

# Generalized Roy Model

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## **Heckman (2008) defines three policy evaluation tasks:**

- ▶ Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- ▶ Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- ▶ Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

## The Generalized Roy Model

### Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

### Cost

$$C = \mu_D(Z) + U_C$$

### Observed Outcomes

$$Y = DY_1 + (1 - D)Y_0$$

### Choice

$$S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

## Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\alpha = \mu_0(X)$$

$$\beta = (Y_1 - Y_0) = \mu_1(X) - \mu_0(X) + (U_1 - U_0)$$

$$\epsilon = U_0$$

## Econometric Problems

- ▶ **Evaluation Problem:** We only observe an individual in either the treated or untreated state.
- ▶ **Selection Problem:** Individuals that select into treatment different from those that do not.

Observed outcome for individual  $i$ :

$$Y_i = Y_{0i} + D_i(Y_{1i} - Y_{0i}) = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$$

## Useful Notation

$$\mu_S(X, Z) = (\mu_1(X) - \mu_0(X)) - \mu_C(Z)$$

$$V = U_C - (U_1 - U_0)$$

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_S(X, Z))$$

$$U_S = F_V(V)$$

Rewriting Choice Equation

$$D = \mathbb{I}[P(X, Z) > U_S]$$

## Individual-specific benefit of treatment:

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

## Sources of Heterogeneity:

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
  - ▶ Uncertainty
  - ▶ Private Information



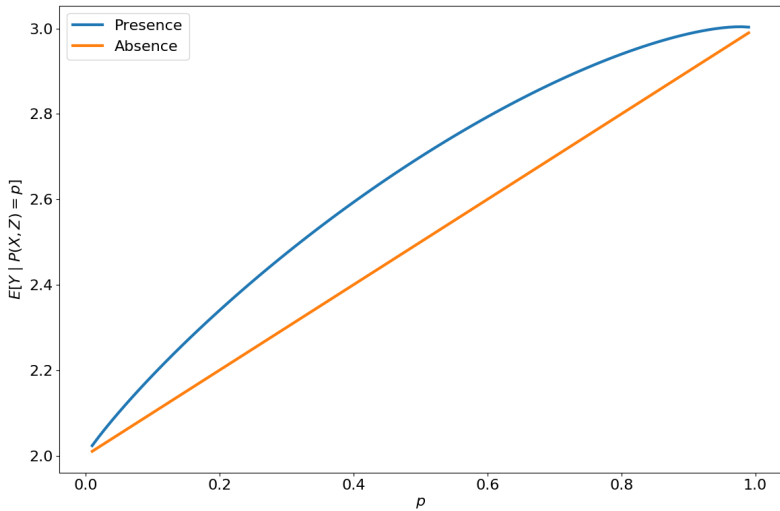
## Essential Heterogeneity

**Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0/D \quad | \quad X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation of Observed Outcome



## Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

⇒ correspond to *extreme* policy alternatives

## Selection Problem

$$\begin{aligned} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0]}_{BATE} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}} \end{aligned}$$

$$\begin{aligned}
 E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} \\
 &\quad + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}
 \end{aligned}$$

$\Rightarrow$  the bias depends on the parameter of interest

Figure: Treatment Effects with Essential Heterogeneity

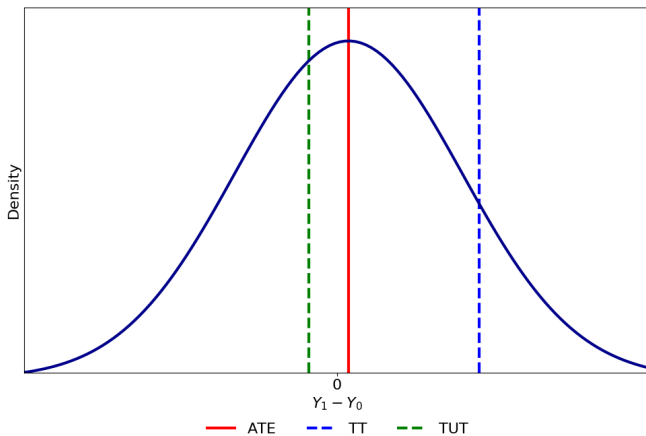
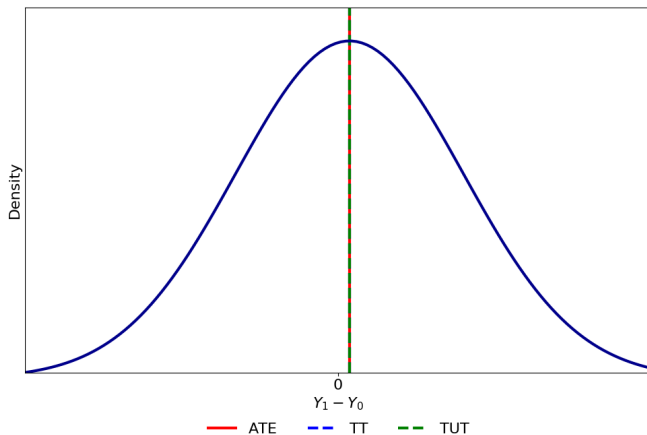


