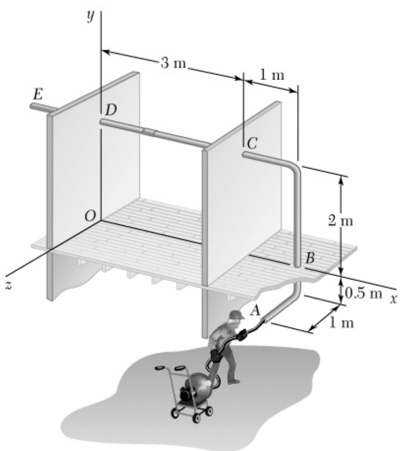
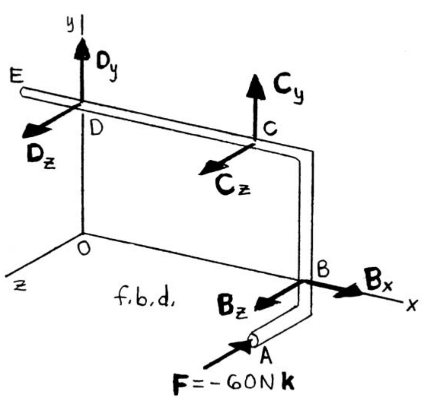
PROBLEM 4.137



Solve Problem 4.136 assuming that the plumber exerts a force $\mathbf{F} = -(60\ \text{N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

P4.136 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

and $\mathbf{B} = (75.0\ \text{N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - B_x(2\ \text{m}) = 0$$

$$\therefore \quad C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\ \text{m}) - (75.0\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20\ \text{N}$$

and $\mathbf{C} = -(20.0\ \text{N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

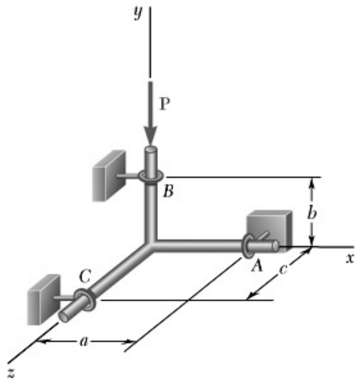
$$\therefore \quad D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 75\ \text{N} - 20\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

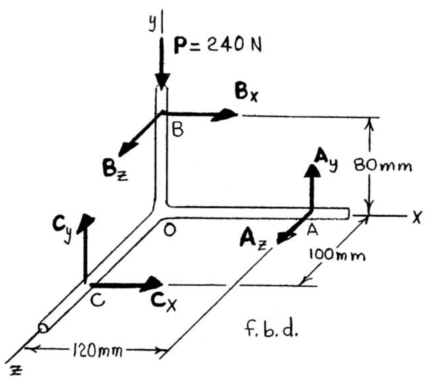
and $\mathbf{D} = (5.00\ \text{N})\mathbf{k}$ ◀



PROBLEM 4.138

Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 120 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 100 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$\left(-120A_z\mathbf{j} + 120A_y\mathbf{k}\right) + \left(80B_z\mathbf{i} - 80B_x\mathbf{k}\right) + \left(-100C_y\mathbf{i} + 100C_x\mathbf{j}\right) = 0$$

From **i**-coefficient $\quad 80B_z - 100C_y = 0$

or $\quad B_z = 1.25C_y \quad (1)$

j-coefficient $\quad -120A_z + 100C_x = 0$

or $\quad C_x = 1.2A_z \quad (2)$

k-coefficient $\quad 120A_y - 80B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad \left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y - 240\text{ N}\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$

From **i**-coefficient $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient $\quad A_y + C_y - 240\text{ N} = 0$

or $\quad A_y + C_y = 240\text{ N} \quad (5)$

k-coefficient $\quad A_z + B_z = 0$

or $\quad A_z = -B_z \quad (6)$

PROBLEM 4.138 CONTINUED

Substituting C_x from Equation (4) into Equation (2)

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7)

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8)

$$C_y = \frac{1.5A_y}{1.5} \qquad \text{or} \qquad C_y = A_y$$

and substituting into Equation (5)

$$2A_y = 240 \text{ N}$$

$$\therefore \quad A_y = C_y = 120 \text{ N} \tag{9}$$

Using Equation (1) and Equation (9)

$$B_z = 1.25(120 \text{ N}) = 150.0 \text{ N}$$

Using Equation (3) and Equation (9)

$$B_x = 1.5(120 \text{ N}) = 180.0 \text{ N}$$

From Equation (4) $\quad C_x = -180.0 \text{ N}$

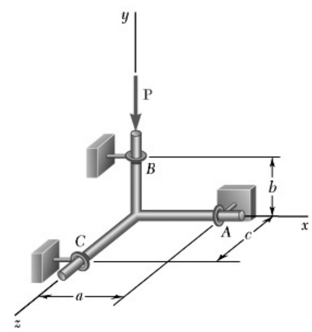
From Equation (6) $\quad A_z = -150.0 \text{ N}$

