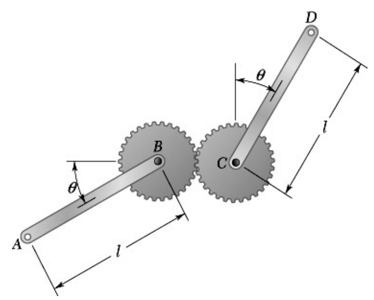
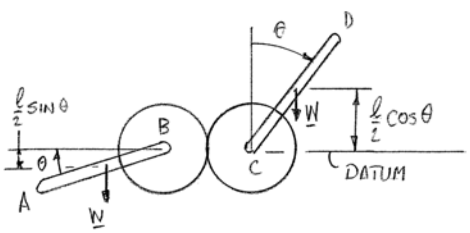


PROBLEM 10.71

Two uniform rods, each of mass *m*, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential Energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \qquad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$

For Equilibrium: $\frac{dV}{d\theta} = 0$: $\sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0°$ and $\theta = 135.0°$

Stability:

At $\theta = -45.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}\left[\sin(-45°) - \cos 45°\right]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

$\therefore \quad \theta = -45.0°$, Unstable ◀

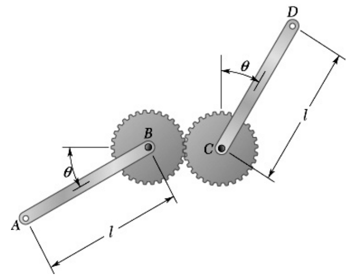
At $\theta = 135.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135° - \cos 135°)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

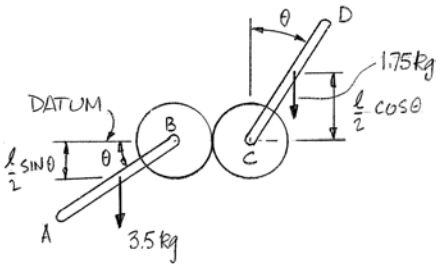
$\therefore \quad \theta = 135.0°$, Stable ◀

PROBLEM 10.72



Two uniform rods, *AB* and *CD*, are attached to gears of equal radii as shown. Knowing that $m_{AB} = 3.5\text{ kg}$ and $m_{CD} = 1.75\text{ kg}$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential Energy

$$V = \left(3.5\text{ kg} \times 9.81\text{ m/s}^2\right)\left(-\frac{l}{2}\sin\theta\right) + \left(1.75\text{ kg} \times 9.81\text{ m/s}^2\right)\left(\frac{l}{2}\cos\theta\right)$$

$$= \left(8.5838\text{ N}\right)l\left(-2\sin\theta + \cos\theta\right)$$

$$\frac{dV}{d\theta} = \left(8.5838\text{ N}\right)l\left(-2\cos\theta - \sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left(2\sin\theta - \cos\theta\right)$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \quad -2\cos\theta - \sin\theta = 0$$

or $$\tan\theta = -2$$

Thus $$\theta = -63.4° \qquad \text{and} \qquad 116.6°$$

Stability

At $\theta = -63.4°$: $$\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(-63.4°\right) - \cos\left(-63.4°\right)\right]$$

$$= \left(8.5838\text{ N}\right)l\left(-1.788 - 0.448\right) < 0$$

$\therefore \quad \theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$: $$\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(116.6°\right) - \cos\left(116.6°\right)\right]$$

$$= \left(8.5838\text{ N}\right)l\left(1.788 + 0.447\right) > 0$$

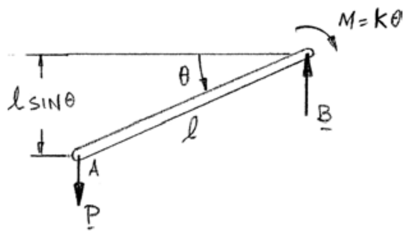
$\therefore \quad \theta = 116.6°$, Stable ◀

PROBLEM 10.73

Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsional spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

SOLUTION

Potential Energy



$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium:
$$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For $\quad P = 400 \text{ lb}, \quad l = 10 \text{ in.}, \quad K = 150 \text{ lb}\cdot\text{ft/rad}$

$$\cos\theta = \frac{150 \text{ lb}\cdot\text{ft/rad}}{(400 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)}\theta$$

$$= 0.450\theta$$

Solving numerically, we obtain

$$\theta = 1.06896 \text{ rad} = 61.247°$$

$\theta = 61.2°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (400 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 61.2° > 0$$

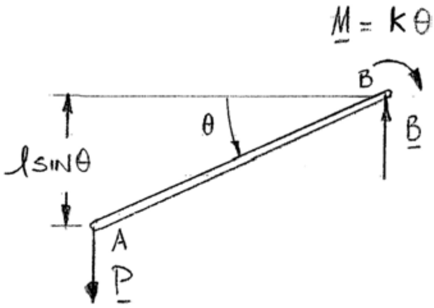
$\therefore$ Stable ◀

PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, of neutral. (See the hint for Problem 10.73.)

