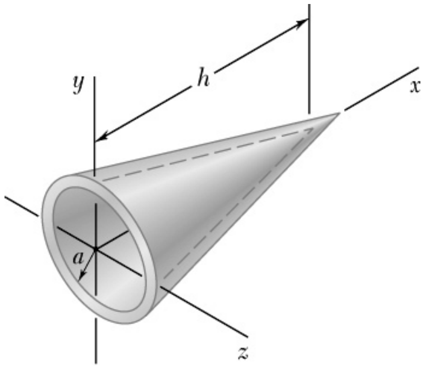
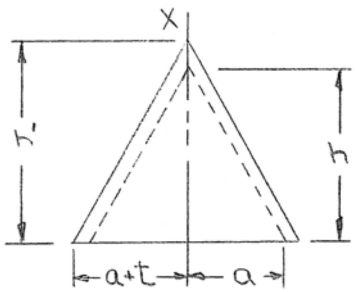
PROBLEM 9.131



Given the dimensions and the mass m of the thin conical shell shown, determine the moment of inertia and the radius of gyration of the shell with respect to the x axis. (*Hint:* Assume that the shell was formed by removing a cone with a circular base of radius a from a cone with a circular base of radius $a+t$. In the resulting expressions, neglect terms containing t^2, t^3, etc. Do not forget to account for the difference in the heights of the two cones.)

SOLUTION



First note $$\frac{h'}{a+t} = \frac{h}{a}$$

or $$h' = \frac{h}{a}(a+t)$$

For a cone of height H whose base has a radius r, have

where $$I_x = \frac{3}{10}mr^2$$

$$m = \rho V$$

$$= \rho \times \frac{\pi}{3} r^2 H$$

Then $$I_x = \frac{3}{10}\left(\frac{\pi}{3}\rho r^2 H\right)r^2$$

$$= \frac{\pi}{10}\rho r^4 H$$

Now, following the hint have

$$m_{\text{shell}} = m_{\text{outer}} - m_{\text{inner}} = \frac{\pi}{3}\rho\left[(a+t)^2 h' - a^2 h\right]$$

$$= \frac{\pi}{3}\rho\left[(a+t)^2 \times \frac{h}{a}(a+t) - a^2 h\right]$$

$$= \frac{\pi}{3}\rho a^2 h\left[\left(1+\frac{t}{a}\right)^3 - 1\right] = \frac{\pi}{3}\rho a^2 h\left(1 + 3\frac{t}{a} + \ldots - 1\right)$$

Neglecting the t^2 and t^3 terms obtain

$$m_{\text{shell}} \approx \pi\rho a h t$$

PROBLEM 9.131 CONTINUED

Also $$\left(I_x\right)_{\text{shell}} = \left(I_x\right)_{\text{outer}} - \left(I_x\right)_{\text{inner}}$$

$$= \frac{\pi}{10}\rho\left[\left(a+t\right)^4 h' - a^4 h\right]$$

$$= \frac{\pi}{10}\rho\left[\left(a+t\right)^4 \times \frac{h}{a}\left(a+t\right) - a^4 h\right]$$

$$= \frac{\pi}{10}\rho a^4 h\left[\left(1+\frac{t}{a}\right)^5 - 1\right]$$

$$= \frac{\pi}{10}\rho a^4 h\left(1 + 5\frac{t}{a} + \ldots - 1\right)$$

Neglecting t^2 and higher order terms, obtain

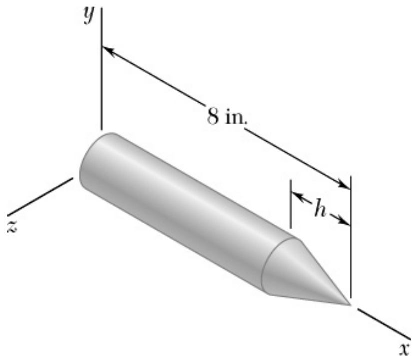
$$\left(I_x\right)_{\text{shell}} \approx \frac{\pi}{2}\rho a^3 ht$$

or $I_x = \frac{1}{2}ma^2$ ◀

Now $$k_x^2 = \frac{I_x}{m} = \frac{\frac{1}{2}ma^2}{m}$$

or $k_x = \frac{a}{\sqrt{2}}$ ◀

PROBLEM 9.132



A portion of an 8-in.-long steel rod of diameter 1.50 in. is turned to form the conical section shown. Knowing that the turning process reduces the moment of inertia of the rod with respect to the x axis by 20 percent, determine the height h of the cone.

