

Economics of Human Capital

Introduction

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Human capital is defined as:

The knowledge, skills, competencies and attributes embodied in individuals that facilitate the creation of personal, social and economic well-being.

- OECD (2001)

Tasks

- ▶ definition and measurement of human capital
- ▶ determining the effect of human capital on a variety of personal, social and economic outcomes
- ▶ understanding the formation of human capital

Tasks

- ▶ identifying the driving forces behind the observed heterogeneity across and within countries
- ▶ search for effective policies to ameliorate disparities
- ▶ ...

Facts

Figure: Years of schooling

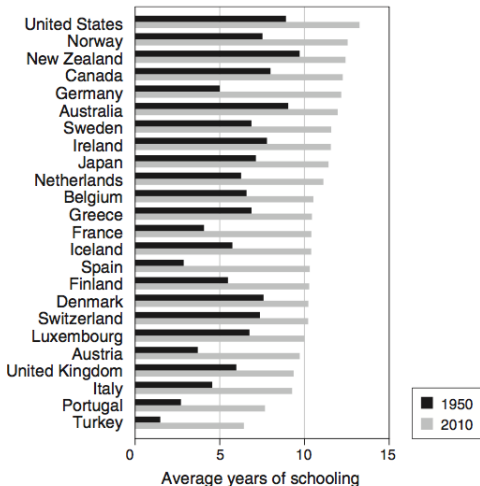


FIGURE 4.4

Years of schooling of the total population aged 25 and older.

Source: Barro and Lee (2010, education data set, available at www.barrolee.com/data).

Figure: Unemployment rates

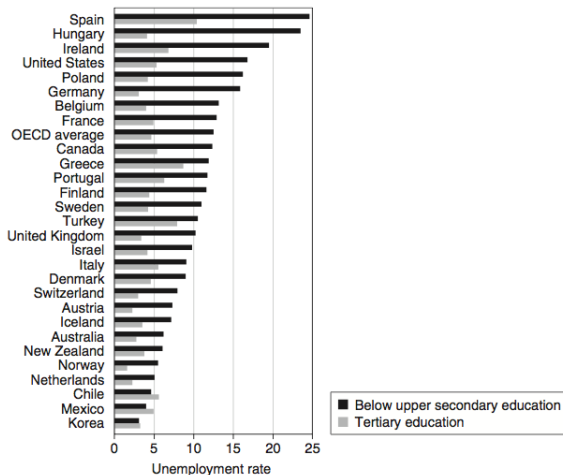


FIGURE 4.6

Unemployment rates by level of educational attainment for 25- to 64-year-olds, 2010. The OECD average is the nonweighted average of the 34 OECD countries, including those not represented on this figure. Data missing for non-OECD countries.

Source: OECD (2012, table A7.4a, p. 133).

Figure: Tertiary education

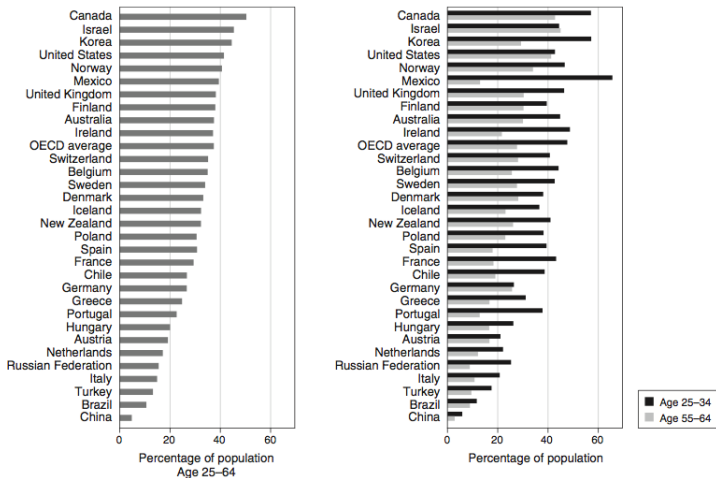


FIGURE 4.3

Percentage of the population that has attained at least tertiary education or advanced research programs, by age group, 2010. The OECD average is the nonweighted average of the 34 OECD countries, including those not represented in this figure. Brazil, China, and the Russian Federation are not part of the OECD.

Source: OECD (2012, table A1.3a, p. 36).

Figure: Secondary education



FIGURE 4.2

Percentage of the population that has attained at least upper secondary education, by age group, 2010. The OECD average is the nonweighted average of the 34 OECD countries, including those not represented in this figure. Brazil, China, and the Russian Federation are not part of the OECD.

Source: OECD (2012, table A1.2a, p. 35).

Figure: Expenditures



FIGURE 4.1

Expenditure on educational institutions as a percentage of GDP, 2009. The OECD average is the nonweighted average of the 34 OECD countries, including those not represented in this figure. Brazil and the Russian Federation are not part of the OECD. Private expenditure is missing for Brazil, Hungary, Norway, and Switzerland. Data are missing for China, Greece and Turkey.

Source: OECD (2012, table B2.3, p. 246).

Figure: Relative earnings



FIGURE 4.5

Relative earnings from employment among 25- to 64-year-olds, by level of educational attainment (2010 or latest available year). Upper secondary and post-secondary nontertiary education = 100. The OECD average is the nonweighted average of the 34 OECD countries, including those not represented in this figure. Brazil is not part of the OECD. Data missing for Chile, China, Iceland, Mexico, and the Russian Federation.

Source: OECD (2012, chart A8.1, p. 140).

Figure: Unemployment rates



FIGURE 4.7

Unemployment rates by level of educational attainment for 25- to 64-year-olds, 2010. The OECD average is the nonweighted average of the 34 OECD countries.

Source: OECD (2012, table A7.4a, p. 133).

Economic models

All models are wrong, but some are useful.

- Box (1987)

Economic models facilitate experiential learning.

- ▶ What question are they designed to address?
- ▶ What are the underlying economic mechanisms?
- ▶ How robust are the conclusions?
- ▶ What is missing?
- ▶ ...

Figure: Modeling process

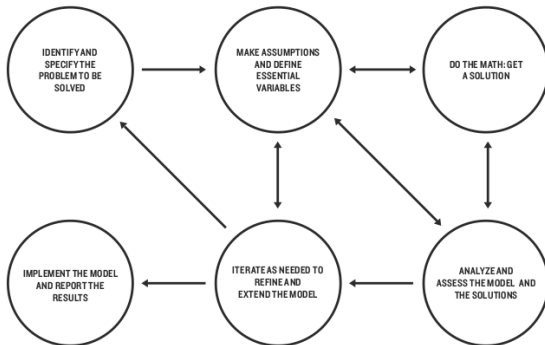


FIGURE 1.2: THE MATH MODELING PROCESS

Famous examples

- ▶ **Lemons model (Akerlof, 1970)**, market unraveling in presence of asymmetric information
- ▶ **Roy model (Roy, 1951)**, static model of self-selection and comparative advantage
- ▶ **Career decisions model (Keane & Wolpin, 1997)**, dynamic model human capital investment with schooling and on-the-job training
- ▶ ...

Life-cycle of earnings

Stylized Facts

- ▶ Life-cycle earnings are increasing at early ages and decline towards the end.
- ▶ Wages tend to increase over the life-cycle with a weak tendency to decline at the end of working life.
- ▶ Hours of work increase at early ages and decline in old age, with the peak occurring earlier than in the wage profiles.

See Weiss (1986) for comprehensive modeling framework that allows to interpret all these facts.

Figure: Wage gains



FIGURE 4.8

Average wage gains for college and high school graduates in the United States in 1996.

Source: Ashenfelter and Rouse (1999).

We study a version of the seminal Ben-Porath Model (Ben-Porath, 1967) that relates human capital accumulation to life-cycle earnings.

Basic Notation

$s(t)$ fraction devoted to training

$h(t)$ stock of human capital

$w(t)$ wage

δ depreciation of knowledge

The individual's objective is to maximize the discounted sum of wages over their life-cycle.

$$\Omega = \int_0^T w(t) e^{-rt} dt$$

Their economic environment is characterized by the production functions for wages and human capital.

$$w(t) = A[1 - s(t)]h(t)dt$$

$$\dot{h} = \theta g(s(t)h(t)) - \delta h(t) \quad g' > 0, g'' < 0$$

Notable Features

- ▶ Individuals cannot work and learn at the same time.
- ▶ There is no individual heterogeneity.
- ▶ There is no direct cost of education but there are the opportunity cost of lost wages.
- ▶ ...

