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Solving the first five cases

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The First Five Cases [1]

A week after his last adventures, Craig was preparing to return home when he suddenly received an emergency call to go to Transylvania to help solve some baffling cases. Transylvania is inhabitated by both vampires and humans; the vampires lie and humans always tell the truth. However, half of the inhabitants, both human and vampire and human, are insane. Sane humans and insane vampires both make only true statements, while insane humans and sane vampires always lie.
Each of Craig's first five cases involved two inhabitants. In each case, it was already known that one of them was a vampire and the other was human, but it was not known which was which. Nothing was known except in case 5, about the sanity of either. The cases are as follow:

The first case involved two sisters named Lucy and Minna, and Craig had to determine which one was a vampire. As indicated above, nothing was known about the sanity of either. Here is the transcript of the investigation:
Craig(to Lucy): Tell me something about yourselves.
Lucy: We are both insane.
Craig(to Minna): Is that true?
Minna: of course not!

The next case was that of the Lugosi brothers. Both had the first name of Bela. Again, one was a vampire and one was not. They made the following statements:
Bela the Elder: I am human.
Bela the Younger: I am human.
Bela the Elder: My brother is sane.

The third case involved another pair of brothers, Michael and Peter Karloff. Here is what they said:
Michael: I am a vampire.
Peter: My brother and I are alike as far as our sanity goes.

The fourth case involved a father and a son whose surname was Turgenief.
Craig(to the father): Are you both sane or both insane, or are you different in this respect?
Father: At least one of us is insane.
Son: That is quite true!
Father: I, of course, am not a vampire.

The fifth case involved a pair of twins, Karl and Martha Dracula. One of them was insane and the other was not, but Craig had no idea which was which.
Karl: My sister is a vampire.
Martha: My brother is insane!

Who were vampires and who were humans?

Solutions

There is a principle that will apply in several of the solutions that follow and which we wil establish in advance - namely that if a Transylvanian says he is human, then he must be sane, and if he says he is a vampire, then he must be insane. The reason is simple: suppose he says he is human. Now, his statement is either true or false. If his statement is true, then he really is a sane human. If the statement is false he is an sane vampire, because only sane vampires tell lies. The same logic applies to the insane. So, the solutions are as follow:

1) Lucy's statement is either true or false. If it is true, then both sisters are really insane, hence Lucy is insane, and the only insane Transylvanian who can make a true statement is an insane vampire. So, if Lucy's statement is true, she is a vampire. Suppose her statement is false, then at least one of the sisters is sane. If Lucy is sane, then, since she has made a false statement she must be a vampire. Supposing Lucy is insane, then it must be Minna who is sane. Also, Minna, by contradicting Lucy's false statement, has made a true statement, so Lucy is the vampire and Minna is human.

2)Both Lugosi brothers claim to be, therefore both must be sane. Bela the Elder makes a true statement when he says that his brother is sane. So Bela the Elder is both sane and makes true statements, hence he is human and Bela the Younger is the vampire

3)Since Michael claims to be a vampire, he is insane, and since Peter claims to be human, he is sane. So Michael is insane and Peter is sane; thus the two brothers are not alike as far as their sanity goes. Therefore, Michael's second statement is false, and since Michael is insane, he must be human. Therefore, Peter is the vampire

4)Father and son agree in answering the question about their sanity. This means that they either both make true statements or both make false statements. But, since only one of the is human, they must necessarily be different as regards their sanity. If they are both sane, the one who is human would make true statements and the vampire would make false statements, and they could never agree; if they are both insane, the human would make false statements and the vampire would make true statements and the vampire would make true statements, and again could never agree. This proves that both of them make true statements. Then, since the father says he is not a vampire, he really isn't. So it is the son who is the vampire.

5)Suppose Martha is the vampire. Then Karl is human and also Karl has made a true statement; hence Karl in this case has to be a sane human. This would make Martha an insane vampire, since, as we have been told, Karl and Martha are different as regards their sanity. But then Martha, an insane vampire, would have made a false statement - that Karl is insane- which insane vampires cannot do. Therefore, the assumption is false, and Karl is the vampire.

