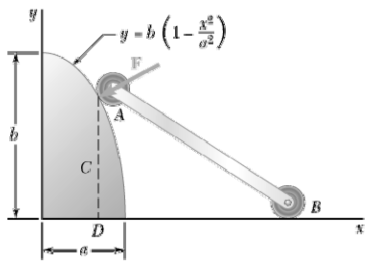
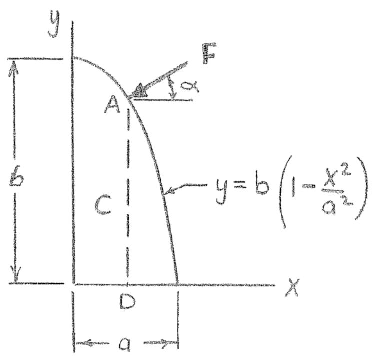
PROBLEM 3.114



As follower AB rolls along the surface of member C, it exerts a constant force **F** perpendicular to the surface. (a) Replace **F** with an equivalent force-couple system at the point D obtained by drawing the perpendicular from the point of contact to the x axis (b) For $a = 1\text{ m}$ and $b = 2\text{ m}$, determine the value of x for which the moment of the equivalent force-couple system at D is maximum.

SOLUTION



(a) The slope of any tangent to the surface of member C is

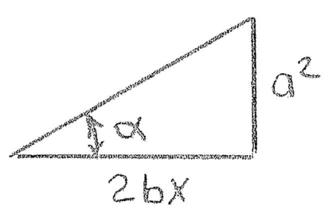
$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$

Since the force **F** is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence

$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$



$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$

where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}, \qquad y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$\therefore \quad M_D = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at D is

$$\mathbf{R} = F \measuredangle \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \circlearrowleft \blacktriangleleft$$

PROBLEM 3.114 CONTINUED

(*b*) To maximize *M*, the value of *x* must satisfy $\frac{dM}{dx} = 0$

where, for $a = 1$ m, $b = 2$ m

$$M = \frac{8F\left(x - x^3\right)}{\sqrt{1 + 16x^2}}$$

$$\therefore \quad \frac{dM}{dx} = 8F\frac{\sqrt{1+16x^2}\left(1 - 3x^2\right) - \left(x - x^3\right)\left[\frac{1}{2}\left(32x\right)\left(1 + 16x^2\right)^{-\frac{1}{2}}\right]}{\left(1 + 16x^2\right)} = 0$$

$$\left(1 + 16x^2\right)\left(1 - 3x^2\right) - 16x\left(x - x^3\right) = 0$$

or $$32x^4 + 3x^2 - 1 = 0$$

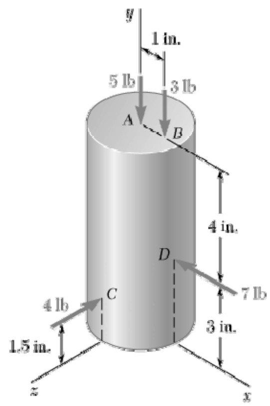
$$x^2 = \frac{-3 \pm \sqrt{9 - 4\left(32\right)\left(-1\right)}}{2\left(32\right)} = 0.136011 \text{ m}^2 \text{ and } -0.22976 \text{ m}^2$$

Using the positive value of x^2,

$$x = 0.36880 \text{ m}$$

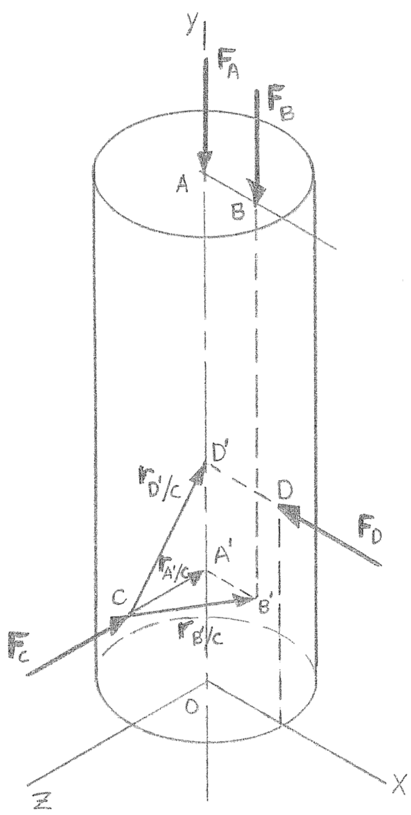
or $x = 369$ mm ◀

PROBLEM 3.115



As plastic bushings are inserted into a 3-in.-diameter cylindrical sheet metal container, the insertion tool exerts the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at *C*.

