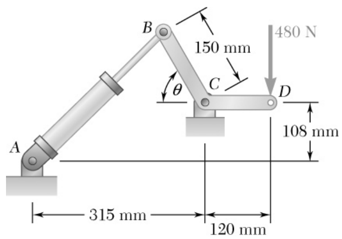
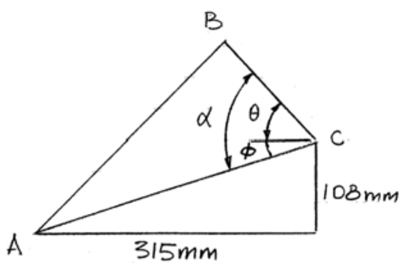
PROBLEM 10.41



The position of crank *BCD* is controlled by the hydraulic cylinder *AB*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* knowing that $\theta = 60°$.

SOLUTION



Have

$$d_{AC} = \sqrt{315^2 + 108^2} = 333 \text{ mm}$$

$$\tan\phi = \frac{108}{315}$$

or $$\phi = 18.9246°$$

Now, let $$\alpha = \theta + \phi$$

Then, by the Law of Cosines

$$d_{AB} = 333^2 + 150^2 - 2(333)(150)\cos\alpha$$

or $$d_{AB} = \sqrt{(13.3389 - 9.990\cos\alpha)} \times 10^2 \ (\text{mm})$$

and

$$\delta d_{AB} = \frac{499.5\sin\alpha}{\sqrt{(13.3389 - 9.990\cos\alpha)}}\delta\alpha \ (\text{mm})$$

With $$\delta\alpha > 0$$

Virtual Work: $$\delta U = 0\text{:} \ P\delta y_D - F_{\text{cyl}}\delta d_{AB} = 0$$

where $P = 480$ N, and

$$\delta y_D = d_{CD}\delta\alpha$$

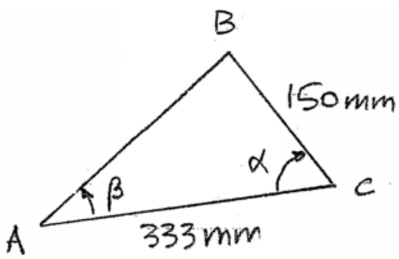
Then

$$(480 \text{ N})(120 \text{ mm})\delta\alpha - F_{\text{cyl}}\left\{\left[\frac{499.5\sin\alpha}{\sqrt{(13.3389 - 9.990\cos\alpha)}}\right] \text{mm}\right\}\delta\alpha = 0$$

or $$(499.5\sin\alpha)F_{\text{cyl}} = (57.6 \times 10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

With $$\theta = 60°\text{:} \quad \alpha = 60° + 18.9246°$$

PROBLEM 10.41 CONTINUED



have

$$\left[499.5\sin\left(60° + 18.9246°\right)\right]F_{\text{cyl}}$$

$$= \left(57.6 \times 10^3\right)\sqrt{13.3389 - 9.990\cos\left(60° + 18.9246°\right)}$$

or $$F_{\text{cyl}} = 397.08 \text{ N}$$

and

$$d_{AB} = 100\sqrt{13.3389 - 9.990\cos 78.9246°} = 337.93 \text{ mm}$$

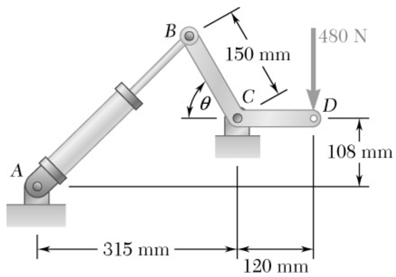
Then, by the Law of Sines

$$\frac{150}{\sin\beta} = \frac{337.93}{\sin 78.9246°}$$

or $$\beta = 25.824°$$

$$\mathbf{F}_{\text{cyl}} = 397 \text{ N} \measuredangle 44.7° \blacktriangleleft$$

PROBLEM 10.42



The position of crank *BCD* is controlled by the hydraulic cylinder *AB*. Determine the angle θ knowing that the hydraulic cylinder exerts a 420-N force on pin *B* when the crank is in the position shown.

