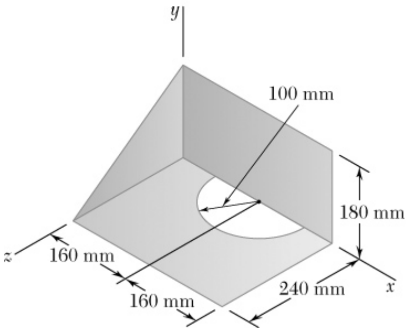
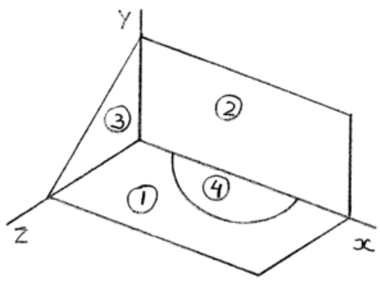
PROBLEM 9.141



A 2-mm-thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



Have $$m = \rho_{st}V = \rho_{st}tA$$

Then $$m_1 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times (0.320 \times 0.240)\text{m}^2$$

$$= 1.20576\text{ kg}$$

$$m_2 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times (0.320 \times 0.180)\text{m}^2$$

$$= 0.90432\text{ kg}$$

$$m_3 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times \left(\frac{1}{2} \times 0.180 \times 0.240\right)\text{m}^2$$

$$= 0.33912\text{ kg}$$

$$m_4 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times \left[\frac{\pi}{2}(0.100\text{ m})^2\right]$$

$$= 0.24662\text{ kg}$$

Using Fig. 9-2B for components 1 and 2 and the equations derived above for components 3 and 4, have

Now $$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

where $$(I_x)_1 = \frac{1}{3}(1.20576\text{ kg})(0.240\text{ m})^2$$

$$= 23.151 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$(I_x)_2 = \frac{1}{3}(0.90432\text{ kg})(0.180\text{ m})^2$$

$$= 9.7667 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$(I_x)_3 = \frac{1}{6}(0.33912\text{ kg})\left[(0.180)^2 + (0.240)^2\right]\text{m}^2$$

$$= 5.0868 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

PROBLEM 9.141 CONTINUED

$$\left(I_x\right)_4 = \frac{1}{4}\left(0.24662 \text{ kg}\right)\left(0.100 \text{ m}\right)^2$$

$$= 0.61655 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Then $I_x = \left[\left(23.151 + 9.7667 + 5.0868 - 0.61655\right) \times 10^{-3}\right] \text{kg} \cdot \text{m}^2$

or $I_x = 37.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

And $I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 - \left(I_y\right)_4$

where $\left(I_y\right)_1 = \frac{1}{3}\left(1.20576 \text{ kg}\right)\left[\left(0.320\right)^2 + \left(0.240\right)^2\right]\text{m}^2$

$$= 64.307 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\left(I_y\right)_2 = \frac{1}{3}\left(0.90432 \text{ kg}\right)\left(0.320 \text{ m}\right)^2$$

$$= 30.867 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\left(I_y\right)_3 = \frac{1}{6}\left(0.33912 \text{ kg}\right)\left(0.240 \text{ m}\right)^2$$

$$= 3.2556 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\left(I_y\right)_4 = \left(0.24662 \text{ kg}\right)\left\{\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(0.100 \text{ m}\right)^2\right] + \left[\left(0.160\right)^2 + \left(\frac{4 \times 0.100}{3\pi}\right)^2\right]\text{m}^2\right\}$$

$$= 7.5466 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Then $I_y = \left[\left(64.307 + 30.067 + 3.2556 - 7.5466\right)10^{-3}\right]\text{kg} \cdot \text{m}^2$

or $I_y = 90.9 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

And $I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3 - \left(I_z\right)_4$

where $\left(I_z\right)_1 = \frac{1}{3}\left(1.20576 \text{ kg}\right)\left(0.320 \text{ m}\right)^2 = 41.157 \times 10^{-3} \text{ kg} \cdot \text{m}^2$

$$\left(I_z\right)_2 = \frac{1}{3}\left(0.90432 \text{ kg}\right)\left[\left(0.320\right)^2 + \left(0.180\right)^2\right] \text{m}^2$$

$$= 40.634 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\left(I_z\right)_3 = \frac{1}{6}\left(0.33912 \text{ kg}\right)\left(0.180 \text{ m}\right)^2 = 1.83125 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 9.141 CONTINUED

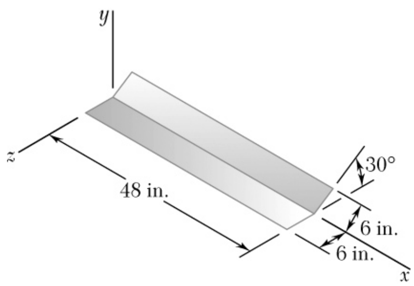
$$(I_z)_4 = (0.24662\text{ kg})\left\{\left[\frac{1}{4}(0.100\text{ m})^2\right] + \left[(0.160\text{ m})^2\right]\right\}$$

$$= 6.9300 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Then $$(I_z)_z = \left[(41.157 + 40.634 + 1.83125 - 6.9300) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2$$

