Estimation of Dynamic Discrete Choice Models by Maximum Likelihood and Simulated Method of Moments

Fit structural model to *observed* data. $\dashrightarrow \hat{\psi}^{obs}$

$\downarrow$

Simulate synthetic dataset using estimates.

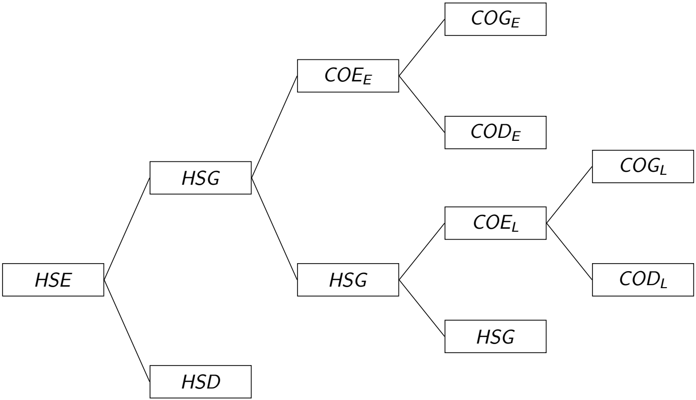
$\downarrow$

Fit structural model to *synthetic* data. $\dashrightarrow \hat{\psi}^{syn}$

$\hat{\psi}^{obs} \xleftrightarrow{\text{Compare}} \hat{\psi}^{syn}$

Model

Figure: Decision Tree



Agent Behavior

- Objective
- Constraints
    - Institution
    - Information

Value Function

$$V(s\,|\,\mathcal{I}(s)) = Y(s) + \max_{s'\in\Omega(s)} \left\{ \frac{1}{1+r}\Big(-C(s',s) + \mathbb{E}[V(s'\,|\,\mathcal{I}(s'))\,\big|\,\mathcal{I}(s)\;]\Big) \right\}$$

Policy Function

$$s' = \begin{cases} \hat{s}' & \text{if} \quad \mathbb{E}\left[V(\hat{s}')\,\big|\,\mathcal{I}(s)\right] - C(\hat{s}',s) > \mathbb{E}\left[V(\tilde{s}')\,\big|\,\mathcal{I}(s)\right] \\ \tilde{s}' & \text{otherwise} \end{cases}$$

Objects of Interest

- Net Return
- Gross Return

Net Return

$$NR(\hat{s}', \tilde{s}', s) = \frac{\mathbb{E}\left[V(\hat{s}') - V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right] - C(\hat{s}', s)}{\mathbb{E}\left[V(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}$$

Gross Return

$$GR(\hat{s}', \tilde{s}', s) = \frac{\mathbb{E}\left[\tilde{V}(\hat{s}') - \tilde{V}(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}{\mathbb{E}\left[\tilde{V}(\tilde{s}') \,\middle|\, \mathcal{I}(s)\right]}$$

Parametrization

- Benefits
- Costs
- Measurements

Functional Forms

$$
\begin{aligned}
Y(s) &= X(s)'\beta_s + \theta'\alpha_s + \epsilon(s) && \forall \quad s \in \mathcal{S} \\
C(\hat{s}', s) &= Q(\hat{s}', s)'\delta_{\hat{s}',s} + \theta'\varphi_{\hat{s}',s} + \eta(\hat{s}', s) && \forall \quad s \in \mathcal{S}^c
\end{aligned}
$$

$$
M(j) = X(j)'\kappa_j + \theta'\gamma_j + \nu(j) \qquad \forall \quad j \in M
$$

Distributions of Unobservables

$$\begin{aligned}
\theta &\sim \mathcal{N}(0, \sigma_\theta) &\forall \quad \theta \in \Theta \\
\epsilon(s) &\sim \mathcal{N}(0, \sigma_{\epsilon(s)}) &\forall \quad s \in \mathcal{S} \\
\eta(\hat{s}', s) &\sim \mathcal{N}(0, \sigma_{\eta(\hat{s}', s)}) &\forall \quad s \in \mathcal{S}^c \\
\nu(j) &\sim \mathcal{N}(0, \sigma_{\nu(j)}) &\forall \quad j \in \mathcal{M}
\end{aligned}$$

