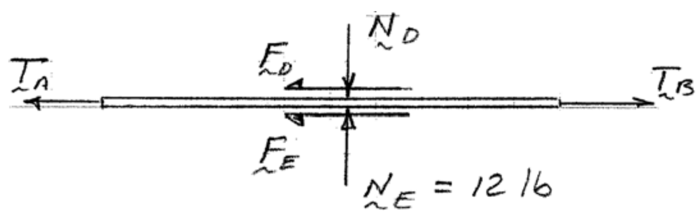
PROBLEM 8.111

A couple $\mathbf{M}_B$ of magnitude $2\text{ lb}\cdot\text{ft}$ is applied to the drive drum *B* of a portable belt sander to maintain the sanding belt *C* at a constant speed. The total downward force exerted on the wooden workpiece *E* is 12 lb, and $\mu_k = 0.10$ between the belt and the sanding platen *D*. Knowing that $\mu_s = 0.35$ between the belt and the drive drum and that the radii of drums *A* and *B* are 1.00 in., determine (*a*) the minimum tension in the lower portion of the belt if no slipping is to occur between the belt and the drive drum, (*b*) the value of the coefficient of kinetic friction between the belt and the workpiece.

SOLUTION

FBD lower portion of belt:



$$\Sigma F_y = 0: \quad N_E - N_D = 0$$

or

$$N_D = N_E = 12 \text{ lb}$$

Slipping:

$$F_D = (\mu_k)_{\text{belt/platen}} N_D$$

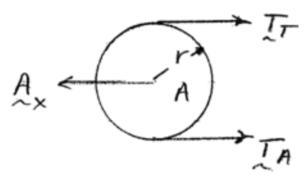
$$F_D = 0.1(12 \text{ lb}) = 1.2 \text{ lb}$$

and

$$F_E = (\mu_k)_{\text{belt/wood}} N_E$$

$$F = (12 \text{ lb})(\mu_k)_{\text{belt/wood}} \qquad (1)$$

FBD drum A: (assumed free to rotate)

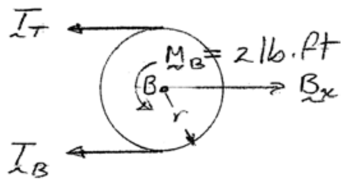


$$\Sigma F_x = 0: \quad T_B - T_A - F_D - F_E = 0 \qquad (2)$$

$$\Sigma M_A = 0: \quad r_A(T_A - T_T) = 0 \qquad \text{or} \qquad T_T = T_A$$

PROBLEM 8.111 CONTINUED

FBD drum B:



$$\circlearrowright \Sigma M_B = 0: \quad M_B + r(T_T - T_B) = 0$$

or $$T_B - T_T = \frac{M_B}{r} = \left(\frac{2\text{ lb}\cdot\text{ft}}{1\text{ in.}}\right)\left(\frac{12\text{ in.}}{\text{ft}}\right) = 24\text{ lb}$$

Impending slipping: $$T_B = T_T e^{\mu_s \beta} = T_T e^{0.35\pi}$$

So $$\left(e^{0.35\pi} - 1\right)T_T = 24\text{ lb} \qquad \text{or} \qquad T_T = 11.983\text{ lb}$$

Now $$T_A = T_T = 11.983\text{ lb} \;\text{ then }\; T_B = (11.983\text{ lb})e^{0.35\pi} = 35.983\text{ lb}$$

From Equation (2): $$35.983\text{ lb} - 11.983\text{ lb} - 1.2\text{ lb} = F_E = 22.8\text{ lb}$$

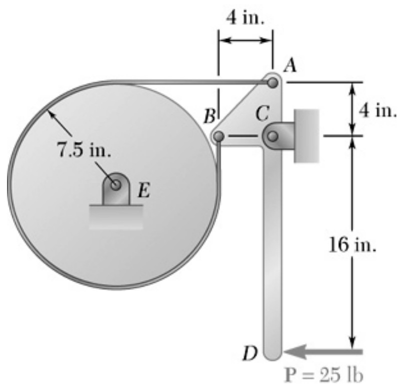
From Equation (1): $$(\mu_k)_{\text{belt/wood}} = \frac{F_E}{12\text{ lb}} = \frac{22.8\text{ lb}}{12\text{ lb}} = 1.900$$

