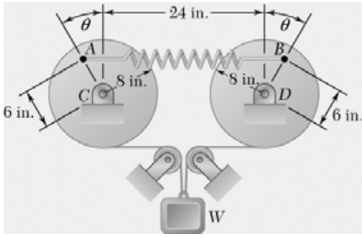
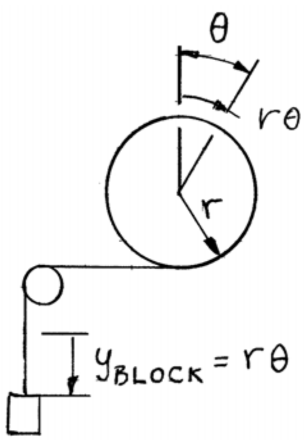


PROBLEM 10.81

Spring *AB* of constant 10 lb/in. is attached to two identical drums as shown. Knowing that the spring is unstretched when $\theta = 0$, determine (*a*) the range of values of the weight *W* of the block for which a position of equilibrium exists, (*b*) the range of values of θ for which the equilibrium is stable.

SOLUTION



Have $$V = \frac{1}{2}kx_{SP}^2 - W_{y_{\text{block}}}$$

where $x_{SP} = 2r_A \sin\theta,\qquad r_A = 6 \text{ in.}$

and $y_{\text{block}} = r\theta,\qquad r = 8 \text{ in.}$

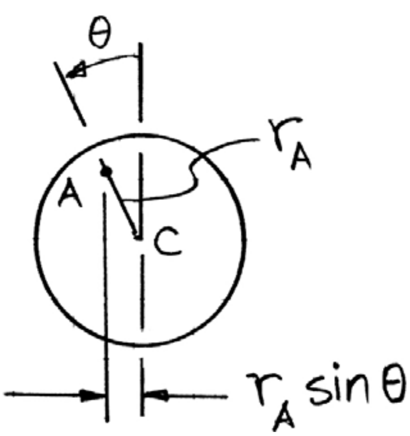
Then $$V = \frac{1}{2}k\left(2r_A \sin\theta\right)^2 - Wr\theta$$

$$= 2kr_A^2 \sin^2\theta - Wr\theta$$

and $$\frac{dV}{d\theta} = 2kr_A^2\left(2\sin\theta\cos\theta\right) - Wr$$

$$= 2kr_A^2 \sin 2\theta - Wr$$

$$\frac{d^2V}{d\theta^2} = 4kr_A^2 \cos 2\theta \qquad (1)$$



For equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad 2kr_A^2 \sin 2\theta - Wr = 0$

Substituting, $\quad 2(10 \text{ lb/in.})(6 \text{ in.})^2 \sin 2\theta - W(8 \text{ in.}) = 0$

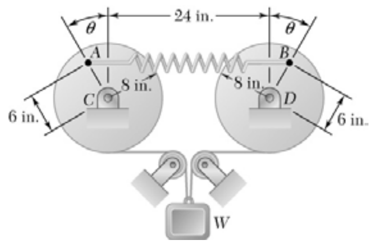
or $\quad W = 90\sin 2\theta \text{ (lb)}$

(*a*) From Equation (2), with $W \geq 0$: $\qquad 0 \leq W \leq 90 \text{ lb}$ ◀

(*b*) From Stable equilibrium $\qquad \dfrac{d^2V}{d\theta^2} > 0$

Then from Equation (1), $\qquad \cos 2\theta > 0$

or $\quad 0 \leq \theta \leq 45°$ ◀



PROBLEM 10.82

Spring AB of constant 10 lb/in. is attached to two identical drums as shown. Knowing that the spring is unstretched when $\theta = 0$ and that $W = 40\text{ lb}$, determine the values of θ less than 180° corresponding to equilibrium. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

See sketch, Problem 10.81.