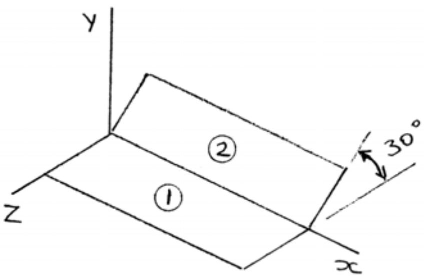
or $I_z = 76.7 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.142



The piece of roof flashing shown is formed from sheet copper that is 0.032 in. thick. Knowing that the specific weight of copper is 558 lb/ft^3, determine the mass moment of inertia of the flashing with respect to each of the coordinate axes.

SOLUTION



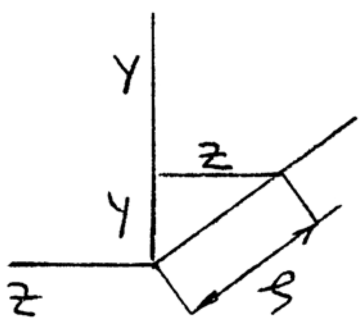
Have $m = \rho_{\text{copper}} V$

$$= \frac{\gamma_{\text{copper}}}{g} tA$$

Then $m_1 = m_2$

$$= \frac{558\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times 0.032\ \text{in.} \times (48 \times 6)\ \text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$

$$= 0.092422\ \text{lb}\cdot\text{s}^2/\text{ft}$$



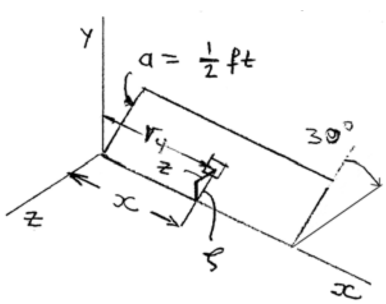
Using Fig. 9-2B for components 1 and 2, have

Now $I_x = (I_x)_1 + (I_x)_2$ and $(I_x)_1 = (I_x)_2$

Then $$I_x = 2\left[\frac{1}{3}(0.092422\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{6}{12}\ \text{ft}\right)^2\right] = 1.54037\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 1.54 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2$$



where $$(I_y)_1 = \frac{1}{3}(0.092422\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{48}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right]\ \text{ft}^2$$

$$= 500.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

and $$(I_y)_2 = \int r_y^2\, dm$$

where $$r_y^2 = x^2 + z^2$$

$$= x^2 + (\zeta \cos 30)^2$$

PROBLEM 9.142 CONTINUED

and $$dm = \rho dV = \frac{\gamma_{\text{copper}}}{g} t d\zeta dx$$

Then $$\left(I_y\right)_2 = \frac{\gamma_{\text{copper}}}{g} t \int_0^L \int_0^a \left(x^2 + \zeta^2 \cos^2 30°\right) d\zeta dx$$

$$= \frac{\gamma_{\text{copper}}}{g} t \int_0^L \left(ax^2 + \frac{1}{3}a^3 \cos^2 30°\right) dx$$

$$= \frac{1}{3}\frac{\gamma_{\text{copper}}}{g} t \left(aL^3 + a^3 L \cos^2 30°\right) \qquad A = aL$$

$$= \frac{1}{3} m_2 \left(L^2 + a^2 \cos^2 30°\right)$$

$$= \frac{1}{3}\left(0.092422 \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left[(4)^2 + \left(\frac{1}{2}\cos 30°\right)^2\right] \text{ft}^2$$

$$= 498.69 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Finally, $$I_y = (500.62 + 498.69) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

or $I_y = 999 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2$$

where $$\left(I_z\right)_1 = \frac{1}{3}\left(0.092422 \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left(\frac{48}{12} \text{ ft}\right)^2$$

$$= 492.92 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

and $$\left(I_z\right)_2 = \int r_z^2 dm$$

where $$r_z^2 = x^2 + y^2$$

and $$y = \zeta \sin 30°$$

Then $$\left(I_z\right)_2 = \int \left[x^2 + \left(\zeta \sin 30°\right)^2\right] dm$$

