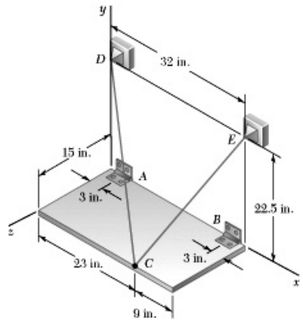
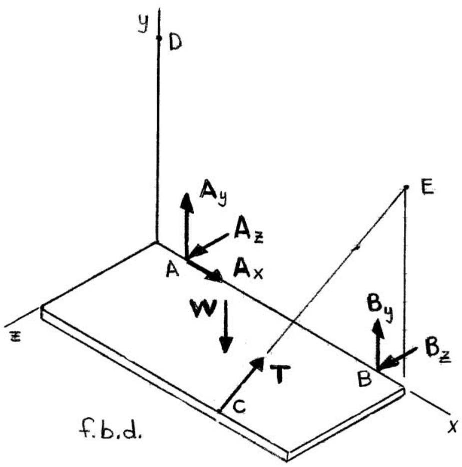
PROBLEM 4.127



Solve Problem 4.126 assuming that cable *DCE* is replaced by a cable attached to point *E* and hook *C*.

P4.126 A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION



First note

$$\lambda_{CE} = \frac{(9\text{ in.})\mathbf{i} + (22.5\text{ in.})\mathbf{j} - (15\text{ in.})\mathbf{k}}{28.5\text{ in.}}$$

$$= \frac{1}{28.5}(9\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\mathbf{W} = -(285\text{ lb})\mathbf{j}$$

From f.b.d. of plate

(*a*)

$$\Sigma M_x = 0: \quad (285\text{ lb})(7.5\text{ in.}) - \left[\left(\frac{22.5}{28.5}\right)T\right](15\text{ in.}) = 0$$

$$\therefore \quad T = 180.500\text{ lb}$$

or $T = 180.5$ lb ◀

(*b*)

$$\Sigma F_x = 0: \quad A_x + T\left(\frac{9}{28.5}\right) = 0$$

$$A_x + 180.5\text{ lb}\left(\frac{9}{28.5}\right) = 0$$

$$\therefore \quad A_x = -57.000\text{ lb}$$

PROBLEM 4.127 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(26 \text{ in.}) + W(13 \text{ in.}) - \left[T\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

$$-A_y(26 \text{ in.}) + (285 \text{ lb})(13 \text{ in.}) - \left[(180.5 \text{ lb})\left(\frac{22.5}{28.5}\right)\right](6 \text{ in.}) = 0$$

$$\therefore \quad A_y = 109.615 \text{ lb}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(26 \text{ in.}) - \left[T\left(\frac{15}{28.5}\right)\right](6 \text{ in.}) + \left[T\left(\frac{9}{28.5}\right)\right](15 \text{ in.}) = 0$$

$$A_z(26 \text{ in.}) + (180.5 \text{ lb})\left(\frac{45}{28.5}\right) = 0$$

$$\therefore \quad A_z = -10.9615 \text{ lb}$$

$$\text{or } \mathbf{A} = -(57.0 \text{ lb})\mathbf{i} + (109.6 \text{ lb})\mathbf{j} - (10.96 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad B_y - W + T\left(\frac{22.5}{28.5}\right) + A_y = 0$$

$$B_y - 285 \text{ lb} + (180.5 \text{ lb})\left(\frac{22.5}{28.5}\right) - 109.615 \text{ lb} = 0$$

$$\therefore \quad B_y = 32.885 \text{ lb}$$

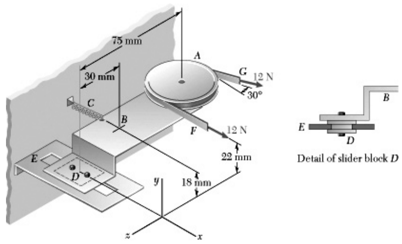
$$\Sigma F_z = 0: \quad B_z + A_z - T\left(\frac{15}{28.5}\right) = 0$$

