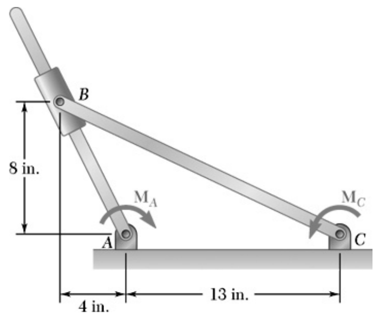
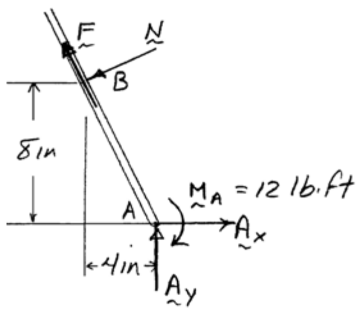
PROBLEM 8.41



In Problem 8.40, determine the smallest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad N\left(\sqrt{8\text{ in}^2 + 4\text{ in}^2}\right) - M_A = 0$$

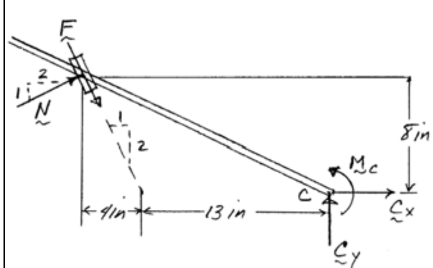
$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\text{ lb})$

$$= 4.830\text{ lb}$$

(Note: For min. M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:

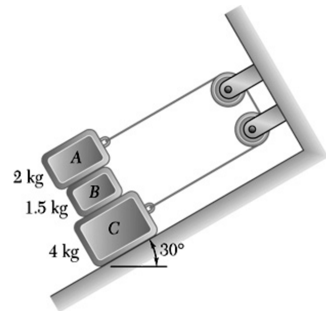


$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N + (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

$$M_C = \frac{1}{\sqrt{5}}\left[(17\text{ in.} + 16\text{ in.})(16.100\text{ lb}) - (26\text{ in.})(4.830\text{ lb})\right]$$

$$= 181.44\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\min} = 15.12\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$



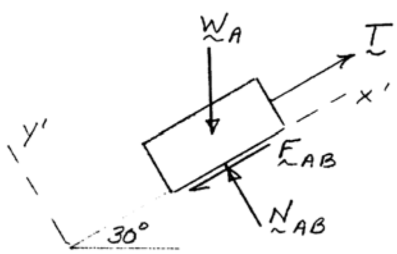
PROBLEM 8.42

Blocks A, B, and C having the masses shown are at rest on an incline. Denoting by μ_s the coefficient of static friction between all surfaces of contact, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

For impending motion, C will start down and A will start up. Since, the normal force between B and C is larger than that between A and B, the corresponding friction force can be larger as well. Thus we assume that motion impends between A and B.

FBD A:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{AB} - W_A \cos 30° = 0; \; N_{AB} = \frac{\sqrt{3}}{2} W_A$$

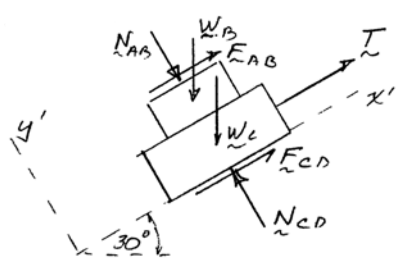
Impending motion: $$F_{AB} = \mu_s N_{AB} = \frac{\sqrt{3}}{2} W_A \mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T - F_{AB} - W_A \sin 30° = 0$$

or $$T = \left(\sqrt{3}\mu_s + 1\right)\frac{W_A}{2}$$

$$\nwarrow \Sigma F_{y'} = 0: \quad N_{CD} - N_{AB} - \left(W_B + W_C\right)\cos 30° = 0$$

FBD B + C:



or $$N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)$$

Impending motion: $$F_{CD} = \mu_s N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)\mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T + F_{AB} + F_{CD} - \left(W_B + W_C\right)\sin 30° = 0$$

$$T = \frac{W_B + W_C}{2} - \frac{\sqrt{3}}{2}\mu_s\left(2W_A + W_B + W_C\right)$$