Individual Likelihood

$$\int_{\underline{\Theta}} \Bigg[\prod_{j \in M} \underbrace{f\Big(M(j) \,\Big|\, D, \theta; \psi\Big)}_{\text{Measurement}} \times$$

$$\prod_{s \in \mathcal{S}} \Bigg\{ \underbrace{f\Big(Y(s) \,\Big|\, D, \theta; \psi\Big)}_{\text{Outcome}} \underbrace{\Pr\Big(G(s) = 1 \,\Big|\, D, \theta; \psi\Big)}_{\text{Transition}} \Bigg\}^{\mathbb{1}\{s \in \Gamma\}} \Bigg] dF(\theta)$$

Results

Table: Cross Section Model Fit

	Average Earnings		State Frequencies	
State	Observed	ML	Observed	ML
High School Graduates	4.29	3.84	0.30	0.32
High School Dropouts	2.29	2.59	0.17	0.14
Early College Graduates	6.73	7.46	0.29	0.29
Early College Dropouts	4.55	3.87	0.12	0.12
Late College Graduates	4.84	6.22	0.06	0.07
Late College Dropouts	4.89	4.88	0.06	0.06

Objects of Interest

- Choice Probabilities
- Gross Return
- Net Return
- Schooling Attainment

Figure: Choice Probability, Early College Enrollment