$$B_z - 10.9615 \text{ lb} - 180.5 \text{ lb}\left(\frac{15}{28.5}\right) = 0$$

$$\therefore \quad B_z = 105.962 \text{ lb}$$

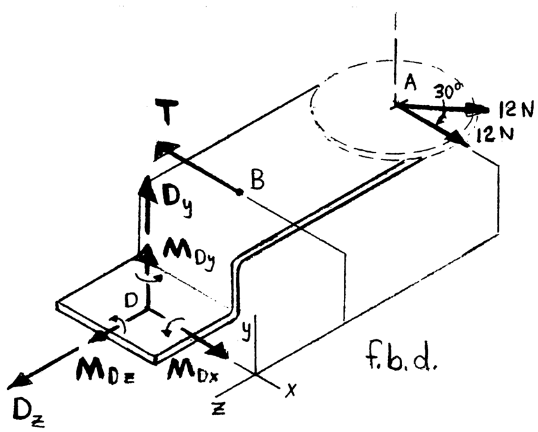
$$\text{or } \mathbf{B} = (32.9 \text{ lb})\mathbf{j} + (106.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.128



The tensioning mechanism of a belt drive consists of frictionless pulley *A*, mounting plate *B*, and spring *C*. Attached below the mounting plate is slider block *D* which is free to move in the frictionless slot of bracket *E*. Knowing that the pulley and the belt lie in a horizontal plane, with portion *F* of the belt parallel to the *x* axis and portion *G* forming an angle of 30° with the *x* axis, determine (*a*) the force in the spring, (*b*) the reaction at *D*.

SOLUTION



From f.b.d. of plate *B*

(*a*) $\Sigma F_x = 0\text{:}\quad 12\text{ N} + (12\text{ N})\cos 30° - T = 0$

$\therefore\quad T = 22.392\text{ N}$ or $T = 22.4\text{ N}$ ◀

(*b*) $\Sigma F_y = 0\text{:}\quad D_y = 0$

$\Sigma F_z = 0\text{:}\quad D_z - (12\text{ N})\sin 30° = 0$

$\therefore\quad D_z = 6\text{ N}$ or $\mathbf{D} = (6.00\text{ N})\mathbf{k}$ ◀

$$\Sigma M_x = 0\text{:}\quad M_{D_x} - \left[(12\text{ N})\sin 30°\right](22\text{ mm}) = 0$$

$$\therefore\quad M_{D_x} = 132.0\text{ N}\cdot\text{mm}$$

$$\Sigma M_{D(y\text{-axis})} = 0\text{:}\quad M_{D_y} + (22.392\text{ N})(30\text{ mm}) - (12\text{ N})(75\text{ mm}) - \left[(12\text{ N})\cos 30°\right](75\text{ mm}) = 0$$

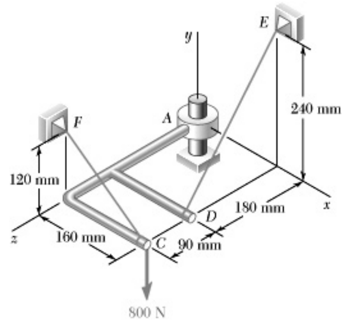
$$\therefore\quad M_{D_y} = 1007.66\text{ N}\cdot\text{mm}$$

$$\Sigma M_{D(z\text{-axis})} = 0\text{:}\quad M_{D_z} + (22.392\text{ N})(18\text{ mm}) - (12\text{ N})(22\text{ mm}) - \left[(12\text{ N})\cos 30°\right](22\text{ mm}) = 0$$