Therefore

$(a)\;\; T_{\min} = T_A = 11.98\text{ lb}$ ◀

$(b)\;\; (\mu_k)_{\text{belt/wood}} = 1.900$ ◀

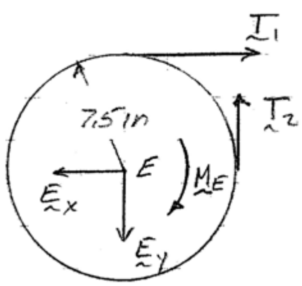
PROBLEM 8.112



A band belt is used to control the speed of a flywheel as shown. Determine the magnitude of the couple being applied to the flywheel knowing that the coefficient of kinetic friction between the belt and the flywheel is 0.25 and that the flywheel is rotating clockwise at a constant speed. Show that the same result is obtained if the flywheel rotates counterclockwise.

SOLUTION

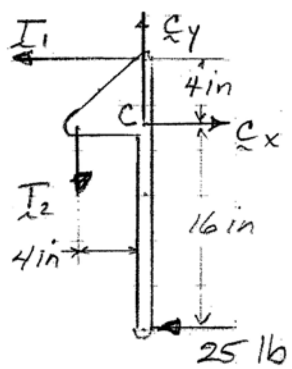
FBD wheel:



$$\circlearrowleft \Sigma M_E = 0: \quad -M_E + (7.5\text{ in.})(T_2 - T_1) = 0$$

or $$M_E = (7.5\text{ in.})(T_2 - T_1)$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (4\text{ in.})(T_1 + T_2) - (16\text{ in.})(25\text{ lb}) = 0$$

or $$T_1 + T_2 = 100\text{ lb}$$

Impending slipping: $$T_2 = T_1 e^{\mu_s \beta}$$

or $$T_2 = T_1 e^{0.25\left(\frac{3\pi}{2}\right)} = 3.2482T_1$$

So $$T_1(1 + 3.2482) = 100\text{ lb}$$

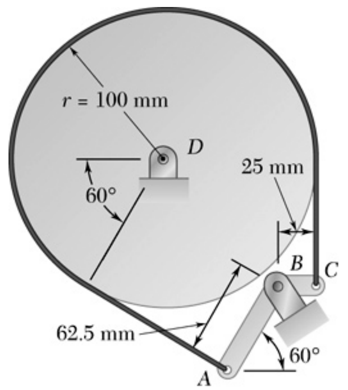
$$T_1 = 23.539\text{ lb}$$

and $$M_E = (7.5\text{ in.})(3.2482 - 1)(23.539\text{ lb}) = 396.9\text{ lb}\cdot\text{in.}$$

$$M_E = 397\text{ lb}\cdot\text{in.} \blacktriangleleft$$

Changing the direction of rotation will change the direction of M_E and will switch the magnitudes of T_1 and T_2.

The magnitude of the couple applied will not change. ◀

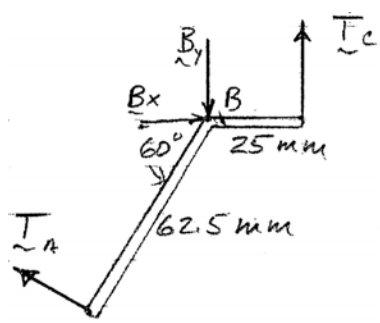


PROBLEM 8.113

The drum brake shown permits clockwise rotation of the drum but prevents rotation in the counterclockwise direction. Knowing that the maximum allowed tension in the belt is 7.2 kN, determine (*a*) the magnitude of the largest counterclockwise couple that can be applied to the drum, (*b*) the smallest value of the coefficient of static friction between the belt and the drum for which the drum will not rotate counterclockwise.

SOLUTION

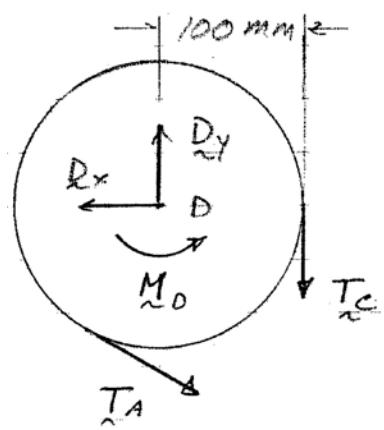
FBD lever:



$$\circlearrowright \Sigma M_B = 0: \quad (25\text{ mm})T_C - (62.5\text{ mm})T_A = 0$$

$$T_C = 2.5T_A$$

FBD lever:



Impending ccw rotation:

