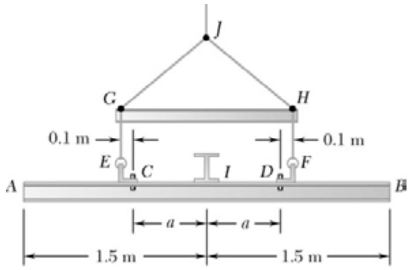
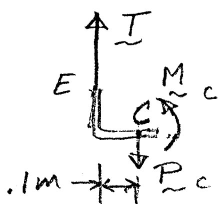


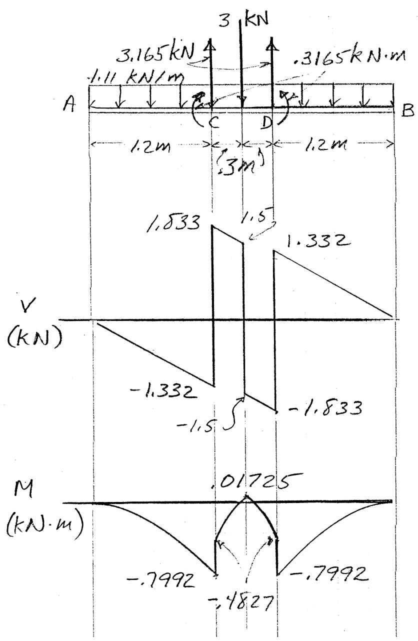
PROBLEM 7.45

Two short angle sections *CE* and *DF* are bolted to the uniform beam *AB* of weight 3.33 kN, and the assembly is temporarily supported by the vertical cables *EG* and *FH* as shown. A second beam resting on beam *AB* at *I* exerts a downward force of 3 kN on *AB*. Knowing that $a = 0.3$ m and neglecting the weight of the ngle sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD angle CE:





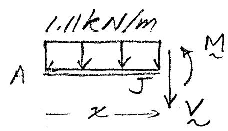
(*a*) By symmetry: $T = \dfrac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$

$$\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165\text{ kN}$$

$$\circlearrowleft \Sigma M_C = 0: M_C - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M_C = 0.3165\text{ kN}\cdot\text{m}$$

By symmetry: $P_D = 3.165\text{ kN};\ M_D = 0.3165\text{ kN}\cdot\text{m}$

Along AC:



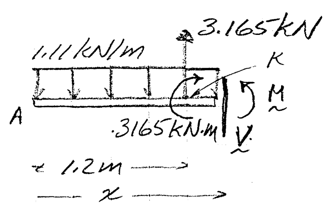
$$\uparrow \Sigma F_y = 0: -x(1.11\text{ kN/m}) - V = 0$$

$$V = -(1.11\text{ kN/m})x \qquad V = -1.332\text{ kN at } C \quad (x = 1.2\text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

$$M = (0.555\text{ kN/m})x^2 \qquad M = -0.7992\text{ kN}\cdot\text{m at } C$$

Along CI:



$$\uparrow \Sigma F_y = 0: -(1.11\text{ kN/m})x + 3.165\text{ kN} - V = 0$$

$$V = 3.165\text{ kN} - (1.11\text{ kN/m})x \qquad V = 1.5\text{ kN at } I \quad (x = 1.5\text{ m})$$

$$\circlearrowleft \Sigma M_k = 0:$$

$$M + (1.11\text{ kN/m})x - (x - 1.2\text{ m})(3.165\text{ kN}) - (0.3165\text{ kN}\cdot\text{m}) = 0$$

PROBLEM 7.45 CONTINUED

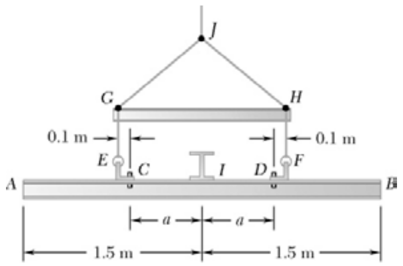
$$M = 3.4815 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})x - (0.555 \text{ kN/m})x^2$$

$$M = -0.4827 \text{ kN}\cdot\text{m at } C \qquad M = 0.01725 \text{ kN}\cdot\text{m at } I$$

Note: At *I*, the downward 3 kN force will reduce the shear *V* by 3 kN, from +1.5 kN to –1.5 kN, with no change in *M*. From *I* to *B*, the diagram can be completed by symmetry.

(*b*) From diagrams: $|V|_{\max} = 1.833 \text{ kN at } C \text{ and } D$ ◀

$|M|_{\max} = 799 \text{ N}\cdot\text{m at } C \text{ and } D$ ◀

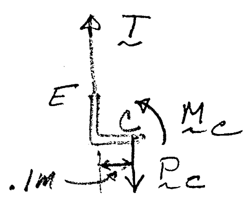


PROBLEM 7.46

Solve Prob. 7.45 when $a = 0.6$ m.

SOLUTION

FBD angle CE:

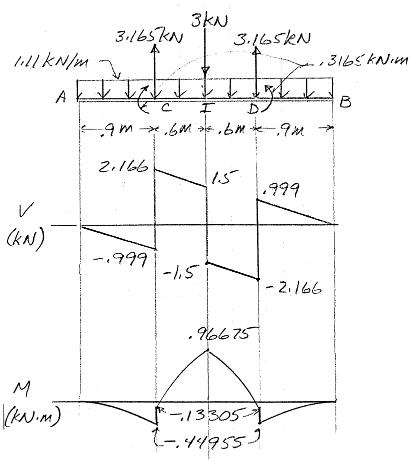


(*a*) By symmetry: $$T = \frac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$$

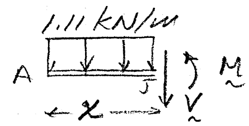
$$\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165\text{ kN}$$

$$\circlearrowleft \Sigma M_C = 0: M_C - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M_C = 0.3165\text{ kN}\cdot\text{m}$$

By symmetry: $P_D = 3.165$ kN $\qquad M_D = 0.3165$ kN·m



Along AC:



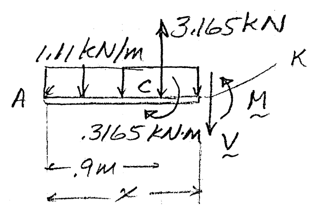
$$\uparrow \Sigma F_y = 0: -(1.11\text{ kN/m})x - V = 0$$

$$V = -(1.11\text{ kN/m})x \qquad V = -0.999\text{ kN at } C \quad (x = 0.9\text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

$$M = -(0.555\text{ kN/m})x^2 \qquad M = -0.44955\text{ kN}\cdot\text{m at } C$$

Along CI:



$$\uparrow \Sigma F_y = 0: -x(1.11\text{ kN/m}) + 3.165\text{ kN} - V = 0$$

$$V = 3.165\text{ kN} - (1.11\text{ kN/m})x \qquad V = 2.166\text{ kN at } C$$

$$V = 1.5\text{ kN at } I \quad (x = 1.5\text{ m})$$

$$\circlearrowleft \Sigma M_K = 0:$$

$$M - 0.3165\text{ kN}\cdot\text{m} + (x - 0.9\text{ m})(3.165\text{ kN}) + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

PROBLEM 7.46 CONTINUED

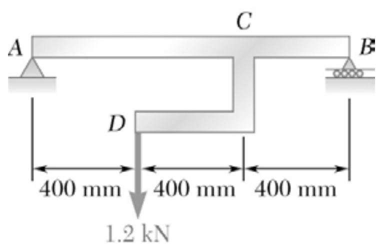
$$M = -2.532 \text{ kN}\cdot\text{m} + (3.165 \text{ kN})x - (0.555 \text{ kN/m})x^2$$

$$M = -0.13305 \text{ kN}\cdot\text{m at } C \qquad M = 0.96675 \text{ kN}\cdot\text{m at } I$$

Note: At I, the downward 3 kN force will reduce the shear V by 3 kN, from +1.5 kN to –1.5 kN, with no change in M. From I to B, the diagram can be completed by symmetry.

(b) From diagrams: $|V|_{\max} = 2.17 \text{ kN at } C \text{ and } D$ ◀

$|M|_{\max} = 967 \text{ N}\cdot\text{m at } I$ ◀

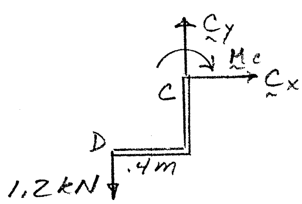


PROBLEM 7.47

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the shear and bending moment (*a*) just to the left of *C*, (*b*) just to the right of *C*.

