0.0.1 Re-framed Particle Swarm Optimization (Re-framed PSO)

- **Objectives**:

Objective Problem	
$f(\mathbf{x}_i)$	fitness of $\mathbf{x}_i$.
n	the dimension of the search space.
$[lb_{\mathbf{x}}, ub_{\mathbf{x}}]$	the interval of objective variable $\mathbf{x}$, in our cases, it is defined in the IOHprofiler, $[lb_{\mathbf{x}}, ub_{\mathbf{x}}] = [-5, +5]$.
Objective Solution	
$\mathbf{x}_i$	it can be imagined as one individual in Swarm-Intelligence Algorithms, $\mathbf{x}_i \in R^n$.

- **Parameters**:

T	maximum iteration, the budget in our cases, in our case, it is defined in IOHprofiler.
M	population size, $M = 25$.
$\mathbf{x}_{i_p}$	the best position that this individual $\mathbf{x}_i$ has found so far.
$\mathbf{x}_g$	the best position that the whole population has found so far.
$\mathbf{y}_i$	the velocity for one individual $\mathbf{y}_i \in R^n$.
$[lb_{\mathbf{y}}, ub_{\mathbf{y}}]$	the interval of assisting variable $\mathbf{y}$, $[lb_{\mathbf{y}}, ub_{\mathbf{y}}] = [0, 0]$.
w_1	scaling strength, here it is specifically called inertia weight, $w_1 = 0.73$, $w_1 \in (0, +\infty)$.
w_2	here it is specifically called personal coefficient, $w_2 = 1.49$, $w_2 \in [0, +\infty)$.
w_3	here it is specifically called global coefficient, $w_3 = 1.49$, $w_3 \in [0, +\infty)$.

- **Functions**:
  - Initialization Process:
    (1) Initialize $\mathbf{x}_i(t=0)$:

$$\mathbf{x}_i(t=0) = \boldsymbol{\mathcal{U}}(lb_{\mathbf{x}}, ub_{\mathbf{x}}), i = 1 \ldots M \tag{1}$$

    (2) Initialize $\mathbf{y}_i(t=0)$:

$$\mathbf{y}_i(t=0) = \boldsymbol{\mathcal{U}}(lb_{\mathbf{y}}, ub_{\mathbf{y}}), i = 1 \ldots M \tag{2}$$

    (3) Initialize $\mathbf{x}_{i_p}(t=0)$:

$$\mathbf{x}_{i_p}(t=0) = \mathbf{x}_i(t) \tag{3}$$

    (4) Initialize $\mathbf{x}_g(t=0)$:

$$\mathbf{x}_g(t=0) = \textbf{Min}(\{\mathbf{x}_i(t)\}), i = 1 \ldots M \tag{4}$$

  - Optimization Process:
    (1) Update $\mathbf{y}_i(t)$ to generate $\mathbf{y}_i(t+1)$:

$$\begin{aligned} \mathbf{y}_i(t+1) &= w_1 \times \mathbf{y}_i(t) + \mathcal{U}(0, w_2) \times (\mathbf{x}_{i_p}(t) - \mathbf{x}_i(t)) \\ &\quad + \mathcal{U}(0, w_3) \times (\mathbf{x}_g(t) - \mathbf{x}_i(t)) \end{aligned} \tag{5}$$

    (2) Update $\mathbf{x}_i(t)$ to generate $\hat{\mathbf{x}}_i(t+1)$:

$$\hat{\mathbf{x}}_i(t+1) = \mathbf{x}_i(t) + \mathbf{y}_i(t+1) \tag{6}$$

    (3) Dealing with outliers C:

$$\mathbf{x}_{i,n}^{\text{fixed}}(t+1) = \begin{cases} ub_x & , \ \mathbf{x}_{i,n}(t+1) > ub_{\mathbf{x}} \\ \mathbf{x}_{i,n}(t+1) & , \ \text{o.w} \\ lb_x & , \ \mathbf{x}_{i,n}(t+1) < lb_{\mathbf{x}} \end{cases} \tag{7}$$

