

# Economics of Human Capital

Dynamic model of human capital accumulation

Philipp Eisenhauer

# Introduction

We build on the following seminal paper:

- ▶ Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3), 473–522.

# Roadmap

- ▶ Economic Model
- ▶ Mathematical Model
- ▶ Data
- ▶ Computational Model
- ▶ Results

# **Economic Model**

## Decision Problem

$t = 1, \dots, T$  decision period

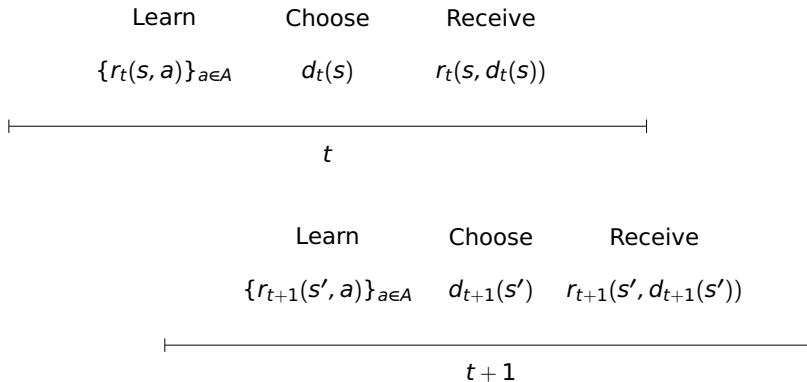
$s \in S$  state

$a \in A$  action

$d_t$  decision rule

$r_t(s, a)$  immediate reward

## Timing of Events



$\pi = (d_1, \dots, d_T)$       policy

$h_t = (s_1, a_1, \dots, s_t)$     history

$\delta$                                   discount factor

$p_t(s, a)$                         conditional distribution



## Individual's Objective under Risk

$$v_1^{\pi^*}(s) = \max_{\pi \in \Pi} E_s^{\pi} \left[ \sum_{t=1}^T \delta^{t-1} r_t(X_t, d_t(X_t)) \right]$$

# **Mathematical Model**

## Policy Evaluation

$$v_t^\pi(s) = \mathbb{E}_s^\pi \left[ \sum_{\tau=t}^T \delta^{\tau-t} r_\tau(X_\tau, d_\tau(X_\tau)) \right]$$

### Inductive Scheme

$$v_t^\pi(s) = r_t(s, d_t(s)) + \delta \mathbb{E}_s^\pi [v_{t+1}^\pi(X_{t+1})]$$

## Optimality Equations

$$v_t^{\pi^*}(s) = \max_{a \in A} \left\{ r_t(s, a) + \delta \mathbb{E}_s^p [v_{t+1}^{\pi^*}(X_{t+1})] \right\}.$$

## Backward Induction Algorithm for MDP

**for**  $t = T, \dots, 1$  **do**

**if**  $t == T$  **then**

$$v_T^{\pi^*}(s) = \max_{a \in A} \left\{ r_T(s, a) \right\} \quad \forall s \in S$$

**else**

        Compute  $v_t^{\pi^*}(s)$  for each  $s \in S$  by

$$v_t^{\pi^*}(s) = \max_{a \in A} \left\{ r_t(s, a) + \delta E_s^p \left[ v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}$$

        and set

$$d_t^{\pi^*}(s) = \arg \max_{a \in A} \left\{ r_t(s, a) + \delta E_s^p \left[ v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}$$

