} &\ddot{z}_1 = 0, \ \ddot{\varphi}_1 = 0 \\ &\ddot{z}_2 = 0, \ \ddot{\varphi}_2 \neq 0 \\ &\ddot{z}_3 = 0, \ \ddot{\varphi}_3 \neq 0 \\ &\ddot{z}_4 = 0, \ \ddot{\varphi}_4 = 0 \end{aligned} \tag{34}$$

By differentiating Equation (23) with respect to time, we obtain the first acceleration loop equation:

$$\begin{cases} -z_1\cos(\varphi_1)\,\dot{\varphi}_1^2 - z_2\sin(\varphi_2)\,\ddot{\varphi}_2 - z_2\cos(\varphi_2)\,\dot{\varphi}_2^2 - z_3\sin(\varphi_3)\,\ddot{\varphi}_3 - z_3\cos(\varphi_3)\,\dot{\varphi}_3^2 = 0 \\ -z_1\sin(\varphi_1)\,\dot{\varphi}_1^2 + z_2\cos(\varphi_2)\,\ddot{\varphi}_2 - z_2\sin(\varphi_2)\,\dot{\varphi}_2^2 + z_3\cos(\varphi_3)\,\ddot{\varphi}_3 - z_3\sin(\varphi_3)\,\dot{\varphi}_3^2 = 0 \end{cases} \tag{35}$$

The unknowns of Equation (35) are the angular accelerations $\ddot{\varphi}_2$ and $\ddot{\varphi}_3$. Equation (35) can be written in matrix form:

$$\begin{bmatrix} -z_2\sin(\varphi_2) & -z_3\sin(\varphi_3) \\ z_2\cos(\varphi_2) & z_3\cos(\varphi_3) \end{bmatrix} \begin{Bmatrix} \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{Bmatrix} = -\begin{Bmatrix} -z_1\cos(\varphi_1)\,\dot{\varphi}_1^2 - z_2\cos(\varphi_2)\,\dot{\varphi}_2^2 - z_3\cos(\varphi_3)\,\dot{\varphi}_3^2 \\ -z_1\sin(\varphi_1)\,\dot{\varphi}_1^2 - z_2\sin(\varphi_2)\,\dot{\varphi}_2^2 - z_3\sin(\varphi_3)\,\dot{\varphi}_3^2 \end{Bmatrix} \tag{36}$$

We can compute the unknowns by exploiting matrix inversion. The numerical computation was done in Matlab and the results are:

$$\begin{Bmatrix} \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{Bmatrix} = \begin{Bmatrix} -50.841\ \mathrm{rad/s^2} \\ -43.587\ \mathrm{rad/s^2} \end{Bmatrix} \tag{37}$$

That means the angular acceleration of link BD is $-50.841\ \mathrm{rad/s^2}$. The angular acceleration of link CE is $-43.587\ \mathrm{rad/s^2}$, as vector $\overline{z}_3$ and $\overline{z}_8$ has the same direction and angular velocity.

Summarization of the acceleration analysis of the first kinematic loop:

$$\begin{gathered} \ddot{z}_1 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_1 = 0\ \mathrm{rad/s^2} \\ \ddot{z}_2 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_2 = -50.841\ \mathrm{rad/s^2} \\ \ddot{z}_3 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_3 = -43.587\ \mathrm{rad/s^2} \\ \ddot{z}_4 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_4 = 0\ \mathrm{rad/s^2} \end{gathered}$$

5.3.2 Acceleration analysis of the second kinematic loop

From the observations and analysis previously made on the magnitude and direction of the position-difference vectors, it can be known:

$$\begin{gathered} \ddot{z}_5 = 0,\ \ddot{\varphi}_5 = 0 \\ \ddot{z}_6 \neq 0,\ \ddot{\varphi}_6 = 0 \\ \ddot{z}_7 = 0,\ \ddot{\varphi}_7 \neq 0 \\ \ddot{z}_8 = 0,\ \ddot{\varphi}_8 = \ddot{\varphi}_3 = -43.587\ \mathrm{rad/s^2} \end{gathered} \tag{38}$$

