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# Parameters of Interest

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Heckman (2008) sets out three tasks for us:

- ▶ Defining the Set of Hypotheticals or Counterfactuals  
⇒ A Scientific Theory
- ▶ Identifying Causal Parameters from Real Data  
⇒ Mathematical Analysis of Data Point or Set Identification
- ▶ Identifying Parameters from Real Data  
⇒ Estimation and Testing Theory

# *Setup*

## The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

## Useful Notation

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$

**Specification** We follow the parameterization in Heckman and Vytlacil (2005):

$$Y_1 = \gamma + \alpha + U_1 \quad U_1 = \sigma_1 \epsilon \quad \gamma = 0.670 \quad \sigma_1 = 0.012$$

$$Y_0 = \gamma + U_0 \quad U_0 = \sigma_0 \epsilon \quad \alpha = 0.200 \quad \sigma_0 = -0.050$$

$$D = \mathbb{I}[Z - V > 0] \quad V = \sigma_V \epsilon \quad \epsilon \sim \mathbb{N}(0, 1) \quad \sigma_V = -1.000$$

$$Z \sim \mathbb{N}(-0.0026, 0.2700) \quad U_D = \Phi\left(\frac{V}{\sigma_V \sigma_\epsilon}\right)$$

# *Individual Heterogeneity*



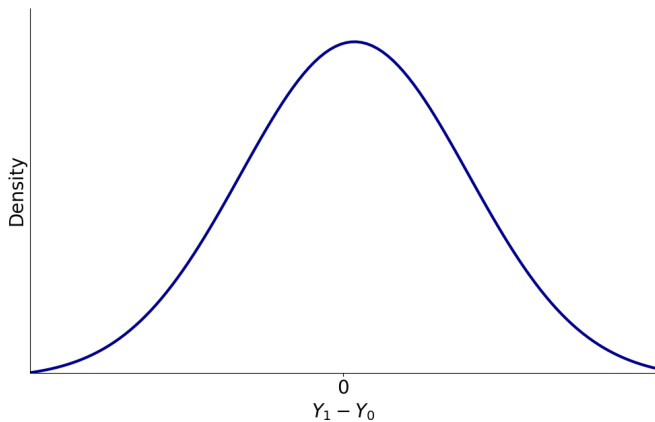
## Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

### **Sources of Heterogeneity**

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
  - ▶ Uncertainty
  - ▶ Private Information

Figure: Distribution of Benefits

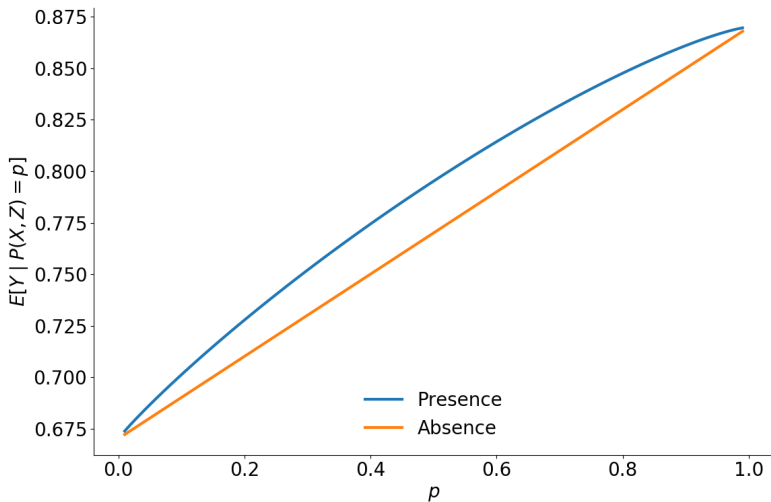


**Essential Heterogeneity Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation and Essential Heterogeneity



# *Conventional Average Treatment Effects*

## Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

⇒ correspond to *extreme* policy alternatives

## Selection Problem

$$\begin{aligned} E[Y | D = 1] - E[Y | D = 0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &+ \underbrace{E[Y_1 - Y_0 | D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 | D = 1] - E[Y_0 | D = 0]}_{\text{Selection Bias}} \end{aligned}$$

$$\begin{aligned}
 E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} \\
 &\quad + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}
 \end{aligned}$$

