Generalized Roy Model

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Heckman (2008) defines three policy evaluation tasks:

- ► Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.

- ► Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.

- ► Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

The Generalized Roy Model

Potential Outcomes

$Y_1 = \mu_1(X) + U_1$

$Y_0 = \mu_0(X) + U_0$

Cost

$C = \mu_D(Z) + U_C$

Observed Outcomes

$Y = DY_1 + (1 - D)Y_0$

Choice

$S = Y_1 - Y_0 - C$

$D = \mathrm{I}[S > 0]$

Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\alpha = \mu_0(X)$$
$$\beta = (Y_1 - Y_0) = \mu_1(X) - \mu_0(X) + (U_1 - U_0)$$
$$\epsilon = U_0$$

Econometric Problems

- ▶ **Evaluation Problem:** We only observe an individual in either the treated or untreated state.

- ▶ **Selection Problem:** Individuals that select into treatment different from those that do not.

Observed outcome for individual i:

$$Y_i = Y_{0i} + D_i(Y_{1i} - Y_{0i}) = \begin{cases} Y_{1i} & \text{if} \quad D_i = 1 \\ Y_{0i} & \text{if} \quad D_i = 0 \end{cases}$$

Useful Notation

$$\mu_S(X, Z) = (\mu_1(X) - \mu_0(X)) - \mu_C(Z)$$
$$V = U_C - (U_1 - U_0)$$
$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_S(X, Z))$$
$$U_s = F_V(V)$$

Rewriting Choice Equation

$$D = \mathrm{I}[P(X, Z) > U_S]$$

Individual-specific benefit of treatment:

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity:

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
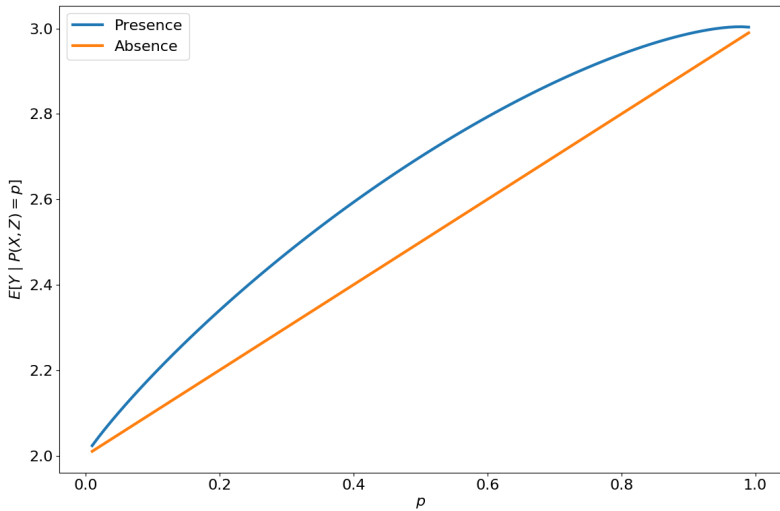   - ▶ Uncertainty
    - ▶ Private Information

Essential Heterogeneity

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 / D \quad | \ X = x.$$

$\Rightarrow$ consequences for the choice of the estimation strategy

Figure: Conditional Expectation of Observed Outcome

Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$
$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$
$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

$\Rightarrow$ correspond to *extreme* policy alternatives

Selection Problem

$$E[Y \mid D = 1] - E[Y \mid D = 0] = \underbrace{E[Y_1 - Y_0]}_{B^{ATE}}$$

$$+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}}$$

$$+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}$$

$$E[Y \mid D = 1] - E[Y \mid D = 0] = \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}}$$

$$+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}$$

$\Rightarrow$ the bias depends on the parameter of interest

Figure: Treatment Effects with Essential Heterogeneity

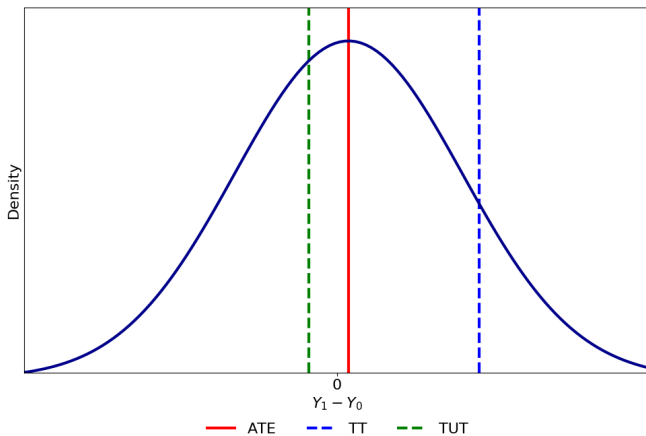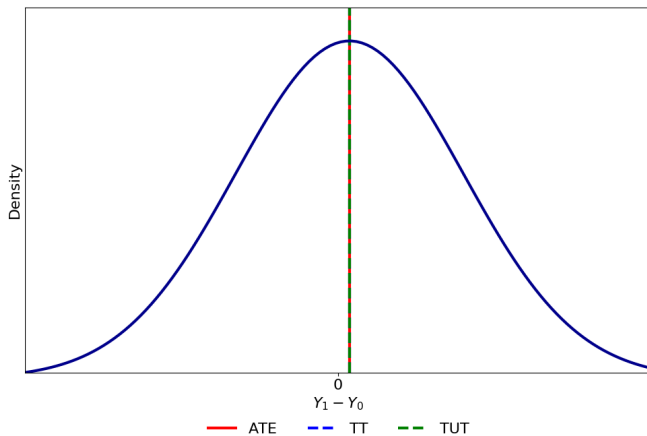Figure: Treatment Effects without Essential Heterogeneity