PROBLEM 9.142 CONTINUED

Similarly, as $(I_y)_2$

$$(I_z)_2 = \frac{1}{3}m_2\left(L^2 + a^2\sin^2 30°\right)$$

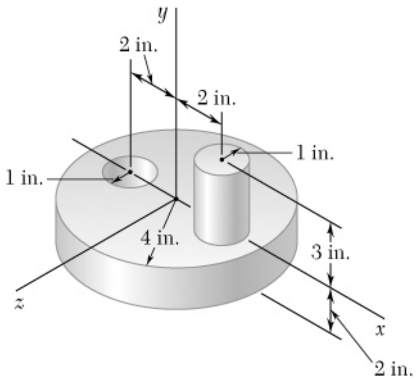
$$= \frac{1}{3}\left(0.092422\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[(4)^2 + \left(\frac{1}{2}\right)^2\sin^2 30°\right]\text{ft}^2$$

$$= 494.84\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_x = (492.92 + 494.84)\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

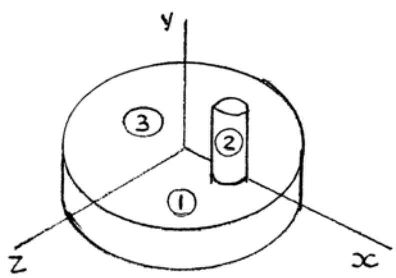
or $I_z = 988\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.143



The machine element shown is fabricated from steel. Determine the mass moment of inertia of the assembly with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The specific weight of steel is $0.284\ \text{lb/in}^3$.)

SOLUTION



Have $$m = \rho V = \frac{\gamma}{g}V = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times V$$

$$= \left(0.0088199\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)V$$

Then $$m_1 = \left(0.0088199\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(4)^2(2)\right]\text{in}^3 = 0.88667\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(0.0088199\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(3)\right]\text{in}^3 = 0.083126\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(0.0088199\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(2)\right]\text{in}^3 = 0.055417\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using Fig. 9-28 and the parallel theorem, have

(*a*)

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

$$= \left(0.88667\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left(0.083126\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + (1.5)^2\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$- \left(0.055417\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + (1)^2\right\}\text{in.} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.034106\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 0.0341\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.143 CONTINUED

(*b*)

$$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(4)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$- \left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 5.0125 \times 10^{-2} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 0.0501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*c*)

$$I_z = (I_z)_1 + (I_z)_2 - (I_z)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + \left[(2)^2 + (1.5)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

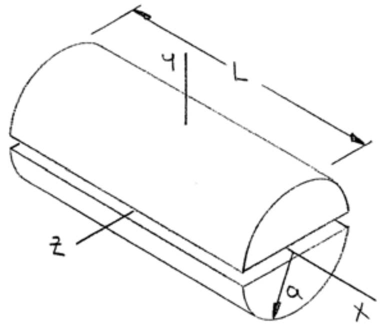
$$- \left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + \left[(2)^2 + (1)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.034876 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 0.0349 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

To the Instructor:

The following formulas for the mass of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.144–9.147.



From Figure 9.28

Cylinder: $$\left(I_x\right)_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

$$\left(I_y\right)_{\text{cyl}} = \left(I_z\right)_{\text{cyl}} = \frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)$$

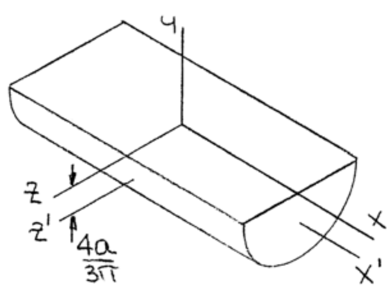
Symmetry and the definition of the mass moment of inertia $\left(I = \int r^2 dm\right)$ imply

$$\left(I\right)_{\text{semicylinder}} = \frac{1}{2}\left(I\right)_{\text{cylinder}}$$

$$\therefore \quad \left(I_x\right)_{\text{sc}} = \frac{1}{2}\left(\frac{1}{2}m_{\text{cyl}}a^2\right)$$

and $$\left(I_y\right)_{\text{sc}} = \left(I_z\right)_{\text{sc}} = \frac{1}{2}\left[\frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)\right]$$

However, $$m_{\text{sc}} = \frac{1}{2}m_{\text{cyl}}$$



Thus, $$\left(I_x\right)_{\text{sc}} = \frac{1}{2}m_{\text{sc}}a^2$$

and $$\left(I_y\right)_{\text{sc}} = \left(I_z\right)_{\text{sc}} = \frac{1}{12}m_{\text{sc}}\left(3a^2 + L^2\right)$$

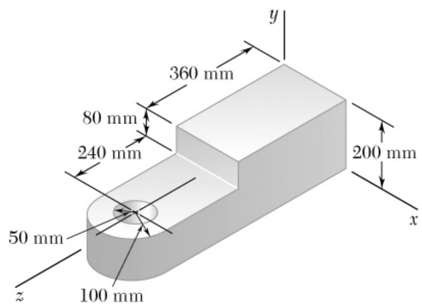
Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}}\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{I}_{z'} = m_{\text{sc}}\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2 + \frac{1}{12}L^2\right]$$

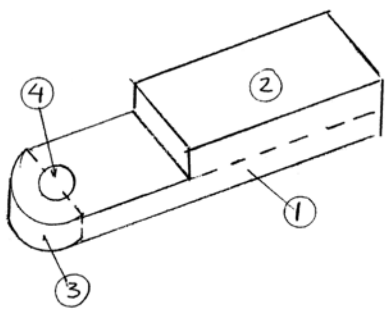
where x' and z' are centroidal axes.

