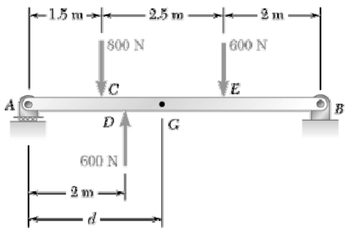
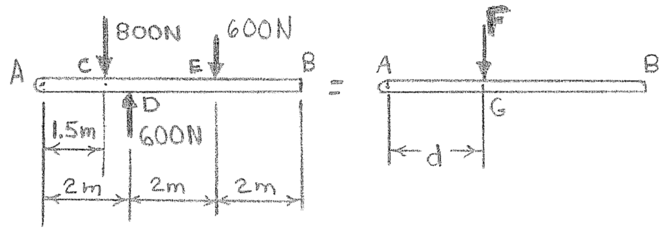


PROBLEM 3.85

A force and a couple are applied to a beam. (*a*) Replace this system with a single force **F** applied at point *G*, and determine the distance *d*. (*b*) Solve part *a* assuming that the directions of the two 600-N forces are reversed.

SOLUTION

(*a*)



Have $+\uparrow \Sigma F_y: \quad F_C + F_D + F_E = F$

$$F = -800 \text{ N} + 600 \text{ N} - 600 \text{ N}$$

$$F = -800 \text{ N}$$ or $\mathbf{F} = 800 \text{ N}\downarrow$ ◀

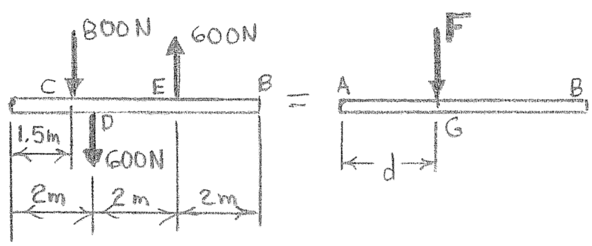
Have $+\circlearrowleft \Sigma M_G: \quad F_C(d - 1.5 \text{ m}) - F_D(2 \text{ m}) = 0$

$$(800 \text{ N})(d - 1.5 \text{ m}) - (600 \text{ N})(2 \text{ m}) = 0$$

$$d = \frac{1200 + 1200}{800}$$

$$d = 3 \text{ m}$$ or $d = 3.00 \text{ m}$ ◀

(*b*)



Changing directions of the two 600 N forces only changes sign of the couple.

$$\therefore \quad F = -800 \text{ N}$$ or $\mathbf{F} = 800 \text{ N}\downarrow$ ◀

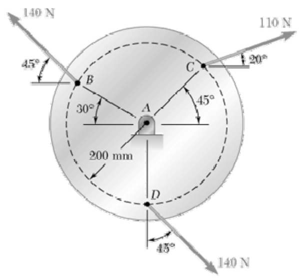
and $+\circlearrowleft \Sigma M_G: \quad F_C(d - 1.5 \text{ m}) + F_D(2 \text{ m}) = 0$

$$(800 \text{ N})(d - 1.5 \text{ m}) + (600 \text{ N})(2 \text{ m})$$

$$d = \frac{1200 - 1200}{800} = 0$$

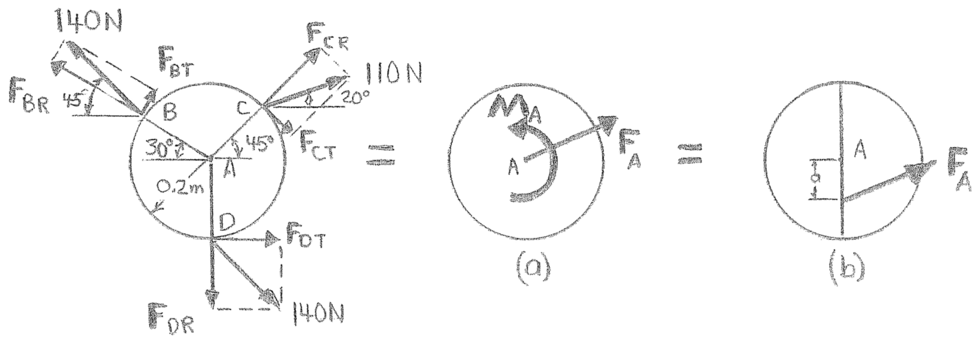
or $d = 0$ ◀

PROBLEM 3.86



Three cables attached to a disk exert on it the forces shown. (*a*) Replace the three forces with an equivalent force-couple system at *A*. (*b*) Determine the single force which is equivalent to the force-couple system obtained in part *a*, and specify its point of application on a line drawn through points *A* and *D*.