Policy-Relevant Average Treatment Effect

Observed Outcomes

$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$
$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Marginal Benefit of Treatment

$$B^{MTE}(x, u_S) = E[Y_1 - Y_0 \mid X = x, U_S = u_S]$$

Intuition: Mean gross return to treatment for persons at quantile u_S of the first-stage unobservable V.
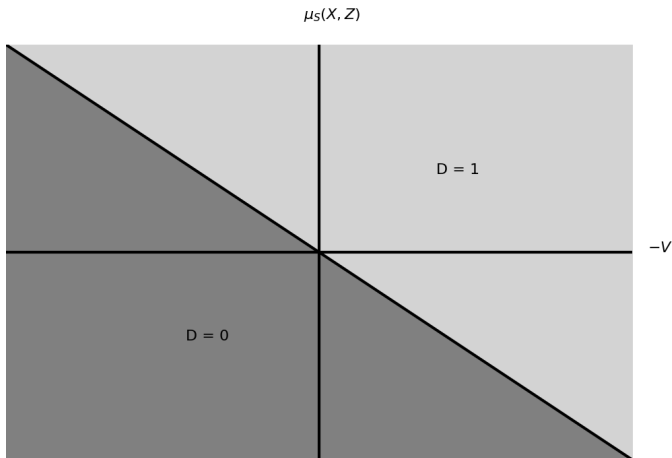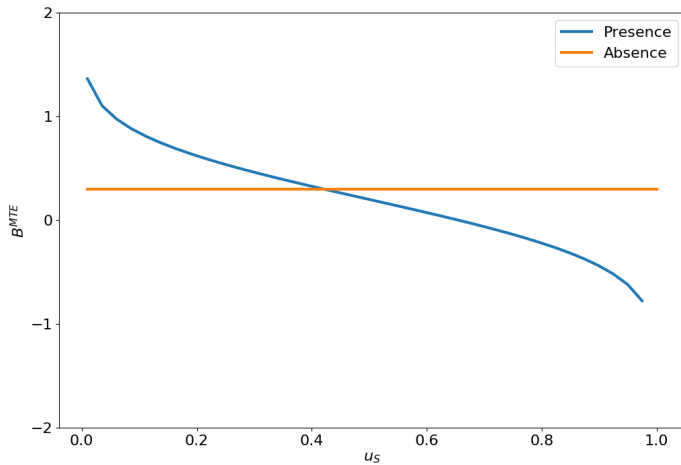
Figure: Margin of Indifference

Figure: Marginal Effect of Treatment

Effects of Treatment as Weighted Averages

Parameter Δ_j, can be written as a weighted average of the $B^{MTE}(x, u_S)$

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_S)\omega^j(x, u_S)du_S,$$

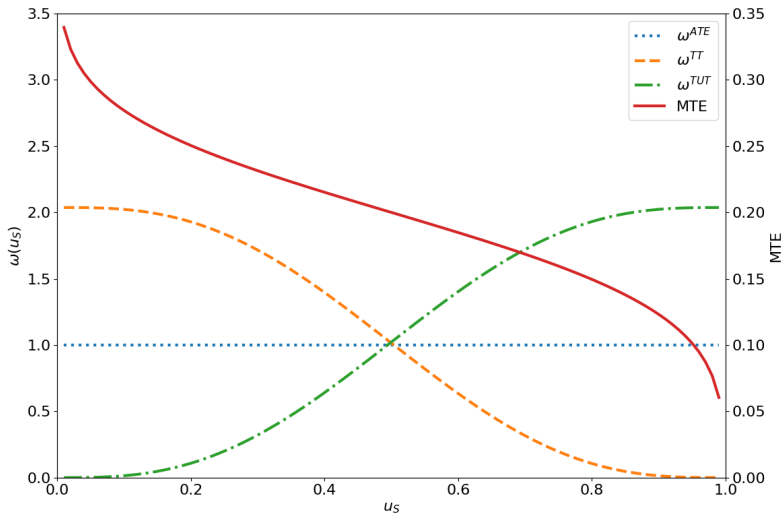where the weights $\omega^j(x, u_S)$ are specific to parameter j and integrate to one.

Weights

$$\omega^{ATE}(x, u_S) = 1$$

$$\omega^{TT}(x, u_S) = \frac{1 - F_{P|X=x}(u_S)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_S) = \frac{F_{P|X=x}(u_S)}{E[1 - P \mid X = x]}$$

Figure: Treatment Effect Weights

- **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.

