Skyrme Neutron Stars

PhD Defence by:

Miguel Huidobro García

Supervisor: Christoph Adam

Doctoral Programme in Nuclear and Particle Physics, USC (IGFAE)

January 25, 2024

IGFAE
Instituto Galego de Física de Altas Enerxías

USC
UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

XUNTA DE GALICIA

Outline

Overview and Motivation

of the physical problem

What is a Neutron Star (NS)?

- At the end of the stellar evolution $\rightarrow$ **core-collapse**.
- If $M_{\text{star}} \gtrsim 8M_{\odot} \Rightarrow p + e^{-} \rightarrow n + \nu_e \Rightarrow$ **supernova**.

What is a Neutron Star (NS)?

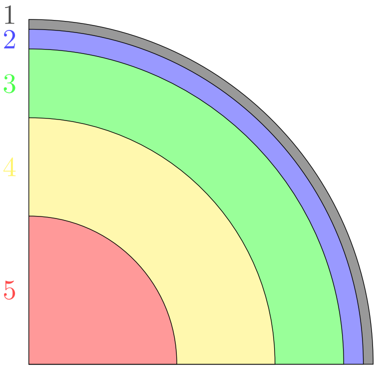
- At the end of the stellar evolution $\rightarrow$ **core-collapse**.
- If $M_{\text{star}} \gtrsim 8M_{\odot} \Rightarrow p + e^{-} \rightarrow n + \nu_e \Rightarrow$ **supernova**.
- NS have the most extreme properties in the Universe:

$$M \sim 2M_{\odot}, \;\; R \sim 10 \text{ km} \longrightarrow C = \frac{GM}{R} \sim 0.2 - 0.3,$$

$$g_{\text{NS}} \sim 10^{10} g_{\oplus}, \;\; \Omega \sim 0.1\Omega_K, \;\; \vec{\mathbf{B}} \lesssim 10^{15} G, \;\; T \lesssim 10^{13} K, \;\; \frac{N_p}{N_n} \lesssim 0.1$$

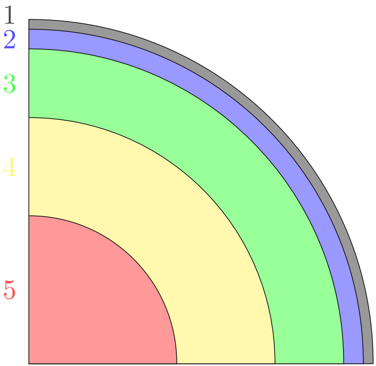
- NS are the only scenarios where the four fundamental interactions are simultaneously important.
  - Source of **GW**.
  - Large amount of neutrino emissions.
  - Production of heavy neutron-rich nuclei (kilonova).
  - Tests for **E**xtended **T**heories of **G**ravity.

Internal composition of a NS



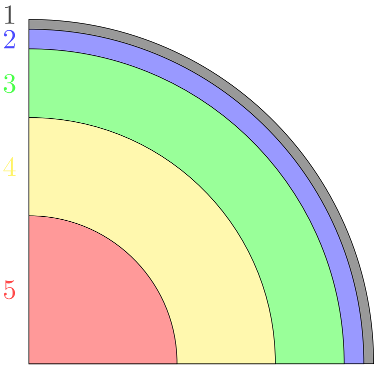
1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.

Internal composition of a NS



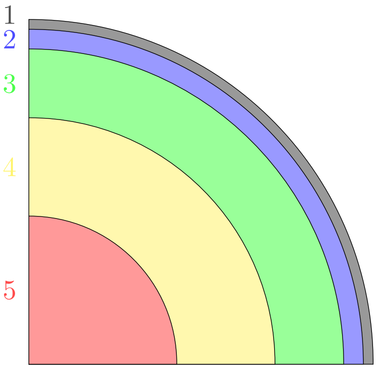
1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.
2. **Outer Crust:** (m) $\sim 10^7 - 10^{14}$ kg/m^3
   Lattice of ^{56}Fe and neutron-rich nuclei.

Internal composition of a NS



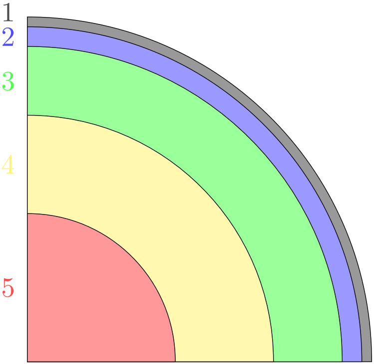
1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.
2. **Outer Crust:** (m) $\sim 10^7 - 10^{14}$ kg/m^3
   Lattice of ^{56}Fe and neutron-rich nuclei.
3. **Inner Crust:** (km) $\leq \rho_0 = 10^{17}$ kg/m^3
   Free neutrons, huge-B nuclei and pasta phases.

Internal composition of a NS



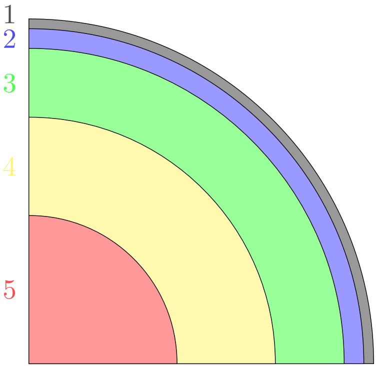
1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.
2. **Outer Crust:** (m) $\sim 10^7 - 10^{14}$ kg/m^3
   Lattice of ^{56}Fe and neutron-rich nuclei.
3. **Inner Crust:** (km) $\leq \rho_0 = 10^{17}$ kg/m^3
   Free neutrons, huge-B nuclei and pasta phases.
4. **Outer Core:** $(2 - 5$ km$) \sim 1 - 2\rho_0$
   Superfluid of neutrons.

