Parameters of Interest

Philipp Eisenhauer

Heckman (2008) sets out three tasks for us:

- Defining the Set of Hypotheticals or Counterfactuals
  $\Rightarrow$ A Scientific Theory
- Identifying Causal Parameters from Real Data
  $\Rightarrow$ Mathematical Analysis of Data Point or Set Identification
- Identifying Parameters from Real Data
  $\Rightarrow$ Estimation and Testing Theory

Setup

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = \mathrm{I}[\mu_D(X, Z) - V > 0]$$

Useful Notation

$$
\begin{aligned}
P(X,Z) &= \Pr(D=1 \mid X,Z) = F_V(\mu_D(X,Z)) \\
U_D &= F_V(V)
\end{aligned}
$$

Specification We follow the parameterization in Heckman and Vytlacil (2005):

$$
\begin{array}{llll}
Y_1 = \gamma + \alpha + U_1 & U_1 = \sigma_1 \epsilon & \gamma = 0.670 & \sigma_1 = 0.012 \\
Y_0 = \gamma + U_0 & U_0 = \sigma_0 \epsilon & \alpha = 0.200 & \sigma_0 = -0.050 \\
D = \mathrm{I}[Z - V > 0] & V \;\; = \sigma_V \epsilon & \epsilon \sim \mathbb{N}(0, 1) & \sigma_V = -1.000
\end{array}
$$

$$
Z \sim \mathbb{N}(-0.0026, 0.2700) \qquad U_D = \Phi\left(\frac{V}{\sigma_V \sigma_\epsilon}\right)
$$

Individual Heterogeneity

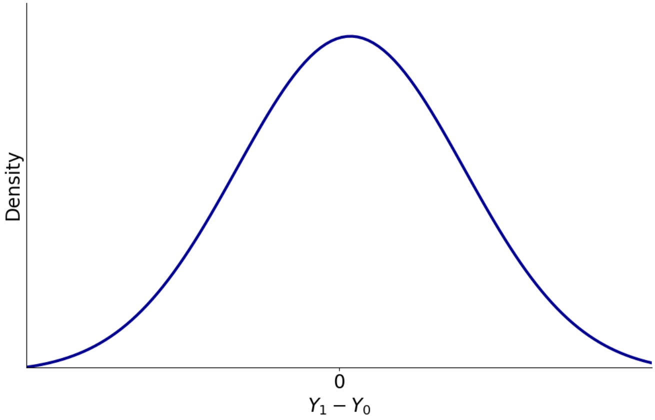
Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity

- Difference in Observable Characteristics
- Difference in Unobservable Characteristics
  - Uncertainty
  - Private Information

Figure: Distribution of Benefits

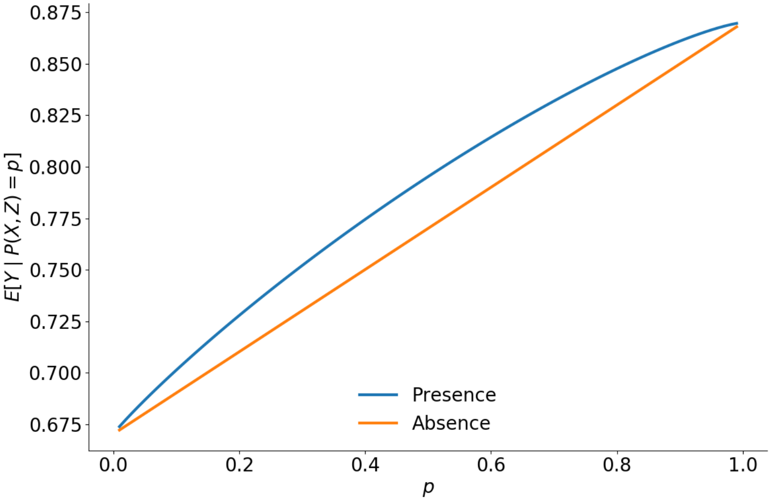


Essential Heterogeneity Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp\!\!\!\perp D \quad | \, X = x.$$

$\Rightarrow$ consequences for the choice of the estimation strategy

Figure: Conditional Expectation and Essential Heterogeneity



Conventional Average Treatment Effects

Conventional Average Treatment Effects

$$
\begin{aligned}
B^{ATE} &= E[Y_1 - Y_0] \\
B^{TT} &= E[Y_1 - Y_0 \mid D = 1] \\
B^{TUT} &= E[Y_1 - Y_0 \mid D = 0]
\end{aligned}
$$

