Economics of Human Capital

Philipp Eisenhauer

Material available on

Visit us!

UNIVERSITÄT BONN

Economics of Human Capital

Returns to schooling

Philipp Eisenhauer

Introduction

I heavily draw on the material presented in:

- Heckman, J. J., Lochner, L. J., & Todd, P. E. (2006). Earnings functions, rates of return and treatment effects: The Mincer equation and beyond. In E. A. Hanushek & F. Welch (Eds.), *Handbook of the economics of education* (1st ed., Vol. 1, pp. 307–458). Amsterdam, Netherlands: North-Holland Publishing Company.

Importance of returns

- explain wage inequality within countries
- explain growth differentials across countries
- assess schooling investment on individual level
- evaluate public policies to foster educational attainment
- ...

Return concepts

- Mincer returns
- internal rate of return
- true rate of return

Mincer returns

Mincer Equation

$$\ln Y(s,x) = \alpha + \rho_s s + \beta_0 x + \beta_1 x^2 + \epsilon$$

$\Rightarrow$ How to interpret the *Mincer Coefficient* ρ_s?

Conceptual Frameworks

- compensating differences
- accounting-identity

Compensating Differences Model

Let $Y(s)$ represent the annual earnings of an individual with s years of education, assumed to be constant over his lifetime. Let r be an externally determined interest rate and T the length of working life, assumed not to depend on s. The present value of earnings associated with schooling level s is

$$V(s) = Y(s) \int_s^T e^{-rt} dt = \frac{Y(s)}{r} (e^{-rs} - e^{-rT}).$$

Figure: Earnings

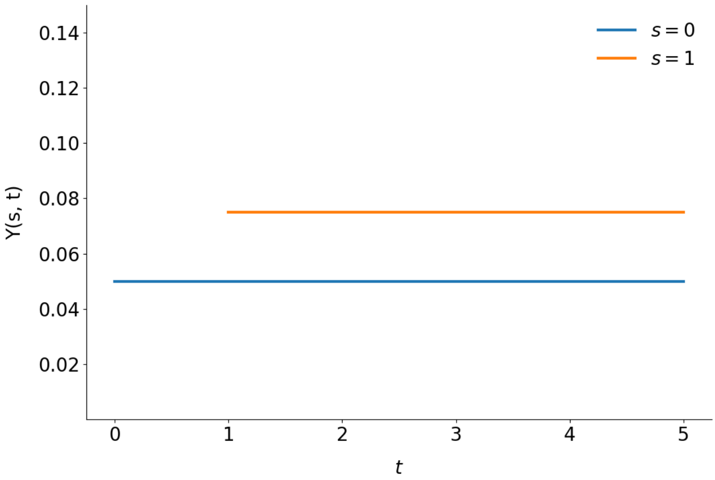
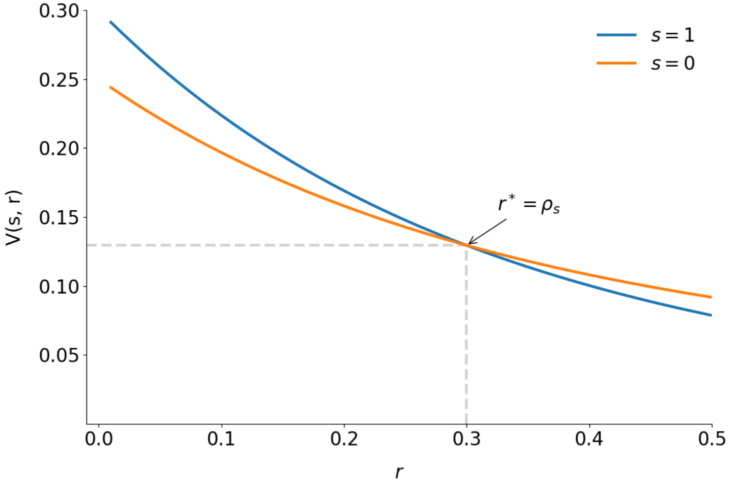


Figure: Value



Equilibrium across heterogeneous schooling levels requires that individuals be indifferent between schooling choices with $V(s) - V(0) = 0$. Equating earnings streams across schooling levels yields:

$$\frac{Y(s)}{r}\left(e^{-rs} - e^{-rT}\right) - \frac{Y(0)}{r}\left(1 - e^{-rT}\right) = 0$$

$$Y(s) = Y(0)\left(\frac{1 - e^{-rT}}{e^{-rs} - e^{-rT}}\right)$$

$$Y(s) = Y(0)\left(\frac{1}{e^{-rs}}\right)\left(\frac{1 - e^{-rT}}{1 - e^{-r(T-s)}}\right)$$

Taking the natural logarithm:

$$\ln Y(s) = \ln Y(0) + rs + \ln\left(\frac{1-e^{-rT}}{1-e^{-r(T-s)}}\right)$$

$\Rightarrow$ ρ_s equals the market interest rate and the internal rate of return to schooling by construction.

Model features

- identical abilities and opportunities
- no credit constraints
- perfect certainty
- no direct cost of schooling
- no nonpecuniary benefits of school and work

Accounting-Identity Model

Model ingredients

P_t	potential earnings at t
$C_t = k_t P_t$	investment cost of training at t
ρ_t	average return to investment at t

$$P_t \equiv P_{t-1}(1 + k_{t-1}\rho_{t-1}) \equiv \prod_{j=0}^{t-1}(1 + \rho_j k_j)P_0$$

Formal schooling is defined as years spent in full-time investment ($k_t = 1$), which is assumed to take place at the beginning of life and to yield a rate of return ρ_s that is constant across all years of schooling.

$$\begin{aligned} \ln P_t &\equiv \ln P_0 + s \ln(1+\rho_s) + \sum_{j=s}^{t-1} \ln(1+\rho_0 k_j) \\ &\approx \ln P_0 + s\rho_s + \rho_0 \sum_{j=s}^{t-1} k_j \end{aligned}$$