Internal composition of a NS



1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.
2. **Outer Crust:** (m) $\sim 10^7 - 10^{14}$ kg/m^3
   Lattice of ^{56}Fe and neutron-rich nuclei.
3. **Inner Crust:** (km) $\leq \rho_0 = 10^{17}$ kg/m^3
   Free neutrons, huge-B nuclei and pasta phases.
4. **Outer Core:** $(2-5$ km$) \sim 1 - 2\rho_0$
   Superfluid of neutrons.
5. **Inner Core:** $(5-10$ km$) \sim 10\rho_0$
   ?

Internal composition of a NS



EOS $\longleftrightarrow \rho(p, n_B, T, \Omega, \vec{\mathbf{B}}, ...)$

1. **Atmosphere:** (mm - cm) $\sim 10^3$ kg/m^3
   Light nuclei and electron cloud.
2. **Outer Crust:** (m) $\sim 10^7 - 10^{14}$ kg/m^3
   Lattice of ^{56}Fe and neutron-rich nuclei.
3. **Inner Crust:** (km) $\leq \rho_0 = 10^{17}$ kg/m^3
   Free neutrons, huge-B nuclei and pasta phases.
4. **Outer Core:** $(2 - 5$ km$) \sim 1 - 2\rho_0$
   Superfluid of neutrons.
5. **Inner Core:** $(5 - 10$ km$) \sim 10\rho_0$
   Mesons condensates, free quark matter...

The Skyrme model

and its generalizations

The Skyrme model

- It is a $SU(2)_L \times SU(2)_R$ **chiral invariant** field theory of pions built on a field:

$$SU(2) \ni U = \sigma \mathbb{I}_2 + \pi_a \tau_a, \quad \sigma^2 + \boldsymbol{\pi}^2 = 1.$$

- The dynamics of the field are determined by the lagrangian:

$$\mathcal{L}_{240} = -\frac{f_\pi^2}{16} \operatorname{Tr}\{L_\mu L^\mu\} + \frac{1}{32e^2} \operatorname{Tr}\{[L_\mu, L_\nu]^2\} + \frac{m_\pi^2 f_\pi^2}{8} \operatorname{Tr}\{U - \mathbb{I}_2\},$$

where $L_\mu = U^\dagger \partial_\mu U$.

- The model has topologically nontrivial solutions (**skyrmions**) identified by a conserved integer number,

$$B^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \operatorname{Tr}\{L_\nu L_\alpha L_\beta\} \longrightarrow \int d^3x\, B^0 = B \equiv \textbf{baryon number}$$

Previous attempts on NS simulations

- In a first approximation, the main contributions in a NS are: nuclear interactions + gravity $\Rightarrow B \sim \frac{M_\odot}{m_N} \sim 10^{57}$ skyrmion.
- A promising result was obtained from a new type of solutions called **Skyrme crystals** $\rightarrow$ **ground state** solutions characterized by a parameter size $L \Rightarrow E(L), p(L), \rho(L)$.
- NS were obtained approximation using the **perfect fluid** stress-energy tensor (TOV formalism):

$$T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu - p g^{\mu\nu} \Longrightarrow \rho_{\text{crystal}}(p).$$

- Correct baryon numbers and radii! Too low maximal mass, $M_{\text{max}} \sim 1.5 M_\odot < 2 M_\odot$.

The BPS Skyrme model

- Following the EFT philosophy, a new term was introduced as an extension to the Skyrme model:

$$\mathcal{L}_6 = -\lambda^2 \pi^4 B_\mu B^\mu$$

- When $\mathcal{L}_6$ is combined with a potential term $(\mathcal{L}_0 = -\mu^2 \mathcal{U})$ $\Rightarrow$ **the BPS model**: $\mathcal{L}_{\text{BPS}} = \mathcal{L}_6 + \mathcal{L}_0$.
  - Solutions have zero binding energies.
  - The model has the symmetries of a fluid.

- Stress-Energy tensor of perfect fluid $\Rightarrow$ full theoretical study when coupled to gravity.

Full theory (FT)

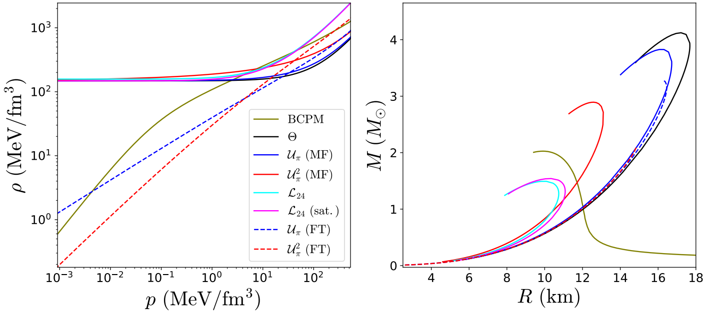
$\rho = p + 2\mu^2 \mathcal{U}$

Mean-field (MF)

$\rho(p) = \frac{\overline{E}(p)}{\overline{V}(p)}$

Skyrme NS

- The **BPS + gravity system** is solved in the FT and MF cases respectively.



- The BPS stars widely surpass the $M_{\text{max}} \geq 2M_\odot$ constraint.
- Different behaviour in the low-mass region (**crust problem**).

