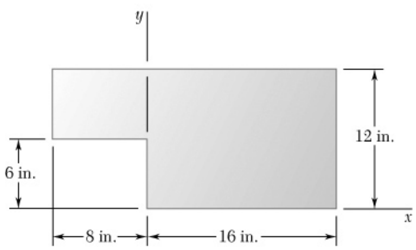
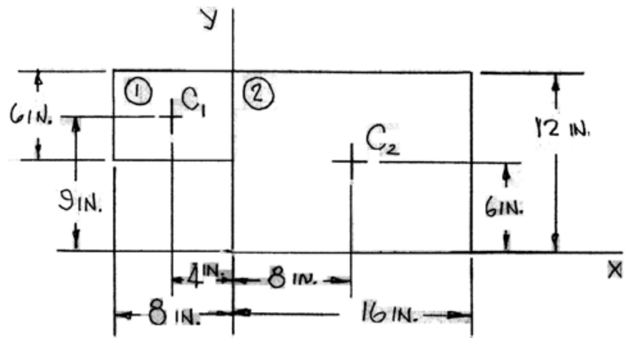
PROBLEM 5.1

Locate the centroid of the plane area shown.



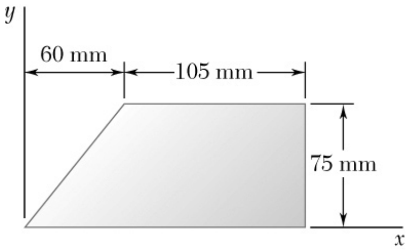
SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$8 \times 6 = 48$	-4	9	-192	432
2	$16 \times 12 = 192$	8	6	1536	1152
Σ	240			1344	1584

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1344 \text{ in}^3}{240 \text{ in}^2}$$ or $\bar{X} = 5.60$ in. ◀

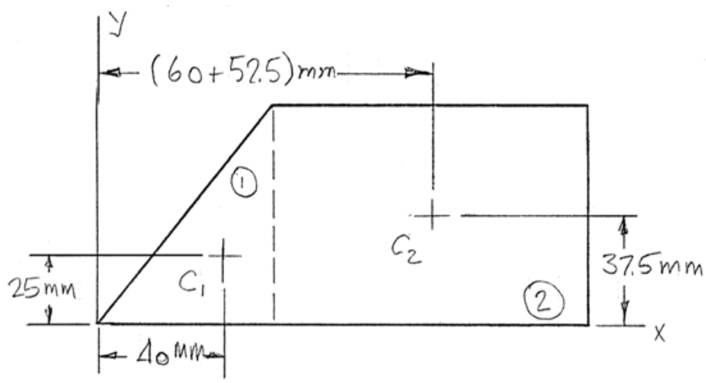
and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{1584 \text{ in}^3}{240 \text{ in}^2}$$ or $\bar{Y} = 6.60$ in. ◀



PROBLEM 5.2

Locate the centroid of the plane area shown.

SOLUTION

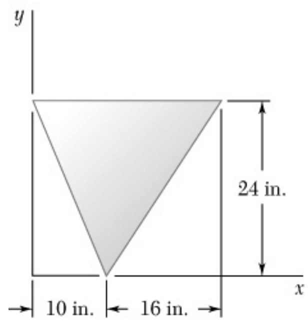


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{2} \times 60 \times 75 = 2250$	40	25	90 000	56 250
2	$105 \times 75 = 7875$	112.5	37.5	885 900	295 300
Σ	10 125			975 900	351 600

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{975\ 900 \text{ mm}^3}{10\ 125 \text{ mm}^2}$$ or $\bar{X} = 96.4$ mm ◀

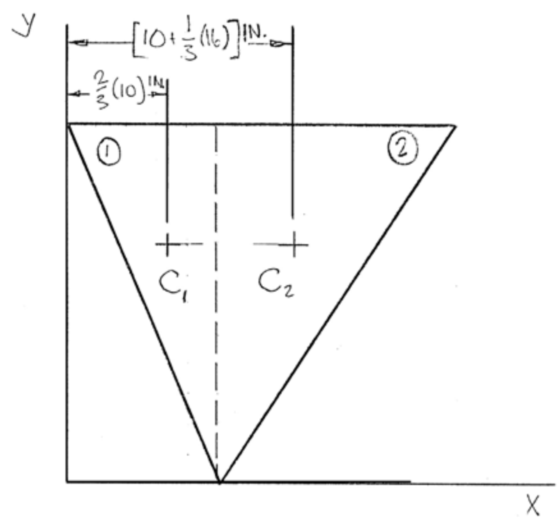
and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{351\ 600 \text{ mm}^3}{10\ 125 \text{ mm}^2}$$ or $\bar{Y} = 34.7$ mm ◀

PROBLEM 5.3



Locate the centroid of the plane area shown.