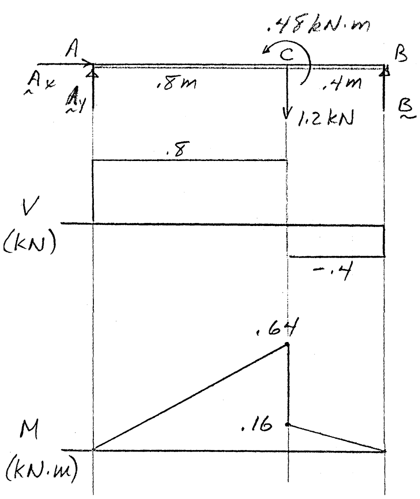
SOLUTION

FBD CD:



$$\uparrow \Sigma F_y = 0: -1.2\text{ kN} + C_y = 0 \qquad \mathbf{C}_y = 1.2\text{ kN}\uparrow$$

$$\circlearrowleft \Sigma M_C = 0: (0.4\text{ m})(1.2\text{ kN}) - M_C = 0 \qquad M_C = 0.48\text{ kN}\cdot\text{m}$$



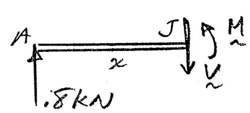
FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (1.2\text{ m})B + 0.48\text{ kN}\cdot\text{m} - (0.8\text{ m})(1.2\text{ kN}) = 0$$

$$\mathbf{B} = 0.4\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 1.2\text{ kN} + 0.4\text{ kN} = 0 \qquad \mathbf{A}_y = 0.8\text{ kN}\uparrow$$

Along AC:

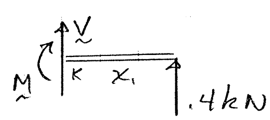


$$\uparrow \Sigma F_y = 0: 0.8\text{ kN} - V = 0 \qquad V = 0.8\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M - x(0.8\text{ kN}) = 0 \qquad M = (0.8\text{ kN})x$$

$$M = 0.64\text{ kN}\cdot\text{m at } x = 0.8\text{ m}$$

Along CB:



$$\uparrow \Sigma F_y = 0: \quad V + 0.4\text{ kN} = 0 \qquad V = -0.4\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: x_1(0.4\text{ kN}) - M = 0 \qquad M = (0.4\text{ kN})x_1$$

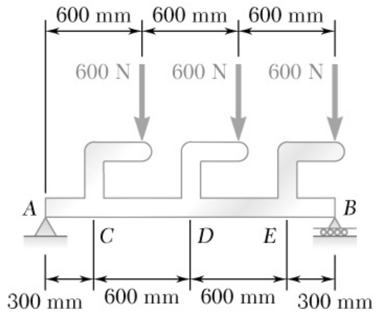
$$M = 0.16\text{ kN}\cdot\text{m at } x_1 = 0.4\text{ m}$$

(*a*) Just left of *C*: $V = 800\text{ N}$ ◀

$M = 640\text{ N}\cdot\text{m}$ ◀

(*b*) Just right of *C*: $V = -400\text{ N}$ ◀

$M = 160.0\text{ N}\cdot\text{m}$ ◀

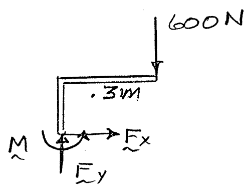


PROBLEM 7.48

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

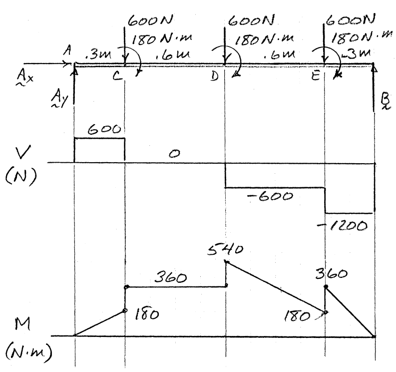
FBD angle:



$$\uparrow \Sigma F_y = 0: F_y - 600 \text{ N} = 0 \qquad F_y = 600 \text{ N}$$

$$\circlearrowleft \Sigma M_{\text{Base}} = 0: \quad M - (0.3 \text{ m})(600 \text{ N}) = 0 \qquad M = 180 \text{ N}\cdot\text{m}$$

All three angles are the same.



FBD Beam:

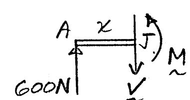
$$\circlearrowleft \Sigma M_A = 0: (1.8 \text{ m})B - 3(180 \text{ N}\cdot\text{m})$$

$$- (0.3 \text{ m} + 0.9 \text{ m} + 1.5 \text{ m})(600 \text{ N}) = 0$$

$$\mathbf{B} = 1200 \text{ N} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 3(600 \text{ N}) + 1200 \text{ N} = 0 \qquad \mathbf{A}_y = 600 \text{ N} \uparrow$$

Along AC:

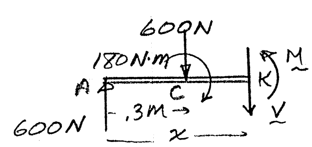


$$\uparrow \Sigma F_y = 0: 600 \text{ N} - V = 0 \qquad V = 600 \text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: M - x(600 \text{ N}) = 0$$

$$M = (600 \text{ N})x = 180 \text{ N}\cdot\text{m at } x = .3 \text{ m}$$

Along CD:



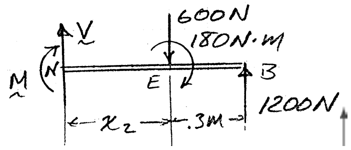
$$\uparrow \Sigma F_y = 0: 600 \text{ N} - 600 \text{ N} - V = 0 \qquad V = 0$$

$$\circlearrowleft \Sigma M_K = 0: M + (x - 0.3 \text{ m})(600 \text{ N}) - 180 \text{ N}\cdot\text{m} - x(600 \text{ N}) = 0$$

$$M = 360 \text{ N}\cdot\text{m}$$

PROBLEM 7.48 CONTINUED

Along DE:



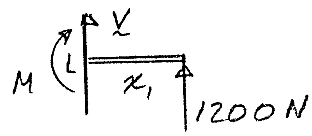
$$\Sigma F_y = 0: V - 600\text{ N} + 1200\text{ N} = 0 \qquad V = -600\text{ N}$$

$$\Sigma M_N = 0: -M - 180\text{ N}\cdot\text{m} - x_2(600\text{ N}) + (x_2 + 0.3\text{ m})(1200\text{ N}) = 0$$

$$M = 180\text{ N}\cdot\text{m} + (600\text{ N})x_2 = 540\text{ N}\cdot\text{m at D, } x_2 = 0.6\text{ m}$$

$$M = 180\text{ N}\cdot\text{m at E } (x_2 = 0)$$

Along EB:



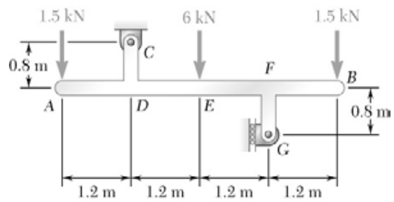
$$\Sigma F_y = 0: V + 1200\text{ N} = 0 \qquad V = -1200\text{ N}$$

$$\Sigma M_L = 0: x_1(1200\text{ N}) - M = 0 \qquad M = (1200\text{ N})x_1$$

$$M = 360\text{ N}\cdot\text{m at } x_1 = 0.3\text{ m}$$

From diagrams: $|V|_{max} = 1200\text{ N on } EB$ ◀

$|M|_{max} = 540\text{ N}\cdot\text{m at } D^+$ ◀

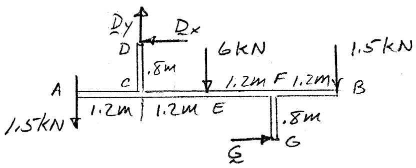


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



$$\circlearrowleft \Sigma M_D = 0: (1.2\text{ m})(1.5\text{ kN}) - (1.2\text{ m})(6\text{ kN})$$

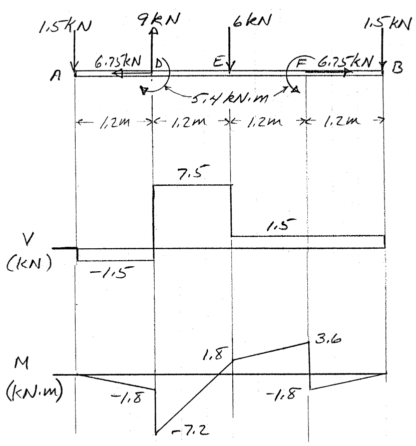
$$-(3.6\text{ m})(1.5\text{ kN}) + (1.6\text{ m})G = 0$$

$$\mathbf{G} = 6.75\text{ kN} \longrightarrow$$

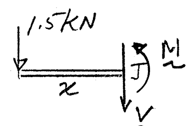
$$\longrightarrow \Sigma F_x = 0: -D_x + G = 0 \qquad \mathbf{D}_x = 6.75\text{ kN} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: D_y - 1.5\text{ kN} - 6\text{ kN} - 1.5\text{ kN} = 0 \qquad \mathbf{D}_y = 9\text{ kN} \uparrow$$

