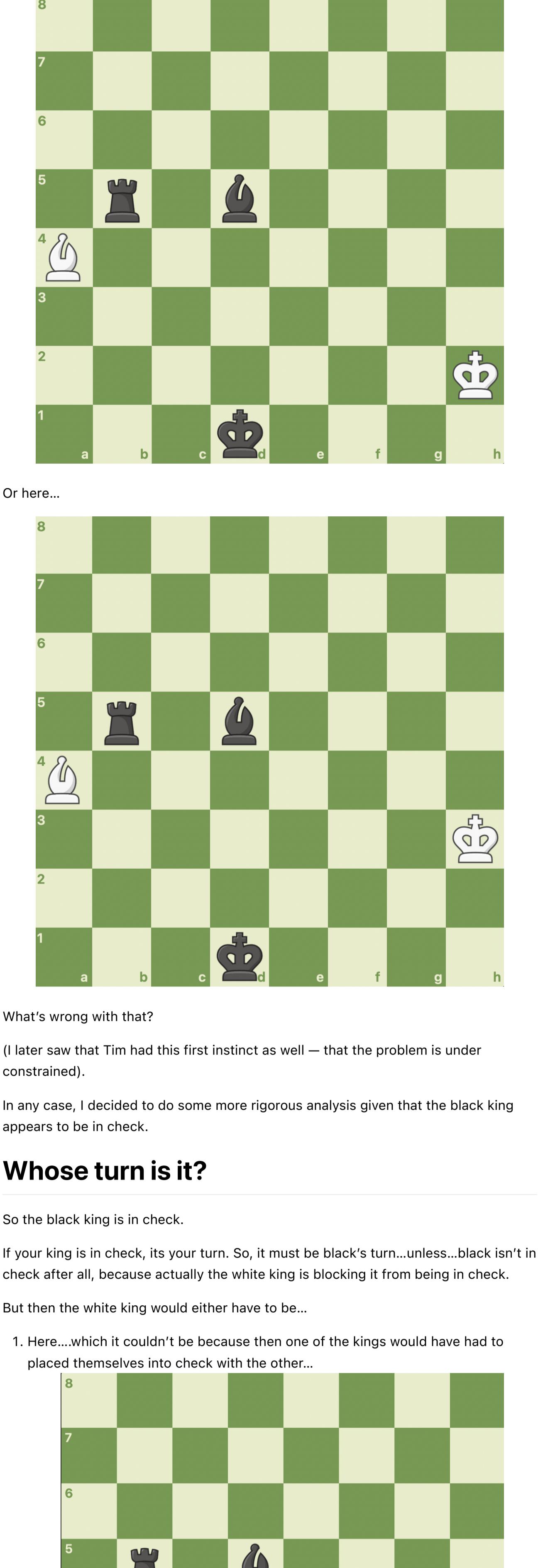


Chess Puzzle: Solution

Mirek introduced us to the following puzzle. Two people are in the middle of a chess game, but suddenly the white king is blown off the board.

- Whose turn is it, and
- Where was the white king?



(Note: white started on lower part of board, black started on upper part of board).

A first instinct...

My first instinct was ...there are way too many options.

For example, it could be black's turn (to get the king out of check), and the white king could be either here...



What's wrong with that?

(I later saw that Tim had this first instinct as well — that the problem is under constrained).

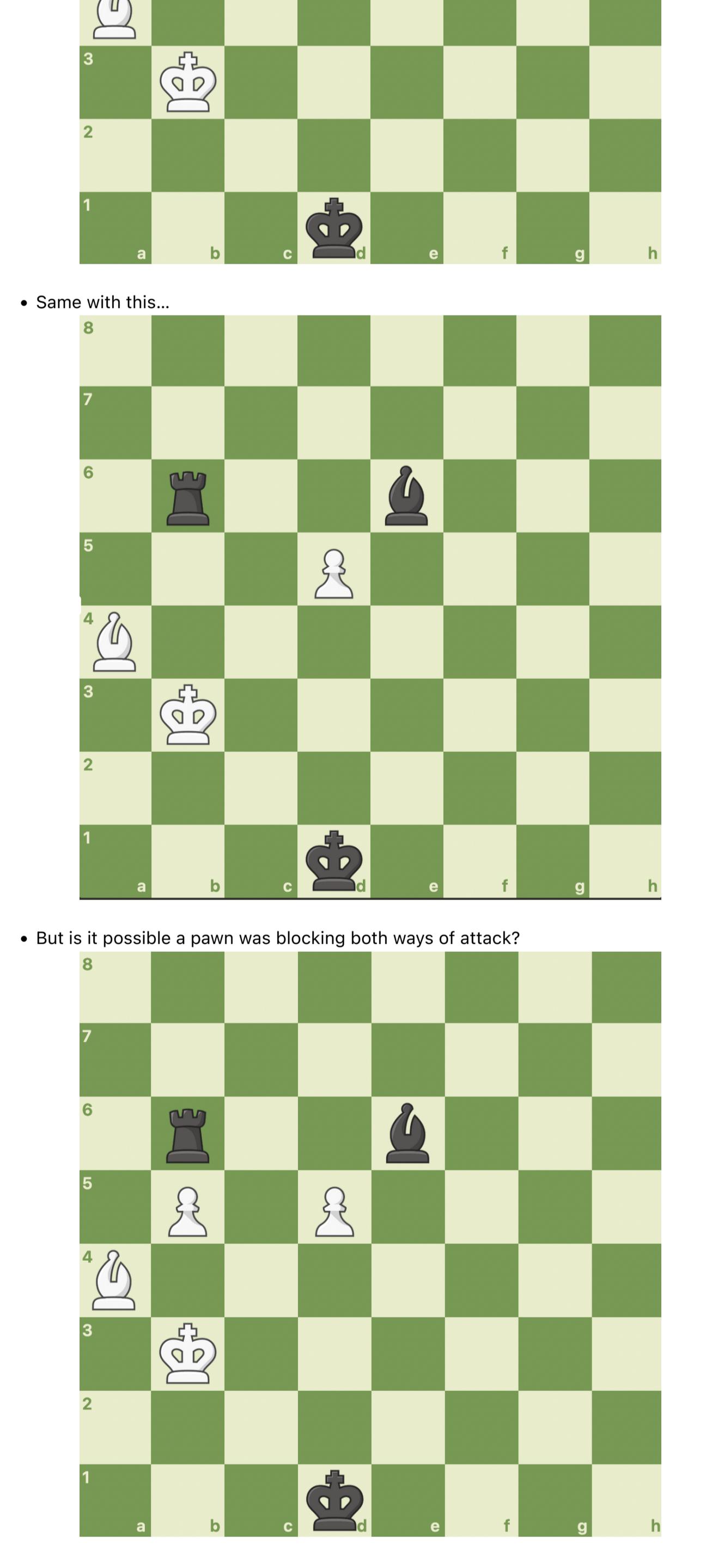
In any case, I decided to do some more rigorous analysis given that the black king appears to be in check.

