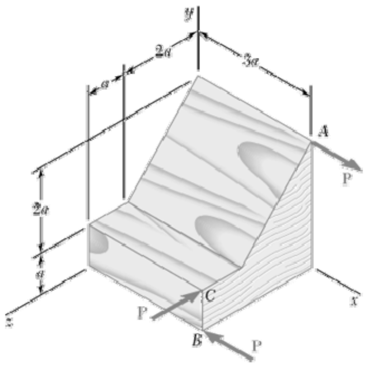
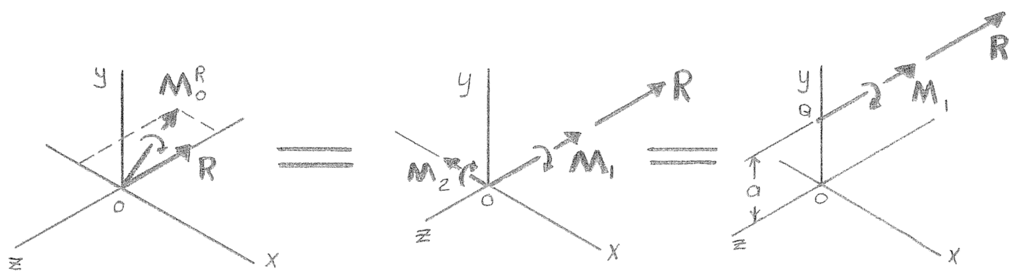
PROBLEM 3.129



A block of wood is acted upon by three forces of the same magnitude P and having the directions shown. Replace the three forces with an equivalent wrench and determine (a) the magnitude and direction of the resultant **R**, (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xy plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma \mathbf{F}: \quad P\mathbf{i} - P\mathbf{i} - P\mathbf{k} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -P\mathbf{k}$$

Have $$\Sigma \mathbf{M}_O: \quad -P(3a)\mathbf{k} - P(3a)\mathbf{j} + P(-a\mathbf{i} + 3a\mathbf{j}) = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = Pa(-\mathbf{i} - 3\mathbf{k})$$

Then let vectors $(\mathbf{R},\ \mathbf{M}_1)$ represent the components of the wrench, where their directions are the same.

(a) $$\mathbf{R} = -P\mathbf{k}$$ or Magnitude of $\mathbf{R} = P$ ◀

Direction of $\mathbf{R}$: $\theta_x = 90°$, $\theta_y = 90°$, $\theta_z = -180°$ ◀

(b) Have $$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R$$

$$= -\mathbf{k} \cdot \left[Pa(-\mathbf{i} - 3\mathbf{k})\right]$$

$$= 3Pa$$

and pitch $$p = \frac{M_1}{R} = \frac{3Pa}{P} = 3a$$ or $p = 3a$ ◀

PROBLEM 3.129 CONTINUED

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = Pa(-\mathbf{i} - 3\mathbf{k}) - (-3Pa\mathbf{k}) = -Pa\mathbf{i}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

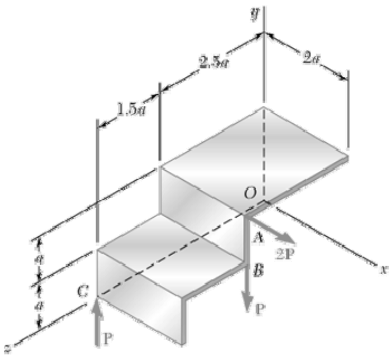
$$-Pa\mathbf{i} = (x\mathbf{i} + y\mathbf{j}) \times (-P)\mathbf{k} = Px\mathbf{j} - Py\mathbf{i}$$

From $$\mathbf{i}: \quad -Pa = -Py \qquad \text{or} \qquad y = a$$

$$\mathbf{j}: \quad x = 0$$

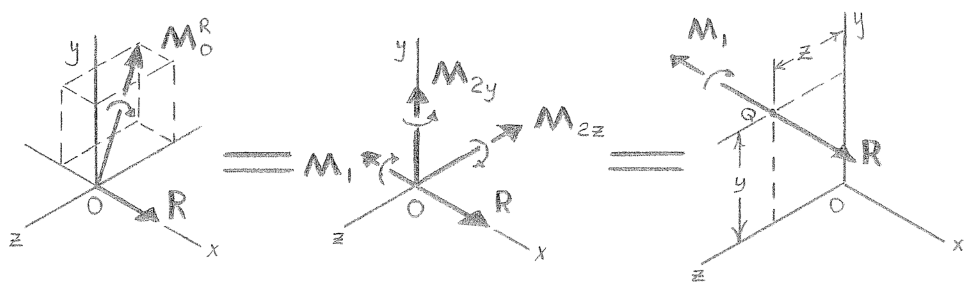
$\therefore$ The axis of the wrench is parallel to the *z*-axis and intersects the *xy* plane at $x = 0,\ y = a$ ◀

PROBLEM 3.130



A piece of sheet metal is bent into the shape shown and is acted upon by three forces. Replace the three forces with an equivalent wrench and determine (*a*) the magnitude and direction of the resultant **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple system at the origin.

