# Economics of Human Capital

## Dynamic model of human capital accumulation

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# Introduction

We build on the following seminal paper:

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## Roadmap

- Economic Model
- Mathematical Model
- Data
- Computational Model
- Results

# Economic Model

## Decision Problem

| | |
|---|---|
| $t = 1, \dots, T$ | decision period |
| $s \in S$ | state |
| $a \in A$ | action |
| $d_t$ | decision rule |
| $r_t(s, a)$ | immediate reward |

# Timing of Events

| Learn | Choose | Receive |
|---|---|---|
| $\{r_t(s,a)\}_{a\in A}$ | $d_t(s)$ | $r_t(s,d_t(s))$ |

$t$

| Learn | Choose | Receive |
|---|---|---|
| $\{r_{t+1}(s',a)\}_{a\in A}$ | $d_{t+1}(s')$ | $r_{t+1}(s',d_{t+1}(s'))$ |

$t+1$

| | |
|---|---|
| $\pi = (d_1, \ldots, d_T)$ | policy |
| $h_t = (s_1, a_1, \ldots, s_t)$ | history |
| $\delta$ | discount factor |
| $p_t(s, a) \in P_t(s, a)$ | conditional distribution |

## Individual's Objective under Risk

$$v_1^{\pi^*}(s) = \max_{\pi \in \Pi} \mathrm{E}_s^{\pi}\left[\sum_{t=1}^{T} \delta^{t-1} r_t(X_t, d_t(X_t))\right]$$

# Mathematical Model

## Policy Evaluation

$$v_t^\pi(s) = \mathrm{E}_s^\pi\left[\sum_{\tau=t}^{T} \delta^{\tau-t} r_\tau(X_\tau, d_\tau(X_\tau))\right]$$

Inductive Scheme

$$v_t^\pi(s) = r_t(s, d_t(s)) + \delta \mathrm{E}_s^\pi\left[v_{t+1}^\pi(X_{t+1})\right]$$

## Optimality Equations

$$v_t^{\pi^*}(s) = \max_{a \in A} \left\{ r_t(s,a) + \delta \mathrm{E}_s^p \left[ v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}.$$

## Backward Induction Algorithm for MDP

**for** $t = T, \ldots, 1$ **do**

  **if** t == T **then**

$$v_T^{\pi^*}(s) = \max_{a \in A}\left\{ r_T(s,a) \right\} \qquad \forall \quad s \in S$$

  **else**

  Compute $v_t^{\pi^*}(s)$ for each $s \in S$ by

$$v_t^{\pi^*}(s) = \max_{a \in A}\left\{ r_t(s,a) + \delta \mathrm{E}_s^p \left[ v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}$$

  and set

$$d_t^{\pi^*}(s) = \arg\max_{a \in A}\left\{ r_t(s,a) + \delta \mathrm{E}_s^p \left[ v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}$$

  **end if**

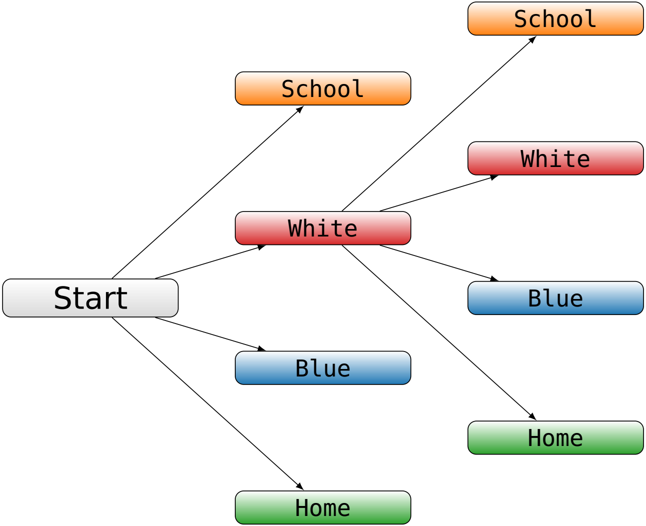
**end for**

# Data

## National Longitudinal Survey of Youth (1979)

- 1,373 white males starting at age 16
- life-cycle histories
  - school attendance
  - occupation-specific work status
  - real wages

