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Metapuzzles

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Metapuzzles [1]

Metapuzzles are puzzles about puzzles where we are given a puzzle without sufficient data to solve it, and then we are given that someone else could or could not solve it given a certain additional information, but we are not always told just what this additional information is. We may, however be given partial information about it, which enables the reader to solve the probem, Solve the following puzzles:

1) The Case of John
This case involved a pair identical twins. It was known that at least one of them never told the truth, but it was not known which. One of the twins was named John, and he had commited a crime.(John was not necessarily the one who always lied.) The purpose of the investigation was to find out which one was John.
"Are you John?" the judge asked the first twin.
"Yes, I am." was the reply.
"Are you John?" the judge asked the second twin.
The second twin then answered either yes or no, and the judge then knew which one was John. Was John the first twin or the second?

2) A Transylvanian metapuzzle
In Transylvania there are 4 types of inhabitants: humans, insane humans, vampires and insane vampires. Humans and insane vampires always tell the truth, vampires and insane humans always lie. Three logicians were discussing their separate trips to Transylvania, where they all met an inhabitant named Igor. The first logician asked Igor if he was a sane human. The second logician asked him if he was a sane vampire. The third asked him if he was a insane vampire. Igor answered either yes or no to all their questions. What type of inhabitant was Igor?

3) A Knight-Knave Metapuzzle A knight always tells the truth and a knave always lies. A logician once visited an island and came across two inhabitants, A and B. He asked A, "Are you both knights?", A answered either yes or no. The logician thought for a while, but did not yet have enough information to determine what they were. The logician then asked A, "Are you two of the same type?"(Same type means both knights or both knaves). A answered either yes or no, and the logician then knew what type each one was. What type is each?

4) Knights, Knaves and Normals
On another island there were also Normals, beside Knights and Knaves. Normals can either lie or tell the truth. A logician once visited this island and came across two inhabitants, A and B. He already knew one of them was a knight and the other was normal, but didn't know which was which. He asked A whether B was normal, and he answered either yes or no. Which of the two was normal?

5)Who is the spy
The case involves a trial of three dependants: A, B and C. It was known that one was a knight,one a knave and one a spy(normal). The purpose of the trial was to find the spy. Each of them made a statement: A said that C is either a knave or a spy, B said that C is either a knight or a knave or a spy and C said that A is either a knight or a knave or a spy. Which one was the spy?

Solutions

The solutions are as follow:

1) If the second twin had also answered yes, the judge obviously could not have known which one was John; hence the second twin must have answered no. This means that either both twins told the truth or both lied, but we know that at least one of them always lies, therefore we can safely assume that both had lied and that the second twin in John.

2) If Igor was a sane human, insane human or sane vampire he would've answered yes to the first logician's question. He would've answered no if he was a insane vampire. If Igor had answered no, then the first logician would have known that Igor was an insane vampire, but since he didn't know what Igor was after his answer we can safely assume that Igor is not an insane vampire.
As to the second logician's question, only an insane human would've answered yes, and the other three types would've answered no. Following the same logic as before, we can safely assume that Igor is not an insane human either.
As to the third logician's question, only a sane human would've answered no and the other three types would've answered yes. So, we can also assume that Igor is not a sane human either.
Finally, by exclusion, Igor is a sane vampire.

3)There are four possible cases:
-they are both knights
-A is a knight and B is a knave
-A is a knave and B is a knight
-they are both knaves
Only case 2 would've answered no to the first question, and since the logician didn't know anything from the answer, we can rule Case 2 out. As to the second question, Case 1 and 3 would've answered yes and Cases 2 and 4 would've answered no. So, if the logician had gotten yes as an answer, he would have had to guess between Case 1 and 3, so he must've gotten no as an answer. Now we have to choose between Case 2 and Case 4, but since we have already ruled out case 2, the answer is that both A and B were knaves.

4) If A had replied yes, then A could have been either a knight or normal since we have no way to prove that he was lying or telling the truth. If A had replied no, then A couldn't be a knight(for the B would be a normal, and A would have lied). So, the only way we can find out which is which, is if A says no. Therefore, A is the normal one.

Human or Vampire	human	human	vampire	vampire
Sane or Insane	sane	insane	sane	insane
Truth or Lie	T	F	F	T

Type	Knight	Knave	Normal
Truth or Lie	T	L	T/L

Solutions

5)There are two possibilities: either the logician was told that A said that C was a knave, or he was told that A said that C was a spy.

