

Labor Economics: Human Capital

Tutorial Session

Contents

1 Quiz	2
2 Introduction	4
3 Static model of educational choice	6
4 Dynamic model of educational choice	8
5 Solutions	10
5.1 Quiz	10
5.2 Introduction	11
5.3 Static model of educational choice	14
5.4 Dynamic model of educational choice	19

1 Quiz

Please mark each claim as either true or false. Make sure to outline your reasoning briefly.

- (1) Consider the seminal model by [Spence \(1973\)](#) as presented in class. High productivity workers always prefer the option to signal their ability.
- (2) Based on the discussion in class, the option value of schooling is always strictly positive for all schooling transitions.
- (3) The accounting-identity model as presented in [Heckman, Lochner, & Todd \(2006\)](#) provides a justification for interpreting the Mincer coefficient as an internal rate of return.
- (4) A college tuition subsidy always leads to an increase in high school graduation rates regardless of the individual time preferences.
- (5) [Heckman, Stixrud, & Urzua \(2006\)](#) report that the effect of cognitive skills on social outcomes is always more pronounced than the effect of noncognitive skills.
- (6) [Keane & Wolpin \(1997\)](#) find that a basic model of human capital investment explains the observed investment patterns just as well as their extended model.
- (7) [Carneiro, Heckman, & Vytalacil \(2011\)](#) find that individuals make their schooling decisions in light of heterogeneous returns.
- (8) [Keane & Wolpin \(1997\)](#) point to heterogeneous schooling levels at age 16 as the main determinant of inequality in expected total lifetime utility.
- (9) [Heckman, Lochner, & Todd \(2006\)](#) compile several pieces of empirical evidence that point towards a rejection of the standard Mincer regression model.
- (10) [Lagakos, Moll, Porzio, Qian, & Schoellman \(2018\)](#) find that wages increase substantially more over the life cycle in poor countries than in rich countries.
- (11) [Lagakos et al. \(2018\)](#) show that their core findings hold up regardless of whether they focus on part or full time male wage workers.
- (12) [Lagakos et al. \(2018\)](#) determine that differences in long-term contracts are an important driver of cross-country differences in life cycle wage growth.
- (13) [Lagakos et al. \(2018\)](#) determine that human capital or search frictions are promising explanations for the cross-country differences in life cycle wage growth.
- (14) [Keane & Wolpin \(1997\)](#) find that the predictions of life cycle choices from a static and dynamic model of human capital investments are in general agreement.

- (15) [Keane & Wolpin \(1997\)](#) find that a \$2,000 college tuition subsidy has a pronounced impact on the expected present value of lifetime utility.
- (16) [Carneiro et al. \(2011\)](#) report point estimates that the marginal benefit of treatment for the average individual remains positive when moving along the distribution of V .
- (17) [Heckman, Lochner, & Todd \(2006\)](#) restrict their analysis to the ex post return to schooling.

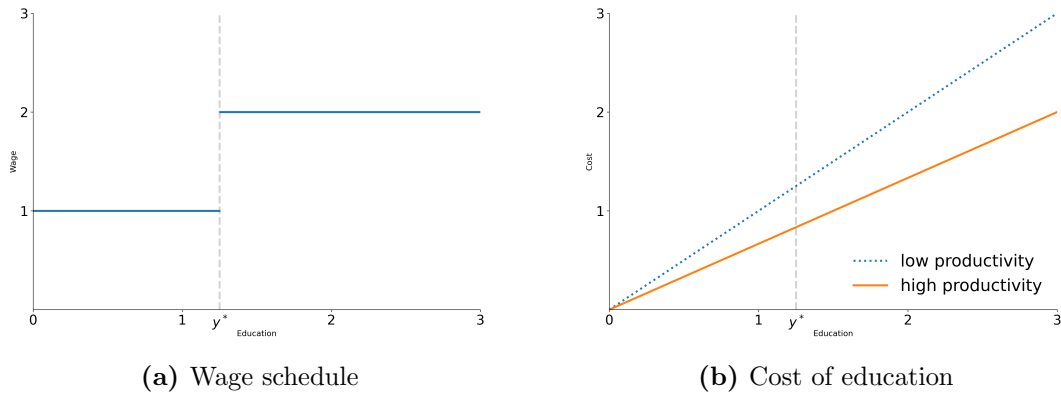
2 Introduction

Consider the motivating models of [Spence \(1973\)](#) and [Ben-Porath \(1967\)](#) as discussed in class.

- (1) For both models, provide a brief description of the question they are designed to address and their key ingredients. Provide two examples of important economic features that are missing from the formal analysis?

Consider the model developed in [Spence \(1973\)](#) in more detail. Figure 1 visualizes the information about the wage schedule and the cost of education in the parametrized model. Please assume throughout that employers believe that individuals with a level of education $y^* \geq \frac{5}{4}$ have a high productivity.

Figure 1: Model parametrization



- (2) Write down the parametrization of the cost (c_L, c_H) and wage (w_L, w_H) functions for the high and low productivity individuals.
- (3) Complete Figure 2 by adding the surplus functions for each of the two groups over the specified range. Also, indicate the optimal level of schooling for each of the two groups.
- (4) Calculate the range of the separating schooling level y^* that confirms the employer's beliefs.