SOLUTION

8-in. rod $\qquad (\bar{I}_x)_0 = \frac{1}{2} m_0 a^2 \qquad$ where $\qquad m_0 = \rho V_0 = \rho(\pi a^2 L)$

and $\qquad a = 0.75 \text{ in.} \qquad$ and $\qquad L = 8 \text{ in.}$

Therefore,

$$(\bar{I}_x)_0 = \frac{1}{2}\rho\pi a^4 L$$

Rod and cone

$$\bar{I}_x = (\bar{I}_x)_{\text{cyl}} + (\bar{I}_x)_{\text{cone}} = \frac{1}{2} m_{\text{cyl}} a^2 + \frac{3}{10} m_{\text{cone}} a^2$$

where

$$m_{\text{cyl}} = \rho V_{\text{cyl}} = \rho\left[\pi a^2 (L - h)\right]$$

and

$$m_{\text{cone}} = \rho V_{\text{cone}} = \rho\left[\frac{1}{3}\pi a^2 h\right]$$

Then

$$\bar{I}_x = \frac{1}{2}\rho\pi a^4 (L - h) + \frac{1}{10}\rho\pi a^4 h$$

Given

$$\bar{I}_x = 0.8(\bar{I}_x)_0$$

Then

$$\frac{1}{2}\rho\pi a^4 (L - h) + \frac{1}{10}\rho\pi a^4 h = 0.8\left(\frac{1}{2}\rho\pi a^4 L\right)$$

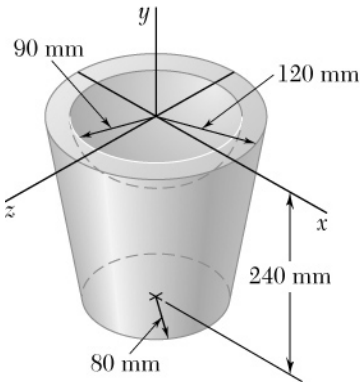
or

$$\frac{5}{10}L - \frac{5}{10}h + \frac{1}{10}h = \frac{4}{10}L$$

or

$$h = \frac{1}{4}L = \frac{1}{4}(8.00 \text{ in.}) = 2.00 \text{ in.}$$

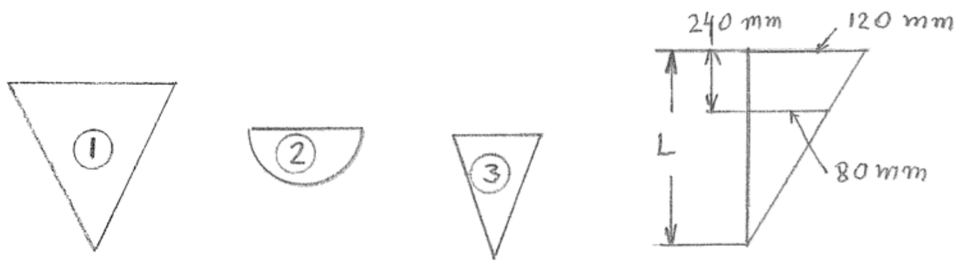
or $h = 2.00$ in. ◀



PROBLEM 9.133

The steel machine component shown is formed by machining a hemisphere into the base of a truncated cone. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to the y axis.

SOLUTION



First note $\quad \dfrac{L}{120} = \dfrac{L-240}{80} \quad$ or $\quad L = 720$ mm

and $\quad m = \rho_{st} V$

Now
$$m_1 = \rho_{st}\left(\frac{1}{3}\pi a_1^2 h_1\right) = \frac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.120 \text{ m})^2 (0.720 \text{ m})$$
$$= 85.230 \text{ kg}$$

$$m_2 = \rho_{st}\left(\frac{2}{3}\pi a_2^2\right) = \frac{2}{3}\pi \times 7850 \text{ kg/m}^3 \times (0.090 \text{ m})^2 = 11.9855 \text{ kg}$$

$$m_3 = \rho_{st}\left(\frac{1}{3}\pi a_3^2 h_3\right) = \frac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.080 \text{ m})^2 (0.720 - 0.240)\text{m}$$
$$= 25.253 \text{ kg}$$

Now
$$\bar{I}_y = (\bar{I}_y)_1 - (\bar{I}_y)_2 - (\bar{I}_y)_3$$

PROBLEM 9.133 CONTINUED

where (using Figure 9.28)

$$\left(\bar{I}_y\right)_1 = \frac{3}{10}m_1a_1^2 = \frac{3}{10}(85.230\text{ kg})(0.120\text{ m})^2 = 0.36819\text{ kg}\cdot\text{m}^2$$

$$\left(\bar{I}_y\right)_2 = \frac{1}{2}\left(\bar{I}_y\right)_{\text{sphere}} = \frac{1}{2}\left(\frac{2}{5}m_{\text{sphere}}a_2^2\right) \qquad \text{where} \qquad m_{\text{sphere}} = 2m_{\text{hemisphere}}$$

$$= \frac{1}{2}\left(\frac{2}{5}\;2\times 11.9855\text{ kg}\right)(0.090\text{ m})^2$$

$$= 0.038833\text{ kg}\cdot\text{m}^2$$

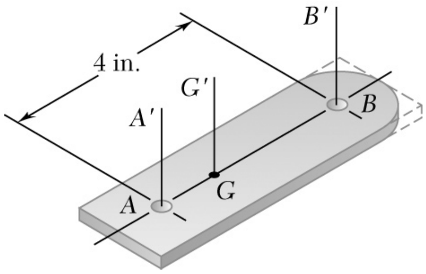
$$\left(\bar{I}_y\right)_3 = \frac{3}{10}m_3a_3^2 = \frac{3}{10}(25.253\text{ kg})(0.080\text{ m})^2 = 0.048486\text{ kg}\cdot\text{m}^2$$

