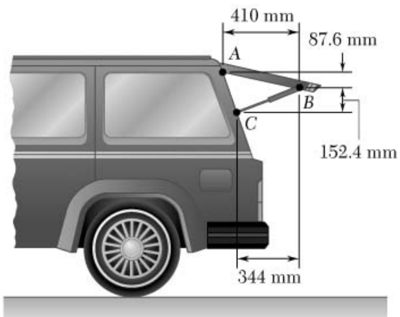
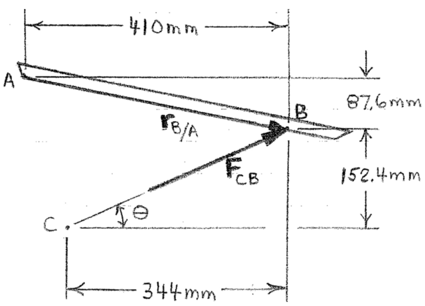


PROBLEM 3.10

The tailgate of a car is supported by the hydraulic lift *BC*. If the lift exerts a 125-N force directed along its center line on the ball and socket at *B*, determine the moment of the force about *A*.

SOLUTION



First note $$d_{CB} = \sqrt{(344 \text{ mm})^2 + (152.4 \text{ mm})^2} = 376.25 \text{ mm}$$

Then $$\cos\theta = \frac{344 \text{ mm}}{376.25 \text{ mm}} \qquad \sin\theta = \frac{152.4 \text{ mm}}{376.25 \text{ mm}}$$

and $$\mathbf{F}_{CB} = (F_{CB}\cos\theta)\mathbf{i} - (F_{CB}\sin\theta)\mathbf{j}$$

$$= \frac{125 \text{ N}}{376.25 \text{ mm}}\left[(344 \text{ mm})\mathbf{i} + (152.4 \text{ mm})\mathbf{j}\right]$$

Now $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

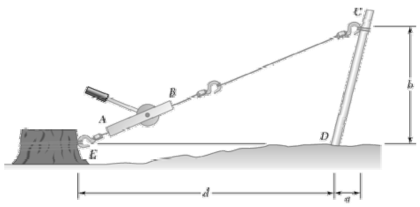
where $$\mathbf{r}_{B/A} = (410 \text{ mm})\mathbf{i} - (87.6 \text{ mm})\mathbf{j}$$

Then $$\mathbf{M}_A = \left[(410 \text{ mm})\mathbf{i} - (87.6 \text{ mm})\mathbf{j}\right] \times \frac{125 \text{ N}}{376.25}(344\mathbf{i} - 152.4\mathbf{j})$$

$$= (30\,770 \text{ N}\cdot\text{mm})\mathbf{k}$$

$$= (30.770 \text{ N}\cdot\text{m})\mathbf{k}$$

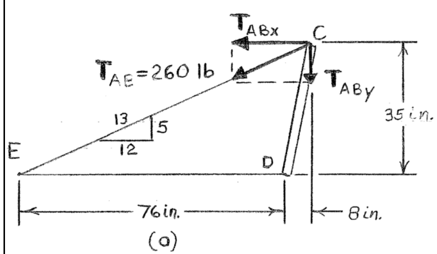
or $\mathbf{M}_A = 30.8 \text{ N}\cdot\text{m}$ ↺ ◀



PROBLEM 3.11

A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 260 lb, length a is 8 in., length b is 35 in., and length d is 76 in., determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (*a*) at point C, (*b*) at point E.

SOLUTION



(*a*) Slope of line $EC = \dfrac{35\text{ in.}}{76\text{ in.} + 8\text{ in.}} = \dfrac{5}{12}$

Then $T_{ABx} = \dfrac{12}{13}(T_{AB})$

$= \dfrac{12}{13}(260\text{ lb}) = 240\text{ lb}$

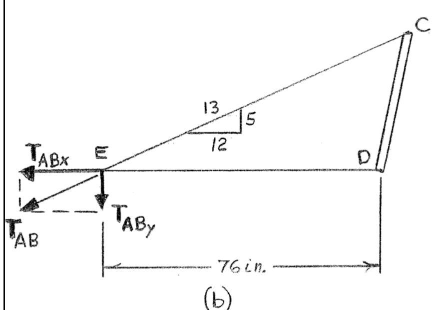
and $T_{ABy} = \dfrac{5}{13}(260\text{ lb}) = 100\text{ lb}$