Figure: Treatment Effects without Essential Heterogeneity



## Policy-Relevant Average Treatment Effect

### Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

### Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$



## Marginal Benefit of Treatment

$$B^{MTE}(x, u_S) = E[Y_1 - Y_0 \mid X = x, U_S = u_S]$$

**Intuition:** Mean gross return to treatment for persons at quantile  $u_S$  of the first-stage unobservable  $V$ .

Figure: Margin of Indifference

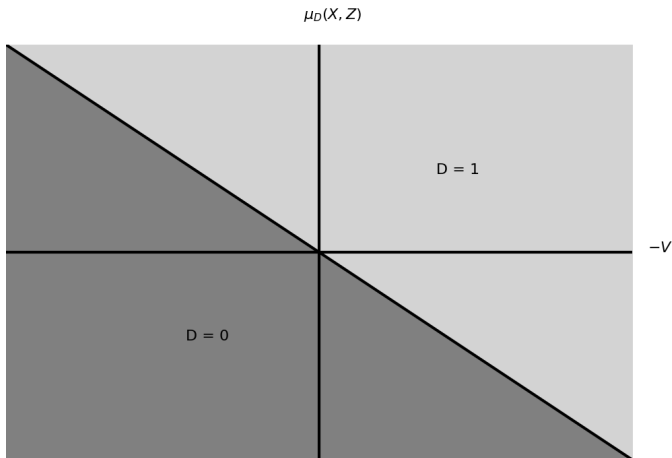
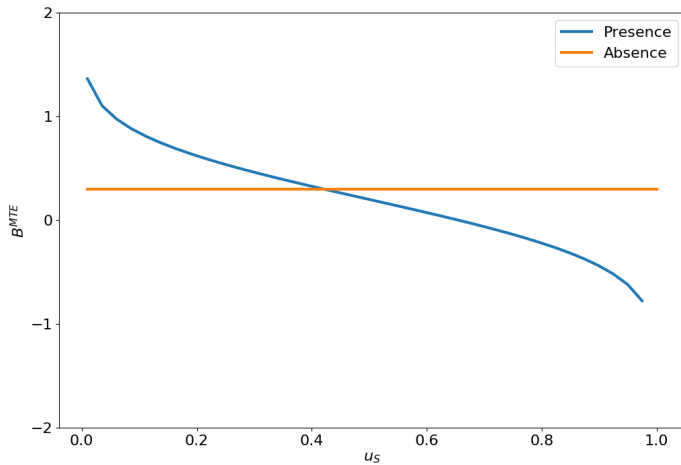


Figure: Marginal Effect of Treatment



## Effects of Treatment as Weighted Averages

Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_S)$

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_S) \omega^j(x, u_S) du_S,$$

where the weights  $\omega^j(x, u_S)$  are specific to parameter  $j$  and integrate to one.

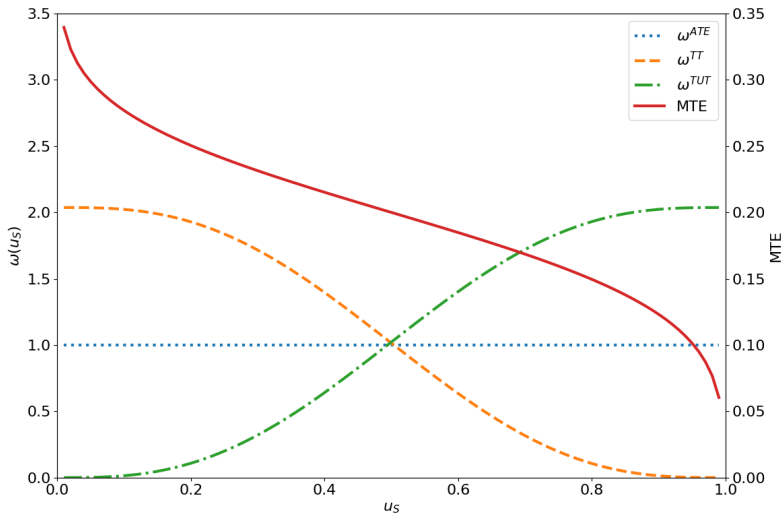
## Weights

$$\omega^{ATE}(x, u_S) = 1$$

$$\omega^{TT}(x, u_S) = \frac{1 - F_{P|X=x}(u_S)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_S) = \frac{F_{P|X=x}(u_S)}{E[1 - P \mid X = x]}$$