Now consider the case where individuals do not have the ability to signal their productivity and the share of individuals with low productivity is denoted by q_L .

- (5) What is the wage for each of the two groups in this scenario as a function of q_L ?
- (6) Complete Figure 3 by adding the surplus and wage for the two groups under the scenario where individuals are able to signal their ability and when they are not.

Figure 2: Surplus of education

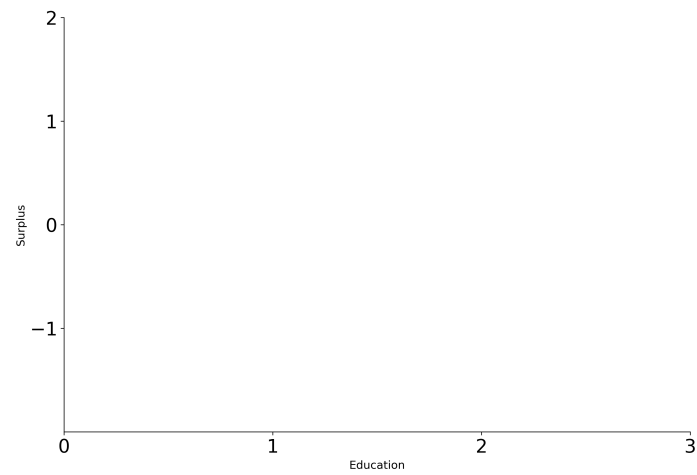
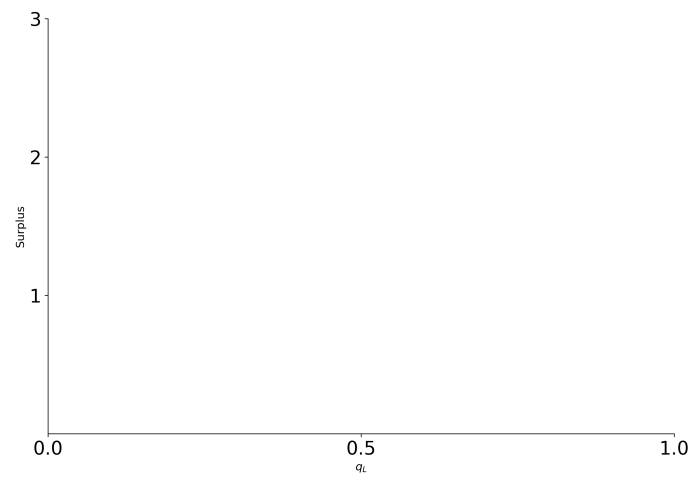


Figure 3: Surplus and market structure



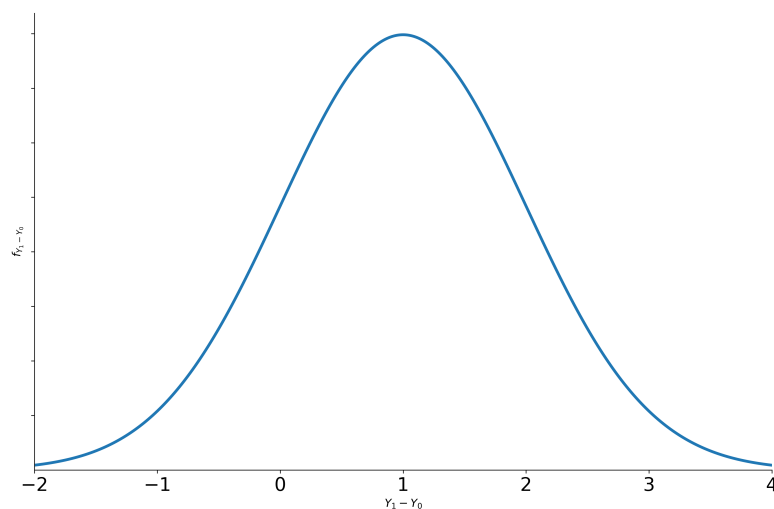
- (7) What scenario do high productivity individuals prefer, what does their assessment depend on, when exactly do they change their mind?

3 Static model of educational choice

Consider the framework of the generalized Roy model presented in class for the static analysis of educational choice.

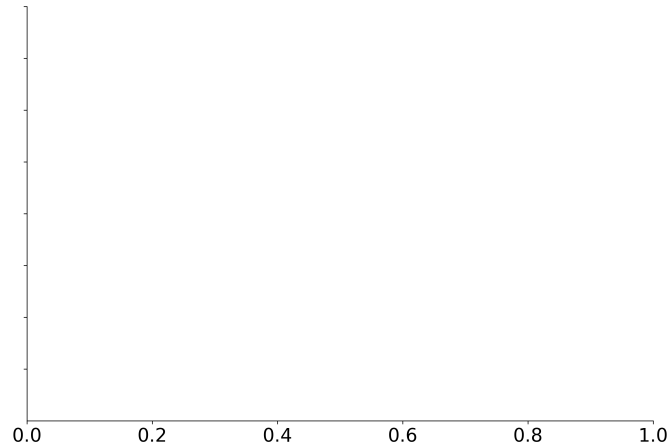
- (1) Write down and briefly describe the key equations of the model.
- (2) Formally define the conventional average treatment effects and describe their limited policy relevance. What is potentially lost by focusing on average effects instead of looking at the whole distribution of individual benefits?
- (3) Define and describe the concept of essential heterogeneity. How does its presence and absence affect the relationship between the conventional average treatment effect parameters. Please integrate the conventional average treatment effects in the absence of essential heterogeneity into Figure 4 which already shows a hypothetical distribution of individual-specific benefits.