(*a*) $$T_C = T_{\max} = 7.2\text{ kN}$$

But $$T_C = 2.5T_A$$

So $$T_A = \frac{7.2\text{ kN}}{2.5} = 2.88\text{ kN}$$

$$\circlearrowright \Sigma M_D = 0: \quad M_D + (100\text{ mm})(T_A - T_C) = 0$$

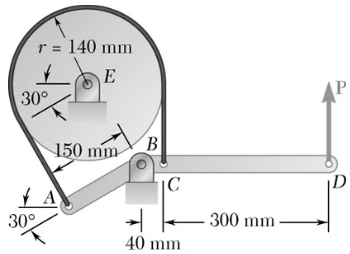
$$M_D = (100\text{ mm})(7.2 - 2.88)\text{ kN}$$

$$M_D = 432\text{ N}\cdot\text{m} \blacktriangleleft$$

(*b*) Also, impending slipping: $$\mu_s\beta = \ln\frac{T_C}{T_A}$$

$$\mu_s = \frac{1}{\beta}\ln\frac{T_C}{T_A} = \frac{1}{\frac{4\pi}{3}}\ln 2.5 = 0.2187$$

Therefore, $$(\mu_s)_{\min} = 0.219 \blacktriangleleft$$

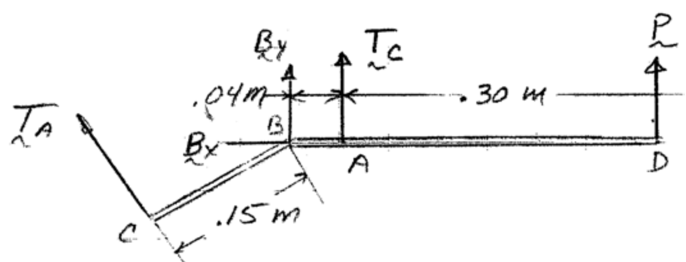


PROBLEM 8.114

A differential band brake is used to control the speed of a drum which rotates at a constant speed. Knowing that the coefficient of kinetic friction between the belt and the drum is 0.30 and that a couple of magnitude is $150\ \text{N}\cdot\text{m}$ applied to the drum, determine the corresponding magnitude of the force **P** that is exerted on end *D* of the lever when the drum is rotating (*a*) clockwise, (*b*) counterclockwise.

SOLUTION

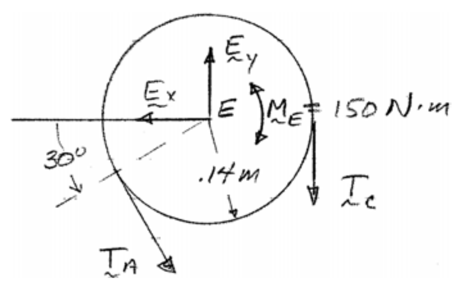
FBD lever:



$$\Sigma M_B = 0: \quad (0.34\ \text{m})P + (0.04\ \text{m})T_C - (0.15\ \text{m})T_A = 0$$

$$P = \frac{15T_A - 4T_C}{34} \qquad (1)$$

FBD drum:



(*a*) For cw rotation, M_E

$$\Sigma M_E = 0: \quad (0.14\ \text{m})(T_A - T_C) - M_E = 0$$

$$T_A - T_C = \frac{150\ \text{N}\cdot\text{m}}{0.14\ \text{m}} = 1071.43\ \text{N}$$

Impending slipping: $\quad T_A = T_C e^{\mu_k \beta} = T_C e^{(0.3)\frac{7\pi}{6}}$

$$T_A = 3.00284 T_C$$

So $\quad (3.00284 - 1)T_C = 1071.43\ \text{N} \quad$ or $\quad T_C = 534.96\ \text{N}$

and $\quad T_A = 1606.39\ \text{N}$

PROBLEM 8.114 CONTINUED

From Equation (1): $$P = \frac{15(1606.39\text{ N}) - 4(534.96\text{ N})}{34}$$

$$P = 646\text{ N} \blacktriangleleft$$

(*b*) For ccw rotation, M_E ↺ and $\Sigma M_E = 0 \Rightarrow T_C - T_A = 1071.43\text{ N}$

