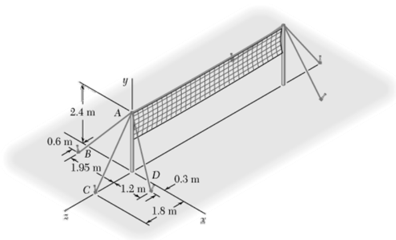
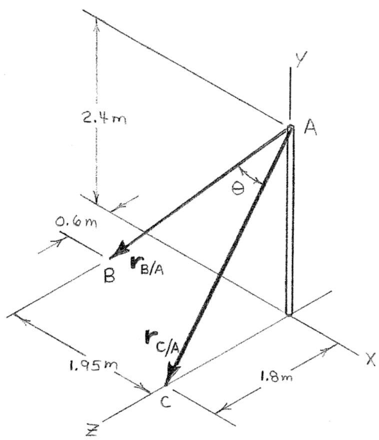
PROBLEM 3.37



Consider the volleyball net shown. Determine the angle formed by guy wires *AB* and *AC*.

SOLUTION



First note

$$AB = |\mathbf{r}_{B/A}| = \sqrt{(-1.95\text{ m})^2 + (-2.4\text{ m})^2 + (0.6\text{ m})^2}$$

$$= 3.15\text{ m}$$

$$AC = |\mathbf{r}_{C/A}| = \sqrt{(0\text{ m})^2 + (-2.4\text{ m})^2 + (1.8\text{ m})^2}$$

$$= 3.0\text{ m}$$

and

$$\mathbf{r}_{B/A} = -(1.95\text{ m})\mathbf{i} - (2.40\text{ m})\mathbf{j} + (0.6\text{ m})\mathbf{k}$$

$$\mathbf{r}_{C/A} = -(2.40\text{ m})\mathbf{j} + (1.80\text{ m})\mathbf{k}$$

By definition

$$\mathbf{r}_{B/A} \cdot \mathbf{r}_{C/A} = |\mathbf{r}_{B/A}||\mathbf{r}_{C/A}|\cos\theta$$

or $$(-1.95\mathbf{i} - 2.40\mathbf{j} + 0.6\mathbf{k}) \cdot (-2.40\mathbf{j} + 1.80\mathbf{k}) = (3.15)(3.0)\cos\theta$$

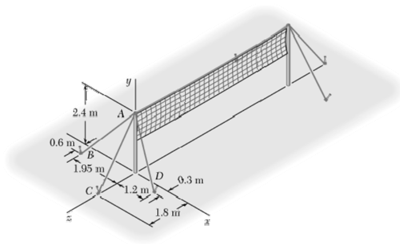
$$(-1.95)(0) + (-2.40)(-2.40) + (0.6)(1.8) = 9.45\cos\theta$$

$$\therefore\ \cos\theta = 0.72381$$

and $$\theta = 43.630°$$

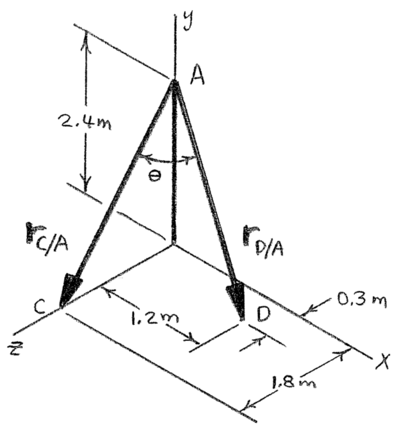
or $\theta = 43.6°$ ◀

PROBLEM 3.38



Consider the volleyball net shown. Determine the angle formed by guy wires *AC* and *AD*.

SOLUTION



First note

$$AC = \left|\mathbf{r}_{C/A}\right| = \sqrt{(-2.4)^2 + (1.8)^2}\ \text{m} = 3\ \text{m}$$

$$AD = \left|\mathbf{r}_{D/A}\right| = \sqrt{(1.2)^2 + (-2.4)^2 + (0.3)^2}\ \text{m} = 2.7\ \text{m}$$

and
$$\mathbf{r}_{C/A} = -(2.4\ \text{m})\,\mathbf{j} + (1.8\ \text{m})\,\mathbf{k}$$

$$\mathbf{r}_{D/A} = (1.2\ \text{m})\,\mathbf{i} - (2.4\ \text{m})\,\mathbf{j} + (0.3\ \text{m})\,\mathbf{k}$$

