Causal Graphs*

Definitions, patterns, and strategies

Definitions

- A **node** represents a random variable labeled by letter. Observed random variables are marked by solid circle • and unobserved - by hollow circle ∘.
- An **edge** shows dependence between joining variables.
- **Adjacent variables** are connected by an edge.
- **Adjacent edges** meet at a variable.
- A **directed edge** represents the cause by a single-headed arrow.
- A **parent/child** is the starting(tail)/ending(head) variable. Therefore, a directed edge represents a direct effect of a parent on a child.
- A **root** is a variable that has no parent. In other words, it is an exogenous variable determined only by forces outside of the graph.
- A **sink** is a variable with no children.
- A **path** is a sequence of adjacent edges.
- A **directed path** is a path traced out entirely along arrows tail-to-head. If there is a directed path from A to B, A is an **ancestor** of B; B is a **descendant** of A.
- A **directed acyclic graph (DAG)** is a graph with only arrows for edges and no feedback loops (i.e. no variable is its own ancestor or its own descendant):
- **Mutual dependence** of two variables on one or more common causes is shown:

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Figure 1: Directed acyclical graph

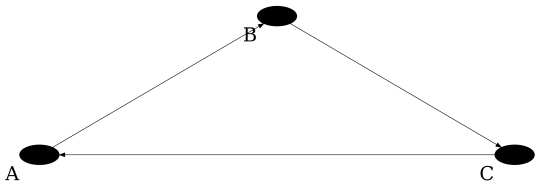
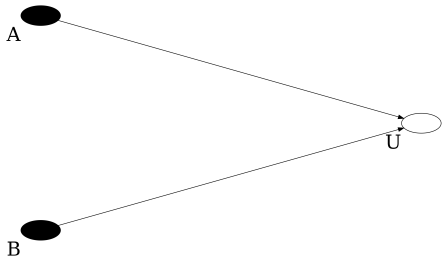


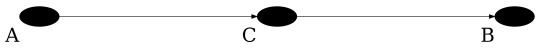
Figure 2: Mutual dependence



Patterns

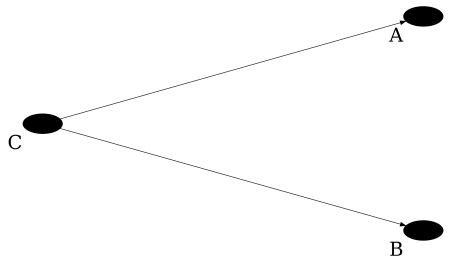
- **Chain of mediation** is a relationship when A affects B through A's causal effect on C and C's causal effect on B.

Figure 3: Mediation



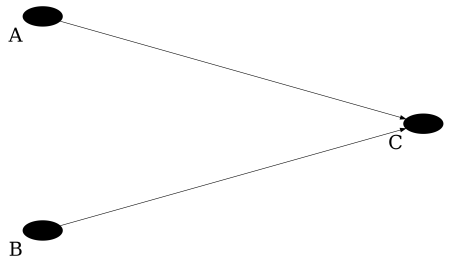
- **Mutual dependence** is a relationship when A and B are both caused by C.
- A **confounding variable** is a variable that affects both the dependent and independent variable.

Figure 4: Mutual dependence



- **Mutual causation** is a relationship when A and B are both causes of C.
- A **collider** is a variable that has two arrows running into it.

Figure 5: Mutual causation



- A **back-door path** is a path between any causally ordered sequence of two variables that include a directed edge that points to the first variable.
- **Conditioning** as a modeling strategy means transforming one graph into a simpler set of component graphs where fewer causes are represented.

Strategies

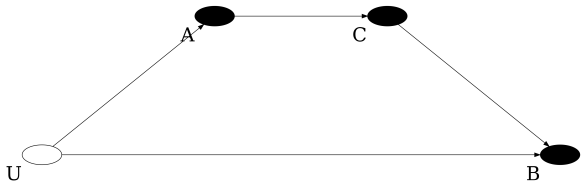
A **back-door criterion** is a set of conditions used to determine whether or not conditioning on a given set of observed variable will identify the causal effect. The causal effect is identified by conditioning on a set of variables Z if and only if all back-door paths between the causal variable and the outcome variable are blocked after conditioning on Z. All back-door paths are blocked by Z if and only if each back-door path:

- contains a chain of mediation A $\rightarrow$ C $\rightarrow$ B where the middle variable C is in Z, or
- contains a fork of mutual dependence A $\leftarrow$ C $\rightarrow$ B, where the middle variable C is in Z, or
- contains an inverted fork of mutual causation A $\rightarrow$ C $\leftarrow$ B, where the middle variable C and all of C's decendents are not in Z.

A **front-door criterion** is an empirical strategy used to identify the causal relationship flowing from A to B if one can find a mechanism C which:

- lies on the causal path between A and B, and
- it is the only such mechanism, and
- it is not affected by the unobserved confounder U.

Figure 6: Front-door criterion



You can find more on front-door criterion application in the Bellemare & Bloem (2020) paper.

References

Bellemare, M., & Bloem, J. (2020). The paper of how: Estimating treatment effects using the front-door criterion. *Working paper*.

Morgan, S. L., & Winship, C. (2014). *Counterfactuals and causal inference*. Cambridge, England: Cambridge University Press.

Pearl, J. (2009). *Causality*. Cambridge, England: Cambridge University Press.