

## Bumpy sequence

**1. Define the entries of your table in words. E.g.  $T(i)$  or  $T(i, j)$  is ...**

$T(i, 0)$  is the maximum length of a bumpy sequence for  $a[1] \dots a[i]$  that the last element has a positive difference with its previous element;

$T(i, 1)$  is the maximum length of a bumpy sequence for  $a[1] \dots a[i]$  that the last element has a negative difference with its previous element;

**2. State a recurrence for the entries of your table in terms of smaller subproblems. Don't forget your base case(s).**

Base case:

for  $i = 1$ ,  $T(i, 0) = 1$ ,  $T(i, 1) = 1$

Recurrence:

for  $2 \leq i \leq n$ :

$T(i, 0) = T(i-1, 1) + 1$ ,  $T(i, 1) = T(i-1, 1)$ , if  $a[i-1] < a[i]$

$T(i, 1) = T(i-1, 0) + 1$ ,  $T(i, 0) = T(i-1, 0)$ , if  $a[i-1] > a[i]$

$T(i, 1) = T(i-1, 1)$ ,  $T(i, 0) = T(i-1, 0)$ , if  $a[i-1] = a[i]$

**3. Write pseudocode for your algorithm to solve this problem.**

when  $i = 1$ :

$T(i, 0) = 1$

$T(i, 1) = 1$

for  $i = 2$  to  $n$ :

if  $a[i] - a[i-1] > 0$ :

$T(i, 0) = T(i-1, 1) + 1$

$T(i, 1) = T(i-1, 1)$

else if  $a[i] - a[i-1] < 0$ :

$T(i, 1) = T(i-1, 0) + 1$

$T(i, 0) = T(i-1, 0)$

else:

$T(i, 1) = T(i-1, 1)$

$T(i, 0) = T(i-1, 0)$

return  $\max\{T(n, 0), T(n, 1)\}$

**4. State and analyze the running time of your algorithm.**

when  $i = 1$ :  $O(1)$

for  $i = 2$  to  $n$ :  $O(n)$

return  $\max\{T(n, 0), T(n, 1)\}$ :  $O(2)$

The running time can be represented by:  $O(1) + O(n) + O(2) \Rightarrow O(n)$