PROBLEMS 5.87 AND 5.88 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 11.32 + 23.4 + 93.6 + 468 = 0$$

or $$A_x = -596.32 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 240 - 240 - 240 + 187.2 = 0$$

or $$A_y = 532.8 \text{ lb} \qquad \therefore \ \mathbf{A} = 800 \text{ lb} \searrow 41.8° ◀$$

5.88 At $h = (d-2)\text{ft}, \quad p_{d-2} = \gamma(d-2) \text{ lb/ft}^2$ where $\gamma = 62.4 \text{ lb/ft}^3$

$$h = d \text{ ft}, \quad p_d = (\gamma d) \text{ lb/ft}^2$$

Then $$P_1 = \frac{1}{2}A_1 p_{d-2} = \frac{1}{2}\left[(d-2)\text{ft} \times (3\text{ ft})\right]\left[\gamma \text{ lb/ft}^3 (d-2)\text{ft}\right] = \frac{3}{2}\gamma(d-2)^2 \text{ lb}$$

(Note: For simplicity, the numerical value of the density γ will be substituted into the equilibrium equations below, rather than at this level of the calculations.)

$$P_2 = A_2 p_{d-2} = \left[(2\text{ ft})(3\text{ ft})\right]\gamma\left[(d-2)\text{ft}\right] = 6\gamma(d-2) \text{ lb}$$

$$P_3 = \frac{1}{2}A_3 p_{d-2} = \frac{1}{2}\left[(2\text{ ft})(3\text{ ft})\right]\gamma\left[(d-2)\text{ft}\right] = 3\gamma(d-2) \text{ lb}$$

$$P_4 = \frac{1}{2}A_4 p_d = \frac{1}{2}\left[(2\text{ ft})(3\text{ ft})\right]\gamma(d \text{ ft}) = (3\gamma d) \text{ lb} = \left[3\gamma(d-2) + 6\gamma\right] \text{ lb}$$

As the gate begins to open, $\mathbf{D} \to 0$

$$\therefore \ +\circlearrowright \Sigma M_A = 0: \quad (2\text{ ft})(240\text{ lb}) + (1\text{ ft})(240\text{ lb}) - \left[2\text{ ft} + \frac{1}{3}(d-2)\text{ft}\right]\left[\frac{3}{2}\gamma(d-2)^2 \text{ lb}\right] +$$

$$-(1\text{ ft})\left[6\gamma(d-2)\text{ lb}\right] - \left[\frac{2}{3}(2\text{ ft})\right]\left[3\gamma(d-2)\text{ lb}\right]$$

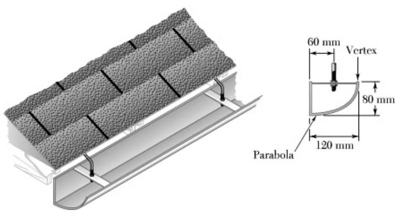
$$-\left[\frac{1}{3}(2\text{ ft})\right]\left[3\gamma(d-2)\text{ lb} + 6\gamma \text{ lb}\right] = 0$$

or $$\frac{1}{2}(d-2)^3 + 3(d-2)^2 + 12(d-2) = \frac{720}{\gamma} - 4$$

$$= \frac{720}{62.4} - 4$$

$$= 7.53846$$

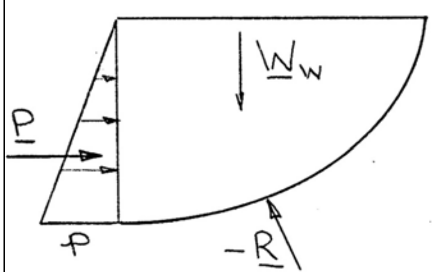
Solving numerically yields $$d = 2.55 \text{ ft} ◀$$



PROBLEM 5.89

A rain gutter is supported from the roof of a house by hangers that are spaced 0.6 m apart. After leaves clog the gutter's drain, the gutter slowly fills with rainwater. When the gutter is completely filled with water, determine (*a*) the resultant of the pressure force exerted by the water on the 0.6-m section of the curved surface of the gutter, (*b*) the force-couple system exerted on a hanger where it is attached to the gutter.