SOLUTION



For equivalence

$$\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}_C$$

$$\mathbf{R}_C = -(5 \text{ lb})\mathbf{j} - (3 \text{ lb})\mathbf{j} - (4 \text{ lb})\mathbf{k} - (7 \text{ lb})\mathbf{i}$$

$$\therefore \quad \mathbf{R}_C = (-7 \text{ lb})\mathbf{i} - (8 \text{ lb})\mathbf{j} - (4 \text{ lb})\mathbf{k} \blacktriangleleft$$

Also for equivalence

$$\Sigma\mathbf{M}_C: \quad \mathbf{r}_{A'/C} \times \mathbf{F}_A + \mathbf{r}_{B'/C} \times \mathbf{F}_B + \mathbf{r}_{D'/C} \times \mathbf{F}_D = \mathbf{M}_C$$

or

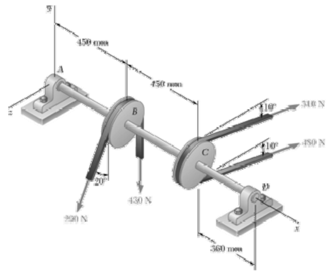
$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1.5 \text{ in.} \\ 0 & 5 \text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 \text{ in.} & 0 & -1.5 \text{ in.} \\ 0 & -3 \text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 \text{ in.} & 1.5 \text{ in.} \\ -7 \text{ lb} & 0 & 0 \end{vmatrix}$$

$$= \left[(-7.50 \text{ lb}\cdot\text{in.} - 0)\mathbf{i}\right] + \left[(0 - 4.50 \text{ lb}\cdot\text{in.})\mathbf{i} + (-3.0 \text{ lb}\cdot\text{in.} - 0)\mathbf{k}\right]$$

$$+ \left[(10.5 \text{ lb}\cdot\text{in.} - 0)\mathbf{j} + (0 + 10.5 \text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$\text{or } \mathbf{M}_C = -(12.0 \text{ lb}\cdot\text{in.})\mathbf{i} + (10.5 \text{ lb}\cdot\text{in.})\mathbf{j} + (7.5 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.116

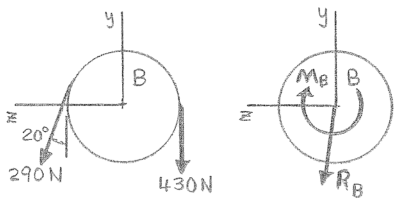


Two 300-mm-diameter pulleys are mounted on line shaft *AD*. The belts *B* and *C* lie in vertical planes parallel to the *yz* plane. Replace the belt forces shown with an equivalent force-couple system at *A*.

SOLUTION

Equivalent force-couple at each pulley

Pulley *B*



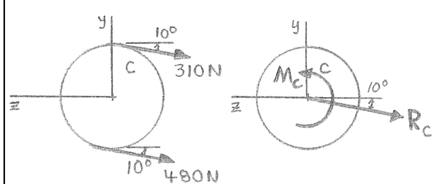
$$\mathbf{R}_B = (290\text{ N})(-\cos 20°\,\mathbf{j} + \sin 20°\,\mathbf{k}) - 430\text{ N}\,\mathbf{j}$$

$$= -(702.51\text{ N})\,\mathbf{j} + (99.186\text{ N})\,\mathbf{k}$$

$$\mathbf{M}_B = -(430\text{ N} - 290\text{ N})(0.15\text{ m})\,\mathbf{i}$$

$$= -(21\text{ N}\cdot\text{m})\,\mathbf{i}$$

Pulley *C*



$$\mathbf{R}_C = (310\text{ N} + 480\text{ N})(-\sin 10°\,\mathbf{j} - \cos 10°\,\mathbf{k})$$

$$= -(137.182\text{ N})\,\mathbf{j} - (778.00\text{ N})\,\mathbf{k}$$

$$\mathbf{M}_C = (480\text{ N} - 310\text{ N})(0.15\text{ m})\,\mathbf{i}$$

$$= (25.5\text{ N}\cdot\text{m})\,\mathbf{i}$$

Then $\quad \mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(839.69\text{ N})\,\mathbf{j} - (678.81\text{ N})\,\mathbf{k}$

$$\text{or}\quad \mathbf{R} = -(840\text{ N})\,\mathbf{j} - (679\text{ N})\,\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