Human or Vampire	human	human	vampire	vampire
Sane or Insane	sane	insane	sane	insane
Truth or Lie	T	L	L	T

Mace4 Solutions

Listing 1: Lucy and Minna

```
formulas(assumptions).
%human/vampire
% h1 : lucy e om
%-h1 : lucy e vampir
% h2 : minna e om
%-h2 : minna e vampir

%sane/insane
% s1 : lucy nu e nebuna
%-s1 : lucy e nebuna
% s2 : minna nu e nebuna
%-s2 : numma e nebuna

%dialog
%d1 = ce spune lucy
%d2 = ce spune minna
%%%%%%%%%%%%%%%%%%%%%%%%%%

%un vampir si un om
(h1 & -h2) | (-h1 & h2).

%lucy: we are both insane
d1 <-> (-s1 & -s2).

%minna: both of us aren't insane
d2 <-> -d1.

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane Human => True
%Insane Human => False
%Sane Vampire => False
%Insane Vampire => True

(-h1 & -s1) ->  d1.
(-h1 &  s1) -> -d1.
( h1 & -s1) -> -d1.
( h1 &  s1) ->  d1.

(-h2 & -s2) ->  d2.
(-h2 &  s2) -> -d2.
( h2 & -s2) -> -d2.
( h2 &  s2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f lucy_mina.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 3363 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [0]),
    relation(d2, [1]),
    relation(h1, [0]),
    relation(h2, [1]),
    relation(s1, [1]),
    relation(s2, [1])]).
interpretation( 2, [number = 2,seconds = 0], [
    relation(d1, [1]),
    relation(d2, [0]),
    relation(h1, [0]),
    relation(h2, [1]),
    relation(s1, [0]),
    relation(s2, [0])]).
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f lugosi.in | interpformat
```

Listing 2: Lugosi

```
formulas(assumptions).
%human/vampire
% h1 : fratele mare e om
%-h1 : fratele mare e vampir
% h2 : fratele mic e om
%-h2 : fratele mic e vampir

%sane/insane
% s1 : fratele mare nu e nebun
%-s1 : fratele mare e nebun
% s2 : fratele mic nu e nebun
%-s2 : fratele mic e nebun

%dialog
%d1 = ce spune fratele mare
%d2 = ce spune fratele mic
%%%%%%%%%%%%%%%%%%%%%%%%%%

%un vampir si un om
(h1 & -h2) | (-h1 & h2).

%fratele mare: i am human, my brother is sane
d1 <-> h1 & s2.

%fratele mic: i am human
d2 <-> h2.

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane Human => True
%Insane Human => False
%Sane Vampire => False
%Insane Vampire => True

(-h1 & -s1) ->  d1.
(-h1 &  s1) -> -d1.
( h1 & -s1) -> -d1.
( h1 &  s1) ->  d1.

(-h2 & -s2) ->  d2.
(-h2 &  s2) -> -d2.
( h2 & -s2) -> -d2.
( h2 &  s2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f lugosi.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 3365 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [0]),
    relation(d2, [1]),
    relation(h1, [0]),
    relation(h2, [1]),
    relation(s1, [1]),
    relation(s2, [1])]).
interpretation( 2, [number = 2,seconds = 0], [
    relation(d1, [1]),
    relation(d2, [0]),
    relation(h1, [1]),
    relation(h2, [0]),
    relation(s1, [1]),
    relation(s2, [1])]).
```

