

(Analysis) Where do we use strengthening in the proof "a tree has at most $n-1$ edges"?

We start here:

- We want to prove a connected, acyclic graph has at most $n - 1$ edges.
- We prove the contrapositive: that given a connected graph with at least n edges, it has a cycle.

Then:

- We notice the average degree of a vertex is at least 2...
- and then we **strengthen** to *all* vertices having degree at least 2. And we notice in that case, we end up with a cycle.

Then we go further:

- We further **strengthen** by removing a hypothesis (the hypothesis that there are no leaves).
- We **learn** then that leaves are crucial to the proof.
- And in fact, removing leaves (1) keep the graph connected and (2) preserves average degree, which simplifies the proof to the case where average degree is greater than 2, which was the strengthening we already proved.