Then

$$\bar{I}_y = (0.36819 - 0.038833 - 0.048486)\text{kg}\cdot\text{m}^2 = 0.28087\text{ kg}\cdot\text{m}^2$$

$$= 0.281\text{ kg}\cdot\text{m}^2$$

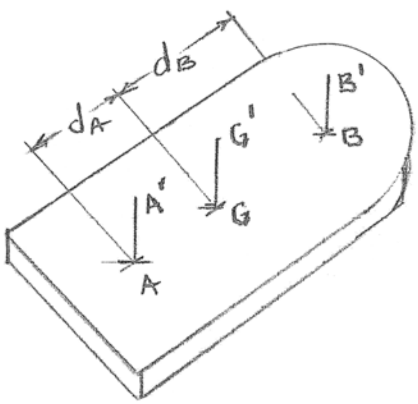
or $\bar{I}_y = 281\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.134

After a period of use, one of the blades of a shredder has been worn to the shape shown and is of weight 0.4 lb. Knowing that the moments of inertia of the blade with respect to the AA' and BB' axes are 0.6×10^{-3} lb · ft · s^2 and 1.26×10^{-3} lb · ft · s^2, respectively, determine (*a*) the location of the centroidal axis GG', (*b*) the radius of gyration with respect to axis GG'.

SOLUTION



Have

(*a*) $$d_B = \frac{4}{12} - d_A = (0.33333 - d_A)\text{ ft}$$

and $$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then $$I_{BB'} - I_{AA'} = m\left(d_B^2 - d_A^2\right)$$

$$= m\left[\left(0.33333 - d_A\right)^2 - d_A^2\right]$$

$$= m\left(0.11111 - 0.66666 d_A\right)$$

Then $$(1.26 - 0.6) \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= \frac{0.40\text{ lb}}{32.2\text{ ft/s}^2}\left(0.11111 - 0.66666 d_A\right)\text{ ft}^2$$

or $$d_A = 0.08697\text{ ft}$$

or $$d_A = 1.044\text{ in.} \blacktriangleleft$$

(*b*) $$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

or $$\bar{I}_{GG'} = 0.6 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$- \frac{0.4\text{ lb}}{32.2\text{ ft/s}^2}\left(0.08697\text{ ft}\right)^2$$

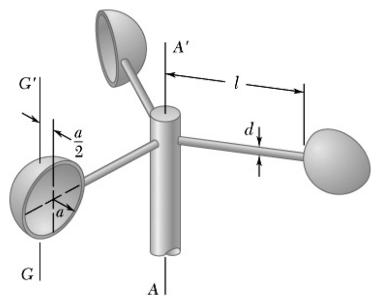
$$= 0.50604 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$\bar{k}_{GG'}^2 = \frac{\bar{I}_{GG'}}{m} = \frac{0.50604 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2}{\frac{0.4\text{ lb}}{32.2\text{ ft/s}^2}}$$

$$= 0.04074\text{ ft}^2$$

$$\bar{k}_{GG'} = 0.20183\text{ ft} = 2.4219\text{ in.}$$

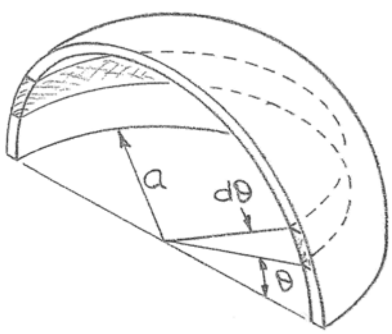
or $$\bar{k}_{GG'} = 2.42\text{ in.} \blacktriangleleft$$



PROBLEM 9.135

The cups and the arms of an anemometer are fabricated from a material of density ρ. Knowing that the moment of inertia of a thin, hemispherical shell of mass m and thickness t with respect to its centroidal axis GG', is $5ma^2/12$, determine (a) the moment of inertia of the anemometer with respect to the axis AA', (b) the ratio of a to l for which the centroidal moment of inertia of the cups is equal to 1 percent of the moment of inertia of the cups with respect to the axis AA'.