SOLUTION



For the area as a whole, it can be concluded by observation that

$$\bar{Y} = \frac{2}{3}(24 \text{ in.})$$

or $\bar{Y} = 16.00$ in. ◀

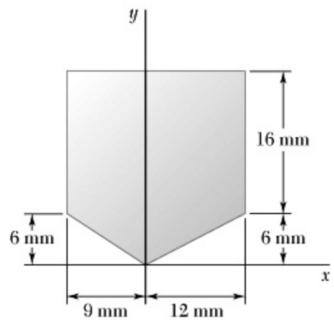
	A, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
1	$\frac{1}{2} \times 24 \times 10 = 120$	$\frac{2}{3}(10) = 6.667$	800
2	$\frac{1}{2} \times 24 \times 16 = 192$	$10 + \frac{1}{3}(16) = 15.333$	2944
Σ	312		3744

Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{3744 \text{ in}^3}{312 \text{ in}^2}$$

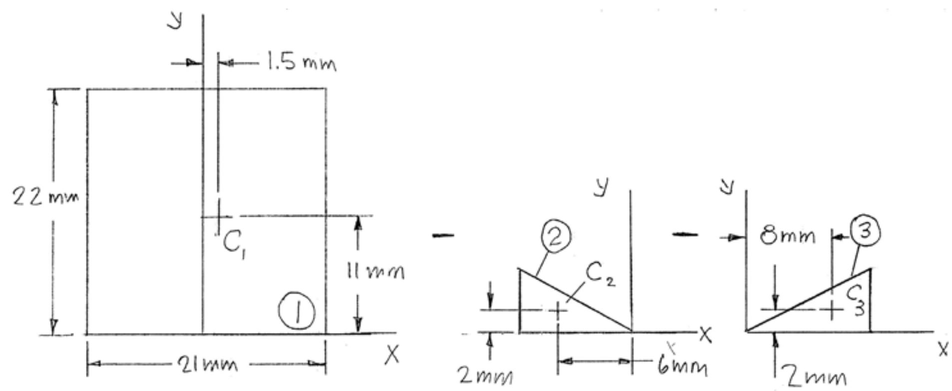
or $\bar{X} = 12.00$ in. ◀

PROBLEM 5.4



Locate the centroid of the plane area shown.

SOLUTION



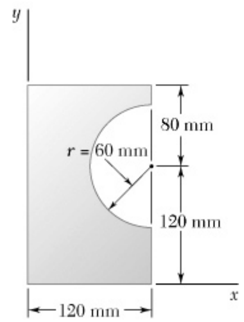
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$21 \times 22 = 462$	1.5	11	693	5082
2	$-\frac{1}{2}(6)(9) = -27$	–6	2	162	–54
3	$-\frac{1}{2}(6)(12) = -36$	8	2	–288	–72
Σ	399			567	4956

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{567 \text{ mm}^3}{399 \text{ mm}^2}$ or $\bar{X} = 1.421$ mm ◀

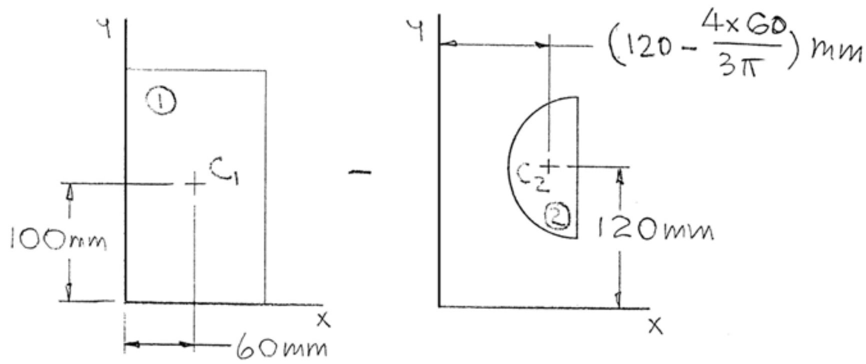
and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{4956 \text{ mm}^3}{399 \text{ mm}^2}$ or $\bar{Y} = 12.42$ mm ◀

PROBLEM 5.5

Locate the centroid of the plane area shown.