$\Rightarrow$ correspond to *extreme* policy alternatives

Selection Problem

$$\begin{aligned} E[Y \mid D=1] - E[Y \mid D=0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D=1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{\text{Selection Bias}} \end{aligned}$$

$$E[Y \mid D=1] - E[Y \mid D=0] = \underbrace{E[Y_1 - Y_0 \mid D=1]}_{B^{TT}} + \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{\text{Selection Bias}}$$

$\Rightarrow$ the bias depends on the parameter of interest

Figure: Distribution of Effects with Essential Heterogeneity

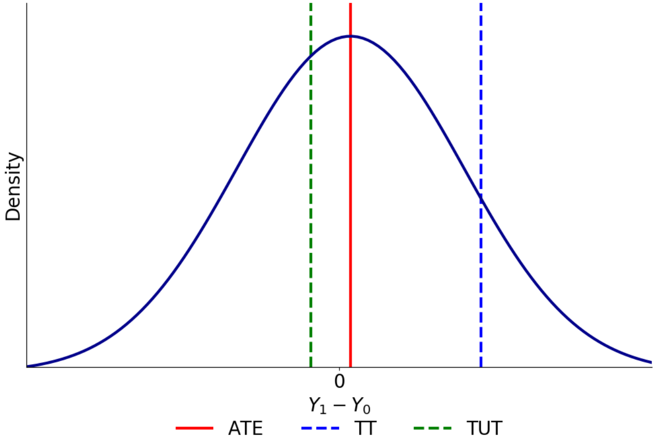


Figure: Distribution of Effects without Essential Heterogeneity

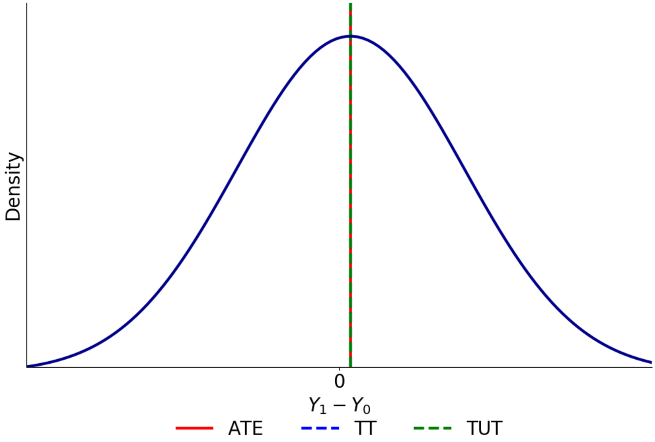
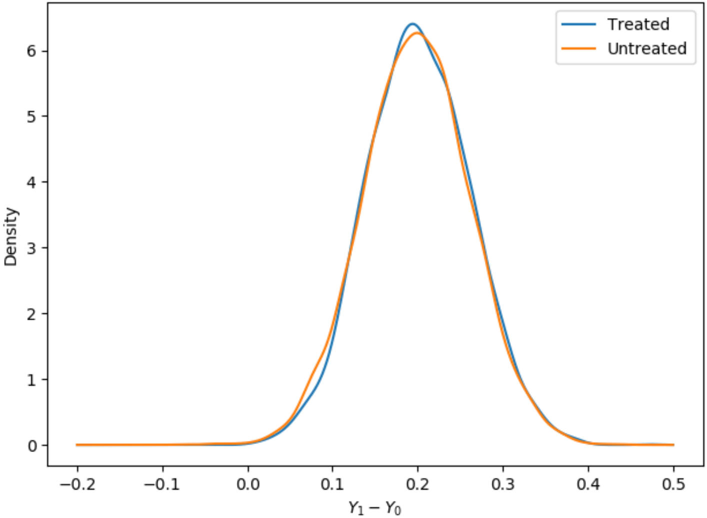


Figure: Distribution of Benefits by Treatment Status



Policy-Relevant Average Treatment Effects

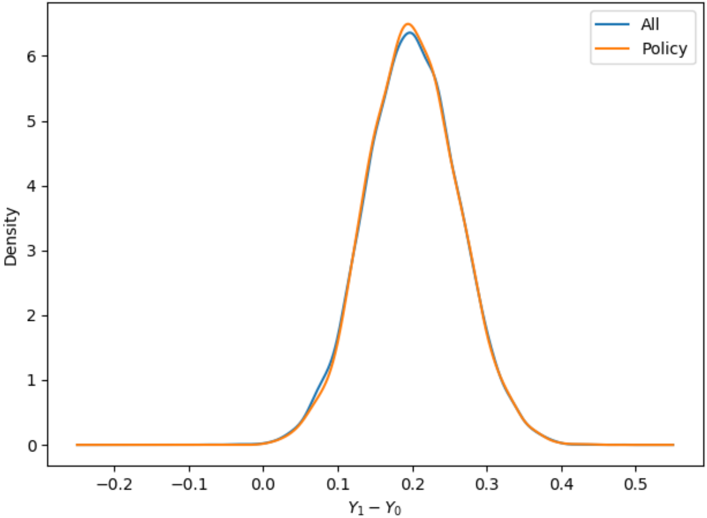
Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$
$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Figure: Distribution of Benefits for Policy



Marginal Effect of Treatment

Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

Intuition: Mean gross return to treatment for persons at quantile u_D of the first-stage unobservable V or a willingness to pay for individuals at the margin of indifference.