**end if**

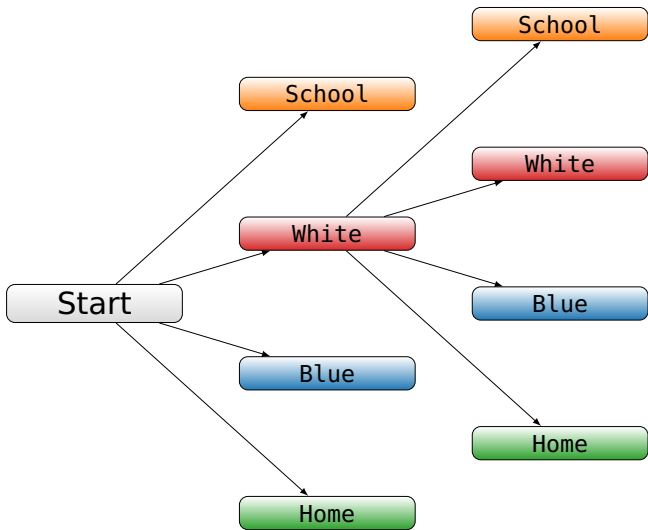
**end for**

# Data

## **National Longitudinal Survey of Youth (1979)**

- ▶ 1,373 white males starting at age 16
- ▶ life-cycle histories
  - ▶ school attendance
  - ▶ occupation-specific work status
  - ▶ real wages

Figure: Decision Tree





# *Descriptives*

Figure: Sample Size

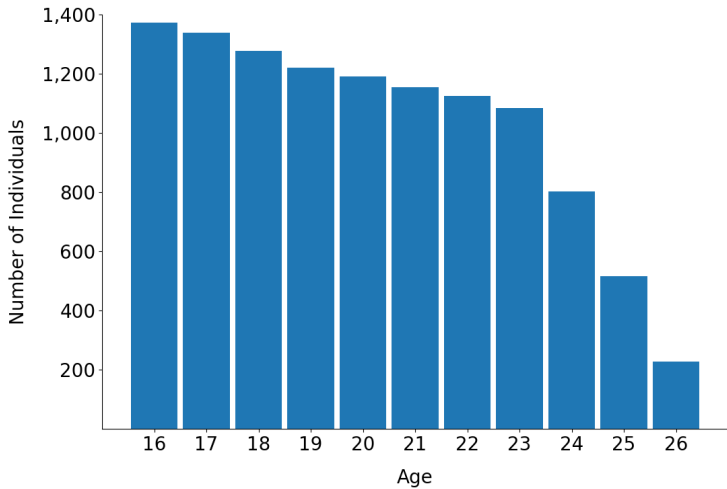


Figure: Observed Choices

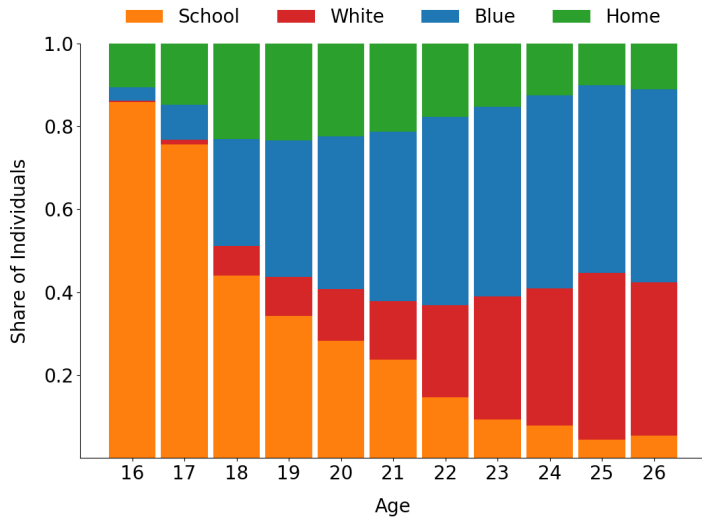


Table: Observed Real Wages

Age	<u>White</u>		<u>Blue</u>	
	Obs.	Mean	Obs.	Mean
16	2	.	26	10,287
20	128	5,499	349	14,432
25	201	16,540	222	21,991

