0.0.1 Re-framed Moth-flame Optimization Algorithm (Re-framed MFO)

- **Objectives**:

Objective Problem	
$f(\mathbf{x}_i)$	fitness of $\mathbf{x}_i$.
n	the dimension of the search space.
$[lb_{\mathbf{x}}, ub_{\mathbf{x}}]$	the interval of objective variable $\mathbf{x}$, in our cases, it is defined in the IOHprofiler, $[lb_{\mathbf{x}}, ub_{\mathbf{x}}] = [-5, +5]$.
Objective Solution	
$\mathbf{x}_i$	it can be imagined as one individual in Swarm-Intelligence Algorithms, $\mathbf{x}_i \in R^n$.

- **Parameters**:

T	maximum iteration, the budget in our cases, in our case, it is defined in IOHprofiler.
M	population size, $M = 30$.
$\mathbf{y}_i$	the best flame position for one individual $\mathbf{y}_i \in R^n$.
z_{1_i}	a weight value.
z_2	a kind of threshold.
w	the shape of spiral, $w = 1$, $w \in (0, +\infty)$.

- **Functions**:
  - Initialization Process:

(1) Initialize $\mathbf{x}_i(t=0)$:

$$\mathbf{x}_i(t=0) = \boldsymbol{\mathcal{U}}(lb_{\mathbf{x}}, ub_{\mathbf{x}}), i = 1 \dots M \quad (1)$$

(2) Initialize $\mathbf{x}_s(t=1)$:

$$\langle \mathbf{x}_i(t) \rangle = \mathbf{Sort}(\{\mathbf{x}_i(t)\}), i = 1 \dots M \quad (2)$$

(3) Initialize a weighting value $z_{1_i}(t)$:

$$z_{1_i}(t) = rand \times (-2 - \frac{t}{T}) + 1 \quad (3)$$

(4) Initialize the threshold $z_2(t)$:

$$z_2(t) = \mathbf{Round}(M - t \times \frac{M-1}{T}) \quad (4)$$

– Optimization Process:

(1) Update $\mathbf{x}_i(t)$ to generate $\hat{\mathbf{x}}_i(t+1)$:

$$\hat{\mathbf{x}}_i(t+1) = \begin{cases} (\mathbf{x}_{s_i}(t) - \mathbf{x}_i(t)) \times e^{w \times z_{1_i}(t)} \times \cos(2\pi \times z_{1_i}(t)) + \mathbf{x}_{s_i}(t), & i \leq z_2(t) \\ (\mathbf{x}_{s_{z_2(t)}}(t) - \mathbf{x}_i(t)) \times e^{w \times z_{1_i}(t)} \times \cos(2\pi \times z_{1_i}(t)) + \mathbf{x}_{s_{z_2(t)}}(t), & \text{o.w.} \end{cases} \quad (5)$$

(2) Dealing with outliers C:

$$\mathbf{x}_{i,n}^{\text{fixed}}(t+1) = \begin{cases} ub_x & , \mathbf{x}_{i,n}(t+1) > ub_{\mathbf{x}} \\ \mathbf{x}_{i,n}(t+1) & , \text{o.w} \\ lb_x & , \mathbf{x}_{i,n}(t+1) < lb_{\mathbf{x}} \end{cases} \quad (6)$$

(3) Select $\mathbf{x}_i(t+1)$ from $\hat{\mathbf{x}}_i(t+1)$:

$$\mathbf{x}_i(t+1) = \hat{\mathbf{x}}_i(t+1) \quad (7)$$

(4) Update $\mathbf{x}_s(t)$ to generate $\mathbf{x}_s(t+1)$:

$$\langle \mathbf{x}_i(t+1) \rangle = \textbf{Sort}(\{\mathbf{x}_i(t)\} \cup \{\mathbf{x}_i(t+1)\}), i = 1 \dots M \quad (8)$$

(5) Update $z_{1_i}(t)$ to generate $z_{1_i}(t+1)$:

$$z_{1_i}(t) = rand \times (-2 - \frac{t}{T}) + 1 \tag{9}$$

(6) Update $z_2(t)$ to generate $z_2(t+1)$:

$$z_2(t) = \mathbf{Round}(M - t \times \frac{M-1}{T}) \tag{10}$$

Algorithm 1 Re-framed MFO with population size M; search space $n, [lb_{\mathbf{x}}, ub_{\mathbf{x}}]$; stop condition T; initialization method $Init_{\mathbf{x}}$, optimization method $Opt_{\mathbf{x}}$, treatment C of outliers, and selection S to objective solutions; initialization method $Init_{\Delta}$ and optimization method Opt_{Δ} to step-size Δ.

$t \leftarrow 0$
$\mathbf{X}(t) \leftarrow Init_{\mathbf{x}}(n, M, [lb_{\mathbf{x}}, ub_{\mathbf{x}}])$ as Eq.**??** ▷ initialize initial population
$F(t) \leftarrow f(\mathbf{X}(t))$ ▷ evaluate
$w \leftarrow Init_{\Delta:w}(w)$ ▷ initialize w-relative step-size
$\mathbf{X}_s(t) \leftarrow Init_{\Delta:\mathbf{x}_s}(\mathbf{X}(t))$ as Eq.**??** ▷ initialize $\mathbf{x}$-relative step-size
$z(t) \leftarrow Init_{\Delta:z}(t, T)$ as Eq.**??**, Eq.**??** ▷ initialize z-relative step-size
while stop condition T **do**
    $\hat{\mathbf{X}}(t+1) \leftarrow Opt_{\mathbf{x}}(\mathbf{X}(t), \mathbf{X}_s(t), z(t), w)$ as Eq.**??**▷ generate temporarily updated population
    $\hat{\mathbf{X}}(t+1) \leftarrow C(\hat{\mathbf{X}}(t+1))$ as Eq.6 ▷ treatment to outliers
    $F(t+1) \leftarrow f(\hat{\mathbf{X}}_i(t+1))$ ▷ evaluate
    $\mathbf{X}(t+1) \leftarrow S(\mathbf{X}_i(t), \hat{\mathbf{X}}_i(t+1))$ as Eq.**??** ▷ select and generate finally updated population
    $\mathbf{X}_s(t+1) \leftarrow Init_{\Delta:\mathbf{x}_s}(\mathbf{X}(t), \mathbf{X}(t+1))$ as Eq.**??** ▷ update dynamic $\mathbf{x}$-relative vector step-size $\mathbf{x}_s$
    $z_1(t+1) \leftarrow Opt_{\Delta:z_1}(t+1, T)$ as Eq.**??**, Eq.**??** ▷ update z-relative step-size
    $t \leftarrow t+1$
end while