

A steel rod is bent into a semicircular ring of radius 0.96 m and is supported in part by cables BD and BE which are attached to the ring at B. Knowing that the tension in cable BE is 250 N, determine the components of this force exerted by the cable on the support at E.

SOLUTION

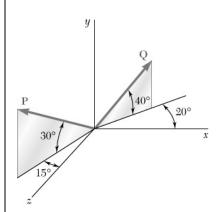
$$\overline{EB} = (0.96 \text{ m})\mathbf{i} - (1.20 \text{ m})\mathbf{j} + (1.28 \text{ m})\mathbf{k}$$

$$EB = \sqrt{(0.96 \text{ m})^2 + (-1.20 \text{ m})^2 + (1.28 \text{ m})^2} = 2.00 \text{ m}$$

$$\mathbf{T}_{EB} = T\lambda_{EB} = T\frac{\overline{EB}}{EB} = \frac{250 \text{ N}}{2.00 \text{ m}} [(0.96 \text{ m})\mathbf{i} - (1.20 \text{ m})\mathbf{j} + (1.28 \text{ m})\mathbf{k}]$$

$$\mathbf{T}_{EB} = (120 \text{ N})\mathbf{i} - (150 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$(T_{EB})_x = +120.0 \text{ N}, \ (T_{EB})_y = -150.0 \text{ N}, \ (T_{EB})_z = +160.0 \text{ N} \blacktriangleleft$$



Find the magnitude and direction of the resultant of the two forces shown knowing that P = 500 N and Q = 600 N.

SOLUTION

$$\mathbf{P} = (500 \text{ lb})[-\cos 30^{\circ} \sin 15^{\circ} \mathbf{i} + \sin 30^{\circ} \mathbf{j} + \cos 30^{\circ} \cos 15^{\circ} \mathbf{k}]$$

=
$$(500 \text{ lb})[-0.2241\mathbf{i} + 0.50\mathbf{j} + 0.8365\mathbf{k}]$$

=
$$-(112.05 \text{ lb})\mathbf{i} + (250 \text{ lb})\mathbf{j} + (418.25 \text{ lb})\mathbf{k}$$

$$\mathbf{Q} = (600 \text{ lb})[\cos 40^{\circ} \cos 20^{\circ} \mathbf{i} + \sin 40^{\circ} \mathbf{j} - \cos 40^{\circ} \sin 20^{\circ} \mathbf{k}]$$

$$= \big(600 \text{ lb}\big) \big[0.71985 \mathbf{i} + 0.64278 \mathbf{j} - 0.26201 \mathbf{k}\big]$$

=
$$(431.91 \text{ lb})\mathbf{i} + (385.67 \text{ lb})\mathbf{j} - (157.206 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} = (319.86 \text{ lb})\mathbf{i} + (635.67 \text{ lb})\mathbf{j} + (261.04 \text{ lb})\mathbf{k}$$

$$R = \sqrt{(319.86 \text{ lb})^2 + (635.67 \text{ lb})^2 + (261.04 \text{ lb})^2} = 757.98 \text{ lb}$$

 $R = 758 \text{ lb} \blacktriangleleft$

$$\cos \theta_x = \frac{R_x}{R} = \frac{319.86 \text{ lb}}{757.98 \text{ lb}} = 0.42199$$

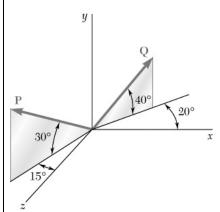
 $\theta_r = 65.0^{\circ} \blacktriangleleft$

$$\cos \theta_y = \frac{R_y}{R} = \frac{635.67 \text{ lb}}{757.98 \text{ lb}} = 0.83864$$

 $\theta_{\rm v} = 33.0^{\circ} \blacktriangleleft$

$$\cos \theta_z = \frac{R_z}{R} = \frac{261.04 \text{ lb}}{757.98 \text{ lb}} = 0.34439$$

 $\theta_z = 69.9^{\circ} \blacktriangleleft$



Find the magnitude and direction of the resultant of the two forces shown knowing that P = 600 N and Q = 400 N.

SOLUTION

Using the results from 2.93:

$$\mathbf{P} = (600 \text{ lb})[-0.2241\mathbf{i} + 0.50\mathbf{j} + 0.8365\mathbf{k}]$$

$$= -(134.46 \text{ lb})\mathbf{i} + (300 \text{ lb})\mathbf{j} + (501.9 \text{ lb})\mathbf{k}$$

$$\mathbf{Q} = (400 \text{ lb})[0.71985\mathbf{i} + 0.64278\mathbf{j} - 0.26201\mathbf{k}]$$

$$= (287.94 \text{ lb})\mathbf{i} + (257.11 \text{ lb})\mathbf{j} - (104.804 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} = (153.48 \text{ lb})\mathbf{i} + (557.11 \text{ lb})\mathbf{j} + (397.10 \text{ lb})\mathbf{k}$$

$$R = \sqrt{(153.48 \text{ lb})^2 + (557.11 \text{ lb})^2 + (397.10 \text{ lb})^2} = 701.15 \text{ lb}$$

