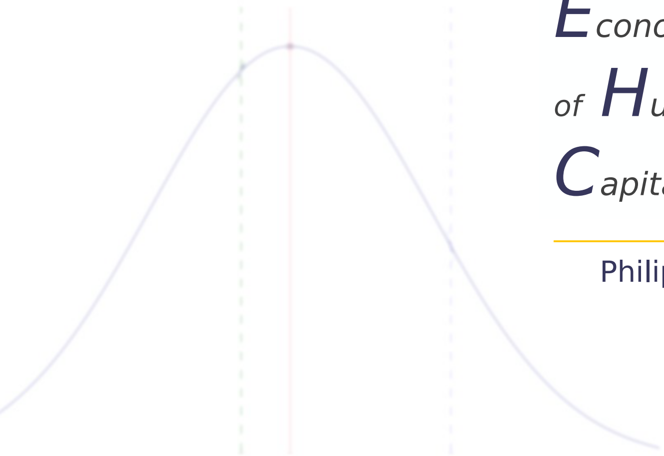

Econometrics of Human Capital

Philipp Eisenhauer

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Generalized Roy Model

Philipp Eisenhauer

Rising wage inequality

- changes in distribution of skills
- changes in relative prices of skills, prices identical across sectors
- comparative advantage, different skills priced different across sectors $\Longrightarrow$ Roy models

- Does the pursuit of comparative advantage increase or decrease earnings inequality within sectors and in the overall economy?
- Do the people with the highest i skill actually work in sector i?
- As people enter a sector in response to an increase in the demand for its services, does the average skill level employed there rise or fall?

Roy (1951) Model

- Individuals are income maximizing, act under perfect information, and possess skills S_1 and S_2.
- The economy offers two employment opportunities associated with skill prices π_1 and π_2 and skill i is only useful in sector i.

An individual chooses sector one if earnings are greater there:

$$w_1 > w_2 \iff \pi_1 S_1 > \pi_2 S_2$$

Some Thoughts on the Distribution of Earnings
Author(s): A. D. Roy
Source: *Oxford Economic Papers*, New Series, Vol. 3, No. 2 (Jun., 1951), pp. 135-146
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2662082
Accessed: 07/10/2009 02:09

Econometric Problems

- **Evaluation Problem** We only observe an individual's wage in the sector they are working in.
- **Selection Problem** As individuals pursue their comparative advantage, we only observe selected samples from the latent skill distribution in either sector.

Key Questions

- What economic concepts are accounted for, which are not?
- What does the individual, what does the econometrician know?
- What gives rise to heterogeneity in skills?

- Skills follow a bivariate normal distribution denoted by $F(s_1, s_2)$.

$$\begin{pmatrix} \ln S_1 \\ \ln S_2 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right)$$

Figure: Joint Distribution of Skills

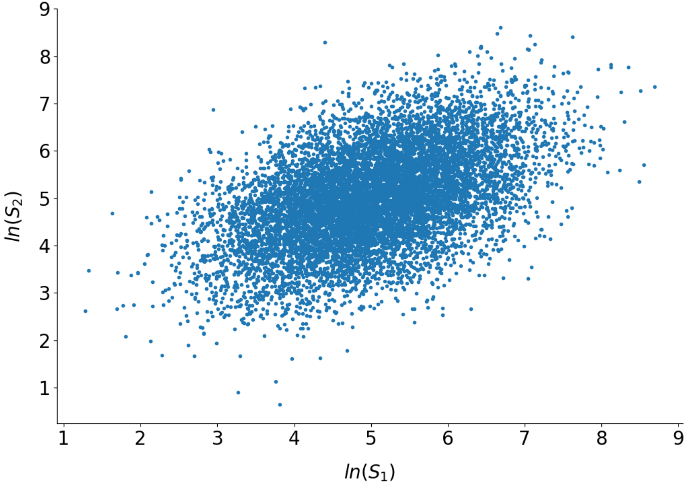
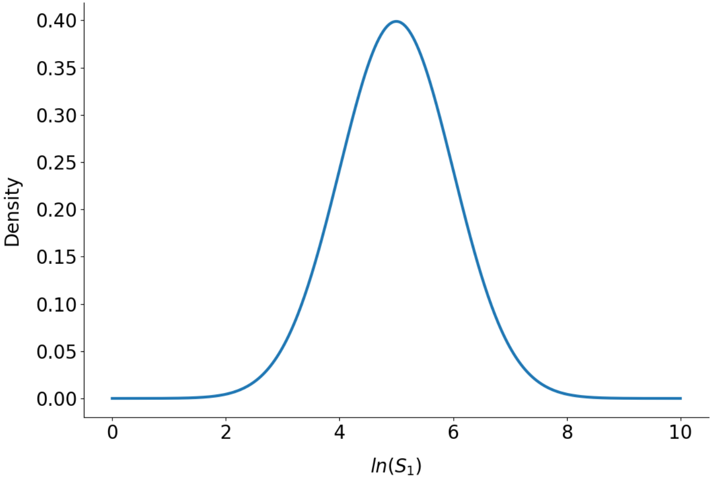


Figure: Marginal Distribution of Skill



The proportion of the population working in sector one P_1

$$P_1 = \int_0^{\infty} \int_0^{\pi_1 s_1/\pi_2} f(s_1, s_s) ds_1 ds_2$$

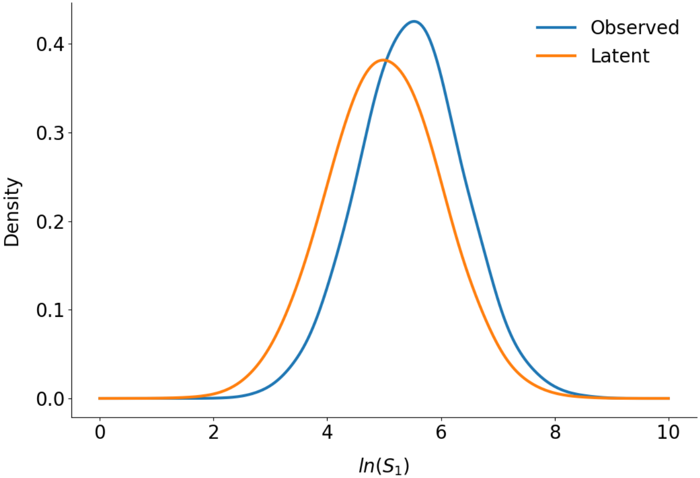
The density of skills employed in sector one differs from the population density of skills.

$$f(s_1) = \int_0^{\infty} f(s_1, s_2) ds_2$$

$$g_1(s_1 \mid \pi_1 S_1 > \pi_2 S_2) = \frac{1}{P_1} \int_0^{\pi_1 s_1/\pi_2} f(s_1, s_2) ds_2$$

The distribution of skills employed in sector 1 differs from the population distribution of skills due to comparative advantage.

Figure: Latent and Observed Distribution of Skill



Truncation and Censoring

Setup

$$\begin{pmatrix} Z \\ I \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1.0 & \rho \\ \rho & 1.0 \end{pmatrix}\right)$$

