The four-digit number aabb is a square. Find it.

Here is Tim's stream-of-consciousness solving this problem:

- OK how are we going to do this? It is equal to $11(100a + b)$, so $100a + b$ must be a multiple of 11, and indeed must be of the form $11c^2$. Writing the number $a0b$, the test for divisibility by 11 tells us that a + b must be a multiple of 11, and it can only be 11. Let me now divide $100a + b$ by 11. Subtracting $99a$ first, I get a + b = 11, so I get the answer $100a + b = 11(9a + 1)$. I need $9a + 1$ to be a perfect square, so $9a = r^2 - 1 = (r + 1)(r - 1)$, which leaves the only possibility as $r = 8$ and $a = 7$. So the number in question is 7744, which turns out to be 88^2.

We could formalize this problem like so:

$$a, b : \mathbb{N}$$

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$$\exists k : \mathbb{N}, k^2 = 11(100a + b)$$

In particular, a and b could be metavariables — we know they are natural numbers, but we want to know more about them, finding equations they satisfy, until we know exactly what a and b are, and therefore what aabb is.

In that case, all the reasoning Tim did was reasoning forward from the target — for example, deducing that because 11(100a+b) was a perfect square, there must be another factor of 11 that divides 100a+b.