Beam AB, with forces **D** and **G** replaced by equivalent force/couples at *C* and *F*



Along AD:

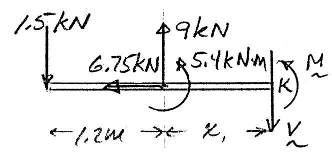


$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} - V = 0 \qquad V = -1.5\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: x(1.5\text{ kN}) + M = 0 \qquad M = -(1.5\text{ kN})x$$

$$M = -1.8\text{ kN at } x = 1.2\text{ m}$$

Along DE:



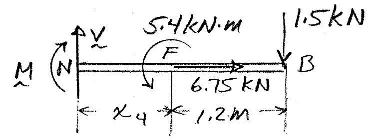
$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} + 9\text{ kN} - V = 0 \qquad V = 7.5\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: M + 5.4\text{ kN}\cdot\text{m} - x_1(9\text{ kN}) + (1.2\text{ m} + x_1)(1.5\text{ kN}) = 0$$

$$M = 7.2\text{ kN}\cdot\text{m} + (7.5\text{ kN})x_1 \qquad M = 1.8\text{ kN}\cdot\text{m at } x_1 = 1.2\text{ m}$$

PROBLEM 7.49 CONTINUED

Along EF:



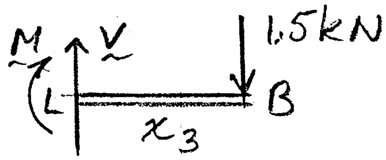
$$\uparrow \Sigma F_y = 0: V - 1.5\text{ kN} = 0 \qquad V = 1.5\text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: -M + 5.4\text{ kN}\cdot\text{m} - \left(x_4 + 1.2\text{ m}\right)\left(1.5\text{ kN}\right)$$

$$M = 3.6\text{ kN}\cdot\text{ m} - \left(1.5\text{ kN}\right)x_4$$

$M = 1.8$ kN·m at $x_4 = 1.2$ m; M = 3.6 kN·m at $x_4 = 0$

Along FB:



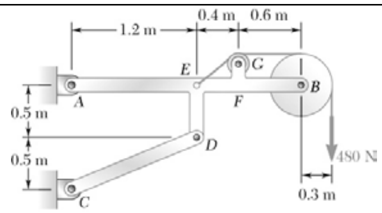
$$\uparrow \Sigma F_y = 0: V - 1.5\text{ kN} = 0 \qquad V = 1.5\text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_3\left(1.5\text{ kN}\right) = 0 \qquad M = \left(-1.5\text{ kN}\right)x_3$$

$$M = -1.8\text{ kN}\cdot\text{m} \text{ at } x_3 = 1.2\text{ m}$$

From diagrams: $|V|_{max} = 7.50$ kN on DE ◀

$|M|_{max} = 7.20$ kN·m at D^+ ◀

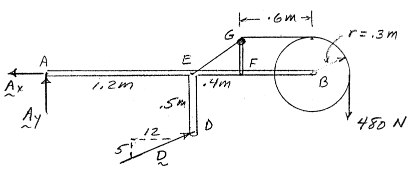


PROBLEM 7.50

Neglecting the size of the pulley at G, (a) draw the shear and bending-moment diagrams for the beam AB, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



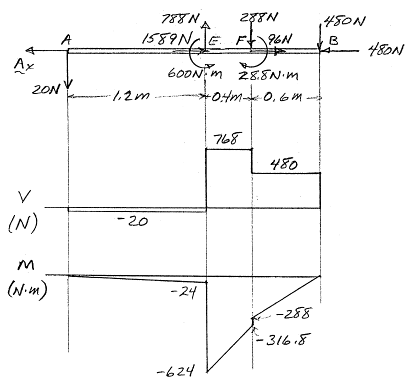
(a) $$\circlearrowleft \Sigma M_A = 0: (0.5\text{ m})\frac{12}{13}D + (1.2\text{ m})\frac{5}{13}D - (2.5\text{ m})(480\text{ N}) = 0$$

$$D = 1300\text{ N}$$

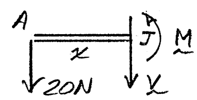
$$\uparrow \Sigma F_y = 0: A_y + \frac{5}{13}(1300\text{ N}) - 480\text{ N} = 0$$

$$A_y = -20\text{ N} \qquad \mathbf{A}_y = 20\text{ N}\downarrow$$

Beam AB with pulley forces and force at D replaced by equivalent force-couples at B, F, E



Along AE:

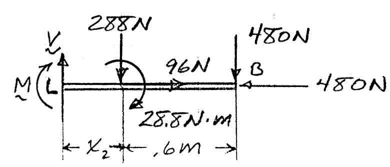


$$\uparrow \Sigma F_y = 0: -20\text{ N} - V = 0 \qquad V = -20\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(20\text{ N}) \qquad M = -(20\text{ N})x$$

$$M = -24\text{ N}\cdot\text{m at } x = 1.2\text{ m}$$

Along EF:



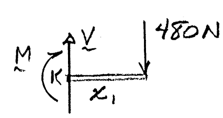
$$\uparrow \Sigma F_y = 0: V - 288\text{ N} - 480\text{ N} = 0 \qquad V = 768\text{ N}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_2(288\text{ N}) - (28.8\text{ N}\cdot\text{m}) - (x_2 + 0.6\text{ m})(480\text{ N}) = 0$$

$$M = -316.8\text{ N}\cdot\text{m} - (768\text{ N})x_2$$

$$M = -316.8\text{ N}\cdot\text{m at } x_2 = 0; \quad M = -624\text{ N}\cdot\text{m at } x_2 = 0.4\text{ m}$$

Along FB:



$$\uparrow \Sigma F_y = 0: V - 480\text{ N} = 0 \qquad V = 480\text{ N}$$

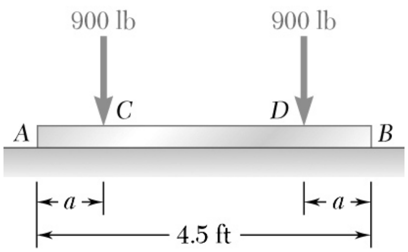
$$\circlearrowleft \Sigma M_K = 0: -M - x_1(480\text{ N}) = 0 \qquad M = -(480\text{ N})x_1$$

$$M = -288\text{ N}\cdot\text{m at } x_1 = 0.6\text{ m}$$

PROBLEM 7.50 CONTINUED

(*b*) From diagrams: $|V|_{max} = 768 \text{ N along } EF$ ◀

$|M|_{max} = 624 \text{ N}\cdot\text{m at } E^{+}$ ◀



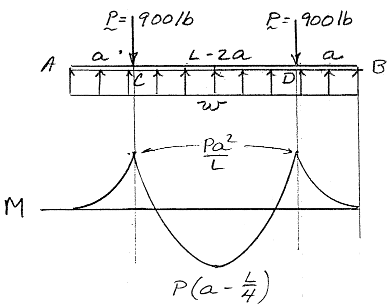
PROBLEM 7.51

For the beam of Prob. 7.43, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$.

(*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

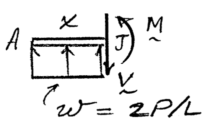
FBD Beam:



$$+\uparrow \Sigma F_y = 0: Lw - 2P = 0$$

$$w = 2\frac{P}{L}$$

Along AC:



$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0 \qquad M = \frac{P}{L}x^2$$

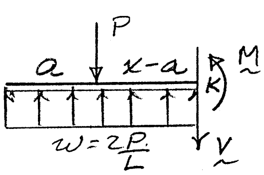
$$M = \frac{P}{L}a^2 \quad \text{at} \quad x = a$$

$$\circlearrowleft \Sigma M_K = 0: M + (x-a)P - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0$$

$$M = P(a-x) + \frac{P}{L}x^2 = \frac{Pa^2}{L} \qquad \text{at} \qquad x = a$$

$$M = P\left(a - \frac{L}{4}\right) \quad \text{at} \quad x = \frac{L}{2}$$

Along CD:



This is *M* min by symmetry–see moment diagram completed by symmetry.

