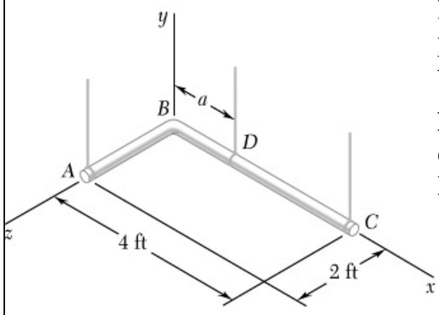
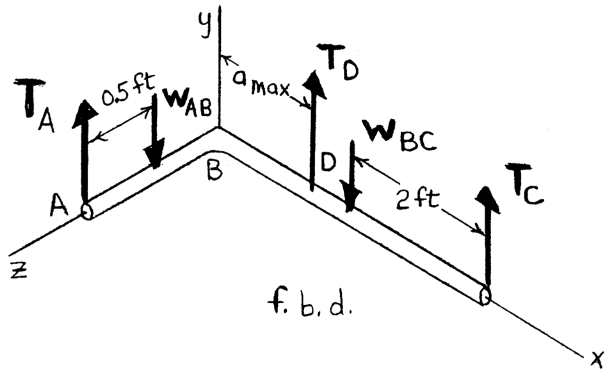
PROBLEM 4.106



For the pile assembly of Problem 4.105, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

P4.105 Two steel pipes *AB* and *BC*, each having a weight per unit length of 5 lb/ft, are welded together at *B* and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note $$W_{AB} = (5\text{ lb/ft})(2\text{ ft}) = 10\text{ lb}$$

$$W_{BC} = (5\text{ lb/ft})(4\text{ ft}) = 20\text{ lb}$$

From f.b.d. of pipe assembly

$$\Sigma F_y = 0\colon\quad T_A + T_C + T_D - 10\text{ lb} - 20\text{ lb} = 0$$

$$\therefore\quad T_A + T_C + T_D = 30\text{ lb} \tag{1}$$

$$\Sigma M_x = 0\colon\quad (10\text{ lb})(1\text{ ft}) - T_A(2\text{ ft}) = 0$$

or $$T_A = 5.00\text{ lb} \tag{2}$$

From Equations (1) and (2) $$T_C + T_D = 25\text{ lb} \tag{3}$$

$$\Sigma M_z = 0\colon\quad T_C(4\text{ ft}) + T_D(a_{\max}) - 20\text{ lb}(2\text{ ft}) = 0$$

or $$(4\text{ ft})T_C + T_D a_{\max} = 40\text{ lb}\cdot\text{ft} \tag{4}$$

PROBLEM 4.106 CONTINUED

Using Equation (3) to eliminate T_C

$$4\left(25 - T_D\right) + T_D a_{max} = 40$$

or
$$a_{max} = 4 - \frac{60}{T_D}$$

By observation, a is maximum when T_D is maximum. From Equation (3), $\left(T_D\right)_{max}$ occurs when $T_C = 0$.
Therefore, $\left(T_D\right)_{max} = 25$ lb and

$$a_{max} = 4 - \frac{60}{25}$$

$$= 1.600 \text{ ft}$$

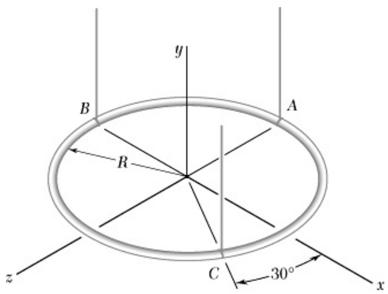
Results: (*a*) $a_{max} = 1.600$ ft ◀

(*b*) $T_A = 5.00$ lb ◀

$T_C = 0$ ◀

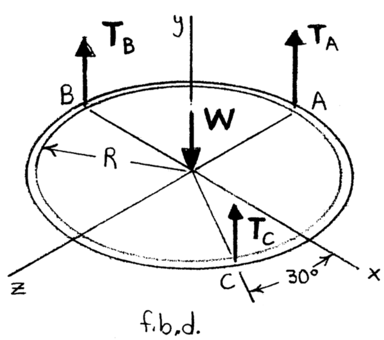
$T_D = 25.0$ lb ◀

PROBLEM 4.107



A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. Determine the tension in each wire.