Model Specification

We study the implementation in Cahuc and Zylberberg (2004).

$$g(h(t), s(t)) = (h(t)s(t))^{0.71}$$

$$A = 0.75 \quad \delta = 0.06 \quad r = 0.05$$

$$h_0 = 5 \quad T = 60 \quad \theta = 0.5$$

Figure: Contour plot of human capital production function

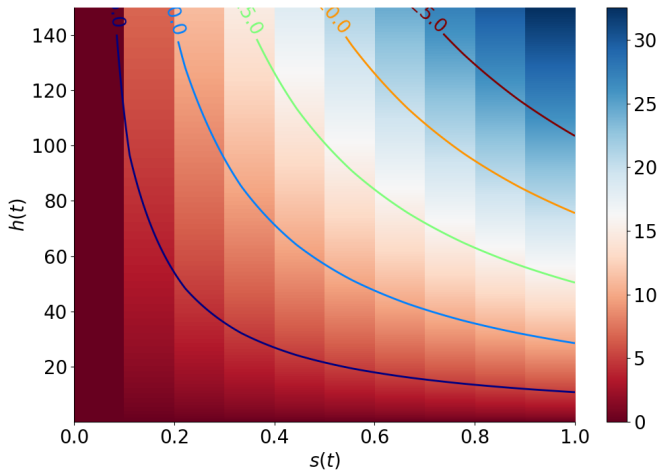


Figure: Surface plot of human capital production function

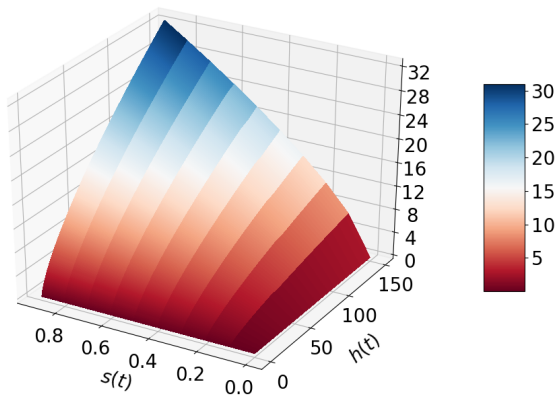


Figure: Wage production

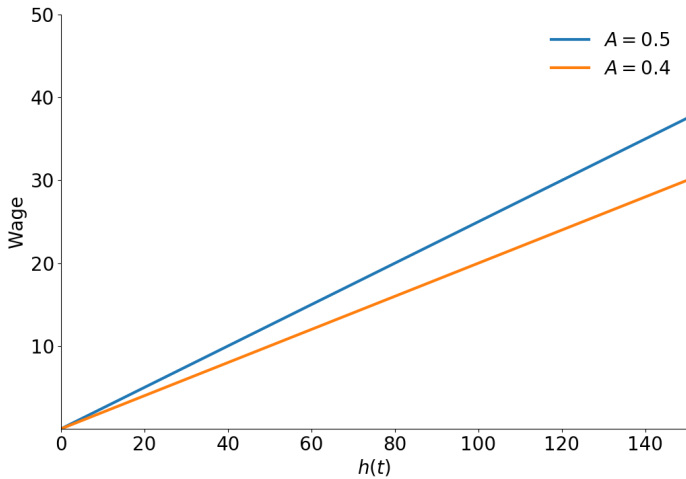


Figure: Wage over the life-cycle

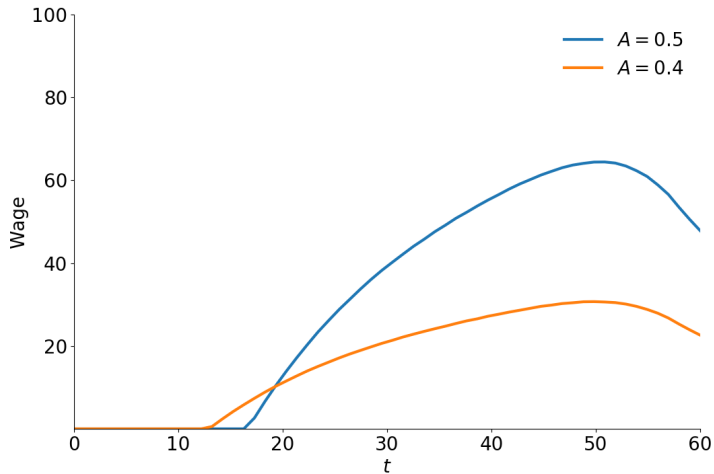


Figure: Stock of human capital over the life-cycle

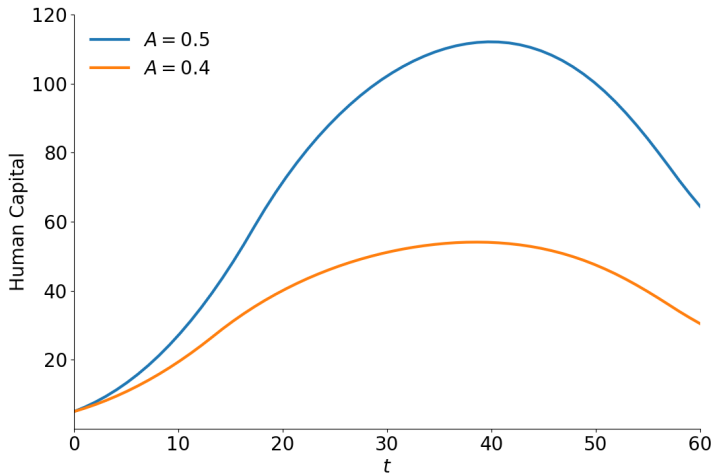
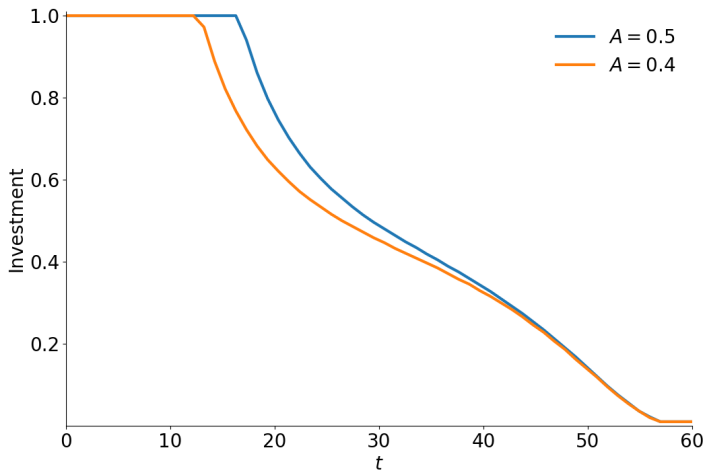


Figure: Human capital investment over the life-cycle



Extensions

Weiss (1986) reviews a host of alternative extensions to the basic model.

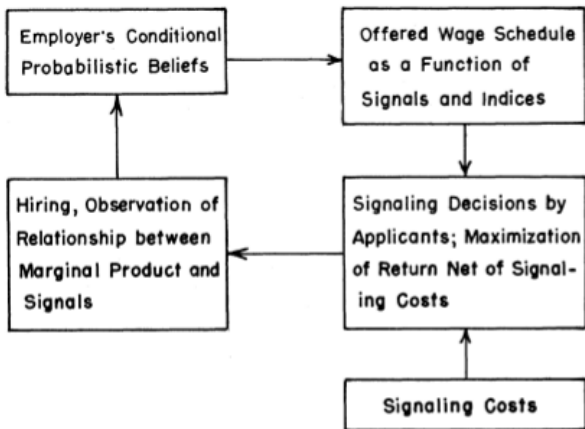
- ▶ general versus specific training
- ▶ hours worked
- ▶ uncertainty
- ▶ borrowing-constraints
- ▶ ...

Job market signaling

We study the seminal model presented in Spence (1973).

- ▶ There are two groups $j \in \{H, L\}$ in the population facing one employer, where $h_{i \in \{L, H\}}$ denotes the respective level of productivity.
- ▶ Group H is a proportion q_H in the population.
- ▶ Education y is measured by an index y of level and achievement and is subject to individual choice.
- ▶ Education costs are both monetary and psychic and differ by group $c_{i \in \{L, H\}}$.

Figure: Informational feedback



We explore the following parameterized version.

