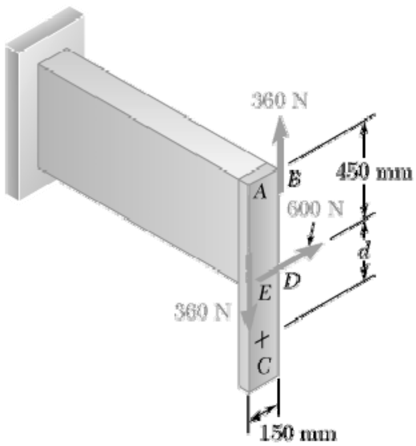
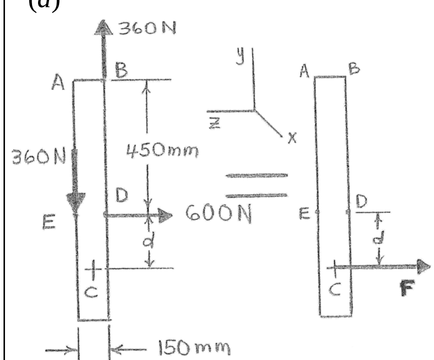


PROBLEM 3.144

A force and a couple are applied as shown to the end of a cantilever beam. (*a*) Replace this system with a single force **F** applied at point *C*, and determine the distance *d* from *C* to a line drawn through points *D* and *E*. (*b*) Solve part *a* if the directions of the two 360-N forces are reversed.

SOLUTION

(*a*)



(*a*) Have $\Sigma \mathbf{F}: \quad \mathbf{F} = (360 \text{ N})\mathbf{j} - (360 \text{ N})\mathbf{j} - (600 \text{ N})\mathbf{k}$

or $\mathbf{F} = -(600 \text{ N})\mathbf{k}$ ◀

and $\Sigma M_D: \quad (360 \text{ N})(0.15 \text{ m}) = (600 \text{ N})(d)$

$\therefore \quad d = 0.09 \text{ m}$

or $d = 90.0$ mm below *ED* ◀

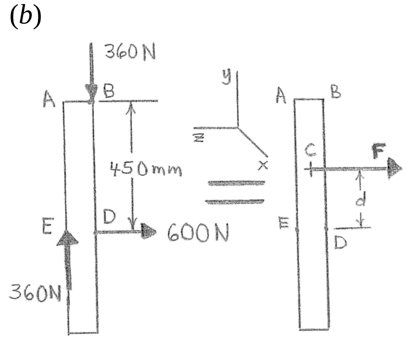
(*b*) Have from part *a* $\mathbf{F} = -(600 \text{ N})\mathbf{k}$ ◀

and $\Sigma M_D: \quad -(360 \text{ N})(0.15 \text{ m}) = -(600 \text{ N})(d)$

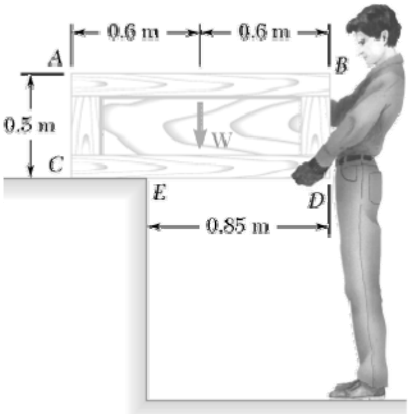
$\therefore \quad d = 0.09 \text{ m}$

or $d = 90.0$ mm above *ED* ◀

(*b*)

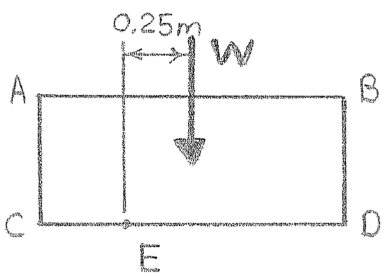


PROBLEM 3.145



A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *B* which creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

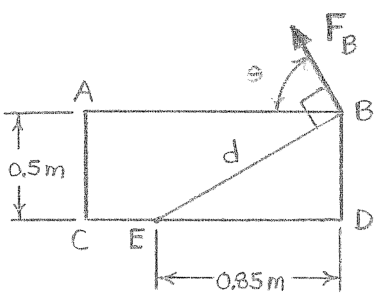


(*a*) By definition $W = mg = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

Have $\Sigma M_E: \quad M_E = (784.8\text{ N})(0.25\text{ m})$

$$\therefore \quad \mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) For the force at *B* to be the smallest, resulting in a moment $(\mathbf{M}_E)$ about *E*, the line of action of force $\mathbf{F}_B$ must be perpendicular to the line connecting *E* to *B*. The sense of $\mathbf{F}_B$ must be such that the force produces a counterclockwise moment about *E*.