$$\therefore\quad M_{D_z} = 89.575\text{ N}\cdot\text{mm}$$

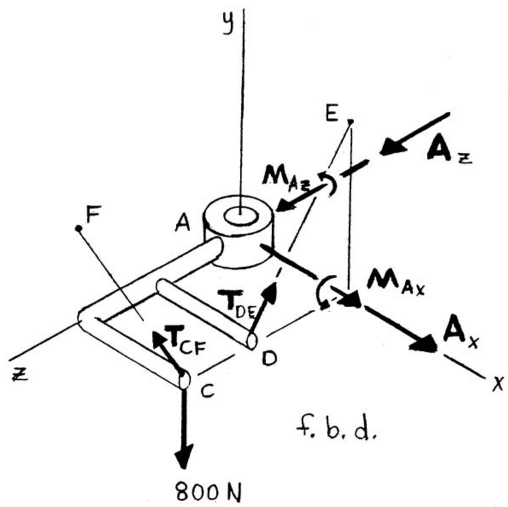
or $\mathbf{M}_D = (0.1320\text{ N}\cdot\text{m})\mathbf{i} + (1.008\text{ N}\cdot\text{m})\mathbf{j} + (0.0896\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 4.129



The assembly shown is welded to collar A which fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{CF} = \lambda_{CF} T_{CF} = \frac{-(0.16 \text{ m})\mathbf{i} + (0.12 \text{ m})\mathbf{j}}{\sqrt{(0.16)^2 + (0.12)^2} \text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE} = \frac{(0.24 \text{ m})\mathbf{j} - (0.18 \text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.18)^2} \text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(*a*) From f.b.d. of assembly

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 800 \text{ N} = 0$$

or

$$0.6T_{CF} + 0.8T_{DE} = 800 \text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.27 \text{ m}) + (0.6T_{DE})(0.16 \text{ m}) = 0$$

or

$$T_{DE} = 2.25T_{CF} \qquad (2)$$

PROBLEM 4.129 CONTINUED

Substituting Equation (2) into Equation (1)

$$0.6T_{CF} + 0.8\left[(2.25)T_{CF}\right] = 800 \text{ N}$$

$$\therefore \quad T_{CF} = 333.33 \text{ N} \qquad \text{or } T_{CF} = 333 \text{ N} \blacktriangleleft$$

and from Equation (2) $\quad T_{DE} = 2.25(333.33 \text{ N}) = 750.00 \text{ N} \qquad$ or $T_{DE} = 750 \text{ N} \blacktriangleleft$

(*b*) From f.b.d. of assembly

$$\Sigma F_z = 0: \quad A_z - (0.6)(750.00 \text{ N}) = 0 \qquad \therefore A_z = 450.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(333.33 \text{ N}) = 0 \qquad \therefore A_x = 266.67 \text{ N}$$

$$\text{or } \mathbf{A} = (267 \text{ N})\mathbf{i} + (450 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_x = 0: \quad M_{A_x} + (800 \text{ N})(0.27 \text{ m}) - \left[(333.33 \text{ N})(0.6)\right](0.27 \text{ m}) - \left[(750 \text{ N})(0.8)\right](0.18 \text{ m}) = 0$$

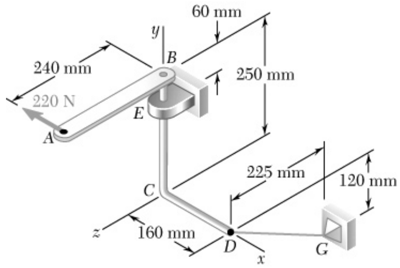
$$\therefore \quad M_{A_x} = -54.001 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (800 \text{ N})(0.16 \text{ m}) + \left[(333.33 \text{ N})(0.6)\right](0.16 \text{ m}) + \left[(750 \text{ N})(0.8)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{A_z} = 0$$

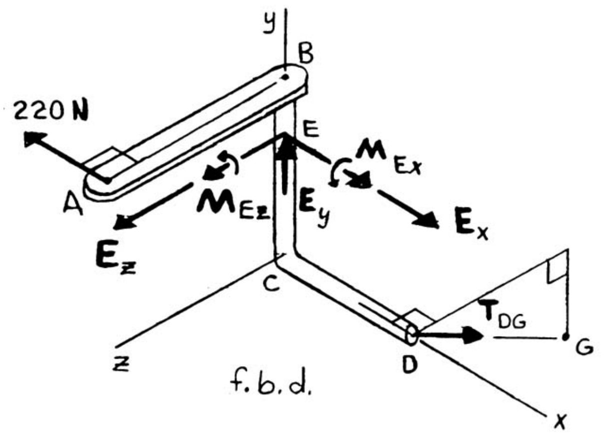
$$\text{or } \mathbf{M}_A = -(54.0 \text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$