Have $$\Sigma \mathbf{F}: \quad (2P)\mathbf{i} - (P)\mathbf{j} + (P)\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = (2P)\mathbf{i}$$

Have $$\Sigma \mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = Pa\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 2.5 \\ 2 & -1 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{vmatrix} = Pa(-1.5\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$$

(*a*) $$\mathbf{R} = 2P\mathbf{i}$$ or Magnitude of $\mathbf{R} = 2P$ ◀

Direction of **R**: $\theta_x = 0°, \quad \theta_y = -90°, \quad \theta_z = 90°$ ◀

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \frac{\mathbf{R}}{R}$$

$$= \mathbf{i} \cdot (-1.5Pa\mathbf{i} + 5Pa\mathbf{j} - 6Pa\mathbf{k})$$

$$= -1.5Pa$$

and pitch $$p = \frac{M_1}{R} = \frac{-1.5Pa}{2P} = -0.75a$$ or $p = -0.75a$ ◀

PROBLEM 3.130 CONTINUED

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (5Pa)\mathbf{j} - (6Pa)\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(5Pa)\mathbf{j} - (6Pa)\mathbf{k} = (y\mathbf{j} + z\mathbf{k}) \times (2P\mathbf{i}) = -(2Py)\mathbf{k} + (2Pz)\mathbf{j}$$

From $$\mathbf{i}: \quad 5Pa = 2Pz$$

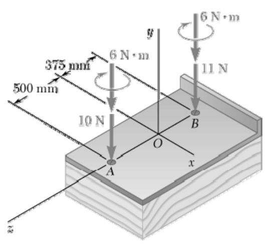
$$\therefore \quad z = 2.5a$$

From $$\mathbf{k}: \quad -6Pa = -2Py$$

$$\therefore \quad y = 3a$$

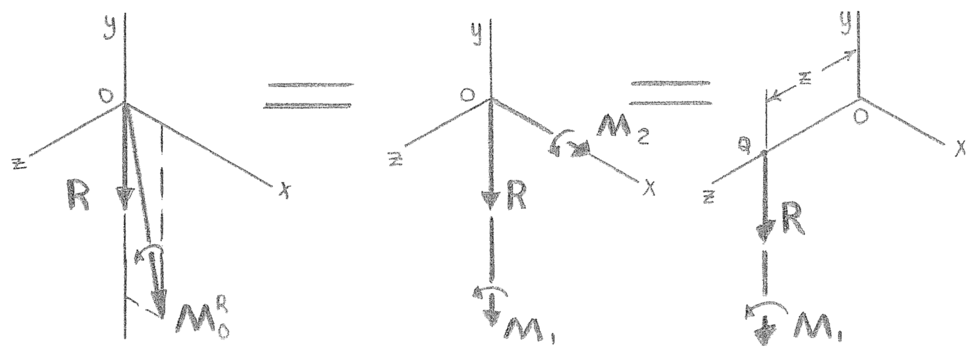
$\therefore$ The axis of the wrench is parallel to the x-axis and intersects the yz-plane at $y = 3a,\ z = 2.5a$ ◀

PROBLEM 3.131



The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma\mathbf{F}: \quad -(10\text{ N})\mathbf{j} - (11\text{ N})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(21\text{ N})\mathbf{j}$$

Have $$\Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.5 \\ 0 & -10 & 0 \end{vmatrix} \text{N}\cdot\text{m} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.375 \\ 0 & -11 & 0 \end{vmatrix} \text{N}\cdot\text{m} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

$$= (0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $$\mathbf{R} = -(21\text{ N})\mathbf{j}$$ or $\mathbf{R} = -(21\text{ N})\mathbf{j}$ ◀