SOLUTION



(a) First note $\quad m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \dfrac{\pi}{4} d^2 l$

and $\quad dm_{\text{cup}} = \rho dV_{\text{cup}}$

$$= \rho\left[(2\pi a\cos\theta)(t)(ad\theta)\right]$$

Then $\quad m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\frac{\pi}{2}} 2\pi\rho a^2 t\cos\theta d\theta$

$$= 2\pi\rho a^2 t\left[\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$= 2\pi\rho a^2 t$$

Now $\quad (I_{AA'})_{\text{anem.}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$

Using the parallel-axis theorem and assuming the arms are slender rods, have

$$(I_{AA'})_{\text{anem.}} = 3\left[(I_{GG'})_{\text{cup}} + m_{\text{cup}} d_{AG}^2\right]$$

$$+ 3\left[\bar{I}_{\text{arm}} + m_{\text{arm}} d_{AG_{\text{arm}}}\right]$$

$$= 3\left\{\frac{5}{12} m_{\text{cup}} a^2 + m_{\text{cup}}\left[(l+a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$+3\left[\frac{1}{2} m_{\text{arm}} l^2 + m_{\text{arm}}\left(\frac{l}{2}\right)^2\right]$$

$$= 3m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) + m_{\text{arm}} l^2$$

$$= 3(2\pi\rho a^2 t)\left(\frac{5}{3}a^2 + 2la + l^2\right) + \left(\frac{\pi}{4}\rho d^2 l\right)(l^2)$$

or $\quad (I_{AA'})_{\text{anem}} = \pi\rho l^2\left[6a^2 t\left(\dfrac{5}{3}\dfrac{a^2}{l^2} + 2\dfrac{a}{l} + 1\right) + \dfrac{d^2 l}{4}\right]$ ◀

PROBLEM 9.135 CONTINUED

(*b*) Have
$$\frac{\left(I_{GG'}\right)_{\text{cup}}}{\left(I_{AA'}\right)_{\text{cup}}} = 0.01$$

or
$$\frac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) \quad \text{(From Part a)}$$

Now let $\zeta = \dfrac{a}{l}$

Then
$$5\zeta^2 = 0.12\left(\frac{5}{3}\zeta^2 + 2\zeta + 1\right)$$

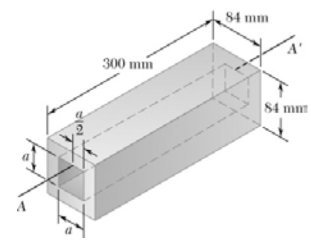
or
$$40\zeta^2 - 2\zeta - 1 = 0$$

Then
$$\zeta = \frac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$$

or
$$\zeta = 0.1851 \qquad \text{and} \qquad \zeta = -0.1351$$

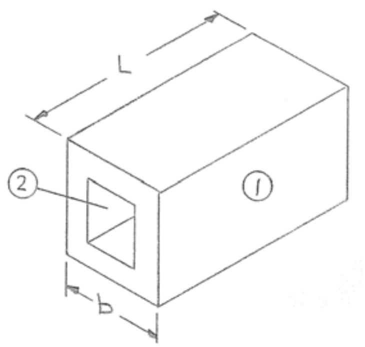
$$\therefore \quad \frac{a}{l} = 0.1851 \blacktriangleleft$$

PROBLEM 9.136



A square hole is centered in and extends through the aluminum machine component shown. Determine (*a*) the value of *a* for which the mass moment of inertia of the component with respect to the axis AA', which bisects the top surface of the hole, is maximum, (*b*) the corresponding values of the mass moment of inertia and the radius of gyration with respect to the axis AA'. (The density of aluminum is $2800\ \text{kg/m}^3$.)

SOLUTION



First note $m_1 = \rho V_1 = \rho b^2 L$

And $m_2 = \rho V_2 = \rho a^2 L$

(*a*) Using Figure 9.28 and the parallel-axis theorem, have

$$I_{AA'} = (I_{AA'})_1 - (I_{AA'})_2$$

$$= \left[\frac{1}{12}m_1\left(b^2 + b^2\right) + m_1\left(\frac{a}{2}\right)^2\right]$$

$$-\left[\frac{1}{12}m_2\left(a^2 + a^2\right) + m_2\left(\frac{a}{2}\right)^2\right]$$

$$= \left(\rho b^2 L\right)\left(\frac{1}{6}b^2 + \frac{1}{4}a^2\right) - \left(\rho a^2 L\right)\left(\frac{5}{12}a^2\right)$$

$$= \frac{\rho L}{12}\left(2b^4 + 3b^2a^2 - 5a^4\right)$$

Then $$\frac{dI_{AA'}}{da} = \frac{\rho L}{12}\left(6b^2 a - 20a^3\right) = 0$$

or $a = 0$ and $a = b\sqrt{\frac{3}{10}}$

Also.. $$\frac{d^2 I_{AA'}}{da^2} = \frac{\rho L}{12}\left(6b^2 - 60a^2\right) = \frac{1}{2}\rho L\left(b^2 - 10a^2\right)$$

Now, for

$a = 0,\ \dfrac{d^2 I_{AA'}}{da^2} > 0$ and for $a = b\sqrt{\dfrac{3}{10}},\ \dfrac{d^2 I_{AA'}}{da^2} < 0$