Figure: Treatment Effect Weights



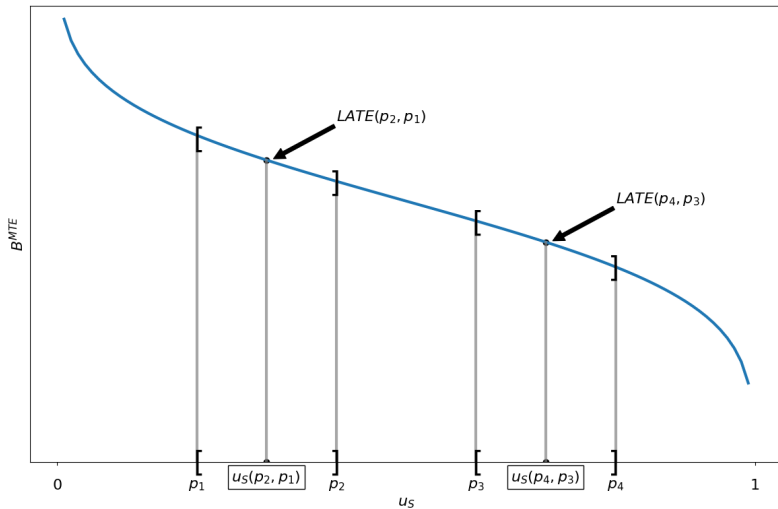
- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.  
⇒ instrument-dependent parameter
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.  
⇒ deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_S, u_{S'}) = \frac{1}{u_S - u_{S'}} \int_{u_S}^{u_{S'}} B^{MTE}(x, u) du$$



Figure: Local Average Treatment Effect



## **Distributional Effects of Treatment:**

- ▶ Marginal Distribution of Benefits
- ▶ Joint Distribution of Potential Outcomes
- ▶ Joint Distribution of Benefits and Surplus

Figure: Marginal Distribution of Benefits

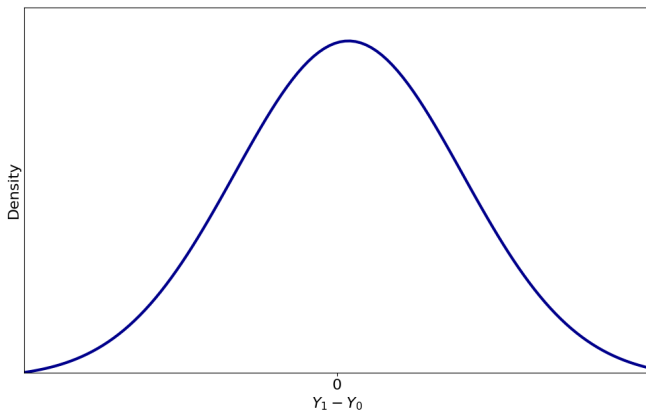


Figure: Joint Distribution of Potential Outcomes

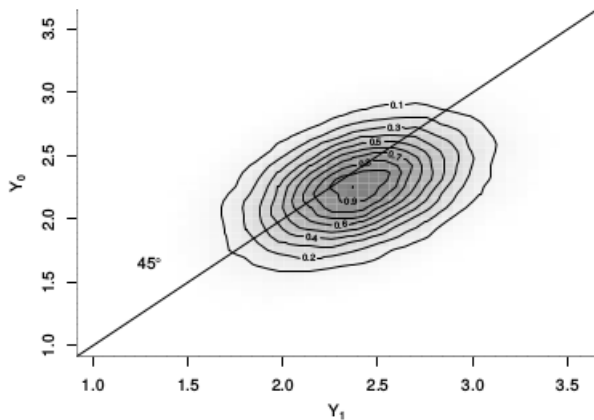
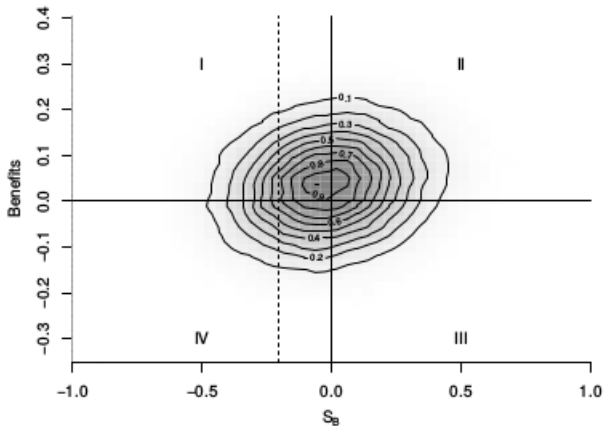



Figure: Joint Distribution of Surplus and Benefits





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`grmpy` is an open-source Python package for the simulation and estimation of generalized Roy Model (Heckman & Vytlacil, 2005 [\[11\]](#)). It's main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model which allows to illustrate a variety of issues in the econometrics of policy evaluation.

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# Appendix

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