SECI1013: DISCRETE STRUCTURE SEM 1 20242025

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ASSIGNMENT 2 – Recurrence Relation, Counting Methods, Permutations and Combinations, Pigeonhole Principle

Group of 3, Due date: 25 November 2024

Recurrence Relation

1. Write the first five terms of the sequence defined by the recurrence relation $a_n = 3a_{n-1} + 2$ with the initial condition $a_0 = 1$.
2. Given the sequence $b_n = b_{n-1} + n^2$ with $b_1 = 1$, determine b_2, b_3, and b_4.
3. Solve the recurrence relation $a_n = 2a_{n-1} + 5$ with the initial condition $a_0 = 3$.
4. The Tower of Hanoi puzzle is described by the recurrence $T_n = 2T_{n-1} + 1$, with $T_1 = 1$. Derive the closed-form solution for T_n.

1. $a_n = 3a_{n-1} + 2$, $a_0 = 1$

$a_1 = 3(1) + 2 = 5$

$a_2 = 3(5) + 2 = 17$

$a_3 = 3(17) + 2 = 53$

$a_4 = 3(53) + 2 = 161$

1, 5, 17, 53, 161

2. $b_n = b_{n-1} + n^2$, $b_1 = 1$

$b_2 = 1 + 2^2 = 5$

$b_3 = 5 + 3^2 = 14$

$b_4 = 14 + 4^2 = 30$

$b_2 = 5$, $b_3 = 14$, $b_4 = 30$

3. $a_n = 2a_{n-1} + 5$, $a_0 = 3$

$a_1 = 2(3) + 5 = 11$

$a_2 = 2(11) + 5 = 27$

$a_3 = 2(27) + 5 = 59$

$a_4 = 2(59) + 5 = 123$

$a_5 = 2(123) + 5 = 251$

3, 11, 27, 59, 123, 251, ...

4. $T_n = 2T_{n-1} + 1$, $T_1 = 1$

$T_2 = 2(1) + 1 = 3$

$T_3 = 2(3) + 1 = 7$

$T_4 = 2(7) + 1 = 15$

$T_5 = 2(15) + 1 = 31$

$T_6 = 2(31) + 1 = 63$

1, 3, 7, 15, 31, 63, ...

$T_n = 2^n - 1$

$T_1 = 2^1 - 1 = 1$

$T_2 = 2^2 - 1 = 3$

$T_3 = 2^3 - 1 = 7$

$T_4 = 2^4 - 1 = 15$

Counting Methods & Probability

1. A security code consists of 5 digits. Each digit can be any number from 0 to 9.
   a. How many codes can be generated if no digit can repeat?
   b. How many codes can be generated if the first digit must be even, and digits cannot be repeated?

2. A group of 10 friends are going on a trip. They decide to take a photo in the following ways:
   a. How many ways can all 10 people line up in a row?
   b. How many ways can they line up if two specific people always stand next to each other?

3. Two six-sided dice are rolled. What is the probability that:
   a. The sum of the numbers is 7.
   b. At least one die shows a 6.
   c. The numbers on the two dice are equal.

4. A class has 15 students. A team of 4 students needs to be selected:
   a. How many different teams can be formed?
   b. If two specific students must be included in the team, how many teams can be formed?

5. In how many ways can the letters of the word **"STATISTICS"** be arranged?
   a. How many of these arrangements start with the letter "S"?
   b. How many arrangements have all the "T"s together?

S = 3 , T = 3 , A = 1, I = 2, C = 1

1. a. $10 \times 9 \times 8 \times 7 \times 6 = 30240$

   b. $5 \times 9 \times 8 \times 7 \times 6 = 15120$

2. a. $10! = 3628800$

   b. $2! \times 9! = 725760$

3. a. Sum of numbers is 7 when (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) → 6 outcomes

   Total number of outcomes when rolling two dice = $6 \times 6 = 36$

   $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$

   b. At least one dice is 6:

   (1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5) → 11 outcomes

   Total number of outcomes when rolling two dice = $6 \times 6 = 36$

   $P(\text{at least one } 6) = \frac{11}{36}$

   c. Numbers are equal: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) → 6 outcomes

   Total number of outcomes when rolling two dice = $6 \times 6 = 36$

   $P(\text{equal numbers}) = \frac{6}{36} = \frac{1}{6}$

4. a. ${}^{15}C_4 = 1365$

   b. $4 - 2 = 2$, $15 - 2 = 13$

   ${}^{13}C_2 = 78$

5. $P(10) = \frac{10!}{3!\,3!\,2!} = 50400$

   a. $\frac{9!}{2!\,3!\,2!} = 15120$

   b. $\frac{8!}{3!\,2!} = 3360$

Permutations and Combinations

1. A password consists of 8 characters, chosen from the set {A, B, C, D, 1, 2, 3, 4, !, @, #, $}.

   1 2 3 4 5 6 7 8 9 10 11 12

   a. How many passwords can be formed if repetition of characters is not allowed?
   b. If the password must start with a letter and end with a digit, how many passwords can be formed?

2. From the word "ENGINEERING," how many unique arrangements can be made:
   a. Without any restrictions? E=3 N=3 G=2 I=2 R=1
   b. If all the "E"s must be together?

