



Computational Intelligence &
Machine Learning Group



UNIVERSITÀ DI PISA



Deep Learning for Graphs – Module II

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DIPARTIMENTO DI INFORMATICA - UNIVERSITA' DI PISA

<https://sites.google.com/view/dl4sd>

Module Outline

- ❖ Convolutional neural network for graphs
 - ❖ Spectral approach
 - ❖ Generalizing spatial convolutions
- ❖ Contextual approaches (layering)
 - ❖ Node2Vec et al
 - ❖ Learning unsupervised graph embedding
- ❖ Research directions
- ❖ Applications

A Nomenclature Nightmare

Deep learning for graphs

Neural networks for graphs

Graph neural networks

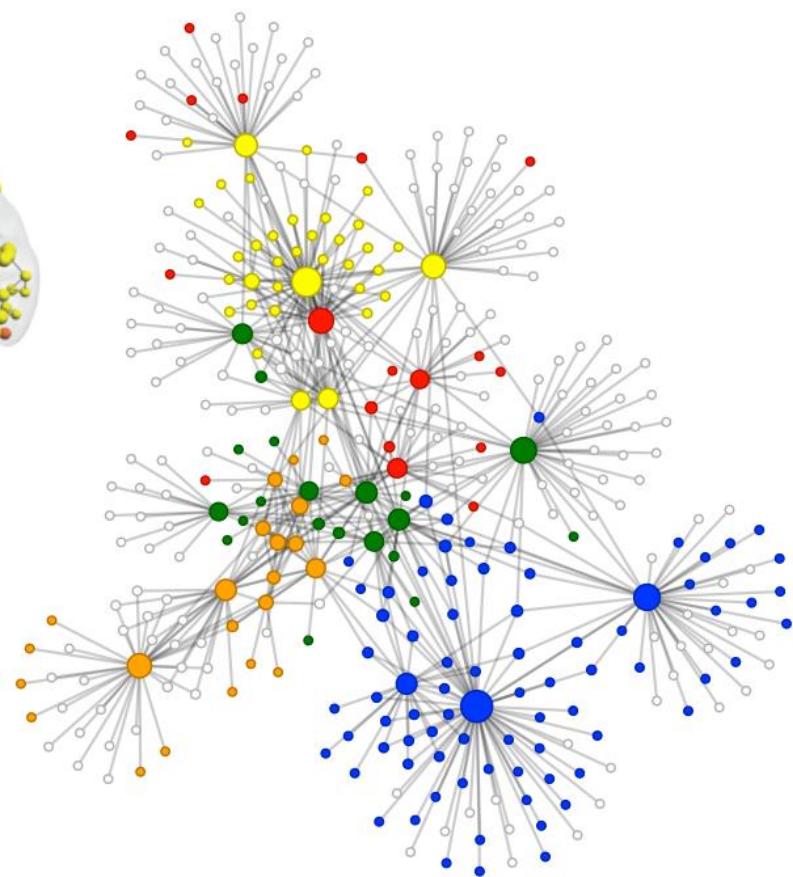
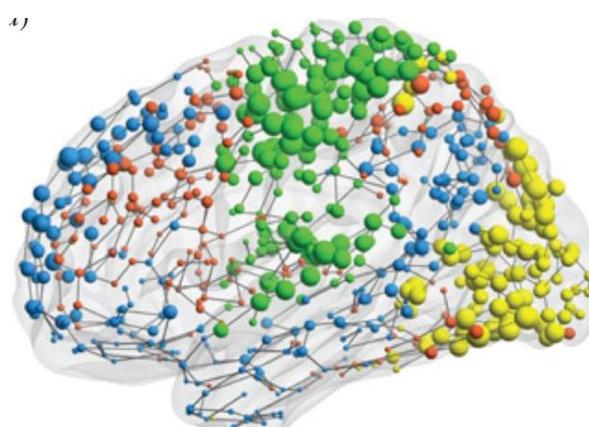
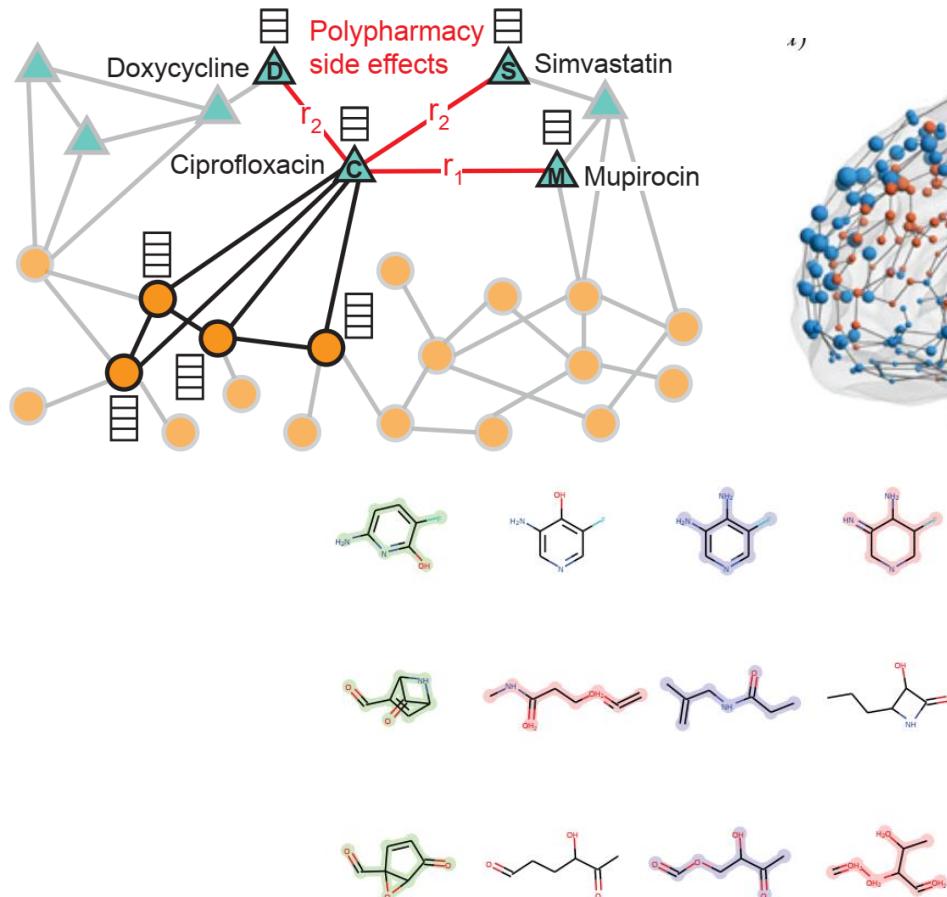
CNN for/on graphs

Graph CNN

Learning graph/node embedding

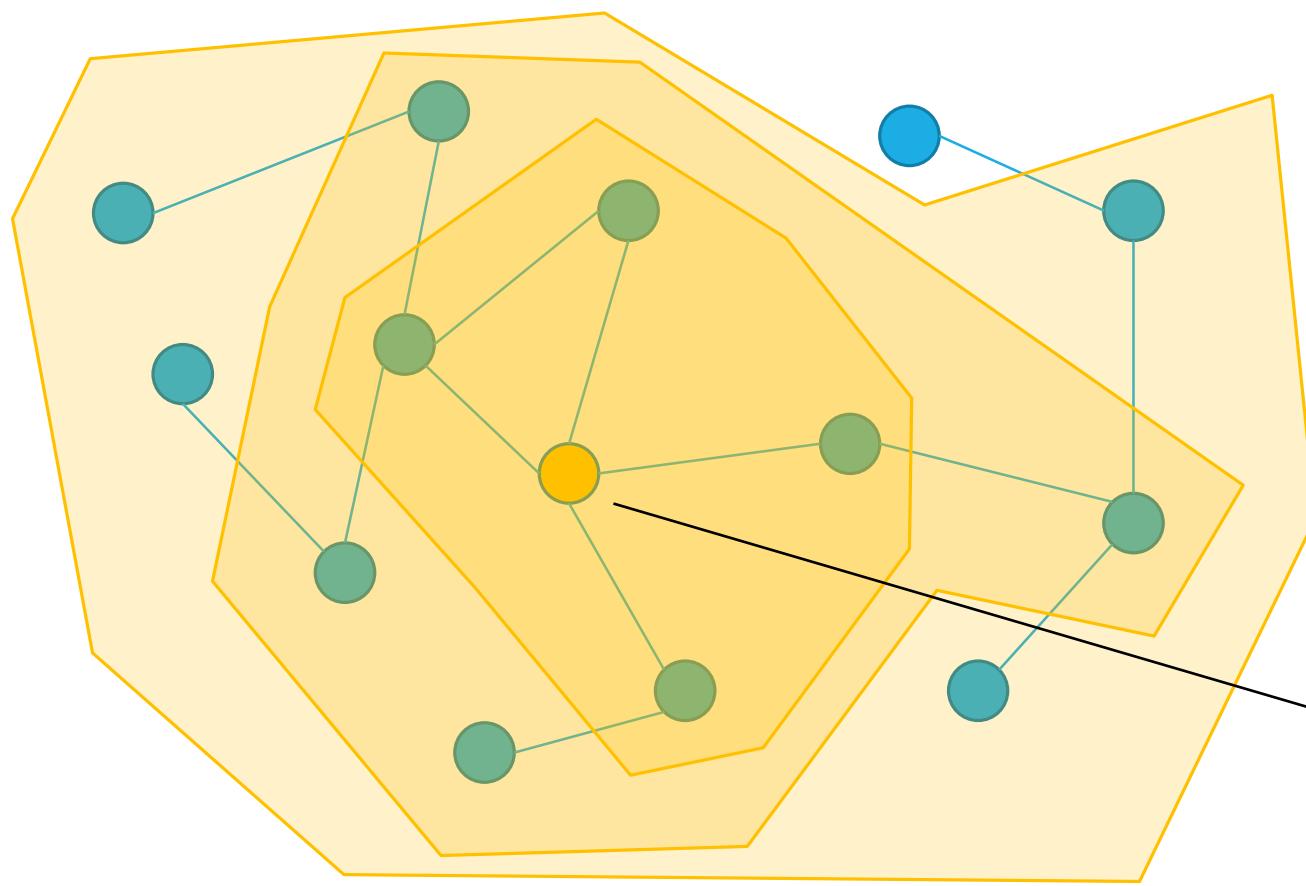
Geometric deep learning

Motivation – Learning with graphs



Motivation – Deep Learning with graphs

Hierarchical representation learning allows to efficiently diffuse information through graph structure

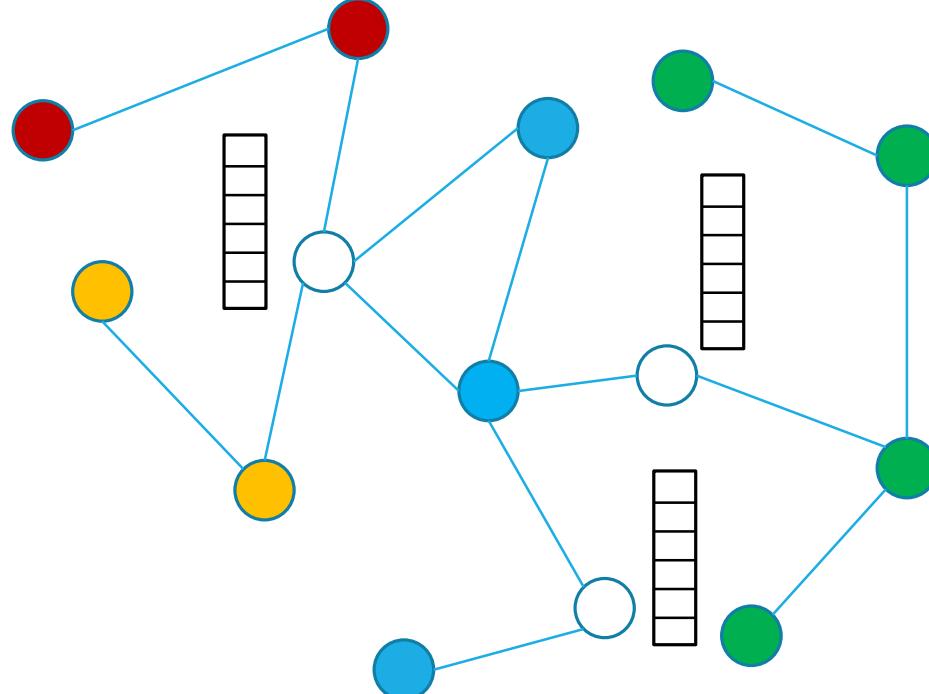


Node representation depends on its context (shorter first-longer later)

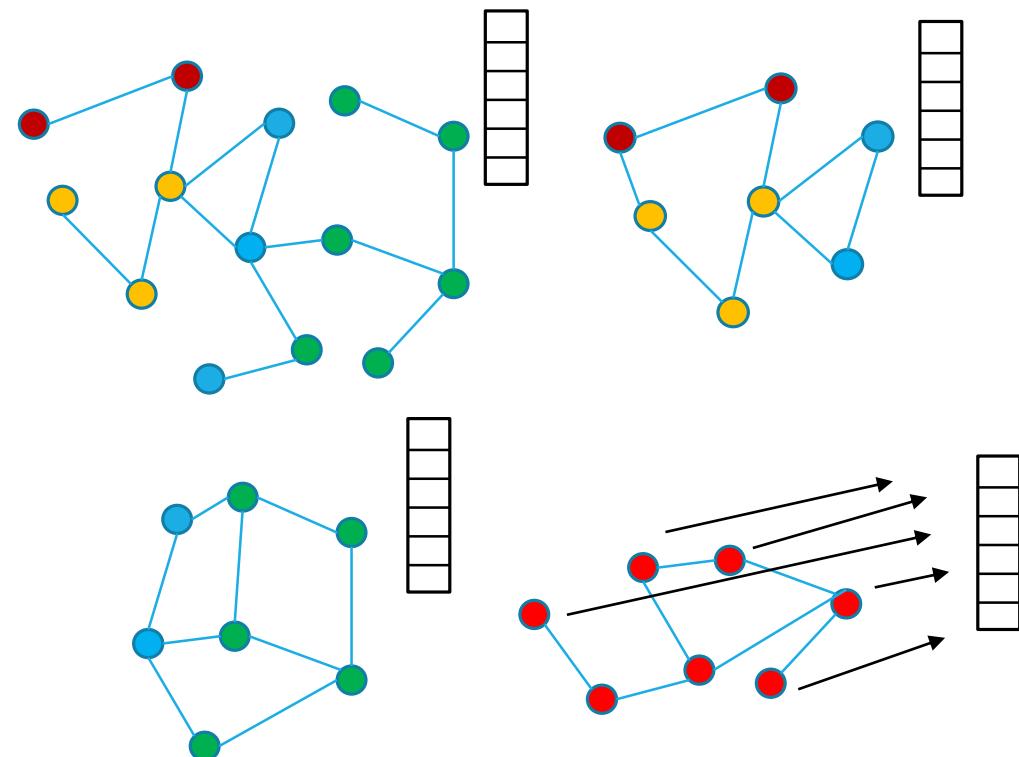


Predictive Tasks

Network data



Graph classification/regression



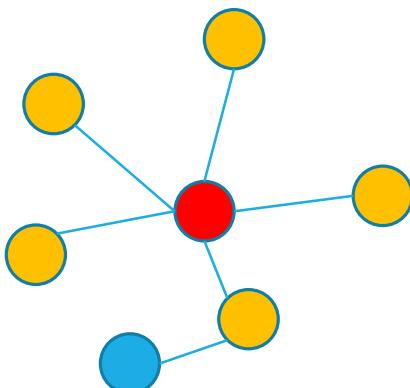
Convolutional Neural Networks for Graphs

How to Perform Convolutions on Graphs

SPATIAL DOMAIN



What is the equivalent of sliding a kernel to aggregate local spatial information?



