Introduction to the generalized Roy model

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Introduction

Heckman (2008) defines three policy evaluation tasks:

- Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

Econometrics of policy evaluation

- is important
- is complicated
- is multifaceted

Numerous applications

- labor economics
- development economics
- industrial economics
- health economics

Numerous effects

- conventional average effects
- policy-relevant average effects
- marginal effects
- distributional effects
- effects on distributions

Numerous estimation strategies

- instrumental variables
- (quasi-)experimental methods
- matching

Model

Generalized Roy model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = \mathrm{I}[\mu_D(X, Z) - V > 0]$$

Individual Heterogeneity

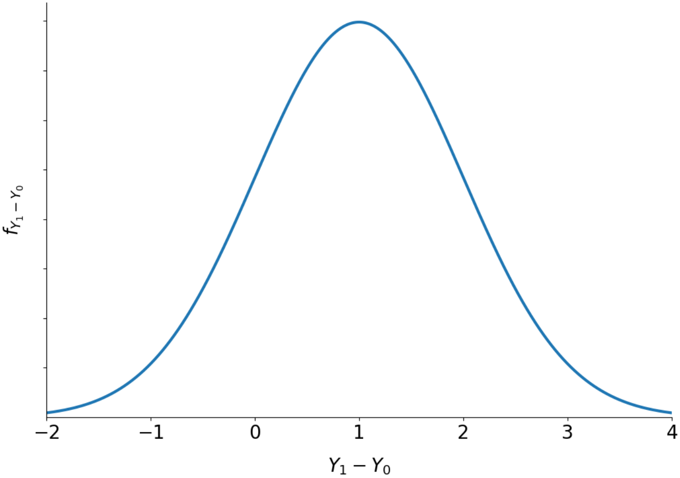
Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity

- Difference in observables
- Difference in unobservables
  - Uncertainty
  - Private information

Figure: Distribution of benefits



Econometric problems

- **Evaluation problem**, we only observe an individual in either the treated or untreated state.
- **Selection problem**, individuals that select into treatment differ from those that do not.

Essential Heterogeneity

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp\!\!\!\perp D \quad | X = x.$$

$\Rightarrow$ consequences for the choice of the estimation strategy

Objects of interest

Useful Notation

$$
\begin{aligned}
P(X,Z) &= \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X,Z)) \\
U_D &= F_V(V)
\end{aligned}
$$

