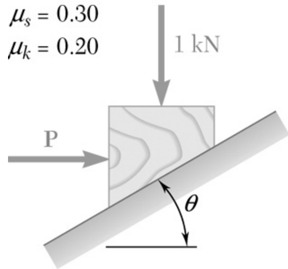
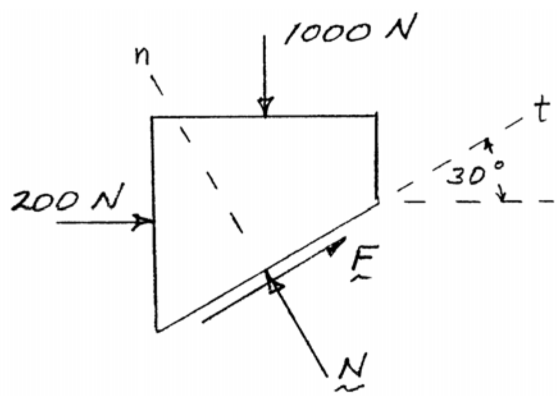
PROBLEM 8.1



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30^{\circ}$ and $P = 200\,\text{N}$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 30° - (200\text{ N})\sin 30° = 0$$

$$N = 966.03\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F + (200\text{ N})\cos 30° - (1000\text{ N})\sin 30° = 0$$

$$F = 326.8\text{ N} = F_{\text{eq.}}$$

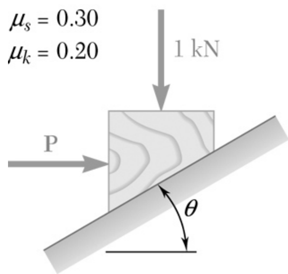
But $$F_{\max} = \mu_s N = (0.3)966\text{ N} = 290\text{ N}$$

$$F_{\text{eq.}} > F_{\max} \qquad \text{impossible} \Rightarrow \text{Block moves} \blacktriangleleft$$

and $$F = \mu_k N$$

$$= (0.2)(966.03\text{ N})$$

Block slides down $$\mathbf{F} = 193.2\text{ N} \nearrow \blacktriangleleft$$

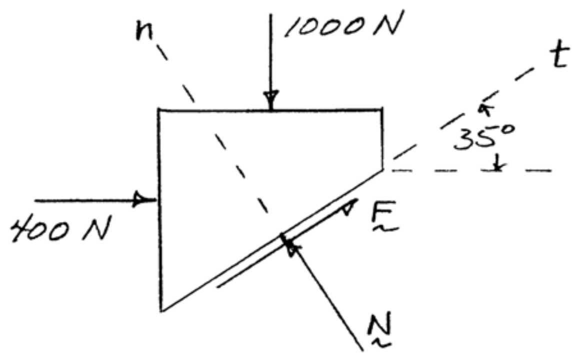


PROBLEM 8.2

Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 35^{\circ}$ and $P = 400$ N.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000 \text{ N})\cos 35^{\circ} - (400 \text{ N})\sin 35^{\circ} = 0$$

$$N = 1048.6 \text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F - (1000 \text{ N})\sin 35^{\circ} + (400 \text{ N})\cos 35^{\circ} = 0$$

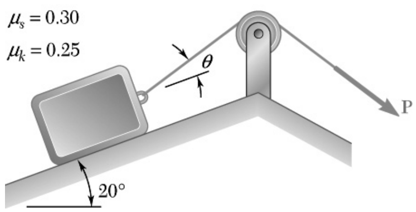
$$F = 246 \text{ N} = F_{\text{eq.}}$$

$$F_{\max} = \mu_s N = (0.3)(1048.6 \text{ N}) = 314 \text{ N}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

$\therefore$ $\mathbf{F} = 246 \text{ N} \nearrow$ ◀

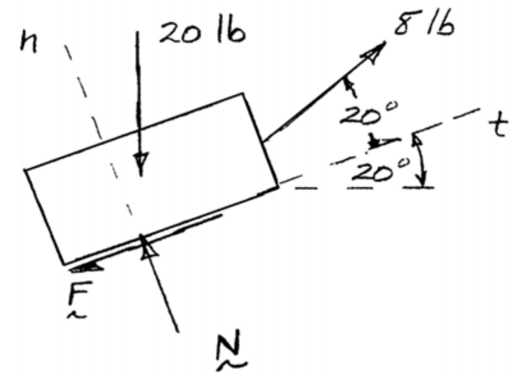
PROBLEM 8.3



Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 8$ lb and $\theta = 20°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + (8\text{ lb})\sin 20° = 0$$

$$N = 16.0577\text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(16.0577\text{ lb}) = 4.817\text{ lb}$$

Assume equilibrium:

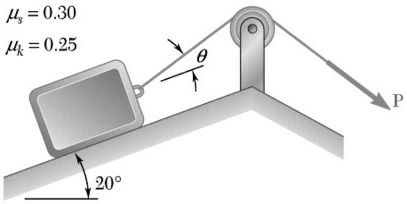
$$\nearrow \Sigma F_t = 0: \quad (8\text{ lb})\cos 20° - (20\text{ lb})\sin 20° - F = 0$$

$$F = 0.6771\text{ lb} = F_{\text{eq.}}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

and $\mathbf{F} = 0.677\text{ lb}$ ↙ ◀

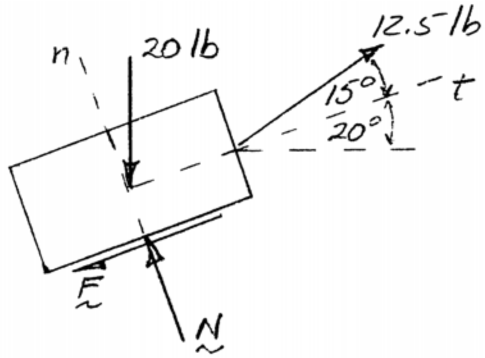
PROBLEM 8.4



Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 12.5$ lb and $\theta = 15°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + (12.5\text{ lb})\sin 15° = 0$$

$$N = 15.559\text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(15.559\text{ lb}) = 4.668\text{ lb}$$

Assume equilibrium:

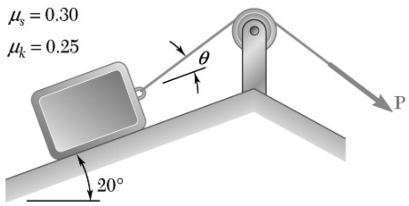
$$\nearrow \Sigma F_t = 0: \quad (12.5\text{ lb})\cos 15° - (20\text{ lb})\sin 20° - F = 0$$

$$F = 5.23\text{ lb} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and $$F = \mu_k N = (0.25)(15.559\text{ lb})$$

$$\mathbf{F} = 3.89\text{ lb} \swarrow$$ ◀

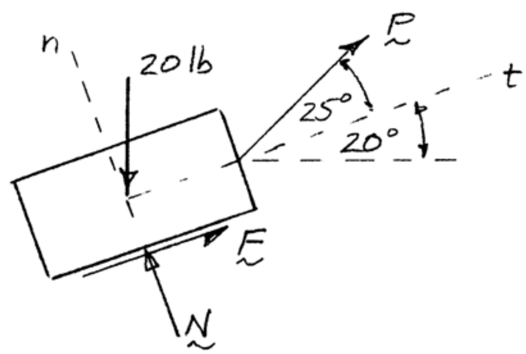


PROBLEM 8.5

Knowing that $\theta = 25°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBD block:



Block is in equilibrium:

$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + P\sin 25° = 0$$

$$N = 18.794 \text{ lb} - P\sin 25°$$

$$\nearrow \Sigma F_t = 0: \quad F - (20 \text{ lb})\sin 20° + P\cos 25° = 0$$

or

$$F = 6.840 \text{ lb} - P\cos 25°$$

Impending motion up: $F = \mu_s N$; Impending motion down: $F = -\mu_s N$

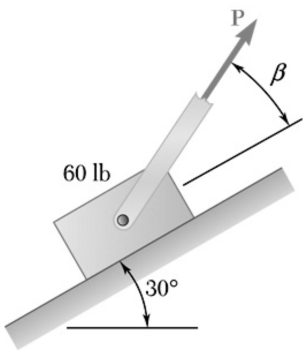
Therefore,

$$6.840 \text{ lb} - P\cos 25° = \pm(0.3)(18.794 \text{ lb} - P\sin 25°)$$

$$P_{\text{up}} = 12.08 \text{ lb} \qquad P_{\text{down}} = 1.542 \text{ lb}$$

$$1.542 \text{ lb} \le P_{\text{eq.}} \le 12.08 \text{ lb} \blacktriangleleft$$

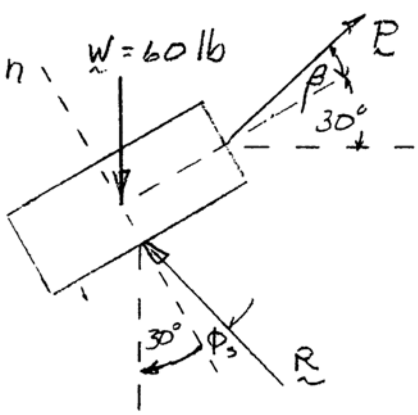
PROBLEM 8.6



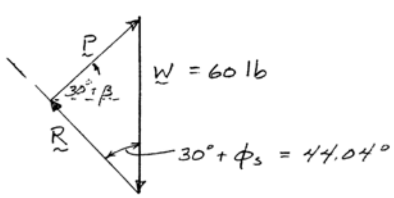
Knowing that the coefficient of friction between the 60-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P for which motion of the block up the incline is impending, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion up)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.04°$$



(*a*) Note: For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$

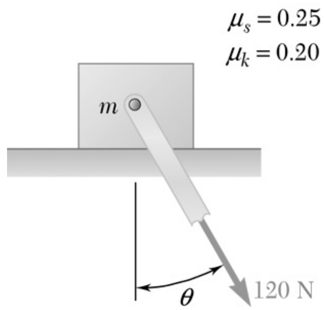
Then $$P = W\sin(30° + \phi_s)$$

$$= (60\text{ lb})\sin 44.04° = 41.71\text{ lb}$$

$P_{\min} = 41.7\text{ lb}$ ◀

(*b*) Have $\beta = \phi_s$

$\beta = 14.04°$ ◀

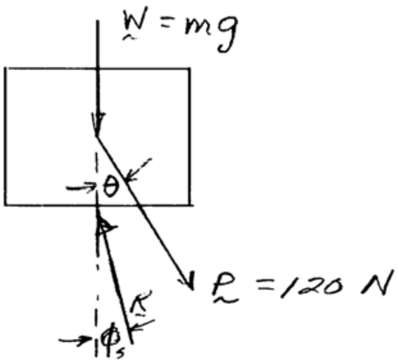


PROBLEM 8.7

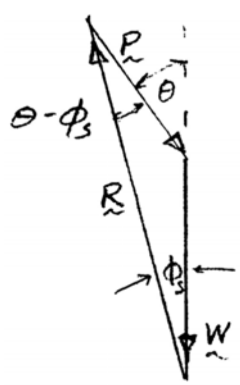
Considering only values of θ less than $90°$, determine the smallest value of θ for which motion of the block to the right is impending when $(a)\, m = 30$ kg, $(b)\, m = 40$ kg.

SOLUTION

FBD block (impending motion to the right)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



$$\frac{P}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W}{P}\sin\phi_s \qquad W = mg$$

(a) $$m = 30 \text{ kg:} \quad \theta - \phi_s = \sin^{-1}\left[\frac{(30 \text{ kg})(9.81 \text{ m/s}^2)}{120 \text{ N}}\sin 14.036°\right]$$

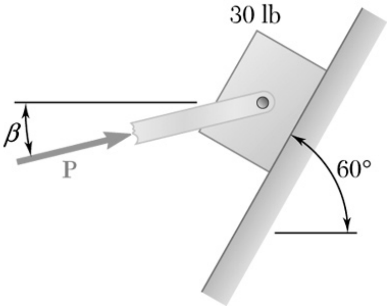
$$= 36.499°$$

$$\therefore \quad \theta = 36.499° + 14.036° \qquad \text{or} \quad \theta = 50.5° \blacktriangleleft$$

(b) $$m = 40 \text{ kg:} \quad \theta - \phi_s = \sin^{-1}\left[\frac{(40 \text{ kg})(9.81 \text{ m/s}^2)}{120 \text{ N}}\sin 14.036°\right]$$

$$= 52.474°$$

$$\therefore \quad \theta = 52.474° + 14.036° \qquad \text{or} \quad \theta = 66.5° \blacktriangleleft$$

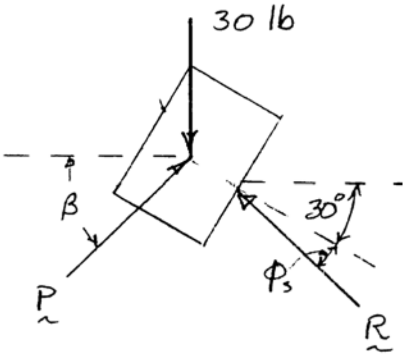


PROBLEM 8.8

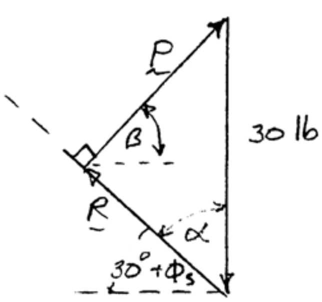
Knowing that the coefficient of friction between the 30-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to maintain the block in equilibrium, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion downward)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



(*a*) Note: For minimum *P*, $\mathbf{P} \perp \mathbf{R}$

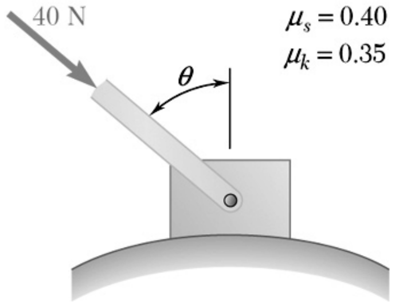
So $\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$

and $P = (30\text{ lb})\sin\alpha = (30\text{ lb})\sin(45.964°) = 21.567\text{ lb}$

$P = 21.6\text{ lb}$ ◀

(*b*) $\beta = 46.0°$ ◀

PROBLEM 8.9

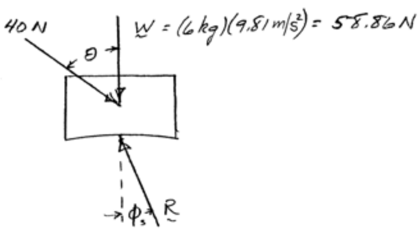


$\mu_s = 0.40$
$\mu_k = 0.35$

A 6-kg block is at rest as shown. Determine the positive range of values of θ for which the block is in equilibrium if (*a*) θ is less than 90°, (*b*) θ is between 90° and 180°.