Figure: Observed Transitions

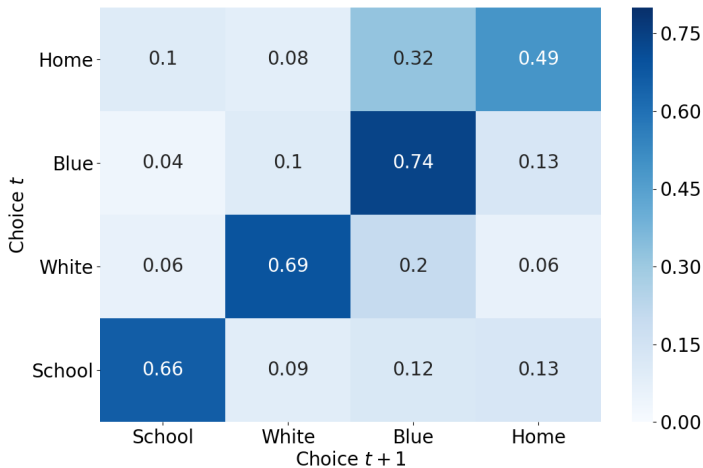


Figure: Initial Schooling

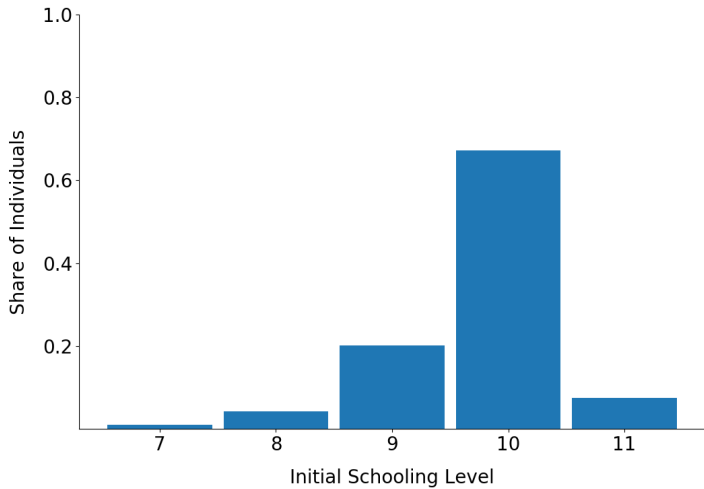


Table: Activities by Initial Schooling

Alternatives	<u>Initial Schooling</u>				
	7	8	9	10	11
School	0.69	0.86	2.48	3.37	2.83
White	0.08	0.38	0.65	1.36	2.04
Blue	3.69	3.62	3.05	2.40	1.98
Home	4.23	4.19	1.91	1.10	1.32
Total	8.69	9.05	8.09	8.24	8.17

# *Reduced-form Analysis*



Table: Mincer Regressions

	Log Real Wages	
Intercept	8.314***	8.329***
Schooling	0.086***	0.077***
	<u>Work Experience</u>	
- linear	0.132***	0.125***
- squared	-0.005***	-0.003***
	<u>Corrected AFQT</u>	
- linear	—	0.002***
Adj- $R^2$	0.21	0.22
Observations	4,420	4,232

Table: Mincer Regressions

	<u>Log Real Wages</u>	
	White	Blue
Intercept	7.748***	8.790***
Schooling	0.128***	0.044***
<u>Own Experience</u>		
- linear	0.146***	0.129***
- squared	-0.003	-0.005***
<u>Other Experience</u>		
- linear	0.096***	0.085***
- squared	0.002	-0.003
Adj-R <sup>2</sup>	0.28	0.17
Observations	1, 468	2, 952

## Open Issues

- ▶ distinction between ex ante and ex post returns
- ▶ role of psychic costs
- ▶ nonlinearities in the return
- ▶ role of uncertainty

# Computational Model

## Additional Structure

$t$  age

$k$  unobserved type

$x_{j,t}$  experience in occupation  $j$  at age  $t$

$a_t$  action at age  $j$

$g_t$  level of schooling at age  $t$

## Skill Production Function

$$\begin{aligned} e_{j,k,t} = \exp \{ & e_{j,k,16} + \underbrace{\alpha_{j,1}g_t + \alpha_{j,2}I[g_t \geq 12] + \alpha_{j,3}I[g_t \geq 16]}_{\text{schooling}} \\ & + \underbrace{\alpha_{j,4}x_{j,t} + \alpha_{j,5}x_{j,t}^2 + \alpha_{j,6}I[x_{j,t} > 0] + \alpha_{j,7}x_{j \neq j',t}}_{\text{work experience}} \\ & + \underbrace{\alpha_{j,8}I[a_{t-1} \neq j]}_{\text{depreciation}} + \alpha_{j,9}(t - 16) + \alpha_{j,10}I[t < 18] + \epsilon_{j,t} \} \end{aligned}$$

