Economics of Human Capital

Dynamic model of human capital accumulation

Philipp Eisenhauer

Introduction

We build on the following seminal paper:

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Roadmap

- Economic Model
- Mathematical Model
- Data
- Computational Model
- Results

Economic Model

Decision Problem

$t = 1, \ldots, T$	decision period
$s \in S$	state
$a \in A$	action
d_t	decision rule
$r_t(s, a)$	immediate reward

Timing of Events

Learn	Choose	Receive
$\{r_t(s,a)\}_{a\in A}$	$d_t(s)$	$r_t(s, d_t(s))$

t

Learn	Choose	Receive
$\{r_{t+1}(s',a)\}_{a\in A}$	$d_{t+1}(s')$	$r_{t+1}(s', d_{t+1}(s'))$

$t+1$

$\pi = (d_1, \ldots, d_T)$	policy
$h_t = (s_1, a_1, \ldots, s_t)$	history
δ	discount factor
$p_t(s, a) \in P_t(s, a)$	conditional distribution

Individual's Objective under Risk

$$v_1^{\pi^*}(s) = \max_{\pi \in \Pi} \mathrm{E}_s^{\pi}\left[\sum_{t=1}^{T} \delta^{t-1} r_t(X_t, d_t(X_t))\right]$$

Mathematical Model

Policy Evaluation

$$v_t^{\pi}(s) = \mathsf{E}_s^{\pi}\left[\sum_{\tau=t}^{T} \delta^{\tau-t} r_\tau(X_\tau, d_\tau(X_\tau))\right]$$

Inductive Scheme

$$v_t^{\pi}(s) = r_t(s, d_t(s)) + \delta \mathsf{E}_s^{\pi}\left[v_{t+1}^{\pi}(X_{t+1})\right]$$

Optimality Equations

$$v_t^{\pi^*}(s) = \max_{a \in A} \left\{ r_t(s,a) + \delta \mathrm{E}_s^p \left[v_{t+1}^{\pi^*}(X_{t+1}) \right] \right\}.$$

Backward Induction Algorithm for MDP

for $t = T, \ldots, 1$ **do**

  if t == T **then**

    $$v_T^{\pi^*}(s) = \max_{a \in A}\Big\{r_T(s,a)\Big\} \qquad \forall \quad s \in S$$

  else

    Compute $v_t^{\pi^*}(s)$ for each $s \in S$ by

$$v_t^{\pi^*}(s) = \max_{a \in A}\Big\{r_t(s,a) + \delta \mathrm{E}_s^p\Big[v_{t+1}^{\pi^*}(X_{t+1})\Big]\Big\}$$

    and set

$$d_t^{\pi^*}(s) = \arg\max_{a \in A}\Big\{r_t(s,a) + \delta \mathrm{E}_s^p\Big[v_{t+1}^{\pi^*}(X_{t+1})\Big]\Big\}$$

  end if

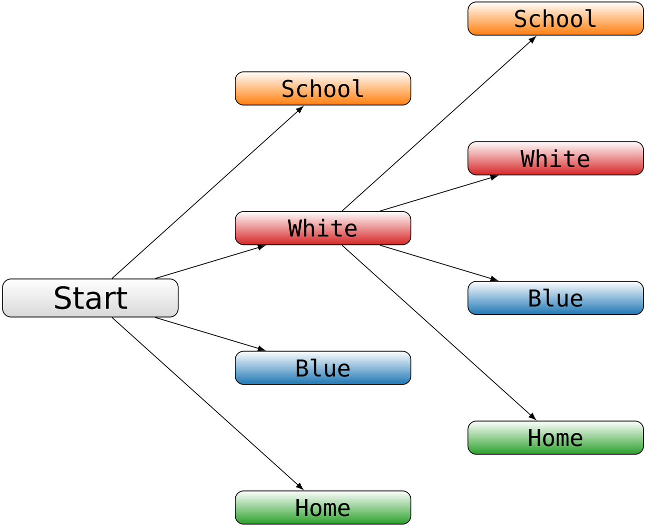
end for

Data

National Longitudinal Survey of Youth (1979)

- 1,373 white males starting at age 16
- life-cycle histories
  - school attendance
  - occupation-specific work status
  - real wages

