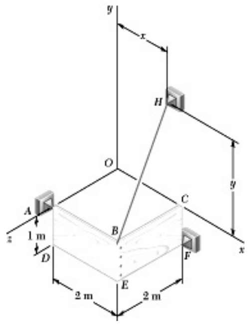
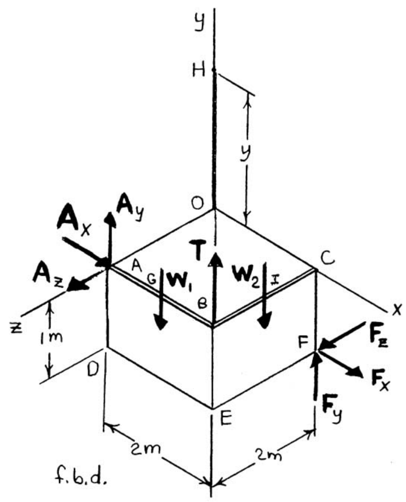
PROBLEM 4.150



Solve Problem 4.149 subject to the restriction that H must lie on the y axis.

P4.149 Two 1×2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION



Let $$\mathbf{W}_1 = \mathbf{W}_2 = -(mg)\mathbf{j} = -(15\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(147.15\text{ N})\mathbf{j}$$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0: \quad \lambda_{AF} \cdot (\mathbf{r}_{G/A} \times \mathbf{W}_1) + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \lambda_{AF} \cdot (\mathbf{r}_{I/A} \times \mathbf{W}_2) = 0$$

where $$\lambda_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{-(2\text{ m})\mathbf{i} + (y)\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (y)^2 + (2)^2}\text{ m}}T$$

$$= \frac{T}{\sqrt{8 + y^2}}(-2\mathbf{i} + y\mathbf{j} - 2\mathbf{k})$$

PROBLEM 4.150 CONTINUED

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ -2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{8 + y^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$2(147.15) + (-4 - 4y)\left(T\sqrt{8 + y^2}\right) + (2)147.15 = 0$$

$$\therefore \quad T = \frac{(147.15)\sqrt{8 + y^2}}{(1 + y)}$$

For $T_{\min}$,

$$\left(\frac{dT}{dy}\right) = 0 \qquad \therefore \quad \frac{(1 + y)\frac{1}{2}\left(8 + y^2\right)^{-\frac{1}{2}}(2y) - \left(8 + y^2\right)^{\frac{1}{2}}(1)}{(1 + y)^2} = 0$$

Setting the numerator equal to zero,

$$(1 + y)y = 8 + y^2$$

$$\therefore \quad y = 8.00 \text{ m}$$

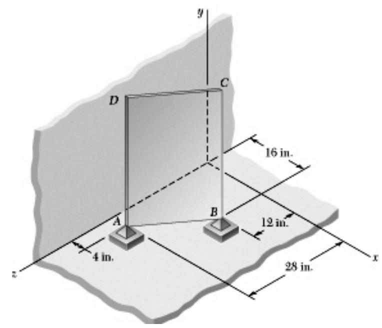
and

$$T_{\min} = \frac{(147.15)\sqrt{8 + (8)^2}}{(1 + 8)} = 138.734 \text{ N}$$

$\therefore$ (*a*) $x = 0, \; y = 8.00$ m ◀

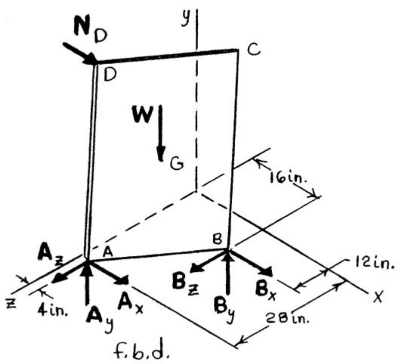
(*b*) $T_{\min} = 138.7$ N ◀

PROBLEM 4.151



A uniform 20 × 30-in. steel plate *ABCD* weighs 85 lb and is attached to ball-and-socket joints at *A* and *B*. Knowing that the plate leans against a frictionless vertical wall at *D*, determine (*a*) the location of *D*, (*b*) the reaction at *D*.