  $\Rightarrow$ instrument-dependent parameter

- **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.

  $\Rightarrow$ deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_S, u_{S'}) = \frac{1}{u_S - u_{S'}} \int_{u_S}^{u_{S'}} B^{MTE}(x, u) du$$

Figure: Local Average Treatment Effect
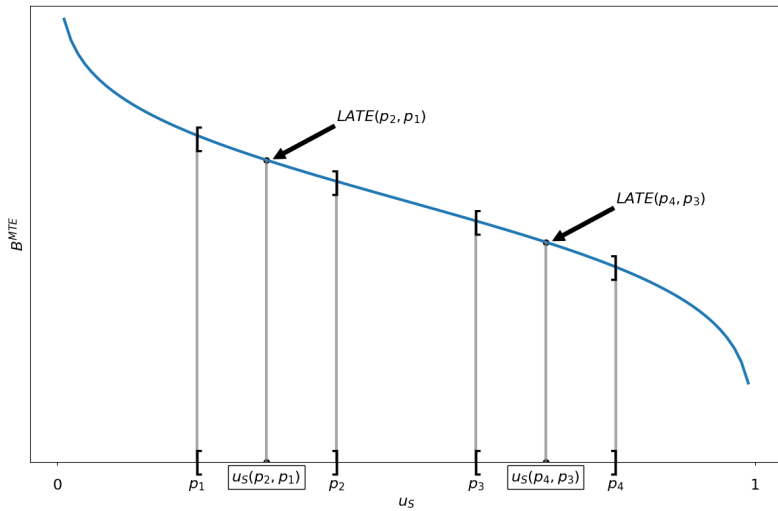
Distributional Effects of Treatment:

- ▶ Marginal Distribution of Benefits
- ▶ Joint Distribution of Potential Outcomes
- ▶ Joint Distribution of Benefits and Surplus
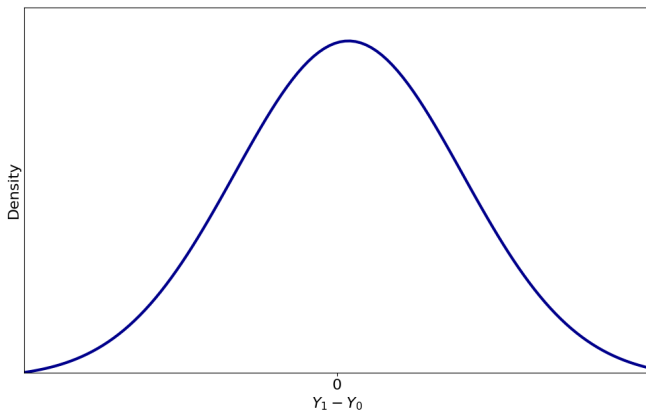
Figure: Marginal Distribution of Benefits
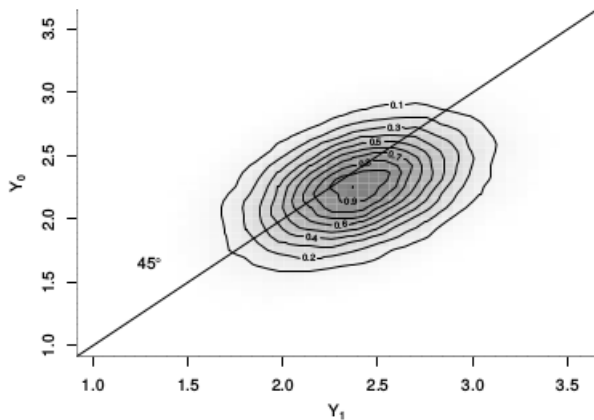
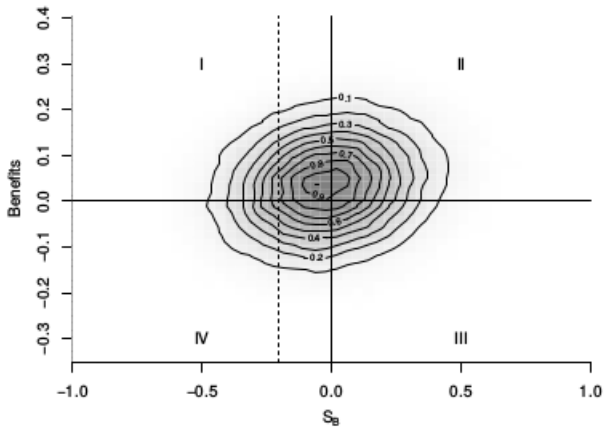# Figure: Joint Distribution of Potential Outcomes

Figure: Joint Distribution of Surplus and Benefits

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`grmpy` is an open-source Python package for the simulation and estimation of generalized Roy Model (Heckman & Vytlacil, 2005 [11]). It's main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model which allows to illustrate a variety of issues in the econometrics of policy evaluation.

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Appendix

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