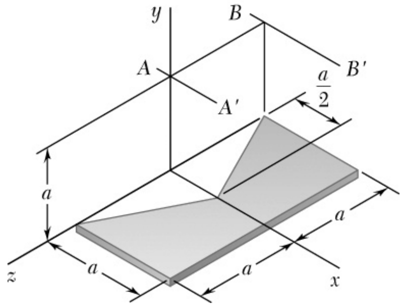


PROBLEM 9.191

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its moment of inertia with respect to (a) the axis AA' (b) the axis BB' where the AA' and BB' axes are parallel to the x axis and lie in a plane parallel to and at a distance a above the zx plane.

SOLUTION

First note that the x axis is a centroidal axis so that

$$I = \bar{I}_{x,\,\text{mass}} + md^2$$

and that from the solution to Problem 9.115,

$$\bar{I}_{x,\,\text{mass}} = \frac{7}{18}ma^2$$

(a) Have

$$I_{AA',\,\text{mass}} = \frac{7}{18}ma^2 + m(a)^2$$

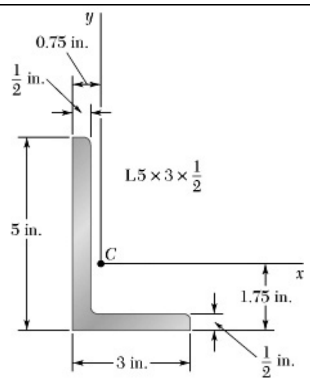
or $I_{AA'} = 1.389ma^2$ ◀

(b) Have

$$I_{BB',\,\text{mass}} = \frac{7}{18}ma^2 + m\left(a\sqrt{2}\right)^2$$

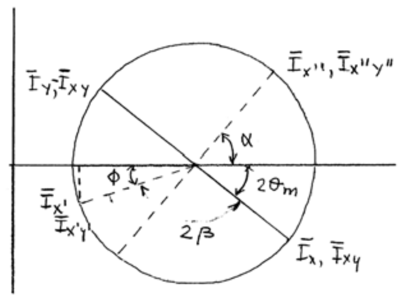
or $I_{BB'} = 2.39ma^2$ ◀

PROBLEM 9.192



For the $5 \times 3 \times \frac{1}{2}$-in. angle cross section shown, use Mohr's circle to determine (*a*) the moments of inertia and the product of inertia with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° counterclockwise, (*b*) the orientation of new centroidal axes for which $\bar{I}_x = 2\text{ in}^4$ and $\bar{I}_{x'y'} < 0$.

SOLUTION



From Figure 9.13A $\qquad \bar{I}_x = 9.45\text{ in}^4$

$$\bar{I}_y = 2.58\text{ in}^4$$

From Problem 9.106 $\qquad \bar{I}_{xy} = -2.8125\text{ in}^4$

Now $\qquad \bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$

$$= \frac{1}{2}(9.45 + 2.58)\text{ in}^4 = 6.015\text{ in}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left(\frac{9.45 - 2.58}{2}\right)^2 + (-2.8125)^2}$$

$$= 4.4395\text{ in}^4$$

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-2.8125)\text{ in}^4}{(9.45 - 2.58)\text{ in}^4} = 0.81877$$

$$2\theta_m = \tan^{-1}(0.81877) = 39.310°$$

PROBLEM 9.192 CONTINUED

(*a*) First note $\alpha = 90° - 39.310° = 50.690°$

Then $\bar{I}_{x''}, \bar{I}_{y''} = \bar{I}_{\text{ave}} \pm R\cos\alpha = (6.015 \pm 4.4395\cos\alpha)\text{ in}^4$

or $\bar{I}_{x''} = 8.83\text{ in}^4$ ◀

and $\bar{I}_{y''} = 3.20\text{ in}^4$ ◀

Also $\bar{I}_{x''y''} = R\sin\alpha = 4.4395\sin\alpha$

or $\bar{I}_{x''y''} = 3.43\text{ in}^4$ ◀

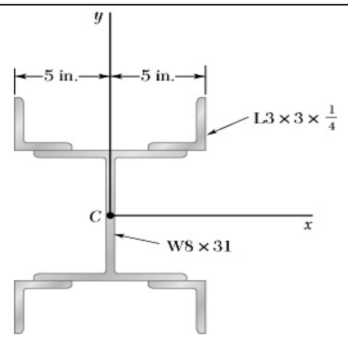
(*b*) Have $\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\phi$