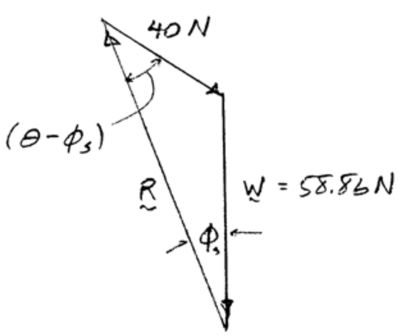
SOLUTION

FBD block (impending motion)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$$

(*a*) $0° \le \theta \le 90°$:



$$\frac{58.86 \text{ N}}{\sin(\theta - \phi_s)} = \frac{40 \text{ N}}{\sin\phi_s}$$

$$\theta - \phi_s = \sin^{-1}\frac{58.86 \text{ N}}{40 \text{ N}}\sin(21.801°)$$

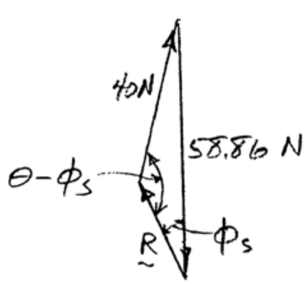
$$= 33.127°,\ 146.873°$$

$$\theta = 54.9° \quad \text{and} \quad \theta = 168.674°$$

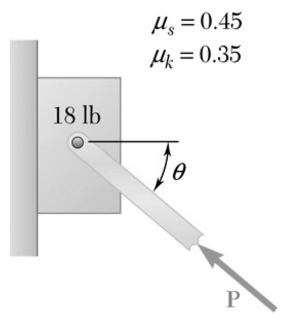
$\therefore$ (*a*) Equilibrium for $0 \le \theta \le 54.9°$ ◀

(*b*) $90° \le \theta \le 180°$:

(*b*) and for $168.7° \le \theta \le 180.0°$ ◀



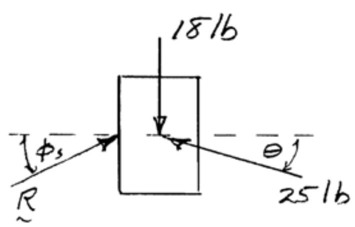
PROBLEM 8.10



Knowing that $P = 25$ lb, determine the range of values of θ for which equilibrium of the 18-lb block is maintained.

SOLUTION

FBD block (impending motion down)

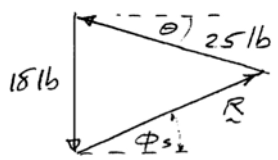


$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.228°$$

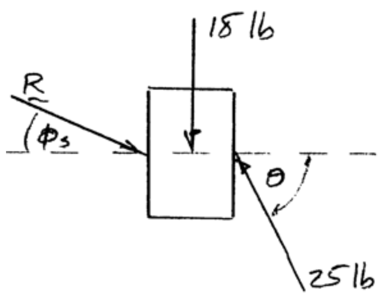
$$\frac{25 \text{ lb}}{\sin(90° - \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta + \phi_s)}$$

$$\theta + \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° - 24.228°)\right] = 41.04°$$

$$\theta = 16.81°$$



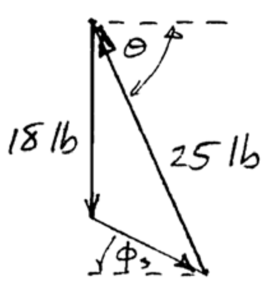
Impending motion up:



$$\frac{25 \text{ lb}}{\sin(90° + \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta - \phi_s)}$$

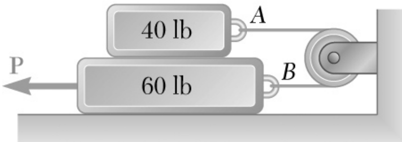
$$\theta - \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° + 24.228°)\right] = 41.04°$$

$$\theta = 65.27°$$



Equilibrium for $16.81° \le \theta \le 65.3°$ ◀

PROBLEM 8.11

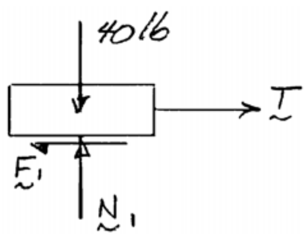


The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable *AB* (*a*) is attached as shown, (*b*) is removed.

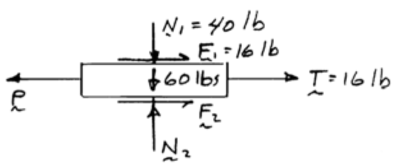
SOLUTION

FBDs

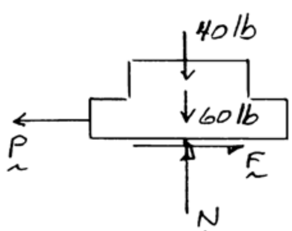
Top block:



Bottom block:



FBD blocks:



(*a*) Note: With the cable, motion must impend at both contact surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad T - F_1 = 0 \qquad T - 16 \text{ lb} = 0 \qquad T = 16 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu_s N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 16 \text{ lb} + 16 \text{ lb} + 40 \text{ lb} = 0$$

$$\mathbf{P} = 72.0 \text{ lb} \longleftarrow \blacktriangleleft$$

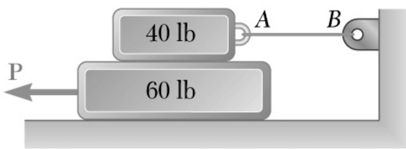
(*b*) Without the cable, both blocks will stay together and motion will impend only at the floor.

$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 40 \text{ lb} - P = 0$$

$$\mathbf{P} = 40.0 \text{ lb} \longleftarrow \blacktriangleleft$$



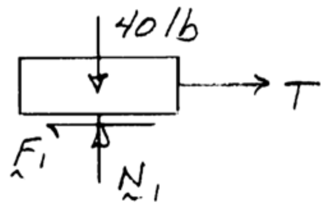
PROBLEM 8.12

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable *AB* (*a*) is attached as shown, (*b*) is removed.

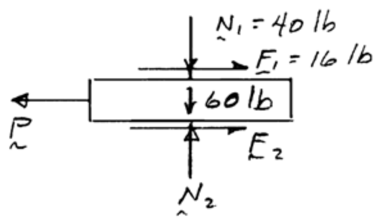
SOLUTION

FBDs

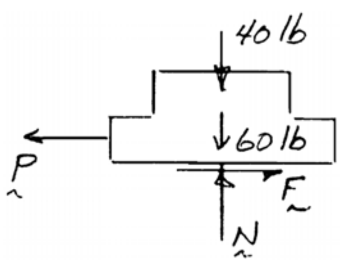
Top block:



Bottom block:



FBD blocks:



(*a*) With the cable, motion must impend at both surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 16 \text{ lb} + 40 \text{ lb} - P = 0 \qquad P = 56 \text{ lb}$$

$\mathbf{P} = 56.0 \text{ lb} \longleftarrow$ ◀

(*b*) Without the cable, both blocks stay together and motion will impend at the floor surface only.