SOLUTION

Potential Energy



$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $\quad \frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$

For $\quad P = 1260 \text{ lb}, \ l = 10 \text{ in., and } K = 150 \text{ lb}\cdot\text{ft/rad}$

$$\cos\theta = \frac{150 \text{ lb}\cdot\text{ft/rad}}{(1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)}\theta$$

or $\quad \cos\theta = \frac{\theta}{7}$

Solving numerically, $\quad \theta = 1.37333 \text{ rad, } 5.652 \text{ rad, and } 6.616 \text{ rad}$

or $\quad \theta = 78.7°, \ 323.8°, \ 379.1°$

Stability At $\theta = 78.7°$: $\quad \frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 78.7°$

$$= 1179.6 \text{ ft}\cdot\text{lb} > 0 \qquad \therefore \quad \theta = 78.7°, \ \text{Stable} ◀$$

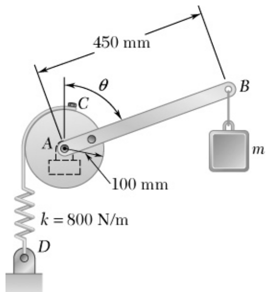
At $\theta = 323.8°$: $\quad \frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 323.8°$

$$= -470 \text{ ft}\cdot\text{lb} < 0 \qquad \therefore \quad \theta = 324°, \ \text{Unstable} ◀$$

At $\theta = 379.1°$: $\quad \frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 379.1°$

$$= 493.5 \text{ ft}\cdot\text{lb} > 0 \qquad \therefore \quad \theta = 379°, \ \text{Stable} ◀$$

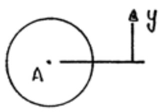
PROBLEM 10.75



Angle θ is equal to 45° after a block of mass m is hung from member AB as shown. Neglecting the mass of AB and knowing that the spring is unstretched when $\theta = 20°$, determine the value of m and state whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Potential Energy



Have $$V = \frac{1}{2}kx_{SP}^2 + mgy_B$$

where $$x_{SP} = r(\theta - \theta_0), \quad r = 100 \text{ mm}, \quad \theta_0 = 20° = \frac{\pi}{9} \text{ rad}$$

$$y_B = L_{AB}\cos\theta, \qquad L_{AB} = 450 \text{ mm}$$

Then $$V = \frac{1}{2}kr^2(\theta - \theta_0)^2 + mgL_{AB}\cos\theta$$

and $$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - mgL_{AB}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = kr^2 - mgL_{AB}\cos\theta$$

With $$k = 800 \text{ N/m}, \qquad \theta = 45°$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \quad (800 \text{ N/m})(0.1 \text{ m})^2\left(\frac{\pi}{4} - \frac{\pi}{9}\right) - m(9.81 \text{ m/s}^2)(0.45 \text{ m})\sin\frac{\pi}{4} = 0$$

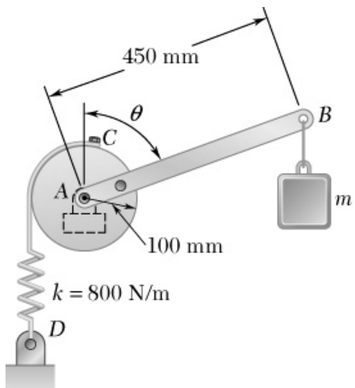
Then $$m = 1.11825 \text{ kg}$$ $$m = 1.118 \text{ kg} \blacktriangleleft$$

Stability

Now $$\frac{d^2V}{d\theta^2} = (800 \text{ N/m})(0.1 \text{ m})^2 - (1.118 \text{ kg})(9.81 \text{ m/s}^2)(0.45 \text{ m})\cos\frac{\pi}{4}$$

$$= 4.51 \text{ J} > 0$$

$\therefore$ Stable $\blacktriangleleft$



PROBLEM 10.76

A block of mass m is hung from member AB as shown. Neglecting the mass of AB and knowing that the spring is unstretched when $\theta = 20°$, determine the value of θ corresponding to equilibrium when $m = 3\text{ kg}$. State whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the general results of Problem 10.76 and noting that now

$$m = 3\text{ kg}, \qquad \text{and} \qquad \theta_0 = 20°$$

we have

Equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad kr^2(\theta - \theta_0) - mgL_{AB}\sin\theta = 0$