Therefore $\quad \mathbf{A} = (120.0 \text{ N})\mathbf{j} - (150.0 \text{ N})\mathbf{k}$ ◀

$$\mathbf{B} = (180.0 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{C} = -(180.0 \text{ N})\mathbf{i} + (120.0 \text{ N})\mathbf{j} \blacktriangleleft$$

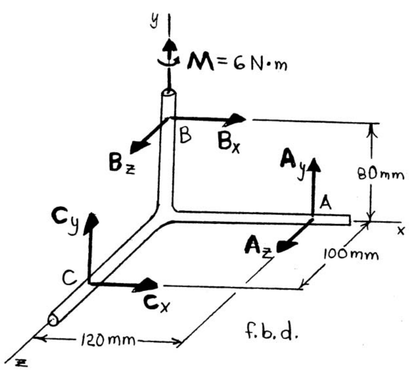
PROBLEM 4.139



Solve Problem 4.138 assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(6\ \text{N}\cdot\text{m})\mathbf{j}$ acting at B.

P4.138 Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.08 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ C_x & C_y & 0 \end{vmatrix} + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

$$\left(-0.12A_z\mathbf{j} + 0.12A_y\mathbf{k}\right) + \left(0.08B_z\mathbf{j} - 0.08B_x\mathbf{k}\right)$$

$$+ \left(-0.1C_y\mathbf{i} + 0.1C_x\mathbf{j}\right) + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

From **i**-coefficient $\qquad 0.08B_z - 0.1C_y = 0$

or $\qquad C_y = 0.8B_z \qquad (1)$

j-coefficient $\qquad -0.12A_z + 0.1C_x + 6 = 0$

or $\qquad C_x = 1.2A_z - 60 \qquad (2)$

k-coefficient $\qquad 0.12A_y - 0.08B_x = 0$

or $\qquad B_x = 1.5A_y \qquad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$\left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$$

From **i**-coefficient $\qquad C_x = -B_x \qquad (4)$

j-coefficient $\qquad C_y = -A_y \qquad (5)$

k-coefficient $\qquad A_z = -B_z \qquad (6)$

Substituting C_x from Equation (4) into Equation (2)

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \qquad (7)$$

PROBLEM 4.139 CONTINUED

Using Equations (1), (6), and (7)

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \qquad (8)$$

From Equations (3) and (8)

$$C_y = A_y - 40$$

Substituting into Equation (5) $\qquad 2A_y = 40$

$$\therefore \quad A_y = 20.0 \text{ N}$$

From Equation (5) $\qquad C_y = -20.0 \text{ N}$

Equation (1) $\qquad B_z = -25.0 \text{ N}$

Equation (3) $\qquad B_x = 30.0 \text{ N}$

Equation (4) $\qquad C_x = -30.0 \text{ N}$

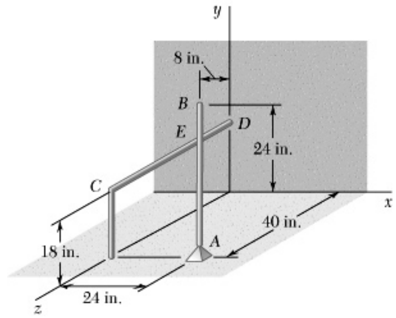
Equation (6) $\qquad A_z = 25.0 \text{ N}$

Therefore $\qquad \mathbf{A} = (20.0 \text{ N})\mathbf{j} + (25.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ N})\mathbf{i} - (25.0 \text{ N})\mathbf{k}$ ◀

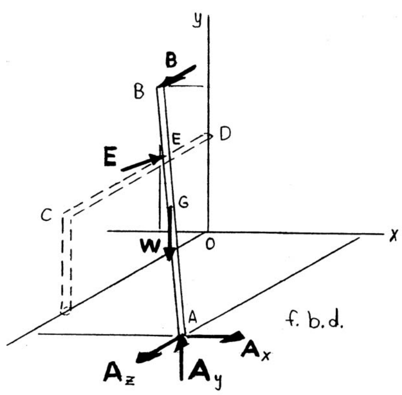
$\mathbf{C} = -(30.0 \text{ N})\mathbf{i} - (20.0 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.140



The uniform 10-lb rod *AB* is supported by a ball-and-socket joint at *A* and leans against both the rod *CD* and the vertical wall. Neglecting the effects of friction, determine (*a*) the force which rod *CD* exerts on *AB*, (*b*) the reactions at *A* and *B*. (*Hint:* The force exerted by *CD* on *AB* must be perpendicular to both rods.)

