Econometrics
of Human
Capital

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Material available on





Estimation Strategies

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Setup

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y = DY_1 + (1 - D)Y_0$$

$$Y_0 = \mu_0(X) + U_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

Key Concept

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp \!\!\!\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

Useful Notation

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

 $U_D = F_V(V)$

Key Assumptions

- \triangleright (U_1, U_0, V) are independent of Z conditional on X
- $\blacktriangleright \mu_D(X,Z)$ is a nondegenerate random variable conditional on X
- $ightharpoonup 0 < \Pr(D = 1 \mid X) < 1$

Evaluation Problem

$$Y = DY_1 + (1 - D)Y_0 = \begin{cases} Y_1 & \text{if } D = 1 \\ Y_0 & \text{if } D = 0 \end{cases}$$

Selection Problem

$$\begin{split} E[Y \mid D=1] - E[Y \mid D=0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D=1] - E[Y_1 - Y_0]}_{\text{selection on gains}} \\ &+ \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{\text{selection on levels}} \end{split}$$

Estimation Strategies

- Randomization
- Matching
- Instrumental Variables
 - conventional and local
- Regression Discontinuity
 - fuzzy and sharp design

Randomization

Treatment Status

D self-selected

 ξ assigned

A actual

Key Identifying Assumptions

$$(Y_1, Y_0) \perp \!\!\! \perp D$$

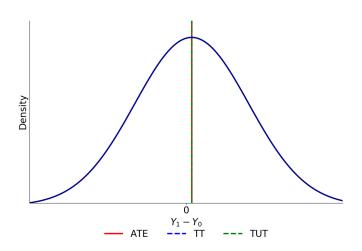
 $(Y_1, Y_0) \perp \!\!\! \perp \xi$
 $(Y_1, Y_0) \perp \!\!\! \perp A$

When do we have to worry about compliance?

$$E(Y | A = 1) - E(Y | A = 0)$$

$$= E(Y_1 | A = 1) - E(Y_0 | A = 0)$$
 (by full compliance)
$$= E(Y_1) - E(Y_0)$$
 (by randomization)
$$= B^{ATE} = B^{TT} = B^{TUT}$$

Figure: Distribution of Effects



What if we can only deny program participation to individuals who are willing to participate?

$$E(Y \mid D = 1, A = 1) - E(Y \mid D = 1, A = 0)$$

$$= E(Y_1 \mid D = 1, A = 1) - E(Y_0 \mid D = 1, A = 0)$$

$$= E(Y_1 \mid D = 1) - E(Y_0 \mid D = 1)$$

$$= B^{TT} \neq B^{ATE} \neq B^{TUT}$$

Issues

- compliance
- ▶ imperfect randomization
- ethical concerns
- feasibility
- expenses
- external validity

Challenges to Scaling Experiments

- market equilibrium effects
- spillovers
- political reactions
- context dependence
- randomization or site-selection bias
- piloting bias

See Banerjee et al. (2017) for a discussion of these challenges and their attempts to address them in their work.

Matching

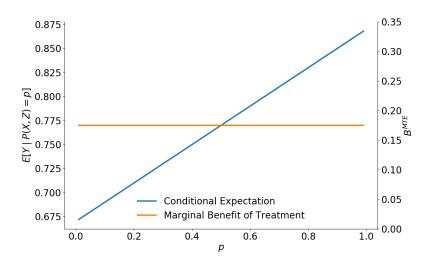
Key Identifying Assumption

$$(Y_1, Y_0) \perp \!\!\!\perp D \mid X$$

What is in the agent's and econometrician's information set?

Heckman and Navarro-Lozano (2004) highlights the sensitivity of results to different conditioning variables.

Figure: Matching and Essential Heterogeneity



Instrumental Variables

Key Identifying Assumption

$$(Y_1, Y_0) \perp\!\!\!\perp Z \mid X$$

Even in the best cases, this is sometimes not as obvious as you think. See Heckman (1997) for a study of implicit behavioral assumptions used in making program evaluations.

Conventional Notation

$$Y = \alpha + \beta D + \epsilon$$
,

where

$$lpha = \mu_0$$
 $eta = (Y_1 - Y_0) = \mu_1 - \mu_0 + (U_1 - U_0)$
 $\epsilon = U_0$

Assume for now that there is no treatment effect heterogeneity, i.e. $Y_1 - Y_0$ is the same for everybody. If we have access to a variable Z with the following properties ...

$$cov(Z, D) \neq 0$$

 $cov(Z, \epsilon) = 0$

then the following holds

$$\operatorname{plim} \hat{\beta}_{IV} = \frac{\operatorname{cov}(Z, Y)}{\operatorname{cov}(Z, D)} = \beta$$

What happens if β varies in the population?

- Do individuals select their treatment status based on gains?
 - ⇒ essential heterogeneity

Let $\beta = \bar{\beta} + \eta$, where $U_1 - U_0 = \eta$, then

$$Y = \alpha + \bar{\beta}D + [\epsilon + \eta D].$$

and

$$\operatorname{plim} \hat{\beta}_{IV} = \bar{\beta} + \frac{\operatorname{cov}(Z, \epsilon + \eta D)}{\operatorname{cov}(D, Z)}$$

So we cannot even learn about the mean effect of treatment unless we rule out essential heterogeneity, i.e. individuals selecting their treatment status based on gains.

