Estimating the technology of cognitive and noncognitive skill formation

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Introduction

The Economics of Human Development

- Multiple periods in the life cycle of childhood and adulthood and the existence of critical and sensitive periods of childhood in the formation of skills.
- Multiple skills for both parents and children that extend traditional notions about the skills required for success in life.
- Multiple forms of investment.

See Heckman and Mosso (2014) for a recent overview.

Paper

This paper formulates and estimates multistage production functions for children cognitive and noncognitive skills. Skills are determined by parental environments and investments at different stages of childhood. We estimate the elasticity of substitution between investments in one period and stocks of skills in that period to assess benefits of early investment in children compared to later remediation. . . . Using the estimated technology, we determine optimal targeting of interventions to children with different parental and personal birth endowments. . . .

- Cunha, F., Heckman, J. J., & Schennach, S. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, *78*(3), 883–931.

Part of a whole sequence ...

- Cunha, F., & Heckman, J. J. (2007). The technology of skill formation. *American Economic Review*, *97*(2), 31–47.
- Cunha, F., & Heckman, J. J. (2008). Formulating, identifying and estimating the technology of cognitive and noncognitive skill formation. *Journal of Human Resources*, *43*(4), 738–782.

Model

$t \in \{1, 2, \ldots, T\}$	childhood periods
$s \in \{1, \ldots, S\}$	developmental stages
$k \in \{C, N\}$	skill type
$\theta_{k,t}$	level of skill
$I_{k,t}$	parental investment

The technology of production of skill k in period t and developmental stage s depends on the stock of skills in period t, investment at t, $I_{k,t}$, parental skills, θ_P, shocks in period t, $\eta_{k,t}$, and the production function at stage s.

$$\theta_{k,t+1} = f_{k,s}(\theta_t, I_{k,t}, \theta_P, \eta_{k,t}),$$

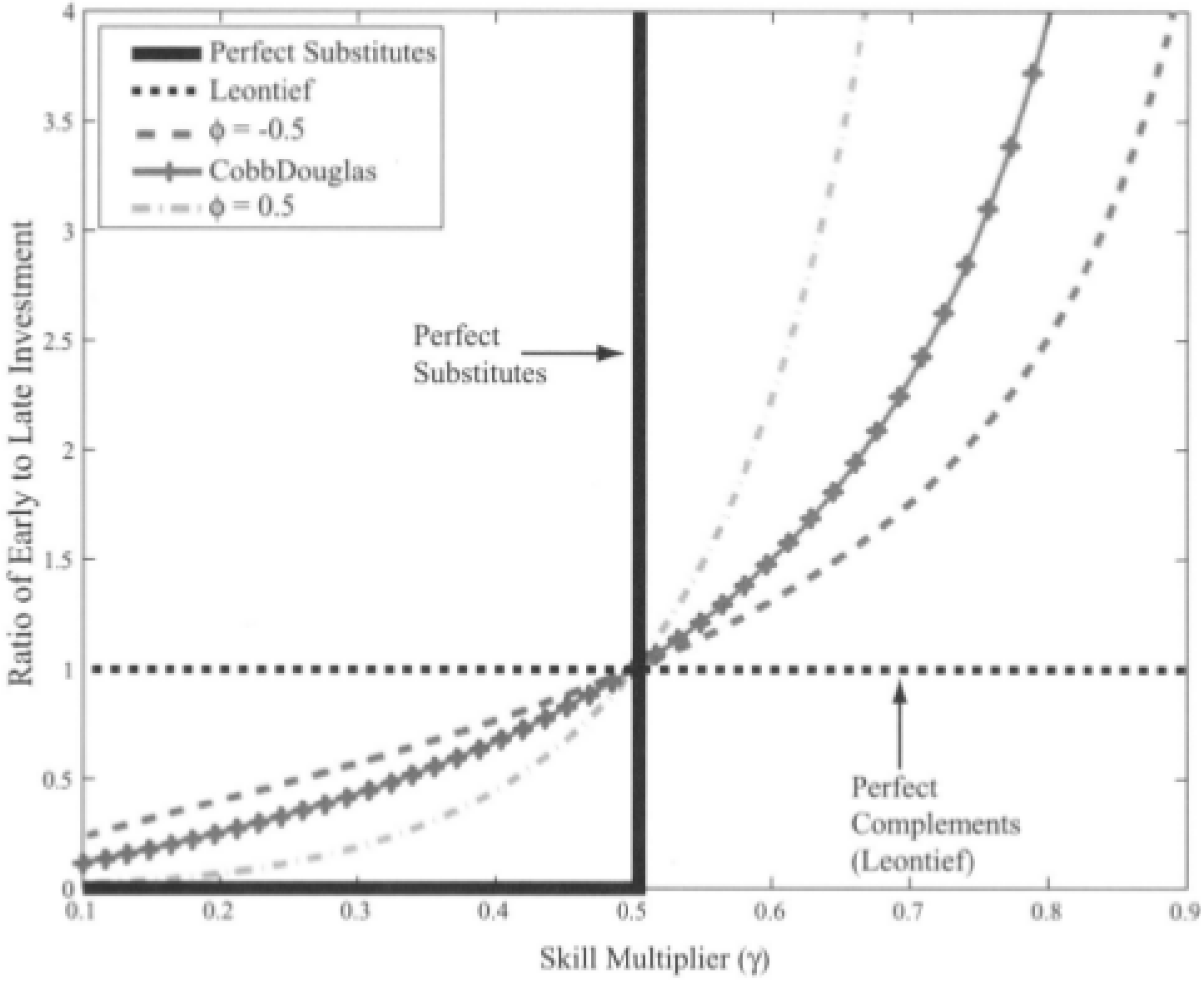
where $k \in \{C, N\}$, $t \in \{1, \ldots, T\}$, and $s \in \{1, \ldots, S\}$.

