*E*conometrics *of* *H*uman *C*apital

Philipp Eisenhauer

Material available on

Visit us!

UNIVERSITÄT BONN

Estimation Strategies

Philipp Eisenhauer

Setup

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = \mathrm{I}[\mu_D(X, Z) - V > 0]$$

Key Concept

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp\!\!\!\perp D \quad | X = x.$$

$\Rightarrow$ consequences for the choice of the estimation strategy

Useful Notation

$$\begin{aligned} P(X,Z) &= \Pr(D=1 \mid X,Z) = F_V(\mu_D(X,Z)) \\ U_D &= F_V(V) \end{aligned}$$

Key Assumptions

- (U_1, U_0, V) are independent of Z conditional on X
- $\mu_D(X, Z)$ is a nondegenerate random variable conditional on X
- $0 < \Pr(D = 1 \mid X) < 1$
- $\ldots$

Evaluation Problem

$$Y = DY_1 + (1 - D)Y_0 = \begin{cases} Y_1 & \text{if} \quad D = 1 \\ Y_0 & \text{if} \quad D = 0 \end{cases}$$

Selection Problem

$$
\begin{aligned}
E[Y \mid D=1] - E[Y \mid D=0] = {} & \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\
& + \underbrace{E[Y_1 - Y_0 \mid D=1] - E[Y_1 - Y_0]}_{\text{selection on gains}} \\
& + \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{\text{selection on levels}}
\end{aligned}
$$

Estimation Strategies

- Randomization
- Matching
- Instrumental Variables
  - conventional and local
- Regression Discontinuity
  - fuzzy and sharp design

Randomization

Treatment Status

D	self-selected
ξ	assigned
A	actual

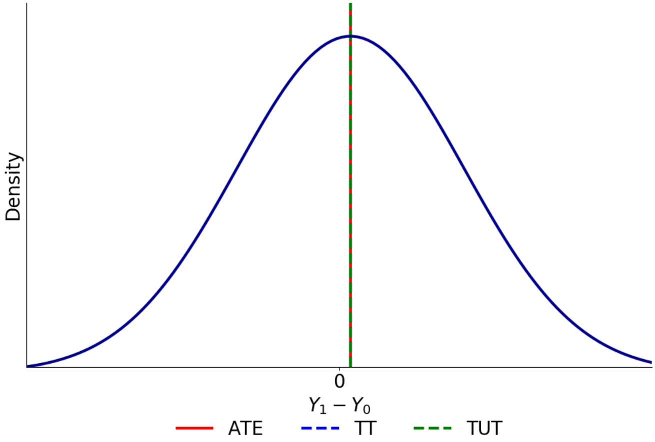
Key Identifying Assumptions

$$(Y_1, Y_0) \perp\!\!\!\perp D$$
$$(Y_1, Y_0) \perp\!\!\!\perp \xi$$
$$(Y_1, Y_0) \perp\!\!\!\perp A$$

When do we have to worry about compliance?

$$
\begin{aligned}
&E(Y \mid A=1) - E(Y \mid A=0) \\
&\qquad = E(Y_1 \mid A=1) - E(Y_0 \mid A=0) \quad \text{(by full compliance)} \\
&\qquad = E(Y_1) - E(Y_0) \quad \text{(by randomization)} \\
&\qquad = B^{ATE} = B^{TT} = B^{TUT}
\end{aligned}
$$

Figure: Distribution of Effects



What if we can only deny program participation to individuals who are willing to participate?

$$\begin{aligned} E(Y \mid D=1, A=1) - E(Y \mid D=1, A=0) \\ &= E(Y_1 \mid D=1, A=1) - E(Y_0 \mid D=1, A=0) \\ &= E(Y_1 \mid D=1) - E(Y_0 \mid D=1) \\ &= B^{TT} \neq B^{ATE} \neq B^{TUT} \end{aligned}$$

Issues

- compliance
- imperfect randomization
- ethical concerns
- feasibility
- expenses
- external validity

Challenges to Scaling Experiments

- market equilibrium effects
- spillovers
- political reactions
- context dependence
- randomization or site-selection bias
- piloting bias

See Banerjee et al. (2017) for a discussion of these challenges and their attempts to address them in their work.