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \frac{\mathbf{R}}{R}$$

$$= (-\mathbf{j}) \cdot \left[(0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}\right]$$

$$= 12\text{ N}\cdot\text{m} \qquad \text{and} \qquad \mathbf{M}_1 = -(12\text{ N}\cdot\text{m})\mathbf{j}$$

and pitch $$p = \frac{M_1}{R} = \frac{12\text{ N}\cdot\text{m}}{21\text{ N}} = 0.57143\text{ m}$$ or $p = 0.571$ m ◀

PROBLEM 3.131 CONTINUED

(*c*) Have

$$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (0.875 \text{ N}\cdot\text{m})\mathbf{i}$$

Require

$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$\therefore \quad (0.875 \text{ N}\cdot\text{m})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times \left[-(21 \text{ N})\mathbf{j}\right]$$

$$0.875\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

From **i**:

$$0.875 = 21z$$

$$\therefore \quad z = 0.041667 \text{ m}$$

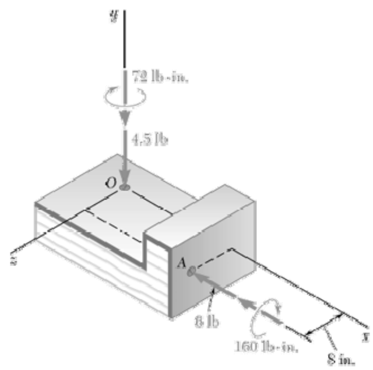
From **k**:

$$0 = -21x$$

$$\therefore \quad z = 0$$

$\therefore$ The axis of the wrench is parallel to the *y*-axis and intersects the *xz*-plane at $x = 0, \; z = 41.7 \text{ mm}$ ◀

PROBLEM 3.132



The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION

First, reduce the given force system to a force-couple system.

Have $\quad \Sigma\mathbf{F}: \quad -(6 \text{ lb})\mathbf{i} - (4.5 \text{ lb})\mathbf{j} = \mathbf{R} \qquad R = 7.5 \text{ lb}$

Have $\quad \Sigma\mathbf{M}_O: \quad \sum(\mathbf{r}_O \times \mathbf{F}) + \sum\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = -6 \text{ lb}(8 \text{ in.})\,\mathbf{j} - (160 \text{ lb}\cdot\text{in.})\mathbf{i} - (72 \text{ lb}\cdot\text{in.})\,\mathbf{j}$$

$$= -(160 \text{ lb}\cdot\text{in.})\mathbf{i} - (120 \text{ lb}\cdot\text{in.})\,\mathbf{j}$$

$$M_O^R = 200 \text{ lb}\cdot\text{in.}$$

(*a*) $\quad \mathbf{R} = -(6 \text{ lb})\mathbf{i} - (4.5 \text{ lb})\,\mathbf{j}$ ◀

(*b*) Have $\quad M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda = \dfrac{\mathbf{R}}{R}$

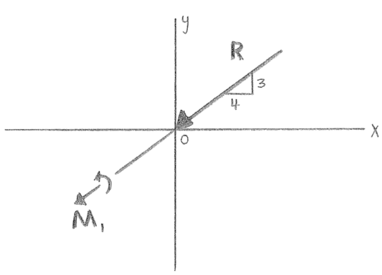
$$= (-0.8\mathbf{i} - 0.6\mathbf{j}) \cdot \left[-(160 \text{ lb}\cdot\text{in.})\mathbf{i} - (120 \text{ lb}\cdot\text{in.})\,\mathbf{j}\right]$$

$$= 200 \text{ lb}\cdot\text{in.}$$

and $\quad \mathbf{M}_1 = 200 \text{ lb}\cdot\text{in.}(-0.8\mathbf{i} - 0.6\mathbf{j})$

Pitch $\quad p = \dfrac{M_1}{R} = \dfrac{200 \text{ lb}\cdot\text{in.}}{7.50 \text{ lb}} = 26.667 \text{ in.}$

or $p = 26.7$ in. ◀



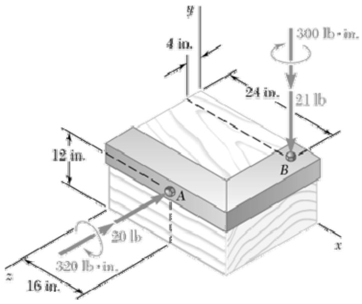
(*c*) From above note that

$$\mathbf{M}_1 = \mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the *xy* plane with a slope of