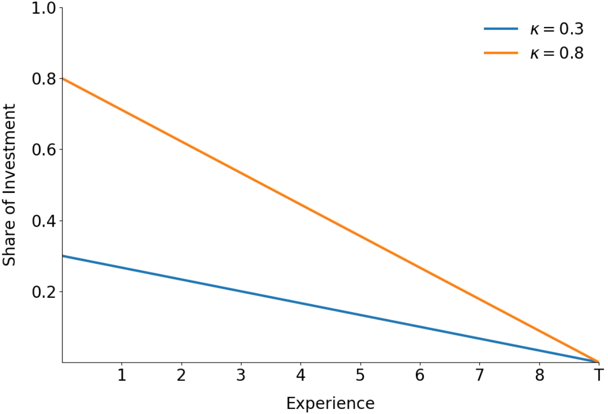
Mincer (1974) assumes a linearly declining rate of post-school investment:

$$k_{s+x} = \kappa\left(1 - x/T\right), \text{where } x = t - s$$

Thus,

$$\ln P_{x+s} = \ln P_0 + s\rho_s + \rho_0 \sum_{j=0}^{x-1} \kappa\left(1 - j/T\right).$$

Figure: Post-School Investment



The derivations draws on the following results for arithmetic series (Chapman & Hall, 2018).

$$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

We need to further decompose the experience addition and use results on arithmetic series.

$$
\begin{aligned}
\rho_0 \sum_{j=0}^{x-1} \kappa\,(1-j/T) &= \rho_0 \sum_{j=0}^{x-1} \kappa - \rho_0 \kappa \sum_{j=0}^{x-1} (j/T) \\
&= \rho_0 \kappa x - \frac{\rho_0 \kappa}{T} \sum_{j=0}^{x-1} j \\
&= \rho_0 \kappa x - \frac{\rho_0 \kappa}{T} \left(\frac{(x-1)((x-1)+1)}{2} \right) \\
&= \rho_0 \kappa x - \frac{\rho_0 \kappa}{T} \left(\frac{x^2 - x}{2} \right) \\
&= \left(\rho_0 \kappa + \frac{\rho_0 \kappa}{2T} \right) x - \frac{\rho_0 \kappa}{2T} x^2
\end{aligned}
$$

Now we can substitute the experience effect back into the baseline equation of potential earnings.

$$\ln P_{x+s} = \ln P_0 + s\rho_s + \left(\rho_0\kappa + \frac{\rho_0\kappa}{2T}\right)x - \frac{\rho_0\kappa}{2T}x^2$$

Accounting for the difference in potential and observed earnings:

$$\begin{aligned}\ln Y(s,x) &\approx \ln P_{x+s} - \kappa\left(1 - x/T\right) \\ &= \left[\ln P_0 - \kappa\right] + \rho_s s + \left(\rho_0 \kappa + \frac{\rho_0 \kappa}{2T} + \frac{\kappa}{T}\right) x - \frac{\rho_0 \kappa}{2T} x^2\end{aligned}$$

$\Rightarrow \rho_s$ is the average earnings increase with schooling

Standard Mincer Equation

$$\ln Y(s,x) = \alpha + \rho_s s + \beta_0 x + \beta_1 x^2,$$

where

$$\alpha = \ln P_0 - \kappa$$
$$\beta_0 = \left(\rho_0 \kappa + \frac{\rho_0 \kappa}{2T} + \frac{\kappa}{T}\right)$$
$$\beta_1 = -\frac{\rho_0 \kappa}{2T}$$

What about heterogeneous returns?

Random Coefficient Version

$$\ln Y(s_i, x_i) = \alpha_i + \rho_{si} s_i + \beta_{0i} x_i + \beta_{1i} x_i^2$$

and let

$$\begin{aligned} \bar{\alpha} &= \mathrm{E}[\alpha_i] \qquad & \bar{\rho}_s &= \mathrm{E}[\rho_{si}] \\ \bar{\beta}_0 &= \mathrm{E}[\beta_{0i}] \qquad & \bar{\beta}_1 &= \mathrm{E}[\beta_{1i}] \end{aligned}$$

Dropping individual subscripts ...

$$
\begin{aligned}
\ln Y(s,x) &= \bar{\alpha} + \bar{\rho}_s s + \bar{\beta}_0 x + \bar{\beta}_1 x^2 \\
&\quad + \underbrace{[(\alpha - \bar{\alpha}) + (\rho_s - \bar{\rho}_s)s + (\beta_0 - \bar{\beta}_0)x + (\beta_1 - \bar{\beta}_1)x^2]}_{\epsilon}
\end{aligned}
$$

$\Rightarrow$ If the schooling decision is determined by individual returns, then we are back in the case of a correlated random coefficient model (Heckman, Urzua, & Vytlacil, 2006).

Table 2: Estimated Coefficients from Mincer Log Earnings Regression for Men

		Whites		Blacks	
		Coefficient	Std. Error	Coefficient	Std. Error
1940	Intercept	4.4771	0.0096	4.6711	0.0298
	Education	0.1250	0.0007	0.0871	0.0022
	Experience	0.0904	0.0005	0.0646	0.0018
	Experience-Squared	-0.0013	0.0000	-0.0009	0.0000
1950	Intercept	5.3120	0.0132	5.0716	0.0409
	Education	0.1058	0.0009	0.0998	0.0030
	Experience	0.1074	0.0006	0.0933	0.0023
	Experience-Squared	-0.0017	0.0000	-0.0014	0.0000
1960	Intercept	5.6478	0.0066	5.4107	0.0220
	Education	0.1152	0.0005	0.1034	0.0016
	Experience	0.1156	0.0003	0.1035	0.0011
	Experience-Squared	-0.0018	0.0000	-0.0016	0.0000
1970	Intercept	5.9113	0.0045	5.8938	0.0155
	Education	0.1179	0.0003	0.1100	0.0012
	Experience	0.1323	0.0002	0.1074	0.0007
	Experience-Squared	-0.0022	0.0000	-0.0016	0.0000
1980	Intercept	6.8913	0.0030	6.4448	0.0120
	Education	0.1023	0.0002	0.1176	0.0009
	Experience	0.1255	0.0001	0.1075	0.0005
	Experience-Squared	-0.0022	0.0000	-0.0016	0.0000
1990	Intercept	6.8912	0.0034	6.3474	0.0144
	Education	0.1292	0.0002	0.1524	0.0011
	Experience	0.1301	0.0001	0.1109	0.0006
	Experience-Squared	-0.0023	0.0000	-0.0017	0.0000

Notes: Data taken from 1940-90 Decennial Censuses. See Appendix B for data description.

