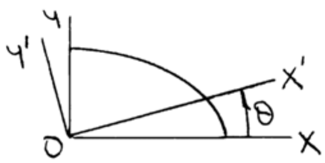
PROBLEM 9.91

Using Mohr's circle, determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

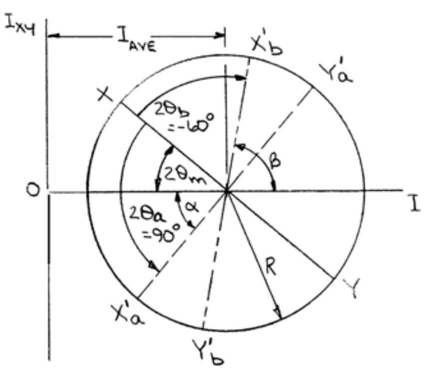
SOLUTION



From Problem 9.79: $$I_x = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy} = \frac{1}{2}a^4$$



The Mohr's circle is defined by the diameter XY, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \qquad \text{and} \qquad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4 = 0.98175a^4$$

and $$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326^{\circ}$$

PROBLEM 9.91 CONTINUED

Then $\alpha = 90° - 40.326°$

$= 49.674°$

$\beta = 180° - (40.326° + 60°)$

$= 79.674°$

(*a*) $I_{x'} = I_{\text{ave}} - R\cos\alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674°$

or $I_{x'} = 0.482a^4$ ◀

$I_{y'} = I_{\text{ave}} + R\cos\alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674°$

or $I_{y'} = 1.482a^4$ ◀

$I_{x'y'} = -R\sin\alpha = -0.77264a^4 \sin 49.674°$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $I_{x'} = I_{\text{ave}} + R\cos\beta = 0.98175a^4 + 0.77264a^4 \cos 79.674°$

or $I_{x'} = 1.120a^4$ ◀

$I_{y'} = I_{\text{ave}} - R\cos\beta = 0.98175a^4 - 0.77264a^4 \cos 79.674°$

or $I_{y'} = 0.843a^4$ ◀

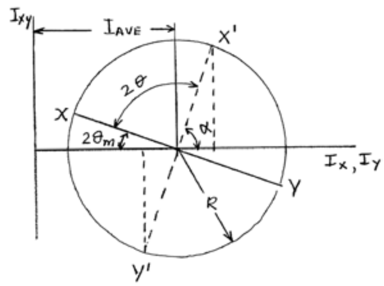
$I_{x'y'} = R\sin\beta = 0.77264a^4 \sin 79.674°$

or $I_{x'y'} = 0.760a^4$ ◀

PROBLEM 9.92

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION



From the solution to

Problem 9.72: $\bar{I}_{xy} = 501.1875 \text{ in}^4$

Problem 9.80: $\bar{I}_x = 865.6875 \text{ in}^4$

$\bar{I}_y = 4758.75 \text{ in}^4$

Now $\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

The Mohr's circle is defined by the points X and Y where

$$X: \ \left(\bar{I}_x, \bar{I}_{xy}\right) \qquad Y: \ \left(\bar{I}_y, -\bar{I}_{xy}\right)$$

Now $I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$

and $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{(-1946.53125)^2 + 501.1875^2} \text{ in}^4$

$$= 2010.0 \text{ in}^4$$

Also, $\tan 2\theta_m = \frac{\bar{I}_{xy}}{\left|\frac{\bar{I}_x - \bar{I}_y}{2}\right|} = \frac{501.1875}{1946.53125} = 0.2575$

or $2\theta_m = 14.4387°$

Then $\alpha = 180° - (14.4387° + 90°) = 75.561°$

PROBLEM 9.92 CONTINUED

Then $\qquad \bar{I}_{x'}, \bar{I}_{y'} = I_{\text{ave}} \pm R\cos\alpha = 2812.2 \pm 2010.0\cos 75.561°$

or $\bar{I}_{x'} = 3.31 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{y'} = 2.31 \times 10^3 \text{ in}^4$ ◀

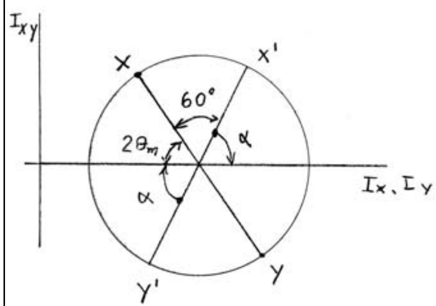
and $\qquad \bar{I}_{x'y'} = R\sin\alpha = 2010.0\sin 75.561°$

or $\bar{I}_{x'y'} = 1.947 \times 10^3 \text{ in}^4$ ◀

PROBLEM 9.93

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the *x* and *y* axes through 30° clockwise.

