PROBLEM 3.20

Determine the moment about the origin O of the force $\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$ which acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = (2.5 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$, (*b*) $\mathbf{r} = (4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$, (*c*) $\mathbf{r} = (4 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (7 \text{ ft})\mathbf{k}$.

SOLUTION

(*a*) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} + (2 \text{ lb})\mathbf{k}$$

$$\mathbf{r} = (2.5 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & -1 & 2 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(2-6)\mathbf{i} + (-3+5)\mathbf{j} + (7.5-1.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

$$\text{or} \quad \mathbf{M}_O = -(4 \text{ lb}\cdot\text{ft})\mathbf{i} + (2 \text{ lb}\cdot\text{ft})\mathbf{j} + (6 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

(*b*) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$$

$$\mathbf{r} = (4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.5 & -9 & 6 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(18-18)\mathbf{i} + (-9+9)\mathbf{j} + (13.5-13.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

$$\text{or} \quad \mathbf{M}_O = 0 \blacktriangleleft$$

This answer is expected since $\mathbf{r}$ and $\mathbf{F}$ are proportional $\left(\mathbf{F} = \frac{-1}{3}\mathbf{r}\right)$.

Therefore, vector $\mathbf{F}$ has a line of action passing through the origin at O.

(*c*) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} - (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$$

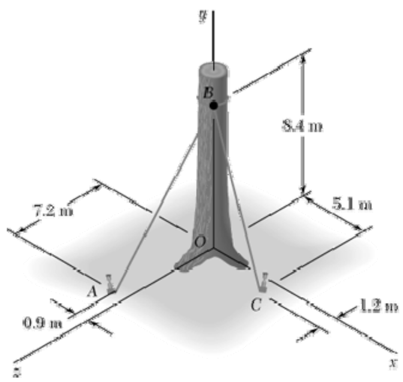
$$\mathbf{r} = (4 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (7 \text{ ft})\mathbf{k}$$

Then

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 7 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(2-21)\mathbf{i} + (-10.5+8)\mathbf{j} + (12-1.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

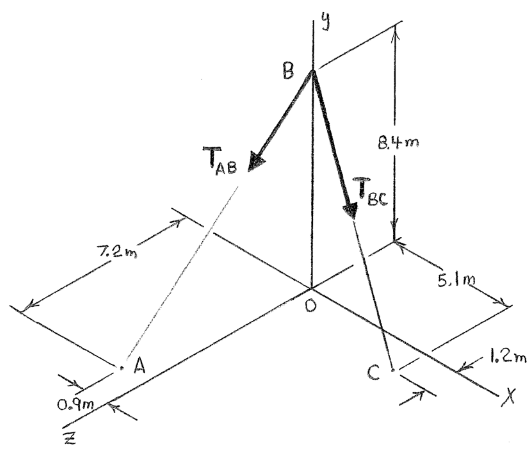
$$\text{or} \quad \mathbf{M}_O = -(19 \text{ lb}\cdot\text{ft})\mathbf{i} - (2.5 \text{ lb}\cdot\text{ft})\mathbf{j} + (10.5 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.21



Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Knowing that the tension in cables *AB* and *BC* are 777 N and 990 N, respectively, determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

SOLUTION



Have $$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}_B$$

where $$\mathbf{r}_{B/O} = (8.4\text{ m})\,\mathbf{j}$$

$$\mathbf{F}_B = \mathbf{T}_{AB} + \mathbf{T}_{BC}$$

$$\mathbf{T}_{AB} = \boldsymbol{\lambda}_{BA} T_{AB} = \frac{-(0.9\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (7.2\text{ m})\mathbf{k}}{\sqrt{(0.9)^2 + (8.4)^2 + (7.2)^2}\text{ m}}(777\text{ N})$$

$$\mathbf{T}_{BC} = \boldsymbol{\lambda}_{BC} T_{BC} = \frac{(5.1\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}}{\sqrt{(5.1)^2 + (8.4)^2 + (1.2)^2}\text{ m}}(990\text{ N})$$