Listing 3: Michael and Peter

```
formulas(assumptions).
%human/vampire
% h1 : michael e om
%-h1 : michael e vampir
% h2 : peter e om
%-h2 : peter e vampir

%sane/insane
% s1 : michael nu e nebun
%-s1 : michael e nebun
% s2 : peter nu e nebun
%-s2 : peter e nebun

%dialog
%d1 = ce spune michael
%d2 = ce spune peter
%%%%%%%%%%%%%%%%%%%%%%%%%%

%un vampir si un om
(h1 & -h2) | (-h1 & h2).

%michael: i am vampire, we have the same sanity
d1 <-> -h1 & ((s1 & s2) | (-s1 & -s2)).

%peter: i am human
d2 <-> h2.

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane Human => True
%Insane Human => False
%Sane Vampire => False
%Insane Vampire => True

(-h1 & -s1) ->  d1.
(-h1 &  s1) -> -d1.
( h1 & -s1) -> -d1.
( h1 &  s1) ->  d1.

(-h2 & -s2) ->  d2.
(-h2 &  s2) -> -d2.
( h2 & -s2) -> -d2.
( h2 &  s2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f michael_peter.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 3367 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [0]),
    relation(d2, [0]),
    relation(h1, [1]),
    relation(h2, [0]),
    relation(s1, [0]),
    relation(s2, [1])]).
```

Listing 4: Turgeniefs

```
formulas(assumptions).
%human/vampire
% h1 : tatal e om
%-h1 : tatal e vampir
% h2 : fiul e om
%-h2 : fiul e vampir

%sane/insane
% s1 : tatal nu e nebun
%-s1 : tatal e nebun
% s2 : fiul nu e nebun
%-s2 : fiul e nebun

%dialog
%d1 = ce spune tatal
%d2 = ce spune fiul
%%%%%%%%%%%%%%%%%%%%%%%%%%

%un vampir si un om
(h1 & -h2) | (-h1 & h2).

%tatal: at least one of us is insane, i am human
d1 <-> ((-s1 & s2) | (s1 & -s2) | (-s1 & -s2)) & h1.

%fiul: father's first statement is true
d2 <-> ((-s1 & s2) | (s1 & -s2) | (-s1 & -s2)).

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane Human => True
%Insane Human => False
%Sane Vampire => False
%Insane Vampire => True

(-h1 & -s1) ->  d1.
(-h1 &  s1) -> -d1.
( h1 & -s1) -> -d1.
( h1 &  s1) ->  d1.

(-h2 & -s2) ->  d2.
(-h2 &  s2) -> -d2.
( h2 & -s2) -> -d2.
( h2 &  s2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f turgeniefs.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 3369 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [1]),
    relation(d2, [1]),
    relation(h1, [1]),
    relation(h2, [0]),
    relation(s1, [1]),
    relation(s2, [0])]).
```

Listing 5: Dracula

```
formulas(assumptions).
%human/vampire
% h1 : karl e om
%-h1 : karl e vampir
% h2 : martha e om
%-h2 : martha e vampir

%sane/insane
% s1 : karl nu e nebun
%-s1 : karl e nebun
% s2 : martha nu e nebuna
%-s2 : martha e nebuna

%dialog
%d1 = ce spune karl
%d2 = ce spune martha
%%%%%%%%%%%%%%%%%%%%%%%%%%

%un vampir si un om
(h1 & -h2) | (-h1 & h2).
%one insane one sane
(s1 & -s2) | (-s1 & s2).

%karl: my sister is a vampire
d1 <-> -h2.

%martha: my brother is insane
d2 <-> -s1.

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane Human => True
%Insane Human => False
%Sane Vampire => False
%Insane Vampire => True

(-h1 & -s1) ->  d1.
(-h1 &  s1) -> -d1.
( h1 & -s1) -> -d1.
( h1 &  s1) ->  d1.

(-h2 & -s2) ->  d2.
(-h2 &  s2) -> -d2.
( h2 & -s2) -> -d2.
( h2 &  s2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f dracula.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 3371 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [0]),
    relation(d2, [0]),
    relation(h1, [0]),
    relation(h2, [1]),
    relation(s1, [1]),
    relation(s2, [0])]).
```

Bibliography

[1] Raymond M Smullyan. *The lady or the tiger?: and other logic puzzles*. Courier Corporation, 2009.