Note: $d = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2} = 0.98615\text{ m}$

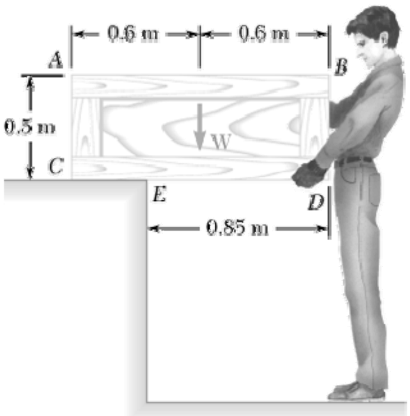
Have $\Sigma M_E: \quad 196.2\text{ N}\cdot\text{m} = F_B(0.98615\text{ m})$

$$\therefore \quad F_B = 198.954\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{0.85\text{ m}}{0.5\text{ m}}\right) = 59.534°$

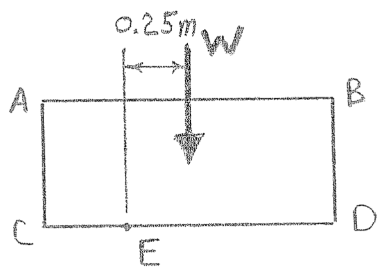
$$\text{or } \mathbf{F}_B = 199.0\text{ N} \nwarrow 59.5° \blacktriangleleft$$

PROBLEM 3.146



A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *A* which creates a moment of equal magnitude and opposite sense about *E*, (*c*) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force which creates a moment of equal magnitude and opposite sense about *E*.

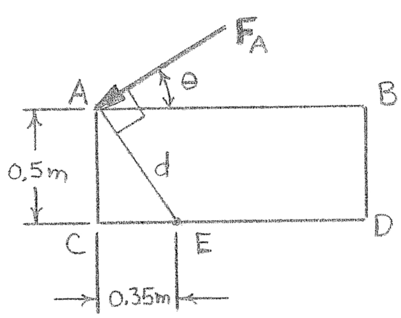
SOLUTION



(*a*) By definition $\quad W = mg = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

Have $\quad \Sigma M_E: \quad M_E = (784.8\text{ N})(0.25\text{ m})$

$$\therefore \quad \mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \;\; ◀$$



(*b*) For the force at *A* to be the smallest, resulting in a moment about *E*, the line of action of force $\mathbf{F}_A$ must be perpendicular to the line connecting *E* to *A*. The sense of $\mathbf{F}_A$ must be such that the force produces a counterclockwise moment about *E*.

Note: $\quad d = \sqrt{(0.35\text{ m})^2 + (0.5\text{ m})^2} = 0.61033\text{ m}$

Have $\quad \Sigma M_E: \quad 196.2\text{ N}\cdot\text{m} = F_A(0.61033\text{ m})$

$$\therefore \quad F_A = 321.47\text{ N}$$

and $\quad \theta = \tan^{-1}\left(\frac{0.35\text{ m}}{0.5\text{ m}}\right) = 34.992°$

$$\text{or } \mathbf{F}_A = 321\text{ N} \measuredangle\ 35.0° \;\; ◀$$

(*c*) The smallest force acting on the bottom of the crate resulting in a moment about *E* will be located at the point on the bottom of the crate farthest from *E* and acting perpendicular to line *CED*. The sense of the force will be such as to produce a counterclockwise moment about *E*. A force acting vertically upward at *D* satisfies these conditions.