or $2 = 6.015 - 4.4395\cos\phi$ or $\phi = 25.260°$

Then $2\beta = 180° - (39.310° + 25.260°) = 115.430°$

or $\beta = 57.715°$

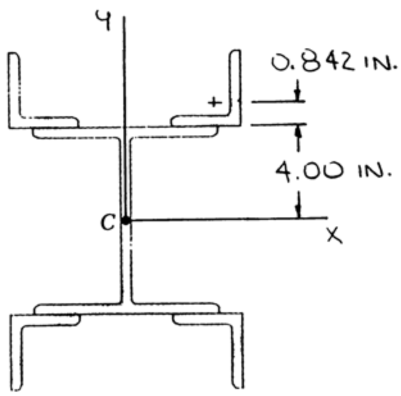
Rotate the centroidal axis 57.7° clockwise. ◀



PROBLEM 9.193

Four $3 \times 3 \times \frac{1}{4}$-in. angles are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION



W section: $A = 9.13\,\text{in}^2$

$$\bar{I}_x = 110\,\text{in}^4$$

$$\bar{I}_y = 37.1\,\text{in}^4$$

Angle: $A = 1.44\,\text{in}^2$

$$\bar{I}_x = \bar{I}_y = 1.24\,\text{in}^4$$

Note: $A_{\text{total}} = A_{\text{W}} + 4A_{\text{angle}}$

$$= \left[9.13 + 4(1.44)\right]\text{in}^2$$

$$= 14.89\,\text{in}^2$$

Now $\bar{I}_x = \left(\bar{I}_x\right)_{\text{W}} + 4\left(I_x\right)_{\text{angle}}$

where $\left(I_x\right)_{\text{angle}} = \bar{I}_x + Ad^2$

$$= 1.24\,\text{in}^4 + \left(1.44\,\text{in}^2\right)\left[(4.00 + 0.842)\,\text{in.}\right]^2$$

$$= 35.0007\,\text{in}^4$$

Then $\bar{I}_x = \left[110 + 4(35.0007)\right]\text{in}^4 = 250.0028\,\text{in}^4$

or $\bar{I}_x = 250\,\text{in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A_{\text{total}}} = \dfrac{250.0028\,\text{in}^4}{14.89\,\text{in}^2}$

or $\bar{k}_x = 4.10\,\text{in.}$ ◀

also $\bar{I}_y = \left(\bar{I}_y\right)_{\text{W}} + 4\left(I_y\right)_{\text{angle}}$

PROBLEM 9.193 CONTINUED

where $\left(I_y\right)_{\text{angle}} = \bar{I}_y + Ad^2$

$$= 1.24\text{ in}^4 + \left(1.44\text{ in}^2\right)\left[\left(5 - 0.842\right)\text{in.}\right]^2$$

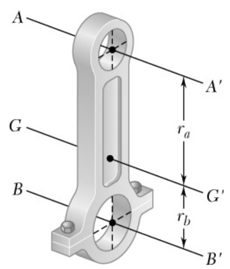
$$= 26.1361\text{ in}^4$$

Then $\bar{I}_y = \left[37.1 + 4\left(26.1361\right)\right]\text{in}^4 = 141.6444\text{ in}^4$

or $\bar{I}_y = 141.6\text{ in}^4$ ◀

and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A_{\text{total}}} = \dfrac{141.6444\text{ in}^4}{14.89\text{ in}^2}$

or $\bar{k}_y = 3.08\text{ in.}$ ◀



PROBLEM 9.194

For the 2-kg connecting rod shown, it has been experimentally determined that the mass moments of inertia of the rod with respect to the center-line axes of the bearings AA' and BB' are, respectively, $I_{AA'} = 78\text{ gm}^2$ and $I_{BB'} = 41\text{ gm}^2$. Knowing that $r_a + r_b = 290\text{ mm}$, determine (*a*) the location of the centroidal axis GG' (*b*) the radius of gyration with respect to axis GG'

