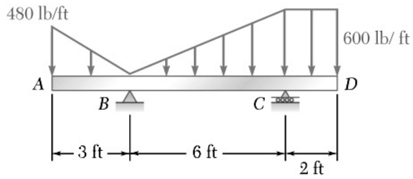
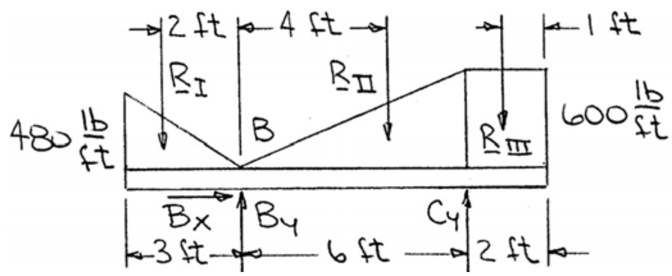
PROBLEM 5.137



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have

$$R_{\mathrm{I}} = \frac{1}{2}(3\text{ ft})(480\text{ lb/ft}) = 720\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(6\text{ ft})(600\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\mathrm{III}} = (2\text{ ft})(600\text{ lb/ft}) = 1200\text{ lb}$$

Then

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb})$$

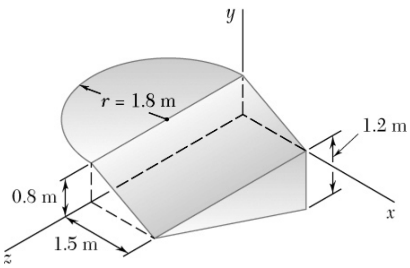
$$+(6\text{ ft})Cy - (7\text{ ft})(1200\text{ lb}) = 0$$

or $\mathrm{C}_y = 2360\text{ lb}$ $\qquad$ $\mathbf{C} = 2360\text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb} - 1200\text{ lb} = 0$$

or $B_y = 1360\text{ lb}$ $\qquad$ $\mathbf{B} = 1360\text{ lb} \uparrow$ ◀

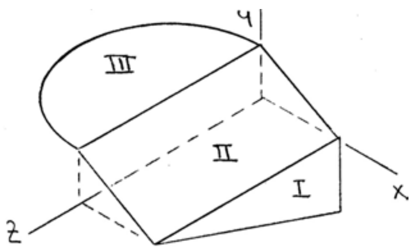
PROBLEM 5.138



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{I}} = -\frac{1}{3}(1.2) = -0.4\text{ m}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(3.6) = 1.2\text{ m}$$

$$\bar{x}_{\mathrm{III}} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi}\text{ m}$$

	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	–0.4	1.2	3.24	–0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	–3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

Have

$$\bar{X}\Sigma V = \Sigma\bar{x}V: \quad \bar{X}\left(13.3694\text{ m}^2\right) = 3.942\text{ m}^3$$

or $\bar{X} = 0.295\text{ m}$ ◀

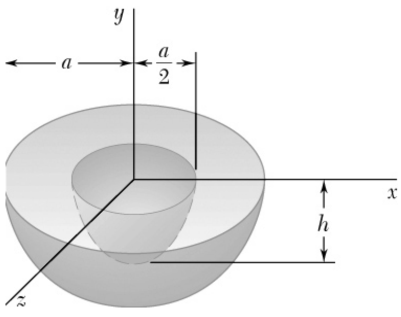
$$\bar{Y}\Sigma V = \Sigma\bar{y}V: \quad \bar{Y}\left(13.3694\text{ m}^2\right) = 5.6555\text{ m}^3$$

or $\bar{Y} = 0.423\text{ m}$ ◀

$$\bar{Z}\Sigma V = \Sigma\bar{z}V: \quad \bar{Z}\left(13.3694\text{ m}^2\right) = 22.769\text{ m}^3$$

or $\bar{Z} = 1.703\text{ m}$ ◀

PROBLEM 5.139



The composite body shown is formed by removing a semiellipsoid of revolution of semimajor axis h and semiminor axis $\frac{a}{2}$ from a hemisphere of radius a. Determine (a) the y coordinate of the centroid when $h = a/2$, (b) the ratio h/a for which $\overline{y} = -0.4a$.