Direct Complementarity

$$\frac{\partial^2 f_{k,s}(\cdot)}{\partial I_{k,t} \partial \theta_{l,t}} > 0, \qquad \text{where } t \in \{1, \ldots, T\},\ l, k \in \{C, N\}$$

Period t stocks of abilities and skills promote the acquisition of skills by making investment more productive.

Figure: Optimal Investment Ratios



Adult Outcomes

Adult outcome j, Q_j, is produced by a combination of different skills at the beginning of period $T+1$.

$$Q_j = g_j(\theta_{C,T+1}, \theta_{N,T+1}), \qquad j \in \{1, \ldots, J\}$$

Technology Equation

$$\theta_{k,t+1} = [\gamma_{s,k,1}\theta_{C,t}^{\phi_{s,k}} + \gamma_{s,k,2}\theta_{N,t}^{\phi_{s,k}} + \gamma_{s,k,3}I_t^{\phi_{s,k}} + \gamma_{s,k,4}\theta_{C,P}^{\phi_{s,k}} + \gamma_{s,k,5}\theta_{N,P}^{\phi_{s,k}}]^{1/\sigma_{s,k}} e^{\eta_{n,k,t+1}},$$

where $\gamma_{s,k,l} \geq 0$, $\sum_{l=1}^{5} \gamma_{s,l,k} = 1$, $k \in \{C, N\}$, $t \in \{1, 2\}$, $s \in \{1, 2\}$, and $\eta_{k,t} \sim \mathbb{N}(0, \sigma_{\eta,s}^2)$.

Measurement Equation

$$Z_{1,k,t,j} = \mu_{1,k,t,j} + \alpha_{1,k,t,j} \ln \theta_{k,t} + \epsilon_{1,k,t,j}$$
$$Z_{2,k,t,j} = \mu_{2,k,t,j} + \alpha_{2,k,t,j} I_{k,t} + \epsilon_{2,k,t,j},$$

where $E(\epsilon_{a,k,t,j}) = 0$, $j \in \{1, \ldots, M_{a,k,t}\}$, $t \in \{1, \ldots, T\}$, $k \in \{C, N\}$, $a \in \{1, 2\}$ and where $\epsilon_{a,k,t,j}$ are uncorrelated across the j.

We estimate the technology on a sample of 2,207 firstborn white children from the Children of the NLSY79 (CNLSY79) sample. Starting in 1986, the children of the NLSY97 female respondents, ages 0-14, have been assessed every 2 years. The assessments measure cognitive ability, temperament, motor and social development, behavior problems, and self-competence of the children as well as their home environments. Data are collected via direct assessment and maternal report during home visits at every biannually.

