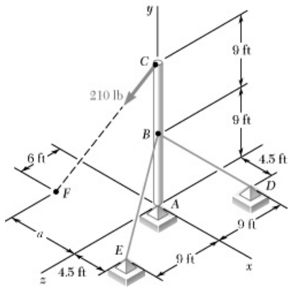
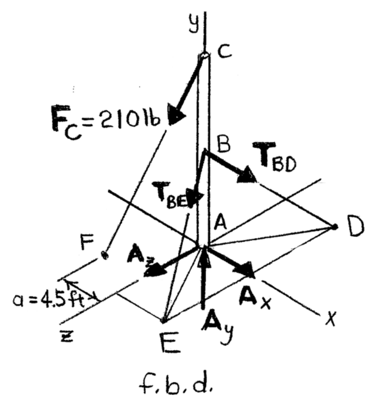


PROBLEM 4.117

Solve Problem 4.116 for $a = 4.5$ ft.

P4.116 The 18-ft pole *ABC* is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 9$ ft, determine the tension in each cable and the reaction at *A*.

SOLUTION



From f.b.d. of pole *ABC*

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where
$$\lambda_{AE} = \frac{(4.5\text{ ft})\mathbf{i} + (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2}\text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} + 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9\text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \frac{(4.5\text{ ft})\mathbf{i} - (9\text{ ft})\mathbf{j} - (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2}\text{ ft}}T_{BD}$$

$$= \left(\frac{T_{BD}}{13.5}\right)(4.5\mathbf{i} - 9\mathbf{j} - 9\mathbf{k})$$

$$\mathbf{F}_C = \lambda_{CF}(210\text{ lb}) = \frac{-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}}{\sqrt{(4.5)^2 + (18)^2 + (6)^2}}(210\text{ lb})$$

$$= \left(\frac{210\text{ lb}}{19.5}\right)(-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k})$$

$$\therefore \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 9 & 0 \\ 4.5 & -9 & -9 \end{vmatrix}\left(\frac{T_{BD}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210\text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

PROBLEM 4.117 CONTINUED

$$\frac{(-364.5 - 364.5)}{13.5\sqrt{101.25}}T_{BD} + \frac{(486 + 729)}{19.5\sqrt{101.25}}(210 \text{ lb}) = 0$$

or $\quad T_{BD} = 242.31 \text{ lb} \quad$ or $T_{BD} = 242 \text{ lb}$ ◀

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BE}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) = 0$$

where $\quad \boldsymbol{\lambda}_{AD} = \dfrac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \dfrac{1}{\sqrt{101.25}}(4.5\mathbf{i} - 9\mathbf{k}),$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE}T_{BE} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{13.5}(4.5\mathbf{i} - 9\mathbf{j} + 9\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 9 & 0 \\ 4.5 & -9 & 9 \end{vmatrix}\left(\frac{T_{BE}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210 \text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

$$\frac{(364.5 + 364.5)}{13.5\sqrt{101.25}}T_{BE} + \frac{(486 - 729)(210 \text{ lb})}{19.5\sqrt{101.25}} = 0$$

or $\quad T_{BE} = 48.462 \text{ lb}$

or $T_{BE} = 48.5 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{BD})_x + (T_{BE})_x - (F_C)_x = 0$$

$$A_x + \left(\frac{4.5}{13.5}\right)242.31 + \left(\frac{4.5}{13.5}\right)48.462 - \left(\frac{4.5}{19.5}\right)210 = 0$$

$$\therefore \quad A_x = -48.459 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - (T_{BD})_y - (T_{BE})_y - (F_C)_y = 0$$

$$A_y - \left(\frac{9}{13.5}\right)242.31 - \left(\frac{9}{13.5}\right)48.462 - \left(\frac{18}{19.5}\right)210 =$$