Figure: Density of truncated standard Normal distribution

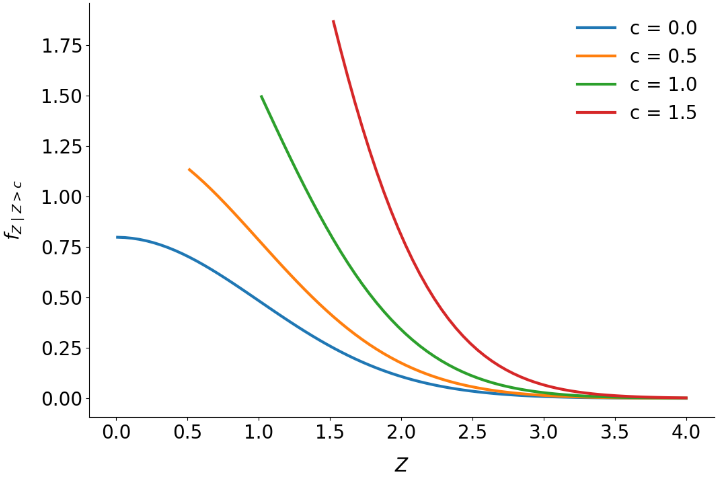


Figure: Expectation of truncated standard Normal distribution

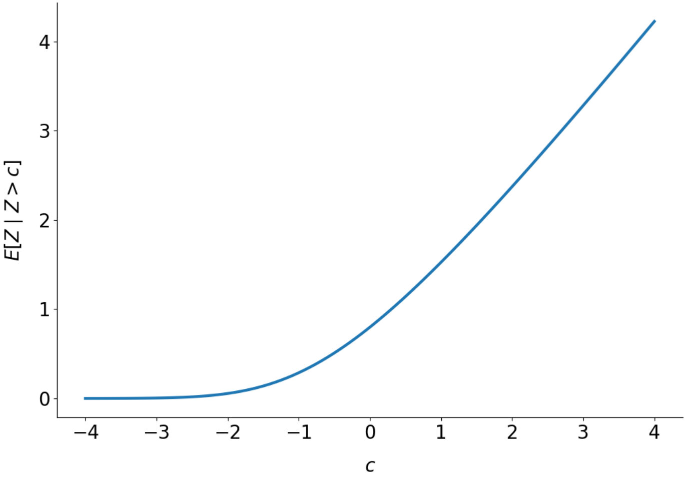


Figure: Variance of truncated standard Normal distribution

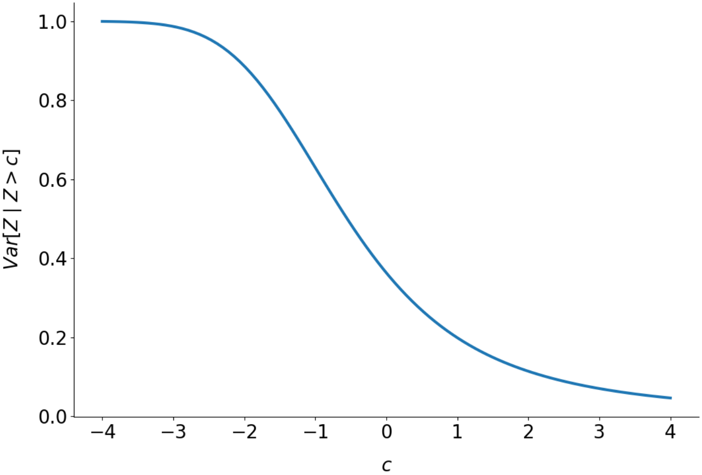
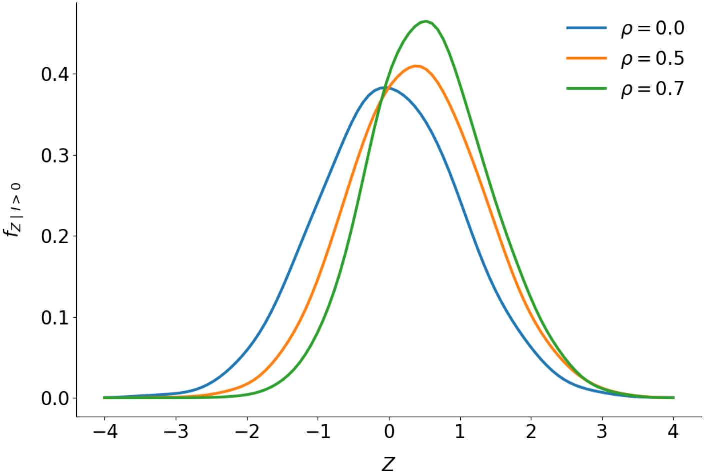


Figure: Density of censored standard Normal distribution



Sorting and selection

Wage Equations

$$\ln W_1 = \ln \pi_1 + \mu_1 + U_1$$
$$\ln W_2 = \ln \pi_2 + \mu_2 + U_2,$$

where $U_i = \ln S_i - \mu_i$.

Some notation

$$\begin{aligned} \sigma^* &= \sigma_{U_1 - U_0} \\ &= \sqrt{(\sigma_{11} - \sigma_{12}) + (\sigma_{22} - \sigma_{12})} \end{aligned}$$

$$c_1^* = (\ln(\pi_1/\pi_2) + \mu_1 - \mu_2)/\sigma^*$$

$$L = U_1 - U_0$$

Selection bias

$$\begin{aligned} E[\ln W_1 \mid \ln W_1 > \ln W_2] &= \ln \pi_1 + \mu_1 + E[U_1 \mid L > -c_1^*] \\ &= \ldots + E[U_1 \mid U_1 - U_0 > -c_1^*] \end{aligned}$$

- What about identification at infinity arguments?

Sorting

$$E[\ln S_1 \mid \ln W_1 > \ln W_2] = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\lambda(-c_1)$$

$$E[\ln S_2 \mid \ln W_2 > \ln W_1] = \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*}\lambda(-c_2)$$

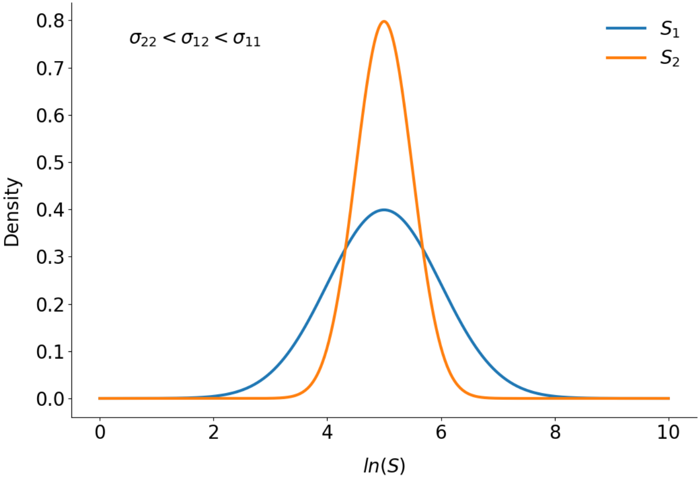
We know the following:

$$\sigma^* = (\sigma_{11} - \sigma_{12}) + (\sigma_{22} - \sigma_{12}) > 0$$

$$\lambda, \lambda' > 0$$

- There must be positive selection into one of the occupations and there can be positive selection into both.

Figure: Marginal Distributions of Skills



What do we know?

- There is positive selection in Sector 1 as $\sigma_{11} > \sigma_{12}$.
- There is negative selection in Sector 2 as $\sigma_{22} < \sigma_{12}$.

We gain further insights into the effect of self-selection on the distribution of earnings for workers in sector 1 by looking at the distribution of $\ln S_1$ conditional on $\ln S_2$.

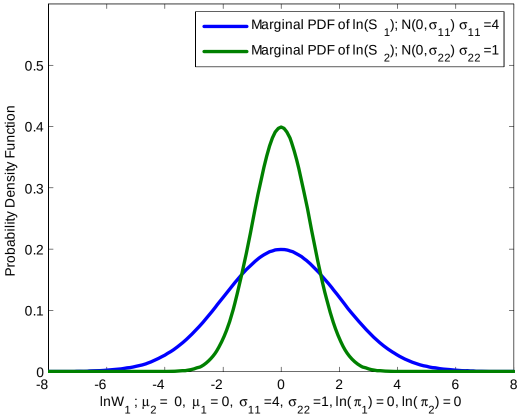
$$\ln S_1 \mid \ln S_2 \sim \mathbb{N}(\mu, \sigma),$$

where

$$\mu = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}\left(\ln S_2 - \mu_2\right) \quad \text{and} \quad \sigma = \sigma_{11}\left(1 - \left(\frac{\sigma_{12}}{\sigma_1 \sigma_2}\right)^2\right)$$

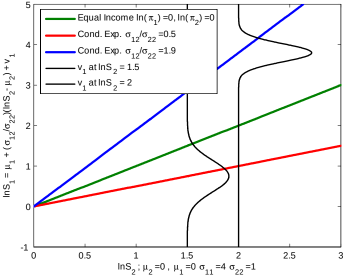
Heckman Productions

Marginal Probability Density Function (PDF) of $\ln S_1$, $\ln S_2$



$$\ln S_1 = \ln(\mu_1) + U_1; \;\; \ln S_2 = \ln(\mu_2) + U_2;$$
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix} \right); \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Graph of $\ln S_1 = f(\ln S_2)$

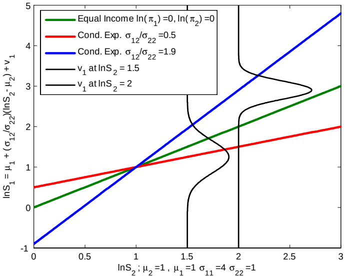


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \;\; \sigma_{12} = 0.5, \;\; 1.9;$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \;\; \sigma_{12} = 0.5, \;\; 1.9;$$

