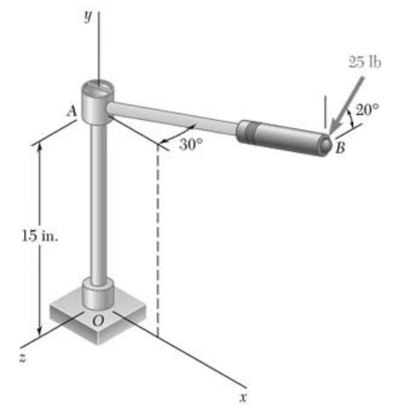
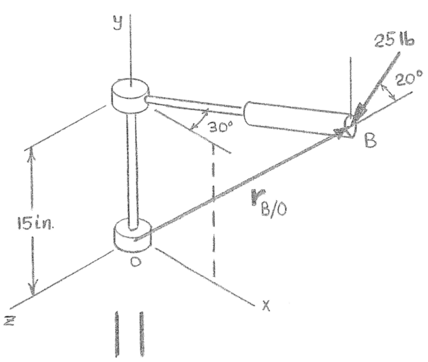
PROBLEM 3.94



A 25-lb force acting in a vertical plane parallel to the *yz* plane is applied to the 8-in.-long horizontal handle *AB* of a socket wrench. Replace the force with an equivalent force-couple system at the origin *O* of the coordinate system.

SOLUTION



Have

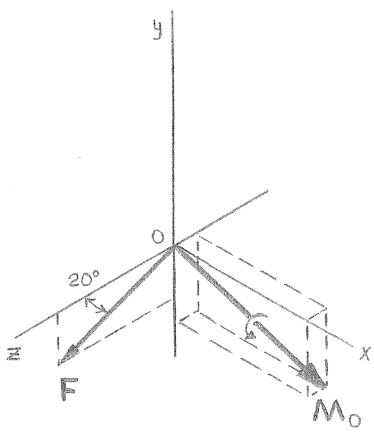
$$\Sigma\mathbf{F}: \quad \mathbf{P}_B = \mathbf{F}$$

where

$$\mathbf{P}_B = 25\text{ lb}\left[-(\sin 20°)\,\mathbf{j} + (\cos 20°)\,\mathbf{k}\right]$$

$$= -(8.5505\text{ lb})\,\mathbf{j} + (23.492\text{ lb})\,\mathbf{k}$$

or $\mathbf{F} = -(8.55\text{ lb})\,\mathbf{j} + (23.5\text{ lb})\,\mathbf{k}$ ◀



Have

$$\Sigma\mathbf{M}_O: \quad \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$$

where

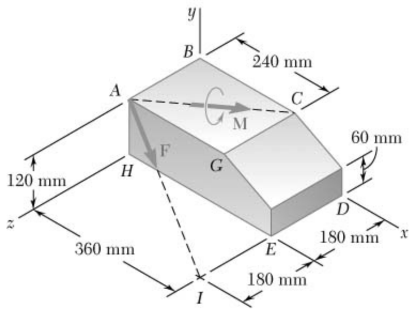
$$\mathbf{r}_{B/O} = \left[(8\cos 30°)\,\mathbf{i} + (15)\,\mathbf{j} - (8\sin 30°)\,\mathbf{k}\right]\text{ in.}$$

$$= (6.9282\text{ in.})\,\mathbf{i} + (15\text{ in.})\,\mathbf{j} - (4\text{ in.})\,\mathbf{k}$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.9282 & 15 & -4 \\ 0 & -8.5505 & 23.492 \end{vmatrix} \text{lb}\cdot\text{in.} = \mathbf{M}_O$$

$$\mathbf{M}_O = \left[(318.18)\,\mathbf{i} - (162.757)\,\mathbf{j} - (59.240)\,\mathbf{k}\right]\text{lb}\cdot\text{in.}$$