By definition

$$\mathbf{r}_{C/A} \cdot \mathbf{r}_{D/A} = \left|\mathbf{r}_{C/A}\right|\left|\mathbf{r}_{D/A}\right|\cos\theta$$

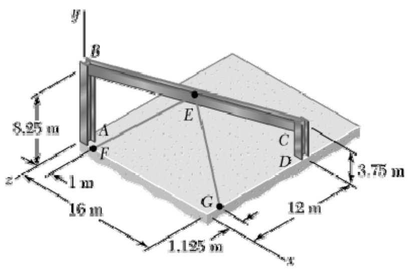
or
$$(-2.4\mathbf{j} + 1.8\mathbf{k}) \cdot (1.2\mathbf{i} - 2.4\mathbf{j} + 0.3\mathbf{k}) = (3)(2.7)\cos\theta$$

$$(0)(1.2) + (-2.4)(-2.4) + (1.8)(0.3) = 8.1\cos\theta$$

and
$$\cos\theta = \frac{6.3}{8.1} = 0.77778$$

$$\theta = 38.942°$$

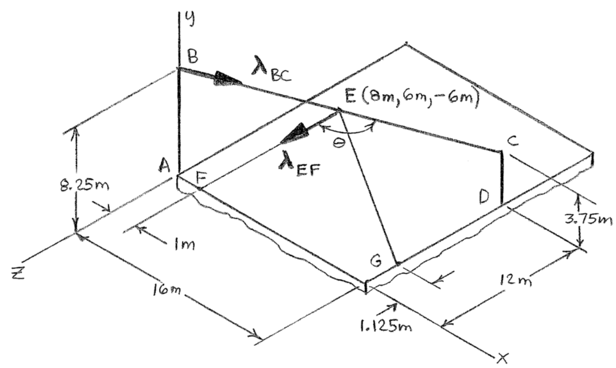
or $\theta = 38.9°$ ◀



PROBLEM 3.39

Steel framing members *AB*, *BC*, and *CD* are joined at *B* and *C* and are braced using cables *EF* and *EG*. Knowing that *E* is at the midpoint of *BC* and that the tension in cable *EF* is 330 N, determine (*a*) the angle between *EF* and member *BC*, (*b*) the projection on *BC* of the force exerted by cable *EF* at point *E*.

SOLUTION



(*a*) By definition $\lambda_{BC} \cdot \lambda_{EF} = (1)(1)\cos\theta$

where
$$\lambda_{BC} = \frac{(16\text{ m})\mathbf{i} - (4.5\text{ m})\mathbf{j} - (12\text{ m})\mathbf{k}}{\sqrt{(16)^2 + (4.5)^2 + (12)^2}\text{ m}} = \frac{1}{20.5}(16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k})$$

$$\lambda_{EF} = \frac{-(7\text{ m})\mathbf{i} - (6\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k}}{\sqrt{(7)^2 + (6)^2 + (6)^2}\text{m}} = \frac{1}{11.0}(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})$$

$$\therefore \frac{(16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k})}{20.5} \cdot \frac{(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})}{11.0} = \cos\theta$$

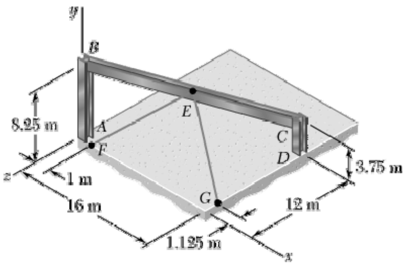
$$(16)(-7) + (-4.5)(-6) + (-12)(6) = (20.5)(11.0)\cos\theta$$

and
$$\theta = \cos^{-1}\left(\frac{-157}{225.5}\right) = 134.125°$$

or $\theta = 134.1°$ ◀

(*b*) By definition
$$(T_{EF})_{BC} = T_{EF}\cos\theta$$
$$= (330\text{ N})\cos 134.125°$$
$$= -229.26\text{ N}$$

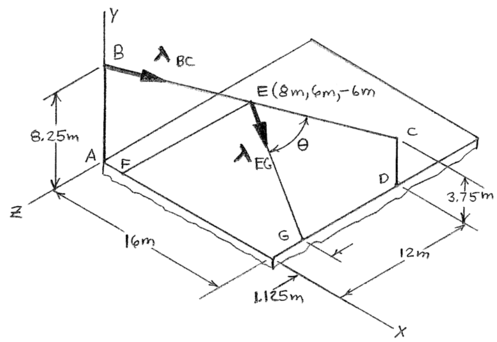
or $(T_{EF})_{BC} = -230\text{ N}$ ◀



PROBLEM 3.40

Steel framing members *AB*, *BC*, and *CD* are joined at *B* and *C* and are braced using cables *EF* and *EG*. Knowing that *E* is at the midpoint of *BC* and that the tension in cable *EG* is 445 N, determine (*a*) the angle between *EG* and member *BC*, (*b*) the projection on *BC* of the force exerted by cable *EG* at point *E*.