SOLUTION

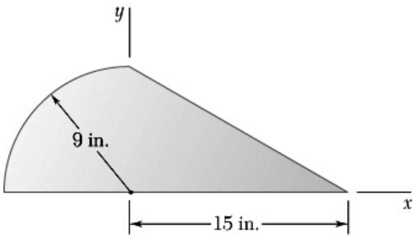


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$120 \times 200 = 24\,000$	60	120	1 440 000	2 880 000
2	$-\dfrac{\pi(60)^2}{2} = -5654.9$	94.5	120	–534 600	–678 600
Σ	18 345			905 400	2 201 400

Then $\bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{905\,400 \text{ mm}^3}{18\,345 \text{ mm}^2}$ or $\bar{X} = 49.4$ mm ◀

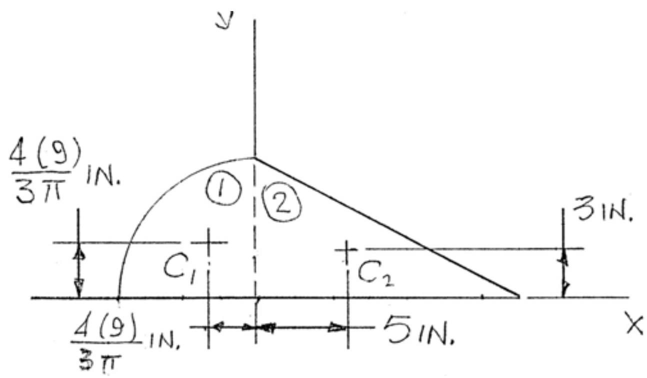
and $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{2\,201\,400 \text{ mm}^3}{18\,345 \text{ mm}^2}$ or $\bar{Y} = 93.8$ mm ◀

PROBLEM 5.6



Locate the centroid of the plane area shown.

SOLUTION

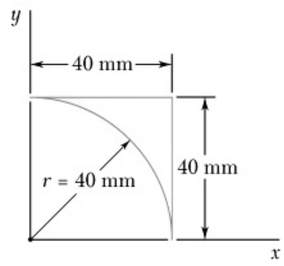


	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\dfrac{\pi(9)^2}{4} = 63.617$	$\dfrac{-4(9)}{(3\pi)} = -3.8917$	3.8917	-243	243
2	$\dfrac{1}{2}(15)(9) = 67.5$	5	3	337.5	202.5
Σ	131.1			94.5	445.5

Then $\quad \bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{94.5 \text{ in}^3}{131.1 \text{ in}^2}$ $\quad$ or $\bar{X} = 0.721$ in. ◀

and $\quad \bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{445.5 \text{ in}^3}{131.1 \text{ in}^2}$ $\quad$ or $\bar{Y} = 3.40$ in. ◀

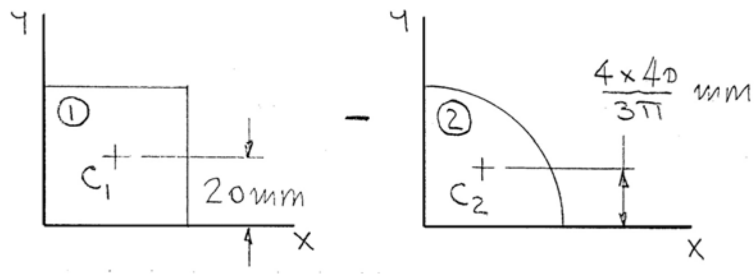
PROBLEM 5.7



Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies $\bar{X} = \bar{Y}$

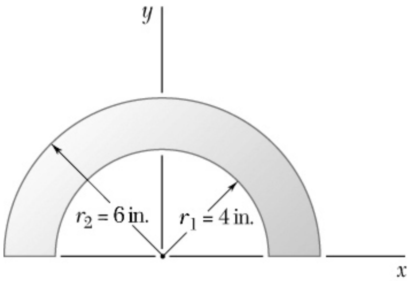


	A, mm^2	$\bar{x}$, mm	$\bar{x}A$, mm^3
1	$40 \times 40 = 1600$	20	32 000
2	$-\dfrac{\pi(40)^2}{4} = -1257$	16.98	−21 330
Σ	343		10 667