with  $j, j' = 1, 2$ ,  $k = 1, \dots, 4$ , and  $t = 16, \dots, 65$

## Labor Market

$$r_{j,k,t} = w_{j,k,t} + \underbrace{\kappa_1 \mathbb{I}[g_t \geq 12] + \kappa_2 \mathbb{I}[g_t \geq 16]}_{\text{common returns}} + \beta_{j,1} \\ + \underbrace{\beta_{j,2} \mathbb{I}[x_{j,t} > 0, a_{t-1} \neq j] + \beta_{j,3} \mathbb{I}[x_{j,t} = 0, a_{t-1} \neq j]}_{\text{entry cost}}$$

with  $w_{j,k,t} = r_j e_{j,k,t}$

## School

$$\begin{aligned} r_{3,k,t} = & e_{3,k,16} + \underbrace{\gamma_1 I[g_t \geq 12] + \gamma_2 I[g_t \geq 16]}_{\text{monetary and psychic cost}} \\ & + \underbrace{\gamma_3 I[a_{t-1} \neq 3, g_t \leq 11] + \gamma_4 I[a_{t-1} \neq 3, g_t \geq 12]}_{\text{reenrollment cost}} \\ & + \gamma_5(t-16) + \gamma_6 I[t \leq 18] + \underbrace{\kappa_1 I[g_t \geq 12] + \kappa_2 I[g_t \geq 16]}_{\text{common returns}} \\ & + \epsilon_{3,t} \end{aligned}$$



## Home

$$r_{4,k,t} = e_{4,k,16} + \zeta_1 I[18 \leq t \leq 20] + \zeta_2 I[t \geq 21] \\ + \underbrace{\kappa_1 I[g_t \geq 12] + \kappa_2 I[g_t \geq 16]}_{\text{common returns}} + \epsilon_{4,t}$$

## State Space

- ▶ at time  $t$

$$s_t = \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}, \{\epsilon_{j,t}\}_{j=1,\dots,4}\}$$

$$\bar{s}_t = \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}\}$$

- ▶ laws of motion

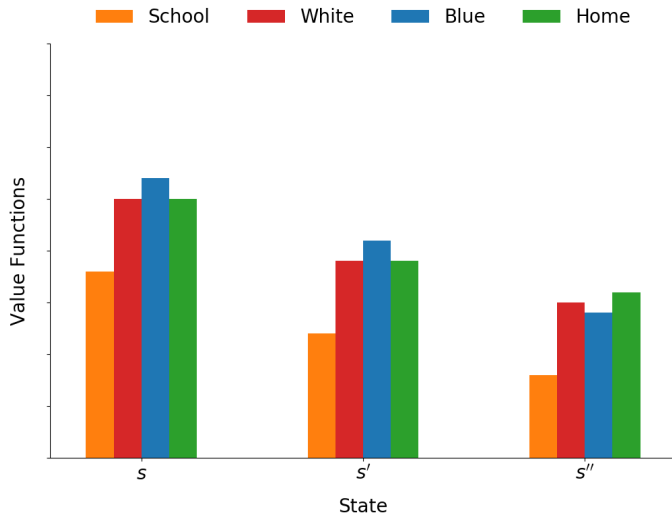
$$x_{j,t+1} = x_{j,t} + \mathbb{I}[a_t = j] \quad \forall \quad j \in \{1, 2\}$$

$$g_{t+1} = g_t + \mathbb{I}[a_t = 3]$$

## Distribution of shocks

$$[\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t}]^T \sim \mathcal{N}_0(\mathbf{0}, \Sigma)$$

Figure: Value Functions



## Computational Tool

<https://respy.readthedocs.io>

- ▶ Technical Documentation
  - ▶ Numerical Methods, Source Codes, Test Suite
- ▶ User Documentation
  - ▶ Tutorial

⇒ Transparency, Recomputability, and Extensibility

# Conclusion

# Appendix

# *References*



Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3), 473–522.