Possibility 1: A said that C is a knave
Now that we have chosen what A said, we have to choose what B and C said. As for B, there are three possible cases:

Case 1: B said that A is a knight. Then, if a is a knight and C is a knave, B will be a spy; if A is a knave, then B would be lying, which means that B is the spy and C is the knight; otherwise if A is the spy, B's statement is still false, so B is the knave and C is the knight. The previous statements result in 3 cases, so we have either:
(1) A knight, B spy, C knave
(2) A knave, B spy, C knight
(3) A spy, B knave, C knight
Now, suppose C said that B is the spy. Then (1) and (3) are ruled out, leaving only (2) open, so we would know that B is a spy. If C said that B is a knight, that would leave only option (1) open, B being the spy again. If C said that B is a knave, the judge wouldn't be able to decide between (1) and (3) because he wouldn't be able to choose between A and B. Therefore, C didn't say that B is a knave and if Case 1 holds, then B is the only one the judge could have convicted.

Case2: B said that A is the spy. We have to choose between the same 3 cases:
(1) A knight, B spy, C knave
(2) A knave, B spy, C knight
(3) A spy, B knave, C knight
If C said that B is the spy, the either (2) or (3) could hold and the judge wouldn't be able to choose between A and B. If C said that B is a knight, then the only viable option would be (1), so the judge would convict B. Also, the same as before, if C said that B is a knave, then either (1) or (3) could hold. Therefore, C must have said that B is a knight, and B would be convicted once again.

Case3: B said that A is a knave. In this case, unlike before, we have 4 cases:
(1) A knight, B spy, C knave
(2) A knave, B spy, C knight
(3) A knave, B knight, C spy
(4) A spy, B knave, C knight
If C said that B is the spy, (2) or (3) could hold, and the judge couldn't have determined which one was guilty, if C said that Bis a knight, (1) or (3) could hold, and the judge, again, couldn't have convicted anyone. If C said that B is a knave, (1), (3), or (4) could hold, and once more the judge could not have determined who was the spy. Therefore, Case 3 is ruled out and we now know that either Case 1 or Case 2 holds, and in both cases, the judge convicted B.
So if A said that C is a knave, Possibility 1 results in B being the spy.

Solutions

5)Possibility 2: A said that C is a spy.
In this case, the logician would be unable to solve the problem because he would be unable to choose between A and B. So, considering what possibility 2 says, there is one way the judge could have convicted A: Suppose B said that A is a knight and C said that B is a knave.
If A is the spy, B could be a knave and C could be a knight. A(the spy) would have falsely claimed that C is the spy. So it really is possible that A,B and C made these three statements and that A is the spy.
Now, if B were the spy, then A would have to be a knave in order to claim that C is the spy, and C would also have to be a knave for claiming that B is a knave, and so this is not possible.
If C were the spy, then A would have to be a knight for truthfully claiming that C is a spy, and B would also have to be a knight for truthfully claiming that A is a knight, which is not possible. Therefore, A must be the spy, so it is possible that a could be convicted.
On the other hand, B could be convicted as well. Suppose B said that A is a knight and C said that B is the spy.
If A is the spy, B is a knave for saying that A is a knight and C is also a knave for saying that B is the spy, which, again, is not possible.
If C is the spy, then A is a knight(since he said C is the spy), and B is also a knight for saying that A is a knight, so this is also not possible. But if B is the spy, there is no contradiction.
So, there is a possibility if A said that C was the spy, the judge could convict B in some cases, and A in others, and there is no way to tell which is the real convict. The only way this problem is solvable is if we're being told that A said that C is a knave. Then, as in Possibility 1, the judge could convict only B, therefore B is the spy.

Mace4 Solutions

Listing 1: The Case of John

```
formulas(assumptions).
% j1 = fratele 1 e john
%-j1 = fratele 1 nu e john
% j2 = fratele 2 e john
%-j2 = fratele 2 nu e john

% t1 = fratele 1 zice adv
%-t1 = fratele 1 nu zice adv
% t2 = fratele 2 zice adv
%-t2 = fratele 2 nu zice

%dialog
%d1 = ce spune fratele 1
%d2 = ce spune fratele 2
%%%%%%%%%%%%%%%%%%%%%%%%%

%doar un john
(j1 & -j2) | (-j1 & j2).
%cel putin un mincinos
(t1 & -t2) | (-t1 & t2) | (-t1 & -t2).

%dialog
d1 <-> j1.
d2 <-> j2 | -j2.

(-j1 & -t1) -> -d1.
(-j1 &  t1) -> -d1.
( j1 & -t1) -> -d1.
( j1 &  t1) ->  d1.

(-j2 & -t2) -> -d2.
(-j2 &  t2) -> -d2.
( j2 & -t2) -> -d2.
( j2 &  t2) ->  d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f john.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 2798 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [0]),
    relation(d2, [1]),
    relation(j1, [0]),
    relation(j2, [1]),
    relation(t1, [0]),
    relation(t2, [1])]).
```