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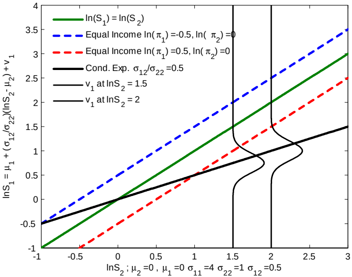

$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$$ln(\pi_1) = -0.5 \quad \text{and} \quad ln(\pi_1) = +0.5$$

Parameters:

$$\begin{bmatrix} \sigma_{11}\, \sigma_{12} \\ \sigma_{12}\, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

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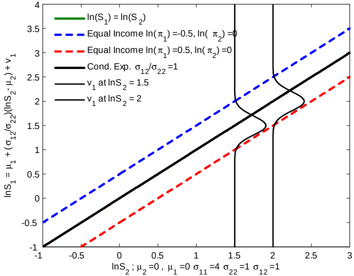

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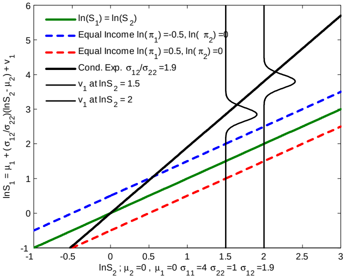
Graph of $\ln S_1 = f(\ln S_2)$



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Parameters:

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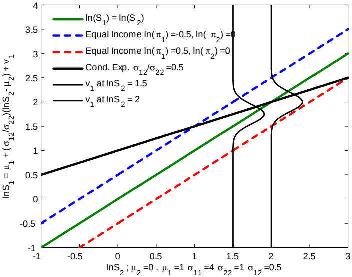
Graph of $\ln S_1 = f(\ln S_2)$



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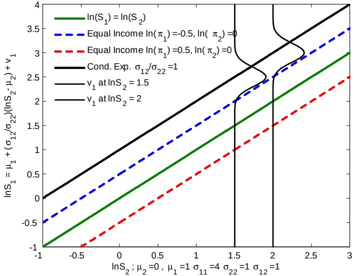
Graph of $\ln S_1 = f(\ln S_2)$



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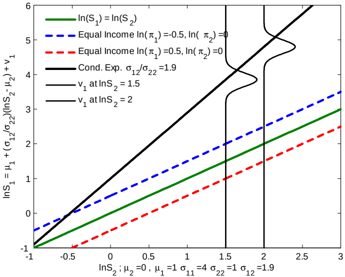
Graph of $\ln S_1 = f(\ln S_2)$



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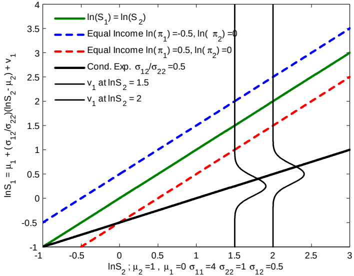
Graph of $\ln S_1 = f(\ln S_2)$



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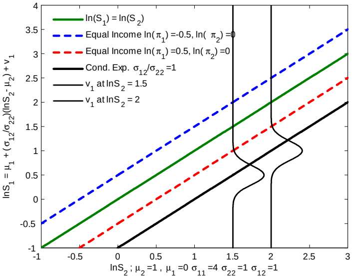
Graph of $\ln S_1 = f(\ln S_2)$



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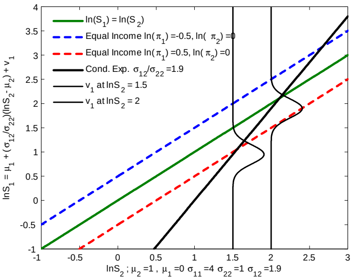
$ln(\pi_1) = -0.5$ and $ln(\pi_1) = +0.5$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



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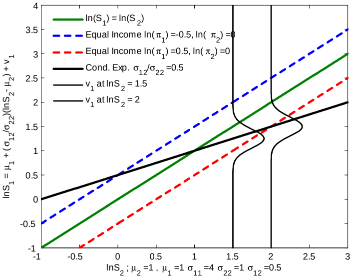
Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1.9 \\ 1 \, 1.9 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



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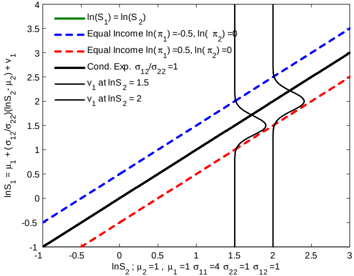
Graph of $\ln S_1 = f(\ln S_2)$



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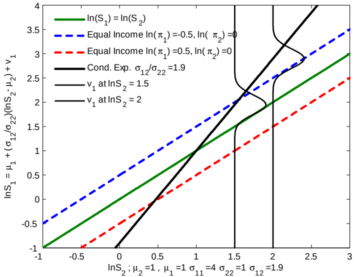
Parameters:

$$\begin{bmatrix} \sigma_{11}\,\sigma_{12} \\ \sigma_{12}\,\sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



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$ln(\pi_1) = -0.5$ and $ln(\pi_1) = +0.5$

Parameters:

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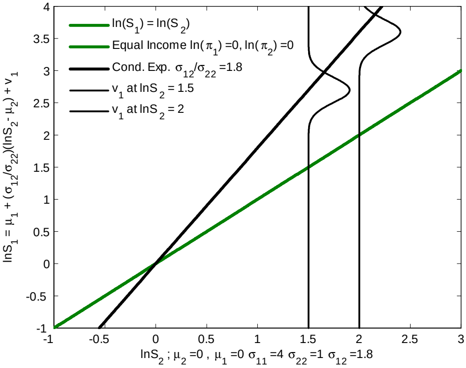
Graph of $\ln S_1 = f(\ln S_2)$



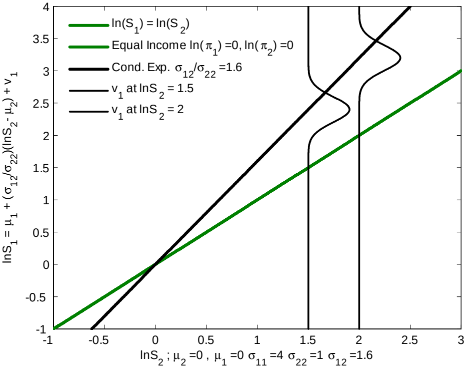
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



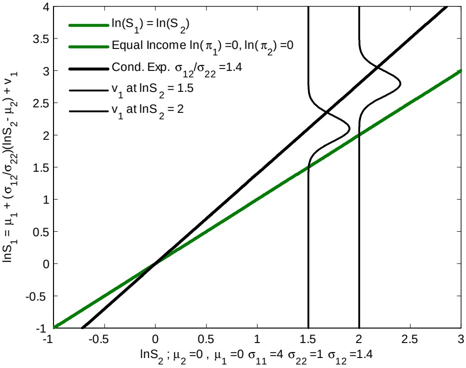
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



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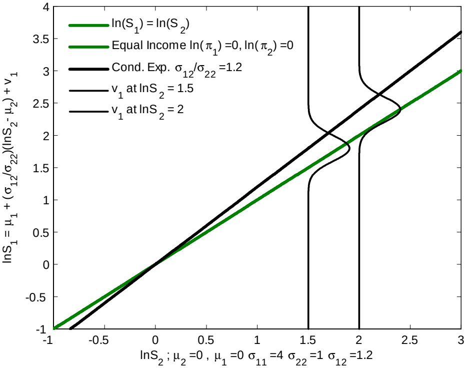
Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



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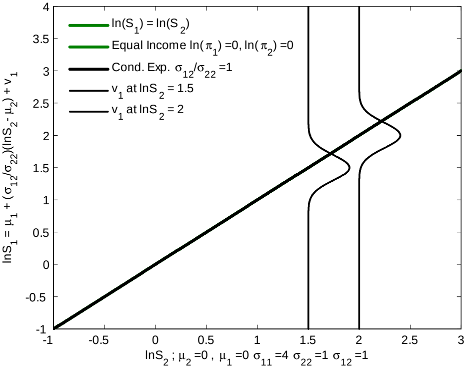
Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