Then $M_D = T_{ABx}(35\text{ in.}) - T_{ABy}(8\text{ in.})$

$= (240\text{ lb})(35\text{ in.}) - (100\text{ lb})(8\text{ in.})$

$= 7600\text{ lb}\cdot\text{in.}$

or $\mathbf{M}_D = 7600\text{ lb}\cdot\text{in.}$ ↻ ◀

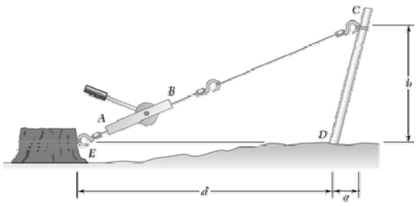


(*b*) Have $M_D = T_{ABx}(y) + T_{ABy}(x)$

$= (240\text{ lb})(0) + (100\text{ lb})(76\text{ in.})$

$= 7600\text{ lb}\cdot\text{in.}$

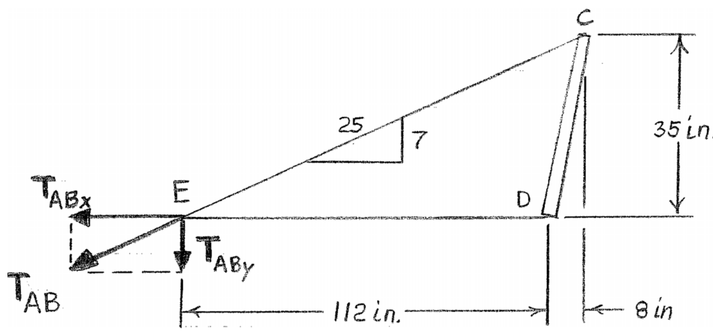
or $\mathbf{M}_D = 7600\text{ lb}\cdot\text{in.}$ ↻ ◀



PROBLEM 3.12

It is known that a force with a moment of 7840 lb·in. about D is required to straighten the fence post CD. If $a = 8$ in., $b = 35$ in., and $d = 112$ in., determine the tension that must be developed in the cable of winch puller AB to create the required moment about point D.

SOLUTION



$$\text{Slope of line } EC = \frac{35\text{ in.}}{112\text{ in.} + 8\text{ in.}} = \frac{7}{24}$$

Then

$$T_{ABx} = \frac{24}{25}T_{AB}$$

and

$$T_{ABy} = \frac{7}{25}T_{AB}$$

Have

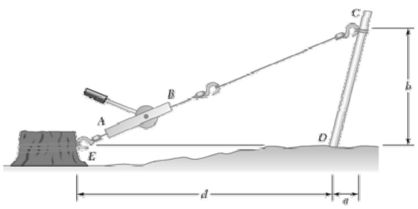
$$M_D = T_{ABx}(y) + T_{ABy}(x)$$

$$\therefore\; 7840\text{ lb}\cdot\text{in.} = \frac{24}{25}T_{AB}(0) + \frac{7}{25}T_{AB}(112\text{ in.})$$

$$T_{AB} = 250\text{ lb}$$

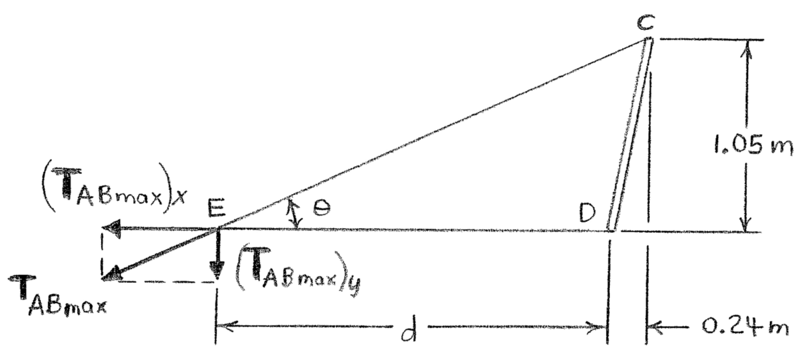
or $T_{AB} = 250$ lb ◀

PROBLEM 3.13



It is known that a force with a moment of $1152\ \text{N}\cdot\text{m}$ about D is required to straighten the fence post CD. If the capacity of the winch puller AB is 2880 N, determine the minimum value of distance d to create the specified moment about point D knowing that $a = 0.24$ m and $b = 1.05$ m.