PROBLEM 9.144



Determine the mass moment of inertia of the steel machine element shown with respect to the y axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



Have $$m = \rho_{\text{steel}} V$$

Then $$m_1 = 7850\ \text{kg/m}^3 (0.200 \times 0.120 \times 0.600)\text{m}^3$$

$$= 113.040\ \text{kg}$$

$$m_2 = 7850\ \text{kg/m}^3 \times (0.200 \times 0.080 \times 0.360)\text{m}^3$$

$$= 45.216\ \text{kg}$$

$$m_3 = 7850\ \text{kg/m}^3 \times \left[\frac{\pi}{2}(0.100)^2(0.120)\right]\text{m}^3$$

$$= 14.7969\ \text{kg}$$

$$m_4 = 7850\ \text{kg/m}^3 \times \left[\pi(0.050)^2(0.120)\right]\text{m}^3$$

$$= 7.3985\ \text{kg}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

Now $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where $$(I_y)_1 = (113.040\ \text{kg})\left\{\frac{1}{12}\left[(0.600)^2 + (0.200)^2\right] + \left[\left(\frac{0.600}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 15.0720\ \text{kg}\cdot\text{m}^2$$

PROBLEM 9.144 CONTINUED

$$\left(I_y\right)_2 = (45.216\text{ kg})\left\{\frac{1}{12}\left[(0.360)^2 + (0.200)^2\right] + \left[\left(\frac{0.360}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 2.5562\text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_3 = (14.7969\text{ kg})\left\{\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.100)^2\right] + \left[(0.100)^2 + \left(0.600 + \frac{4\times 0.100}{3\pi}\right)^2\right]\right\}\text{m}^2$$

$$= 6.3024\text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_4 = (7.3985\text{ kg})\left\{\left[\frac{1}{2}(0.050)^2\right] + \left[(0.100)^2 + (0.600)^2\right]\right\}\text{m}^2$$

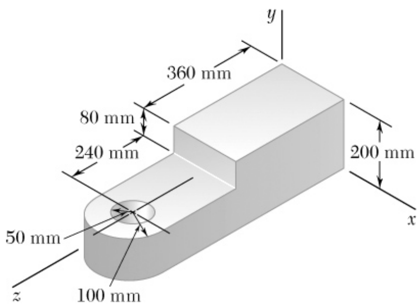
$$= 2.7467\text{ kg}\cdot\text{m}^2$$

Then
$$I_y = (15.0720 + 2.5562 + 6.3024 - 2.7467)\text{kg}\cdot\text{m}^2$$

$$= 21.1839\text{ kg}\cdot\text{m}^2$$

or $I_y = 21.2\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.145



Determine the mass moment of inertia of the steel machine element shown with respect to the z axis. (The density of steel is 7850 kg/m^3.)

SOLUTION

See machine elements shown in Problem 9.145

Also note

$$m_1 = 113.040\text{ kg}$$

$$m_2 = 45.216\text{ kg}$$

$$m_3 = 14.7969\text{ kg}$$

$$m_4 = 7.3985\text{ kg}$$

Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

Now

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where

$$(I_z)_1 = (113.040\text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.120)^2\right] + \left[\left(\frac{0.200}{2}\right)^2 + \left(\frac{0.120}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 2.0498\text{ kg}\cdot\text{m}^2$$

$$(I_z)_2 = (45.216\text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.080)^2\right] + \left[(0.100)^2 + (0.160)^2\right]\right\}\text{m}^2$$

$$= 1.78453\text{ kg}\cdot\text{m}^2$$

$$(I_z)_3 = (14.7969\text{ kg})\left\{\frac{1}{12}\left[3(0.100)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$

$$= 0.25599\text{ kg}\cdot\text{m}^2$$

$$(I_z)_4 = (7.3985\text{ kg})\left\{\frac{1}{12}\left[3(0.050)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$

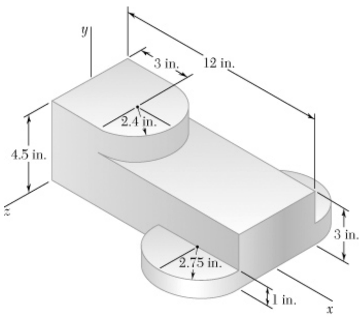
$$= 0.114122\text{ kg}\cdot\text{m}^2$$

Then

$$I_z = (2.0498 + 1.78453 + 0.25599 - 0.114122)\text{ kg}\cdot\text{m}^2$$

$$= 3.97629\text{ kg}\cdot\text{m}^2$$

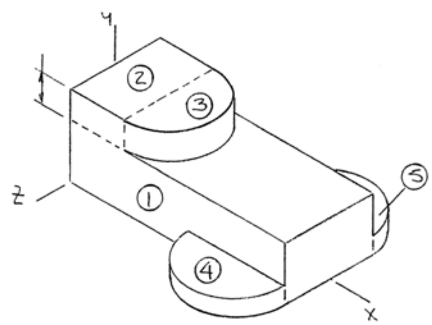
or $I_z = 3.98\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.146