SOLUTION



From Problems 9.73 and 9.81

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

$$\bar{I}_x = 51.84\pi \times 10^6 \text{ mm}^4$$

$$= 162.86 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 103.68\pi \times 10^6 \text{ mm}^4$$

$$= 325.72 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$$

$$= 244.29 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 160.4405 \times 10^6 \text{ mm}^4$$

From Problem 9.87 $$2\theta_m = 59.5^{\circ}$$

Then $$\alpha = 180 - 60^{\circ} - 2\theta_m = 60.5^{\circ}$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = 244.29 + 160.4405\cos 60.5^{\circ}$$

$$= 323.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 323 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = 244.24 - 160.4405\cos 60.5^{\circ}$$

$$= 165.29 \times 10^6 \text{ mm}^4$$

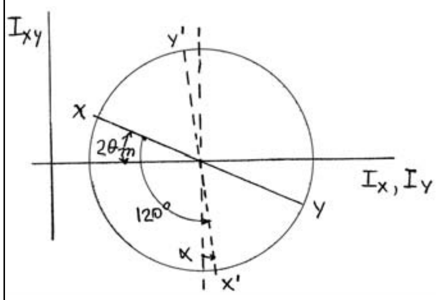
or $\bar{I}_{y'} = 165.3 \times 10^6 \text{ mm}^4$ ◀

$\bar{I}_{x'y'} = R\sin\alpha = 160.44\sin 60.5^{\circ} = 139.6 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.94

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes through 60° counterclockwise.

SOLUTION



From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 7.728 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = 3.8494 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.12°$$

and $$\alpha = 120° - 24.12° - 90 = 5.88°$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\sin\alpha = (4.2147 + 3.8494\sin 5.88°) \times 10^6 \text{ mm}^4$$

$$= 4.6091 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 4.61 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\sin\alpha = (4.2147 - 3.8494\sin 5.88°) \times 10^6 \text{ mm}^4$$

$$= 3.8203 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 3.82 \times 10^6 \text{ mm}^4$ ◀

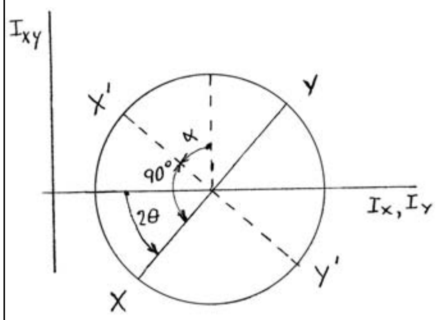
$$\bar{I}_{x'y'} = -R\cos\alpha = -3.8494\cos 5.88° = -3.8291 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = -3.83 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.95

Using Mohr's circle, determine the moments of inertia and the product of inertia of the L76 × 51 × 6.4-mm angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes through 45° clockwise.

SOLUTION



From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 0.21463 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04^{\circ}$$

and $$\alpha + 90^{\circ} - 2\theta = 90^{\circ}; \ \alpha = 2\theta_m$$

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\sin\alpha = \left(0.3095 - 0.21463\sin 48.04^{\circ}\right) \times 10^6 \text{ mm}^4$$

$$= 0.14989 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4$ ◀

and

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\sin\alpha = \left(0.3095 + 0.21463\sin 48.04^{\circ}\right) \times 10^6 \text{ mm}^4$$

$$= 0.46910 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 0.4690 \times 10^6 \text{ mm}^4$ ◀

and

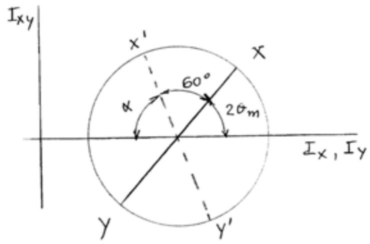
$$\bar{I}_{x'y'} = R\cos\alpha = 0.21463\cos 48.04^{\circ} = 0.1435 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = 0.1435 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.96

Using Mohr's circle, determine the moments of inertia and the product of inertia of the L5 × 3 × $\frac{1}{2}$ -in. angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes through 30° counterclockwise.