Figure 4: Distribution of effects



- (4) Define and describe the marginal benefit of treatment. What exactly is the conditioning set? Complete the empty canvas below by sketching the marginal benefit of treatment in the presence and absence of essential heterogeneity. Ensure that both axes are properly labeled.
- (5) What are the main findings in [Carneiro et al. \(2011\)](#) on the marginal benefit of a college education?
- (6) Briefly outline the shortcomings of a static model of educational choice compared to a dynamic model.

Figure 5: Marginal benefit of treatment



Consider the following parameterization of the generalized Roy model presented in class for the static analysis of educational choice.

$$\begin{aligned} Y_1 &= 0.25 & D &= \mathbb{1}[0.50 > U] \\ Y_0 &= U \end{aligned}$$

Assume that U is unobservable and follows a uniform distribution between zero and one. Please be careful about correctly labeling all graphs that you decide to include in your answers.

- (1) Define the individual effect of treatment. What are the sources of heterogeneity in the model. What fraction of individuals have a positive benefit of treatment? How many do select into treatment?
- (2) Formally define the conventional average treatment effects and describe their limited policy relevance. How does the distribution of benefits for the model above look like? What is its exact range? Please mark the part of the distribution conditional on treatment status. Calculate the conventional effects of treatment.
- (3) Define and describe the concept of essential heterogeneity. Does the parameterized model exhibit essential heterogeneity? Please explain your answer.
- (4) Define and describe the marginal benefit of treatment B^{MTE} . How exactly does the B^{MTE} for the parameterized model above look like?

4 Dynamic model of educational choice

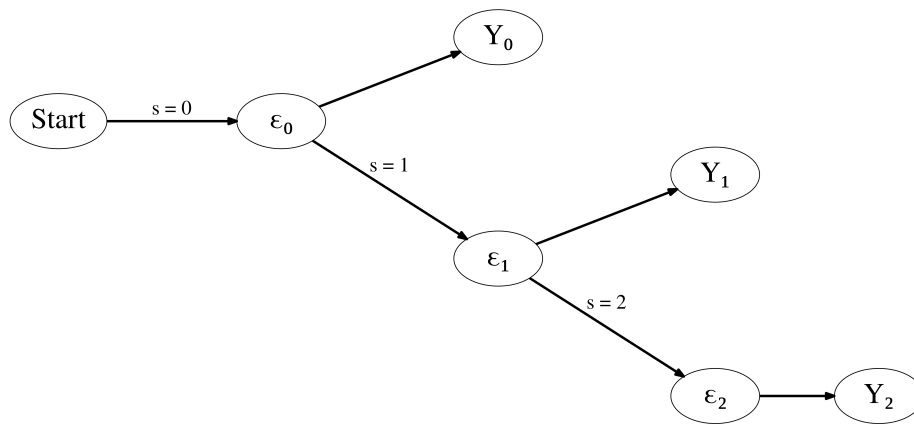
Please consider the standard setup of the Mincer returns as presented in class.

- (1) Please state and briefly describe the Mincer equation ([Mincer, 1974](#)). What are its key features? What important characteristics of schooling choices are lost when studying Mincer returns? What alternative return concepts exist?
- (2) [Heckman, Lochner, & Todd \(2006\)](#) derive several implications of the Mincer equation and put them to the empirical test. Please provide their formal statement and a brief verbal explanation.

Now please consider the sequential model of schooling decisions introduced in class to determine the true return to schooling.

Figure 6 shows the two period decision problem. Individuals enter the model without any schooling and can decide to pursue up to two additional years if they desire to do so. At any point, they can drop out of school and enter the labor market with the following earnings.

Figure 6: Decision problem



$$Y_0 = 0.5$$

$$Y_1 = 1.5 + \epsilon_1$$

$$Y_2 = 1.5$$

Note that not all earnings have a random component. In addition, there is no time-discounting and ϵ_1 follows a uniform distribution between zero and one.

- (4) What share of individuals will continue their schooling at each decision node? How does the final distribution of schooling levels look like?

- (5) Please provide the formal definition and a brief verbal explanation for the value V_s and true return $R_{s,s-1}$ of a schooling level. For an individual entering the model, what is the value of no schooling V_0 and the true return of the first year $R_{1,0}$?
- (6) Please provide the formal definition and a brief verbal explanation of the option value of a schooling level. For an individual entering the model, what is the option value $O_{1,0}$ of the first year of schooling?