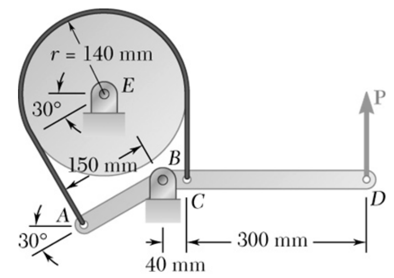
Also, impending slip $\Rightarrow$ $T_C = 3.00284T_A$, so $T_A = 534.96\text{ N}$

and $T_C = 1606.39\text{ N}$

And Equation (1) $\Rightarrow$ $$P = \frac{15(534.96\text{ N}) - 4(1606.39\text{ N})}{34}$$

$$P = 47.0\text{ N} \blacktriangleleft$$

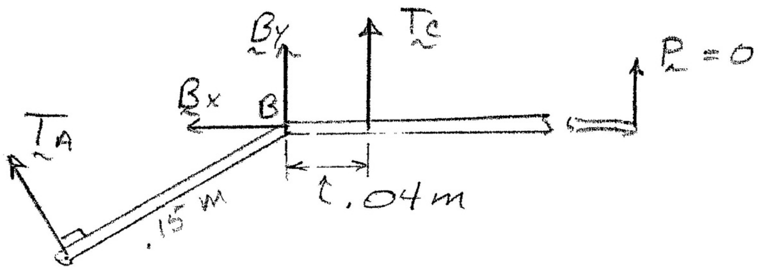
PROBLEM 8.115



A differential band brake is used to control the speed of a drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking when the drum rotates counterclockwise.

SOLUTION

FBD lever:

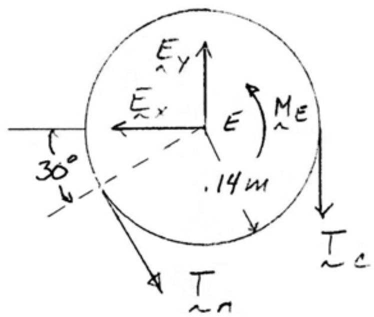


For self-locking $\mathbf{P} = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.04\text{ m})T_C - (0.15\text{ m})T_A = 0$$

$$T_C = 3.75T_A$$

FBD drum:



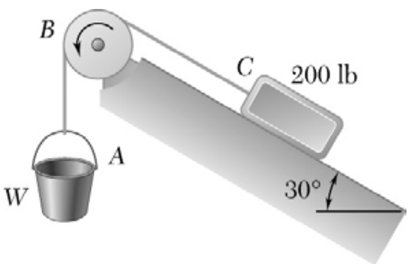
For impending slipping of belt $\quad T_C = T_A e^{\mu_s \beta}$

or $\quad \mu_s \beta = \ln \dfrac{T_C}{T_A}$

Then $\quad \mu_s = \dfrac{1}{\beta} \ln \dfrac{T_C}{T_A} = \dfrac{1}{\frac{7\pi}{6}} \ln 3.75 = 0.3606$

$$(\mu_s)_{\text{req}} = 0.361 \blacktriangleleft$$

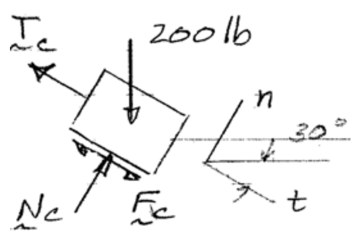
PROBLEM 8.116



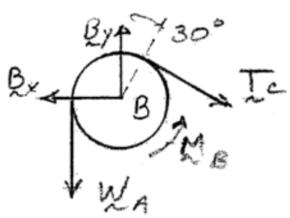
Bucket *A* and block *C* are connected by a cable that passes over drum *B*. Knowing that drum *B* rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined weight *W* of the bucket and its contents for which block *C* will (*a*) remain at rest, (*b*) be about to move up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

FBD block:



FBD drum:



$$\nearrow \Sigma F_n = 0: \quad N_C - (200\text{ lb})\cos 30° = 0; \;\; N = 100\sqrt{3}\text{ lb}$$

$$\nwarrow \Sigma F_t = 0: \quad T_C - (200\text{ lb})\sin 30° \mp F_C = 0$$

$$T_C = 100\text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts $\searrow$

(*a*) For impending motion of block $\searrow$, F_C $\nwarrow$, and

$$F_C = \mu_s N_C = 0.35\left(100\sqrt{3}\text{ lb}\right) = 35\sqrt{3}\text{ lb}$$