$$h_L = 1 \quad h_H = 2$$

$$c_L = y \quad c_H = \frac{1}{2}y$$

Figure: Benefit of education

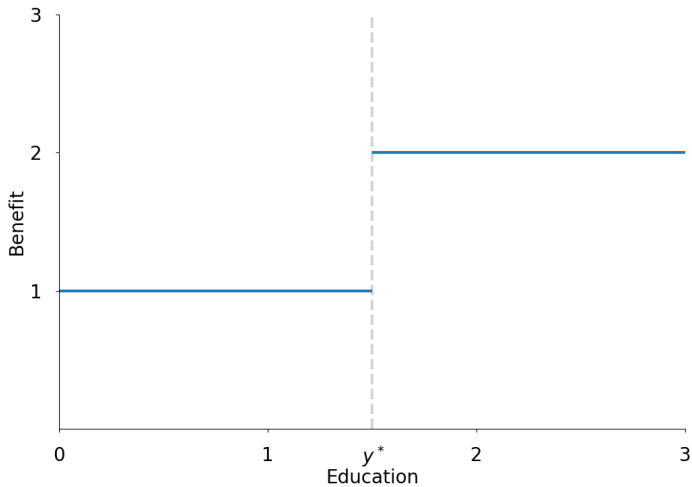


Figure: Cost of education

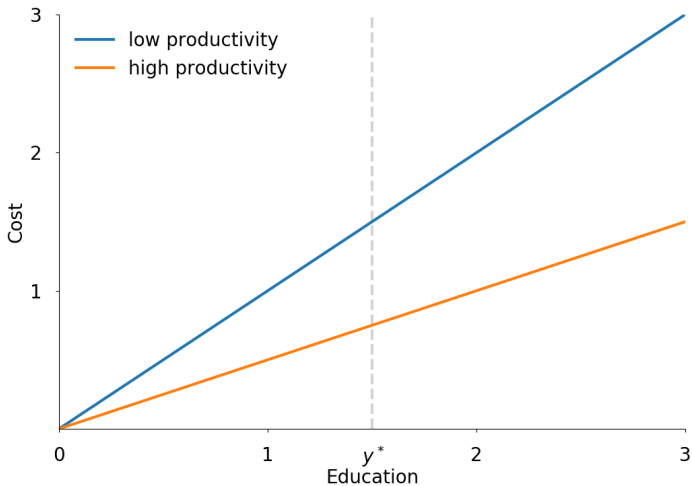
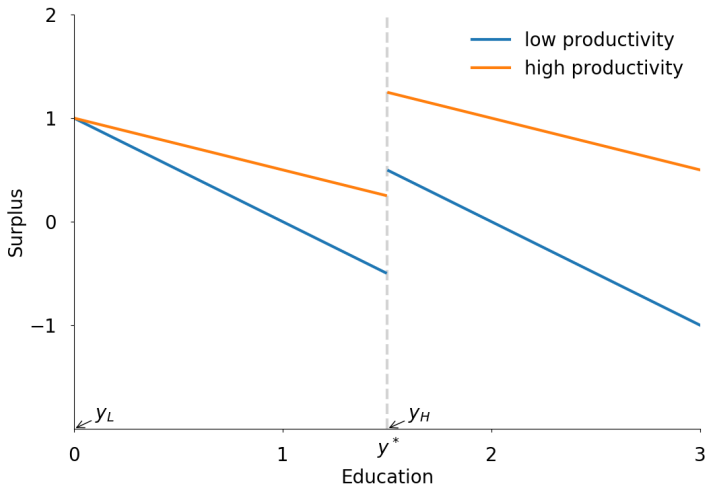


Figure: Surplus of education I



- ▶ For $y^* = 1.5$ the employer's beliefs are confirmed. More generally, L chooses $y_L = 0$ if $1 > 2 - y^*$ and H acquires $y_H = y^*$ provided that $2 - 0.5y^* > 1$.
- ▶ Beliefs are confirmed provided that the following holds:

$$1 < y^* < 2$$

Figure: Surplus of education II

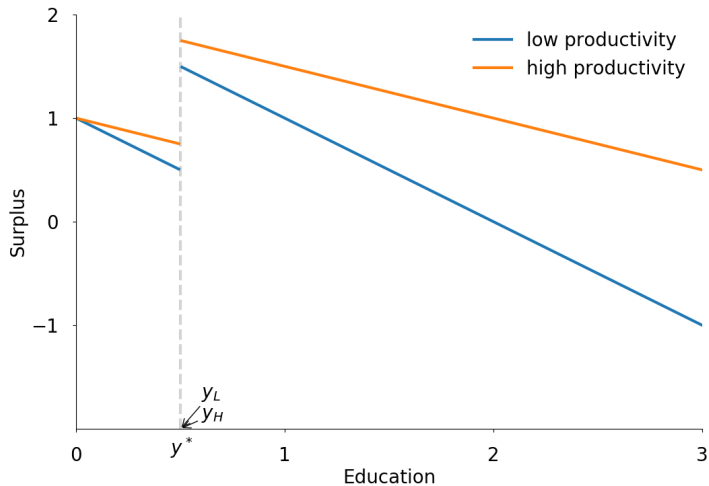
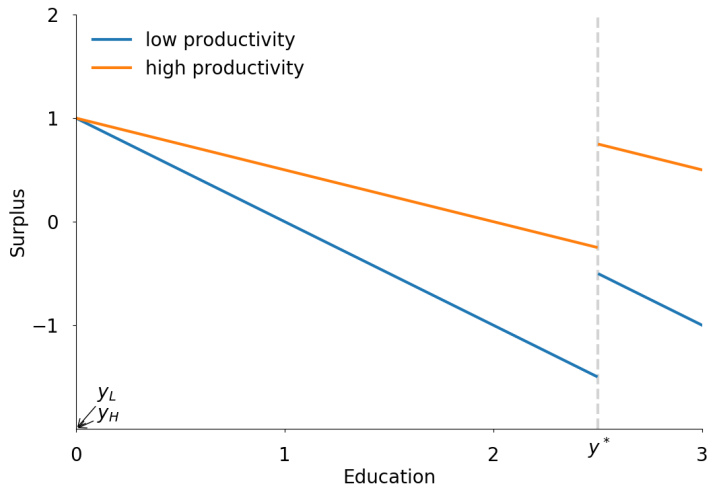


Figure: Surplus of education III



- ▶ From the outside, education appears to be productive and is for the individual. However, there is no real effect on the marginal product.

- ▶ In the absence of signaling, both groups are paid the unconditional expected marginal product.

$$q_L \times 1 + (1 - q_L) \times 2$$

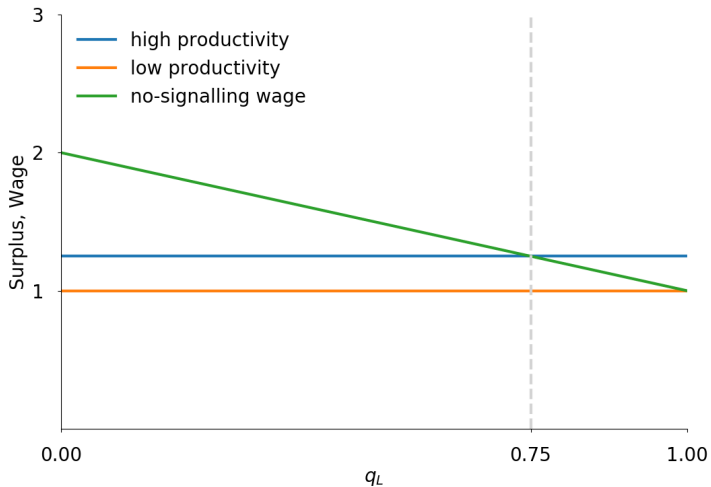
- ▶ It depends on the share of low ability individuals whether high ability individuals actually prefer a no-signaling case. The surplus is determined as follows:

$$\text{signaling} \quad 2 - \frac{1}{2}y^*$$

$$\text{no-signaling} \quad 2 - q_L$$

- ▶ High ability individual prefer the signaling case as long as $y^* \geq 2q_L$.

Figure: Market structure



- ▶ The ability to signal has a detrimental effect on low ability workers, while the consequences are ambiguous for high ability workers.
- ▶ High ability workers benefit from their ability to send a signal if their proportion is sufficiently small with respect to the ability gap to low ability individuals.

Appendix

References

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- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy*, 75(4), 352–365.
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Economics of Human Capital

Returns to schooling

Philipp Eisenhauer


Housekeeping

Course Website

You find all information about the course on our website.

<https://github.com/HumanCapitalEconomics>

This includes the lecture dates, topics, reading list, and the slides.

If you have further questions, please feel free to contact us using .

?

Multidimensionality of skills

- ▶ Heckman, J. J., Stixrud, J., & Urzua, S. (2006). The effects of cognitive and noncognitive abilities on labor market outcomes and social behavior. *Journal of Labor Economics*, 24(3), 411–482.
- ▶ Eisenhauer, P., Heckman, J. J., & Mosso, S. (2015). Estimation of dynamic discrete choice models by maximum likelihood and the simulated method of moments. *International Economic Review*, 56(2), 331–357.

Introduction

I heavily draw on the material presented in:

- ▶ Heckman, J. J., Lochner, L. J., & Todd, P. E. (2006). Earnings functions, rates of return and treatment effects: The Mincer equation and beyond. In E. A. Hanushek & F. Welch (Eds.), *Handbook of the economics of education* (1st ed., Vol. 1, pp. 307–458). Amsterdam, Netherlands: North-Holland Publishing Company.

Importance of returns

- ▶ explain wage inequality within countries
- ▶ explain growth differentials across countries
- ▶ assess schooling investment on individual level
- ▶ evaluate public policies to foster educational attainment
- ▶ ...

Return concepts

- ▶ Mincer returns
- ▶ internal rate of return
- ▶ true rate of return

Mathematical models

Lecture

All models are designed to shed light on the process of schooling decisions.

- ▶ Compensating differences
- ▶ Accounting-identity
- ▶ Option value

Economic models facilitate experiential learning.

- ▶ What question are they designed to address?
- ▶ What are the underlying economic mechanisms?
- ▶ How robust are the conclusions?
- ▶ What is missing?
- ▶ ...