SOLUTION



(*a*) Consider a 0.6 m long parabolic section of water.

Then $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g h)$

$$= \frac{1}{2}(0.08\text{ m})(0.6\text{ m})\left[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.08\text{ m})\right]$$

$$= 18.84\text{ N}$$

$$W_w = \rho g V$$

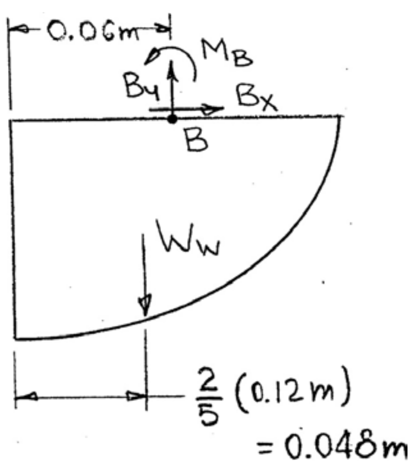
$$= (10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[\frac{2}{3}(0.12\text{ m})(0.08\text{ m})(0.6\text{ m})\right]$$

$$= 37.67\text{ N}$$

Now $\Sigma\mathbf{F} = 0\text{:} \quad (-\mathbf{R}) + \mathbf{P} + \mathbf{W}_w = 0$

So that $R = \sqrt{P^2 + W_w{}^2}, \qquad \tan\theta = \dfrac{W_w}{P}$

$$= 42.12\text{ N}, \quad \theta = 63.4° \qquad \mathbf{R} = 42.1\text{ N} \nwarrow 63.4° \blacktriangleleft$$



(*b*) Consider the free-body diagram of a 0.6 m long section of water and gutter.

Then $\xrightarrow{+} \Sigma F_x = 0\text{:} \quad B_x = 0$

$$+\uparrow \Sigma F_y = 0\text{:} \quad B_y - 37.67\text{ N} = 0$$

or $B_y = 37.67\text{ N}$

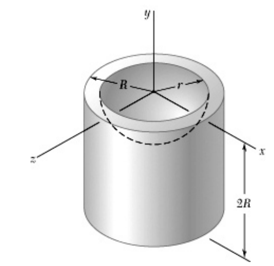
$$+\circlearrowright \Sigma M_B = 0\text{:} \quad M_B + \left[(0.06 - 0.048)\text{ m}\right](37.67\text{ N}) = 0$$

or $M_B = -0.4520\text{ N}\cdot\text{m}$

The force-couple system exerted on the hanger is then

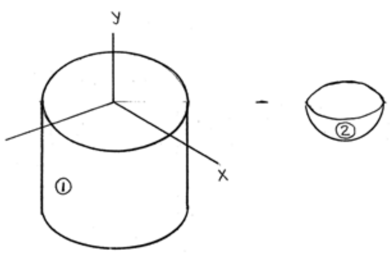
$$37.7\text{ N}\downarrow,\ 0.452\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

PROBLEM 5.90



The composite body shown is formed by removing a hemisphere of radius r from a cylinder of radius R and height $2R$. Determine (*a*) the y coordinate of the centroid when $r = 3R/4$, (*b*) the ratio r/R for which $\bar{y} = -1.2R$.