Figure: Decision Tree



Descriptives

Figure: Sample Size

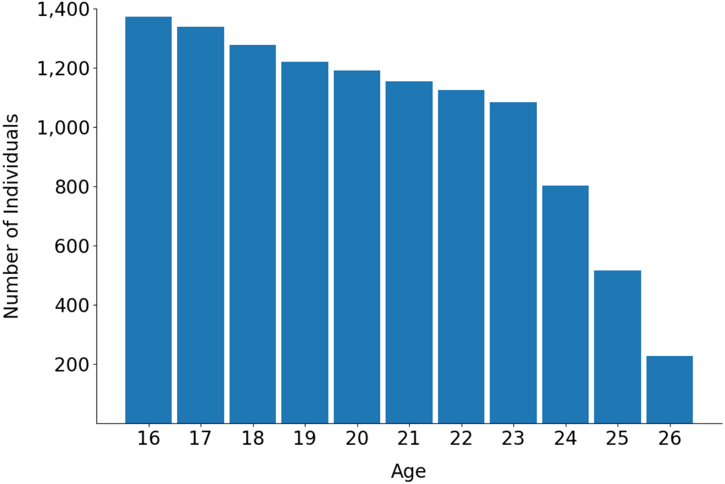


Figure: Observed Choices

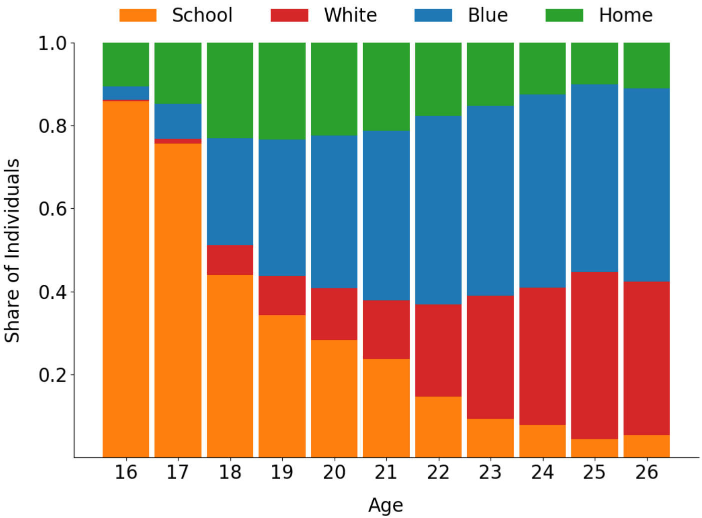


Table: Observed Real Wages

	White		Blue	
Age	Obs.	Mean	Obs.	Mean
16	2	.	26	10,287
20	128	5,499	349	14,432
25	201	16,540	222	21,991

Figure: Observed Transitions

Figure: Initial Schooling

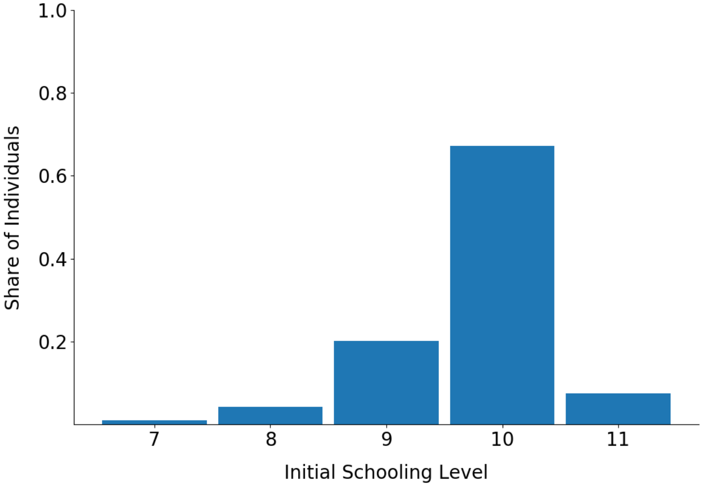


Table: Activities by Initial Schooling

	Initial Schooling				
Alternatives	7	8	9	10	11
School	0.69	0.86	2.48	3.37	2.83
White	0.08	0.38	0.65	1.36	2.04
Blue	3.69	3.62	3.05	2.40	1.98
Home	4.23	4.19	1.91	1.10	1.32
Total	8.69	9.05	8.09	8.24	8.17

Reduced-form Analysis

Table: Mincer Regressions

	Log Real Wages	
Intercept	8.314***	8.329***
Schooling	0.086***	0.077***
	Work Experience	
- linear	0.132***	0.125***
- squared	−0.005***	−0.003***
	Corrected AFQT	
- linear	—	0.002***
Adj-R^2	0.21	0.22
Observations	4,420	4,232

Table: Mincer Regressions

	Log Real Wages	
	White	Blue
Intercept	7.748***	8.790***
Schooling	0.128***	0.044***
	Own Experience	
- linear	0.146***	0.129***
- squared	−0.003	−0.005***
	Other Experience	
- linear	0.096***	0.085***
- squared	0.002	−0.003
Adj-R^2	0.28	0.17
Observations	1,468	2,952

Open Issues

- distinction between ex ante and ex post returns
- role of psychic costs
- nonlinearities in the return
- role of uncertainty

Computational Model

Additional Structure

t	age
k	unobserved type
$x_{j,t}$	experience in occupation j at age t
a_t	action at age j
g_t	level of schooling at age t

