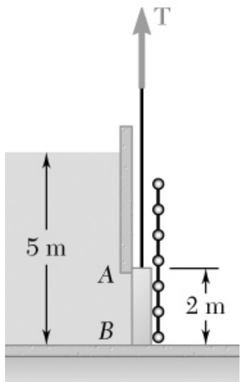
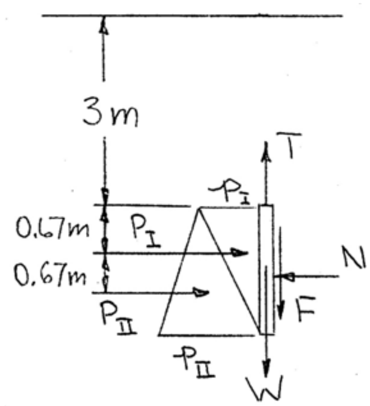
PROBLEM 5.79



The friction force between a 2×2-m square sluice gate *AB* and its guides is equal to 10 percent of the resultant of the pressure forces exerted by the water on the face of the gate. Determine the initial force needed to lift the gate that its mass is 500 kg.

SOLUTION



Consider the free-body diagram of the gate.

Now
$$P_{\mathrm{I}} = \frac{1}{2}Ap_{\mathrm{I}} = \frac{1}{2}\left[(2 \times 2)\,\mathrm{m}^2\right]\left[(10^3\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(3\ \mathrm{m})\right]$$
$$= 58.86\ \mathrm{kN}$$

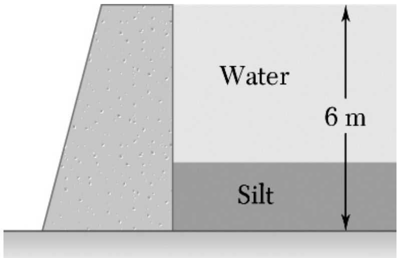
$$P_{\mathrm{II}} = \frac{1}{2}Ap_{\mathrm{II}} = \frac{1}{2}\left[(2 \times 2)\,\mathrm{m}^2\right]\left[(10^3\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(5\ \mathrm{m})\right]$$
$$= 98.10\ \mathrm{kN}$$

Then
$$F = 0.1P = 0.1(P_{\mathrm{I}} + P_{\mathrm{II}})$$
$$= 0.1(58.86 + 98.10)\ \mathrm{kN}$$
$$= 15.696\ \mathrm{kN}$$

Finally
$$+\uparrow \Sigma F_y = 0: \quad T - 15.696\ \mathrm{kN} - (500\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 0$$

or $\mathbf{T} = 20.6\ \mathrm{kN}\uparrow$ ◀

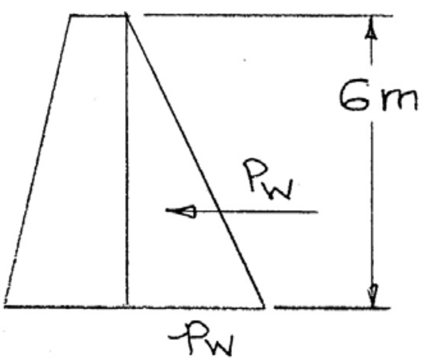
PROBLEM 5.80



The dam for a lake is designed to withstand the additional force caused by silt which has settled on the lake bottom. Assuming that silt is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3$ kg/m^3 and considering a 1-m-wide section of dam, determine the percentage increase in the force acting on the dam face for a silt accumulation of depth 1.5 m.

SOLUTION

First, determine the force on the dam face without the silt.

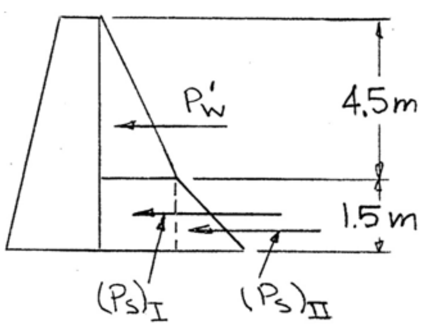


Have $$P_w = \frac{1}{2}Ap_w = \frac{1}{2}A(\rho g h)$$

$$= \frac{1}{2}\left[(6\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6\text{ m})\right]$$

$$= 176.58\text{ kN}$$

Next, determine the force on the dam face with silt.