SOLUTION

(a) Have $\quad I_{AA'} = \bar{I}_{GG'} + mr_a^2 \qquad r_a + r_b = 290\text{ mm}$

$$= 0.29\text{ m}$$

and $\quad I_{BB'} = \bar{I}_{GG'} + mr_b^2$

Subtracting $\quad I_{BB'} - I_{AA'} = m\left(r_b^2 - r_a^2\right)$

$$(41 - 78)\text{g}\cdot\text{m}^2 = (2000\text{ g})(r_b + r_a)(r_b - r_a)$$

or $\quad -37 = (2000)(0.29)(r_b - r_a)$

or $\quad r_a - r_b = 63.793\times 10^{-3}\text{ m}$

now $\quad r_a + r_b = 0.29\text{ m}$

so that $\quad 2r_a = 0.35379\text{ m}$

$$r_a = 0.17689\text{ m}$$

or $r_a = 176.9\text{ mm}$ ◀

(b) Have $\quad I_{AA'} = \bar{I}_{GG'} + mr_a^2$

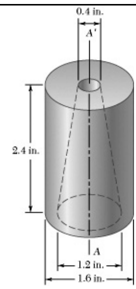
Then $\quad \bar{I}_{GG'} = 78\text{ g}\cdot\text{m}^2 - (2000\text{ g})(0.17689\text{ m})^2$

$$= 15.420\text{ g}\cdot\text{m}^2$$

Finally, $\quad \bar{k}_{GG'}^2 = \dfrac{\bar{I}_{GG'}}{m} = \dfrac{15.420\text{ g}\cdot\text{m}^2}{2000\text{ g}} = 0.007710\text{ m}^2$

$$\bar{k}_{GG'} = 0.08781\text{ m}$$

$\bar{k}_{GG'} = 87.8\text{ mm}$ ◀

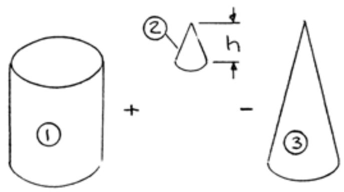


PROBLEM 9.195

Determine the mass moment of inertia of the 0.9-lb machine component shown with respect to the axis AA'.

SOLUTION

First note that the given shape can be formed adding a small cone to a cylinder and then removing a larger cone as indicated.



Now $$\frac{h}{0.4} = \frac{h+2.4}{1.2} \quad \text{or} \quad h = 1.2 \text{ in.}$$

The weight of the body is given by

$$W = mg = g(m_1 + m_2 - m_3) = \rho g(V_1 + V_2 - V_3)$$

or $$0.9 \text{ lb} = \rho \times 32.2 \text{ ft/s}^2 \left[\pi(0.8)^2(2.4) + \frac{\pi}{3}(0.2)^2(1.2) - \frac{\pi}{3}(0.6)^2(3.6)\right] \text{in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \rho \times 32.2 \text{ ft/s}^2 (2.79253 + 0.02909 - 0.78540) \times 10^{-3} \text{ ft}^3$$

or $$\rho = 13.7266 \text{ lb}\cdot\text{s}^2/\text{ft}^4$$

Then $$m_1 = (13.7266)\left(2.79253 \times 10^{-3}\right) = 0.038332 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = (13.7266)\left(0.02909 \times 10^{-3}\right) = 0.000399 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = (13.7266)\left(0.78540 \times 10^{-3}\right) = 0.010781 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Finally, using Figure 9.28 have,

$$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2 - (I_{AA'})_3$$

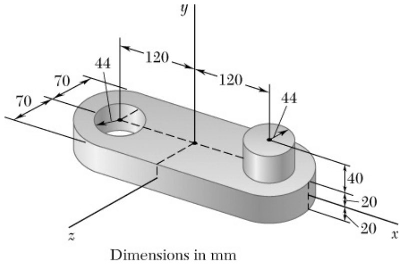
$$= \frac{1}{2}m_1 a_1^2 + \frac{3}{10}m_2 a_2^2 - \frac{3}{10}m_3 a_3^2$$

$$= \left[\frac{1}{2}(0.038332)(0.8)^2 + \frac{3}{10}(0.000399)(0.2)^2 - \frac{3}{10}(0.010781)(0.6)^2\right](\text{lb}\cdot\text{s}^2/\text{ft}) \times \text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= (85.1822 + 0.0333 - 8.0858) \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