PROBLEM 3.146 CONTINUED

Have $$\Sigma\mathbf{M}_E: \quad \mathbf{M}_E = \mathbf{r}_{D/E} \times \mathbf{F}_D$$

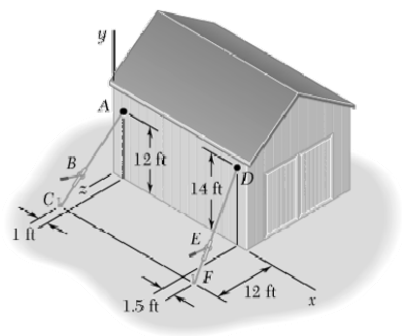
$$(196.2\text{ N}\cdot\text{m})\mathbf{k} = (0.85\text{ m})\mathbf{i} \times (F_D)\mathbf{j}$$

$$(196.2\text{ N}\cdot\text{m})\mathbf{k} = (0.85F_D)\mathbf{k}$$

$$\therefore \quad F_D = 230.82\text{ N}$$

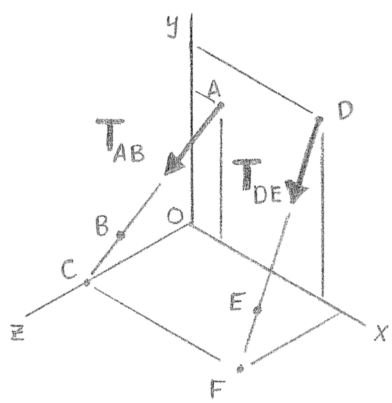
or $\mathbf{F}_D = 231\text{ N} \uparrow$ ◀

PROBLEM 3.147



A farmer uses cables and winch pullers B and E to plumb one side of a small barn. Knowing that the sum of the moments about the x axis of the forces exerted by the cables on the barn at points A and D is equal to $4728\text{ lb}\cdot\text{ft}$, determine the magnitude of $\mathbf{T}_{DE}$ when $\mathbf{T}_{AB} = 255$ lb.

SOLUTION



The moment about the x-axis due to the two cable forces can be found using the z-components of each force acting at their intersection with the xy-plane (A and D). The x-components of the forces are parallel to the x-axis, and the y-components of the forces intersect the x-axis. Therefore, neither the x or y components produce a moment about the x-axis.

Have $$\Sigma M_x: \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$$

where $$(T_{AB})_z = \mathbf{k}\cdot\mathbf{T}_{AB} = \mathbf{k}\cdot(T_{AB}\boldsymbol{\lambda}_{AB})$$

$$= \mathbf{k}\cdot\left[255\text{ lb}\left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17}\right)\right] = 180\text{ lb}$$

$$(T_{DE})_z = \mathbf{k}\cdot\mathbf{T}_{DE} = \mathbf{k}\cdot(T_{DE}\boldsymbol{\lambda}_{DE})$$

$$= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5}\right)\right] = 0.64865T_{DE}$$

$$y_A = 12\text{ ft}$$

$$y_D = 14\text{ ft}$$

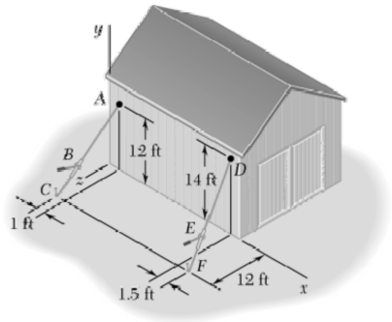
$$M_x = 4728\text{ lb}\cdot\text{ft}$$

$$\therefore \quad (180\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\text{ lb}\cdot\text{ft}$$

and $$T_{DE} = 282.79\text{ lb}$$

or $T_{DE} = 283$ lb ◀

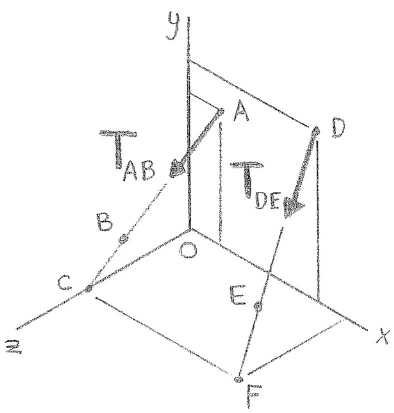
PROBLEM 3.148



Solve Problem 3.147 when the tension in cable AB is 306 lb.