Figure: First-stage unobservable

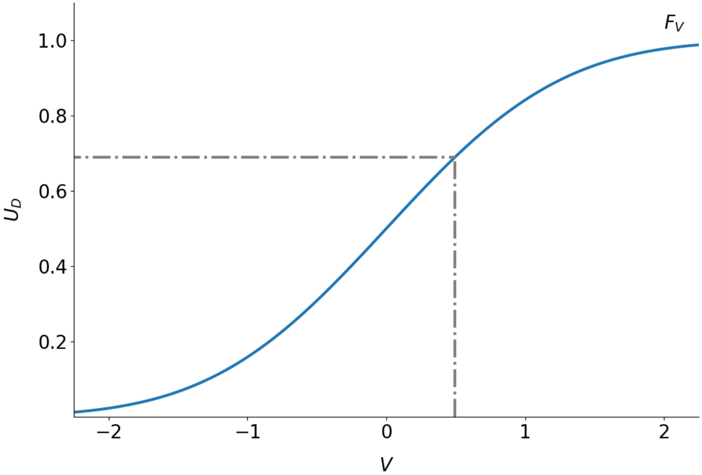


Figure: Support

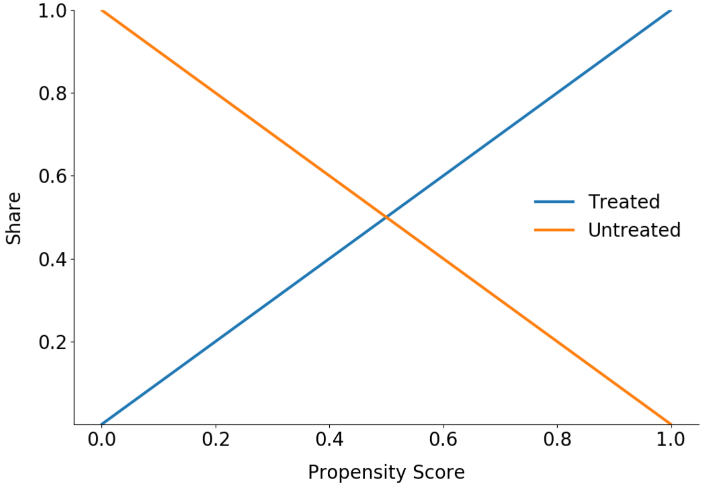


Figure: Distribution of benefits

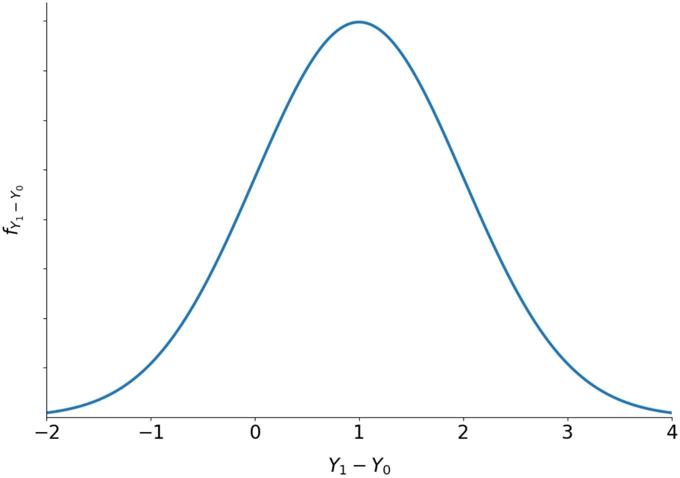
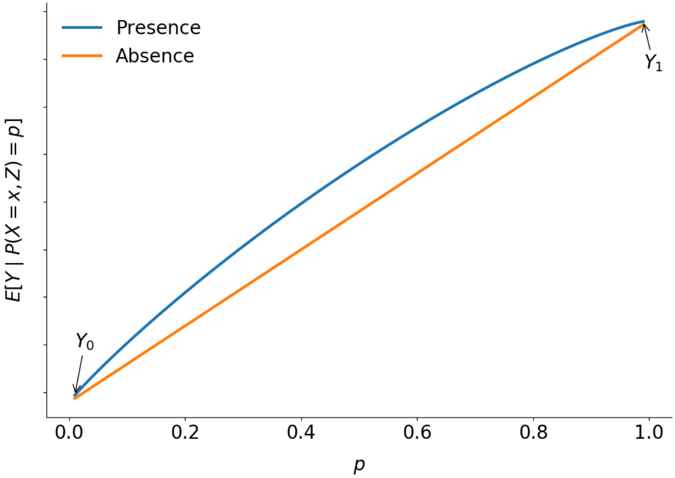


Figure: Conditional expectation and essential heterogeneity



Conventional Average Treatment Effects

Conventional Average Treatment Effects

$$\begin{aligned} B^{ATE} &= E[Y_1 - Y_0] \\ B^{TT} &= E[Y_1 - Y_0 \mid D = 1] \\ B^{TUT} &= E[Y_1 - Y_0 \mid D = 0] \end{aligned}$$

$\Rightarrow$ correspond to *extreme* policy alternatives

Selection Problem

$$
\begin{aligned}
E[Y \mid D=1]-E[Y \mid D=0] = {} & \underbrace{E[Y_1-Y_0]}_{B^{ATE}} \\
& + \underbrace{E[Y_1-Y_0 \mid D=1]-E[Y_1-Y_0]}_{\text{Selection on gains}} \\
& + \underbrace{E[Y_0 \mid D=1]-E[Y_0 \mid D=0]}_{\text{Selection on levels}}
\end{aligned}
$$

$$
\begin{aligned}
E[Y \mid D=1]-E[Y \mid D=0] = & \underbrace{E[Y_1 - Y_0 \mid D=1]}_{B^{TT}} \\
& + \underbrace{E[Y_0 \mid D=1]-E[Y_0 \mid D=0]}_{\text{Selection Bias}}
\end{aligned}
$$