SOLUTION



(*a*) The force acting at *E* on the f.b.d. of rod *AB* is perpendicular to *AB* and *CD*. Letting $\lambda_E =$ direction cosines for force **E**,

$$\lambda_E = \frac{\mathbf{r}_{B/A} \times \mathbf{k}}{|\mathbf{r}_{B/A} \times \mathbf{k}|}$$

$$= \frac{\left[-(32\text{ in.})\mathbf{i} + (24\text{ in.})\mathbf{j} - (40\text{ in.})\mathbf{k}\right] \times \mathbf{k}}{\sqrt{(32)^2 + (24)^2}\text{ in.}}$$

$$= 0.6\mathbf{i} + 0.8\mathbf{j}$$

Also, $\mathbf{W} = -(10\text{ lb})\mathbf{j}$

$\mathbf{B} = B\mathbf{k}$

$\mathbf{E} = E(0.6\mathbf{i} + 0.8\mathbf{j})$

From f.b.d. of rod *AB*

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{G/A} \times \mathbf{W} + \mathbf{r}_{E/A} \times \mathbf{E} + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -16 & 12 & -20 \\ 0 & -1 & 0 \end{vmatrix}(10\text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -24 & 18 & -30 \\ 0.6 & 0.8 & 0 \end{vmatrix} E + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -32 & 24 & -40 \\ 0 & 0 & 1 \end{vmatrix} B = 0$$

$$(-20\mathbf{i} + 16\mathbf{k})(10\text{ lb}) + (24\mathbf{i} - 18\mathbf{j} - 30\mathbf{k})E + (24\mathbf{i} + 32\mathbf{j})B = 0$$

From **k**-coefficient $\quad 160 - 30E = 0$

$\therefore \quad E = 5.3333\text{ lb}$

and $\quad \mathbf{E} = 5.3333\text{ lb}(0.6\mathbf{i} + 0.8\mathbf{j})$

or $\quad \mathbf{E} = (3.20\text{ lb})\mathbf{i} + (4.27\text{ lb})\mathbf{j}$ ◀

(*b*) From **j**-coefficient $\quad -18(5.3333\text{ lb}) + 32B = 0$

$\therefore \quad B = 3.00\text{ lb}$

or $\quad \mathbf{B} = (3.00\text{ lb})\mathbf{k}$ ◀

PROBLEM 4.140 CONTINUED

From f.b.d. of rod *AB*

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{E} + \mathbf{B} = 0$$

$$A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} - (10 \text{ lb})\mathbf{j} + \left[(3.20 \text{ lb})\mathbf{i} + (4.27 \text{ lb})\mathbf{j}\right] + (3.00 \text{ lb})\mathbf{k} = 0$$

From **i**-coefficient $\quad A_x + 3.20 \text{ lb} = 0$

$$\therefore \quad A_x = -3.20 \text{ lb}$$

j-coefficient $\quad A_y - 10 \text{ lb} + 4.27 \text{ lb} = 0$

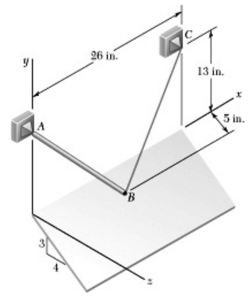
$$\therefore \quad A_y = 5.73 \text{ lb}$$

k-coefficient $\quad A_z + 3.00 \text{ lb} = 0$

$$\therefore \quad A_z = -3.00 \text{ lb}$$

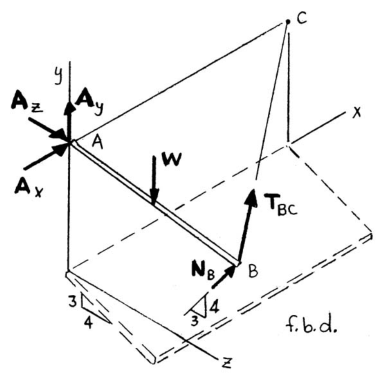
Therefore $\quad \mathbf{A} = -(3.20 \text{ lb})\mathbf{i} + (5.73 \text{ lb})\mathbf{j} - (3.00 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.141



A 21-in.-long uniform rod AB weighs 6.4 lb and is attached to a ball-and-socket joint at A. The rod rests against an inclined frictionless surface and is held in the position shown by cord BC. Knowing that the cord is 21 in. long, determine (a) the tension in the cord, (b) the reactions at A and B.