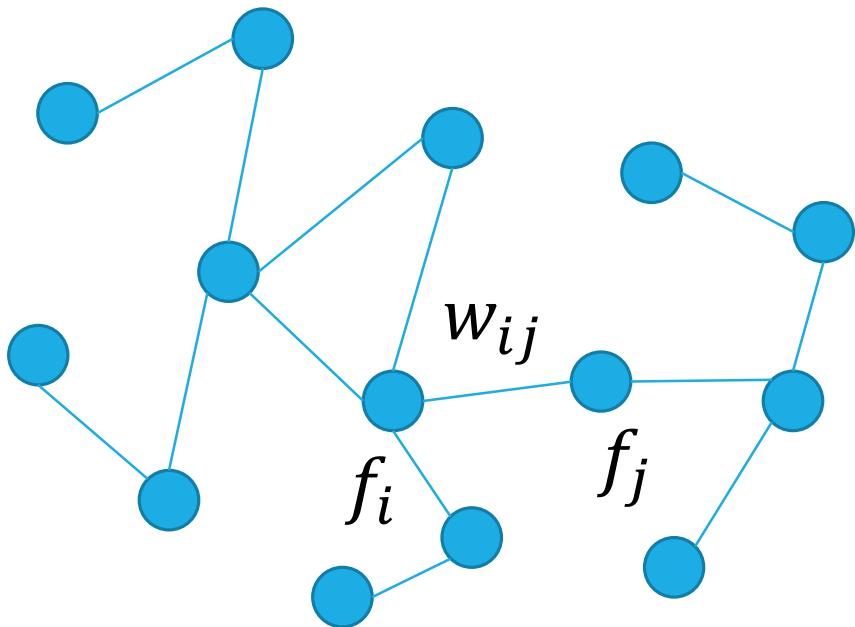
SPECTRAL DOMAIN

$$\mathcal{F}(f * g) = \mathcal{F}(f) \times \mathcal{F}(g)$$

Exploit the **Convolution Theorem** and **Fourier analysis** to perform convolutions in the spectral domain

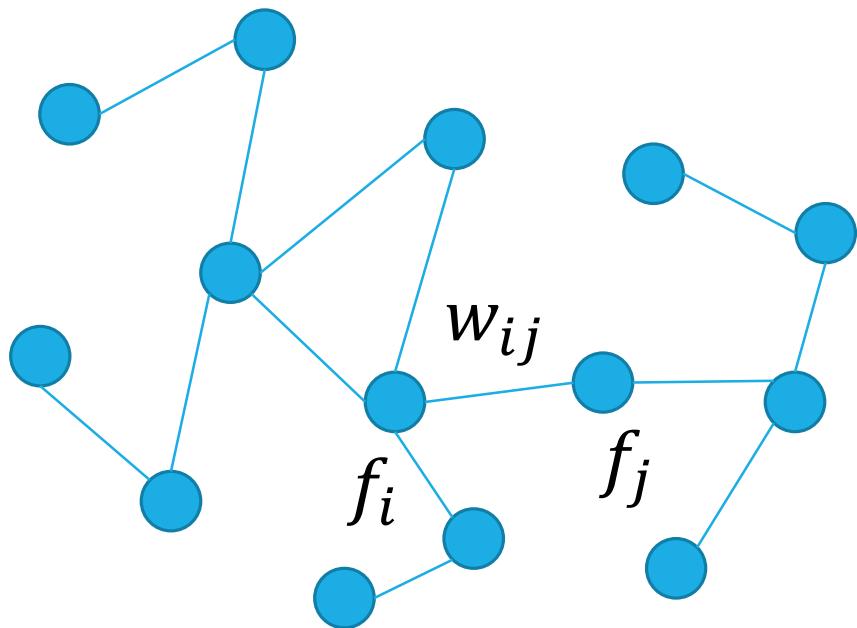
Spectral Domain Convolutions

The Scenario



- ❖ Single weighted undirected graph
- ❖ $w_{ij} > 0$ weight of the i-j edge
- ❖ Functions f_i attaching values (i.e. labels/signals x_i) to nodes i
- ❖ Task: process the signals defined on the graph structure

Graph Fundamentals (I)



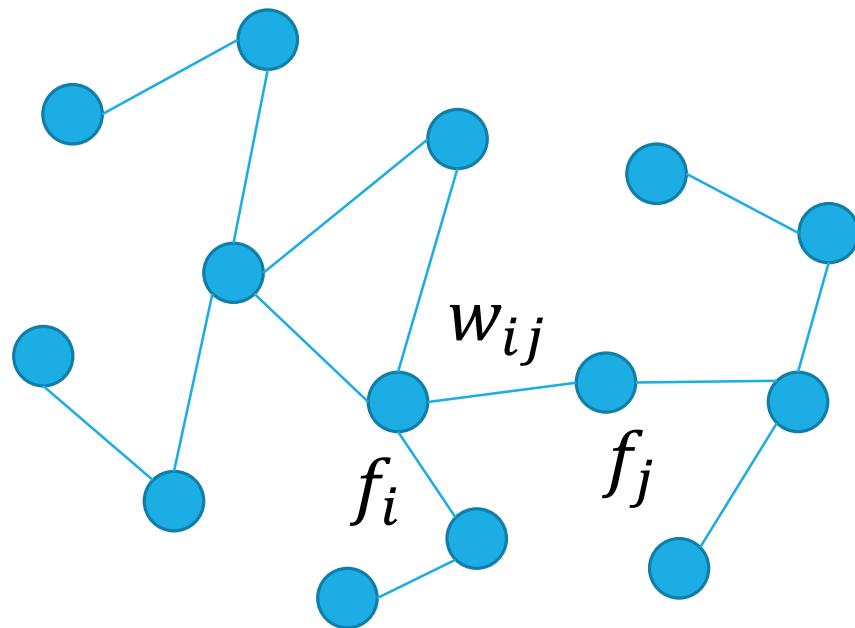
Adjacency matrix

$$A = \begin{bmatrix} 0 & \cdots & w_{1N} \\ \vdots & \ddots & \vdots \\ w_{N1} & \cdots & 0 \end{bmatrix}$$

Degree matrix

$$D = \begin{bmatrix} D_{11} = \sum_{j \in \mathcal{N}_1} w_{1j} & 0 & 0 & 0 \\ 0 & D_{22} & 0 & 0 \\ 0 & 0 & D_{33} & 0 \\ 0 & 0 & 0 & D_{44} \end{bmatrix}$$

Graph Fundamentals (II)



Difference between vertex i information
and the weighted contribution from its
neighbors

We are interested in the **Laplacian**

$$L = D - A$$

L is **symmetric and positive semidefinite**

L is an **operator on the vector space of functions** $f = [f_1 \dots f_N]$

$$(Lf)(i) = \sum_{j \in \mathcal{N}_i} w_{ij} (f_i - f_j)$$

Normalized Laplacian

In practice the **normalized (graph) Laplacian** is often used

$$L = I_n - D^{-1/2}AD^{-1/2}$$

Normalization ensures that application of the Laplacian operator is scale invariant (performs an actual local averaging of the neighbors)

I_n is **the nxn identity matrix** allowing a node to consider its signal in the weighted local average

One last bit

Since the Laplacian is real symmetric and positive semidefinite

$$L = U\Lambda U^T$$

Λ diagonal matrix of eigenvalues $\lambda_k \geq 0$

U matrix containing complete orthonormal basis $\{u_k\}_k$ associated to each λ_k

So where are the graph convolutions?

The Convolution Theorem

The Fourier transform of the $f * g$ convolution can be transformed into a matrix multiplication of the transform of the functions f and g

$$\mathcal{F}(f * g) = \mathcal{F}(f) \times \mathcal{F}(g)$$

Fourier Analysis (in 1 ugly slide)

Decompose a function f as a combination of vectors e_k from an orthonormal basis

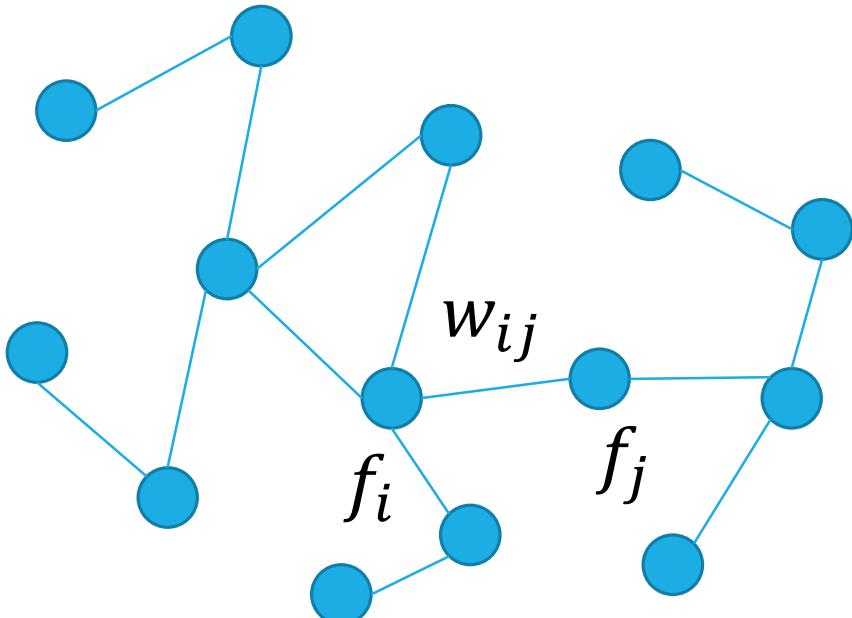
$$f = \sum_{k \geq 0} \tilde{f}_k e_k$$

A signal is a superposition of basis functions at different frequencies k

\tilde{f}_k are the Fourier coefficients (at frequency k), obtained by projecting f on the basis (easily sometimes, e.g. FFT)

Back on the Graph

The eigendecomposition of the graph Laplacian L gives us an orthonormal basis U for the graph



The graph Fourier transform of a signal $\mathcal{F}(f_i)$ is $\tilde{f}_i = U^T f_i$

Can be inverted using $f_i = U \tilde{f}_i$

Eigenvalues λ_k are the graph frequencies

Spectral Graph Convolution

Given a graph G and the associated Fourier basis we can compute the graph convolution of its node signals f with a filter

$$(f *_G g) = \mathcal{F}^{-1}(\mathcal{F}(f) \mathcal{F}(g)) = UU^T g U^T f$$

Convolutional filter in spectral domain

Graph equivalent of the learnable CNN
filter matrix W

$$h = UWU^T f$$

Parameterized graph convolution,
with parameter matrix W

About the Spectral Filter Parameters

$$\mathbf{h} = \mathbf{U}\mathbf{W}\mathbf{U}^T\mathbf{f}$$

- ❖ \mathbf{f} vertex vector function (e.g. the matrix holding vector numerical labels)
- ❖ \mathbf{W} is a $[N \times N]$ diagonal matrix of learnable filter coefficients
- ❖ Like in spatial convolution \mathbf{W} can contain all free parameters (easiest choice)

- ❖ Filters are basis dependent ($\mathbf{W} = \mathbf{U}^T \mathbf{g}$), hence they are graph specific
- ❖ With unconstrained \mathbf{W}
 - ❖ Convolution has $O(N)$ parameters
 - ❖ Forward and inverse transform cost $O(N^2)$
 - ❖ No guarantee of spatial localization of the resulting filter

Need smart ways of
constraining \mathbf{W}

Spectral Graph Convolution Models (I)

Bruna, Zaremba, Szlam, LeCun, ICLR 2014; Henaff, Bruna, LeCun, Arxiv 2015

- ❖ Filter localization in space by filter **smoothness** in frequency domain
- ❖ Parametrize the filter using a **smooth spectral transfer function** $W(\Lambda)$

Defferrard, Bresson, Vandergheynst, NIPS 2016 (ChebNet)

- ❖ Represent spectral transfer function as a **Chebychev polynomial** of order r
- ❖ $O(r)$ parameters θ_j and $O(Nr)$ computational cost

$$W(\Lambda) = \sum_{j=0}^r \theta_j \Lambda^j$$

ChebNet Architecture

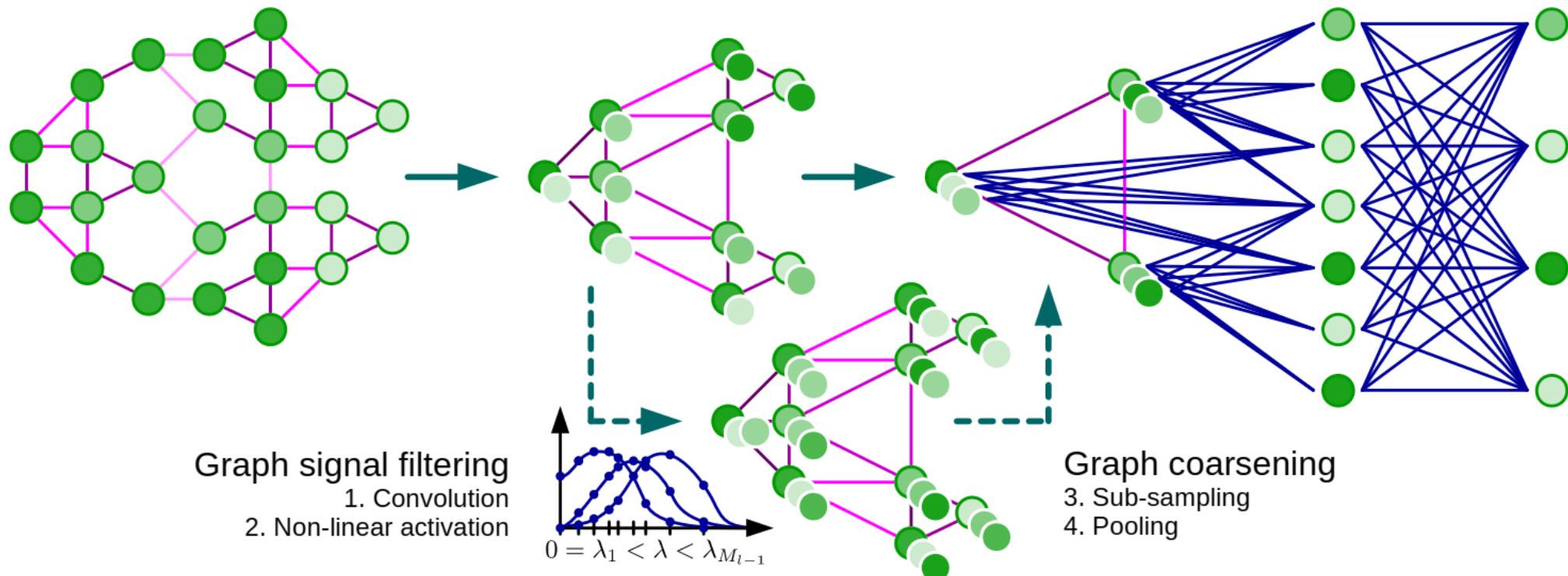


Figure from Defferrard, Bresson, Vandergheynst, NIPS 2016

Spectral Graph Convolution Models (II)

Kipf, Welling, ICLR 2017

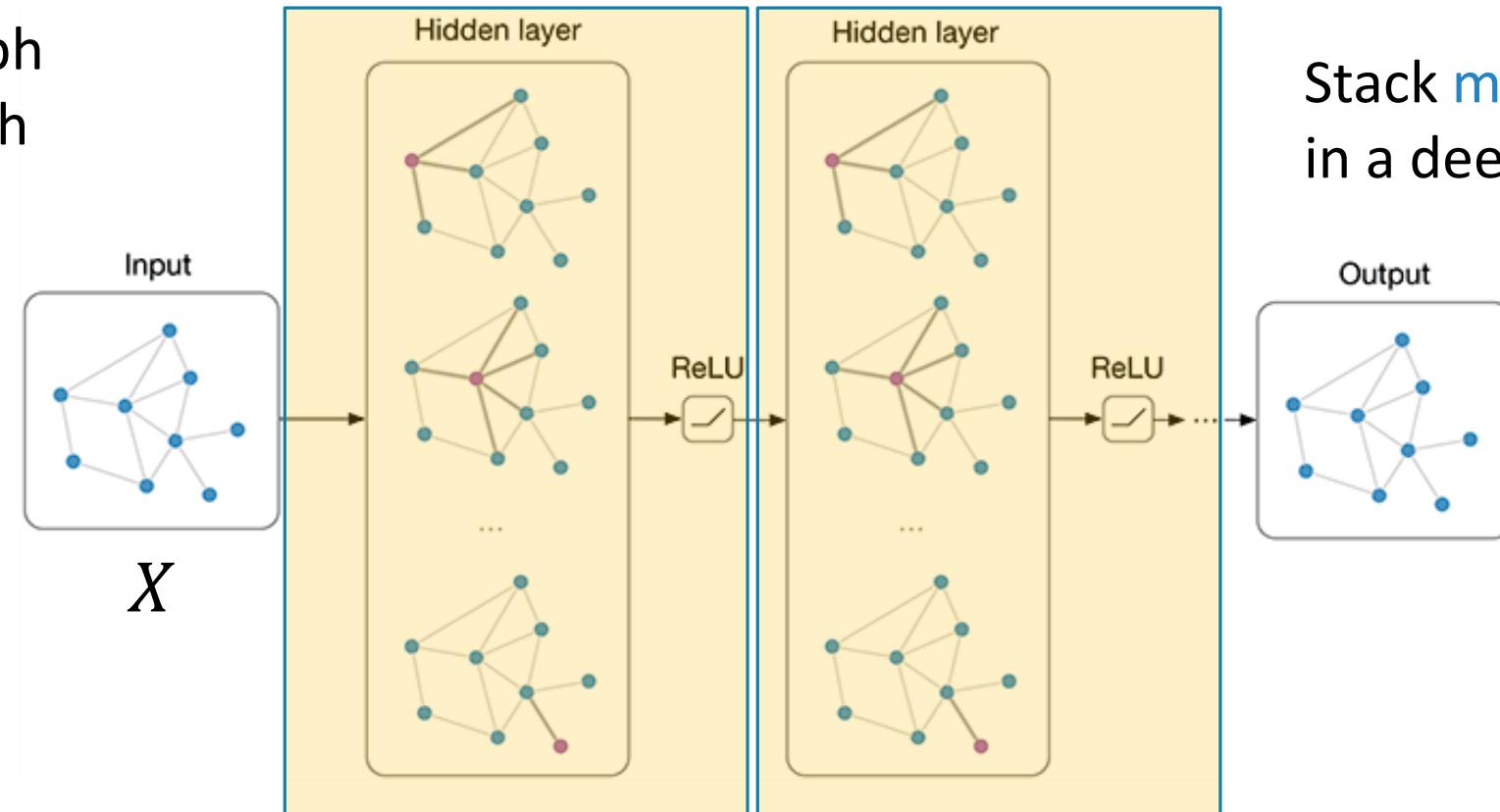
- ❖ Simplify Chebychev polynomials to degree $r = 2$ and approximate largest Laplacian eigenvalue to ≈ 2 (using renormalization to control numerical stability)
- ❖ Constrain parameters so that $\theta = \theta_0 = \theta_1$ (single parameter)

$$\mathbf{h} = \theta(I_n + D^{-1/2}AD^{-1/2}) \mathbf{f}$$

Graph Convolutional Networks (GCN) framework

GCN Neural Architecture

Relabel the graph structure at each layer with the newly obtained embedding
 (Weisfeiler-Lehman like)

