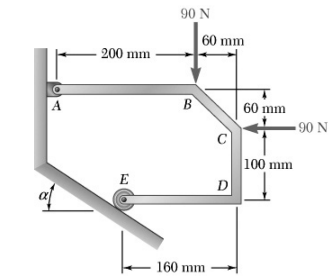
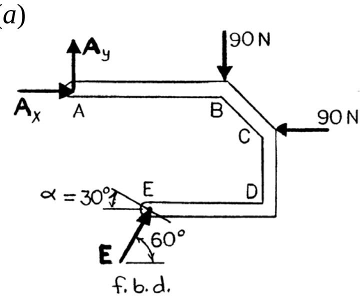
PROBLEM 4.27



For the frame and loading shown, determine the reactions at A and E when (a) $\alpha = 30^{\circ}$, (b) $\alpha = 45^{\circ}$.

SOLUTION

(*a*)



(*a*) Given $\alpha = 30°$

From f.b.d. of frame

$$+\circlearrowleft \Sigma M_A = 0: \quad -(90\text{ N})(0.2\text{ m}) - (90\text{ N})(0.06\text{ m})$$

$$+(E\cos 60°)(0.160\text{ m}) + (E\sin 60°)(0.100\text{ m}) = 0$$

$$\therefore \quad E = 140.454\text{ N}$$

or $\mathbf{E} = 140.5\text{ N} \measuredangle 60°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90\text{ N} + (140.454\text{ N})\cos 60° = 0$$

$$\therefore \quad A_x = 19.7730\text{ N}$$

or $\mathbf{A}_x = 19.7730\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90\text{ N} + (140.454\text{ N})\sin 60° = 0$$

$$\therefore \quad A_y = -31.637\text{ N}$$

or $\mathbf{A}_y = 31.6\text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(19.7730)^2 + (31.637)^2}$

$$= 37.308\text{ lb}$$

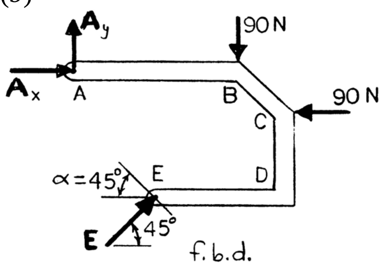
and $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \tan^{-1}\left(\dfrac{-31.637}{19.7730}\right)$

$$= -57.995°$$

or $\mathbf{A} = 37.3\text{ N} \searrow 58.0°$ ◀

PROBLEM 4.27 CONTINUED

(*b*)



(*b*) Given $\alpha = 45°$

From f.b.d. of frame

$$+\circlearrowleft \Sigma M_A = 0: \quad -(90\text{ N})(0.2\text{ m}) - (90\text{ N})(0.06\text{ m})$$

$$+(E\cos 45°)(0.160\text{ m}) + (E\sin 45°)(0.100\text{ m}) = 0$$

$$\therefore\ E = 127.279\text{ N}$$

or $\mathbf{E} = 127.3\text{ N} \measuredangle\ 45°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90 + (127.279\text{ N})\cos 45° = 0$$

$$\therefore\ A_x = 0$$

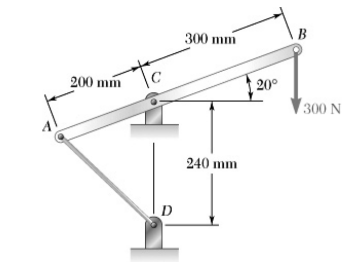
$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 + (127.279\text{ N})\sin 45° = 0$$

$$\therefore\ A_y = 0$$

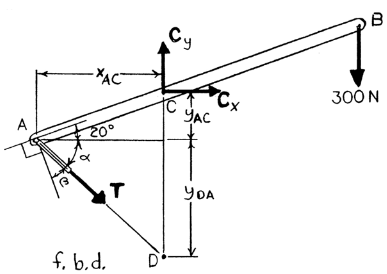
or $\mathbf{A} = 0$ ◀

PROBLEM 4.28

A lever AB is hinged at C and is attached to a control cable at A. If the lever is subjected to a 300-N vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.



