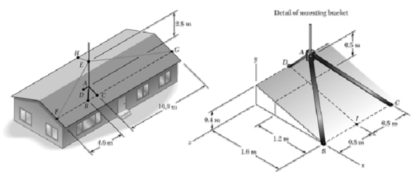
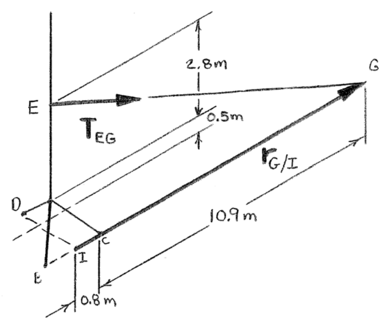


PROBLEM 3.56

A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EG* at *E* is 61.5 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION



Have $$M_{DI} = \boldsymbol{\lambda}_{DI} \cdot \left[\mathbf{r}_{G/I} \times \mathbf{T}_{EG}\right]$$

where $$\boldsymbol{\lambda}_{DI} = \frac{(1.6\text{ m})\mathbf{i} - (0.4\text{ m})\mathbf{j}}{0.4\sqrt{17}\text{ m}}$$

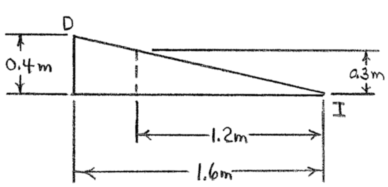
$$= \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

$$\mathbf{r}_{G/I} = -(10.9\text{ m} + 0.8\text{ m})\mathbf{k} = -(11.7\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \boldsymbol{\lambda}_{EG} T_{EG}$$

$$= \frac{(1.2\text{ m})\mathbf{i} - (3.6\text{ m})\mathbf{j} - (11.7\text{ m})\mathbf{k}}{12.3\text{ m}}(61.5\text{ N})$$

$$= 5\left[(1.2\text{ N})\mathbf{i} - (3.6\text{ N})\mathbf{j} - (11.7\text{ N})\mathbf{k}\right]$$



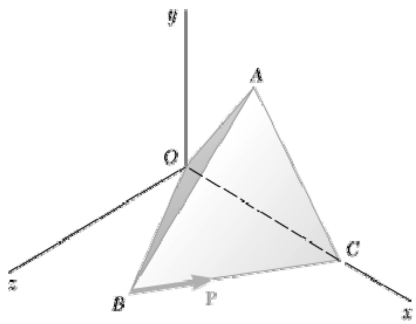
$$\therefore \quad M_{DI} = \frac{5\text{ N}(11.7\text{ m})}{\sqrt{17}}\begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & -1 \\ 1.2 & -3.6 & -11.7 \end{vmatrix}$$

$$= (14.1883\text{ N}\cdot\text{m})\left\{\left[0 - (4)(-1)(-3.6)\right] + \left[(-1)(-1)(1.2) - 0\right]\right\}$$

$$= -187.286\text{ N}\cdot\text{m}$$

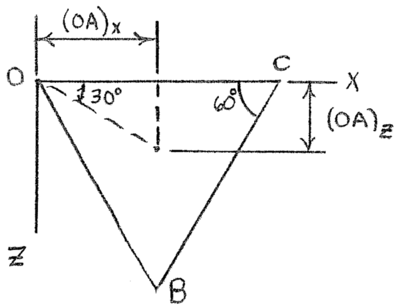
or $M_{DI} = -187.3\text{ N}\cdot\text{m}$ ◀

PROBLEM 3.57



A rectangular tetrahedron has six edges of length a. A force $\mathbf{P}$ is directed as shown along edge BC. Determine the moment of $\mathbf{P}$ about edge OA.