SOLUTION

	V	$\overline{y}$	$\overline{y}V$
Hemisphere	$\frac{2}{3}\pi a^3$	$-\frac{3}{8}a$	$-\frac{1}{4}\pi a^4$
Semiellipsoid	$-\frac{2}{3}\pi\left(\frac{a}{2}\right)^2 h = -\frac{1}{6}\pi a^2 h$	$-\frac{3}{8}h$	$+\frac{1}{16}\pi a^2 h^2$

Then $$\Sigma V = \frac{\pi}{6}a^2(4a - h) \qquad \Sigma\overline{y}V = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

Now $$\overline{Y}\Sigma V = \Sigma\overline{y}V$$

so that $$\overline{Y}\left[\frac{\pi}{6}a^2(4a - h)\right] = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

or $$\overline{Y}\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right] \qquad (1)$$

(a) $\overline{Y} = ?$ when $h = \frac{a}{2}$

Substituting $\frac{h}{a} = \frac{1}{2}$ into Eq. (1)

$$\overline{Y}\left(4 - \frac{1}{2}\right) = -\frac{3}{8}a\left[4 - \left(\frac{1}{2}\right)^2\right]$$

or $$\overline{Y} = -\frac{45}{112}a$$

$\overline{Y} = -0.402a$ ◀

PROBLEM 5.139 CONTINUED

(*b*) $\dfrac{h}{a} = ?$ when $\overline{Y} = -0.4a$

Substituting into Eq. (1)

$$(-0.4a)\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right]$$

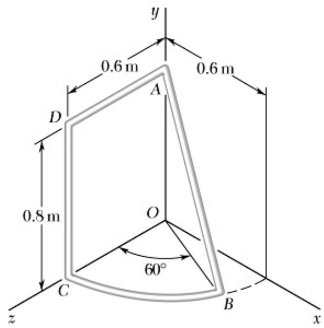
or
$$3\left(\frac{h}{a}\right)^2 - 3.2\left(\frac{h}{a}\right) + 0.8 = 0$$

Then
$$\frac{h}{a} = \frac{3.2 \pm \sqrt{(-3.2)^2 - 4(3)(0.8)}}{2(3)}$$

$$= \frac{3.2 \pm 0.8}{6}$$

or $\dfrac{h}{a} = \dfrac{2}{5}$ and $\dfrac{h}{a} = \dfrac{2}{3}$ ◀

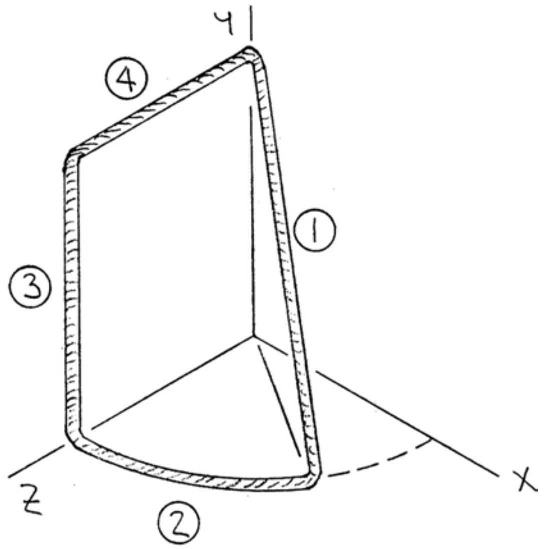
PROBLEM 5.140



A thin steel wire of uniform cross section is bent into the shape shown. Locate its center of gravity.

