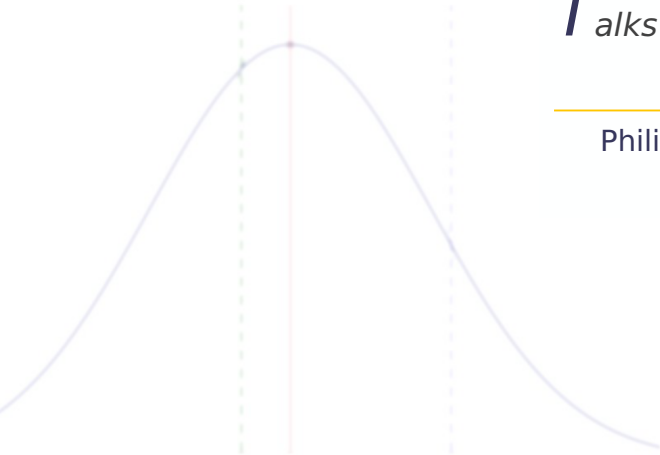


# $T_{alks}$

---

Philipp Eisenhauer



Material available on



Visit us!

# Simulation of choice probabilities

Philipp Eisenhauer

I draw on the material presented in:

- ▶ Train, K. (2009). *Discrete choice models with simulation*. Cambridge, New York: Cambridge University Press.

## Probit setup

$$U_{nj} = V_{nj} + \epsilon_{nj} \quad \forall j \in J$$

$$\epsilon'_n = (\epsilon_{n1}, \dots, \epsilon_{nj})$$

The approaches we discuss (mostly) have general applicability.

## Choice probability

$$\begin{aligned} P_{ni} &= P(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i) \\ &= \int I(V_{ni} + \epsilon_{ni} > V_{nj} + \epsilon_{nj} \quad \forall j \neq i) \phi(\epsilon_n) d\epsilon_n \end{aligned}$$

The choice probabilities do not have a closed form expression and must be approximated numerically.

- ▶ Quadrature
- ▶ Monte Carlo methods
  - ▶ crude accept-reject simulator
  - ▶ smoothed accept-reject simulator

## Crude accept-reject simulator

1. Draw  $J$  values from the multivariate normal distribution to sample  $\epsilon_n^r$ .
2. Calculate the simulated utilities  $U_{nj}^r$  for all alternatives.
3. Set  $I^r = 1$  if  $U_{nj}^r$  is the maximum and zero otherwise.

## Crude accept-reject simulator

4. Repeat the steps above  $R$  times

The simulated probability is the number of accepts divided by the number of repetitions:  $\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^R I^r$ .



## Issues

- ▶ simulation for low probability events
- ▶ simulated probability is step function

We will explore these issues further in the accompanying notebook.

## Smooth accept-reject simulator

- ▶ The smoothed AR simulator mitigates these difficulties with is to replace the 1 - 0 AR indicator with a smooth, strictly positive function.
- ▶ McFadden (1989) suggested the logit-smoothed AR simulator.

We replace step 3 with the following transformation:

$$S^r = \frac{\exp \frac{U_{ni}^r}{\lambda}}{\sum_j \exp \frac{U_{nj}^r}{\lambda}}$$

The simulated probability is the number of accepts divided by the number of repetitions:  $\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^R S^r$ .

We will explore these issues further in the accompanying notebook.

# **Appendix**

# *References*

- Hahn, J., Todd, P. E., & van der Klaauw, W. (2001). Identification and estimation of treatment effects with a regression-discontinuity design. *Econometrica*, 69(1), 201–209.
- McFadden, D. L. (1989). A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, 57(5), 995–1026.
- Thistlethwaite, D. L., & Campbell, D. T. (1960). Regression-discontinuity analysis: An alternative to the ex-post facto experiment. *Journal of Educational Psychology*, 51(6), 309–317.

Train, K. (2009). *Discrete choice models with simulation*.  
Cambridge, New York: Cambridge University Press.