⇒ the bias depends on the parameter of interest



Figure: Distribution of Effects with Essential Heterogeneity

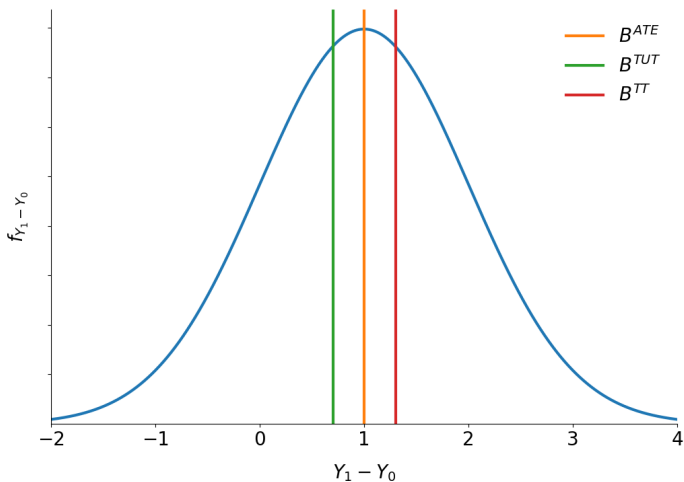


Figure: Distribution of Effects without Essential Heterogeneity

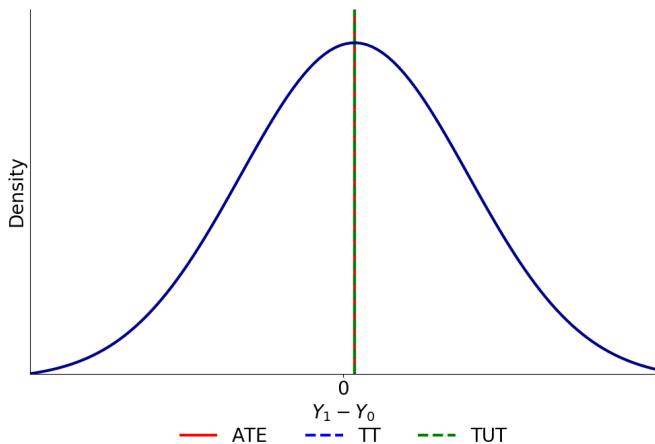
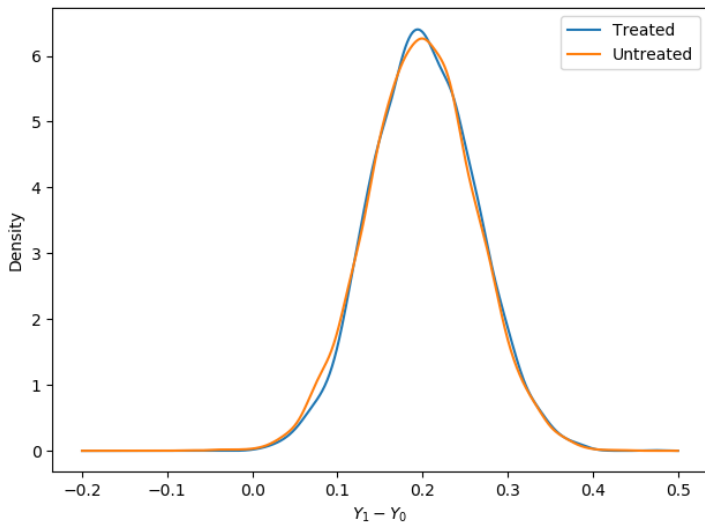


Figure: Distribution of Benefits by Treatment Status



# *Policy-Relevant Average Treatment Effects*

## Observed Outcomes

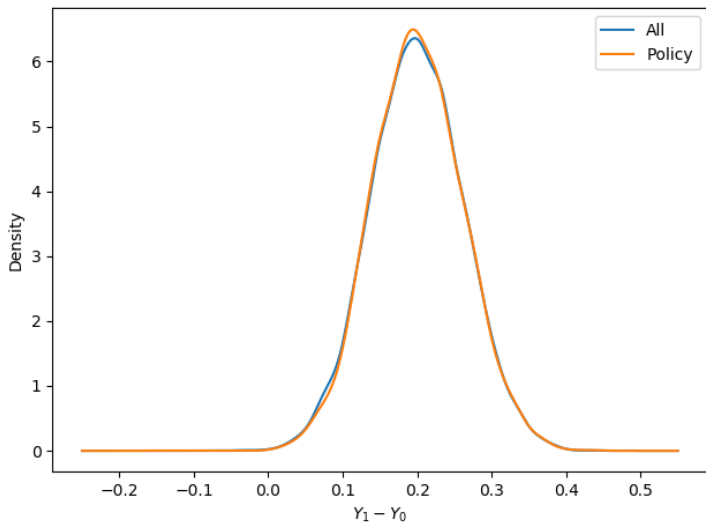
$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

## Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Figure: Distribution of Benefits for Policy



## *Marginal Effect of Treatment*

## Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

**Intuition:** Mean gross return to treatment for persons at quantile  $u_D$  of the first-stage unobservable  $V$  or a willingness to pay for individuals at the margin of indifference.