Whose turn is it?

So the black king is in check.

If your king is in check, it's your turn. So, it must be black's turn...unless...black isn't in check after all, because actually the white king is blocking it from being in check.

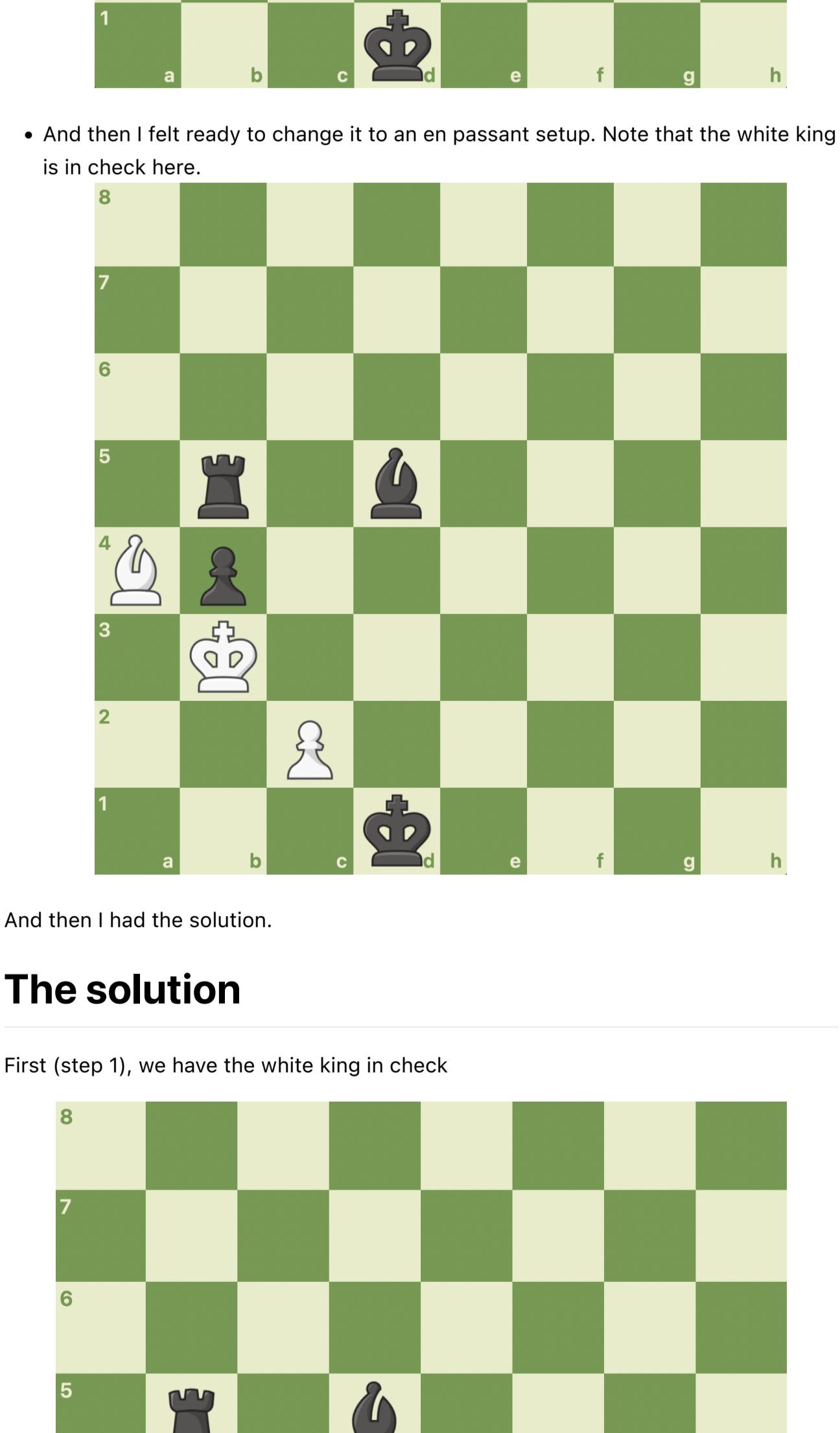
But then the white king would either have to be...



...which I thought it couldn't be because then it means it is about to be white's turn, and the turn before, black would have moved either...

- the black rook to re-check the white king after the king didn't move out of the way of the bishop-check (not possible)
- the black bishop to re-check the white king after the king didn't move out of the way of the rook-check (not possible)

So I concluded it must be black's turn right now, and the black king is in check. And we've eliminated two possible places for the white king to be.



I ultimately decided that the white bishop couldn't have been moved this past turn (white just finished its move before the white king blew away). So the white bishop had to have been there before. That's because if the white bishop moved there this move, that means it either came from...

- the top right (where the rook was blocking its way...not possible)
- the bottom right (where it was checking the king, but the king didn't move out of the way...not possible).

That is, the check couldn't have happened by moving the white bishop, because the bishop was there the move before.

- But it couldn't have been the white king that moved out of the way (as shown before, the white king couldn't be blocking that diagonal in either b3 or c2).
- And it couldn't have been another white piece that moved out of the way, because the king is the only white piece not involved in the check.
- So a black piece must have moved out of the way to get that check. But black isn't allowed to put itself into check, so that's not possible either.

So we have no sensible theory for how we came to this position...

(I later saw that Tim also had the same "proof" that this position is impossible to reach...and wasn't certain how to deal with that either.)

Eventually, I gave up, and looked for a hint.

