0.0.1 Re-framed Butterfly Optimization Algorithm (Re-framed BOA)

- **Objectives**:

Objective Problem	
$f(\mathbf{x}_i)$	fitness of $\mathbf{x}_i$, $(f : R^n \to R)$.
n	the dimensionality of the search space.
$[lb_{\mathbf{x}}, ub_{\mathbf{x}}]$	the interval of objective variable $\mathbf{x}$, in our case, it is defined in IOHprofiler, $[lb_{\mathbf{x}}, ub_{\mathbf{x}}] = [-5, +5]$.
Objective Solution	
$\mathbf{x}_i$	it can be imagined as one individual in Swarm-Intelligence Algorithms, $\mathbf{x}_i \in R^n$.

- **Parameters**:

T	maximum iteration, the budget in our cases, in our case, it is defined in IOHprofiler.
M	population size, $M = 5$.
$\mathbf{x}_g$	the best position that the whole population has found so far.
z	sensory modality.
z^0	the initial value of sensory modality z, $z^0 = 0.01$, $z^0 \in [0, 1]$.
w_1	power exponent, $w_1 = 0.1$, $w_1 \in [0, 1]$.
w_2	switch probability, $w_2 = 0.8$.

- **Functions**:
  - Initialization Process:

(1) Initialize $\mathbf{x}_i(t=0)$:

$$\mathbf{x}_i(t=0) = \boldsymbol{\mathcal{U}}(lb_{\mathbf{x}}, ub_{\mathbf{x}}), i = 1\dots M \tag{1}$$

(2) Initialize $\mathbf{x}_g(t=0)$:

$$\mathbf{x}_g(t=0) = \textbf{Min}(\{\mathbf{x}_i(t)\}), i = 1\dots M \tag{2}$$

(3) Initialize $z(t=0)$:

$$z(t=0) = z^0 \tag{3}$$

– Optimization Process:

(1) Update $\mathbf{x}(t)$ to generate $\hat{\mathbf{x}}_i(t+1)$:

$$\hat{\mathbf{x}}_i(t+1) = \begin{cases} \mathbf{x}_i(t) + (rand^2 \times \mathbf{x}_g(t) - \mathbf{x}_i(t)) \times z_1(t) \times f(\mathbf{x}_i(t))^{w_1} & , \; rand > w_2 \\ \mathbf{x}_i(t) + (rand^2 \times \mathbf{x}_j(t) - \mathbf{x}_k(t)) \times z_1(t) \times f(\mathbf{x}_i(t))^{w_1} & , \; \text{o.w} \end{cases} \tag{4}$$

where $\mathbf{x}_j$ and $\mathbf{x}_k$ are any two neighbors around $\mathbf{x}_i$.

(2) Dealing with outliers C:

$$\mathbf{x}_{i,n}^{\text{fixed}}(t+1) = \begin{cases} ub_x & , \; \mathbf{x}_{i,n}(t+1) > ub_{\mathbf{x}} \\ \mathbf{x}_{i,n}(t+1) & , \; \text{o.w} \\ lb_x & , \; \mathbf{x}_{i,n}(t+1) < lb_{\mathbf{x}} \end{cases} \tag{5}$$

(3) Select $\mathbf{x}_i(t+1)$ from $\hat{\mathbf{x}}_i(t+1)$:

$$\mathbf{x}_i(t+1) = \begin{cases} \hat{\mathbf{x}}_i(t+1) & , \; f(\hat{\mathbf{x}}_i(t+1)) < f(\mathbf{x}_i(t)) \\ \mathbf{x}_i(t) & , \; \text{o.w} \end{cases} \tag{6}$$

(4) Update $\mathbf{x}_g(t)$ to generate $\mathbf{x}_g(t+1)$:

$$\mathbf{x}_g(t+1) = \mathbf{Min}(\mathbf{x}_g(t) \cup \{\mathbf{x}_i(t+1)\}), i = 1\dots M \tag{7}$$

(5) Update $z(t)$ to generate $z(t+1)$:

$$z(t+1) = z(t) + \frac{0.025}{z(t) \times T} \tag{8}$$

Algorithm 1 Re-framed BOA with population size M; search space $n, [lb_{\mathbf{x}}, ub_{\mathbf{x}}]$; stop condition T; initialization method $Init_{\mathbf{x}}$, optimization method $Opt_{\mathbf{x}}$, treatment C of outliers, and selection S to objective solutions; initialization method $Init_{\Delta}$ and optimization method Opt_{Δ} to step-size Δ.

$t \leftarrow 0$
$\mathbf{X}(t) \leftarrow Init_{\mathbf{x}}(n, M, [lb_{\mathbf{x}}, ub_{\mathbf{x}}])$ as Eq.1 ▷ initialize initial population
$F(t) \leftarrow f(\mathbf{X}(t))$ ▷ evaluate
$w, z^0 \leftarrow Init_{\Delta:w}(w, z^0)$ ▷ initialize w-relative step-size
$\mathbf{x}_g(t) \leftarrow Init_{\Delta:\mathbf{x}}(\mathbf{X}(t))$ as Eq.2 ▷ initialize $\mathbf{x}$-relative step-size
$z(t) \leftarrow Init_{\Delta:z}(z^0)$ as Eq.3 ▷ initialize z-relative step-size
while stop condition T **do**
  $\hat{\mathbf{X}}(t+1) \leftarrow Opt_{\mathbf{x}}(\mathbf{X}(t), \mathbf{x}_g(t), z(t), w)$ as Eq.4 ▷ generate temporarily updated population
  $\hat{\mathbf{X}}(t+1) \leftarrow C(\hat{\mathbf{X}}(t+1))$ as Eq.5 ▷ treatment to outliers
  $F(t+1) \leftarrow f(\hat{\mathbf{X}}(t+1))$ ▷ evaluate
  $\mathbf{X}(t+1) \leftarrow S(\mathbf{X}(t), \hat{\mathbf{X}}(t+1))$ as Eq.6 ▷ select and generate finally updated population
  $\mathbf{x}_g(t+1) \leftarrow Init_{\Delta:\mathbf{x}}(\mathbf{X}(t), \mathbf{X}(t+1))$ as Eq.7 ▷ update $\mathbf{x}$-relative step-size
  $z(t+1) \leftarrow Opt_{\Delta:z}(z(t), t+1)$ as Eq.8 ▷ update z-relative step-size
  $t \leftarrow t+1$
end while