SOLUTION

From Problem 10.41, we have

$$(499.5\sin\alpha)F_{\text{cyl}} = (57.6\times10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

Then, with $$F_{\text{cyl}} = 420 \text{ N}$$

We have

$$499.5\sin\alpha(420) = (57.6\times10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

or

$$(3.64219\sin\alpha)^2 = 13.3389 - 9.990\cos\alpha$$

or

$$13.2655(1-\cos^2\alpha) = 13.3389 - 9.990\cos\alpha$$

or

$$13.2655\cos^2\alpha - 9.990\cos\alpha + 0.0734 = 0$$

Then

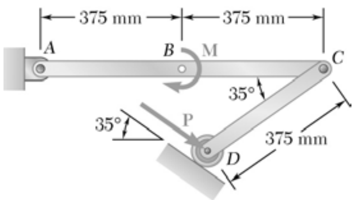
$$\cos\alpha = \frac{9.990 \pm \sqrt{(-9.990)^2 - 4(13.2655)(0.0734)}}{2(13.2655)}$$

or

$$\alpha = 41.7841° \qquad \text{and} \qquad \alpha = 89.5748°$$

Now $$\theta = \alpha - \phi \qquad \text{and} \qquad \phi = 18.9246°$$

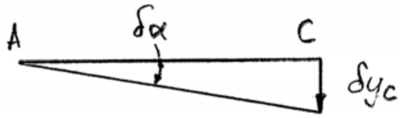
so that $$\theta = 22.9° \text{ and } \theta = 70.7° \blacktriangleleft$$



PROBLEM 10.43

For the linkage shown, determine the force **P** required for equilibrium when $M = 40 \text{ N}\cdot\text{m}$.

SOLUTION

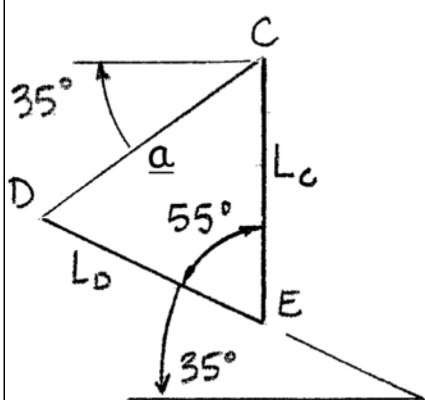


For bar *ABC*, we have

$$\delta\alpha = \frac{\delta y_c}{2a} \qquad \text{where} \qquad a = 375 \text{ mm}$$

and for bar *CD*, using the Law of Cosines

$$a^2 = L_C^2 + L_D^2 - 2L_C L_D \cos 55°$$



Then, noting that $a = \text{constant}$, we have

$$0 = 2L_C\delta L_C + 2L_D\delta L_D - 2(\delta L_C)L_D\cos 55° - 2L_C(\delta L_D)\cos 55°$$

Then, because $\delta L_C = -\delta y_C$:

$$(L_C - L_D\cos 55°)\delta y_C = (L_D - L_C\cos 55°)\delta L_D$$

For the given position of member *CD*, ΔCDE is isosceles.

$$\therefore \quad L_D = a \qquad \text{and} \qquad L_C = 2a\cos 55°$$

Then

$$(2a\cos 55° - a\cos 55°)\delta y_C = \left(a - 2a\cos^2 55°\right)\delta L_D$$

or

$$\delta L_D = \frac{\cos 55°}{1 - 2\cos^2 55°}\delta y_C$$

Now, Virtual Work: $\qquad \delta U = 0\text{:} \quad M\delta a - P\delta L_D = 0$

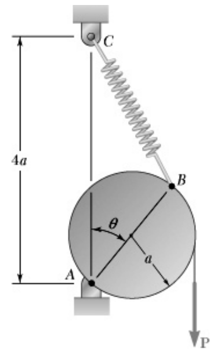
or

$$M\left(\frac{\delta y_C}{2a}\right) - P\left(\frac{\cos 55°}{1 - 2\cos^2 55°}\right)\delta y_C = 0$$

which gives $\qquad P = \dfrac{M}{2a}\dfrac{1 - 2\cos^2 55°}{\cos 55°}$

Then $\qquad P = \dfrac{40 \text{ N}\cdot\text{m}}{2(0.375 \text{ m})}\dfrac{1 - 2\cos^2 55°}{\cos 55°}$