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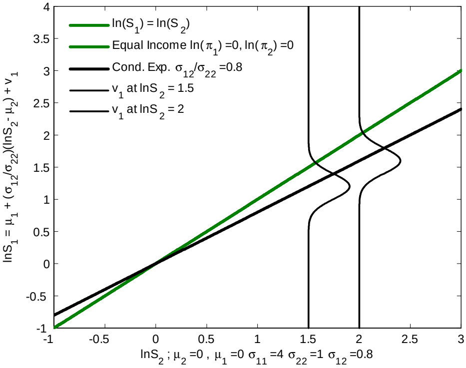
Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

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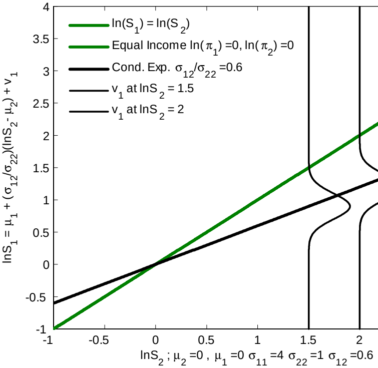
Graph of $\ln S_1 = f(\ln S_2)$



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Parameters:

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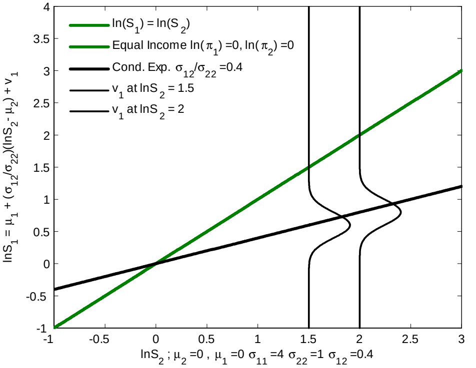

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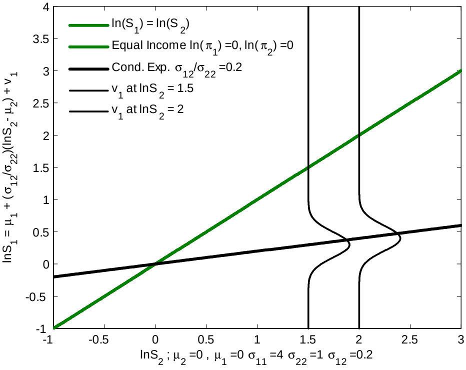

$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

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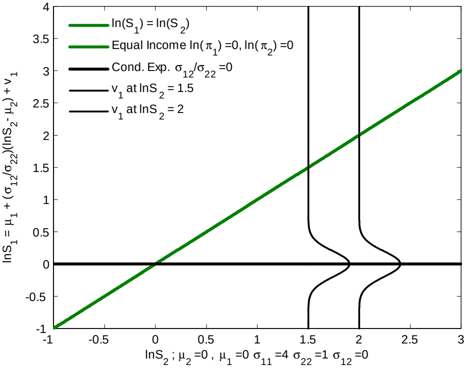

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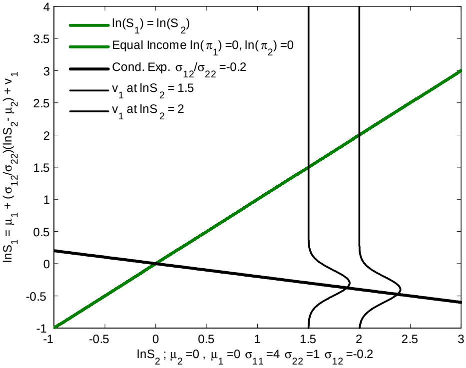
Graph of $\ln S_1 = f(\ln S_2)$



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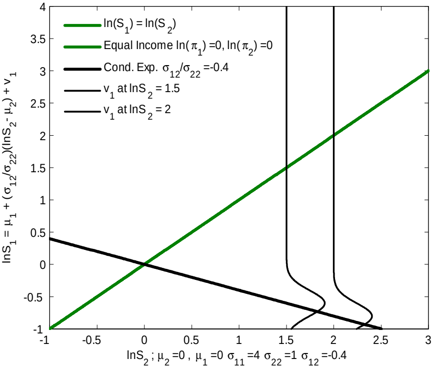

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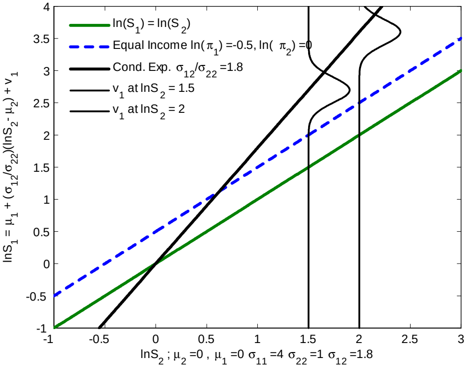

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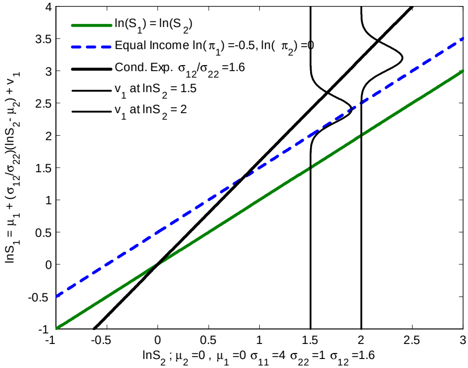

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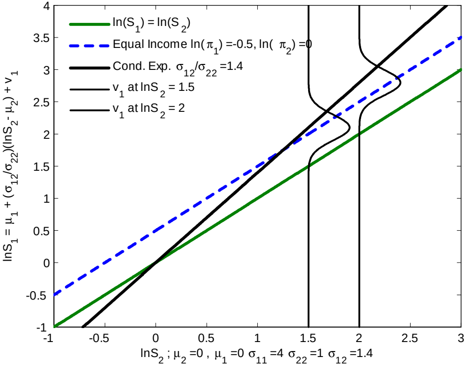

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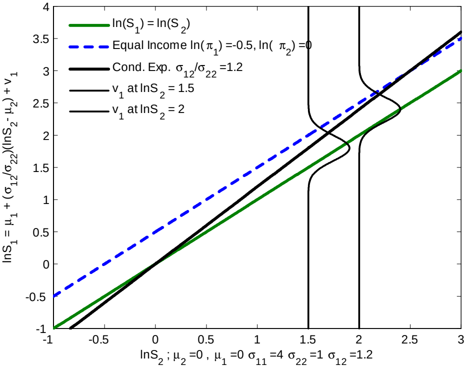

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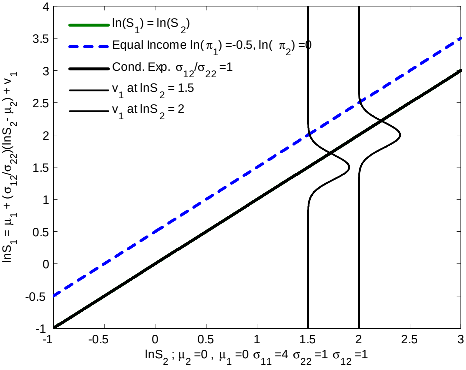

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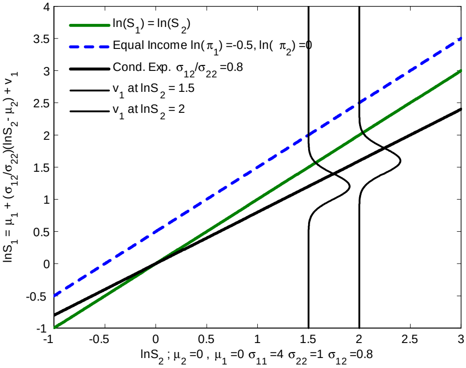

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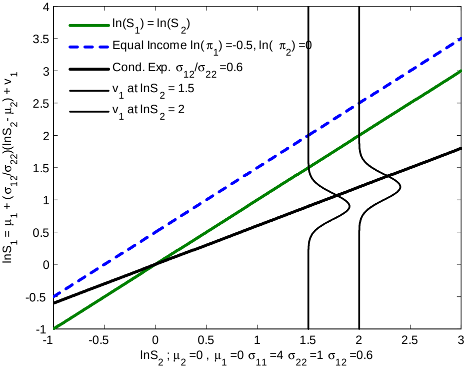

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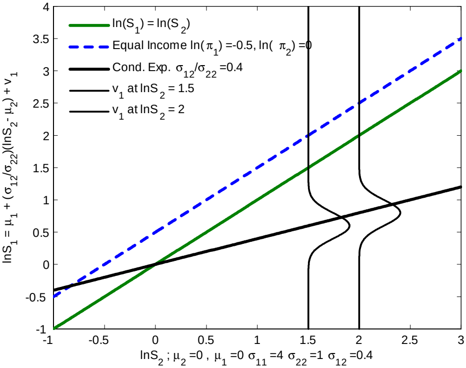

