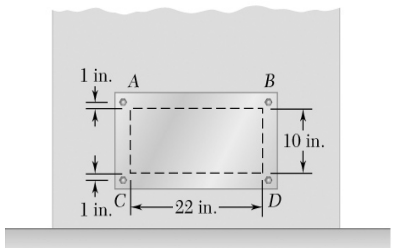
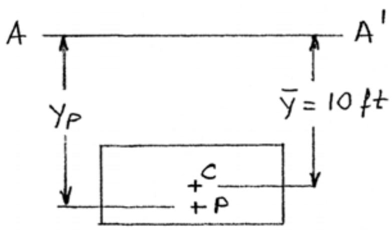


PROBLEM 9.61

The cover for a 10 × 22-in. access hole in an oil storage tank is attached to the outside of the tank with four bolts as shown. Knowing that the specific weight of the oil is 57.4 lb/ft^3 and that the center of the cover is located 10 ft below the surface of the oil, determine the additional force on each bolt because of the pressure of the oil.

SOLUTION



Using the equation developed on page 491 of the text have

$$y_P = \frac{I_{AA'}}{\bar{y}A} \qquad R = \gamma \bar{y} A$$

Then $$R = 57.4 \text{ lb/ft}^3 \times 10 \text{ ft} \times (22 \times 10) \text{ in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2}$$

$$= 876.94 \text{ lb}$$

and $$I_{AA'} = \frac{1}{12}(22 \text{ in.})(10 \text{ in.})^3 + [(22 \text{ in.})(10 \text{ in.})](120 \text{ in.})^2$$

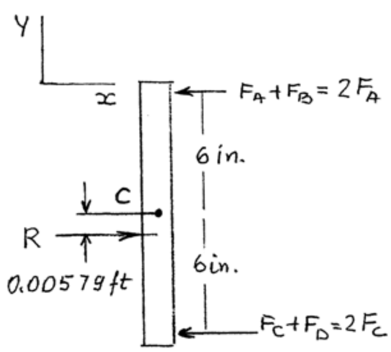
$$= 3.169833 \times 10^6 \text{ in}^4 = 152.8662 \text{ ft}^4$$

and $$\bar{y}A = 10 \text{ ft} \times (22 \times 10) \text{ in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 15.27778 \text{ ft}^3$$

Then $$y_p = \frac{152.8662 \text{ ft}^4}{15.27778 \text{ ft}^2} = 10.00579 \text{ ft}$$

Now symmetry implies $F_A = F_B$ and $F_C = F_D$

Equilibrium



$$\Sigma M_{CD} = 0: \quad (1 \text{ ft})(2FAa) - (0.5 - 0.00579) \text{ ft} \times 876.94 \text{ lb} = 0$$

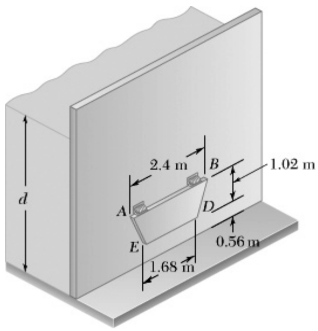
$$F_A = 216.70 \text{ lb}$$

or $F_A = F_B = 217 \text{ lb}$ ◀

Also $\Sigma F_x = 0$:

or $$-2(216.70) + 876.94 - 2F_C = 0$$

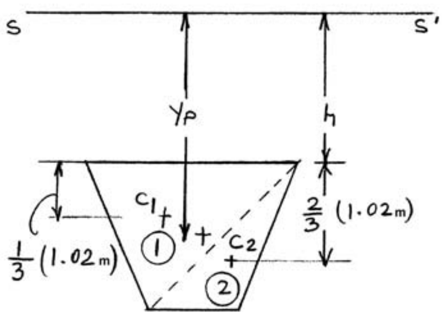
or $F_C = F_D = 222 \text{ lb}$ ◀



PROBLEM 9.62

A vertical trapezoidal gate that is used as an automatic valve is held shut by two springs attached to hinges located along edge *AB*. Knowing that each spring exerts a couple of magnitude $8.50\ \text{kN}\cdot\text{m}$, determine the depth *d* of water for which the gate will open.