Using Equation (2) of Problem 10.81, with $W = 40$ lb

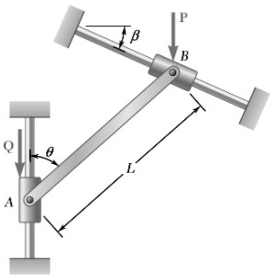
$$40 = 90\sin 2\theta \quad \text{(for equilibrium)}$$

Solving $\theta = 13.1939°$ and $\theta = 76.806°$

Using Equation (1) of Problem 10.81, we have

At $\theta = 13.1939°$: $\dfrac{d^2V}{d\theta^2} = 4kr_A^2\cos(2\times 13.1939°) > 0$ $\quad\therefore\quad \theta = 13.19°,$ Stable ◀

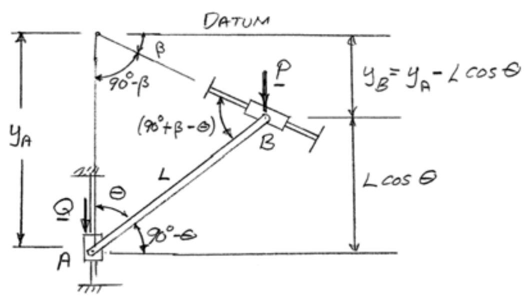
At $\theta = 76.806°$: $\dfrac{d^2V}{d\theta^2} = 4kr_A^2\cos(2\times 76.806°) < 0$ $\quad\therefore\quad \theta = 76.8°,$ Unstable ◀



PROBLEM 10.83

A slender rod AB of negligible weight is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30^{\circ}$ and $P = Q = 100\text{ lb}$, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines

$$\frac{y_A}{\sin(90^{\circ} + \beta - \theta)} = \frac{L}{\sin(90 - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos\beta}$$

or

$$y_A = L\frac{\cos(\theta - \beta)}{\cos\beta}$$

From the figure:

$$y_B = L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta$$

Potential Energy:

$$V = -Py_B - Qy_A = -P\left[L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta\right] - QL\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -PL\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + \sin\theta\right] + QL\frac{\sin(\theta - \beta)}{\cos\beta}$$

$$= L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta = 0$$

or

$$(P + Q)\sin(\theta - \beta) = P\sin\theta\cos\beta$$

$$(P + Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

PROBLEM 10.83 CONTINUED

or
$$-(P+Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$$

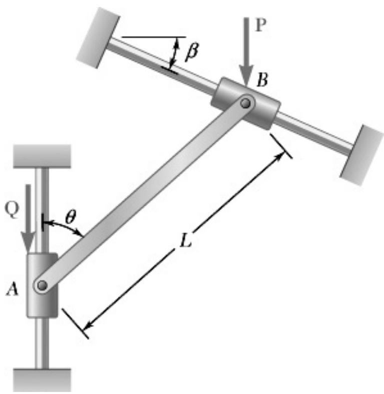
$$-\frac{P+Q}{Q}\frac{\sin\beta}{\cos\beta} + \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = \frac{P+Q}{Q}\tan\beta \qquad (2)$$

With
$$P = Q = 100 \text{ lb}, \qquad \beta = 30^\circ$$

$$\tan\theta = \frac{200 \text{ lb}}{100 \text{ lb}}\tan 30^\circ = 1.1547$$

$$\theta = 49.1^\circ \blacktriangleleft$$



PROBLEM 10.84

A slender rod AB of negligible weight is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30^\circ$, $P = 40$ lb, and $Q = 10$ lb, determine the value of the angle θ corresponding to equilibrium.

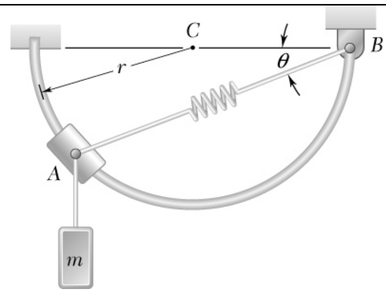
SOLUTION

Using Equation (2) of Problem 10.83, with $P = 40$ lb, $Q = 10$ lb, and $\beta = 30°$, we have

$$\tan\theta = \frac{(40\text{ lb})(10\text{ lb})}{(10\text{ lb})}\tan 30° = 2.88675$$