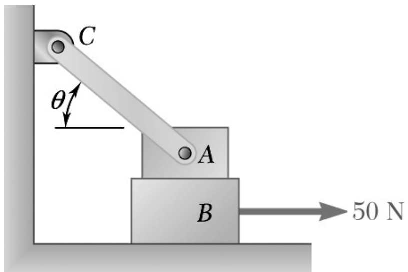
$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 40 \text{ lb} = 0 \qquad P = 40 \text{ lb}$$

$\mathbf{P} = 40.0 \text{ lb} \longleftarrow$ ◀

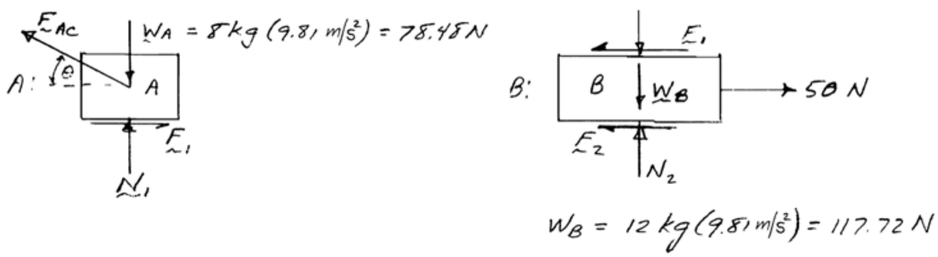
PROBLEM 8.13



The 8-kg block A is attached to link AC and rests on the 12-kg block B. Knowing that the coefficient of static friction is 0.20 between all surfaces of contact and neglecting the mass of the link, determine the value of θ for which motion of block B is impending.

SOLUTION

FBDs:



Motion must impend at both contact surfaces

Block A: $\uparrow \Sigma F_y = 0: \quad N_1 - W_A = 0 \qquad N_1 = W_A$

Block B: $\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_B = 0$

$$N_2 = N_1 + W_B = W_A + W_B$$

Impending motion: $F_1 = \mu_s N_1 = \mu_s W_A$

$$F_2 = \mu_s N_2 = \mu_s (N_1 + W_B)$$

Block B: $\longrightarrow \Sigma F_x = 0: \quad 50 \text{ N} - F_1 - F_2 = 0$

or $50 \text{ N} = \mu_s (N_1 + N_1 + W_B) = 0.2(2N_1 + 117.72 \text{ N})$

$$N_1 = 66.14 \text{ N} \qquad F_1 = 0.2(66.14 \text{ N}) = 13.228 \text{ N}$$

Block A: $\longrightarrow \Sigma F_x = 0: \quad 13.228 \text{ N} - F_{AC}\cos\theta = 0$

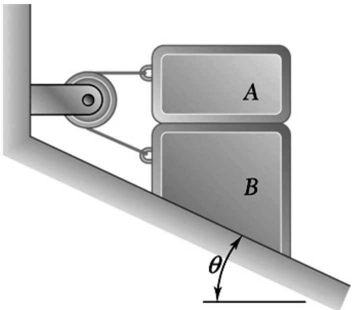
or $F_{AC}\cos\theta = 13.228 \text{ N}$ (1)

$$\uparrow \Sigma F_y = 0: \quad 66.14 \text{ N} - 78.48 \text{ N} + F_{AC}\sin\theta = 0$$

or $F_{AC}\sin\theta = 78.48 \text{ N} - 66.14 \text{ N}$ (2)

Then, $\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$ $\tan\theta = \dfrac{78.48 \text{ N} - 66.14 \text{ N}}{13.228 \text{ N}}$

$\theta = 43.0° \blacktriangleleft$

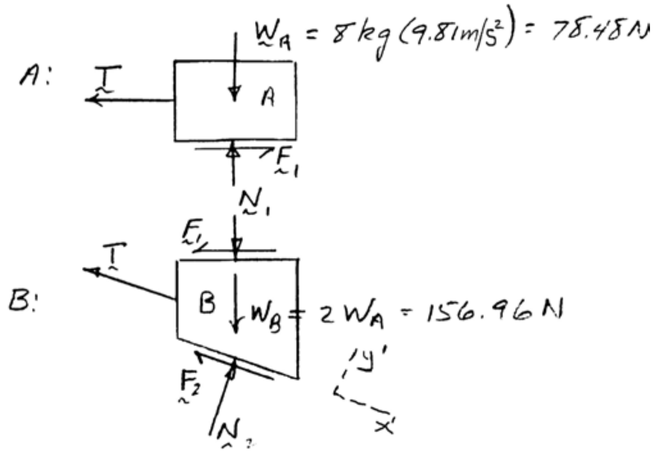


PROBLEM 8.14

The 8-kg block A and the 16-kg block B are at rest on an incline as shown. Knowing that the coefficient of static friction is 0.25 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBDs:



Block A: $\qquad \uparrow \Sigma F_y = 0{:}\quad N_1 - W_A = 0 \qquad N_1 = W_A$

Impending motion: $\qquad F_1 = \mu_s N_1 = \mu_s W_A$

$$\rightarrow \Sigma F_x = 0{:}\quad F_1 - T = 0 \qquad T = F_1 = \mu_s W_A$$

Block B: $\qquad \nearrow \Sigma F_{y'} = 0{:}\quad N_2 - (N_1 + W_B)\cos\theta - F_1 \sin\theta = 0$

$$N_2 = 3W_A \cos\theta + \mu_s W_A \sin\theta$$

$$= W_A(3\cos\theta + 0.25\sin\theta)$$

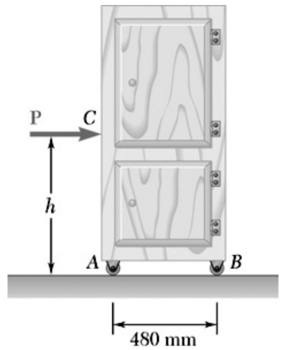
Impending motion: $\qquad F_2 = \mu_s N_2 = 0.25 W_A(3\cos\theta + 0.25\sin\theta)$

$$\searrow \Sigma F_{x'} = 0{:}\quad -T - F_2 - F_1\cos\theta + (N_1 + W_B)\sin\theta = 0$$

$$\left[-0.25 - 0.25(3\cos\theta + 0.25\sin\theta) - 0.25\cos\theta + 3\sin\theta\right]W_A = 0$$

or $\qquad 47\sin\theta - 16\cos\theta - 4 = 0$

Solving numerically $\qquad \theta = 23.4° \blacktriangleleft$

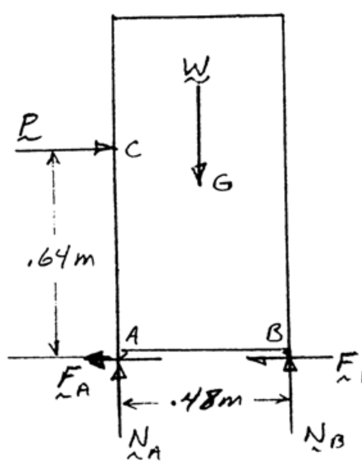


PROBLEM 8.15

A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Knowing that $h = 640$ mm, determine the magnitude of the force **P** required for impending motion of the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet:



$$W = 48\text{ kg}\left(9.81\text{ m/s}^2\right)$$

$$= 470.88\text{ N}$$

$$\mu_s = 0.3$$

Note: For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.24\text{ m})W - (0.64\text{ m})P_{\text{tip}} = 0 \qquad P_{\text{tip}} = 0.375W$$

(*a*) All casters locked: Impending slip: $F_A = \mu_s N_A$, $F_B = \mu_s N_B$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W$$

So $F_A + F_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$$

$$\therefore \quad P = 0.3(470.88\text{ N}) \qquad \text{or} \qquad P = 141.3\text{ N} \blacktriangleleft$$

$$\left(P = 0.3W < P_{\text{tip}} \qquad \text{OK}\right)$$

(*b*) Casters at *A* free, so $F_A = 0$

Impending slip: $F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.64\text{ m})P + (0.24\text{ m})W - (0.48\text{ m})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \qquad P = 0.25W$$

$$\left(P = 0.25W < P_{\text{tip}} \qquad \text{OK}\right)$$

$$\therefore \quad P = 0.25(470.88\text{ N}) \qquad P = 117.7\text{ N} \blacktriangleleft$$