PROBLEM 4.130



The lever *AB* is welded to the bent rod *BCD* which is supported by bearing *E* and by cable *DG*. Assuming that the bearing can exert an axial thrust and couples about axes parallel to the *x* and *z* axes, determine (*a*) the tension in cable *DG*, (*b*) the reaction at *E*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \frac{-(0.12\text{ m})\,\mathbf{j} - (0.225\text{ m})\,\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}} T_{DG}$$

$$= \frac{T_{DG}}{0.255}(-0.12\mathbf{j} - 0.225\mathbf{k})$$

(*a*) From f.b.d. of weldment

$$\Sigma M_y = 0: \quad \left[\left(\frac{0.225}{0.255}\right) T_{DG}\right](0.16\text{ m}) - (220\text{ N})(0.24\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N}$$

or $T_{DG} = 374$ N ◀

(*b*) From f.b.d. of weldment

$$\Sigma F_x = 0: \quad E_x - 220\text{ N} = 0$$

$$\therefore \quad E_x = 220.00\text{ N}$$

$$\Sigma F_y = 0: \quad E_y - (374.00\text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad E_y = 176.000\text{ N}$$

PROBLEM 4.130 CONTINUED

$$\Sigma F_z = 0: \quad E_z - (374.00 \text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

$$\therefore \quad E_z = 330.00 \text{ N}$$

$$\text{or } \mathbf{E} = (220 \text{ N})\mathbf{i} + (176.0 \text{ N})\mathbf{j} + (330 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_x = 0: \quad M_{E_x} + (330.00 \text{ N})(0.19 \text{ m}) = 0$$

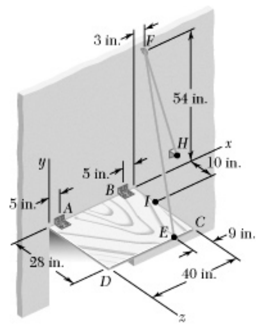
$$\therefore \quad M_{E_x} = -62.700 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad (220 \text{ N})(0.06 \text{ m}) + M_{E_z} - \left[(374.00 \text{ N})\left(\frac{0.12}{0.255}\right)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{E_z} = -14.9600 \text{ N}\cdot\text{m}$$

$$\text{or } \mathbf{M}_E = -(62.7 \text{ N}\cdot\text{m})\mathbf{i} - (14.96 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

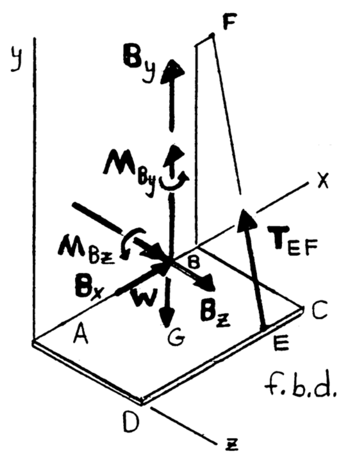
PROBLEM 4.131



Solve Problem 4.124 assuming that the hinge at *A* is removed and that the hinge at *B* can exert couples about the *y* and *z* axes.

P4.124 A small door weighing 16 lb is attached by hinges *A* and *B* to a wall and is held in the horizontal position shown by rope *EFH*. The rope passes around a small, frictionless pulley at *F* and is tied to a fixed cleat at *H*. Assuming that the hinge at *A* does not exert any axial thrust, determine (*a*) the tension in the rope, (*b*) the reactions at *A* and *B*.

