MS&E 448 Problem Set 1

Multi-time Scale Feedback Model

Part 1.

By choosing T = 300, Δt = 1/252, P0 = 26.84, and σ0 to be 0.015, I did a total of 2768 simulation (~11 years)[1]. Simulated price as well as the empirical distribution of price histogram are shown below. In addition, I also plotted changes in log return for comparison purpose.

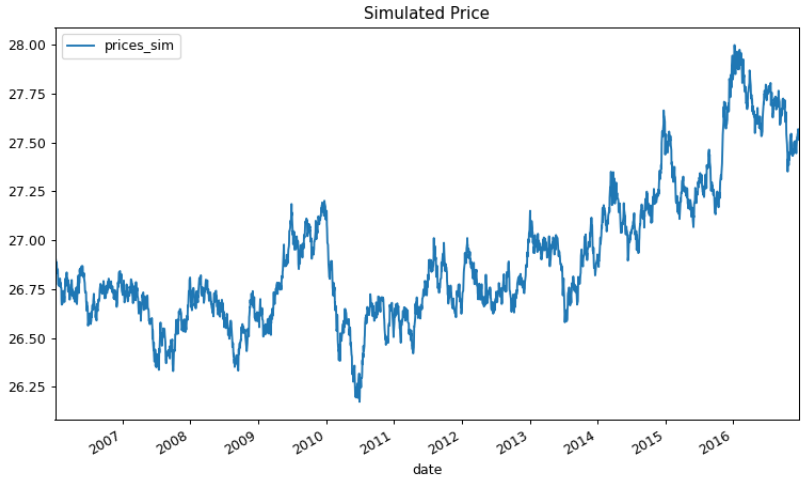

Fig 1. Simulated Price

Comments:

This feedback model looks at volatility fluctuations, therefore the simulation formula does not guarantee that price increases overall (occasionally the simulation gives negative trend). The above simulation result shows that price increases slowly over the time span of simulated ten years, which is not consistent with real world, but this is not of our interest to look at in this simulation.

[1] For details of implementation, please visit https://github.com/HujiaYuYoyo/448_hws/blob/master/hw1.ipynb. Any suggestions for improvements are welcomed.

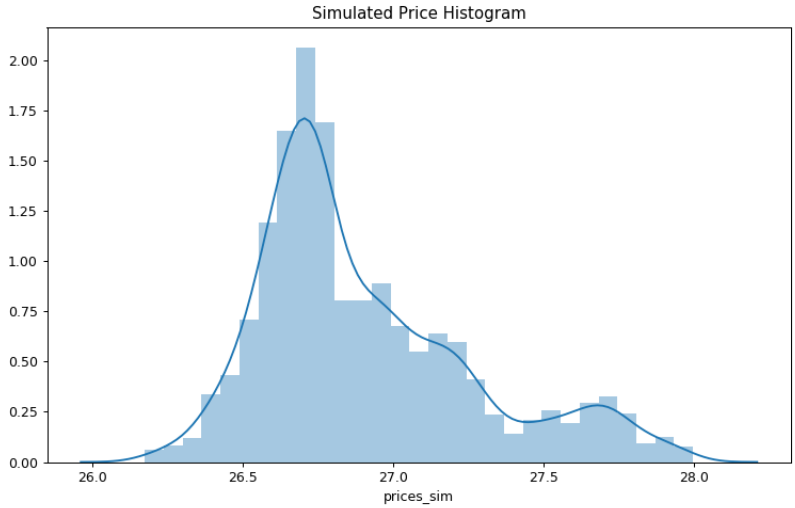

Fig 2. Simulated Price Histogram

Comment:

The shape of the simulated price histogram aligns very closely with that of MSFT empirical price histogram distribution in Figure 9 below.

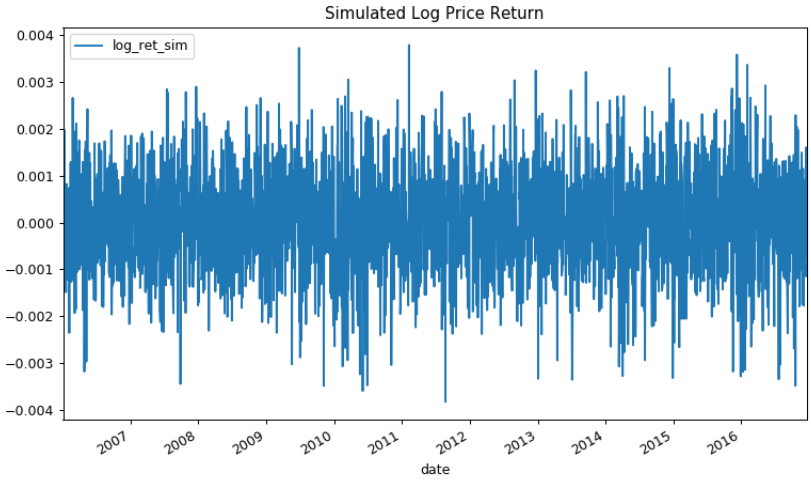

Fig 3. Simulated Log Price Return

Part 2.

