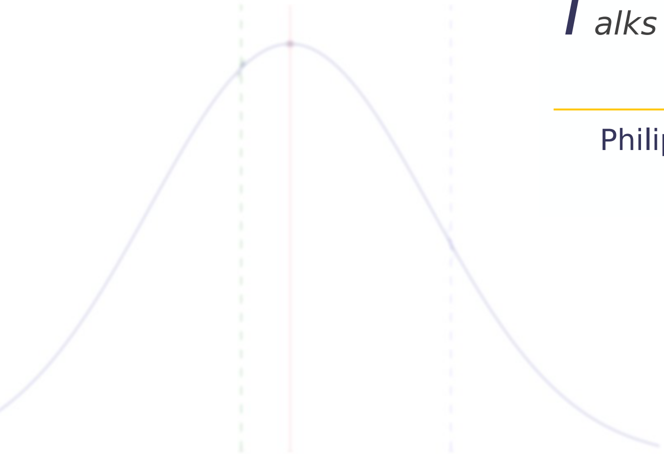

Talks

Philipp Eisenhauer

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Sensitivity analysis

Philipp Eisenhauer

I draw on the material presented in:

- Saltelli, A., Tarantola, S., Campolongo, F., & Ratto, M. (2004). *Sensitivity analysis in practice: A guide to assessing scientific models*. John Wiley & Sons.
- Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., . . . Tarantola, S. (2008). *Global sensitivity analysis: The primer*. John Wiley & Sons.

Definitions

Sensitivity analysis The study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input

- uncertainty propagation

Selected issues

- computational challenges
- deterministic vs. probabilistic
- independent vs. dependent
- global vs. local

Selected issues

- quantitative vs. qualitative
- interaction vs. dependence
- full model vs. surrogate

Settings

- *factor prioritization*, i.e. which factor is the one that, if determined (i.e., fixed to its true, albeit unknown, value), would lead to the greatest reduction in the variance of the output.
- *factor fixing*, i.e. which factor or the subset of input factors that we can fix at any given value over their range of uncertainty without significantly reducing the output variance.

Sensitivity analysis in economics

- Harenberg, D., Marelli, S., Sudret, B., & Winschel, V. (2019). Uncertainty quantification and global sensitivity analysis for economic models. *Quantitative Economics*, *10*(1), 1–41.

Notation

The model input vector $\mathbf{X} = (X_1, \ldots, X_N) \in \mathbb{R}^d$. The quantity of interest y of the model $f(\cdot)$:

$$Y = f(\mathbf{X})$$

Following the literature, all parameters x_i are scaled to take on values in the interval $[0, 1]$, and the region of interest Ω is the k- dimensional unit hypercube.

- We collect all parameter in $\mathbf{x} = [x_1, \ldots, x_n]$. x_i denotes one

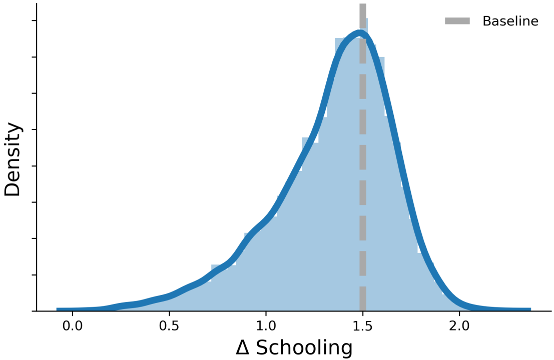
- particular value for input parameter i and $\mathbf{x}_{\sim i} = [x_1, \ldots, x_{i-}$
as the complementary set of inputs.

- We use the notation x_i and $\bar{x}_i$ to distinguish a random vector x_i generated from a joint probability density function $p(x_i, x_{\sim i})$ and a random vector $\bar{x}_i$ generated from a conditional probability distribution $p(\bar{x}_i, x_{\sim i} \mid x_{\sim i})$.

Uncertainty propagation

- We sample from the distribution of input parameters and assess the distribution of the quantity of interest.

Figure: Uncertainty propagation



Qualitative

Morris method for independent and dependent factors.

- Morris, M. D. (1991). Factorial sampling plans for preliminary computational experiments. *Technometrics*, *33*(2), 161–174.
- Ge, Q., & Menendez, M. (2017). Extending morris method for qualitative global sensitivity analysis of models with dependent inputs. *Reliability Engineering & System Safety*, *162*, 28–39.

- The approach segments the model input ranges $[x_i^-, x_i^+]$ in l levels. Given l levels with m inputs, there are l^n points in the grid from which a subset of r points is drawn at random. For each of the r, the model is evaluated performing a series of OAT sensitivities.

$$ee_i = \frac{f(x_1, \ldots, x_{i-1}, x_i + \Delta, x_{i+1}, \ldots, x_n) - f(\mathbf{x})}{\Delta}$$

$$ee_i^{ind} = \frac{f(\bar{x}_i, \mathbf{x}_{\sim i}) - f(x_i, \mathbf{x}_{\sim i})}{\Delta}$$

$$ee_i^{dep} = \frac{f(\bar{x}_i, \bar{\mathbf{x}}_{\sim i}) - f(x_i, \mathbf{x}_{\sim i})}{\Delta}$$

$$\mu_i^j = \frac{1}{N}\sum_{r=1}^{n} |ee_{ir}^j|$$

$$\sigma_i^j = \frac{1}{N-1}\sum_{r=1}^{n} (ee_{ir}^j - \mu_i)^2$$

Quantitative

$$V[Y] = V_{X_i}[E_{X_{\sim i}}[Y \mid X_i]] + E_{X_i}[V_{\mathbf{X}_{\sim i}}[Y \mid X_i]] \tag{1}$$

Main effect

- We rank all based on the smallest conditional variance $V[Y \mid X_i = x_i]$ evaluated over all possible vales x_i of X_i. Following Equation (1), this is equivalent to ranking factors by the largest $V_{X_i}[E_{\mathbf{X}_{\sim i}}[Y \mid X_i]]$ and so the main effect is defined as:

$$S_i^M = \frac{V_{X_i}[E_{\mathbf{X}_{\sim i}}[Y \mid X_i]]}{V[Y]}$$

Total effect

- We want to identify the factors that we can fix at their value without significantly reducing the output variance.

$$S_i^T = \frac{E_{\mathbf{X}_{\sim i}}[V_{X_i}[Y \mid \mathbf{X}_{\sim i}]]}{V[Y]} = 1 - S_{\sim i}^M$$

Appendix

References

Ge, Q., & Menendez, M. (2017). Extending morris method for qualitative global sensitivity analysis of models with dependent inputs. *Reliability Engineering & System Safety*, *162*, 28–39.

Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, *69*(1), 201–209.

Harenberg, D., Marelli, S., Sudret, B., & Winschel, V. (2019). Uncertainty quantification and global sensitivity analysis for economic models. *Quantitative Economics*, *10*(1), 1–41.

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Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex-post facto experiment. *Journal of Educational Psychology*, *51*(6), 309–317.