5 Solutions

5.1 Quiz

- (1) **False**, this depends on the share of low productivity workers.
- (2) **False**, it is zero for the last transition.
- (3) **False**, it only allows to interpret the coefficient as an average growth rate of wages with schooling. This is only true in the compensating-differences model by construction.
- (4) **False**, this depends on whether individuals are forward-looking or not.
- (5) **Ambiguous**, this depends on the outcome under investigation. In particular for some social outcomes, the effect of noncognitive skills is more pronounced.
- (6) **False**, the extended model does a much better job at explaining the observed choice patterns. In particular, the persistence in choices is better captured. For example, the extended version includes mobility costs that discourage switching between occupations.
- (7) **True**, the test and reject the hypothesis that individuals do not.
- (8) **False**, it is initial unobserved heterogeneity that accounts for most of the observed inequality in expected total lifetime utility.
- (9) **True**, this is their conclusion.
- (10) **False**, their finding is the exact opposite. Instead, it is rich countries where wages increases stantially more over the life cycle.
- (11) **False**, they restrict their analysis to full time workers throughout.
- (12) **False**, instead they point to human capital and search as reasonable explanations for the observed patterns. Long-term contracts can explain flatter experience-wage profiles in poor countries in two scenarios. The first scenario is that wages are more front-loaded in poor countries. In this case we would expect day laborers in poor countries to have steeper profiles than the rest of the workforce. The second scenario is that wages are more back-loaded in rich countries. In this case, we would expect day laborers in rich countries to have flatter profiles. We find no evidence of either of these scenarios.
- (13) **True**, they only rule out long-term contracts.
- (14) **False**, while the withing-sample model fit is similar the predictions of the models differ considerably. For example, the static model does indicate that all individuals will work in a white collar occupation. The dynamic model points towards an equal allocation across occupations.

- (15) **False**, they document a considerable effect on High School and College graduation rate instead. Individuals induced to pursue increased schooling due to the sidy do end up working in the same occupations in the end and thus there is no real effect on their lifetime utility.
- (16) **False**, at least the point estimates indicate a negative effect for individuals with a large dislike for treatment participation.
- (17) **False**, they present models and return concept that allow to distinguish between ex ante and ex post return to schooling.

5.2 Introduction

- (1) Both models are designed as to explain why individuals invest in their human capital. The key difference is in the productivity effect of human capital. In [Ben-Porath \(1967\)](#) human capital does increase productivity, while in [Spence \(1973\)](#) it does not. Numerous features are missing from both models. Among those discussed in class are a lack of distinction between general and specific training, uncertainty, and borrowing constraints.
- (2) From the graphs it follows that:

$$w_L(y) = w_H(z) = \begin{cases} 1 & \text{if } y < y^* \\ 2 & \text{if } y \geq y^* \end{cases}$$

and

$$\begin{aligned} c_L(y) &= y \\ c_H(y) &= \frac{2}{3}y \end{aligned}$$

- (3) Individuals receive a wage for working but also incur a cost of investing in human capital.

$$s_j(y) = \begin{cases} w_L - c_j(y) & \text{if } y < y^* \\ w_H - c_j(y) & \text{if } y \geq y^* \end{cases}$$

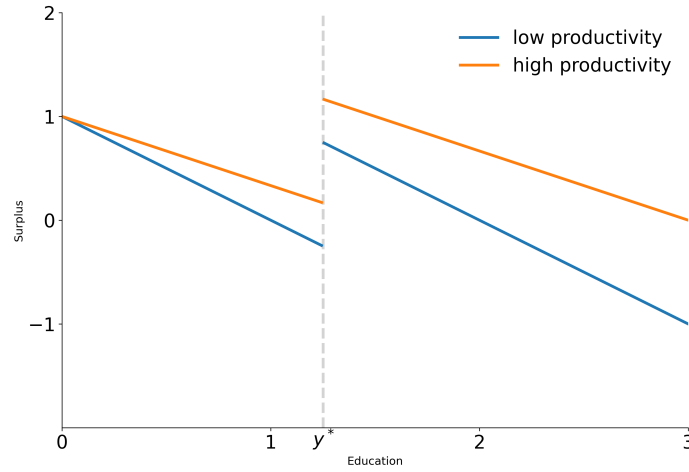
For the parametrization in this particular case:

$$s_L(y) = \begin{cases} 1 - y & \text{if } y < y^* \\ 2 - y & \text{if } y \geq y^* \end{cases}$$

$$s_H(y) = \begin{cases} 1 - \frac{2}{3}y & \text{if } y < y^* \\ 2 - \frac{2}{3}y & \text{if } y \geq y^* \end{cases}$$

Figure 7 shows the visual presentation.

Figure 7: Surplus



- (4) The separating level of schooling needs to be high enough that low productivity individuals do not pursue any schooling and still low enough so that high productivity individuals find it fruitful to do so. So, these two equations have to be fulfilled at the same time.

$$\begin{aligned} 1 &> 2 - y^* && \rightarrow y^* > 1 \\ 1 &< 2 - \frac{2}{3}y^* && \rightarrow y^* < \frac{3}{2} \end{aligned}$$

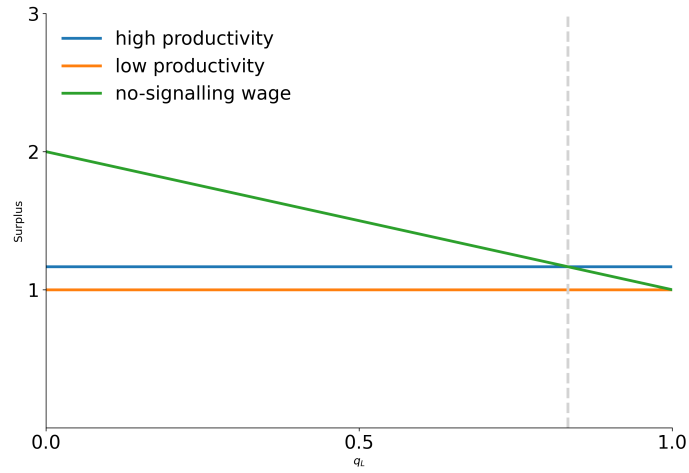
And thus the employer's beliefs are confirmed if $1 < y^* < \frac{3}{2}$.