SOLUTION

First assume that the wire is homogeneous so that its center of gravity will coincide with the centroid of the corresponding line.



$$\bar{x}_1 = 0.3\sin 60° = 0.15\sqrt{3}\text{ m}$$

$$\bar{z}_1 = 0.3\cos 60° = 0.15\text{ m}$$

$$\bar{x}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\sin 30°$$

$$= \frac{0.9}{\pi}\text{ m}$$

$$\bar{z}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\cos 30°$$

$$= \frac{0.9}{\pi}\sqrt{3}\text{ m}$$

$$L_2 = \left(\frac{\pi}{3}\right)(0.6) = (0.2\pi)\text{ m}$$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	1.0	$0.15\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2π	$\frac{0.9}{\pi}$	0	$\frac{0.9\sqrt{3}}{\pi}$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.48	0.18
Σ	3.0283				0.43981	1.20	1.12177

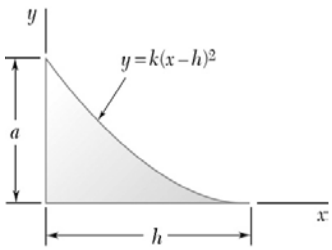
Have

$$\bar{X}\Sigma L = \Sigma\bar{x}L: \quad \bar{X}(3.0283\text{ m}) = 0.43981\text{ m}^2 \qquad \text{or } \bar{X} = 0.1452\text{ m} \blacktriangleleft$$

$$\bar{Y}\Sigma L = \Sigma\bar{y}L: \quad \bar{Y}(3.0283\text{ m}) = 1.20\text{ m}^2 \qquad \text{or } \bar{Y} = 0.396\text{ m} \blacktriangleleft$$

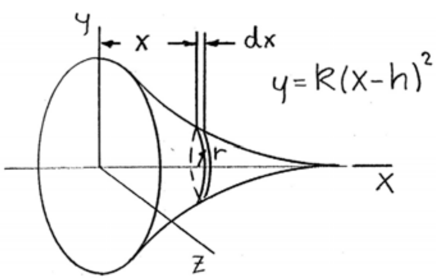
$$\bar{Z}\Sigma L = \Sigma\bar{z}L: \quad \bar{Z}(3.0283\text{ m}) = 1.12177\text{ m}^2 \qquad \text{or } \bar{Z} = 0.370\text{ m} \blacktriangleleft$$

PROBLEM 5.141



Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

SOLUTION



First note that symmetry implies $\bar{y} = 0$ ◀

and $\bar{z} = 0$ ◀

Have $\quad y = k(X - h)^2$

At $\quad x = 0, y = a: \; a = k(-h)^2$

or $\quad k = \dfrac{a}{h^2}$

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \;\; \bar{X}_{EL} = x$$

Now $\quad r = \dfrac{a}{h^2}(x - h)^2$

so that $\quad dV = \pi \dfrac{a^2}{h^4}(x - h)^4 \, dx$

Then
$$V = \int_0^h \pi \frac{a^2}{h^4}(x - h)^4 \, dx = \frac{\pi}{5}\frac{a^2}{h^4}\Big[(x - h)^5\Big]_0^h$$
$$= \frac{1}{5}\pi a^2 h$$

and
$$\int \bar{x}_{EL} \, dV = \int_0^h x\left[\pi \frac{a^2}{h^4}(x - h)^4 \, dx\right]$$
$$= \pi \frac{a^2}{h^4}\int_0^h \left(x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x\right)dx$$
$$= \pi \frac{a^2}{h^4}\left[\frac{1}{6}x^6 - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2\right]_0^h$$
$$= \frac{1}{30}\pi a^2 h^2$$

Now $\quad \bar{x}V = \int \bar{x}_{EL} \, dV: \quad \bar{x}\left(\dfrac{\pi}{5}a^2 h\right) = \dfrac{\pi}{30}a^2 h^2$

or $\bar{x} = \dfrac{1}{6}h$ ◀