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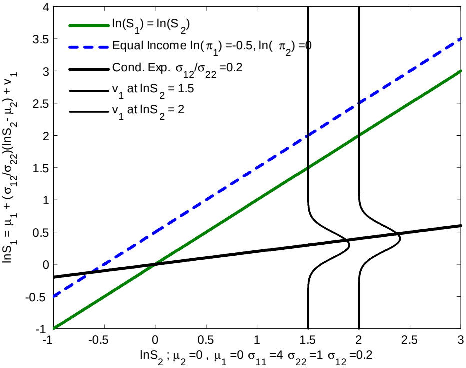

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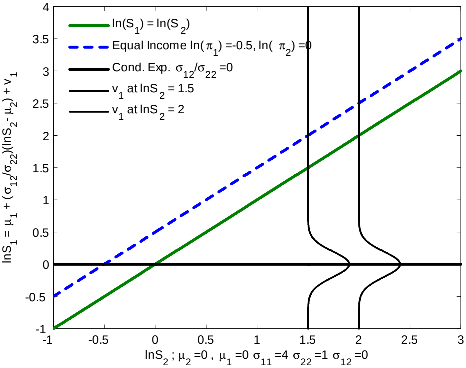

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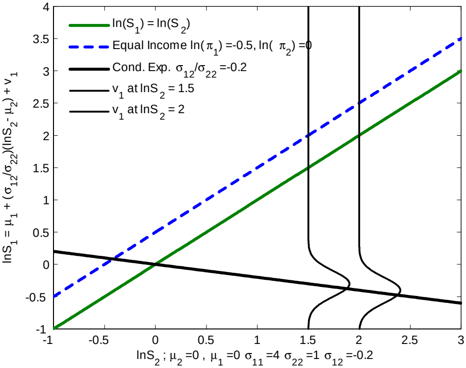

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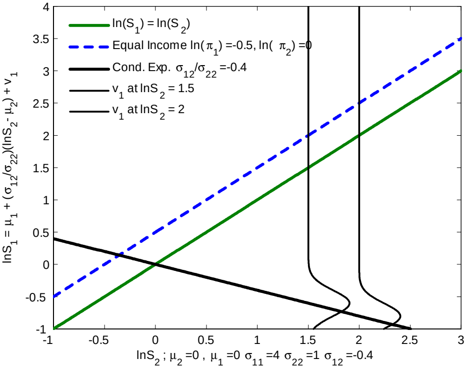

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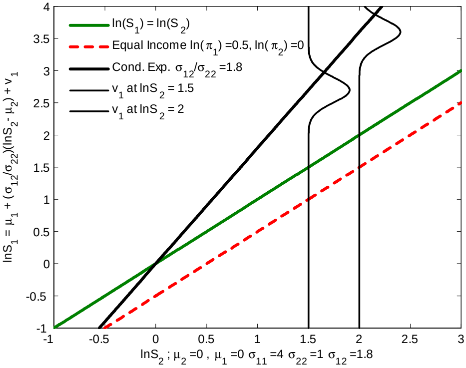

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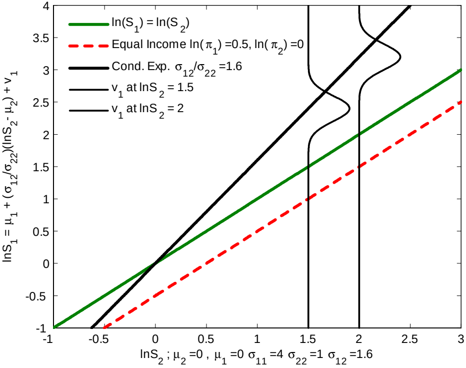
Graph of $\ln S_1 = f(\ln S_2)$



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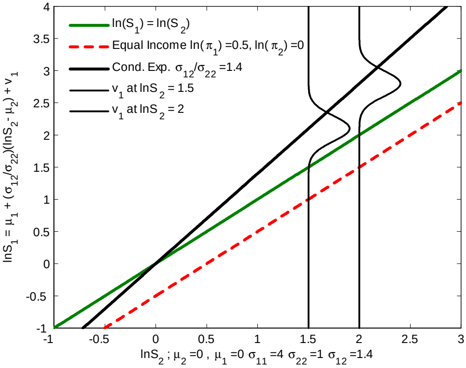

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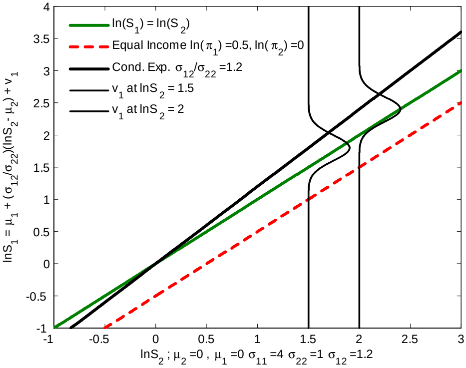

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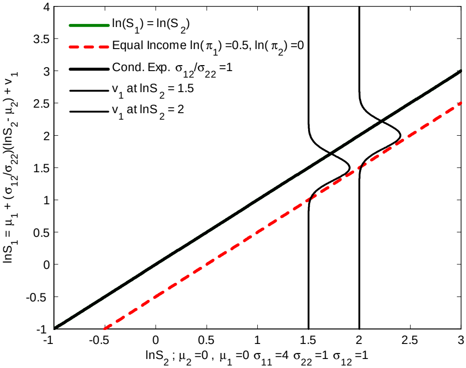

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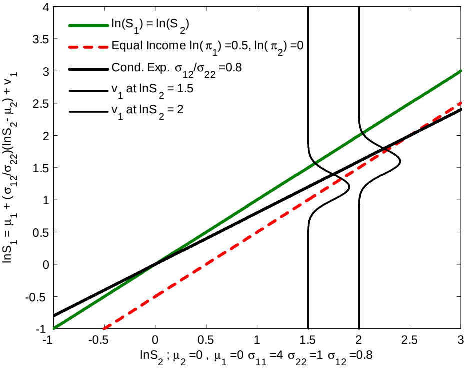

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$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

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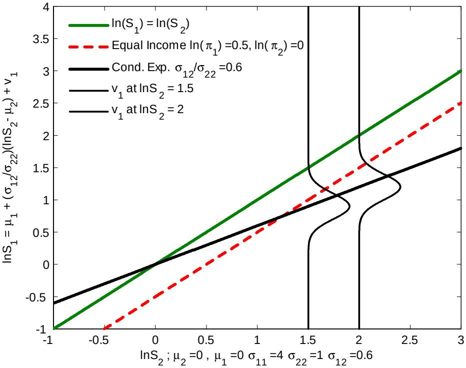

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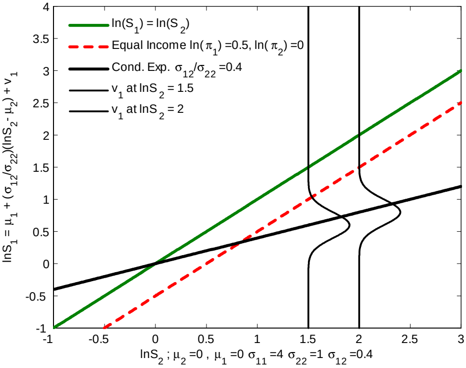

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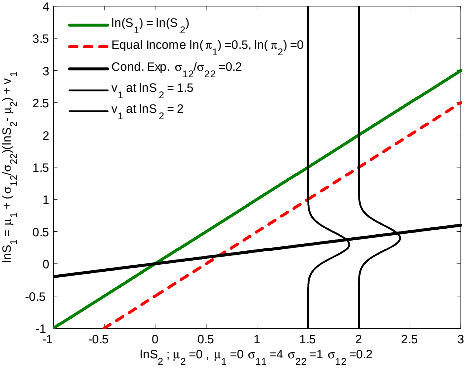

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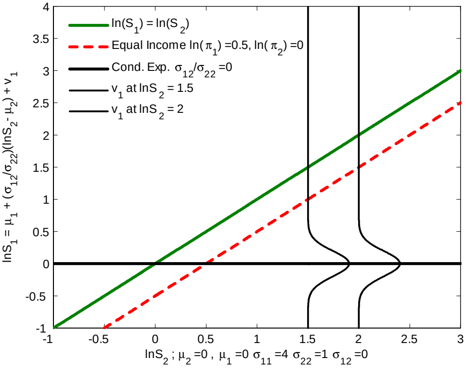