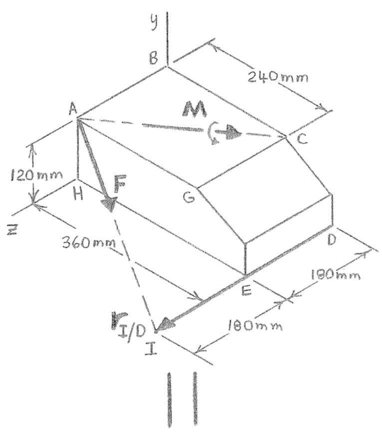
or $\mathbf{M}_O = (318\text{ lb}\cdot\text{in.})\,\mathbf{i} - (162.8\text{ lb}\cdot\text{in.})\,\mathbf{j} - (59.2\text{ lb}\cdot\text{in.})\,\mathbf{k}$ ◀

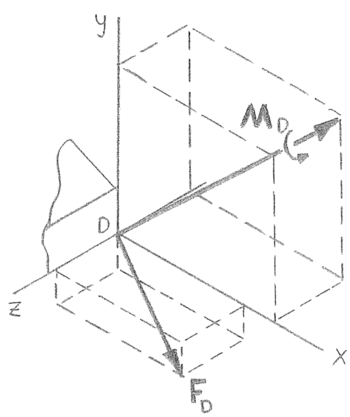


PROBLEM 3.95

A 315-N force **F** and 70-N · m couple **M** are applied to corner *A* of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner *D*.

SOLUTION





Have

$$\Sigma\mathbf{F}: \quad \mathbf{F} = \mathbf{F}_D$$

$$= \lambda_{AI} F$$

$$= \frac{(0.360\text{ m})\mathbf{i} - (0.120\text{ m})\mathbf{j} + (0.180\text{ m})\mathbf{k}}{0.420\text{ m}}(315\text{ N})$$

$$= (750\text{ N})(0.360\mathbf{i} - 0.120\mathbf{j} + 0.180\mathbf{k})$$

$$\text{or } \mathbf{F}_D = (270\text{ N})\mathbf{i} - (90.0\text{ N})\mathbf{j} + (135.0\text{ N})\mathbf{k} \blacktriangleleft$$

Have

$$\Sigma\mathbf{M}_D: \quad \mathbf{M} + \mathbf{r}_{I/D} \times \mathbf{F} = \mathbf{M}_D$$

where

$$\mathbf{M} = \lambda_{AC} M$$

$$= \frac{(0.240\text{ m})\mathbf{i} - (0.180\text{ m})\mathbf{k}}{0.300\text{ m}}(70.0\text{ N}\cdot\text{m})$$

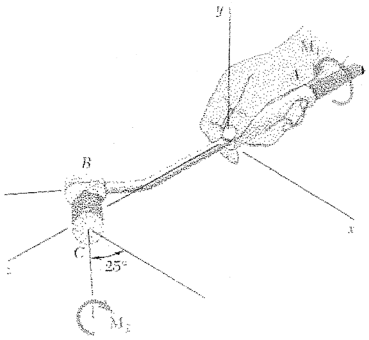
$$= (70.0\text{ N}\cdot\text{m})(0.800\mathbf{i} - 0.600\mathbf{k})$$

$$\mathbf{r}_{I/D} = (0.360\text{ m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_D = (70.0\text{ N}\cdot\text{m})(0.8\mathbf{i} - 0.6\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.36 \\ 0.36 & -0.12 & 0.18 \end{vmatrix}(750\text{ N}\cdot\text{m})$$

$$= (56.0\text{ N}\cdot\text{m})\mathbf{i} - (42.0\text{ N}\cdot\text{m})\mathbf{k} + \left[(32.4\text{ N}\cdot\text{m})\mathbf{i} + (97.2\text{ N}\cdot\text{m})\mathbf{j}\right]$$

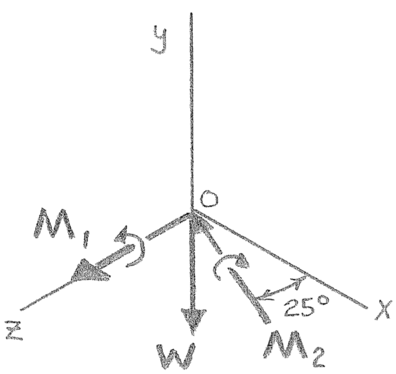
$$\text{or } \mathbf{M}_D = (88.4\text{ N}\cdot\text{m})\mathbf{i} + (97.2\text{ N}\cdot\text{m})\mathbf{j} - (42.0\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.96