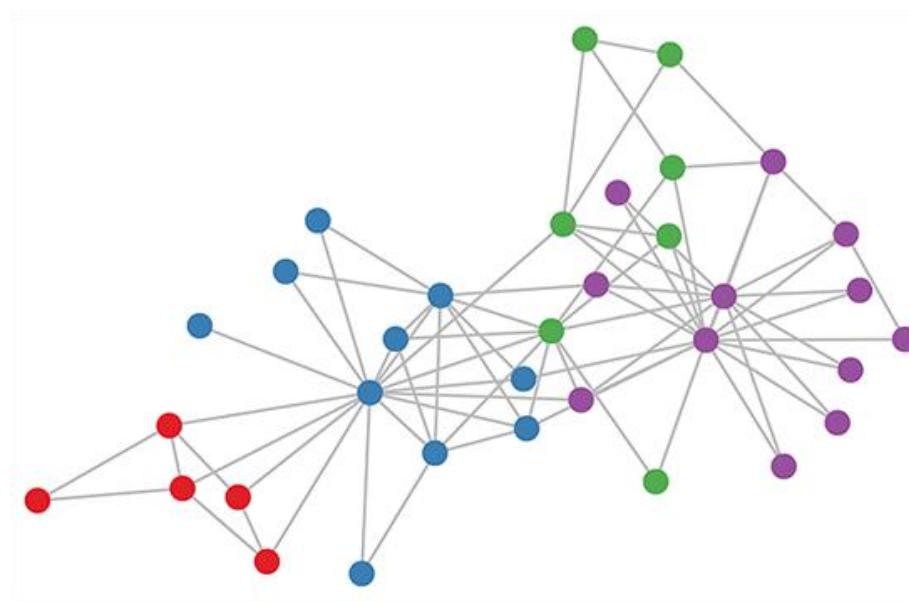


$$h^1 = \sigma(\hat{D}^{-1/2} \hat{A} \hat{D}^{-1/2} X W^1)$$

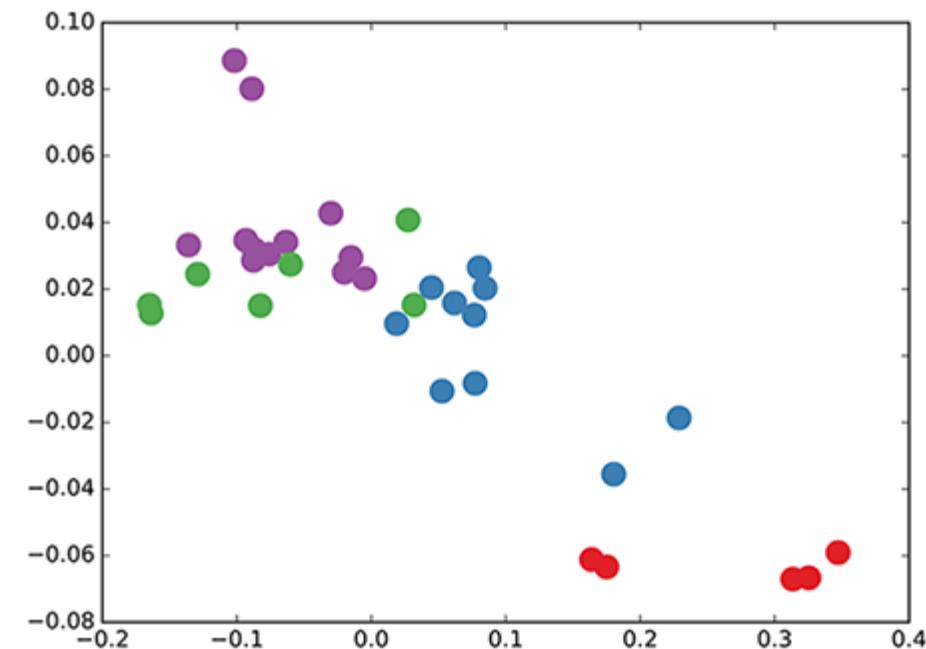
$$h^2 = \sigma(\hat{D}^{-\frac{1}{2}} \hat{A} \hat{D}^{-\frac{1}{2}} h^1 W^2)$$

Stack **many simple layers** in a deep-like fashion

How Good Are the Vertices Embeddings?

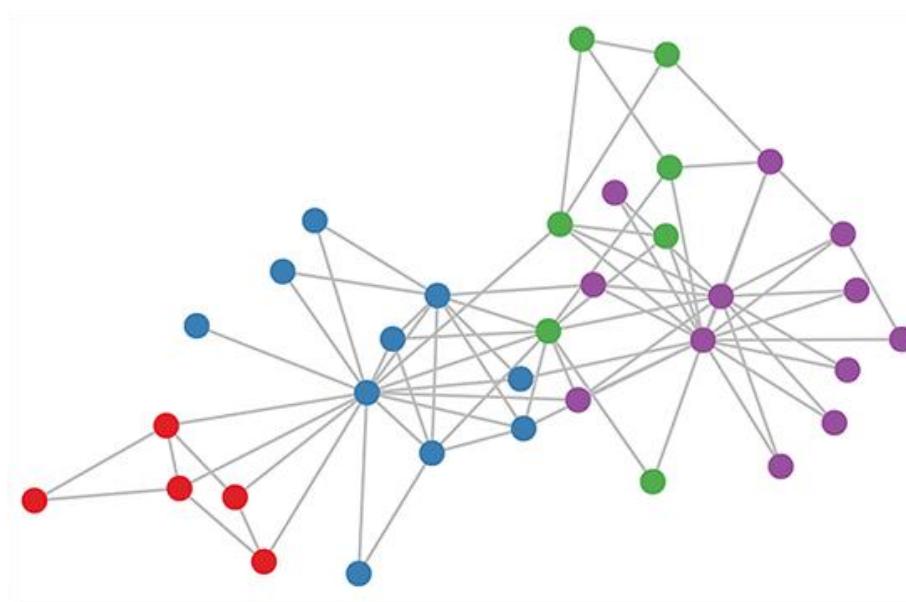


Karate club community
detection network

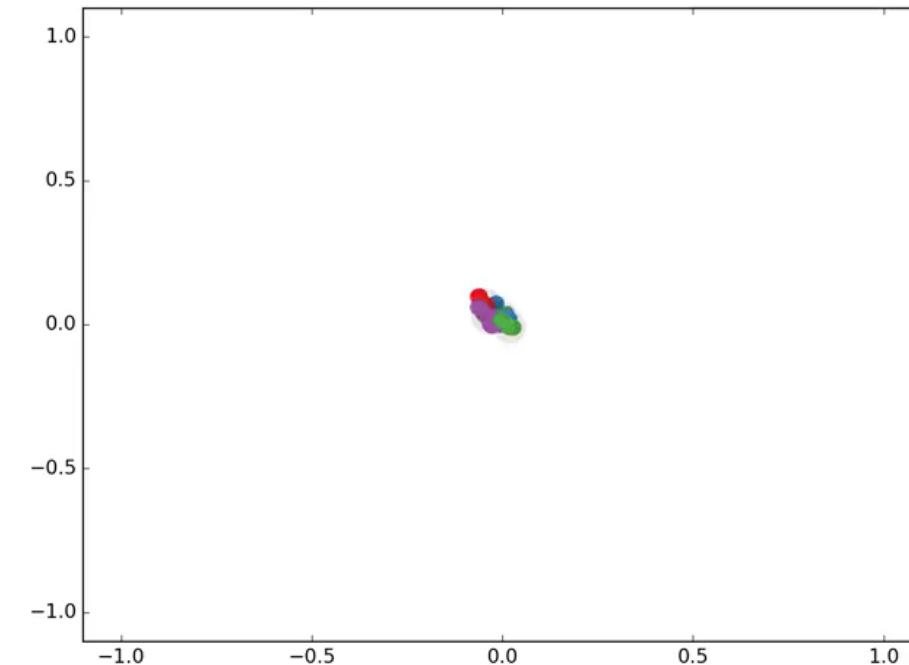


GCN embedding of vertices with randomly
initialized untrained weights

How Good Are the Vertices Embeddings?



Karate club community
detection network



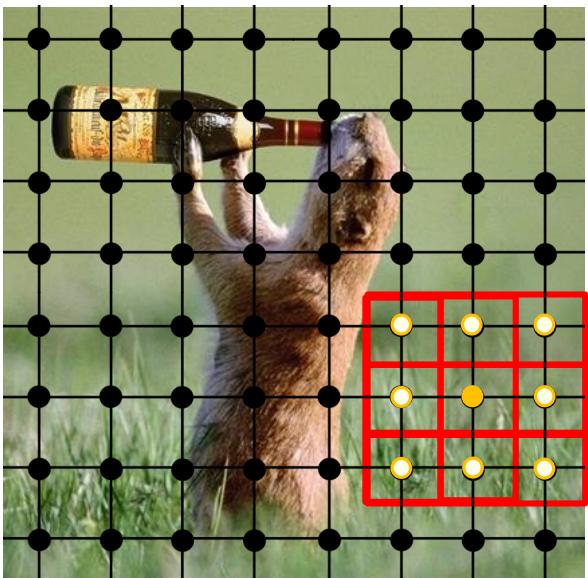
GCN embedding of vertices evolution
during training

A Final Word on Spectral Approaches

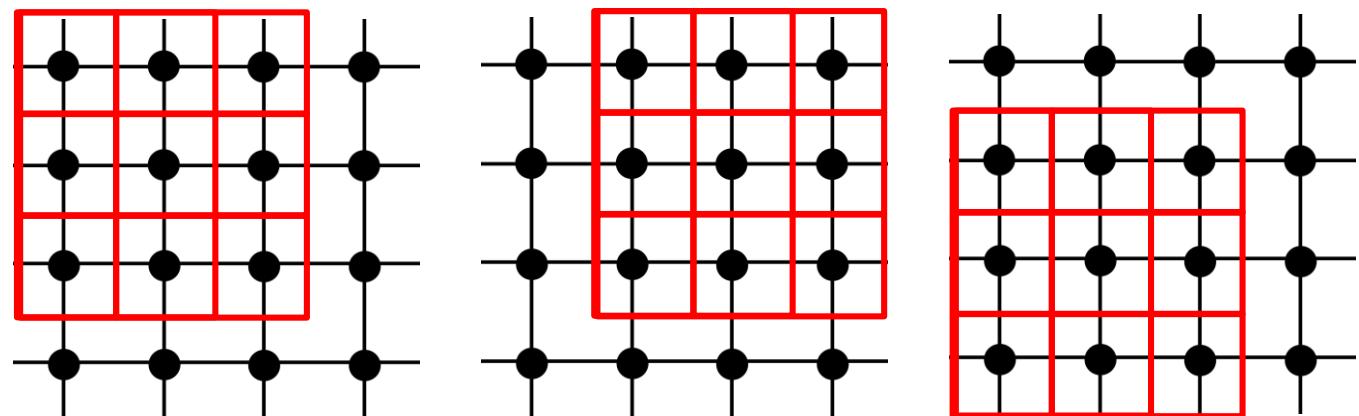
- ❖ Cannot handle multiple graphs due to convolution dependency on Laplacian (use on network data tasks)
- ❖ Mostly limited to undirected graphs with unlabeled edges
 - ❖ Extension to directed graphs using Laplacian block structure and triangular motifs (Benson et al 2016; Monti, Otness, Bronstein 2018)
- ❖ Difficult control on context diffusion through the graph structure
- ❖ Working with the Laplacian can be impractical for large graphs

Spatial Domain Convolutions

A Graph View on Convolutions



Visual convolutions are graph convolutions on a **regular grid**

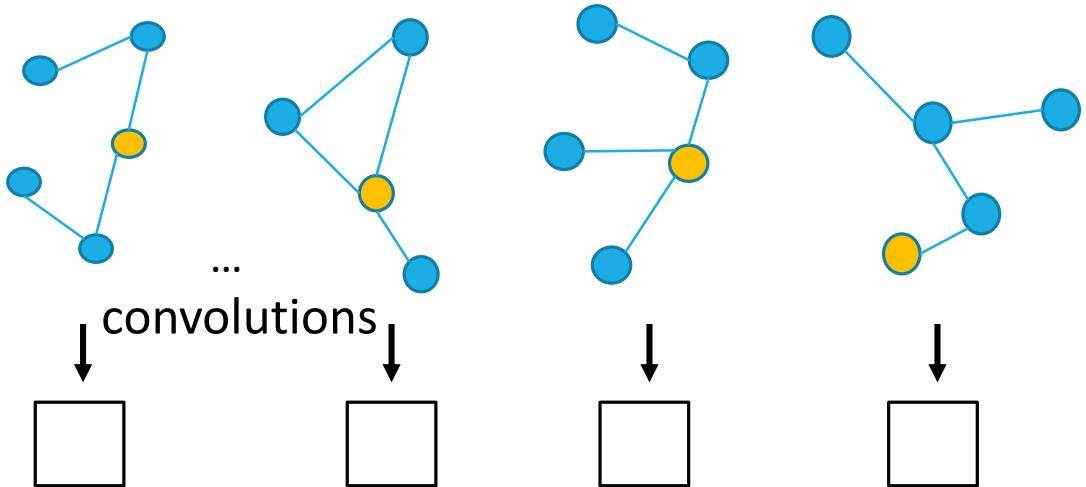
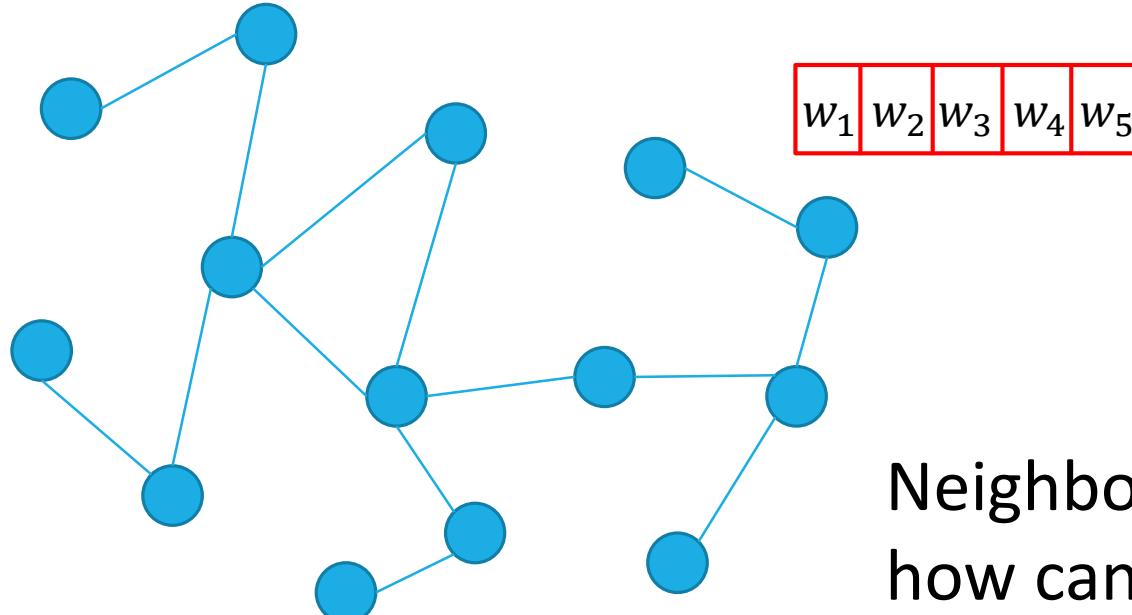


Plus some **key assumptions** which make it difficult to directly apply them to graphs

- ❖ Regular neighborhood
- ❖ Existence of a total node ordering

Node Neighborhoods

Example of 4-neighborhoods



Neighborhoods depend on node ordering:
how can I get coherent node ordering across
multiple graphs?

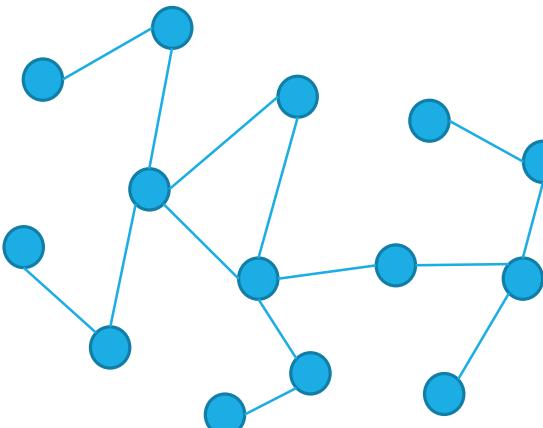
PATCHY-SAN (I)

Niepert, Ahmed, Kutzkov, ICML 2016

Leverage graph labelling techniques (e.g. Weisfeiler-Lehman) to determine a coherent ordering within the graph and between the graphs

1. Node Sequence Selection

Apply graph labelling to reorder the adjacency matrix and visit the graph in the given order

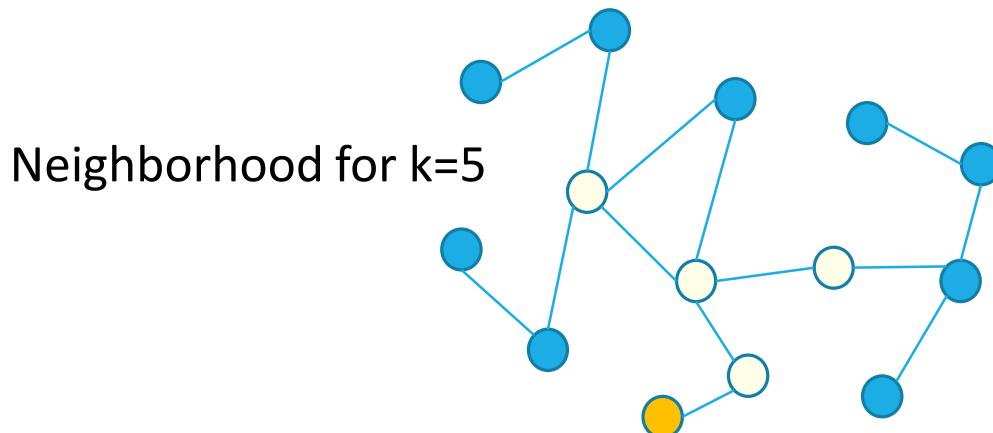


PATCHY-SAN (II)

Niepert, Ahmed, Kutzkov, ICML 2016

2. Assemble the neighbourhood

Given the current vertex assemble its neighbourhood (of fixed size k) by breadth first search, taking the first k vertices (pad with the neighbours if necessary to reach k nodes).