PROBLEM 3.21 CONTINUED

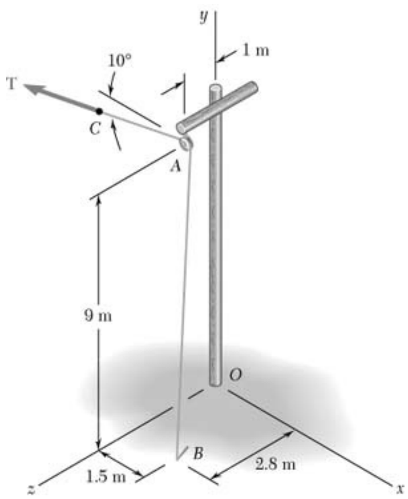
$$\therefore\quad \mathbf{F}_B = \left[-(63.0\text{ N})\mathbf{i} - (588\text{ N})\mathbf{j} + (504\text{ N})\mathbf{k}\right] + \left[(510\text{ N})\mathbf{i} - (840\text{ N})\mathbf{j} + (120\text{ N})\mathbf{k}\right]$$

$$= (447\text{ N})\mathbf{i} - (1428\text{ N})\mathbf{j} + (624\text{ N})\mathbf{k}$$

and

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ 447 & -1428 & 624 \end{vmatrix} \text{N}\cdot\text{m} = (5241.6\text{ N}\cdot\text{m})\mathbf{i} - (3754.8\text{ N}\cdot\text{m})\mathbf{k}$$

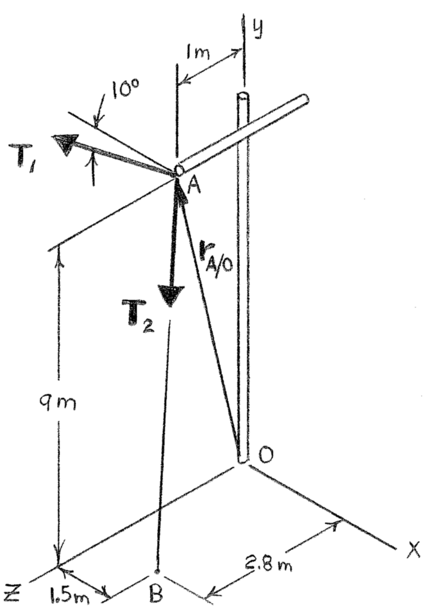
or $\mathbf{M}_O = (5.24\text{ kN}\cdot\text{m})\mathbf{i} - (3.75\text{ kN}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.22

Before a telephone cable is strung, rope *BAC* is tied to a stake at *B* and is passed over a pulley at *A*. Knowing that portion *AC* of the rope lies in a plane parallel to the *xy* plane and that the tension **T** in the rope is 124 N, determine the moment about *O* of the resultant force exerted on the pulley by the rope.

SOLUTION



Have $$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{R}$$

where $$\mathbf{r}_{A/O} = (0\text{ m})\mathbf{i} + (9\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2$$

$$\mathbf{T}_1 = -\left[(124\text{ N})\cos 10°\right]\mathbf{i} - \left[(124\text{ N})\sin 10°\right]\mathbf{j}$$

$$= -(122.116\text{ N})\mathbf{i} - (21.532\text{ N})\mathbf{j}$$

$$\mathbf{T}_2 = \lambda T_2 = \left[\frac{(1.5\text{ m})\mathbf{i} - (9\text{ m})\mathbf{j} + (1.8\text{ m})\mathbf{k}}{\sqrt{(1.5\text{ m})^2 + (9\text{ m})^2 + (1.8\text{ m})^2}}\right](124\text{ N})$$

$$= (20\text{ N})\mathbf{i} - (120\text{ N})\mathbf{j} + (24\text{ N})\mathbf{k}$$

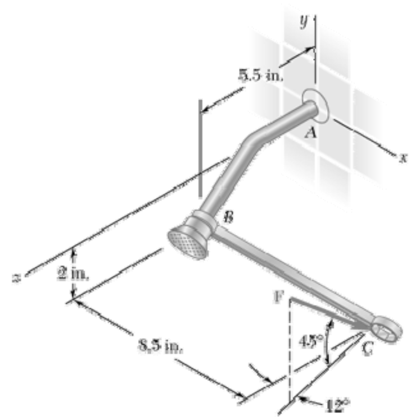
$$\therefore\quad \mathbf{R} = -(102.116\text{ N})\mathbf{i} - (141.532\text{ N})\mathbf{j} + (24\text{ N})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 9 & 1 \\ -102.116 & -141.532 & 24 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (357.523\text{ N}\cdot\text{m})\mathbf{i} - (102.116\text{ N}\cdot\text{m})\mathbf{j} + (919.044\text{ N}\cdot\text{m})\mathbf{k}$$