Figure: Decision Tree



*Descriptives*

## Figure: Sample Size

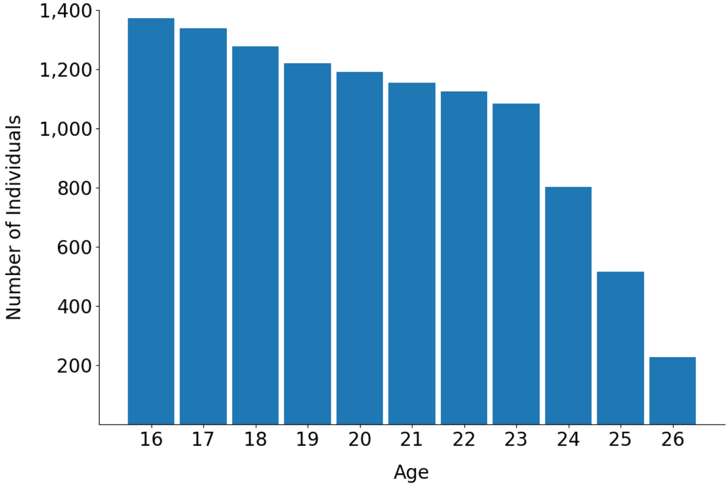


**Figure:** Observed Choices

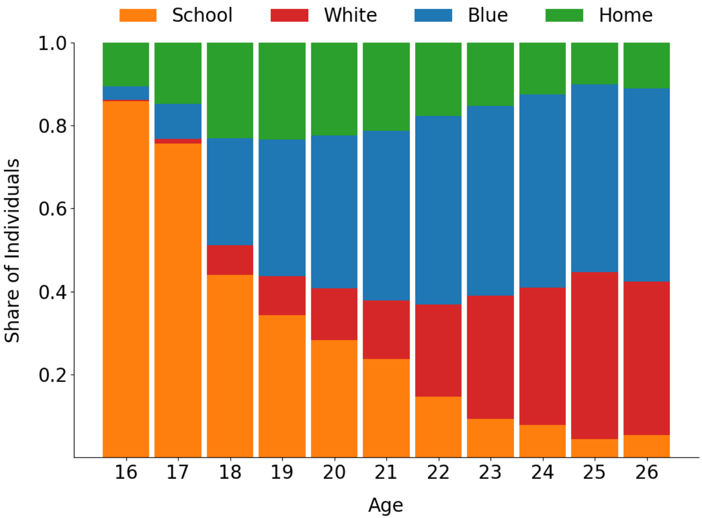


Table: Observed Real Wages

| | White | | Blue | |
| --- | --- | --- | --- | --- |
| Age | Obs. | Mean | Obs. | Mean |
| 16 | 2 | . | 26 | 10,287 |
| 20 | 128 | 5,499 | 349 | 14,432 |
| 25 | 201 | 16,540 | 222 | 21,991 |