Figure: Modeling process

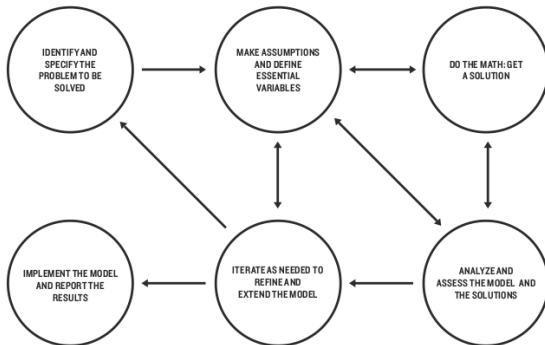


FIGURE 1.2: THE MATH MODELING PROCESS

Mincer returns

Mincer Equation

$$\ln Y(s, x) = \alpha + \rho_s s + \beta_0 x + \beta_1 x^2 + \epsilon$$

⇒ How to interpret the *Mincer Coefficient* ρ_s ?

Conceptual Frameworks

- ▶ compensating differences (Mincer, 1958)
- ▶ accounting-identity (Mincer, 1974)

Compensating Differences Model

Let $Y(s)$ represent the annual earnings of an individual with s years of education, assumed to be constant over his lifetime. Let r be an externally determined interest rate and T the length of working life, assumed not to depend on s . The present value of earnings associated with schooling level s is

$$V(s) = Y(s) \int_s^T e^{-rt} dt = \frac{Y(s)}{r} (e^{-rs} - e^{-rT}).$$

Figure: Earnings

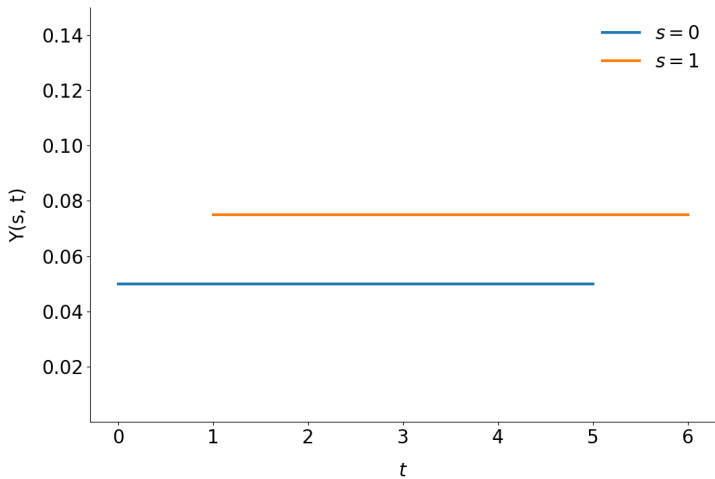
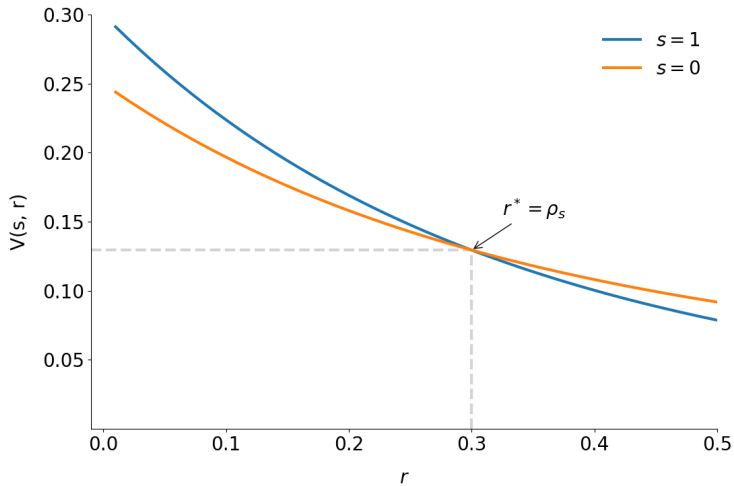


Figure: Value



Equilibrium across heterogeneous schooling levels requires that individuals be indifferent between schooling choices, with allocations being driven by demand conditions. Equating earnings streams across schooling levels and taking logs yields

$$\ln Y(s) = \ln Y(0) + rs + \ln \left(\frac{1 - e^{-rs}}{1 - e^{-r(T-s)}} \right).$$

$\Rightarrow \rho_s$ equals the market interest rate and the internal rate of return to schooling by construction.

Model features

- ▶ identical abilities and opportunities
- ▶ no credit constraints
- ▶ perfect certainty
- ▶ no direct cost of schooling
- ▶ no nonpecuniary benefits of school and work

Accounting-Identity Model

Model ingredients

P_t potential earnings at t

$C_t = k_t P_t$ investment cost of training at t

ρ_t average return to investment at t

$$P_t \equiv P_{t-1}(1 + k_{t-1}\rho_{t-1}) \equiv \prod_{j=0}^{t-1} (1 + \rho_j k_j) P_0$$

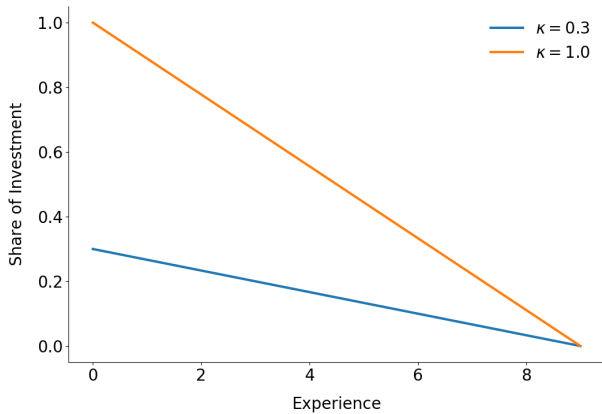
Formal schooling is defined as years spent in full-time investment ($k_t = 1$), which is assumed to take place at the beginning of life and to yield a rate of return ρ_s that is constant across all years of schooling.

$$\begin{aligned}\ln P_t &\equiv \ln P_0 + s \ln(1 + \rho_s) + \sum_{j=s}^{t-1} \ln(1 + \rho_0 k_j) \\ &\approx \ln P_0 + s \rho_s + \rho_0 \sum_{j=s}^{t-1} k_j\end{aligned}$$

Mincer (1974) assumes a linearly declining rate of post-school investment:

$$k_{s+x} = \kappa (1 - x/T), \text{ where } x = t - s$$

Figure: Post-School Investment



The derivations draws on the following results for arithmetic series.

$$\sum_{i=0}^n i = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\ln P_{x+s} \approx \ln P_0 + s\rho_s + \underbrace{\left(\rho_0 K + \frac{\rho_0 K}{2T} \right) x - \frac{\rho_0 K}{2T} x^2}_{(1)}$$

You can derive (1) using the previous results.

Accounting for the difference in potential and observed earnings:

$$\begin{aligned}\ln Y(s, x) &\approx \ln P_{x+s} - \kappa (1 - x/T) \\ &= [\ln P_0 - \kappa] + \rho_s s + \left(\rho_0 \kappa + \frac{\rho_0 \kappa}{2T} + \frac{\kappa}{T} \right) x - \frac{\rho_0 \kappa}{2T} x^2\end{aligned}$$

$\Rightarrow \rho_s$ is the average earnings increase with schooling

Standard Mincer Equation

$$\ln Y(s, x) = \alpha + \rho_s s + \beta_0 x + \beta_1 x^2,$$

where

$$\alpha = \ln P_0 - \kappa$$

$$\beta_0 = \left(\rho_0 \kappa + \frac{\rho_0 \kappa}{2T} + \frac{\kappa}{T} \right)$$

$$\beta_1 = -\frac{\rho_0 \kappa}{2T}$$

What about heterogeneous returns?

Random Coefficient Version

$$\ln Y(s_i, x_i) = \alpha_i + \rho_{si}s_i + \beta_{0i}x_i + \beta_{1i}x_i^2$$

and let

$$\begin{aligned}\bar{\alpha} &= E[\alpha_i] & \bar{\rho}_s &= E[\rho_{si}] \\ \bar{\beta}_0 &= E[\beta_{0i}] & \bar{\beta}_1 &= E[\beta_{1i}]\end{aligned}$$

Dropping individual subscripts ...

$$\ln Y(s, x) = \bar{\alpha} + \bar{\rho}_s s + \bar{\beta}_0 x + \bar{\beta}_1 x^2 \\ + \underbrace{[(\alpha - \bar{\alpha}) + (\rho_s - \bar{\rho}_s)s + (\beta_0 - \bar{\beta}_0)x + (\beta_1 - \bar{\beta}_1)x^2]}_{\epsilon}$$

⇒ If the schooling decision is determined by individual returns, then we are back in the case of a correlated random coefficient model (Heckman, Urzua, & Vytlačil, 2006).