Parametric convolutional
filter of size k

w_1	w_2	w_3	w_4	w_5
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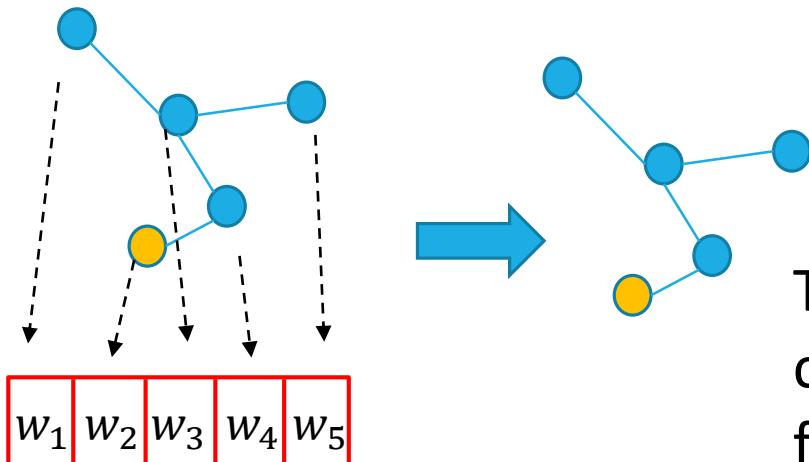
In what order do I map nodes to
filter parameters?

PATCHY-SAN (III)

Niepert, Ahmed, Kutzkov, ICML 2016

3. Neighbourhood normalization

Given the current neighbourhood apply labelling techniques to reorder nodes before applying them to the convolutional filter



Graph normalization is
NP-complete

The result of the convolution is the encoding for the target vertex

PATCHY-SAN Summary

- ❖ Can handle multiple graphs, undirected and directed, with labels on both edges and nodes
- ❖ Can reuse CNN machinery: striding, pooling, ...
- ❖ Performance relies heavily on quality of the ordering
- ❖ Edge labels are used only for computing node ordering
- ❖ How to choose neighborhood size?
- ❖ Worst case complexity is exponential due to graph normalization

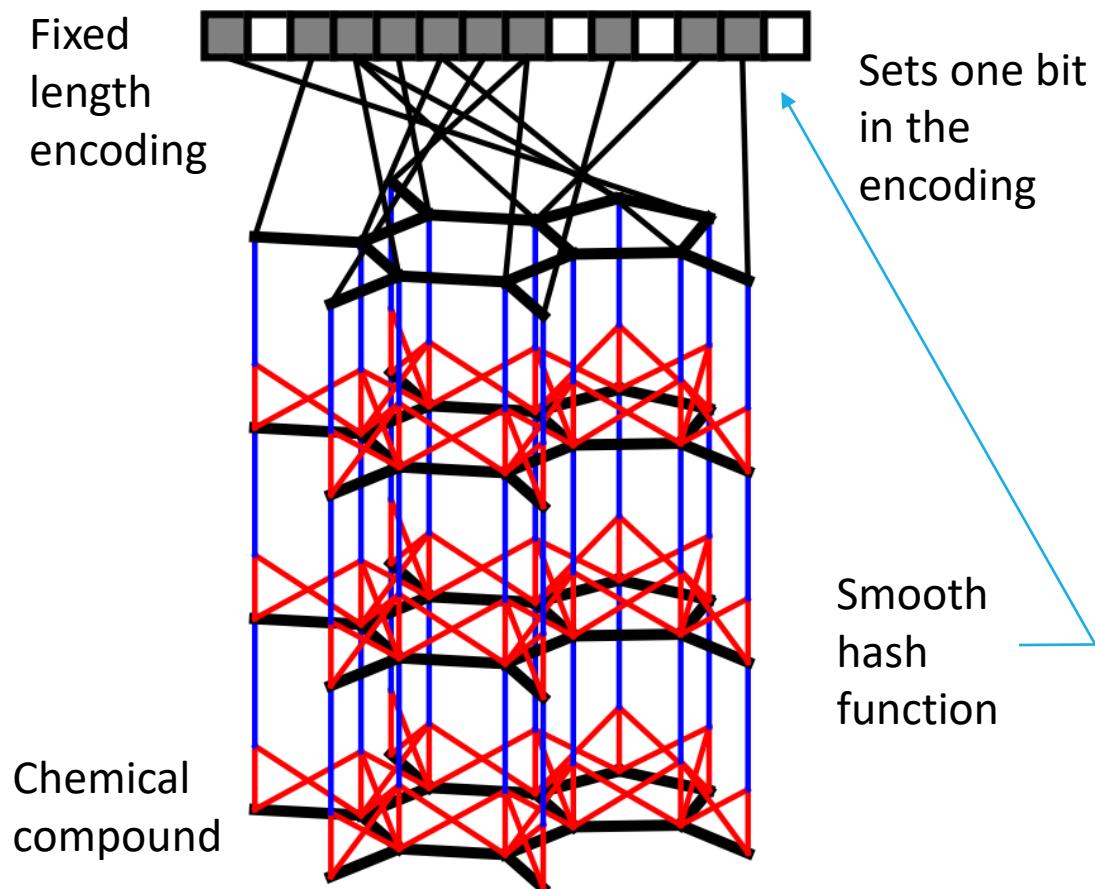
Contextual Graph Processing (Layering)

On the Need of Efficient Context Diffusion

- ❖ Defining **explicit vertex neighborhoods** on multi-graphs is **difficult** and requires **computationally intensive** solutions
- ❖ An **implicit** approach exploiting the available graph structure while handling **loops**
 - ❖ **Contractive** approach – Use dynamical systems with guaranteed attractors to obtain stable vertex encodings (GNN)
 - ❖ **Contextual** approaches – Use layering to avoid direct dependencies on cycles

Contextual Graph Convolutions

Molecular (Circular) Fingerprints

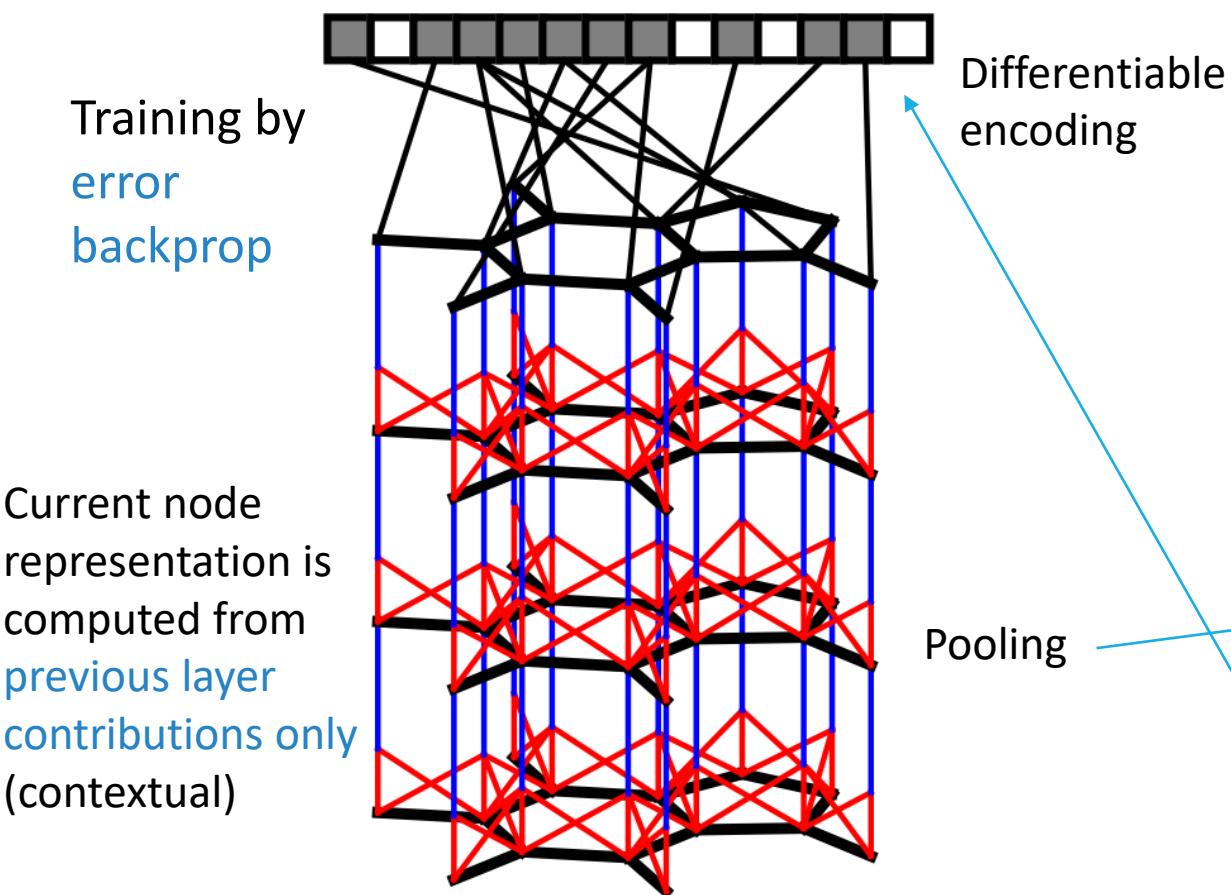


Algorithm 1 Circular fingerprints

```

1: Input: molecule, radius  $R$ , fingerprint length  $S$ 
2: Initialize: fingerprint vector  $\mathbf{f} \leftarrow \mathbf{0}_S$ 
3: for each atom  $a$  in molecule
4:    $\mathbf{r}_a \leftarrow g(a)$             $\triangleright$  lookup atom features
5: for  $L = 1$  to  $R$             $\triangleright$  for each layer
6:   for each atom  $a$  in molecule
7:      $\mathbf{r}_1 \dots \mathbf{r}_N = \text{neighbors}(a)$ 
8:      $\mathbf{v} \leftarrow [\mathbf{r}_a, \mathbf{r}_1, \dots, \mathbf{r}_N]$      $\triangleright$  concatenate
9:      $\mathbf{r}_a \leftarrow \text{hash}(\mathbf{v})$             $\triangleright$  hash function
10:     $i \leftarrow \text{mod}(r_a, S)$             $\triangleright$  convert to index
11:     $\mathbf{f}_i \leftarrow 1$                     $\triangleright$  Write 1 at index
12: Return: binary vector  $\mathbf{f}$ 
  
```

Neural (differentiable) Fingerprints



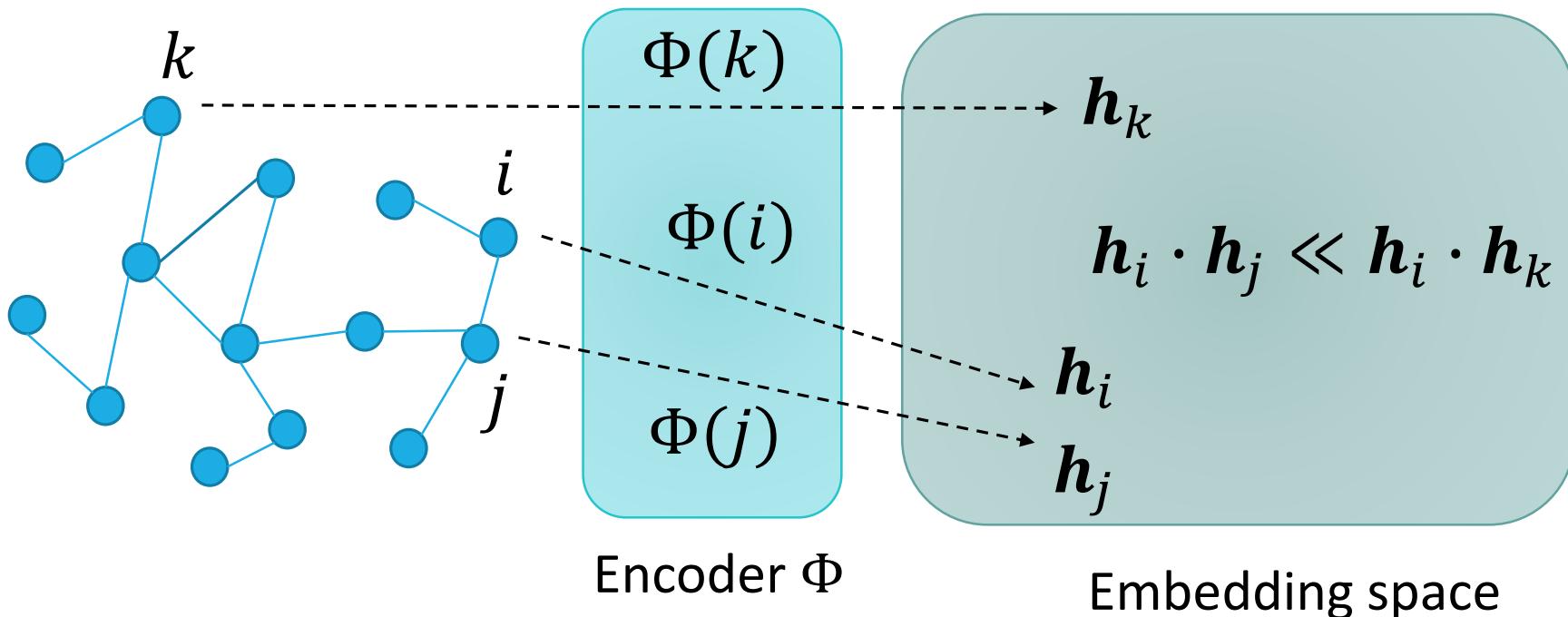
Algorithm 2 Neural graph fingerprints

- 1: **Input:** molecule, radius R , **hidden weights** $H_1^1 \dots H_R^5$, **output weights** $W_1 \dots W_R$
- 2: **Initialize:** fingerprint vector $\mathbf{f} \leftarrow \mathbf{0}_S$
- 3: **for** each atom a in molecule
- 4: $\mathbf{r}_a \leftarrow g(a)$ \triangleright lookup atom features
- 5: **for** $L = 1$ to R \triangleright for each layer
- 6: **for** each atom a in molecule
- 7: $\mathbf{r}_1 \dots \mathbf{r}_N = \text{neighbors}(a)$
- 8: $\mathbf{v} \leftarrow \mathbf{r}_a + \sum_{i=1}^N \mathbf{r}_i$ \triangleright sum
- 9: $\mathbf{r}_a \leftarrow \sigma(\mathbf{v} H_L^N)$ \triangleright smooth function
- 10: $\mathbf{i} \leftarrow \text{softmax}(\mathbf{r}_a W_L)$ \triangleright sparsify
- 11: $\mathbf{f} \leftarrow \mathbf{f} + \mathbf{i}$ \triangleright add to fingerprint
- 12: **Return:** real-valued vector \mathbf{f}

Node Embedding

The Key Idea

- ❖ Encode graph vertices into a vector space where vertex similarities (however defined) are preserved



Vertex Encoder

- ❖ A complex function which can **take into account node context** when generating the vectorial encoding

$$\Phi(k) = \Phi(k|G) \text{ or } \Phi(k|N_k)$$

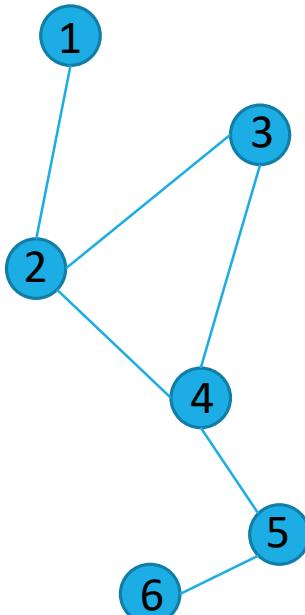
- ❖ **Parameterized function** so that embedding can be learned to minimize an error (supervised or unsupervised)

$$\mathbf{h}_k = \Phi(k|G, \theta) \text{ and } \min_{\theta} L_{\theta}(\mathbf{h}_1, \dots, \mathbf{h}_k, \dots, Y)$$

