Chess Puzzle: Analysis

How does our solution to this chess puzzle utilize conflict-driven learning (or, learning from failure)?

Defining conflict-driven learning

The process that seems to be present in most conflict-driven learning proofs is the following:

1. **Make a risky move** (e.g. prove the opposite)
2. **Fail** (e.g. derive a contradiction)
3. **Come up with a conflict-inspired lemma** (e.g. find the key property that caused the contradiction)
4. **Apply the conflict-inspired lemma** to prove or make progress on the theorem.

Identifying instances of conflict-driven learning in this puzzle

I used the above steps numerous times while solving this chess puzzle.

One part that is interesting is that this proof consisted of lots of mini-proofs-by-contradiction, each of which added a conflict-inspired lemma e.g. (roughly in order of discovery)

- lemma: it is black's turn
- lemma: the bishop didn't move to check the king
- lemma: there was a revealed check on the black king
- lemma: the white king created the revealed check on the black king
- lemma: "the white king moved to c3 to create that revealed check

That is to say, *oscillation* played a key role.

Anyways, here are a few instances of this lemma creation.

Proving it is black's turn

This was a pretty straightforward proof-by-contradiction, so there was not really an extra "conflict-inspired lemma -> proof" step needed.

1. (Take a risk) I think maybe it's black's turn. So I try to prove the opposite - that it's white's turn.
2. (Fail) We fail - if it was white's turn, the black king couldn't have been in check, so the white king needed to be at b3, but then black must have started its turn the turn before with the white king in check, contradiction. So, it is black's turn.

This process does add a useful lemma to the overall proof though — that it is black's turn.

Proving the white king created the revealed check

1. (Take a risk) I thought it might be possible to have the king on the far-right side of the board. So I tried to prove it was possible — and played a chess game down to those pieces, and then tried to keep that arrangement.
2. (Fail) I failed — I found there was some difficulty in getting both the bishop is in place first (at a4) or the black king is in place first (at d1).
3. (Create a hint) And then I had a hint — I should do case analysis that hinged around whether the king or bishop was in place first.
4. (Apply the hint) The case analysis helped prove that the king-on-the-far-right wouldn't be possible, because the white king is the only piece that could allow that configuration of white bishop and black king, by creating a revealed check.

Proving there has to be a piece that takes a piece in a different place than it lands (and the only way that happens is en passant).

I don't think I actually used this conflict-driven process when I solved the problem (instead, I took a hint from Tim), but I think I see how we make an algorithm do it.

1. (Take a risk) We think it's not possible for black to put the white king in double-check in one move. But this is the only non-watertight part of the argument, so we think it's false. So we try to prove it's true.
2. (Fail) We fail to prove it's true to not create a double check...but we do realize it involves clearing out two pathways for two pieces in one move...we can't find a way to do that, but we can't prove we can't either.
3. (Create a lemma) We add a conflict-inspired lemma: black must have created the double-check on white by moving two pieces in one move: one piece is the piece that moved, and one piece is the piece that gets taken.
4. (Apply the lemma) We work out only way this could happen is with a piece that captures from a different location than where it is. And the only way this could happen is with *en passant*.