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$:

$$P\frac{a^2}{L} = -P\left(a - \frac{L}{4}\right)$$

or

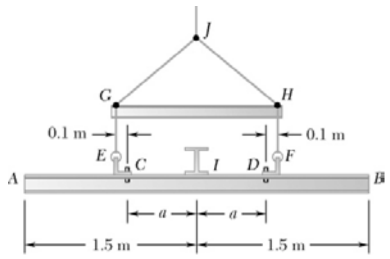
$$a^2 + La - \frac{L^2}{4} = 0$$

Solving:

$$a = \frac{-1 \pm \sqrt{2}}{2}L$$

Positive answer (*a*) $a = 0.20711L = 0.932 \text{ ft}$ ◀

(*b*) $|M|_{\max} = 0.04289PL = 173.7 \text{ lb}\cdot\text{ft}$ ◀

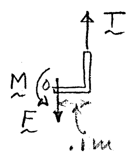


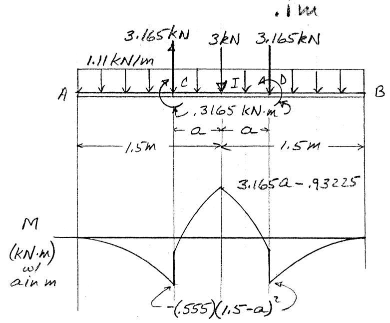
PROBLEM 7.52

For the assembly of Prob. 7.45, determine (a) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (b) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.51.)

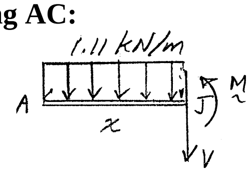
SOLUTION

FBD Angle:

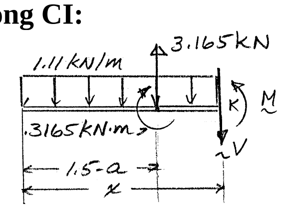




Along AC:



Along CI:



By symmetry of whole arrangement:

$$T = \frac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$$

$$\uparrow \Sigma F_y = 0: T - F = 0 \qquad F = 3.165\text{ kN}$$

$$\circlearrowleft \Sigma M_0 = 0: M - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M = 0.3165\text{ kN}\cdot\text{m}$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

$$M = -(0.555\text{ kN/m})x^2 = -(0.555\text{ kN/m})(1.5\text{ m} - a)^2$$

at C (this is M_{min})

$$\circlearrowleft \Sigma M_K = 0: M - 0.3165\text{ kN}\cdot\text{m} + \frac{x}{2}(1.11\text{ kN/m})x$$
$$- [x - (1.5\text{ m} - a)](3.165\text{ kN}) = 0$$

$$M = -4.431\text{ kN}\cdot\text{m} + (3.165\text{ kN})(x + a) - (0.555\text{ kN/m})x^2$$

$$M_{max}(\text{at } x = 1.5\text{ m}) = -0.93225\text{ kN}\cdot\text{m} + (3.165\text{ kN})a$$

For minimum $|M|_{max}$, set $M_{max} = -M_{min}$:

$$-0.93225\text{ kN}\cdot\text{m} + (3.165\text{ kN})a = (0.555\text{ kN/m})(1.5\text{ m} - a)^2$$

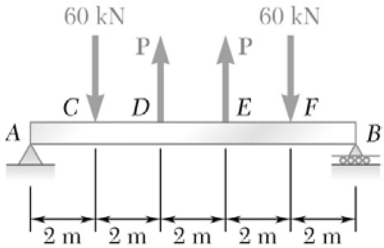
Yielding: $\quad a^2 - (8.7027\text{ m})a + 3.92973\text{ m}^2 = 0$

Solving: $\quad a = 4.3514 \pm \sqrt{13.864} = 0.4778\text{ m},\ 8.075\text{ m}$

Second solution out of range, so (a) $\quad a = 0.478\text{ m}$ ◀

$$M_{max} = 0.5801\text{ kN}\cdot\text{m}$$ ◀

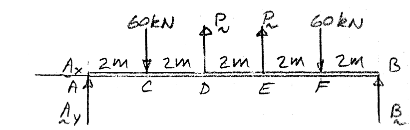
(b) $\quad M_{max} = 580\text{ N}\cdot\text{m}$ ◀

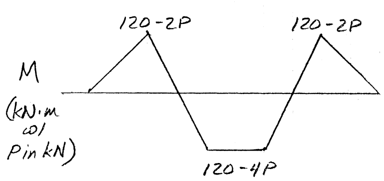


PROBLEM 7.53

For the beam shown, determine (*a*) the magnitude *P* of the two upward forces for which the maximum value of the bending moment is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

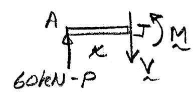
SOLUTION





By symmetry: $A_y = B = 60 \text{ kN} - P$

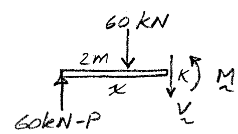
Along AC:



$$\circlearrowleft \Sigma M_J = 0{:}\ M - x(60 \text{ kN} - P) = 0 \qquad M = (60 \text{ kN} - P)x$$

$$M = 120 \text{ kN}\cdot\text{m} - (2 \text{ m})P \quad \text{at} \quad x = 2 \text{ m}$$

Along CD:

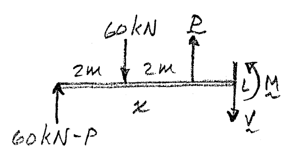


$$\circlearrowleft \Sigma M_K = 0{:}\ M + (x - 2 \text{ m})(60 \text{ kN}) - x(60 \text{ kN} - P) = 0$$

$$M = 120 \text{ kN}\cdot\text{m} - Px$$

$$M = 120 \text{ kN}\cdot\text{m} - (4 \text{ m})P \quad \text{at} \quad x = 4 \text{ m}$$

Along DE:



$$\circlearrowleft \Sigma M_L = 0{:}\ M - (x - 4 \text{ m})P + (x - 2 \text{ m})(60 \text{ kN})$$
$$- x(60 \text{ kN} - P) = 0$$

$$M = 120 \text{ kN}\cdot\text{m} - (4 \text{ m})P \qquad \text{(const)}$$

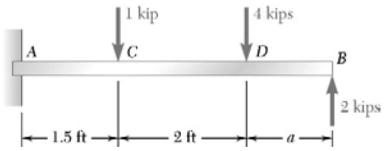
Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120 \text{ kN}\cdot\text{m} - (2 \text{ m})P = -[120 \text{ kN}\cdot\text{m} - (4 \text{ m})P]$$

(*a*) $P = 40.0 \text{ kN}$ ◀

$M_{\min} = 120 \text{ kN}\cdot\text{m} - (4 \text{ m})P$ (*b*) $|M|_{\max} = 40.0 \text{ kN}\cdot\text{m}$ ◀

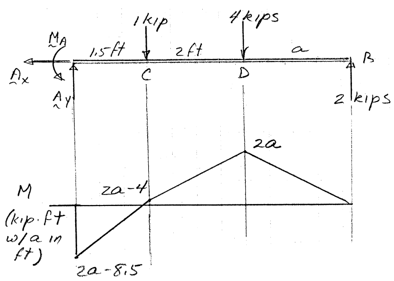


PROBLEM 7.54

For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

SOLUTION

FBD Beam:



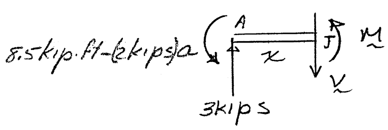
$$\Sigma M_A = 0\!: M_A - (1.5\text{ ft})(1\text{ kip}) - (3.5\text{ ft})(4\text{ kips}) + (3.5\text{ ft} + a)(2\text{ kips}) = 0$$

$$\mathbf{M}_A = [8.5\text{ kip}\cdot\text{ft} - (2\text{ kips})a]$$

$$\uparrow \Sigma F_y = 0\!: A_y - 1\text{ kip} - 4\text{ kips} + 2\text{ kips} = 0$$

$$\mathbf{A}_y = 3\text{ kips} \uparrow$$

Along AC:



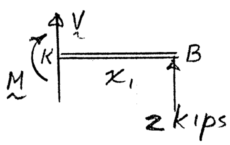
$$\Sigma M_J = 0\!: M - x(3\text{ kips}) + 8.5\text{ kip}\cdot\text{ft} - (2\text{ kips})a = 0$$

$$M = (3\text{ kips})x + (2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft}$$

$$M = (2\text{ kips})a - 4\text{ kip}\cdot\text{ft} \;\text{ at }\; C(x = 1.5\text{ ft})$$

$$M = (2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft} \;\text{ at }\; A(M_{\min})$$

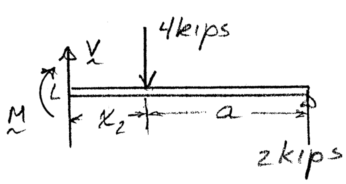
Along DB:



$$\Sigma M_K = 0\!: -M + x_1(2\text{ kips}) = 0 \qquad M = (2\text{ kips})x_1$$

$$M = (2\text{ kips})a \;\text{ at }\; D$$

Along CD:



$$\Sigma M_L = 0\!: (x_2 + a)(2\text{ kips}) - x_2(4\text{ kips}) - M = 0$$

$$M = (2\text{ kips})a - (2\text{ kips})x_2$$

$$M = (2\text{ kips})a - 4\text{ kip}\cdot\text{ft} \;\text{ at } C \;\;\text{(see above)}$$