SOLUTION



From f.b.d. of door

(*a*) $$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times \mathbf{W} + \mathbf{r}_{E/B} \times \mathbf{T}_{EF} + \mathbf{M}_B = 0$$

where

$$\mathbf{W} = -(16 \text{ lb})\,\mathbf{j}$$

$$\mathbf{M}_B = M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \frac{(12 \text{ in.})\mathbf{i} + (54 \text{ in.})\mathbf{j} - (28 \text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (54)^2 + (28)^2} \text{ in.}} T_{EF}$$

$$= \frac{T_{EF}}{31}(6\mathbf{i} + 27\mathbf{j} - 14\mathbf{k})$$

$$\mathbf{r}_{G/B} = -(15 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{E/B} = -(4 \text{ in.})\mathbf{i} + (28 \text{ in.})\mathbf{k}$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -15 & 0 & 14 \\ 0 & -1 & 0 \end{vmatrix}(16 \text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 28 \\ 6 & 27 & -14 \end{vmatrix}\left(\frac{T_{EF}}{31}\right) + \left(M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}\right) = 0$$

or $$(224 - 24.387T_{EF})\mathbf{i} + \left(3.6129T_{EF} + M_{B_y}\right)\mathbf{j}$$

$$+ \left(240 - 3.4839T_{EF} + M_{B_z}\right)\mathbf{k} = 0$$

From **i**-coefficient $$224 - 24.387T_{EF} = 0$$

$$\therefore \quad T_{EF} = 9.1852 \text{ lb}$$

or $T_{EF} = 9.19$ lb ◀

(*b*) From **j**-coefficient $$3.6129(9.1852) + M_{B_y} = 0$$

$$\therefore \quad M_{B_y} = -33.185 \text{ lb}\cdot\text{in.}$$

PROBLEM 4.131 CONTINUED

From **k**-coefficient $240 - 3.4839(9.1852) + M_{B_z} = 0$

$$\therefore \quad M_{B_z} = -208.00 \text{ lb}\cdot\text{in.}$$

$$\text{or } \mathbf{M}_B = -(33.2 \text{ lb}\cdot\text{in.})\mathbf{j} - (208 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad B_x + \frac{6}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_x = -1.77778 \text{ lb}$$

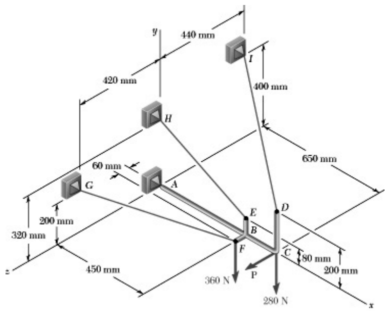
$$\Sigma F_y = 0: \quad B_y - 16 \text{ lb} + \frac{27}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_y = 8.0000 \text{ lb}$$

$$\Sigma F_z = 0: \quad B_z - \frac{14}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_z = 4.1482 \text{ lb}$$

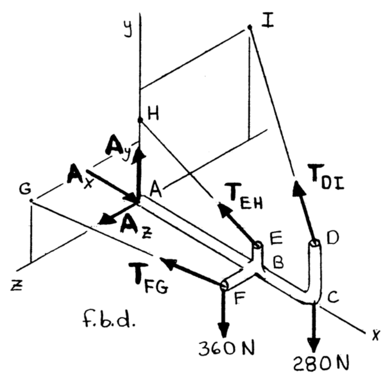
$$\text{or } \mathbf{B} = -(1.778 \text{ lb})\mathbf{i} + (8.00 \text{ lb})\mathbf{j} + (4.15 \text{ lb})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.132

The frame shown is supported by three cables and a ball-and-socket joint at A. For $\mathbf{P} = 0$, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI}T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}}T_{DI}$$

$$= \frac{T_{DI}}{0.81}(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH}T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}}T_{EH}$$

$$= \frac{T_{EH}}{0.51}(-0.45\mathbf{i}) + (0.24\,\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG}T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45\,)^2 + (0.2)^2 + (0.36)^2}\text{ m}}T_{FG}$$

$$= \frac{T_{FG}}{0.61}(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times (-280\text{ N})\mathbf{j} + \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j} = 0$$