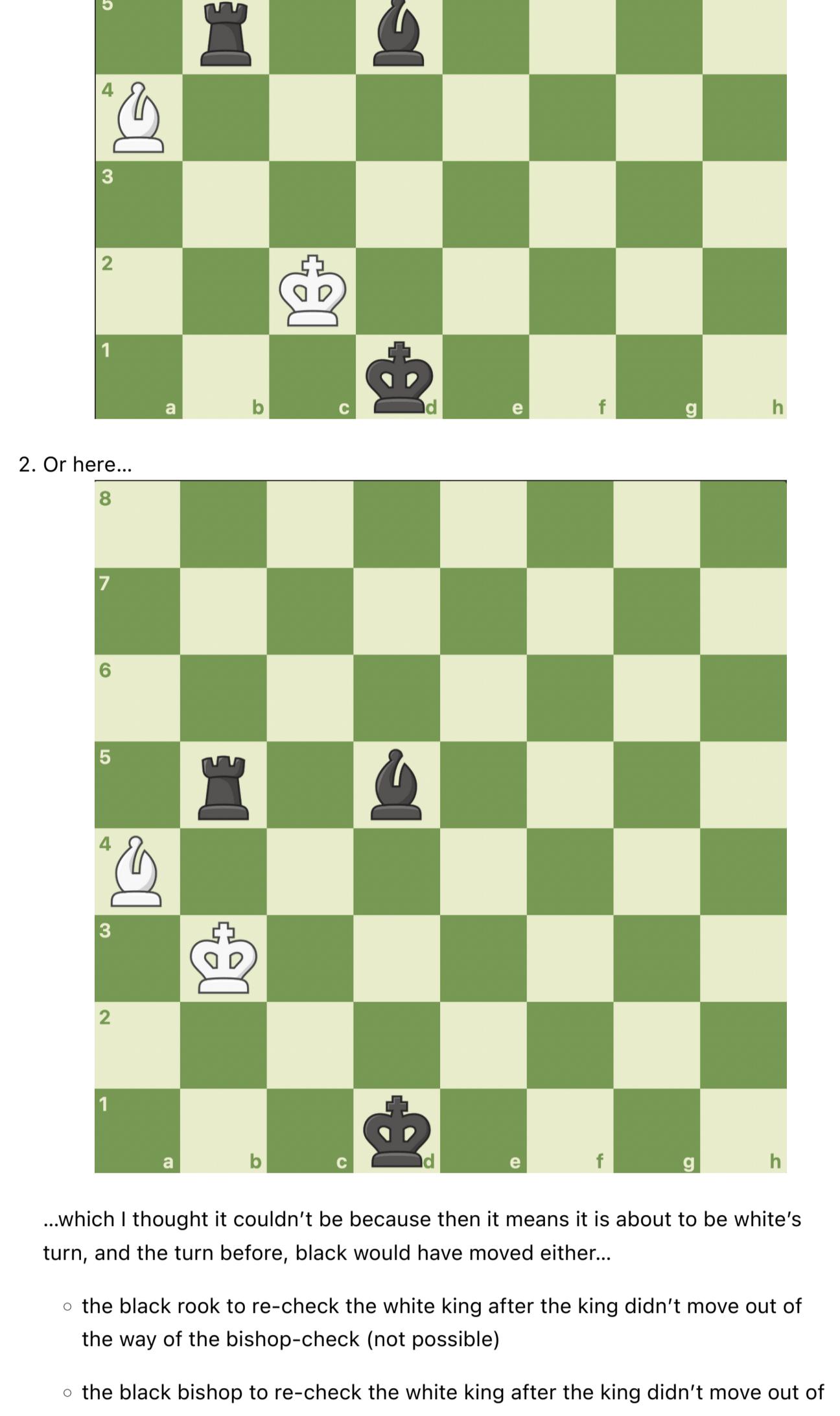
A crucial hint: en passant

I started to read Tim's attempt at the problem, and saw he mentioned there could be some "en passant" going on. As soon as I saw that, I tried again.

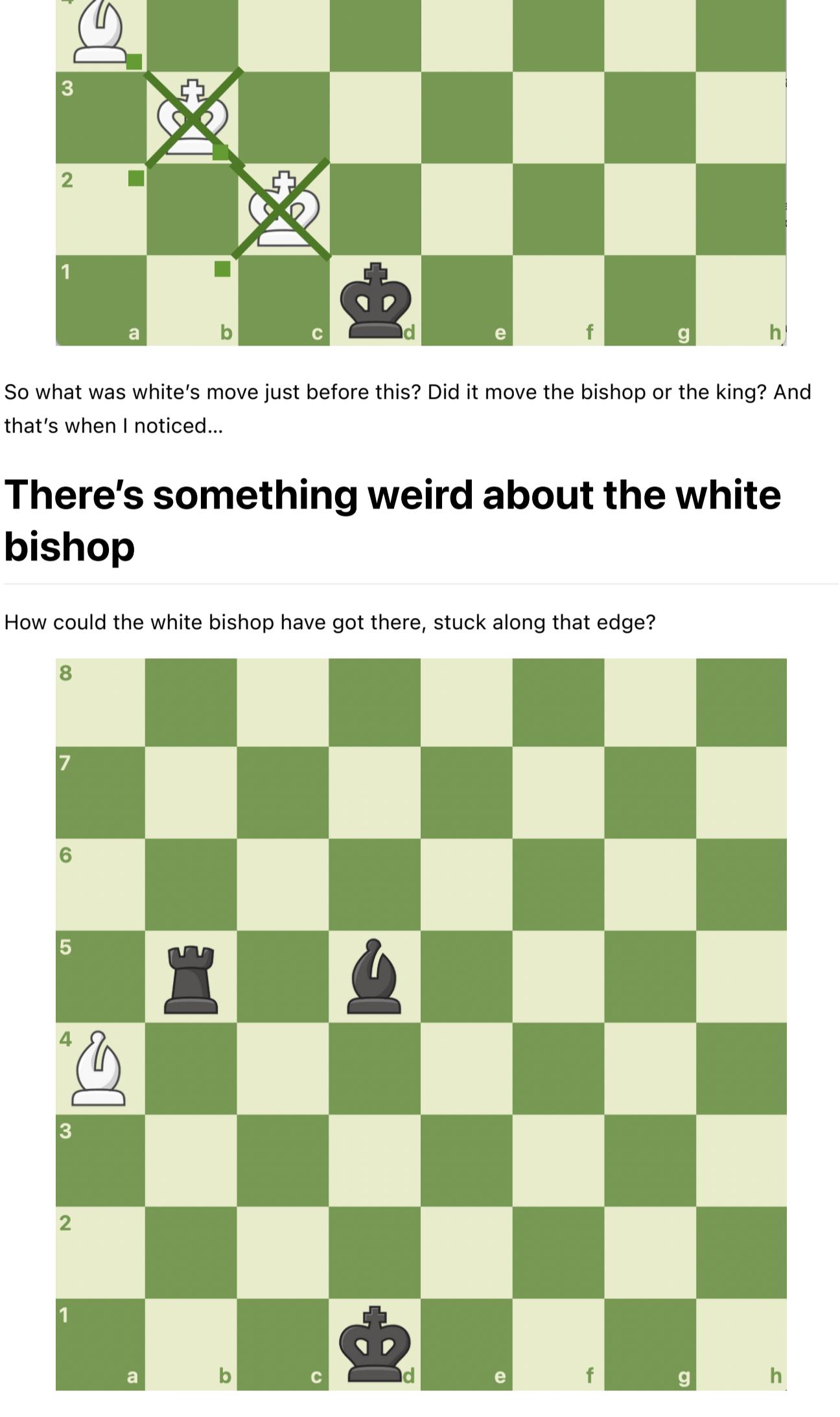
The least watertight part of the "proof there is no solution" is the part about "there's no way to double-check the king". Tim noticed this too.

- Same with this...