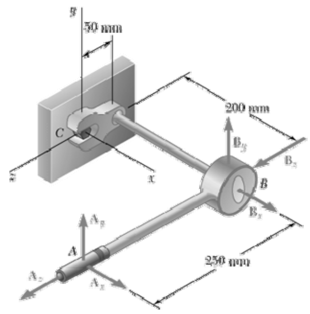
$$= -(21\text{ N}\cdot\text{m})\,\mathbf{i} + (25.5\text{ N}\cdot\text{m})\,\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -702.51 & 99.186 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.90 & 0 & 0 \\ 0 & -137.182 & -778.00 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (4.5\text{ N}\cdot\text{m})\,\mathbf{i} + (655.57\text{ N}\cdot\text{m})\,\mathbf{j} - (439.59\text{ N}\cdot\text{m})\,\mathbf{k}$$

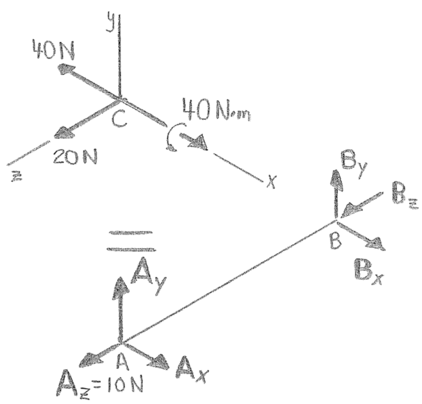
$$\text{or}\quad \mathbf{M}_A = (4.50\text{ N}\cdot\text{m})\,\mathbf{i} + (656\text{ N}\cdot\text{m})\,\mathbf{j} - (440\text{ N}\cdot\text{m})\,\mathbf{k} \blacktriangleleft$$

PROBLEM 3.117



A mechanic uses a crowfoot wrench to loosen a bolt at C. The mechanic holds the socket wrench handle at points A and B and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at C consisting of the force $\mathbf{C} = -(40\text{ N})\mathbf{i} + (20\text{ N})\mathbf{k}$ and the couple $\mathbf{M}_C = (40\text{ N}\cdot\text{m})\mathbf{i}$, determine the forces applied at A and B when $A_z = 10\text{ N}$.

SOLUTION



Have $\Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$

or $F_x: \quad A_x + B_x = -40\text{ N}$

$$\therefore \quad B_x = -(A_x + 40\text{ N}) \tag{1}$$

$$\Sigma F_y: \quad A_y + B_y = 0$$

or
$$A_y = -B_y \tag{2}$$

$$\Sigma F_z: \quad 10\text{ N} + B_z = 20\text{ N}$$

or
$$B_z = 10\text{ N} \tag{3}$$

Have $\Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & -0.05 \\ B_x & B_y & 10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0.2 \\ A_x & A_y & 10 \end{vmatrix} \text{N}\cdot\text{m} = (40\text{ N}\cdot\text{m})\mathbf{i}$$

or
$$(0.05B_y - 0.2A_x)\mathbf{i} + (-0.05B_x - 2 + 0.2A_x - 2)\mathbf{j}$$

$$+(0.2B_y + 0.2A_y)\mathbf{k} = (40\text{ N}\cdot\text{m})\mathbf{i}$$

From $\mathbf{i}$ - coefficient $\quad 0.05B_y - 0.2A_y = 40\text{ N}\cdot\text{m} \quad (4)$

$\mathbf{j}$ - coefficient $\quad -0.05B_x + 0.2A_x = 4\text{ N}\cdot\text{m} \quad (5)$