SOLUTION



Note, for the axes shown

	V	$\bar{y}$	$\bar{y}V$
1	$(\pi R^2)(2R) = 2\pi R^3$	$-R$	$-2\pi R^4$
2	$-\frac{2}{3}\pi r^3$	$-\frac{3}{8}r$	$\frac{1}{4}\pi r^4$
Σ	$2\pi\left(R^3 - \frac{r^3}{3}\right)$		$-2\pi\left(R^4 - \frac{r^4}{8}\right)$

Then
$$\bar{Y} = \frac{\Sigma \bar{y}V}{\Sigma V} = -\frac{R^4 - \frac{1}{8}r^4}{R^3 - \frac{1}{3}r^3}$$

$$= \frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}$$

(*a*)
$$r = \frac{3}{4}R: \quad \bar{y} = -\frac{1 - \frac{1}{3}\left(\frac{3}{4}\right)^4}{1 - \frac{1}{3}\left(\frac{3}{4}\right)^3}R$$

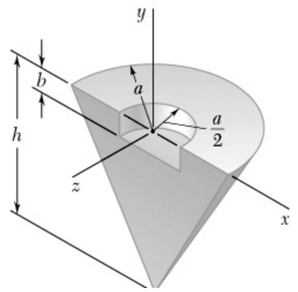
or $\bar{y} = -1.118R$ ◀

(*b*)
$$\bar{y} = -1.2R: \quad -1.2R = -\frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}R$$

or
$$\left(\frac{r}{R}\right)^4 - 3.2\left(\frac{r}{R}\right)^3 + 1.6 = 0$$

Solving numerically $\frac{r}{R} = 0.884$ ◀

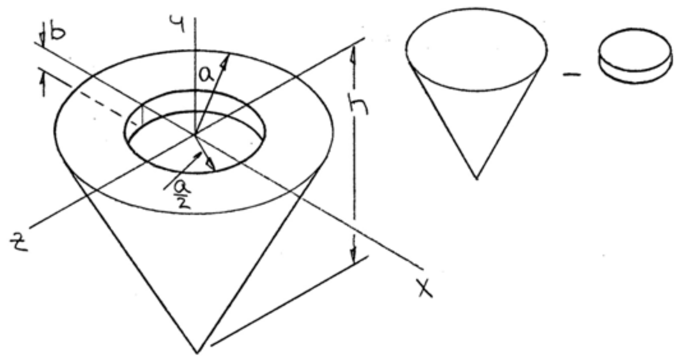
PROBLEM 5.91



Determine the y coordinate of the centroid of the body shown.

SOLUTION

First note that the values of $\overline{Y}$ will be the same for the given body and the body shown below. Then



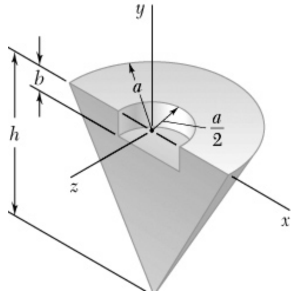
	V	$\overline{y}$	$\overline{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi\left(\frac{a}{2}\right)^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h - 3b)$		$-\frac{\pi}{24}a^2\left(2h^2 - 3b^2\right)$

Have $$\overline{Y}\Sigma V = \Sigma\overline{y}V$$

Then $$\overline{Y}\left[\frac{\pi}{12}a^2(4h - 3b)\right] = -\frac{\pi}{24}a^2\left(2h^2 - 3b^2\right)$$

or $$\overline{Y} = -\frac{2h^2 - 3b^2}{2(4h - 3b)} \blacktriangleleft$$

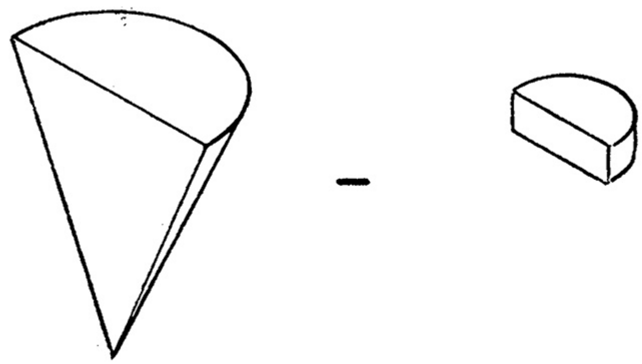
PROBLEM 5.92



Determine the z coordinate of the centroid of the body shown. (*Hint:* Use the result of Sample Problem 5.13.)

SOLUTION

First note that the body can be formed by removing a “half-cylinder” from a “half-cone,” as shown.