PROBLEM 8.15 CONTINUED

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$\rightarrow \Sigma F_x = 0$: $P - F_A = 0$ $\quad P = F_A = \mu_s N_A$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

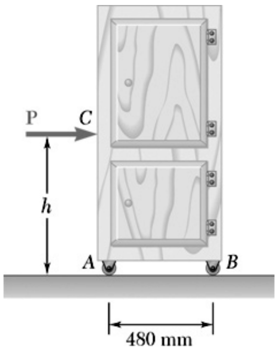
$\circlearrowleft \Sigma M_B = 0$: $(0.24\text{ m})W - (0.64\text{ m})P - (0.48\text{ m})N_A = 0$

$$3W - 8P - 6\frac{P}{0.3} = 0 \qquad P = 0.10714W = 50.45\text{ N}$$

$$\left(P < P_{\text{tip}} \qquad \text{OK}\right)$$

$P = 50.5\text{ N}$ ◀

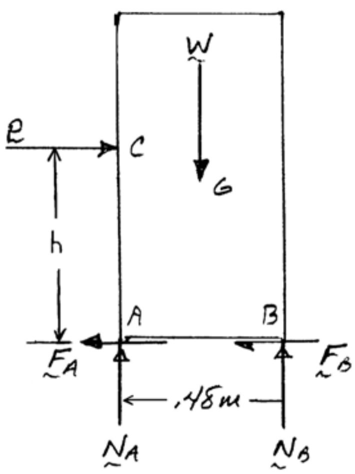
PROBLEM 8.16



A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at A and B are locked, determine (a) the force **P** required for impending motion of the cabinet to the right, (b) the largest allowable height h if the cabinet in not to tip over.

SOLUTION

FBD cabinet:



$$W = 48\text{ kg}\left(9.81\text{ m/s}^2\right)$$

$$= 470.88\text{ N}$$

(a) $\uparrow \Sigma F_y = 0\text{:}\quad N_A + N_B - W = 0; \qquad N_A + N_B = W$

Impending slip: $\quad F_A = \mu_s N_A,\ F_B = \mu_s N_B$

So $\quad F_A + F_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0\text{:}\quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$$

$$P = 0.3\left(470.88\text{ N}\right) = 141.26\text{ N}$$

$$\mathbf{P} = 141.3\text{ N} \longrightarrow ◀$$

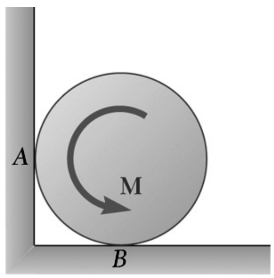
(b) For tipping, $\quad N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0\text{:}\quad hP - \left(0.24\text{ m}\right)W = 0$$

$$h_{\text{max}} = \left(0.24\text{ m}\right)\frac{W}{P} = \left(0.24\text{ m}\right)\frac{1}{\mu_s} = \frac{0.24\text{ m}}{0.3}$$

$$h_{\text{max}} = 0.800\text{ m} ◀$$

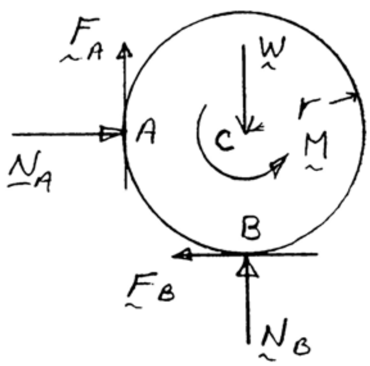
PROBLEM 8.17



The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A,\ F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B + \mu_s^2 N_B = W$$

or $$N_B = \frac{W}{1+\mu_s^2}$$

and $$F_B = \frac{\mu_s W}{1+\mu_s^2}$$

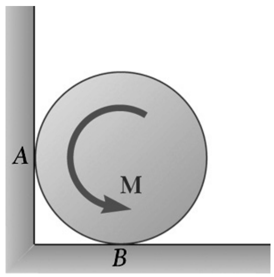
$$F_A = \frac{\mu_s^2 W}{1+\mu^2}$$

$$\circlearrowleft \Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0$$

$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1+\mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1+\mu_s}{1+\mu_s^2} \blacktriangleleft$$

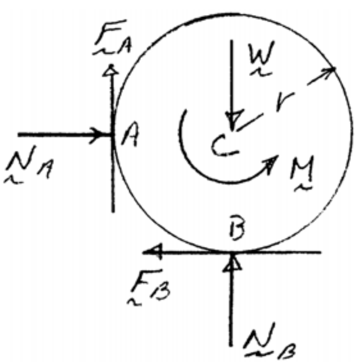
PROBLEM 8.18



The cylinder shown is of weight W and radius r. Express in terms of W and r the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate assuming that the coefficient of static friction is (*a*) zero at A and 0.36 at B, (*b*) 0.30 at A and 0.36 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A; \qquad F_B = \mu_B N_B$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B\left(1 + \mu_A \mu_B\right) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

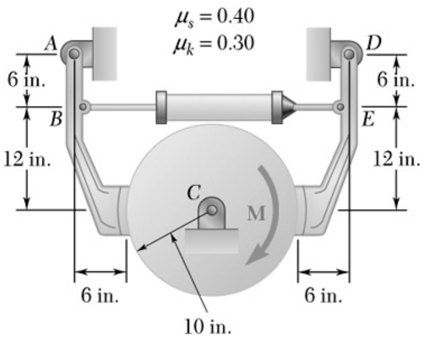
$$\circlearrowleft \Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0 \qquad M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.36$

$$M = 0.360Wr \blacktriangleleft$$

(*b*) For $\mu_A = 0.30$ and $\mu_B = 0.36$

$$M = 0.422Wr \blacktriangleleft$$

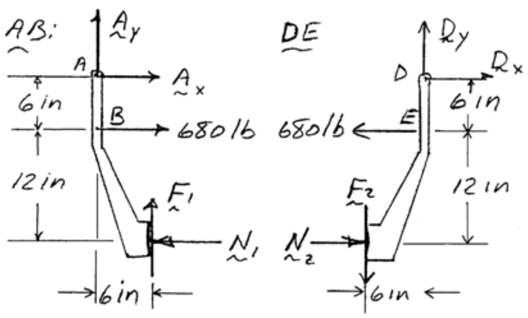


PROBLEM 8.19

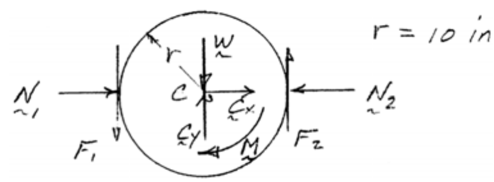
The hydraulic cylinder shown exerts a force of 680 lb directed to the right on point *B* and to the left on point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

FBDs



Drum:



Rotating drum ⇒ slip at both sides; constant speed ⇒ equilibrium

$$\therefore \quad F_1 = \mu_k N_1 = 0.3N_1; \qquad F_2 = \mu_k N_2 = 0.3N_2$$

AB: $\circlearrowleft \Sigma M_A = 0\text{:} \quad (6\text{ in.})(680\text{ lb}) + (6\text{ in.})(F_1) - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.3} - 6\text{ in.}\right) = (6\text{ in.})(680\text{ lb}) \qquad \text{or} \qquad F_1 = 75.555\text{ lb}$$