An aluminum casting has the shape shown. Knowing that the specific weight of aluminum is $0.100\ \text{lb/in}^3$, determine the moment of inertia of the casting with respect to the z axis.

SOLUTION



Have
$$m = \rho V = \frac{\gamma}{g}V = \frac{0.10\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}V$$

$$= \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)V$$

Then
$$m_1 = \left(0.0031056\ \text{lb}\,\text{s}^2/\text{ft}\cdot\text{in}^3\right)(12\ \text{in.})(3\ \text{in.})(4.8\ \text{in.})$$

$$= 0.53665\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)(1.5\ \text{in.})(4.8\ \text{in.})(3\ \text{in.}) = 0.06708\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\frac{\pi}{2}(2.4\ \text{in.})^2 \times (1.5\ \text{in.})\right]$$

$$= 0.042148\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_4 = m_5 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\frac{\pi}{2}(2.75\ \text{in.})^2 \times (1.0\ \text{in.})\right]$$

$$= 0.036892\ \text{lb}\cdot\text{s}^2/\text{ft}$$

PROBLEM 9.146 CONTINUED

Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3, 4, and 5, have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 + (I_z)_5 \qquad \text{where} \qquad (I_z)_4 = (I_z)_5$$

$$I_z = \left(0.53665 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(12 \text{ in.})^2 + (3 \text{ in.})^2\right] + \left(\frac{12 \text{ in.}}{2}\right)^2 + \left(\frac{3 \text{ in.}}{2}\right)^2\right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(0.06708 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3 \text{ in.})^2 + (1.5 \text{ in.})^2\right] + (1.5 \text{ in.})^2 + (4.5 \text{ in.} - 0.75 \text{ in.})^2\right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+\left(0.042148 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left(\frac{1}{4} - \frac{16}{4\pi}\right)(2.4 \text{ in.})^2 + \frac{1}{12}(1.5 \text{ in.})^2\right.$$

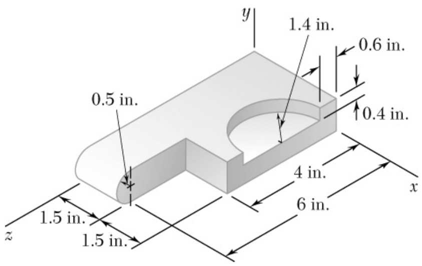
$$\left. + \left(3 \text{ in.} + \frac{4 \times 2.4 \text{ in.}}{3\pi}\right)^2 + (4.5 \text{ in.} - 0.75 \text{ in.})^2\right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ 2\left(0.036892 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(2.75 \text{ in.})^2 + (1.0 \text{ in.})^2\right] + \left[(12 \text{ in} - 2.75 \text{ in})^2 + (0.5 \text{ in})^2\right]\right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.252096 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

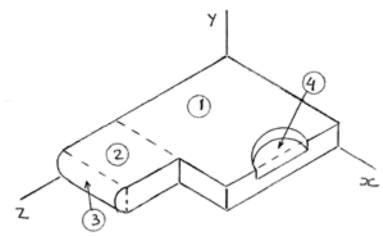
or $I_z = 0.252 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.147



Determine the moment of inertia of the steel machine element shown with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION



Have $$m = \rho_{ST} V = \frac{\delta_{ST}}{g} V$$

Then $$m_1 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (3 \times 1 \times 4)\text{ in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 105.676 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (1.5 \times 1 \times 2)\text{ in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3 = 26.419 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left[\frac{\pi}{2}(0.5)^2 \times 1.5\right]\text{in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3 = 5.1874 \times 10^3\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_4 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left[\frac{\pi}{2}(1.4)^2 \times 0.4\right]\text{in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 10.8491 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