By differentiating Equation (29) with respect to time, we obtain the second acceleration loop equation:

$$\begin{cases} \ddot{z}_6\cos(\varphi_6) - z_7\sin(\varphi_7)\,\ddot{\varphi}_7 - z_7\cos(\varphi_7)\,\dot{\varphi}_7^2 - z_8\sin(\varphi_8)\,\ddot{\varphi}_8 - z_8\cos(\varphi_8)\,\dot{\varphi}_8^2 = 0 \\ \ddot{z}_6\sin(\varphi_6) + z_7\cos(\varphi_7)\,\ddot{\varphi}_7 - z_7\sin(\varphi_7)\,\dot{\varphi}_7^2 + z_8\cos(\varphi_8)\,\ddot{\varphi}_8 - z_8\sin(\varphi_8)\,\dot{\varphi}_8^2 = 0 \end{cases} \tag{39}$$

The unknowns of Equation (39) are $\ddot{z}_6$ and $\ddot{\varphi}_7$. Equation (39) can be written in matrix form:

$$\begin{bmatrix}\cos(\varphi_6) & -z_7\sin(\varphi_7)\\ \sin(\varphi_6) & z_7\cos(\varphi_7)\end{bmatrix}\begin{Bmatrix}\ddot{z}_6\\ \ddot{\varphi}_7\end{Bmatrix} = -\begin{Bmatrix}-z_7\cos(\varphi_7)\,\dot{\varphi}_7^2 - z_8\sin(\varphi_8)\,\ddot{\varphi}_8 - z_8\cos(\varphi_8)\,\dot{\varphi}_8^2\\ -z_7\sin(\varphi_7)\,\dot{\varphi}_7^2 + z_8\cos(\varphi_8)\,\ddot{\varphi}_8 - z_8\sin(\varphi_8)\,\dot{\varphi}_8^2\end{Bmatrix} \quad (40)$$

We can compute the unknowns by exploiting matrix inversion. The numerical computation was done in Matlab and the results are:

$$\begin{Bmatrix}\ddot{z}_6\\ \ddot{\varphi}_7\end{Bmatrix} = \begin{Bmatrix}-342.675\ \mathrm{m/s^2}\\ -236.818\ \mathrm{rad/s^2}\end{Bmatrix} \quad (41)$$

Summarization of the acceleration analysis of the second kinematic loop:

$$\ddot{z}_5 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_5 = 0\ \mathrm{rad/s^2}$$
$$\ddot{z}_6 =- 342.675\ \mathrm{m/s^2},\ \ddot{\varphi}_6 = 0\ \mathrm{rad/s^2}$$
$$\ddot{z}_7 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_7 = -236.818\ \mathrm{rad/s^2}$$
$$\ddot{z}_8 = 0\ \mathrm{m/s^2},\ \ddot{\varphi}_8 = -43.587\ \mathrm{rad/s^2}$$

6. Calculate the velocity and acceleration of the slider in F

6.1 Velocity of the slider in F

Vector $\overline{z}_6$ is defined as:

$$\overline{z}_6 = \overline{F-G} = \begin{Bmatrix}x_F - x_G\\ y_F - y_G\end{Bmatrix} = \begin{Bmatrix}z_6\cos(\varphi_6)\\ z_6\sin(\varphi_6)\end{Bmatrix} = \begin{Bmatrix}0\\ -z_6\end{Bmatrix}, \quad (42)$$

where $\varphi_6 = (3/2)\pi\ \mathrm{rad},\ \cos(\varphi_6) = 0,\ \sin(\varphi_6) =- 1$, as known from Equation (18).

Then by differentiating Equation (42) with respect to time, we obtain:

$$\begin{Bmatrix}\dot{x}_F - \dot{x}_G\\ \dot{y}_F - \dot{y}_G\end{Bmatrix} = \begin{Bmatrix}0\\ -\dot{z}_6\end{Bmatrix}, \quad (43)$$

where $\dot{x}_G$ and $\dot{y}_G$ both are equal to zero as G is a fixed point.

