

*M*icroeconometrics

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Generalized method of moments

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I heavily draw on the material presented in:

- ▶ Whitney Newey, course materials for 14.385 Non-linear Econometric Analysis, Fall 2007. MIT OpenCourseWare (<http://ocw.mit.edu>), Massachusetts Institute of Technology.

General idea

- ▶ The generalized method of moments (GMM) is a general estimation principle, where the estimators are derived from so-called moment conditions. It provides a unifying framework for the comparison of alternative estimators.

Structure

- ▶ Setup
- ▶ Identification
- ▶ Asymptotic distribution
- ▶ Testing

Setup

Notation

| | | |
|---------|--------------|------------------|
| β | $p \times 1$ | parameter vector |
|---------|--------------|------------------|

| | | |
|-------|-------------------|-------------|
| w_i | $i = 1, \dots, n$ | data points |
|-------|-------------------|-------------|

| | | |
|-------------------|--------------|--------|
| $g_i(w_i, \beta)$ | $m \times 1$ | moment |
|-------------------|--------------|--------|

- ▶ The GMM estimator is based on a model where, for the true parameter value β_0 the moment conditions $E[g_i(\beta_0)] = 0$ are satisfied.
- ▶ The estimator is formed by choosing β so that the sample average of $g_i(\beta)$ is close to its zero population value.

The estimator is formed by choosing β so that the sample average of $g_i(\beta)$ is close to its zero population value. Let

$$\hat{g}(\beta) = \frac{1}{n} \sum_{i=1}^n g_i(\beta)$$

- ▶ theoretical moments
- ▶ empirical moments

Let \hat{A} denote a $m \times m$ positive semi-definite matrix, then the GMM estimator is given by

$$\hat{\beta} = \arg \min_{\beta} \hat{g}(\beta)' \hat{A} \hat{g}(\beta)$$

The GMM estimator chooses $\hat{\beta}$ so the sample average $\hat{g}(\beta)$ is close to zero.

Instrumental variables

Let's work through an example on the blackboard.

Unifying framework

Many other popular estimation strategies can be analyzed in a GMM setup.

Ordinary least squares $E[x_i(y_i - x_i\beta_0)] = 0$

Instrumental variables $E[z_i(y_i - x_i\beta_0)] = 0$

Maximum likelihood $E[\partial \ln f(x_i, \beta_0)/\partial \beta] = 0$

If moments cannot be evaluated analytically then we have an application of the method of simulated moments.

Distance and weighing matrix

Let's look at the role of the weighing matrix for a two dimensional example.

- ▶ identity matrix

$$Q(\beta) = \begin{pmatrix} g_1 & g_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = g_1^2 + g_2^2$$

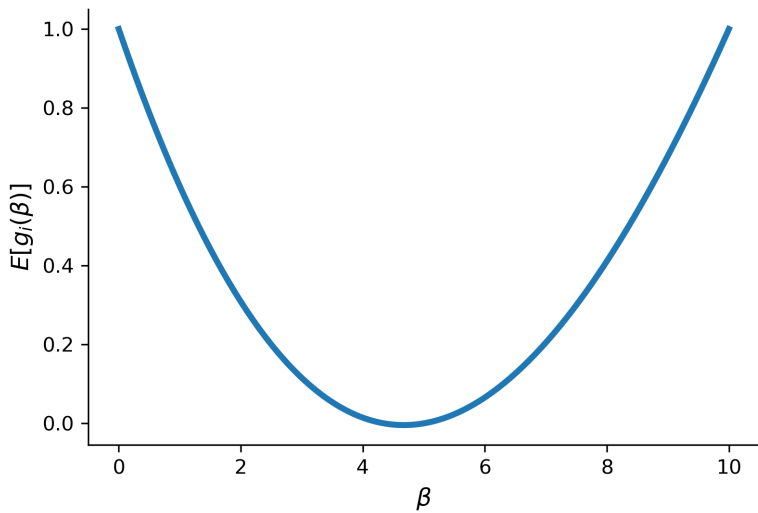
► alternative

$$Q(\beta) = \begin{pmatrix} g_1 & g_2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = 2\dot{g}_1^2 + g_2^2$$

Our alternative attaches more weight to the first coordinate in the distance.

Identification

The parameters β_0 are identified if β_0 is the only solution to $E[g_i(\beta)] = 0$.





- ▶ Necessary condition for identification is that $m \geq p$. When $m \leq p$, i.e. there are fewer equations to solve than parameters, there will typically be multiple solutions to the moment conditions.

- ▶ Let $G = E[\partial g_i(\beta_0)/\partial \beta]$. Rank condition is $\text{rank}(G) = p$. Necessary and sufficient for identification when $g_i(\beta)$ is linear in β .
- ▶ In the general nonlinear case it is difficult to specify conditions for uniqueness of the solution to $E[g_i(\beta)] = 0$.