So, from Equation (1): $\quad T_C = \left(100 - 35\sqrt{3}\right)\text{ lb}$

But belt slips on drum, so $\quad T_C = W_A e^{\mu_k \beta}$

$$W_A = \left[\left(100 - 35\sqrt{3}\right)\text{lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$$

$W_A = 23.3\text{ lb}$ ◀

(*b*) For impending motion of block $\nwarrow$, F_C $\searrow$ and $F_C = \mu_s N_C = 35\sqrt{3}\text{ lb}$

From Equation (1): $\quad T_C = \left(100 + 35\sqrt{3}\right)\text{ lb}$

Belt still slips, so $\quad W_A = T_C e^{-\mu_k \beta} = \left[\left(100 + 35\sqrt{3}\right)\text{ lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$

$W_A = 95.1\text{ lb}$ ◀

PROBLEM 8.116 CONTINUED

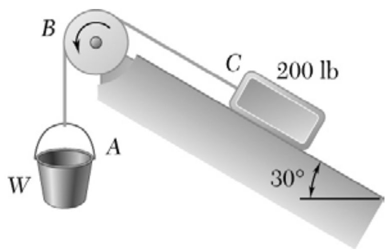
(*c*) For steady motion of block $\nwarrow$, F_C $\searrow$, and $F_C = \mu_k N_C = 25\sqrt{3}$ lb

Then, from Equation (1): $T = \left(100 + 25\sqrt{3}\right)$ lb.

Also, belt is not slipping on drum, so

$$W_A = T_C e^{-\mu_s \beta} = \left[\left(100 + 25\sqrt{3}\right) \text{lb}\right] e^{-0.35\left(\frac{2\pi}{3}\right)}$$

$W_A = 68.8$ lb ◀

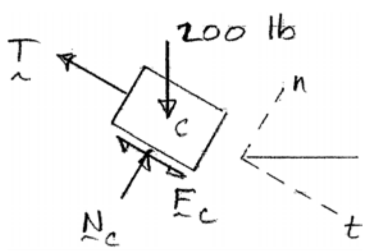


PROBLEM 8.117

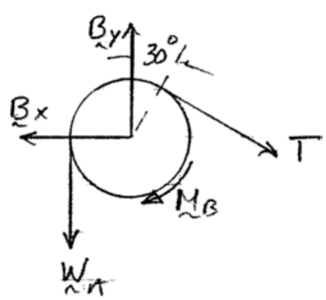
Solve Problem 8.116 assuming that drum B is frozen and cannot rotate.

SOLUTION

FBD block:



FBD drum:



$$\nearrow \Sigma F_n = 0: \quad N_C - (200\text{ lb})\cos 30° = 0; \;\; N_C = 100\sqrt{3}\text{ lb}$$

$$\searrow \Sigma F_t = 0: \quad \pm F_C + (200\text{ lb})\sin 30° - T = 0$$

$$T = 100\text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts $\searrow$

(*a*) For impending motion of block $\searrow$, F_C $\nwarrow$ and $F_C = \mu_s N_C$

So $$F_C = 0.35\left(100\sqrt{3}\text{ lb}\right) = 35\sqrt{3}\text{ lb}$$

and $$T = 100\text{ lb} - 35\sqrt{3}\text{ lb} = 39.375\text{ lb}$$

Also belt slipping is impending so $$T = W_A e^{\mu_s \beta}$$

or $$W_A = Te^{-\mu_s \beta} = (39.378\text{ lb})e^{-0.35\left(\frac{2\pi}{3}\right)}$$

$$W_A = 18.92\text{ lb} \blacktriangleleft$$

(*b*) For impending motion of block $\nwarrow$, F_C $\searrow$, and

$$F_C = \mu_s N_C = 35\sqrt{3}\text{ lb}$$

But $$T = \left(100 + 35\sqrt{3}\right)\text{ lb} = 160.622\text{ lb}.$$

Also belt slipping is impending

So $$W_A = Te^{+\mu_s \beta} = (160.622\text{ lb})e^{0.35\left(\frac{2\pi}{3}\right)};$$

$$W_A = 334\text{ lb} \blacktriangleleft$$

(*c*) For steady motion of block $\nwarrow$, F_C $\searrow$, and $F_C = \mu_k N_C = 25\sqrt{3}\text{ lb}$