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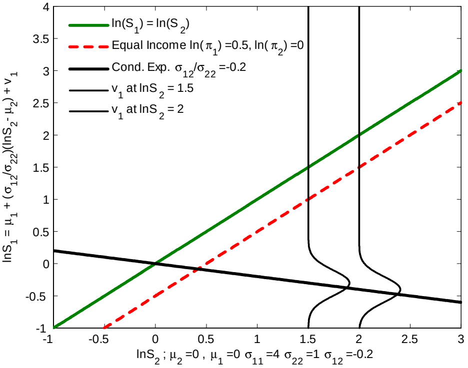
$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



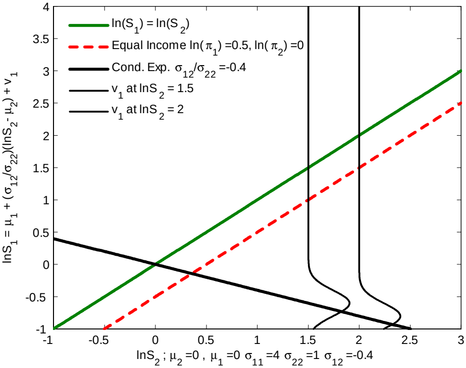
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



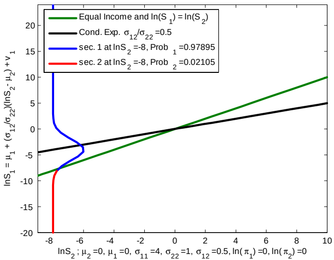
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

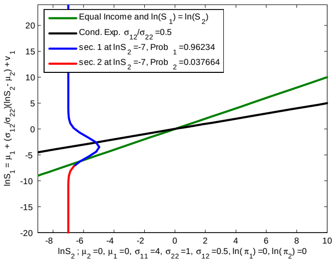
$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -8) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -8) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

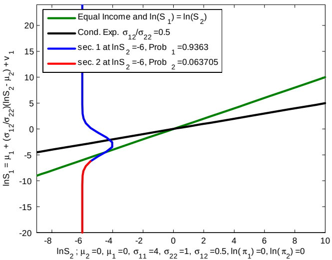
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -7) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -7) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

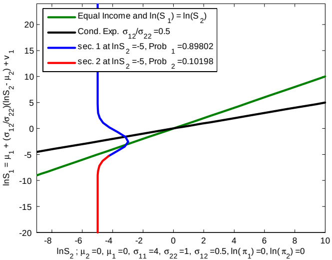
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



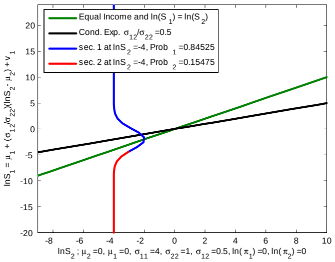
$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}}\left(\ln S_2 - \mu_2\right) + v_1$$
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{12} & \sigma_{22}\end{bmatrix} = \begin{bmatrix}4 & 0.5\\ 0.5 & 1\end{bmatrix}, \begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}; \begin{bmatrix}\ln(\pi_1)\\ \ln(\pi_2)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$

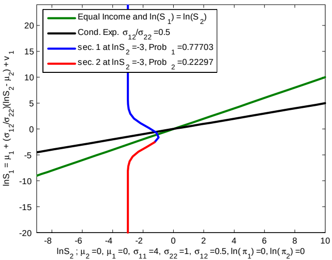


$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

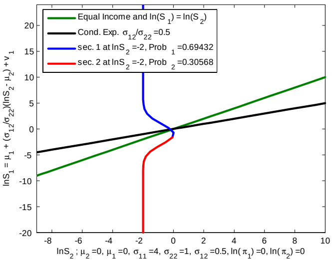
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

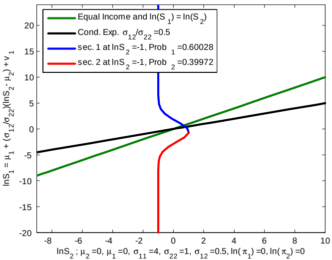
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

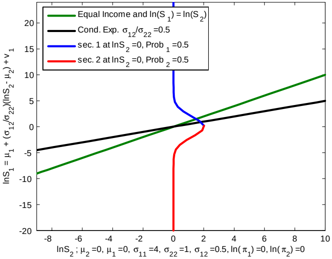
$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -1) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -1) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

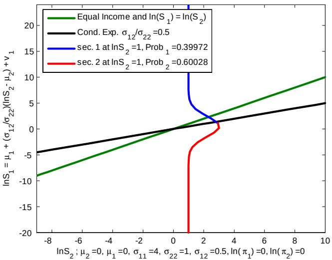
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

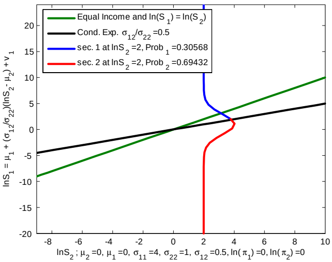
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} \left(\ln S_2 - \mu_2\right) + v_1$$

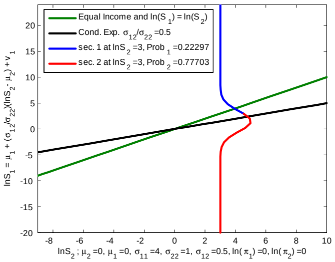
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 \middle| \ln S_2 = 2\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 \middle| \ln S_2 = 2\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

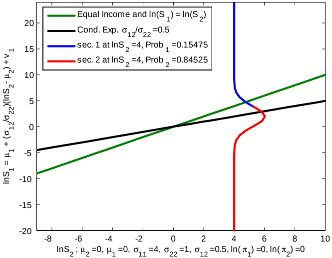
$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow$ Pr. of Working at Sector 1

$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

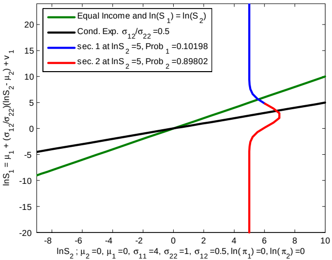
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

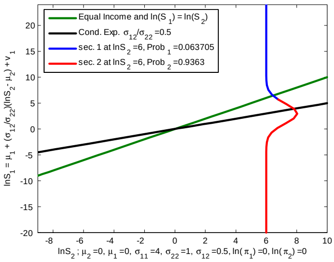
$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = 5\right) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = 5\right) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

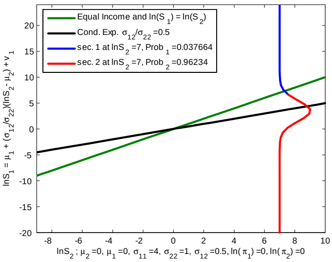
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

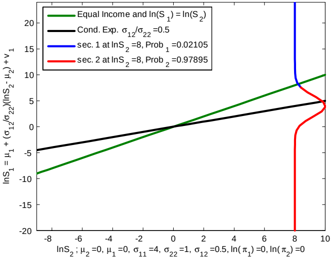
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

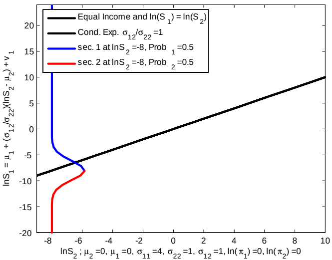
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 8) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 8) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$

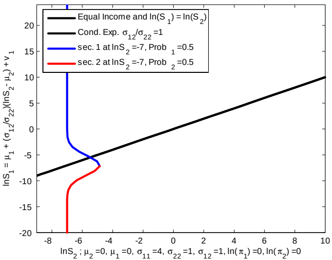


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} \left(\ln S_2 - \mu_2\right) + v_1$$

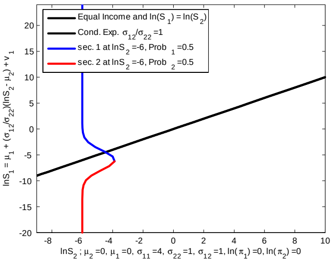
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -7\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -7\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