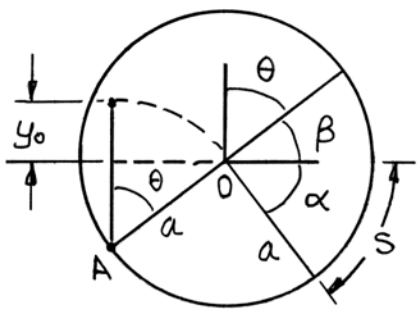
or $\mathbf{P} = 31.8 \text{ N} \searrow 35.0°$ ◀

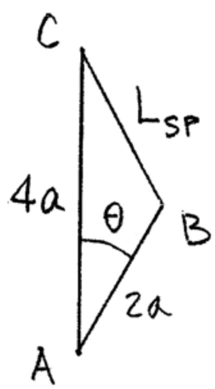


PROBLEM 10.44

A cord is wrapped around a drum of radius a that is pinned at A. The constant of the spring is $3\,\text{kN/m}$, and the spring is unstretched when $\theta = 0$. Knowing that $a = 150\,\text{mm}$ and neglecting the mass of the drum, determine the value of θ corresponding to equilibrium when a downward force **P** of magnitude 48 N is applied to the end of the cord.

SOLUTION





First note

$$\theta + \beta = 90°$$

$$\alpha + \beta = 90° \qquad \Rightarrow \qquad \alpha = \theta$$

$$\therefore \quad s = a\theta \qquad \text{Length of cord unwound for rotation } \theta$$

Now $y_0 = a(1 - \cos\theta)$, the distance O moves down for rotation θ

$$y_P = y_O + s$$

$\therefore \quad y_P = a\theta + a(1 - \cos\theta)$ is the distance P moves down for rotation θ

Then $$\delta y_P = (a + a\sin\theta)\delta\theta$$

Now, by the Law of Cosines

$$L_{SP}^2 = (4a)^2 + (2a)^2 - 2(4a)(2a)\cos\theta$$

or

$$L_{SP} = 2a\sqrt{5 - 4\cos\theta}$$

Then

$$\delta L_{SP} = 2a\frac{4\sin\theta}{2\sqrt{5 - 4\cos\theta}}\delta\theta$$

$$= \frac{4a\sin\theta}{\sqrt{5 - 4\cos\theta}}\delta\theta$$

Finally

$$F_{SP} = k\left[L_{SP} - (L_{SP})_0\right]$$

$$= k\left(2a\sqrt{5 - 4\cos\theta} - 2a\right)$$

$$= 2ka\left(\sqrt{5 - 4\cos\theta} - 1\right)$$

Thus, by Virtual Work: $$\delta U = 0: \; P\delta y_P - F_{SP}\delta L_{SP} = 0$$

PROBLEM 10.44 CONTINUED

or

$$Pa(1+\sin\theta)\delta\theta - 2ka\left(\sqrt{5-4\cos\theta} - 1\right)\left(\frac{4a\sin\theta}{\sqrt{5-4\cos\theta}}\delta\theta\right) = 0$$

or

$$\left[\frac{P}{8ka}(1+\sin\theta) - \sin\theta\right]\sqrt{5-4\cos\theta} + \sin\theta = 0$$

Substituting given values:

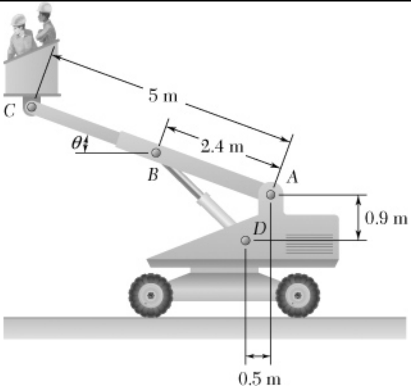
$$\left[\frac{48\text{ N}}{8(3000\text{ N/m})(0.15\text{ m})}(1+\sin\theta) - \sin\theta\right]\sqrt{5-4\cos\theta} + \sin\theta = 0$$

or

$$\left[\frac{1}{75}(1+\sin\theta) - \sin\theta\right]\sqrt{5-4\cos\theta} + \sin\theta = 0$$