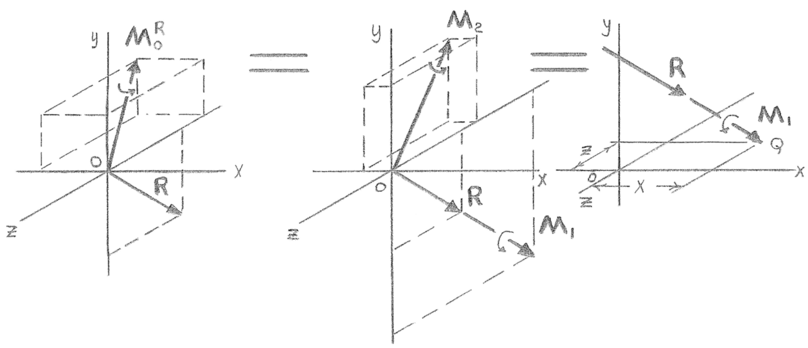
$$\frac{dy}{dx} = \frac{3}{4} \text{ ◀}$$

PROBLEM 3.133



Two bolts A and B are tightened by applying the forces and couple shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $\Sigma \mathbf{F}: \quad -(20\text{ lb})\mathbf{k} - (21\text{ lb})\mathbf{j} = -(21\text{ lb})\mathbf{j} - (20\text{ lb})\mathbf{k} = \mathbf{R} \qquad R = 29\text{ lb}$

and $\Sigma \mathbf{M}_O: \quad \sum(\mathbf{r}_O \times \mathbf{F}) + \sum \mathbf{M}_C = \mathbf{M}_O^R$

$$20\text{ lb}(4\text{ in.})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 0 \\ 0 & 0 & -1 \end{vmatrix} + 21\text{ lb}(4\text{ in.})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} + (-300\mathbf{j} - 320\mathbf{k})\text{lb}\cdot\text{in.} = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = -(156\text{ lb}\cdot\text{in.})\mathbf{i} + (20\text{ lb}\cdot\text{in.})\mathbf{j} - (824\text{ lb}\cdot\text{in.})\mathbf{k}$$

(*a*) $\mathbf{R} = -(21\text{ lb})\mathbf{j} - (20\text{ lb})\mathbf{k}$ ◀

(*b*) Have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \dfrac{\mathbf{R}}{R}$

$$= -\frac{-21\mathbf{j} - 20\mathbf{k}}{29} \cdot \left[-(156\text{ lb}\cdot\text{in.})\mathbf{i} + (20\text{ lb}\cdot\text{in.})\mathbf{j} - (824\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= 553.80\text{ lb}\cdot\text{in.}$$

PROBLEM 3.133 CONTINUED

and $$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R = -(401.03 \text{ lb}\cdot\text{in.})\mathbf{j} - (381.93 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $$p = \frac{M_1}{R} = \frac{553.80 \text{ lb}\cdot\text{in.}}{29 \text{ lb}} = 19.0964 \text{ in.}$$ or $p = 19.10$ in. ◀

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \left[(-156\mathbf{i} + 20\mathbf{j} - 824\mathbf{k}) - (-401.03\mathbf{j} - 381.93\mathbf{k})\right]\text{lb}\cdot\text{in.}$$

$$= -(156.0 \text{ lb}\cdot\text{in.})\mathbf{i} + (421.03 \text{ lb}\cdot\text{in.})\mathbf{j} - (442.07 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(-156\mathbf{i} + 421.03\mathbf{j} - 442.07\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (-21\mathbf{j} - 20\mathbf{k})$$

$$= (21z)\mathbf{i} + (20x)\mathbf{j} - (21x)\mathbf{k}$$

From **i**: $$-156 = 21z$$

$$\therefore \quad z = -7.4286 \text{ in.}$$

or $$z = -7.43 \text{ in.}$$

From **k**: $$-442.07 = -21x$$

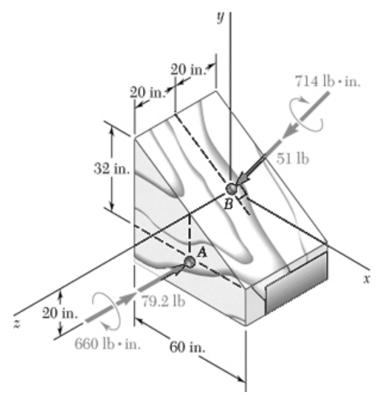
$$\therefore \quad x = 21.051 \text{ in.}$$