Figure: Signal and Noise for Skills

	%Signal	%Noise		%Signal	%Noise
Measurement of Child's Cognitive Skills			*Measurement of Child's Noncognitive Skills*		
Gestation Length	0.501	0.499	Difficulty at Birth	0.151	0.849
Weight at Birth	0.557	0.443	Friendliness at Birth	0.165	0.835
Motor-Social Development at Birth	0.045	0.955	Compliance at Ages 1–2	0.232	0.768
Motor-Social Development at Ages 1–2	0.275	0.725	Insecure at Ages 1–2	0.080	0.920
Body Parts at Ages 1–2	0.308	0.692	Sociability at Ages 1–2	0.075	0.925
Memory for Locations at Ages 1–2	0.160	0.840	Difficulty at Ages 1–2	0.382	0.618
Motor-Social Development at Ages 3–4	0.410	0.590	Friendliness at Ages 1–2	0.189	0.811
Picture Vocabulary at Ages 3–4	0.431	0.569	Compliance at Ages 3–4	0.133	0.867
Picture Vocabulary at Ages 5–6	0.225	0.775	Insecure at Ages 3–4	0.122	0.878
PIAT Mathematics at Ages 5–6	0.314	0.686	Sociability at Ages 3–4	0.008	0.992
PIAT Reading Recognition at Ages 5–6	0.958	0.042	Behavior Problem Index Antisocial at Ages 3–4	0.405	0.595
PIAT Reading Comprehension at Ages 5–6	0.938	0.062	Behavior Problem Index Anxiety at Ages 3–4	0.427	0.573
PIAT Mathematics at Ages 7–8	0.465	0.535	Behavior Problem Index Headstrong at Ages 3–4	0.518	0.482
PIAT Reading Recognition at Ages 7–8	0.869	0.131	Behavior Problem Index Hyperactive at Ages 3–4	0.358	0.642
PIAT Reading Comprehension at Ages 7–8	0.797	0.203	Behavior Problem Index Conflict at Ages 3–4	0.336	0.664
PIAT Mathematics at Ages 9–10	0.492	0.508	Behavior Problem Index Antisocial at Ages 5–6	0.435	0.565
PIAT Reading Recognition at Ages 9–10	0.817	0.183	Behavior Problem Index Anxiety at Ages 5–6	0.409	0.591
PIAT Reading Comprehension at Ages 9–10	0.666	0.334	Behavior Problem Index Headstrong at Ages 5–6	0.611	0.389
PIAT Mathematics at Ages 11–12	0.516	0.484	Behavior Problem Index Hyperactive at Ages 5–6	0.481	0.519
PIAT Reading Recognition at Ages 11–12	0.781	0.219	Behavior Problem Index Conflict at Ages 5–6	0.290	0.710
PIAT Reading Comprehension at Ages 11–12	0.614	0.386	Behavior Problem Index Antisocial Ages 7–8	0.446	0.554
PIAT Mathematics at Ages 13–14	0.537	0.463	Behavior Problem Index Anxiety Ages 7–8	0.475	0.525
PIAT Reading Recognition at Ages 13–14	0.735	0.265	Behavior Problem Index Headstrong Ages 7–8	0.605	0.395
PIAT Reading Comprehension at Ages 13–14	0.549	0.451	Behavior Problem Index Hyperactive Ages 7–8	0.497	0.503

Figure: Signal and Noise for Investment

	%Signal	%Noise		%Signal	%Noise
Measurements of Parental Investments			*Measurements of Parental Investments*		
How Often Child Goes on Outings During Year of Birth	0.329	0.671	Child Has Musical Instruments Ages 7–8	0.022	0.978
Number of Books Child Has During Year of Birth	0.209	0.791	Family Subscribes to Daily Newspapers Ages 7–8	0.023	0.977
How Often Mom Reads to Child During Year of Birth	0.484	0.516	Child Has Special Lessons Ages 7–8	0.018	0.982
Number of Soft Toys Child Has During Year of Birth	0.126	0.874	How Often Child Goes to Musical Shows Ages 7–8	0.266	0.734
Number of Push/Pull Toys Child Has During Year of Birth	0.019	0.981	How Often Child Attends Family Gatherings Ages 7–8	0.125	0.875
How Often Child Eats With Mom/Dad During Year of Birth	0.511	0.489	How Often Child Is Praised Ages 7–8	0.046	0.954
How Often Mom Calls From Work During Year of Birth	0.119	0.881	How Often Child Gets Positive Encouragement Ages 7–8	0.053	0.947
How Often Child Goes on Outings at Ages 1–2	0.148	0.852	Number of Books Child Has Ages 9–10	0.013	0.987
Number of Books Child Has Ages 1–2	0.055	0.945	Mom Reads to Child Ages 9–10	0.137	0.863
How Often Mom Reads to Child Ages 1–2	0.186	0.814	Eats With Mom/Dad Ages 9–10	0.162	0.838
Number of Soft Toys Child Has Ages 1–2	0.240	0.760	How Often Child Goes to Museum Ages 9–10	0.219	0.781
Number of Push/Pull Toys Child Has Ages 1–2	0.046	0.954	Child Has Musical Instruments Ages 9–10	0.019	0.981
How Often Child Eats With Mom/Dad Ages 1–2	0.194	0.806	Family Subscribes to Daily Newspapers Ages 9–10	0.019	0.981
Mom Calls From Work Ages 1–2	0.070	0.930	Child Has Special Lessons Ages 9–10	0.015	0.985
How Often Child Goes on Outings Ages 3–4	0.123	0.877	How Often Child Goes to Musical Shows Ages 9–10	0.242	0.758
Number of Books Child Has Ages 3–4	0.012	0.988	How Often Child Attends Family Gatherings Ages 9–10	0.115	0.885
How Often Mom Reads to Child Ages 3–4	0.088	0.912	How Often Child Is Praised Ages 9–10	0.036	0.964
How Often Child Eats With Mom/Dad Ages 3–4	0.170	0.830	How Often Child Gets Positive Encouragement Ages 9–10	0.041	0.959
Number of Magazines at Home Ages 3–4	0.193	0.807	Number of Books Child Has Ages 11–12	0.016	0.984