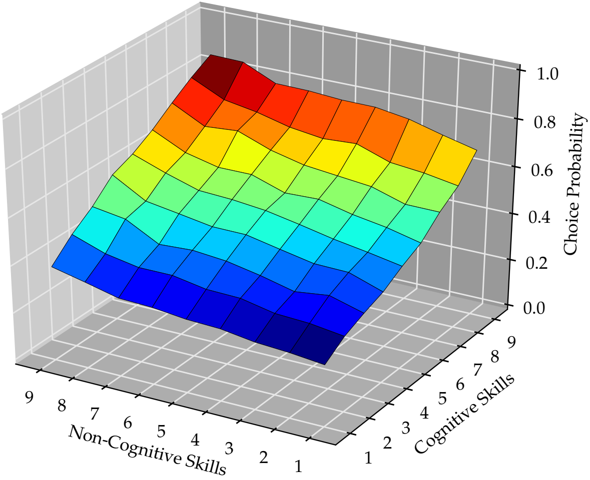


Figure: Gross Return, Early College Enrollment

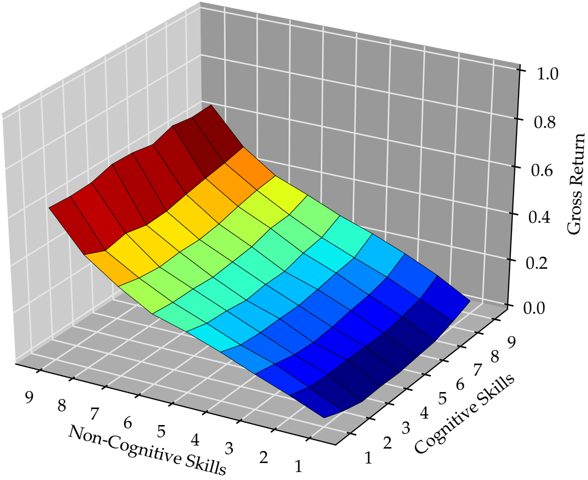


Figure: Net Return, Early College Enrollment

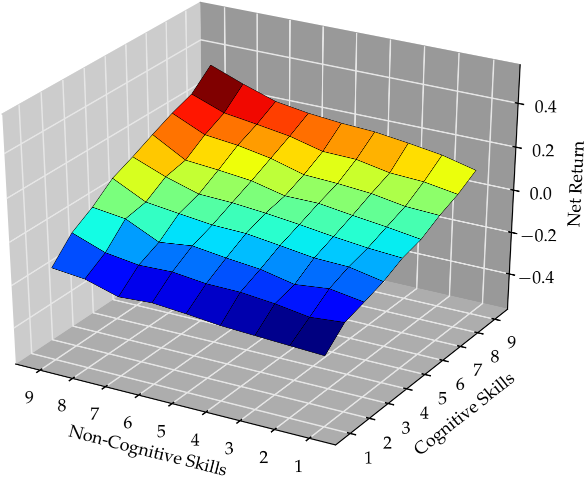


Figure: Schooling Attainment by Cognitive Skills

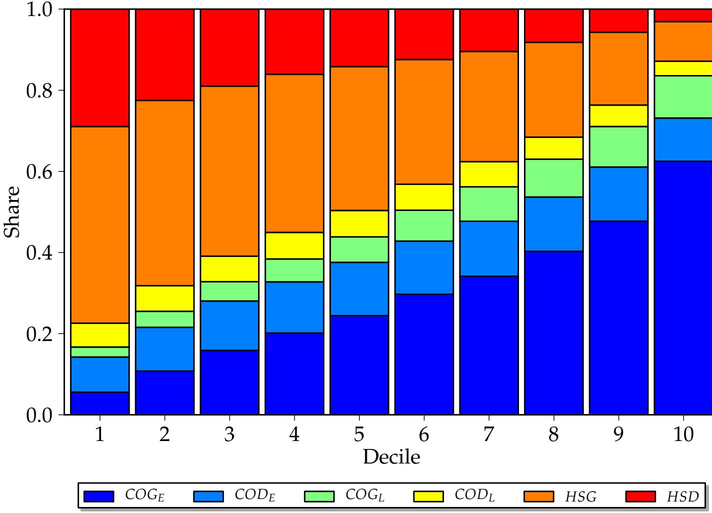
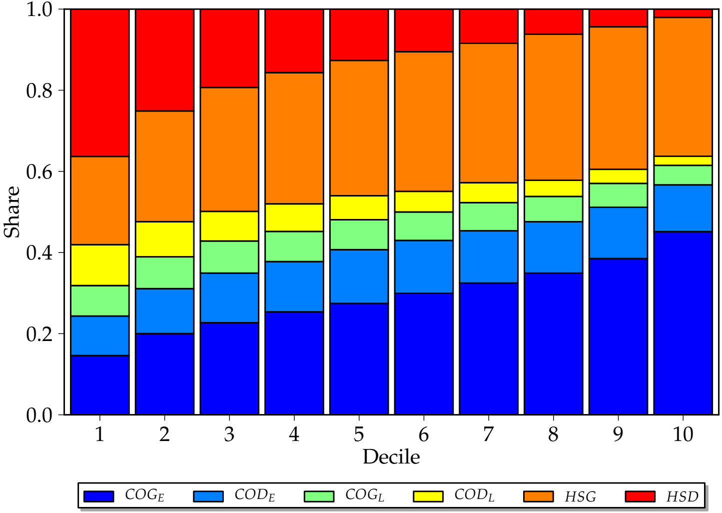


Figure: Schooling Attainment by Non-Cognitive Skills



Monte Carlo Study

Synthetic Agents	5,000
Structural Parameters	192
Free Parameters	138

Simulated Method of Moments

Criterion Function

$$\arg\min_{\psi} \quad \Lambda(\psi) = \left[\check{f} - \hat{f}(\psi)\right]' W^{-1} \left[\check{f} - \hat{f}(\psi)\right]$$

$$, \text{ where} \quad \hat{f}(\psi) = \frac{1}{R}\sum_{r=1}^{R} \hat{f}_r(u_r;\psi)$$

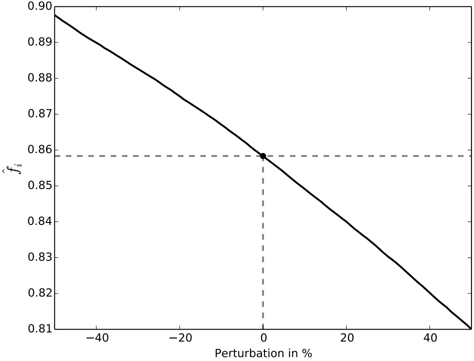
Number of Moments	250
Number of Replications	50
Weighting Matrix	Diagonal Matrix with Variances
Optimization Algorithm	TAO POUNDerS