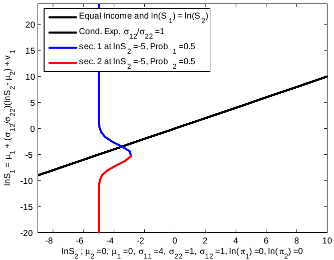
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11}\,\sigma_{12} \\ \sigma_{12}\,\sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} \left(\ln S_2 - \mu_2\right) + v_1$$

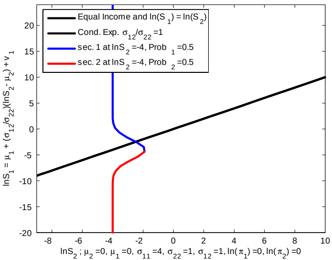
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -5\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11}\, \sigma_{12} \\ \sigma_{12}\, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

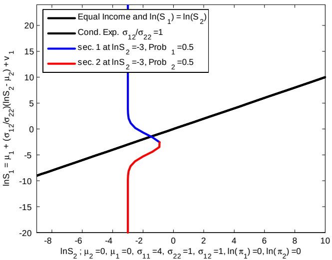
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

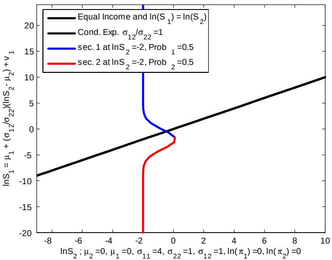
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11}\, \sigma_{12} \\ \sigma_{12}\, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

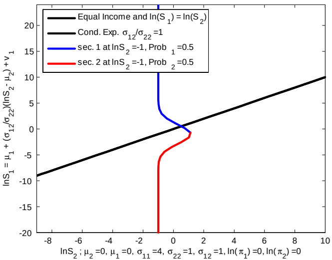
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 \middle| \ln S_2 = -2\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 \middle| \ln S_2 = -2\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

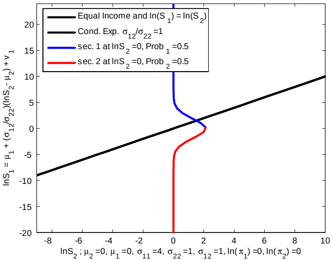
$$\begin{bmatrix} \sigma_{11}\,\sigma_{12} \\ \sigma_{12}\,\sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -1\right) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -1\right) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

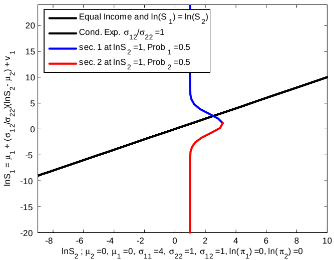
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11}\,\sigma_{12} \\ \sigma_{12}\,\sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

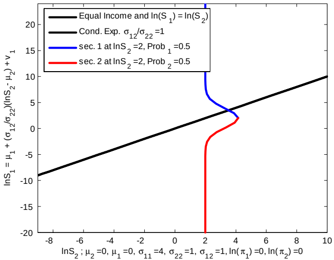
$$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

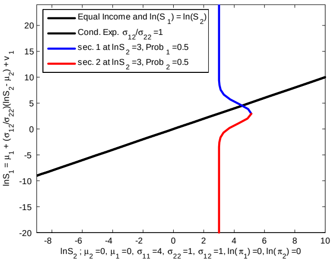
$$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

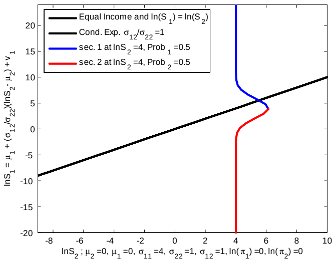
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

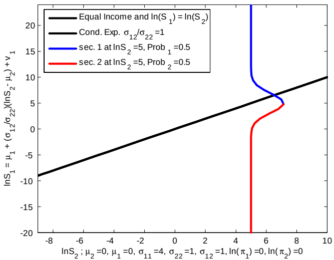
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

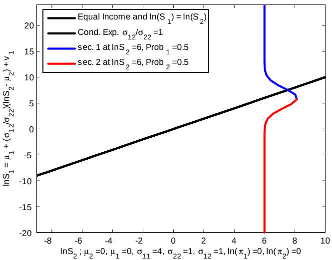
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = 5\right) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = 5\right) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11}\, \sigma_{12} \\ \sigma_{12}\, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4\,1 \\ 1\,1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

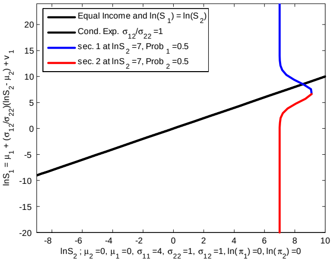
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

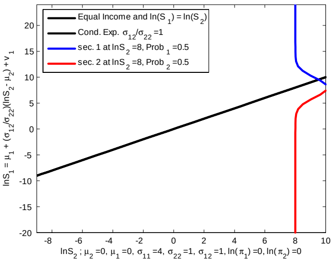
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



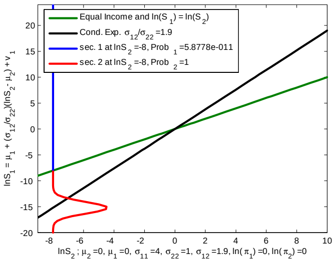
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\mathrm{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\mathrm{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 \, 1 \\ 1 \, 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

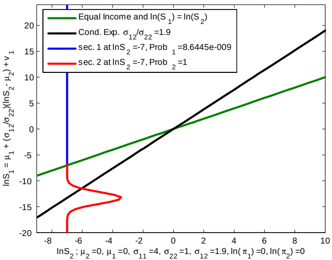
$$\text{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -8\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -8\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

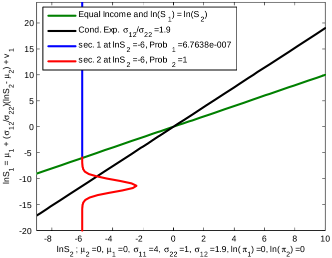
$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -7) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -7) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

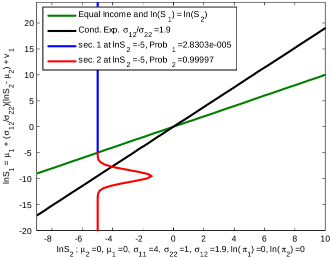
$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow$ Pr. of Working at Sector 1

$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

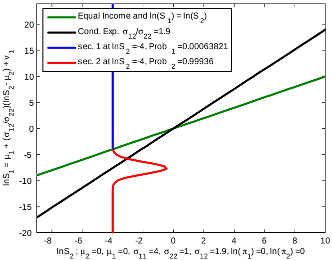
$$\mathrm{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\mathrm{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

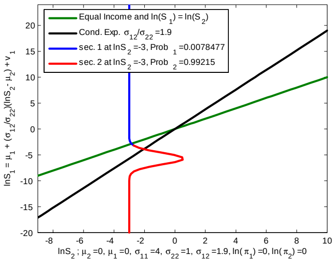
$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow$ Pr. of Working at Sector 1

$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

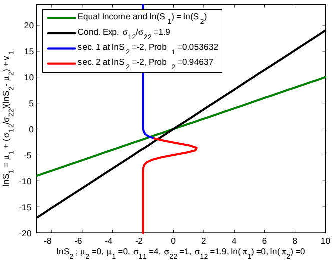
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

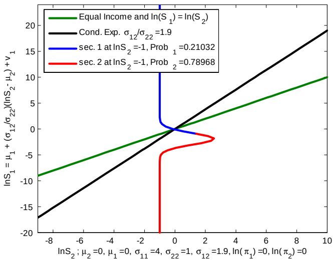
$$\text{Prob}_1 = \Pr (W_1 > W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr (W_1 < W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

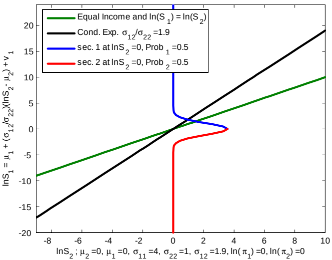
$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} \left(\ln S_2 - \mu_2\right) + v_1$$
$$\mathrm{Prob}_1 = \Pr\left(W_1 > W_2 | \ln S_2 = -1\right) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\mathrm{Prob}_2 = \Pr\left(W_1 < W_2 | \ln S_2 = -1\right) \Rightarrow \text{Pr. of Working at Sector 2}$$
Parameters:
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