Have $$P'_w = \frac{1}{2}\left[(4.5\text{ m})(1\text{m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 99.326\text{ kN}$$

$$(P_s)_{\text{I}} = \left[(1.5\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 66.218\text{ kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[(1.5\text{ m})(1\text{ m})\right]\left[\left(1.76\times10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.5\text{ m})\right]$$

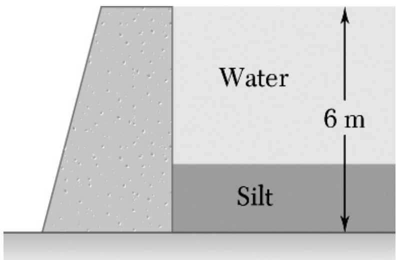
$$= 19.424\text{ kN}$$

Then $$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = 184.97\text{ kN}$$

The percentage increase, % inc., is then given by

$$\%\text{ inc.} = \frac{P' - P_w}{P_w}\times 100\% = \frac{(184.97 - 176.58)}{176.58}\times 100\% = 4.7503\%$$

$$\%\text{ inc.} = 4.75\% \blacktriangleleft$$



PROBLEM 5.81

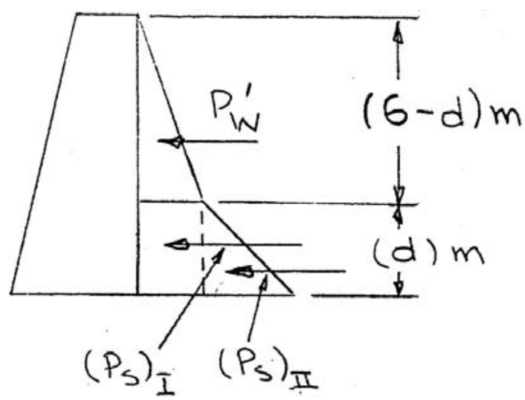
The base of a dam for a lake is designed to resist up to 150 percent of the horizontal force of the water. After construction, it is found that silt (which is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3 \text{ kg/m}^3$) is settling on the lake bottom at a rate of 20 mm/y. Considering a 1-m-wide section of dam, determine the number of years until the dam becomes unsafe.

SOLUTION

From Problem 5.80, the force on the dam face before the silt is deposited, is $P_w = 176.58 \text{ kN}$. The maximum allowable force P_{allow} on the dam is then:

$$P_{\text{allow}} = 1.5P_w = (1.5)(176.58 \text{ kN}) = 264.87 \text{ kN}$$

Next determine the force P' on the dam face after *a* depth *d* of silt has settled.



Have

$$P_w' = \frac{1}{2}\left[(6-d)\text{m} \times (1\text{ m})\right]\left[\left(10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(6-d)\text{m}\right]$$

$$= 4.905(6-d)^2 \text{ kN}$$

$$(P_s)_{\text{I}} = \left[d(1\text{ m})\right]\left[\left(10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(6-d)\text{m}\right]$$

$$= 9.81\left(6d - d^2\right) \text{kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[d(1\text{ m})\right]\left[\left(1.76 \times 10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(d)\text{m}\right]$$

$$= 8.6328d^2 \text{ kN}$$

$$P' = P_w' + (P_s)_{\text{I}} + (P_s)_{\text{II}} = \left[4.905\left(36 - 12d + d^2\right) + 9.81\left(6d - d^2\right) + 8.6328d^2\right]\text{kN}$$

$$= \left[3.7278d^2 + 176.58\right]\text{kN}$$

PROBLEM 5.81 CONTINUED

Now required that $P' = P_{\text{allow}}$ to determine the maximum value of d.