Matching

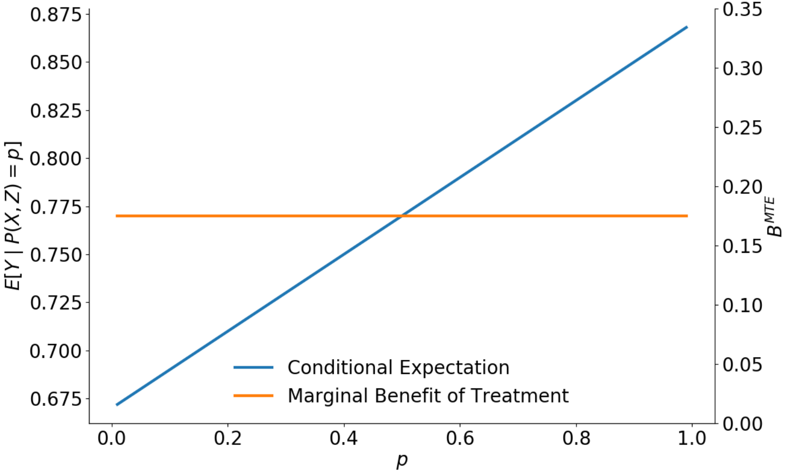
Key Identifiying Assumption

$$(Y_1, Y_0) \perp\!\!\!\perp D \mid X$$

What is in the agent's and econometrician's information set?

- Heckman and Navarro-Lozano (2004) highlights the sensitivity of results to different conditioning variables.

Figure: Matching and Essential Heterogeneity



Instrumental Variables

Key Identifiying Assumption

$$(Y_1, Y_0) \perp\!\!\!\perp Z \mid X$$

Even in the best cases, this is sometimes not as obvious as you think. See Heckman (1997) for a study of implicit behavioral assumptions used in making program evaluations.

Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\begin{aligned} \alpha &= \mu_0 \\ \beta &= (Y_1 - Y_0) = \mu_1 - \mu_0 + (U_1 - U_0) \\ \epsilon &= U_0 \end{aligned}$$

Assume for now that there is no treatment effect heterogeneity, i.e. $Y_1 - Y_0$ is the same for everybody. If we have access to a variable Z with the following properties ...

$$\text{cov}(Z, D) \neq 0$$
$$\text{cov}(Z, \epsilon) = 0$$

then the following holds

$$\text{plim}\,\hat{\beta}_{IV} = \frac{\text{cov}(Z, Y)}{\text{cov}(Z, D)} = \beta$$

What happens if β varies in the population?

- Do individuals select their treatment status based on gains?

  $\Rightarrow$ essential heterogeneity

Let $\beta = \bar{\beta} + \eta$, where $U_1 - U_0 = \eta$, then

$$Y = \alpha + \bar{\beta}D + [\epsilon + \eta D].$$

and

$$\operatorname{plim} \hat{\beta}_{IV} = \bar{\beta} + \frac{\operatorname{cov}(Z, \epsilon + \eta D)}{\operatorname{cov}(D, Z)}$$

So we cannot even learn about the mean effect of treatment unless we rule out essential heterogeneity, i.e. individuals selecting their treatment status based on gains.

Local Average Treatment Effect

- Average effect for those induced to change treatment because of a change in the instrument.

  ⇒ instrument-dependent parameter

$$\frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')} =$$
$$E(Y_1 - Y_0 \mid D(z) = 1, D(z') = 0)$$

Local Instrumental Variables

Local Instrumental Variable

$$\left.\frac{\partial E(Y \mid P(Z)=p)}{\partial p}\right|_{p=u_D} = E(Y_1 - Y_0 | U_D = u_D)$$
$$= B^{MTE}(u_D)$$

$\Rightarrow$ we can only identify the $B^{MTE}(u_D)$ over the support of p in our data