SOLUTION



(*a*) Have $\Sigma \mathbf{F}: \quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{F}_A$

Since $\mathbf{F}_B = -\mathbf{F}_D$

$$\therefore \quad \mathbf{F}_A = \mathbf{F}_C = 110 \text{ N} \measuredangle 20°$$

or $\mathbf{F}_A = 110.0 \text{ N} \measuredangle 20.0°$ ◀

Have $\Sigma M_A: \quad -F_{BT}(r) - F_{CT}(r) + F_{DT}(r) = M_A$

$$-\left[(140 \text{ N})\sin 15°\right](0.2 \text{ m}) - \left[(110 \text{ N})\sin 25°\right](0.2 \text{ m}) + \left[(140 \text{ N})\sin 45°\right](0.2 \text{ m}) = M_A$$

$$M_A = 3.2545 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 3.25 \text{ N}\cdot\text{m}$ ↺ ◀

(*b*) Have $\Sigma \mathbf{F}: \quad \mathbf{F}_A = \mathbf{F}_E$ or $\mathbf{F}_E = 110.0 \text{ N} \measuredangle 20.0°$ ◀

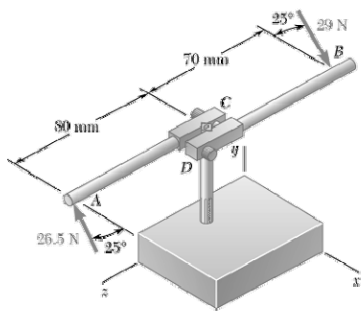
$$\Sigma M: \quad M_A = \left[F_E \cos 20°\right](a)$$

$$\therefore \quad 3.2545 \text{ N}\cdot\text{m} = \left[(110 \text{ N})\cos 20°\right](a)$$

$$a = 0.031485 \text{ m}$$

or $a = 31.5$ mm below A ◀

PROBLEM 3.87



While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

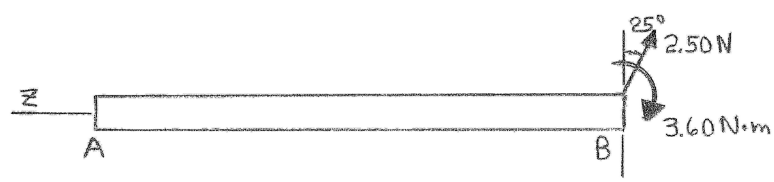
Since the forces at *A* and *B* are parallel, the force at *B* can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at *A* except with an opposite sense, resulting in a force-couple.

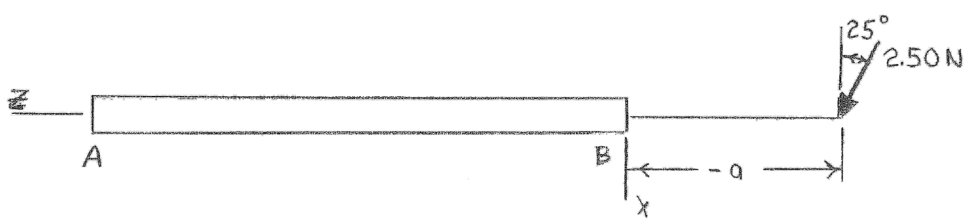
Have $F_B = 26.5 \text{ N} + 2.5 \text{ N},$ where the 26.5 N force be part of the couple. Combining the two parallel forces,

$$M_{\text{couple}} = (26.5 \text{ N})\left[(0.080 \text{ m} + 0.070 \text{ m})\cos 25°\right]$$

$$= 3.60 \text{ N}\cdot\text{m}$$

and, $\mathbf{M}_{\text{couple}} = 3.60 \text{ N}\cdot\text{m}$





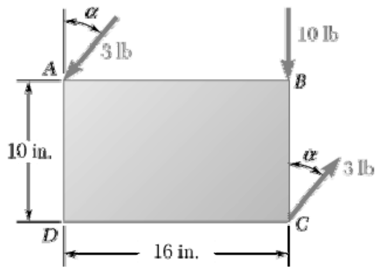
A single equivalent force will be located in the negative *z*-direction.