 $R = 701 \text{ lb} \blacktriangleleft$

$$\cos \theta_x = \frac{R_x}{R} = \frac{153.48 \text{ lb}}{701.15 \text{ lb}} = 0.21890$$

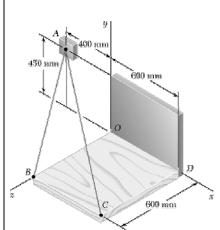
 $\theta_x = 77.4^{\circ} \blacktriangleleft$

$$\cos \theta_y = \frac{R_y}{R} = \frac{557.11 \text{ lb}}{701.15 \text{ lb}} = 0.79457$$

 $\theta_v = 37.4^{\circ} \blacktriangleleft$

$$\cos \theta_z = \frac{R_z}{R} = \frac{397.10 \text{ lb}}{701.15 \text{ lb}} = 0.56637$$

 $\theta_z = 55.5^{\circ} \blacktriangleleft$



Knowing that the tension is 850 N in cable AB and 1020 N in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overrightarrow{AB} = (400 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$AB = \sqrt{(400 \text{ mm})^2 + (-450 \text{ mm})^2 + (600 \text{ mm})^2} = 850 \text{ mm}$$

$$\overrightarrow{AC} = (1000 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$AC = \sqrt{(1000 \text{ mm})^2 + (-450 \text{ mm})^2 + (600 \text{ mm})^2} = 1250 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \boldsymbol{\lambda}_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (850 \text{ N}) \left[\frac{(400 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}}{850 \text{ mm}} \right]$$

$$\mathbf{T}_{AB} = (400 \text{ N})\mathbf{i} - (450 \text{ N})\mathbf{j} + (600 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \boldsymbol{\lambda}_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (1020 \text{ N}) \left[\frac{(1000 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}}{1250 \text{ mm}} \right]$$

$$\mathbf{T}_{AC} = (816 \text{ N})\mathbf{i} - (367.2 \text{ N})\mathbf{j} + (489.6 \text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (1216 \text{ N})\mathbf{i} - (817.2 \text{ N})\mathbf{j} + (1089.6 \text{ N})\mathbf{k}$$

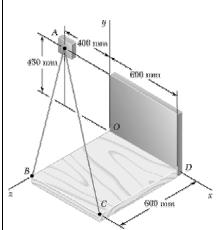
Then:
$$R = 1825.8 \text{ N}$$
 $R = 1826 \text{ N}$

and
$$\cos \theta_x = \frac{1216}{1825.8} = 0.66601$$
 $\theta_x = 48.2^{\circ} \blacktriangleleft$

$$\cos \theta_y = \frac{-817.2}{1825.8} = -0.44758 \qquad \theta_y = 116.6^{\circ} \blacktriangleleft$$

$$\cos \theta_z = \frac{1089.6}{1825.8} = 0.59678$$

$$\theta_z = 53.4^{\circ} \blacktriangleleft$$



Assuming that in Problem 2.95 the tension is 1020 N in cable AB and 850 N in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overrightarrow{AB} = (400 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$AB = \sqrt{(400 \text{ mm})^2 + (-450 \text{ mm})^2 + (600 \text{ mm})^2} = 850 \text{ mm}$$

$$\overrightarrow{AC} = (1000 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$AC = \sqrt{(1000 \text{ mm})^2 + (-450 \text{ mm})^2 + (600 \text{ mm})^2} = 1250 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \mathbf{\lambda}_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (1020 \text{ N}) \left[\frac{(400 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}}{850 \text{ mm}} \right]$$

$$\mathbf{T}_{AB} = (480 \text{ N})\mathbf{i} - (540 \text{ N})\mathbf{j} + (720 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \boldsymbol{\lambda}_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (850 \text{ N}) \left[\frac{(1000 \text{ mm})\mathbf{i} - (450 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}}{1250 \text{ mm}} \right]$$

$$\mathbf{T}_{AC} = (680 \text{ N})\mathbf{i} - (306 \text{ N})\mathbf{j} + (408 \text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (1160 \text{ N})\mathbf{i} - (846 \text{ N})\mathbf{j} + (1128 \text{ N})\mathbf{k}$$

Then:
$$R = 1825.8 \text{ N}$$
 $R = 1826 \text{ N}$

and
$$\cos \theta_x = \frac{1160}{1825.8} = 0.6353$$
 $\theta_x = 50.6^{\circ} \blacktriangleleft$

$$\cos \theta_y = \frac{-846}{1825.8} = -0.4634 \qquad \theta_y = 117.6^{\circ} \blacktriangleleft$$

$$\cos \theta_z = \frac{1128}{1825.8} = 0.6178$$

$$\theta_z = 51.8^{\circ} \blacktriangleleft$$