SOLUTION



Have $\bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\bar{I}_{xy} = 2.8125 \text{ in}^4$

Now $$\bar{I}_{\text{ave}} = \frac{\bar{I}_x + \bar{I}_y}{2} = 6.015 \text{ in}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2}$$

$$= 4.43952 \text{ in}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(2.8125)}{9.45 - 2.58}\right] = -39.31^{\circ}$$

$$2\theta_m + 60 + \alpha = 180^{\circ}, \qquad \alpha = 80.69^{\circ}$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 6.015 \text{ in}^4 - \left(4.43952 \text{ in}^4\right)\cos 80.69^{\circ}$$

$$= 5.29679 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = 6.015 \text{ in}^4 + \left(4.43952 \text{ in}^4\right)\cos 80.69^{\circ}$$

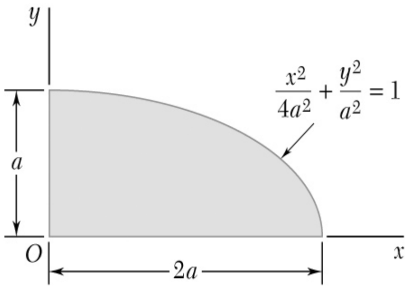
$$= 6.73321 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = \left(4.43952 \text{ in}^4\right)\sin 80.69^{\circ} = 4.38104 \text{ in}^4$$

or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀

PROBLEM 9.97



For the quarter ellipse of Problem 9.67, use Mohr's circle to determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $\quad I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $\quad I_{xy} = \frac{1}{2}a^4$

The Mohr's circle is defined by the diameter XY, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

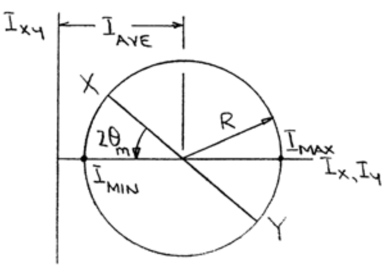
Now $\quad I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$

and

$$R = \sqrt{\left[\frac{1}{2}\left(I_x - I_y\right)\right]^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

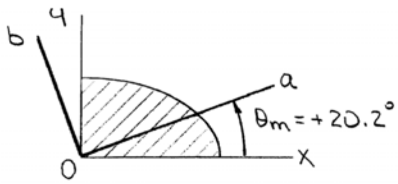
$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $\quad 2\theta_m = 40.326°$

and $\quad \theta_m = 20.2°$

PROBLEM 9.97 CONTINUED



$\therefore$ The principal axes are obtained by rotating the *xy* axes through

20.2° counterclockwise ◀

About O.

Now $$I_{\text{max, min}} = I_{\text{ave}} \pm R = 0.98175a^4 \pm 0.77264a^4$$

or $I_{\text{max}} = 1.754a^4$ ◀

and $I_{\text{min}} = 0.209a^4$ ◀

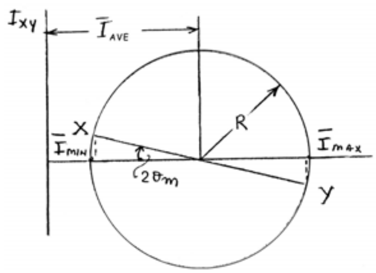
From the Mohr's circle it is seen that the *a* axis corresponds to I_{min} and the *b* axis corresponds to I_{max}.

PROBLEM 9.98

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.72

SOLUTION



From the solution to Problem 9.72:

$$\bar{I}_{xy} = 501.1875 \text{ in}^4$$

From the solution to Problem 9.80:

$$\bar{I}_x = 865.6875 \text{ in}^4$$

$$\bar{I}_y = 4758.75 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^2$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

The Mohr's circle is defined by the point $X: \ \left(\bar{I}_x, \bar{I}_{xy}\right), \qquad Y: \ \left(\bar{I}_y, -\bar{I}_{xy}\right)$

Now $I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$

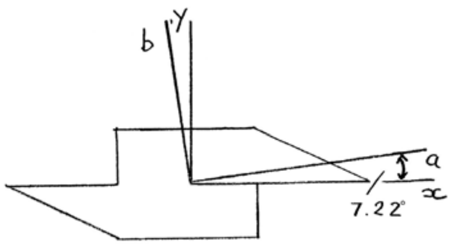
and $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}} = \sqrt{\left(-1946.53125\right)^2 + 501.1875^2} = 2010.0 \text{ in}^4$

PROBLEM 9.98 CONTINUED

$$\tan 2\theta_m = -\frac{\bar{I}_{xy}}{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)} = -\frac{501.1875}{-1946.53125} = 0.2575, \qquad 2\theta_m = 14.4387°$$

or $\theta_m = 7.22°$ counterclockwise ◀

Then $\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (2812.2 \pm 2010.0)\text{in}^4$



or $\bar{I}_{\max} = 4.82 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 802 \text{ in}^4$ ◀