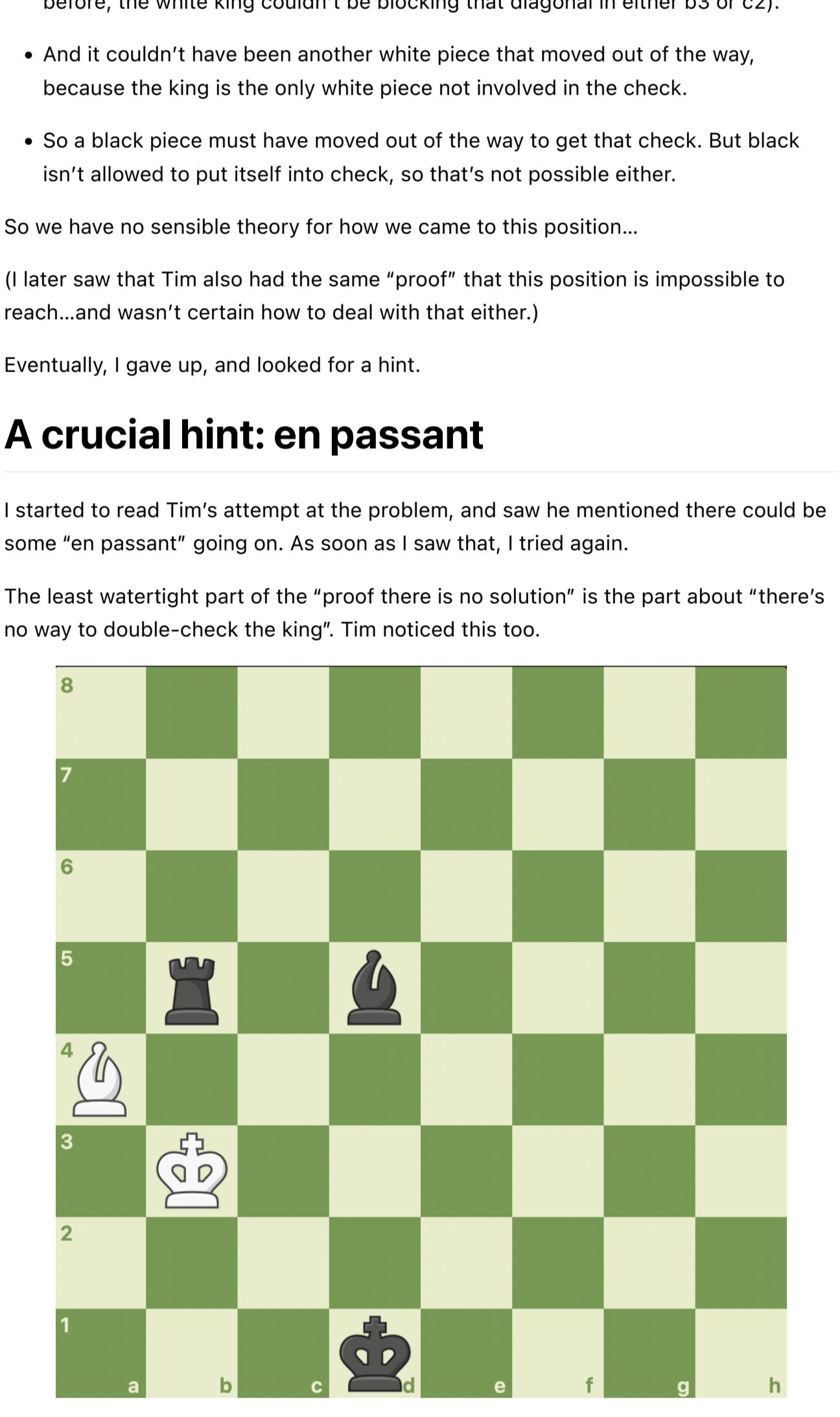
- But is it possible a pawn was blocking both ways of attack?



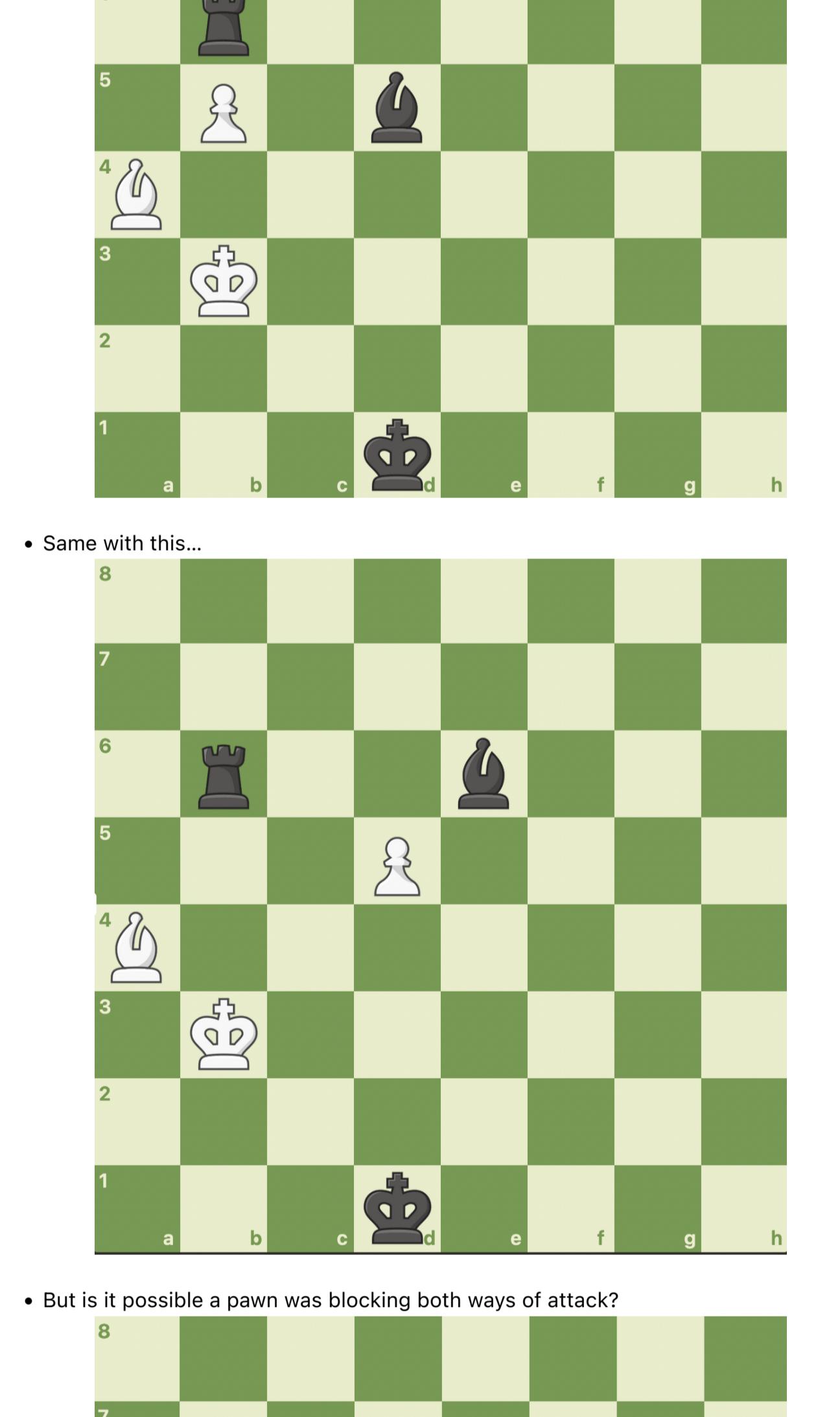
- Then I thought maybe the configuration was more like this, so the bishop and rook are in the same place as when the king is blown away.