Finally, we get the linear velocity of the slider in F:

$$\begin{Bmatrix}\dot{x}_F - 0\\ \dot{y}_F - 0\end{Bmatrix} = \begin{Bmatrix}\dot{x}_F\\ \dot{y}_F\end{Bmatrix} = \begin{Bmatrix}0\\ -\dot{z}_6\end{Bmatrix} = \begin{Bmatrix}0\ m/s\\ 88.031\ m/s\end{Bmatrix} \quad (44)$$

6.2 Acceleration of the slider in F

The linear acceleration of the slider in F can be found by differentiating Equation (43) with respect to time:

$$\begin{Bmatrix} \ddot{x}_F - \ddot{x}_G \\ \ddot{y}_F - \ddot{y}_G \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\ddot{z}_6 \end{Bmatrix} , \tag{45}$$

where $\ddot{x}_G$ and $\ddot{y}_G$ both are equal to zero as G is a fixed point.

We then get the linear acceleration of the slider in F:

$$\begin{Bmatrix} \ddot{x}_F - 0 \\ \ddot{y}_F - 0 \end{Bmatrix} = \begin{Bmatrix} \ddot{x}_F \\ \ddot{y}_F \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\ddot{z}_6 \end{Bmatrix} = \begin{Bmatrix} 0 \text{ m/s}^2 \\ 342.675 \text{ m/s}^2 \end{Bmatrix} \tag{46}$$

7. Calculate the first-order kinematic coefficient which relates the velocity of the slider in F and the velocity of the crank AB

As the linkage has only one degree of freedom, and we already obtained the velocity of the slider in F, the first-order kinematic coefficient K_{y_F,φ_1} can be computed as:

$$K_{y_F,\varphi_1} = \frac{\dot{y}_F}{\dot{\varphi}_1} = \frac{88.031}{-20} = -4.402 \tag{47 - 1}$$

Note: The direction of the velocity of vector $\overline{z}_6$ is opposite as the direction of the velocity of the slider in F, therefore, if we would like to calculate K_{z_6,φ_1}, the sign of the result is opposite:

$$K_{z_6,\varphi_1} = \frac{\dot{z}_6}{\dot{\varphi}_1} = \frac{-88.031}{-20} = 4.402 \tag{47 - 2}$$

8. Create a Simscape Multibody™ model of this linkage (moving with a constant speed). Describe briefly the model and what does each block represent

Figure 7 provides the model of the linkage created using Simscape Multibody™. The model is composed by 5 bodies (corresponding to link AB, BD, CE, EF and the slider in F), 6 revolute joints and one prismatic joint.

Block Rigid Transform 1

To connect link BD and CE as in the description, a translation of -70 cm along the x axis should be made between Port R of link CE and the revolute joint in D, as CE = 520 cm and CD = 190 cm, the distance between Port R and point D is 520/2 – 190 = 70. The minus sign of the translation is because in Simscape, when CE is in the rest position (when the position of the revolute joint in C is 0), to translate Port R to point D, the translation should be made along the -x axis. The translation was done by the block Rigid Transform 1.

Block Rigid Transform 2

Between the world frame and the revolute joint in C, a translation of -230 cm along the y axis of the world frame was made by the block Rigid Transform 2.

Block Rigid Transform 3

The z axis of port R in the slider F is not aligned with the direction of the prismatic pair. To address this, the block Rigid Transform 3 was inserted between port R of the slider and the base port of the Prismatic Joint 1. This block applies a - 90° rotation along the y axis, aligning the z axis with the longitudinal direction of the slider.

Block Rigid Transform 4

The slider F operates along a vertical axis that intersects point G, with coordinates (x_G, y_G) = (-630 cm, 0 cm). According to the analytical solution, the initial position of the slider is at coordinates (-630 cm, -790 cm). To correctly align the system, the block Rigid Transform 4 was inserted between the world frame and the follower port of the Prismatic Joint 1. It first applies a 90° rotation around the x axis, followed by a 90° rotation around the z axis. Additionally, it translates the system by -630 cm along the x axis and -790 cm along the y axis.