SOLUTION



From f.b.d. of ring

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = W \qquad (1)$$

$$\Sigma M_x = 0: \quad T_A(R) - T_C(R\sin 30°) = 0$$

$$\therefore \quad T_A = 0.5T_C \qquad (2)$$

$$\Sigma M_z = 0: \quad T_C(R\cos 30°) - T_B(R) = 0$$

$$\therefore \quad T_B = 0.86603T_C \qquad (3)$$

Substituting T_A and T_B from Equations (2) and (3) into Equation (1)

$$0.5T_C + 0.86603T_C + T_C = W$$

$$\therefore \quad T_C = 0.42265W$$

From Equation (2)

$$T_A = 0.5(0.42265W) = 0.21132W$$

From Equation (3)

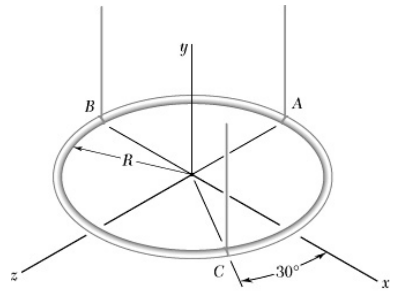
$$T_B = 0.86603(0.42265W) = 0.36603W$$

or $T_A = 0.211W$ ◀

$T_B = 0.366W$ ◀

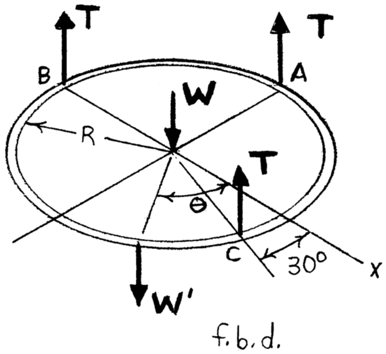
$T_C = 0.423W$ ◀

PROBLEM 4.108



A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. A small collar of weight W' is then placed on the ring and positioned so that the tensions in the three wires are equal. Determine (a) the position of the collar, (b) the value of W', (c) the tension in the wires.

SOLUTION



Let θ = angle from x-axis to small collar of weight W'

From f.b.d. of ring

$$\Sigma F_y = 0: \quad 3T - W - W' = 0 \tag{1}$$

$$\Sigma M_x = 0: \quad T(R) - T(R\sin 30°) + W'(R\sin\theta) = 0$$

or

$$W'\sin\theta = -\frac{1}{2}T \tag{2}$$

$$\Sigma M_z = 0: \quad T(R\cos 30°) - W'(R\cos\theta) - T(R) = 0$$

or

$$W'\cos\theta = -\left(1 - \frac{\sqrt{3}}{2}\right)T \tag{3}$$

Dividing Equation (2) by Equation (3)

$$\tan\theta = \left(\frac{1}{2}\right)\left[1 - \left(\frac{\sqrt{3}}{2}\right)\right]^{-1} = 3.7321$$

$$\therefore \quad \theta = 75.000° \qquad \text{and} \qquad \theta = 255.00°$$

Based on Equations (2) and (3), $\theta = 75.000°$ will give a negative value for W', which is not acceptable.

(a) $\therefore$ W' is located at $\theta = 255°$ from the x-axis or $15°$ from A towards B. ◀

(b) From Equation (1) and Equation (2)

$$W' = 3(-2W')(\sin 255°) - W$$

$$\therefore \quad W' = 0.20853W$$

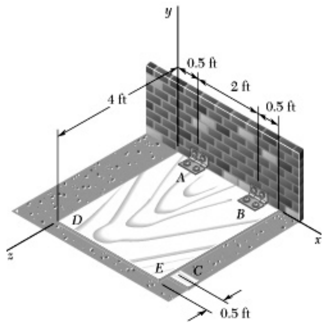
or $W' = 0.209W$ ◀

(c) From Equation (1)