Figure: Observed Transitions

## Figure: Initial Schooling

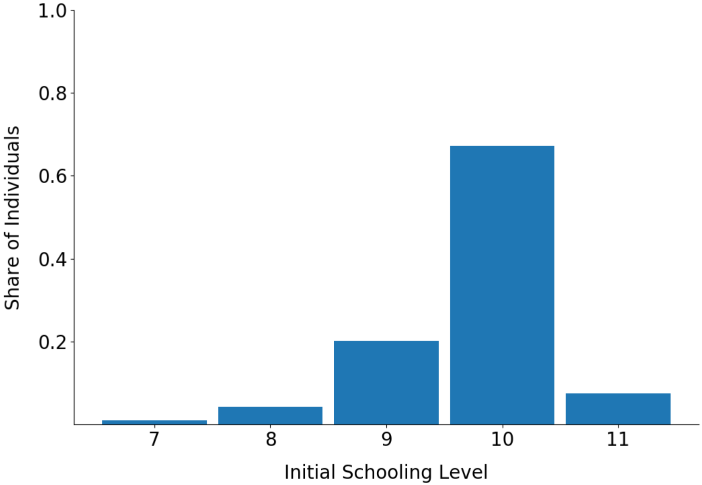


Table: Activities by Initial Schooling

| | Initial Schooling | | | | |
|---|---|---|---|---|---|
| Alternatives | 7 | 8 | 9 | 10 | 11 |
| School | 0.69 | 0.86 | 2.48 | 3.37 | 2.83 |
| White | 0.08 | 0.38 | 0.65 | 1.36 | 2.04 |
| Blue | 3.69 | 3.62 | 3.05 | 2.40 | 1.98 |
| Home | 4.23 | 4.19 | 1.91 | 1.10 | 1.32 |
| Total | 8.69 | 9.05 | 8.09 | 8.24 | 8.17 |

*Reduced-form Analysis*

## Table: Mincer Regressions

| | Log Real Wages | |
|---|---|---|
| Intercept | 8.314*** | 8.329*** |
| Schooling | 0.086*** | 0.077*** |
| | *Work Experience* | |
| - linear | 0.132*** | 0.125*** |
| - squared | −0.005*** | −0.003*** |
| | *Corrected AFQT* | |
| - linear | — | 0.002*** |
| Adj-$R^2$ | 0.21 | 0.22 |
| Observations | 4,420 | 4,232 |

## Table: Mincer Regressions

| | Log Real Wages | |
|---|---|---|
| | White | Blue |
| Intercept | 7.748*** | 8.790*** |
| Schooling | 0.128*** | 0.044*** |
| | *Own Experience* | |
| - linear | 0.146*** | 0.129*** |
| - squared | −0.003 | −0.005*** |
| | *Other Experience* | |
| - linear | 0.096*** | 0.085*** |
| - squared | 0.002 | −0.003 |
| Adj-$R^2$ | 0.28 | 0.17 |
| Observations | 1,468 | 2,952 |

## Open Issues

- distinction between ex ante and ex post returns
- role of psychic costs
- nonlinearities in the return
- role of uncertainty

# Computational Model

## Additional Structure

| | |
|---|---|
| $t$ | age |
| $k$ | unobserved type |
| $x_{j,t}$ | experience in occupation $j$ at age $t$ |
| $a_t$ | action at age $j$ |
| $g_t$ | level of schooling at age $t$ |

## Skill Production Function

$$
\begin{aligned}
e_{j,k,t} = \exp\{ e_{j,k,16} &+ \underbrace{\alpha_{j,1} g_t + \alpha_{j,2} \mathrm{I}[g_t \geq 12] + \alpha_{j,3} \mathrm{I}[g_t \geq 16]}_{\text{schooling}} \\
&+ \underbrace{\alpha_{j,4} x_{j,t} + \alpha_{j,5} x_{j,t}^2 + \alpha_{j,6} \mathrm{I}[x_{j,t} > 0] + \alpha_{j,7} x_{j \neq j',t}}_{\text{work experience}} \\
&+ \underbrace{\alpha_{j,8} \mathrm{I}[a_{t-1} \neq j]}_{\text{depreciation}} + \alpha_{j,9}(t-16) + \alpha_{j,10} \mathrm{I}[t < 18] + \epsilon_{j,t} \}
\end{aligned}
$$