SOLUTION



First

$$x_{AC} = (0.200 \text{ m})\cos 20° = 0.187\,939 \text{ m}$$

$$y_{AC} = (0.200 \text{ m})\sin 20° = 0.068\,404 \text{ m}$$

Then

$$y_{DA} = 0.240 \text{ m} - y_{AC}$$

$$= 0.240 \text{ m} - 0.068404 \text{ m}$$

$$= 0.171596 \text{ m}$$

and

$$\tan\alpha = \frac{y_{DA}}{x_{AC}} = \frac{0.171\,596}{0.187\,939}$$

$$\therefore\quad \alpha = 42.397°$$

and

$$\beta = 90° - 20° - 42.397° = 27.603°$$

(a) From f.b.d. of lever AB

$$+\circlearrowleft \Sigma M_C = 0: \quad T\cos 27.603°(0.2 \text{ m})$$

$$- 300 \text{ N}\left[(0.3 \text{ m})\cos 20°\right] = 0$$

$$\therefore\quad T = 477.17 \text{ N} \qquad \text{or } T = 477 \text{ N} \blacktriangleleft$$

(b) From f.b.d. of lever AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (477.17 \text{ N})\cos 42.397° = 0$$

$$\therefore\quad C_x = -352.39 \text{ N}$$

or

$$\mathbf{C}_x = 352.39 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 300 \text{ N} - (477.17 \text{ N})\sin 42.397° = 0$$

$$\therefore\quad C_y = 621.74 \text{ N}$$

or

$$\mathbf{C}_y = 621.74 \text{ N} \uparrow$$

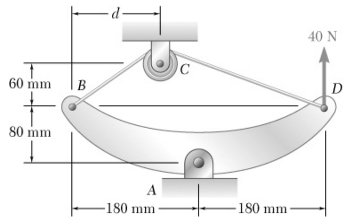
PROBLEM 4.28 CONTINUED

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(352.39)^2 + (621.74)^2} = 714.66 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{621.74}{-352.39}\right) = -60.456^\circ$$

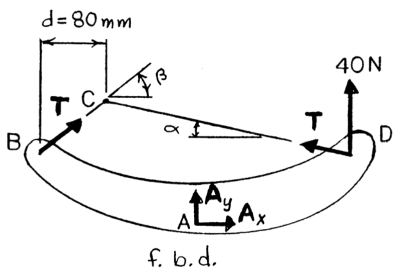
or $\mathbf{C} = 715 \text{ N} \measuredangle 60.5^\circ$ ◀

PROBLEM 4.29



Neglecting friction and the radius of the pulley, determine the tension in cable *BCD* and the reaction at support *A* when $d = 80$ mm.

SOLUTION



First

$$\alpha = \tan^{-1}\left(\frac{60}{280}\right) = 12.0948°$$

$$\beta = \tan^{-1}\left(\frac{60}{80}\right) = 36.870°$$

From f.b.d. of object *BAD*

$$+\circlearrowleft\ \Sigma M_A = 0:\quad (40\text{ N})(0.18\text{ m}) + (T\cos\alpha)(0.08\text{ m})$$

$$+(T\sin\alpha)(0.18\text{ m}) - (T\cos\beta)(0.08\text{ m})$$

$$-(T\sin\beta)(0.18\text{ m}) = 0$$

$$\therefore\quad T = \left(\frac{7.2\text{ N}\cdot\text{m}}{0.056061}\right) = 128.433\text{ N}$$

or $T = 128.4$ N ◀

$$\xrightarrow{+}\ \Sigma F_x = 0:\quad (128.433\text{ N})(\cos\beta - \cos\alpha) + A_x = 0$$

$$\therefore\quad A_x = 22.836\text{ N}$$

or $\mathbf{A}_x = 22.836\text{ N} \longrightarrow$

$$+\uparrow\ \Sigma F_y = 0:\quad A_y + (128.433\text{ N})(\sin\beta + \sin\alpha) + 40\text{ N} = 0$$

$$\therefore\quad A_y = -143.970\text{ N}$$

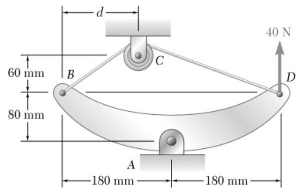
or $\mathbf{A}_y = 143.970\text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(22.836)^2 + (143.970)^2} = 145.770\text{ N}$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-143.970}{22.836}\right) = -80.987°$