Local Average Treatment Effect

- ► Average effect for those induced to change treatment because of a change in the instrument.
 - ⇒ instrument-dependent parameter

$$\frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')} = E(Y_1 - Y_0 \mid D(z) = 1, D(z') = 0)$$

Local Instrumental Variables

Local Instrumental Variable

$$\frac{\partial E(Y \mid P(Z) = p)}{\partial p} \bigg|_{p=u_D} = E(Y_1 - Y_0 | U_D = u_D)$$
$$= B^{MTE}(u_D)$$

 \Rightarrow we can only identify the $B^{MTE}(u_D)$ over the support of p in our data

Figure: Observed Outcome and Essential Heterogeneity

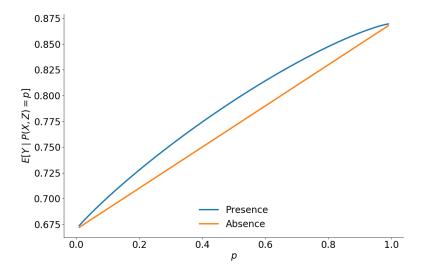
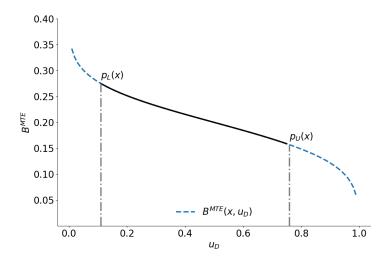


Figure: Identification I



Making X = x explicit

$$E(Y_1 - Y_0 | X = x, U_D = u_D)$$

= $(\mu_1(x) - \mu_0(x)) + E(U_1 - U_0 | X = x, U_D = u_D)$

but if we are willing to assume $(U_1 - U_0) \perp \!\!\! \perp X$ then

$$E(Y_1 - Y_0 | X = x, U_D = u_D)$$

= $(\mu_1(x) - \mu_0(x)) + E(U_1 - U_0 | U_D = u_D)$

Figure: Identification II

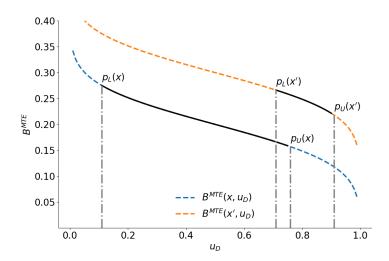
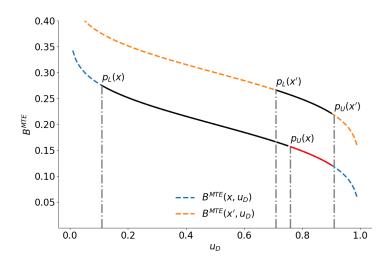


Figure: Identification III



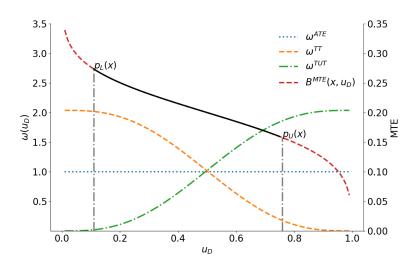
Effects of Treatment as Weighted Averages

Parameter Δ_j , can be written as a weighted average of the $B^{MTE}(x, u_D)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights $\omega^{j}(x,u_{D})$ are specific to parameter j and integrate to one.

Figure: Identification IV



Regression Discontinuity Design

Suppose D = 1 if $X \ge x_0$, and D = 0 otherwise

$$\Rightarrow \begin{cases} E(Y \mid X = x) = E(Y_0 \mid X = x) & \text{for } x < x_0 \\ E(Y \mid X = x) = E(Y_1 \mid X = x) & \text{for } x \ge x_0 \end{cases}$$

Suppose $E(Y_1 \mid X = x)$, $E(Y_0 \mid X = x)$ are continuous in x.

$$\Rightarrow \begin{cases} \lim_{\epsilon \searrow 0} E(Y_0 \mid X = x_0 - \epsilon) = E(Y_0 \mid X = x_0) \\ \lim_{\epsilon \searrow 0} E(Y_1 \mid X = x_0 + \epsilon) = E(Y_1 \mid X = x_0) \end{cases}$$

$$\lim_{\epsilon \searrow 0} E(Y \mid X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y \mid X = x_0 - \epsilon)
= \lim_{\epsilon \searrow 0} E(Y_1 \mid X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y_0 \mid X = x_0 - \epsilon)
= E(Y_1 \mid X = x_0) - E(Y_0 \mid X = x_0)
= E(Y_1 - Y_0 \mid X = x_0)$$

Figure: Probability

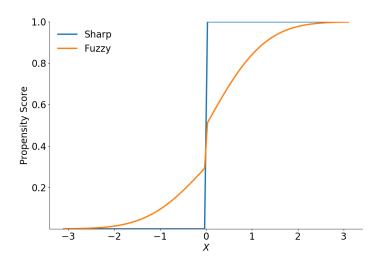


Figure: Observed Outcome in a Sharp Design

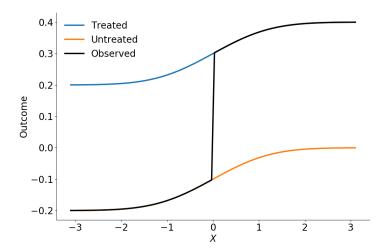
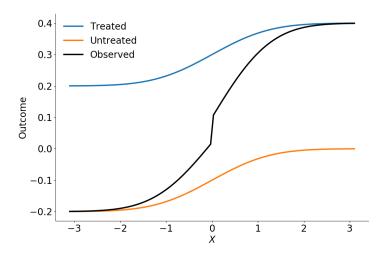


Figure: Observed Outcome in a Fuzzy Design



Conclusion

Appendix

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