or $$x = 21.1 \text{ in.}$$

∴ The axis of the wrench intersects the *xz*-plane at

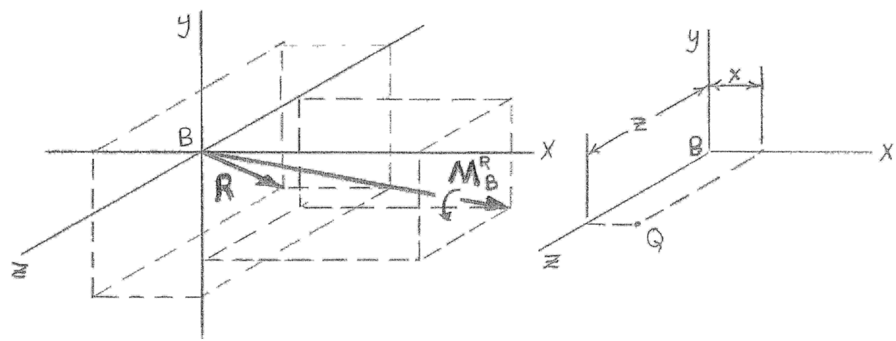
$x = 21.1$ in., $z = -7.43$ in. ◀

PROBLEM 3.134



Two bolts A and B are tightened by applying the forces and couple shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant $\mathbf{R}$, (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz plane.

SOLUTION



First reduce the given force system to a force-couple at the origin at B.

(a) Have $$\Sigma\mathbf{F}: \quad -(79.2\text{ lb})\mathbf{k} - (51\text{ lb})\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(24.0\text{ lb})\mathbf{i} - (45.0\text{ lb})\mathbf{j} - (79.2\text{ lb})\mathbf{k} \blacktriangleleft$$

and $$R = 94.2\text{ lb}$$

Have $$\Sigma\mathbf{M}_B: \quad \mathbf{r}_{A/B} \times \mathbf{F}_A + \mathbf{M}_A + \mathbf{M}_B = \mathbf{M}_B^R$$

$$\mathbf{M}_B^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -20 & 0 \\ 0 & 0 & -79.2 \end{vmatrix} - 660\mathbf{k} - 714\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = 1584\mathbf{i} - 660\mathbf{k} - 42(8\mathbf{i} + 15\mathbf{j})$$

$$\therefore \quad \mathbf{M}_B^R = (1248\text{ lb}\cdot\text{in.})\mathbf{i} - (630\text{ lb}\cdot\text{in.})\mathbf{j} - (660\text{ lb}\cdot\text{in.})\mathbf{k}$$

(b) Have $$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$

$$= \frac{-24.0\mathbf{i} - 45.0\mathbf{j} - 79.2\mathbf{k}}{94.2} \cdot \left[(1248\text{ lb}\cdot\text{in.})\mathbf{i} - (630\text{ lb}\cdot\text{in.})\mathbf{j} - (660\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= 537.89\text{ lb}\cdot\text{in.}$$

PROBLEM 3.134 CONTINUED

and $\mathbf{M}_1 = M_1 \boldsymbol{\lambda}_R$

$$= -(137.044 \text{ lb}\cdot\text{in.})\mathbf{i} - (256.96 \text{ lb}\cdot\text{in.})\mathbf{j} - (452.24 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $p = \dfrac{M_1}{R} = \dfrac{537.89 \text{ lb}\cdot\text{in.}}{94.2 \text{ lb}} = 5.7101 \text{ in.}$ or $p = 5.71 \text{ in.}$ ◀

(*c*) Have $\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\therefore\ \mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (1248\mathbf{i} - 630\mathbf{j} - 660\mathbf{k}) - (-137.044\mathbf{i} - 256.96\mathbf{j} - 452.24\mathbf{k})$$

$$= (1385.04 \text{ lb}\cdot\text{in.})\mathbf{i} - (373.04 \text{ lb}\cdot\text{in.})\mathbf{j} - (207.76 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$

$$1385.04\mathbf{i} - 373.04\mathbf{j} - 207.76\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -24 & -45 & -79.2 \end{vmatrix}$$

$$= (45z)\mathbf{i} - (24z)\mathbf{j} + (79.2x)\mathbf{j} - (45x)\mathbf{k}$$

From **i**: $1385.04 = 45z \qquad \therefore\ z = 30.779 \text{ in.}$

From **k**: $-207.76 = -45x \qquad \therefore\ x = 4.6169 \text{ in.}$

$\therefore$ The axis of the wrench intersects the *xz*-plane at

$x = 4.62 \text{ in.},\ \ z = 30.8 \text{ in.}$ ◀