

# Introduction to the generalized Roy model

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# Introduction

## **Heckman (2008) defines three policy evaluation tasks:**

- ▶ Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- ▶ Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- ▶ Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

## **Econometrics of policy evaluation**

- ▶ is important
- ▶ is complicated
- ▶ is multifaceted

## **Numerous applications**

- ▶ labor economics
- ▶ development economics
- ▶ industrial economics
- ▶ health economics

## Numerous effects

- ▶ conventional average effects
- ▶ policy-relevant average effects
- ▶ marginal effects
- ▶ distributional effects
- ▶ effects on distributions

## **Numerous estimation strategies**

- ▶ instrumental variables
- ▶ (quasi-)experimental methods
- ▶ matching

# Model



## Generalized Roy model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

# *Individual Heterogeneity*

## Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

### **Sources of Heterogeneity**

- ▶ Difference in observables
- ▶ Difference in unobservables
  - ▶ Uncertainty
  - ▶ Private information

Figure: Distribution of benefits



## Econometric problems

- ▶ **Evaluation problem**, we only observe an individual in either the treated or untreated state.
- ▶ **Selection problem**, individuals that select into treatment differ from those that do not.

## Essential Heterogeneity

**Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

# **Objects of interest**

## Useful Notation

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$



Figure: First-stage unobservable

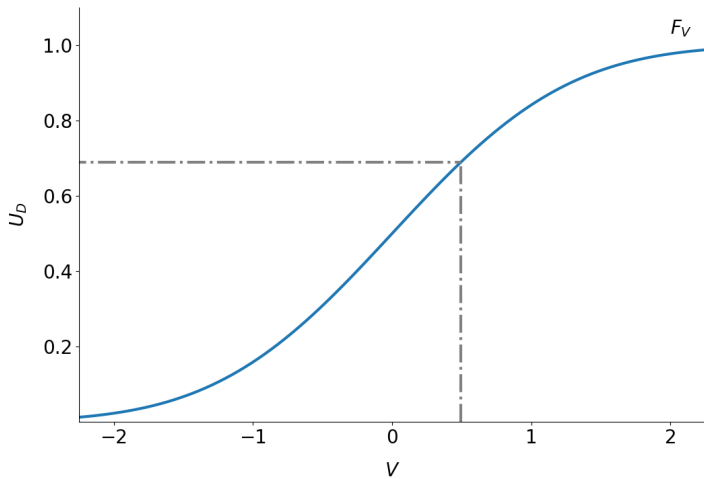


Figure: Support

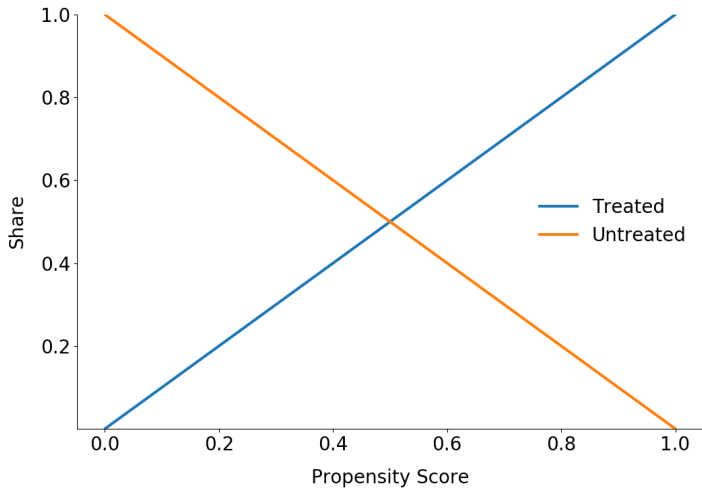
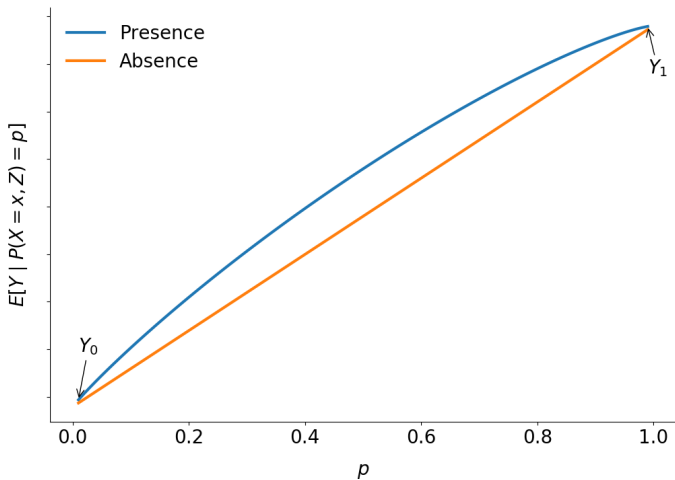


