••P62 A ball of mass 0.05 kg moves with a velocity $\langle 17,0,0\rangle$ m/s. It strikes a ball of mass 0.1 kg that is initially at rest. After the collision, the heavier ball moves with a velocity of $\langle 3,3,0\rangle$ m/s. **(a)** What is the velocity of the lighter ball after impact? **(b)** What is the impulse delivered to the 0.05 kg ball by the heavier ball? **(c)** If the time of contact between the balls is 0.03 s, what is the force exerted by the heavier ball on the lighter ball?

••P63 Suppose that all the people of the Earth go to the North Pole and, on a signal, all jump straight up. Estimate the recoil speed of the Earth. The mass of the Earth is 6×10^{24} kg, and there are about 6 billion people (6×10^9).

COMPUTATIONAL PROBLEMS

More detailed and extended versions of some of these computational modeling problems may be found in the lab activities included in the *Matter & Interactions*, Fourth Edition, resources for instructors.

••P64 To use an `arrow` object to visualize a force, it is usually necessary to scale the length of the arrow in order to make it fit on the screen with the objects exerting and experiencing the force. Watch VPython Instructional Video 5: Scalefactors, at vpython.org/video05.html to learn how to do this.

Now, write a VPython program to do the following: **(a)** Create a sphere representing a planet at the origin, with radius 6.4×10^6 m. The mass of this planet is 6×10^{24} kg. **(b)** Create five spheres representing 5 spacecraft, at locations $\langle -13 \times 10^7, 6.5 \times 10^7, 0\rangle$ m, $\langle -6.5 \times 10^7, 6.5 \times 10^7, 0\rangle$ m, $\langle 0, 6.5 \times 10^7, 0\rangle$ m, $\langle 6.5 \times 10^7, 6.5 \times 10^7, 0\rangle$ m, and $\langle 13 \times 10^7, 6.5 \times 10^7, 0\rangle$ m. You will have to exaggerate the radius of each spacecraft to make it visible; try 3×10^6 m. The mass of each spacecraft is 15×10^3 kg. **(c)** For each spacecraft, have your program calculate the gravitational force exerted on the spacecraft by the planet, and visualize it with an arrow whose tail is at the center of the spacecraft. Use the same scalefactor for all arrows. Check that the display your program produces makes physical sense. **(d)** For each spacecraft, have your program calculate the gravitational force exerted on the planet by the spacecraft, and visualize it with an arrow whose tail is at the center of the planet. Use the same scalefactor as you used in the previous part. Check that the display your program produces makes physical sense.

••P65 Write a computer program to model the motion of a spacecraft of mass 15000 kg that is launched from a location 10 Earth radii from the center of the Earth (Figure 3.67). (Data for the Earth are given on the inside back cover of the textbook. Start with a Δt of 60 s and an initial speed of 2×10^3 m/s in a direction perpendicular to the line between the spacecraft and the Earth.) **(a)** Vary the initial speed (but not the direction), and have the spacecraft leave a trail. What trajectories can you produce? **(b)** Find an initial speed that produces an elliptical orbit. **(c)** For the elliptical orbit, display arrows indicating the directions of the momentum of the spacecraft and the net force on the spacecraft as it moves. **(d)** Find an initial speed that produces a circular orbit. **(e)** Experiment by increasing and decreasing the time step Δt. What is the largest value of Δt that gives enough accuracy to produce a closed circular orbit?

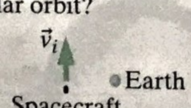

Figure 3.67

••P66 Extend the program you wrote in the previous problem by including the effect of the Moon, placing the Moon on the opposite side of the Earth from the spacecraft's initial location (Figure 3.68). (Relevant data are given on the inside back cover of the textbook.) To simplify the model, keep both the Earth and the Moon fixed. (This is called a "restricted three-body problem.") **(a)** Find an initial speed for the spacecraft that results in an orbit around both Earth and Moon. **(b)** By adjusting the initial speed of the spacecraft, can you produce a figure-eight trajectory? **(c)** What other interesting trajectories can you produce by varying the initial speed? (Small variations may produce large effects.)

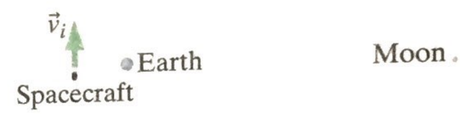

Figure 3.68

•••P67 Once you have completed the previous problem, allow the Earth and Moon to move. Use what you know about the period of their orbits to determine appropriate initial velocities for the Earth and the Moon.

••P68 Model the motion of Mars around the Sun. You can find data for masses and distances online. Use the known period of Mars's orbit to determine an approximate initial velocity. Display a trail, so you can see the shape of the orbit. Determine an appropriate value for Δt. Display arrows representing the momentum of Mars and the net force on Mars. Determine the period of the orbit of the planet in your model. How can you produce noncircular orbits?

••P69 About a half of the visible "stars" are actually systems consisting of two stars orbiting each other, called "binary stars." In your computer model of a planet and Sun (Problem P68), replace the planet with a star whose mass is half the mass of our Sun, and take into account the gravitational effects that the second star has on the Sun. **(a)** Give the second star the speed of the actual Earth, and give the Sun zero initial momentum. What happens? Try a variety of other initial conditions. What kinds of orbits do you find? **(b)** Choose initial conditions so that the total momentum of the two-star system is zero, but the stars are not headed directly at each other. What is special about the motion you observe in this case?