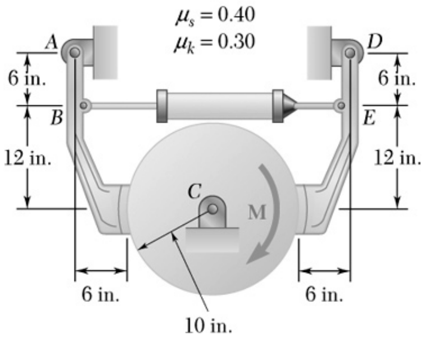
DE: $\circlearrowleft \Sigma M_D = 0\text{:} \quad (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})(680\text{ lb}) = 0$

$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.3}\right) = (6\text{ in.})(680\text{ lb}) \qquad \text{or} \qquad F_2 = 61.818\text{ lb}$$

Drum: $\circlearrowleft \Sigma M_C = 0\text{:} \quad r(F_1 + F_2) - M = 0$

$$M = (10\text{ in.})(75.555 + 61.818)\text{lb}$$

$$M = 1374\text{ lb}\cdot\text{in.} \blacktriangleleft$$

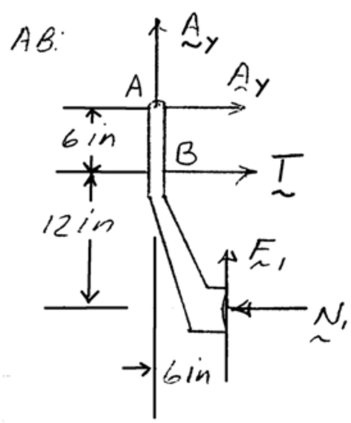


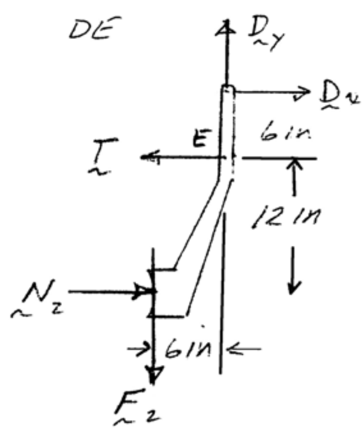
PROBLEM 8.20

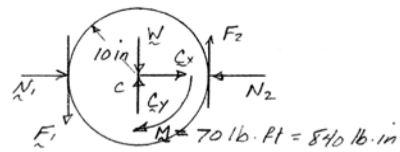
A couple **M** of magnitude $70\ \text{lb}\cdot\text{ft}$ is applied to the drum as shown. Determine the smallest force which must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

SOLUTION

FBDs **DE:** **Drum:**







For minimum *T*, slip impends at both sides, so

$$F_1 = \mu_s N_1 = 0.4N_1 \qquad F_2 = \mu_s N_2 = 0.4N_2$$

AB: $\circlearrowleft \Sigma M_A = 0\text{:}\quad (6\text{ in.})T + (6\text{ in.})F_1 - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.4} - 6\text{ in.}\right) = (6\text{ in.})T \qquad \text{or} \qquad F_1 = \frac{T}{6.5}$$

DE: $\circlearrowleft \Sigma M_D = 0\text{:}\quad (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})T = 0$

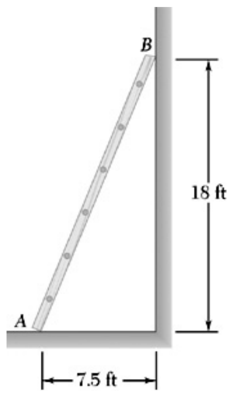
$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.4}\right) = (6\text{ in.})T \qquad \text{or} \qquad F_2 = \frac{T}{8.5}$$

Drum: $\circlearrowleft \Sigma M_C = 0\text{:}\quad (10\text{ in.})(F_1 + F_2) - 840\text{ lb}\cdot\text{in.} = 0$

$$T\left(\frac{1}{6.5} + \frac{1}{8.5}\right) = 84\text{ lb}$$

$T = 309\text{ lb}$ ◀

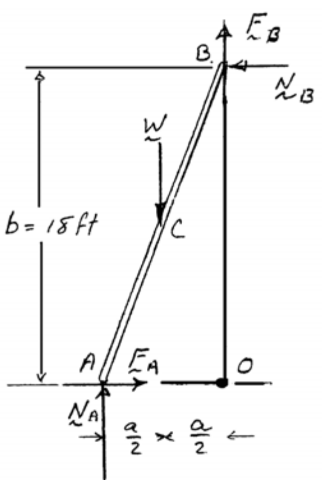
PROBLEM 8.21



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5$ ft

$b = 18$ ft

Motion impends at both A and B.

$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = \mu_s N_A$$

Then

$$F_B = \mu_s N_B = \mu_s^2 N_A$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + F_B = 0 \quad \text{or} \quad N_A\left(1 + \mu_s^2\right) = W$$

$$\Sigma M_O = 0: \quad bN_B + \frac{a}{2}W - aN_A = 0$$

or

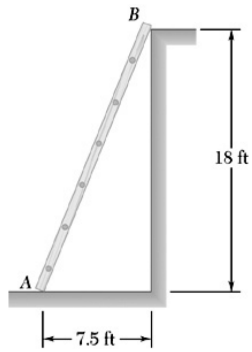
$$aN_A - b\mu_s N_A = \frac{a}{2}W = \frac{a}{2}N_A\left(1 + \mu_s^2\right)$$

$$\mu_s^2 + \frac{2b}{a}\mu_s - 1 = 0$$

$$\mu_s = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 + 1} = -2.4 \pm 2.6$$

The positive root is physically possible. Therefore, $\mu_s = 0.200$ ◀

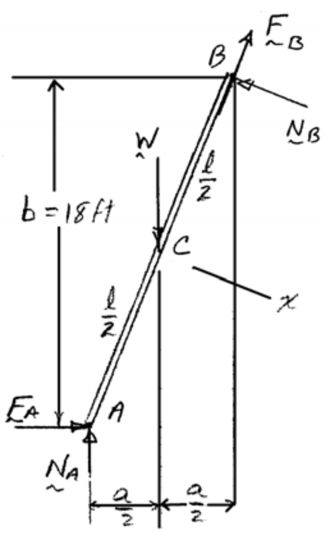
PROBLEM 8.22



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$$a = 7.5 \text{ ft}$$

$$l = 19.5 \text{ ft}$$

$$\frac{a}{l} = \frac{5}{13}$$

$$\frac{b}{l} = \frac{12}{13}$$

Motion impends at both A and B, so

$$F_A = \mu_s N_A \qquad \text{and} \qquad F_B = \mu_s N_B$$

$$\Sigma M_A = 0: \quad lN_B - \frac{a}{2}W = 0 \qquad \text{or} \qquad N_B = \frac{a}{2l}W = \frac{7.5 \text{ ft}}{39 \text{ ft}}W$$

or
$$N_B = \frac{2.5}{13}W$$

Then
$$F_B = \mu_s N_B = \mu_s \frac{2.5W}{13}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_A + \frac{5}{13}F_B - \frac{12}{13}N_B = 0$$

$$\mu_s N_A + \frac{12.5}{(13)^2}\mu_s W - \frac{30}{(13)^2}W = 0$$

$$N_A - \frac{W}{(13)^2}\frac{(30 - 12.5\mu_s)}{\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + \frac{12}{13}F_B + \frac{5}{13}N_B = 0$$

$$\left(\frac{30 - 12.5\mu_s}{\mu_s} + 30\mu_s + 12.5\right)\frac{W}{(13)^2} = W$$

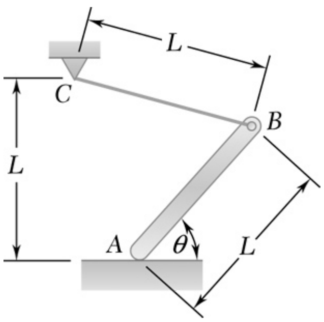
or
$$\mu_s^2 - 5.6333\mu_s + 1 = 0$$

$$\mu_s = 2.8167 \pm 2.6332$$

or
$$\mu_s = 0.1835 \qquad \text{and} \qquad \mu_s = 5.45$$

The larger value is very unlikely unless the surface is treated with some "non-skid" material.