SOLUTION



The minimum value of d can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about D:

$$M_D = \left(T_{AB\max}\right)_y (d)$$

where
$$M_D = 1152\ \text{N}\cdot\text{m}$$

$$\left(T_{AB\max}\right)_y = T_{AB\max}\sin\theta = (2880\ \text{N})\sin\theta$$

Now
$$\sin\theta = \frac{1.05\ \text{m}}{\sqrt{(d+0.24)^2 + (1.05)^2}\ \text{m}}$$

$$\therefore\ 1152\ \text{N}\cdot\text{m} = 2880\ \text{N}\left[\frac{1.05}{\sqrt{(d+0.24)^2+(1.05)^2}}\right](d)$$

or
$$\sqrt{(d+0.24)^2 + (1.05)^2} = 2.625d$$

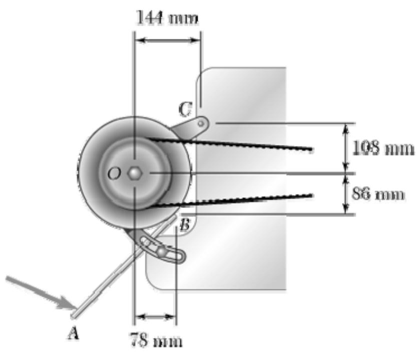
or
$$(d+0.24)^2 + (1.05)^2 = 6.8906d^2$$

or
$$5.8906d^2 - 0.48d - 1.1601 = 0$$

Using the quadratic equation, the minimum values of d are 0.48639 m and –0.40490 m.
Since only the positive value applies here, $d = 0.48639$ m

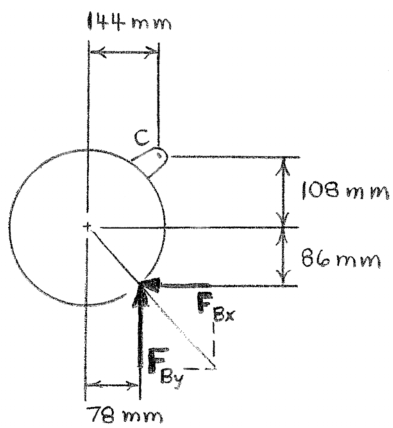
or $d = 486$ mm ◀

PROBLEM 3.14



A mechanic uses a piece of pipe *AB* as a lever when tightening an alternator belt. When he pushes down at *A*, a force of 580 N is exerted on the alternator *B*. Determine the moment of that force about bolt *C* if its line of action passes through *O*.

SOLUTION



Have $$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$$

Noting the direction of the moment of each force component about *C* is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

where $$x = 144 \text{ mm} - 78 \text{ mm} = 66 \text{ mm}$$

$$y = 86 \text{ mm} + 108 \text{ mm} = 194 \text{ mm}$$

and $$F_{Bx} = \frac{78}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 389.65 \text{ N}$$

$$F_{By} = \frac{86}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 429.62 \text{ N}$$

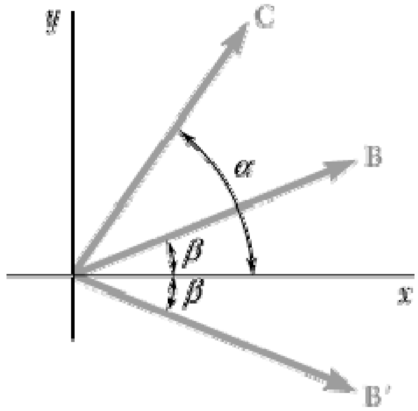
$$\therefore \quad M_C = (66 \text{ mm})(429.62 \text{ N}) + (194 \text{ mm})(389.65 \text{ N})$$

$$= 103947 \text{ N}\cdot\text{mm}$$

$$= 103.947 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 103.9 \text{ N}\cdot\text{m}$ ◀

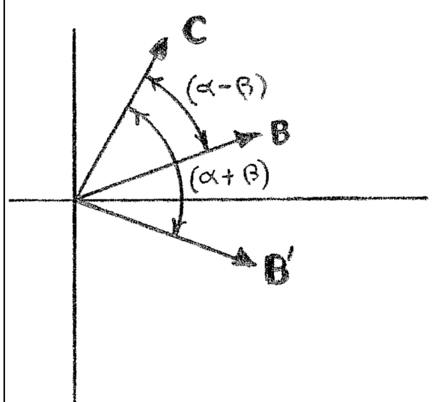
PROBLEM 3.15



Form the vector products $\mathbf{B} \times \mathbf{C}$ and $\mathbf{B}' \times \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha+\beta) + \tfrac{1}{2}\sin(\alpha-\beta).$$