or $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -0.65 & 0.2 & -0.44 \end{vmatrix}\left(\frac{T_{DI}}{0.81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}(280\text{ N}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -0.45 & 0.24 & 0 \end{vmatrix}\left(\frac{T_{EH}}{0.51}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -0.45 & 0.2 & 0.36 \end{vmatrix}\left(\frac{T_{FG}}{0.61}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix}(360\text{ N}) = 0$$

or $$(-0.088\mathbf{i} + 0.286\mathbf{j} + 0.26\mathbf{k})\frac{T_{DI}}{0.81} + (-0.65\mathbf{k})280\text{ N} + (0.144\mathbf{k})\frac{T_{EH}}{0.51}$$

$$+ (-0.012\mathbf{i} - 0.189\mathbf{j} + 0.09\mathbf{k})\frac{T_{FG}}{0.61} + (0.06\mathbf{i} - 0.45\mathbf{k})(360\text{ N}) = 0$$

PROBLEM 4.132 CONTINUED

From **i**-coefficient $-0.088\left(\frac{T_{DI}}{0.81}\right) - 0.012\left(\frac{T_{FG}}{0.61}\right) + 0.06(360 \text{ N}) = 0$

$$\therefore \quad 0.108642T_{DI} + 0.0196721T_{FG} = 21.6 \qquad (1)$$

From **j**-coefficient $0.286\left(\frac{T_{DI}}{0.81}\right) - 0.189\left(\frac{T_{FG}}{0.61}\right) = 0$

$$\therefore \quad T_{FG} = 1.13959T_{DI} \qquad (2)$$

From **k**-coefficient

$$0.26\left(\frac{T_{DI}}{0.81}\right) - 0.65(280 \text{ N}) + 0.144\left(\frac{T_{EH}}{0.51}\right) + 0.09\left(\frac{T_{FG}}{0.61}\right)$$

$$-0.45(360 \text{ N}) = 0$$

$$\therefore \quad 0.32099T_{DI} + 0.28235T_{EH} + 0.147541T_{FG} = 344 \text{ N} \qquad (3)$$

Substitution of Equation (2) into Equation (1)

$$0.108642T_{DI} + 0.0196721(1.13959T_{DI}) = 21.6$$

$$\therefore \quad T_{DI} = 164.810 \text{ N}$$

or $T_{DI} = 164.8 \text{ N}$ ◀

Then from Equation (2)

$$T_{FG} = 1.13959(164.810 \text{ N}) = 187.816 \text{ N}$$

or $T_{FG} = 187.8 \text{ N}$ ◀

And from Equation (3)

$$0.32099(164.810 \text{ N}) + 0.28235T_{EH} + 0.147541(187.816 \text{ N}) = 344 \text{ N}$$

$$\therefore \quad T_{EH} = 932.84 \text{ N}$$

or $T_{EH} = 933 \text{ N}$ ◀

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{164.810 \text{ N}}{0.81}(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k})$$

$$= -(132.25 \text{ N})\mathbf{i} + (40.694 \text{ N})\mathbf{j} - (89.526 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{932.84 \text{ N}}{0.51}(-0.45\mathbf{i} + 0.24\mathbf{j}) = -(823.09 \text{ N})\mathbf{i} + (438.98 \text{ N})\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{187.816 \text{ N}}{0.61}(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k})$$

$$= -(138.553 \text{ N})\mathbf{i} + (61.579 \text{ N})\mathbf{j} + (110.842 \text{ N})\mathbf{k}$$

PROBLEM 4.132 CONTINUED

Then, from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 132.25 - 823.09 - 138.553 = 0$$

$$\therefore \quad A_x = 1093.89 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + 40.694 + 438.98 + 61.579 - 360 - 280 = 0$$

$$\therefore \quad A_y = 98.747 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 89.526 + 110.842 = 0$$

$$\therefore \quad A_z = -21.316 \text{ N}$$

or

$$\mathbf{A} = (1094 \text{ N})\mathbf{i} + (98.7 \text{ N})\mathbf{j} - (21.3 \text{ N})\mathbf{k} \blacktriangleleft$$