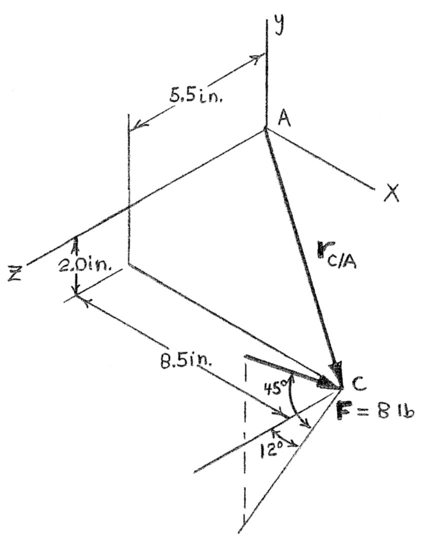
$$\text{or}\quad \mathbf{M}_O = (358\text{ N}\cdot\text{m})\mathbf{i} - (102.1\text{ N}\cdot\text{m})\mathbf{j} + (919\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.23



An 8-lb force is applied to a wrench to tighten a showerhead. Knowing that the centerline of the wrench is parallel to the *x* axis, determine the moment of the force about *A*.

SOLUTION



Have
$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}$$

where
$$\mathbf{r}_{C/A} = (8.5 \text{ in.})\mathbf{i} - (2.0 \text{ in.})\mathbf{j} + (5.5 \text{ in.})\mathbf{k}$$

$$F_x = -(8\cos 45° \sin 12°)\text{lb}$$

$$F_y = -(8\sin 45°)\text{lb}$$

$$F_z = -(8\cos 45° \cos 12°)\text{lb}$$

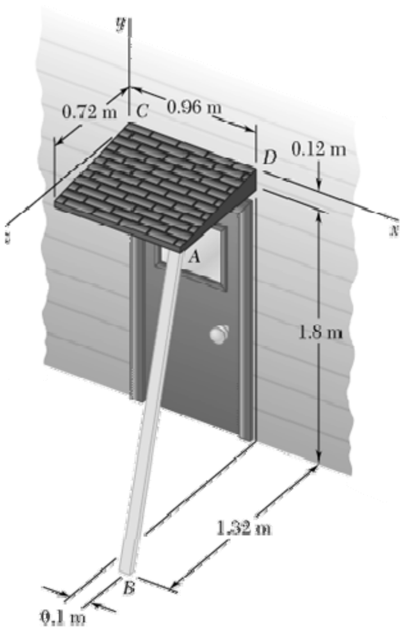
$$\therefore \quad \mathbf{F} = -(1.17613 \text{ lb})\mathbf{i} - (5.6569 \text{ lb})\mathbf{j} - (5.5332 \text{ lb})\mathbf{k}$$

and
$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8.5 & -2.0 & 5.5 \\ -1.17613 & -5.6569 & -5.5332 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= (42.179 \text{ lb} \cdot \text{in.})\mathbf{i} + (40.563 \text{ lb} \cdot \text{in.})\mathbf{j} - (50.436 \text{ lb} \cdot \text{in.})\mathbf{k}$$

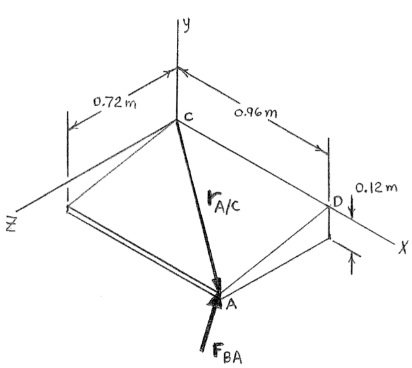
$$\text{or} \quad \mathbf{M}_A = (42.2 \text{ lb} \cdot \text{in.})\mathbf{i} + (40.6 \text{ lb} \cdot \text{in.})\mathbf{j} - (50.4 \text{ lb} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.24



A wooden board *AB*, which is used as a temporary prop to support a small roof, exerts at point *A* of the roof a 228 N force directed along *BA*. Determine the moment about *C* of that force.