$$\therefore \quad \left(3.7278d^2 + 176.58\right)\text{kN} = 264.87\text{ kN}$$

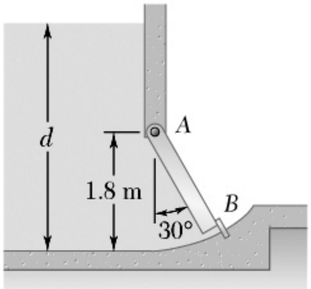
or

$$d = 4.8667\text{ m}$$

Finally

$$4.8667\text{ m} = 20 \times 10^{-3}\,\frac{\text{m}}{\text{year}} \times N$$

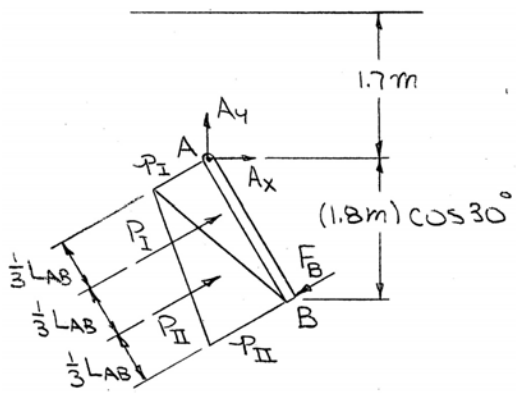
or $N = 243$ years ◀



PROBLEM 5.82

The square gate AB is held in the position shown by hinges along its top edge A and by a shear pin at B. For a depth of water $d = 3.5$ m, determine the force exerted on the gate by the shear pin.

SOLUTION



First consider the force of the water on the gate.

Have
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho gh)$$

Then
$$P_{\mathrm{I}} = \frac{1}{2}(18\text{ m})^2\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.7\text{ m})$$
$$= 26.99\text{ kN}$$
$$P_{\mathrm{II}} = \frac{1}{2}(18\text{ m})^2\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.7 \times 1.8\cos 30°)\text{m}$$
$$= 51.74\text{ kN}$$

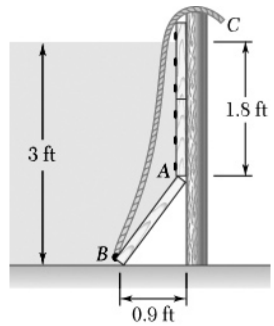
Now
$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad \frac{1}{3}(L_{AB})P_{\mathrm{I}} + \frac{2}{3}(L_{AB})P_{\mathrm{II}} - L_{AB}F_B = 0$$

or
$$\frac{1}{3}(26.99\text{ kN}) + \frac{2}{3}(51.74\text{ kN}) - F_B = 0$$

or
$$F_B = 43.49\text{ kN}$$

$$\mathbf{F}_B = 4.35\text{ kN} \measuredangle 30.0° \blacktriangleleft$$

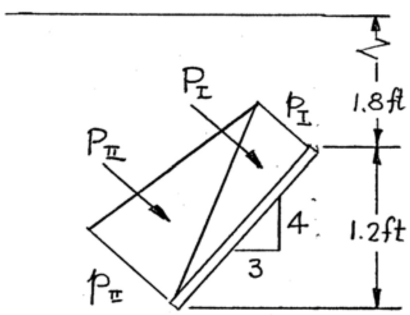
PROBLEMS 5.83 AND 5.84



Problem 5.83: A temporary dam is constructed in a 5-ft-wide fresh water channel by nailing two boards to pilings located at the sides of the channel and propping a third board *AB* against the pilings and the floor of the channel. Neglecting friction, determine the reactions at *A* and *B* when rope *BC* is slack.

Problem 5.84: A temporary dam is constructed in a 5-ft-wide fresh water channel by nailing two boards to pilings located at the sides of the channel and propping a third board *AB* against the pilings and the floor of the channel. Neglecting friction, determine the magnitude and direction of the minimum tension required in rope *BC* to move board *AB*.