(*a*) Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

where $$(I_x)_1 = (105.676 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left\{\frac{1}{12}\left[(1)^2 + (4)^2\right] + \left[\left(\frac{1}{2}\right)^2 + \left(\frac{4}{2}\right)^2\right]\text{in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2\right\}$$

$$= 4.1585 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_x)_2 = (26.419 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left\{\frac{1}{12}\left[(1)^2 + (2)^2\right] + \left[(0.5)^2 + (5)^2\right]\right\}\text{in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 4.7089 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.147 CONTINUED

$$\left(I_x\right)_3 = \left(5.1874\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(0.5)^2\right]+\left[(0.5)^2+\left(6+\frac{4\times0.5}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.40209\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_x\right)_4 = \left(10.8451\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1.4)^2+(0.4)^2\right]+\left(6\times\frac{4\times0.5}{3\pi}\right)^2\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.38736\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_x = \left[(4.1585+4.7089+1.40209-0.38736)\times10^{-3}\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 9.8821\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 9.88\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*b*) Have $$I_y = \left(I_y\right)_1+\left(I_y\right)_2+\left(I_y\right)_3-\left(I_y\right)_4$$

where

$$\left(I_y\right)_1 = \left(105.676\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2+(4)^2\right]+\left[\left(\frac{3}{2}\right)^2+\left(\frac{4}{2}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 6.1155\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_2 = \left(26.419\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2+(2)^2\right]+\left[(0.75)^2+(5)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 4.7854\times10^{-5}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_3 = \left(5.1874\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)(0.5)^2+\frac{1}{12}(1.5)^2\right]+\left[(0.75)^2+\left(6+\frac{4.05}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.4178\text{s}\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_4 = \left(10.8451\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(1.4)^2\right]+\left[\left(3-\frac{4\times1.4}{3\pi}\right)^2+(2)^2\right]\right\}\text{in}^2\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.78438\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.147 CONTINUED

Then $$I_y = \left[\left(16.1155 + 4.7854 + 1.41785 - 0.78438\right) \times 10^{-3}\right] \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 11.5344 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 11.53 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*c*) Have $$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3 - \left(I_z\right)_4$$

where

$$\left(I_z\right)_1 = \left(105.676 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2 + (1)^2\right] + \left[\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 2.4462 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_z\right)_2 = \left(26.419 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2 + (1)^2\right] + \left[\left(\frac{1.5}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.198754 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_z\right)_3 = \left(5.1874 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(0.5)^2 + (1.5)^2\right] + \left[(0.75)^2 + (0.5)^2\right]\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.038275 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_z\right)_4 = \left(10.8451 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(1.4)^2 + \frac{1}{12}(0.4)^2\right] + \left[\left(3 - \frac{4 \times 1.4}{3\pi}\right)^2 + (0.8)^2\right]\right\}\text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.49543 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_z = \left[\left(2.4462 + 0.198754 + 0.038275 - 0.49543\right) \times 10^{-3}\right] \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

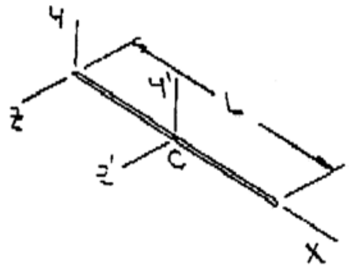
$$= 2.1878 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 2.19 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

To the instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of problems 9.148–9.150

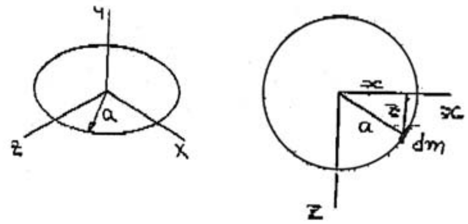
Slender Rod



$$I_x = 0 \qquad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Fig. 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$

Circle



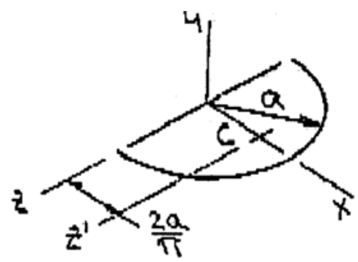
Have $\qquad \bar{I}_y = \int r^2 dm = ma^2$

Now $\qquad \bar{I}_y = \bar{I}_x + \bar{I}_z$

And symmetry implies $\qquad \bar{I}_x = \bar{I}_z$

$$\therefore \quad \bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$

Semicircle



Following the above arguments for a circle, have

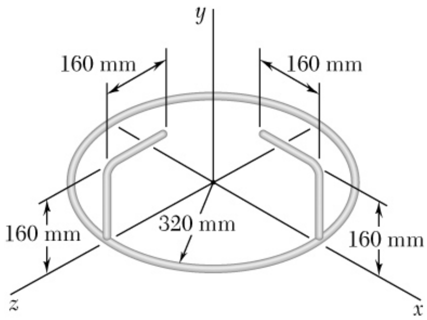
$$\bar{I}_x = I_z = \frac{1}{2}ma^2 \qquad I_y = ma^2$$

Using the parallel-axis theorem

$$I_z = \bar{I}_{z'} + m\bar{x}^2 \qquad x = \frac{2a}{\pi}$$

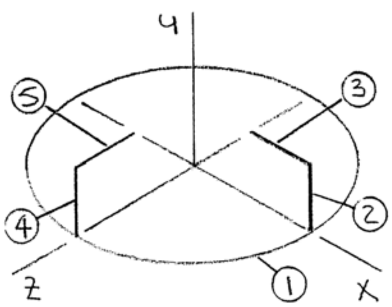
or $\qquad I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$

PROBLEM 9.148



Aluminum wire with a mass per unit length of 0.049 kg/m is used to form the circle and the straight members of the figure shown. Determine the mass moment of inertia of the assembly with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component.