For minimum $|M|_{\max}$, set $M_{\max}(\text{at } D) = -M_{\min}(\text{at } A)$

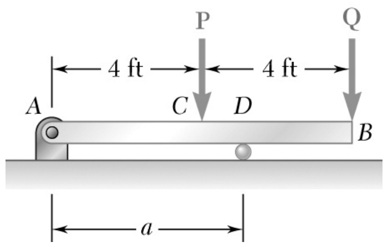
$$(2\text{ kips})a = -[(2\text{ kips})a - 8.5\text{ kip}\cdot\text{ft}]$$

$$4a = 8.5\text{ ft} \qquad a = 2.125\text{ ft}$$

(*a*) $a = 2.13\text{ ft}$ ◀

So $M_{\max} = (2\text{ kips})a = 4.25\text{ kip}\cdot\text{ft}$

(*b*) $|M|_{\max} = 4.25\text{ kip}\cdot\text{ft}$ ◀



PROBLEM 7.55

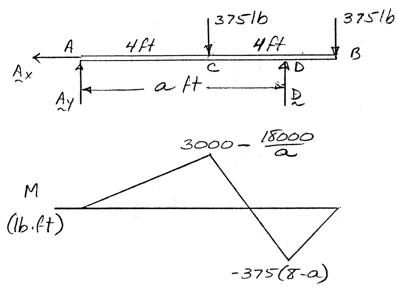
Knowing that $P = Q = 375$ lb, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Prob. 7.51.)

SOLUTION

$$\circlearrowleft \Sigma M_A = 0: (a \text{ ft})D - (4 \text{ ft})(375 \text{ lb}) - (8 \text{ ft})(375 \text{ lb}) = 0$$

FBD Beam:

$$\mathbf{D} = \frac{4500}{a} \text{ lb} \uparrow$$

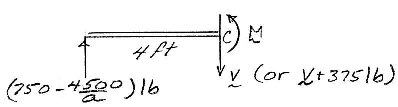


$$\uparrow \Sigma F_y = 0: A_y - 2(375 \text{ lb}) + \frac{4500}{a} \text{ lb} = 0$$

$$\mathbf{A}_y = \left(750 - \frac{4500}{a}\right) \text{lb} \uparrow$$

It is apparent that $M = 0$ at *A* and *B*, and that all segments of the *M* diagram are straight, so the max and min values of *M* must occur at *C* and *D*

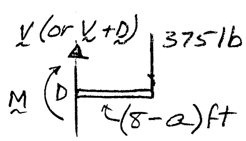
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4 \text{ ft})\left(750 - \frac{4500}{a}\right) \text{lb} = 0$$

$$M = \left(3000 - \frac{18000}{a}\right) \text{lb} \cdot \text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: -[(8-a)\text{ft}](375 \text{ lb}) - M = 0$$

$$M = -375(8-a) \text{lb} \cdot \text{ft}$$

For minimum $|M|_{max}$, set $M_{max} = -M_{min}$

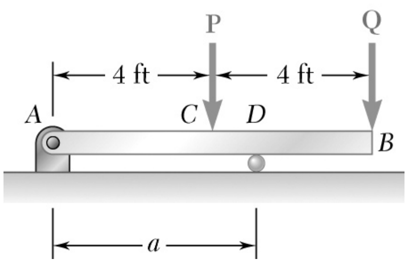
So
$$3000 - \frac{18000}{a} = 375(8-a)$$

$$a^2 = 48 \qquad a = 6.9282 \text{ ft}$$

(*a*) $a = 6.93$ ft ◀

$$M_{max} = 375(8-a) = 401.92 \text{ lb} \cdot \text{ft}$$

(*b*) $|M|_{max} = 402$ lb·ft ◀

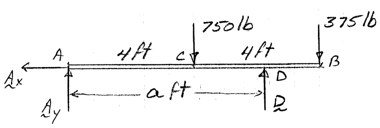


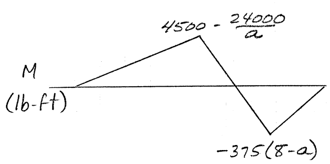
PROBLEM 7.56

Solve Prob. 7.55 assuming that $P = 750$ lb and $Q = 375$ lb.

SOLUTION

FBD Beam:





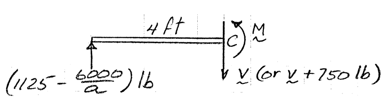
$$\circlearrowleft \Sigma M_D = 0: -(a\text{ ft})A_y + \left[(a-4)\text{ft}\right](750\text{ lb})$$

$$-\left[(8-a)\text{ft}\right](375\text{ lb}) = 0$$

$$\mathbf{A}_y = \left(1125 - \frac{6000}{a}\right)\text{lb} \uparrow$$

It is apparent that $M = 0$ at A and B, and that all segments of the M-diagram are straight, so $M_{\max}$ and $M_{\min}$ occur at C and D.

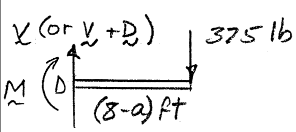
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4\text{ ft})\left(1125 - \frac{6000}{a}\right)\text{lb} = 0$$

$$M = \left(4500 - \frac{24000}{a}\right)\text{lb}\cdot\text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: \ -M - \left[(8-a)\text{ft}\right](375\text{ lb}) = 0$$

$$M = -375(8-a)\text{lb}\cdot\text{ft}$$

For minimum $M_{\max}$, set $M_{\max} = -M_{\min}$

$$4500 - \frac{24000}{a} = 375(8-a)$$

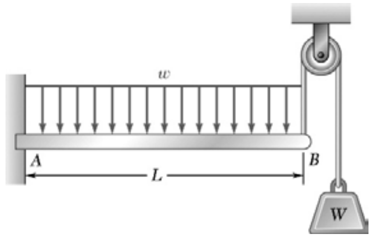
$$a^2 + 4a - 64 = 0 \qquad a = -2 \pm \sqrt{68}\,(\text{need } +)$$

$$a = 6.2462 \text{ ft} \qquad (a) \qquad a = 6.25 \text{ ft} \blacktriangleleft$$

Then $$M_{\max} = 375(8-a) = 657.7 \text{ lb}\cdot\text{ft}$$

$$(b) \qquad |M|_{\max} = 658 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

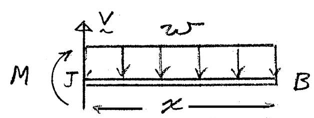
PROBLEM 7.57



In order to reduce the bending moment in the cantilever beam *AB*, a cable and counterweight are permanently attached at end *B*. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{\max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

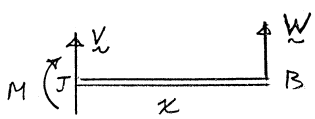
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

M due to counter weight:

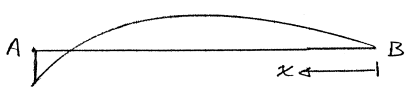


$$\circlearrowleft \Sigma M_J = 0: -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**

$$M = W_x - \frac{w}{2}x^2 \qquad \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$



And here $M = \dfrac{W^2}{2w} > 0$ so $M_{\max}$; $M_{\min}$ must be at $x = L$

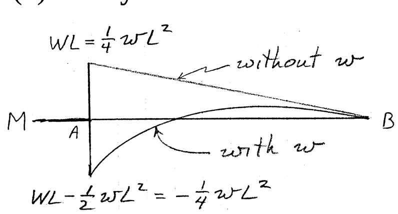
So $M_{\min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{\max}$ set $M_{\max} = -M_{\min}$, so

$$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \text{ or } W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need +)} \qquad W = \left(\sqrt{2} - 1\right)wL = 0.414wL \blacktriangleleft$$

$$M_{\max} = \frac{W^2}{2w} = \frac{\left(\sqrt{2} - 1\right)^2}{2}wL^2 \qquad M_{\max} = 0.858wL^2 \blacktriangleleft$$

(*b*) ***w*** **may be removed:**



Without w, $\qquad M = Wx,\ M_{\max} = WL$ at A

With w (see part *a*) $\qquad M = Wx - \dfrac{w}{2}x^2, \qquad M_{\max} = \dfrac{W^2}{2w}$ at $x = \dfrac{W}{w}$

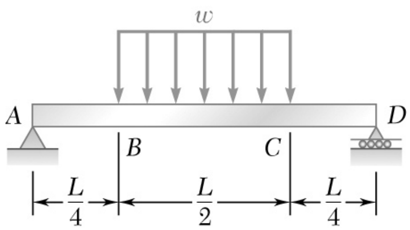
$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

PROBLEM 7.57 CONTINUED

For minimum $M_{\max}$, set $M_{\max}(\text{no } w) = -M_{\min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $$W = \frac{1}{4}wL \blacktriangleleft$$



PROBLEM 7.58

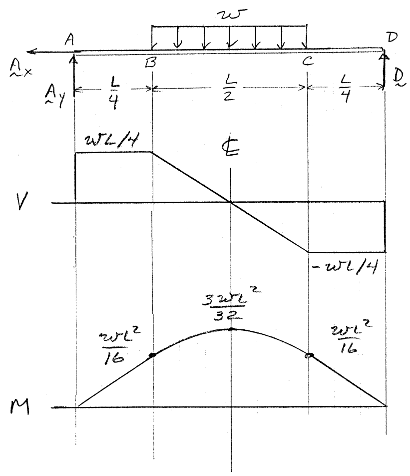
Using the method of Sec. 7.6, solve Prob. 7.29.