We can analyze this model in a Jupyter Noteboook. Visit

`http://bit.ly/2kAtcyg`

for the implementation.

Implications

- Log-earnings experience profiles are parallel across schooling levels.

$$\frac{\partial \ln Y(s,x)}{\partial s \partial x} = 0$$

- Log-earnings age profiles diverge with age across schooling levels.

$$\frac{\partial \ln Y(s,x)}{\partial s \partial t} = \frac{\rho_0 \kappa}{T} > 0$$

Figure: Experience profiles

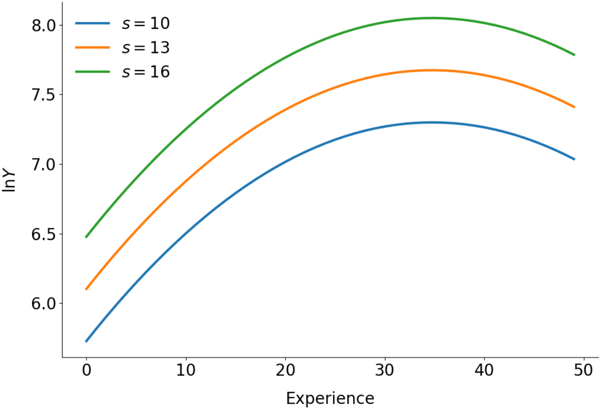
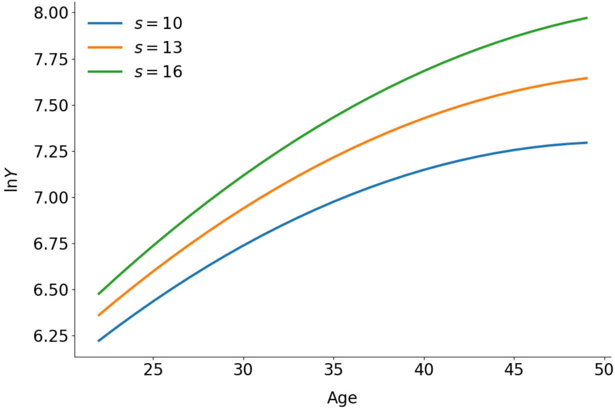


Figure: Age profiles



- The variance of earnings over the life cycle has a U-shaped pattern.

Derivation for minimizing variance

$$
\begin{aligned}
\ln Y(s,x) &= \ln P_{s+x} + \ln(1 - k_{s+x}) \\
&\approx \ln P_s + \rho_0 \sum_{j=0}^{x-1} k_{s+j} - k_{s+x}
\end{aligned}
$$

Further, using the assumption of linearly declining investment yields

$$
\ln Y(s,x) \approx \ln P_s + \kappa \left(\rho_0 \sum_{j=0}^{x-1} (1 - j/T) - (1 - x/T) \right)
$$

Assuming only initial earnings potential P_s and investment levels κ vary in the population, the variance of log earnings is given by

$$
\begin{aligned}
\operatorname{var}(\ln Y(s,x)) = {} & \operatorname{var}(\ln P_s) \\
& + \left(\rho_0 \sum_{j=0}^{x-1}(1-j/T) - (1-x/T)\right)^2 \operatorname{var}(\kappa) \\
& + 2\left(\rho_0 \sum_{j=0}^{x-1}(1-j/T) - (1-x/T)\right) \operatorname{cov}(\ln P_s, \kappa).
\end{aligned}
$$

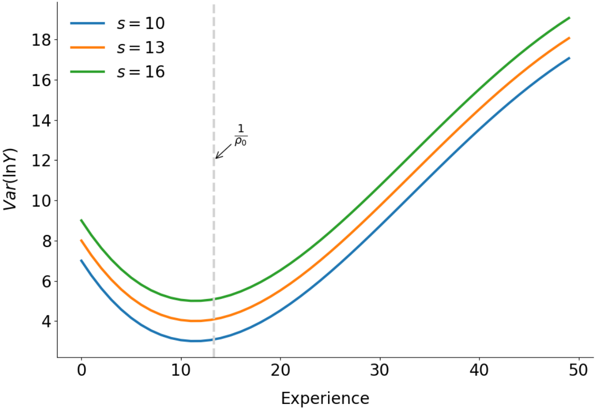
If κ and $\ln P_s$ are uncorrelated, then earnings are minimized (and equal to $Var(\ln P_s)$) when

$$\rho_0 \sum_{j=0}^{x-1} (1 - j/T) = 1 - x/T, or$$

$$\rho_0 \left(x - \frac{x(x-1)}{2T} \right) = (1 - x/T).$$

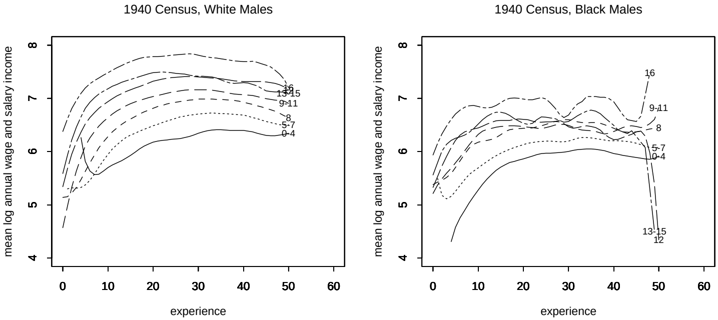
Clearly, $\lim_{T\to\infty} x^* = \frac{1}{\rho_0}$, so the variance minimizing age is $\frac{1}{\rho_0}$ when the work-life is long.

