

Generalized Roy Model

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... background material available at
<https://github.com/policyMetrics/talks>

Policy Evaluation Tasks

Heckman (2008) defines three policy evaluation tasks:

- ▶ Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- ▶ Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- ▶ Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Cost

$$C = \mu_D(Z) + U_C$$

Observed Outcomes

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

The Generalized Roy Model

Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\alpha = \mu_0(X)$$

$$\beta = (Y_1 - Y_0) = \mu_1(X) - \mu_0(X) + (U_1 - U_0)$$

$$\epsilon = U_0$$

Econometric Problems

- ▶ **Evaluation Problem:** We only observe an individual in either the treated or untreated state.
- ▶ **Selection Problem:** Individuals that select into treatment different from those that do not.

Econometric Problems

Observed outcome for individual i :

$$Y_i = Y_{0i} + D_i(Y_{1i} - Y_{0i}) = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$$

Econometric Problems

$$\mu_D(X, Z) = (\mu_1(X) - \mu_0(X)) - \mu_C(Z)$$

$$V = U_C - (U_1 - U_0)$$

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$

Rewriting Choice Equation

$$D = \mathbb{I}[P(X, Z) > U_D]$$

Econometric Problems

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
 - ▶ Uncertainty
 - ▶ Private Information

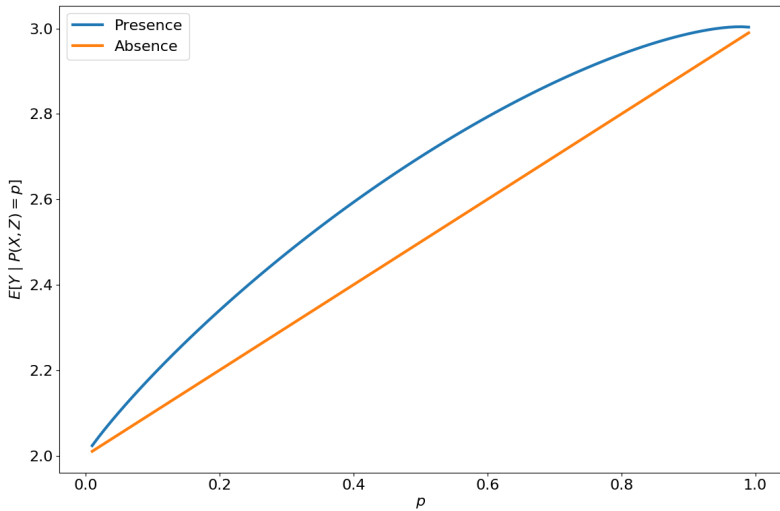
Essential Heterogeneity

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation



Conventional Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

\Rightarrow correspond to *extreme* policy alternatives

Selection Problem

$$\begin{aligned} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0]}_{BATE} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}} \end{aligned}$$

Selection Problem

$$\begin{aligned} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} \\ &\quad + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}} \end{aligned}$$

\Rightarrow the bias depends on the parameter of interest

Figure: Treatment Effects with Essential Heterogeneity

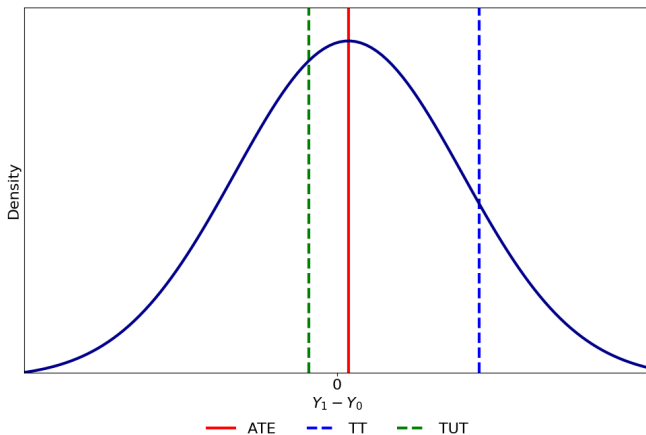
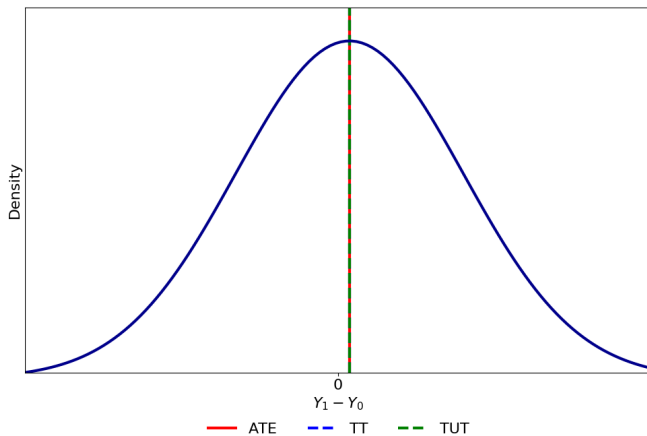