- (5) In the absence of signaling, the wage is simply the unconditional expected marginal product and there is no reason to invest in one's human capital ($y = 0$).

$$\bar{w} = q_L \times 1 + (1 - q_L) \times 2$$

(6) Figure 8 shows the market structure.

Figure 8: Market Structure



(7) The high productivity individuals' preferences depend on the share of the low productivity individuals. The wage, which corresponds to the surplus as $y = 0$, is easily computed.

$$s_h = w_h = 2 - q_L.$$

The surplus in the case of signaling and $y^* = \frac{5}{4}$ is computed as:

$$\begin{aligned} s_h &= 2 - \frac{2}{3} \frac{5}{4} \\ &= 2 - \frac{10}{12} = \frac{7}{6}. \end{aligned}$$

The surplus for the high productivity individuals is equal in both scenarios at $q_L = \frac{5}{6}$.

5.3 Static model of educational choice

- (1) Please see below for the key equations of the model.

Potential Outcomes	Observed Outcome
$Y_1 = \mu_1(X) + U_1$	$Y = DY_1 + (1 - D)Y_0$
$Y_0 = \mu_0(X) + U_0$	
Choice	
$D = \mathbb{I}[\mu_D(X, Z) - V > 0]$	

- (2) Given the notation above, the definition of the conventional average effects of treatment is straightforward.

$$\begin{aligned}
 B^{ATE} &= E[Y_1 - Y_0] \\
 B^{TT} &= E[Y_1 - Y_0 \mid D = 1] \\
 B^{TUT} &= E[Y_1 - Y_0 \mid D = 0]
 \end{aligned}$$

Their policy-relevance is limited, as they correspond to extreme policy alternatives. For example a stylized version of the Affordable Care Act, where health insurance coverage was universal after the implementation of the reform. As a considerable share of Americans was already covered before, this corresponds to the B^{TT} . A focus on average affect might mask considerable heterogeneity in benefits.

- (3) The concept of essential heterogeneity describes the idea that individuals select their treatment status based on unobservable gains from treatment even after conditioning on unobservables. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

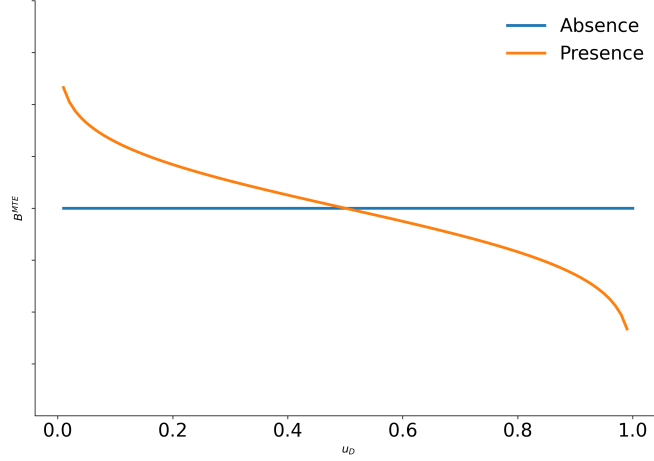
In the presence of essential heterogeneity, the conventional average treatment effects differ. They coincide in the absence of essential heterogeneity. Thus the figure is a simple replication from the lecture slides.

- (4) The marginal benefit is the average effect of treatment for individuals at different quantiles of the first-stage unobservable V . More formally,

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D].$$

Figure 9 shows the marginal benefit of treatment in the presence and absence of essential heterogeneity.

Figure 9: Marginal Benefit of Treatment



- (5) [Carneiro et al. \(2011\)](#) document considerable heterogeneity in the returns to a college education. They range from 40% to -20% evaluated at the mean values of the sample's observables.
- (6) The static model does not account for the dynamic nature of educational choices, where uncertainty reveals information about its benefits and cost throughout one's educational career.
- (1) The individual effect of treatment B is defined as the difference in potential outcomes $Y_1 - Y_0$. The unobservable U is the only source of heterogeneity in the model. Given its distribution and the selection equation, about 50% of individuals will select into treatment. However, only about 25% have a positive benefit of treatment.
- (2) The conventional average treatment effects are defined below.