SOLUTION



(*a*) By definition

$$\boldsymbol{\lambda}_{BC} \cdot \boldsymbol{\lambda}_{EG} = (1)(1)\cos\theta$$

where

$$\boldsymbol{\lambda}_{BC} = \frac{(16\text{ m})\mathbf{i} - (4.5\text{ m})\mathbf{j} - (12\text{ m})\mathbf{k}}{\sqrt{(16\text{ m})^2 + (4.5)^2 + (12)^2}\text{ m}} = \frac{16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k}}{20.5}$$

$$= 0.78049\mathbf{i} - 0.21951\mathbf{j} - 0.58537\mathbf{k}$$

$$\boldsymbol{\lambda}_{EG} = \frac{(8\text{ m})\mathbf{i} - (6\text{ m})\mathbf{j} + (4.875\text{ m})\mathbf{k}}{\sqrt{(8)^2 + (6)^2 + (4.875)^2}\text{m}} = \frac{8\mathbf{i} - 6\mathbf{j} + 4.875\mathbf{k}}{11.125}$$

$$= 0.71910\mathbf{i} - 0.53933\mathbf{j} + 0.43820\mathbf{k}$$

$$\therefore\ \boldsymbol{\lambda}_{BC} \cdot \boldsymbol{\lambda}_{EG} = \frac{16(8) + (-4.5)(-6) + (-12)(4.875)}{(20.5)(11.25)} = \cos\theta$$

and

$$\theta = \cos^{-1}\left(\frac{96.5}{228.06}\right) = 64.967°$$

or $\theta = 65.0°$ ◀

(*b*) By definition

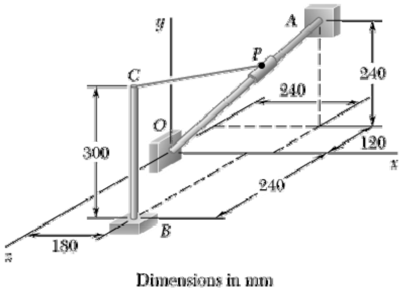
$$(T_{EG})_{BC} = T_{EG}\cos\theta$$

$$= (445\text{ N})\cos 64.967°$$

$$= 188.295\text{ N}$$

or $(T_{EG})_{BC} = 188.3\text{ N}$ ◀

PROBLEM 3.41



Slider P can move along rod OA. An elastic cord PC is attached to the slider and to the vertical member BC. Knowing that the distance from O to P is 0.12 m and the tension in the cord is 30 N, determine (*a*) the angle between the elastic cord and the rod OA, (*b*) the projection on OA of the force exerted by cord PC at point P.

SOLUTION

(*a*) By definition $$\lambda_{OA} \cdot \lambda_{PC} = (1)(1)\cos\theta$$

where $$\lambda_{OA} = \frac{(0.24\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.24)^2 + (0.12)^2}\text{ m}}$$

$$= \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

Knowing that $|\mathbf{r}_{A/O}| = L_{OA} = 0.36\text{ m}$ and that P is located 0.12 m from O, it follows that the coordinates of P are $\frac{1}{3}$ the coordinates of A.

$$\therefore\ P(0.08\text{ m}, 0.08\text{ m}, -0.040\text{ m})$$

Then $$\lambda_{PC} = \frac{(0.10\text{ m})\mathbf{i} + (0.22\text{ m})\mathbf{j} + (0.28\text{ m})\mathbf{k}}{\sqrt{(0.10)^2 + (0.22)^2 + (0.28)^2}\text{ m}}$$

$$= 0.27037\mathbf{i} + 0.59481\mathbf{j} + 0.75703\mathbf{k}$$

$$\therefore \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \cdot (0.27037\mathbf{i} + 0.59481\mathbf{j} + 0.75703\mathbf{k}) = \cos\theta$$