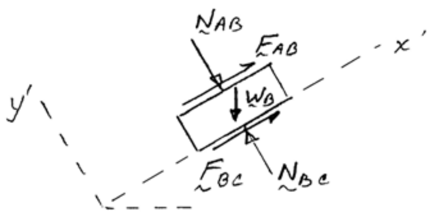
Equating T's: $$\sqrt{3}\mu_s\left(3W_A + W_B + W_C\right) = W_B + W_C - W_A$$

$$\mu_s = \frac{m_B + m_C - m_A}{\left(3m_A + m_B + m_C\right)\sqrt{3}} = \frac{1.5 \text{ kg} + 4 \text{ kg} - 2 \text{ kg}}{\left(6 \text{ kg} + 1.5 \text{ kg} + 4 \text{ kg}\right)\sqrt{3}}$$

$$\mu_s = 0.1757 \blacktriangleleft$$

PROBLEM 8.42 CONTINUED

FBD B:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{BC} - N_{AB} - W_B \cos 30° = 0$$

or
$$N_{BC} = \frac{\sqrt{3}}{2}\left(W_A + W_B\right)$$

$$\left(F_{BC}\right)_{\text{max}} = \mu_s N_{BC} = 0.1757\frac{\sqrt{3}}{2}\left(W_A + W_B\right)$$

$$= 0.1522\left(m_A + m_B\right)g = 0.1522\left(3.5 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)$$

$$= 5.224 \text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB} + F_{BC} - W_B \sin 30° = 0$$

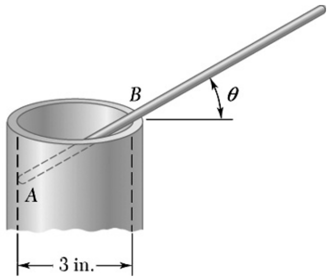
or
$$F_{BC} = -F_{AB} + \frac{1}{2}W_B = -\frac{\sqrt{3}}{2}W_A\left(0.1757\right) + \frac{W_B}{2}$$

$$= \left(-0.1522 m_A + 0.5 m_B\right)g$$

$$= \left[-0.1522\left(2 \text{ kg}\right) + 0.5\left(1.5 \text{ kg}\right)\right]\left(9.81 \text{ m/s}^2\right)$$

$$= 4.37 \text{ N}$$

$$F_{BC} < F_{BC\,\text{max}} \qquad \text{OK}$$

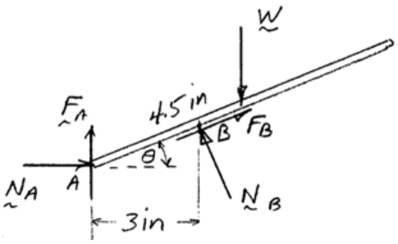


PROBLEM 8.43

A slender steel rod of length 9 in. is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right] W = 0$$

or $$N_B = (1.5\cos^2\theta)W$$

Impending motion: $$F_B = \mu_s N_B = (1.5\mu_s\cos^2\theta)W$$

$$= (0.3\cos^2\theta)W$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - N_B\sin\theta + F_B\cos\theta = 0$$

or $$N_A = (1.5\cos^2\theta)W(\sin\theta - 0.2\cos\theta)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= (0.3\cos^2\theta)W(\sin\theta - 0.2\cos\theta)$$

$$\uparrow \Sigma F_y = 0: \quad F_A + N_B\cos\theta + F_B\sin\theta - W = 0$$

or $$F_A = W\left(1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta\right)$$