Figure: Treatment Effects without Essential Heterogeneity



Policy - Relevant Average Treatment Effect

Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

Intuition: Mean gross return to treatment for persons at quantile u_D of the first-stage unobservable V .

Figure: Treatment Effects: Margin of Indifference

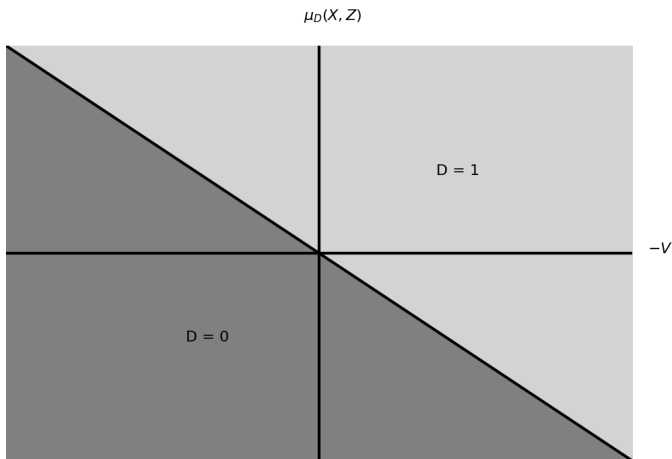
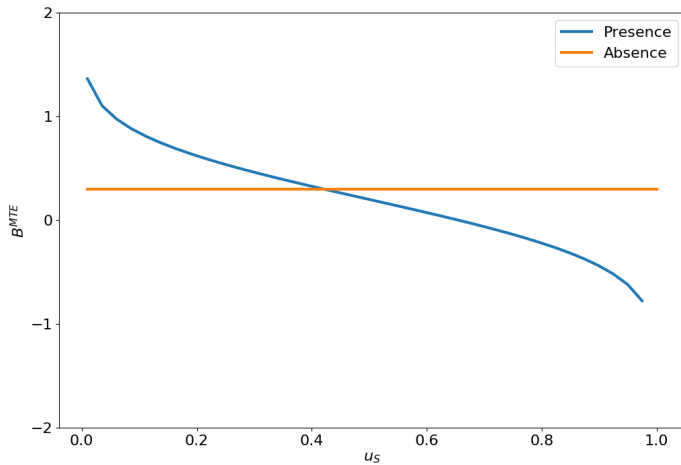


Figure: Marginal Effect of Heterogeneity



Effects of Treatment as Weighted Averages

Parameter Δ_j , can be written as a weighted average of the $B^{MTE}(x, u_D)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights $\omega^j(x, u_D)$ are specific to parameter j and integrate to one.

Effects of Treatment as Weighted Averages

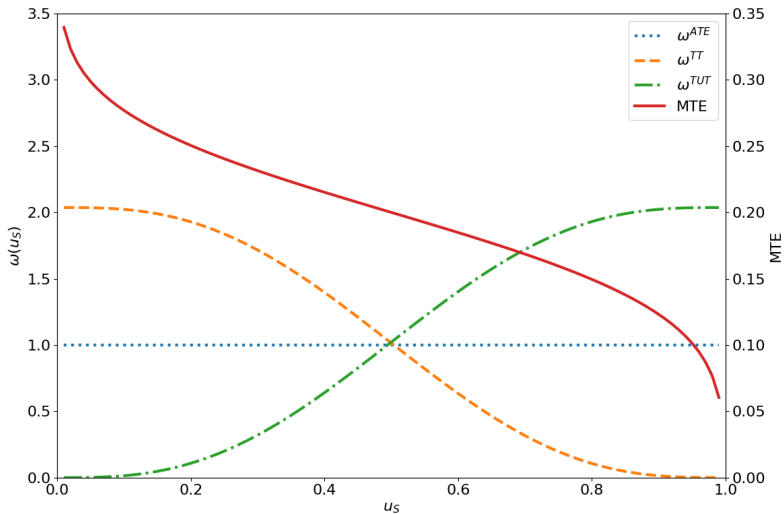
Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P \mid X = x]}$$