To show that simulated distribution tails are fatter than the ones I would have gotten if simulated prices were normally distributed. I plot normal distribution and the simulated log return data in the same plot and observe closely at the tail.

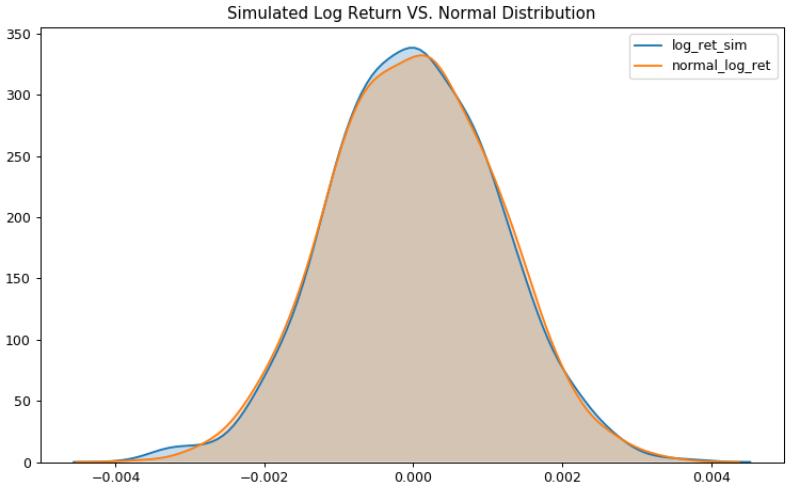

Fig. 4. Simulated Log Returns vs. Normal Distributed Log Returns

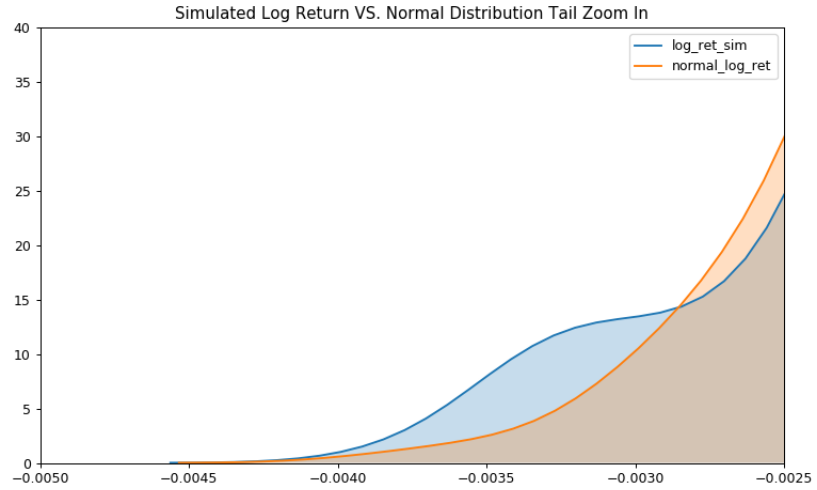

Fig. 5. Tail of Simulated Log Returns vs. Normal Distributed Log Returns

Comments:

The simulated log return distribution (blue) shows a significantly fatter tail than that of normal distribution (yellow) according to Figure 5. However, according to the paper [1], the simulated distribution is also supposed to capture the higher peak as a financial data fact, which is only slightly shown in the plot here. This is maybe due to under-calibrated parameters.

Part 3.

I computed the autocorrelation of the squares of the log returns to evaluate the presence of volatility clustering.

The reason why autocorrelation can be used to evaluate volatility clustering is because volatility clustering essentially indicates a stylized financial data fact that nearby volatilities (σt) are correlated, which suggests that nearby |r_t| or nearby squared r_t would be correlated also. Therefore, autocorrelation can also be used to evaluated volatility clustering.

Quantitatively, volatility clustering means that the series of squared returns will exhibit a significant and slowly decaying pattern, which is shown in the plots below.