Problem 3.147: A farmer uses cables and winch pullers B and E to plumb one side of a small barn. Knowing that the sum of the moments about the x axis of the forces exerted by the cables on the barn at points A and D is equal to $4728\ \text{lb}\cdot\text{ft}$, determine the magnitude of $\mathbf{T}_{DE}$ when $\mathbf{T}_{AB} = 255$ lb.

SOLUTION



The moment about the x-axis due to the two cable forces can be found using the z components of each force acting at the intersection with the xy plane (A and D). The x components of the forces are parallel to the x axis, and the y components of the forces intersect the x axis. Therefore, neither the x or y components produce a moment about the x axis.

Have $$\Sigma M_x: \quad (T_{AB})_z (y_A) + (T_{DE})_z (y_D) = M_x$$

where $$(T_{AB})_z = \mathbf{k}\cdot\mathbf{T}_{AB} = \mathbf{k}\cdot(T_{AB}\boldsymbol{\lambda}_{AB})$$

$$= \mathbf{k}\cdot\left[306\text{ lb}\left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17}\right)\right] = 216\text{ lb}$$

$$(T_{DE})_z = \mathbf{k}\cdot\mathbf{T}_{DE} = \mathbf{k}\cdot(T_{DE}\boldsymbol{\lambda}_{DE})$$

$$= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5}\right)\right] = 0.64865T_{DE}$$

$$y_A = 12\text{ ft}$$

$$y_D = 14\text{ ft}$$

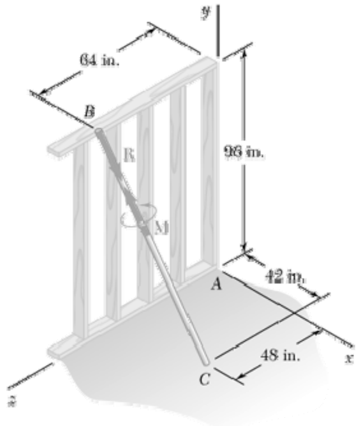
$$M_x = 4728\ \text{lb}\cdot\text{ft}$$

$$\therefore \quad (216\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\ \text{lb}\cdot\text{ft}$$

and $$T_{DE} = 235.21\text{ lb}$$

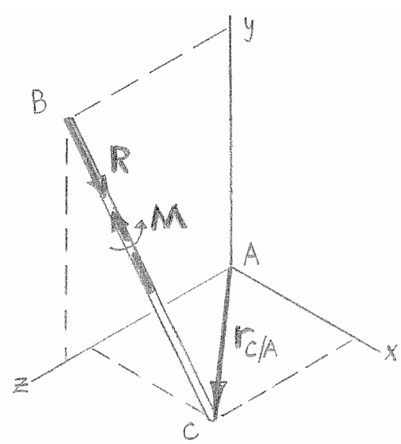
or $T_{DE} = 235$ lb ◀

PROBLEM 3.149



As an adjustable brace BC is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at A knowing that $R = 21.2$ lb and $M = 13.25$ lb·ft.

SOLUTION



Have $\Sigma\mathbf{F}$: $\mathbf{R} = \mathbf{R}_A = \mathbf{R}\lambda_{BC}$

where $$\lambda_{BC} = \frac{(42\text{ in.})\mathbf{i} - (96\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}}{106\text{ in.}}$$

$$\therefore\quad \mathbf{R}_A = \frac{21.2\text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or $\mathbf{R}_A = (8.40\text{ lb})\mathbf{i} - (19.20\text{ lb})\mathbf{j} - (3.20\text{ lb})\mathbf{k}$ ◀

Have $\Sigma\mathbf{M}_A$: $\mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where $$\mathbf{r}_{C/A} = (42\text{ in.})\mathbf{i} + (48\text{ in.})\mathbf{k} = \frac{1}{12}(42\mathbf{i} + 48\mathbf{k})\text{ ft}$$

$$= (3.5\text{ ft})\mathbf{i} + (4.0\text{ ft})\mathbf{k}$$

$$\mathbf{R} = (8.40\text{ lb})\mathbf{i} - (19.20\text{ lb})\mathbf{j} - (3.20\text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\lambda_{BC}M$$

$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25\text{ lb}\cdot\text{ft})$$

$$= -(5.25\text{ lb}\cdot\text{ft})\mathbf{i} + (12\text{ lb}\cdot\text{ft})\mathbf{j} + (2\text{ lb}\cdot\text{ft})\mathbf{k}$$