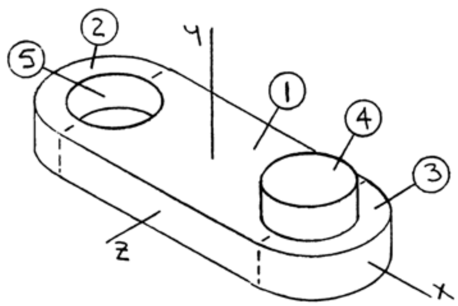
or $I_{AA'} = 77.1 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.196



Determine the moments of inertia and the radii of gyration of the steel machine element shown with respect to the x and y axes. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. Have

$$m = \rho_{st} V$$

Then
$$m_1 = (7850\ \text{kg/m}^3)(0.24 \times 0.04 \times 0.14)\,\text{m}^3$$

$$= 10.5504\ \text{kg}$$

$$m_2 = m_3 = (7850\ \text{kg/m}^3)\left[\frac{\pi}{2}(0.07)^2 \times 0.04\right]\text{m}^3 = 2.41683\ \text{kg}$$

$$m_4 = m_5 = (7850\ \text{kg/m}^3)\left[\pi(0.044)^2 \times (0.04)\right]\text{m}^3 = 1.90979\ \text{kg}$$

Using Figure 9.28 for components 1, 4, and 5 and the equations derived above (before the solution to Problem 9.144) for a semicylinder, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 - (I_x)_5 \qquad \text{where} \qquad (I_x)_2 = (I_x)_3$$

$$= \left[\frac{1}{12}(10.5504\ \text{kg})(0.04^2 + 0.14^2)\text{m}^2\right] + 2\left\{\frac{1}{12}(2.41683\ \text{kg})\left[3(0.07\ \text{m})^2 + (0.04\ \text{m})^2\right]\right\}$$

$$+ \left\{\frac{1}{12}(1.90979\ \text{kg})\left[3(0.044\ \text{m})^2 + (0.04\ \text{m})^2\right] + (1.90979\ \text{kg})(0.04\ \text{m})^2\right\}$$

$$- \left\{\frac{1}{12}(1.90979\ \text{kg})\left[3(0.044\ \text{m})^2 + (0.04\ \text{m})^2\right]\right\}$$

$$= \left[(0.0186390) + 2(0.0032829) + (0.0011790 + 0.0030557) - (0.0011790)\right]\text{kg}\cdot\text{m}^2$$

$$= 0.0282605\ \text{kg}\cdot\text{m}^2$$

or $I_x = 28.3 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.196 CONTINUED

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 - (I_y)_5$$

where $$(I_y)_2 = (I_y)_3 \qquad (I_y)_4 = |(I_y)_5|$$

Then $$I_y = \left[\frac{1}{12}(10.5504 \text{ kg})(0.24^2 + 0.14^2)\text{m}^2\right]$$

$$+ 2\left[(2.41683 \text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.07 \text{ m}^2) + (2.41683 \text{ kg})\left(0.12 + \frac{4 \times 0.07}{3\pi}\right)^2 \text{m}^2\right]$$

$$= [(0.0678742) + 2(0.0037881 + 0.0541678)]\text{kg}\cdot\text{m}^2$$

$$= 0.1837860 \text{ kg}\cdot\text{m}^2$$

or $I_y = 183.8 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

Also $$m = m_1 + m_2 + m_3 + m_4 - m_5 \text{ where } m_2 = m_3, \; m_4 = |m_5|$$

$$= (10.5504 + 2 \times 2.41683)\text{kg} = 15.38406 \text{ kg}$$

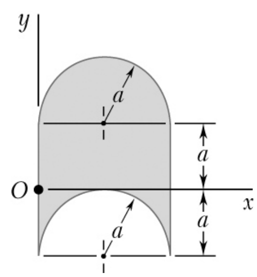
Then $$k_x^2 = \frac{I_x}{m} = \frac{0.0282605 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

or $k_x = 42.9 \text{ mm}$ ◀

and $$k_y^2 = \frac{I_y}{m} = \frac{0.1837860 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

