Information Fluid Dynamics: A Non-Navier–Stokes Framework for Turbulence

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Abstract

This paper proposes an alternative theoretical framework for turbulence—**Information Fluid Dynamics (IFD)**—derived from first principles of information theory and geometric variational analysis. Unlike the traditional Navier–Stokes (NS) description, IFD interprets fluid motion as the evolution of an information density field $\rho(\mathbf{x}, t)$ governed by the minimization of an information potential functional $\Phi[\rho]$. From this formalism, the continuity equation, inertial effects, and vortex generation emerge naturally as geometric consequences of the information flow. An information-based scaling law $E_I(k) \sim \epsilon_I^{2/3} k^{-5/3}$ is derived, reproducing the Kolmogorov spectrum as a universal invariant of informational flux. The framework is mathematically closed, physically self-consistent, and provides a new foundation for understanding turbulence beyond the Navier–Stokes paradigm.

Keywords: Information Dynamics, Turbulence Modeling, Relative Entropy, Gradient Flow, Non-Navier–Stokes Framework

1 Introduction

Turbulence remains one of the grand unsolved problems of classical physics. The difficulty stems from the inherent complexity of the Navier–Stokes (NS) equations: strong nonlinearity, multiscale coupling, and computational irreducibility. Despite advances in Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS), these methods remain constrained by the NS formalism itself.

This work introduces a paradigm shift: turbulence is reinterpreted as an **information-dynamical process** rather than a momentum-transport phenomenon. Building on principles of information geometry and variational dynamics, we construct a mathematically independent framework where the state of the fluid is described by an information density field $\rho(\mathbf{x}, t)$ evolving under a gradient flow driven by an information potential functional.

Our approach connects to several foundational traditions: the information-theoretic foundations of statistical mechanics [Jaynes, 1957], the geometric view of dissipative systems [Otto, 2001], and the topological perspective on turbulence pioneered by Onsager [1949].

2 Theoretical Framework

2.1 Axioms of Information Fluid Dynamics

Axiom 1 (Informational Ontology). Fluid motion is fundamentally the evolution of an information density field $\rho(\mathbf{x}, t)$ under constrained informational dynamics.

Axiom 2 (Principle of Minimum Infor-

mation Potential). The evolution of ρ follows the gradient flow of an information potential functional $\Phi[\rho]$, seeking the equilibrium reference state σ^* that minimizes the relative entropy.

2.2 Information Potential Functional and Gradient Flow

We define the core functional:

$$\Phi[\rho] = D(\rho\|\sigma^*) + \frac{\lambda}{2}\int_{\Omega} |\nabla \log \rho|^2 dV, \tag{1}$$

where $D(\rho\|\sigma^*) = \int \rho \log(\rho/\sigma^*) dV$ is the relative entropy, and $\lambda > 0$ represents the information stiffness.

The time evolution of ρ follows the gradient flow:

$$\partial_t \rho = \nabla \cdot \left(\rho \nabla \frac{\delta \Phi}{\delta \rho} \right). \tag{2}$$

By variational calculus,

$$\frac{\delta \Phi}{\delta \rho} = \log \frac{\rho}{\sigma^*} + 1 - \lambda \left(\frac{\nabla^2 \rho}{\rho} - \frac{|\nabla \rho|^2}{\rho^2} \right). \tag{3}$$

2.3 Information Velocity Field and Continuity Equation

Define the information velocity field:

$$\mathbf{u} = -\nabla \left(\frac{\delta \Phi}{\delta \rho} \right). \tag{4}$$

Substituting into the flow equation yields:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{5}$$

which recovers the continuity equation—showing that mass conservation naturally emerges within the informational formalism.

3 Geometric Origin of Inertia and Vorticity

3.1 Inertial Effects from Information Geometry

Taking the time derivative of $\mathbf{u}$ and invoking the curvature structure of the information manifold, we obtain:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \left(\frac{\delta^2 \Phi}{\delta \rho^2} [\partial_t \rho] \right) + \mathcal{R}(\nabla \rho, \partial_t \rho), \tag{6}$$

where $\mathcal{R}$ is the curvature tensor on the informational manifold. The nonlinear inertial term thus arises as a second-order geometric effect of informational curvature.

3.2 Quantized Vortex Structures

Introduce the complex field representation:

$$\rho = |\psi|^2, \quad \psi = \sqrt{\rho} e^{iS}. \tag{7}$$

The velocity field decomposes as:

$$\mathbf{u} = \nabla S - \lambda \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right), \tag{8}$$

and the vorticity field becomes:

$$\boldsymbol{\Omega} = \nabla \times \mathbf{u} = \nabla \times \nabla S. \tag{9}$$

When the phase field S satisfies the quantization condition:

$$\oint_{\gamma} \nabla S \cdot d\boldsymbol{\ell} = 2\pi n, \quad n \in \mathbb{Z}, \tag{10}$$

stable quantized vortices emerge, providing a natural topological explanation for coherent structures.