$\Rightarrow$ the bias depends on the parameter of interest

Figure: Distribution of effects with essential heterogeneity

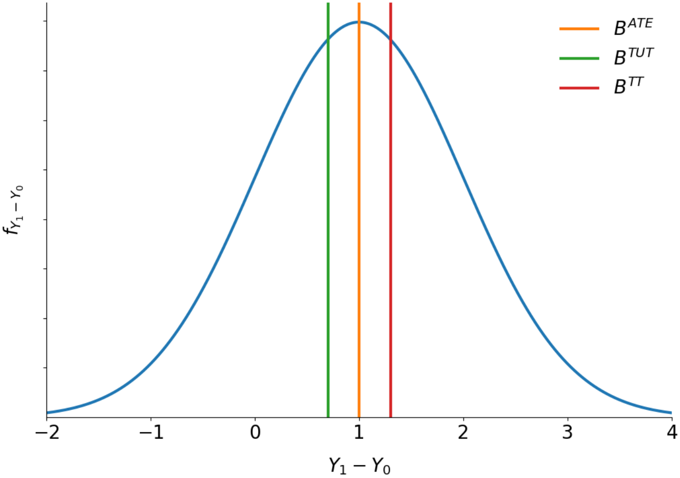
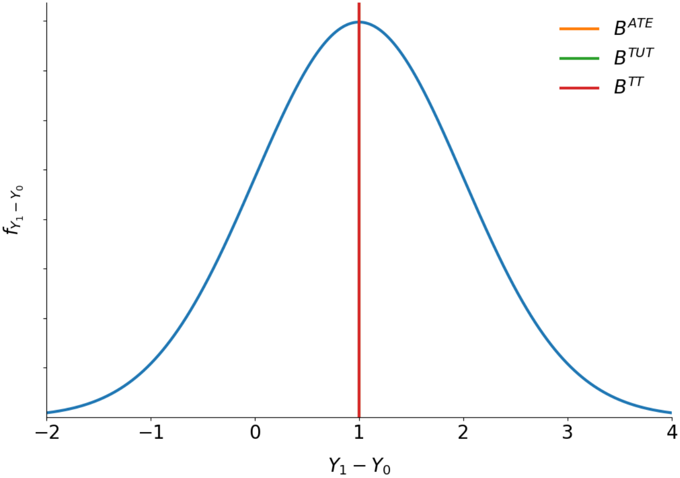


Figure: Distribution of effects without essential heterogeneity



Policy-Relevant Average Treatment Effects

Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$
$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Marginal Effect of Treatment

Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

Intuition: Mean gross return to treatment for persons at quantile u_D of the first-stage unobservable V or a willingness to pay for individuals at the margin of indifference.

Figure: Margin of indifference

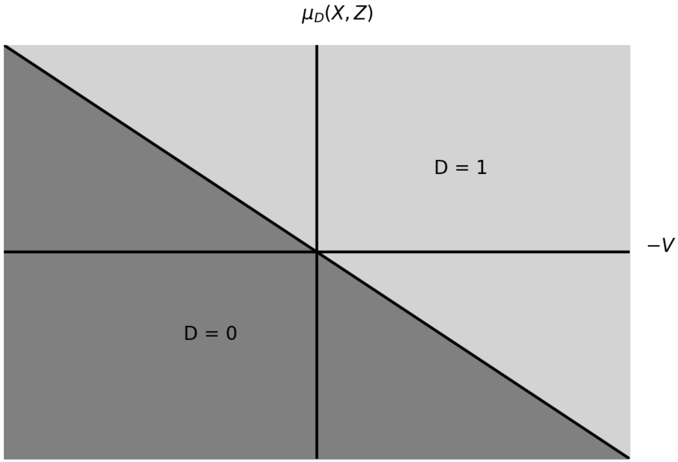
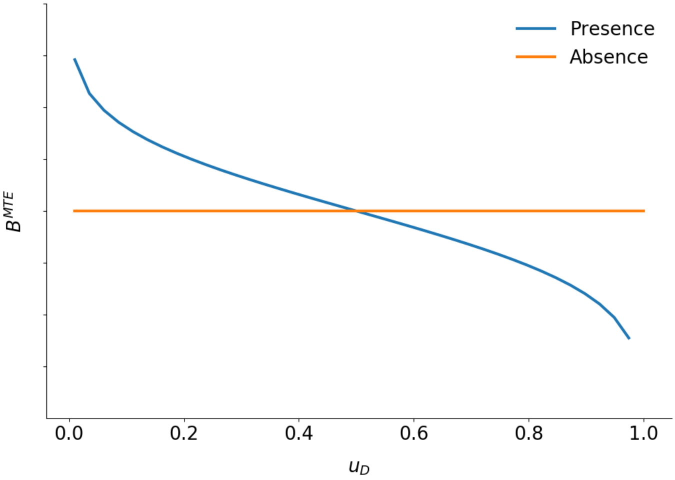