and $$\theta = \cos^{-1}(0.32445) = 71.068°$$

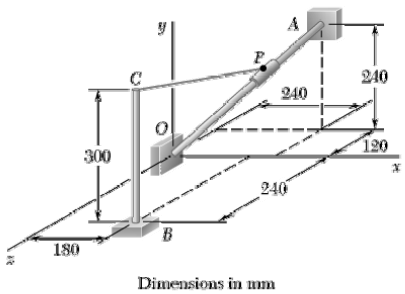
or $\theta = 71.1°$ ◀

(*b*) $$(T_{PC})_{OA} = T_{PC}\cos\theta = (30\text{ N})\cos 71.068°$$

$$(T_{PC})_{OA} = 9.7334\text{ N}$$

or $(T_{PC})_{OA} = 9.73\text{ N}$ ◀

PROBLEM 3.42



Slider P can move along rod OA. An elastic cord PC is attached to the slider and to the vertical member BC. Determine the distance from O to P for which cord PC and rod OA are perpendicular.

SOLUTION

The requirement that member OA and the elastic cord PC be perpendicular implies that

$$\lambda_{OA} \cdot \lambda_{PC} = 0 \qquad \text{or} \qquad \lambda_{OA} \cdot \mathbf{r}_{C/P} = 0$$

where

$$\lambda_{OA} = \frac{(0.24\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.24)^2 + (0.12)^2}\text{ m}}$$

$$= \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

Letting the coordinates of P be $P(x, y, z)$, we have

$$\mathbf{r}_{C/P} = \left[(0.18 - x)\mathbf{i} + (0.30 - y)\mathbf{j} + (0.24 - z)\mathbf{k}\right]\text{m}$$

$$\therefore \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \cdot \left[(0.18 - x)\mathbf{i} + (0.30 - y)\mathbf{j} + (0.24 - z)\mathbf{k}\right] = 0 \qquad (1)$$

Since

$$\mathbf{r}_{P/O} = \lambda_{OA} d_{OP} = \frac{d_{OP}}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

Then

$$x = \frac{2}{3}d_{OP}, \quad y = \frac{2}{3}d_{OP}, \quad z = \frac{-1}{3}d_{OP} \qquad (2)$$

Substituting the expressions for x, y, and z from Equation (2) into Equation (1),

$$\frac{1}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \left[\left(0.18 - \frac{2}{3}d_{OP}\right)\mathbf{i} + \left(0.30 - \frac{2}{3}d_{OP}\right)\mathbf{j} + \left(0.24 + \frac{1}{3}d_{OP}\right)\mathbf{k}\right] = 0$$

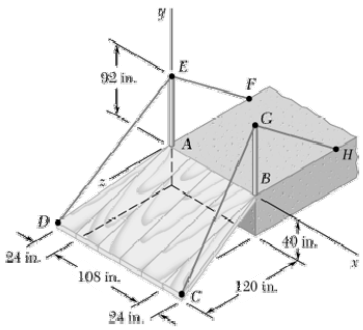
or

$$3d_{OP} = 0.36 + 0.60 - 0.24 = 0.72$$

$$\therefore d_{OP} = 0.24\text{ m}$$

or $d_{OP} = 240$ mm ◀

PROBLEM 3.43



Determine the volume of the parallelepiped of Figure 3.25 when (*a*) **P** = −(7 in.)**i** − (1 in.)**j** + (2 in.)**k**, **Q** = (3 in.)**i** − (2 in.)**j** + (4 in.)**k**, and **S** = −(5 in.)**i** + (6 in.)**j** − (1 in.)**k**, (*b*) **P** = (1 in.)**i** + (2 in.)**j** − (1 in.)**k**, **Q** = −(8 in.)**i** − (1 in.)**j** + (9 in.)**k**, and **S** = (2 in.)**i** + (3 in.)**j** + (1 in.)**k**.

SOLUTION

Volume of a parallelepiped is found using the mixed triple product.

(*a*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} -7 & -1 & 2 \\ 3 & -2 & 4 \\ -5 & 6 & -1 \end{vmatrix} \text{in}^3 = (-14 + 168 + 20 - 3 + 36 - 20)\,\text{in}^3$$

$$= 187\ \text{in}^3$$

$$\text{or Volume} = 187\ \text{in}^3 \blacktriangleleft$$

(*b*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} 1 & 2 & -1 \\ -8 & -1 & 9 \\ 2 & 3 & 1 \end{vmatrix} \text{in}^3 = (-1 - 27 + 36 + 16 + 24 - 2)\,\text{in}^3$$

$$= 46\ \text{in}^3$$

$$\text{or Volume} = 46\ \text{in}^3 \blacktriangleleft$$

PROBLEM 3.44

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + P_z\mathbf{k}$, $\mathbf{Q} = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, determine the value of P_z for which the three vectors are coplanar.