SOLUTION

(*a*) and (*b*)

By symmetry: $A_y = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{wL}{4}$ or $\mathbf{A}_y = \mathbf{D} = \frac{wL}{4}\uparrow$

Shear Diag: V jumps to $A_y = \frac{wL}{4}$ at A,



and stays constant (no load) to B. From B to C, V is linear $\left(\frac{dV}{dx} = -w\right)$, and it becomes $\frac{wL}{4} - w\frac{L}{2} = -\frac{wL}{4}$ at C.

(Note: $V = 0$ at center of beam. From C to D, V is again constant.)

Moment Diag: M starts at zero at A

and increases linearly $\left(\frac{dM}{dV} = \frac{wL}{4}\right)$ to B.

$$M_B = 0 + \frac{L}{4}\left(\frac{wL}{4}\right) = \frac{wL^2}{16}.$$

From B to C M is parabolic

$\left(\frac{dM}{dx} = V,\ \text{which decreases to zero at center and } -\frac{wL}{4} \text{ at } C\right)$,

M is maximum at center. $M_{\max} = \frac{wL^2}{16} + \frac{1}{2}\left(\frac{L}{4}\right)\left(\frac{wL}{4}\right)$

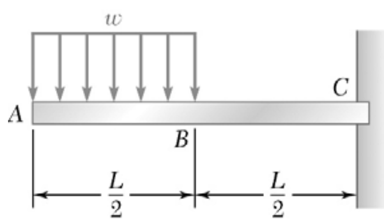
Then, M is linear with $\frac{dM}{dy} = -\frac{wL}{4}$ to D

$$M_D = 0$$

$$|V|_{\max} = \frac{wL}{4} \blacktriangleleft$$

$$|M|_{\max} = \frac{3wL^2}{32} \blacktriangleleft$$

Notes: Symmetry could have been invoked to draw second half.
Smooth transitions in M at B and C, as no discontinuities in V.

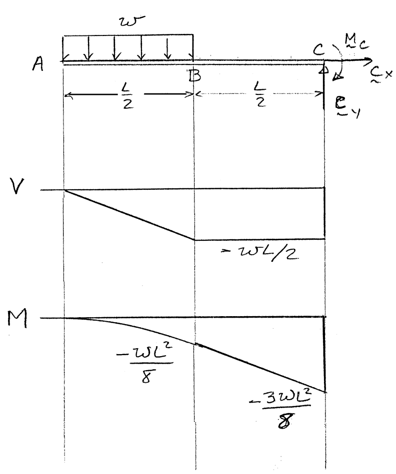


PROBLEM 7.59

Using the method of Sec. 7.6, solve Prob. 7.30.

SOLUTION

(a) and (b)



Shear Diag: $V = 0$ at A and is linear

$\left(\frac{dV}{dx} = -w\right)$ to $-w\left(\frac{L}{2}\right) = -\frac{wL}{2}$ at B. V is constant $\left(\frac{dV}{dx} = 0\right)$ from B to C.

$$|V|_{\max} = \frac{wL}{2} ◀$$

Moment Diag: $M = 0$ at A and is

parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$ to B.

$$M_B = \frac{1}{2}\left(\frac{L}{2}\right)\left(-\frac{wL}{2}\right) = -\frac{wL^2}{8}$$

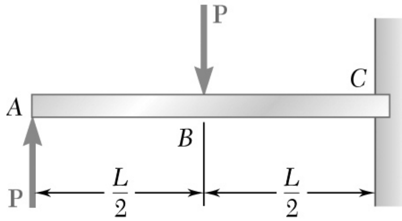
From B to C, M is linear $\left(\frac{dM}{dx} = -\frac{wL}{2}\right)$

$$M_C = -\frac{wL^2}{8} - \left(\frac{L}{2}\right)\left(\frac{wL}{2}\right) = -\frac{3wL^2}{8}$$

$$|M|_{\max} = \frac{3wL^2}{8} ◀$$

Notes: Smooth transition in M at B, as no discontinuity in V.

It was not necessary to predetermine reactions at C.
In fact they are given by $-V_C$ and $-M_C$.

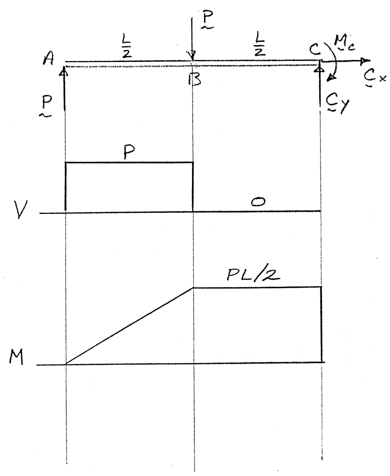


PROBLEM 7.60

Using the method of Sec. 7.6, solve Prob. 7.31.

SOLUTION

(*a*) and (*b*)



Shear Diag:

V jumps to P at A, then is constant $\left(\frac{dV}{dx} = 0\right)$ to B. V jumps down P to zero at B, and is constant (zero) to C.

$$|V|_{\max} = P \blacktriangleleft$$

Moment Diag:

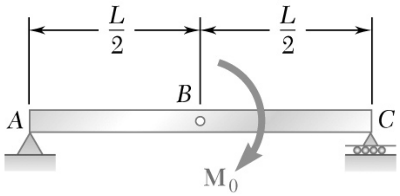
M is linear $\left(\frac{dM}{dy} = V = P\right)$ to B.

$$M_B = 0 + \left(\frac{L}{2}\right)(P) = \frac{PL}{2}.$$

M is constant $\left(\frac{dM}{dx} = 0\right)$ at $\frac{PL}{2}$ to C

$$|M|_{\max} = \frac{PL}{2} \blacktriangleleft$$

Note: It was not necessary to predetermine reactions at C. In fact they are given by $-V_C$ and $-M_C$.

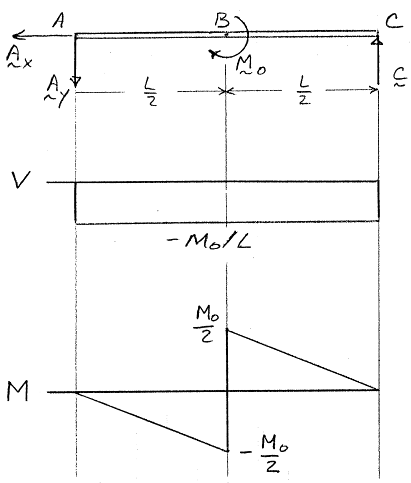


PROBLEM 7.61

Using the method of Sec. 7.6, solve Prob. 7.32.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_C = 0\text{: } LA_y - M_0 = 0 \qquad \mathbf{A}_y = \frac{M_0}{L}\downarrow$$

Shear Diag:

V jumps to $-\dfrac{M_0}{L}$ at A and is constant $\left(\dfrac{dV}{dx} = 0\right)$ all the way to C

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$

Moment Diag:

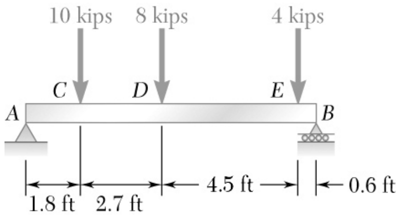
M is zero at A and linear $\left(\dfrac{dM}{dx} = V = -\dfrac{M_0}{L}\right)$ throughout.

$$M_{B^-} = -\frac{L}{2}\left(\frac{M_0}{L}\right) = -\frac{M_0}{2},$$

but M jumps by $+M_0$ to $+\dfrac{M_0}{2}$ at B.

$$M_C = \frac{M_0}{2} - \frac{L}{2}\left(\frac{M_0}{L}\right) = 0$$

$$|M|_{\max} = \frac{M_0}{2} \blacktriangleleft$$

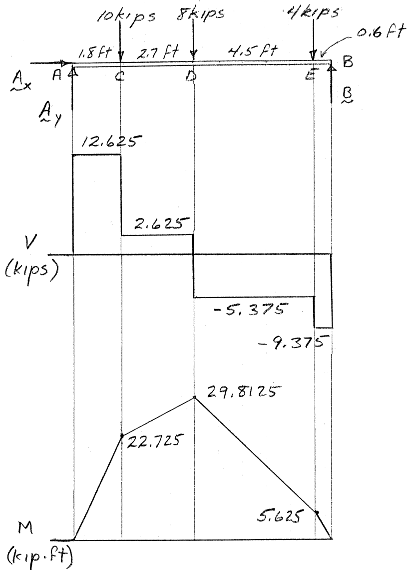


PROBLEM 7.62

Using the method of Sec. 7.6, solve Prob. 7.33.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_B = 0: (0.6 \text{ ft})(4 \text{ kips}) + (5.1 \text{ ft})(8 \text{ kips}) + (7.8 \text{ ft})(10 \text{ kips}) - (9.6 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625 \text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0\right)$ with discontinuities at each concentrated force. (equal to force)

$$|V|_{\max} = 12.63 \text{ kips} \blacktriangleleft$$

Moment Diag:

M is zero at A, and piecewise linear $\left(\dfrac{dM}{dx} = V\right)$ throughout.