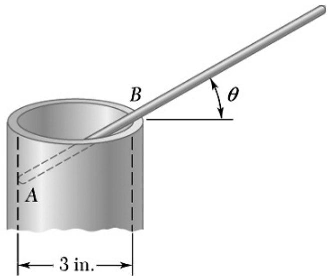
Equating F_A's

$$0.3\cos^2\theta(\sin\theta - 0.2\cos\theta) = 1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta$$

$$0.6\cos^2\theta\sin\theta + 1.44\cos^3\theta = 1$$

Solving numerically $\theta = 35.8°$ ◀

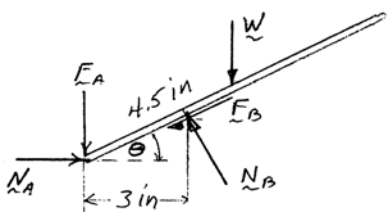
PROBLEM 8.44



In Problem 8.43, determine the smallest value of θ for which the rod will not fall out of the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right] W = 0$$

or $$N_B = 1.5W\cos^2\theta$$

Impending motion: $$F_B = \mu_s N_B = 0.2\left(1.5W\cos^2\theta\right)$$

$$= 0.3W\cos^2\theta$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta - F_B\cos\theta = 0$$

or $$N_A = W\cos^2\theta\left(1.5\sin\theta + 0.3\cos\theta\right)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= W\cos^2\theta\left(0.3\sin\theta + 0.06\cos\theta\right)$$

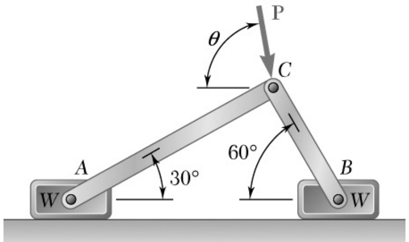
$$\uparrow \Sigma F_y = 0: \quad N_B\cos\theta - F_B\sin\theta - W - F_A = 0$$

or $$F_A = W\left[\cos^2\theta\left(1.5\cos\theta - 0.3\sin\theta\right) - 1\right]$$

Equating F_A's:

$$\cos^2\theta\left(1.44\cos\theta - 0.6\sin\theta\right) = 1$$

Solving numerically $$\theta = 20.5° \blacktriangleleft$$

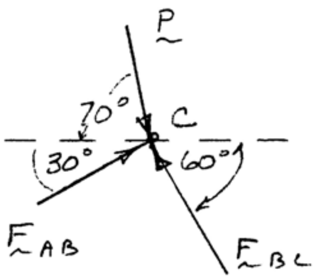


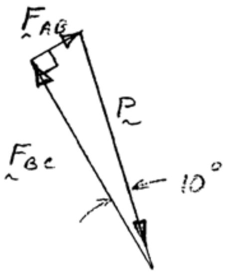
PROBLEM 8.45

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 70^\text{o}$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:





$$F_{AB} = P\sin 10° = 0.173648P$$

$$F_{BC} = P\cos 10° = 0.98481P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AB}\sin 30° = 0$$

or $$N_A = W + 0.173648P\sin 30° = W + 0.086824P$$

$$\longrightarrow \Sigma F_x = 0: \quad F_A - F_{AB}\cos 30° = 0$$

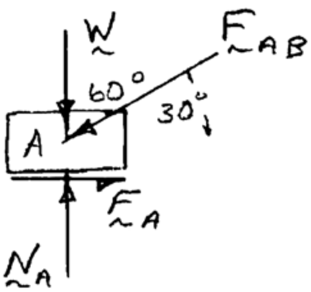
or $$F_A = 0.173648P\cos 30° = 0.150384P$$

For impending motion at A: $$F_A = \mu_s N_A$$

Then $$N_A = \frac{F_A}{\mu_s}: \quad W + 0.086824P = \frac{0.150384}{0.3}P$$

or $$P = 2.413W$$

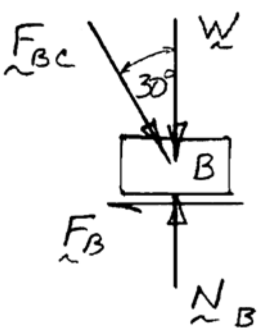
FBD block A:



$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30° = 0$$

$$N_B = W + 0.98481P\cos 30° = W + 0.85287P$$

FBD block B:



$$\longrightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30° - F_B = 0$$

$$F_B = 0.98481P\sin 30° = 0.4924P$$

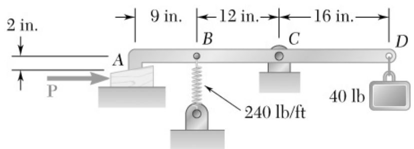
For impending motion at B: $$F_B = \mu_s N_B$$