Solving numerically,

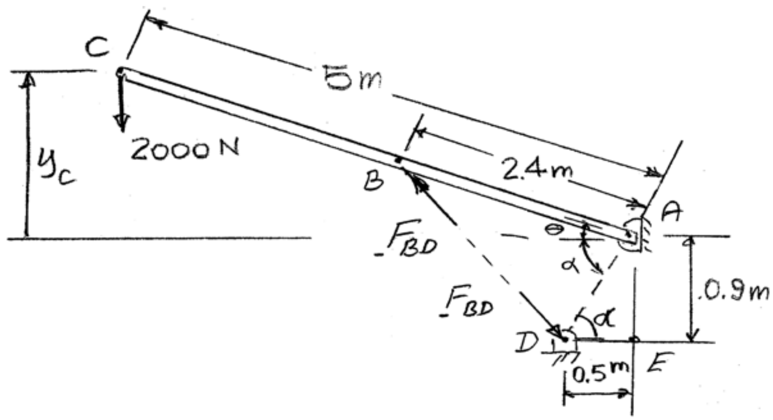
$\theta = 15.27°$ ◀



PROBLEM 10.45

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 204 kg, and their combined center of gravity is located directly above *C*. For the position when $\theta = 20^{\circ}$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



In ΔADE:

$$\tan\alpha = \frac{AE}{DE} = \frac{0.9 \text{ m}}{0.5 \text{ m}}$$

$$\alpha = 60.945^{\circ}$$

$$AD = \frac{0.9 \text{ m}}{\sin 60.945^{\circ}} = 1.0296 \text{ m}$$

From the geometry:

$$y_C = (5 \text{ m})\sin\theta, \qquad \delta y_C = (5 \text{ m})\cos\theta\delta\theta$$

Then, in triangle *BAD*: Angle $BAD = \alpha + \theta$

Law of Cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha + \theta)$$

or

$$BD^2 = (2.4 \text{ m})^2 + (1.0296 \text{ m})^2 - 2(2.4 \text{ m})(1.0296 \text{ m})\cos(\alpha + \theta)$$

$$BD^2 = 6.82 \text{ m}^2 - \left(4.942\cos(\alpha + \theta)\right) \text{ m}^2 \tag{1}$$

PROBLEM 10.45 CONTINUED

And then

$$2(BD)(\delta BD) = (4.942\sin(\alpha + \theta))\delta\theta$$

$$\delta BD = \frac{4.942\sin(\alpha + \theta)}{2(BD)}\delta\theta$$

Virtual work: $\delta U = 0$: $-P\delta y_C + F_{BD}\delta BD = 0$ Substituting $-(2000\text{ N})(5\text{ m})\cos\theta\delta\theta + F_{\text{}}$

or $$F_{BD} = \left[4047\frac{\cos\theta}{\sin(\alpha + \theta)}BD\right]\text{N/m} \qquad (2)$$

Now, with $\theta = 20°$ and $\alpha = 60.945°$

Equation (1):

$$BD^2 = 6.82 - 4.942\cos(60.945° + 20°)$$

$$BD^2 = 6.042$$

$$BD = 2.46\text{ m}$$

Equation (2)

$$F_{BD} = \left[4047\frac{\cos 20°}{\sin(60.945° + 20°)}(2.46\text{ m})\right]\text{N/m}$$

or $F_{BD} = 9473\text{ N}$ $\qquad$ $\mathbf{F}_{BD} = 9.47\text{ kN}$ ◀

PROBLEM 10.46

Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20^\circ$.