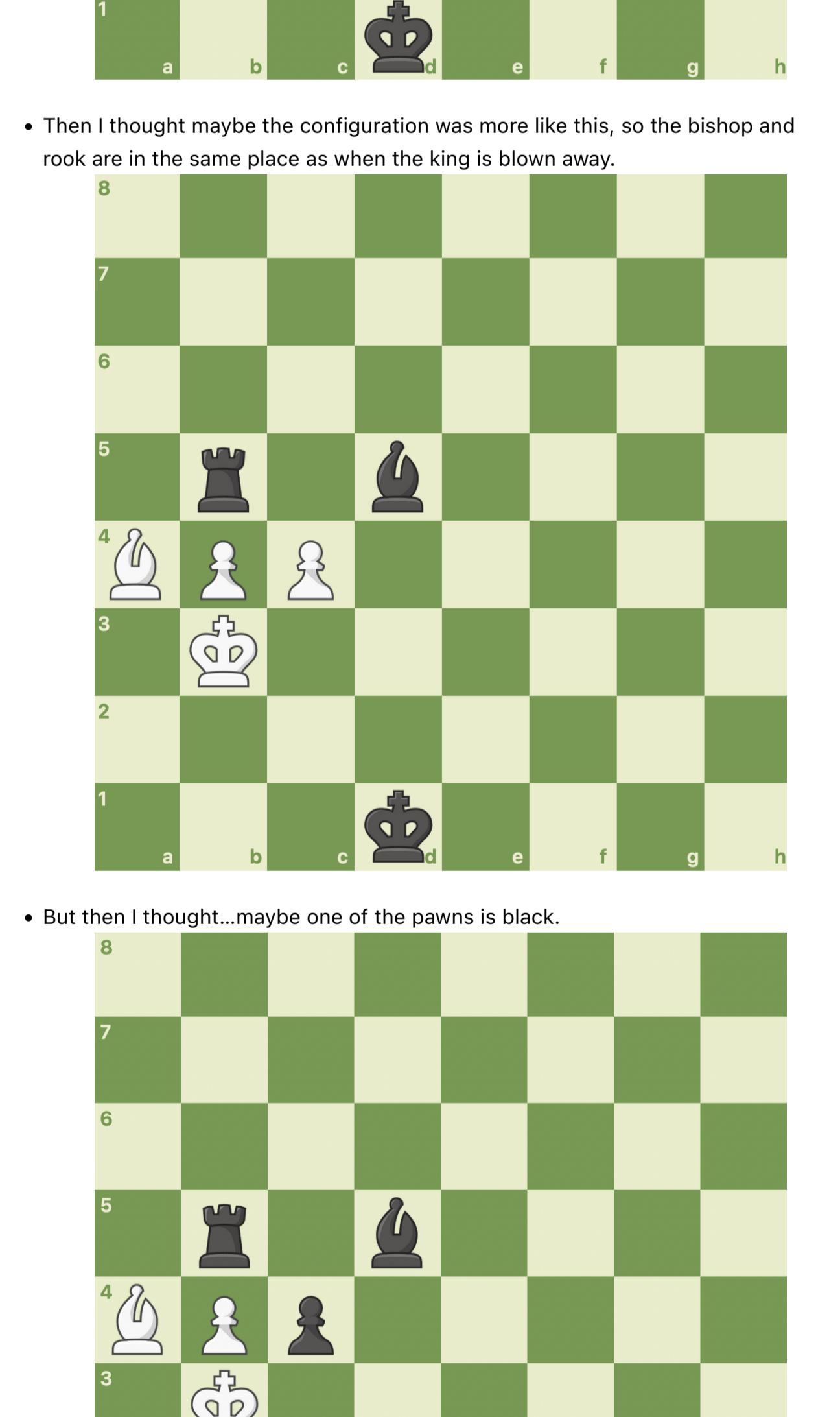
- But then I thought...maybe one of the pawns is black.