SOLUTION



$$d = (h + 1.58)\text{ m}$$

From page 491 $\quad y_p = \dfrac{I_{ss'}}{\bar{y}A} \qquad \gamma = \rho g \qquad R = \gamma \bar{y}A$

Now $\quad \bar{y}A = \Sigma \bar{y}A$

$$= \left[(h + 0.34\text{ m})\right]\left(\frac{1}{2} \times 2.4\text{ m} \times 1.02\text{ m}\right) + \left[(h + 0.68\text{ m})\right]\left(\frac{1}{2} \times 1.68\text{ m} \times 1.02\text{ m}\right)$$

$$= (2.0808h + 0.99878)\text{m}^3$$

Also, $\quad R = (10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(2.0808h + 0.99878)\text{m}^3$

$$= 20\ 413(h + 0.480)\text{ N}$$

And $\quad I_{ss'} = (I_{ss'})_1 + (I_{ss'})_2 = \left\{\frac{1}{36}(2.4\text{ m})(1.02\text{ m})^3 + \left[\frac{1}{2}(2.4\text{ m})(1.02\text{ m})\right](h + 0.34)^2\text{ m}^2\right\}$

$$+ \left\{\frac{1}{36}(1.68\text{ m})(1.08\text{ m})^3 + \left[\frac{1}{2}(1.68\text{ m})(1.02\text{ m})\right](h + 0.68)^2\text{ m}^2\right\}$$

$$= \left[0.07075 + 1.224(h + 0.34)^2 + 0.04952 + 0.8568(h + 0.68)^2\right]\text{m}^4$$

$$= \left[0.12027 + 1.224(h^2 + 0.68h + 0.1156) + 0.8568(h^2 + 1.36h + 0.4624)\right]\text{m}^4$$

$$= (2.0808h^2 + 1.9976h + 0.65795)\text{ m}^4$$

PROBLEM 9.62 CONTINUED

Then
$$y_p = \frac{\left(2.0808h^2 + 1.9976h + 0.65795\right)\text{ m}^4}{\left(2.0808h + 0.99878\right)\text{ m}^3}$$

$$= \frac{h^2 + 0.960h + 0.3162}{h + 0.480}\text{ m}$$

For gate to open
$$\Sigma M_{AB} = 0: \quad M_{\text{open}} - \left(y_p - h\right)R = 0$$

$$2\left(8500\text{ N}\cdot\text{m}\right) - \left[\left(\frac{h^2 + 0.960h + 0.3162}{h + 0.480} - h\right)\text{m}\right]\left[\left(20\ 413\right)\left(h + 0.48\right)\right]\text{N} = 0$$

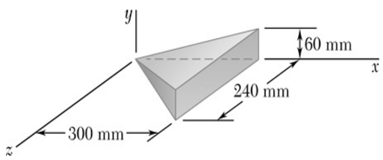
or
$$17\ 000 = 20\ 413\left(h^2 + 0.96h + 0.3162 - h^2 - 0.480h\right)$$

or
$$0.48h - 0.5166 = 0$$

$$h = 1.0763\text{ m}$$

Now
$$d = h + 1.58\text{ m} = \left(1.0763 + 1.58\right)\text{m} = 2.6563\text{ m}$$

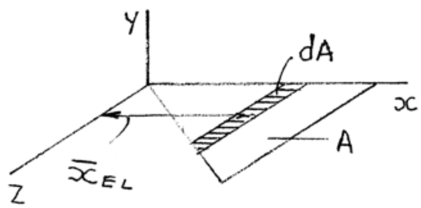
or $d = 2.66\text{ m}$ ◀



PROBLEM 9.63

Determine the x coordinate of the centroid of the volume shown. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



Have $$\bar{x} = \frac{\int \bar{x}_{EL} dV}{\int dV}$$

where $$dV = y dA \qquad \text{and} \qquad \bar{x}_{EL} = x$$

Now $$y = \frac{60}{300}x = \frac{1}{5}x$$

Then $$\bar{x} = \frac{\int x\left(\frac{1}{5}x\right)dA}{\int\left(\frac{1}{5}x\right)dA}$$

$$= \frac{\int x^2 dA}{\int x dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

where $(I_z)_A$ is the moment of inertia of the area with respect to the z axis, and $\bar{x}$ is analogous to y_p

