(Analysis) Fill-in-the-dice

Solving this problem required a few unsafe moves to make any progress.

1. It all started with a **strengthening** — conjecturing a part of the existential — that is, that the "low" dice contains a 4.

   - **Why it helped:** I think that If you don't guess there's a 4, there are too many possibilities to brute-force, and it's difficult to make progress.

2. From there, every time we got stuck, we got unstuck by **strengthening** the statement (via conjecturing an existential), and learning from any resulting failures.

   - In particular, the process was to:

     - (Take a risk) Guessing a number was on a certain dice face

     - (Failing) Realizing we ended up with a higher probability of rolling a certain value than you were allowed

     - (Learning from failure) Realizing you needed fewer of that number on that dice face

   - **Why it helped:** There were too many possibilities in the search space, and strengthening artificially narrowed it down. Since we just had to construct an object, we know we would either (1) end up with a correct construction, or (2) end up with a contradiction, and so we could go back to our "unsafe move" and make a different choice.

   - **When it helped:** When there were too many degrees of freedom in creating a correct construction otherwise.

Note: This usage of "conjecturing the existential" has the same flavour as the usage of conjecturing the existential while solving a chess puzzle e.g. as Tim said, "boldly conjecturing" that the winning strategy will involve moving the bishop.

Note: It's also worth noting that Tim didn't solve this problem using these unsafe moves — rather, he made one really big "unsafe" move (that turned out to be safe) to narrow down the search space — he did assume the number on the low-die is a 4 (like I did), but he also assumed the distributions on the low die would be symmetric. That is, he immediately guessed that the distribution would be 1,2,2,3,3,4. And that assumption allowed him to construct the dice faces without any backtracking (which did mean he didn't necessarily learn from failure).