Figure: Margin of Indifference

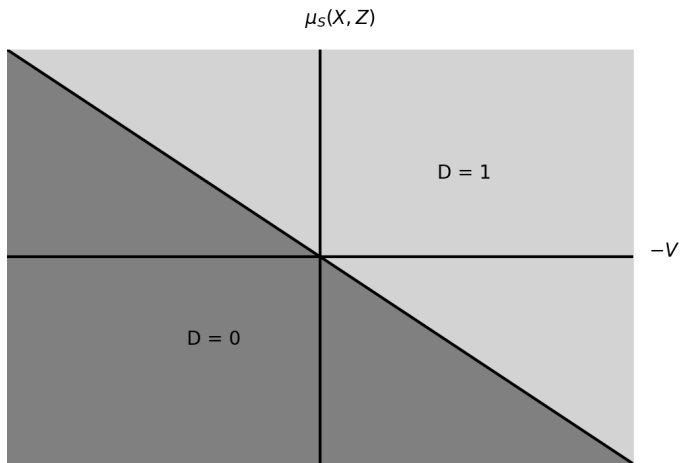
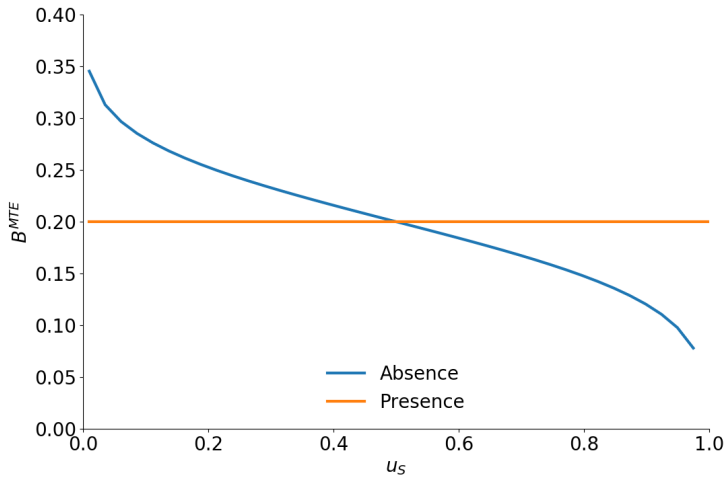


Figure: Marginal Benefit of Treatment



**Effects of Treatment as Weighted Averages** Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_D)$ .

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights  $\omega^j(x, u_D)$  are specific to parameter  $j$  and integrate to one.

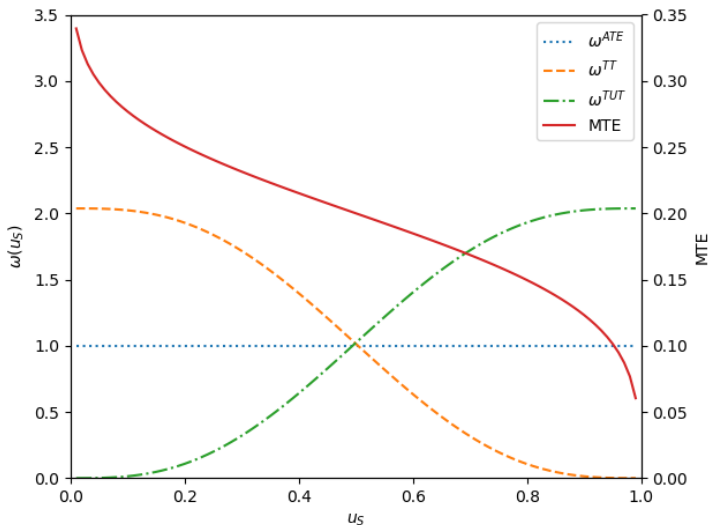
## Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P \mid X = x]}$$