$$T = -2(0.20853W)\sin 255°$$

$$= 0.40285W$$

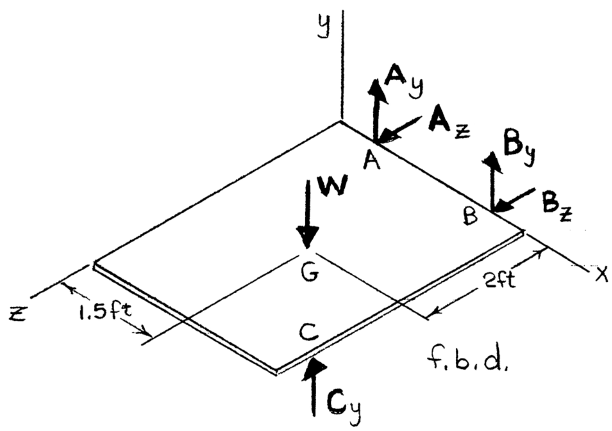
or $T = 0.403W$ ◀



PROBLEM 4.109

An opening in a floor is covered by a 3×4-ft sheet of plywood weighing 12 lb. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



From f.b.d. of plywood sheet

$$\Sigma M_x = 0: \quad (12 \text{ lb})(2 \text{ ft}) - C_y(3.5 \text{ ft}) = 0$$

$$\therefore \quad C_y = 6.8571 \text{ lb} \qquad \text{or} \qquad C_y = 6.86 \text{ lb}$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad (12 \text{ lb})(1 \text{ ft}) + (6.8571 \text{ lb})(0.5 \text{ ft}) - A_y(2 \text{ ft}) = 0$$

$$\therefore \quad A_y = 7.7143 \text{ lb} \qquad \text{or} \qquad A_y = 7.71 \text{ lb}$$

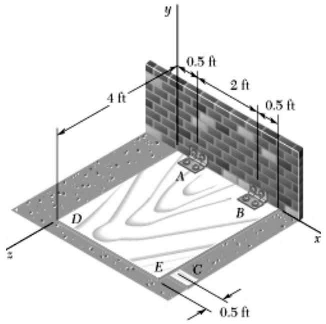
$$\Sigma M_{A(z\text{-axis})} = 0: \quad -(12 \text{ lb})(1 \text{ ft}) + B_y(2 \text{ ft}) + (6.8571 \text{ lb})(2.5 \text{ ft}) = 0$$

$$\therefore \quad B_y = 2.5714 \text{ lb} \qquad \text{or} \qquad B_y = 2.57 \text{ lb}$$

(*a*) $A_y = 7.71 \text{ lb}$ ◀

(*b*) $B_y = 2.57 \text{ lb}$ ◀

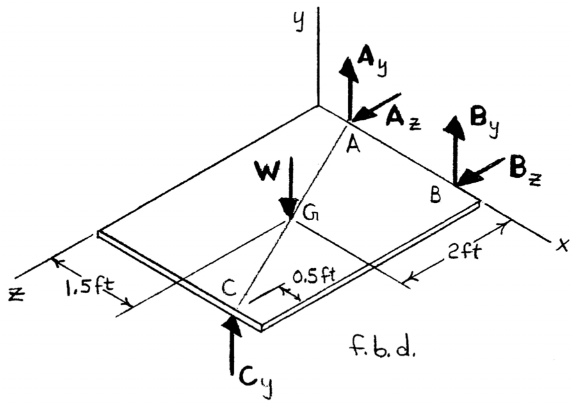
(*c*) $C_y = 6.86 \text{ lb}$ ◀



PROBLEM 4.110

Solve Problem 4.109 assuming that the small block C is moved and placed under edge DE at a point 0.5 ft from corner E.

SOLUTION



First,

$$\mathbf{r}_{B/A} = (2 \text{ ft})\mathbf{i}$$

$$\mathbf{r}_{C/A} = (2 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (1 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{k}$$

From f.b.d. of plywood sheet

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \left(B_y\mathbf{j} + B_z\mathbf{k}\right) + \mathbf{r}_{C/A} \times C_y\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(2 \text{ ft})\mathbf{i} \times B_y\mathbf{j} + (2 \text{ ft})\mathbf{i} \times B_z\mathbf{k} + [(2 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{k}] \times C_y\mathbf{j}$$

$$+ [(1 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{k}] \times (-12 \text{ lb})\mathbf{j} = 0$$

$$2B_y\mathbf{k} - 2B_z\mathbf{j} + 2C_y\mathbf{k} - 4C_y\mathbf{i} - 12\mathbf{k} + 24\mathbf{i} = 0$$