SOLUTION



First note

$$\mathbf{W} = -(6.4 \text{ lb})\mathbf{j}$$

$$\mathbf{N}_B = N_B(0.8\mathbf{j} + 0.6\mathbf{k})$$

$$L_{AB} = 21 \text{ in.}$$

$$= \sqrt{(x_B)^2 + (13+3)^2 + (4)^2} = \sqrt{x_B^2 + (16)^2 + (4)^2}$$

$$\therefore \; x_B = 13 \text{ in.}$$

$$\mathbf{T}_{BC} = \lambda_{BC} T_{BC} = \frac{(13 \text{ in.})\mathbf{i} + (16 \text{ in.})\mathbf{j} - (4 \text{ in.})\mathbf{k}}{21 \text{ in.}} T_{BC}$$

$$= \frac{T_{BC}}{21}(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k})$$

From f.b.d. of rod AB

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{G/A} \times \mathbf{W} + \mathbf{r}_{B/A} \times \mathbf{N}_B + \mathbf{r}_{C/A} \times \mathbf{T}_{BC} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -8 & 2 \\ 0 & -6.4 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & -16 & 4 \\ 0 & 0.8 & 0.6 \end{vmatrix} N_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 13 & 16 & -4 \end{vmatrix} \frac{26T_{BC}}{21} = 0$$

$$(12.8\mathbf{i} - 41.6\mathbf{k}) + (-12.8\mathbf{i} - 7.8\mathbf{j} + 10.4\mathbf{k})N_B + (4\mathbf{j} + 16\mathbf{k})\frac{26T_{BC}}{21} = 0$$

PROBLEM 4.141 CONTINUED

From **i**-coeff. $12.8 - 12.8N_B = 0 \qquad \therefore \quad N_B = 1.00 \text{ lb}$

or $\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$

From **j**-coeff. $-7.8N_B + 4\left(\dfrac{26}{21}\right)T_{BC} = 0 \qquad \therefore \quad T_{BC} = 1.575 \text{ lb}$

From f.b.d. of rod *AB*

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{N}_B + \mathbf{T}_{BC} = 0$$

$$\left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right) - (6.4 \text{ lb})\mathbf{j} + (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k} + \left(\frac{1.575}{21}\right)(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k}) = 0$$

From **i**-coefficient $A_x = -0.975 \text{ lb}$

j-coefficient $A_y = 4.40 \text{ lb}$

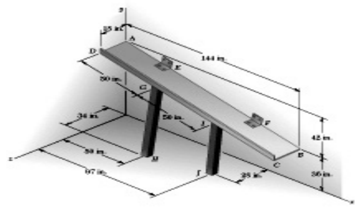
k-coefficient $A_z = -0.3 \text{ lb}$

$\therefore$ (*a*) $T_{BC} = 1.575 \text{ lb}$ ◀

(*b*) $\mathbf{A} = -(0.975 \text{ lb})\mathbf{i} + (4.40 \text{ lb})\mathbf{j} - (0.300 \text{ lb})\mathbf{k}$ ◀

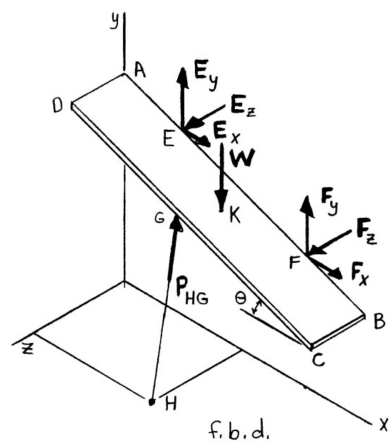
$\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.142



While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *GH* if prop *IJ* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42 \text{ in.}}{144 \text{ in.}}\right) = 16.2602°$$

$$x_G = (50 \text{ in.})\cos 16.2602° = 48 \text{ in.}$$

$$y_G = 78 \text{ in.} - (50 \text{ in.})\sin 16.2602° = 64 \text{ in.}$$

$$\boldsymbol{\lambda}_{BA} = \frac{-(144 \text{ in.})\mathbf{i} + (42 \text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2} \text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (48 \text{ in.})\mathbf{i} - (78 \text{ in.} - 64 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k} = (48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56 \text{ lb})\mathbf{j}$$

$$\mathbf{P}_{HG} = \boldsymbol{\lambda}_{HG} P_{HG}$$

$$= \frac{-(2 \text{ in.})\mathbf{i} + (64 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{\sqrt{(2)^2 + (64)^2 + (16)^2} \text{ in.}} P_{HG}$$

$$= \frac{P_{HG}}{33}(-\mathbf{i} + 32\mathbf{j} - 8\mathbf{k})$$

PROBLEM 4.142 CONTINUED

From the f.b.d. of the chute

$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{P}_{HG}\right) = 0$$

$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 48 & -14 & 18 \\ -1 & 32 & -8 \end{vmatrix} \left[\frac{P_{HG}}{33(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-14)(-8) - (-24)(18)(32) + 7(18)(-1) - (7)(48)(-8)\right]\frac{P_{HG}}{33(25)} = 0$$

$$\therefore \quad P_{HG} = 29.141 \text{ lb}$$

or $P_{HG} = 29.1 \text{ lb}$ ◀