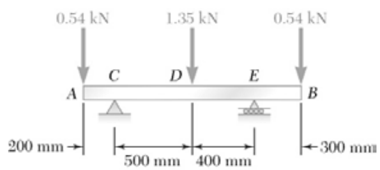
$$M_C = (1.8 \text{ ft})(12.625 \text{ kips}) = 22.725 \text{ kip}\cdot\text{ft}$$

$$M_D = 22.725 \text{ kip}\cdot\text{ft} + (2.7 \text{ ft})(2.625 \text{ kips})$$
$$= 29.8125 \text{ kip}\cdot\text{ft}$$

$$M_E = 29.8125 \text{ kip}\cdot\text{ft} - (4.5 \text{ ft})(5.375 \text{ kips})$$
$$= 5.625 \text{ kip}\cdot\text{ft}$$

$$M_B = 5.625 \text{ kip}\cdot\text{ft} - (0.6 \text{ ft})(9.375 \text{ kips}) = 0$$

$$|M|_{\max} = 29.8 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

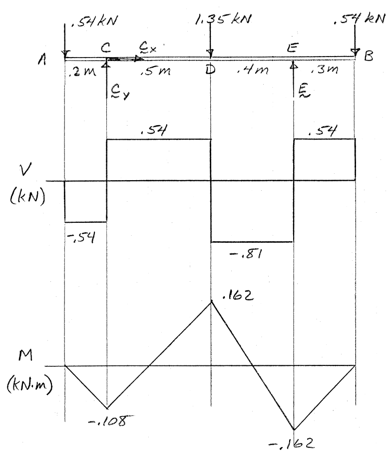


PROBLEM 7.63

Using the method of Sec. 7.6, solve Prob. 7.36.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_E = 0: (1.1\text{ m})(0.54\text{ kN}) - (0.9\text{ m})C_y$$
$$+ (0.4\text{ m})(1.35\text{ kN}) - (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$\mathbf{C}_y = 1.08\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -0.54\text{ kN} + 1.08\text{ kN} - 1.35\text{ kN} + E - 0.54\text{ kN} = 0$$

$$\mathbf{E} = 1.35\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0 \text{ everywhere}\right)$ with discontinuities at each concentrated force. (equal to the force)

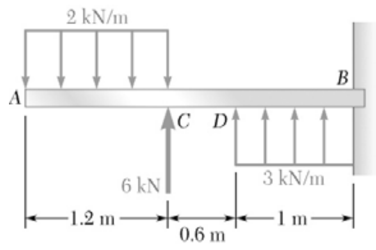
$$|V|_{\max} = 810\text{ N} \blacktriangleleft$$

Moment Diag:

M is piecewise linear starting with $M_A = 0$

$$M_C = 0 - 0.2\text{ m}(0.54\text{ kN}) = 0.108\text{ kN}\cdot\text{m}$$
$$M_D = 0.108\text{ kN}\cdot\text{m} + (0.5\text{ m})(0.54\text{ kN}) = 0.162\text{ kN}\cdot\text{m}$$
$$M_E = 0.162\text{ kN}\cdot\text{m} - (0.4\text{ m})(0.81\text{ kN}) = -0.162\text{ kN}\cdot\text{m}$$
$$M_B = 0.162\text{ kN}\cdot\text{m} + (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$|M|_{\max} = 0.162\text{ kN}\cdot\text{m} = 162.0\text{ N}\cdot\text{m} \blacktriangleleft$$

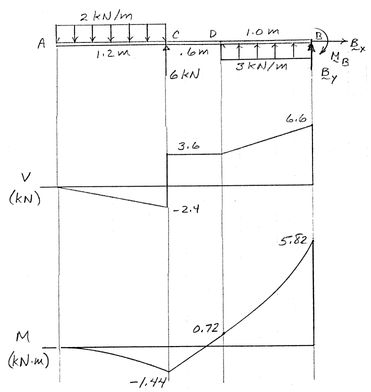


PROBLEM 7.64

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



Shear Diag:

$V = 0$ at A and linear $\left(\dfrac{dV}{dx} = -2\text{ kN/m}\right)$ to C

$$V_C = -1.2\text{ m}(2\text{ kN/m}) = -2.4\text{ kN}.$$

At C, V jumps 6 kN to 3.6 kN, and is constant to D. From there, V is linear $\left(\dfrac{dV}{dx} = +3\text{ kN/m}\right)$ to B

$$V_B = 3.6\text{ kN} + (1\text{ m})(3\text{ kN/m}) = 6.6\text{ kN}$$

$$|V|_{\max} = 6.60\text{ kN} \blacktriangleleft$$

Moment Diag: $M_A = 0.$

From A to C, M is parabolic, $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$.

$$M_C = -\frac{1}{2}(1.2\text{ m})(2.4\text{ kN}) = -1.44\text{ kN}\cdot\text{m}$$

From C to D, M is linear $\left(\dfrac{dM}{dx} = 3.6\text{ kN}\right)$

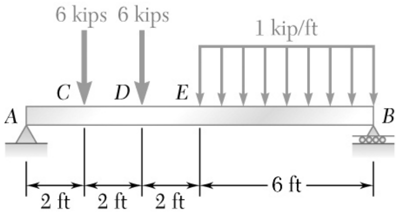
$$M_D = -1.44\text{ kN}\cdot\text{m} + (0.6\text{ m})(3.6\text{ kN})$$
$$= 0.72\text{ kN}\cdot\text{m}.$$

From D to B, M is parabolic $\left(\dfrac{dM}{dx}\text{ increasing with } V\right)$

$$M_B = 0.72\text{ kN}\cdot\text{m} + \frac{1}{2}(1\text{ m})(3.6 + 6.6)\text{ kN}$$
$$= 5.82\text{ kN}\cdot\text{m}$$

$$|M|_{\max} = 5.82\text{ kN}\cdot\text{m} \blacktriangleleft$$

Notes: Smooth transition in M at D. It was unnecessary to predetermine reactions at B, but they are given by $-V_B$ and $-M_B$

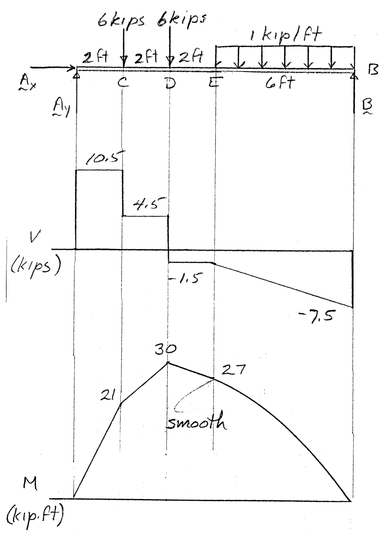


PROBLEM 7.65

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_B = 0: (3\text{ ft})(1\text{ kip/ft})(6\text{ ft}) + (8\text{ ft})(6\text{ kips}) + (10\text{ ft})(6\text{ kips}) - (12\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 10.5\text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant from A to E, with discontinuities at A, C, and E equal to the forces. $V_E = -1.5\text{ kips}$. From E to B, V is linear

$$\left(\frac{dV}{dx} = -1\text{ kip/ft}\right),$$

so

$$V_B = -1.5\text{ kips} - (6\text{ ft})(1\text{ kip/ft}) = -7.5\text{ kips}$$

$$|V|_{\max} = 10.50\text{ kips} \blacktriangleleft$$

Moment Diag: $M_A = 0$, then M is piecewise linear to E

$$M_C = 0 + 2\text{ ft}(10.5\text{ kips}) = 21\text{ kip}\cdot\text{ft}$$

$$M_D = 21\text{ kip}\cdot\text{ft} + (2\text{ ft})(4.5\text{ kips}) = 30\text{ kip}\cdot\text{ft}$$