Figure: Distribution of benefits



Figure: Conditional expectation and essential heterogeneity



# *Conventional Average Treatment Effects*

## Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

⇒ correspond to *extreme* policy alternatives

## Selection Problem

$$\begin{aligned} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Selection on gains}} \\ &+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection on levels}} \end{aligned}$$

$$\begin{aligned}
 E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} \\
 &\quad + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}
 \end{aligned}$$

⇒ the bias depends on the parameter of interest



Figure: Distribution of effects with essential heterogeneity

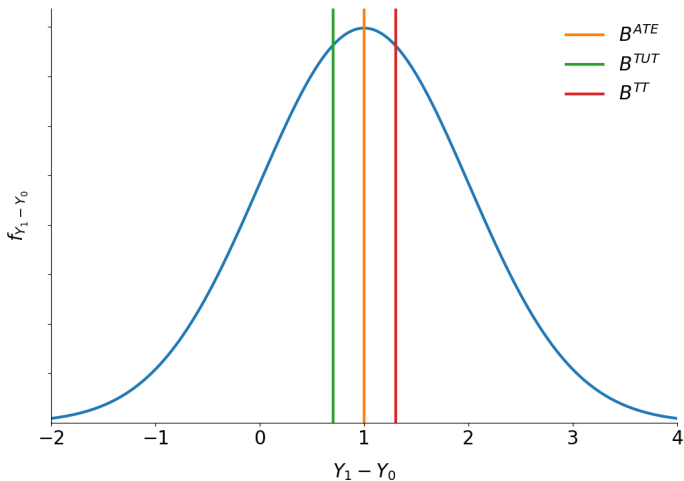
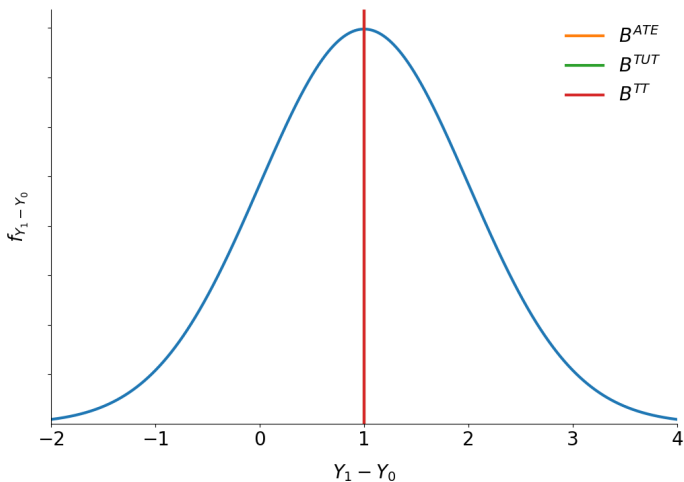


Figure: Distribution of effects without essential heterogeneity



# *Policy-Relevant Average Treatment Effects*

## Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

## Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

# *Marginal Effect of Treatment*

## Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

**Intuition:** Mean gross return to treatment for persons at quantile  $u_D$  of the first-stage unobservable  $V$  or a willingness to pay for individuals at the margin of indifference.