SOLUTION



Have $$M_{OA} = \lambda_{OA} \cdot \left(\mathbf{r}_{C/O} \times \mathbf{P}\right)$$

where

From triangle OBC

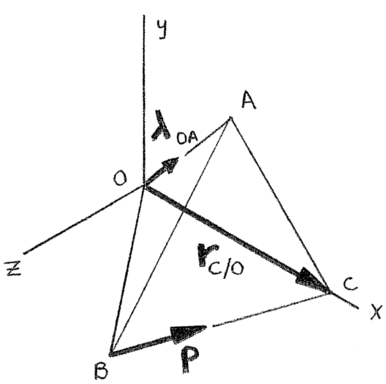
$$\left(OA\right)_x = \frac{a}{2}$$

$$\left(OA\right)_z = \left(OA\right)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $$\left(OA\right)^2 = \left(OA\right)_x^2 + \left(OA\right)_y^2 + \left(OA_z\right)^2$$

or $$a^2 = \left(\frac{a}{2}\right)^2 + \left(OA\right)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$

$$\therefore \quad \left(OA\right)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$



Then $$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and $$\lambda_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

$$\mathbf{P} = \lambda_{BC} P$$

$$= \frac{\left(a \sin 30°\right)\mathbf{i} - \left(a \cos 30°\right)\mathbf{k}}{a}\left(P\right)$$

$$= \frac{P}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$

$$\mathbf{r}_{C/O} = a\mathbf{i}$$

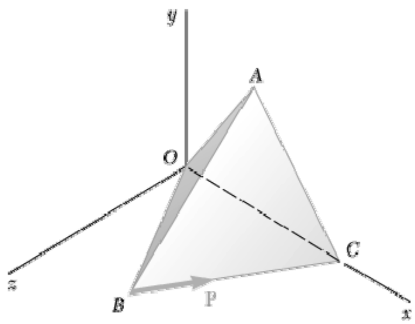
PROBLEM 3.57 CONTINUED

$$\therefore \quad M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix} (a)\left(\frac{P}{2}\right)$$

$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)\left(-\sqrt{3}\right)$$

$$= \frac{aP}{\sqrt{2}}$$

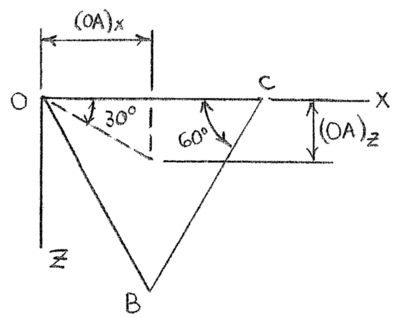
$$M_{OA} = \frac{aP}{\sqrt{2}} \blacktriangleleft$$



PROBLEM 3.58

A rectangular tetrahedron has six edges of length a. (a) Show that two opposite edges, such as OA and BC, are perpendicular to each other. (b) Use this property and the result obtained in Problem 3.57 to determine the perpendicular distance between edges OA and BC.

SOLUTION



(a) For edge OA to be perpendicular to edge BC,

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

where

From triangle OBC

$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

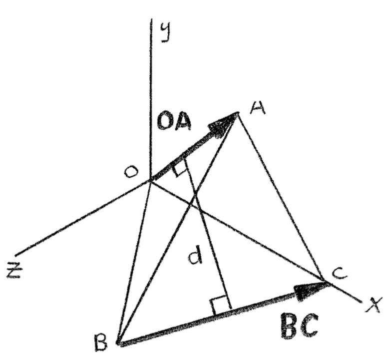
$$\therefore \quad \overrightarrow{OA} = \left(\frac{a}{2}\right)\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$

and

$$\overrightarrow{BC} = (a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}$$

$$= \frac{a}{2}\mathbf{i} - \frac{a\sqrt{3}}{2}\mathbf{k}$$

$$= \frac{a}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$



Then

$$\left[\frac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}\right] \cdot \left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)\frac{a}{2} = 0$$

or

$$\frac{a^2}{4} + (OA)_y(0) - \frac{a^2}{4} = 0$$

$$\therefore \quad \overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

so that

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

PROBLEM 3.58 CONTINUED

(*b*) Have $M_{OA} = Pd$, with P acting along BC and d the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