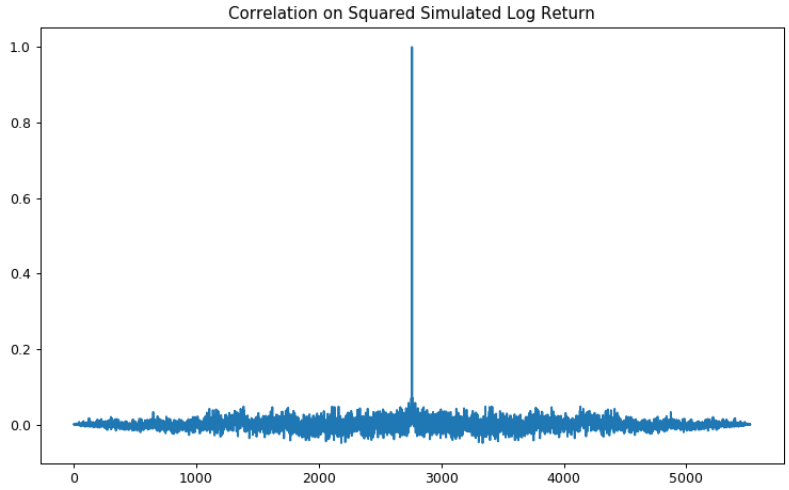

Fig. 6. Correlation on Squared Simulated Log Return

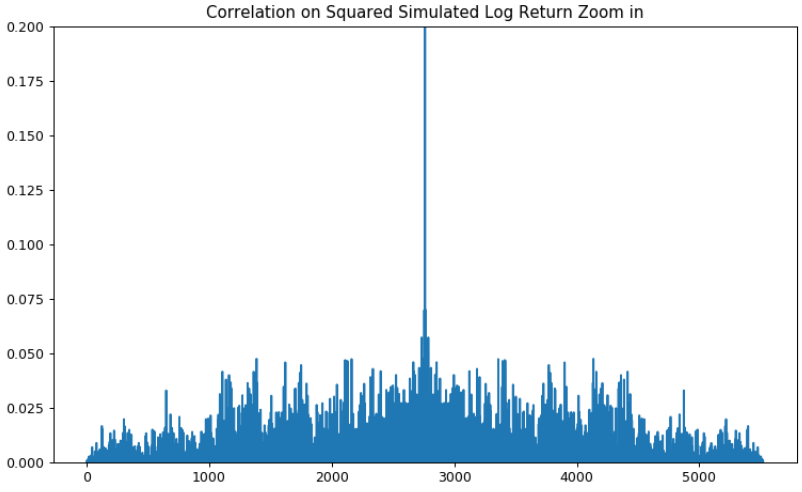

Fig. 7. Rescaled Correlation on Squared Simulated Log Return

Comment:

The series of squared returns exhibits a significant and slowly decaying pattern, which suggests volatility clustering.

Part 4.