Figure: Baseline Estimates

		First Stage Parameters		Second Stage Parameters
The Technology of Cognitive Skill Formation				
Current Period Cognitive Skills (Self-Productivity)	$\gamma_{1,C,1}$	0.487 (0.030)	$\gamma_{2,C,1}$	0.902 (0.014)
Current Period Noncognitive Skills (Cross-Productivity)	$\gamma_{1,C,2}$	0.083 (0.026)	$\gamma_{2,C,2}$	0.011 (0.005)
Current Period Investments	$\gamma_{1,C,3}$	0.231 (0.024)	$\gamma_{2,C,3}$	0.020 (0.006)
Parental Cognitive Skills	$\gamma_{1,C,4}$	0.050 (0.013)	$\gamma_{2,C,4}$	0.047 (0.008)
Parental Noncognitive Skills	$\gamma_{1,C,5}$	0.148 (0.030)	$\gamma_{2,C,5}$	0.020 (0.010)
Complementarity Parameter	$\phi_{1,C}$	0.611 (0.240)	$\phi_{2,C}$	−1.373 (0.168)
Implied Elasticity of Substitution	$1/(1-\phi_{1,C})$	2.569	$1/(1-\phi_{2,C})$	0.421
Variance of Shocks $\eta_{C,t}$	$\delta^2_{1,C}$	0.165 (0.007)	$\delta^2_{2,C}$	0.097 (0.003)
The Technology of Noncognitive Skill Formation				
Current Period Cognitive Skills (Cross-Productivity)	$\gamma_{1,N,1}$	0.000 (0.025)	$\gamma_{2,N,1}$	0.008 (0.010)
Current Period Noncognitive Skills (Self-Productivity)	$\gamma_{1,N,2}$	0.649 (0.034)	$\gamma_{2,N,2}$	0.868 (0.011)
Current Period Investments	$\gamma_{1,N,3}$	0.146 (0.027)	$\gamma_{2,N,3}$	0.055 (0.013)
Parental Cognitive Skills	$\gamma_{1,N,4}$	0.022 (0.011)	$\gamma_{2,N,4}$	0.000 (0.007)
Parental Noncognitive Skills	$\gamma_{1,N,5}$	0.183 (0.031)	$\gamma_{2,N,5}$	0.069 (0.017)
Complementarity Parameter	$\phi_{1,N}$	−0.674 (0.324)	$\phi_{2,N}$	−0.695 (0.274)
Implied Elasticity of Substitution	$1/(1-\phi_{1,N})$	0.597	$1/(1-\phi_{2,N})$	0.590
Variance of Shocks $\eta_{N,t}$	$\delta^2_{1,N}$	0.189 (0.012)	$\delta^2_{2,N}$	0.103 (0.004)

Usefulness of Structural Models

- assess relative importance of competing economic mechanisms
- conduct ex ante policy evaluation

See Wolpin (2013) and Rust (2014) for an instructive discussion of the usefulness and limits of structural econometric methods.

Heterogeneity in Educational Attainment

- 34% skills
  - 16% cognitive
  - 12% noncognitive
- 15% parental investment

Policy Evaluation

$h \in \{1 \ldots, H\}$	children
$\theta_{1,h}$	initial condition
S_h	schooling attainment
$B = 2H$	budget

Optimization Problem

$$\max \bar{S} = \frac{1}{H} \sum_{h \in H} S_h(\theta_{C,3,h}, \theta_{N,3,h})$$

$$s.t.$$

$$\sum_{h \in H} (I_{1,h} + I_{2,h}) = 2H$$

$$\theta_{k,t+1,h} = f_{k,t}(\theta_{C,t,h}, \theta_{N,t,h}, \theta_{C,P}, \theta_{N,P})$$

$$\text{for} \quad k \in \{C, N\} \quad \text{and} \quad t \in \{1, 2\}$$

There is an analogous constraint minimization problem for crime.