The handpiece of a miniature industrial grinder weighs 2.4 N, and its center of gravity is located on the y axis. The head of the handpiece is offset in the xz plane in such a way that line BC forms an angle of $25°$ with the x direction. Show that the weight of the handpiece and the two couples $\mathbf{M}_1$ and $\mathbf{M}_2$ can be replaced with a single equivalent force. Further assuming that $M_1 = 0.068\ \text{N}\cdot\text{m}$ and $M_2 = 0.065\ \text{N}\cdot\text{m}$, determine ($a$) the magnitude and the direction of the equivalent force, (b) the point where its line of action intersects the xz plane.

SOLUTION



First assume that the given force $\mathbf{W}$ and couples $\mathbf{M}_1$ and $\mathbf{M}_2$ act at the origin.

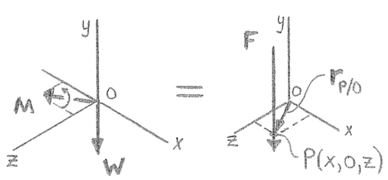
Now $$\mathbf{W} = -W_{\mathbf{j}}$$

and $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

Note that since $\mathbf{W}$ and $\mathbf{M}$ are perpendicular, it follows that they can be replaced with a single equivalent force.

(a) Have $\quad F = \mathbf{W} \quad$ or $\quad \mathbf{F} = -W_{\mathbf{j}} = -\left(2.4\ \text{N}\right)\mathbf{j}$

or $\mathbf{F} = -\left(2.40\ \text{N}\right)\mathbf{j}$ ◀



(b) Assume that the line of action of $\mathbf{F}$ passes through point $P\ (x, 0, z)$.

Then for equivalence

$$\mathbf{M} = \mathbf{r}_{P/O} \times \mathbf{F}$$

where $$\mathbf{r}_{P/O} = x\mathbf{i} + z\mathbf{k}$$

$$\therefore \quad -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & -W & 0 \end{vmatrix} = \left(Wz\right)\mathbf{i} - \left(Wx\right)\mathbf{k}$$

PROBLEM 3.96 CONTINUED

Equating the **i** and **k** coefficients,

$$z = \frac{-M_z \cos 25°}{W} \qquad \text{and} \qquad x = -\left(\frac{M_1 - M_2 \sin 25°}{W}\right)$$

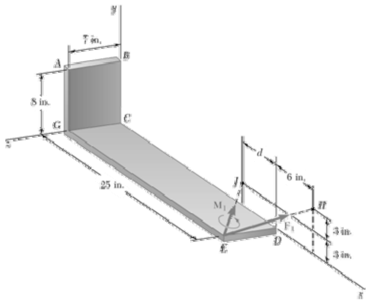
(*b*) For $W = 2.4\text{ N},\ M_1 = 0.068\text{ N}\cdot\text{m},\ M_2 = 0.065\text{ N}\cdot\text{m}$