Figure: Effects of Treatment as Weighted Averages



# *Local Average Treatment Effect*

## Local Average Treatment Effect

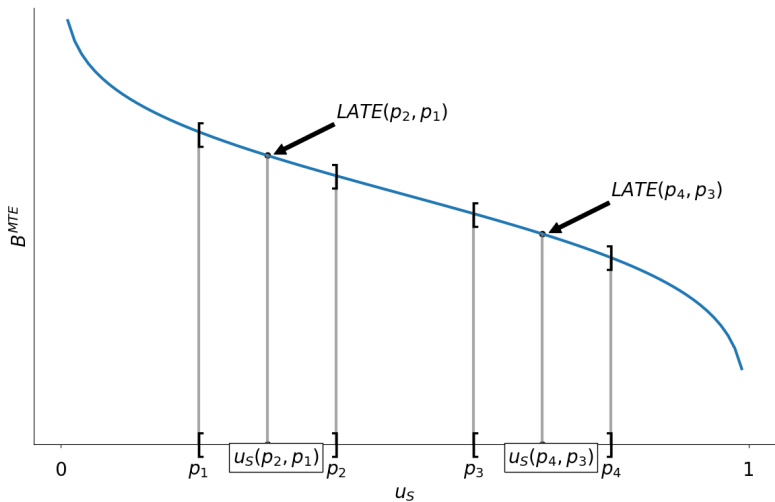
- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.  $\Rightarrow$  instrument-dependent parameter
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.  
 $\Rightarrow$  deep economic parameter

$$B^{LATE} = \frac{E(Y | Z = z) - E[Y | Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_D, u_{S'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{S'}} B^{MTE}(x, u) du,$$



Figure: Local Average Treatment Effect



## *Distributions of Effects*

Figure: Distribution of Potential Outcomes

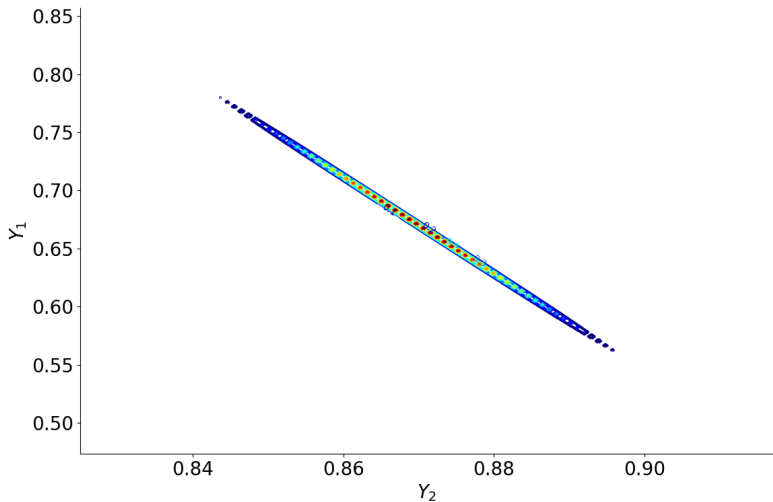
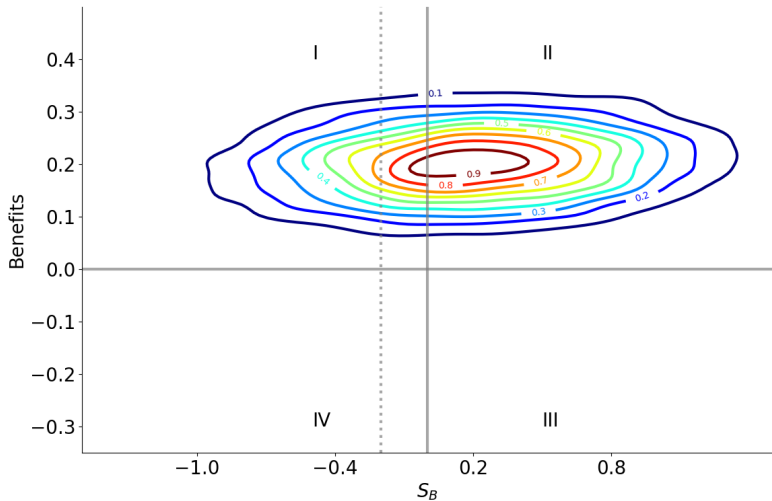


Figure: Distribution of Benefits and Surplus



# Appendix

# *References*

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