Based on $\qquad \Sigma M_B: \quad -3.60 \text{ N}\cdot\text{m} = \left[(2.5 \text{ N})\cos 25°\right](a)$

$$a = -1.590 \text{ m}$$

$$\mathbf{F}' = (2.5 \text{ N})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$$

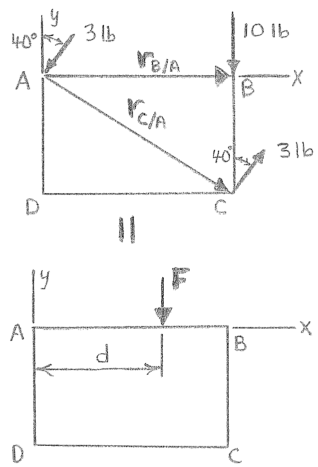
and is applied on an extension of handle *BD*
at a distance of 1.590 m to the right of *B* ◀



PROBLEM 3.88

A rectangular plate is acted upon by the force and couple shown. This system is to be replaced with a single equivalent force. (a) For $\alpha = 40°$, specify the magnitude and the line of action of the equivalent force. (b) Specify the value of α if the line of action of the equivalent force is to intersect line CD 12 in. to the right of D.

SOLUTION



(a) Have $\quad \Sigma F_x: \quad -(3 \text{ lb})\sin 40° + (3 \text{ lb})\sin 40° = F_x$

$$\therefore \quad F_x = 0$$

Have $\quad \Sigma F_y: \quad -(3 \text{ lb})\cos 40° - 10 \text{ lb} + (3 \text{ lb})\cos 40° = F_y$

$$\therefore \quad F_y = -10 \text{ lb}$$

or $F = 10.00$ lb ◀

Note: The two 3-lb forces form a couple

and $\quad \Sigma \mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{P}_C + \mathbf{r}_{B/A} \times \mathbf{P}_B = \mathbf{r}_{X/A} \times \mathbf{F}$

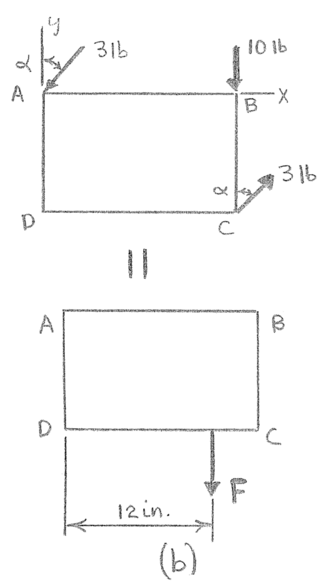
$$3\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 16 & -10 & 0 \\ \sin 40° & \cos 40° & 0 \end{vmatrix} + 160\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 10\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ d & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\mathbf{k}: \quad 3(16)\cos 40° - (-10)3\sin 40° - 160 = -10d$$

$$36.770 + 19.2836 - 160 = -10d$$

$$\therefore \quad d = 10.3946 \text{ in.}$$

or $\mathbf{F} = 10.00 \text{ lb} \downarrow$ at 10.39 in. right of A or at 5.61 in. left of B ◀



(b) From part (a), $\quad \mathbf{F} = 10.00 \text{ lb} \downarrow$

Have $\quad \Sigma \mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{P}_C + \mathbf{r}_{B/A} \times \mathbf{P}_B = (12 \text{ in.})\mathbf{i} \times \mathbf{F}$

$$3\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 16 & -10 & 0 \\ \sin\alpha & \cos\alpha & 0 \end{vmatrix} + 160\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 120\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\mathbf{k}: \quad 48\cos\alpha + 30\sin\alpha - 160 = -120$$

$$24\cos\alpha = 20 - 15\sin\alpha$$

PROBLEM 3.88 CONTINUED

Squaring both sides of the equation, and

using the identity $\cos^2\alpha = 1 - \sin^2\alpha$, results in

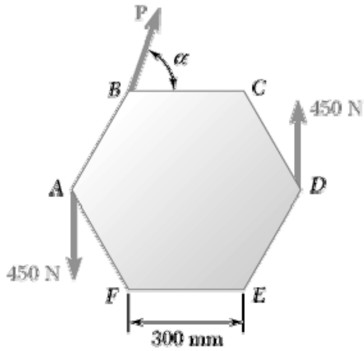
$$\sin^2\alpha - 0.74906\sin\alpha - 0.21973 = 0$$