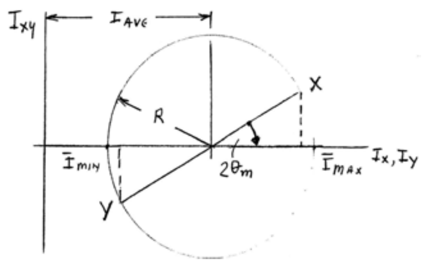
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

PROBLEM 9.99

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.76

SOLUTION



From the solution to Problem 9.76 $\quad \bar{I}_{xy} = 576 \text{ in}^4$

Now
$$\bar{I}_x = (I_x)_1 - (I_x)_2 - (I_x)_3, \quad \text{where} \quad (I_x)_2 = (I_x)_3$$

$$= \frac{\pi}{4}(15 \text{ in.})^4 - 2\left[\frac{1}{12}(9 \text{ in.})(6 \text{ in.})^3\right] = (39761 - 324)\text{in}^4$$

$$= 39{,}437 \text{ in}^4$$

and
$$\bar{I}_y = (I_y)_1 - (I_y)_2 - (I_y)_3, \quad \text{where} \quad (I_y)_2 = (I_y)_3$$

$$= \frac{\pi}{4}(15 \text{ in.})^4 - 2\left[\frac{1}{36}(6 \text{ in.})(9 \text{ in.})^3 + \frac{1}{2}(9 \text{ in.})(6 \text{ in.})(6 \text{ in.})^2\right]$$

$$= (39{,}761 - 243 - 1944)\text{in}^4 = 37{,}574 \text{ in}^4$$

The Mohr's circle is defined by the point (X, Y) where

$$X: \quad (\bar{I}_x, \bar{I}_{xy}) \qquad Y: \quad (\bar{I}_y, -\bar{I}_{xy})$$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(39{,}437 + 37{,}574)\text{in}^4 = 38{,}506 \text{ in}^4$$

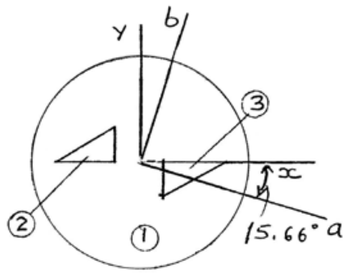
and
$$R = \sqrt{\frac{\bar{I}_x - \bar{I}_y}{2} + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(39{,}437 - 37{,}574)\right]^2 + 567^2} = 1090.5 \text{ in}^4$$

PROBLEM 9.99 CONTINUED

$$\tan 2\theta_m = \frac{-\bar{I}_{xy}}{\frac{\bar{I}_x - \bar{I}_y}{2}} = \frac{-567}{\frac{1}{2}(39{,}437 - 37{,}574)} = -0.6087$$

or $\theta_m = -15.66°$ clockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (38{,}506 \pm 1090.50)\text{in}^4$$



or $\bar{I}_{\text{max}} = 39.6 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 37.4 \times 10^3 \text{ in}^4$ ◀

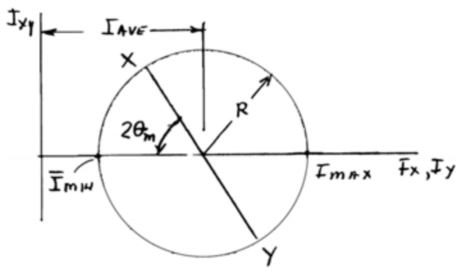
Note: From the Mohr's circle it is seen that the *a* axis corresponds to the $\bar{I}_{\text{max}}$ and the *b* axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.100

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION



From Problems 9.73 and 9.81 $\bar{I}_x = 162.86 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 325.72 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

Define points $X(162.86, 138.24) \times 10^6 \text{ mm}^4 \qquad Y(325.72, -138.24) \times 10^6 \text{ mm}^4$

Now $I_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(162.86 + 325.72) \times 10^6 \text{ mm}^4$

$$= 244.29 \times 10^6 \text{ mm}^4$$

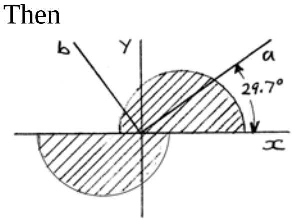
and $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{(162.86 - 325.72)}{2} \times 10^6\right]^2 + (138.24 \times 10^6)^2}$

$$= 160.44 \times 10^6 \text{ mm}^4$$

and $2\theta_m = \tan^{-1}\left[\frac{-2(138.24) \times 10^6}{(162.86 - 325.72) \times 10^6}\right] = 59.4999°$

or $\theta_m = 29.7°$ counterclockwise ◀

Then $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (244.29 \times 10^6 \pm 160.44 \times 10^6) \text{ mm}^4$



or $\bar{I}_{\text{max}} = 405 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 83.9 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.