Stock: MSFT

Date range: 2006/1/1 - 2016/12/31

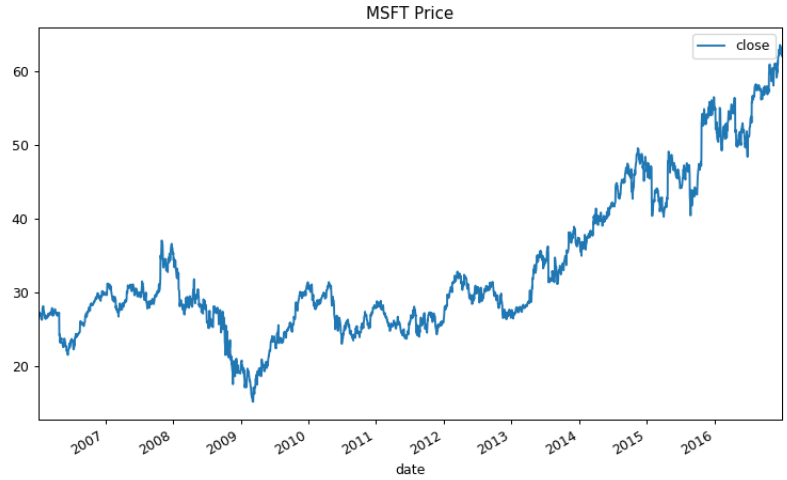

Fig. 8. MSFT Price Data From 2016/1/1-2016/12/31

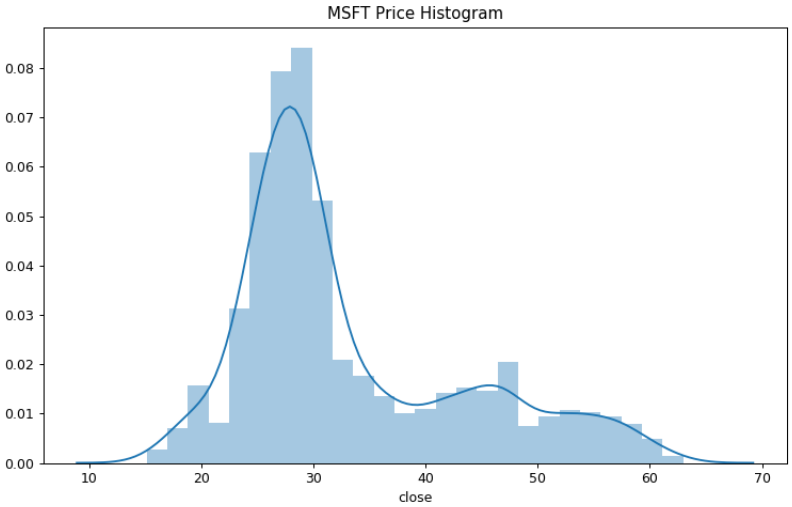

Fig. 9. MSFT Price Histogram From 2016/1/1-2016/12/31

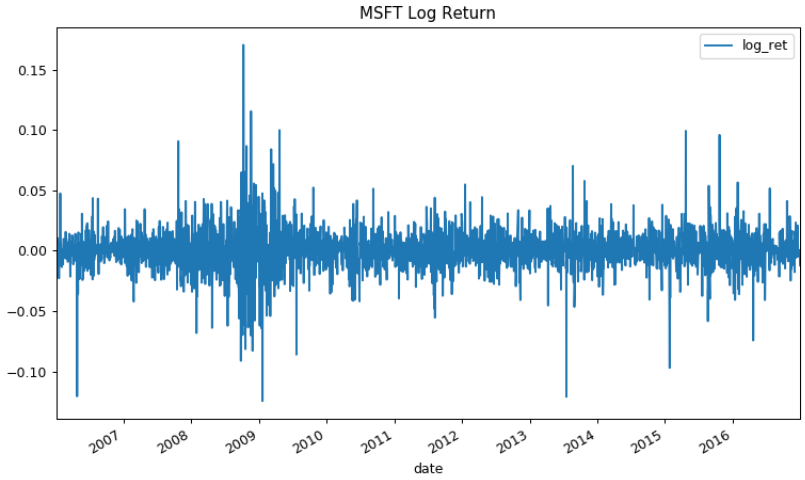

Fig. 10. MSFT Log Price Return Data From 2016/1/1-2016/12/31

- **Fatter tail**

I plotted normal distribution and the MSFT log return data in the same plot and observe closely at the tail.