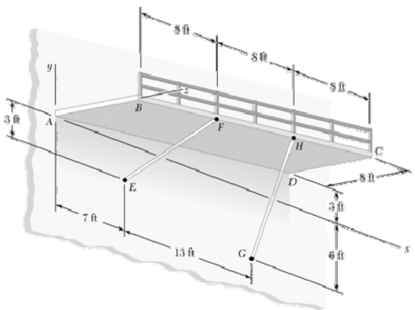
From the results of Problem 3.57,

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

$$\therefore \quad \frac{Pa}{\sqrt{2}} = Pd$$

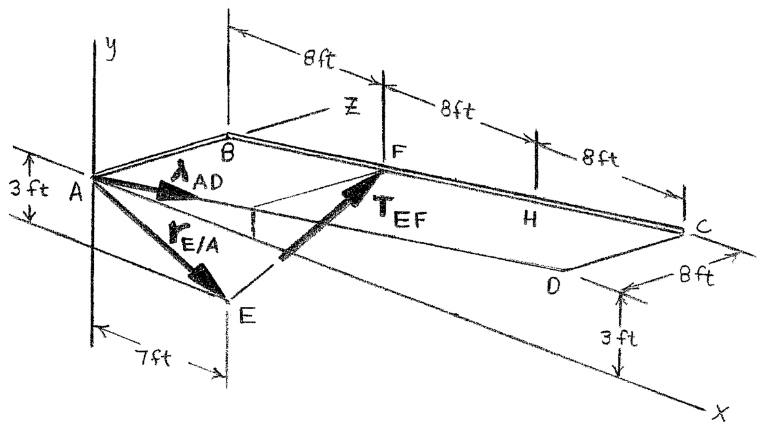
or $$d = \frac{a}{\sqrt{2}} \blacktriangleleft$$

PROBLEM 3.59



The 8-ft-wide portion *ABCD* of an inclined, cantilevered walkway is partially supported by members *EF* and *GH*. Knowing that the compressive force exerted by member *EF* on the walkway at *F* is 5400 lb, determine the moment of that force about edge *AD*.

SOLUTION



Having $$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{E/A} \times \mathbf{T}_{EF}\right)$$

where $$\boldsymbol{\lambda}_{AD} = \frac{(24\text{ ft})\mathbf{i} + (3\text{ ft})\mathbf{j}}{\sqrt{(24)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{65}}(8\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{E/A} = (7\text{ ft})\mathbf{i} - (3\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{EF} = \boldsymbol{\lambda}_{EF} T_{EF} = \frac{(8\text{ ft} - 7\text{ ft})\mathbf{i} + \left[3\text{ ft} + \left(\frac{8}{24}\right)(3\text{ ft})\right]\mathbf{j} + (8\text{ ft})\mathbf{k}}{\sqrt{(1)^2 + (4)^2 + (8)^2}\text{ ft}}(5400\text{ lb})$$

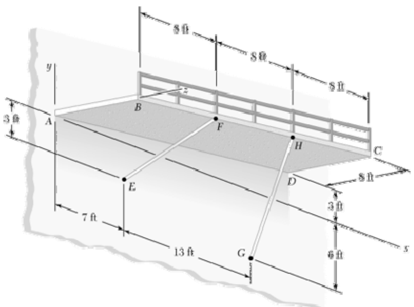
$$= 600\left[(1\text{ lb})\mathbf{i} + (4\text{ lb})\mathbf{j} + (8\text{ lb})\mathbf{k}\right]$$

$$\therefore\ M_{AD} = \frac{600}{\sqrt{65}}\begin{vmatrix} 8 & 1 & 0 \\ 7 & -3 & 0 \\ 1 & 4 & 8 \end{vmatrix}\text{lb}\cdot\text{ft} = \frac{600}{\sqrt{65}}(-192 - 56)\text{lb}\cdot\text{ft}$$