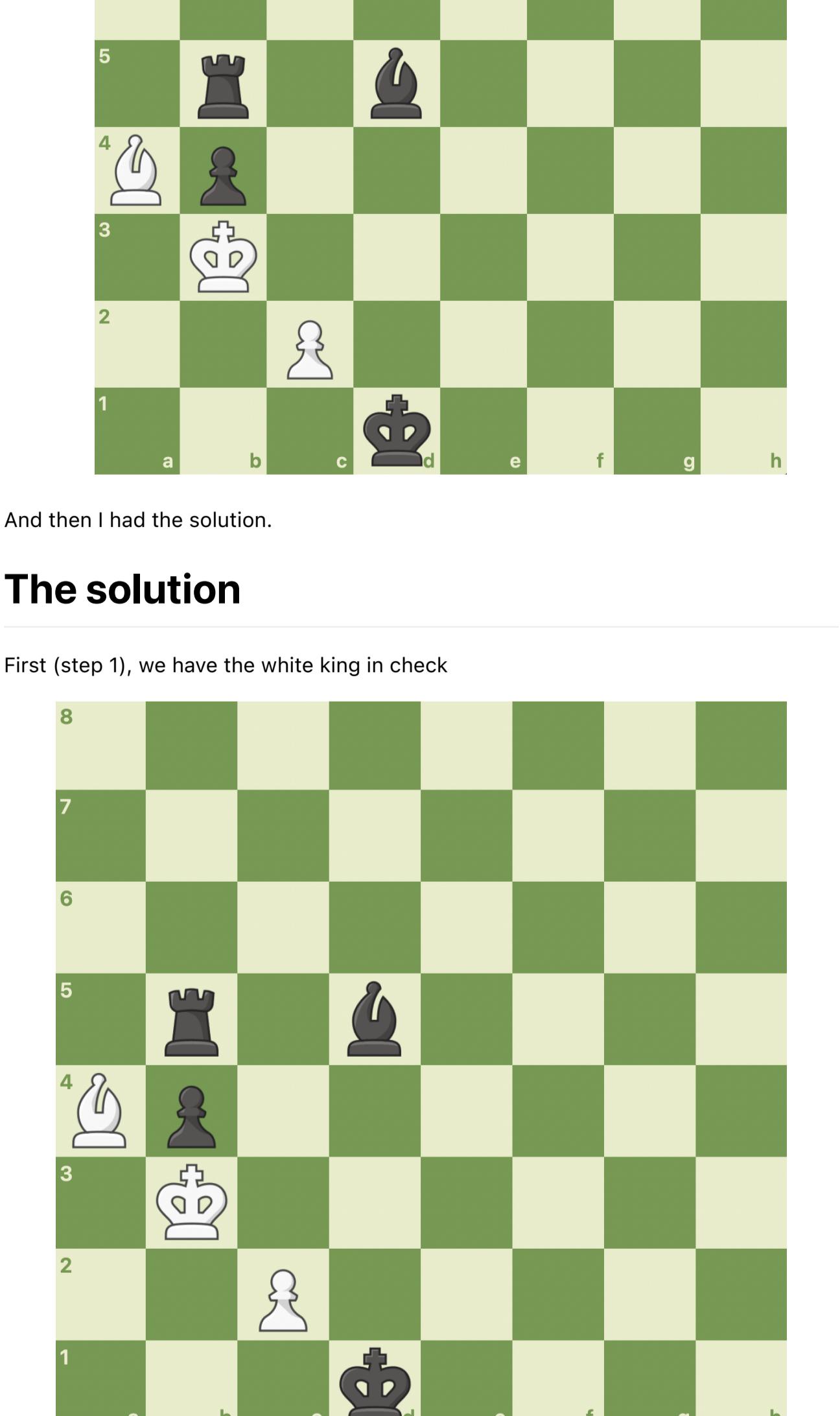
- And then I felt ready to change it to an en passant setup. Note that the white king is in check here.



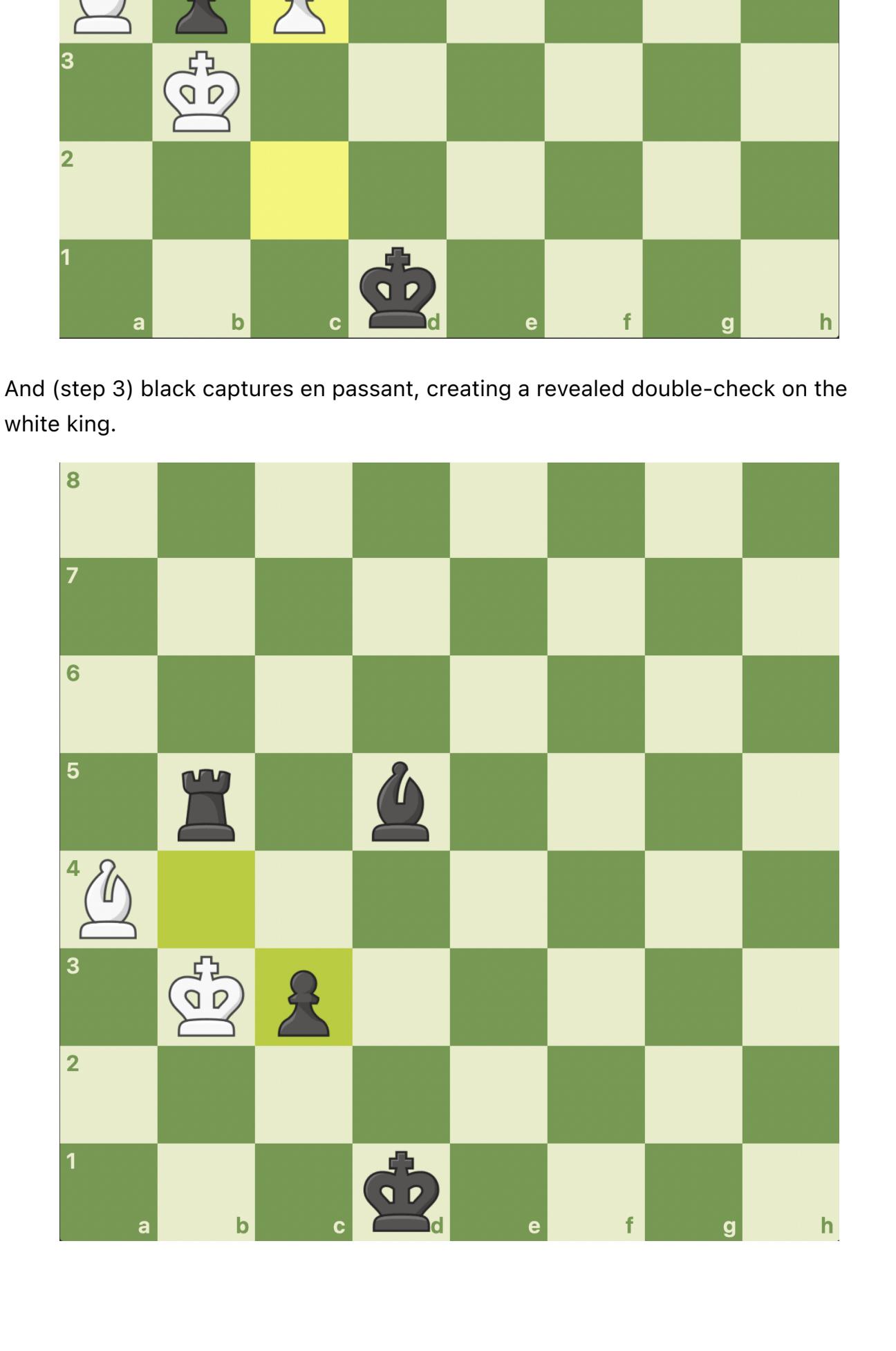
- Then I thought maybe the configuration was more like this, so the bishop and rook are in the same place as when the king is blown away.



- But then I thought...maybe one of the pawns is black.



- And then I felt ready to change it to an en passant setup. Note that the white king is in check here.



And then I had the solution.

The solution

First (step 1), we have the white king in check

Then (step 2) white moves pawn up 2. This gets the white king out of check.