Figure: Observed Outcome and Essential Heterogeneity

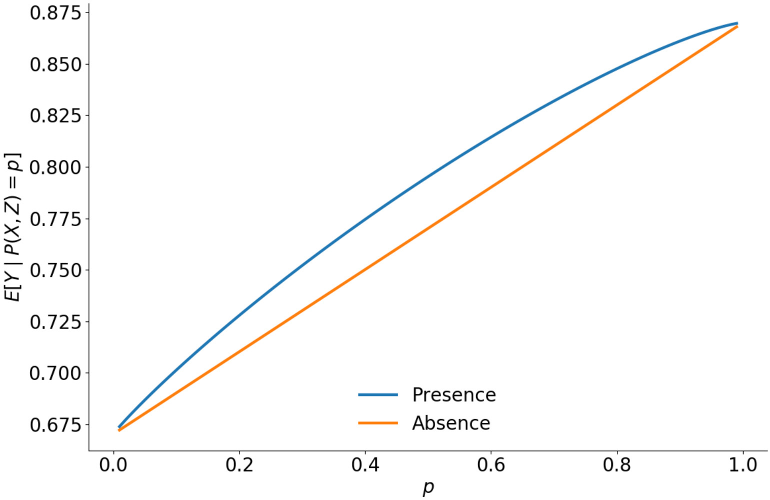
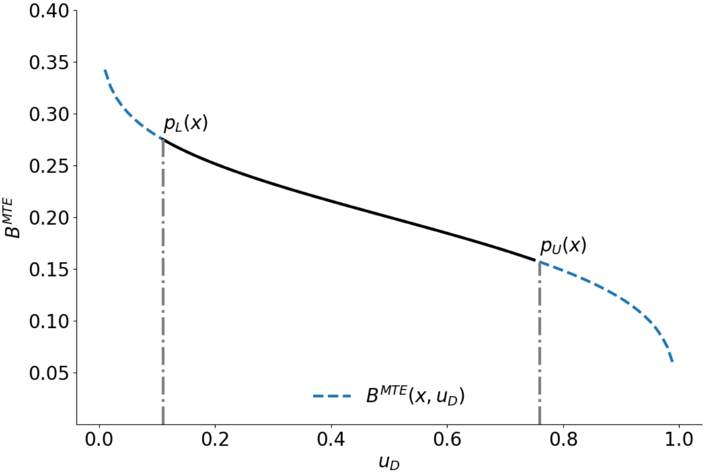


Figure: Identification I



Making $X = x$ explicit

$$E(Y_1 - Y_0|X = x, U_D = u_D)$$
$$= (\mu_1(x) - \mu_0(x)) + E(U_1 - U_0|X = x, U_D = u_D)$$

but if we are willing to assume $(U_1 - U_0) \perp\!\!\!\perp X$ then

$$E(Y_1 - Y_0|X = x, U_D = u_D)$$
$$= (\mu_1(x) - \mu_0(x)) + E(U_1 - U_0|U_D = u_D)$$

Figure: Identification II

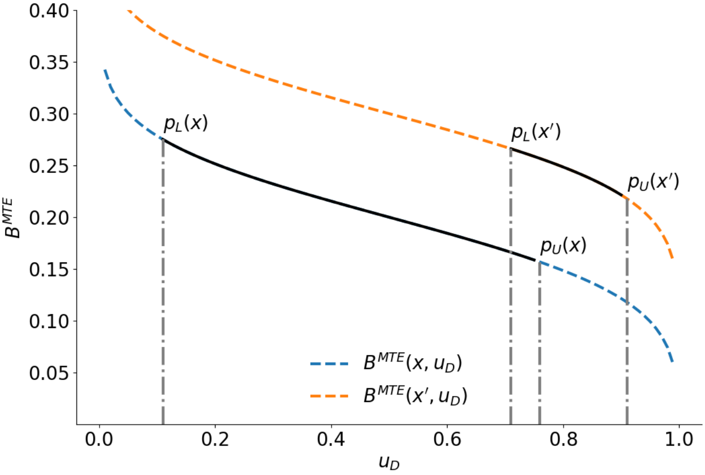
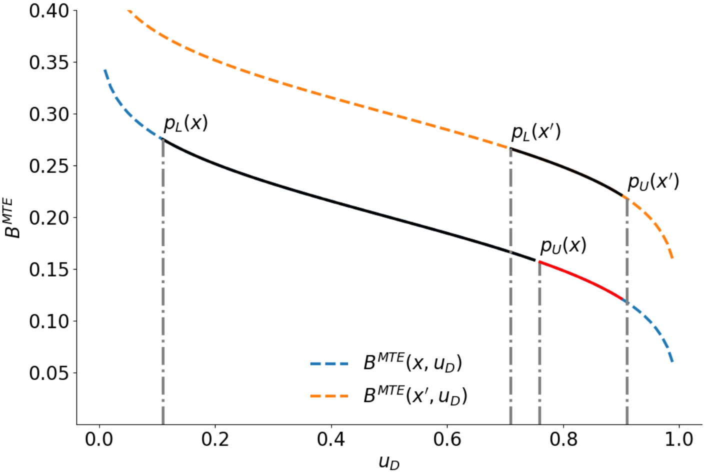