SOLUTION



First, consider the force of the water on the gate.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma h)$$

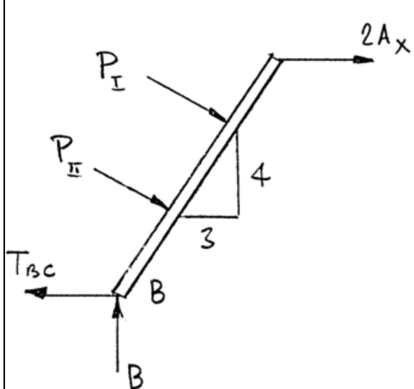
So that $$P_{\mathrm{I}} = \frac{1}{2}\left[(1.5\text{ ft})(5\text{ ft})\right]\left[\left(62.4\text{ lb/ft}^3\right)(1.8\text{ ft})\right]$$

$$= 421.2\text{ lb}$$

$$P_{\mathrm{II}} = \frac{1}{2}\left[(1.5\text{ ft})(5\text{ ft})\right]\left[\left(62.4\text{ lb/ft}^3\right)(3\text{ ft})\right]$$

$$= 702\text{ lb}$$

5.83 Find the reactions at *A* and *B* when rope is slack.



$$+\circlearrowright \Sigma M_A = 0: \quad -(0.9\text{ ft})B + (0.5\text{ ft})(421.2\text{ lb}) + (1.0\text{ ft})(702\text{ lb}) = 0$$

or $$B = 1014\text{ lb}$$

$$\mathbf{B} = 1014\text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2A_x + \frac{4}{5}(421.2\text{ lb}) + \frac{4}{5}(702\text{ lb}) = 0$$

or $$A_x = -449.28\text{ lb}$$

Note that the factor 2 $(2A_x)$ is included since A_x is the horizontal force exerted by the board on each piling.

$$+\uparrow \Sigma F_y = 0: \quad 1014\text{ lb} - \frac{3}{5}(421.2\text{ lb}) - \frac{3}{5}(702\text{ lb}) + A_y = 0$$

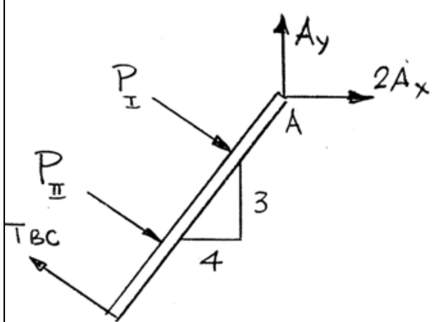
or $$A_y = -340.08\text{ lb}$$

$$\therefore \quad \mathbf{A} = 563\text{ lb} \swarrow 37.1° \blacktriangleleft$$

PROBLEMS 5.83 AND 5.84 CONTINUED

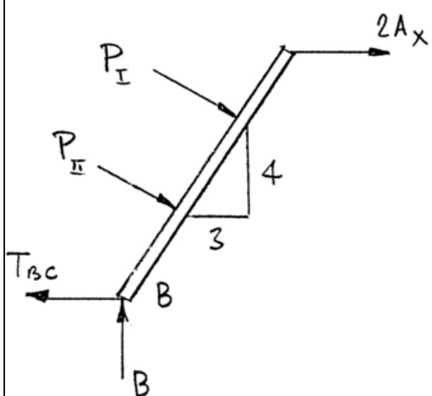
5.84 Note that there are two ways to move the board:

1. Pull upward on the rope fastened at *B* so that the board rotates about *A*. For this case $\mathbf{B} \to 0$ and $\mathbf{T}_{BC}$ is perpendicular to *AB* for minimum tension.
2. Pull horizontally at *B* so that the edge *B* of the board moves to the left. For this case $A_y \to 0$ and the board remains against the pilings because of the force of the water.



Case (1) $$\overset{+}{\circlearrowleft} \Sigma M_A = 0: \quad -1.5T_{BC} + (0.5 \text{ ft})(421.2 \text{ lb}) + (1.0 \text{ ft})(702 \text{ lb}) = 0$$

or $$T_{BC} = 608.4 \text{ lb}$$



Case (2) $$\overset{+}{\circlearrowleft} \Sigma M_B = 0: \quad -(1.2 \text{ ft})(2A_x) - (0.5 \text{ ft})(702 \text{ lb}) - (1.0 \text{ ft})(421.2 \text{ lb}) = 0$$

