HW6: Find the Flower

In order to prove the Flower Problem is NP-complete, I will prove that a solution can be verified in polynomial time and I can reduce Clique → Flower Problem in polynomial time. We know that Clique is NP-complete.

NP Proof:

Given a graph G = (V, E) and a natural number N >0, we look for a set of N+4 vertices such that the induced subgraph is a flower, or return NO if not. (We assume that N >2)

First, we need to show that the verification of this problem is in NP. Given a solution S, we need to Iterate all vertices in S and identify those that are only connected by a single edge, check whether only 3 vertices v1, v2, v3 are connected by a single edge and they all connected to a same vertex v_c. This process takes O(|V| + |E|) time. Then remove v1, v2, v3 and the edges connected to them, check if only one edge is connected to v_c now, this process takes O(1) time. Till here, we can verify the "Star" structure exists.

Second, remove v_c and the edges connected to it as well. Then, we check whether the remaining of S (we donated it as S') is clique. We can verify it by checking that all pairs of vertices in S' are connected in O(n^2) time, and to check that |S'| >= N in O(n) time.

Overall, it takes O(n^2) to verify, which is polynomial.

Input:

Let G1 = (V1, E1), N be the input to the Clique problem. We need to add the "Star" structure to each vertex in G1. The "Star" has 4 vertices, 3 of them are connected to the another one (called v_center) by a single edge. We add a "Star" to each vertex in G1 by creating an edge between the v_center of each "Star" and every vertex in G1, which means we add |V| "Star" in G1 (A total of 4 |V| vertices and 4|V| edges are added.). Now the new flower graph G2= (V2, E2) = (V1+V', E1+E') is formed. We use G2, N as the inputs to the Flower Problem input. This takes O(n) time by adding the vertices and edges to establish the graph containing "Star" structure, which is polynomial.

Output:

If the algorithm for Flower Problem returns NO then the Clique algorithm also returns NO.

If the algorithm for Flower Problem returns a satisfying set of N+4 vertices, we remove any vertex and edges we might have added to remove the "Star" part. The rest is the output of Clique.

This takes O(n) to check for a vertex and remove it, and O(1) to return NO or the Clique, which is polynomial.

Correctness:

If there is a solution in the Flower Problem, because we added the "Star" Structure to form G2, then the clique was in the original G1. This also means that if there is a clique in the G1, we have added the "Star" needed to force a solution in Flower Problem. Therefore, a clique of N exists in G1 IFF it corresponds to a N+4 Flower in G2. We could remove any vertex and edges we might have added, and the pass it to the output of Clique.

If a set of N+4 vertices does not exist in G2, then the Clique of size N also does not exist in G1. So, if Flower Problem returns NO, then so does the Clique Algorithm.

Thus, as Flower Problem is the class NP, and at least as hard as Clique, we conclude that Flower Problem is NP-complete.