Figure: Variance profiles



Empirical Evidence

Figure 1a: Experience-Earnings Profiles, 1940-1960



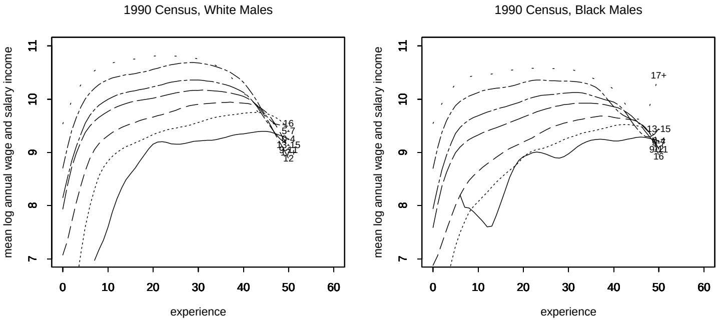
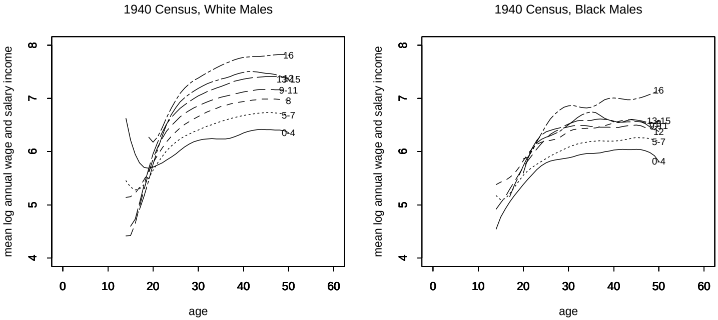

1990 Census, White Males
mean log annual wage and salary income
experience
16
5-7
0-4
13-15
9-11
12
1990 Census, Black Males
mean log annual wage and salary income
experience
17+
13-15
9-11
16

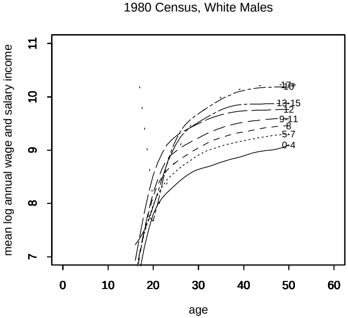
Table 1: Tests of Parallelism in Log Earnings Experience Profiles for Men

Sample	Experience Level	Estimated Difference Between College and High School Log Earnings at Different Experience Levels 1940	1950	1960	1970	1980	1990
Whites	10	0.54	0.30	0.46	0.41	0.37	0.59
	20	0.40	0.40	0.43	0.49	0.45	0.54
	30	0.54	0.27	0.46	0.48	0.43	0.52
	40	0.58	0.21	0.50	0.45	0.27	0.30
	p-value	0.32	0.70	<0.001	<0.001	<0.001	<0.001
Blacks	10	0.20	0.58	0.48	0.38	0.70	0.77
	20	0.38	0.05	0.25	0.22	0.48	0.69
	30	-0.11	0.24	0.08	0.33	0.36	0.53
	40	-0.20	0.00	0.73	0.26	0.22	-0.04
	p-value	0.46	0.55	0.58	0.91	<0.001	<0.001

Notes: Data taken from 1940-90 Decennial Censuses without adjustment for inflation. Because there are very few blacks in the 1940 and 1950 samples with college degrees, especially at higher experience levels, the test results for blacks in those years refer to a test of the difference between earnings for high school graduates and persons with 8 years of education. See Appendix B for data description. See Appendix C for the formulae used for the test statistics.

Figure 2: Age-Earnings Profiles, 1940,1960,1980




1980 Census, White Males
mean log annual wage and salary income
age
17+
16
13-15
12
9-11
8
5-7
0-4
0
10
20
30
40
50
60
7
8
9
10
11

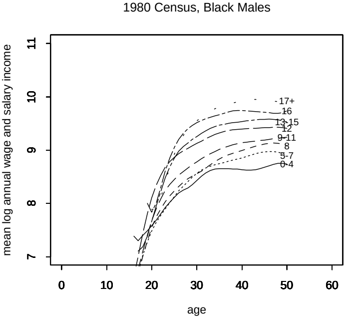
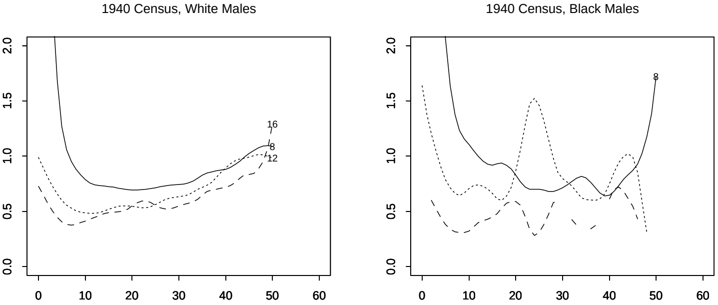
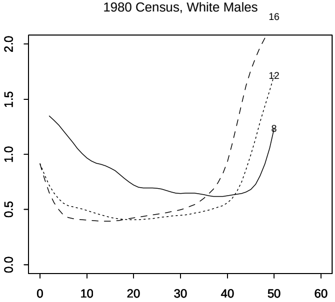
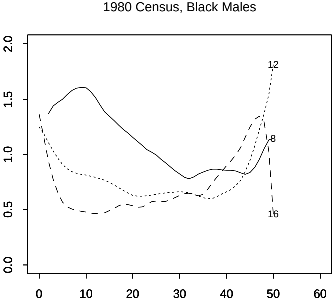

1980 Census, Black Males
mean log annual wage and salary income
age
17+
16
13-15
12
9-11
8
5-7
0-4
0
10
20
30
40
50
60
7
8
9
10
11

Figure 3: Experience-Variance Log Earnings





1980 Census, White Males
16
12
8
0.0
0.5
1.0
1.5
2.0
0
10
20
30
40
50
60



1980 Census, Black Males
12
8
16
0.0
0.5
1.0
1.5
2.0
0
10
20
30
40
50
60

In the end, Heckman, Lochner, and Todd (2006) conclude:

> *In common usage, the coefficient on schooling in a regression of log earnings on years of schooling is often called a rate of return. In fact, it is a price of schooling from a hedonic market wage equation. It is a growth rate of market earnings with years of schooling and not an internal rate of return measure, except under stringent conditions which we specify, test and reject in this chapter.*

Internal rate of returns

We study two different concepts of the rate of return in schooling:

- marginal differences
- non-marginal differences

Income Maximization under Perfect Certainty

s	schooling level
x	experience level
$Y(s, x)$	wage income
$T(s)$	last age of earnings
v	tuition and psychic cost of schooling
τ	proportional tax rate
r	before-tax interest rate