Figure: Margin of Indifference

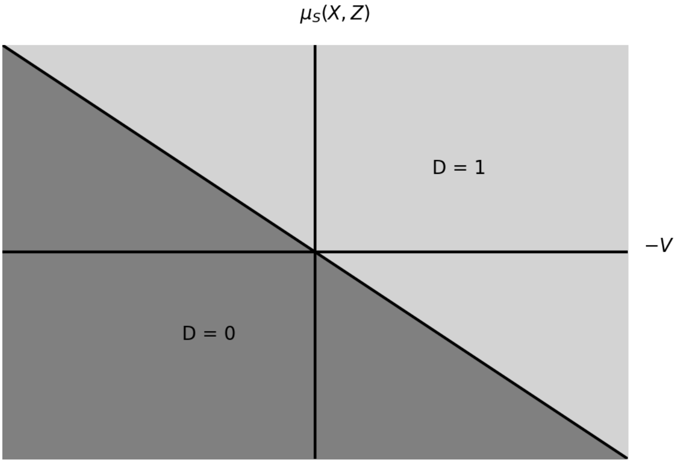
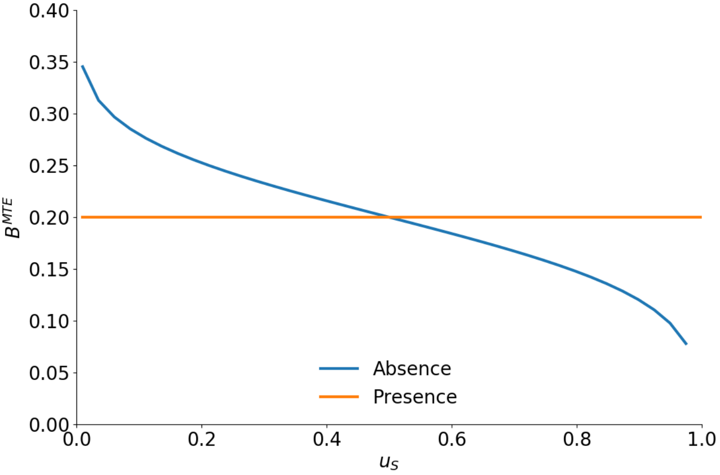


Figure: Marginal Benefit of Treatment



Effects of Treatment as Weighted Averages Parameter Δ_j, can be written as a weighted average of the $B^{MTE}(x, u_D)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D)\omega^j(x, u_D)du_D,$$

where the weights $\omega^j(x, u_D)$ are specific to parameter j and integrate to one.

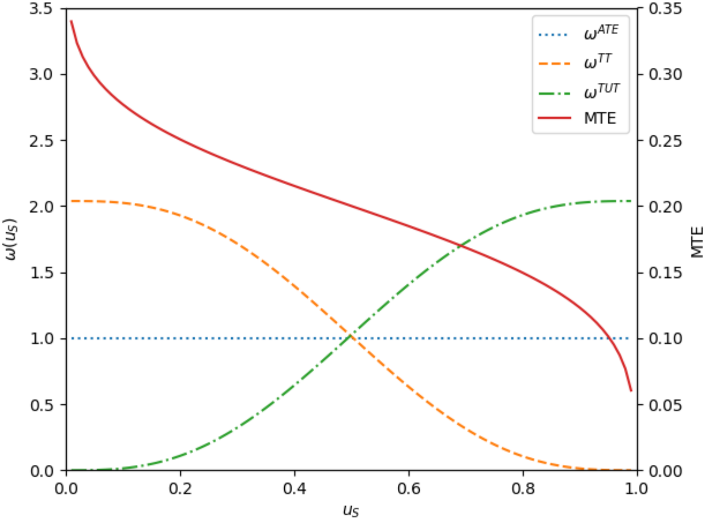
Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P \mid X = x]}$$