	V	$\bar{z}$	$\bar{z}V$
Half-Cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$ *	$-\frac{1}{6}a^3 h$
Half-Cylinder	$-\frac{\pi}{2}\left(\frac{a}{2}\right)^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}\left(\frac{a}{2}\right) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h - 3b)$		$-\frac{1}{12}a^3(2h - b)$

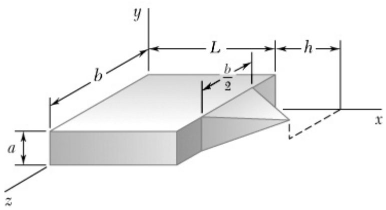
From Sample Problem 5.13

Have $$\bar{Z}\Sigma V = \Sigma\bar{z}V$$

Then $$\bar{Z}\left[\frac{\pi}{24}a^2(4h - 3b)\right] = -\frac{1}{12}a^3(2h - b)$$

$$\text{or } \bar{Z} = -\frac{2a}{\pi}\frac{2h - b}{4h - 3b} \blacktriangleleft$$

PROBLEM 5.93



Consider the composite body shown. Determine (*a*) the value of $\bar{x}$ when $h = L/2$, (*b*) the ratio h/L for which $\bar{x} = L$.

SOLUTION

	V	$\bar{x}$	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L+\frac{1}{4}h$	$\frac{1}{6}abh\left(L+\frac{1}{4}h\right)$

Then $$\Sigma V = ab\left(L+\frac{1}{6}h\right) \qquad \Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L+\frac{1}{4}h\right)\right]$$

Now $$\bar{X}\Sigma V = \Sigma\bar{x}V \qquad \text{so that}$$

$$\bar{X}\left[ab\left(L+\frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2 + hL + \frac{1}{4}h^2\right)$$

or $$\bar{X}\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right) \qquad (1)$$

(*a*) $\bar{X} = ?$ when $h = \frac{1}{2}L$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1)

$$\bar{X}\left[1+\frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3+\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

or $$\bar{X} = \frac{57}{104}L \qquad \bar{X} = 0.548L \blacktriangleleft$$

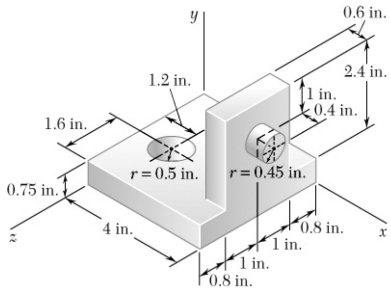
(*b*) $\frac{h}{L} = ?$ when $\bar{X} = L$

Substituting into Eq. (1) $$L\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right)$$

or $$1+\frac{1}{6}\frac{h}{L} = \frac{1}{2}+\frac{1}{6}\frac{h}{L}+\frac{1}{24}\frac{h^2}{L^2}$$

or $$\frac{h^2}{L^2} = 12 \qquad \therefore\ \frac{h}{L} = 2\sqrt{3} \blacktriangleleft$$

PROBLEMS 5.94 AND 5.95

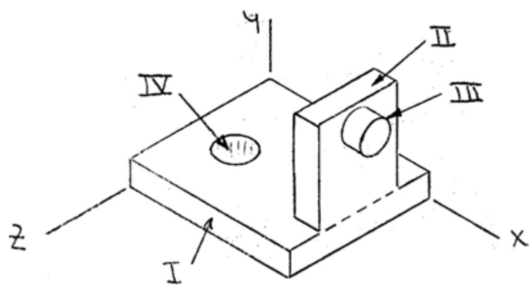


Problem 5.94: For the machine element shown, determine the *x* coordinate of the center of gravity.

Problem 5.95: For the machine element shown, determine the *y* coordinate of the center of gravity.

