

## Case study - I

Determine whether  $X$  is a prime number or not.

Solution-1:

Algorithm:

if  $(X == 1) \parallel (X == 2)$

print  $\rightarrow$  "X is a prime number";

for(int  $i = 2$ ;  $i < X$ ;  $i++$ )

{ if  $(X \% i == 0)$

{ print  $\rightarrow$  "X is not a prime number";

return; } }

~~print~~ print  $\rightarrow$  "X is a prime number"; }

Let  $X = 31$

prime( $X$ ) = search space

= 2 to 30

Time complexity:

Best case : a)  $X$  is an even number

b)  $\Omega(\cdot) = 1$

Worst case : a)  $X$  is a prime number

b)  $O(\cdot) = X$

Solution - 2:

Algorithm:

if  $(X == 1) || (X == 2)$

{ print  $\rightarrow$  "X is a prime number";  
return ; }

for  $(i = 2; i \leq (\frac{X}{2}); i++)$

{ if  $(X \% i == 0)$

{ print  $\rightarrow$  "X is not a prime  
number"

return ; } }

~~else~~

print  $\rightarrow$  "X is a prime  
number"; }

Let  $X = 31$

prime  $(X)$  = search space

= 2 to 15

Search space is reduced to half

in solution 2 as compared to solution

1.

Time complexity:

Best case: a)  $X$  is an even number

b)  $\Omega(\cdot) = 1$

Worst case: a)  $X$  is a prime number

b)  $O(\cdot) = (X/2)$

Solution - 3:

Algorithm:

if  $((X == 1) \vee (X == 2))$

print  $\rightarrow$  " $X$  is a prime number"  
return;

if  $(X \% 2 == 0)$

print  $\rightarrow$  " $X$  is not a prime number. In fact,  $X$  is an even number";

return;

for  $(i = 3; i \leq (X/2); i = i + 2)$

if  $(X \% i == 0)$

print  $\rightarrow$  " $X$  is not a prime number";

return;

Print  $\rightarrow$  "X is a prime number";  
return;

$$X = 31$$

Prime(X) = search space

$$\text{prime}(X) = 3, 5, 7, 9, 11, 13, 15$$

In solution 3, the half remaining <sup>search space</sup> solution 2 is further reduced by removing the even numbers from it.

Time complexity:

Best case: a) X is an even number or equal to 1 or 2

$$b) \Omega(\cdot) = 1$$

Worst case: a) X is a prime number

$$b) O(\cdot) = \lfloor X/4 \rfloor$$

Conclusion:

Solution 2 is twice faster than solution 1.

$$\text{Sol}_2 = 2 \times \text{Sol}_1$$

Solution 3 is two times faster

than solution 2, and four times  
faster than solution 1

$$sol_3 = 2 \times sol_2$$

$$sol_3 = 4 \times sol_1$$