SOLUTION



(*a*) Since $\mathbf{r}_{D/A}$ is perpendicular to $\mathbf{r}_{B/A}$,

$$\mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = 0$$

where coordinates of *D* are $(0, y, z)$, and

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (y)\mathbf{j} + (z - 28\text{ in.})\mathbf{k}$$

$$\mathbf{r}_{B/A} = (12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}$$

$$\therefore\ \mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = -48 - 16z + 448 = 0$$

or $$z = 25\text{ in.}$$

Since $$L_{AD} = 30\text{ in.}$$

$$30 = \sqrt{(4)^2 + (y)^2 + (25 - 28)^2}$$

$$900 = 16 + y^2 + 9$$

or $$y = \sqrt{875}\text{ in.} = 29.580\text{ in.}$$

$\therefore$ Coordinates of *D*: $x = 0,\ y = 29.6\text{ in.},\ z = 25.0\text{ in.}$ ◀

(*b*) From f.b.d. of steel plate *ABCD*

$$\Sigma M_{AB} = 0: \quad \lambda_{AB} \cdot (\mathbf{r}_{D/A} \times \mathbf{N}_D) + \lambda_{AB} \cdot (\mathbf{r}_{G/B} \times \mathbf{W}) = 0$$

where $$\lambda_{AB} = \frac{(12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (16)^2}\text{ in.}} = \frac{1}{5}(3\mathbf{i} - 4\mathbf{k})$$

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} - (3\text{ in.})\mathbf{k}$$

$$\mathbf{N}_D = N_D\mathbf{i}$$

PROBLEM 4.151 CONTINUED

$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{D/B} = \frac{1}{2}\left[-(16\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} + (25\text{ in.} - 12\text{ in.})\mathbf{k}\right]$$

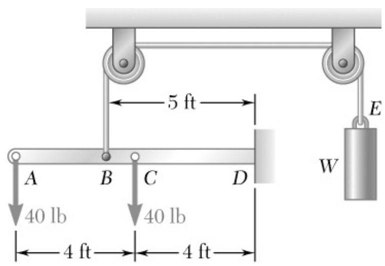
$$\mathbf{W} = -(85\text{ lb})\mathbf{j}$$

$$\therefore \begin{vmatrix} 3 & 0 & -4 \\ -4 & 29.580 & -3 \\ 1 & 0 & 0 \end{vmatrix}\left(\frac{N_D}{5}\right) + \begin{vmatrix} 3 & 0 & -4 \\ -16 & 29.580 & 13 \\ 0 & -1 & 0 \end{vmatrix}\left[\frac{85}{2(5)}\right] = 0$$

$$118.32N_D + (39 - 64)42.5 = 0$$

$$\therefore\ N_D = 8.9799\text{ lb}$$

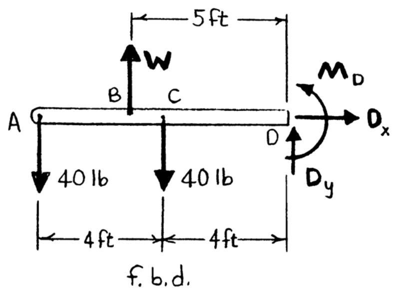
$$\text{or}\quad \mathbf{N}_D = (8.98\text{ lb})\mathbf{i} \blacktriangleleft$$



PROBLEM 4.152

Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE which is attached to the counter-weight W. Determine the reaction at D when (a) $W = 100$ lb, (b) $W = 90$ lb.

SOLUTION



(a) $W = 100$ lb

From f.b.d. of beam AD

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 40\text{ lb} - 40\text{ lb} + 100\text{ lb} = 0$$

$$\therefore\ D_y = -20.0\text{ lb}$$

$$\text{or } \mathbf{D} = 20.0\text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (100\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft})$$

$$+(40\text{ lb})(4\text{ ft}) = 0$$

$$\therefore\ M_D = 20.0\text{ lb}\cdot\text{ft}$$

$$\text{or } \mathbf{M}_D = 20.0\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

(b) $W = 90$ lb

From f.b.d. of beam AD

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + 90\text{ lb} - 40\text{ lb} - 40\text{ lb} = 0$$

$$\therefore\ D_y = -10.00\text{ lb}$$

$$\text{or } \mathbf{D} = 10.00\text{ lb} \downarrow \blacktriangleleft$$

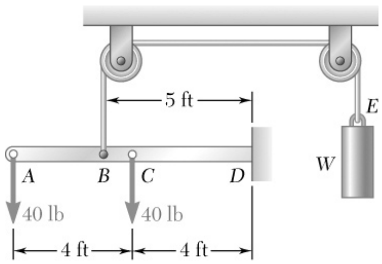
$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (90\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft})$$

$$+(40\text{ lb})(4\text{ ft}) = 0$$

$$\therefore\ M_D = -30.0\text{ lb}\cdot\text{ft}$$

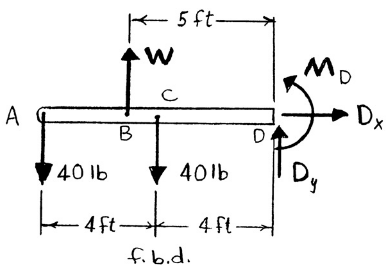
$$\text{or } \mathbf{M}_D = 30.0\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

PROBLEM 4.153



For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed $40\ \text{lb}\cdot\text{ft}$.