$\mathbf{k}$ - coefficient $\quad 0.2B_y + 0.2A_y = 0 \quad (6)$

PROBLEM 3.117 CONTINUED

From Equations (2) and (4): $0.05B_y - 0.2(-B_y) = 40$

$$B_y = 160 \text{ N}, A_y = -160 \text{ N}$$

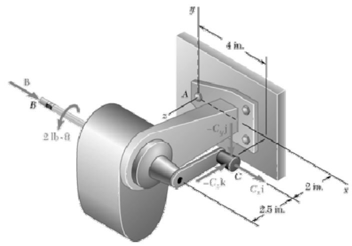
From Equations (1) and (5): $-0.05(-A_x - 40) + 0.2A_x = 4$

$$A_x = 8 \text{ N}$$

From Equation (1): $B_x = -(8 + 40) = -48 \text{ N}$

$$\therefore \quad \mathbf{A} = (8 \text{ N})\mathbf{i} - (160 \text{ N})\mathbf{j} + (10 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -(48 \text{ N})\mathbf{i} + (160 \text{ N})\mathbf{j} + (10 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.118

While using a pencil sharpener, a student applies the forces and couple shown. (*a*) Determine the forces exerted at *B* and *C* knowing that these forces and the couple are equivalent to a force-couple system at *A* consisting of the force $\mathbf{R} = (3.9\text{ lb})\mathbf{i} + R_y\mathbf{j} - (1.1\text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.5\text{ lb}\cdot\text{ft})\mathbf{j} - (1.1\text{ lb}\cdot\text{ft})\mathbf{k}$.. (*b*) Find the corresponding values of R_y and M_x.

SOLUTION

Have $$\Sigma\mathbf{F}: \quad \mathbf{B} + \mathbf{C} = \mathbf{R}$$

$$\Sigma F_x: \quad B_x + C_x = 3.9\text{ lb} \qquad \text{or} \qquad B_x = 3.9\text{ lb} - C_x \tag{1}$$

$$\Sigma F_y: \quad C_y = R_y \tag{2}$$

$$\Sigma F_z: \quad C_z = -1.1\text{ lb} \tag{3}$$

Have $$\Sigma\mathbf{M}_A: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C} + \mathbf{M}_B = \mathbf{M}_A^R$$

$$\therefore \quad \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & 4.5 \\ B_x & 0 & 0 \end{vmatrix} + \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 2.0 \\ C_x & C_y & -1.1 \end{vmatrix} + (2\text{ lb}\cdot\text{ft})\mathbf{i} = M_x\mathbf{i} + (1.5\text{ lb}\cdot\text{ft})\mathbf{j} - (1.1\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$(2 - 0.166667C_y)\mathbf{i} + (0.375B_x + 0.166667C_x + 0.36667)\mathbf{j} + (0.33333C_y)\mathbf{k}$$

$$= M_x\mathbf{i} + (1.5)\mathbf{j} - (1.1)\mathbf{k}$$

From

$$\mathbf{i}\text{ - coefficient} \qquad 2 - 0.166667C_y = M_x \tag{4}$$

$$\mathbf{j}\text{ - coefficient} \qquad 0.375B_x + 0.166667C_x + 0.36667 = 1.5 \tag{5}$$

$$\mathbf{k}\text{ - coefficient} \qquad 0.33333C_y = -1.1 \qquad \text{or} \qquad C_y = -3.3\text{ lb} \tag{6}$$

(*a*) From Equations (1) and (5):

$$0.375(3.9 - C_x) + 0.166667C_x = 1.13333$$

$$C_x = \frac{0.32917}{0.20833} = 1.58000\text{ lb}$$

From Equation (1): $$B_x = 3.9 - 1.58000 = 2.32\text{ lb}$$

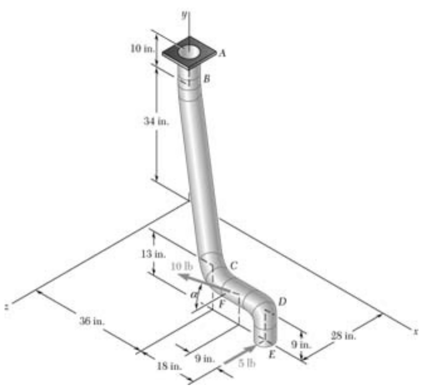
$$\therefore \quad \mathbf{B} = (2.32\text{ lb})\mathbf{i} \blacktriangleleft$$

$$\mathbf{C} = (1.580\text{ lb})\mathbf{i} - (3.30\text{ lb})\mathbf{j} - (1.1\text{ lb})\mathbf{k} \blacktriangleleft$$