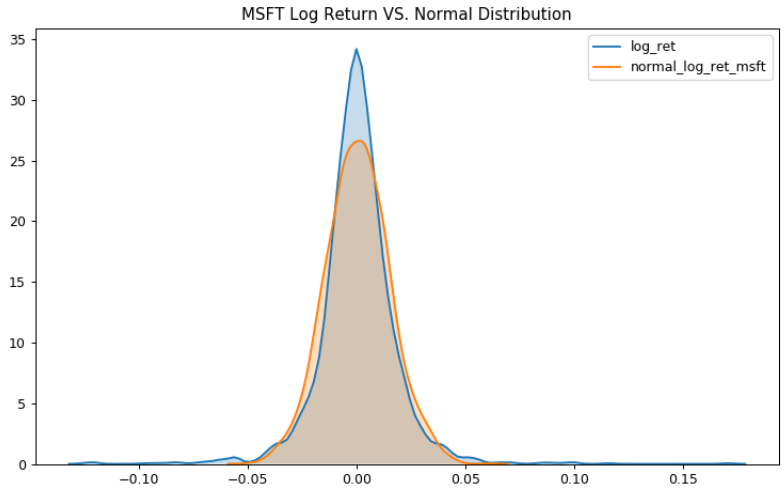

Fig. 11. MSFT Log Return vs. Normal Distributed Log Return

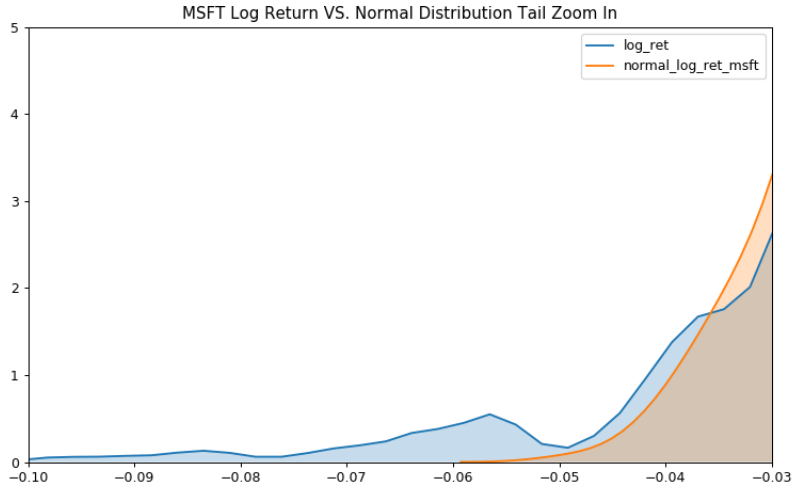

Fig. 12. Tail of MSFT Log Return vs. Normal Distributed Log Return

Comments:

The MSFT log price return (blue) shows a significantly fatter tail than that of normal distribution (yellow) according to Figure 12. Also, MSFT empirical distribution successfully demonstrates the higher peak than that of a Gaussian distribution in Figure 11. This further indicates that the assumption of log normal return of stock price is inaccurate.

• Volatility clustering

Below I have plotted correlation on squared MSFT log price return and its rescaled plot for better observation.

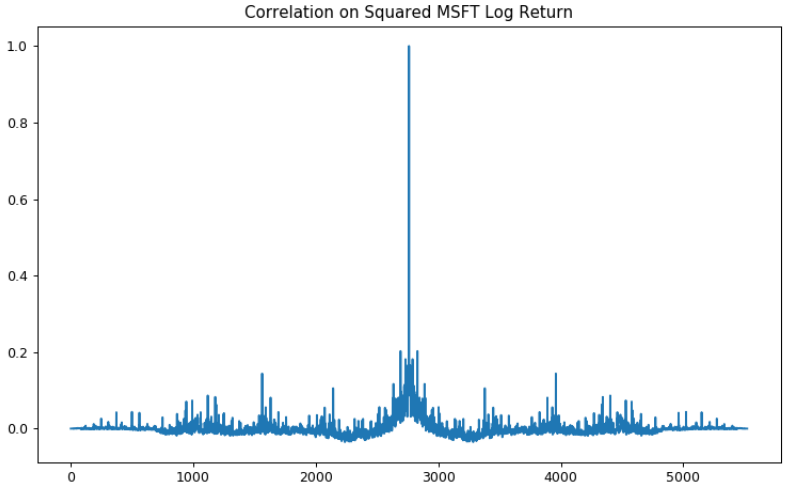

Fig. 13. Correlation on MSFT Log Return

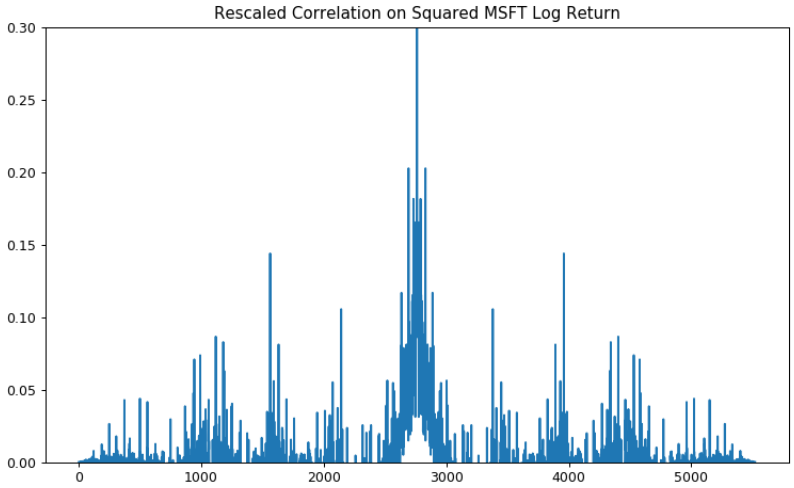

Fig. 14. Rescaled Correlation on MSFT Log Return

Comments:

Like the pattern of simulated log return data, the series of empirical squared returns on MSFT data exhibits a significant and slowly decaying pattern, which suggests volatility clustering.

Reference:

[1] Lisa Borland and J-Ph Bouchaud. On a multi-timescale statistical feed- back model for volatility fluctuations. *arXiv preprint physics/0507073,* 2005.