Have $$m = \rho L$$

Then $$m_1 = (0.049 \text{ kg/m})\left[2\pi(0.32 \text{ m})\right]$$

$$= 0.09852 \text{ kg}$$

$$m_2 = m_3 = m_4 = m_5$$

$$= (0.049 \text{ kg/m})(0.160 \text{ m}) = 0.00784 \text{ kg}$$

Using the equation given above and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_5$$

$$= (0.09852 \text{ kg})\left[\left(\frac{1}{2}\right)(0.32 \text{ m})^2\right] + (0.00784 \text{ kg})\left[\left(\frac{1}{3}\right)(0.160 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[0 + (0.160 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[\left(\frac{1}{12}\right)(0.16 \text{ m})^2 + (0.08 \text{ m})^2 + (0.32 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[\frac{1}{12}(0.16 \text{ m})^2 + (0.16 \text{ m})^2 + (0.32 \text{ m} - 0.08 \text{ m})^2\right]$$

$$= \left[(5.0442 + 0.06690 + 0.2007 + 0.86972 + 0.66901) \times 10^{-3}\right] \text{kg} \cdot \text{m}^2$$

$$= 6.8505 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_x = 6.85 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

PROBLEM 9.148 CONTINUED

Have $$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 + \left(I_y\right)_4 + \left(I_y\right)_5$$

where $\left(I_y\right)_2 = \left(I_y\right)_4$ and $\left(I_y\right)_3 = \left(I_y\right)_5$

Then $$I_y = \left(0.09852\,\text{kg}\right)\left[\left(0.32\text{ m}\right)^2\right] + 2\left(0.00784\,\text{kg}\right)\left[0 + \left(0.32\,\text{m}\right)^2\right]$$

$$+ 2\left(0.00784\,\text{kg}\right)\left[\frac{1}{12}\left(0.16\,\text{m}\right)^2 + \left(0.24\,\text{m}\right)^2\right]$$

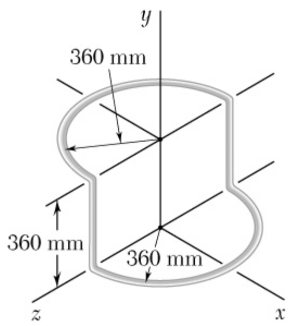
$$= \left[10.088 + 2\left(0.80282\right) + 2\left(0.46831\right)\right] \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

$$= 12.6303 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

or $I_y = 12.63 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

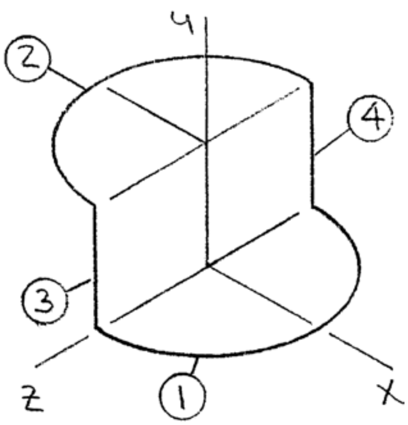
By symmetry $I_z = I_x$ or $I_z = 6.85 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.149



The figure shown is formed of 3-mm-diameter steel wire. Knowing that the density of the steel is 7850 kg/m^3, determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



Have $$m = \rho V = \rho AL$$

Then

$$m_1 = m_2 = \left(7850 \text{ kg/m}^3\right)\left[\pi\left(0.0015 \text{ m}\right)^2\right] \times \left(\pi \times 0.36 \text{ m}\right)$$

$$m_2 = m_1 = 0.062756 \text{ kg}$$

$$m_3 = m_4 = \left(7850 \text{ kg/m}^3\right)\left[\pi\left(0.0015 \text{ m}\right)^2\right] \times \left(0.36 \text{ m}\right)$$

$$= 0.019976 \text{ kg}$$

Using the equations given above and the parallel axis theorem, have

$$I_x = \left(I_x\right)_1 + \left(I_x\right)_2 + \left(I_x\right)_3 + \left(I_x\right)_4$$

where $$\left(I_x\right)_3 = \left(I_x\right)_4$$

Then

$$I_x = \left(0.062756 \text{ kg}\right)\left[\frac{1}{2}\left(0.36 \text{ m}\right)^2\right] + \left(0.062756\right)\left[\frac{1}{2}\left(0.36 \text{ m}\right)^2 + \left(0.36 \text{ m}\right)^2\right]$$

$$+ 2\left(0.019976 \text{ kg}\right)\left[\frac{1}{12}\left(0.36 \text{ m}\right)^2 + \left(0.18 \text{ m}\right)^2 + \left(0.36 \text{ m}\right)^2\right]$$

$$= \left[4.06659 + 12.19977 + 2\left(3.45185\right)\right] \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$= 23.1701 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_x = 23.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