SOLUTIONS

First, assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



	V, in^3	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}V$, in^4	$\bar{y}V$, in^4
I	$(4)(3.6)(0.75) = 10.8$	2.0	0.375	21.6	4.05
II	$(2.4)(2.0)(0.6) = 2.88$	3.7	1.95	10.656	5.616
III	$\pi(0.45)^2(0.4) = 0.2545$	4.2	2.15	1.0688	0.54711
IV	$-\pi(0.5)^2(0.75) = -0.5890$	1.2	0.375	–0.7068	–0.22089
Σ	13.3454			32.618	9.9922

5.94

Have $$\bar{X}\Sigma V = \Sigma \bar{x} V$$

$$\bar{X}\left(13.3454 \text{ in}^3\right) = 32.618 \text{ in}^4$$

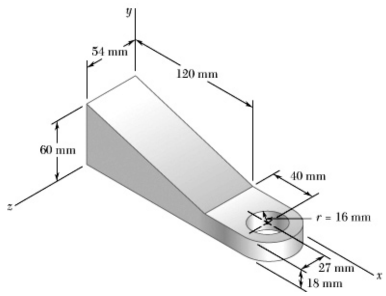
or $\bar{X} = 2.44$ in. ◀

5.95

Have $$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}\left(13.3454 \text{ in}^3\right) = 9.9922 \text{ in}^4$$

or $\bar{Y} = 0.749$ in. ◀



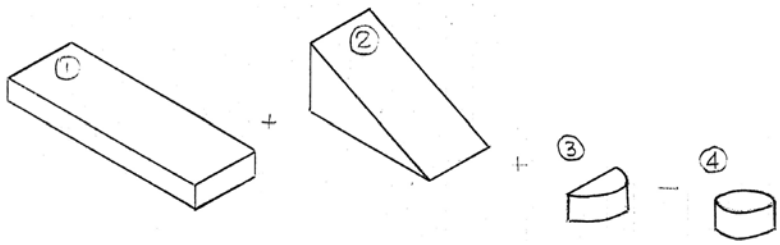
PROBLEMS 5.96 AND 5.97

Problem 5.96: For the machine element shown, locate the *x* coordinate of the center of gravity.

Problem 5.97: For the machine element shown, locate the *y* coordinate of the center of gravity.

SOLUTIONS

First, assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
1	$(160)(54)(18) = 155\ 520$	80	9	12 441 600	1 399 680
2	$\frac{1}{2}(120)(42)(54) = 136\ 080$	40	32	5 443 200	4 354 560
3	$\frac{\pi}{2}(27)^2(18) = 6561\pi$	$160 + \frac{36}{\pi}$	9	3 534 114	185 508
4	$-\pi(16)^2(18) = -4608\pi$	160	9	–2 316 233	–130 288
Σ	297 736			19 102 681	5 809 460

5.96

Have $$\bar{X}\Sigma V = \Sigma\bar{x}V$$

$$\bar{X}\left(297\ 736\ \text{mm}^3\right) = 19\ 102\ 681\ \text{mm}^4$$

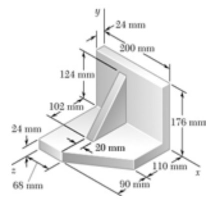
or $\bar{X} = 64.2$ mm ◀

5.97

Have $$\bar{Y}\Sigma V = \Sigma\bar{y}V$$

$$\bar{Y}\left(297\ 736\ \text{mm}^3\right) = 5\ 809\ 460\ \text{mm}^4$$

or $\bar{Y} = 19.51$ mm ◀



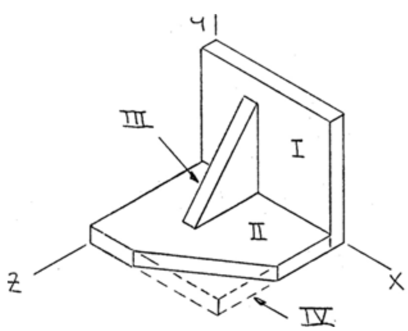
PROBLEMS 5.98 AND 5.99

Problem 5.98: For the stop bracket shown, locate the *x* coordinate of the center of gravity.