$\therefore\ (I_{AA'})_{\max}$ occurs when $a = b\sqrt{\frac{3}{10}}$

$$a = 84\sqrt{\frac{3}{10}} = 46.009 \text{ mm}$$

or $a = 46.0$ mm ◀

PROBLEM 9.136 CONTINUED

(b) From part (a)

$$\left(I_{AA'}\right)_{\text{mass}} = \frac{\rho L}{12}\left[2b^4 + 3b^2\left(b\sqrt{\frac{3}{10}}\right)^2 - 5\left(b\sqrt{\frac{3}{10}}\right)^4\right]$$

$$= \frac{49}{240}\rho L b^4 = \frac{49}{240}\left(2800\ \text{kg/m}^3\right)\left(0.3\ \text{m}\right)\left(0.4\ \text{m}\right)^4$$

$$= 8.5385 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $\left(I_{AA'}\right)_{\text{mass}} = 8.54 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

and $$k_{AA'}^2 = \frac{\left(I_{AA'}\right)_{\text{mass}}}{m}$$

where

$$m = m_1 - m_2 = \rho L\left(b^2 - a^2\right) = \rho L\left[b^2 - \left(b\sqrt{\frac{3}{10}}\right)^2\right] = \frac{7}{10}\rho L b^2$$

Then

$$k_{AA'}^2 = \frac{\frac{49}{240}\rho L b^4}{\frac{7}{10}\rho L b^2} = \frac{7}{24}b^2 = \frac{7}{24}\left(84\ \text{mm}\right)^2 = 2058\ \text{mm}^2$$

$$k_{AA'} = 45.3652\ \text{mm}$$

or $k_{AA'} = 45.4\ \text{mm}$ ◀

To the instructor:

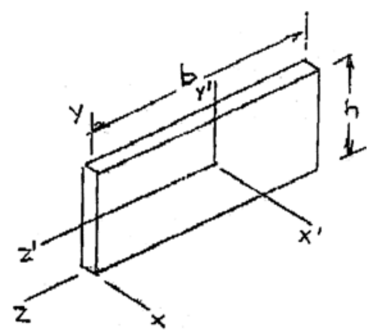
The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.137–9.142

Thin rectangular plate

$$\left(I_x\right)_m = \left(\bar{I}_{x'}\right)_m + md^2$$

$$= \frac{1}{12}m\left(b^2 + h^2\right) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]$$

$$= \frac{1}{3}m\left(b^2 + h^2\right)$$



$$\left(I_y\right)_m = \left(\bar{I}_{y'}\right)_m + md^2$$

$$= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2$$

$$= \frac{1}{3}mb^2$$

$$I_z = \left(\bar{I}_{z'}\right)_m + md^2$$

$$= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{3}mh^2$$

Thin triangular plate

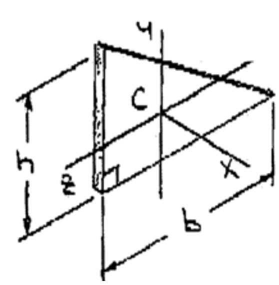
Have $\quad m = \rho V = \rho\left(\frac{1}{2}bht\right)$

and $\quad \bar{I}_{z,\text{ area}} = \frac{1}{36}bh^3$

Then $\quad \bar{I}_{z,\text{ mass}} = \rho t I_{z,\text{ area}}$

$$= \rho t \times \frac{1}{36}bh^3$$

$$= \frac{1}{18}mh^2$$

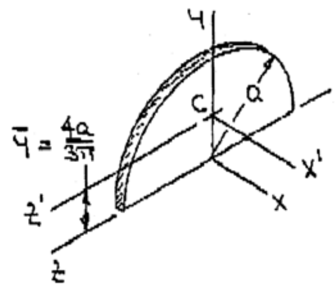


Similarly, $$\bar{I}_{y,\text{ mass}} = \frac{1}{18}mb^2$$

Now $$\bar{I}_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + \bar{I}_{z,\text{ mass}} = \frac{1}{18}m\left(b^2 + h^2\right)$$

Thin semicircular plate

Have $$m = \rho V = \rho\left(\frac{\pi}{2}a^2t\right)$$



And $$\bar{I}_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{8}a^4$$

Then $$\bar{I}_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t\bar{I}_{y,\text{ area}}$$

$$= \rho t \times \frac{\pi}{8}a^4$$

$$= \frac{1}{4}ma^2$$

Now $$I_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$$

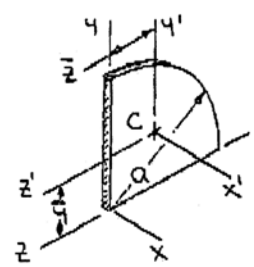
Also $$I_{x,\text{ mass}} = \bar{I}_{x',\text{ mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{x',\text{ mass}} = m\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

And $$I_{z,\text{ mass}} = \bar{I}_{z',\text{ mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{z',\text{ mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$

Thin Quarter-Circular Plate

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Have $$m = \rho V = \rho\left(\frac{\pi}{4}a^2t\right)$$



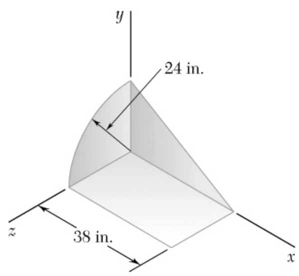
and $$I_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{16}a^4$$

