Classification of Data (Worked Examples)

Frequency Distribution

Q: 1 The raw data given below show the weights of 50 students in pounds

114	120	103	110	124	135	132	125	124	119	116
105	122	125	145	87	136	117	117	119	131	134
104	102	109	126	119	131	127	146	122	120	118
114	119	129	124	99	92	120	143	138	119	118
96	140	126	121	95	112					

a) Construct a frequency distribution using 8 equal classes
b) Find mid points and class boundaries of the interval
c) Construct histogram and frequency polygon.

Solution:

Construction of Grouped Frequency distribution

Step 1: Determine number of classes (m)

Here number of classes is given i.e. **m= 8**

Step 2: Determine the class width (h)

h= $\frac{Range}{m}$; Range = Max value – Min value = 146- 87 = 59

h= $\frac{\text{Max value – Min value}}{m}$

h= $\frac{59}{8}$ = 7.375 ~ 8 , so **h = 8**

Step 3: Deciding the starting point (i.e. lower limit of first class)

The starting point should be the minimum value of less than the minimum value of the data. Here minimum value is 87 so we start with 85.

Step: 4 Determine the remaining class

Remaining class can be obtained by adding the certain amount of width repeatedly.

Step 5: Distribute the data and total frequency column

This step can be done by using tally column. Then add the tally marks accordingly to fill up the frequency column

(a)

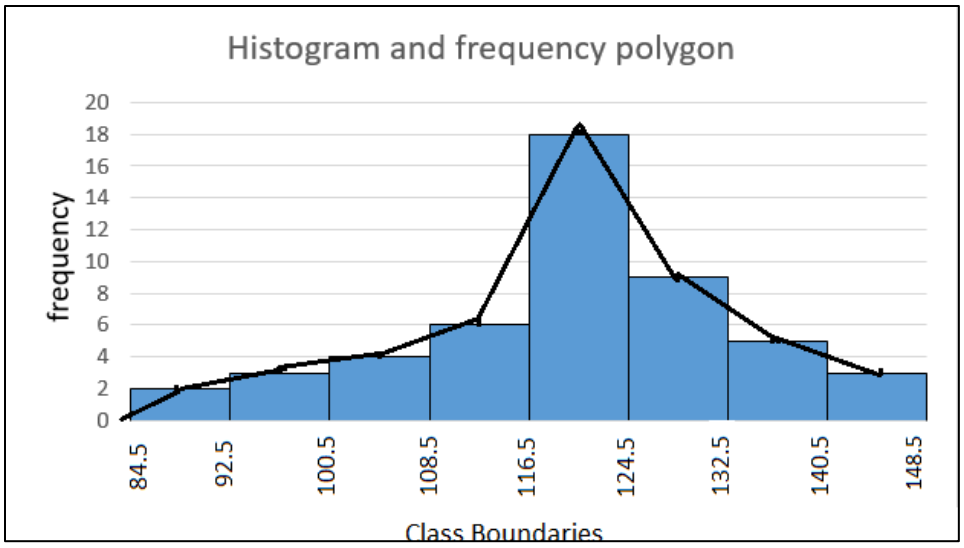
Class interval	Tally Marks	Frequency
85-92	II	2
93- 100	III	3
101- 108	IIII	4
109- 116	𝍸 I	6
117- 124	𝍸 𝍸 𝍸 III	18
125-132	𝍸 IIII	9
133-140	𝍸	5
141-148	III	3
	Total	50

Now find class boundaries and mid- values (x) .The class boundaries can be obtained by adding 0.5 in the upper limit and subtracting 0.5 in the lower limit. For mid points we take the average of each class interval.

(b)

Class interval (C.I)	Frequency (f)	Class Boundaries (C.B)	Mid values (x)
85-92	2	84.5 - 92.5	88.5
93- 100	3	92.5 - 100.5	96.5
101- 108	4	100.5 - 108.5	104.5
109- 116	6	108.5 - 116.5	112.5
117- 124	18	116.5 - 124.5	120.5
125-132	9	124.5 - 132.5	128.5
133-140	5	132.5 -140.5	136.5
141-148	3	140.5 - 148.5	144.5

(c) Histogram and frequency polygon



Mean , Variance and Standard Deviation for Grouped data

Q: 2 Find an estimate of the variance and standard deviation of the following data for the marks obtained in a test by 88 students.

Marks (x)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	16	24	25	17

Solution:

Marks	Mid Interval values (x)	f	fx	x^2	fx^2
0 – 10	5	6	30	25	150
10 – 20	15	16	240	225	3600
20 – 30	25	24	600	625	15000
30 – 40	35	25	875	1225	30625
40 – 50	45	17	765	2025	34425
		88	2510		83800

Find out the mean first,

$$\text{Mean } \bar{x} = \frac{\sum fx}{n} = \frac{2510}{88}$$

$$\text{Variance } \sigma^2 = \frac{\sum fx^2}{n} - \bar{x}^2 = \frac{83800}{88} - \left(\frac{2510}{88}\right)^2 = 952.273 - 813.546 = 138.727 = 138.73 \text{ (2 dp)}$$

$$\text{Standard deviation} = \sqrt{138.727} = 11.78$$

Practice question

The grouped frequency table shows the length of service in years of employees who have been working for a company for at least ten years. Calculate an estimate of the standard deviation of the length of service of these employees.

Length of Service (x)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Frequency (f)	30	42	23	13	8	4

Mean , Variance and Standard Deviation for UnGrouped data

Q: 3 Find the variance of 6, 7, 10, 11, 11, 13, 16, 18, 25.

Solution

Firstly we find the mean,

$$\bar{x} = \frac{\sum x}{n}$$

$\bar{x} = \frac{117}{9} = 13$

Then find the variance, the formula of finding variance for ungrouped data is,

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

x	6	7	10	11	11	13	16	18	25	Total
x^2	36	49	100	121	121	169	256	324	625	1801

$$\begin{aligned} \sigma^2 &= \frac{\sum x^2}{n} - \bar{x}^2 \\ &= \frac{1801}{9} - 13^2 \\ &= 200.11 - 169 \\ &= 31.11 \end{aligned}$$

Standard Deviation (σ)

Since the variance is measured in terms of x^2, we often wish to use the standard deviation where $\sigma = \sqrt{\text{variance}}$. The standard deviation, unlike the variance, will be measured in the same units as the original data.

In the above example $\sigma = \sqrt{31.11} = 5.58$

Exercises

Find the variance and standard deviation of the following

1. a) $10, 16, 12, 15, 9, 16, 10, 17, 12, 15$

   b) $74, 72, 83, 96, 64, 79, 88, 69$

   c) £326, £438, £375, £366, £419, £424

Answers

1. a) 7.76, 2.79 b) 97.36, 9.87 c) £^{2}1 531.22, £39.13