SOLUTION



First note $$\mathbf{B} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})$$

$$\mathbf{B}' = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})$$

$$\mathbf{C} = C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

By definition $$|\mathbf{B} \times \mathbf{C}| = BC\sin(\alpha - \beta) \tag{1}$$

$$|\mathbf{B}' \times \mathbf{C}| = BC\sin(\alpha + \beta) \tag{2}$$

Now $$\mathbf{B} \times \mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) \times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k} \tag{3}$$

$$\mathbf{B} \times \mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j}) \times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k} \tag{4}$$

Equating magnitudes of $\mathbf{B} \times \mathbf{C}$ from Equations (1) and (3), (5)

$$\sin(\alpha - \beta) = \cos\beta\sin\alpha - \sin\beta\cos\alpha$$

Similarly, equating magnitudes of $\mathbf{B}' \times \mathbf{C}$ from Equations (2) and (4),

$$\sin(\alpha + \beta) = \cos\beta\sin\alpha + \sin\beta\cos\alpha \tag{6}$$

Adding Equations (5) and (6)

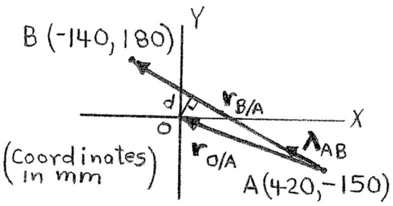
$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\cos\beta\sin\alpha$$

$$\therefore \quad \sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta) \blacktriangleleft$$

PROBLEM 3.16

A line passes through the points (420 mm, –150 mm) and (–140 mm, 180 mm). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

SOLUTION



Have $$d = \left|\lambda_{AB} \times \mathbf{r}_{O/A}\right|$$

where $$\lambda_{AB} = \frac{\mathbf{r}_{B/A}}{\left|\mathbf{r}_{B/A}\right|}$$

and $$\mathbf{r}_{B/A} = (-140\text{ mm} - 420\text{ mm})\mathbf{i} + [180\text{ mm} - (-150\text{ mm})]\mathbf{j}$$

$$= -(560\text{ mm})\mathbf{i} + (330\text{ mm})\mathbf{j}$$

$$\left|\mathbf{r}_{B/A}\right| = \sqrt{(-560)^2 + (330)^2}\text{mm} = 650\text{ mm}$$

$$\therefore \quad \lambda_{AB} = \frac{-(560\text{ mm})\mathbf{i} + (330\text{ mm})\mathbf{j}}{650\text{ mm}} = \frac{1}{65}(-56\mathbf{i} + 33\mathbf{j})$$

$$\mathbf{r}_{O/A} = (0 - x_A)\mathbf{i} + (0 - y_A)\mathbf{j} = -(420\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}$$

$$\therefore \quad d = \left|\frac{1}{65}(-56\mathbf{i} - 33\mathbf{j}) \times [-(420\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}]\right| = 84.0\text{ mm}$$

$d = 84.0$ mm ◀

PROBLEM 3.17

A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) 4**i** − 2**j** + 3**k** and −2**i** + 6**j** − 5**k**, (*b*) 7**i** + **j** − 4**k** and −6**i** − 3**k** + 2**k**.

SOLUTION

(*a*) Have
$$\lambda = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where
$$\mathbf{A} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$
$$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$$

Then
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & 3 \\ -2 & 6 & -5 \end{vmatrix} = (10-18)\mathbf{i} + (-6+20)\mathbf{j} + (24-4)\mathbf{k} = 2(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})$$

and
$$|\mathbf{A}\times\mathbf{B}| = 2\sqrt{(-4)^2 + (7)^2 + (10)^2} = 2\sqrt{165}$$

$$\therefore\quad \lambda = \frac{2(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})}{2\sqrt{165}} \qquad \text{or}\quad \lambda = \frac{1}{\sqrt{165}}(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k}) \blacktriangleleft$$

(*b*) Have
$$\lambda = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where
$$\mathbf{A} = 7\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$
$$\mathbf{B} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