Figure: Margin of indifference

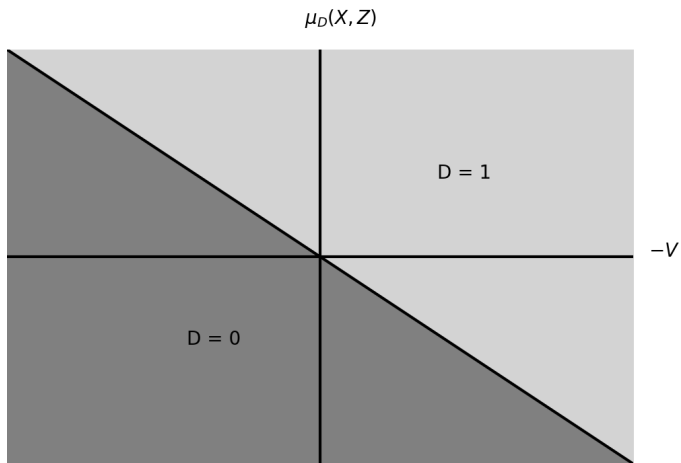
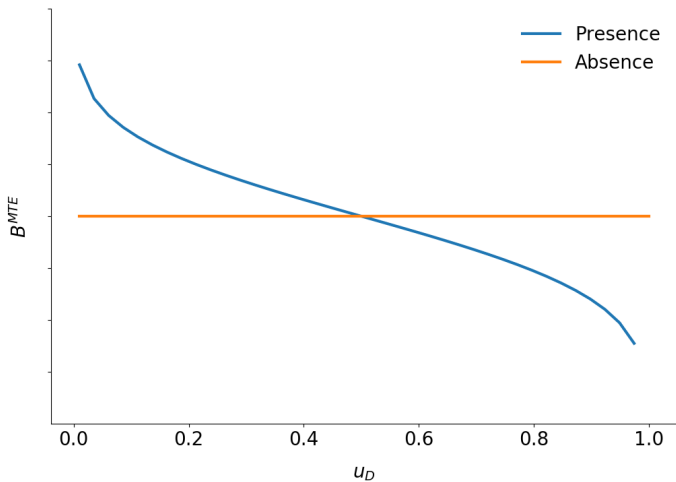


Figure:  $B^{MTE}$  and essential heterogeneity





**Effects of treatment as weighted averages** Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_D)$ .

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights  $\omega^j(x, u_D)$  are specific to parameter  $j$  and integrate to one.

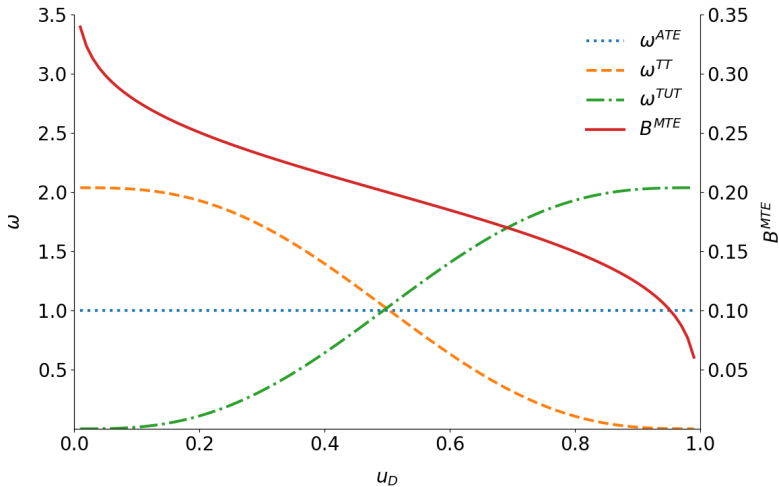
## Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P \mid X = x]}$$

Figure: Effects of treatment as weighted averages



# *Local Average Treatment Effect*

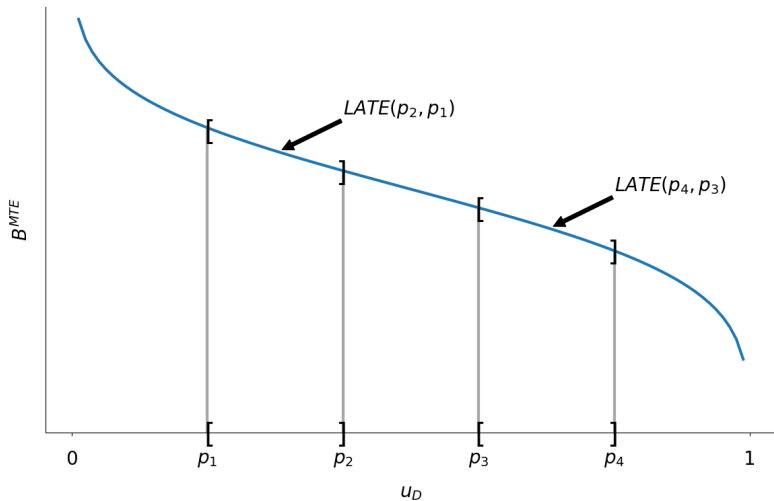
## Local Average Treatment Effect

- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.  
⇒ instrument-dependent parameter
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.  
⇒ deep economic parameter

$$B^{LATE} = \frac{E[Y | Z = z] - E[Y | Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_D, u_{D'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{D'}} B^{MTE}(x, u) du,$$