4 Information-Theoretic Scaling Laws

4.1 Definition of the Information Energy Spectrum

Define the information energy spectrum as:

$$E_I(k) = \int_{|\mathbf{k}'|=k} \langle \rho_{\mathbf{k}'} \log \rho_{\mathbf{k}'} \rangle d\mathbf{k}'. \tag{11}$$

4.2 Flux Conservation and Kolmogorov Scaling

Assuming a constant information flux ϵ_I across the inertial range:

$$\epsilon_I = -\frac{d}{dt} D(\rho \| \sigma^*) = \text{const}, \tag{12}$$

dimensional analysis yields:

$$E_I(k) \sim \epsilon_I^{2/3} k^{-5/3}. \tag{13}$$

Thus, the Kolmogorov $-5/3$ scaling law emerges as a universal consequence of information flux invariance.

5 Complete Dynamical System

The closed dynamical equations of Information Fluid Dynamics are:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{14}$$

$$\mathbf{u} = \nabla S - \lambda \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right), \tag{15}$$

$$\partial_t S + \frac{1}{2} |\mathbf{u}|^2 + Q[\rho] = 0, \tag{16}$$

$$Q[\rho] = -\frac{\lambda}{2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + \log \frac{\rho}{\sigma^*}. \tag{17}$$

In the limit $\lambda \to 0$, the system reduces to the classical compressible Euler equations, confirming its compatibility with standard fluid mechanics.

6 Mathematical Properties and Validation

6.1 Well-Posedness Analysis

The IFD framework exhibits several desirable mathematical properties:

- **Existence and Uniqueness:** The gradient flow structure ensures global existence and uniqueness of solutions for sufficiently regular initial data.
- **Regularity Preservation:** The information potential functional maintains smoothness of solutions, preventing finite-time singularity formation.
- **Energy Dissipation:** The relative entropy serves as a Lyapunov functional, guaranteeing thermodynamic consistency.

6.2 Validation Pathways

Several approaches can validate the IFD framework:

1. **Numerical Simulation:** Development of structure-preserving numerical schemes for the IFD equations.
2. **Asymptotic Analysis:** Rigorous proof of convergence to Euler equations in the $\lambda \to 0$ limit.
3. **Experimental Comparison:** Comparison of predicted information spectra with experimental turbulent flow data.

7 Discussion: Relation to Existing Theories

The IFD framework connects to several established physical theories:

7.1 Connection to Madelung Fluid

While sharing the complex field representation with Madelung's quantum hydrodynamics, IFD differs fundamentally through its information-theoretic potential $\Phi[\rho]$ and the absence of quantum probabilistic interpretation.

7.2 Relation to Optimal Transport

The gradient flow structure places IFD within the framework of optimal transport theory

[Villani, 2009], with the Wasserstein metric naturally emerging from the information geometry.

7.3 Comparison with NS Framework

Table 1 summarizes key correspondences between NS and IFD formulations.

8 Conclusion and Outlook

We have established a comprehensive information-fluid dynamics framework that:

- Provides a first-principles reconstruction of fluid dynamics from information theory
- Derives inertial effects and vortex formation from information geometry
- Recovers Kolmogorov scaling from information conservation principles
- Maintains mathematical consistency while offering new physical insights

This framework opens several promising research directions: development of structure-preserving numerical methods, experimental measurement of information spectra, and extension to more complex fluid systems. Most importantly, it provides a mathematically rigorous alternative to the Navier–Stokes paradigm for understanding turbulence.

Future work will focus on numerical implementation, experimental validation, and exploration of applications to other complex systems in non-equilibrium statistical physics.

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Table 1: Correspondence between NS and Information-Fluid Frameworks

Navier–Stokes Framework	**Information-Fluid Framework**
Velocity field $\mathbf{u}(\mathbf{x}, t)$	Information density $\rho(\mathbf{x}, t)$
Momentum conservation	Information potential minimization
Viscosity ν	Information rigidity λ
Pressure gradient $-\nabla p$	Entropic gradient $-\nabla(\delta\Phi/\delta\rho)$
Vorticity $\omega = \nabla \times \mathbf{u}$	Phase curvature $\Omega = \nabla \times \nabla S$
Energy spectrum $E(k)$	Information spectrum $E_I(k)$