Then $$I_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t I_{y,\text{ area}}$$

$$= \rho t \times \frac{\pi}{16}a^4$$

$$= \frac{1}{4}ma^2$$

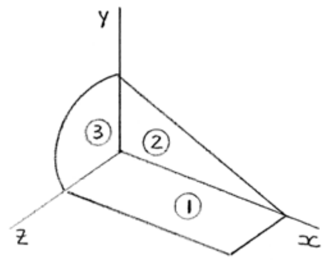
Now	$I_{x,\text{ mass}} = I_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$
Also	$I_{x,\text{ mass}} = \bar{I}_{x',\text{ mass}} + m\left(\bar{y}^2 + \bar{z}^2\right)$
or	$\bar{I}_{x',\text{ mass}} = m\left(\frac{1}{2} - \frac{32}{9\pi^2}\right)a^2$
and	$I_{y,\text{ mass}} = \bar{I}_{y',\text{ mass}} + m\bar{z}^2$
or	$\bar{I}_{y',\text{ mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$



PROBLEM 9.137

A 0.1-in-thick piece of sheet metal is cut and bent into the machine component shown. Knowing that the specific weight of steel is 0.284 lb/in^3, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



$$m_1 = \rho V = \frac{\gamma}{g} tA$$

$$= \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2}(0.10 \text{ in.})(38 \text{ in.})(24 \text{ in.})$$

$$= 0.80438 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{0.284}{32.2}(0.1 \text{ in.})\left[\left(\frac{1}{2}\right)(38 \text{ in.})(24 \text{ in.})\right]$$

$$= 0.402186 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{0.284}{32.2}(0.1 \text{ in.})\left[\left(\frac{\pi}{4}\right)(24 \text{ in.})^2\right] = 0.39900 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Using Fig. 9.28 for component 1 and the equations derived for components 2 and 3, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= (0.80438 \text{ lb}\cdot\text{s}^2/\text{ft})\left[\frac{1}{12}\left(\frac{24}{12}\right)^2 + \left(\frac{24}{2\times 12}\right)^2\right]\text{ft}^2 + (0.402186 \text{ lb}\cdot\text{s}^2/\text{ft})\left[\frac{1}{18}\left(\frac{24}{12}\right)^2 + \left(\frac{24}{3\times 12}\right)^2\right]\text{ft}^2$$

$$+ (0.39900 \text{ lb}\cdot\text{s}^2/\text{ft})\left[\frac{1}{2}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= (1.0725 + 0.26812 + 0.7980) \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 2.13862 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 2.14 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.137 CONTINUED

Also

$$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

$$= \left(0.80438 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[\left(\frac{38}{12}\right)^2 + \left(\frac{24}{12}\right)^2\right]\text{ft}^2 + \left[\left(\frac{38}{2\times 12}\right)^2 + \left(\frac{24}{2\times 12}\right)^2\right]\text{ft}^2\right\}$$

$$+ \left(0.402186 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{18}\left(\frac{38}{12}\right)^2 + \left(\frac{38}{3\times 12}\right)^2\right]\text{ft}^2$$

$$+ \left(0.39900 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{4}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= \left(3.76122 + 0.672172 + 0.3990\right)\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.83234 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 4.83 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3$$

$$= \left(0.80438 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{12}\left(\frac{38}{12}\right)^2 + \left(\frac{38}{2\times 12}\right)^2\right]\text{ft}^2$$

$$+ \left(0.401286 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{18}\left[\left(\frac{38}{12}\right)^2 + \left(\frac{24}{12}\right)^2\right] + \left[\left(\frac{38}{3\times 12}\right)^2 + \left(\frac{24}{3\times 12}\right)^2\right]\right\}\text{ft}^2$$

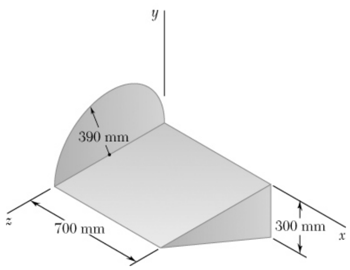
$$+ \left(0.3990 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{4}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= \left(2.68871 + 0.940296 + 0.3990\right)\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.0280 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

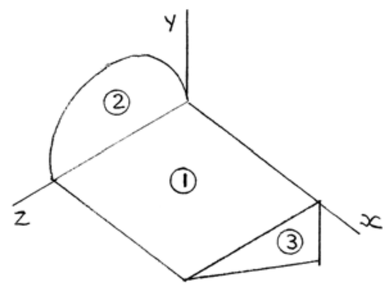
or $I_z = 4.03 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.138



A 3-mm-thick piece of sheet metal is cut and bent into the machine component shown. Knowing that the density of steel is $7850\ \text{kg/m}^3$, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component.