The generalized Skyrme model

- The combination of the four term seems a natural generalization of the Skyrme model,

$$\mathcal{L}_{2460} = -\frac{f_\pi^2}{16}\,\mathrm{Tr}\{L_\mu L^\mu\} + \frac{1}{32e^2}\,\mathrm{Tr}\{[L_\mu, L_\nu]^2\}$$
$$- \lambda^2\pi^4 B_\mu B^\mu + \frac{m_\pi^2 f_\pi^2}{8}\,\mathrm{Tr}\{U - \mathbb{I}_2\}.$$

- In a first approach we consider an **effective** description of the generalized model to compute NS observables.

- It is convenient to redefine the energy and length scales,

$$E_s = \frac{3\pi^2 f_\pi}{e}, \quad x_s = \frac{1}{f_\pi e} \qquad \text{(Skyrme units)}$$

Generalized Skyrme EOS

- Under a scale transformation $U_{\text{sol}}(x) \to U_{\text{sol}}(x/\sigma_s)$ we find,

$$E(\sigma_s) = \sigma_s E_2 + \frac{E_4}{\sigma_s} + \frac{E_6}{\sigma_s^3} + \sigma_s^3 E_0.$$

- σ_s is inversely related to the pressure of the system
  $\Rightarrow$ the $\mathcal{L}_{240}$ model becomes less dominant at high densities.
- We construct an effective **generalized Skyrme EOS**,

$$\rho_{\text{Gen}}(p) = [1 - \alpha(p)]\,\rho_{\text{crystal}}(p) + \alpha(p)\,[p + \rho_{\text{crystal}}(p_{PT})]\,,$$

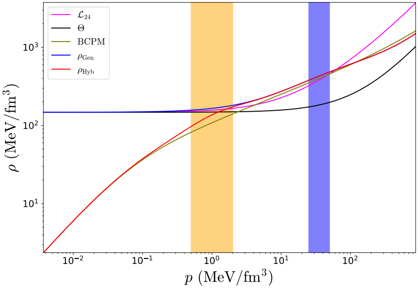
$$\alpha(p; p_{PT}, \beta) = \frac{(p/p_{PT})^{\beta}}{1 + (p/p_{PT})^{\beta}}.$$

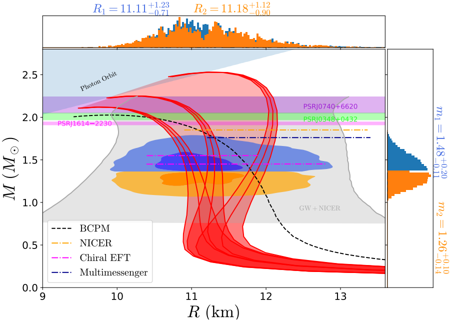
- We must be careful to maintain $c_s^2 = \frac{\partial p}{\partial \rho} < 1$ in the transition
  $\Rightarrow$ constraint on β.

Generalized Skyrme Neutron Stars

- We complete the description in the whole range of densities with a standard nuclear physics EOS at some p_*

$$\Rightarrow \rho_{\text{Hyb}} = [1 - \alpha(p)]\, \rho_{\text{BCPM}} + \alpha(p)\rho_{\text{Gen}}.$$





- We vary the values of p_{PT} and p_* to map the mass-radius diagram.

Skyrme Crystals

Crystalline configuration of skyrmions

- Infinitely extended solutions $\Rightarrow$ finite-size unit cells + **periodic boundary conditions**.

Crystalline configuration of skyrmions

- Infinitely extended solutions $\Rightarrow$ finite-size unit cells + **periodic boundary conditions**.
- The Skyrme fields are expanded in Fourier series:

$$\overline{\sigma} = \sum_{a,b,c}^{\infty} \beta_{abc} \cos \frac{a\pi x}{L} \cos \frac{b\pi y}{L} \cos \frac{c\pi z}{L}.$$

  L is the size parameter of the unit cell, which extends from $-L$ to L in each dimension.

- Different **symmetries** may be imposed on the fields $\Rightarrow$ **constraints** on the Fourier coefficients β_{abc}.
- The series are truncated and the rest of non-vanishing coefficients are varied to minimize the energy functional.

Different symmetries for cubic crystals

- First, **cubic symmetry** is imposed on the unit cell:

$$\mathsf{A}_1 : (x, y, z) \to (-x, y, z), \qquad \mathsf{A}_2 : (x, y, z) \to (y, z, x),$$

$$(\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3) \quad (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_2, \pi_3, \pi_1),$$

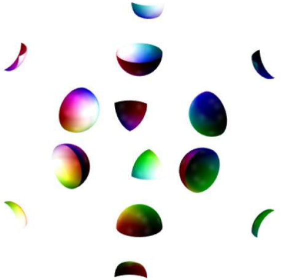
Different symmetries for cubic crystals

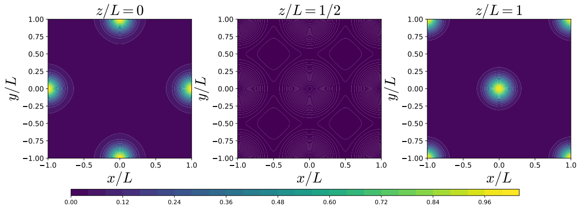
- First, **cubic symmetry** is imposed on the unit cell:

$$\mathsf{A}_1 : (x, y, z) \to (-x, y, z), \qquad \mathsf{A}_2 : (x, y, z) \to (y, z, x),$$

$$(\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, -\pi_1, \pi_2, \pi_3) \quad (\sigma, \pi_1, \pi_2, \pi_3) \to (\sigma, \pi_2, \pi_3, \pi_1),$$

- Further symmetries are imposed to construct different unit cells: **FCC, BCC, FCC$_{1/2}$**.





