When does this exponential function equal 17?

The Problem

Problem: Is there a solution to $e^{\sqrt{x}} + e^{\sqrt[3]{x}} = 17$?

The Solution

- Tim mentioned it might be a problem that someone, coming straight out of high school, might struggle with.
- But it helps to weaken the hypothesis (and therefore strengthen the statement) by turning the whole left hand side to a continuous function $f(x)$ that attains all real values greater than 2.
- And then you can realize you can use intermediate value theorem to realize there must be a solution, since at some point the continuous function takes on a value less than 17, and at some point it takes on a value greater than 17.
- That is, the information about "e" just distracts you.

The (Motivated) Point-and-Click Solution

So you start with

$$================$$

$$\exists x : \mathbb{R}, e^{\sqrt{x}} + e^{\sqrt[3]{x}} = 17$$

The target a very close syntactic match with the intermediate value theorem, which looks in the library like this:

$$f : \mathbb{R} \to \mathbb{R}$$

$$f : continuous$$

$$y : \mathbb{R}$$

$$\exists x s.t. f(x) < y$$

$$\exists x s.t. f(x) > y$$

$$================$$

$$\exists x : \mathbb{R}, f(x) = y$$

So now, we backward-reason using that library result (by clicking **apply lemma**).

Then, the **library matching forces us to automatically generalize the function** to a generic continuous function f(x):

$$f(x) = e^{\sqrt{x}} + e^{\sqrt[3]{x}}$$

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$$\exists x : \mathbb{R}, f(x) = 17$$

But now we have some additional targets to prove:

$$f(x) = e^{\sqrt{x}} + e^{\sqrt[3]{x}}$$

$$================$$

$$f : continuous$$

And:

$$f(x) = e^{\sqrt{x}} + e^{\sqrt[3]{x}}$$

$$================$$

$$\exists x, f(x) < 17$$

And:

$$f(x) = e^{\sqrt{x}} + e^{\sqrt[3]{x}}$$

$$================$$

$$\exists x, f(x) > 17$$

All of which should be able to be discharged using routine reasoning moves.