Problem 5.99: For the stop bracket shown, locate the *z* coordinate of the center of gravity.

SOLUTIONS

First, assume that the bracket is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



Have..

$$\bar{Z}_{\mathrm{II}} = 24\text{ mm} + \frac{1}{2}(90+86)\text{mm} = 112\text{ mm}$$

$$\bar{Z}_{\mathrm{III}} = 24\text{ mm} + \frac{1}{3}(102)\text{mm} = 58\text{ mm}$$

$$\bar{X}_{\mathrm{III}} = 68\text{ mm} + \frac{1}{2}(20)\text{mm} = 78\text{ mm}$$

$$\bar{Z}_{\mathrm{IV}} = 110\text{ mm} + \frac{2}{3}(90)\text{mm} = 170\text{ mm}$$

$$\bar{X}_{\mathrm{IV}} = 60\text{ mm} + \frac{2}{3}(132)\text{mm} = 156\text{ mm}$$

	V, mm^3	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}V$, mm^4	$\bar{z}V$, mm^4
I	$(200)(176)(24) = 844\,800$	100	12	84 480 000	1 013 760
II	$(200)(24)(176) = 844\,800$	100	112	84 480 000	94 617 600
III	$\frac{1}{2}(20)(124)(102) = 126\,480$	78	58	9 865 440	733 840
IV	$-\frac{1}{2}(90)(132)(24) = -142\,560$	156	170	–22 239 360	–24 235 200
Σ	1 673 520			156 586 080	8 785 584

5.98

Have $\bar{X}\Sigma V = \Sigma\bar{x}V$

$$\bar{X}\left(1\,673\,520\text{ mm}^3\right) = 156\,586\,080\text{ mm}^4$$

or $\bar{X} = 93.6\text{ mm}$ ◀

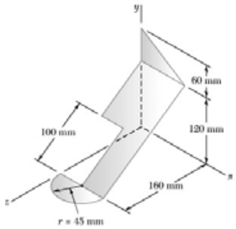
5.99

Have $\bar{Z}\Sigma V = \Sigma\bar{z}V$

$$\bar{Z}\left(1\,673\,520\text{ mm}^3\right) = 8\,785\,584\text{ mm}^4$$

or $\bar{Z} = 52.5\text{ mm}$ ◀

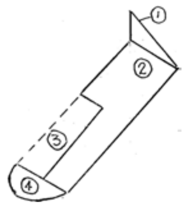
PROBLEM 5.100



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity coincides with the centroid of the corresponding area.



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
1	$\frac{1}{2}(90)(60) = 2700$	30	$120 + 20 = 140$	0	81 000	378 000	0
2	$(90)(200) = 18\,000$	45	60	80	810 000	1 080 000	1 440 000
3	$-(45)(100) = -4500$	22.5	30	120	–101 250	–135 000	–540 000
4	$\frac{\pi}{2}(45)^2 = 1012.5\pi$	45	0	$160 + \frac{(4)(45)}{3\pi} = 179.1$	143 139	0	569 688
Σ	19 380.9				932 889	1 323 000	1 469 688

Have $\bar{X}\Sigma A = \Sigma\bar{x}A$:

$$\bar{X}\left(19\,380.9 \text{ mm}^2\right) = 932\,889 \text{ mm}^3$$

or $$= 48.1 \text{ mm}$$ $\bar{X} = 48.1 \text{ mm}$ ◀

$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\left(19\,380.9 \text{ mm}^2\right) = 1\,323\,000 \text{ mm}^3$$

or $$\bar{Y} = 68.3 \text{ mm}$$ $\bar{Y} = 68.3 \text{ mm}$ ◀

$$\bar{Z}\Sigma A = \Sigma\bar{z}A$$

$$\bar{Z}\left(19\,380.9 \text{ mm}^2\right) = 1\,469\,688 \text{ mm}^3$$

or $$\bar{Z} = 75.8 \text{ mm}$$ $\bar{Z} = 75.8 \text{ mm}$ ◀