Figure: Effects of Treatment as Weighted Averages



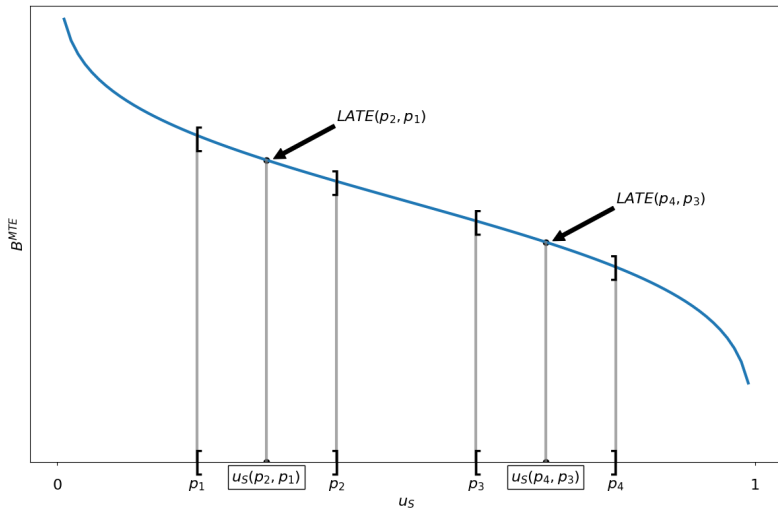
Local Average Treatment Effect

- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument. \Rightarrow instrument-dependent parameter
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.
 \Rightarrow deep economic parameter

$$B^{LATE} = \frac{E(Y | Z = z) - E[Y | Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_D, u_{D'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{D'}} B^{MTE}(x, u) du,$$

Figure: Local Average Treatment Effect



Distributional Effects of Treatment

- ▶ Marginal Distribution of Benefits
- ▶ Joint Distribution of Potential Outcomes
- ▶ Joint Distribution of Benefits and Surplus

