Talks

Philipp Eisenhauer

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Simulation of choice probabilities

Philipp Eisenhauer

I draw on the material presented in:

- Train, K. (2009). *Discrete choice models with simulation*. Cambridge, New York: Cambridge University Press.

Probit setup

$$
\begin{aligned}
U_{nj} &= V_{nj} + \epsilon_{nj} \quad \forall j \in J \\
\epsilon_n' &= (\epsilon_{n1}, \ldots, \epsilon_{nJ})
\end{aligned}
$$

The approaches we discuss (mostly) have general applicability.

Choice probability

$$
\begin{aligned}
P_{ni} &= P(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i) \\
&= \int \mathrm{I}(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i) \phi(\epsilon_n) d\epsilon_n
\end{aligned}
$$

The choice probabilities do not have a closed form expression and must be approximated numerically.

- Quadrature
- Monte Carlo methods
  - crude accept-reject simulator
  - smoothed accept-reject simulator

Crude accept-reject simulator

1. Draw J values from the multivariate normal distribution to sample ϵ_n^r.

2. Calculate the simulated utilities U_{nj}^r for all alternatives.

3. Set $I^r = 1$ if U_{nj}^r is the maximum and zero otherwise.

Crude accept-reject simulator

4. Repeat the steps above R times

The simulated probability is the number of accepts divided by the number of repetitions: $\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^{R} I^r$.

Issues

- simulation for low probability events
- simulated probability is step function

We will explore these issues further in the accompanying notebook.

Smooth accept-reject simulator

- The smoothed AR simulator mitigates these difficulties with is to replace the 1 - 0 AR indicator with a smooth, strictly positive function.

- McFadden (1989) suggested the logit-smoothed AR simulator.

We replace step 3 with the following transformation:

$$S^r = \frac{\exp^{\frac{U_{ni}^r}{\lambda}}}{\sum_J \exp^{\frac{U_{nj}^r}{\lambda}}}$$

The simulated probability is the number of accepts divided by the number of repetitions: $\hat{P}_{ni} = \frac{1}{R}\sum_{r=1}^{R} S^r$.

We will explore these issues further in the accompanying notebook.

Appendix

References

Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, *69*(1), 201–209.

McFadden, D. L. (1989). A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, *57*(5), 995–1026.

Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex-post facto experiment. *Journal of Educational Psychology*, *51*(6), 309–317.

Train, K. (2009). *Discrete choice models with simulation*. Cambridge, New York: Cambridge University Press.