•••P70 Modify your orbit computation to use a different force, such as a force that is proportional to $1/r$ or $1/r^3$, or a constant force, or a force proportional to r (this represents the force of a spring whose relaxed length is nearly zero). How do orbits with these forces differ from the circles and ellipses that result from a $1/r^2$ force? If you want to keep the magnitude of the force roughly the same as before, you will need to adjust the force constant G.

••P71 The first U.S. spacecraft to photograph the Moon close up was the unmanned *Ranger 7* photographic mission in 1964. The spacecraft, shown in Figure 3.69, contained television cameras that transmitted close-up pictures of the Moon back to Earth as the spacecraft approached the Moon. The spacecraft did not have retro-rockets to slow itself down, and it eventually simply crashed onto the Moon's surface, transmitting its last photos immediately before impact.

Figure 3.69 (Image courtesy of NASA)

Figure 3.70 is the first image of the Moon taken by a U.S. spacecraft, *Ranger 7*, on July 31, 1964, about 17 minutes before impact on the lunar surface. To find out more about the actual *Ranger* lunar missions, see http://nssdc.gsfc.nasa.gov/planetary/lunar/ranger.html.

Figure 3.70 (Image courtesy of NASA)

Create a computational model of the *Ranger 7* mission. Start your model when the spacecraft, whose mass is = 173 kg, has been brought to 50 km above the Earth's surface (5×10^4 m) by several stages of large rockets, and has a speed of around 1×10^4 m/s. All fuel has been used up, and the spacecraft now coasts toward the Moon. Data for the Earth and Moon may be found on the inside back cover of the textbook.

For this simple model, keep the Earth and Moon fixed in space during the mission, and ignore the effect of the Sun. **(a)** Compute and display the path of the spacecraft, having it leave a trail. **(b)** Determine experimentally the approximate *minimum* initial speed needed to reach the Moon, to three significant figures (this is the speed that the spacecraft obtained from the multistage rocket, at the time of release above the Earth's atmosphere). **(c)** Check your result by decreasing the time step size until your results do not change significantly. **(d)** Use a launch speed 10% larger than the approximate minimum value found in part (b). How long does it take to go to the Moon, in hours or days? **(e)** What is the "impact speed" of the spacecraft (its speed just before it hits the Moon's surface)? Make sure that your spacecraft crashes on the surface of the Moon, and not at its center!

•••**P72** In the *Ranger 7* model, take into account the motion of the Moon around the Earth and the motion of the Earth around the Sun. In addition, the Sun and other planets exert gravitational forces on the spacecraft.

•••**P73** In the *Ranger 7* analysis (the Moon voyage), you used a simplified model in which you neglected among other things the effect of Venus. An important aspect of physical modeling is making estimates of how large the neglected effects might be. Venus and the Earth have similar size and mass. At its closest approach to the Earth, Venus is about 40 million km away (4×10^{10} m). In the real world, Venus would attract the Earth and the Moon as well as the spacecraft, but to get an idea of the size of the effects, imagine that the Earth, the Moon, and Venus are all fixed in position. (See Figure 3.71.)

Figure 3.71

If we take Venus into account, make a rough estimate of whether the spacecraft will miss the Moon entirely. How large a sideways deflection of the crash site will there be? Explain your reasoning and approximations. If you expect a significant effect, modify your program to include the effects of Venus.

•••**P74** Create a computational model of the motion of a three-body gravitational system, with all three objects free to move, and plot the trajectories, leaving trails behind the objects. Calculate all of the forces before using these forces to update the momenta and positions of the objects. Otherwise the calculations of gravitational forces would mix positions corresponding to different times.

Try different initial positions and initial momenta. Find at least one set of initial conditions that produces a long-lasting orbit, one set of initial conditions that results in a collision with a massive object, and one set of initial conditions that allows one of the objects to wander off without returning. Report the masses and initial conditions that you used.

ANSWERS TO CHECKPOINTS

1 (a) 4×10^{25} N, **(b)** 8×10^{25} N, **(c)** 4.4×10^{24} N
2 (a) (1) $\langle -1,0,0 \rangle$, (2) $\langle 1,0,0 \rangle$, (3) $\langle 1,0,0 \rangle$, **(b)** (1) $\langle 1,0,0 \rangle$, (2) $\langle -1,0,0 \rangle$, (3) $\langle -1,0,0 \rangle$
3 (a) 0.285 N/kg, **(b)** 19.9 N, **(c)** 0.029 as much (the astronaut's weight on the asteroid is about 3% of what it would be on Earth)
4 (a) 588 N, **(b)** 588 N
5 The magnitude of $\vec{p}_{\text{future}}$ will be less than the magnitude of $\vec{p}_{\text{now}}$ (so speed decreases) because part of $\vec{F}_{\text{net,now}}$ is opposite to $\vec{p}_{\text{now}}$. The direction of $\vec{p}_{\text{future}}$ will also be different (it will have a larger $+y$ component).
6 (a) 2.02×10^{-9} N, **(b)** 2.02×10^{-9} N
7 (a) Si: 14 p and 14 n, **(b)** Sn: 50 p and 69 n, **(c)** Au: 79 p and 118 n, **(d)** Th: 90 p and 142 n, **(e)** The farther you go in the periodic table, the more "excess" neutrons with their strong interactions are needed to offset the proton repulsions.
8 $\langle 0.57, 5.43, 0 \rangle$ kg·m/s
9 (a) $\langle 0.04, 0.04, -0.1 \rangle$ kg·m/s, **(b)** $\langle 0.04, 0.04, -0.1 \rangle$ kg·m/s, **(c)** $\langle 1, 1, -2.5 \rangle$ m/s
10 6, 6