(4) Select $\mathbf{x}_i(t+1)$ from $\hat{\mathbf{x}}_i(t+1)$:

$$\mathbf{x}_i(t+1) = \hat{\mathbf{x}}_i(t+1) \tag{8}$$

(5) Optimize $\mathbf{x}_{i_p}(t)$ to generate $\mathbf{x}_{i_p}(t+1)$:

$$\mathbf{x}_{i_p}(t+1) = \mathbf{Min}(\{\mathbf{x}_{i_p}(t), \mathbf{x}_i(t+1)\}) \tag{9}$$

(6) Optimize $\mathbf{x}_g(t)$ to generate $\mathbf{x}_g(t+1)$:

$$\mathbf{x}_g(t+1) = \mathbf{Min}(\mathbf{x}_g(t) \cup \{\mathbf{x}_i(t+1)\}), i = 1 \ldots M \tag{10}$$

Algorithm 1 Re-framed PSO with population size M; search space $n, [lb_{\mathbf{x}}, ub_{\mathbf{x}}]$; stop condition T; initialization method $Init_{\mathbf{x}}$, optimization method $Opt_{\mathbf{x}}$, treatment C of outliers, and selection S to objective solutions; initialization method $Init_{\Delta}$ and optimization method Opt_{Δ} to step-size Δ.

$t \leftarrow 0$
$\mathbf{X}(t) \leftarrow Init_{\mathbf{x}}(n, M, [lb_{\mathbf{x}}, ub_{\mathbf{x}}])$ as Eq.1 ▷ initialize initial population
$F(t) \leftarrow f(\mathbf{X}(t))$ ▷ evaluate
$w, [lb_{\mathbf{y}}, ub_{\mathbf{y}}] \leftarrow Init_{\Delta:w}(w, [lb_{\mathbf{y}}, ub_{\mathbf{y}}])$ ▷ initialize w-relative step-size
$\mathbf{Y}(t) \leftarrow Init_{\Delta:\mathbf{y}}(n, M, [lb_{\mathbf{y}}, ub_{\mathbf{y}}])$ as Eq.2 ▷ initialize $\mathbf{y}$-relative step-size
$X_p(t), \mathbf{x}_g(t) \leftarrow Init_{\Delta:\mathbf{x}}(\mathbf{X}(t))$ as Eq.3, Eq.4 ▷ initialize $\mathbf{x}$-relative step-size
while stop condition T **do**
  $\mathbf{Y}(t+1) \leftarrow Opt_{\Delta:\mathbf{y}}(\mathbf{Y}(t)$ as Eq.5 ▷ update $\mathbf{y}$-relative step-size
  $\hat{\mathbf{X}}(t+1) \leftarrow Opt_{\mathbf{x}}(\mathbf{X}(t), \mathbf{Y}(t), \mathbf{x}_g(t), \mathbf{X}_p(t), w)$ as Eq.6 ▷ generate temporarily updated population
  $\hat{\mathbf{X}}(t+1) \leftarrow C(\hat{\mathbf{X}}(t+1))$ as Eq.7 ▷ treatment to outliers
  $F(t+1) \leftarrow f(\hat{\mathbf{X}}(t+1))$ ▷ evaluate
  $\mathbf{X}(t+1) \leftarrow S(\mathbf{X}(t), \hat{\mathbf{X}}(t+1))$ as Eq.8 ▷ select and generate finally updated population
  $\mathbf{X}_p(t+1), \mathbf{x}_g(t+1) \leftarrow Init_{\Delta:\mathbf{x}}(\mathbf{X}(t), \mathbf{X}(t+1))$ as Eq.9, Eq.10 ▷ update $\mathbf{x}$-relative step-size
  $t \leftarrow t+1$
end while