$$(800\text{ N/m})(0.1\text{ m})^2\left(\theta - \frac{\pi}{9}\right) - (3\text{ kg})(9.81\text{ m/s}^2)(0.45\text{ m})\sin\theta = 0$$

or

$$\left(\theta - \frac{\pi}{9}\right) - 1.65544\sin\theta = 0$$

Solving numerically, $\quad \theta = 1.91011\text{ rad}$

$$= 109.441°$$

or $\theta = 109.4°$ ◀

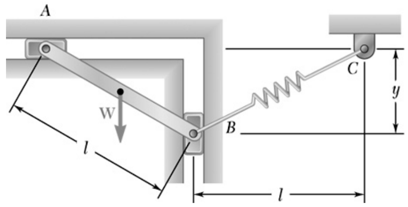
Stability

$$\frac{d^2v}{d\theta^2} = kr^2 - mgL_{AB}\cos\theta$$

$$= (800\text{ N/m})(0.1\text{ m})^2 - (3\text{ kg})(9.81\text{ m/s})(0.45\text{ m})\cos(109.4°)$$

$$= 12.41\text{ J} > 0$$

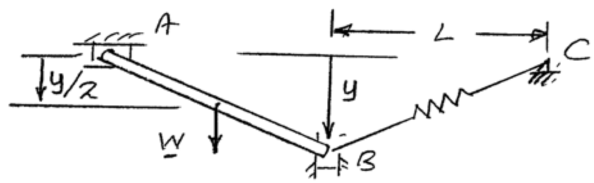
$\therefore$ Stable ◀



PROBLEM 10.77

A slender rod AB, of mass m, is attached to two blocks A and B which can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $m = 12$ kg, $l = 750$ mm, and $k = 900$ N/m.

SOLUTION



Deflection of spring = s, where $$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential Energy: $$V = \frac{1}{2}ks^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium $$\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$$

Now $$W = mg = (12\text{ kg})(9.81\text{ m/s}^2) = 117.72\text{ N}, \; l = 0.75\text{ m, and } k = 900\text{ N/m}$$

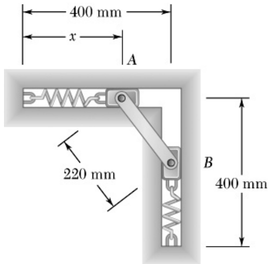
Then $$\left(1 - \frac{0.75\text{ m}}{\sqrt{(0.75\text{ m})^2 + y^2}}\right)y = \frac{1}{2}\frac{(117.72\text{ N})}{(900\text{ N/m})}$$

or $$\left(1 - \frac{0.75}{\sqrt{0.5625 + y^2}}\right)y = 0.6540$$

Solving numerically, $$y = 0.45342\text{ m}$$

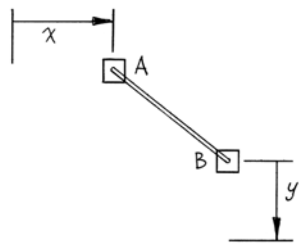
$$y = 453\text{ mm} \blacktriangleleft$$

PROBLEM 10.78



The slender rod *AB* of negligible mass is attached to two 4-kg blocks *A* and *B* that can move freely in the guides shown. Knowing that the constant of the springs is 160 N/m and that the unstretched length of each spring is 150 mm, determine the value of *x* corresponding to equilibrium.

SOLUTION



First note
$$y = \left[0.4 - \sqrt{(0.22)^2 - (0.4 - x)^2}\right]\text{m}$$
$$= \left(0.4 - \sqrt{-x^2 + 0.8x - 0.1116}\right)\text{m}$$

Now, the Potential Energy is
$$V = \frac{1}{2}k(x - 0.15)^2 + \frac{1}{2}k(y - 0.15)^2 + 0.4m_A g + m_B g y$$
$$= \frac{1}{2}k(x - 0.15)^2 + \frac{1}{2}k\left(0.25 - \sqrt{-x^2 + 0.8x - 0.1116}\right)^2$$
$$+0.4m_A g + m_B g\left(0.4 - \sqrt{-x^2 + 0.8x - 0.1116}\right)$$

For Equilibrium
$$\frac{dV}{d\theta} = 0: \quad k(x - 0.15) + k\left(0.25 - \sqrt{-x^2 + 0.8x - 0.1116}\right)\left(-\frac{0.8 - 2x}{2\sqrt{-x^2 + 0.8x - 0.1116}}\right)$$
$$- m_B g\frac{0.8 - 2x}{2\sqrt{-x^2 + 0.8x - 0.1116}} = 0$$

