population activity $A_N^\alpha(t) = \frac{1}{N^\alpha}\sum_{i=1}^{N^\alpha} s_i^\alpha(t)$

spike train $s_i^\alpha(t) = \sum_k \delta(t - t_{i,k}^\alpha)$

coarse-grained population activity $A_N^\alpha(t) = \frac{\Delta n^\alpha(t)}{N^\alpha \Delta t}$

membrane potential $u_i^\alpha(t)$

dynamic threshold $\theta_i^\alpha(t) = u_{th} + \int_{-\infty}^{t} \theta^\alpha(t-t') s_i^\alpha(t') dt'$

θ^α is a spike-triggered adaptation kernel

θ_i^α is the threshold of neuron i

Calligraphic fonts

$\theta_i^\alpha(t) \equiv \theta^\alpha(t, t_{i,k}^\alpha < t)$ another form for simplicity

They are difference

conditional intensity $\lambda_i^\alpha(t) = f^\alpha(u_i^\alpha(t) - \theta^\alpha(t, t_{i,k}^\alpha < t))$ for individual

effective conditional intensity $\lambda_A^\alpha(t|\hat{t}) = f^\alpha(u_A^\alpha(t,\hat{t}) - \underbrace{\theta_A^\alpha(t,\hat{t})}_{\text{effective threshold.}})$ for population after mean-field approximation

survival propability $S(t|\hat{t})$

total population activity given by integral $A(t) = \int_{-\infty}^{t} \lambda_A(t|\hat{t}) S(t|\hat{t}) A(\hat{t}) d\hat{t}$

survival propability decay $\frac{\partial S(t|\hat{t})}{\partial t} = -\lambda_A(t|\hat{t}) S(t|\hat{t})$

the density of last spike times $\hat{t}$:

$Q_\infty(t,\hat{t}) = S(t|\hat{t}) A(\hat{t})$ with initial condition $S(\hat{t}|\hat{t}) = 1$

For finite-size case

$$1 = \int_{-\infty}^{t} S_N(t|\hat{t}) A_N(\hat{t}) d\hat{t}$$

$$A_N(t) = -\int_{-\infty}^{t} \partial_t S_N(t|\hat{t}) A_N(\hat{t}) d\hat{t}$$

$$S_N(t+\Delta t|\hat{t}_k) = S_N(t|\hat{t}_k) - X(t,\hat{t}_k)/\Delta n(\hat{t}_k)$$

$X(t,\hat{t}_k)$: the number of neurons that fire in the group with last spike time $\hat{t}_k$.

$$A_N(t) = \frac{\Delta n(t)}{N \Delta t}, \quad \Delta n(t) = \sum_{\hat{t}_k < t} X(t,\hat{t}_k)$$

$$Q_N(t,\hat{t}) = S_N(t|\hat{t}) A_N(\hat{t})$$

~~pseud~~ the pseudo density $Q(t,\hat{t}_k)=S(t|\hat{t}_k)A_N(\hat{t}_k)$

the macroscopic density $Q_\infty(t,\hat{t})=S(t|\hat{t})A(\hat{t})$

the microscopic density $Q_N(t,\hat{t}_k)=S_N(t|\hat{t}_k)A_N(\hat{t}_k)$

$$\underline{S_N(t|\hat{t}_k)=S(t|\hat{t}_k)+\delta S(t|\hat{t}_k)}$$

density normalization $1=\int_{-\infty}^{t} S_N(t|\hat{t})A_N(\hat{t})d\hat{t}$

expected population rate $\bar{A}(t)=\int_{-\infty}^{t}\lambda_A(t|\hat{t})S_N(t|\hat{t})A_N(\hat{t})d\hat{t}$

$$1=\int_{-\infty}^{t}S(t|\hat{t})A_N(\hat{t})d\hat{t}+\underline{\int_{-\infty}^{t}\delta S(t|\hat{t})A_N(\hat{t})d\hat{t}}$$

for infinitely large net, this term is zero?

$$\bar{A}(t)=\int_{-\infty}^{t}\lambda_A(t|\hat{t})S(t|\hat{t})A_N(\hat{t})d\hat{t}+\underline{\int_{-\infty}^{t}\lambda_A(t|\hat{t})\delta S(t|\hat{t})A_N(\hat{t})d\hat{t}}$$

$$\approx \Lambda(t)\int_{-\infty}^{t}\delta S(t|\hat{t})A_N(\hat{t})d\hat{t}$$

$$\bar{A}(t)=\int_{-\infty}^{t}\lambda_A(t|\hat{t})S(t|\hat{t})A_N(\hat{t})d\hat{t}+\Lambda(t)\left(1-\int_{-\infty}^{t}S(t|\hat{t})A_N(\hat{t})d\hat{t}\right) \quad \text{Eq(13)}$$

$$\Lambda=\frac{\int_{-\infty}^{t}\lambda_A(t|\hat{t})v(t,\hat{t})d\hat{t}}{\int_{-\infty}^{t}v(t,\hat{t})d\hat{t}} \quad \text{Eq(11)}$$

$$\frac{\partial v}{\partial t}=-2\lambda_A(t|\hat{t})v+\lambda_A(t|\hat{t})S(t|\hat{t})A_N(\hat{t})$$