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

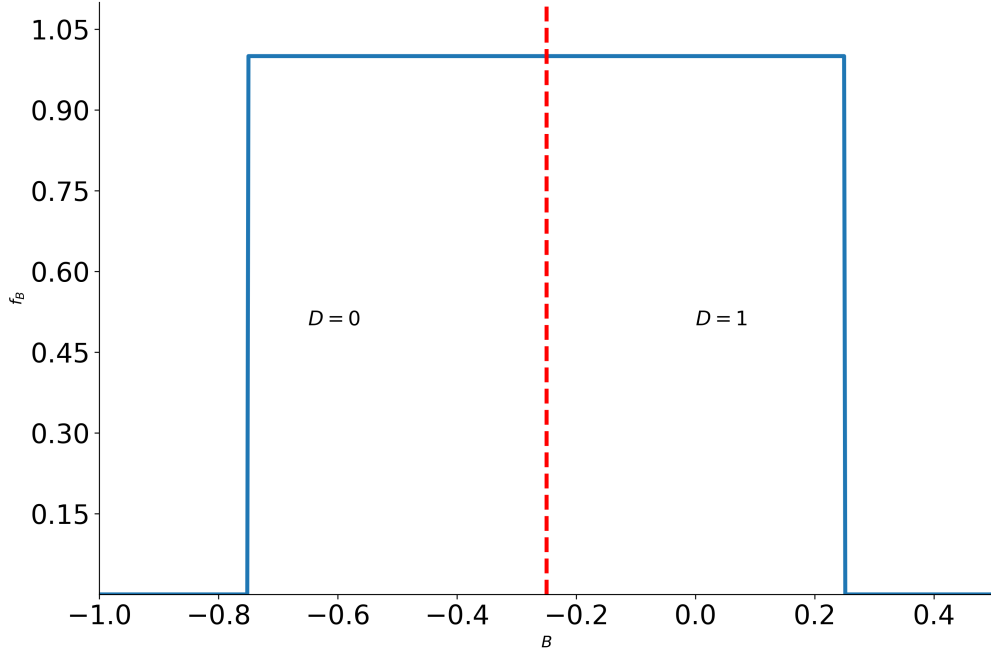
$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

Their policy-relevance is limited, as they correspond to extreme policy alternatives. For example a stylized version of the Affordable Care Act, where health insurance coverage

was universal after the implementation of the reform. As a considerable share of Americans was already covered before, this corresponds to the B^{TT} . A focus on average effects might mask considerable heterogeneity in benefits.

Figure 12 shows the distribution of benefits. All individuals with a benefit above -0.25 select into treatment because it still holds that $0.50 > U$.

Figure 10: Distribution of benefits



The conventional effects of treatment are calculated as:

$$B^{ATE} = -0.25$$

$$B^{TT} = 0.00$$

$$B^{TUT} = -0.50$$

- (3) The concept of essential heterogeneity describes the idea that individuals select their treatment status based on unobservable gains from treatment even after conditioning on unobservables. More formally,

$$Y_1 - Y_0 \not\propto D \quad | \quad X = x.$$

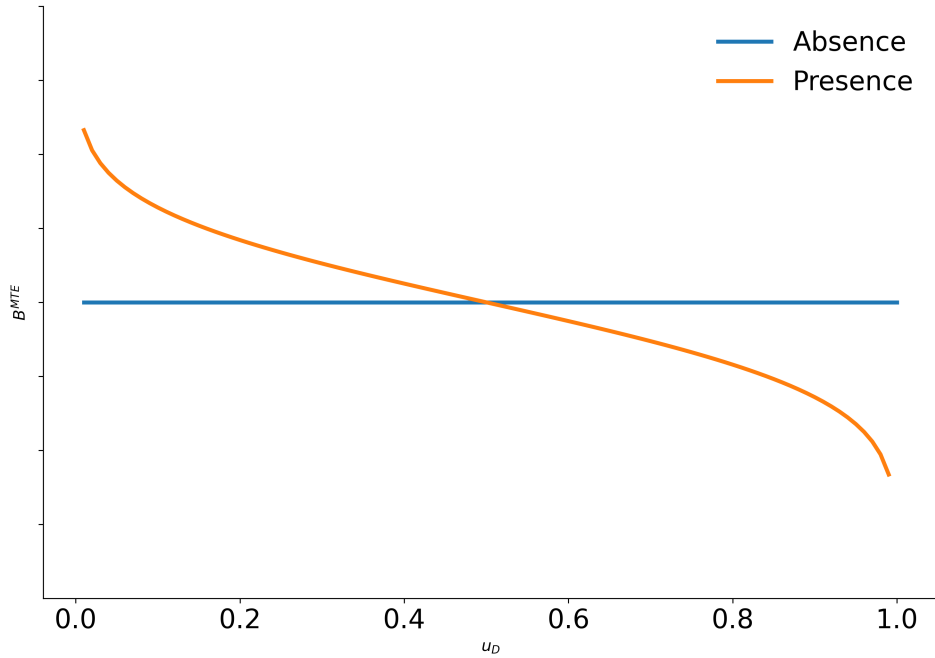
The parameterized model does exhibit essential heterogeneity as individuals with the highest dislike to select into treatment U have the least to gain.

- (4) The marginal benefit is the average effect of treatment for individuals at different quantiles of the first-stage unobservable V . More formally,

$$B^{MTE}(u_D) = E[Y_1 - Y_0 \mid U_D = u_D].$$

Figure 13 shows the marginal benefit of treatment for the parameterized model.

Figure 11: Marginal benefit of treatment



- (1) The individual effect of treatment B is defined as the difference in potential outcomes $Y_1 - Y_0$. The unobservable U is the only source of heterogeneity in the model. Given its distribution and the selection equation, about 50% of individuals will select into treatment. However, only about 25% have a positive benefit of treatment.
- (2) The conventional average treatment effects are defined below.