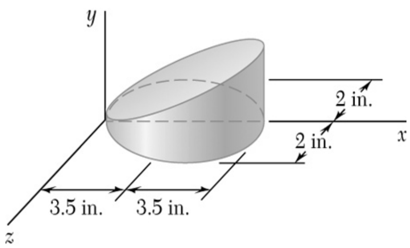
Now $$(I_z)_A = \frac{1}{36}(240\text{ mm})(300\text{ mm})^3 + \frac{1}{2}\left[(240\text{ mm})(300\text{ mm})\right](200\text{ mm})^2$$

$$= 1.620 \times 10^9\text{ mm}^4$$

and $$\bar{x}A = (200\text{ mm})\left[\frac{1}{2}(240\text{ mm})(300\text{ mm})\right] = 7.20 \times 10^6\text{ mm}^3$$

Then $$\bar{x} = \frac{1.620 \times 10^9\text{ mm}^4}{7.20 \times 10^6\text{ mm}^3}$$

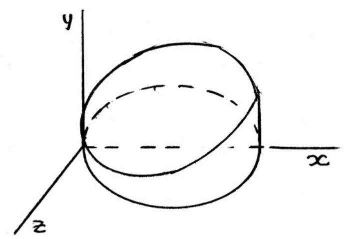
or $\bar{x} = 225\text{ mm}$ ◀



PROBLEM 9.64

Determine the x coordinate of the centroid of the volume shown; this volume was obtained by intersecting an elliptic cylinder with an oblique plane. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



Have
$$y = \frac{h}{a}x$$

$$\bar{x}\int dV = \int \bar{x}_{EL} dV, \qquad \text{where} \qquad \bar{x}_{EL} = x$$

And
$$dV = ydA = \left(\frac{h}{a}x\right)dA$$

Now
$$\bar{x} = \frac{\int x\left(\frac{h}{a}x\right)dA}{\int\left(\frac{h}{a}x\right)dA} = \frac{\int x^2 dA}{\int x dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

For the given volume

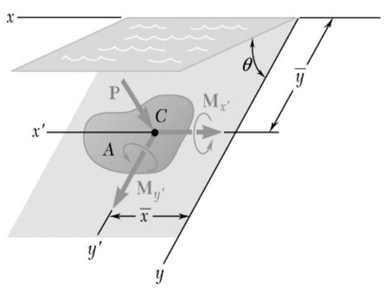
$$(I_z)_A = \frac{\pi}{4}(2\text{ in.})(3.5\text{ in.})^3 + \left[\pi(3.5\text{ in.})(2\text{ in.})\right](3.5\text{ in.})^2$$

$$= (21.4375\pi + 85.7500\pi)\text{ in}^4 = 336.74\text{ in}^4$$

and
$$(\bar{x}A)_A = 3.5\text{ in.}\left[\pi(3.5\text{ in.})(2\text{ in.})\right] = 76.969\text{ in}^3$$

Then
$$\bar{x} = \frac{336.74\text{ in}^4}{76.969\text{ in}^3} = 4.375\text{ in.}$$

or $\bar{x} = 4.38\text{ in.}$ ◀



PROBLEM 9.65

Show that the system of hydrostatic forces acting on a submerged plane area A can be reduced to a force **P** at the centroid C of the area and two couples. The force **P** is perpendicular to the area and is of magnitude $P = \gamma A\bar{y}\sin\theta$, where γ is the specific weight of the liquid, and the couples are $\mathbf{M}_{x'} = \left(\gamma \bar{I}_{x'} \sin\theta\right)\mathbf{i}$ and $\mathbf{M}_{y'} = \left(\gamma \bar{I}_{x'y'} \sin\theta\right)\mathbf{j}$, where $\bar{I}_{x'y'} = \int x'y'\,dA$ (see Sec. 9.8). Note that the couples are independent of the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = (\gamma y\sin\theta)dA$$

Equivalence of the force **P** and the system of infinitesimal forces dF requires

$$\Sigma F:\quad P = \int dF = \int \gamma y\sin\theta dA = \gamma\sin\theta \int y dA$$

or $P = \gamma A\bar{y}\sin\theta$ ◀

Equivalence of the force and couple $\left(\mathbf{P},\ \mathbf{M}_{x'} + \mathbf{M}_{y'}\right)$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x:\quad -\bar{y}P - M_{x'} = \int(-ydF)$$