Figure: Identification III



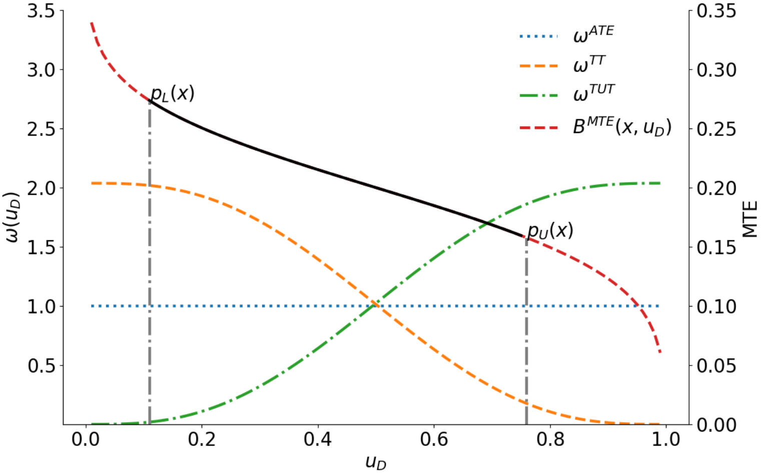
Effects of Treatment as Weighted Averages

Parameter Δ_j, can be written as a weighted average of the $B^{MTE}(x, u_D)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D)\omega^j(x, u_D)du_D,$$

where the weights $\omega^j(x, u_D)$ are specific to parameter j and integrate to one.

Figure: Identification IV



Regression Discontinuity Design

Suppose $D = 1$ if $X \geq x_0$, and $D = 0$ otherwise

$$\Rightarrow \begin{cases} E(Y \mid X = x) = E(Y_0 \mid X = x) & \text{for} \quad x < x_0 \\ E(Y \mid X = x) = E(Y_1 \mid X = x) & \text{for} \quad x \geq x_0 \end{cases}$$

Suppose $E(Y_1 \mid X = x), E(Y_0 \mid X = x)$ are continuous in x.

$$\Rightarrow \begin{cases} \lim_{\epsilon \searrow 0} E(Y_0 \mid X = x_0 - \epsilon) = E(Y_0 \mid X = x_0) \\ \lim_{\epsilon \searrow 0} E(Y_1 \mid X = x_0 + \epsilon) = E(Y_1 \mid X = x_0) \end{cases}$$

$$
\begin{aligned}
&\lim_{\epsilon \searrow 0} E(Y \mid X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y \mid X = x_0 - \epsilon) \\
&\qquad = \lim_{\epsilon \searrow 0} E(Y_1 \mid X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y_0 \mid X = x_0 - \epsilon) \\
&\qquad = E(Y_1 \mid X = x_0) - E(Y_0 \mid X = x_0) \\
&\qquad = E(Y_1 - Y_0 \mid X = x_0)
\end{aligned}
$$