Table 2: Estimated Coefficients from Mincer Log Earnings Regression for Men

		Whites		Blacks	
		Coefficient	Std. Error	Coefficient	Std. Error
1940	Intercept	4.4771	0.0096	4.6711	0.0298
	Education	0.1250	0.0007	0.0871	0.0022
	Experience	0.0904	0.0005	0.0646	0.0018
	Experience-Squared	-0.0013	0.0000	-0.0009	0.0000
1950	Intercept	5.3120	0.0132	5.0716	0.0409
	Education	0.1058	0.0009	0.0998	0.0030
	Experience	0.1074	0.0006	0.0933	0.0023
	Experience-Squared	-0.0017	0.0000	-0.0014	0.0000
1960	Intercept	5.6478	0.0066	5.4107	0.0220
	Education	0.1152	0.0005	0.1034	0.0016
	Experience	0.1156	0.0003	0.1035	0.0011
	Experience-Squared	-0.0018	0.0000	-0.0016	0.0000
1970	Intercept	5.9113	0.0045	5.8938	0.0155
	Education	0.1179	0.0003	0.1100	0.0012
	Experience	0.1323	0.0002	0.1074	0.0007
	Experience-Squared	-0.0022	0.0000	-0.0016	0.0000
1980	Intercept	6.8913	0.0030	6.4448	0.0120
	Education	0.1023	0.0002	0.1176	0.0009
	Experience	0.1255	0.0001	0.1075	0.0005
	Experience-Squared	-0.0022	0.0000	-0.0016	0.0000
1990	Intercept	6.8912	0.0034	6.3474	0.0144
	Education	0.1292	0.0002	0.1524	0.0011
	Experience	0.1301	0.0001	0.1109	0.0006
	Experience-Squared	-0.0023	0.0000	-0.0017	0.0000

Notes: Data taken from 1940-90 Decennial Censuses. See Appendix B for data description.

We can analyze this model in a Jupyter Notebook. Visit

<http://bit.ly/2IVfMHA>

for the implementation.

Implications

- ▶ Log-earnings profiles are parallel across schooling levels.

$$\frac{\partial \ln Y(s, x)}{\partial s \partial x} = 0$$

- ▶ Log-earnings age profiles diverge with age across schooling levels.

$$\frac{\partial \ln Y(s, x)}{\partial s \partial t} = \frac{\rho_0 K}{T} > 0$$

Figure: Experience profiles

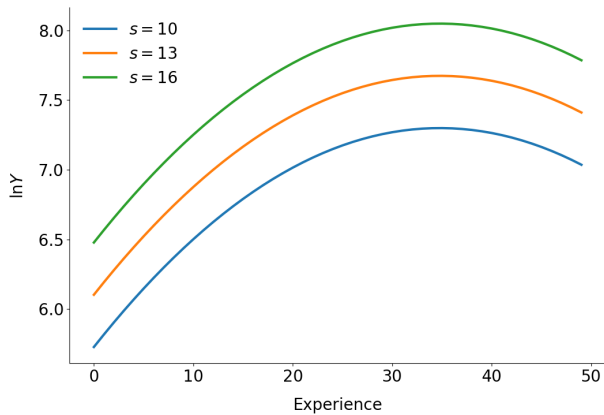
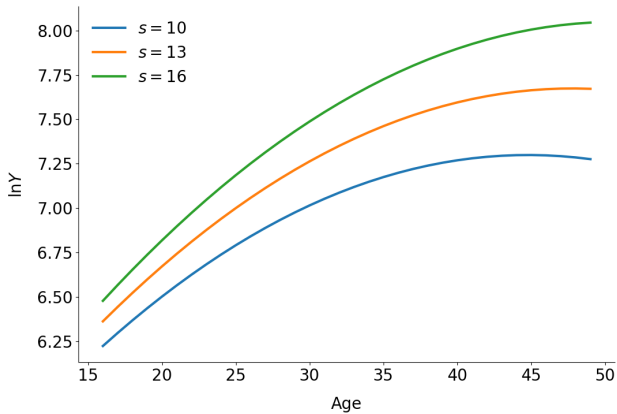


Figure: Age profiles



- ▶ The variance of earnings over the life cycle has a U-shaped pattern.

Derivation for minimizing variance

$$\begin{aligned}\ln Y(s, x) &= \ln P_{s+x} + \ln(1 - k_{s+x}) \\ &\approx \ln P_s + \rho_0 \sum_{j=0}^{x-1} k_{s+j} - k_{s+x}\end{aligned}$$

Further, using the assumption of linearly declining investment yields

$$\ln Y(s, x) \approx \ln P_s + \kappa \left(\rho_0 \sum_{j=0}^{x-1} (1 - j/T) - (1 - x/T) \right)$$

Assuming only initial earnings potential (P_s) and investment levels (κ) vary in the population, the variance of log earnings is given by

$$\begin{aligned} \text{Var}(\ln Y(s, x)) &= \text{Var}(\ln P_s) \\ &+ \left(\rho_0 \sum_{j=0}^{x-1} (1 - j/T) - (1 - x/T) \right)^2 \text{Var}(\kappa) \\ &+ 2 \left(\rho_0 \sum_{j=0}^{x-1} (1 - j/T) - (1 - x/T) \right) \text{COV}(\ln P_s, k). \end{aligned}$$

If κ and $\ln P_s$ are uncorrelated, then earnings are minimized (and equal to $\text{Var}(\ln P_s)$) when

$$\rho_0 \sum_{j=0}^{x-1} (1 - j/T) = 1 - x/T, \text{ or}$$

$$\rho_0 \left(x - \frac{x(x-1)}{2T} \right) = (1 - x/T).$$

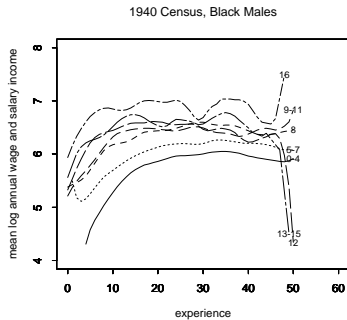
Clearly, $\lim_{T \rightarrow \infty} x^* = \frac{1}{\rho_0}$, so the variance minimizing age is $\frac{1}{\rho_0}$ when the work-life is long.

Figure: Variance profiles

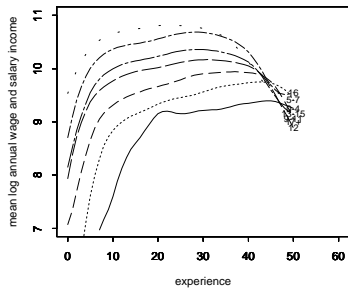


Empirical Evidence

Figure 1a: Experience-Earnings Profiles, 1940-1960



1990 Census, White Males



1990 Census, Black Males

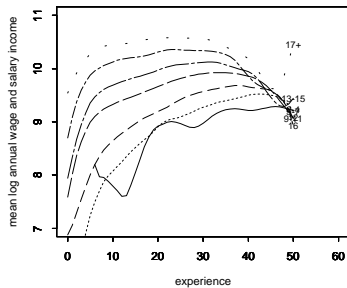
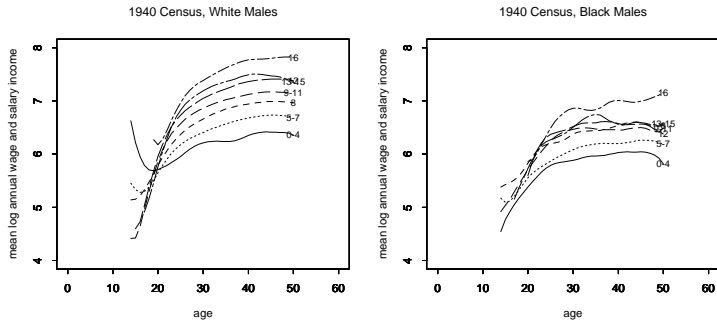


Table 1: Tests of Parallelism in Log Earnings Experience Profiles for Men

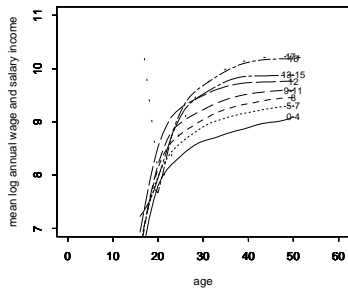
Sample	Experience Level	Estimated Difference Between College and High School Log Earnings at Different Experience Levels					
		1940	1950	1960	1970	1980	1990
Whites	10	0.54	0.30	0.46	0.41	0.37	0.59
	20	0.40	0.40	0.43	0.49	0.45	0.54
	30	0.54	0.27	0.46	0.48	0.43	0.52
	40	0.58	0.21	0.50	0.45	0.27	0.30
	p-value	0.32	0.70	<0.001	<0.001	<0.001	<0.001
Blacks	10	0.20	0.58	0.48	0.38	0.70	0.77
	20	0.38	0.05	0.25	0.22	0.48	0.69
	30	-0.11	0.24	0.08	0.33	0.36	0.53
	40	-0.20	0.00	0.73	0.26	0.22	-0.04
	p-value	0.46	0.55	0.58	0.91	<0.001	<0.001

Notes: Data taken from 1940-90 Decennial Censuses without adjustment for inflation. Because there are very few blacks in the 1940 and 1950 samples with college degrees, especially at higher experience levels, the test results for blacks in those years refer to a test of the difference between earnings for high school graduates and persons with 8 years of education. See Appendix B for data description. See Appendix C for the formulae used for the test statistics.