Infinite Nuclear Matter (INM)

- **Symmetric INM** is an idealized system of infinite nucleons without surface, Coulomb and isospin-asymmetry effects.

- The $E(n_B)$ has been experimentally constrained for $n_B \gtrsim n_0$ by measuring the multipoles in the expansion around n_0,

$$E(n_B) = E_0 + \frac{1}{2}K_0\frac{(n_B - n_0)^2}{9n_0^2} + \mathcal{O}(n_B^3)$$

Infinite Nuclear Matter (INM)

- **Symmetric INM** is an idealized system of infinite nucleons without surface, Coulomb and isospin-asymmetry effects.

- The $E(n_B)$ has been experimentally constrained for $n_B \gtrsim n_0$ by measuring the multipoles in the expansion around n_0,

$$E(n_B) = E_0 + \frac{1}{2}K_0\frac{(n_B - n_0)^2}{9n_0^2} + \mathcal{O}(n_B^3)$$

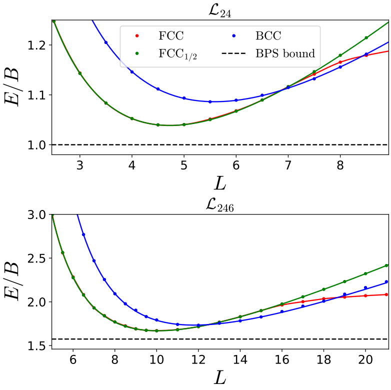
- The semi-empirical mass formula reproduces the binding energies of **isolated nuclei**:

$$\mathrm{BE}_B := m_N B - E_B = a_V B - a_S B^{2/3} - a_A \frac{(N_n - N_p)^2}{B} - a_C \frac{N_p\,(N_p - 1)}{B^{1/3}}$$

$$E_0 = m_N - a_V = 923\ \text{MeV}, \quad n_0 = 0.16\ \text{fm}^{-3}$$

Comparison between the symmetries

- Skyrme crystals naturally generate the $E(n_B) \leftrightarrow E(L)$ curve $\Rightarrow n_B = B_{\text{cell}}/(8L^3)$.

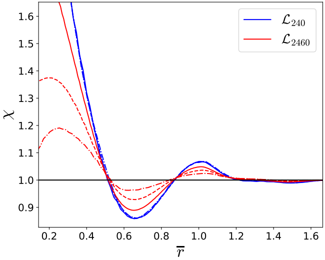


- The ground state may be attained for each range of densities by different symmetries $\Rightarrow$ **phase transitions**.

High-density phase transitions

The dominance of the sextic term at high densities $\Rightarrow$ **transition to a fluid**.

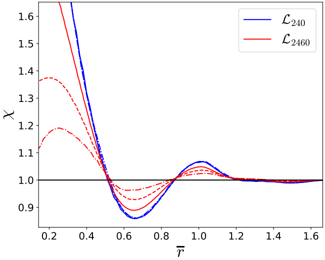
$$\chi(r) = \frac{1}{\mathcal{E}_{\text{mean}}} \int_0^r \mathcal{E}\, d^3x.$$



High-density phase transitions

The dominance of the sextic term at high densities $\Rightarrow$ **transition to a fluid**.

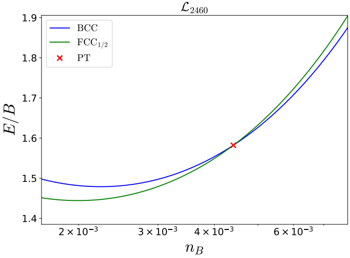
$$\chi(r) = \frac{1}{\mathcal{E}_{\text{mean}}} \int_0^r \mathcal{E}\, d^3x.$$



The BCC symmetry becomes the ground state at sufficiently high densities $\rightarrow$ first order transition.

$\Rightarrow$ **Maxwell construction**

Mixed phase: $p^{\text{I}} = p^{\text{II}}$, $\mu_B^{\text{I}} = \mu_B^{\text{II}}$.

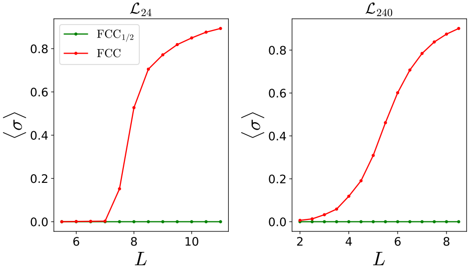


Low-density phase transition

The FCC and $\mathrm{FCC}_{1/2}$ symmetries split at large L.

The $\mathrm{FCC}_{1/2}$ is invariant under $\sigma \to -\sigma \implies \langle\sigma\rangle = 0$.

Reduces the freedom.

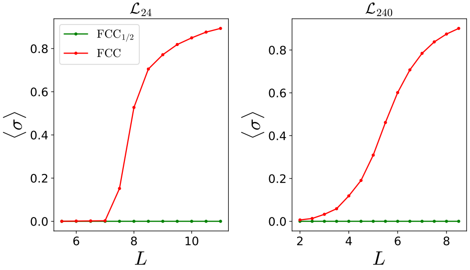


Low-density phase transition

The FCC and $\text{FCC}_{1/2}$ symmetries split at large L.

The $\text{FCC}_{1/2}$ is invariant under $\sigma \to -\sigma \implies \langle\sigma\rangle = 0$.