Now $$-\int ydF = -\int y(\gamma y\sin\theta)dA = -\gamma\sin\theta\int y^2 dA$$

$$= -(\gamma\sin\theta)I_x$$

Then $$-\bar{y}P - M_{x'} = -(\gamma\sin\theta)I_x$$

or $$M_{x'} = (\gamma\sin\theta)I_x - \bar{y}(\gamma A\bar{y}\sin\theta)$$

$$= \gamma\sin\theta\left(I_x - A\bar{y}^2\right)$$

or $M_{x'} = \gamma \bar{I}_{x'}\sin\theta$ ◀

$$\Sigma M_y:\quad \bar{x}P + M_{y'} = \int xdF$$

Now $$\int xdF = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xydA$$

$$= (\gamma\sin\theta)I_{xy} \qquad \text{(Equation 9.12)}$$

PROBLEM 9.65 CONTINUED

Then

$$\bar{x}P + M_{y'} = (\gamma \sin\theta) I_{xy}$$

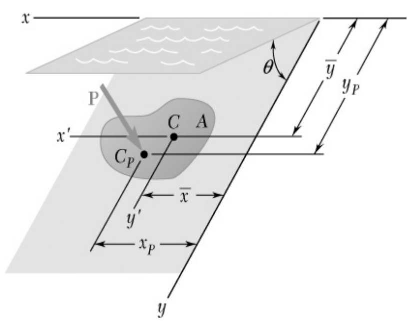
or

$$M_{y'} = (\gamma \sin\theta) I_{xy} - \bar{x}(\gamma A\bar{y}\sin\theta)$$

$$= \gamma \sin\theta \left(I_{xy} - A\bar{x}\,\bar{y}\right)$$

or, using Equation 9.13,

or $M_{y'} = \gamma \bar{I}_{x'y'} \sin\theta$ ◀



PROBLEM 9.66

Show that the resultant of the hydrostatic forces acting on a submerged plane area A is a force **P** perpendicular to the area and of magnitude $P = \gamma A\bar{y}\sin\theta = \bar{p}A$, where γ is the specific weight of the liquid and $\bar{p}$ is the pressure at the centroid C of the area. Show that **P** is applied at a point C_p, called the center of pressure, whose coordinates are $x_p = I_{xy}/A\bar{y}$ and $y_p = I_x/A\bar{y}$, where $\bar{I}_{xy} = \int xy\,dA$ (see Sec. 9.8). Show also that the difference of ordinates $y_p - \bar{y}$ is equal to $\bar{k}_{x'}^2/\bar{y}$ and thus depends upon the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y\sin\theta)dA$$

The magnitude **P** of the resultant force acting on the plane area is then

$$P = \int dP = \int \gamma y\sin\theta dA = \gamma\sin\theta\int ydA$$

$$= \gamma\sin\theta(\bar{y}A)$$

Now $$\bar{p} = \gamma\bar{y}\sin\theta \qquad \therefore\ P = \bar{p}A \blacktriangleleft$$

Next observe that the resultant **P** is equivalent to the system of infinitesimal forces d**P**. Equivalence then requires

$$\Sigma M_x:\quad -y_P P = -\int ydP$$

Now $$\int ydP = \int y(\gamma y\sin\theta)dA = \gamma\sin\theta\int y^2 dA$$

$$= (\gamma\sin\theta)I_x$$

Then $$y_P P = (\gamma\sin\theta)I_x$$

or $$y_P = \frac{(\gamma\sin\theta)I_x}{\gamma\sin\theta(\bar{y}A)}$$

$$\text{or}\quad y_P = \frac{I_x}{A\bar{y}} \blacktriangleleft$$

$$\Sigma M_y:\quad x_P P = \int xdP$$

Now $$\int xdP = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xydA$$

$$= (\gamma\sin\theta)I_{xy} \qquad \text{(Equation 9.12)}$$

PROBLEM 9.66 CONTINUED

Then
$$x_P P = (\gamma \sin\theta) I_{xy}$$

or
$$x_P = \frac{(\gamma \sin\theta) I_{xy}}{\gamma \sin\theta (\bar{y} A)}$$

$$\text{or} \quad x_P = \frac{I_{xy}}{A\bar{y}} \blacktriangleleft$$

Now
$$I_x = \bar{I}_{x'} + A\bar{y}^2$$

From above
$$I_x = (A\bar{y}) y_P$$

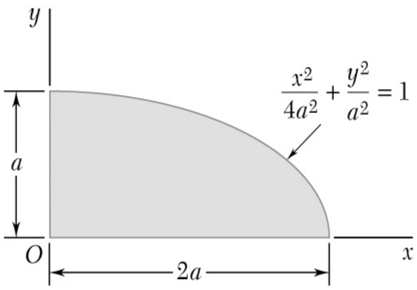
By definition
$$\bar{I}_{x'} = \bar{k}_{x'}^2 A$$