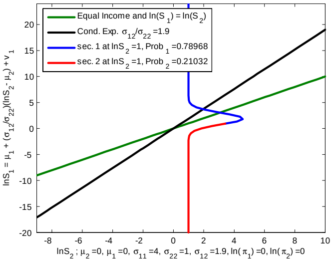
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \tfrac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

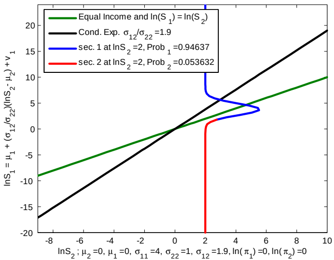
$$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

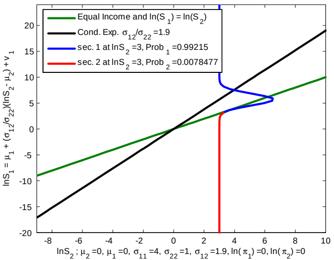
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11}\, \sigma_{12} \\ \sigma_{12}\, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

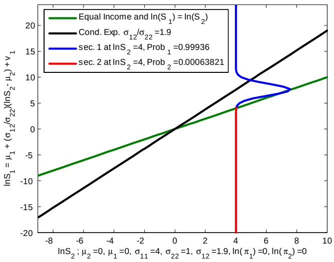
$\mathrm{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow$ Pr. of Working at Sector 1

$\mathrm{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

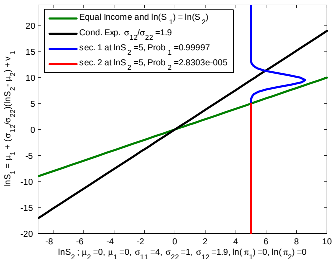
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow$ Pr. of Working at Sector 1

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow$ Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$

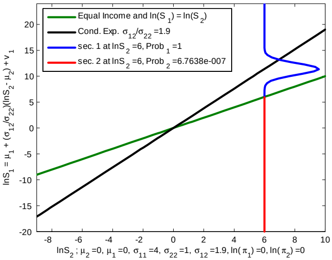


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$
$$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 1}$$
$$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} \left(\ln S_2 - \mu_2\right) + v_1$$

$$\mathrm{Prob}_1 = \Pr\left(W_1 > W_2 \middle| \ln S_2 = 6\right) \Rightarrow \text{Pr. of Working at Sector 1}$$

$$\mathrm{Prob}_2 = \Pr\left(W_1 < W_2 \middle| \ln S_2 = 6\right) \Rightarrow \text{Pr. of Working at Sector 2}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} \, \sigma_{12} \\ \sigma_{12} \, \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Change in skill prices

$$E[\ln S_1 \mid \ln W_1 > \ln W_2] = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*}\lambda(-c_1) > \mu_1$$

$\rightarrow$ positive selection

$$E[\ln S_2 \mid \ln W_2 > \ln W_1] = \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*}\lambda(-c_2) < \mu_2$$

$\rightarrow$ negative selection

, where $c_i = \ln(\pi_i/\pi_j) + \mu_i - \mu_j$

How does the skill composition react to a change in prices?

$$\frac{E[\ln S_1 \mid \ln W_1 > \ln W_2]}{\partial \ln \pi_1} < 0$$

$$\frac{E[\ln S_2 \mid \ln W_2 > \ln W_1]}{\partial \ln \pi_2} > 0$$

- What are the underlying economics?

Importance of Assignment Mechanism

Heckman and Honore (1990) show that ...

> *For a log normal Roy economy, any random assignment of persons to sectors with the same proportion of persons in each sector as in the Roy economy has higher variance of log earnings provided the proportions lie strictly in the unit interval. This is true whether or not skill prices in the two economies are the same.*

Choices over Time

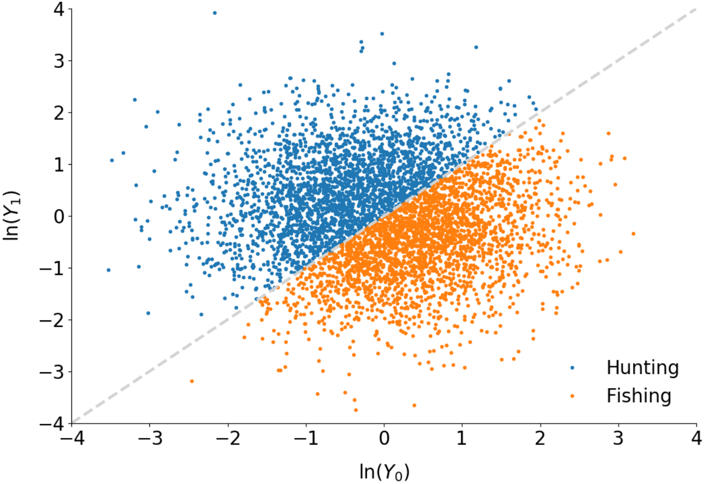
Incarnations of the Roy Model

Original Roy model

Potential Outcomes	Cost
$W_1 = \pi_1 S_1$	$C = 0$
$W_2 = \pi_2 S_2$	

Observed Outcomes	Choice
$W = DW_1 + (1-D)W_2$	$S = W_1 - W_2$
	$D = \mathrm{I}[S > 0]$

Figure: Occupational sorting in the original Roy model

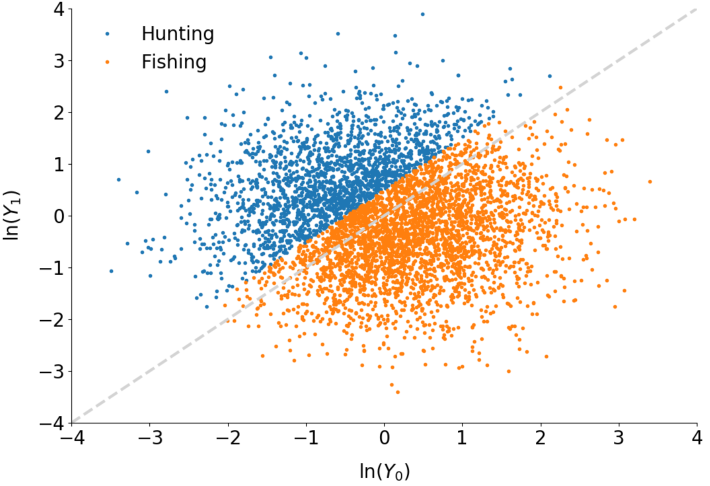


Extended Roy model

Potential Outcomes	Cost
$Y_1 = \mu_1(X) + U_1$	$C = \mu_D(Z)$
$Y_0 = \mu_0(X) + U_0$	

Observed Outcomes	Choice
$Y = DY_1 + (1-D)Y_0$	$S = Y_1 - Y_0 - C$
	$D = \mathrm{I}[S > 0]$

Figure: Occupational sorting in the extended Roy model



Generalized Roy model

Potential Outcomes

$Y_1 = \mu_1(X) + U_1$

$Y_0 = \mu_0(X) + U_0$

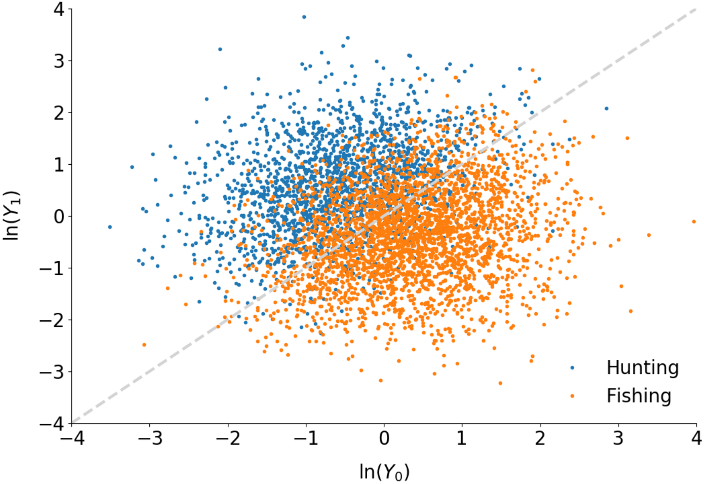
Observed Outcome

$Y = DY_1 + (1 - D)Y_0$

Choice

$D = \mathrm{I}[\mu_D(X, Z) - V > 0]$

Figure: Occupational sorting in the generalized Roy model



Appendix

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