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

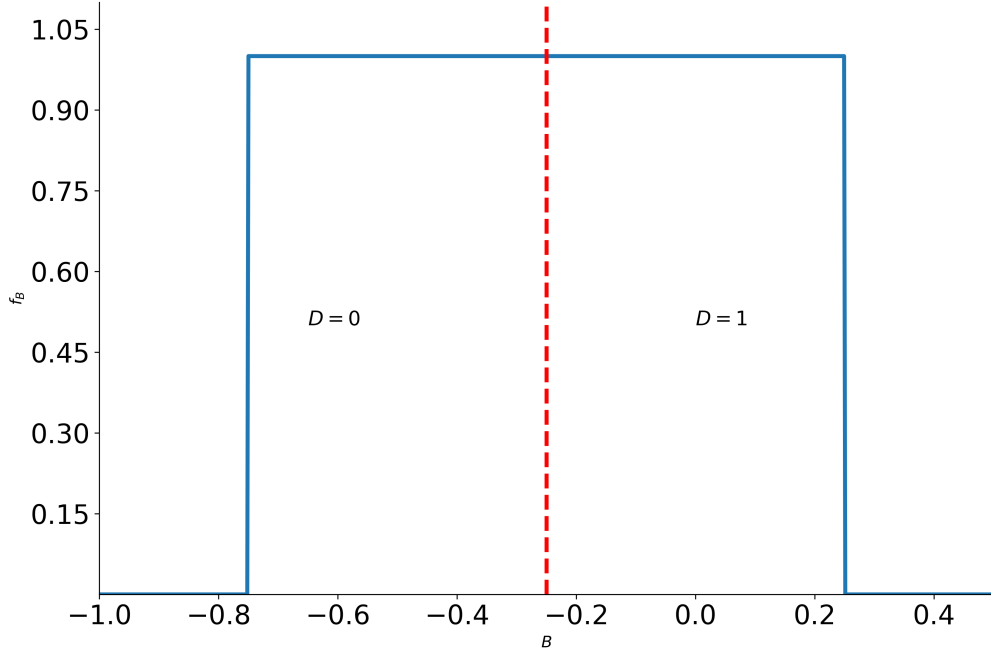
$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

Their policy-relevance is limited, as they correspond to extreme policy alternatives. For example a stylized version of the Affordable Care Act, where health insurance coverage

was universal after the implementation of the reform. As a considerable share of Americans was already covered before, this corresponds to the B^{TT} . A focus on average effects might mask considerable heterogeneity in benefits.

Figure 12 shows the distribution of benefits. All individuals with a benefit above -0.25 select into treatment because it still holds that $0.50 > U$.

Figure 12: Distribution of benefits



The conventional effects of treatment are calculated as:

$$B^{ATE} = -0.25$$

$$B^{TT} = 0.00$$

$$B^{TUT} = -0.50$$

- (3) The concept of essential heterogeneity describes the idea that individuals select their treatment status based on unobservable gains from treatment even after conditioning on unobservables. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

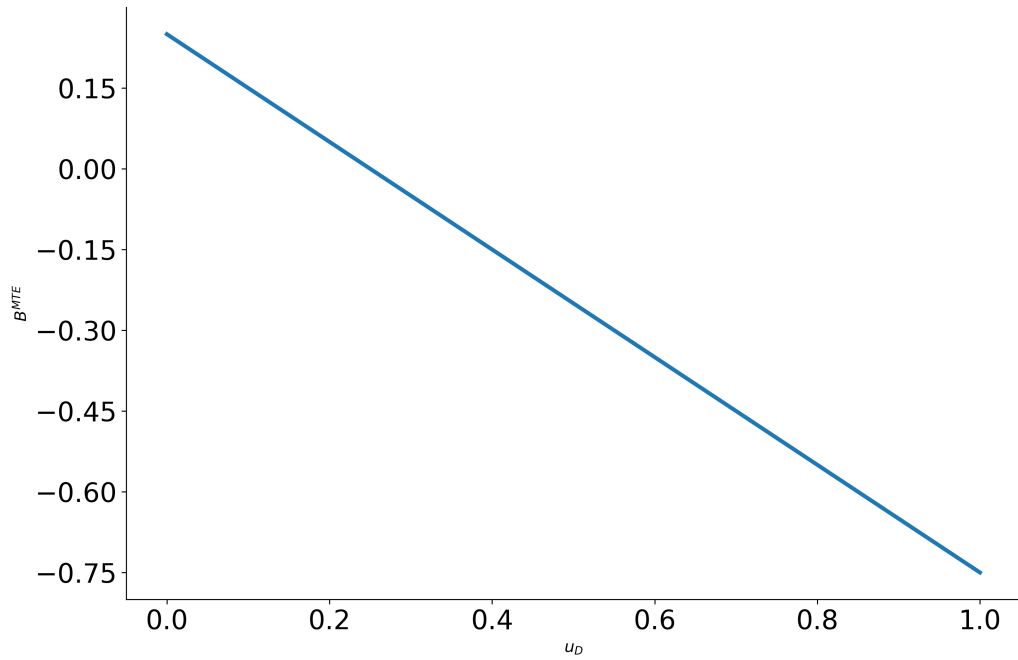
The parameterized model does exhibit essential heterogeneity as individuals with the highest dislike to select into treatment U have the least to gain.