Listing 2: A Transylvanian Metapuzzle

```
formulas(assumptions).
%s1/vampire
% s1 : igor nu e nebun
%-s1 : igor e nebun

%sane/insane
% h1 : igor e om
%-h1 : igor e vampir

%dialog
%d1 = ce spune logicianul 1
%d2 = ce spune logicianul 2
%d3 = ce spune logicianul 3
%%%%%%%%%%%%%%%%%%%%%%%%%

%log1: sane s1?
d1 <-> (h1 & s1) | -(h1 & s1).

%log2 : sane vamp?
d2 <-> (h1 & -s1) | -(h1 & -s1).

%log3 : insane vamp?
d3 <-> (-h1 & -s1) | -(-h1 & -s1).

%aici luam toate cazurile posibile de vampirism si nebunie in urmatorul fel:
%Sane s1 => True
%Insane s1 => False
%Sane Vampire => False
%Insane Vampire => True

( s1 &  h1) ->  d1.
( s1 & -h1) ->  d1.
(-s1 &  h1) ->  d1.
(-s1 & -h1) -> -d1.


( s1 &  h1) -> -d2.
( s1 & -h1) ->  d2.
(-s1 &  h1) -> -d2.
(-s1 & -h1) -> -d2.

( s1 &  h1) -> -d3.
( s1 & -h1) ->  d3.
(-s1 &  h1) ->  d3.
(-s1 & -h1) ->  d3.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f igor.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 22783 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(d1, [1]),
    relation(d2, [1]),
    relation(d3, [1]),
    relation(h1, [0]),
    relation(s1, [1])]).
hulea@hulea-VirtualBox:~/Desktop$
```

Listing 3: A Knight-Knave Metapuzzle

```
formulas(assumptions).
% a1 : a knave
%-a1 : a knight
% b1 : b knave
%-b1 : b knight

%dialog
%d1 = ce spune logicianul 1
%%%%%%%%%%%%%%%%%%%%%%%%%

%are you both knights
d1 <-> (a1 & b1) | -(a1 & b1).

%are you two of the same type
d2 <-> ((a1 & b1) | (-a1 & -b1)) | ((-a1 & b1) | (a1 & -b1)).

%aici luam toate cazurile posibile :
%1) a and b knight
%2) a knight, b knave
%3) a knave, b knight
%4) a and b knave

(-a1 & -b1) -> d1.
(-a1 &  b1) -> d1.
( a1 & -b1) ->-d1.
( a1 &  b1) -> d1.

(-a1 & -b1) -> d2.
(-a1 &  b1) ->-d2.
( a1 & -b1) -> d2.
( a1 &  b1) ->-d2.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f knightknave.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 2796 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(a1, [0]),
    relation(b1, [0]),
    relation(d1, [1]),
    relation(d2, [1])]).
```

Listing 4: Knights, Knaves and Normals

```
formulas(assumptions).
% a1 : a knight
%-a1 : a normal
% b1 : b knight
%-b1 : b normal

%one knight one knave
(a1 & -b1) | (-a1 & b1).

%is B normal? yes or no
d1 <-> b1 | -d1.

a1 -> d1.
-a1 -> -d1 | d1.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f normals.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 2792 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(a1, [0]),
    relation(b1, [1]),
    relation(d1, [1])]).
```

Listing 5: Who is the spy?

```
formulas(assumptions).
% a1 : a knight
% a2 : a knave
% a3 : a normal
% b1 : b knight
% b2 : b knave
% b3 : b normal
% c1 : c knight
% c2 : c knave
% c3 : c normal

%1 knight 1 knave 1 spy

(a1 & b2 & c3) |
(a1 & b3 & c2) |
(a2 & b1 & c3) |
(a2 & b3 & c1) |
(a3 & b1 & c2) |
(a3 & b2 & c1) .

a1 -> -a2.
a1 -> -a3.
a2 -> -a3.

b1 -> -b2.
b1 -> -b3.
b2 -> -b3.

c1 -> -c2.
c1 -> -c3.
```

```
c2 -> -c3.

%a said that c is a knave or c is a spy
da <-> c2 | c3.

%b said that a is a knight or a is a knave or a is the spy
db <-> a1 | a2 | a3.

%c said that b is a knight or b is a knave or b is the spy
dc <-> b1 | b2 | b3.

a1 -> da.
a2 -> -da.
a3 -> da | -da.

b1 -> db.
b2 -> -db.
b3 -> db | -db.

c1 -> dc.
c2 -> -dc.
c3 -> dc | -dc.

end_of_list.
```

```
hulea@hulea-VirtualBox:~/Desktop$ mace4 -c -n 2 -m -1 -f spy.in | interpformat

=== Mace4 starting on domain size 2. ===

------ process 2794 exit (all_models) ------
interpretation( 2, [number = 1,seconds = 0], [
    relation(a1, [0]),
    relation(a2, [1]),
    relation(a3, [0]),
    relation(b1, [0]),
    relation(b2, [0]),
    relation(b3, [1]),
    relation(c1, [1]),
    relation(c2, [0]),
    relation(c3, [0]),
    relation(da, [0]),
    relation(db, [1]),
    relation(dc, [1])]).
```

Bibliography

[1] Raymond M Smullyan. *The lady or the tiger?: and other logic puzzles*. Courier Corporation, 2009.