SOLUTION



Have $$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

where $$\mathbf{r}_{A/C} = (0.96\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

and $$\mathbf{F}_{BA} = \lambda_{BA} F_{BA}$$

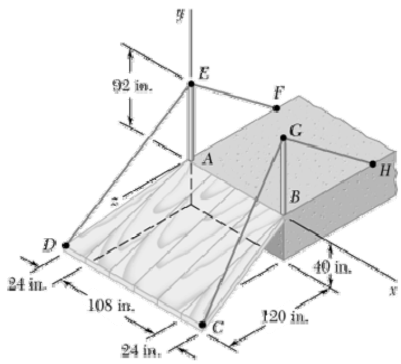
$$= \left[\frac{-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k}}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}\right](228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} + (60.480\text{ N}\cdot\text{m})\mathbf{j} + (205.92\text{ N}\cdot\text{m})\mathbf{k}$$

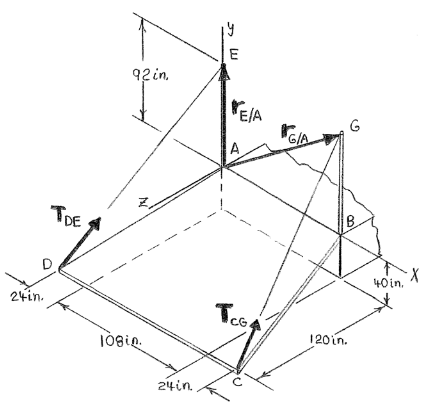
or $$\mathbf{M}_C = -(146.9\text{ N}\cdot\text{m})\mathbf{i} + (60.5\text{ N}\cdot\text{m})\mathbf{j} + (206\text{ N}\cdot\text{m})\mathbf{k}\ \blacktriangleleft$$



PROBLEM 3.25

The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



(*a*) Have $\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$

where $\mathbf{r}_{E/A} = (92 \text{ in.})\,\mathbf{j}$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE}$$

$$= \frac{(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= (48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.} = -(22{,}080 \text{ lb}\cdot\text{in.})\mathbf{i} - (4416 \text{ lb}\cdot\text{in})\mathbf{k}$$

or $\mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} - (368 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

(*b*) Have $\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$

where $\mathbf{r}_{G/A} = (108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$

$$\mathbf{T}_{CG} = \lambda_{CG} T_{CG} = \frac{-(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

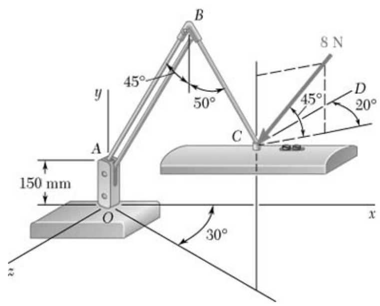
$$= -(48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 108 & 92 & 0 \\ -48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= -(22{,}080 \text{ lb}\cdot\text{in.})\mathbf{i} + (25{,}920 \text{ lb}\cdot\text{in.})\mathbf{j} + (32{,}928 \text{ lb}\cdot\text{in.})\mathbf{k}$$

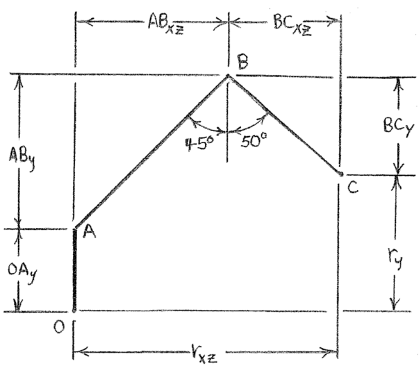
or $\mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} + (2160 \text{ lb}\cdot\text{ft})\mathbf{j} + (2740 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

PROBLEM 3.26



The arms *AB* and *BC* of a desk lamp lie in a vertical plane that forms an angle of 30° with the *xy* plane. To reposition the light, a force of magnitude 8 N is applied at *C* as shown. Determine the moment of the force about *O* knowing that $AB = 450$ mm, $BC = 325$ mm, and line *CD* is parallel to the *z* axis.

SOLUTION



Have $\quad \mathbf{M}_O = \mathbf{r}_{C/O} \times \mathbf{F}_C$

where $\quad \left(r_{C/O}\right)_x = \left(AB_{xz} + BC_{xz}\right)\cos 30^{\circ}$

$$AB_{xz} = (0.450 \text{ m})\sin 45^{\circ} = 0.31820 \text{ m}$$

$$BC_{xz} = (0.325 \text{ m})\sin 50^{\circ} = 0.24896 \text{ m}$$

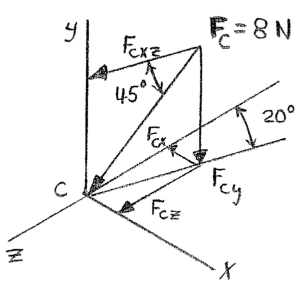
$$\left(r_{C/O}\right)_y = \left(OA_y + AB_y - BC_y\right) = 0.150 \text{ m} + (0.450 \text{ m})\cos 45^{\circ}$$