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20°$, we have

Equation (1):
$$BD^2 = 6.82 - 4.942\cos(60.945° - 20°)$$

$$BD^2 = 3.087$$

$$BD = 1.757 \text{ m}$$

Equation (2):
$$F_{BD} = 4047\frac{\cos(-20°)}{\sin(60.945° - 20°)}(1.757)$$

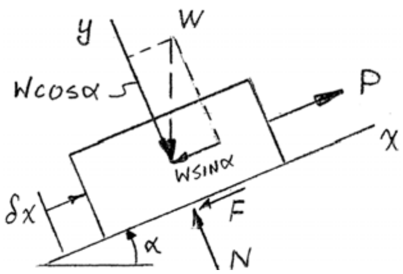
$$F_{BD} = 10196 \text{ N}$$

or $\mathbf{F}_{BD} = 10.20$ kN ◀

PROBLEM 10.47

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force **P** directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force **P** is removed.

SOLUTION



$$\text{Input work } = P\delta x$$

$$\text{Output work } = (W\sin\alpha)\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\delta x}{P\delta x} \quad \text{or} \quad \eta = \frac{W\sin\alpha}{P} \tag{1}$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \quad \text{or} \quad P = W\sin\alpha + F \tag{2}$$

$$+\nwarrow \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \quad \text{or} \quad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2): $\quad P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$

Equation (1): $\quad \eta = \dfrac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)} \quad \text{or} \quad \eta = \dfrac{1}{1 + \mu\cot\alpha}$ ◀

If block is to remain in place when $P = 0$, we know (see page 416) that $\phi_s \geq \alpha$ or, since

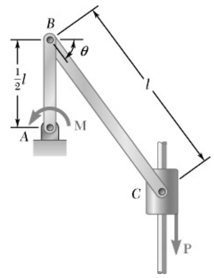
$$\mu = \tan\phi_s, \qquad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$: $\quad \mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$

Add 1 to each side: $\quad 1 + \mu\cot\alpha \geq 2$

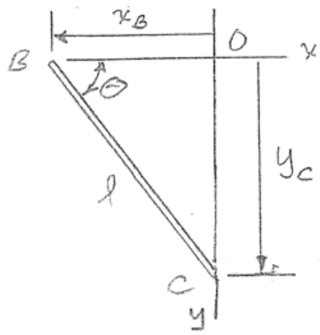
Recalling the expression for η, we find $\quad \eta \leq \dfrac{1}{2}$ ◀

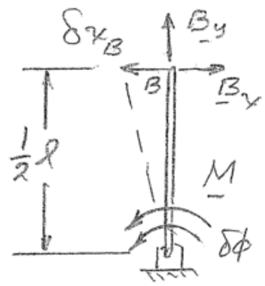
PROBLEM 10.48

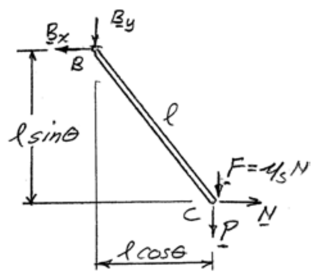


Denoting by μ_s the coefficient of static friction between collar C and the vertical rod, derive an expression for the magnitude of the largest couple **M** for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan\theta$.

SOLUTION







Member BC: Have $\quad x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

and $\quad y_C = l\sin\theta$

$$\delta y_C = l\cos\theta\delta\theta \qquad (2)$$

Member AB: Have $\quad \delta x_B = \frac{1}{2}l\delta\phi$

Substituting from Equation (1),

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or

$$\delta\phi = -2\sin\theta\delta\theta \qquad (3)$$

Free body of rod BC

For $M_{\max}$, motion of collar C impends upward

$$+\circlearrowright \Sigma M_B = 0: \quad Nl\sin\theta - (P + \mu_s N)(l\cos\theta) = 0$$

$$N\tan\theta - \mu_s N = P$$

$$N = \frac{P}{\tan\theta - \mu_s}$$

Virtual Work

$$\delta U = 0: \quad M\delta\phi + (P + \mu_s N)\delta y_C = 0$$

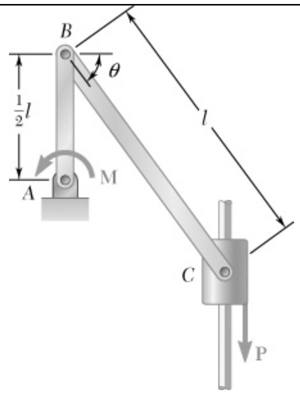
$$M(-2\sin\theta\delta\theta) + (P + \mu_s N)l\cos\theta\delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2\tan\theta}l = \frac{P + \mu_s \frac{P}{\tan\theta - \mu_s}}{2\tan\theta}l$$

or

$$M_{\max} = \frac{Pl}{2(\tan\theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan\theta$, $M = \infty$, system becomes *self-locking*