Present Discounted Value of Lifetime Earnings

$$V(s) = \int_0^{T(s)-s} (1-\tau)e^{-(1-\tau)r(x+s)}Y(s,x)dx - \int_0^{s} ve^{-(1-\tau)rz}dz$$

First-Order Condition

$$
\begin{aligned}
&[T'(s)-1]e^{-(1-\tau)r(T(s)-s)}Y(s,T(s)-s) \\
&-(1-\tau)r\int_0^{T(s)-s} e^{-(1-\tau)rx}Y(s,x)dx \\
&+\int_0^{T(s)-s} e^{-(1-\tau)rx}\frac{\partial Y(s,x)}{\partial s}dx \\
&-\frac{v}{1-\tau}=0
\end{aligned}
$$

Rearranging and defining $\tilde{r} = (1 - \tau) r$ ...

$$\tilde{r} = \frac{[T'(s) - 1] e^{-\tilde{r}(T(s)-s)} Y(s, T(s) - s)}{\int_0^{T(s)-s} e^{-\tilde{r}x} Y(s, x) dx} \tag{1}$$

$$+ \frac{\int_0^{T(s)-s} e^{-\tilde{r}x} \left[\frac{\partial Y(s,x)}{\partial s} \right] dx}{\int_0^{T(s)-s} e^{-\tilde{r}x} Y(s, x) dx} \tag{2}$$

$$- \frac{\frac{v}{1-\tau}}{\int_0^{T(s)-s} e^{-\tilde{r}x} Y(s, x) dx} \tag{3}$$

Interpretation

- (1) ... the change in the present value of earnings due to a change in working-life with additional schooling
- (2) ... weighted average effect of schooling on log earnings by experience
- (3) ... tuition and psychic costs

All components are expressed as a fraction of the present value of earnings measured at age s

Getting back to Mincer

- no tuition and psychic costs of schooling

  $\Rightarrow v = 0$

- no loss of working life from schooling

  $\Rightarrow T'(s) = 1$

- multiplicative separability between schooling and experience component of earnings

  $\Rightarrow Y(s, x) = \mu(s)\psi(x)$

$$\tilde{r} = \frac{\mu'(s)}{\mu(s)} \quad \forall \quad s$$

Thus, wage growth must be log linear in schooling and $\mu(s) = \mu(0)e^{\rho_s s}$ and $\tilde{r} = \rho_s$.

Heckman, Lochner, and Todd (2006) thus establish ...

> *After allowing for taxes, tuition, variable length of working life, and a flexible relationship between earnings, schooling and experience, the coefficient on years of schooling in a log earnings regression need no longer equal the internal rate of return.*

Structural Approach for the IRR

The internal rate of return for schooling level s_1 versus s_2, $r(s_1, s_2)$ solves ...

$$\int_0^{T(s_1)-s_1} (1-\tau)e^{-r(x+s_1)}Y(s_1,x)dx - \int_0^{s_1} ve^{-rz}dz$$
$$= \int_0^{T(s_2)-s_2} (1-\tau)e^{-r(x+s_2)}Y(s_2,x)dx - \int_0^{s_2} ve^{-rz}dz$$

Back to Mincer ....

- no taxes and no direct or psychic costs of schooling

$\Rightarrow v = 0$ and $\tau = 0$

$$\int_0^{T(s_1)-s_1} e^{-r(x+s_1)} Y(s_1, x) dx = \int_0^{T(s_2)-s_2} e^{-r(x+s_2)} Y(s_2, x) dx$$

- equal work-lives irrespective of years of schooling

$$\Rightarrow T = T(s_1) - s_1 = T(s_2) - s_2$$

$$\int_0^T e^{-r(x+s_1)} Y(s_1, x) dx = \int_0^T e^{-r(x+s_2)} Y(s_2, x) dx$$

- parallelism in experience across schooling categories

$$\Rightarrow Y(s,x) = \mu(s)\psi(x)$$

$$\int_0^T e^{-r(x+s_1)}\mu(s)\psi(x)dx = \int_0^T e^{-r(x+s_2)}\mu(s)\psi(x)dx$$

- linearity of log earnings in schooling

$$\Rightarrow \quad \mu(s) = \mu(0)e^{\rho_s s}$$

$$\int_0^T e^{-r(x+s_1)}\mu(0)e^{\rho_s s_1}\psi(x)dx = \int_0^T e^{-r(x+s_2)}\mu(0)e^{\rho_s s_2}\psi(x)dx$$

After some further rearranging ...

$$e^{(\rho_s - r)s_1} = e^{(\rho_s - r)s_2}$$

$$\Rightarrow \rho_s = r$$

Empirical Evidence

Specifications

- **relax linearity in** S, including indicator variables for each level of schooling
- **relax linearity and parallelism**, nonparametrically estimated functions of experience, separately within each schooling level

Table 3a: Internal Rates of Return for White Men: Earnings Function Assumptions
(Specifications Assume Work Lives of 47 Years)

	Schooling Comparisons					
	6-8	8-10	10-12	12-14	12-16	14-16
1940						
Mincer Specification	13	13	13	13	13	13
Relax Linearity in S	16	14	15	10	15	21
Relax Linearity in S & Quad. in Exp.	16	14	17	10	15	20
Relax Lin. in S & Parallelism	12	14	24	11	18	26
1950						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	13	13	18	0	8	16
Relax Linearity in S & Quad. in Exp.	14	12	16	3	8	14
Relax Linearity in S & Parallelism	26	28	28	3	8	19
1960						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	9	7	22	6	13	21
Relax Linearity in S & Quad. in Exp.	10	9	17	8	12	17
Relax Linearity in S & Parallelism	23	29	33	7	13	25
1970						
Mincer Specification	13	13	13	13	13	13
Relax Linearity in S	2	3	30	6	13	20
Relax Linearity in S & Quad. in Exp.	5	7	20	10	13	17
Relax Linearity in S & Parallelism	17	29	33	7	13	24
1980						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	3	-11	36	5	11	18
Relax Linearity in S & Quad. in Exp.	4	-4	28	6	11	16
Relax Linearity in S & Parallelism	16	66	45	5	11	21
1990						
Mincer Specification	14	14	14	14	14	14
Relax Linearity in S	-7	-7	39	7	15	24
Relax Linearity in S & Quad. in Exp.	-3	-3	30	10	15	20