Then $$N_B = \frac{F_B}{\mu_s}: \quad W + 0.85287P = \frac{0.4924P}{0.3}$$

or $$P = 1.268W$$

Thus, maximum P for equilibrium $$P_{max} = 1.268W \blacktriangleleft$$

PROBLEM 8.46

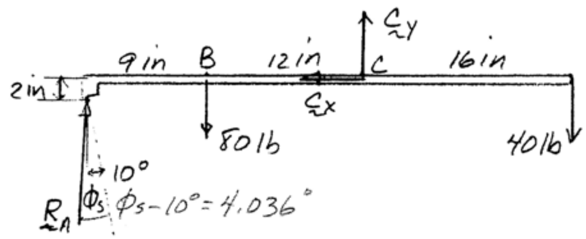


A 40-lb weight is hung from a lever which rests against a 10° wedge at *A* and is supported by a frictionless hinge at *C*. Knowing that the coefficient of static friction is 0.25 at both surfaces of the wedge and that for the position shown the spring is stretched 4 in., determine (*a*) the magnitude of the force **P** for which motion of the wedge is impending, (*b*) the components of the corresponding reaction at *C*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos(\phi_s - 10°)$$

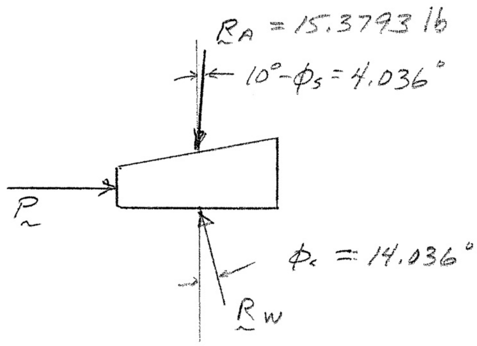
$$+ (2 \text{ in.})R_A\sin(\phi_s - 10°) = 0$$

or $\qquad R_A = 15.3793 \text{ lb}$

(*b*) $\qquad \rightarrow \Sigma F_x = 0: \quad (15.379 \text{ lb})\sin(4.036°) - C_x = 0 \qquad\qquad \mathbf{C}_x = 1.082 \text{ lb} \leftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad (15.379 \text{ lb})\cos(4.036°) - 80 \text{ lb} - 40 \text{ lb} + C_y = 0 \qquad\qquad \mathbf{C}_y = 104.7 \text{ lb} \uparrow$ ◀

FBD wedge:



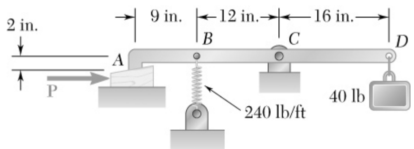
$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (15.3793 \text{ lb})\cos 4.036° = 0$$

or $\qquad R_W = 15.8133 \text{ lb}$

(*a*) $\qquad \rightarrow \Sigma F_x = 0: \quad P - (15.3793 \text{ lb})\sin 4.036° - (15.8133 \text{ lb})\sin 14.036° = 0$

$P = 4.92 \text{ lb}$ ◀

PROBLEM 8.47

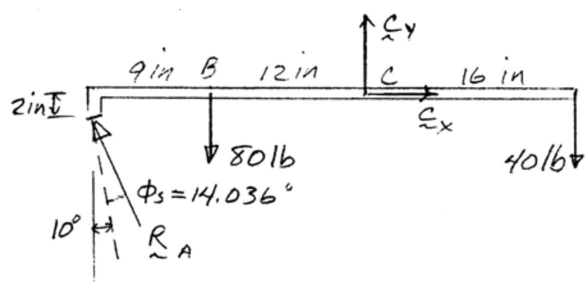


Solve Problem 8.46 assuming that force **P** is directed to the left.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos 24.036°$$