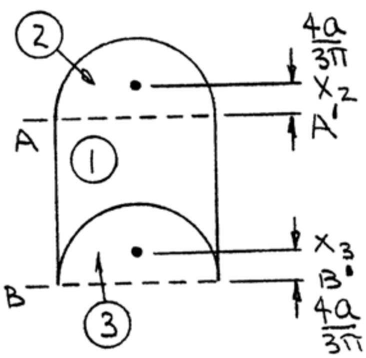
or $k_y = 109.3 \text{ mm}$ ◀

PROBLEM 9.197



Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION



Have $$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4$$

Now $$(I_{AA})_2 = (I_{BB})_3 = \frac{\pi}{8}a^4$$

and $$(I_{AA})_2 = (\bar{I}_{xz})_2 + Ad^2$$

or $$(\bar{I}_{x_2})_2 = (\bar{I}_{x_3})_3 = \frac{\pi}{8}a^4 - \left(\frac{\pi}{2}a^2\right)\left(\frac{4a}{3\pi}\right)^2$$

$$= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4$$

Then $$(I_x)_2 = (\bar{I}_{x_2})_2 + Ad_2^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a + \frac{4a}{3\pi}\right)^2$$

$$= \left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

$$(I_x)_3 = (\bar{I}_{x_3})_3 + Ad_3^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a - \frac{4a}{3\pi}\right)^2$$

$$= \left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

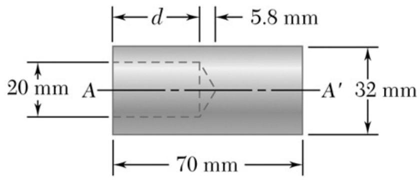
Finally $$I_x = \frac{4}{3}a^4 + \left[\left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right] - \left[\left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right]$$

or $I_x = 4a^4$ ◀

Also $$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

where $$(I_y)_1 = \frac{1}{12}(2a)(2a)^3 + (2a)^2(a)^2 = \frac{16}{3}a^4$$

$$(I_y)_2 = (I_y)_3 \qquad \left[= \frac{\pi}{8}a^4 + \left(\frac{\pi}{2}a^2\right)(a)^2\right] \qquad \therefore\ I_y = \frac{16}{3}a^4$$ ◀



PROBLEM 9.198

A 20-mm-diameter hole is bored in a 32-mm-diameter rod as shown. Determine the depth d of the hole so that the ratio of the moments of inertia of the rod with and without the hole with respect to the axis AA' is 0.96.

SOLUTION

First note

Cylinder:
$$I_{AA'} = \frac{1}{2}ma^2 \qquad m = \rho V = \rho \times \left(\pi a^2 L\right)$$
$$= \frac{1}{2}\pi\rho a^4 L$$

Cone:
$$I_{AA'} = \frac{3}{10}ma^2 \qquad m = \rho V = \rho \times \left(\frac{\pi}{3}a^2 h\right)$$
$$= \frac{1}{10}\pi\rho a^4 h$$

Now
$$\frac{\left(I_{AA'}\right)_{\text{bored}}}{\left(I_{AA'}\right)_{\text{solid}}} = 0.96$$

or
$$0.96\left(I_{AA'}\right)_{\text{solid}} = \left(I_{AA'}\right)_{\text{solid}} - \left(I_{AA'}\right)_{\text{hole}}$$

or
$$0.04\left(I_{AA'}\right)_{\text{solid}} = \left[\left(I_{AA'}\right)_{\text{cylinder}} + \left(I_{AA'}\right)_{\text{cone}}\right]_{\text{hole}}$$

Then
$$0.04\left(\frac{1}{2}\pi\rho a_{\text{rod}}^4 L_{\text{rod}}\right) = \frac{1}{2}\pi\rho a_{\text{hole}}^4 d + \frac{1}{10}\pi\rho a_{\text{hole}}^4 h_{\text{cone}}$$

or
$$d = 0.04\left(\frac{a_{\text{rod}}}{a_{\text{hole}}}\right)^4 L_{\text{rod}} - \frac{1}{5}h_{\text{cone}}$$
$$= 0.04\left(\frac{16\text{ mm}}{10\text{ mm}}\right)^4 (70\text{ mm}) - \frac{1}{5}(5.8\text{ mm})$$

or $d = 17.19$ mm ◀