Have $m = \rho_{st}V = \rho_{st}tA$

$$m_1 = (7850\ \text{kg/m}^3)(0.003\ \text{m})(0.70\ \text{m})(0.780\ \text{m}) = 12.858\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)(0.003\ \text{m})\left[\left(\frac{\pi}{2}\right)(0.39\ \text{m})^2\right] = 5.6265\ \text{kg}$$

$$m_3 = (7850\ \text{kg/m}^3)(0.003\ \text{m})\left[\left(\frac{1}{2}\right)(0.780\ \text{m})(0.3\ \text{m})\right] = 2.7554\ \text{kg}$$

Using Fig. 9-28 for component 1 and the equations derived above for components 2 and 3 have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= (12.858\ \text{kg})\left[\frac{1}{12}(0.78)^2 + \left(\frac{0.78}{2}\right)^2\right]\text{m}^2$$

$$+ (5.6265\ \text{kg})\left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.39)^2 + \left[\left(\frac{4\times 0.39}{3\pi}\right)^2 + (0.39)^2\right]\right\}\text{m}^2$$

$$+ (2.7554\ \text{kg})\left\{\frac{1}{18}\left[(0.78)^2 + (0.30)^2\right] + \left[\left(\frac{0.78}{3}\right)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\}\text{m}^2$$

$$= (2.6076 + 1.2836 + 0.3207)\ \text{kg}\cdot\text{m}^2$$

$$= 4.2119\ \text{kg}\cdot\text{m}^2$$

or $I_x = 4.21\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.138 CONTINUED

And

$$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

$$= \left(12.858\text{ kg}\right)\left\{\frac{1}{12}\left[\left(0.7\right)^2 + \left(0.78\right)^2\right] + \frac{1}{4}\left[\left(0.7\right)^2 + \left(0.78\right)^2\right]\right\}\text{m}^2$$

$$+\left(5.6265\text{ kg}\right)\left[\frac{1}{4}\left(0.39\right)^2 + \left(0.39\right)^2\right]\text{m}^2$$

$$+\left(2.7554\text{ kg}\right)\left\{\frac{1}{18}\left(0.78\right)^2 + \left[\left(0.7\right)^2 + \left(\frac{0.78}{3}\right)^2\right]\right\}\text{m}^4$$

$$= \left(4.7077 + 1.0697 + 1.6295\right)\text{kg}\cdot\text{m}^2$$

$$= 7.4069\text{ kg}\cdot\text{m}^2$$

or $I_y = 7.41\text{ kg}\cdot\text{m}^2$ ◀

And

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3$$

$$= \left(12.858\text{ kg}\right)\left[\left(0.7\right)^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]\text{m}^2$$

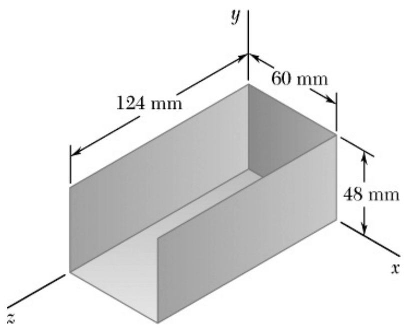
$$+\left(5.6265\text{ kg}\right)\left[\frac{1}{4}\left(0.39\right)^2\right]\text{m}^2$$

$$+\left(2.7554\text{ kg}\right)\left\{\frac{1}{18}\left(0.3\right)^2 + \left[\left(0.70\right)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\}\text{m}^2$$

$$= \left(2.1001 + 0.21395 + 1.39145\right)\text{kg}\cdot\text{m}^2 = 3.7055\text{ kg}\cdot\text{m}^2$$

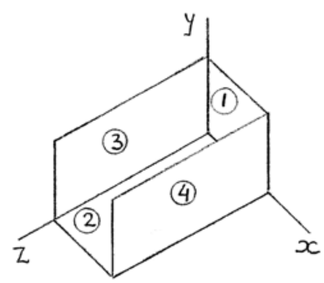
or $I_z = 3.71\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.139



The cover of an electronic device is formed from sheet aluminum that is 2 mm thick. Determine the mass moment of inertia of the cover with respect to each of the coordinate axes. (The density of aluminum is 2770 kg/m^3.)

SOLUTION



Have $$m = \rho V$$

Now
$$m_1 = (2770 \text{ kg/m}^3)(0.002 \text{ m})(0.048 \text{ m})(0.06 \text{ m})$$
$$= 0.015955 \text{ kg}$$
$$m_2 = (2770 \text{ kg/m}^3)(0.002 \text{ m})(0.06 \text{ m})(0.124 \text{ m})$$
$$= 0.041218 \text{ kg}$$
$$m_3 = (2770 \text{ kg/m}^3)(0.002 \text{ m})(0.048 \text{ m})(0.124 \text{ m})$$
$$= 0.032974 \text{ kg}$$

Using Fig. 9.28 and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \qquad \text{and} \qquad (I_x)_3 = (I_x)_4$$
$$= (0.015955 \text{ kg})\left[(0.048)^2\left(\frac{1}{12}+\frac{1}{4}\right)\right]\text{m}^2 + (0.041218 \text{ kg})\left[(0.124)^2\left(\frac{1}{12}+\frac{1}{4}\right)\right]\text{m}^2$$
$$+ 2(0.032974 \text{ kg})\left\{\left[(0.048)^2 + (0.124)^2\right]\left(\frac{1}{12}+\frac{1}{4}\right)\right\}\text{m}^2$$
$$= (0.012253\times10^{-3} + 0.211256\times10^{-3} + 0.388654\times10^{-3})\text{kg}\cdot\text{m}^2 = 0.61216\times10^{-3} \text{ kg}\cdot\text{m}^2$$

or $I_x = 0.612\times10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.139 CONTINUED

