Create the number 19 by adding 4s and 7s

The Problem

Tim proposed the following problem: make the number 19 as a sum of numbers taken from the set $\{4, 7\}$.

The Solution

The solution is 4+4+4+7=19.

The Motivated Solution

Tim pointed out that we can arrive at this by reasoning forward from the (existential) target.

We want to prove

$$\exists a, b : \mathbb{N} \; \textit{such that} \; 4a + 7b = 19.$$

We know $4a$ is even, and 19 is odd, so $7b$ must be odd, and therefore b must be odd.

We can then see that $b > 1$ gives us a number that is at least 21, which is impossible.

So, we have $b = 1$.

And now all we need is to find a such that $4a + 7 = 19$, and so $a = 3$.

In summary, parameterization via "reasoning forward from target" reduced the problem from finding (a, b)… to finding a…to the solution.