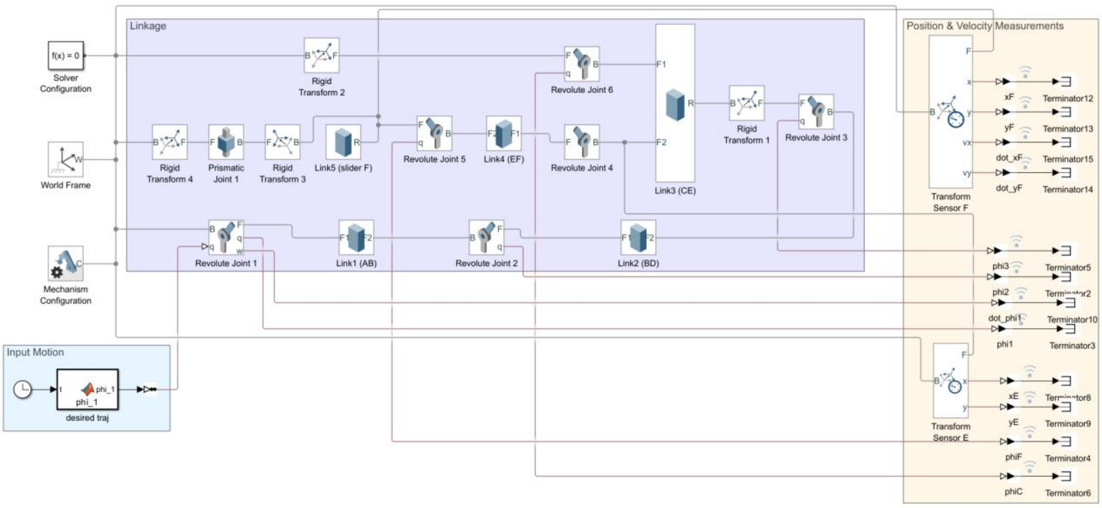


Figure 7. *The created model of the linkage using Simscape Multibody™.*

Visualization and Animation

To animate the linkage, we can apply a constant velocity as the input motion to the first revolute joint, which serves as the generalized coordinate. This constant velocity motion is defined by the equation:

$$\varphi_1 = \varphi_{1_0} + kt \quad , \tag{48}$$

where φ_{1_0} represents the initial displacement (φ_{1_0} = 180°), k denotes the velocity, and t stands for time. The input motion is implemented using a MATLAB Function within the designated area named "Input Motion" . In addition, an initial state target for the first revolute joint is specified as 180° .

Figure 8 demonstrates the visualization of the simulation from the Mechanics Explorer. A demo video of the movement of the simulated linkage is provided in the link: https://youtu.be/NFoRHApSNnc . It can be seen that the linkage has been correctly modeled and its movement coincides with the requirements, i.e. the crank AB rotates in the clockwise direction with a constant speed of 20 rad/s and the slider in F moves along the y axis of the absolute reference frame.

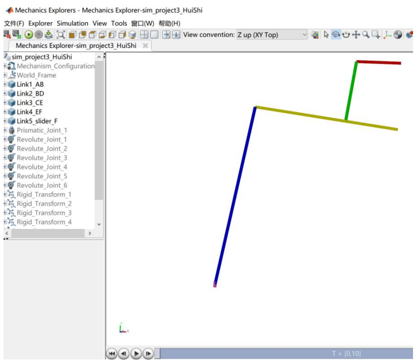
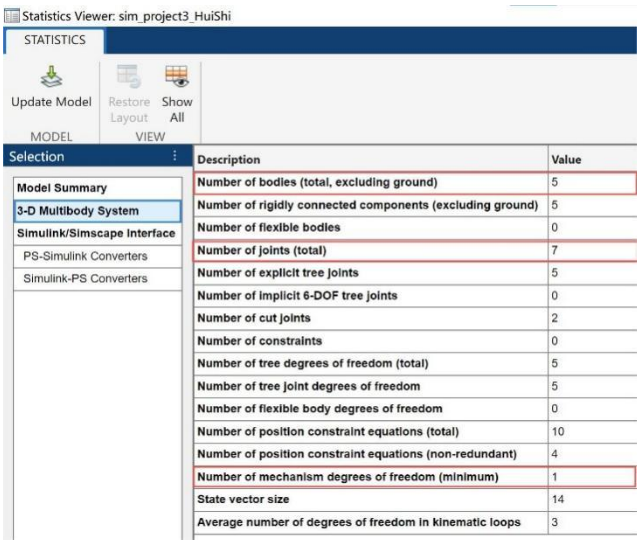


Figure 8. *Visualization of the simulation in the Mechanics Explorer.*

9. Check the correctness of the model (number of degrees of freedom, number of links and kinematic pairs)

The correctness of the model can be verified using the "Statistics Viewer." For this linkage, as shown in Figure 9, the Simscape™ simulation correctly predicts the number of degrees of freedom N = 1, the number of bodies (links) is 5 when excluding the ground link, and the number of joints (kinematic pairs) is 7.