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

$$= (0.015955\text{ kg})\left[\frac{1}{3}(0.06)^2\right]\text{m}^2 + (0.041218\text{ kg})\left\{\left[(0.06)^2 + (0.124)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2$$

$$+ (0.032974\text{ kg})\left[(0.124)^2\left(\frac{1}{3}\right)\right]\text{m}^2 + (0.032974\text{ kg})\left[\frac{1}{12}(0.124)^2 + (0.06)^2 + \frac{1}{4}(0.124)^2\right]\text{m}^2$$

$$= \left[(0.019146 + 0.260718 + 0.16900 + 0.287709)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.73657\times 10^{-3}\text{kg}\cdot\text{m}^2$$

or $I_y = 0.737\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= (0.015955\text{ kg})\left\{\left[(0.06)^2 + (0.048)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2 + (0.041218\text{ kg})\left[(0.06)^2\left(\frac{1}{3}\right)\right]\text{m}^2$$

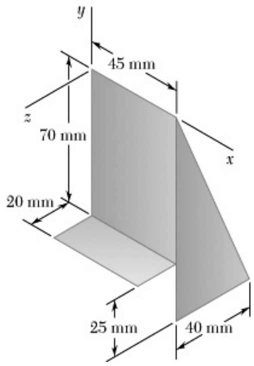
$$+ (0.032974\text{ kg})\left[(0.048)^2\left(\frac{1}{3}\right)\right]\text{m}^2 + (0.032974\text{ kg})\left[\left(\frac{1}{3}\right)(0.048)^2 + (0.06)^2\right]\text{m}^2$$

$$= \left[(0.031399 + 0.049462 + 0.025324 + 0.14403)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2$$

$$= 0.250215\times 10^{3}\text{ kg}\cdot\text{m}^2$$

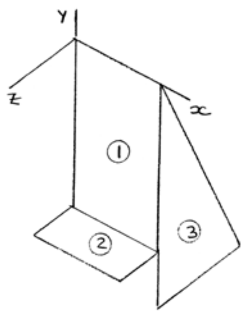
or $I_z = 0.250\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.140



A framing anchor is formed of 2-mm-thick galvanized steel. Determine the mass moment of inertia of the anchor with respect to each of the coordinate exes. (The density of galvanized steel is 7530 kg/m^3.)

SOLUTION



First compute the mass of each component

Have $$m = \rho V = \rho A t$$

Now $$m_1 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m})(0.045 \text{ m})(0.070 \text{ m}) = 0.047439 \text{ kg}$$

$$m_2 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m})(0.045 \text{ m})(0.020 \text{ m}) = 0.013554 \text{ kg}$$

$$m_3 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m}) \times \frac{1}{2}(0.04 \text{ m})(0.095 \text{ m}) = 0.028614 \text{ kg}$$

Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= (0.047439 \text{ kg})\left[\frac{1}{3}(0.07)^2\right]\text{m}^2 + (0.013554 \text{ kg})\left\{\frac{1}{12}(0.020)^2 + \left[(0.07)^2 + (0.01)^2\right]\right\}\text{m}^2$$

$$+ (0.028614 \text{ kg})\left\{\frac{1}{18}\left[(0.095)^2 + (0.04)^2\right] + \frac{1}{9}\left[(2 \times 0.095)^2 + (0.040)^2\right]\right\}\text{m}^2$$

$$= \left[(0.077484 + 0.068222 + 0.136751) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.282457 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

or $I_x = 0.2825 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.140 CONTINUED

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= (0.047439 \text{ kg})\left[\frac{1}{3}(0.045)^2\right]\text{m}^2 + (0.013554 \text{ kg})\left\{\left[(0.045)^2 + (0.02)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2$$

$$+ (0.028614 \text{ kg})\left[\frac{1}{18}(0.04)^2(0.045)^2 + \frac{1}{9}(0.04)^2\right]\text{m}^2$$

$$= \left[(0.03202 + 0.010956 + 0.065574) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.10855 \times 10^3 \text{ kg}\cdot\text{m}^2$$

$$\text{or } I_y = 0.1086 \times 10^3 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= (0.047439 \text{ kg})\left\{\left[(0.045)^2 + (0.070)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2 + (0.013554 \text{ kg})\left[(0.045)^2\left(\frac{1}{3}\right) + (0.070)^2\right]\text{m}^2$$

$$+ (0.028614 \text{ kg})\left[\frac{1}{18}(0.095)^2 + 0.045^2 + \left(\frac{2}{3}0.095\right)^2\right]\text{m}^2$$

$$= \left[(0.0109505 + 0.075564 + 0.187064) \times 10^{-3}\right]\text{kg}\cdot\text{m}^3$$

$$= 0.37213 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$\text{or } I_z = 0.372 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$