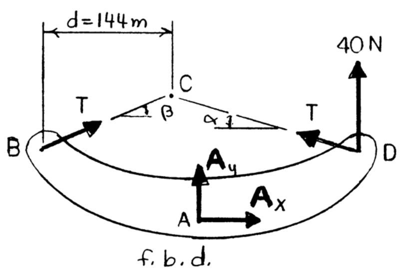
or $\mathbf{A} = 145.8$ N ⦪ 81.0° ◀

PROBLEM 4.30



Neglecting friction and the radius of the pulley, determine the tension in cable *BCD* and the reaction at support *A* when $d = 144$ mm.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{60}{216}\right) = 15.5241°$$

$$\beta = \tan^{-1}\left(\frac{60}{144}\right) = 22.620°$$

From f.b.d. of member *BAD*

$$+\circlearrowleft \Sigma M_A = 0: \quad (40 \text{ N})(0.18 \text{ m}) + (T\cos\alpha)(0.08 \text{ m})$$

$$+(T\sin\alpha)(0.18 \text{ m}) - (T\cos\beta)(0.08 \text{ m})$$

$$-(T\sin\beta)(0.18 \text{ m}) = 0$$

$$\therefore \quad T = \left(\frac{7.2 \text{ N}\cdot\text{m}}{0.0178199\,\text{m}}\right) = 404.04 \text{ N}$$

or $T = 404$ N ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (404.04 \text{ N})(\cos\beta - \cos\alpha) = 0$$

$$\therefore \quad A_x = 16.3402 \text{ N}$$

or $\mathbf{A}_x = 16.3402 \text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (404.04 \text{ N})(\sin\beta + \sin\alpha) + 40 \text{ N} = 0$$

$$\therefore \quad A_y = -303.54 \text{ N}$$

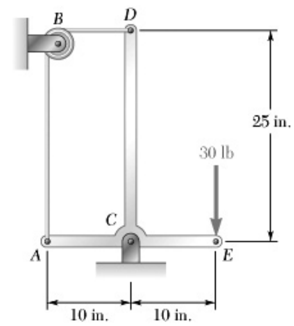
or $\mathbf{A}_y = 303.54 \text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(16.3402)^2 + (303.54)^2} = 303.98 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-303.54}{16.3402}\right) = -86.919°$

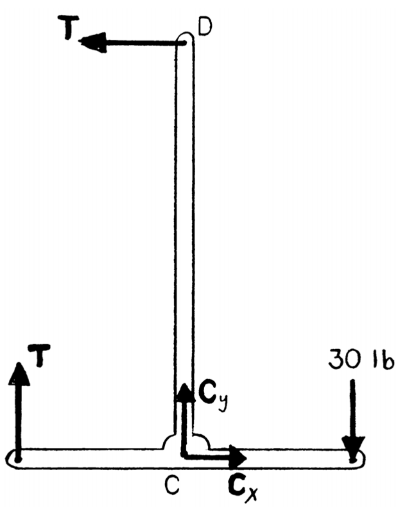
or $\mathbf{A} = 304$ N ⦪ 86.9° ◀

PROBLEM 4.31



Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

SOLUTION



From f.b.d. of inverted T-member

$$+\circlearrowleft\ \Sigma M_C = 0:\quad T(25\text{ in.}) - T(10\text{ in.}) - (30\text{ lb})(10\text{ in.}) = 0$$

$$\therefore\ T = 20\text{ lb}$$

or $T = 20.0\text{ lb}$ ◀

$$\xrightarrow{+}\ \Sigma F_x = 0:\quad C_x - 20\text{ lb} = 0$$

$$\therefore\ C_x = 20\text{ lb}$$

or $\mathbf{C}_x = 20.0\text{ lb} \longrightarrow$

$$+\uparrow\ \Sigma F_y = 0:\quad C_y + 20\text{ lb} - 30\text{ lb} = 0$$

$$\therefore\ C_y = 10\text{ lb}$$

or $\mathbf{C}_y = 10.00\text{ lb} \uparrow$

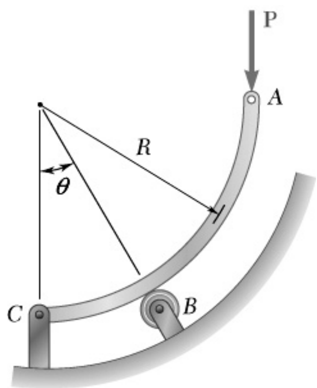
Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(20)^2 + (10)^2} = 22.361\text{ lb}$

and $\theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right) = \tan^{-1}\left(\dfrac{10}{20}\right) = 26.565°$

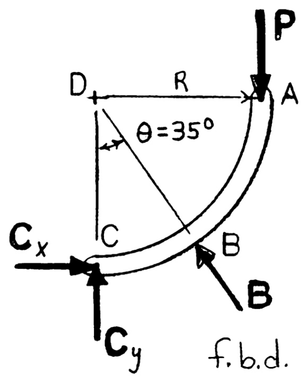
or $\mathbf{C} = 22.4\text{ lb} \measuredangle 26.6°$ ◀

PROBLEM 4.32

Rod *ABC* is bent in the shape of a circular arc of radius *R*. Knowing that $\theta = 35^\circ$, determine the reaction (*a*) at *B*, (*b*) at *C*.