Substituting
$$(A\bar{y}) y_P = \bar{k}_{x'}^2 A + A\bar{y}^2$$

Rearranging yields
$$y_P - \bar{y} = \frac{\bar{k}_{x'}^2}{\bar{y}} \blacktriangleleft$$

Although $\bar{k}_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), $\bar{y}$ is dependent on the depth.

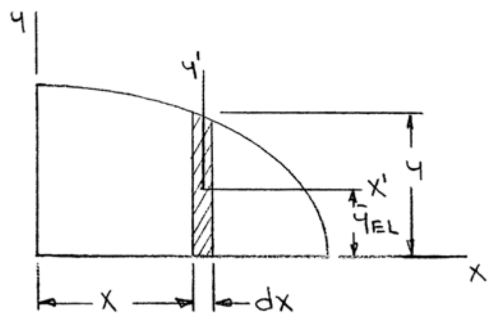
$$\therefore \; (y_P - \bar{y}) = f(\text{depth})$$



PROBLEM 9.67

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



First note

$$y = a\sqrt{1 - \frac{x^2}{4a^2}}$$

$$= \frac{1}{2}\sqrt{4a^2 - x^2}$$

Have

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where

$$d\bar{I}_{x'y'} = 0 \qquad (\text{symmetry}) \qquad \bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2 - x^2}$$

$$dA = ydx = \frac{1}{2}\sqrt{4a^2 - x^2 dx}$$

Then

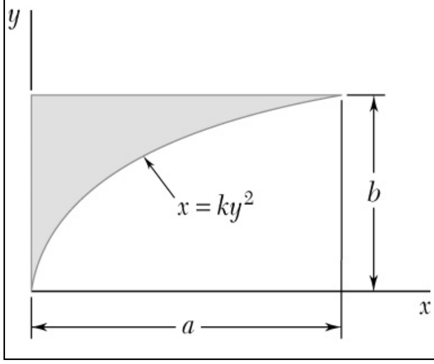
$$I_{xy} = \int dI_{xy} = \int_0^{2a} x\left(\frac{1}{4}\sqrt{4a^2 - x^2}\right)\left(\frac{1}{2}\sqrt{4a^2 - x^2}\right)dx$$

$$= \frac{1}{8}\int_0^{2a}\left(4a^2x - x^3\right)dx = \frac{1}{8}\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^{2a}$$

$$= \frac{a^4}{8}\left[2(2)^2 - \frac{1}{4}(2)^4\right]$$

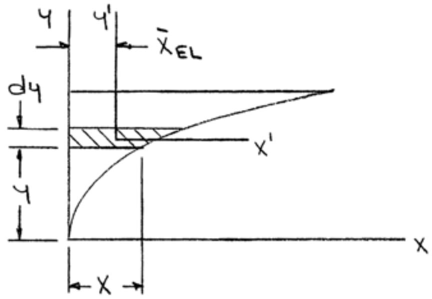
or $I_{xy} = \frac{1}{2}a^4$ ◀

PROBLEM 9.68



Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



At $\qquad x = a,\ y = b: \quad a = kb^2$

or $\qquad k = \dfrac{a}{b^2}$

Then $\qquad x = \dfrac{a}{b^2} y^2$

Have $\qquad dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$

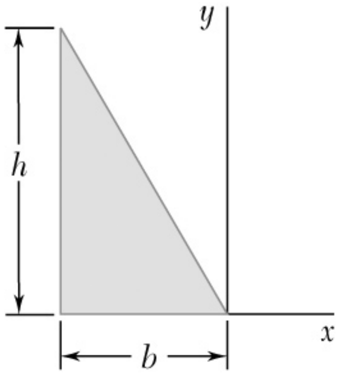
where $\qquad d\bar{I}_{x'y'} = 0 \quad (\text{symmetry}) \qquad \bar{x}_{EL} = \dfrac{1}{2}x = \dfrac{a}{2b^2}y^2$