Have $$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 + \left(I_y\right)_4$$

where $\left(I_y\right)_1 = \left(I_y\right)_2$ and $\left(I_y\right)_3 = \left(I_y\right)_4$

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PROBLEM 9.149 CONTINUED

Then

$$I_y = 2(0.062756\text{ kg})\left[(0.36\text{ m})^2\right] + 2(0.019976\text{ kg})\left[0 + (0.36\text{ m})^2\right]$$

$$= 2\left(8.13318 \times 10^{-3}\text{ kg}\cdot\text{m}^2\right) + 2\left(2.58889 \times 10^{-3}\text{ kg}\cdot\text{m}^2\right)$$

$$= 21.44414 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

or $I_y = 21.4 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

Have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

where $(I_z)_3 = (I_z)_4$

Then

$$I_z = (0.062756\text{ kg})\left[\frac{1}{2}(0.36\text{ m})^2\right]$$

$$+ (0.062756\text{ kg})\left[\left(\frac{1}{2} - \frac{4}{\pi^2}\right)(0.36\text{ m})^2 + \left(\frac{2 \times 0.36\text{ m}^2}{\pi}\right) + (0.36\text{ m})^2\right]$$

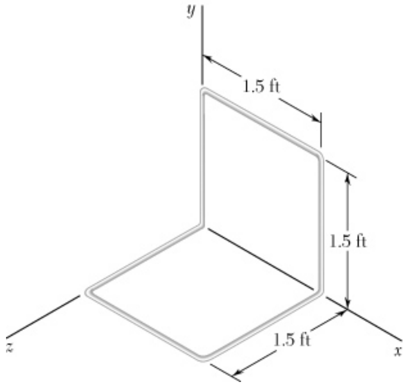
$$+ 2(0.019976\text{ kg})\left[\frac{1}{3}(0.36\text{ m})^2\right]$$

$$= \left[4.06659 + 12.1998 + 2(0.86296)\right]10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 17.9923 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

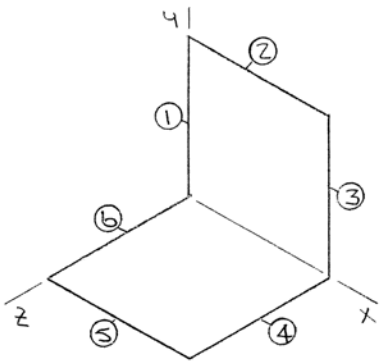
or $I_z = 17.99 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.150



A homogeneous wire with a weight per unit length of 0.041 lb/ft is used to form the figure shown. Determine the moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. Mass of each component is

indentical

$$m = \frac{(m/L)}{g}L$$

Have

$$= \frac{(0.041\text{ lb/ft})(1.5\text{ ft})}{32.2\text{ ft/s}^2}$$

$$= 0.00190994\text{ lb}\cdot\text{s}^2\text{/ft}$$

Using the equations given above and the parallel axis theorem, have

$$(I_x)_1 = (I_x)_3 + (I_x)_4 = (I_x)_6 \qquad \text{and} \qquad (I_x)_2 = (I_x)_5$$

Then

$$I_x = 4(I_x)_1 + 2(I_x)_2$$

$$I_x = 4(0.00190994\text{ lb}\cdot\text{s}^2\text{/ft})\left[\frac{1}{3}(5\text{ ft})^2\right]$$

$$+ 2(0.00190994\text{ lb}\cdot\text{s}^2\text{/ft})\left[0 + (1.5\text{ ft})^2\right]$$

$$= 0.0143246\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 14.32 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now

$$(I_y)_1 = 0 \qquad (I_y)_2 = (I_y)_6 \qquad (I_y)_4 = (I_y)_5$$

PROBLEM 9.150 CONTINUED

Then

$$I_y = 2(I_y)_2 + (I_y)_3 + 2(I_y)_4$$

$$= \left(0.0019094 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[2\left(\frac{1}{3}\right)(1.5 \text{ ft})^2\right] + \left[0 + (1.5 \text{ ft})^2\right]\right.$$

$$\left. + 2\left[\frac{1}{12}(1.5 \text{ ft})^2 + (1.5 \text{ ft})^2 + (0.75 \text{ ft})^2\right]\right\}$$

$$= 0.0019094(1.5 + 2.25 + 6) \text{ lb}\cdot\text{ft}\cdot\text{s}^2 = 0.0186219 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 18.62 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

By symmetry $I_z = I_y$

$$I_z = 18.62 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$