PROBLEM 10.49

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple **M** for which equilibrium is maintained in the position shown when $\theta = 35^{\text{o}}$, $l = 30$ in., and $P = 1.2$ kips.

SOLUTION

From the analysis of Problem 10.48, we have $$M_{\max} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

With $\theta = 35°$, $\quad l = 30$ in., $\quad P = 1.25$ kips

$$M_{\max} = \frac{(1200\text{ lb})(30\text{ in.})}{2(\tan 35° - 0.4)} = 59{,}958.5\text{ lb}\cdot\text{in.}$$

$$= 4996.5\text{ lb}\cdot\text{ft}$$

$$= 4.9965\text{ kip}\cdot\text{ft}$$

$$M_{\max} = 5.00\text{ kip}\cdot\text{ft} \blacktriangleleft$$

For $M_{\min}$, motion of C impends downward and F acts upward. The equations of Problem 10.48 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\min} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

Substituting, $$M_{\min} = \frac{(1200\text{ lb})(30\text{ in.})}{2(\tan 35° + 0.4)} = 16{,}360.5\text{ lb}\cdot\text{in.}$$

$$= 1363.4\text{ lb}\cdot\text{ft}$$

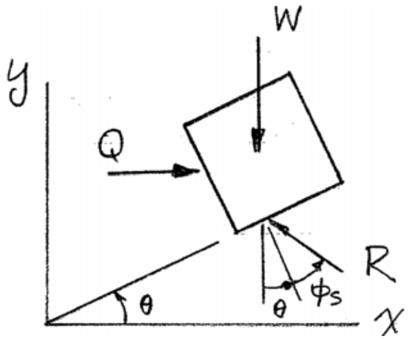
$$= 1.3634\text{ kip}\cdot\text{ft}$$

$$M_{\min} = 1.363\text{ kip}\cdot\text{ft} \blacktriangleleft$$

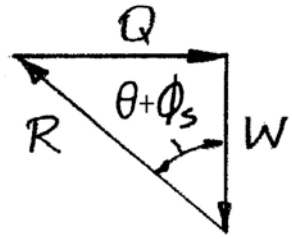
PROBLEM 10.50

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION



Recall Figure 8.9a. Draw force triangle



$$Q = W\tan(\theta + \phi_s)$$

$$y = x\tan\theta \text{ so that } \delta y = \delta x\tan\theta$$

$$\text{Input work } = Q\delta x = W\tan(\theta + \phi_s)\delta x$$

$$\text{Output work } = W\delta y = W(\delta x)\tan\theta$$

Efficiency: $\eta = \dfrac{W\tan\theta\delta x}{W\tan(\theta + \phi_s)\delta x}$; $\qquad \eta = \dfrac{\tan\theta}{\tan(\theta + \phi_s)}$ ◀

From page 432, we know the jack is self-locking if

$$\phi_s \ge \theta$$

Then $\quad \theta + \phi_s \ge 2\theta$

so that $\quad \tan(\theta + \phi_s) \ge \tan 2\theta$

From above $\quad \eta = \dfrac{\tan\theta}{\tan(\theta + \phi_s)}$

It then follows that $\quad \eta \le \dfrac{\tan\theta}{\tan 2\theta}$

But $\quad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Then $\quad \eta \le \dfrac{\tan\theta(1 - \tan^2\theta)}{2\tan\theta} = \dfrac{1 - \tan^2\theta}{2} \qquad \therefore \quad \eta \le \dfrac{1}{2}$ ◀