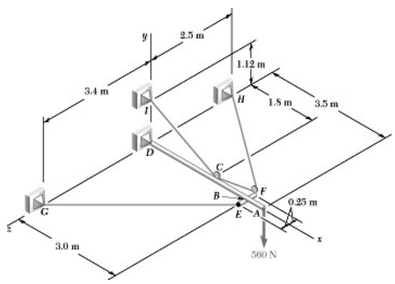
$$\therefore \quad A_y = 387.69 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - (T_{BD})_z + (T_{BE})_z + (F_C)_z = 0$$

$$A_z - \left(\frac{9}{13.5}\right)242.31 + \left(\frac{9}{13.5}\right)48.462 + \left(\frac{6}{19.5}\right)2$$

$$\therefore \quad A_z = 64.591 \text{ lb}$$

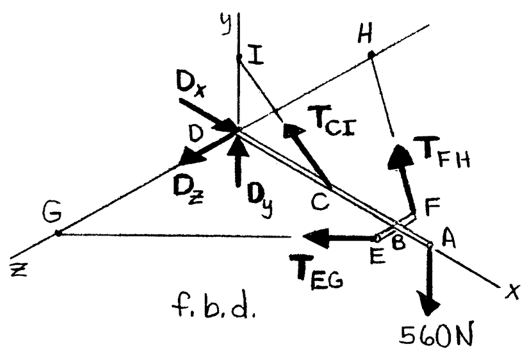
or $\mathbf{A} = -(48.5 \text{ lb})\mathbf{i} + (388 \text{ lb})\mathbf{j} + (64.6 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.118

Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CI}\right) + \mathbf{k} \cdot \left(\mathbf{r}_{A/D} \times \mathbf{F}_A\right) = 0$$

where

$$\mathbf{r}_{C/D} = (1.8 \text{ m})\mathbf{i}$$

$$\mathbf{T}_{CI} = \lambda_{CI} T_{CI} = \frac{-(1.8 \text{ m})\mathbf{i} + (1.12 \text{ m})\mathbf{j}}{\sqrt{(1.8)^2 + (1.12)^2} \text{ m}} T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)(-1.8\mathbf{i} + 1.12\mathbf{j})$$

$$\mathbf{r}_{A/D} = (3.5 \text{ m})\mathbf{i}$$

$$\mathbf{F}_A = -(560 \text{ N})\mathbf{j}$$

$$\therefore \quad \Sigma M_z = \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix} \left(\frac{T_{CI}}{2.12}\right) + \begin{vmatrix} 0 & 0 & 1 \\ 3.5 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} (560 \text{ N}) = 0$$

$$(2.016)\frac{T_{CI}}{2.12} + (-3.5)560 = 0$$

or

$$T_{CI} = T_{FH} = 2061.1 \text{ N}$$

$T_{ICFH} = 2.06$ kN ◀

PROBLEM 4.118 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j}\cdot\left(\mathbf{r}_{G/D} \times \mathbf{T}_{EG}\right) + \mathbf{j}\cdot\left(\mathbf{r}_{H/D} \times \mathbf{T}_{FH}\right) = 0$$

where $\mathbf{r}_{G/D} = (3.4 \text{ m})\mathbf{k}$

$$\mathbf{r}_{H/D} = -(2.5 \text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \frac{-(3.0 \text{ m})\mathbf{i} + (3.15 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (3.15)^2} \text{ m}} T_{EG} = \left(\frac{T_{EG}}{4.35}\right)(-3\mathbf{i} + 3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH} T_{FH} = \frac{-(3.0 \text{ m})\mathbf{i} - (2.25 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (2.25)^2} \text{ m}}(2061.1 \text{ N}) = \frac{2061.1 \text{ N}}{3.75}(-3\mathbf{i} - 2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix}\left(\frac{T_{EG}}{4.35}\right) + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix}\left(\frac{2061.1 \text{ N}}{3.75}\right) = 0$$

$$-(10.2)\frac{T_{EG}}{4.35} + (7.5)\frac{2061.1 \text{ N}}{3.75} = 0$$

or $$T_{EG} = 1758.00 \text{ N}$$

$$T_{EG} = 1.758 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad D_x - \left(T_{CI}\right)_x - \left(T_{FH}\right)_x - \left(T_{EG}\right)_x = 0$$

$$D_x - \left(\frac{1.8}{2.12}\right)(2061.1 \text{ N}) - \left(\frac{3.0}{3.75}\right)(2061.1 \text{ N}) - \left(\frac{3}{4.35}\right)(1758 \text{ N}) = 0$$