Figure 2: Age-Earnings Profiles, 1940,1960,1980



1980 Census, White Males



1980 Census, Black Males

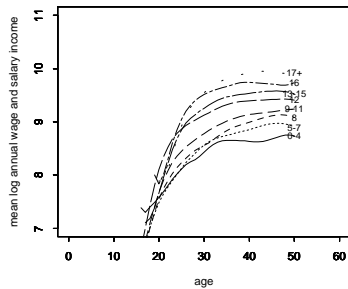
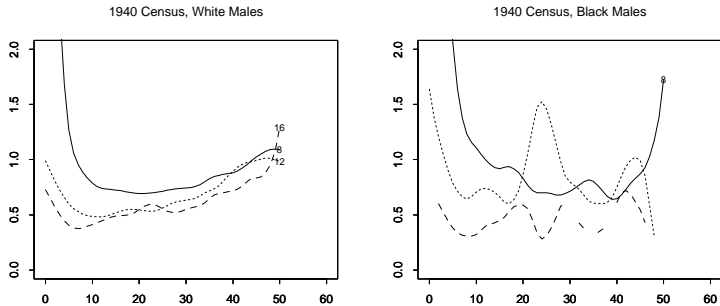
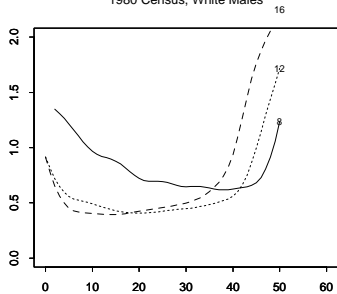


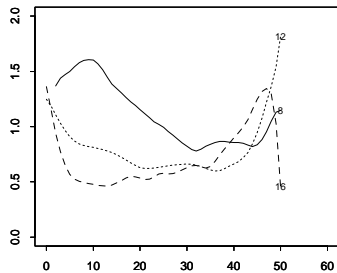
Figure 3: Experience-Variance Log Earnings



1980 Census, White Males



1980 Census, Black Males



In the end, (Heckman, Lochner, & Todd, 2006) conclude:

In common usage, the coefficient on schooling in a regression of log earnings on years of schooling is often called a rate of return. In fact, it is a price of schooling from a hedonic market wage equation. It is a growth rate of market earnings with years of schooling and not an internal rate of return measure, except under stringent conditions which we specify, test and reject in this chapter.

Table 2
Estimated coefficients from Mincer log earnings regression for men

		Whites		Blacks	
		Coefficient	Std. Error	Coefficient	Std. Error
1940	Intercept	4.4771	0.0096	4.6711	0.0298
	Education	0.1250	0.0007	0.0871	0.0022
	Experience	0.0904	0.0005	0.0646	0.0018
	Experience-squared	-0.0013	0.0000	-0.0009	0.0000
1950	Intercept	5.3120	0.0132	5.0716	0.0409
	Education	0.1058	0.0009	0.0998	0.0030
	Experience	0.1074	0.0006	0.0933	0.0023
	Experience-squared	-0.0017	0.0000	-0.0014	0.0000

Internal rate of returns

Income Maximization under Perfect Certainty

s	schooling level
x	experience level
$Y(s, x)$	wage income
$T(s)$	last age of earnings
v	tuition and psychic cost of schooling
τ	proportional tax rate
r	before-tax interest rate

Present Discounted Value of Lifetime Earnings

$$V(s) = \int_0^{T(s)-s} (1-\tau)e^{-(1-\tau)r(x+s)}Y(s, x)dx \\ - \int_0^s ve^{-(1-\tau)rz}dz$$

First-Order Condition

$$\begin{aligned} & [T'(s) - 1]e^{-(1-\tau)r(T(s)-s)}Y(s, T(s) - s) \\ & - (1 - \tau)r \int_0^{T(s)-s} e^{-(1-\tau)rx} Y(s, x) dx \\ & + \int_0^{T(s)-s} e^{-(1-\tau)rx} \frac{\partial Y(s, x)}{\partial s} dx \\ & - \frac{v}{1 - \tau} = 0 \end{aligned}$$

Rearranging and defining $\tilde{r} = (1 - \tau)r \dots$

$$\tilde{r} = \frac{[T'(s) - 1]e^{-\tilde{r}(T(s)-s)}Y(s, T(s) - s)}{\int_0^{T(s)-s} e^{-\tilde{r}x}Y(s, x)dx} \quad (1)$$

$$+ \frac{\int_0^{T(s)-s} e^{-\tilde{r}x} \left[\frac{\partial Y(s, x)}{\partial s} \right] dx}{\int_0^{T(s)-s} e^{-\tilde{r}x}Y(s, x)dx} \quad (2)$$

$$- \frac{\frac{v}{1-\tau}}{\int_0^{T(s)-s} e^{-\tilde{r}x}Y(s, x)dx} \quad (3)$$

Interpretation

- ▶ (1) ... the change in the present value of earnings due to a change in working-life with additional schooling
- ▶ (2) ... weighted average effect of schooling on log earnings by experience
- ▶ (3) ... tuition and psychic costs expressed as a fraction of lifetime income measured at age s

All components are expressed as a fraction of the present value of earnings measured at age s

Getting back to Mincer ...

- ▶ no tuition and psychic costs of schooling
 $\Rightarrow v = 0$
- ▶ no loss of working life from schooling
 $\Rightarrow T'(s) = 1$
- ▶ multiplicative separability between schooling and experience component of earnings
 $\Rightarrow Y(s, x) = \mu(s)\psi(x)$

$$\tilde{r} = \frac{\mu'(s)}{\mu(s)} \quad \forall \quad s$$

Thus, wage growth must be log linear in schooling and
 $\mu(s) = \mu(0)e^{\rho_s s}$

(Heckman, Lochner, & Todd, 2006) thus establish ...

After allowing for taxes, tuition, variable length of working life, and a flexible relationship between earnings, schooling and experience, the coefficient on years of schooling in a log earnings regression need no longer equal the internal rate of return.

Structural Approach for the IRR

The internal rate of return for schooling level s_1 versus s_2 , $r(s_1, s_2)$ solves ...

$$\begin{aligned} & \int_0^{T(s_1)-s_1} (1-\tau)e^{-r(x+s_1)}Y(s_1, x)dx - \int_0^{s_1} ve^{-rz}dz \\ &= \int_0^{T(s_2)-s_2} (1-\tau)e^{-r(x+s_2)}Y(s_2, x)dx - \int_0^{s_2} ve^{-rz}dz \end{aligned}$$

Back to Mincer

- ▶ no taxes and no direct or psychic costs of schooling

$$\Rightarrow v = 0 \text{ and } \tau = 0$$

$$\int_0^{T(s_1)-s_1} e^{-r(x+s_1)} Y(s_1, x) dx = \int_0^{T(s_2)-s_2} e^{-r(x+s_2)} Y(s_2, x) dx$$

- ▶ equal work-lives irrespective of years of schooling

$$\Rightarrow T = T(s_1) - s_1 = T(s_2) - s_2$$

$$\int_0^T e^{-r(x+s_1)} Y(s_1, x) dx = \int_0^T e^{-r(x+s_2)} Y(s_2, x) dx$$

- ▶ parallelism in experience across schooling categories

$$\Rightarrow Y(s, x) = \mu(s)\psi(x)$$

$$\int_0^T e^{-r(x+s_1)} \mu(s) \psi(x) dx = \int_0^T e^{-r(x+s_2)} \mu(s) \psi(x) dx$$

- ▶ linearity of log earnings in schooling

$$\Rightarrow \mu(s) = \mu(0)e^{\rho_s s}$$

$$\int_0^T e^{-r(x+s_1)} \mu(0) e^{\rho_s s_1} \psi(x) dx = \int_0^T e^{-r(x+s_2)} \mu(0) e^{\rho_s s_2} \psi(x) dx$$

After some further rearranging ...

$$e^{(\rho_s - r)S_1} = e^{(\rho_s - r)S_2}$$

$$\Rightarrow \rho_s = r$$

Empirical Evidence

Table 3a: Internal Rates of Return for White Men: Earnings Function Assumptions
(Specifications Assume Work Lives of 47 Years)

	Schooling Comparisons					
	6-8	8-10	10-12	12-14	12-16	14-16
1940						
Mincer Specification	13	13	13	13	13	13
Relax Linearity in S	16	14	15	10	15	21
Relax Linearity in S & Quad. in Exp.	16	14	17	10	15	20
Relax Lin. in S & Parallelism	12	14	24	11	18	26
1950						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	13	13	18	0	8	16
Relax Linearity in S & Quad. in Exp.	14	12	16	3	8	14
Relax Linearity in S & Parallelism	26	28	28	3	8	19
1960						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	9	7	22	6	13	21
Relax Linearity in S & Quad. in Exp.	10	9	17	8	12	17
Relax Linearity in S & Parallelism	23	29	33	7	13	25
1970						
Mincer Specification	13	13	13	13	13	13
Relax Linearity in S	2	3	30	6	13	20
Relax Linearity in S & Quad. in Exp.	5	7	20	10	13	17
Relax Linearity in S & Parallelism	17	29	33	7	13	24
1980						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	3	-11	36	5	11	18
Relax Linearity in S & Quad. in Exp.	4	-4	28	6	11	16
Relax Linearity in S & Parallelism	16	66	45	5	11	21
1990						
Mincer Specification	14	14	14	14	14	14
Relax Linearity in S	-7	-7	39	7	15	24
Relax Linearity in S & Quad. in Exp.	-3	-3	30	10	15	20