$$-(0.325 \text{ m})\cos 50^{\circ} = 0.25929 \text{ m}$$

$$\left(r_{C/O}\right)_z = \left(AB_{xz} + BC_{xz}\right)\sin 30^{\circ}$$

$$= (0.31820 \text{ m} + 0.24896 \text{ m})\sin 30^{\circ} = 0.28358 \text{ m}$$

or $\quad \mathbf{r}_{C/O} = (0.49118 \text{ m})\mathbf{i} + (0.25929 \text{ m})\mathbf{j} + (0.28358 \text{ m})\mathbf{k}$



$$\left(F_C\right)_x = -(8 \text{ N})\cos 45^{\circ}\sin 20^{\circ} = -1.93476 \text{ N}$$

$$\left(F_C\right)_y = -(8 \text{ N})\sin 45^{\circ} = -5.6569 \text{ N}$$

$$\left(F_C\right)_z = (8 \text{ N})\cos 45^{\circ}\cos 20^{\circ} = 5.3157 \text{ N}$$

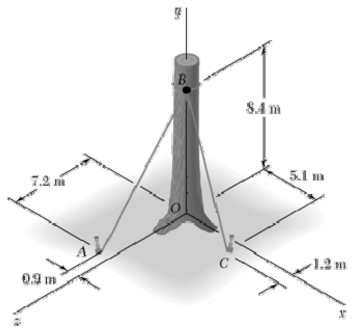
or $\quad \mathbf{F}_C = -(1.93476 \text{ N})\mathbf{i} - (5.6569 \text{ N})\mathbf{j} + (5.3157 \text{ N})\mathbf{k}$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.49118 & 0.25929 & 0.28358 \\ -1.93476 & -5.6569 & 5.3157 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (2.9825 \text{ N}\cdot\text{m})\mathbf{i} - (3.1596 \text{ N}\cdot\text{m})\mathbf{j} - (2.2769 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\quad \mathbf{M}_O = (2.98 \text{ N}\cdot\text{m})\mathbf{i} - (3.16 \text{ N}\cdot\text{m})\mathbf{j} - (2.28 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

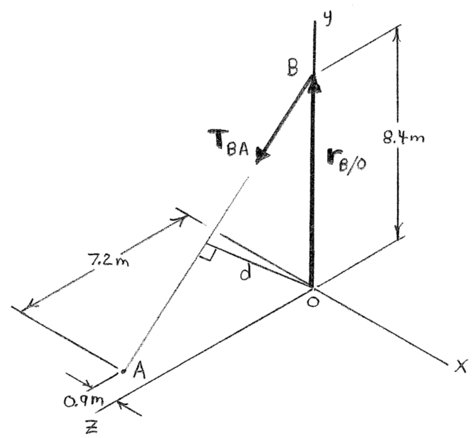
PROBLEM 3.27



In Problem 3.21, determine the perpendicular distance from point O to cable AB.

Problem 3.21: Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tension in cables AB and BC are 777 N and 990 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

SOLUTION



Have $$|\mathbf{M}_O| = T_{BA}d$$

where $$d = \text{perpendicular distance from O to line } AB.$$

Now $$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BA}$$

and $$\mathbf{r}_{B/O} = (8.4\text{ m})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{AB} = \frac{-(0.9\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (7.2\text{ m})\mathbf{k}}{\sqrt{(0.9)^2 + (8.4)^2 + (7.2)^2}\text{ m}}(777\text{ N})$$

$$= -(63.0\text{ N})\mathbf{i} - (588\text{ N})\mathbf{j} + (504\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ -63.0 & -588 & 504 \end{vmatrix}\text{N}\cdot\text{m} = (4233.6\text{ N}\cdot\text{m})\mathbf{i} + (529.2\text{ N}\cdot\text{m})\mathbf{k}$$

and $$|\mathbf{M}_O| = \sqrt{(4233.6)^2 + (529.2)^2} = 4266.5\text{ N}\cdot\text{m}$$

$$\therefore\ 4266.5\text{ N}\cdot\text{m} = (777\text{ N})d$$

or $$d = 5.4911\text{ m}$$

or $d = 5.49$ m ◀