Then
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 1 & -4 \\ -6 & -3 & 2 \end{vmatrix} = (2-12)\mathbf{i} + (24-14)\mathbf{j} + (-21+6)\mathbf{k} = 5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

and
$$|\mathbf{A}\times\mathbf{B}| = 5\sqrt{(-2)^2 + (2)^2 + (-3)^2} = 5\sqrt{17}$$

$$\therefore\quad \lambda = \frac{5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})}{5\sqrt{17}} \qquad \text{or}\quad \lambda = \frac{1}{\sqrt{17}}(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \blacktriangleleft$$

PROBLEM 3.18

The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) **P** = (8 in.)**i** + (2 in.)**j** − (1 in.)**k** and **Q** = −(3 in.)**i** + (4 in.)**j** + (2 in.)**k**, (*b*) **P** = −(3 in.)**i** + (6 in.)**j** + (4 in.)**k** and **Q** = (2 in.)**i** + (5 in.)**j** − (3 in.)**k**.

SOLUTION

(*a*) Have $$A = |\mathbf{P} \times \mathbf{Q}|$$

where $$\mathbf{P} = (8\text{ in.})\mathbf{i} + (2\text{ in.})\mathbf{j} - (1\text{ in.})\mathbf{k}$$

$$\mathbf{Q} = -(3\text{ in.})\mathbf{i} + (4\text{ in.})\mathbf{j} + (2\text{ in.})\mathbf{k}$$

Then $$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & -1 \\ -3 & 4 & 2 \end{vmatrix} \text{in}^2 = \left[(4+4)\mathbf{i} + (3-16)\mathbf{j} + (32+6)\mathbf{k}\right]\text{in}^2$$

$$= (8\text{ in}^2)\mathbf{i} - (13\text{ in}^2)\mathbf{j} + (38\text{ in}^2)\mathbf{k}$$

$$\therefore\ A = \sqrt{(8)^2 + (-13)^2 + (38)^2}\,\text{in}^2 = 40.951\text{ in}^2$$

or $A = 41.0\text{ in}^2$ ◀

(*b*) Have $$A = |\mathbf{P} \times \mathbf{Q}|$$

where $$\mathbf{P} = -(3\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} + (4\text{ in.})\mathbf{k}$$

$$\mathbf{Q} = (2\text{ in.})\mathbf{i} + (5\text{ in.})\mathbf{j} - (3\text{ in.})\mathbf{k}$$

Then $$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 6 & 4 \\ 2 & 5 & -3 \end{vmatrix} \text{in}^2 = \left[(-18-20)\mathbf{i} + (8-9)\mathbf{j} + (-15-12)\mathbf{k}\right]\text{in}^2$$

$$= -(38\text{ in}^2)\mathbf{i} - (1\text{ in}^2)\mathbf{j} - (27\text{ in}^2)\mathbf{k}$$

$$\therefore\ A = \sqrt{(-38)^2 + (-1)^2 + (-27)^2}\,\text{in}^2 = 46.626\text{ in}^2$$

or $A = 46.6\text{ in}^2$ ◀

PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$ which acts at a point A. Assume that the position vector of A is (a) $\mathbf{r} = (4\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$, ($b$) $\mathbf{r} = -(8\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} + (4\text{ m})\mathbf{k}$, ($c$) $\mathbf{r} = (7.5\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4.5\text{ m})\mathbf{k}$.

SOLUTION

(a) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = (4\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & -1 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(-6-2)\mathbf{i} + (5-12)\mathbf{j} + (-8-10)\mathbf{k}\right]\text{N}\cdot\text{m}$$

$$= (-8\mathbf{i} - 7\mathbf{j} - 18\mathbf{k})\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = -(8\text{ N}\cdot\text{m})\mathbf{i} - (7\text{ N}\cdot\text{m})\mathbf{j} - (18\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(b) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = -(8\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4\text{ m})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 3 & 4 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(9+8)\mathbf{i} + (-20+24)\mathbf{j} + (16+15)\mathbf{k}\right]\text{N}\cdot\text{m}$$

$$= (17\mathbf{i} + 4\mathbf{j} + 31\mathbf{k})\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = (17\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{j} + (31\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(c) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = (7.5\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4.5\text{ m})\mathbf{k}$$

PROBLEM 3.19 CONTINUED

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7.5 & 3 & -4.5 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(9-9)\mathbf{i} + (22.5-22.5)\mathbf{j} + (-15+15)\mathbf{k}\right]\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = 0$ ◀

This answer is expected since **r** and **F** are proportional $\left(\mathbf{F} = \dfrac{-2}{3}\mathbf{r}\right)$. Therefore, vector **F** has a line of action passing through the origin at *O*.