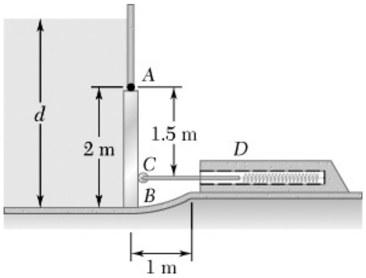
or $$2A_x = -643.5 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -T_{BC} - 643.5 \text{ lb} + \frac{4}{5}(421.2 \text{ lb}) + \frac{4}{5}(702 \text{ lb}) = 0$$

or $$T_{BC} = 255.06 \text{ lb}$$

$$\therefore \; (\mathbf{T}_{BC})_{\min} = 255 \text{ lb} \longleftarrow \blacktriangleleft$$

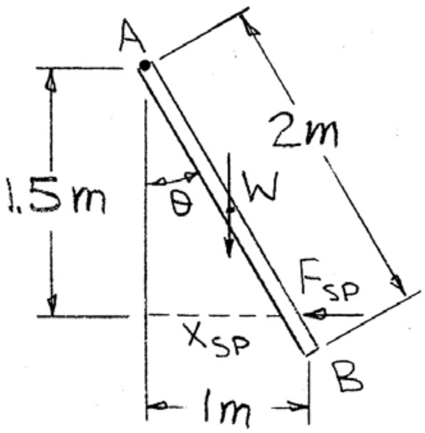
PROBLEMS 5.85 AND 5.86

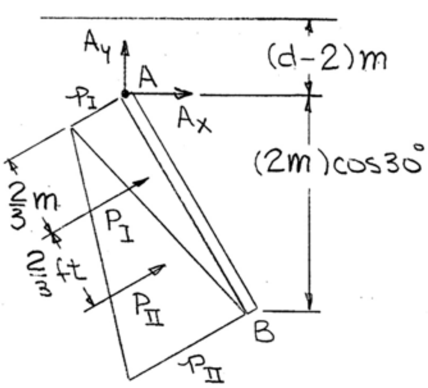


Problem 5.85: A 2×3-m gate is hinged at *A* and is held in position by rod *CD*. End *D* rests against a spring whose constant is 12 kN/m. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod *CD* on the gate remains horizontal, determine the minimum depth of water *d* for which the bottom *B* of the gate will move to the end of the cylindrical portion of the floor.

Problem 5.86: Solve Problem 5.85 if the mass of the gate is 500 kg.

SOLUTION





First, determine the forces exerted on the gate by the spring and the water when *B* is at the end of the cylindrical portion of the floor.

Have $\sin\theta = \frac{1}{2}$ $\quad \therefore \quad \theta = 30°$

Then $x_{sp} = (1.5\text{ m})\tan 30°$

and $F_{sp} = kx_{sp}$

$= (12\text{ kN/m})(1.5\text{ m})\tan 30°$

$= 10.39\text{ kN}$

Assume $d \geq 2\text{ m}$

Have $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g)h$

Then $P_{\text{I}} = \frac{1}{2}\left[(2\text{ m})(3\text{ m})\right]\left[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(d-2)\text{m}\right]$

$= 29.43(d-2)\text{ kN}$

$P_{\text{II}} = \frac{1}{2}\left[(2\text{ m})(3\text{ m})\right]\left[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(d-2+2\cos 30°)\text{m}\right]$

$= 29.43(d-0.2679)\text{ kN}$

PROBLEMS 5.85 AND 5.86 CONTINUED

5.85 Find $d_{\min}$ so that gate opens, $W = 0$.

Using the above free-body diagrams of the gate, we have

$$\circlearrowright + \Sigma M_A = 0: \quad \left(\frac{2}{3}\text{m}\right)\left[29.43(d-2)\text{kN}\right]$$

$$+\left(\frac{4}{3}\text{m}\right)\left[29.43(d-0.2679)\text{kN}\right]$$

$$-(1.5\text{ m})(10.39\text{ kN}) = 0$$

or $$19.62(d-2) + 39.24(d-0.2679) = 15.585$$

$$58.86d = 65.3374$$

or $$d = 1.1105\text{ m}$$ $d = 1.110\text{ m}$ ◀

5.86 Find $d_{\min}$ so that the gate opens.

$$W = \left(9.81\text{ m/s}^2\right)(500\text{ kg}) = 4.905\text{ kN}$$

Using the above free-body diagrams of the gate, we have

$$\circlearrowright + \Sigma M_A = 0: \quad \left(\frac{2}{3}\text{m}\right)\left[29.43(d-2)\text{kN}\right]$$