Figure: Distribution of Benefits

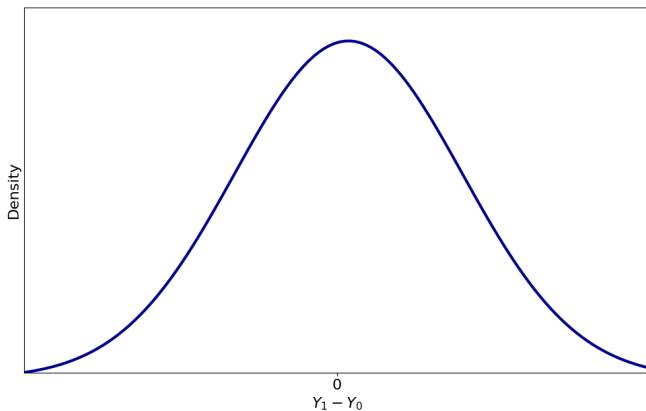


Figure: Joint Distribution of Potential Outcomes

Figure 7: Joint Distribution of Potential Outcomes

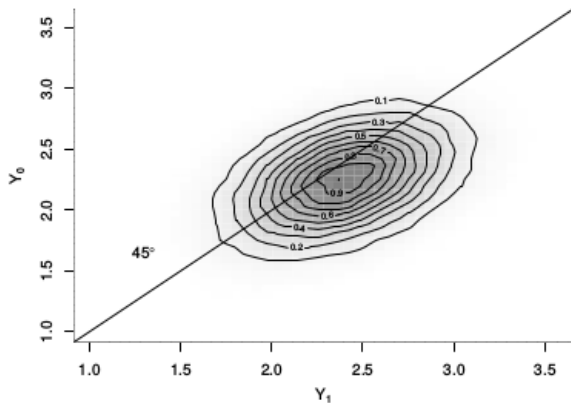
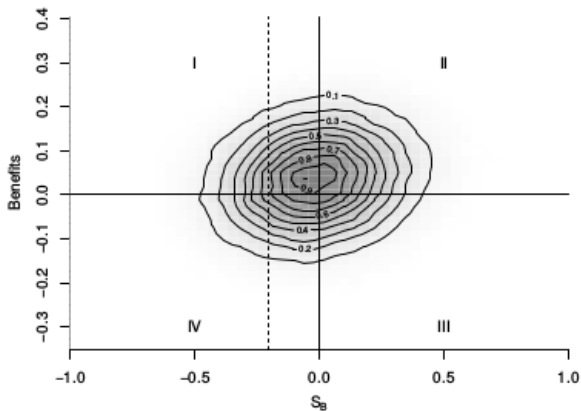



Figure: Joint Distribution of Surplus and Benefits

Figure 9: Joint Distribution of Surplus and Benefits




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`grmpy` is an open-source Python package for the simulation and estimation of generalized Roy Model (Heckman & Vytlacil, 2005 [\[11\]](#)). It's main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model which allows to illustrate a variety of issues in the econometrics of policy evaluation.

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References

Abbring, J. H. and Heckman, J. J. (2007). Econometric evaluation of social programs, part iii: Distributional treatment effects, dynamic treatment effects, dynamic discrete choice, and general equilibrium policy evaluation. In Heckman, J. J. and Leamer, E. E., editors, *Handbook of econometrics*, volume 6, pages 5145–5303. Elsevier Science, Amsterdam, Netherlands.

Carneiro, P., Hansen, K., and Heckman, J. J. (2003). Estimating distributions of treatment effects with an application to the returns to schooling and measurement of the effects of uncertainty on college choice. *International Economic Review*, 44(2):361–422.

Carneiro, P., Heckman, J. J., and Vytlačil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6):2754–2781.

Heckman, J. J. (2001). Micro data, heterogeneity, and the evaluation of public policy: Nobel lecture. *Journal of Political Economy*, 109(4):673–748.

Heckman, J. J. (2008). Econometric causality. *International Statistical Review*, 76(1):1–27.

Heckman, J. J., Smith, J., and Clements, N. (1997). Making the most out of programme evaluations and social experiments: Accounting for heterogeneity in programme impacts. *The Review of Economic Studies*, 64(4):487–535.

- Heckman, J. J., Urzua, S., and Vytlacil, E. J. (2006). Understanding instrumental variables in models with essential heterogeneity. *The Review of Economics and Statistics*, 88(3):389–432.
- Heckman, J. J. and Vytlacil, E. J. (2001a). Local instrumental variables. In Hsiao, C., Morimune, K., and Powell, J., editors, *Nonlinear Statistical Modeling: Proceedings of the Thirteenth International Symposium in Economic Theory and Econometrics: Essays in Honor of Takeshi Amemiya*, pages 1–46. Cambridge University Press, Cambridge, United Kingdom.
- Heckman, J. J. and Vytlacil, E. J. (2001b). Policy-relevant treatment effects. *American Economic Review*, 91(2):107–111.

- Heckman, J. J. and Vytlačil, E. J. (2005). Structural equations, treatment effects, and econometric policy evaluation. *Econometrica*, 73(3):669–739.
- Heckman, J. J. and Vytlačil, E. J. (2007a). Econometric evaluation of social programs, part i: Causal effects, structural models and econometric policy evaluation. In Heckman, J. J. and Leamer, E. E., editors, *Handbook of Econometrics*, volume 6B, pages 4779–4874. Elsevier Science, Amsterdam, Netherlands.

Heckman, J. J. and Vytlačil, E. J. (2007b). Econometric evaluation of social programs, part ii: Using the marginal treatment effect to organize alternative economic estimators to evaluate social programs and to forecast their effects in new environments. In Heckman, J. J. and Leamer, E. E., editors, *Handbook of Econometrics*, volume 6B, pages 4875–5144. Elsevier Science, Amsterdam, Netherlands.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford Economic Papers*, 3(2):135–146.