Table 3b: Internal Rates of Return for Black Men: Earnings Function Assumptions
(Specifications Assume Work Lives of 47 Years)

	Schooling Comparisons					
	6-8	8-10	10-12	12-14	12-16	14-16
1940						
Mincer Specification	9	9	9	9	9	9
Relax Linearity in S	18	7	5	3	11	18
Relax Linearity in S & Quad. in Exp.	18	8	6	2	10	19
Relax Linearity in S & Parallelism	11	0	10	5	12	20
1950						
Mincer Specification	10	10	10	10	10	10
Relax Linearity in S	16	14	18	-2	4	9
Relax Linearity in S & Quad. in Exp.	16	14	18	0	3	6
Relax Linearity in S & Parallelism	35	15	48	-3	6	34
1960						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	13	12	18	5	8	11
Relax Linearity in S & Quad. in Exp.	13	11	18	5	7	10
Relax Linearity in S & Parallelism	22	15	38	5	11	25
1970						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	5	11	30	7	10	14
Relax Linearity in S & Quad. in Exp.	6	11	24	10	11	12
Relax Linearity in S & Parallelism	15	27	44	9	14	23
1980						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	-4	1	35	10	15	19
Relax Linearity in S & Quad. in Exp.	-4	6	29	11	14	17
Relax Linearity in S & Parallelism	10	44	48	8	16	31
1990						
Mincer Specification	16	16	16	16	16	16
Relax Linearity in S	-5	-5	41	15	20	25
Relax Linearity in S & Quad. in Exp.	-3	-3	35	17	19	22

Table 4: Internal Rates of Return for White & Black Men: Accounting for Taxes and Tuition
(General Non-Parametric Specification Assuming Work Lives of 47 Years)

		Schooling Comparisons					
		Whites			Blacks		
		12-14	12-16	14-16	12-14	12-16	14-16
1940	No Taxes or Tuition	11	18	26	5	12	20
	Including Tuition Costs	9	15	21	4	10	16
	Including Tuition & Flat Taxes	8	15	21	4	9	16
	Including Tuition & Prog. Taxes	8	15	21	4	10	16
1950	No Taxes or Tuition	3	8	19	-3	6	34
	Including Tuition Costs	3	8	16	-3	5	25
	Including Tuition & Flat Taxes	3	8	16	-3	5	24
	Including Tuition & Prog. Taxes	3	7	15	-3	5	21
1960	No Taxes or Tuition	7	13	25	5	11	25
	Including Tuition Costs	6	11	21	5	9	18
	Including Tuition & Flat Taxes	6	11	20	4	8	17
	Including Tuition & Prog. Taxes	6	10	19	4	8	15
1970	No Taxes or Tuition	7	13	24	9	14	23
	Including Tuition Costs	6	12	20	7	12	18
	Including Tuition & Flat Taxes	6	11	20	7	11	17
	Including Tuition & Prog. Taxes	5	10	18	7	10	16
1980	No Taxes or Tuition	5	11	21	8	16	31
	Including Tuition Costs	4	10	18	7	13	24
	Including Tuition & Flat Taxes	4	9	17	6	12	21
	Including Tuition & Prog. Taxes	4	8	15	6	11	20
1990	No Taxes or Tuition	10	16	26	18	25	35
	Including Tuition Costs	9	14	20	14	18	25
	Including Tuition & Flat Taxes	8	13	19	13	17	22
	Including Tuition & Prog. Taxes	8	12	18	13	17	22

Notes: Data taken from 1940-90 Decennial Censuses. See discussion in text and Appendix B for a description of tuition and tax amounts.

Figure 4a: Average College Tuition Paid (in 2000 dollars)

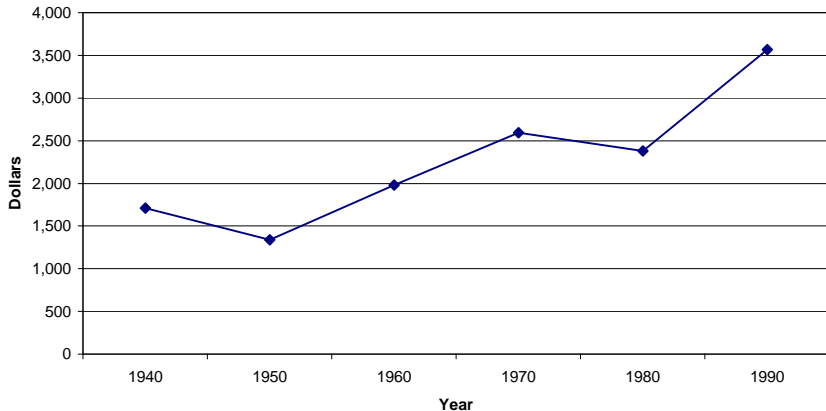


Figure 4b: Marginal Tax Rates
(from Barro & Sahasakul, 1983, Mulligan & Marion, 2000)

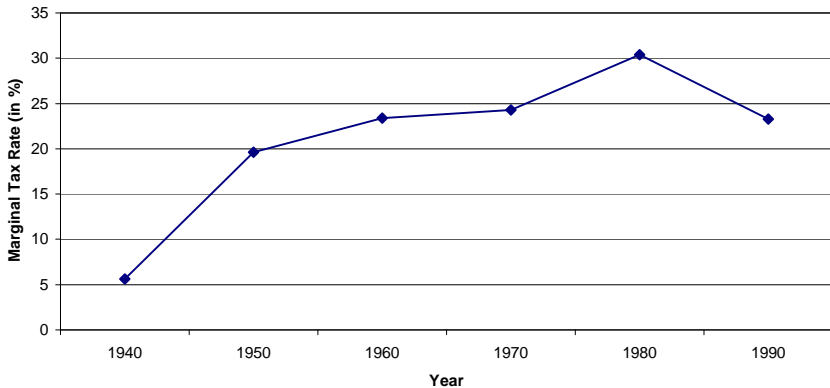


Figure 5: IRR for High School Completion (White and Black Men)

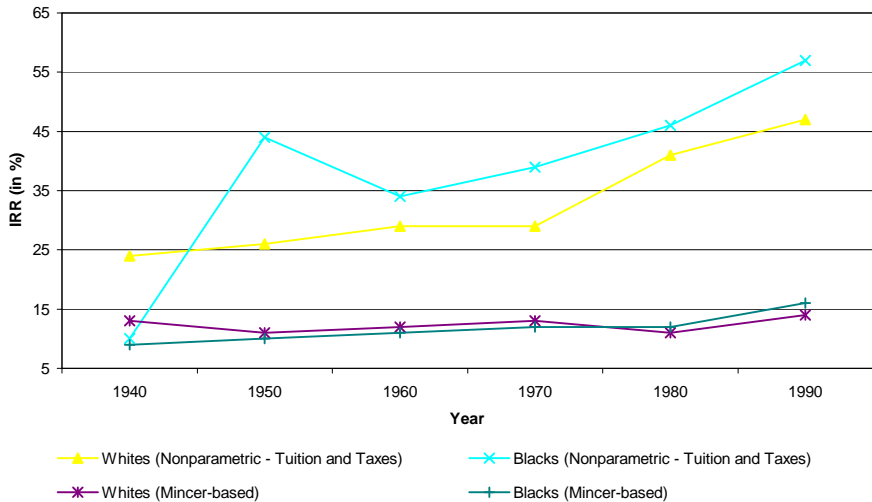
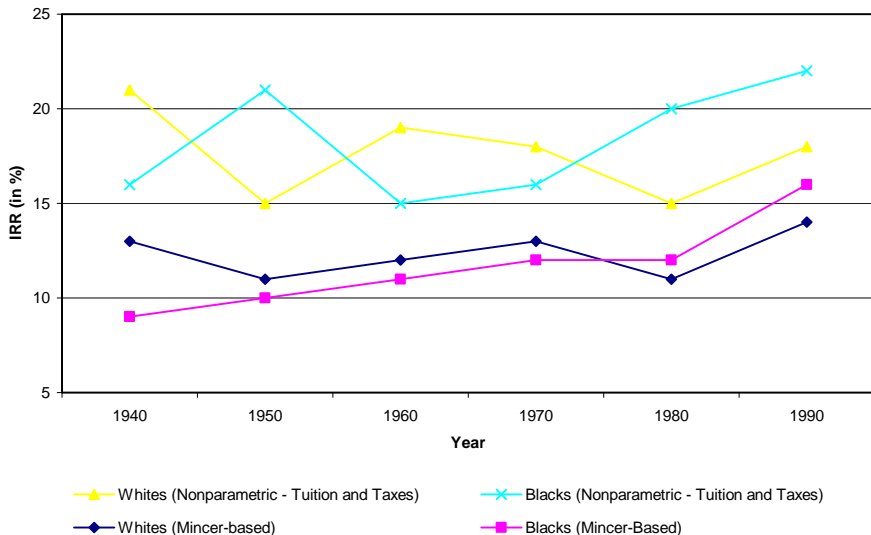