$$\bar{y}_{EL} = y \qquad dA = xdy = \frac{a}{b^2}y^2dy$$

Then $\qquad I_{xy} = \int dI_{xy} = \int_0^b \left(\dfrac{a}{2b^2}y^2\right)(y)\left(\dfrac{a}{b^2}y^2dy\right)$

$$= \frac{a^2}{2b^4}\int_0^b y^5dy = \frac{a^2}{2b^4}\left[\frac{1}{6}y^6\right]_0^b$$

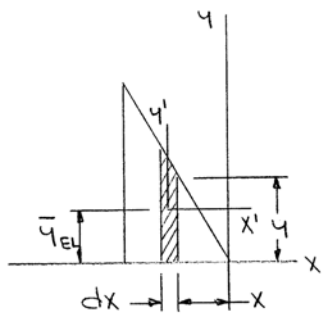
or $I_{xy} = \dfrac{1}{12}a^2b^2$ ◀



PROBLEM 9.69

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



First note that

$$y = -\frac{h}{b}x$$

Now $$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where $$d\bar{I}_{x'y'} = 0 \;\left(\text{symmetry}\right)$$

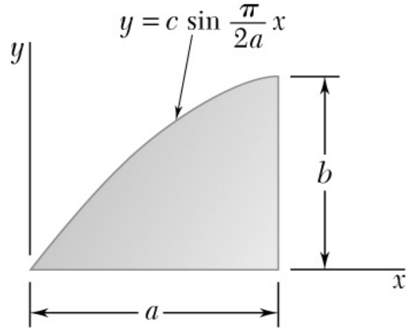
$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y = -\frac{1}{2}\frac{h}{b}x$$

$$dA = ydx = -\frac{h}{b}xdx$$

Then $$I_{xy} = \int dI_{xy} = \int_{-b}^{0} x\left(-\frac{1}{2}\frac{h}{b}x\right)\left(-\frac{h}{b}xdx\right)$$

$$= \frac{1}{2}\frac{h^2}{b^2}\int_{-b}^{0} x^3dx = \frac{1}{2}\frac{h^2}{b^2}\left[\frac{1}{4}x^4\right]_{-b}^{0}$$

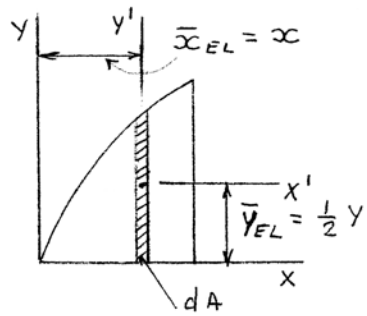
or $I_{xy} = -\frac{1}{8}b^2h^2$ ◀

	**PROBLEM 9.70**

PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



At $\qquad x = a, y = b: \quad b = c\sin\left(\frac{\pi}{2a}a\right) \qquad$ or $\qquad c = b$

Now $\qquad y = b\sin\frac{\pi}{2a}x$

Have $\qquad d\bar{I}_{x'y'} = 0 \qquad$ (symmetry)

$$dA = ydx$$

Now $\qquad dI_{xy} = \cancelto{0}{d\bar{I}_{x'y'}} + \bar{x}_{\text{EL}}\bar{y}_{\text{EL}}dA$

or $\qquad I_{xy} = \int_0^a x\left(\frac{1}{2}y\right)(ydx) = \frac{1}{2}\int_0^a xb^2\sin^2\frac{\pi}{2a}xdx$

$$= \frac{b^2}{2}\left[\frac{x^2}{4} - \frac{x\sin\frac{\pi}{a}x}{4\left(\frac{\pi}{2a}\right)} - \frac{\cos\frac{\pi}{a}x}{8\left(\frac{\pi}{2a}\right)^2}\right]_0^a$$

$$= \frac{b^2}{2}\left(\frac{a^2}{4} + \frac{4a^2}{8\pi^2} + \frac{4a^2}{8\pi^2}\right)$$

or $\qquad I_{xy} = \frac{a^2b^2}{8\pi^2}\left(4 + \pi^2\right) ◀$