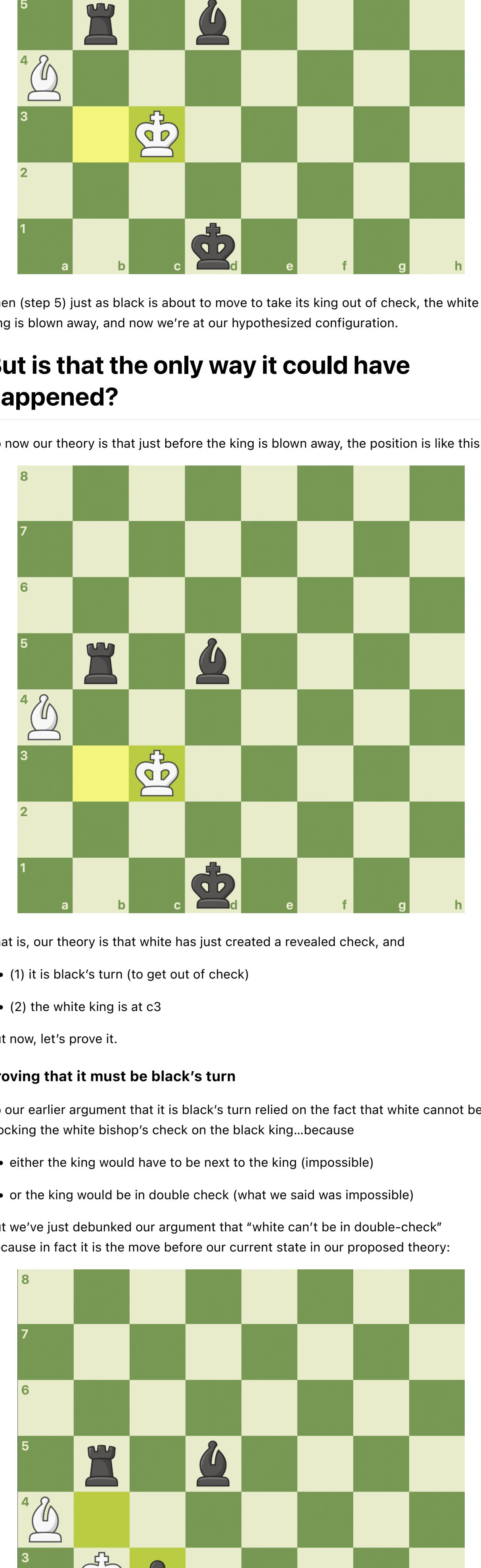
- And then I felt ready to change it to an en passant setup. Note that the white king is in check here.

- Then I thought maybe the configuration was more like this, so the bishop and rook are in the same place as when the king is blown away.

- But then I thought...maybe one of the pawns is black.

- And then I felt ready to change it to an en passant setup. Note that the white king is in check here.

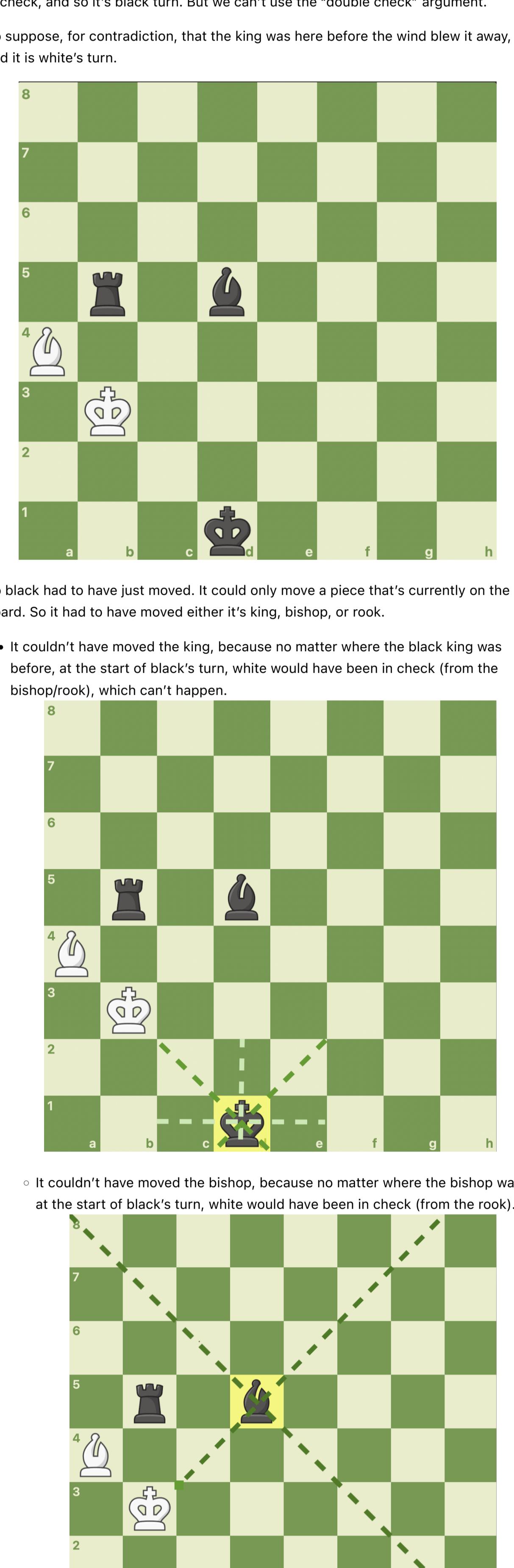
Then (step 4) white king captures the pawn, placing the black king in revealed-check.



Then (step 5) just as black is about to move to take its king out of check, the white king is blown away, and now we're at our hypothesized configuration.

But is that the only way it could have happened?

So now our theory is that just before the king is blown away, the position is like this...



That is, our theory is that white has just created a revealed check, and

- (1) it is black's turn (to get out of check)

- (2) the white king is at c3

But now, let's prove it.