- ❖ Model differ in how they compute the **encoding and the loss**

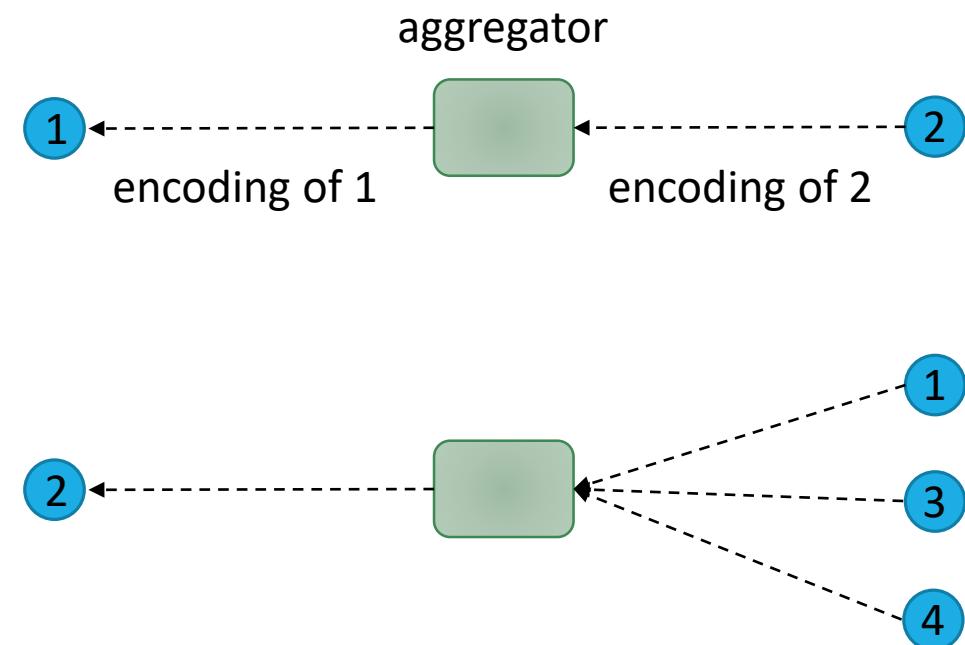
Neighborhood Aggregation

Vertex **encoding** is generated (again) using the **node neighborhood**



Neighborhood can
be made

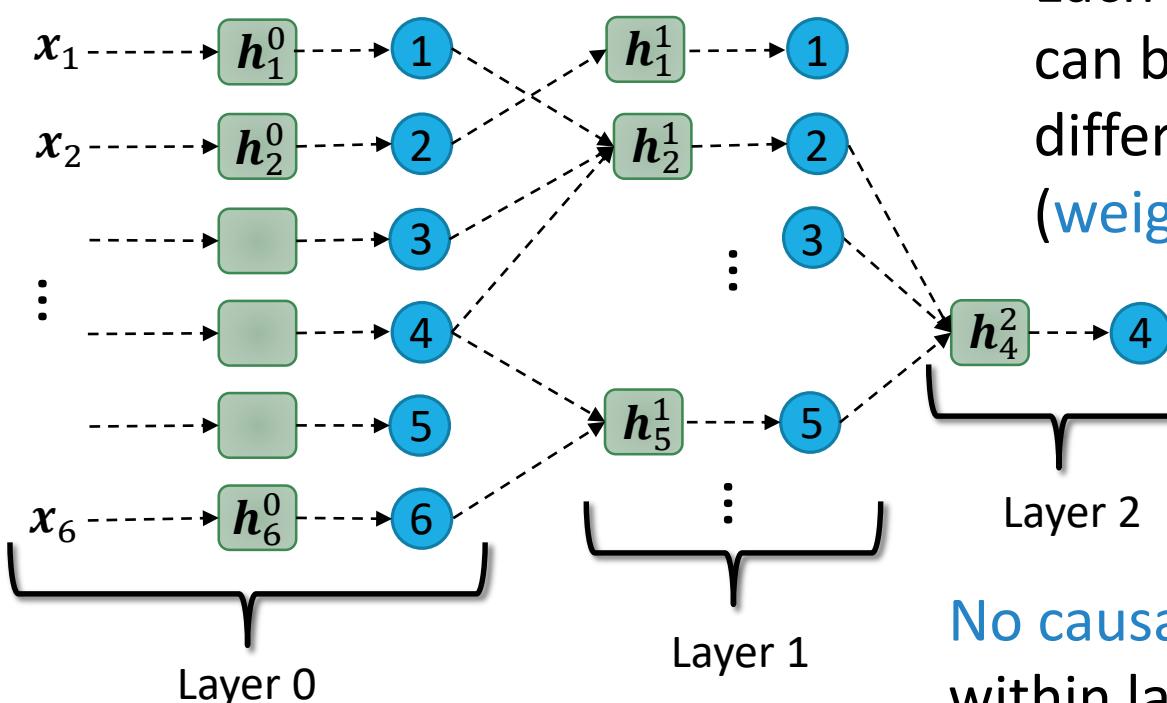
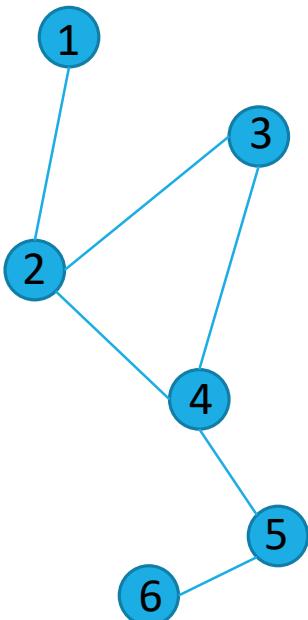
- By adjacent
nodes
- By sampling a
fixed set of nodes
in a predefined
neighborhood



The encoding of 1 and 2 are **mutually
dependent**: how do we solve it?

Layered Encoding

Compute embedding in a layered fashion (contextual)



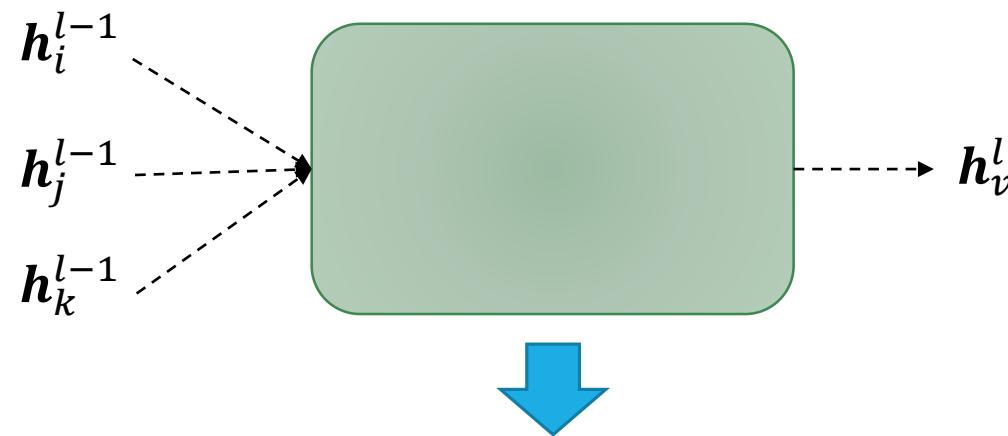
Each layers' aggregator can be parameterized differently or tied (weight sharing)

Layer 2

No causal dependencies within layer

What is inside of the Box?

A **learning model** of course (e.g. a neural network) including an aggregation function to **handle size-varying neighborhoods**



A simple model

$$\mathbf{h}_v^l = \sigma \left(\mathbf{W}_l \sum_{i \in N(v)} \frac{\mathbf{h}_i^{l-1}}{|N(v)|} + \widehat{\mathbf{W}}_l \mathbf{h}_v^{l-1} \right)$$

Simple feedforward
layer

Neighborhood average

Can also include contribution
from vertex label at each level

Generalizing Aggregation - GraphSage

Hamilton, Ying, Leskovec, NIPS 2017

$$\mathbf{h}_v^l = \sigma(\mathbf{W}_l AGG(\{\mathbf{h}_i^{l-1} : i \in N(v)\}), \widehat{\mathbf{W}}_l \mathbf{h}_v^{l-1})$$

Can use any reasonable **aggregation strategy**

Mean

$$\sum_{i \in N(v)} \frac{\mathbf{h}_i^{l-1}}{|N(v)|}$$

Pool

$$\max(\{\sigma(\mathbf{Q}\mathbf{h}_i^{l-1}) : i \in N(v)\})$$

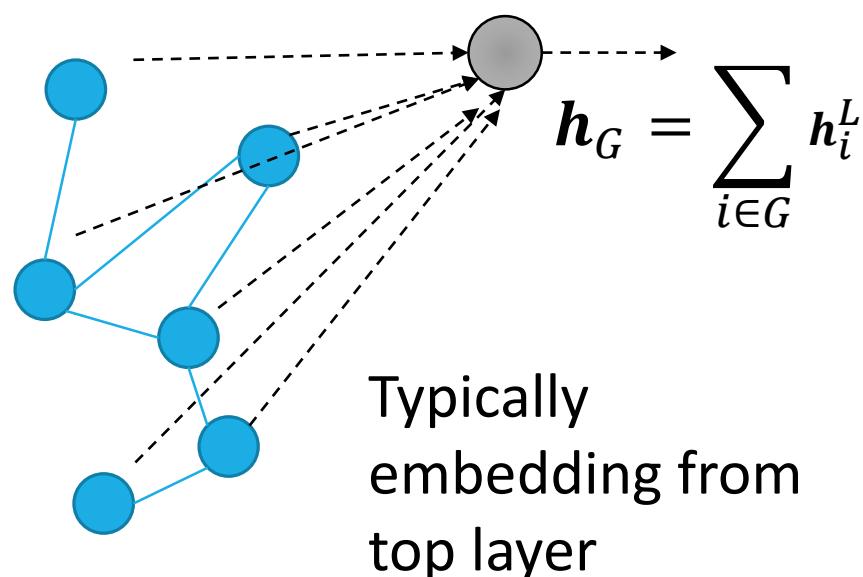
Transform embedding and
apply element-wise max
pooling

Sequential Aggregation

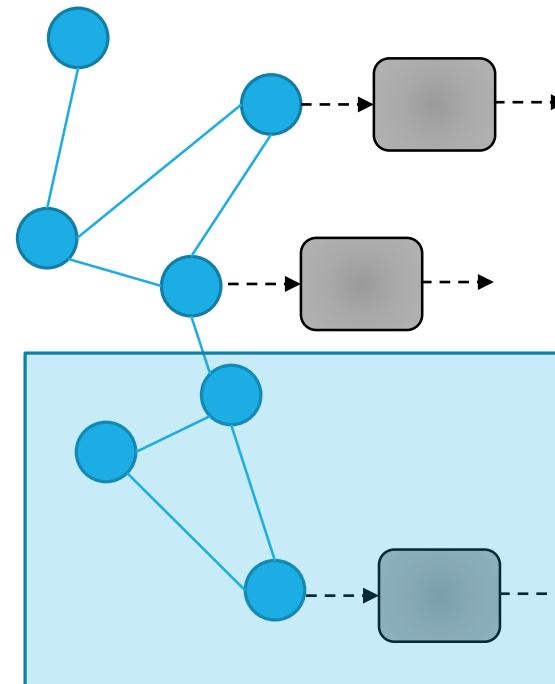
Feed random permutations of
the neighbor embeddings \mathbf{h}_i^{l-1} to
a recurrent layer (LSTM)

Using Node Embedding

Aggregate all node embeddings
to compute **graph level**
predictions



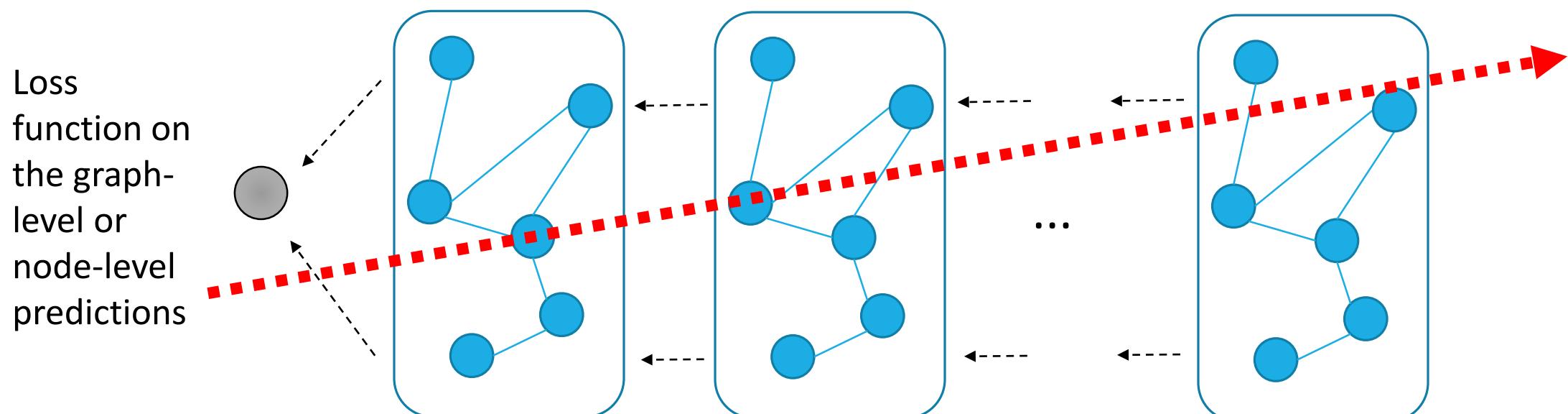
Train **node level predictors**



Works also for
inductive
learning

Training the Embedding

Backpropagate from the (graph or node level) error computed from the **top layer embeddings** to the early layers



Can also be **unsupervisedly trained** by using structure induced notions of **node similarity** (e.g. Node2Vec)

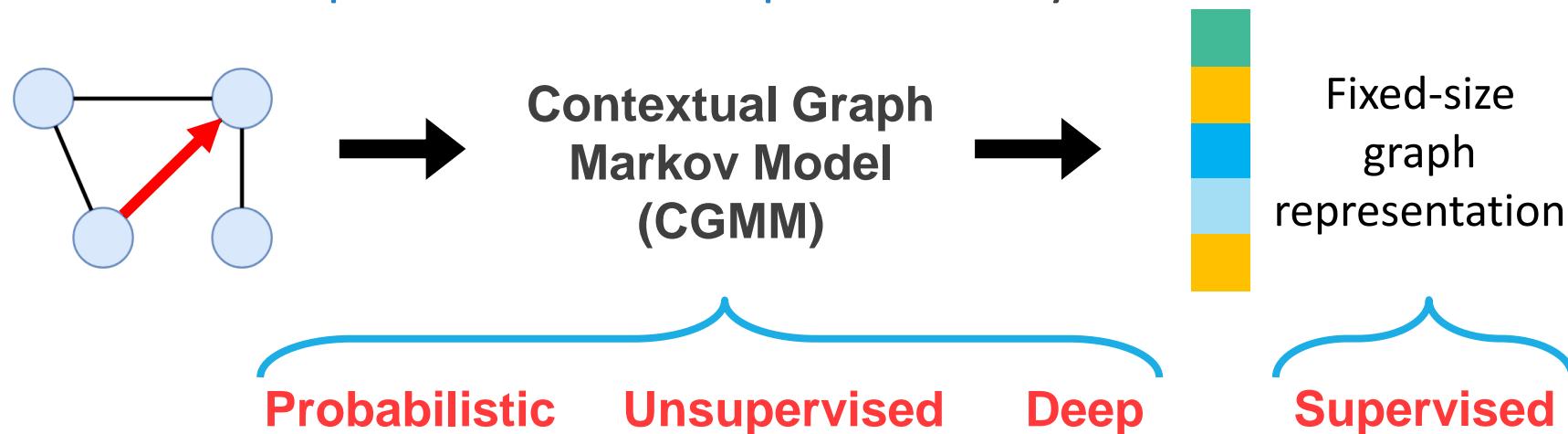
Node Embedding Recap

- ❖ Exploit contextual approach to avoid complex neighborhood construction strategies
- ❖ No causal dependencies within layers, hence need no fixed-point recurrence
- ❖ Restrict context to the preceding layer alone (less general than NN4G)
- ❖ Number of layers is typically small due to computational and parameterization issues (end-to-end training requires backpropagation through all layers)
- ❖ Embedding are either task-dependent (supervised learning) or need to hand-define similarity in node space (unsupervised learning)

Layer-wise Unsupervised Learning of Embedding

Generative learning for graphs

- ❖ General, efficient and scalable architecture
- ❖ Handle arbitrary structure (directed, undirected or mixed), labelled edges and nodes
- ❖ Learn in both supervised and unsupervised way