$$\therefore \quad D_x = 4611.3 \text{ N}$$

$$\Sigma F_y = 0: \quad D_y + \left(T_{CI}\right)_y - 560 \text{ N} = 0$$

$$D_y + \left(\frac{1.12}{2.12}\right)(2061.1 \text{ N}) - 560 \text{ N} = 0$$

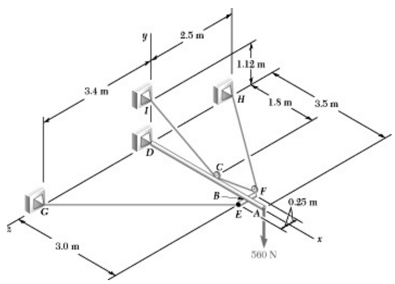
$$\therefore \quad D_y = -528.88 \text{ N}$$

$$\Sigma F_z = 0: \quad D_z + \left(T_{EG}\right)_z - \left(T_{FH}\right)_z = 0$$

$$D_z + \left(\frac{3.15}{4.35}\right)(1758 \text{ N}) - \left(\frac{2.25}{3.75}\right)(2061.1 \text{ N}) = 0$$

$$\therefore \quad D_z = -36.374 \text{ N}$$

and $\mathbf{D} = (4610 \text{ N})\mathbf{i} - (529 \text{ N})\mathbf{j} - (36.4 \text{ N})\mathbf{k} \blacktriangleleft$

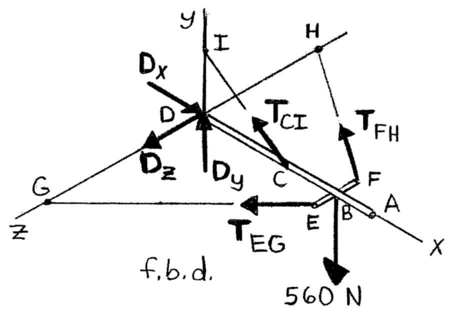


PROBLEM 4.119

Solve Problem 4.118 assuming that the 560-N load is applied at *B*.

P4.118 Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CI}\right) + \mathbf{k} \cdot \left(\mathbf{r}_{B/D} \times \mathbf{F}_B\right) = 0$$

where

$$\mathbf{r}_{C/D} = (1.8 \text{ m})\mathbf{i}$$

$$\mathbf{T}_{CI} = \lambda_{CI} T_{CI} = \frac{-(1.8 \text{ m})\mathbf{i} + (1.12 \text{ m})\mathbf{j}}{\sqrt{(1.8)^2 + (1.12)^2} \text{ m}} T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)(-1.8\mathbf{i} + 1.12\mathbf{j})$$

$$\mathbf{r}_{B/D} = (3.0 \text{ m})\mathbf{i}$$

$$\mathbf{F}_B = -(560 \text{ N})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix} \left(\frac{T_{CI}}{2.12}\right) + \begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} (560 \text{ N}) = 0$$

$$(2.016)\frac{T_{CI}}{2.12} + (-3)560 = 0$$

or

$$T_{CI} = T_{FH} = 1766.67 \text{ N}$$

$$T_{ICFH} = 1.767 \text{ kN} \blacktriangleleft$$

PROBLEM 4.119 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j}\cdot\left(\mathbf{r}_{G/D}\times\mathbf{T}_{EG}\right) + \mathbf{j}\cdot\left(\mathbf{r}_{H/D}\times\mathbf{T}_{FH}\right) = 0$$

where $\mathbf{r}_{G/D} = (3.4\text{ m})\mathbf{k}$

$$\mathbf{r}_{H/D} = -(2.5\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \lambda_{EG}T_{EG} = \frac{-(3.0\text{ m})\mathbf{i} + (3.15\text{ m})\mathbf{k}}{\sqrt{(3)^2 + (3.15)^2}\text{ m}}T_{EG} = \frac{T_{EG}}{4.35}(-3\mathbf{i} + 3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH}T_{FH} = \frac{-(3.0\text{ m})\mathbf{i} - (2.25\text{ m})\mathbf{k}}{\sqrt{(3)^2 + (2.25)^2}\text{ m}}T_{FH} = \frac{1766.67\text{ N}}{3.75}(-3\mathbf{i} - 2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix}\left(\frac{T_{EG}}{4.35}\right) + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix}\left(\frac{1766.67}{3.75}\right) = 0$$