- ▶ $m = p$, exact identification, $\hat{g}(\hat{\beta}) = 0$ asymptotically
- ▶ $m > p$, overidentification, $\hat{g}(\hat{\beta}) > 0$ asymptotically

In the case of overidentification, the choice of A matters and affects the estimator's asymptotic distribution.

Asymptotic distribution

Asymptotic distribution

Under some regularity conditions, the GMM estimator has the following asymptotic distribution.

$$\sqrt{n}(\hat{\beta} - \beta_0) \xrightarrow{d} \mathbb{N}(0, V),$$

where $V = (G'AG)^{-1}G'A\Omega AG(G'AG)^{-1}$ with $G = E[\partial g_i(\beta_0)/\partial \beta]$ and $\Omega = E[g_i(\beta_0)g_i(\beta_0)']$.

\Rightarrow asymptotic variance depends on the choice of the weighing matrix A

The optimal weighing matrix $A = \Omega^{-1}$ the asymptotic variance simplifies to

$$V = (G' \Omega^{-1} G)^{-1}$$

What makes a good moment?

- ▶ small Ω , small sample variation of the moment
- ▶ large G , moment informative on true value

Figure: Weak identification



Figure: Sharp identification



Instrumental variables

Let's continue our example on the blackboard.

Testing

An important statistic for GMM is the test of overidentifying restrictions that is given by

$$T = n \hat{g}(\hat{\beta})' \hat{\Sigma}^{-1} \hat{g}(\hat{\beta})$$

which converges in distribution to

$$T \xrightarrow{d} \chi^2(m - p)$$

under H_0 that the model is correctly specified.

Figure: Density of $\chi^2(2)$

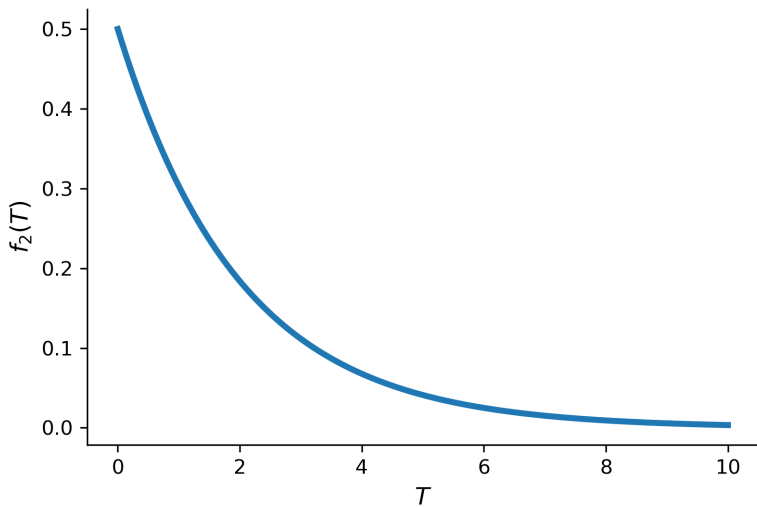
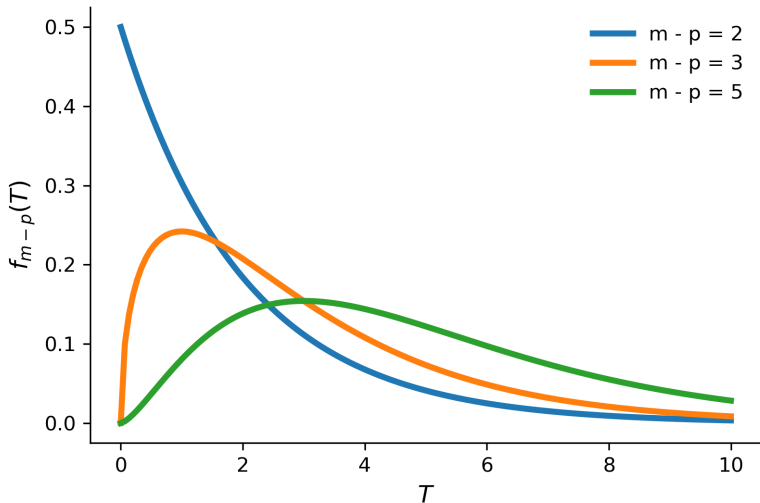


Figure: Density of $\chi^2(m-p)$



Wrapping up

Feasible efficient GMM

The optimal weighing matrix depends on moment evaluations at β_0 which is unknown.

- ▶ iterated feasible GMM
- ▶ continuously updating GMM

Let's turn to our Jupyter notebooks ...



Appendix

References

- Davidson, R., & MacKinnon, J. G. (2003). *Econometric theory and methods*. New York: Oxford University Press.
- Hall, A. A. (2005). *Generalized method of moments*. New York: Oxford University Press.