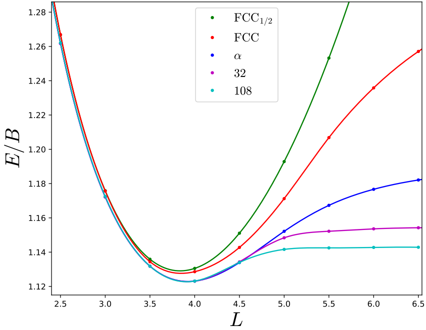
Reduces the freedom.



- The energy of the FCC crystal reaches a constant value in the $L \to \infty$ limit $\Rightarrow$ **interpolates INM** and **isolated nuclei**.
- This transition and the asymptotic behaviour motivated the search for a **new crystal** with lower energy at $L \to \infty$.

New branch of solutions: the $B = 4N_\alpha^3$ lattices

- It is known that the $B = 4$ skyrmion has cubic symmetry $\Rightarrow$ impose p.b.c. on a skyrmion inside a box.
- Indeed, the $B = 32$ and 108 skyrmions also have cubic symmetry and have lower energies.



- These new lattices are the **ground state** at low densities.

Surface energy in the Skyrme model

- The three curves converge at (E_0, n_0) and split for $L \to \infty$.
- We may extract the surface contributions from a geometric analysis of the $B = 4N_\alpha^3$ lattices.

$$E_B = E_0 B + E_{\text{surface}}$$

$$E_{\text{surface}} = 6N_\alpha^2 E_{\text{face}} = \frac{6}{4^{2/3}} E_{\text{face}} B^{2/3}$$

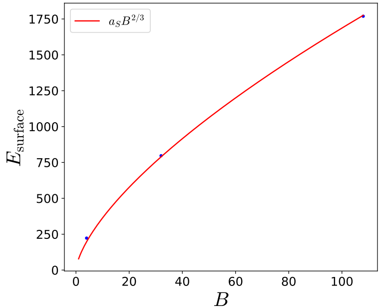
Surface energy in the Skyrme model

- The three curves converge at (E_0, n_0) and split for $L \to \infty$.
- We may extract the surface contributions from a geometric analysis of the $B = 4N_\alpha^3$ lattices.

$$E_B = E_0 B + E_{\text{surface}}$$

$$E_{\text{surface}} = 6N_\alpha^2 E_{\text{face}} = \frac{6}{4^{2/3}} E_{\text{face}} B^{2/3}$$

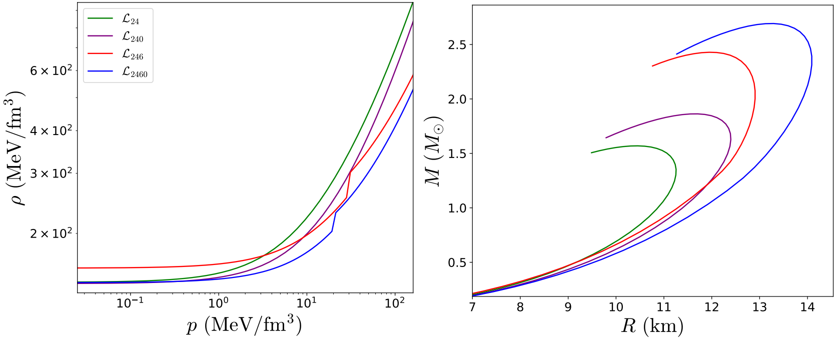
$$a_S = 78.3 \text{ MeV} \gg 18.3 \text{ MeV}$$



- The large value of a_S reflects the **too high binding energies** of the solutions in the Skyrme model.

NS from generalized Skyrme crystals

- Compute the EOS: $\rho(L) = \frac{E}{V}$, $p(L) = -\frac{dE}{dV}$ and solve the TOV system of equations.



- Sufficiently high masses but slightly large radii.
- Wrong behaviour at $p \to 0 \Rightarrow$ **symmetric INM** $(N_p = N_n)$.

Realistic Neutron Star matter

in the Skyrme model

Isospin quantization

- Isospin asymmetric effects are crucial in NS $(N_n \gg N_p)$
  $\Rightarrow$ **isospin quantization** on top of Skyrme crystals.

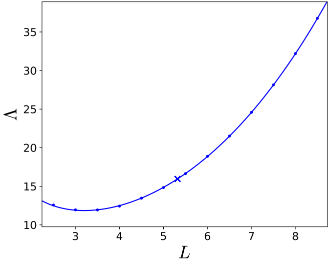
  Procedure: $U(x) \to A(t)U(x)A^\dagger(t) \Longrightarrow \mathcal{L} = \mathcal{T} - \mathcal{E}$.

$$T = \frac{1}{24\pi^2}\int d^3x \left[-\frac{1}{2}\,\mathrm{Tr}\{L_0^2\} - \frac{1}{2}\,\mathrm{Tr}\{[L_0, L_i]^2\} + 4\pi^4 c_6 \left(B^i\right)^2\right].$$

- We extract the isospin inertia tensor Λ_{ab} from the isorotational kinetic energy,

$$T = \frac{1}{2}\omega_a \Lambda_{ab}\omega_b,$$

$$\Lambda_{ab} = \Lambda(L)\delta_{ab}.$$



Isospin energy contribution

- The energy is obtained from the expectation value of the Hamiltonian on a specific quantum state

  $\to B_{\text{cell}} = 4 \Rightarrow i_3 = 0, \pm 1, \pm 2 \longrightarrow \gamma := \frac{N_p}{B} \geq 1/4.$