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{10\,667 \text{ mm}^3}{343 \text{ mm}^2}$$ or $\bar{X} = 31.1$ mm ◀

and $\bar{Y} = \bar{X} = 31.1$ mm ◀

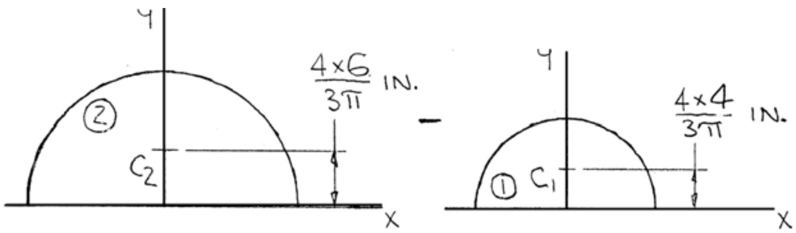
PROBLEM 5.8



Locate the centroid of the plane area shown.

SOLUTION

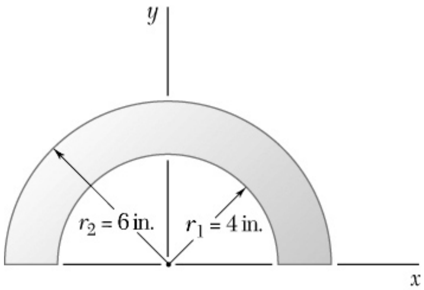
First note that symmetry implies $\bar{X} = 0$ ◀



	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$-\frac{\pi(4)^2}{2} = -25.13$	1.6977	–42.67
2	$\frac{\pi(6)^2}{2} = 56.55$	2.546	144
Σ	31.42		101.33

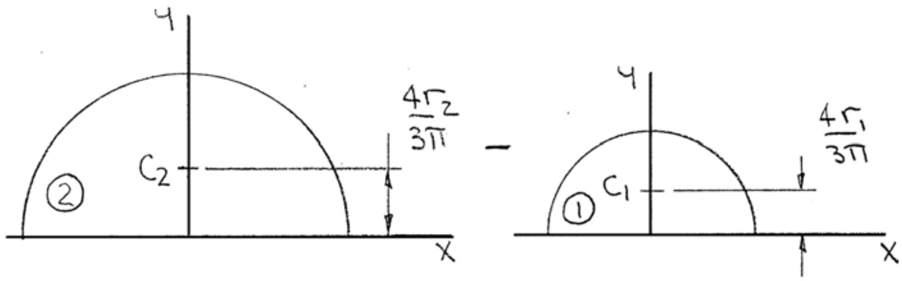
Then $\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{101.33 \text{ in}^3}{31.42 \text{ in}^2}$ or $\bar{Y} = 3.23$ in. ◀

PROBLEM 5.9



For the area of Problem 5.8, determine the ratio r_2/r_1 so that $\bar{y} = 3r_1/4$.

SOLUTION



	A	$\bar{y}$	$\bar{y}A$
1	$-\frac{\pi}{2}r_1^2$	$\frac{4r_1}{3\pi}$	$-\frac{2}{3}r_1^3$
2	$\frac{\pi}{2}r_2^2$	$\frac{4r_2}{3\pi}$	$\frac{2}{3}r_2^3$
Σ	$\frac{\pi}{2}\left(r_2^2 - r_1^2\right)$		$\frac{2}{3}\left(r_2^3 - r_1^3\right)$

Then $$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

or $$\frac{3}{4}r_1 \times \frac{\pi}{2}\left(r_2^2 - r_1^2\right) = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$\frac{9\pi}{16}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] = \left(\frac{r_2}{r_1}\right)^3 - 1$$

Let $$p = \frac{r_2}{r_1}$$

$$\frac{9\pi}{16}\left[(p+1)(p-1)\right] = (p-1)(p^2 + p + 1)$$

or $$16p^2 + (16 - 9\pi)p + (16 - 9\pi) = 0$$

PROBLEM 5.9 CONTINUED

Then
$$p = \frac{-(16 - 9\pi) \pm \sqrt{(16 - 9\pi)^2 - 4(16)(16 - 9\pi)}}{2(16)}$$

or
$$p = -0.5726 \qquad p = 1.3397$$

Taking the positive root
$$\frac{r_2}{r_1} = 1.340 \blacktriangleleft$$