Solving Ax = 0 and Ax = b

* This section goes onward from U to a **reduced form R**
  + the simplest matrix that elimination can give
    - R reveals all solutions immediately
* For an invertible matrix,
  + the nullspace contains only x = 0 (multiply Ax = 0 by A −1 )
  + The column space is the whole space (Ax = b has a solution for every b)
* The new questions appear when
  + the nullspace contains more than the zero vector
  + the column space contains less than all vectors:
* complete solution Axp = b and Axn = 0 produce A(xp + xn) = b
  + all linear equations have this from x = xp +xn
* extra conditions needed to make Ax = b solvable
  + when column space doesn’t contain every b in Rm
* The 1 by 1 system 0x = b, **one equation** and **one unknown**, shows two possibilities
  + 1)
    - 0x = b has no solution unless b = 0
    - The column space of the 1 by 1 zero matrix contains only b = 0
  + 2)
    - 0x = 0 has infinitely many solutions
    - The nullspace contains all x
  + A particular solution is x p = 0
    - the complete solution is x = x p + x n = 0 + (any x)