$$= -18{,}456.4\text{ lb}\cdot\text{ft}$$

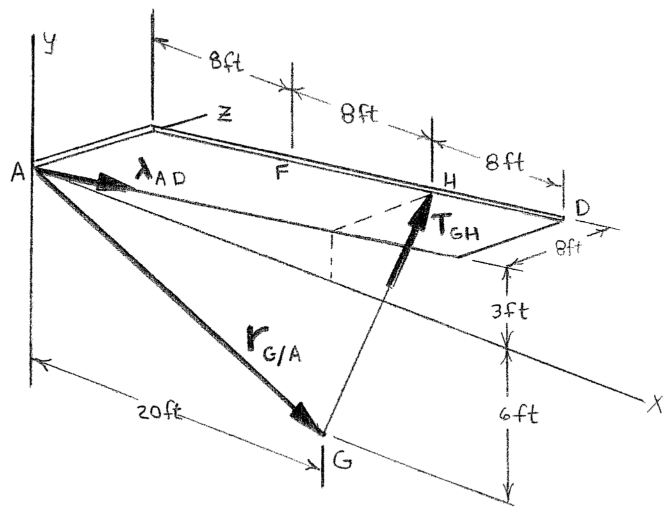
or $M_{AD} = -18.46\text{ kip}\cdot\text{ft}$ ◀

PROBLEM 3.60



The 8-ft-wide portion *ABCD* of an inclined, cantilevered walkway is partially supported by members *EF* and *GH*. Knowing that the compressive force exerted by member *GH* on the walkway at *H* is 4800 lb, determine the moment of that force about edge *AD*.

SOLUTION



Having

$$M_{AD} = \lambda_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{T}_{GH}\right)$$

where

$$\lambda_{AD} = \frac{(24 \text{ ft})\mathbf{i} + (3 \text{ ft})\mathbf{j}}{\sqrt{(24)^2 + (3)^2} \text{ ft}} = \frac{1}{\sqrt{65}}(8\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{G/A} = (20 \text{ ft})\mathbf{i} - (6 \text{ ft})\mathbf{j} = 2\left[(10 \text{ ft})\mathbf{i} - (3 \text{ ft})\mathbf{j}\right]$$

$$\mathbf{T}_{GH} = \lambda_{GH} T_{GH} = \frac{(16 \text{ ft} - 20 \text{ ft})\mathbf{i} + \left[6 \text{ ft} + \left(\tfrac{16}{24}\right)(3 \text{ ft})\right]\mathbf{j} + (8 \text{ ft})\mathbf{k}}{\sqrt{(4)^2 + (8)^2 + (8)^2} \text{ ft}}(4800 \text{ lb})$$

$$= 1600\left[-(1 \text{ lb})\mathbf{i} + (2 \text{ lb})\mathbf{j} + (2 \text{ lb})\mathbf{k}\right]$$

$$\therefore \quad M_{AD} = \frac{(1600 \text{ lb})(2 \text{ ft})}{\sqrt{65}}\begin{vmatrix} 8 & 1 & 0 \\ 10 & -3 & 0 \\ -1 & 2 & 2 \end{vmatrix} = \frac{3200 \text{ lb}\cdot\text{ft}}{\sqrt{65}}(-48 - 20)$$

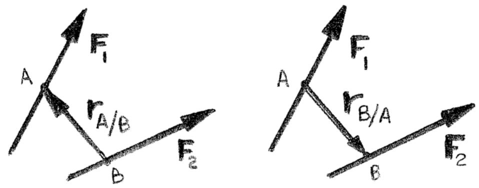
$$= -26{,}989 \text{ lb}\cdot\text{ft}$$

or $M_{AD} = -27.0 \text{ kip}\cdot\text{ft}$ ◀

PROBLEM 3.61

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in space have the same magnitude F. Prove that the moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$ is equal to the moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$.