- Realistic values of $\gamma(n_B) \in \mathbb{R} \sim 10^{-2} - 10^{-1}$

  $\Rightarrow$ consider a **chunk of crystal** enclosing N unit cells,

$$i_3 := \frac{N_p - N_n}{2} = -N \frac{B_{\text{cell}}}{2} (1 - 2\gamma) \, .$$

- The isospin energy in this generic quantum state is:

$$\frac{E_{\text{iso}}}{B} = \frac{\hbar^2 B_{\text{cell}}}{8\Lambda} (1 - 2\gamma)^2 = \frac{\hbar^2 B_{\text{cell}}}{8\Lambda} \delta^2 .$$

Symmetry energy of INM

- **Isospin asymmetry** ($\delta := \frac{N_n - N_p}{B} = 1 - 2\gamma$) is introduced in the energy curve of nuclear matter as an expansion,

$$\frac{E(n_B, \delta)}{B} = E(n_B) + S(n_B)\delta^2 + \mathcal{O}(\delta^3),$$

$$S(n_B) = S_0 + \frac{1}{3} L_{\rm sym} \left(\frac{n_B - n_0}{n_0} \right) + \frac{1}{18} K_{\rm sym} \left(\frac{n_B - n_0}{n_0} \right)^2 + \mathcal{O}(n_B^3)$$

- We consider the estimations from a combined analysis between **astrophysics + nuclear experiments**,

S_0 (MeV)	$L_{\rm sym}$ (MeV)	$K_{\rm sym}$ (MeV)
31.7 ± 3.2	57.7 ± 19	-107 ± 88

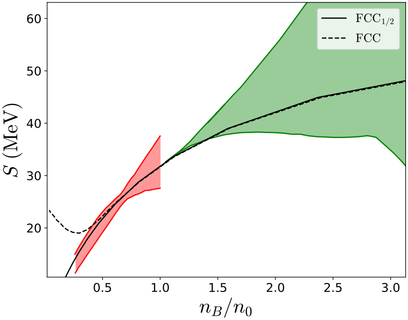
- We identify $S(n_B)$ from the energy of the Skyrme crystal,

$$\frac{E_{\rm iso}}{B} = \frac{\hbar^2 B_{\rm cell}}{8\Lambda}\delta^2 \Longrightarrow S_{\rm Skyrme}(n_B) = \frac{\hbar^2}{2\Lambda}$$

Symmetry energy in the Skyrme model

The sextic term may be tunned to fit S_0, L_{sym} and K_{sym}.

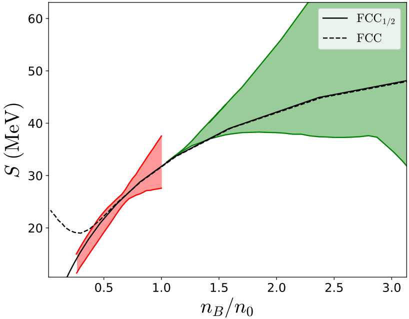
We find difficulties in the simultaneous fit of E_0, n_0 and S_0 with arbitrary accuracy ($\leq 15\%$).



Symmetry energy in the Skyrme model

The sextic term may be tunned to fit S_0, L_{sym} and K_{sym}.

We find difficulties in the simultaneous fit of E_0, n_0 and S_0 with arbitrary accuracy ($\leq 15\%$).



- The red constraint applies for INM models, but the FCC crystal yields isolated nuclear matter at large L.

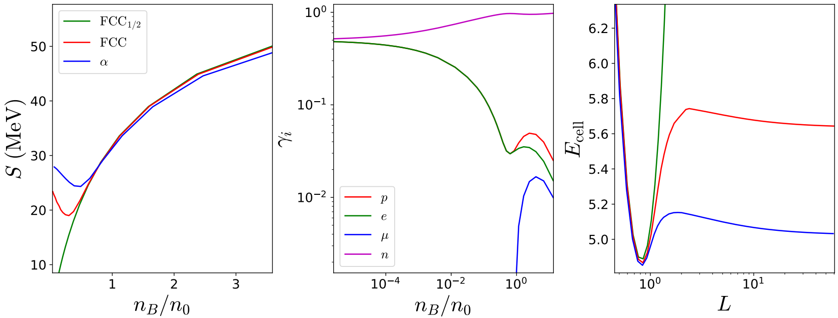
$$S_{\mathsf{FCC}}(n_B \to 0) = a_A = 24.2 \text{ MeV} \approx 23.2 \text{ MeV}$$

$npe\mu$ matter in Skyrme crystals

- The non-zero γ introduces electric charges $\Rightarrow$ insert a Fermi gas of leptons to ensure charge neutrality.

$$(\beta - \text{equilibrium}) \longrightarrow \mu_I = \frac{\hbar^2 B_{\text{cell}}}{2\Lambda}(1-2\gamma) = \mu_l, \quad l = e, \mu$$

$$(\text{charge neutrality}) \longrightarrow n_p = n_e + \Theta\left(\mu_e - m_\mu\right) n_\mu.$$



New degrees of freedom in NS cores

- The appearance of muons due to the increase of μ_e motivates the introduction of additional particles.
- Kaons are expected to appear in NS cores

  $\Rightarrow$ no degeneracy pressure and form **condensates**.