SOLUTION



For $W_{\min}$, $\quad M_D = -40\ \text{lb}\cdot\text{ft}$

From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0:\quad (40\ \text{lb})(8\ \text{ft}) - W_{\min}(5\ \text{ft}) + (40\ \text{lb})(4\ \text{ft}) - 40\ \text{lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\min} = 88.0\ \text{lb}$$

For $W_{\max}$, $\quad M_D = 40\ \text{lb}\cdot\text{ft}$

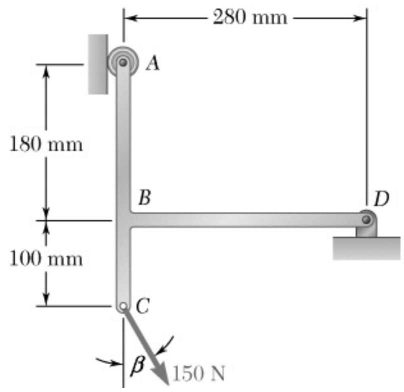
From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0:\quad (40\ \text{lb})(8\ \text{ft}) - W_{\max}(5\ \text{ft}) + (40\ \text{lb})(4\ \text{ft}) + 40\ \text{lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\max} = 104.0\ \text{lb}$$

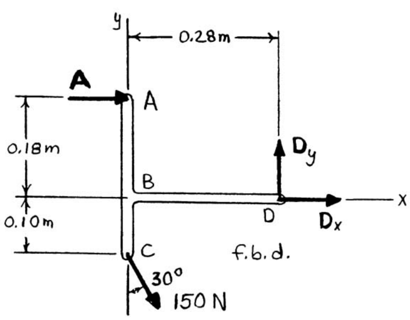
or $88.0\ \text{lb} \le W \le 104.0\ \text{lb}$ ◀

PROBLEM 4.154



Determine the reactions at A and D when $\beta = 30°$.

SOLUTION



From f.b.d. of frame *ABCD*

$$+\circlearrowleft\ \Sigma M_D = 0: \quad -A(0.18\text{ m}) + [(150\text{ N})\sin 30°](0.10\text{ m})$$

$$+[(150\text{ N})\cos 30°](0.28\text{ m}) = 0$$

$$\therefore\ A = 243.74\text{ N}$$

or $\mathbf{A} = 244\text{ N} \longrightarrow$ ◀

$$\xrightarrow{+}\ \Sigma F_x = 0: \quad (243.74\text{ N}) + (150\text{ N})\sin 30° + D_x = 0$$

$$\therefore\ D_x = -318.74\text{ N}$$

$$+\uparrow\ \Sigma F_y = 0: \quad D_y - (150\text{ N})\cos 30° = 0$$

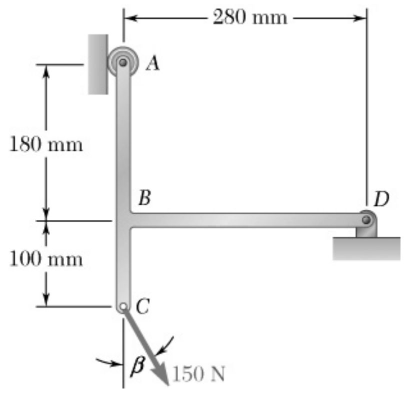
$$\therefore\ D_y = 129.904\text{ N}$$

Then $\quad D = \sqrt{(D_x)^2 + D_x^2} = \sqrt{(318.74)^2 + (129.904)^2} = 344.19\text{ N}$

and $\quad \theta = \tan^{-1}\left(\dfrac{D_y}{D_x}\right) = \tan^{-1}\left(\dfrac{129.904}{-318.74}\right) = -22.174°$

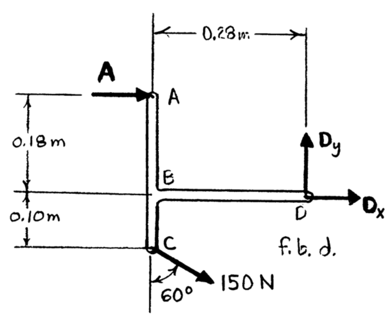
or $\mathbf{D} = 344\text{ N} \searrow 22.2°$ ◀

PROBLEM 4.155



Determine the reactions at A and D when $\beta = 60°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + \left[(150 \text{ N})\sin 60°\right](0.10 \text{ m})$$

$$+\left[(150 \text{ N})\cos 60°\right](0.28 \text{ m}) = 0$$

$$\therefore\ A = 188.835 \text{ N}$$

or $\mathbf{A} = 188.8 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (188.835 \text{ N}) + (150 \text{ N})\sin 60° + D_x = 0$$

$$\therefore\ D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 60° = 0$$

$$\therefore\ D_y = 75.0 \text{ N}$$

Then $$D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(318.74)^2 + (75.0)^2} = 327.44 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{75.0}{-318.74}\right) = -13.2409°$$

or $\mathbf{D} = 327 \text{ N} \measuredangle\ 13.24°$ ◀