Figure: Local average treatment effect



## *Distributions of Effects*



## **Distributions of Effects**

- ▶ marginal distribution of benefits
- ▶ joint distribution of potential outcomes
- ▶ joint distribution of benefits and surplus

Figure: Distribution of benefits

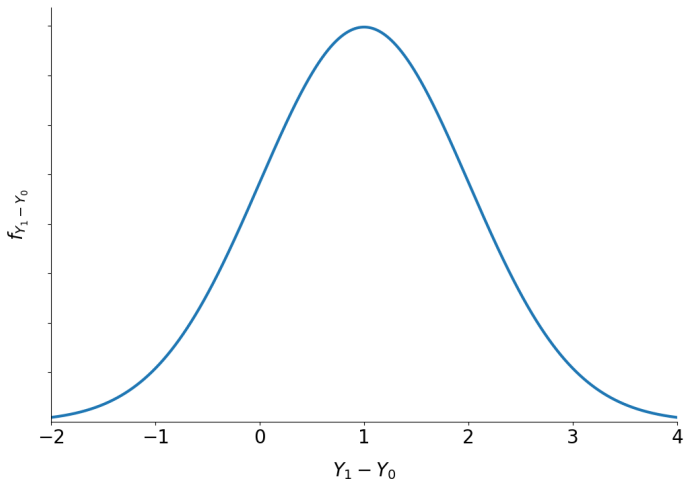


Figure: Distribution of potential outcomes

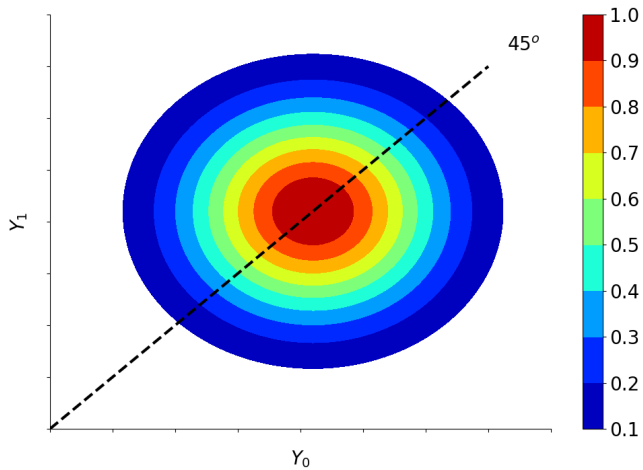
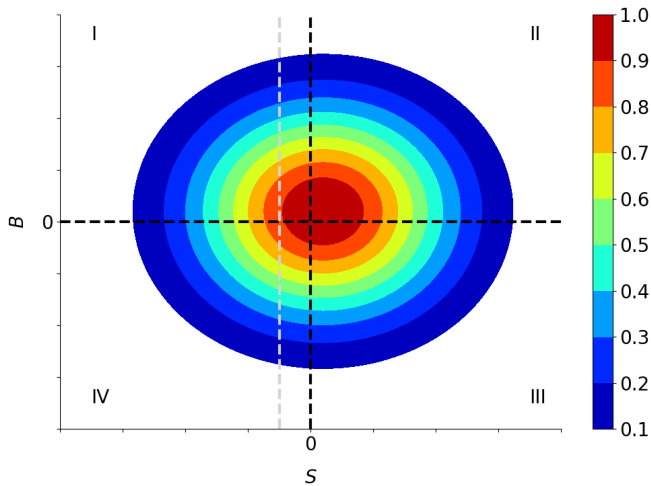


Figure: Distribution of benefits and surplus



# Conclusion

# Appendix

# *References*

Heckman, J. J. (2008). Schools, skills, and synapses. *Economic Inquiry*, 46, 289–324.