Figure 6: IRR for College Completion (White and Black Men)

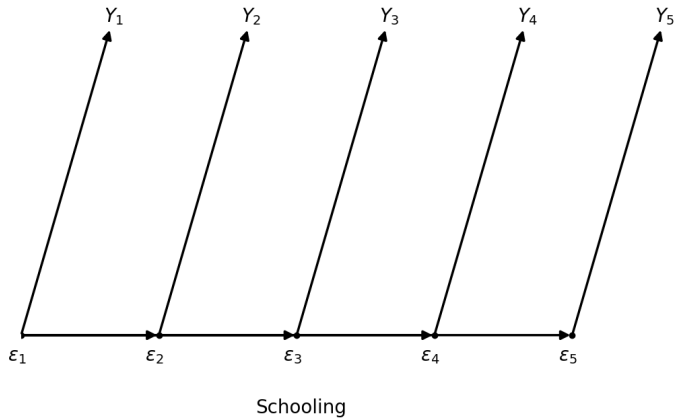


True rate of return

Suppose there is uncertainty about net earnings conditional on s and actual lifetime earnings for someone with s years of schooling are:

$$Y_s = \left[\sum_{x=0}^T (1+r)^{-1} Y(s, x) \right] \epsilon_s$$

Figure: Model structure



We define expected earnings associated with schooling s conditional on current schooling $s - 1$,

$$\bar{Y}_s = E_{s-1}(Y_s)$$

The decision problem for a person with s years of schooling given the sequential revelation of information is to complete another year of schooling if

$$Y_s \leq \frac{E_s(V_{s+1})}{1+r}.$$

So the value of schooling level s , V_s , is

$$V_s = \max \left\{ Y_s, \frac{E_s(V_{s+1})}{1+r} \right\}$$

for $s \leq \bar{S}$. At the maximum schooling level, \bar{S} , after all information is revealed, we obtain $V_{\bar{S}} = Y_{\bar{S}} = \bar{Y}_{\bar{S}} \epsilon_{\bar{S}}$.

The endogenously determined probability of going on from school level s to $s + 1$ is

$$p_{s+1,s} = \Pr\left(\epsilon_s \leq \frac{E_s(V_{s+1})}{(1+r)Y_s}\right),$$

where $E_s(V_{s+1})$ may depend on ϵ_s because it enters the agent's information set.

The average earnings of a person who stops at schooling level s are

$$\begin{aligned}
 E_{s-1}(V_s) = & \underbrace{(1 - p_{s+1,s})Y_s E_{s-1}\left(\epsilon \mid \epsilon \geq \frac{E_s(V_{s+1})}{(1+r)Y_s}\right)}_{\text{direct return}} \\
 & + \underbrace{p_{s+1,s}\left(\frac{E_{s-1}(V_{s+1})}{1+r}\right)}_{\text{option value}}.
 \end{aligned}$$

Objects of interest

- ▶ Option value

$$O_{s,s-1} = E_{s-1} [V_s - Y_s]$$

- ▶ True rate of return

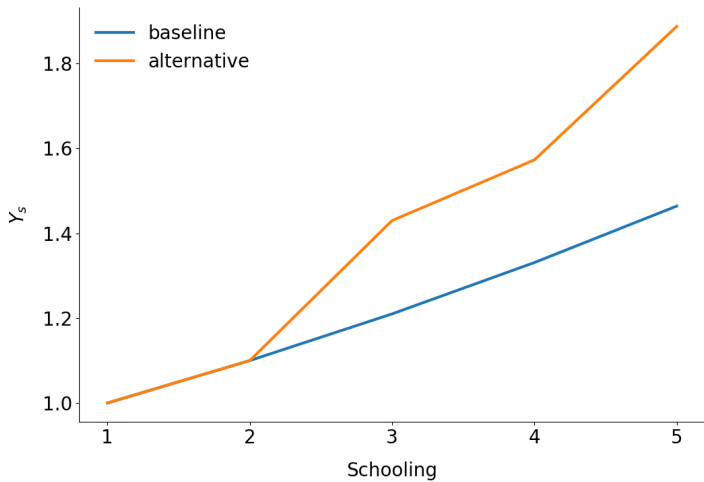
$$R_{s,s-1} = \frac{E_{s-1} [V_s] - Y_{s-1}}{Y_{s-1}}$$

Model specification

$$\log(\epsilon_s) \sim \mathbb{N}(0, \sigma) \quad r = 0.1$$

$$Y_{s+1} = (1 + \rho_{s+1})Y_s \quad \sigma = 0.1$$

Figure: Scenarios



We can analyze this model in a Jupyter Notebook. Visit

<http://bit.ly/2smCj9p>

for the implementation.

Figure: Option values and uncertainty

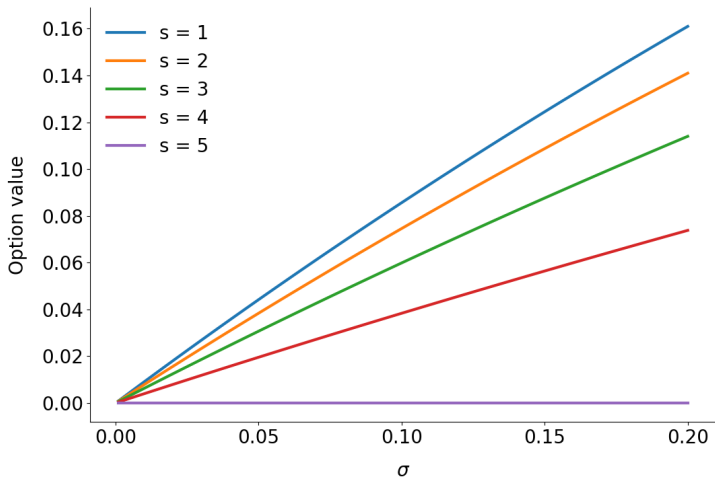


Figure: Option values and sheepskin effects

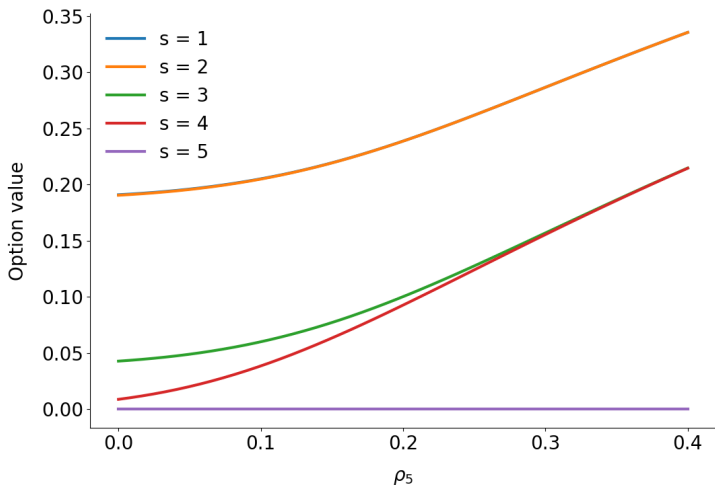
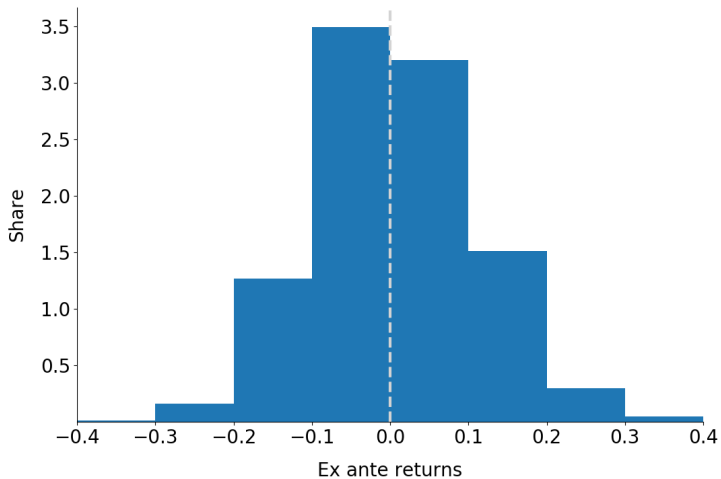


Figure: Returns



Conclusion

Appendix

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Multidimensionality of skills

Philipp Eisenhauer

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Static model of educational choice

Philipp Eisenhauer

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Dynamic model of human capital accumulation

Philipp Eisenhauer

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Intergenerational transmission of skills

Philipp Eisenhauer

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Human capital policy

Philipp Eisenhauer

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