$$-(10.2)\frac{T_{EG}}{4.35} + (7.5)\frac{1766.67}{3.75} = 0$$

or $T_{EG} = 1506.86\text{ N}$

$$T_{EG} = 1.507\text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad D_x - (T_{CI})_x - (T_{FH})_x - (T_{EG})_x = 0$$

$$D_x - \left(\frac{1.8}{2.12}\right)(1766.67\text{ N}) - \left(\frac{3}{3.75}\right)(1766.67\text{ N}) - \left(\frac{3}{4.35}\right)(1506.86\text{ N}) = 0$$

$$\therefore \quad D_x = 3952.5\text{ N}$$

$$\Sigma F_y = 0: \quad D_y + (T_{CI})_y - 560\text{ N} = 0$$

$$D_y + \left(\frac{1.12}{2.12}\right)(1766.67\text{ N}) - 560\text{ N} = 0$$

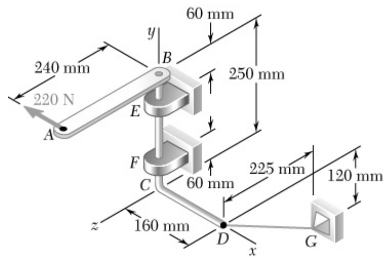
$$\therefore \quad D_y = -373.34\text{ N}$$

$$\Sigma F_z = 0: \quad D_z + (T_{EG})_z - (T_{FH})_z = 0$$

$$D_z + \left(\frac{3.15}{4.35}\right)(1506.86\text{ N}) - \left(\frac{2.25}{3.75}\right)(1766.67\text{ N}) = 0$$

$$\therefore \quad D_z = -31.172\text{ N}$$

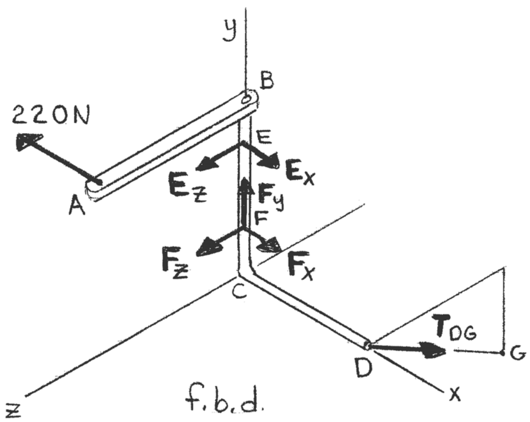
$$\mathbf{D} = (3950\text{ N})\mathbf{i} - (373\text{ N})\mathbf{j} - (31.2\text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.120

The lever *AB* is welded to the bent rod *BCD* which is supported by bearings at *E* and *F* and by cable *DG*. Knowing that the bearing at *E* does not exert any axial thrust, determine (*a*) the tension in cable *DG*, (*b*) the reactions at *E* and *F*.

SOLUTION



(*a*) From f.b.d. of assembly

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \left[\frac{-(0.12\text{ m})\mathbf{j} - (0.225\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}}\right] = \frac{T_{DG}}{0.255}\left[-(0.12)\mathbf{j} - (0.225)\mathbf{k}\right]$$

$$\Sigma M_y = 0: \quad -(220\text{ N})(0.24\text{ m}) + \left[T_{DG}\left(\frac{0.225}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N}$$

or $T_{DG} = 374\text{ N}$ ◀

(*b*) From f.b.d. of assembly

$$\Sigma M_{F(z\text{-axis})} = 0: \quad (220\text{ N})(0.19\text{ m}) - E_x(0.13\text{ m}) - \left[374\text{ N}\left(\frac{0.120}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad E_x = 104.923\text{ N}$$

$$\Sigma F_x = 0: \quad F_x + 104.923\text{ N} - 220\text{ N} = 0$$

$$\therefore \quad F_x = 115.077\text{ N}$$

$$\Sigma M_{F(x\text{-axis})} = 0: \quad E_z(0.13\text{ m}) + \left[374\text{ N}\left(\frac{0.225}{0.255}\right)\right](0.06\text{ m}) = 0$$

$$\therefore \quad E_z = -152.308\text{ N}$$

PROBLEM 4.120 CONTINUED

$$\Sigma F_z = 0: \quad F_z - 152.308 \text{ N} - (374 \text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

