# **T**alks

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Material available on





### Simulation of choice probabilities

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#### I draw on the material presented in:

► Train, K. (2009). *Discrete choice models with simulation*. Cambridge, New York: Cambridge University Press.

#### **Probit setup**

$$U_{nj} = V_{nj} + \epsilon_{nj} \quad \forall j \in J$$
  
 $\epsilon'_n = (\epsilon_{n1}, \dots, \epsilon_{nJ})$ 

The approaches we discuss (mostly) have general applicability.

#### Choice probability

$$P_{ni} = P(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i)$$

$$= \int I(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i) \phi(\epsilon_n) d\epsilon_n$$

The choice probabilities do not have a closed form expression and must be approximated numerically.

- Quadrature
- Monte Carlo methods
  - crude accept-reject simulator
  - smoothed accept-reject simulator

#### **Crude accept-reject simulator**

- 1. Draw J values from the multivariate normal distribution to sample  $\epsilon_n^r$ .
- 2. Calculate the simulated utilities  $U_{nj}^r$  for all alternatives.
- 3. Set  $I^r = 1$  if  $U^r_{nj}$  is the maximum and zero otherwise.

#### Crude accept-reject simulator

4. Repeat the steps above R times

The simulated probability is the number of accepts divided by the number of repetitions:  $\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^{R} I^{r}$ .

#### Issues

- simulation for low probability events
- simulated probability is step function

We will explore these issues further in the accompanying notebook.

#### Smooth accept-reject simulator

- ► The smoothed AR simulator mitigates these difficulties with is to replace the 1 0 AR indicator with a smooth, strictly positive function.
- ► McFadden (1989) suggested the logit-smoothed AR simulator.

We replace step 3 with the following transformation:

$$S^{r} = \frac{\exp \frac{U_{ni}^{r}}{\lambda}}{\sum_{J} \exp \frac{U_{nj}^{r}}{\lambda}}$$

The simulated probability is the number of accepts divided by the number of repetitions:  $\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^{R} S^{r}$ .

We will explore these issues further in the accompanying notebook.

## **Appendix**

### References

- Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, 69(1), 201–209.
- McFadden, D. L. (1989). A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, 57(5), 995–1026.
- Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex-post facto experiment. *Journal of Educational Psychology*, *51*(6), 309–317.

Train, K. (2009). *Discrete choice models with simulation*. Cambridge, New York: Cambridge University Press.