Figure: B^{MTE} and essential heterogeneity



Effects of treatment as weighted averages Parameter Δ_j, can be written as a weighted average of the $B^{MTE}(x, u_D)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D)\omega^j(x, u_D)du_D,$$

where the weights $\omega^j(x, u_D)$ are specific to parameter j and integrate to one.

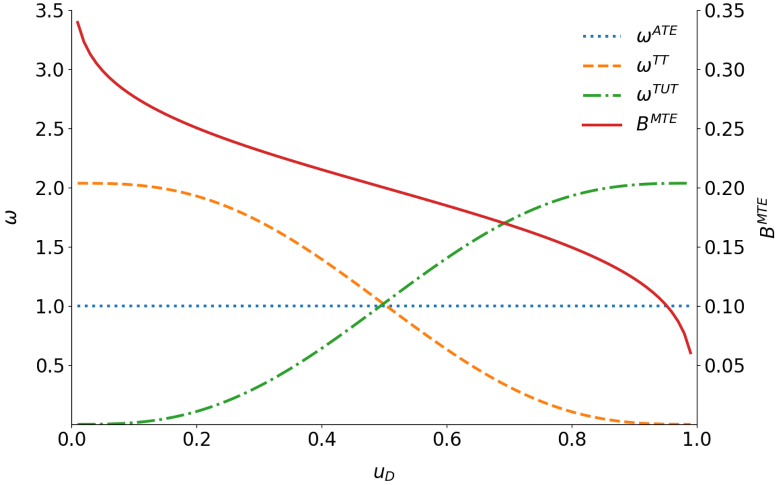
Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P \mid X = x]}$$

Figure: Effects of treatment as weighted averages



Local Average Treatment Effect

Local Average Treatment Effect

- **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.

  ⇒ instrument-dependent parameter

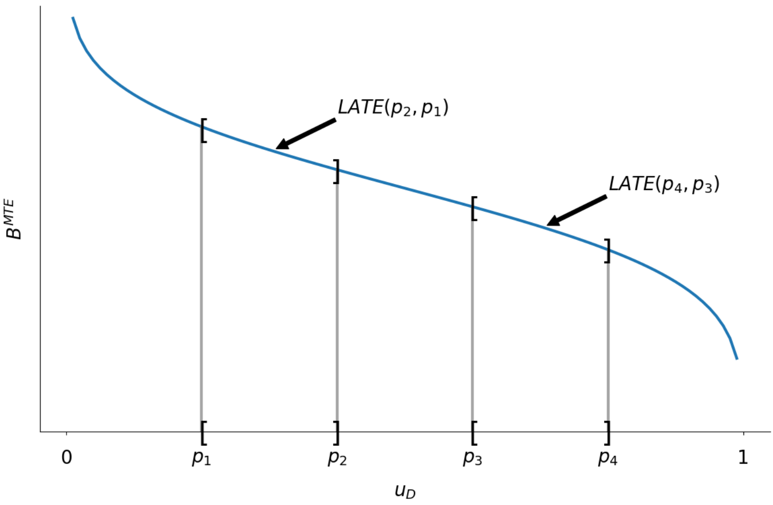
- **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.