Table 3b: Internal Rates of Return for Black Men: Earnings Function Assumptions
(Specifications Assume Work Lives of 47 Years)

	Schooling Comparisons					
	6-8	8-10	10-12	12-14	12-16	14-16
1940						
Mincer Specification	9	9	9	9	9	9
Relax Linearity in S	18	7	5	3	11	18
Relax Linearity in S & Quad. in Exp.	18	8	6	2	10	19
Relax Linearity in S & Parallelism	11	0	10	5	12	20
1950						
Mincer Specification	10	10	10	10	10	10
Relax Linearity in S	16	14	18	-2	4	9
Relax Linearity in S & Quad. in Exp.	16	14	18	0	3	6
Relax Linearity in S & Parallelism	35	15	48	-3	6	34
1960						
Mincer Specification	11	11	11	11	11	11
Relax Linearity in S	13	12	18	5	8	11
Relax Linearity in S & Quad. in Exp.	13	11	18	5	7	10
Relax Linearity in S & Parallelism	22	15	38	5	11	25
1970						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	5	11	30	7	10	14
Relax Linearity in S & Quad. in Exp.	6	11	24	10	11	12
Relax Linearity in S & Parallelism	15	27	44	9	14	23
1980						
Mincer Specification	12	12	12	12	12	12
Relax Linearity in S	-4	1	35	10	15	19
Relax Linearity in S & Quad. in Exp.	-4	6	29	11	14	17
Relax Linearity in S & Parallelism	10	44	48	8	16	31
1990						
Mincer Specification	16	16	16	16	16	16
Relax Linearity in S	-5	-5	41	15	20	25
Relax Linearity in S & Quad. in Exp.	-3	-3	35	17	19	22

Table 4: Internal Rates of Return for White & Black Men: Accounting for Taxes and Tuition (General Non-Parametric Specification Assuming Work Lives of 47 Years)

		Schooling Comparisons					
		Whites			Blacks		
		12-14	12-16	14-16	12-14	12-16	14-16
1940	No Taxes or Tuition	11	18	26	5	12	20
	Including Tuition Costs	9	15	21	4	10	16
	Including Tuition & Flat Taxes	8	15	21	4	9	16
	Including Tuition & Prog. Taxes	8	15	21	4	10	16
1950	No Taxes or Tuition	3	8	19	-3	6	34
	Including Tuition Costs	3	8	16	-3	5	25
	Including Tuition & Flat Taxes	3	8	16	-3	5	24
	Including Tuition & Prog. Taxes	3	7	15	-3	5	21
1960	No Taxes or Tuition	7	13	25	5	11	25
	Including Tuition Costs	6	11	21	5	9	18
	Including Tuition & Flat Taxes	6	11	20	4	8	17
	Including Tuition & Prog. Taxes	6	10	19	4	8	15
1970	No Taxes or Tuition	7	13	24	9	14	23
	Including Tuition Costs	6	12	20	7	12	18
	Including Tuition & Flat Taxes	6	11	20	7	11	17
	Including Tuition & Prog. Taxes	5	10	18	7	10	16
1980	No Taxes or Tuition	5	11	21	8	16	31
	Including Tuition Costs	4	10	18	7	13	24
	Including Tuition & Flat Taxes	4	9	17	6	12	21
	Including Tuition & Prog. Taxes	4	8	15	6	11	20
1990	No Taxes or Tuition	10	16	26	18	25	35
	Including Tuition Costs	9	14	20	14	18	25
	Including Tuition & Flat Taxes	8	13	19	13	17	22
	Including Tuition & Prog. Taxes	8	12	18	13	17	22

Notes: Data taken from 1940-90 Decennial Censuses. See discussion in text and Appendix B for a description of tuition and tax amounts.

Figure 4a: Average College Tuition Paid (in 2000 dollars)

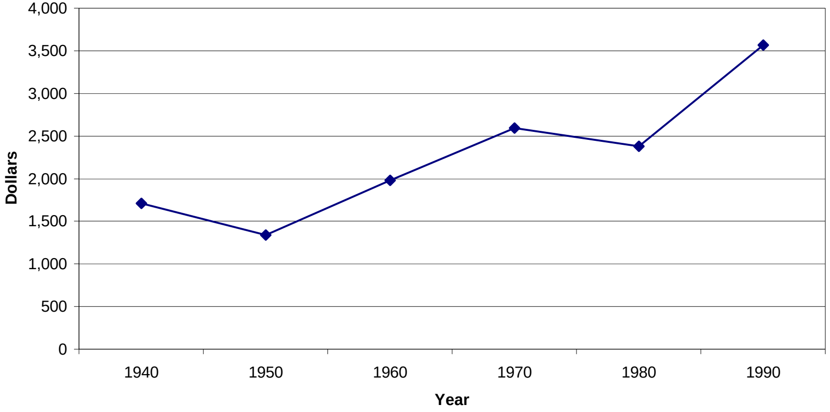


Figure 4b: Marginal Tax Rates
(from Barro & Sahasakul, 1983, Mulligan & Marion, 2000)

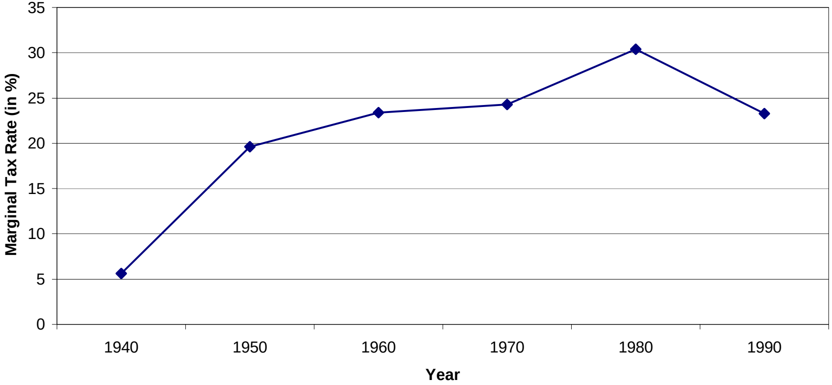


Figure 5: IRR for High School Completion (White and Black Men)