(*b*) From Equation (2): $$R_y = C_y = -3.30\text{ lb} \qquad \text{or} \quad \mathbf{R}_y = -(3.30\text{ lb})\mathbf{j} \blacktriangleleft$$

From Equation (4): $$M_x = -0.166667(-3.30) + 2.0 = 2.5500\text{ lb}\cdot\text{ft}$$

$$\text{or} \quad \mathbf{M}_x = (2.55\text{ lb}\cdot\text{ft})\mathbf{i} \blacktriangleleft$$



PROBLEM 3.119

A portion of the flue for a furnace is attached to the ceiling at *A*. While supporting the free end of the flue at *F*, a worker pushes in at *E* and pulls out at *F* to align end *E* with the furnace. Knowing that the 10-lb force at *F* lies in a plane parallel to the *yz* plane, determine (*a*) the angle α the force at *F* should form with the horizontal if duct *AB* is not to tend to rotate about the vertical, (*b*) the force-couple system at *B* equivalent to the given force system when this condition is satisfied.

SOLUTION

(*a*) Duct *AB* will not have a tendency to rotate about the vertical or *y*-axis if:

$$M_{By}^R = \mathbf{j} \cdot \Sigma \mathbf{M}_B^R = \mathbf{j} \cdot \left(\mathbf{r}_{F/B} \times \mathbf{F}_F + \mathbf{r}_{E/B} \times \mathbf{F}_E \right) = 0$$

where

$$\mathbf{r}_{F/B} = (45 \text{ in.})\mathbf{i} - (23 \text{ in.})\mathbf{j} + (28 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{E/B} = (54 \text{ in.})\mathbf{i} - (34 \text{ in.})\mathbf{j} + (28 \text{ in.})\mathbf{k}$$

$$\mathbf{F}_F = 10 \text{ lb}\left[(\sin\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k} \right]$$

$$\mathbf{F}_E = -(5 \text{ lb})\mathbf{k}$$

$$\therefore \quad \Sigma \mathbf{M}_B^R = (10 \text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 \text{ in.} & -23 \text{ in.} & 28 \text{ in.} \\ 0 & \sin\alpha & \cos\alpha \end{vmatrix} + (5 \text{ lb})(2 \text{ in.})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 27 & -17 & 14 \\ 0 & 0 & -1 \end{vmatrix}$$

$$= \left[(-230\cos\alpha - 280\sin\alpha + 170)\mathbf{i} - (450\cos\alpha - 270)\mathbf{j} + (450\sin\alpha)\mathbf{k} \right] \text{lb} \cdot \text{in.}$$

Thus,

$$M_{By}^R = -450\cos\alpha + 270 = 0$$

$$\cos\alpha = 0.60$$

$$\alpha = 53.130°$$

or $\alpha = 53.1° \blacktriangleleft$

PROBLEM 3.119 CONTINUED

(*b*) $\mathbf{R} = \mathbf{F}_E + \mathbf{F}_F$

where

$$\mathbf{F}_E = -(5\text{ lb})\mathbf{k}$$

$$\mathbf{F}_F = (10\text{ lb})(\sin 53.130°\mathbf{j} + \cos 53.130°\mathbf{k}) = (8\text{ lb})\mathbf{j} + (6\text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{R} = (8\text{ lb})\mathbf{j} + (1\text{ lb})\mathbf{k} \blacktriangleleft$$

and $$\mathbf{M} = \Sigma\mathbf{M}_B^R = -[230(0.6) + 280(0.8) - 170]\mathbf{i} - [450(0.6) - 270]\mathbf{j} + [450(0.8)]\mathbf{k}$$

$$= -(192\text{ lb}\cdot\text{in.})\mathbf{i} - (0)\mathbf{j} + (360\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\text{or } \mathbf{M} = -(192\text{ lb}\cdot\text{in.})\mathbf{i} + (360\text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$