  ⇒ deep economic parameter

$$B^{LATE} = \frac{E[Y \mid Z = z] - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_D, u_{D'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{D'}} B^{MTE}(x, u) du,$$

Figure: Local average treatment effect



Distributions of Effects

Distrbutions of Effects

- marginal distribution of benefits
- joint distribution of potential outcomes
- joint distribution of benefits and surplus

Figure: Distribution of benefits

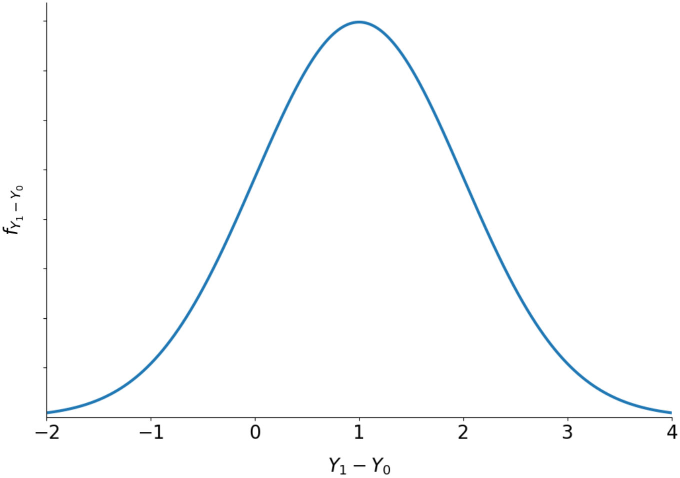


Figure: Distribution of potential outcomes

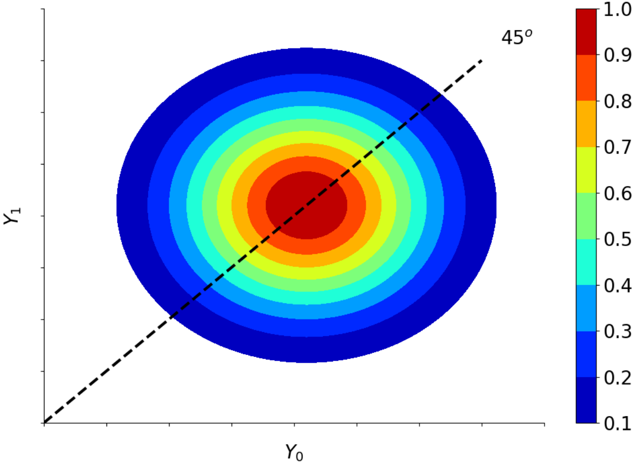
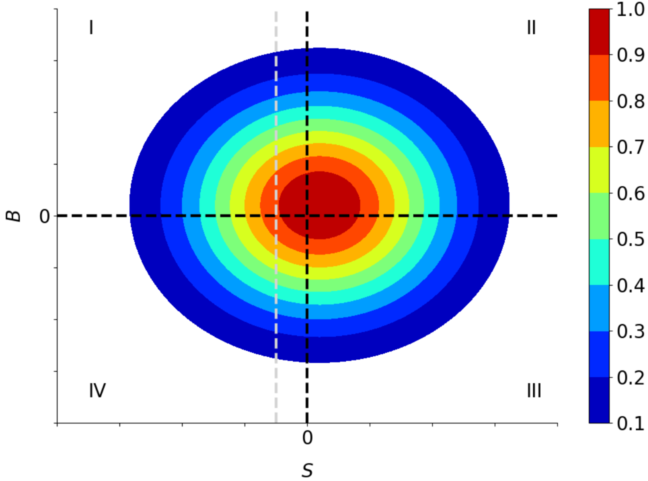


Figure: Distribution of benefits and surplus



Conclusion

Appendix

References

Heckman, J. (2008). Schools, skills, and synapses. *Economic Inquiry*, *46*, 289–324.