Generalized Roy Model

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. . . background material available at
https://github.com/policyMetrics/talks

Policy Evaluation Tasks

Heckman (2008) defines three policy evaluation tasks:

- Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Cost

$$C = \mu_D(Z) + U_C$$

Observed Outcomes

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$S = Y_1 - Y_0 - C$$

$$D = \mathrm{I}[S > 0]$$

The Generalized Roy Model

Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\begin{aligned}
\alpha &= \mu_0(X) \\
\beta &= (Y_1 - Y_0) = \mu_1(X) - \mu_0(X) + (U_1 - U_0) \\
\epsilon &= U_0
\end{aligned}$$

Econometric Problems

- **Evaluation Problem:** We only observe an individual in either the treated or untreated state.
- **Selection Problem:** Individuals that select into treatment different from those that do not.

Econometric Problems

Observed outcome for individual i:

$$Y_i = Y_{0i} + D_i(Y_{1i} - Y_{0i}) = \begin{cases} Y_{1i} & \text{if} \quad D_i = 1 \\ Y_{0i} & \text{if} \quad D_i = 0 \end{cases}$$

Econometric Problems

$$\begin{aligned}
\mu_S(X,Z) &= (\mu_1(X) - \mu_0(X)) - \mu_C(Z) \\
V &= U_C - (U_1 - U_0) \\
P(X,Z) &= \Pr(D = 1 \mid X, Z) = F_V(\mu_S(X,Z)) \\
U_s &= F_V(V)
\end{aligned}$$

Rewriting Choice Equation

$$D = \mathrm{I}[P(X,Z) > U_S]$$

Econometric Problems

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity

- Difference in Observable Characteristics
- Difference in Unobservable Characteristics
  - Uncertainty
  - Private Information

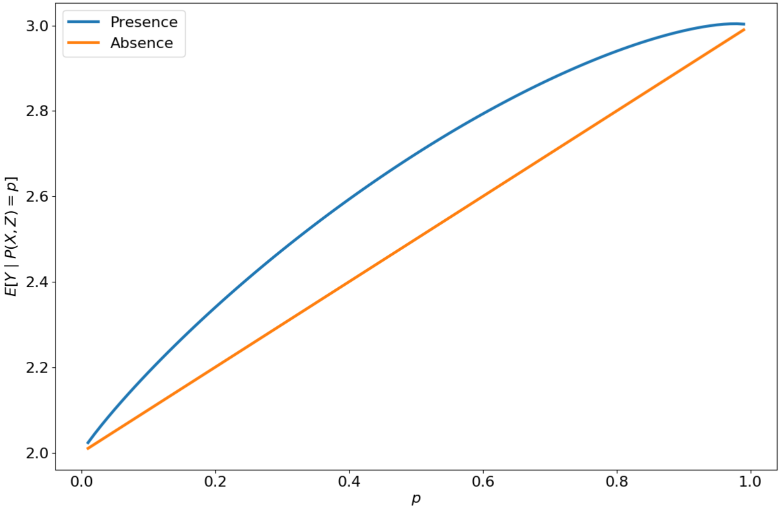
Essential Heterogeneity

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 / D \quad | X = x.$$

$\Rightarrow$ consequences for the choice of the estimation strategy

Figure: Conditional Expectation



Conventional Treatment Effects

$$
\begin{aligned}
B^{ATE} &= E[Y_1 - Y_0] \\
B^{TT} &= E[Y_1 - Y_0 \mid D = 1] \\
B^{TUT} &= E[Y_1 - Y_0 \mid D = 0]
\end{aligned}
$$

$\Rightarrow$ correspond to *extreme* policy alternatives

Selection Problem

$$
\begin{aligned}
E[Y \mid D=1] - E[Y \mid D=0] = &\underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\
&+ \underbrace{E[Y_1 - Y_0 \mid D=1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\
&+ \underbrace{E[Y_0 \mid D=1] - E[Y_0 \mid D=0]}_{\text{Selection Bias}}
\end{aligned}
$$

Selection Problem

$$E[Y \mid D = 1] - E[Y \mid D = 0] = \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}}$$
$$+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}$$