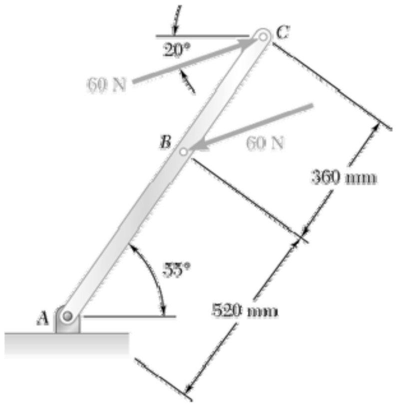
PROBLEM 3.149 CONTINUED

Then
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb}\cdot\text{ft} + \left(-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k}\right)\text{lb}\cdot\text{ft} = \mathbf{M}_A$$

$$\therefore \quad \mathbf{M}_A = \left(71.55 \text{ lb}\cdot\text{ft}\right)\mathbf{i} + \left(56.80 \text{ lb}\cdot\text{ft}\right)\mathbf{j} - \left(65.20 \text{ lb}\cdot\text{ft}\right)\mathbf{k}$$

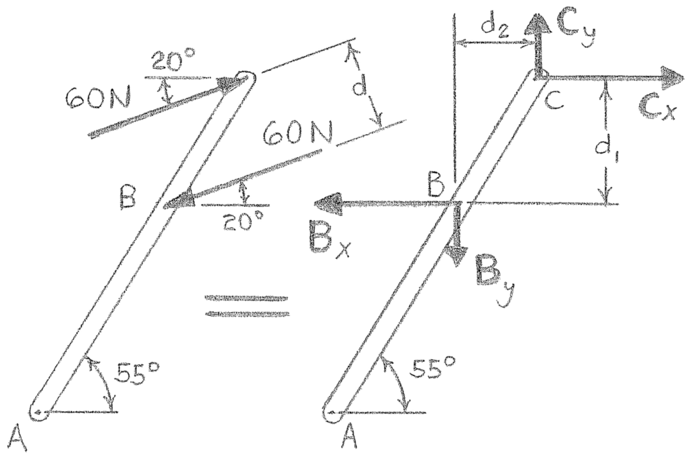
$$\text{or } \mathbf{M}_A = \left(71.6 \text{ lb}\cdot\text{ft}\right)\mathbf{i} + \left(56.8 \text{ lb}\cdot\text{ft}\right)\mathbf{j} - \left(65.2 \text{ lb}\cdot\text{ft}\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.150



Two parallel 60-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about point *A*.

SOLUTION



(*a*) Have $$\Sigma \mathbf{M}_B: \quad -d_1 C_x + d_2 C_y = \mathbf{M}$$

where $$d_1 = (0.360 \text{ m})\sin 55^\circ = 0.29489 \text{ m}$$

$$d_2 = (0.360 \text{ m})\cos 55^\circ = 0.20649 \text{ m}$$

$$C_x = (60 \text{ N})\cos 20^\circ = 56.382 \text{ N}$$

$$C_y = (60 \text{ N})\sin 20^\circ = 20.521 \text{ N}$$

$$\therefore \quad \mathbf{M} = -(0.29489 \text{ m})(56.382 \text{ N})\mathbf{k} + (0.20649 \text{ m})(20.521 \text{ N})\mathbf{k} = -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \curvearrowright$ ◀

(*b*) Have $$\mathbf{M} = Fd(-\mathbf{k}) = 60 \text{ N}\left[(0.360 \text{ m})\sin(55^\circ - 20^\circ)\right](-\mathbf{k})$$

$$= -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \curvearrowright$ ◀

PROBLEM 3.150 CONTINUED

(*c*) Have $\Sigma \mathbf{M}_A: \quad \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

$$\therefore \quad M = (0.520 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ -\cos 20^\circ & -\sin 20^\circ & 0 \end{vmatrix} + (0.880 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ \cos 20^\circ & \sin 20^\circ & 0 \end{vmatrix}$$

$$= (17.8956 \text{ N}\cdot\text{m} - 30.285 \text{ N}\cdot\text{m})\mathbf{k} = -(12.3892 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m}$ ◀