SOLUTION



First note that $\mathbf{F}_1 = F_1\boldsymbol{\lambda}_1$ and $\mathbf{F}_2 = F_2\boldsymbol{\lambda}_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{M}_1$

and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{M}_2$

Now, by definition

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \mathbf{F}_2\right) = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F_2$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \mathbf{F}_1\right) = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1\right) F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right) F$$

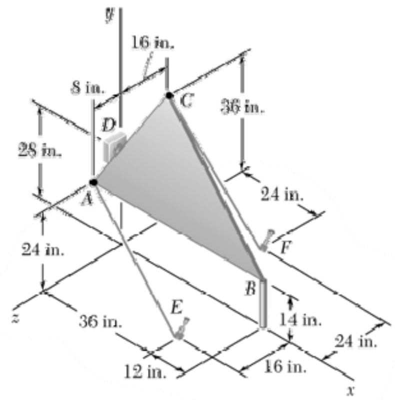
Using Equation (3.39)

$$\boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right)$$

so that $M_2 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$

$$\therefore \quad M_{12} = M_{21} \blacktriangleleft$$

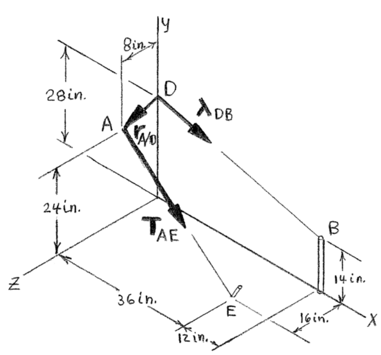
PROBLEM 3.62



In Problem 3.53, determine the perpendicular distance between cable *AE* and the line joining points *D* and *B*.

Problem 3.53: The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 220 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \left(\mathbf{r}_{A/D} \times \mathbf{T}_{AE}\right)$$

where $$\boldsymbol{\lambda}_{DB} = \frac{(48\text{ in.})\mathbf{i} - (14\text{ in.})\mathbf{j}}{50\text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{A/D} = -(4\text{ in.})\mathbf{j} + (8\text{ in.})\mathbf{k}$$

$$\mathbf{T}_{AE} = \boldsymbol{\lambda}_{AE} T_{AE}$$

$$= \frac{(36\text{ in.})\mathbf{i} - (24\text{ in.})\mathbf{j} + (8\text{ in.})\mathbf{k}}{44\text{ in.}}(220\text{ lb})$$

$$= (180\text{ lb})\mathbf{i} - (120\text{ lb})\mathbf{j} + (40\text{ lb})\mathbf{k}$$

$$\therefore \quad M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & -4 & 8 \\ 180 & -120 & 40 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= 364.8\text{ lb}\cdot\text{in.}$$

Only the perpendicular component of $\mathbf{T}_{AE}$ contributes to the moment of $\mathbf{T}_{AE}$ about line *DB*. The parallel component of $\mathbf{T}_{AE}$ will be used to find the perpendicular component.

PROBLEM 3.62 CONTINUED

Have

$$(T_{AE})_{\text{parallel}} = \lambda_{DB} \cdot \mathbf{T}_{AE}$$

$$= (0.96\mathbf{i} - 0.28\mathbf{j}) \cdot \left[(180 \text{ lb})\mathbf{i} - (120 \text{ lb})\mathbf{j} + (40 \text{ lb})\mathbf{k}\right]$$

$$= \left[(0.96)(180) + (-0.28)(-120) + (0)(40)\right]\text{lb}$$

$$= (172.8 + 33.6)\text{lb}$$

$$= 206.4 \text{ lb}$$

Since $\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{perpendicular}} + (\mathbf{T}_{AE})_{\text{parallel}}$

$$\therefore \quad (T_{AE})_{\text{perpendicular}} = \sqrt{(T_{AE})^2 - (T_{AE})^2_{\text{parallel}}}$$

$$= \sqrt{(220)^2 - (206.41)^2}$$

$$= 76.151 \text{ lb}$$

Then $\qquad M_{DB} = (T_{AE})_{\text{perpendicular}}(d)$

$$364.8 \text{ lb}\cdot\text{in.} = (76.151 \text{ lb})d$$

$$d = 4.7905 \text{ in.}$$

or $d = 4.79$ in. ◀