In any event, the smallest value for equilibrium is $\mu_s = 0.1835$ ◀

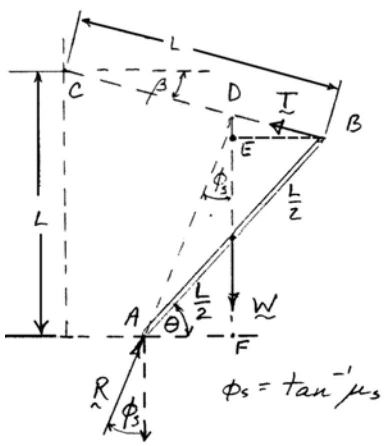


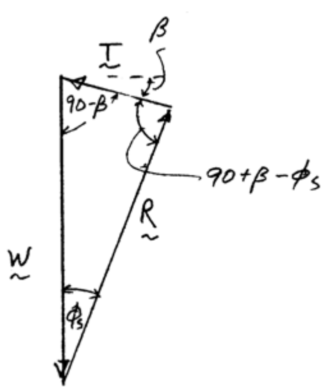
PROBLEM 8.23

End A of a slender, uniform rod of weight W and length L bears on a horizontal surface as shown, while end B is supported by a cord BC of length L. Knowing that the coefficient of static friction is 0.40, determine (a) the value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

FBD rod:





(a) Geometry: $\quad BE = \dfrac{L}{2}\cos\theta \qquad DE = \left(\dfrac{L}{2}\cos\theta\right)\tan\beta$

$$EF = L\sin\theta \qquad DF = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

So

$$L\left(\frac{1}{2}\cos\theta\tan\beta + \sin\theta\right) = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

or

$$\tan\beta + 2\tan\theta = \frac{1}{\tan\phi_s} = \frac{1}{\mu_s} = \frac{1}{0.4} = 2.5 \qquad (1)$$

Also,

$$L\sin\theta + L\sin\beta = L$$

or

$$\sin\theta + \sin\beta = 1 \qquad (2)$$

Solving Eqs. (1) and (2) numerically $\quad \theta_1 = 4.62° \qquad \beta_1 = 66.85°$

$$\theta_2 = 48.20° \qquad \beta_2 = 14.75°$$

Therefore, $\quad \theta = 4.62°$ and $\theta = 48.2°$ ◀

(b) Now

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$$

and

$$\frac{T}{\sin\phi_s} = \frac{W}{\sin(90 + \beta - \phi_s)}$$

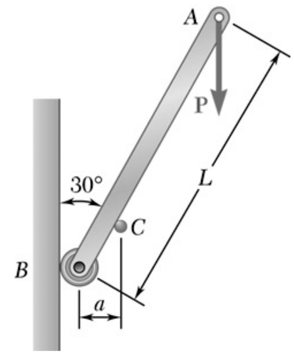
or

$$T = W\frac{\sin\phi_s}{\sin(90 + \beta - \phi_s)}$$

For $\quad \theta = 4.62° \qquad T = 0.526W$ ◀

$\theta = 48.2° \qquad T = 0.374W$ ◀

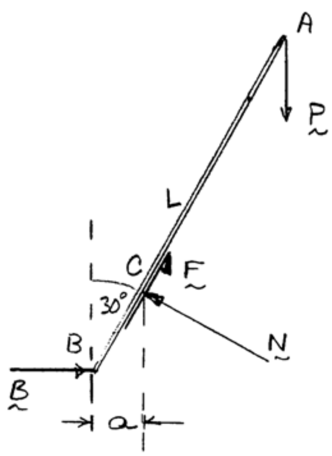
PROBLEM 8.24



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.25 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_B = 0: \quad \frac{a}{\sin 30°} N - L \sin 30° P = 0$$

$$N = \frac{L}{a} \sin^2 30° P = \frac{L}{a} \frac{P}{4}$$

$$\text{Impending motion at } C\text{: down } \rightarrow F = \mu_s N \atop \text{up } \rightarrow F = -\mu_s N \Bigg\} F = \pm \frac{N}{4}$$

$$\uparrow \Sigma F_y = 0: \quad F \cos 30° + N \sin 30° - P = 0$$

$$\pm \frac{L}{a} \frac{P}{16} \frac{\sqrt{3}}{2} + \frac{L}{a} \frac{P}{4} \frac{1}{2} = P$$

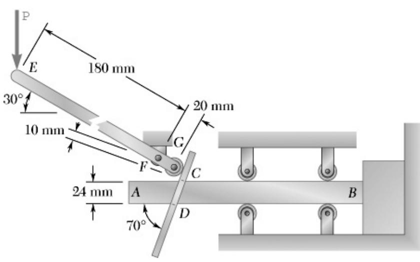
$$\frac{L}{a} \left[\frac{1}{8} \pm \frac{\sqrt{3}}{32} \right] = 1$$

$$\frac{L}{a} = \frac{32}{4 \pm \sqrt{3}}$$

or $\quad \dfrac{L}{a} = 5.583 \quad$ and $\quad \dfrac{L}{a} = 14.110$

For equilibrium: $\quad 5.58 \le \dfrac{L}{a} \le 14.11$ ◀

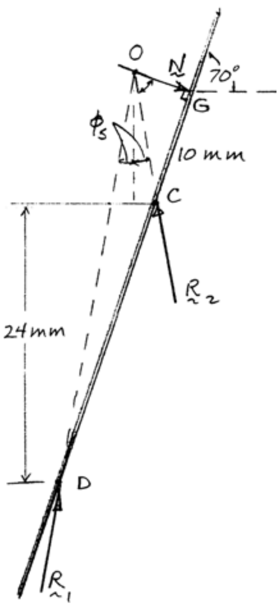
PROBLEM 8.25



The basic components of a clamping device are bar *AB*, locking plate *CD*, and lever *EFG*; the dimensions of the slot in *CD* are slightly larger than those of the cross section of *AB*. To engage the clamp, *AB* is pushed against the workpiece, and then force **P** is applied. Knowing that $P = 160$ N and neglecting the friction force between the lever and the plate, determine the smallest allowable value of the static coefficient of friction between the bar and the plate.

SOLUTION

FBD Plate:



DC is three-force member and motion impends at *C* and *D* (for minimum μ_s).

$$\measuredangle\ OCG = 20° + \phi_s \qquad \measuredangle\ ODG = 20° - \phi_s$$

$$\overline{OG} = (10\text{ mm})\tan(20° + \phi_s) = \left(\frac{24\text{ mm}}{\sin 70°} + 10\text{ mm}\right)\tan(20° - \phi_s)$$

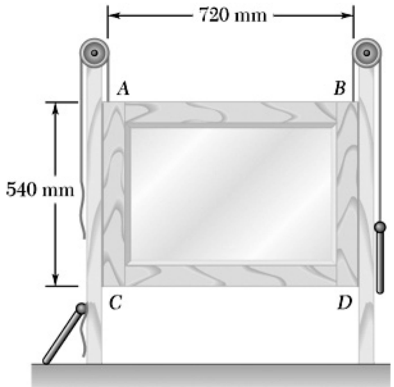
or
$$\tan(20° + \phi_s) = 3.5540\tan(20° - \phi_s)$$

Solving numerically $\phi_s = 10.565°$

Now $\mu_s = \tan\phi_s$

so that $\mu_s = 0.1865$ ◀

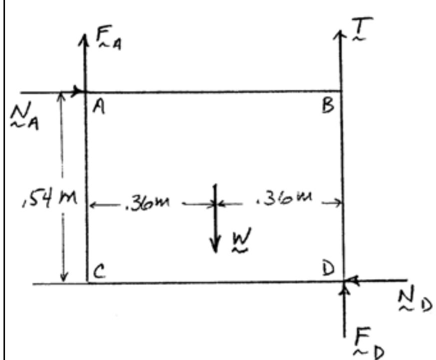
PROBLEM 8.26



A window sash having a mass of 4 kg is normally supported by two 2-kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller that the frame and will bind only at points A and D.)