Figure: Effects of Treatment as Weighted Averages



Local Average Treatment Effect

Local Average Treatment Effect

- **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.$\Rightarrow$ instrument-dependent

  parameter

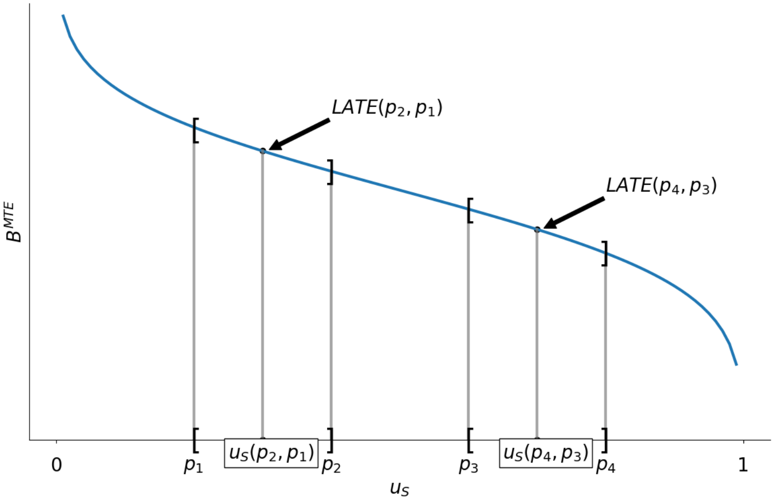
- **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.

  $\Rightarrow$ deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_D, u_{S'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{S'}} B^{MTE}(x, u)du,$$

Figure: Local Average Treatment Effect



Distributions of Effects

Figure: Distribution of Potential Outcomes

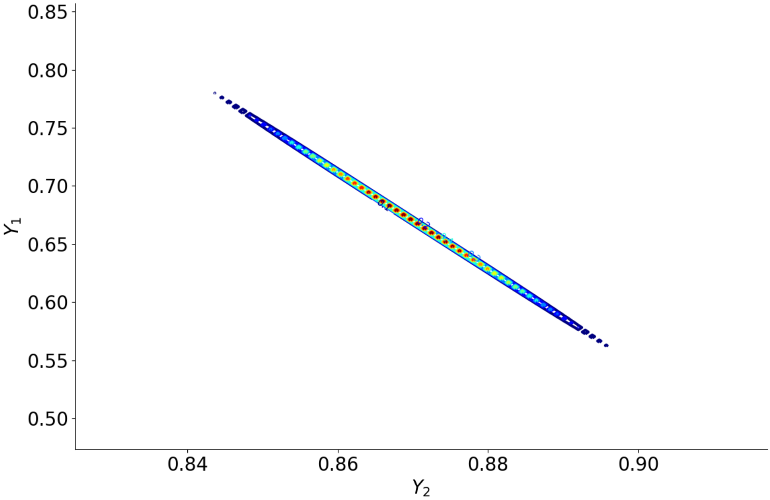
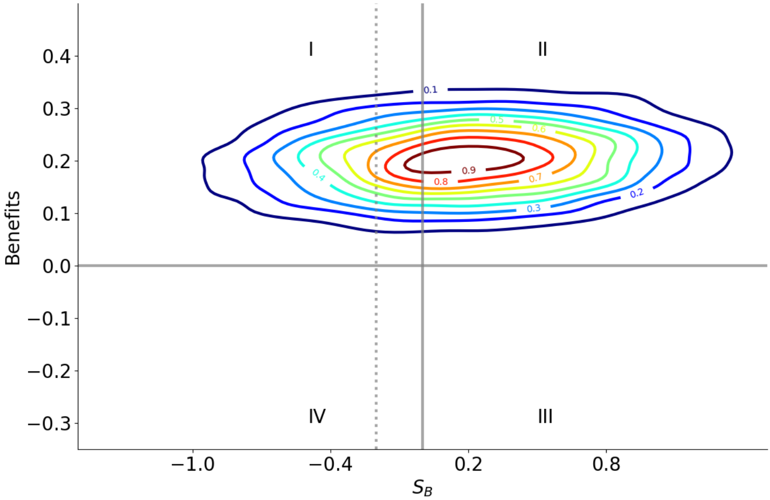


Figure: Distribution of Benefits and Surplus



Appendix

References

Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, *101*(6), 2754–2781.

Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.

Heckman, J. J. (2008). Econometric causality. *International Statistical Review*, *76*(1), 1–27.

Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconometrics* (pp. 221–228). London: Palgrave Macmillan.

Heckman, J. J., & Vytlacil, E. J. (2005). Structural equations, treatment effects, and econometric policy evaluation. *Econometrica*, *73*(3), 669–739.