with $j, j' = 1, 2$, $k = 1, \dots, 4$, and $t = 16, \dots, 65$

## Labor Market

$$r_{j,k,t} = w_{j,k,t} + \underbrace{\kappa_1 \mathbb{I}[g_t \geq 12] + \kappa_2 \mathbb{I}[g_t \geq 16]}_{\text{common returns}} + \beta_{j,1}$$
$$+ \underbrace{\beta_{j,2} \mathbb{I}[x_{j,t} > 0, a_{t-1} \neq j] + \beta_{j,3} \mathbb{I}[x_{j,t} = 0, a_{t-1} \neq j]}_{\text{entry cost}}$$

with $w_{j,k,t} = r_j e_{j,k,t}$

## School

$$r_{3,k,t} = e_{3,k,16} + \underbrace{\gamma_1 \mathrm{I}[g_t \geq 12] + \gamma_2 \mathrm{I}[g_t \geq 16]}_{\text{monetary and psychic cost}}$$

$$+ \underbrace{\gamma_3 \mathrm{I}[a_{t-1} \neq 3, g_t \leq 11] + \gamma_4 \mathrm{I}[a_{t-1} \neq 3, g_t \geq 12]}_{\text{reenrollment cost}}$$

$$+ \gamma_5 (t - 16) + \gamma_6 \mathrm{I}[t \leq 18] + \underbrace{\kappa_1 \mathrm{I}[g_t \geq 12] + \kappa_2 \mathrm{I}[g_t \geq 16]}_{\text{common returns}}$$

$$+ \epsilon_{3,t}$$

**Home**

$$r_{4,k,t} = e_{4,k,16} + \zeta_1 \mathrm{I}[18 \leq t \leq 20] + \zeta_2 \mathrm{I}[t \geq 21] + \underbrace{\kappa_1 \mathrm{I}[g_t \geq 12] + \kappa_2 \mathrm{I}[g_t \geq 16]}_{\text{common returns}} + \epsilon_{4,t}$$

## State Space

- at time $t$

$$s_t = \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}, \{\epsilon_{j,t}\}_{j=1,\dots,4}\}$$
$$\bar{s}_t = \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}\}$$

- laws of motion

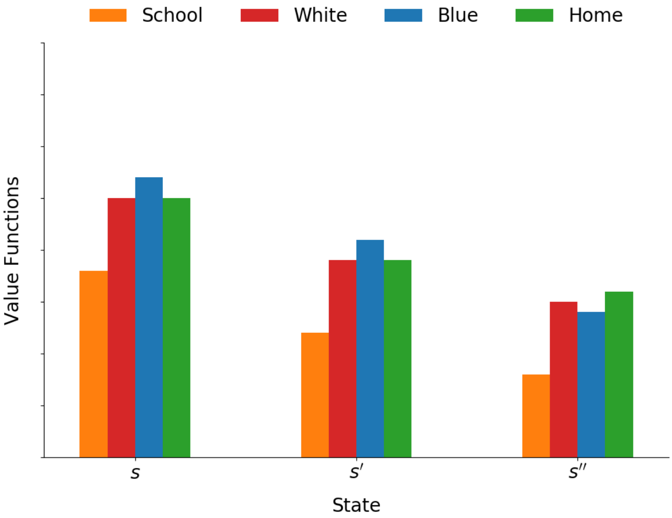
$$x_{j,t+1} = x_{j,t} + \mathrm{I}[a_t = j] \qquad \forall \quad j \in \{1, 2\}$$

$$g_{t+1} = g_t + \mathrm{I}[a_t = 3]$$

**Distribution of shocks**

$$[\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t}]^T \sim \mathcal{N}_0(\mathbf{0}, \Sigma)$$

Figure: Value Functions



## Computational Tool

https://respy.readthedocs.io

- Technical Documentation
  - Numerical Methods, Source Codes, Test Suite
- User Documentation
  - Tutorial

⇒ Transparency, Recomputability, and Extensibility

# Conclusion

# Appendix

*References*

Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, *105*(3), 473–522.