Moment Conditions

- Benefit Equations
    - Means
    - Standard Deviations
    - Ordinary Least Squares Models
- Cost Equations
    - State Frequencies
    - Linear Probability Models

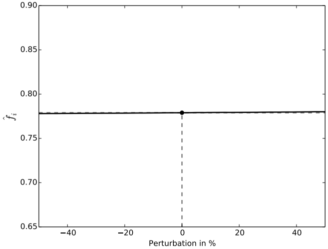
Parameter Perturbations

Figure: Parameter Perturbation, Cost

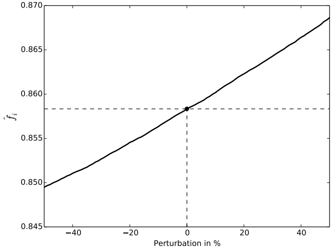


High School Graduation, State Frequency

Figure: Parameter Perturbation, Benefit



HS Graduates, Average Wages



HS Graduation, State Frequency

Model Fit

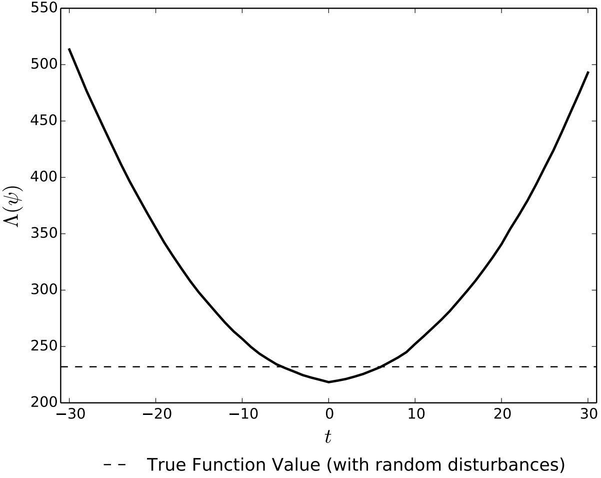
Table: Cross Section Model Fit

	Average Earnings		
State	True	ML	SMM
High School Graduates	3.87	3.87	3.86
High School Dropouts	2.51	2.52	2.53
Early College Graduates	6.80	6.78	6.86
Early College Dropouts	3.90	3.95	3.92
Late College Graduates	6.03	6.14	6.24
Late College Dropouts	5.10	5.07	5.08
RMSE		0.05	0.07

Table: Cross Section Model Fit

	State Frequencies		
State	True	ML	SMM
High School Graduates	0.32	0.32	0.33
High School Dropouts	0.14	0.14	0.13
Early College Graduates	0.29	0.29	0.29
Early College Dropouts	0.12	0.12	0.13
Late College Graduates	0.07	0.07	0.05
Late College Dropouts	0.06	0.06	0.07
RMSE		0.00	0.00

Figure: Function Value



Economic Implications

Table: Economic Implications

	Gross Return		
State	True	ML	SMM
High School Graduation	30%	41%	42%
Early College Graduation	88%	96%	96%
Late College Graduation	29%	28%	36%
RMSE		0.09	0.13

Table: Economic Implications

	Net Return		
State	True	ML	SMM
High School Graduation	63%	61%	212%
Early College Graduation	57%	51%	125%
Late College Graduation	14%	12%	38%
RMSE		0.05	0.75

Table: Standard Deviations

		$\hat{\sigma}_{\eta_{(\hat{s}',s)}}$	
State	True	ML	SMM
High School Graduation	0.27	0.24	0.85
Early College Graduation	0.61	0.59	1.49
Late College Graduation	0.61	0.59	1.49
RMSE		0.01	0.68

Tuning Parameters

- Moment Conditions
- Replications
- Optimization Algorithm

Moment Conditions

Table: Set of Moments

Sets	*Dynamic (Panel) Moments* Base	Alt. A	Alt. B	*Cross Section Moments* Base
Benefit Models				
Means	✓	✓	✓	✓
Standard Deviations	✓	✓	✓	✓
Ordinary Least Squares	✓	✓	✓	✓
Correlations			✓	
Choice Models				
State Frequencies	✓	✓	✓	✓
Linear Probability				
- cross section				✓
- dynamic	✓	✓	✓	
Probit				
- dynamic		✓	✓	
Correlations			✓	