New degrees of freedom in NS cores

- The appearance of muons due to the increase of μ_e motivates the introduction of additional particles.
- Kaons are expected to appear in NS cores

  $\Rightarrow$ no degeneracy pressure and form **condensates**.
- The kaon field is introduced as a fluctuation in the $SU(3)$ direction of the Skyrme field,

$$U = \sqrt{U_K} U_\pi \sqrt{U_K},$$

$$U_\pi = \begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix}, \quad u \in SU(2), \quad U_K = \exp\left\{ i \frac{2\sqrt{2}}{f_\pi} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right\}$$

- Add the kaon mass term within a generalized potential,

$$\mathcal{L}_0^{\text{new}} = \frac{f_\pi^2}{24}\left(m_\pi^2 + 2m_K^2\right) \mathrm{Tr}\{U - \mathbb{I}_3\} + \frac{\sqrt{3}}{24}\left(m_\pi^2 - m_K^2\right) \mathrm{Tr}\{\lambda_8 \left(U + U^\dagger\right)\}.$$

Condensation of the kaon field

- A condensed field is given by the vacuum expectation value,

$$\langle K^{\pm} \rangle = \phi e^{\pm i \mu_K t},$$

where μ_K is the kaon chemical potential.

- The kaon field induces a new potential term $V_K(\mu_K, \phi)$.
- The quantization procedure may be performed as before $\Rightarrow$ the isospin inertia tensor depends on μ_K and ϕ.

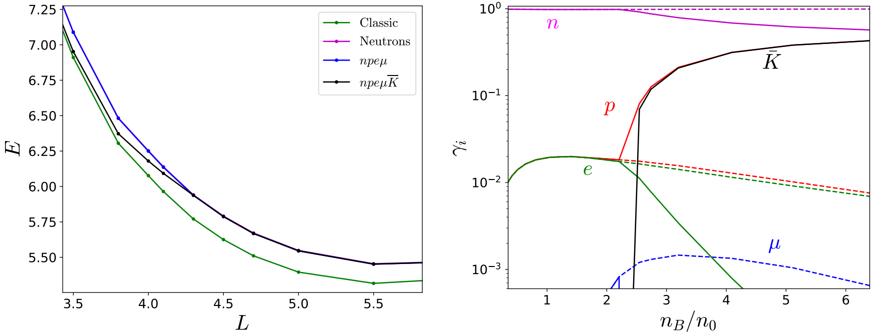
$$E = E_{\text{class}}(L) + E_{\text{iso}}(\gamma, \mu_K, \phi) + E_e(N_e) + E_\mu(N_\mu) + V_K(\mu_K, \phi).$$

- We search for minimal energy solutions under β-equilibrium and the conservation of electric charge conditions,

$$\Omega(\mu_e, \phi) = E(\gamma, \phi) + \mu_e \left(N_p - N_e - N_\mu\right) \longrightarrow \mu_e = \mu_\mu = \mu_K, \quad \frac{\partial \Omega}{\partial \mu_e} = \frac{\partial \Omega}{\partial \phi} = 0.$$

Impact on the energy curve

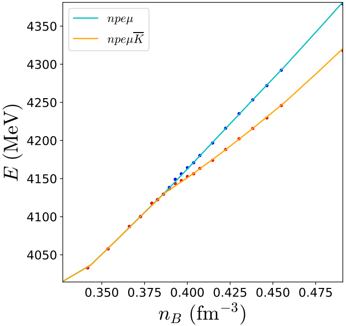
- Kaons condense at $n_B = 2.3 n_0$ and a new branch of lower energy is developed.
- The appearance of kaons sharply increase the proton fraction.

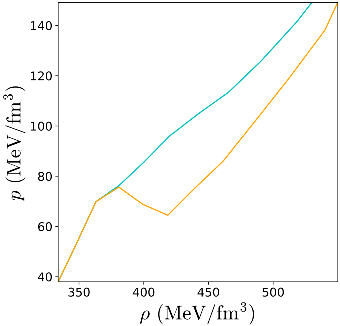


- We must be careful about the thermodynamical magnitudes around the phase transition.

First order transition

- The pressure cannot decrease as a function of ρ
  $\Rightarrow$ **first order phase transition**.

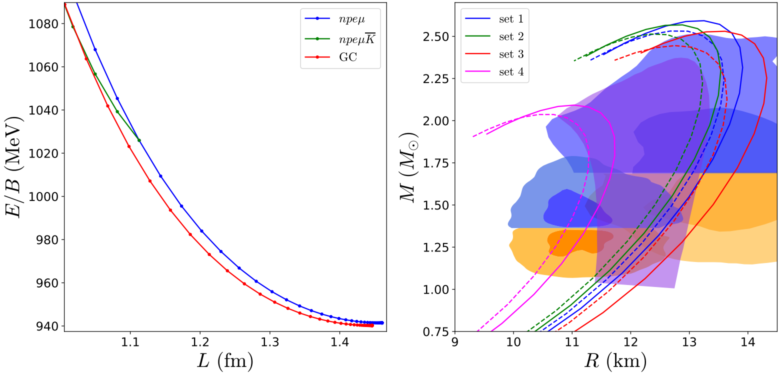




- Two charges are conserved in this case: B, Q
  $\Rightarrow$ **Gibbs construction** (GC).

$$p^{\mathsf{I}} = p^{\mathsf{II}}, \quad \mu_B^{\mathsf{I}} = \mu_B^{\mathsf{II}}, \quad \mu_e^{\mathsf{I}} = \mu_e^{\mathsf{II}}.$$

Skyrme NS with kaon-condensed core



- The GC produces a lower energy branch and reaches the minimum of energy.
- The kaon condensation **reduces the masses and radii** of the stars ⇒ better results.