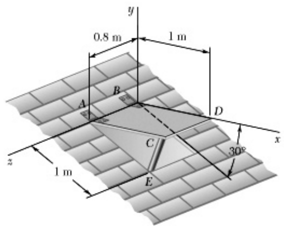
$$\therefore \quad F_z = 482.31 \text{ N}$$

$$\Sigma F_y = 0: \quad F_y - (374 \text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad F_y = 176.0 \text{ N}$$

$$\mathbf{E} = (104.9 \text{ N})\mathbf{i} - (152.3 \text{ N})\mathbf{k} \blacktriangleleft$$

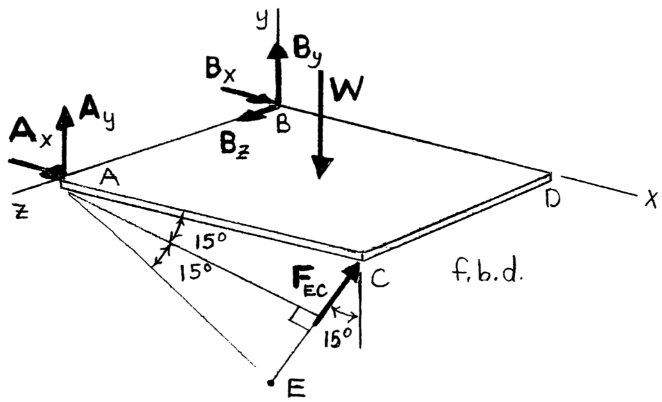
$$\mathbf{F} = (115.1 \text{ N})\mathbf{i} + (176.0 \text{ N})\mathbf{j} + (482 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.121

A 30-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

SOLUTION



First note $$W = mg = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$\mathbf{F}_{EC} = \lambda_{EC}F_{EC} = [(\sin 15^\circ)\mathbf{i} + (\cos 15^\circ)\mathbf{j}]F_{EC}$$

From f.b.d. of cover

(*a*) $$\Sigma M_z = 0: \quad (F_{EC}\cos 15^\circ)(1.0\text{ m}) - W(0.5\text{ m}) = 0$$

or $$F_{EC}\cos 15^\circ(1.0\text{ m}) - (294.3\text{ N})(0.5\text{ m}) = 0$$

$$\therefore \quad F_{EC} = 152.341\text{ N}$$ or $F_{EC} = 152.3\text{ N}$ ◀

(*b*) $$\Sigma M_x = 0: \quad W(0.4\text{ m}) - A_y(0.8\text{ m}) - (F_{EC}\cos 15^\circ)(0.8\text{ m}) = 0$$

or $$(294.3\text{ N})(0.4\text{ m}) - A_y(0.8\text{ m}) - [(152.341\text{ N})\cos 15^\circ](0.8\text{ m}) = 0$$

$$\therefore \quad A_y = 0$$

$$\Sigma M_y = 0: \quad A_x(0.8\text{ m}) + (F_{EC}\sin 15^\circ)(0.8\text{ m}) = 0$$

or $$A_x(0.8\text{ m}) + [(152.341\text{ N})\sin 15^\circ](0.8\text{ m}) = 0$$

$$\therefore \quad A_x = -39.429\text{ N}$$

$$\Sigma F_x = 0: \quad A_x + B_x + F_{EC}\sin 15^\circ = 0$$

$$-39.429\text{ N} + B_x + (152.341\text{ N})\sin 15^\circ = 0$$

$$\therefore \quad B_x = 0$$

PROBLEM 4.121 CONTINUED

$$\Sigma F_y = 0: \quad F_{EC}\cos 15° - W + B_y = 0$$

or

$$(152.341\text{ N})\cos 15° - 294.3\text{ N} + B_y = 0$$

$$\therefore \quad B_y = 147.180\text{ N}$$

or $\mathbf{A} = -(39.4\text{ N})\mathbf{i}$ ◀

$\mathbf{B} = (147.2\text{ N})\mathbf{j}$ ◀