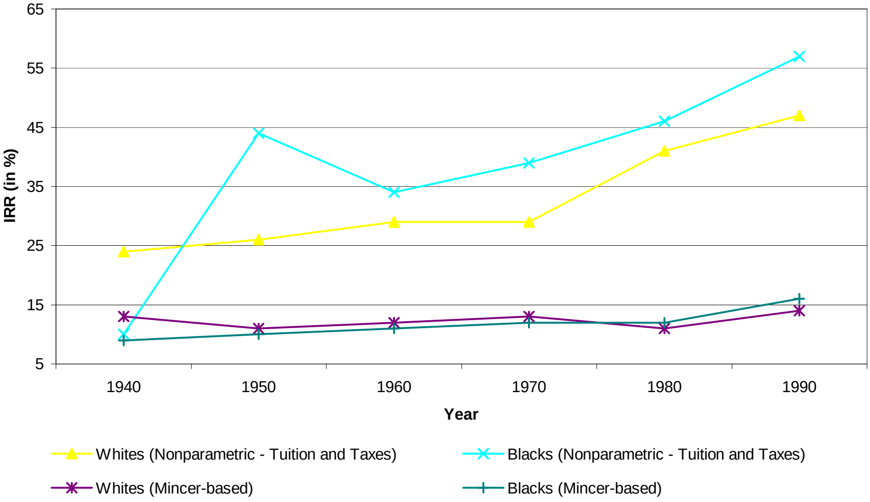
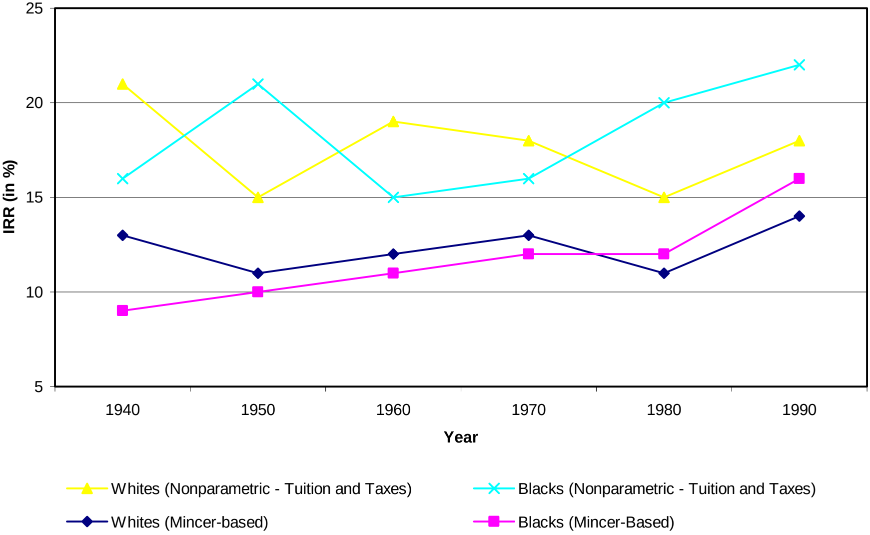


Figure 6: IRR for College Completion (White and Black Men)

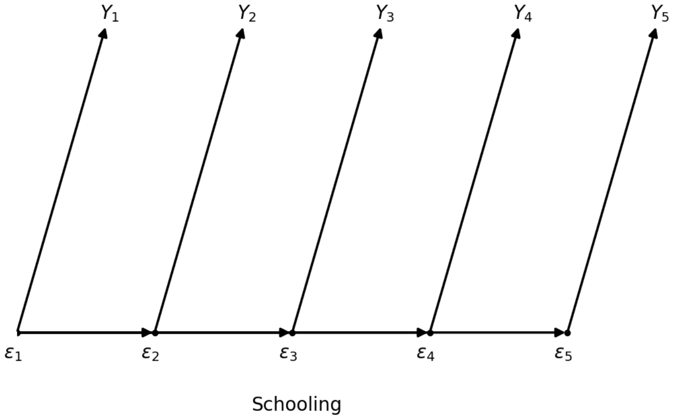


True rate of return

Suppose their is uncertainty about net earnings conditional on s and actual lifetime earnings for someone with s years of schooling are:

$$Y_s = \underbrace{\left[\sum_{x=0}^{T} (1+r)^{-x} Y(s,x)\right]}_{\bar{Y}_s} \epsilon_s$$

Figure: Model structure



The decision problem for a person with s years of schooling given the sequential revelation of information is to complete another year of schooling if

$$Y_s \leq \frac{E_s(V_{s+1})}{1+r}.$$

So the value of schooling level s, V_s, is

$$V_s = \max\left\{Y_s, \frac{E_s(V_{s+1})}{1+r}\right\}$$

for $s \leq \bar{S}$. At the maximum schooling level, $\bar{S}$, after all information is revealed, we obtain $V_{\bar{s}} = Y_{\bar{s}} = \bar{Y}_{\bar{s}}\epsilon_{\bar{s}}$.

The endogenously determined probability of going on from school level s to $s+1$ is

$$p_{s+1,s} = \Pr\left(\epsilon_s \leq \frac{E_s(V_{s+1})}{(1+r)\bar{Y}_s}\right),$$

where $E_s(V_{s+1})$ may depend on ϵ_s because it enters the agent's information set.

Thus, the expected value of schooling level s as perceived at current schooling $s-1$ is:

$$E_{s-1}(V_s) = \underbrace{(1-p_{s+1,s})\bar{Y}_s E_{s-1}\left(\epsilon \mid \epsilon \geq \frac{E_s(V_{s+1})}{(1+r)\bar{Y}_s}\right)}_{\text{direct return}}$$
$$+ \underbrace{p_{s+1,s}\left(\frac{E_{s-1}(V_{s+1})}{1+r}\right)}_{\text{option value}}.$$

