Write an integer as sum of coprime integers

In general, there are a few classes for "for all" problems where weakening helps.

This is one of them.

Problem

Given a positive integer n > 6, prove it can be written as the sum of 2 positive integers greater than 1 with no common divisor.

Solution

First Tim noticed this can't be done with 6. The options there are:

- 6=1+5. (Not both > 1)
- 6=2+4. (Not coprime)
- 6=3+3. (Not coprime)

Then Tim noticed it could be done **when n is odd (n=2k+1)**.

- We can write it as (k) + (k+1).

But then we thought about why it could be done **when n is even (n=2k)**.

- We first tried writing it as (k+1) + (k-1). (This ends up working when **k is even**).
    - Then you can notice GCD between two consecutive odd numbers is 1, since gcd(k, k+2) = gcd(k, k+2-k) = gcd(k, 2)= 1 (since a=k-1 is odd).
    - But that only works when k is even.
- Otherwise, you can divide into (k+2)+(k-2). (This ends up working when **k is odd**).
    - Now you have odds that are 4 apart. We might simplify this, using an analogous proof of the proof in the library that we used to prove gcd(k+1,k-1)=gcd(k+2, k)=gcd(k,2)=1.
    - So now we have gcd(k+2,k-2)=gcd(4,k-2)<=4. So it's either 1, 2, or 4 (the only divisors of 4). But it can't be 2 or 4, because k-2 is odd. So, the gcd is 1.

Analysis

Tim **weakened** here when he said: "I don't see why this is true in general. But let me see why it's true for odd numbers."

And then indeed, that weaker statement (why it's true for odd numbers), generalized/analogized nicely to even numbers.

Indeed, it might be the case in general that *in for-all statements*,

- **proving a weaker statement is helpful, because it provides an analogue for the rest of the proof**.

This is a contrast to where proving a weaker statement is helpful **in existence problems**, where

- **proving a weaker statement is helpful, because it parameterizes the existential object**.