$\Rightarrow$ the bias depends on the parameter of interest

Figure: Treatment Effects with Essential Heterogeneity

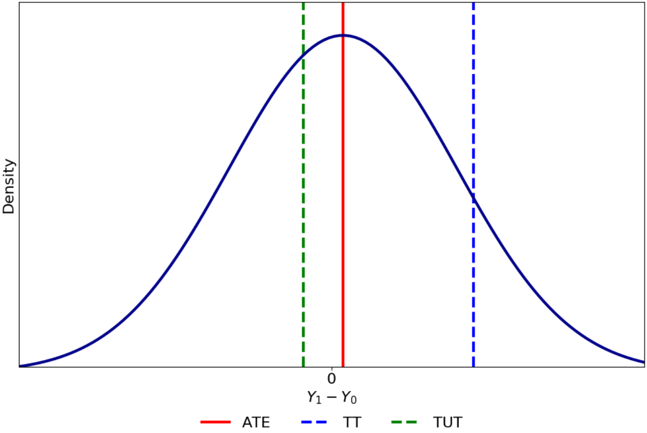
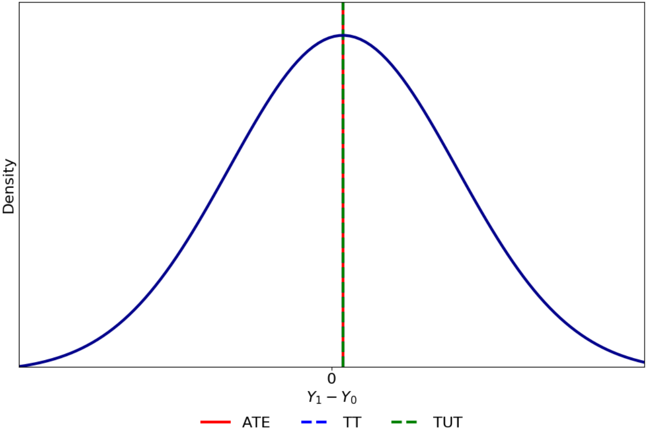


Figure: Treatment Effects without Essential Heterogeneity



Policy - Relevant Average Treatment Effect

Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$
$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Marginal Benefit of Treatment

$$B^{MTE}(x, u_S) = E[Y_1 - Y_0 \mid X = x, U_S = u_S]$$

Intuition: Mean gross return to treatment for persons at quantile u_S of the first-stage unobservable V.

Figure: Treatment Effects: Margin of Indifference

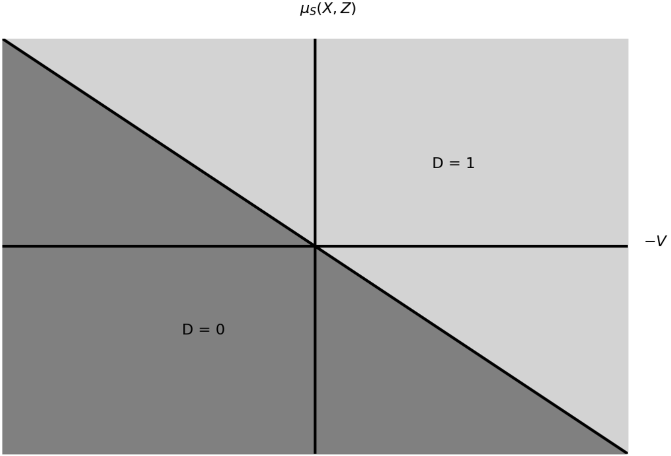
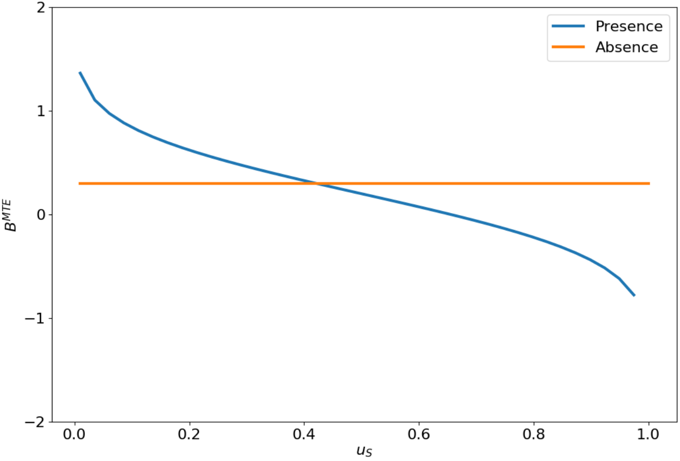