The compression modulus problem

- The compression modulus of INM has been estimated
  $\Rightarrow K_0 = 240 \pm 20$ MeV $\longrightarrow \frac{K_0}{E_0} \approx 0.25$
- We obtained the values from Skyrme crystals,

$$K_0 = 9 n_0^2 \left. \frac{\partial^2 E(n_B)}{\partial n_B^2} \right|_{n_0}$$

Model	K_0/E_0
$\mathcal{L}_{24}$	0.96
$\mathcal{L}_{240}$	1.45
$\mathcal{L}_{246}$	1.98
$\mathcal{L}_{2460}$	2.53

- It may be formally proved that for Skyrme crystals we have:

$$\frac{K_0}{E_0} \gtrsim 1.$$

Including ρ mesons in the Skyrme model

- We consider the Skyrme lagrangian (without $\mathcal{L}_6$) and the standard ρ mesons theory,

$$\mathcal{L} = \mathcal{L}_{240} - \frac{1}{8}\,\mathrm{Tr}\left\{R^{\dagger}_{\mu\nu}R^{\mu\nu}\right\} + \frac{m_\rho^2}{4}\,\mathrm{Tr}\left\{R^{\dagger}_{\mu}R^{\mu}\right\}$$

with the interaction term:

$$\mathcal{L}_I = -\frac{\alpha}{2}\,\mathrm{Tr}\{R_{\mu\nu}\,[L^\mu, L^\nu]\}$$

Including ρ mesons in the Skyrme model

- We consider the Skyrme lagrangian (without $\mathcal{L}_6$) and the standard ρ mesons theory,

$$\mathcal{L} = \mathcal{L}_{240} - \frac{1}{8}\,\mathrm{Tr}\left\{R^{\dagger}_{\mu\nu}R^{\mu\nu}\right\} + \frac{m_\rho^2}{4}\,\mathrm{Tr}\left\{R^{\dagger}_{\mu}R^{\mu}\right\}$$

with the interaction term:

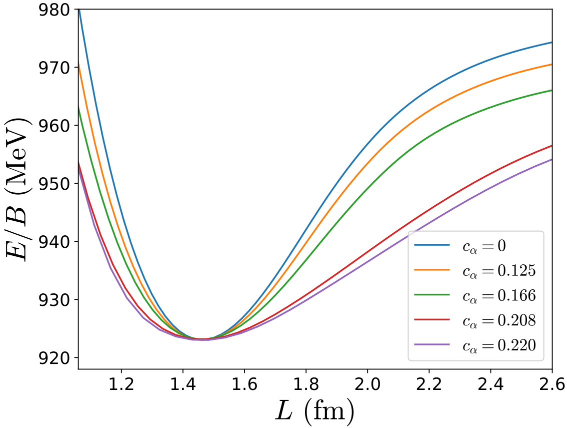
$$\mathcal{L}_I = -\frac{\alpha}{2}\,\mathrm{Tr}\{R_{\mu\nu}\,[L^\mu, L^\nu]\}$$

- This interaction term induces a negative quartic term $\Rightarrow$ **reduces the curvature** of the energy curve around the minimum but **affects the stability** of the solitons.

$$c_\alpha := \alpha e < 1/4.$$

Results for K_0

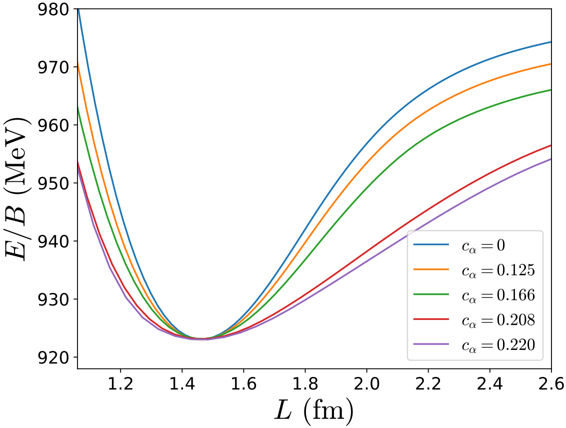
c_α	K_0/E_0
0	1.170
0.125	0.985
0.166	0.778
0.208	0.461
0.220	0.381



- The compression modulus gets smaller for larger $c_\alpha = \alpha e$.

Bonus! Better binding energies

c_α	K_0 (MeV)	BE (%)
0	1080	5.54
0.125	909	5.36
0.166	718	5.00
0.208	425	4.25
0.220	351	3.85



- Binding energies slightly improve, but still not enough $\Rightarrow \frac{m_\alpha/4 - E_0}{E_0} \lesssim 1\%$.

Conclusions and Prospects

Conclusions

- The **sextic term** is a crucial contribution for a correct description of NS within the Skyrme model.
- Skyrme crystals are not only good candidates to describe INM but also predict several **phase transitions** inside NS with physical interest.
- Standard isospin quantization may be performed in Skyrme crystals and yield realistic results for the **particle fractions** and **symmetry energy**.
- **Kaon condensation** has been accomplished purely within the Skyrme model with interesting impact on NS.
- The inclusion of vector mesons seems a viable solution to both the **compression modulus** and **binding energies** problems.

Open problems

- The Skyrme model is still not able to reproduce the low density regime of the EOS:

  crust problem $\longleftrightarrow$ **binding energies**.

- We enhanced the value of K_0 but we are still a $\sim 50\%$ above the experimental value.

- The electrostatic coefficient a_C in the semi-empirical mass may be reproduce introducing Coulomb effects

  $\Rightarrow$ **the pasta phases**.

- Inclusion of magnetic field in the EOS

  $\Rightarrow$ deformation, maximal $\vec{\mathbf{B}}$...

Skyrme Neutron Stars

PhD Defence by:

Miguel Huidobro García

Doctoral Programme in Nuclear and Particle Physics, USC (IGFAE)

THANK YOU