Using quadratic formula

$$\sin\alpha = 0.97453 \qquad \sin\alpha = -0.22547$$

so that

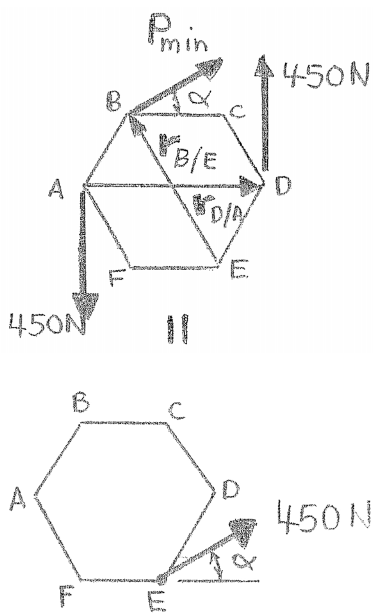
$$\alpha = 77.0° \qquad \text{and} \qquad \alpha = -13.03° \blacktriangleleft$$



PROBLEM 3.89

A hexagonal plate is acted upon by the force **P** and the couple shown. Determine the magnitude and the direction of the smallest force **P** for which this system can be replaced with a single force at E.

SOLUTION



Since the minimum value of P acting at B is realized when $P_{\min}$ is perpendicular to a line connecting B and E, $\alpha = 30°$

Then,

$$\Sigma \mathbf{M}_E: \quad \mathbf{r}_{B/E} \times \mathbf{P}_{\min} + \mathbf{r}_{D/A} \times \mathbf{P}_D = 0$$

where

$$\mathbf{r}_{B/E} = -(0.30 \text{ m})\mathbf{i} + \left[2(0.30 \text{ m})\cos 30°\right]\mathbf{j}$$

$$= -(0.30 \text{ m})\mathbf{i} + (0.51962 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{D/A} = \left[0.30 \text{ m} + 2(0.3 \text{ m})\sin 30°\right]\mathbf{i}$$

$$= (0.60 \text{ m})\mathbf{i}$$

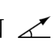
$$\mathbf{P}_D = (450 \text{ N})\mathbf{j}$$

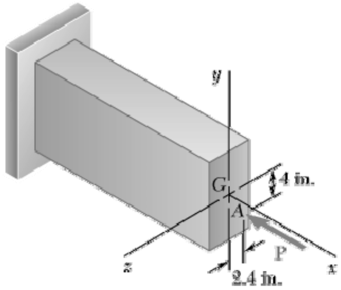
$$\mathbf{P}_{\min} = P_{\min}\left[(\cos 30°)\mathbf{i} + (\sin 30°)\mathbf{j}\right]$$

$$\therefore \quad P_{\min}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.30 & 0.51962 & 0 \\ 0.86603 & 0.50 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.60 & 0 & 0 \\ 0 & 450 & 0 \end{vmatrix} \text{N} \cdot \text{m} = 0$$

$$P_{\min}(-0.15 \text{ m} - 0.45 \text{ m})\mathbf{k} + (270 \text{ N} \cdot \text{m})\mathbf{k} = 0$$

$$\therefore \quad P_{\min} = 450 \text{ N}$$

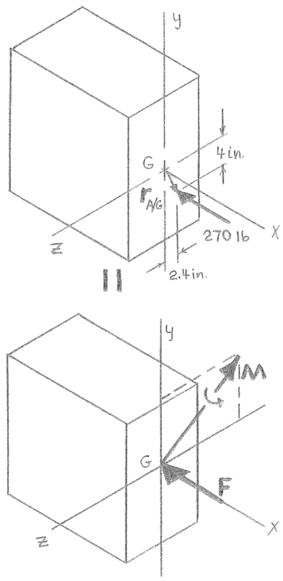
or $\mathbf{P}_{\min} = 450 \text{ N}$  $30°$ ◀



PROBLEM 3.90

An eccentric, compressive 270-lb force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at G.

SOLUTION



Have

$$\Sigma \mathbf{F}: \quad -(270 \text{ lb})\mathbf{i} = \mathbf{F}$$

$$\therefore \quad \mathbf{F} = -(270 \text{ lb})\mathbf{i} \blacktriangleleft$$

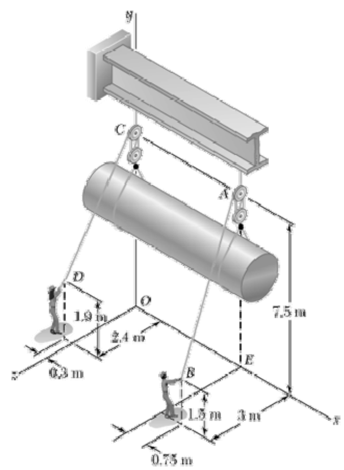
Also, have

$$\Sigma \mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

$$270\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & -2.4 \\ -1 & 0 & 0 \end{vmatrix} \text{lb}\cdot\text{in.} = \mathbf{M}$$

$$\therefore \quad \mathbf{M} = (270 \text{ lb}\cdot\text{in.})\left[(-2.4)(-1)\mathbf{j} - (-4)(-1)\mathbf{k}\right]$$