$$+\left(\frac{4}{3}\text{m}\right)\left[29.43(d-0.2679)\text{kN}\right]$$

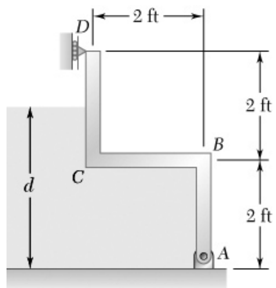
$$-(1.5\text{ m})(10.39\text{ kN}) +$$

$$-(0.5\text{ m})(4.905\text{ kN}) = 0$$

or $$19.62(d-2) + 39.24(d-0.2679) = 18.0375$$

or $$d = 1.15171\text{ m}$$ $d = 1.152\text{ m}$ ◀

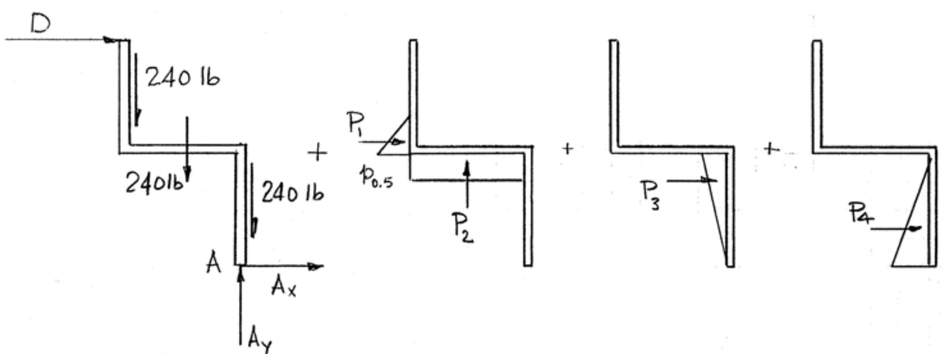
PROBLEMS 5.87 AND 5.88



Problem 5.87: The gate at the end of a 3-ft-wide fresh water channel is fabricated from three 240-lb, rectangular steel plates. The gate is hinged at A and rests against a frictionless support at D. Knowing that $d = 2.5$ ft, determine the reactions at A and D.

Problem 5.88: The gate at the end of a 3-ft-wide fresh water channel is fabricated from three 240-lb, rectangular steel plates. The gate is hinged at A and rests against a frictionless support at D. Determine the depth of water d for which the gate will open.

SOLUTION



5.87 Note that in addition to the weights of the gate segments, the water exerts pressure on all submerged surfaces $(p = \gamma h)$.

Thus, at $$h = 0.5\text{ ft},\ \ p_{0.5} = \left(62.4\text{ lb/ft}^3\right)(0.5)\text{ ft} = 31.2\text{ lb/ft}^2$$

$$h = 2.5\text{ ft},\ \ p_{2.5} = \left(6.24\text{ lb/ft}^3\right)(2.5)\text{ ft} = 156.0\text{ lb/ft}^2$$

Then $$P_1 = \frac{1}{2}\big[(0.5\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 23.4\text{ lb}$$

$$P_2 = \big[(2\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 187.2\text{ lb}$$

$$P_3 = \frac{1}{2}\big[(2\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 93.6\text{ lb}$$

$$P_4 = \frac{1}{2}\big[(2\text{ ft})(3\text{ ft})\big]\left(156\text{ lb/ft}^2\right) = 468\text{ lb}$$

and $$+\circlearrowleft \Sigma M_A = 0\!:\ \ -4D + (2\text{ ft})(240\text{ lb}) + (1\text{ ft})(240\text{ lb}) - \left[\left(2 + \frac{1}{3}\times 0.5\right)\text{ft}\,(23.4\text{ lb})\right] - (1\text{ ft})(187.2\text{ lb})$$

$$-\frac{2}{3}(2\text{ ft})(93.6\text{ lb}) - \frac{1}{3}(2\text{ ft})(468\text{ lb}) = 0$$

or $$D = 11.325\text{ lb}$$ $\therefore\ \ \mathbf{D} = 11.33\text{ lb} \longrightarrow$ ◀