Bacciu, Errica, Micheli , ICML 2018

CGMM in a nutshell

The single layer graphical model

- ❖ **Extension** of a standard mixture model

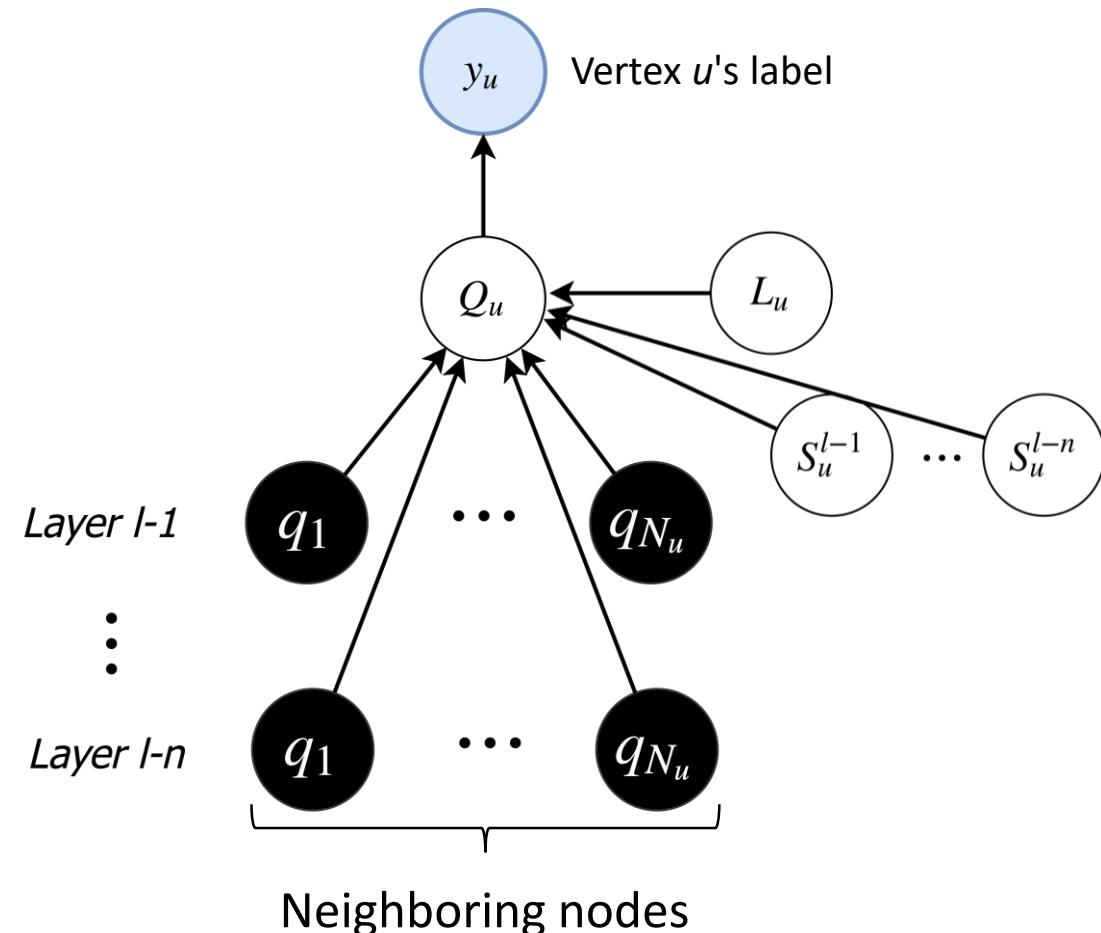
Discrete variables

- ❖ Likelihood becomes **intractable**

Exploit a **Switching Parent approximation**

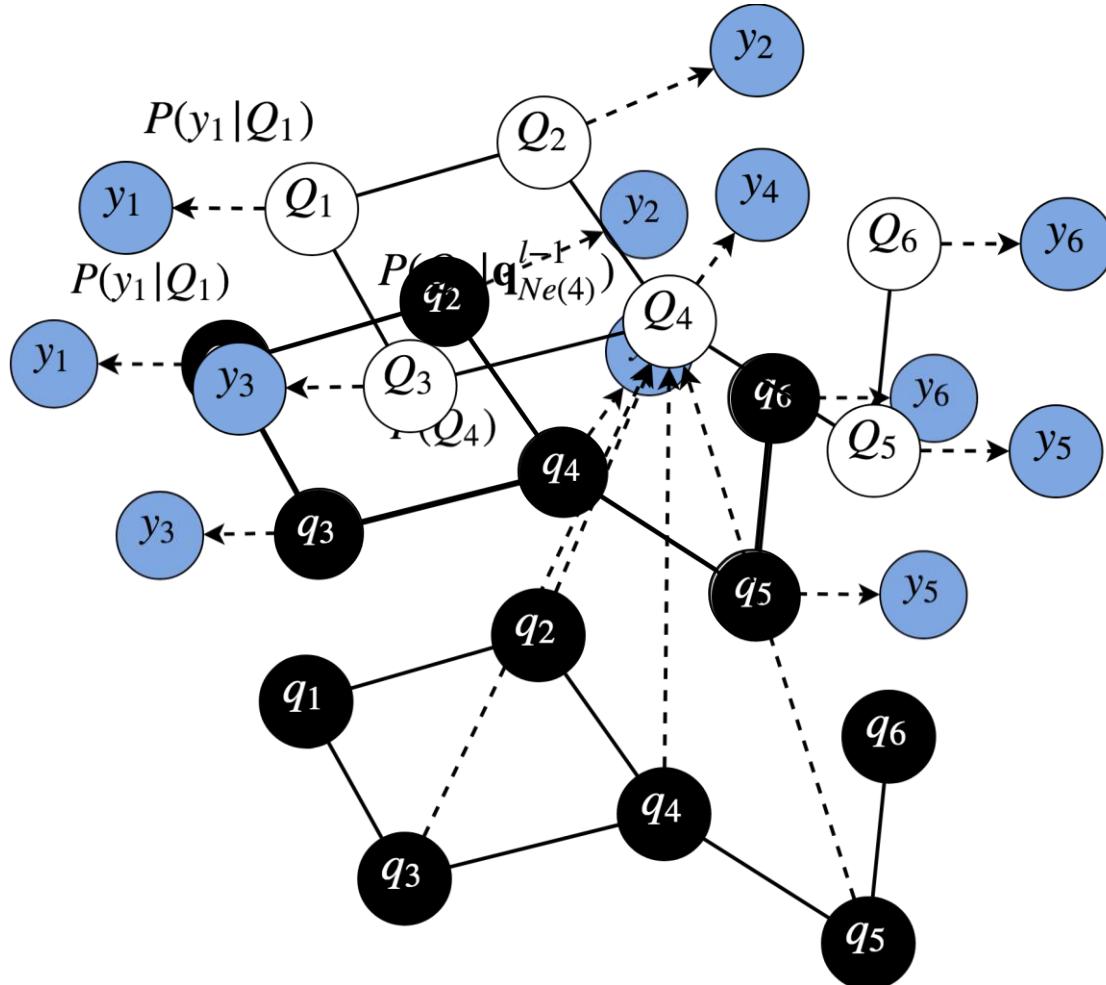
- ❖ Consider only the **direct neighborhood**

- ❖ **Full** stationarity



How to build the model

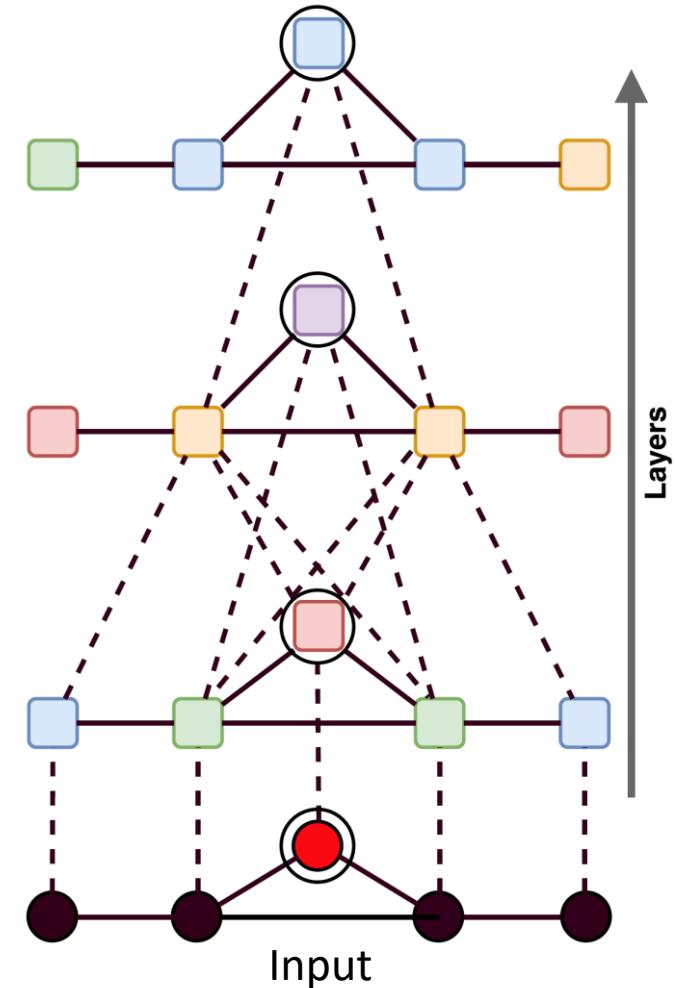
1. Map the graph to the model (base case)
 2. Perform inference and freeze states
 3. Add a new layer and use frozen states as observed variables in the graphical model
- Go back to step 2



CGMM in a nutshell

Symmetric context spreading between vertexes

- ❖ Each layer is trained in **isolation**
- ❖ Inference computes hidden states' assignments
Variables encode information
- ❖ The **architecture** diffuses information
Deeper net → Wider context window



Learning phase

A **maximum likelihood** approach to learning

$$\mathcal{L} = \prod_{g \in G} \prod_{u \in g} \sum_i^C P^l(y_u | Q_u = i) P^l(Q_u = i | \mathbf{q}_{\mathcal{N}(\mathbf{u})}^{\mathbf{L}_{\text{prec}}}(\mathbf{g}))$$

Assumption: i.i.d. graphs

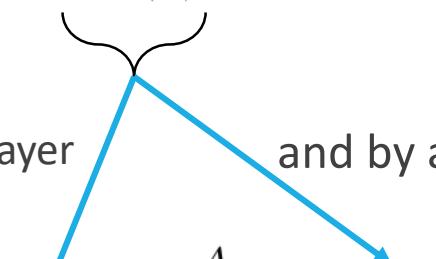
 Emission distrib.

 Switching Parents distrib.

 Transition distrib.

$$= \prod_{g \in G} \prod_{u \in g} \sum_i^C P^l(y_u | Q_u = i) \sum_{\bar{l} \in \bar{L}} P(L_u = \bar{l}) \sum_{a=1}^A P^{\bar{l}}(S_u = a) \underbrace{\frac{\sum_{v \in \mathcal{N}^a(u)} P^{\bar{l}, a}(Q_u = i | q_v)}{|\mathcal{N}^a(u)|}}$$

Split by layer and by arc



Average of the remaining contributions

Trained by **EM**

Inference

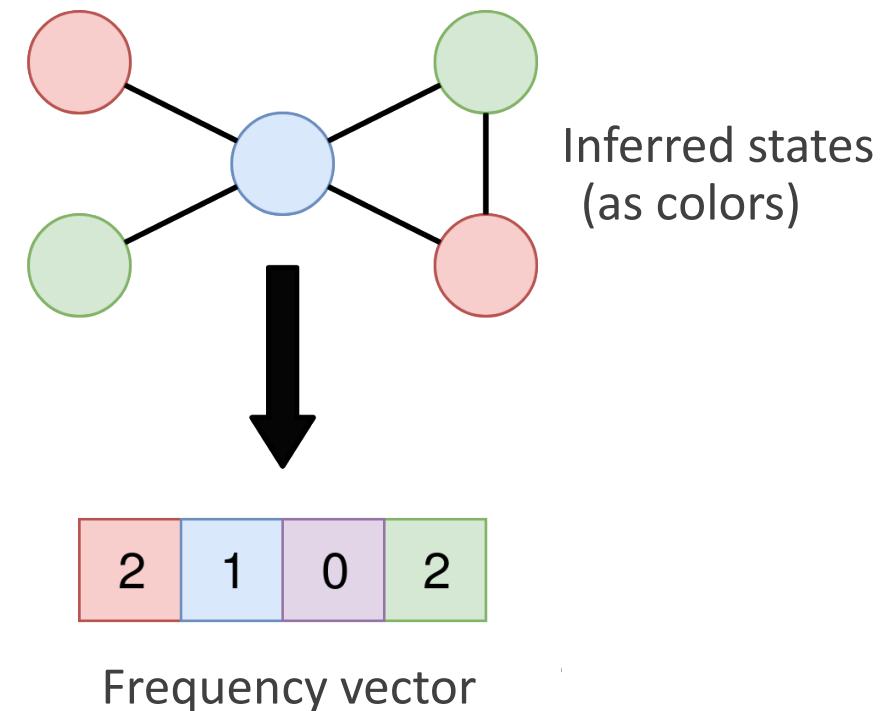
- ❖ Finding the most likely **state assignment**

$$\max_i P(y_u | Q_u = i) P(Q_u = i | \mathbf{q}_{\mathcal{N}(u)})$$

- ❖ The inferred latent states are used as observable variables in subsequent layers

- ❖ A **fixed-size vector of states frequencies** as graph encoding

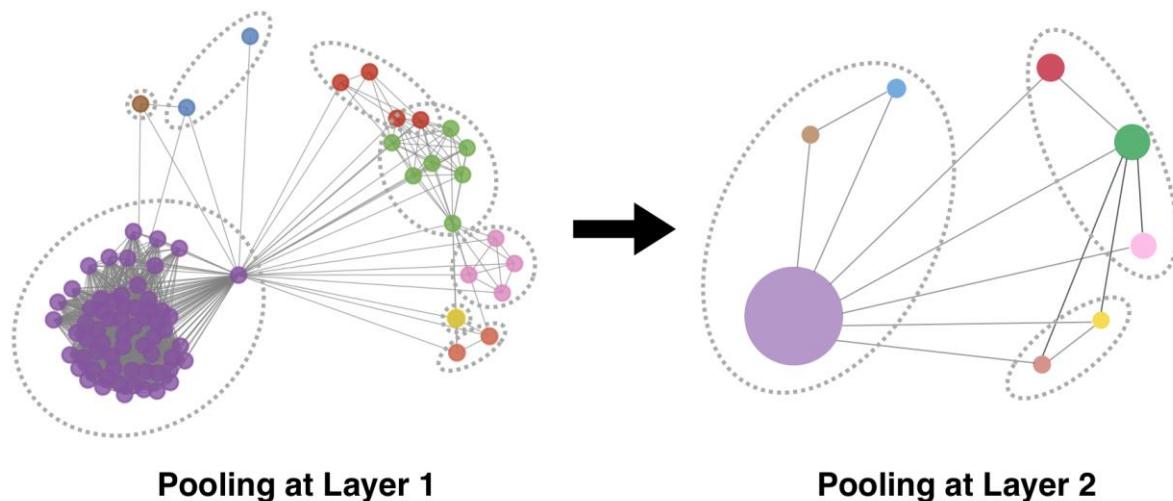
Hidden states = 4



Research Directions

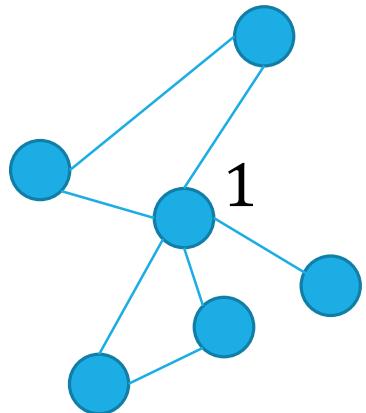
What About Pooling?

- ❖ Standard aggregation operates on **predefined node subsets** (whole graph, vertex neighborhoods)
- ❖ Ignore **community/hierarchical structure** in the graph
- ❖ Advanced (differentiable) pooling operators

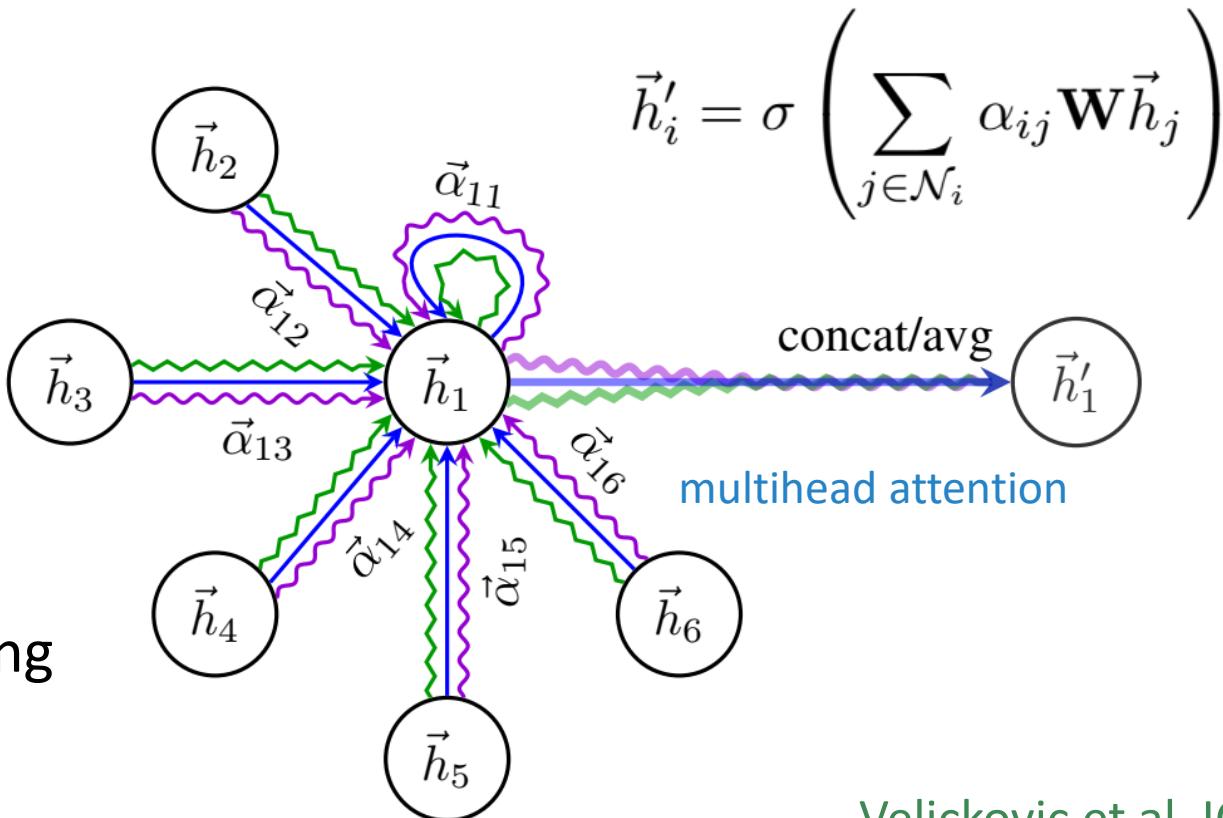


Rex Ying et al, Arxiv 2018

Graph Attention



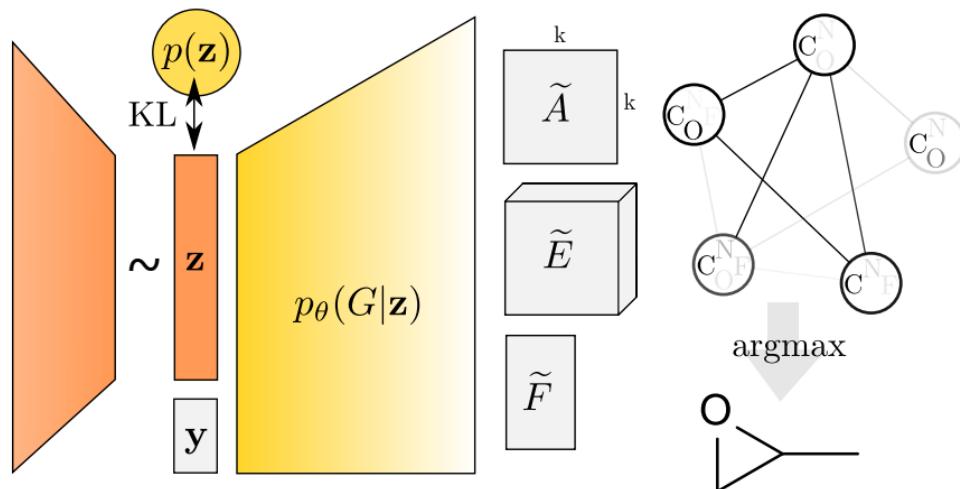
Learning to **weight contribution** of other nodes when aggregating to form the node embedding