Description	Value
Number of bodies (total, excluding ground)	5
Number of rigidly connected components (excluding ground)	5
Number of flexible bodies	0
Number of joints (total)	7
Number of explicit tree joints	5
Number of implicit 6-DOF tree joints	0
Number of cut joints	2
Number of constraints	0
Number of tree degrees of freedom (total)	5
Number of tree joint degrees of freedom	5
Number of flexible body degrees of freedom	0
Number of position constraint equations (total)	10
Number of position constraint equations (non-redundant)	4
Number of mechanism degrees of freedom (minimum)	1
State vector size	14
Average number of degrees of freedom in kinematic loops	3

Figure 9. *Statistics Viewer of the Simscape™ simulation of the linkage.*

10. Check the results of the position analysis calculated analytically and the results provided by Simscape Multibody™

The unknowns of the position analysis are the directions φ_2 and φ_3 of the vectors $\overline{z}_2$ and $\overline{z}_3$ respectively, the magnitude z_6 of vector $\overline{z}_6$ and the direction φ_7 of vector $\overline{z}_7$. To cross check the results of the analytical analysis and the results of the simulation, the "Sensing" functionality of the joints was used. Figure 10 provides the plot of the data exported from the Data Inspector, demonstrating the trend of φ_1^*, φ_2^*, φ_3^*, φ_F^* and position of F over time. The $*$ is for distinguishing the angle values from Simscape and from the analytical solution. Note that we would like to compare the values with the analytical solution at the initial configuration, i.e. at time t = 0 and φ_1 = 180° = π rad .

Check for φ_2

In Figure 10 (a), The value of φ_2^* at t = 0 is 1.4336 which is measured with respect to the first link AB. To convert the value to be in the absolute reference frame, the value of φ_1^* should be added to it. Thus, the final value of φ_2 = 3.1416 + 1.4336 = 4.575 rad which coincides with the result of the analytical solution.

Check for φ_3

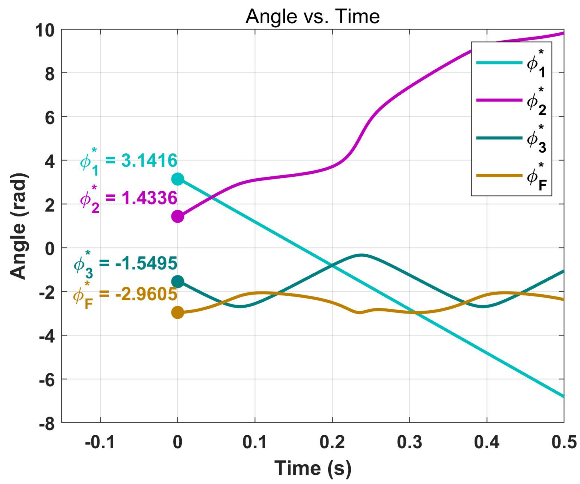
The value of φ_3^* is - 1.5495 which is measured with respect to the second link BD. To cross check it with the analytical solution, there are some math to do. The vector $\overline{z}_3$ is a vector from point D pointing to point C, as shown in Figure 3. Therefore, the direction of $\overline{z}_3$ should be an addition of the values of φ_1^*, φ_2^* and $(\pi - \varphi_3^*)$ which is 6.167 rad (3.1416 + 1.4336 + (3.1416 - 1.5495)).

Check for z_6

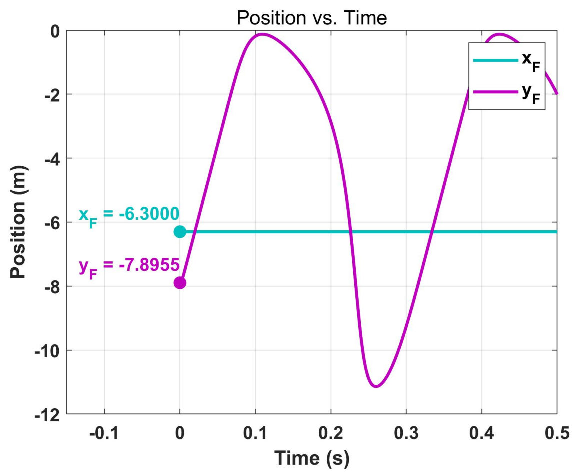
It can be seen from Figure 10 (b), the position of the slider F at time t = 0 is(- 6.3, - 7.8955) which is the same as the analytical calculation z5 = 6.3, z6 = 7.896. Note that z5 and z6 are magnitudes, these values are always positive. The coordinates of point F (represented by frame R of link 5) was calculated by the block Transform Sensor F. This block calculates the coordinates of the origin of the frame R of link 5 relative to the world frame.