SOLUTION

FBD window:



$$W = (4\text{ kg})(9.81\text{ m/s}^2) = 39.24\text{ N}$$

$$T = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N} = \frac{W}{2}$$

$$\rightarrow \Sigma F_x = 0: \qquad N_A - N_D = 0 \qquad N_A = N_D$$

Impending motion: $\qquad F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$$\Sigma M_D = 0: \quad (0.36\text{ m})W - (0.54\text{ m})N_A - (0.72\text{ m})F_A = 0$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

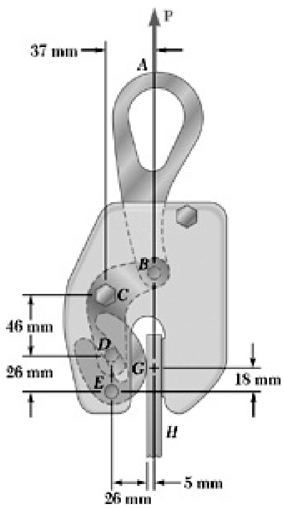
$$F_A + F_D = W - T$$

$$= \frac{W}{2}$$

Now $\qquad F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$

Then $\qquad \dfrac{W}{2} = 2\mu_s \dfrac{2W}{3 + 4\mu_s}$

or $\qquad \mu_s = 0.750$ ◀



PROBLEM 8.27

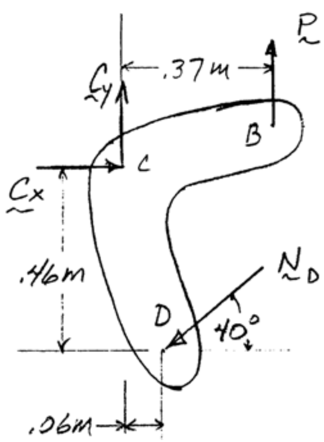
The steel-plate clamp shown is used to lift a steel plate H of mass 250 kg. Knowing that the normal force exerted on steel cam EG by pin D forms an angle of 40° with the horizontal and neglecting the friction force between the cam and the pin, determine the smallest allowable value of the coefficient of static friction.

SOLUTION

(Note: **P** is vertical as AB is two force member; also $P = W$ since clamp + plate is a two force FBD)

FBDs:

BCD:

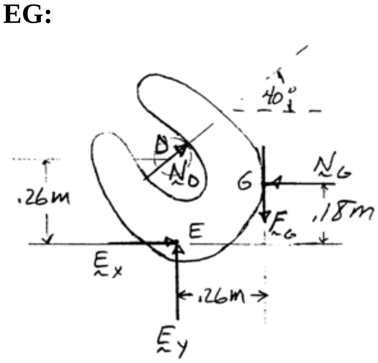


$$\circlearrowleft \Sigma M_C = 0: \quad (0.37\text{ m})P - (0.46\text{ m})N_D\cos 40°$$

$$-(0.06\text{ m})N_D\sin 40° = 0$$

or $$N_D = 0.94642P = 0.94642W$$

EG:



$$\circlearrowleft \Sigma M_E = 0: \quad (0.18\text{ m})N_G - (0.26\text{ m})F_G - (0.26\text{ m})N_D\cos 40° = 0$$

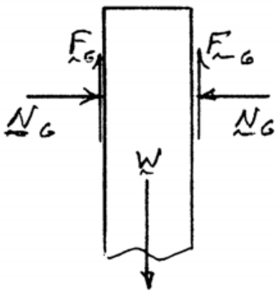
Impending motion: $$F_G = \mu_s N_G$$

Combining $$(18 + 26\mu_s)N_G = 19.9172N_D$$

$$= 18.850W$$

PROBLEM 8.27 CONTINUED

Plate:

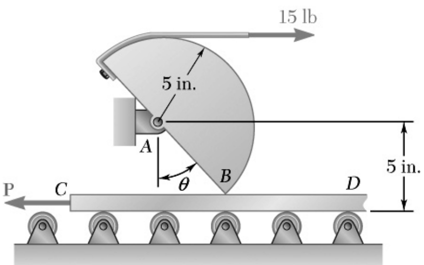


From plate: $F_G = \dfrac{W}{2}$ so that $N_G = \dfrac{W}{2\mu_s}$

Then $(18 + 26\mu_s)\dfrac{W}{2\mu_s} = 18.85W$

$$\mu_s = 0.283 \blacktriangleleft$$

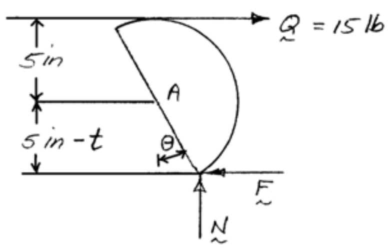
PROBLEM 8.28

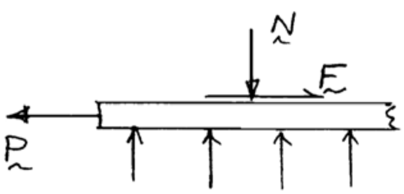


The 5-in.-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** for which motion of the plate is impending knowing that the plate is 1 in. thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking, (that is, for which the plate cannot be moved however large the force **P** may be).

SOLUTION

FBDs:





From plate: $\xrightarrow{} \Sigma F_x = 0: \quad F - P = 0 \qquad F = P$

From cam geometry: $\cos\theta = \dfrac{5\text{ in.} - t}{5\text{ in.}}$

$$\circlearrowleft \Sigma M_A = 0: \quad \left[(5\text{ in.})\sin\theta\right]N - \left[(5\text{ in.})\cos\theta\right]F - (5\text{ in.})Q = 0$$

Impending motion: $F = \mu_s N$

So $N\sin\theta - \mu_s N\cos\theta = Q = 15\text{ lb}$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta}$$

So $P = F = \mu_s N = \dfrac{\mu_s Q}{\sin\theta - \mu_s\cos\theta}$

(*a*) $t = 1\text{ in.} \Rightarrow \cos\theta = \dfrac{4\text{ in.}}{5\text{ in.}} = 0.8; \; \sin\theta = 0.6$

$$P = \frac{(0.45)(15\text{ lb})}{0.6 - (0.45)(0.8)} = 28.125\text{ lb}; \; \mathbf{P} = 28.1\text{ lb} \longleftarrow \blacktriangleleft$$

(*b*) $P \to \infty: \; \sin\theta - \mu_s\cos\theta = \dfrac{\mu_s Q}{P} \longrightarrow 0$

Thus $\tan\theta \to \mu_s = 0.45$ so that $\theta = 24.228°$

But $(5\text{ in.})\cos\theta = 5\text{ in.} - t$ or $t = (5\text{ in.})(1 - \cos\theta)$

$$t = 0.440\text{ in.} \blacktriangleleft$$