$$\theta = 70.89°$$

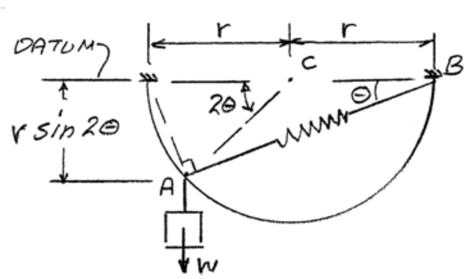
$\theta = 70.9°$ ◀



PROBLEM 10.85

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $m = 20$ kg, $r = 180$ mm, and $k = 3$ N/mm.

SOLUTION



Stretch of Spring

$$s = AB - r$$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now

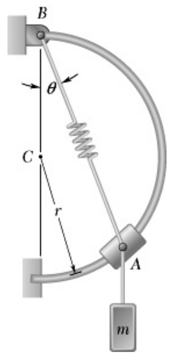
$$\frac{W}{kr} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{(3000\text{ N/m})(0.180\text{ m})} = 0.36333$$

Then

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.36333$$

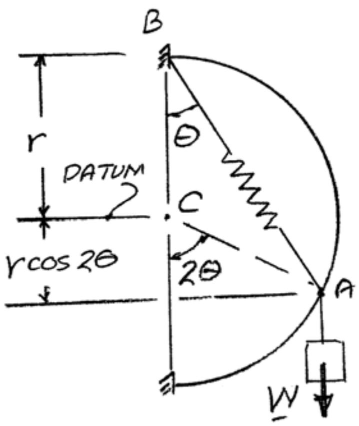
Solving numerically, $\theta = 0.9580\text{ rad} = 54.9°$ $\qquad \theta = 54.9°$ ◀

PROBLEM 10.86



Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $m = 20$ kg, $r = 180$ mm, and $k = 3$ N/mm.

SOLUTION



Stretch of spring

$$s = AB - r = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$

$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$

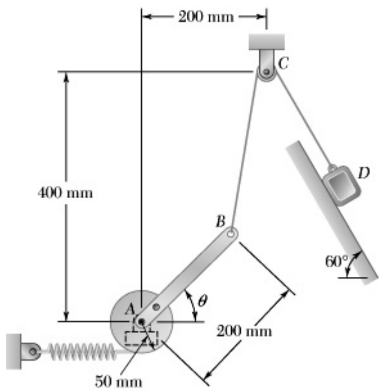
$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

or $$\frac{(2\cos\theta - 1)\sin\theta}{2\cos\theta} = \frac{W}{kr}$$

Now $$\frac{W}{kr} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{(3000\text{ N/m})(0.180\text{ m})} = 0.36333$$

Then $$\frac{2\cos\theta - 1}{2\cos\theta} = 0.36333$$

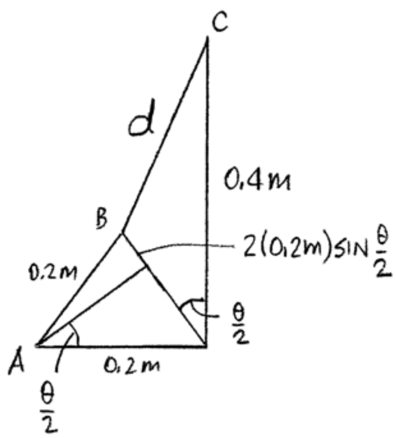
Solving $\theta = 38.2482°$ $\qquad\qquad \theta = 38.2° \blacktriangleleft$



PROBLEM 10.87

The 12-kg block D can slide freely on the inclined surface. Knowing that the constant of the spring is 480 N/m and that the spring is unstretched when $\theta = 0$, determine the value of θ corresponding to equilibrium.