Skill Production Function

$$
\begin{aligned}
e_{j,k,t} = \exp\{e_{j,k,16} + &\underbrace{\alpha_{j,1} g_t + \alpha_{j,2} \mathrm{I}[g_t \geq 12] + \alpha_{j,3} \mathrm{I}[g_t \geq 16]}_{\text{schooling}} \\
&+ \underbrace{\alpha_{j,4} x_{j,t} + \alpha_{j,5} x_{j,t}^2 + \alpha_{j,6} \mathrm{I}[x_{j,t} > 0] + \alpha_{j,7} x_{j \neq j',t}}_{\text{work experience}} \\
&+ \underbrace{\alpha_{j,8} \mathrm{I}[a_{t-1} \neq j]}_{\text{depreciation}} + \alpha_{j,9}(t-16) + \alpha_{j,10} \mathrm{I}[t < 18] + \epsilon_{j,t}\}
\end{aligned}
$$

with $j, j' = 1, 2$, $k = 1, \ldots, 4$, and $t = 16, \ldots, 65$

Labor Market

$$r_{j,k,t} = w_{j,k,t} + \underbrace{\kappa_1 \mathrm{I}[g_t \geq 12] + \kappa_2 \mathrm{I}[g_t \geq 16]}_{\text{common returns}} + \beta_{j,1}$$
$$+ \underbrace{\beta_{j,2} \mathrm{I}[x_{j,t} > 0, a_{t-1} \neq j] + \beta_{j,3} \mathrm{I}[x_{j,t} = 0, a_{t-1} \neq j]}_{\text{entry cost}}$$

with $w_{j,k,t} = r_j e_{j,k,t}$

School

$$
\begin{aligned}
r_{3,k,t} = e_{3,k,16} &+ \underbrace{\gamma_1 \mathrm{I}[g_t \geq 12] + \gamma_2 \mathrm{I}[g_t \geq 16]}_{\text{monetary and psychic cost}} \\
&+ \underbrace{\gamma_3 \mathrm{I}[a_{t-1} \neq 3, g_t \leq 11] + \gamma_4 \mathrm{I}[a_{t-1} \neq 3, g_t \geq 12]}_{\text{reenrollment cost}} \\
&+ \gamma_5(t-16) + \gamma_6 \mathrm{I}[t \leq 18] + \underbrace{\kappa_1 \mathrm{I}[g_t \geq 12] + \kappa_2 \mathrm{I}[g_t \geq 16]}_{\text{common returns}} \\
&+ \epsilon_{3,t}
\end{aligned}
$$

Home

$$r_{4,k,t} = e_{4,k,16} + \zeta_1 \mathrm{I}[18 \leq t \leq 20] + \zeta_2 \mathrm{I}[t \geq 21] + \underbrace{\kappa_1 \mathrm{I}[g_t \geq 12] + \kappa_2 \mathrm{I}[g_t \geq 16]}_{\text{common returns}} + \epsilon_{4,t}$$

State Space

- at time t

$$\begin{aligned} s_t =& \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}, \{\epsilon_{j,t}\}_{j=1,\dots,4}\} \\ \bar{s}_t =& \{g_t, \{x_{j,t}\}_{j=1,2}, a_{t-1}\} \end{aligned}$$

- laws of motion

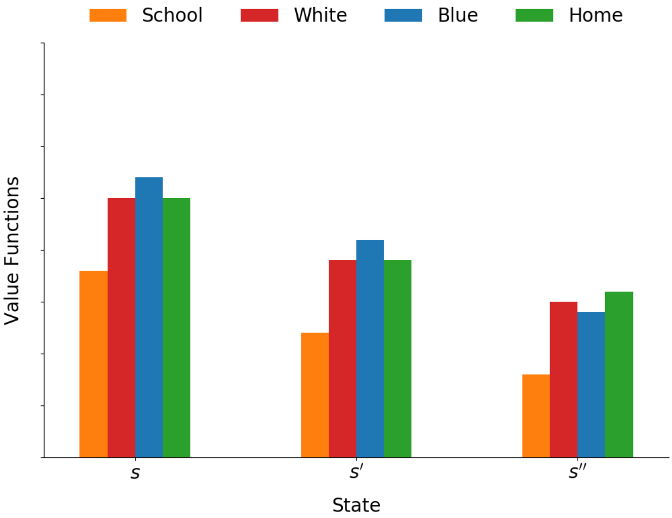
$$x_{j,t+1} = x_{j,t} + \mathrm{I}[a_t = j] \qquad \forall \quad j \in \{1, 2\}$$

$$g_{t+1} = g_t + \mathrm{I}[a_t = 3]$$

Distribution of shocks

$$[\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t}]^T \sim \mathcal{N}_0(\mathbf{0}, \Sigma)$$

Figure: Value Functions



Computational Tool

https://respy.readthedocs.io

- Technical Documentation
  - Numerical Methods, Source Codes, Test Suite
- User Documentation
  - Tutorial

⇒ Transparency, Recomputability, and Extensibility

Conclusion

Appendix

References

Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. *Journal of Political Economy*, *105*(3), 473–522.