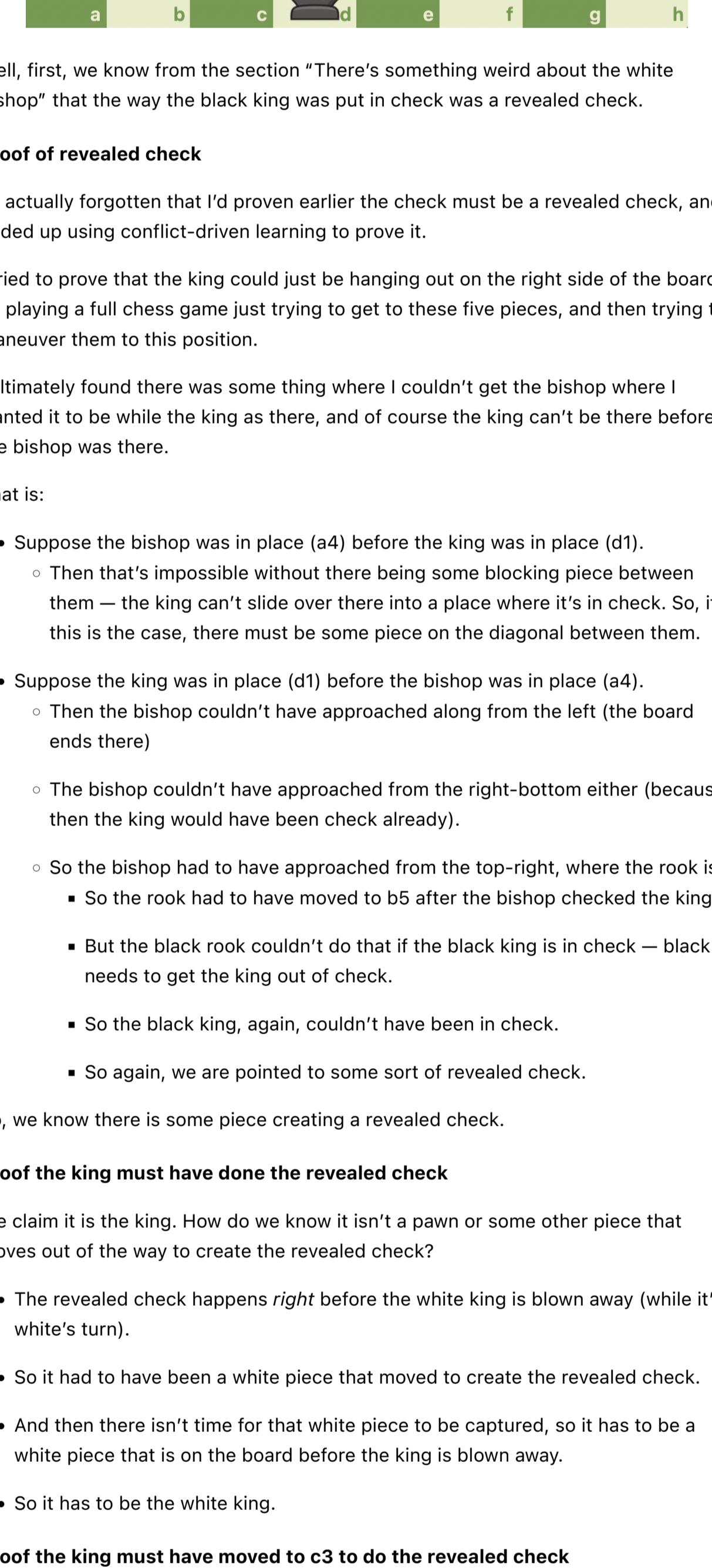
Proving that it must be black's turn

So our earlier argument that it is black's turn relied on the fact that white cannot be blocking the white bishop's check on the black king...because

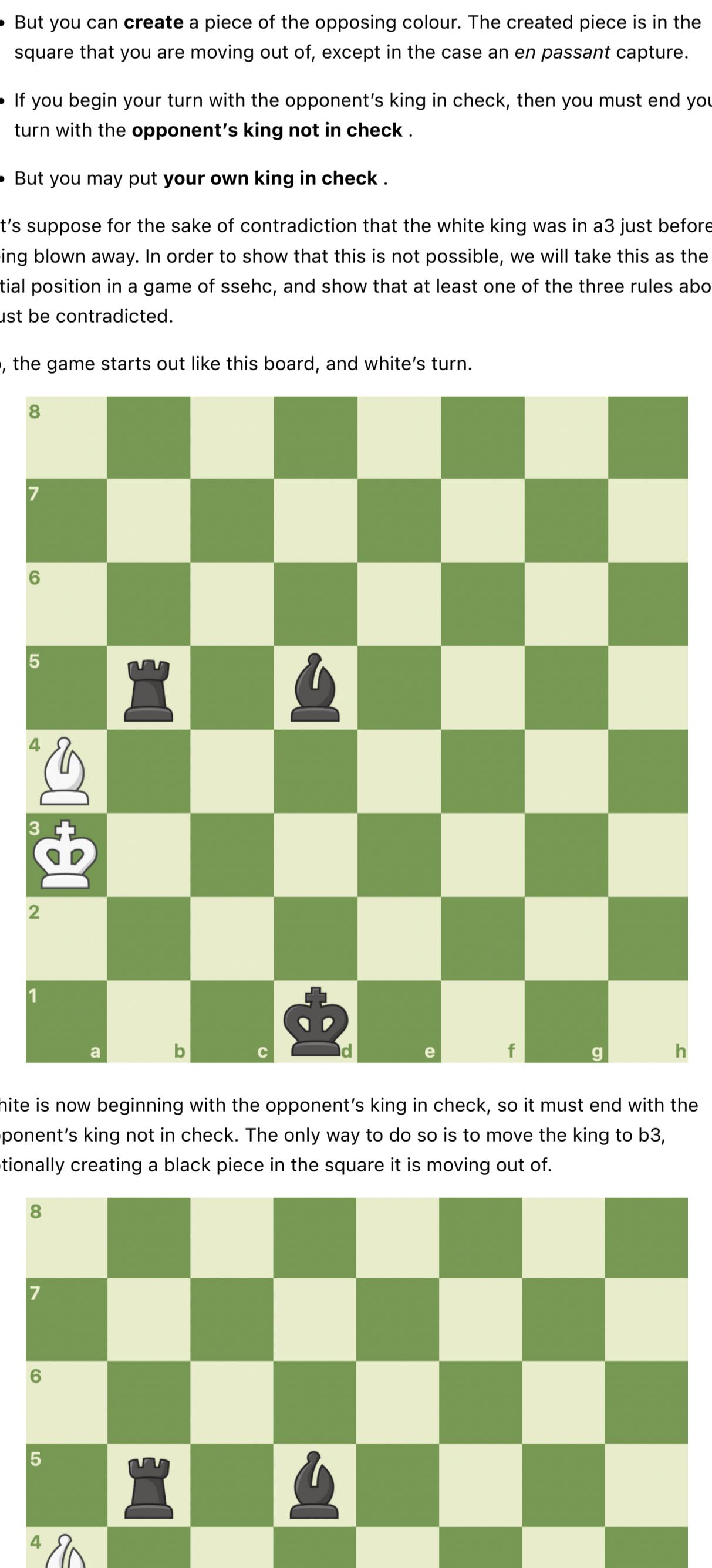
- either the king would have to be next to the king (impossible)

- or the king would be in double check (what we said was impossible)

But we've just debunked our argument that "white can't be in double-check" because in fact it is the move before our current state in our proposed theory:



So why couldn't the king have just been there all along, meaning it's actually white's turn?



We want to rule out the fact that king could be there, thus proving that black king is in check, and so it's black turn. But we can't use the "double check" argument.

So suppose, for contradiction, that the king was here before the wind blew it away, and it is white's turn.



Then (step 4) white king captures the pawn, placing the black king in revealed-check.

Takeaway

So indeed, not only do we have a *solution* where when the wind picked up (1) it was black's turn and (2) the white king was at c3, but we also have proof of its uniqueness.