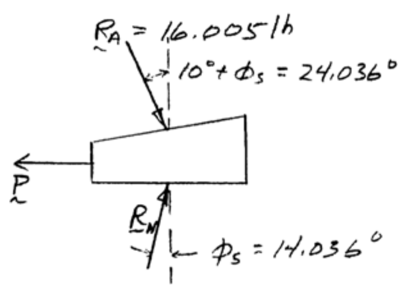
$$- (2 \text{ in.})R_A \sin 24.036° = 0$$

or $\qquad R_A = 16.005 \text{ lb}$

(*b*) $\quad \xrightarrow{\quad} \Sigma F_x = 0: \quad C_x - (16.005 \text{ lb})\sin 24.036° = 0 \qquad \mathbf{C}_x = 6.52 \text{ lb} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad C_y - 80 \text{ lb} - 40 \text{ lb} + (16.005 \text{ lb})\cos(24.036°) = 0 \qquad \mathbf{C}_y = 105.4 \text{ lb} \uparrow$ ◀

FBD wedge:



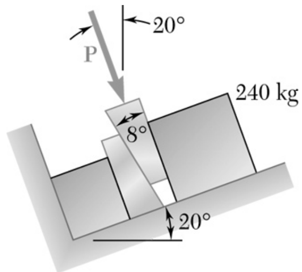
$\uparrow \Sigma F_y = 0: \quad R_W \cos 14.036° - (16.005 \text{ lb})\cos 24.036° = 0$

or $\qquad R_W = 15.067 \text{ lb}$

(*a*) $\quad \xrightarrow{\quad} \Sigma F_x = 0: \quad (16.005 \text{ lb})\sin 24.036° + (15.067 \text{ lb})\sin 14.036° - P = 0$

$P = 10.17 \text{ lb}$ ◀

PROBLEM 8.48

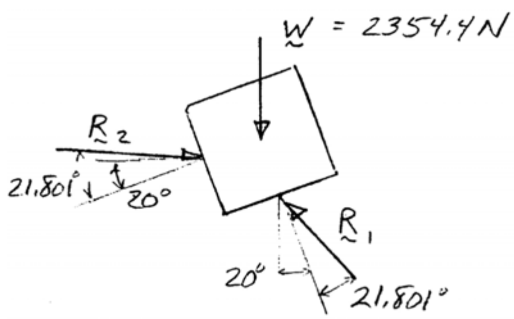


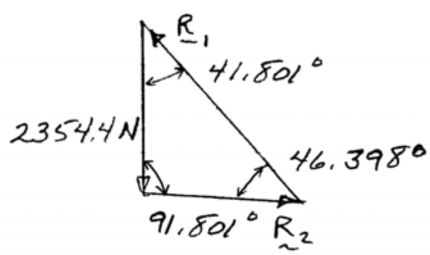
Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801^\circ \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

FBD block:

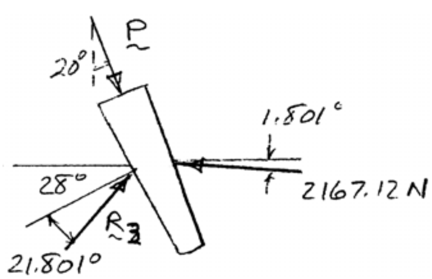


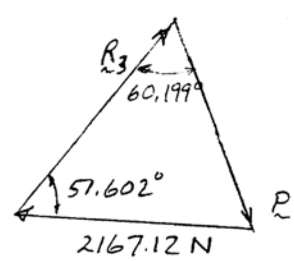


$$\frac{R_2}{\sin 41.801^\circ} = \frac{2354.4\text{ N}}{\sin 46.398^\circ}$$

$$R_2 = 2167.12\text{ N}$$

FBD wedge:



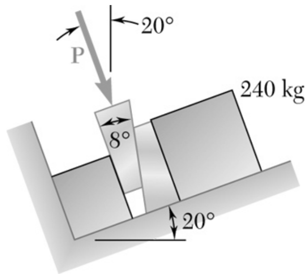


$$\frac{P}{\sin 51.602^\circ} = \frac{2167.12\text{ N}}{\sin 60.199^\circ}$$

$$P = 1957\text{ N}$$

$P = 1.957$ kN ◀

PROBLEM 8.49

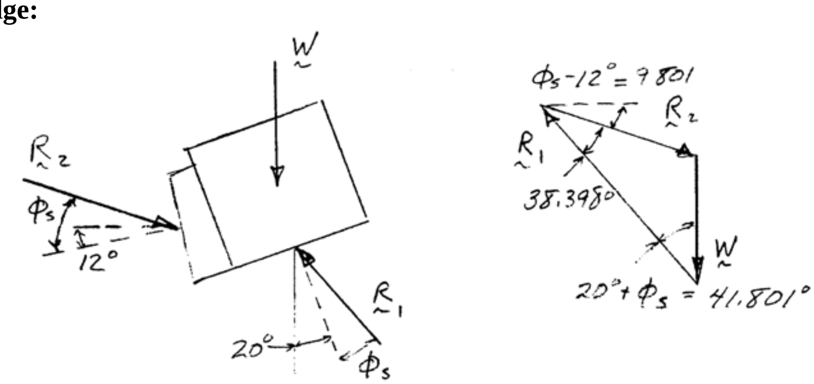


Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

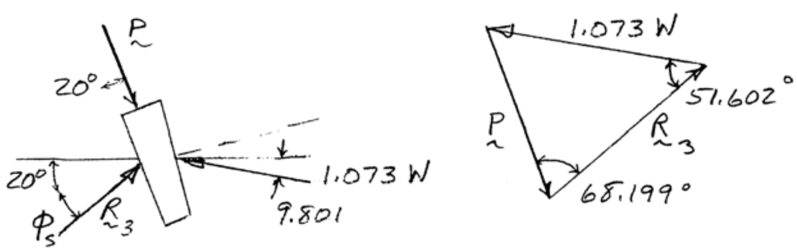
FBD block + wedge:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 38.398°}$$

$$R_2 = 2526.6\text{ N}$$

FBD wedge:

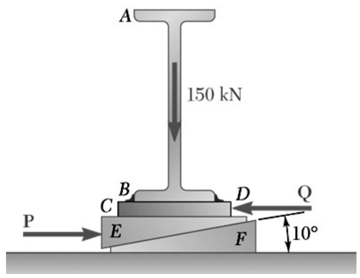


$$\frac{P}{\sin 51.602°} = \frac{2526.6\text{ N}}{\sin 68.199°}$$

$$P = 2132.7\text{ N}$$

$P = 2.13\text{ kN}$ ◀

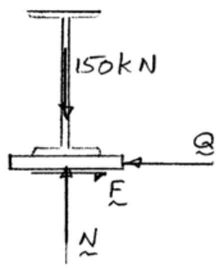
PROBLEM 8.50



The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

FBD AB + CD:



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

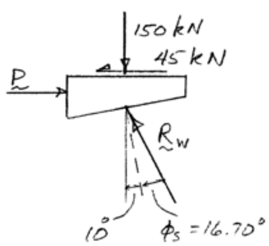
$$\uparrow \Sigma F_y = 0: \quad N - 150\text{ kN} = 0 \qquad N = 150\text{ kN}$$

Impending motion: $F = \mu_s N = 0.3(150\text{ kN}) = 45\text{ kN}$

$$\longrightarrow \Sigma F_x = 0: \quad F - Q = 0$$

$$(b)\ \mathbf{Q} = 45.0\text{ kN} \longleftarrow \blacktriangleleft$$

FBD top wedge:



Assume bottom wedge doesn't move:

$$\uparrow \Sigma F_y = 0: \quad R_W \cos(10° + 16.70°) - 150\text{ kN} = 0$$

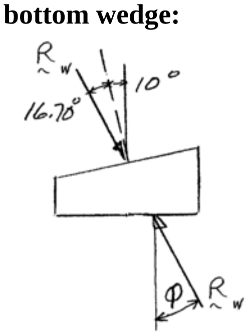
$$R_W = 167.9\text{ kN}$$

$$\longrightarrow \Sigma F_x = 0: \quad P - 45\text{ kN} - (167.9\text{ kN})\sin 26.70° = 0$$

$$P = 120.44\text{ kN}$$

$$(a)\ \mathbf{P} = 120.4\text{ kN} \longrightarrow \blacktriangleleft$$

FBD bottom wedge:



Bottom wedge is two-force member, so $\phi = 26.70°$ for equilibrium, but

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 31.0° \ (\text{steel on concrete})$$

So $\phi < \phi_s$ OK.