Simplifying,
$$k(x - 0.4) + \sqrt{-x^2 + 0.8x - 0.1116} + 4m_B g(x - 0.4) = 0$$

Substituting the masses, $m_A = m_B = 0.4$ kg, and the spring constant $k = 160$ N/m:
$$(160\text{ N/m})\left(x - 0.4 + \sqrt{-x^2 + 0.8x - 0.1116}\right)\text{m}^2 + 4(4\text{ kg})(9.81\text{ m/s}^2)(x - 0.4)\text{m} = 0$$

PROBLEM 10.78 CONTINUED

or
$$\left(x - 0.4 + \sqrt{-x^2 + 0.8x - 0.1116}\right) + 0.981(x - 0.4) = 0$$

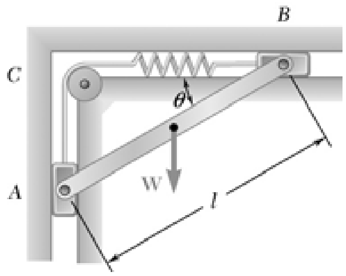
Simplifying,
$$\left(0.8x - x^2 - 0.1116\right)^2 = (0.7924 - 1.981x)^2$$

or
$$4.92436^2 - 3.93949x + 0.739498 = 0$$

Then
$$x = \frac{3.93949 \pm \sqrt{(-3.93949)^2 - 4(4.92436)(0.739498)}}{2(4.92436)}$$

or
$$x = 0.49914 \text{ m} \qquad \text{and} \qquad x = 0.30086 \text{ m}$$

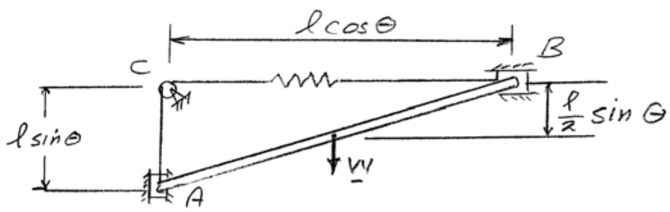
Now
$$x \leq 0.4 \text{ m} \Rightarrow x = 301 \text{ mm} \blacktriangleleft$$



PROBLEM 10.79

A slender rod *AB*, of mass *m*, is attached to two blocks *A* and *B* that can move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when *AB* is horizontal. Neglecting the weight of the blocks, derive an equation in θ, m, l, and k that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of Spring:

$$s = l\sin\theta + l\cos\theta - l$$

$$s = l(\sin\theta + \cos\theta - 1)$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - W\frac{l}{2}\sin\theta \qquad W = mg$$

$$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$$

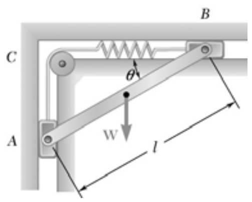
$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta \tag{1}$$

Equilibrium:

$$\frac{dV}{d\theta} = 0{:}\ (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$$

or

$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0 \blacktriangleleft$$



PROBLEM 10.80

A slender rod AB, of mass m, is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when AB is horizontal, determine three values of θ corresponding to equilibrium when $m = 125$ kg, $l = 320$ mm, and $k = 15$ kN/mm. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.79, particularly the condition of equilibrium

$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0$$

Now, with $W = mg = (125 \text{ kg})(9.81 \text{ m/s}^2) = 1226.25 \text{ N}$, $l = 320$ mm, and $k = 15$ kN/m,

Now $$\frac{W}{2kl} = \frac{1226.25 \text{ N}}{2(15000 \text{ N/m})(0.32 \text{ m})} = 1.2773$$

so that $$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - 1.2773\right] = 0$$

By inspection, one solution is $\cos\theta = 0$ or $\theta = 90.0°$

Solving numerically: $\theta = 0.38338 \text{ rad} = 9.6883°$ and $\theta = 0.59053 \text{ rad} = 33.8351°$

Stability

$$\frac{d^2V}{d\theta^2} = kl^2\left[(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta - 1)(-\sin\theta - \cos\theta)\right] + \frac{1}{2}mgl\sin\theta$$

$$= kl^2\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta - 2\sin\theta\cos\theta + \sin\theta + \cos\theta + \frac{mg}{2kl}\sin\theta\right]$$

$$= kl^2\left[\left(1 + \frac{mg}{2kl}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$= (15 \text{ N/m})(0.32 \text{ m})^2\left[(1 - 127.73)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

Thus, at

At $\theta = 90°$: $\frac{d^2V}{d\theta^2} = 89.7 > 0$ $\therefore\ \theta = 90.0°$, Stable ◀

At $\theta = 9.6883°$: $\frac{d^2V}{d\theta^2} = 0.512 > 0$ $\therefore\ \theta = 9.69°$, Stable ◀

At $\theta = 33.8351°$: $\frac{d^2V}{d\theta^2} = -0.391 < 0$ $\therefore\ \theta = 33.8°$, Unstable ◀