SOLUTION



First note, by Law of Cosines

$$d^2 = (0.4)^2 + \left(0.4\sin\frac{\theta}{2}\right)^2 - 2(0.4)\left(0.4\sin\frac{\theta}{2}\right)\cos\frac{\theta}{2}$$

or
$$d = 0.4\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta} \text{ m}$$

Now

$$V = \frac{1}{2}kx_{SP}^2 - m_D g y_D$$

$$= \frac{1}{2}k(r_A\theta)^2 - m_D g\left[(y_D)_0 + (0.4 - d)\sin 60°\right]$$

$$= \frac{1}{2}kr_A^2\theta^2 - m_D g\left[(y_D)_0 + \left(0.4 - 0.4\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}\right)\sin 60°\right]$$

For equilibrium $\quad \dfrac{dV}{d\theta} = 0$:

$$kr_A^2\theta + 0.4m_D g\sin 60° \frac{2\left(\frac{1}{2}\right)\left(\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos\theta\right)}{2\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

or
$$kr_A^2\theta + 0.1m_D g\sin 60° \frac{\sin\theta - 2\cos\theta}{\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

PROBLEM 10.87 CONTINUED

Substituting,

$$(480\text{ N/m})(0.050\text{ m})^2\theta\sqrt{1+\sin^2\frac{\theta}{2}-\sin\theta}$$

$$+(0.1\text{ m})(12\text{ kg})(9.81\text{ m/s}^2)\frac{\sqrt{3}}{2}(\sin\theta-2\cos\theta)=0$$

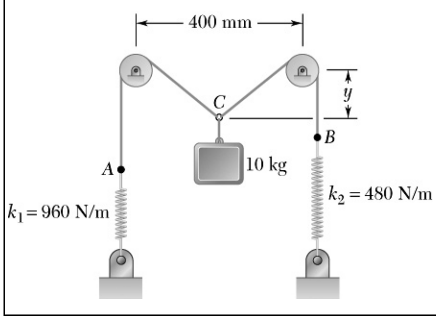
or

$$\theta\sqrt{1+\sin^2\frac{\theta}{2}-\sin\theta}+8.4957(\sin\theta-2\cos\theta)=0$$

Solving numerically, $\theta = 1.07223\text{ rad}$

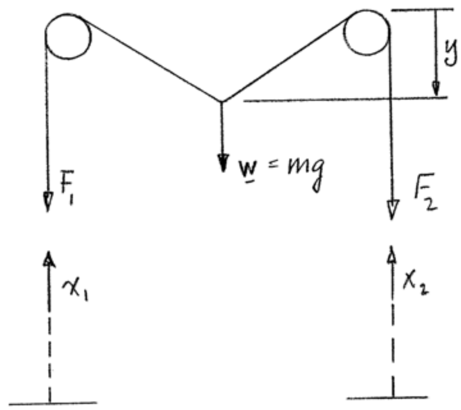
or $\theta = 61.4°$ ◀

PROBLEM 10.88



Cable *AB* is attached to two springs and passes through a ring at *C*. Knowing that the springs are unstretched when $y = 0$, determine the distance *y* corresponding to equilibrium.

SOLUTION



First note that the tension in the cable is the same throughout.

$$\therefore \; F_1 = F_2$$

or

$$k_1 x_1 = k_2 x_2$$

or

$$x_2 = \frac{k_1}{k_2} x_1$$

$$= \frac{960 \text{ N/m}}{480 \text{ N/m}} x_1$$

$$= 2x_1$$

Now, point *C* is midway between the pulleys.

$$\therefore \; y^2 = \left[0.2 + \frac{1}{2}(x_1 + x_2)^2\right] - (0.2)^2$$

$$= 0.2(x_1 + x_2) + \frac{1}{4}(x_1 + x_2)^2$$

$$= 0.2(x_1 + 2x_1) + \frac{1}{4}(x_1 + 2x_1)^2$$

$$= 0.6x_1 + \frac{9}{4}x_1^2 \; (\text{m}^2)$$

PROBLEM 10.88 CONTINUED

Now

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 - mgy$$

$$= \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(2x_1)^2 - mg\left(\frac{1}{4}\sqrt{2.4x_1 + 9x_1^2}\right)$$

$$= \frac{1}{2}(k_1 + 4k_2)x_1^2 - mg\left(\frac{1}{4}\sqrt{2.4x_1 + 9x_1^2}\right)$$