- (4) The marginal benefit is the average effect of treatment for individuals at different quantiles of the first-stage unobservable V . More formally,

$$B^{MTE}(u_D) = E[Y_1 - Y_0 \mid U_D = u_D].$$

Figure 13 shows the marginal benefit of treatment for the parameterized model.

Figure 13: Marginal benefit of treatment



5.4 Dynamic model of educational choice

- (1) The Mincer equation is stated below.

$$\ln Y(s, x) = \alpha + \rho_s s + \beta_0 x + \beta_1 x^2 + \epsilon$$

The Mincer equation imposes a linear and homogeneous return to schooling and additive separability between schooling and work experience. There is no role for uncertainty and thus no distinction between ex ante and ex post returns. There exist numerous alternative return concepts such as the internal rate of return and the true return.

- Log-earnings experience profiles are parallel across schooling levels.

$$\frac{\partial \ln Y(s, x)}{\partial s \partial x} = 0$$

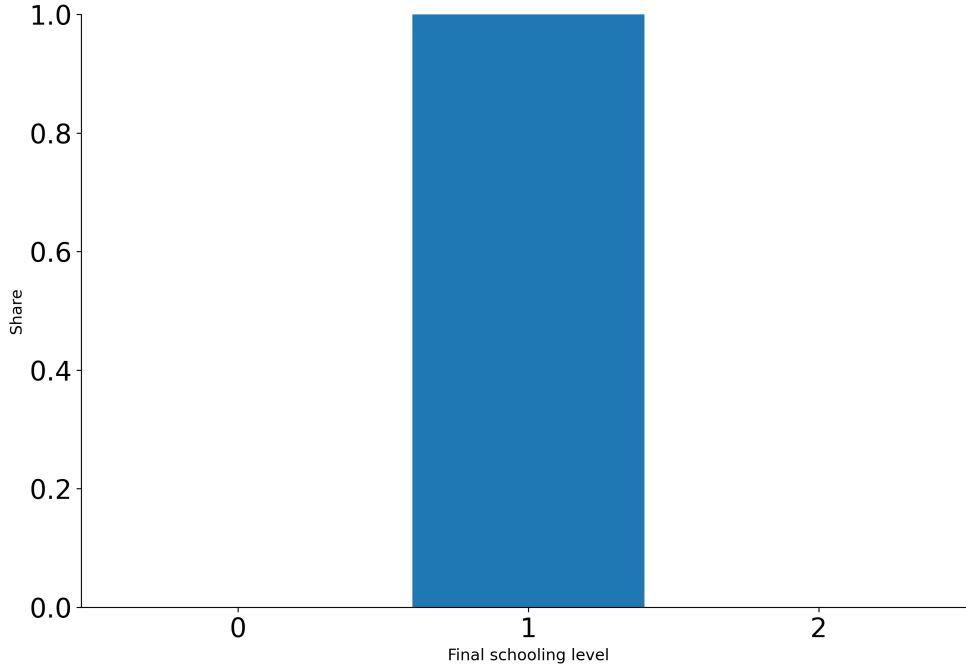
- Log-earnings age profiles diverge with age across schooling levels.

$$\frac{\partial \ln Y(s, x)}{\partial s \partial t} > 0$$

- The variance of earnings over the life cycle has a U-shaped pattern.

(2) Everybody will go to school for one more year.

Figure 14: Final schooling level



(3) The value and true return are defined below.

$$V_s = \max \{Y_s, E_s(V_{s+1})\}$$

$$R_{s,s-1} = \frac{E_{s-1}[V_s] - Y_{s-1}}{Y_{s-1}}$$

The value of zero schooling is $V_0 = 2$ and the true return amounts to $R_{1,0} = 3$

(4) The option value is defined below.

$$O_{s,s-1} = E_{s-1}[V_s - Y_s]$$

The option value captures the possibilities to continue schooling even further after completing a schooling level. Given the model parameterization, the option value amounts is

zero.

References

- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy*, 75(4), 352–365.
- Carneiro, P., Heckman, J. J., & Vytlačil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754–2781.
- Heckman, J. J., Lochner, L. J., & Todd, P. E. (2006). Earnings functions, rates of return and treatment effects: The Mincer equation and beyond. In E. A. Hanushek & F. Welch (Eds.), *Handbook of the economics of education* (1st ed., Vol. 1, pp. 307–458). Amsterdam, Netherlands: North-Holland Publishing Company.
- Heckman, J. J., Stixrud, J., & Urzua, S. (2006). The effects of cognitive and noncognitive abilities on labor market outcomes and social behavior. *Journal of Labor Economics*, 24(3), 411–482.
- Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, 105(3), 473–522.
- Lagakos, D., Moll, B., Porzio, T., Qian, N., & Schoellman, T. (2018). Life cycle wage growth across countries. *Journal of Political Economy*, 126(2), 797–849.
- Mincer, J. (1974). *Schooling, experience and earnings*. New York, NY: National Bureau of Economic Research.
- Spence, M. (1973). Job market signaling. *The Quarterly Journal of Economics*, 87(3), 355–374.