3. 8 beads of different colors are to be strung together to form a circular necklace.
   a. How many distinct arrangements can be made?
   b. If the necklace is symmetrical (indistinguishable when flipped), how many arrangements are possible?

4. A committee of 6 members is to be formed from a group of 10 men and 8 women. $10+8=18$
   a. How many committees can be formed if there are no restrictions?
   b. How many committees can be formed if the committee must have at least 2 women?

5. A class of 30 students is divided into three groups for a project: one group of 10 students, one group of 12 students, and one group of 8 students.
   a. How many ways can the class be divided into these groups?
   b. If one specific student must be in the group of 10, how many ways can the division be done?

6. A bakery sells 10 different types of cakes. A customer wants to buy 5 cakes.
   a. How many ways can the customer choose the cakes if the selection is such that at least one of each type must be chosen?
   b. How many ways can the cakes be chosen if there is no restriction on the number of cakes of each type?

1. a. ${}^{12}P_8 = 19958400$

   b. $\underbrace{4}_{\text{letter}} \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \underbrace{4}_{\text{digit}} = 2419200$

2. a. $\dfrac{11!}{3!\,3!\,2!\,2!} = 277200$

   b. $\dfrac{9!}{3!\,2!\,2!} = 15120$

3. a. $(8-1)! = 7! = 5040$

   b. $\dfrac{5040}{2} = 2520$

4. a. ${}^{18}C_6 = 18564$

   b. Ways if less than 2 women are selected: $\underbrace{{}^{10}C_6}_{\text{all men}} + \underbrace{{}^{10}C_5 \cdot {}^{8}C_1}_{\text{5 men \& 1 woman}} = 2226$

   $18564 - 2226 = 16338$

5. a. $\dfrac{30!}{10!\,12!\,8!} = 3.78 \times 10^{12}$

   b. $\dfrac{29!}{9!\,12!\,8!} = 1.26 \times 10^{12}$

6. a. ${}^{10}C_5 = 252$

   b. ${}^{10+5-1}C_5 = 2002$

Pigeonhole Principle

1. Show that if 50 integers are chosen from the set {1,2,3,...,99}, at least two of them are consecutive.
2. Prove that in any group of 9 positive integers, there are two integers whose difference is divisible by 8.
3. In a set of 30 students, prove that at least two students were born in the same day of the week.
4. Prove that if 9 socks are drawn from a drawer containing 4 red socks, 4 blue socks, and 4 green socks, at least three socks must be of the same color.

1. Split the set {1,2,3,...,99} into 49 pairs which each contains two consecutive numbers, such as {1,2}, {3,4}, ..., {97, 98}, and one leftover number {99}.

Pigeons - 50 integers are chosen

Pigeonhole - 49 pairs of consecutive number

At least one pair that the two number both must be selected, so at least two of them are consecutive.

2. When an integers is divided by 8, the possible remainder are : 0,1,2,3,4,5,6,7

Pigeons - 9 integers

Pigeonhole - 8 possible remainder

There are at least two integers must have the same remainder when divided by 8.

When the remainder is the same, difference of two integers will be the multiples of 8, so it is divisible by 8.

Let $x_1 = 73$

$x_2 = 65$

Both have same remainder = 1

when $x_1 - x_2 = 73 - 65$

$= 8$

8 is divisible by 8.

3. Pigeons - 30 students

Pigeonhole - 7 days in a week

Because of 30 students can't fit into 7 days without some sharing, so at least two students were born in the same day of the week.

4. Let X = number of socks to drawn

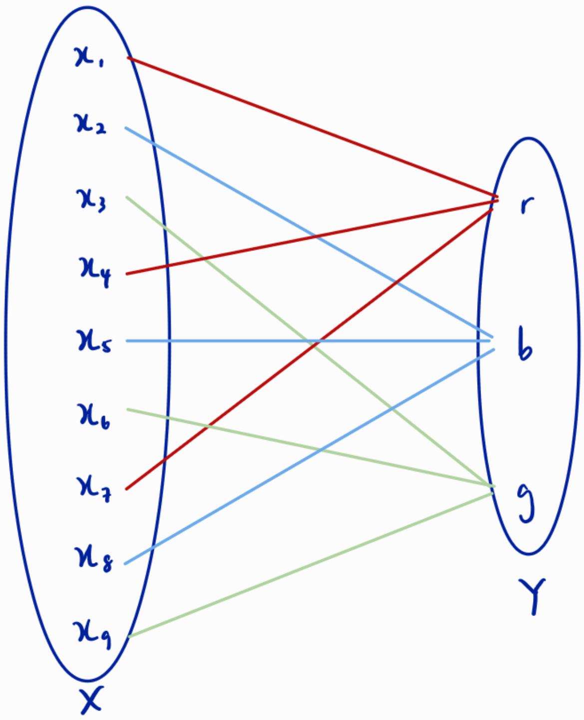
$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$

$|X| = 9$

Let Y = colour of socks

$Y = \{\text{red}, \text{blue}, \text{green}\}$

$|Y| = 3$



There are at least 3 socks must be the same colour.