Velickovic et al, ICLR 2018

Graph Generation

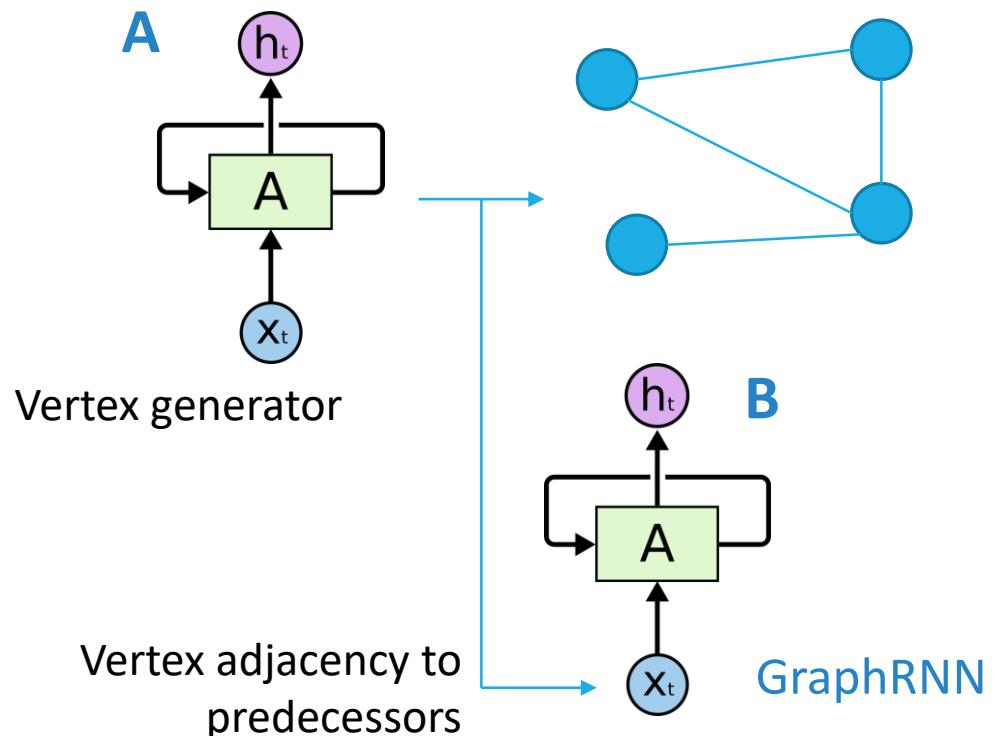
Generate a prediction that is **itself a graph**



GraphVAE generates **adjacency matrix** up to k vertices

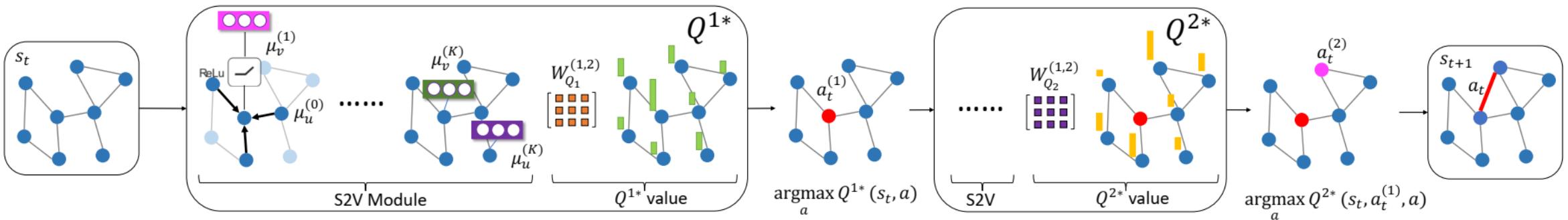
Simonovsky, Komodakis, ICLR-WS 2018

You et al, ICML 2018



Adversarial Attacks

Learn an attack policy by Q-learning (edge addition or deletion)

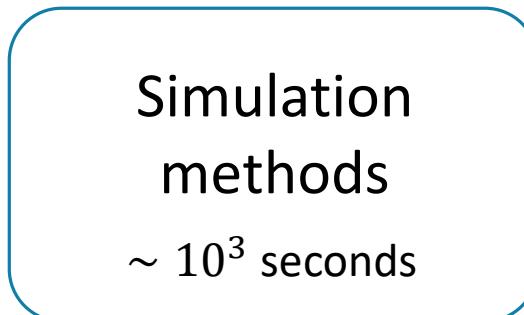
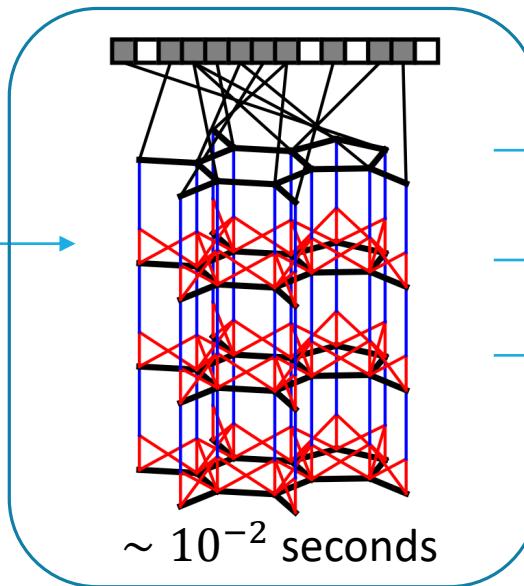
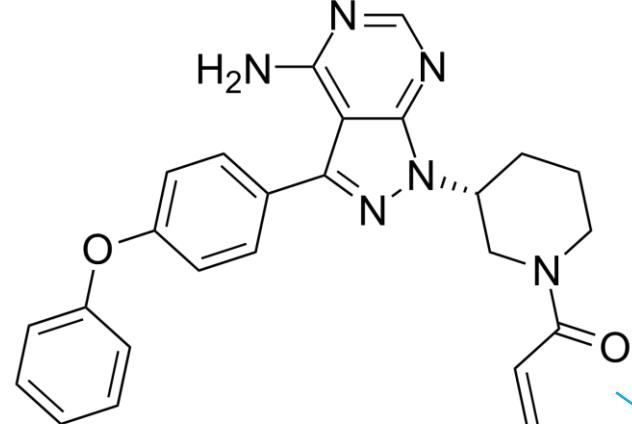


Show GraphRNN vulnerability to both **black-box** and **white-box** attacks (attack edges with maximum gradient)

H. Dai et al, ICML 2018

Applications

Predicting Properties of Chemical Compounds

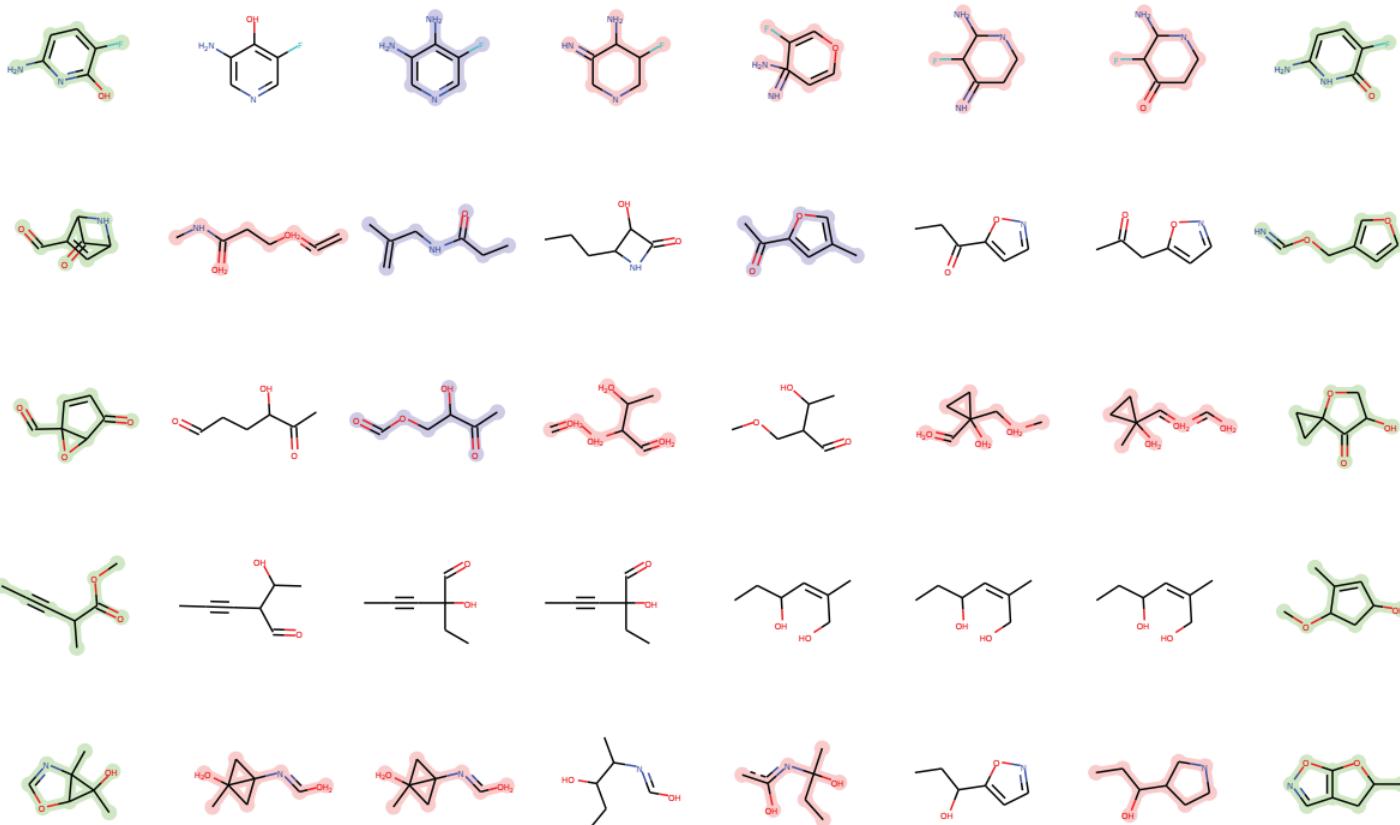


Micheli et al, JCICS 2001

Duvenaud, Maclaurin et al, NIPS 2015

Gilmer et al, ICML 2017

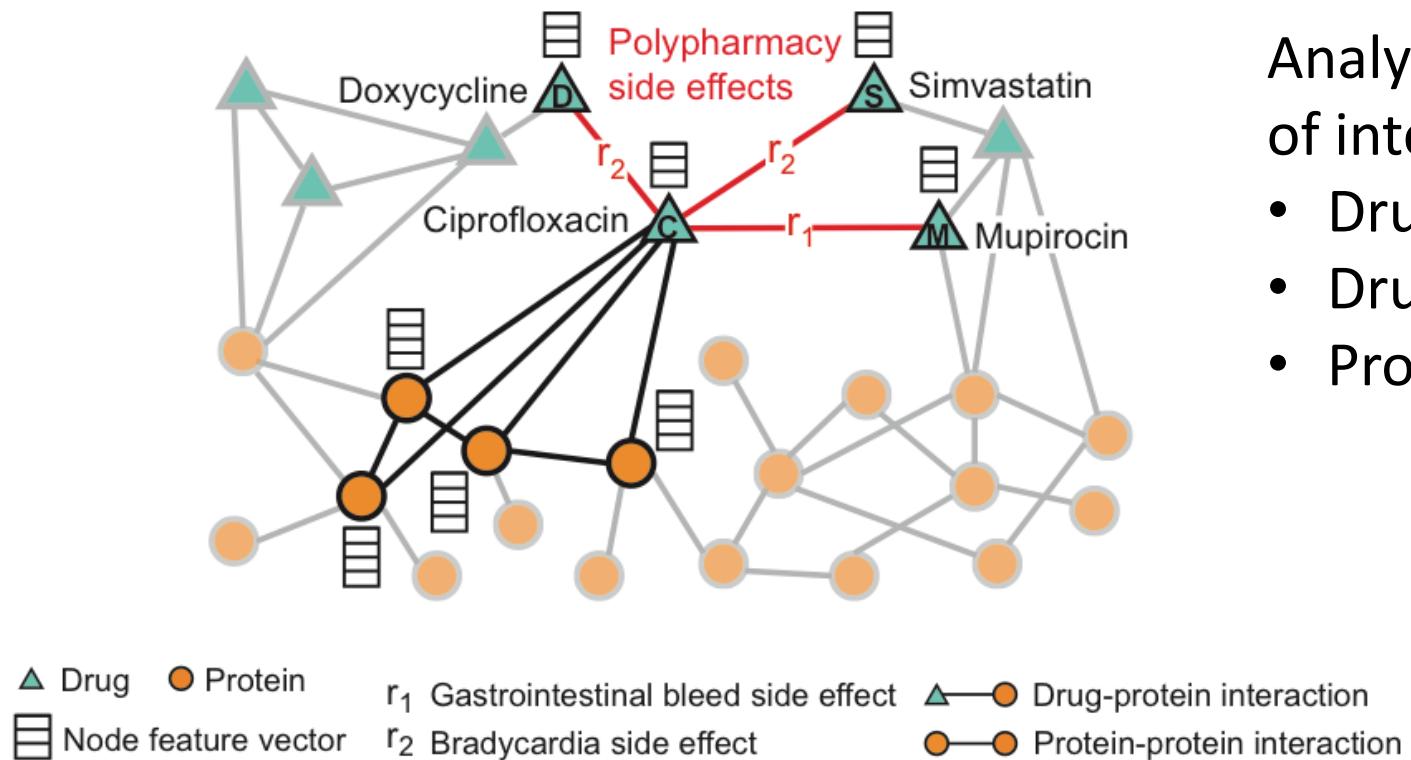
Generating Molecules



Molecule generation
by linear interpolation
in the latent space of
a **Graph Variational
Autoencoder**