$$x = \frac{0.068 - 0.065\sin 25°}{-2.4} = -0.0168874\text{ m}$$

or $x = -16.89$ mm ◀

$$z = \frac{-0.065\cos 25°}{2.4} = -0.024546\text{ m}$$

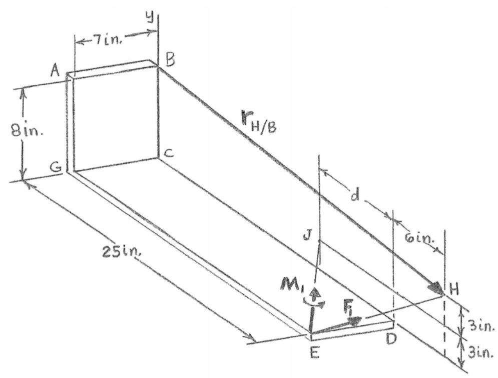
or $z = -24.5$ mm ◀



PROBLEM 3.97

A 20-lb force $\mathbf{F}_1$ and a 40-$\text{lb}\cdot\text{ft}$ couple $\mathbf{M}_1$ are applied to corner E of the bent plate shown. If $\mathbf{F}_1$ and $\mathbf{M}_1$ are to be replaced with an equivalent force-couple system $(\mathbf{F}_2, \mathbf{M}_2)$ at corner B and if $(M_2)_z = 0$, determine (a) the distance d, (b) $\mathbf{F}_2$ and $\mathbf{M}_2$.

SOLUTION



(a) Have

$$\Sigma M_{Bz}: \quad M_{2z} = 0$$

$$\mathbf{k}\cdot\left(\mathbf{r}_{H/B}\times\mathbf{F}_1\right) + M_{1z} = 0 \qquad (1)$$

where

$$\mathbf{r}_{H/B} = (31\text{ in.})\mathbf{i} - (2\text{ in.})\mathbf{j}$$

$$\mathbf{F}_1 = \lambda_{EH}F_1$$

$$= \frac{(6\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{11.0\text{ in.}}(20\text{ lb})$$

$$= \frac{20\text{ lb}}{11.0}(6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$$

$$M_{1z} = \mathbf{k}\cdot\mathbf{M}_1$$

$$\mathbf{M}_1 = \lambda_{EJ}M_1$$

$$= \frac{-d\mathbf{i} + (3\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{\sqrt{d^2+58}\text{ in.}}(480\text{ lb}\cdot\text{in.})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix}\frac{20\text{ lb}\cdot\text{in.}}{11.0} + \frac{(-7)(480\text{ lb}\cdot\text{in.})}{\sqrt{d^2+58}} = 0$$

PROBLEM 3.97 CONTINUED

Solving for d, Equation (1) reduces to

$$\frac{20 \text{ lb}\cdot\text{in.}}{11.0}(186 + 12) - \frac{3360 \text{ lb}\cdot\text{in.}}{\sqrt{d^2 + 58}} = 0$$

From which $d = 5.3955$ in.

or $d = 5.40$ in. ◀

(b)

$$\mathbf{F}_2 = \mathbf{F}_1 = \frac{20 \text{ lb}}{11.0}(6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$$

$$= (10.9091\mathbf{i} + 10.9091\mathbf{j} - 12.7273\mathbf{k})\text{lb}$$

or $\mathbf{F}_2 = (10.91 \text{ lb})\mathbf{i} + (10.91 \text{ lb})\mathbf{j} - (12.73 \text{ lb})\mathbf{k}$ ◀

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

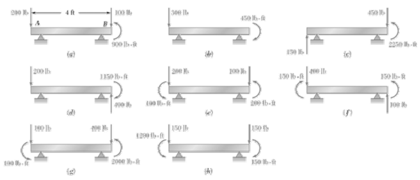
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix} \frac{20 \text{ lb}\cdot\text{in.}}{11.0} + \frac{(-5.3955)\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}}{9.3333}(480 \text{ lb}\cdot\text{in.})$$

$$= (25.455\mathbf{i} + 394.55\mathbf{j} + 360\mathbf{k})\text{lb}\cdot\text{in.}$$

$$+ (-277.48\mathbf{i} + 154.285\mathbf{j} - 360\mathbf{k})\text{lb}\cdot\text{in.}$$

$$\mathbf{M}_2 = -(252.03 \text{ lb}\cdot\text{in.})\mathbf{i} + (548.84 \text{ lb}\cdot\text{in.})\mathbf{j}$$

or $\mathbf{M}_2 = -(21.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (45.7 \text{ lb}\cdot\text{ft})\mathbf{j}$ ◀

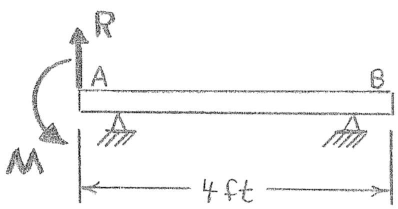


PROBLEM 3.98

A 4-ft-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?