Table: Set of Moments

	Dynamic (Panel) Moments			*Cross Section Moments*
Sets	Base	Alt. A	Alt. B	Base
Overall Statistics				
Number of Moments	250	339	545	222
Number of Replications	50	50	50	50
Weighting Matrix	Diagonal Matrix with Variances			
Optimization Algorithm	TAO POUNDerS			
Quality of Fit Measures				
$\Lambda(\hat{\psi})$	218.32	277.80	447.60	632.07
$\Lambda(\psi^*)$	232.66	291.56	471.95	215.12

Table: Robustness of Economic Implications

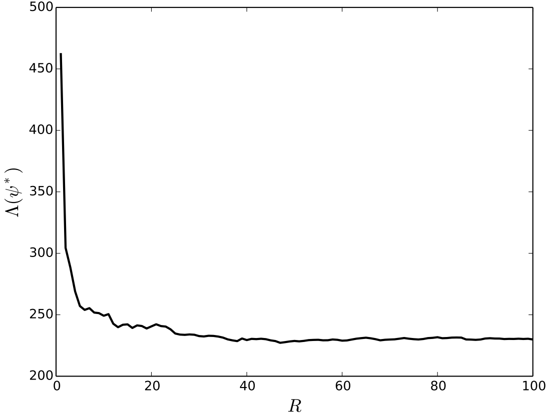
		Dynamic (Panel) Moments		
State	True	Base	Alt. A	Alt. B
Gross Return				
High School Graduation	30%	42%	37%	37%
Early College Graduation	88%	96%	72%	73%
Late College Graduation	29%	36%	18%	18%
Net Return				
High School Graduation	63%	212%	203%	194%
Early College Graduation	57%	125%	107%	112%
Late College Graduation	14%	38%	30%	35%

Table: Robustness of Economic Implications

		Cross Section Moments
State	True	Base
	Gross Return	
High School Graduation	30%	16%
Early College Graduation	88%	57%
Late College Graduation	29%	16%
	Net Return	
High School Graduation	63%	215%
Early College Graduation	57%	79%
Late College Graduation	14%	26%

Replications

Figure: Role of Replications



Optimization Algorithm

TAO POUNDerS

Practical Optimization Using No Derivatives for Sums of Squares

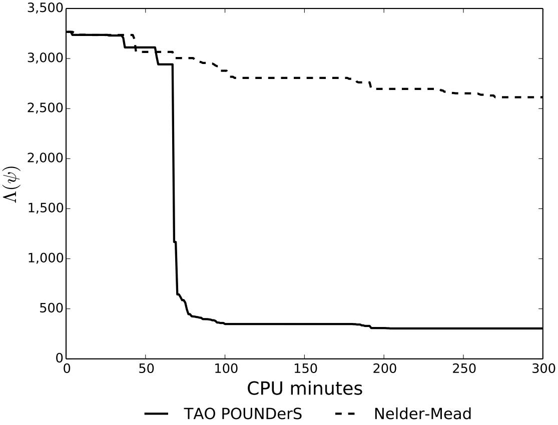
Nonlinear Least-Squares

$$\Lambda(\psi) = \sum_{i=1}^{I} \bar{f}_i(\psi)^2 = \sum_{i=1}^{I} \left(\frac{\check{f}_i - \hat{f}_i(\psi)}{\hat{\sigma}_i} \right)^2$$

Derivative-Free Trust-Region Algorithm

$$\min\{m_k(\psi) : ||\psi - \psi_k||_\infty \leq \Delta_k\}$$

Figure: Optimization Algorithms



Quality Assurance

Transparency, Recomputability, and Extensibility

http://www.policy-lab.org/sim-methods

- Documentation
  - Source Code
  - Test Suite
- Download

Conclusion

Philipp Eisenhauer

Mail	eisenhauer@policy-lab.org
Web	`http://www.policy-lab.org/peisenha`
Repository	`https://github.com/peisenha`

Project	`http://www.policy-lab.org/sim-methods`

Appendix

References