SOLUTION



For $\theta = 35^\circ$

(*a*) From the f.b.d. of rod *ABC*

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - B\sin 35^\circ = 0$$

$$\therefore \quad B = \frac{P}{\sin 35^\circ} = 1.74345P$$

or $\mathbf{B} = 1.743P \searrow 55.0^\circ$ ◀

(*b*) From the f.b.d. of rod *ABC*

$$+\uparrow \Sigma F_y = 0: \quad C_y + (1.74345P)\cos 35^\circ - P = 0$$

$$\therefore \quad C_y = -0.42815P$$

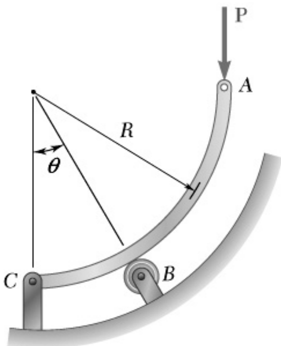
or $$\mathbf{C}_y = 0.42815P \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.42815P)^2} = 1.08780P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-0.42815P}{P}\right) = -23.178^\circ$$

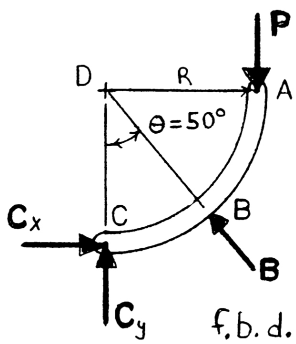
or $\mathbf{C} = 1.088P \searrow 23.2^\circ$ ◀

PROBLEM 4.33



Rod *ABC* is bent in the shape of a circular arc of radius *R*. Knowing that $\theta = 50^{\circ}$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



For $\theta = 50^{\circ}$

(*a*) From the f.b.d. of rod *ABC*

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - B\sin 50^{\circ} = 0$$

$$\therefore \quad B = \frac{P}{\sin 50^{\circ}} = 1.30541P$$

or $\mathbf{B} = 1.305P$ ⦨ 40.0° ◀

(*b*) From the f.b.d. of rod *ABC*

$$+\uparrow \Sigma F_y = 0: \quad C_y - P + (1.30541P)\cos 50^{\circ} = 0$$

$$\therefore \quad C_y = 0.160900P$$

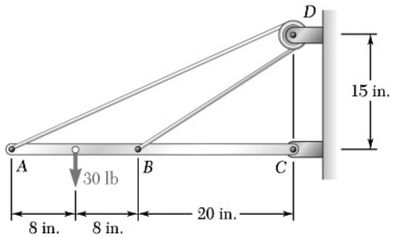
or $$\mathbf{C}_y = 0.1609P \uparrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.1609P)^2} = 1.01286P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{0.1609P}{P}\right) = 9.1405^{\circ}$$

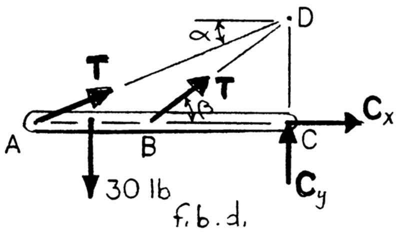
or $\mathbf{C} = 1.013P$ ⦣ 9.14° ◀

PROBLEM 4.34



Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ABD*, (*b*) the reaction at *C*.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{15}{36}\right) = 22.620°$$

$$\beta = \tan^{-1}\left(\frac{15}{20}\right) = 36.870°$$

(*a*) From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (30 \text{ lb})(28 \text{ in.}) - (T\sin 22.620°)(36 \text{ in.})$$

$$-(T\sin 36.870°)(20 \text{ in.}) = 0$$

$$\therefore \quad T = 32.500 \text{ lb}$$

or $T = 32.5$ lb ◀

(*b*) From f.b.d. of member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (32.500 \text{ lb})(\cos 22.620° + \cos 36.870°) = 0$$

$$\therefore \quad C_x = -56.000 \text{ lb}$$

or $\quad \mathbf{C}_x = 56.000 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 30 \text{ lb} + (32.500 \text{ lb})(\sin 22.620° + \sin 36.870°) = 0$$

$$\therefore \quad C_y = -2.0001 \text{ lb}$$

or $\quad \mathbf{C}_y = 2.0001 \text{ lb} \downarrow$

Then $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(56.0)^2 + (2.001)^2} = 56.036 \text{ lb}$

and $\quad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-2.0}{-56.0}\right) = 2.0454°$

or $\mathbf{C} = 56.0$ lb $\measuredangle$ 2.05° ◀