Figure: Marginal Effect of Heterogeneity



Effects of Treatment as Weighted Averages

Parameter Δ_j, can be written as a weighted average of the $B^{MTE}(x, u_S)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_S)\omega^j(x, u_S)du_S,$$

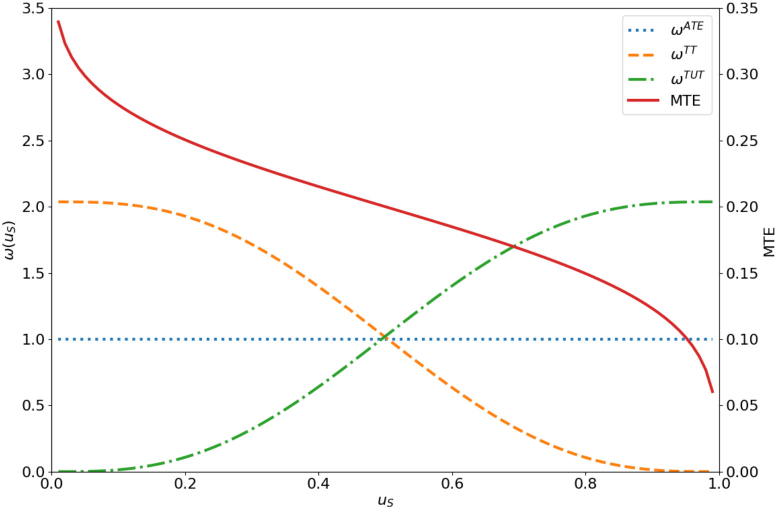
where the weights $\omega^j(x, u_S)$ are specific to parameter j and integrate to one.

Effects of Treatment as Weighted Averages

Weights

$$
\begin{aligned}
\omega^{ATE}(x, u_S) &= 1 \\
\omega^{TT}(x, u_S) &= \frac{1 - F_{P|X=x}(u_S)}{E[P \mid X = x]} \\
\omega^{TUT}(x, u_S) &= \frac{F_{P|X=x}(u_S)}{E[1 - P \mid X = x]}
\end{aligned}
$$

Figure: Effects of Treatment as Weighted Averages



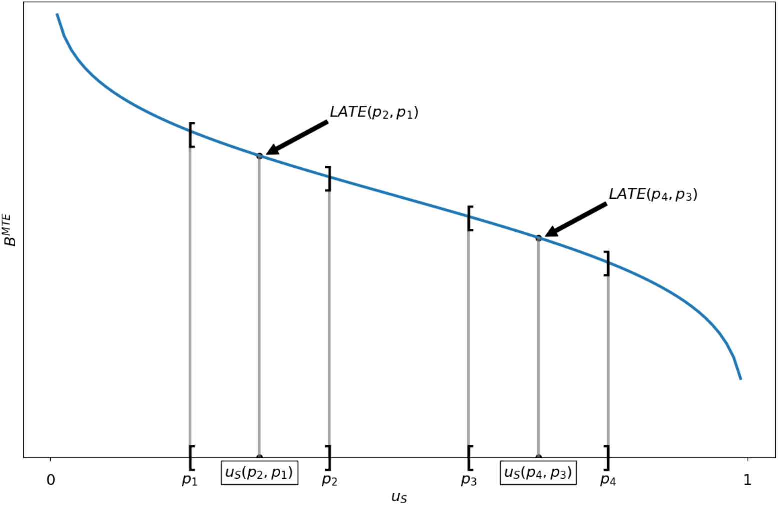
Local Average Treatment Effect

- **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument. $\Rightarrow$ instrument-dependet parameter
- **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment. $\Rightarrow$ deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_S, u_{S'}) = \frac{1}{u_S - u_{S'}} \int_{u_S}^{u_{S'}} B^{MTE}(x, u) du,$$

Figure: Local Average Treatment Effect



Distributional Effects of Treatment

- Marginal Distribution of Benefits
- Joint Distribution of Potential Outcomes
- Joint Distribution of Benefits and Surplus