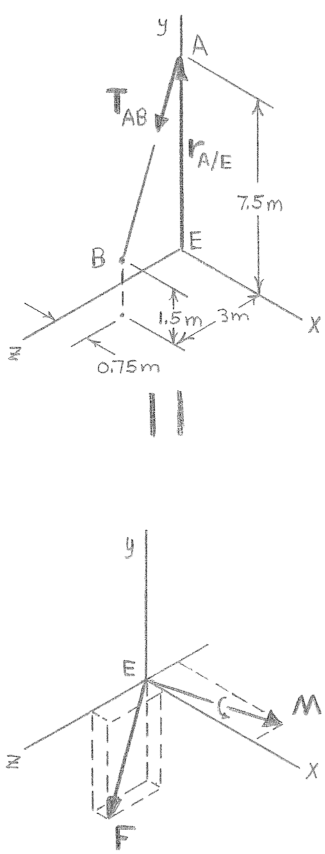
$$\text{or} \quad \mathbf{M} = (648 \text{ lb}\cdot\text{in.})\mathbf{j} - (1080 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.91

Two workers use blocks and tackles attached to the bottom of an I-beam to lift a large cylindrical tank. Knowing that the tension in rope *AB* is 324 N, replace the force exerted at *A* by rope *AB* with an equivalent force-couple system at *E*.

SOLUTION



Have

$$\Sigma\mathbf{F}: \quad \mathbf{T}_{AB} = \mathbf{F}$$

where

$$\mathbf{T}_{AB} = \lambda_{AB} T_{AB}$$

$$= \frac{(0.75 \text{ m})\mathbf{i} - (6.0 \text{ m})\mathbf{j} + (3.0 \text{ m})\mathbf{k}}{6.75 \text{ m}}(324 \text{ N})$$

$$\therefore \quad \mathbf{T}_{AB} = 36 \text{ N}(\mathbf{i} - 8\mathbf{j} + 4\mathbf{k})$$

so that

$$\mathbf{F} = (36.0 \text{ N})\mathbf{i} - (288 \text{ N})\mathbf{j} + (144.0 \text{ N})\mathbf{k} \blacktriangleleft$$

Have

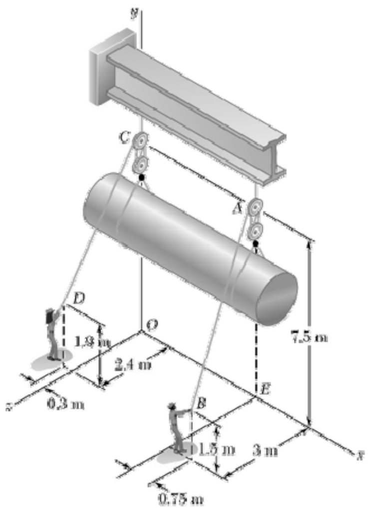
$$\Sigma\mathbf{M}_E: \quad \mathbf{r}_{A/E} \times \mathbf{T}_{AB} = \mathbf{M}$$

or

$$(7.5 \text{ m})(36 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & -8 & 4 \end{vmatrix} = \mathbf{M}$$

$$\therefore \quad \mathbf{M} = (270 \text{ N}\cdot\text{m})(4\mathbf{i} - \mathbf{k})$$

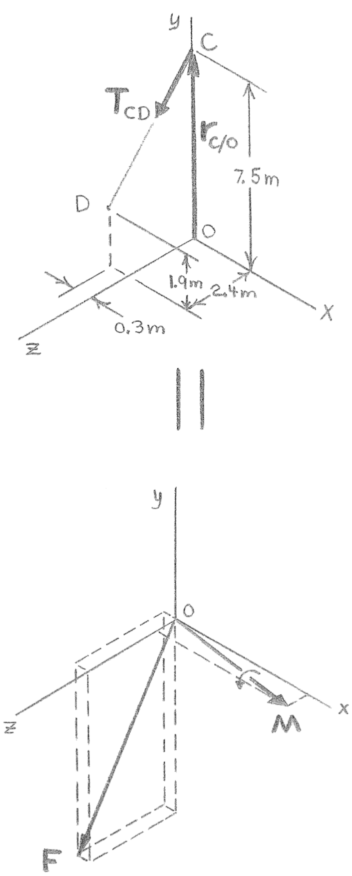
$$\text{or } \mathbf{M} = (1080 \text{ N}\cdot\text{m})\mathbf{i} - (270 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.92

Two workers use blocks and tackles attached to the bottom of an I-beam to lift a large cylindrical tank. Knowing that the tension in rope *CD* is 366 N, replace the force exerted at *C* by rope *CD* with an equivalent force-couple system at *O*.