Figure: Aggregate Education by Initial Conditions

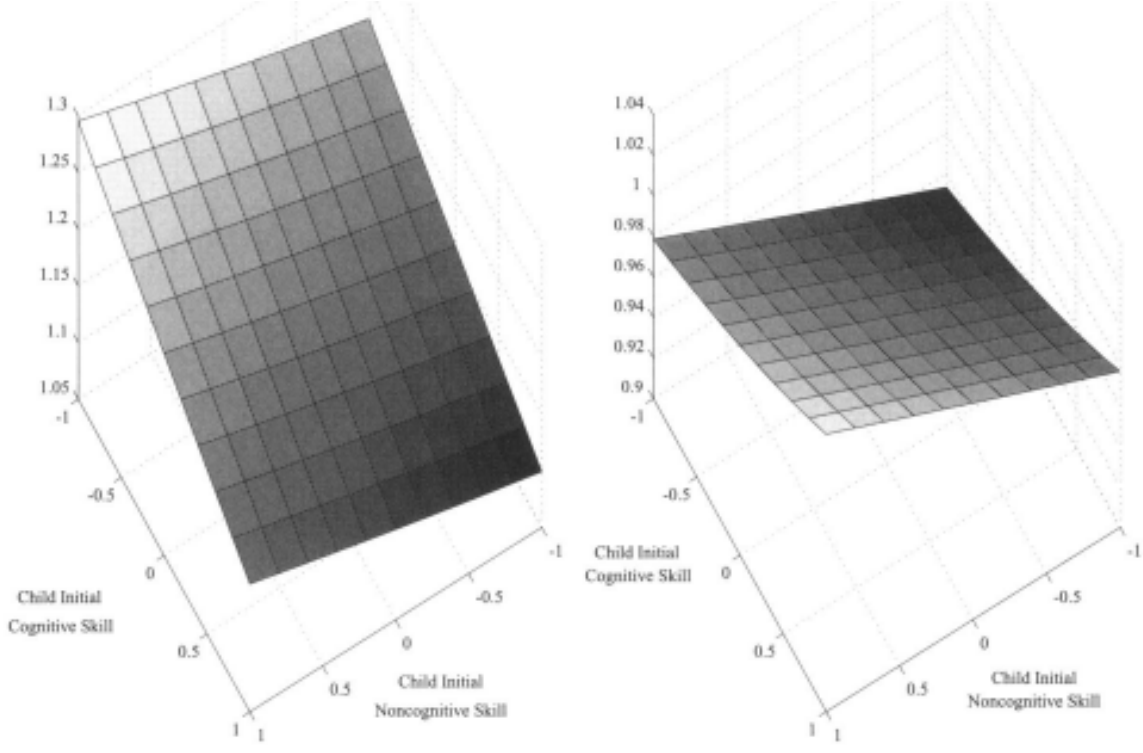


Figure: Aggregate Education by Maternal Endowment

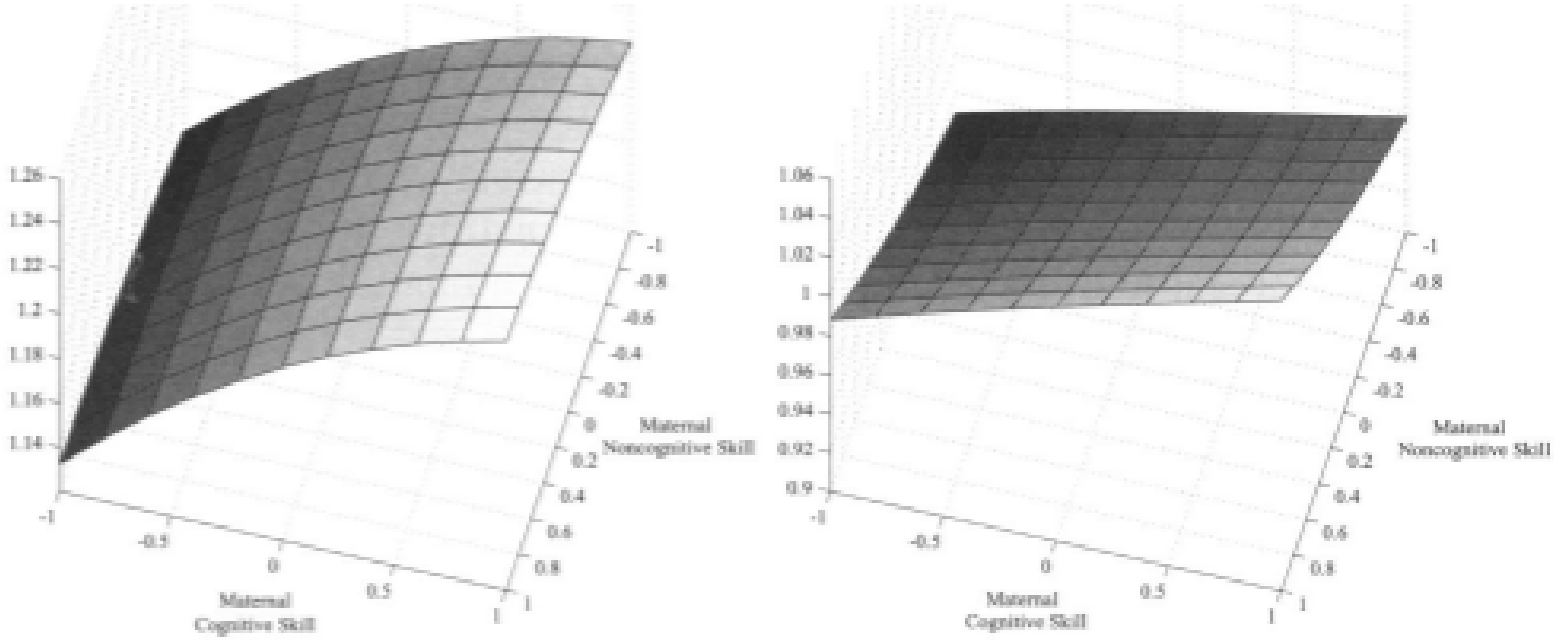


Figure: Aggregate Crime by Initial Conditions

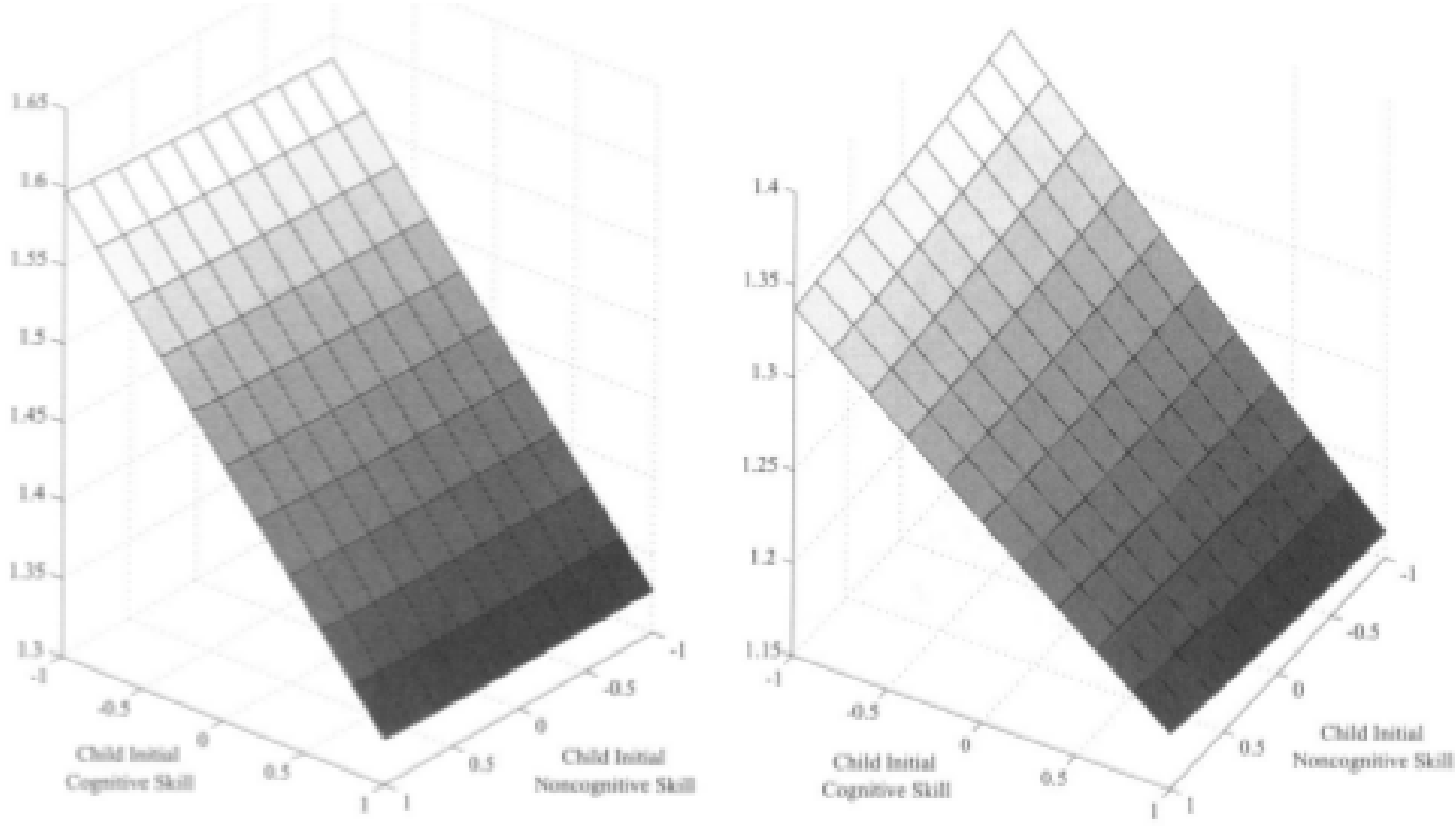


Figure: Aggregate Crime by Maternal Endowment

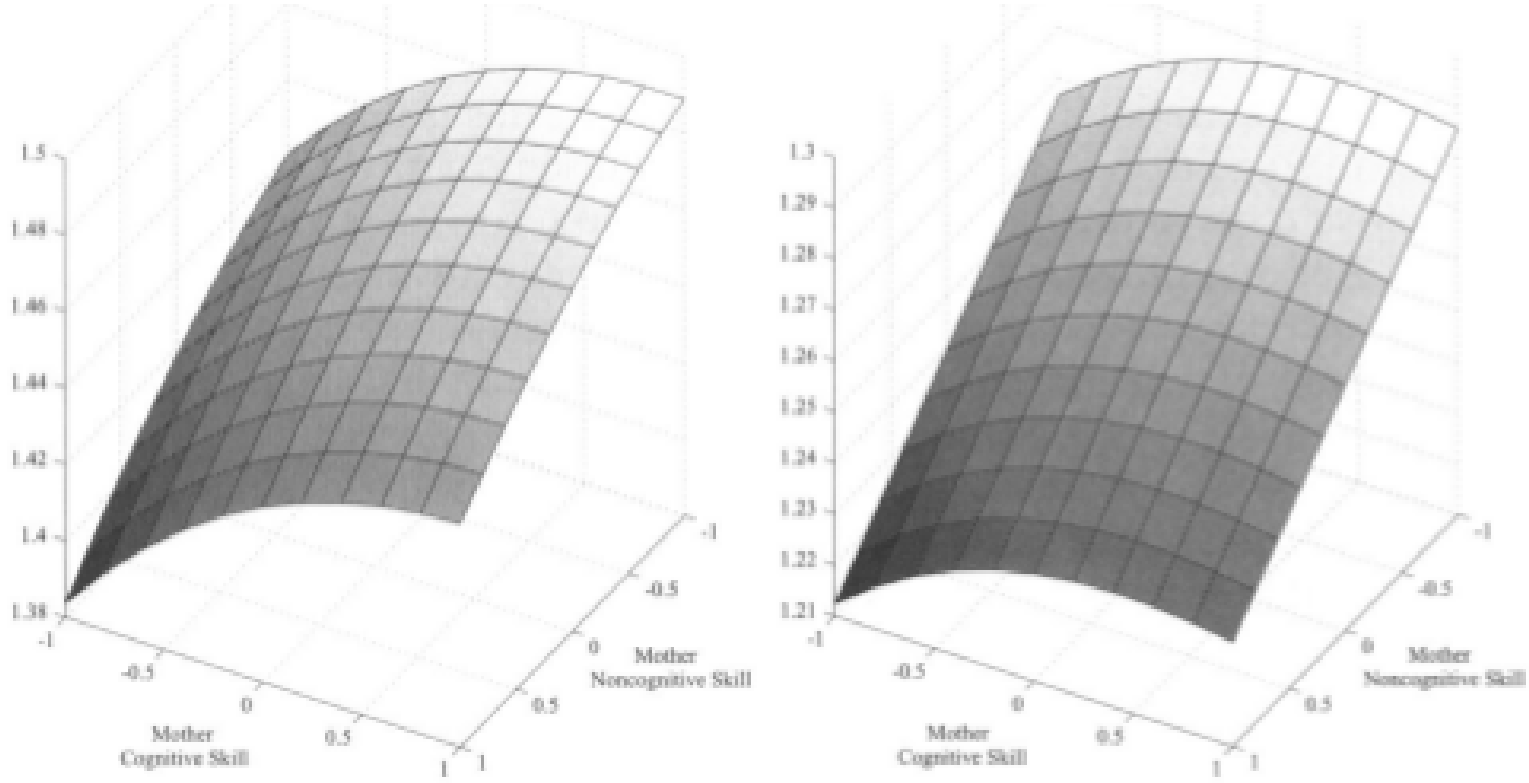
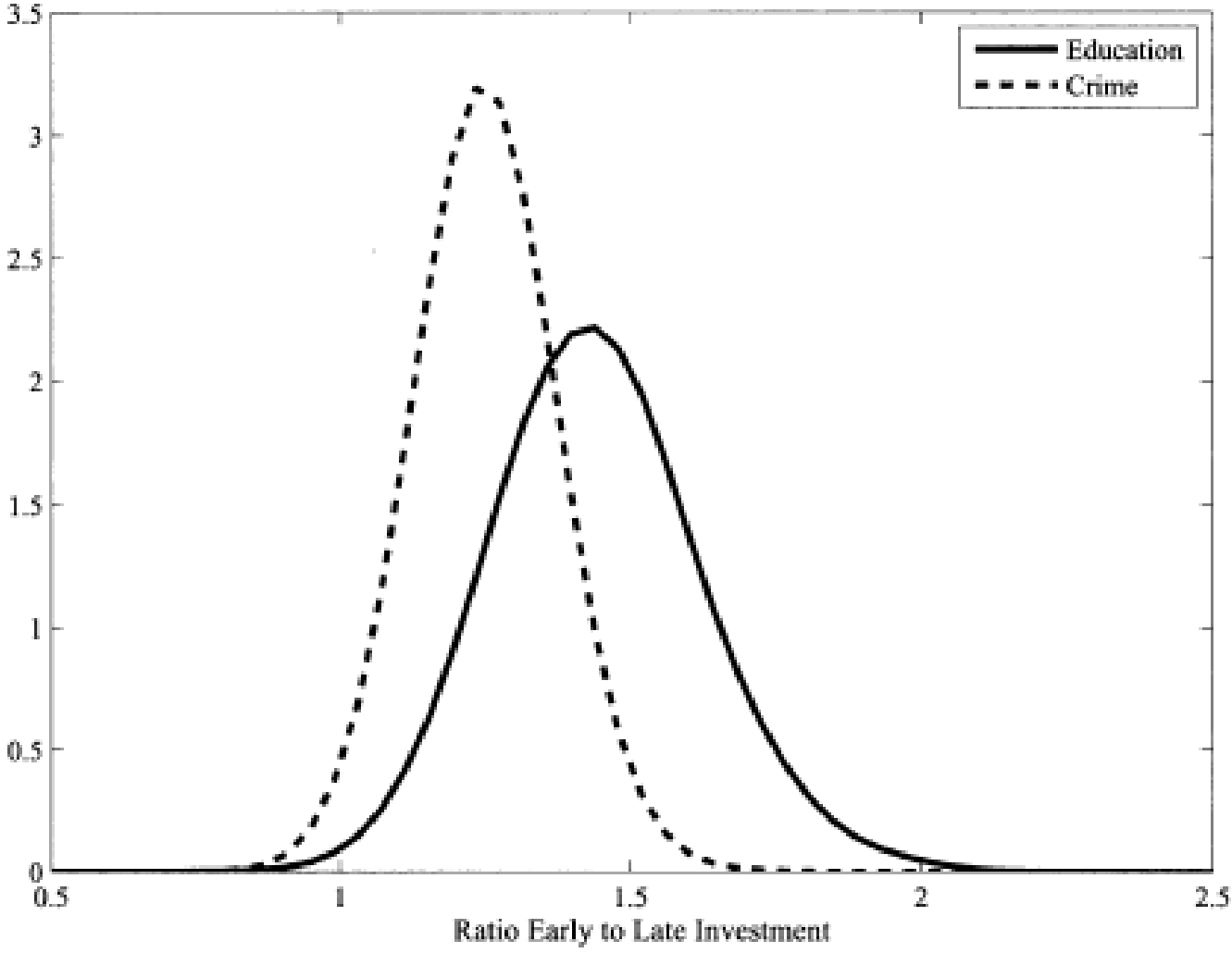


Figure: Density Ratios



These simulations suggest that the timing and level of optimal interventions for disadvantaged children depend on the condition for disadvantage and the nature of desired outcomes. Targeted strategies are likely to be effective especially for different targets that weight cognitive and noncognitive skills differently.

Key Contributions

- formulation and estimation of a multistage model with multidimensional skills
- illustration of the importance of distinguishing between early and late investments
- nonparametric identification of production technology

What is next?

Conclusion

Appendix

References

Cunha, F., & Heckman, J. J. (2007). The technology of skill formation. *American Economic Review*, *97*(2), 31–47.

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Cunha, F., Heckman, J. J., & Schennach, S. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, *78*(3), 883–931.

Heckman, J. J., & Mosso, S. (2014). The economics of human development and social mobility. *Annual Review of Economics*, *6*(1), 689–733.

Rust, J. (2014). The limits of inference with theory: A review of Wolpin (2013). *Journal of Economic Literature*, *52*(3), 820–850.

Wolpin, K. I. (2013). *Limits to inference without theory*. Cambridge, MA: MIT University Press.