For equilibrium

$$\frac{dV}{dx_1} = 0: \quad (k_1 + 4k_2)x_1 - mg\left(\frac{2.4 + 18x_1}{2\sqrt{2.4x_1 + 9x_1^2}}\right) = 0$$

or $$(980 + 4 \times 490)\,\text{N/m} \times (x_1)(\text{m})\left(\sqrt{2.4x_1 + 9x_1^2}\right)(\text{m}) - \frac{1}{2}(10\text{ kg})(9.81\text{ m/s}^2)(1.2 + 9x_1)(\text{m}) = 0$$

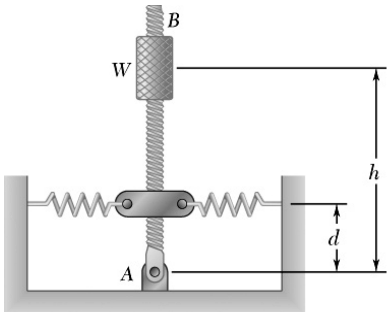
or $$288x_1\sqrt{2.4x_1 + 9x_1^2} - 5.886(1 + 7.5x_1) = 0$$

Solving, $$x_1 = 0.068151\text{ m}$$

Then $$y^2 = 0.6(0.068151) + \frac{9}{4}(0.068151)^2$$

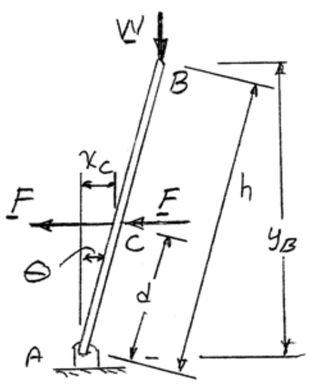
or $y = 227$ mm ◀

PROBLEM 10.89



Rod *AB* is attached to a hinge at *A* and to two springs, each of constant *k*. If $h = 50$ in., $d = 24$ in., and $W = 160$ lb, determine the range of values of *k* for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION



Have $\quad x_C = d\sin\theta \qquad y_B = h\cos\theta$

Potential Energy: $\quad V = 2\left(\frac{1}{2}kx_C^2 + Wy_B\right)$

$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then $$\frac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$$

$$= kd^2\sin 2\theta - Wh\sin\theta$$

and $$\frac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or $$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^2} = -8kd^2\cos 2\theta + Wh\cos\theta$$

PROBLEM 10.89 CONTINUED

For $\theta = 0$:
$$\frac{d^4V}{d\theta^4} = -8kd^2 + Wh$$

Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

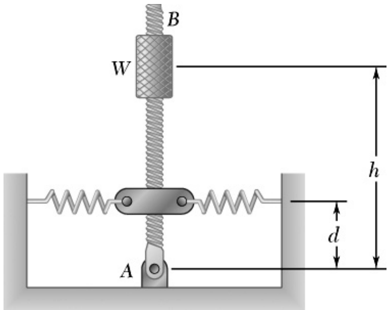
With
$$W = 160 \text{ lb},\ h = 50 \text{ in., and } d = 24 \text{ in.}$$

Equation (2) gives
$$k(24 \text{ in.})^2 > \frac{1}{2}(160 \text{ lb})(50 \text{ in.})$$

or
$$k > 6.944 \text{ lb/in.}$$

$k > 6.94$ lb/in. ◀

PROBLEM 10.90



Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 30$ in., $k = 4$ lb/in., and $W = 40$ lb, determine the smallest distance d for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.89 with

$$h = 30 \text{ in.}, k = 4 \text{ lb/in., and } W = 40 \text{ lb}$$

$$(4 \text{ lb/in.})d^2 > \frac{1}{2}(40 \text{ lb})(30 \text{ in.})$$

or

$$d^2 > 150 \text{ in}^2$$

$$d > 12.247 \text{ in.}$$

$$\text{smallest } d = 12.25 \text{ in.} \blacktriangleleft$$