Check for φ_7

To cross check the analytical solution of φ_7 with the simulation, first we got the value of φ_F^* (angle of the revolute joint in F) is - 2.96 which is measured with respect to the link EF, then there are some math to do. The vector $\overline{z}_7$ is a vector from point F pointing to point E, as shown in Figure 3. Therefore, the direction φ_7 of $\overline{z}_7$ can be calculated as $\pi/2 - (\pi + \varphi_F^*)$, obtaining 1.389 ($\pi/2 - (\pi - 2.96$)) which is the same as the outcome of the analytical solution.



(a) Trend of φ_1^*, φ_2^*, φ_3^* and φ_F^* over time.



(b) Trend of position of F over time.

Figure 10. *Trend of* φ_1^*, φ_2^*, φ_3^*, φ_F^* *and position of F over time.*

11. Create a graph showing the trend over time of the angle θ and of the position (x, y) of point E

To determine the coordinates of point E (represented by frame F2 of link CE), the block Transform Sensor E is utilized. This block calculates the coordinates of the origin of the frame F2 of link CE relative to the world frame. Figure 11 shows the created graph of the trend over time of the angle φ_1 and of the position of point E. The data used for the plotting was exported from the Data Inspector.

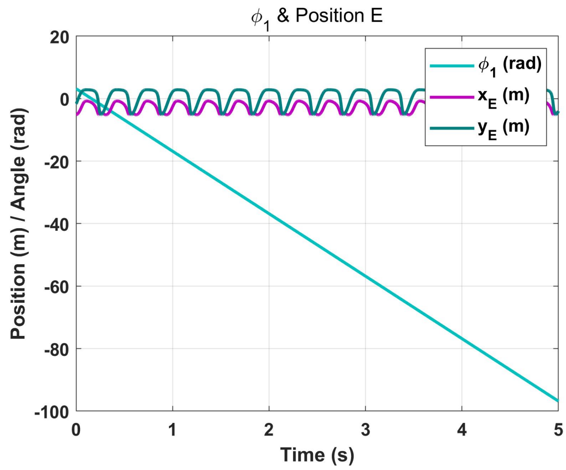


Figure 11. *The trend over time of the angle $\varphi_1$and of the position of point E*

12. Create a graph showing the trend over time of the velocity of the crank AB and the resulting velocity of the slider F

To determine the velocity of the slider in F, the block Transform Sensor F described in section 10 is utilized. This block also provides the velocity of point F. Figure 12 shows the created graph of the trend over time of the velocity of the crank AB and of the resulting velocity of the slider F. The data used for the plotting was exported from the Data Inspector.

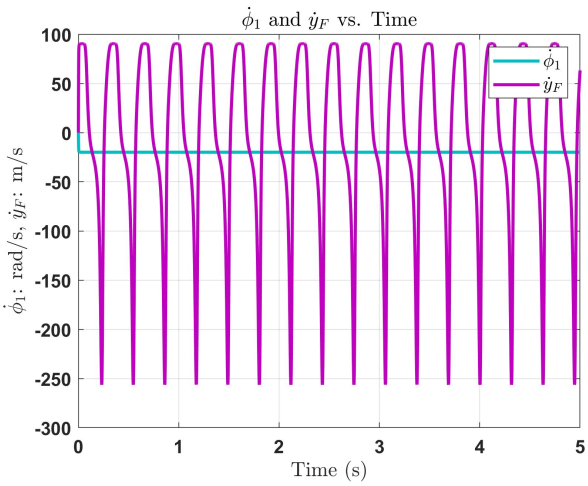


Figure 12. *Trend over time of $\dot{\varphi}_1$ and of the resulting velocity of the slider F.*