SOLUTION



Have
$$\Sigma \mathbf{F}: \quad \mathbf{T}_{CD} = \mathbf{F}$$

where

$$\mathbf{T}_{CD} = \lambda_{CD} T_{CD}$$

$$= \frac{-(0.3\text{ m})\mathbf{i} - (5.6\text{ m})\mathbf{j} + (2.4\text{ m})\mathbf{k}}{6.1\text{ m}}(366\text{ N})$$

$$\therefore \quad \mathbf{T}_{CD} = (6.0\text{ N})(-3\mathbf{i} - 56\mathbf{j} + 24\mathbf{k})$$

so that
$$\mathbf{F} = -(18.00\text{ N})\mathbf{i} - (336\text{ N})\mathbf{j} + (144.0\text{ N})\mathbf{k} \blacktriangleleft$$

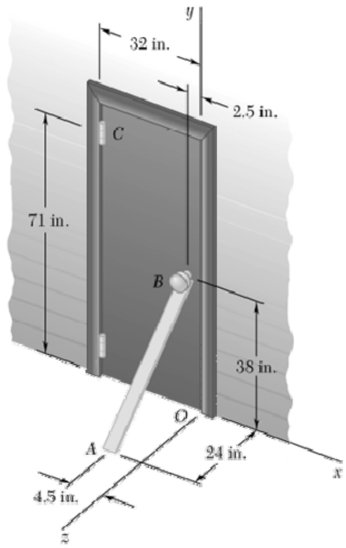
Have

$$\Sigma \mathbf{M}_O: \quad \mathbf{r}_{C/O} \times \mathbf{T}_{CD} = \mathbf{M}$$

or
$$(7.5\text{ m})(6\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ -3 & -56 & 24 \end{vmatrix} = \mathbf{M}$$

$$\therefore \quad \mathbf{M} = (45\text{ N}\cdot\text{m})(24\mathbf{i} + 3\mathbf{k})$$

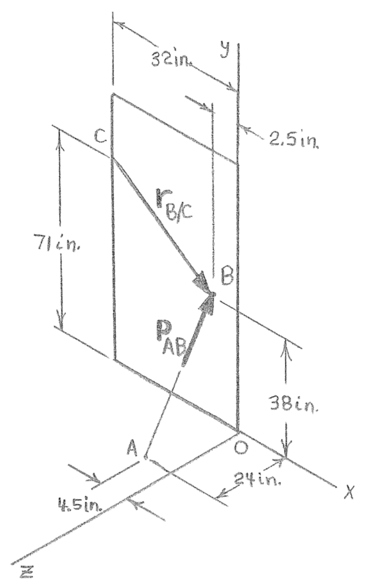
$$\text{or} \quad \mathbf{M} = (1080\text{ N}\cdot\text{m})\mathbf{i} + (135.0\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.93

To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 45-lb force directed along line AB. Replace that force with an equivalent force-couple system at C.

SOLUTION



Have

$$\Sigma\mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \lambda_{AB}P_{AB}$$

$$= \frac{(2.0 \text{ in.})\mathbf{i} + (38 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}}{44.989 \text{ in.}}(45 \text{ lb})$$

$$\text{or } \mathbf{F}_C = (2.00 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j} - (24.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Have

$$\Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$$

$$\mathbf{M}_C = 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 29.5 & -33 & 0 \\ 1 & 19 & -12 \end{vmatrix} \text{ lb}\cdot\text{in.}$$

$$= (2 \text{ lb}\cdot\text{in.})\{(-33)(-12)\mathbf{i} - (29.5)(-12)\mathbf{j}$$

$$+ [(29.5)(19) - (-33)(1)]\mathbf{k}\}$$

$$\text{or } \mathbf{M}_C = (792 \text{ lb}\cdot\text{in.})\mathbf{i} + (708 \text{ lb}\cdot\text{in.})\mathbf{j} + (1187 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$