Simonovsky, Komodakis, ICLR-WS 2018

Side Effects of Drug Combinations

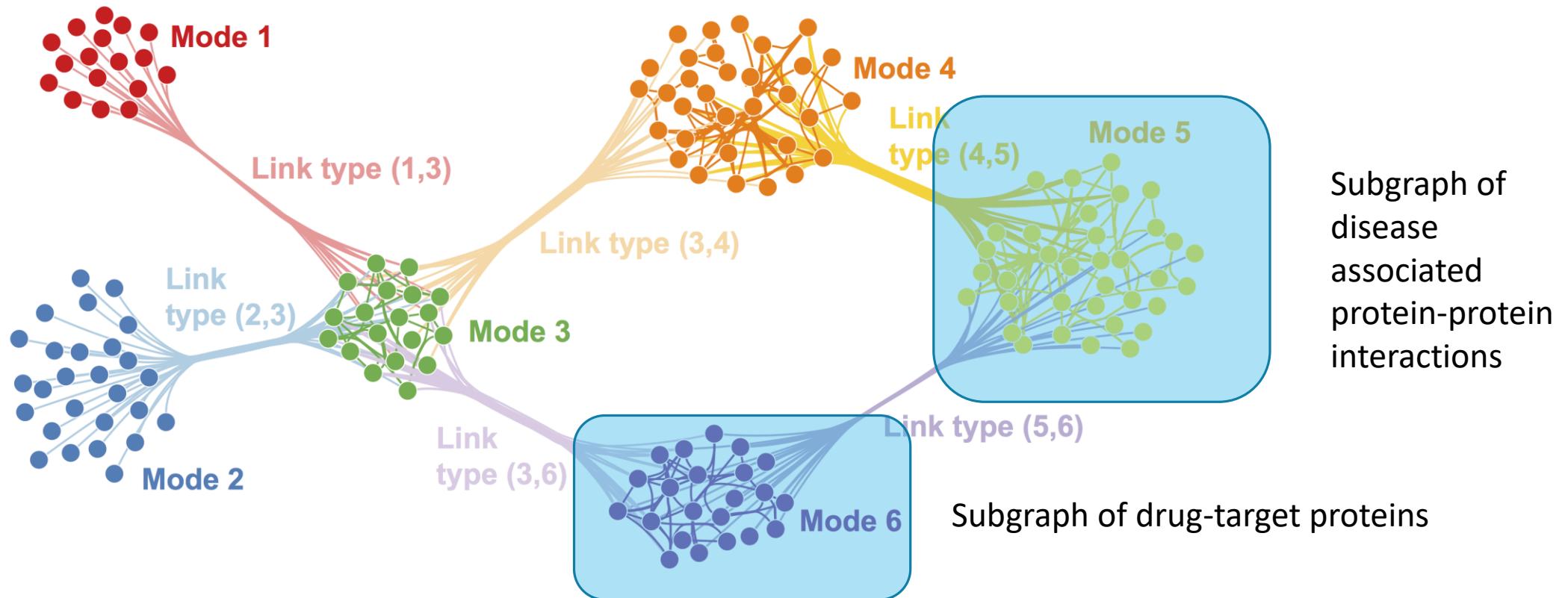


Analyzing a multimodal graph of interactions

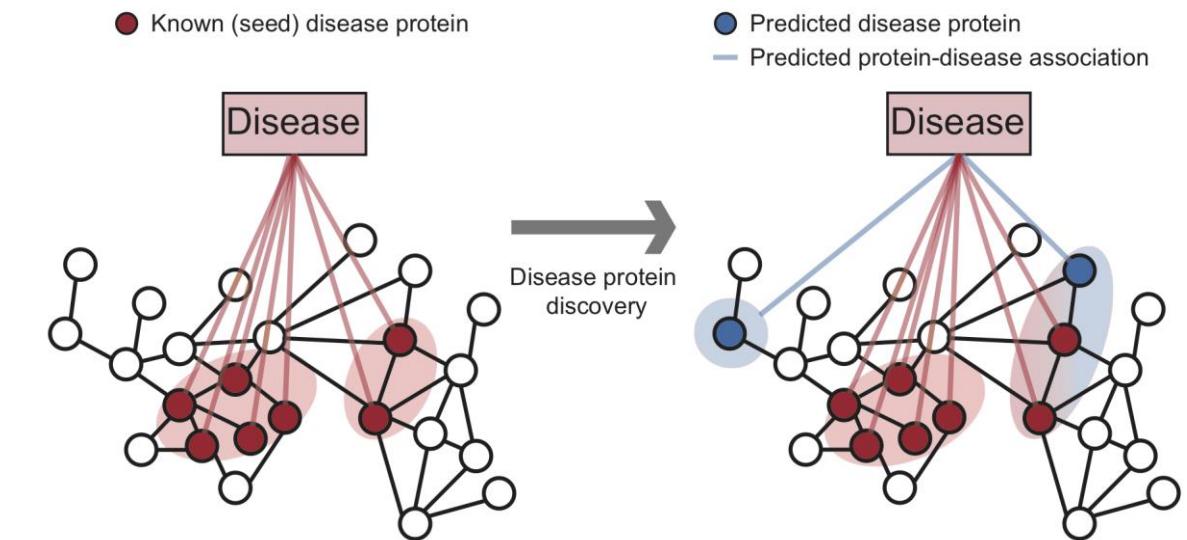
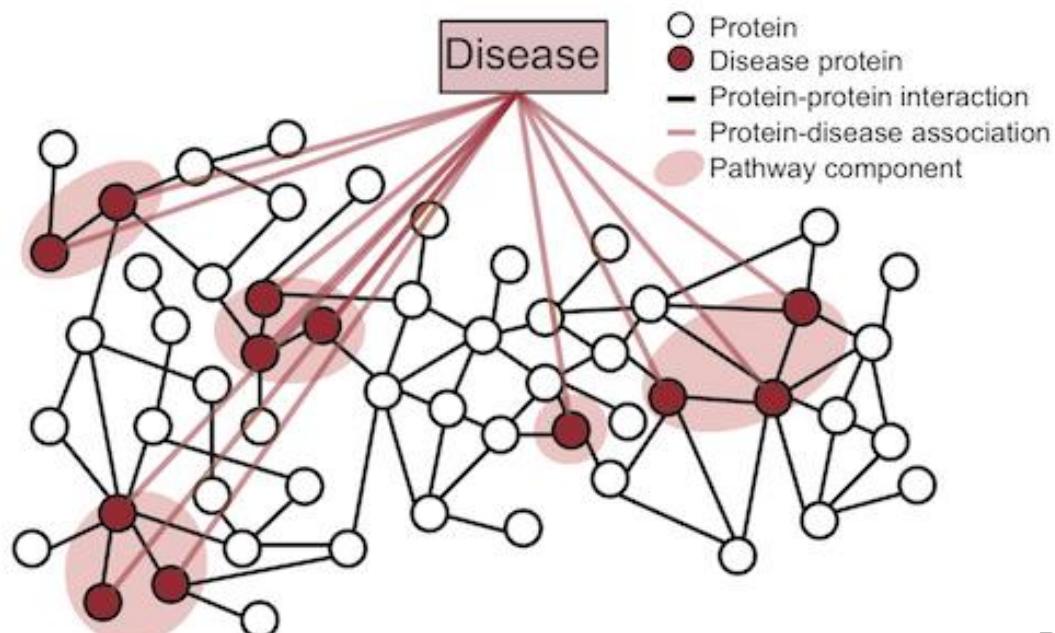
- Drug-drug
- Drug-protein
- Protein-protein

Drug Repurposing

Predict unknown disease-drug interactions



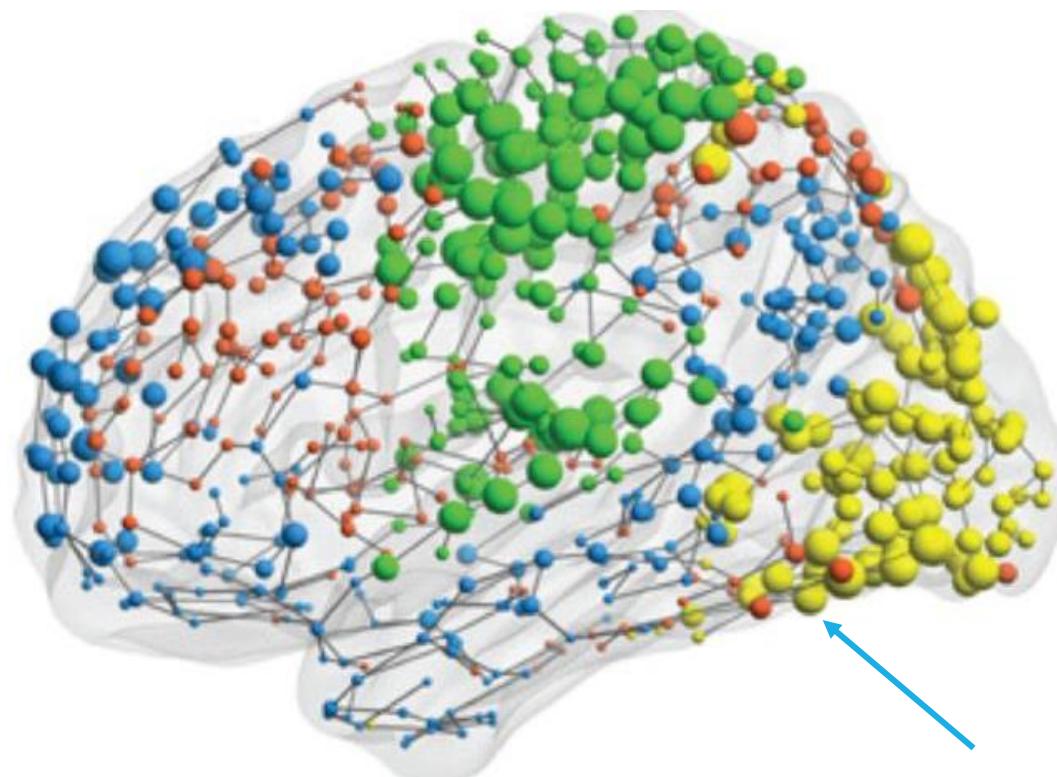
Pathway Prediction



Predicting unknown protein-disease associations from interaction networks

Zitnik, Agrawal, Leskovec, PSB 2018

Functional Brain Images



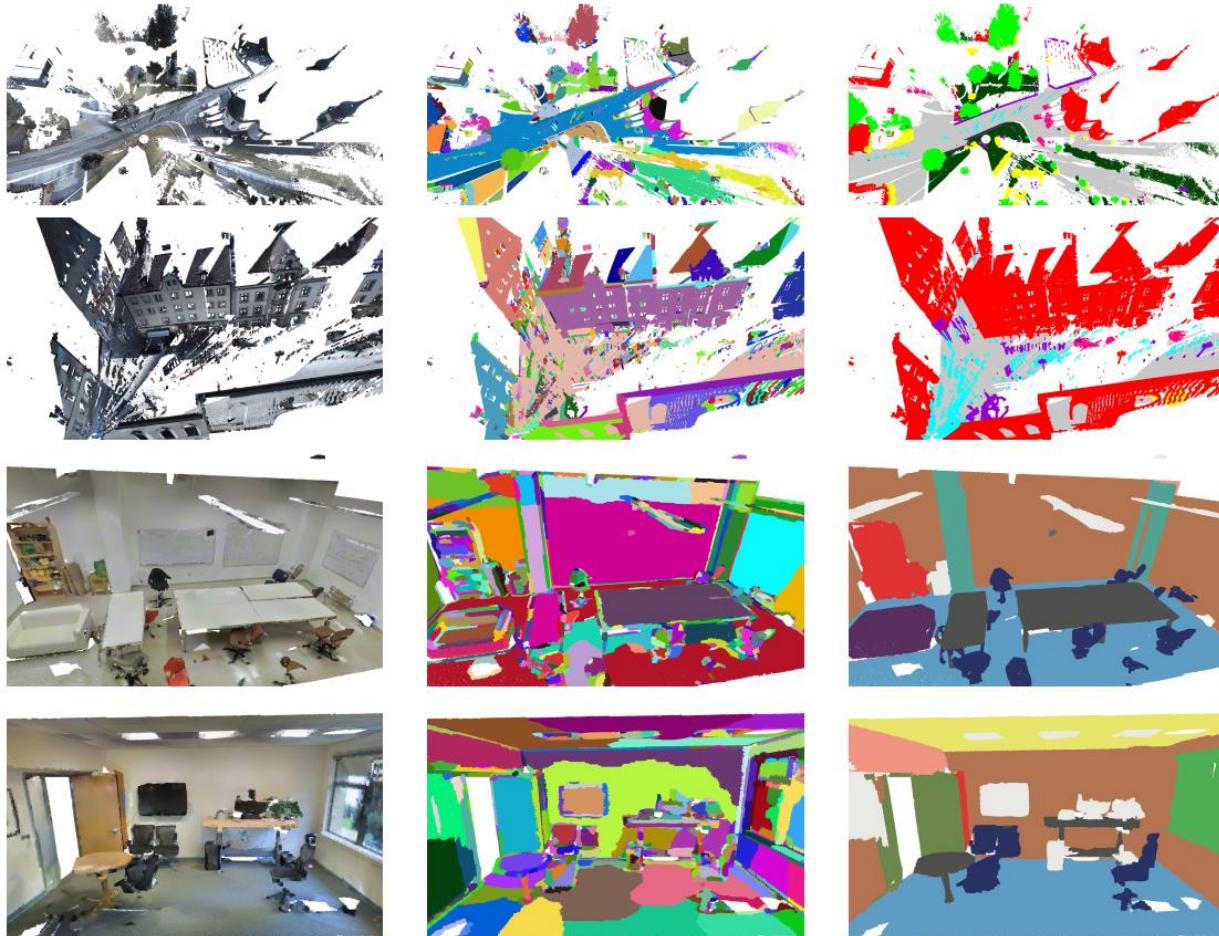
Brain state prediction

Disease prediction

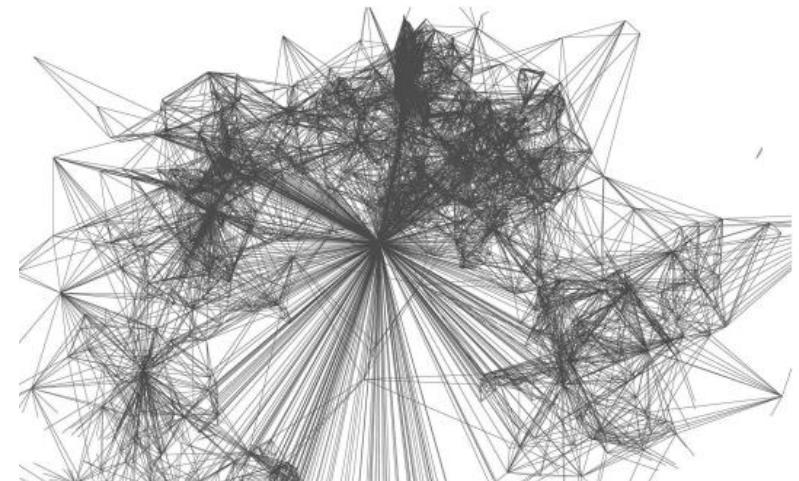
Brain reading

Timeseries data attached to nodes

Point Clouds – Semantic Segmentation



Build [point cloud graphs](#) and
train [semantic class predictors](#)
based on vertex embeddings



Landrieu, Simonovsky, CVPR 2018

Object Part Prediction



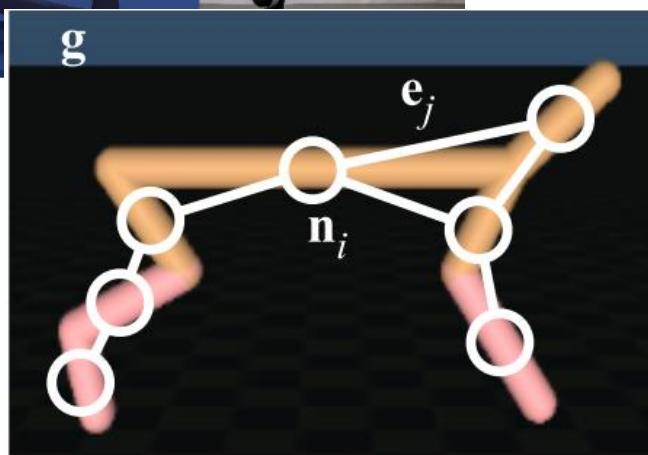
Te et al, ACM MultiMedia, 2018

Graphs in Physics



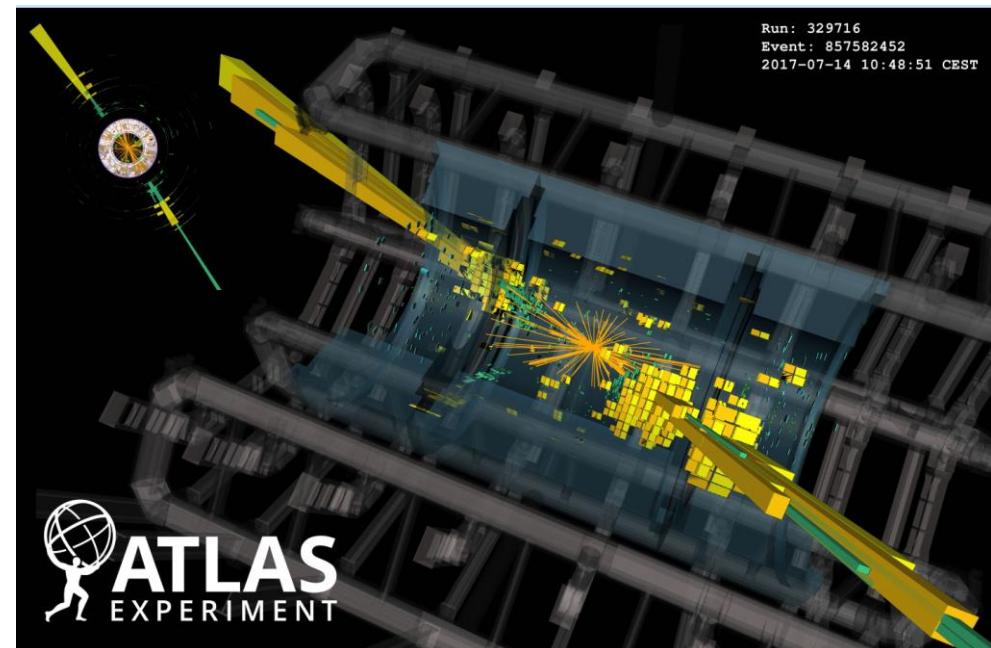
Learning **dynamics** of a **physical system** through a graph-based representation

Sanchez-Gonzalez et al, ICML 2018



Particle interactions represented as graphs

Henrion et al, NIPS WS 2017



Conclusions

- ❖ Deep learning for graphs is an exploding research topic
 - ❖ Many works sharing same underlying idea ([adjacency](#), [contractive](#), [contextual](#))
 - ❖ Much early work left [unacknowledged](#)
- ❖ What should we focus on?
 - ❖ Unifying frameworks for graph processing ([neural message passing](#))
 - ❖ Theoretical characterization ([VC dimension](#))
 - ❖ Efficiency and efficacy of context propagation ([unsupervised](#), [gradient issues](#))
 - ❖ Killer [applications](#) still missing
 - ❖ Research directions ([pooling](#), [generative](#))

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4. Davide Bacciu, Federico Errica, Alessio Micheli , Contextual Graph Markov Model: A Deep and Generative Approach to Graph Processing, ICML 2018

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3. Jiaxuan You, Rex Ying, Xiang Ren, William L. Hamilton, Jure Leskovec, GraphRNN: Generating Realistic Graphs with Deep Auto-regressive Models, ICML 2018 ([Graph Generation](#))
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5. Hanjun Dai, Hui Li, Tian Tian, Xin Huang, Lin Wang, Jun Zhu, Le Song, Adversarial Attack on Graph Structured Data, ICML 2018 ([Adversarial Attacks](#))

Contact and Ack's

Deep Learning for Graphs

- ❖ (Web) <https://sites.google.com/view/dl4sd>

Contextual graph Markov Model

- ❖ (Code) <https://github.com/diningphil/CGMM>
- ❖ (Paper) <http://arxiv.org/abs/1805.10636>

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New tutorial on DL4Graphs

INNS BIG DATA AND DEEP LEARNING 2019
April 16 – 18, Sestri Levante, Italy

<https://innsbddl2019.org/>

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Deep Learning for Graphs

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<https://sites.google.com/view/dl4sd>