SOLUTION

For the vectors to all be in the same plane, the mixed triple product is zero.

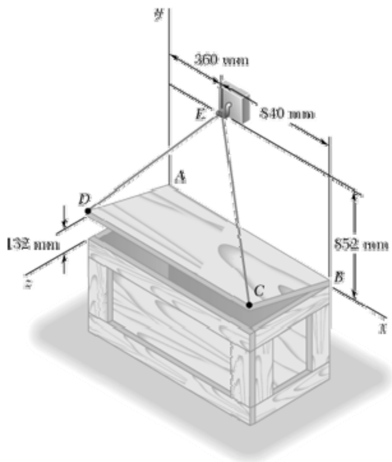
$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

$$\therefore\ O = \begin{vmatrix} 4 & -2 & P_z \\ 1 & 3 & -5 \\ -6 & 2 & -1 \end{vmatrix} = -12 + 40 - 60 - 2 + P_z(2 + 18)$$

so that

$$P_z = \frac{34}{20} = 1.70$$

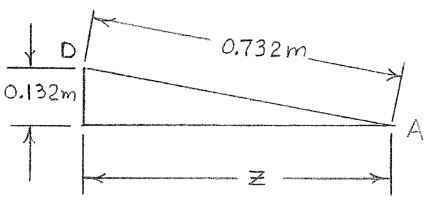
or $P_z = 1.700$ ◀



PROBLEM 3.45

The 0.732×1.2-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 54 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *D*.

SOLUTION



First note $$z = \sqrt{(0.732)^2 - (0.132)^2} \text{ m}$$

$$= 0.720 \text{ m}$$

Then $$d_{DE} = \sqrt{(0.360)^2 + (0.720)^2 + (0.720)^2} \text{ m}$$

$$= 1.08 \text{ m}$$

and $$\mathbf{r}_{E/D} = (0.360 \text{ m})\mathbf{i} + (0.720 \text{ m})\mathbf{j} - (0.720 \text{ m})\mathbf{k}$$

Have $$\mathbf{T}_{DE} = \frac{T_{OE}}{d_{DE}}(\mathbf{r}_{E/D})$$

$$= \frac{54 \text{ N}}{1.08}(0.360\mathbf{i} + 0.720\mathbf{j} - 0.720\mathbf{k})$$

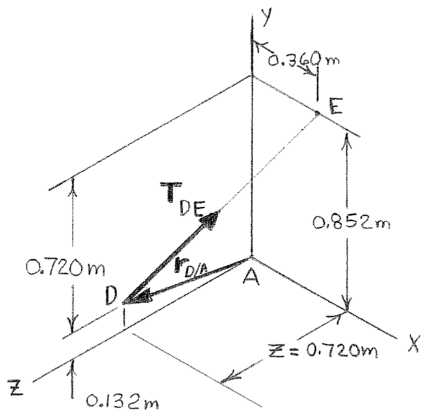
$$= (18.0 \text{ N})\mathbf{i} + (36.0 \text{ N})\mathbf{j} - (36.0 \text{ N})\mathbf{k}$$

Now $$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{T}_{DE}$$

where $$\mathbf{r}_{D/A} = (0.132 \text{ m})\mathbf{j} + (0.720 \text{ m})\mathbf{k}$$

Then $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.132 & 0.720 \\ 18.0 & 36.0 & -36.0 \end{vmatrix} \text{ N}\cdot\text{m}$$

PROBLEM 3.45 CONTINUED



$$\therefore\ \mathbf{M}_A = \Big\{\big[(0.132)(-36.0) - (0.720)(36.0)\big]\mathbf{i} + \big[(0.720)(18.0) - 0\big]\mathbf{j}$$

$$+\big[0 - (0.132)(18.0)\big]\mathbf{k}\Big\}\ \text{N}\cdot\text{m}$$

or $$\mathbf{M}_A = -(30.7\ \text{N}\cdot\text{m})\mathbf{i} + (12.96\ \text{N}\cdot\text{m})\mathbf{j} - (2.38\ \text{N}\cdot\text{m})\mathbf{k}$$

$$\therefore\ M_x = -30.7\ \text{N}\cdot\text{m},\ M_y = 12.96\ \text{N}\cdot\text{m},\ \ M_z = -2.38\ \text{N}\cdot\text{m}\ \blacktriangleleft$$