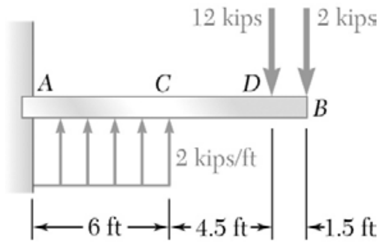
$$M_E = 30\text{ kip}\cdot\text{ft} - (2\text{ ft})(1.5\text{ kips}) = 27\text{ kip}\cdot\text{ft}$$

From E to B, M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$, and

$$M_B = 27\text{ kip}\cdot\text{ft} - \frac{1}{2}(6\text{ ft})(1.5\text{ kips} + 7.5\text{ kips}) = 0$$

$$|M|_{\max} = 30.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

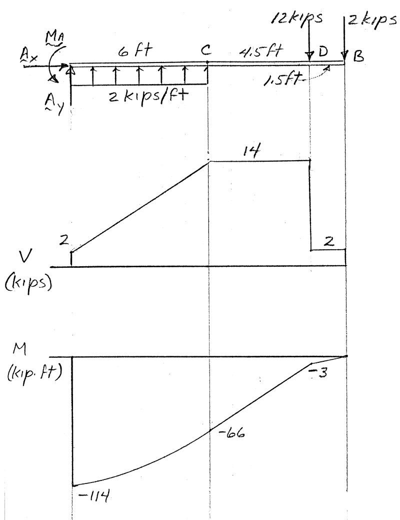
PROBLEM 7.66



Using the method of Sec. 7.6, solve Prob. 7.37.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$+\uparrow \Sigma F_y = 0: A_y + (6 \text{ ft})(2 \text{ kips/ft}) - 12 \text{ kips} - 2 \text{ kips} = 0$$

$$\mathbf{A}_y = 2 \text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: M_A + (3 \text{ ft})(2 \text{ kips/ft})(6 \text{ ft})$$

$$-(10.5 \text{ ft})(12 \text{ kips}) - (12 \text{ ft})(2 \text{ kips}) = 0$$

$$\mathbf{M}_A = 114 \text{ kip}\cdot\text{ft} \circlearrowright$$

Shear Diag:

$V_A = A_y = 2$ kips. Then V is linear $\left(\dfrac{dV}{dx} = 2 \text{ kips/ft}\right)$ to C, where

$$V_C = 2 \text{ kips} + (6 \text{ ft})(2 \text{ kips/ft}) = 14 \text{ kips}.$$

V is constant at 14 kips to D, then jumps down 12 kips to 2 kips and is constant to B

$$|V|_{\max} = 14.00 \text{ kips} \blacktriangleleft$$

Moment Diag: $M_A = -114 \text{ kip}\cdot\text{ft}.$

From A to C, M is parabolic $\left(\dfrac{dM}{dx} \text{ increasing with } V\right)$ and

$$M_C = -114 \text{ kip}\cdot\text{ft} + \frac{1}{2}(2 \text{ kips} + 14 \text{ kips})(6 \text{ ft})$$

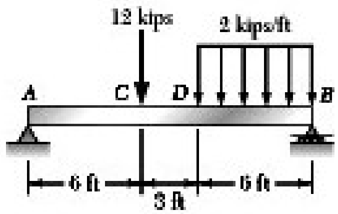
$$M_C = -66 \text{ kip}\cdot\text{ft}.$$

Then M is piecewise linear.

$$M_D = -66 \text{ kip}\cdot\text{ft} + (14 \text{ kips})(4.5 \text{ ft}) = -3 \text{ kip}\cdot\text{ft}$$

$$M_B = -3 \text{ kip}\cdot\text{ft} + (2 \text{ kips})(1.5 \text{ ft}) = 0$$

$$|M|_{\max} = 114.0 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

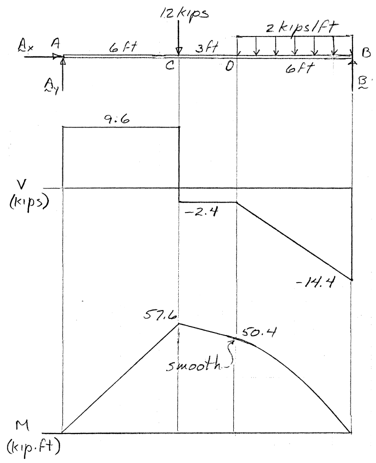


PROBLEM 7.67

Using the method of Sec. 7.6, solve Prob. 7.38.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0{:}\ (3\text{ ft})\left(2\frac{\text{kips}}{\text{ft}}\right)(6\text{ ft}) + (9\text{ ft})(12\text{ kips}) - (15\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 9.6\text{ kips} \uparrow$$

Shear Diag:

V jumps to $A_y = 9.6$ kips at A, is constant to C, jumps down 12 kips to -2.4 kips at C, is constant to D, and then is linear $\left(\frac{dV}{dx} = -2\text{ kips/ft}\right)$ to B

$$V_B = -2.4\text{ kips} - (2\text{ kips/ft})(6\text{ ft})$$

$$= -14.4\text{ kips}$$

$$|V|_{\max} = 14.40\text{ kips} \blacktriangleleft$$

Moment Diag:

M is linear from A to C $\quad \left(\frac{dM}{dx} = 9.6\text{ kips/ft}\right)$

$$M_C = 9.6\text{ kips}(6\text{ ft}) = 57.6\text{ kip}\cdot\text{ft},$$

M is linear from C to D $\quad \left(\frac{dM}{dx} = -2.4\text{ kips/ft}\right)$

$$M_D = 57.6\text{ kip}\cdot\text{ft} - 2.4\text{ kips}(3\text{ ft})$$

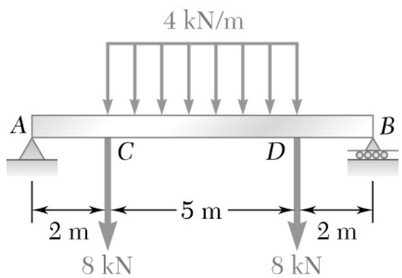
$$M_D = 50.4\text{ kip}\cdot\text{ft}.$$

M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ to B.

$$M_B = 50.4\text{ kip}\cdot\text{ft} - \frac{1}{2}(2.4\text{ kips} + 14.4\text{ kips})(6\text{ ft}) = 0$$

$$= 0$$

$$|M|_{\max} = 57.6\text{ kip}\cdot\text{ft} \blacktriangleleft$$

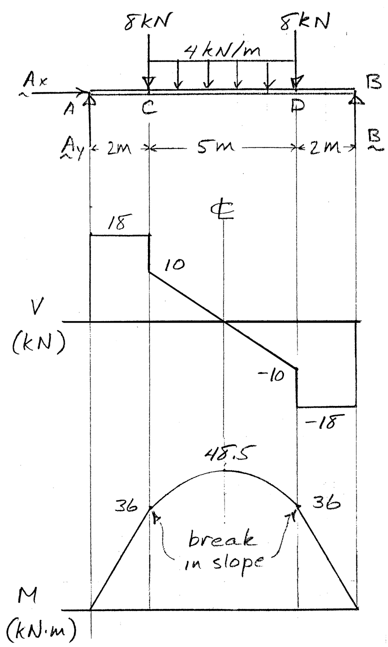


PROBLEM 7.68

Using the method of Sec. 7.6, solve Prob. 7.39.

SOLUTION

(*a*) and (*b*)



FBD Beam:

By symmetry: $A_y = B = \frac{1}{2}(5\text{ m})(4\text{ kN/m}) + 8\text{ kN}$

or $\mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$

Shear Diag:

V jumps to 18 kN at A, and is constant to C, then drops 8 kN to 10 kN.

After C, V is linear $\left(\frac{dV}{dx} = -4\text{ kN/m}\right)$, reaching -10 kN at $D\left[V_D = 10\text{ kN} - (4\text{ kN/m})(5\text{ m})\right]$ passing through zero at the beam center. At D, V drops 8 kN to -18 kN and is then constant to B

$|V|_{\text{max}} = 18.00\text{ kN}$ ◀

Moment Diag:

$M_A = 0$. Then M is linear $\left(\frac{dM}{dx} = 18\text{ kN/m}\right)$ to C

$M_C = (18\text{ kN})(2\text{ m}) = 36\text{ kN}\cdot\text{m}$, M is parabolic to D $\left(\frac{dM}{dx}\text{ decreases with } V \text{ to zero at center}\right)$

$$M_{\text{center}} = 36\text{ kN}\cdot\text{m} + \frac{1}{2}(10\text{ kN})(2.5\text{ m}) = 48.5\text{ kN}\cdot\text{m} = M_{\text{max}}$$

$|M|_{\text{max}} = 48.5\text{ kN}\cdot\text{m}$ ◀

Complete by invoking symmetry.