SOLUTION

(*a*)



(*a*) Have $\Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R_a$

or $\mathbf{R}_a = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 900 \text{ lb}\cdot\text{ft} - (100 \text{ lb})(4 \text{ ft}) = M_a$

or $\mathbf{M}_a = 500 \text{ lb}\cdot\text{ft}$ ◀

(*b*) Have $\Sigma F_y: \quad -300 \text{ lb} = R_b$

or $\mathbf{R}_b = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad -450 \text{ lb}\cdot\text{ft} = M_b$

or $\mathbf{M}_b = 450 \text{ lb}\cdot\text{ft}$ ◀

(*c*) Have $\Sigma F_y: \quad 150 \text{ lb} - 450 \text{ lb} = R_c$

or $\mathbf{R}_c = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 2250 \text{ lb}\cdot\text{ft} - (450 \text{ lb})(4 \text{ ft}) = M_c$

or $\mathbf{M}_c = 450 \text{ lb}\cdot\text{ft}$ ◀

(*d*) Have $\Sigma F_y: \quad -200 \text{ lb} + 400 \text{ lb} = R_d$

or $\mathbf{R}_d = 200 \text{ lb}\uparrow$ ◀

and $\Sigma M_A: \quad (400 \text{ lb})(4 \text{ ft}) - 1150 \text{ lb}\cdot\text{ft} = M_d$

or $\mathbf{M}_d = 450 \text{ lb}\cdot\text{ft}$ ◀

(*e*) Have $\Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R_e$

or $\mathbf{R}_e = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 100 \text{ lb}\cdot\text{ft} + 200 \text{ lb}\cdot\text{ft} - (100 \text{ lb})(4 \text{ ft}) = M_e$

or $\mathbf{M}_e = 100 \text{ lb}\cdot\text{ft}$ ◀

PROBLEM 3.98 CONTINUED

(*f*) Have $\Sigma F_y: \quad -400 \text{ lb} + 100 \text{ lb} = R_f$

or $\mathbf{R}_f = 300 \text{ lb} \downarrow$ ◀

and $\Sigma M_A: \quad -150 \text{ lb}\cdot\text{ft} + 150 \text{ lb}\cdot\text{ft} + (100 \text{ lb})(4 \text{ ft}) = M_f$

or $\mathbf{M}_f = 400 \text{ lb}\cdot\text{ft}$ ↻ ◀

(*g*) Have $\Sigma F_y: \quad -100 \text{ lb} - 400 \text{ lb} = R_g$

or $\mathbf{R}_g = 500 \text{ lb} \downarrow$ ◀

and $\Sigma M_A: \quad 100 \text{ lb}\cdot\text{ft} + 2000 \text{ lb}\cdot\text{ft} - (400 \text{ lb})(4 \text{ ft}) = M_g$

or $\mathbf{M}_g = 500 \text{ lb}\cdot\text{ft}$ ↻ ◀

(*h*) Have $\Sigma F_y: \quad -150 \text{ lb} - 150 \text{ lb} = R_h$

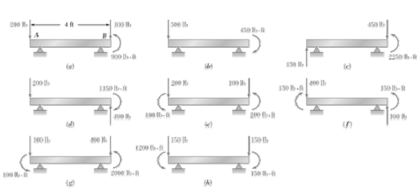
or $\mathbf{R}_h = 300 \text{ lb} \downarrow$ ◀

and $\Sigma M_A: \quad 1200 \text{ lb}\cdot\text{ft} - 150 \text{ lb}\cdot\text{ft} - (150 \text{ lb})(4 \text{ ft}) = M_h$

or $\mathbf{M}_h = 450 \text{ lb}\cdot\text{ft}$ ↻ ◀

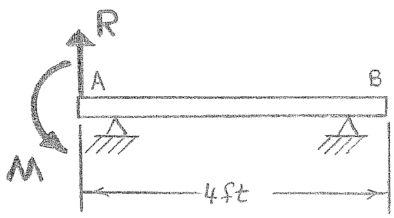
(*b*) Therefore, loadings (*c*) and (*h*) are equivalent ◀

PROBLEM 3.99



A 4-ft-long beam is loaded as shown. Determine the loading of Problem 3.98 which is equivalent to this loading.