Figure: Probability

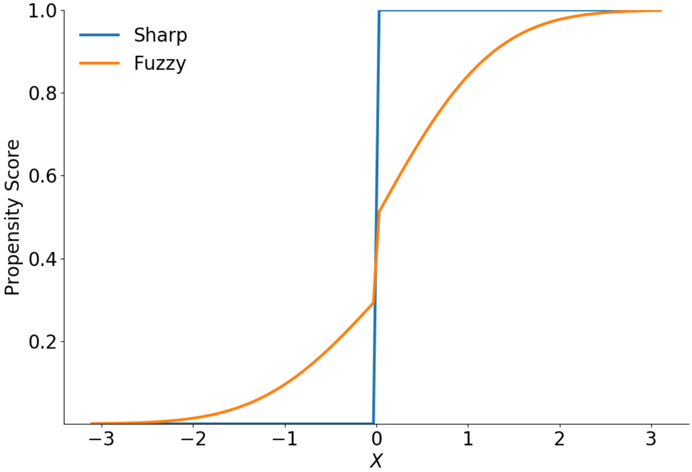


Figure: Observed Outcome in a Sharp Design

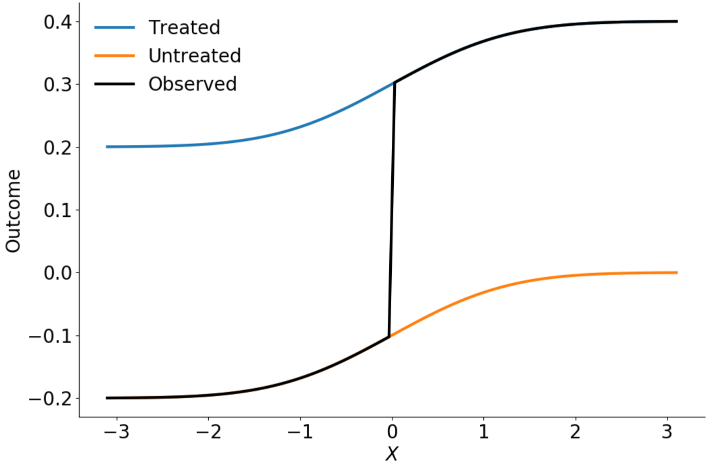
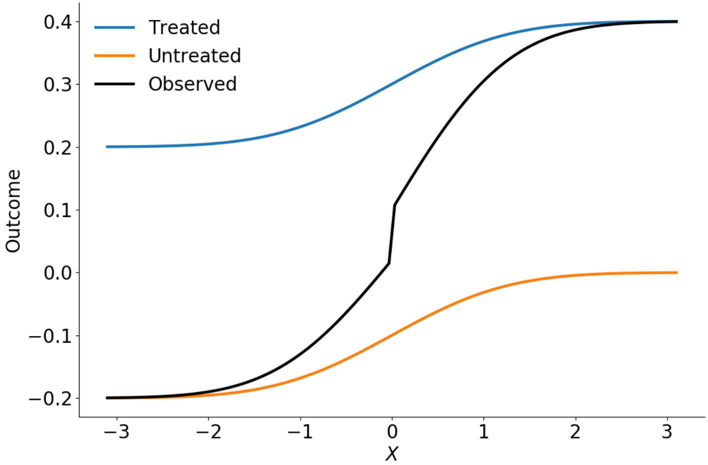


Figure: Observed Outcome in a Fuzzy Design



Conclusion

Appendix

References

Banerjee, A., Banerji, R., Berry, J., Duflo, E., Kannan, H., Mukerji, S., . . . Walton, M. (2017). From proof of concept to scalable policies: Challenges and solutions, with an application. *Journal of Economic Perspectives*, *31*(4), 73–102.

Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, *101*(6), 2754–2781.

Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.

Heckman, J. J. (1997). Instrumental variables: A study of implicit behavioral assumptions used in making program evaluations. *The Journal of Human Resources*, *32*(3), 441.

Heckman, J. J., & Navarro-Lozano, S. (2004). Using matching, instrumental variables, and control functions to estimate economic choice models. *Review of Economics and Statistics*, *86*(1), 30–57.

Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconometrics* (pp. 221–228). London: Palgrave Macmillan.