Objects of interest

- Option value

$$O_{s,s-1} = \mathrm{E}_{s-1}\left[V_s - Y_s\right]$$

- True rate of return

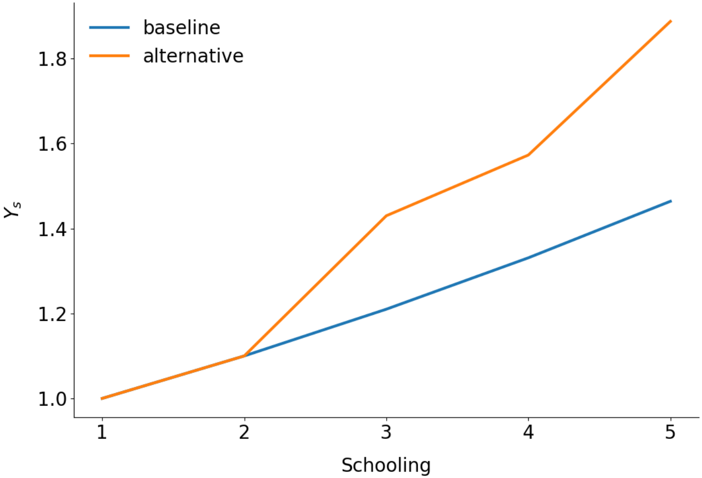
$$R_{s,s-1} = \frac{\mathrm{E}_{s-1}\left[V_s\right] - Y_{s-1}}{Y_{s-1}}$$

Model specification

$$\ln(\epsilon_s) \sim \mathbb{N}(0, \sigma) \qquad r = 0.1$$

$$Y_{s+1} = (1 + \rho_{s+1}) Y_s \qquad \sigma = 0.1$$

Figure: Scenarios



We can analyze this model in a Jupyter Noteboook. Visit

`http://bit.ly/2skwwli`

for the implementation.

Figure: Option values and uncertainty

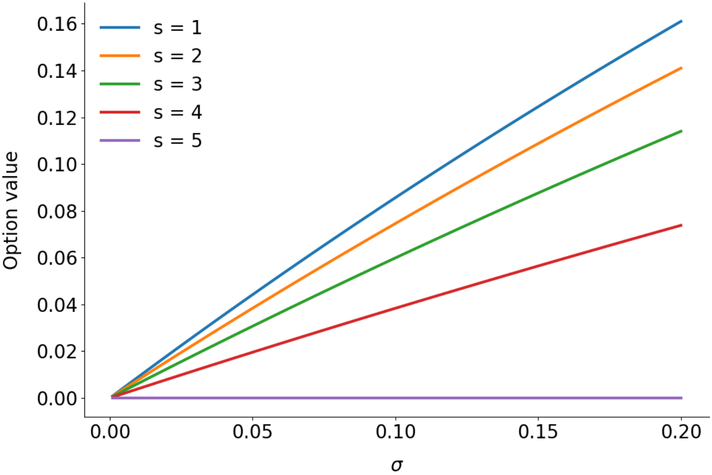


Figure: Option values and sheepskin effects

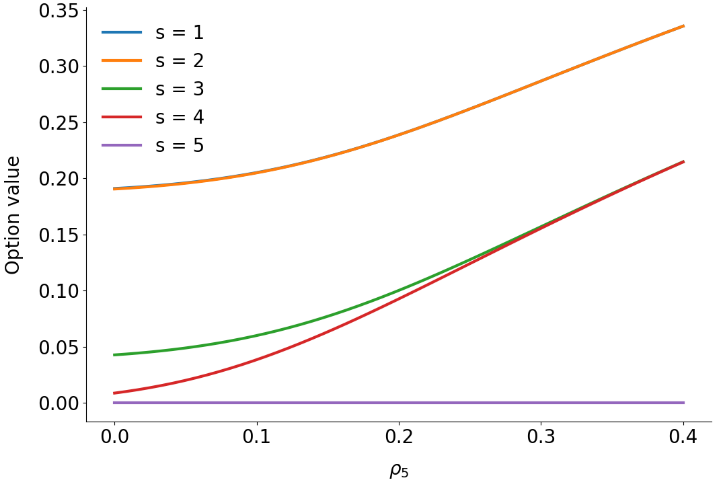
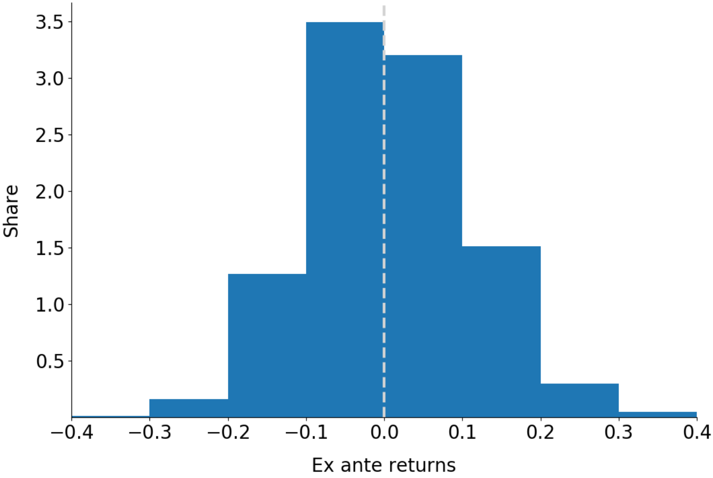


Figure: Returns



Conclusion

Appendix

References

Becker, G. S. (1964). *Human capital* (1st ed.). New York City, NY: Columbia University Press.

Chapman, & Hall. (2018). *Crc standard mathematical tables and formulas* (33rd Edition ed.; D. Zwillinger, Ed.). Florida, FL: CRC Press.

Heckman, J. J., Lochner, L. J., & Todd, P. E. (2006). Earnings functions, rates of return and treatment effects: The Mincer equation and beyond. In E. A. Hanushek & F. Welch (Eds.), *Handbook of the economics of education* (1st ed., Vol. 1, pp. 307–458). Amsterdam, Netherlands: North-Holland Publishing Company.

Heckman, J. J., Urzua, S., & Vytlacil, E. J. (2006). Understanding instrumental variables in models with essential heterogeneity. *Review of Economics and Statistics*, *88*(3), 389–432.

Mincer, J. (1974). *Schooling, experience and earnings*. New York, NY: National Bureau of Economic Research.

Rosen, S. (1977). Human capital: A survey of empirical research. In R. G. Ehrenberg (Ed.), *Research in labor economics* (Vol. 1, pp. 3–40). Greenwich, CT: JAI Press.

Willis, R., & Rosen, S. (1979). Education and self-selection. *Journal of Political Economy*, *87*(5, Part 2), S7–S36.