Figure: Distribution of Benefits

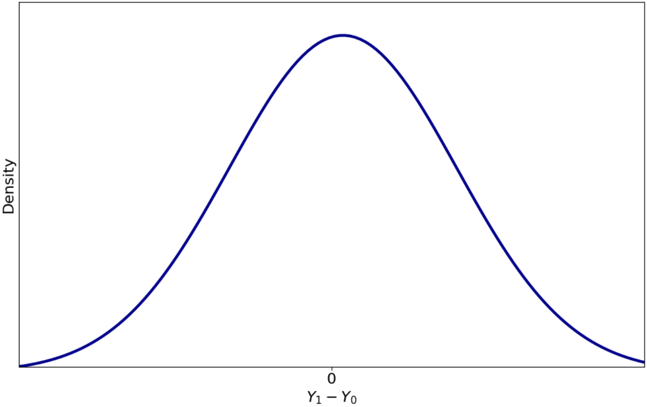


Figure: Joint Distribution of Potential Outcomes

Figure 7: Joint Distribution of Potential Outcomes

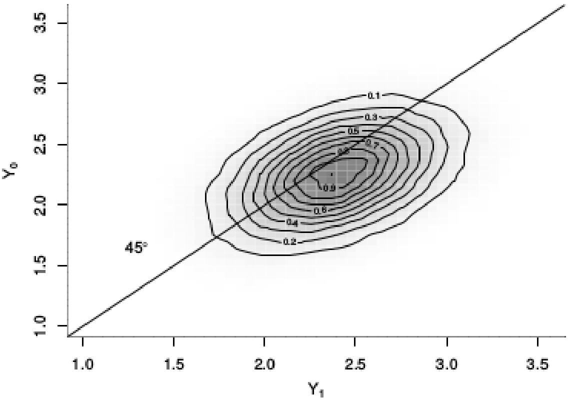
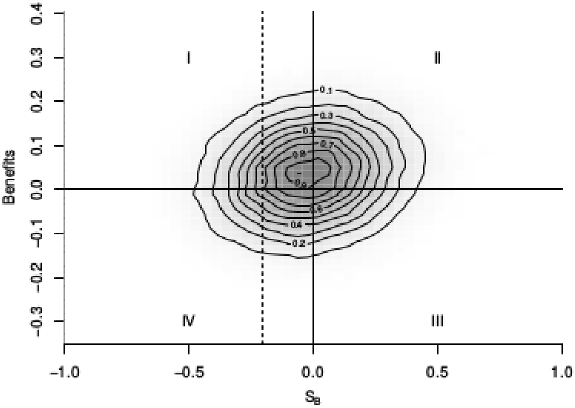
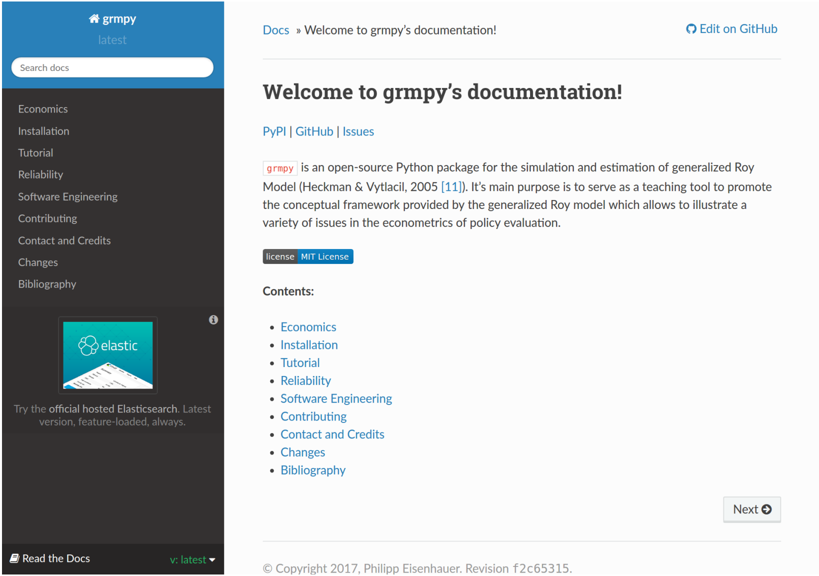


Figure: Joint Distribution of Surplus and Benefits

Figure 9: Joint Distribution of Surplus and Benefits



Teaching Tool



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