i-coeff. $\quad -4C_y + 24 = 0 \quad \therefore \; C_y = 6.00 \text{ lb}$

j-coeff. $\quad -2B_z = 0 \quad \therefore \; B_z = 0$

k-coeff. $\quad 2B_y + 2C_y - 12 = 0$

or $\quad 2B_y + 2(6) - 12 = 0 \quad \therefore \; B_y = 0$

PROBLEM 4.110 CONTINUED

$$\Sigma\mathbf{F} = 0: \quad A_y\mathbf{j} + A_z\mathbf{k} + B_y\mathbf{j} + B_z\mathbf{k} + C_y\mathbf{j} - W\mathbf{j} = 0$$

$$A_y\mathbf{j} + A_z\mathbf{k} + 0\mathbf{j} + 0\mathbf{k} + 6\mathbf{j} - 12\mathbf{j} = 0$$

j-coeff. $\quad A_y + 6 - 12 = 0 \qquad \therefore \; A_y = 6.00 \text{ lb}$

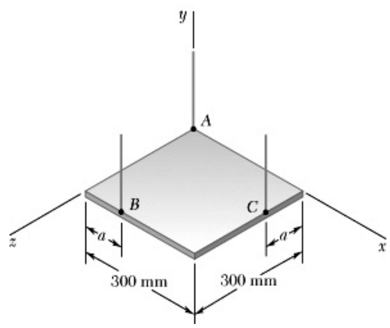
k-coeff. $\quad A_z = 0 \qquad A_z = 0$

$\therefore$ *a*) $A_y = 6.00 \text{ lb}$ ◀

b) $B_y = 0$ ◀

c) $C_y = 6.00 \text{ lb}$ ◀

PROBLEM 4.111

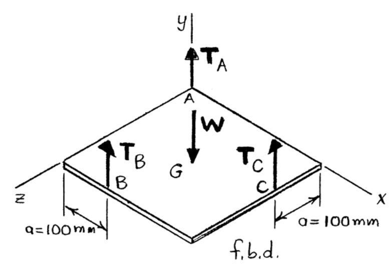


The 10-kg square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 100$ mm, (*b*) the value of *a* for which tensions in the three wires are equal.

SOLUTION

First note $\qquad W = mg = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$

(*a*)



(*a*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = 98.1\text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad W(150\text{ mm}) - T_B(300\text{ mm}) - T_C(100\text{ mm}) = 0$$

$$\therefore \quad 6T_B + 2T_C = 294.3 \qquad (2)$$

$$\Sigma M_z = 0: \quad T_B(100\text{ mm}) + T_C(300\text{ mm}) - (98.1\text{ N})(150\text{ mm}) = 0$$

$$\therefore \quad -6T_B - 18T_C = -882.9 \qquad (3)$$

Equation (2) + Equation (3)

$$-16T_C = -588.6$$

$$\therefore \quad T_C = 36.788\text{ N}$$

or $\qquad T_C = 36.8\text{ N}$ ◀

Substitution into Equation (2)

$$6T_B + 2(36.788\text{ N}) = 294.3\text{ N}$$

$$\therefore \quad T_B = 36.788\text{ N} \qquad \text{or} \qquad T_B = 36.8\text{ N}$$ ◀

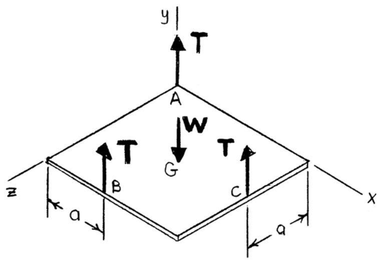
From Equation (1)

$$T_A + 36.788 + 36.788 = 98.1\text{ N}$$

$$\therefore \quad T_A = 24.525\text{ N} \qquad \text{or} \qquad T_A = 24.5\text{ N}$$ ◀

PROBLEM 4.111 CONTINUED

(*b*)



(*b*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad 3T - W = 0$$

$$\therefore \quad T = \frac{1}{3}W \qquad (1)$$

$$\Sigma M_x = 0: \quad W(150 \text{ mm}) - T(a) - T(300 \text{ mm}) = 0$$

$$\therefore \quad T = \frac{150W}{a + 300} \qquad (2)$$

Equating Equation (1) to Equation (2)

$$\frac{1}{3}W = \frac{150W}{a + 300}$$

or

$$a + 300 = 3(150)$$

or $a = 150.0$ mm ◀