SOLUTION



Have $\Sigma \mathbf{F}_y: \quad -100 \text{ lb} - 200 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$

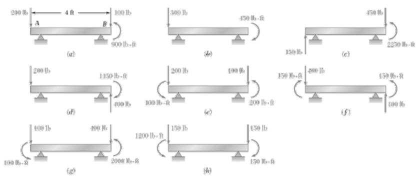
and $\Sigma M_A: \quad -200 \text{ lb}\cdot\text{ft} + 1400 \text{ lb}\cdot\text{ft} - (200 \text{ lb})(4 \text{ ft}) = M$

or $\mathbf{M} = 400 \text{ lb}\cdot\text{ft} \circlearrowleft$

Equivalent to case (*f*) of Problem 3.98 ◀

Problem 3.98 Equivalent force-couples at *A*

case	**R**	**M**	
(*a*)	300 lb ↓	500 lb·ft ↺	
(*b*)	300 lb ↓	450 lb·ft ↻	
(*c*)	300 lb ↓	450 lb·ft ↺	
(*d*)	200 lb ↑	450 lb·ft ↺	
(*e*)	300 lb ↓	100 lb·ft ↻	
(*f*)	300 lb ↓	400 lb·ft ↺	◀
(*g*)	500 lb ↓	500 lb·ft ↺	
(*h*)	300 lb ↓	450 lb·ft ↺	



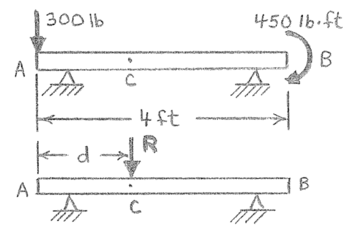
PROBLEM 3.100

Determine the single equivalent force and the distance from point A to its line of action for the beam and loading of (a) Problem 3.98b, (b) Problem 3.98d, (c) Problem 3.98e.

Problem 3.98: A 4-ft-long beam is subjected to a variety of loadings. (a) Replace each loading with an equivalent force-couple system at end A of the beam. (b) Which of the loadings are equivalent?

SOLUTION

(a)



For equivalent single force at distance d from A

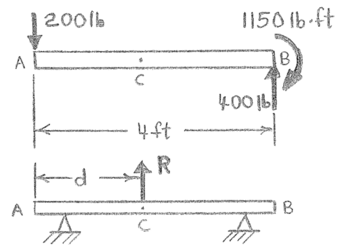
Have $\qquad \Sigma F_y: \quad -300 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$ ◀

and $\qquad \Sigma M_C: \quad (300 \text{ lb})(d) - 450 \text{ lb}\cdot\text{ft} = 0$

or $d = 1.500 \text{ ft}$ ◀

(b)



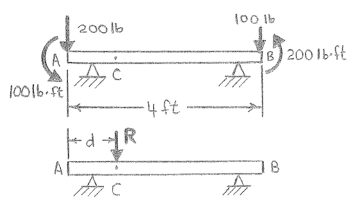
Have $\qquad \Sigma F_y: \quad -200 \text{ lb} + 400 \text{ lb} = R$

or $\mathbf{R} = 200 \text{ lb} \uparrow$ ◀

and $\qquad \Sigma M_C: \quad (200 \text{ lb})(d) + (400 \text{ lb})(4 - d) - 1150 \text{ lb}\cdot\text{ft} = 0$

or $d = 2.25 \text{ ft}$ ◀

(c)



Have $\qquad \Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$ ◀

and $\qquad \Sigma M_C: \quad 100 \text{ lb}\cdot\text{ft} + (200 \text{ lb})(d) - (100 \text{ lb})(4 - d) + 200 \text{ lb}\cdot\text{ft} = 0$

or $d = 0.333 \text{ ft}$ ◀