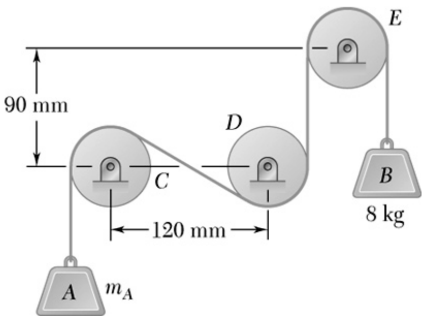
Then $$T = \left(100\text{ lb} + 25\sqrt{3}\text{ lb}\right) = 143.301\text{ lb}.$$

Now belt is slipping

So $$W_A = Te^{\mu_k \beta} = (143.301\text{ lb})e^{0.25\left(\frac{2\pi}{3}\right)}$$

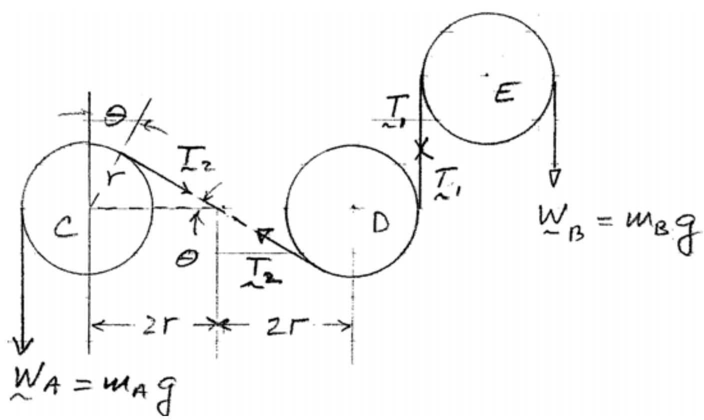
$$W_A = 242\text{ lb} \blacktriangleleft$$

PROBLEM 8.118



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (a) if pulley D is locked, (b) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \beta_D = \frac{2\pi}{3} \qquad \text{and} \qquad \beta_E = \pi$$

(a) All pulleys locked $\Rightarrow$ slipping impends at all surface simultaneously.

If A impends $\uparrow$, $$T_2 = W_A e^{\mu_s\beta_C};\ T_1 = T_2 e^{\mu_s\beta_D};\ W_B = T_1 e^{\mu_s\beta_E}$$

So $$W_B = W_A e^{\mu_s(\beta_C+\beta_D+\beta_E)} \qquad \text{or} \qquad W_A = W_B e^{-\mu_s(\beta_C+\beta_D+\beta_E)}$$

Then $$m_A = m_B e^{-\mu_s(\beta_C+\beta_D+\beta_E)} = (8\text{ kg})e^{-0.2\left(\frac{2\pi}{3}+\frac{2\pi}{3}+\pi\right)} = 1.847\text{ kg}$$

If A impends $\downarrow$, $$W_A = T_2 e^{\mu_s\beta_C} = T_1 e^{\mu_s\beta_D} e^{\mu_s\beta_C} = W_B e^{\mu_s(\beta_E+\beta_D+\beta_C)}$$

So $$m_A = m_B e^{\mu_s(\beta_E+\beta_D+\beta_C)} = (8\text{ kg})e^{0.2\left(\pi+\frac{2\pi}{3}+\frac{2\pi}{3}\right)} = 34.7\text{ kg}$$

Equilibrium for $1.847\text{ kg} \le m_A \le 34.7\text{ kg}$ ◀

PROBLEM 8.118 CONTINUED

(*b*) Pulleys C & E locked, pulley D free $\Rightarrow T_1 = T_2$, other relations remain the same.

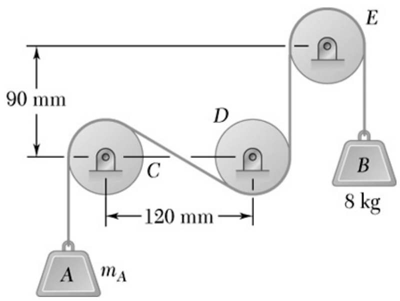
If A impends $\uparrow$, $\quad T_2 = W_A e^{\mu_s \beta_C} = T_1 \qquad W_B = T_1 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$

So $\quad m_A = m_B e^{-\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{-0.2\left(\frac{2\pi}{3}+\pi\right)} = 2.807\text{ kg}$

If A impends $\downarrow$ slipping is reversed, $\quad W_A = W_B e^{+\mu_s(\beta_C + \beta_E)}$

Then $\quad m_A = m_B e^{\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{0.2\left(\frac{5\pi}{3}\right)} = 22.8\text{ kg}$

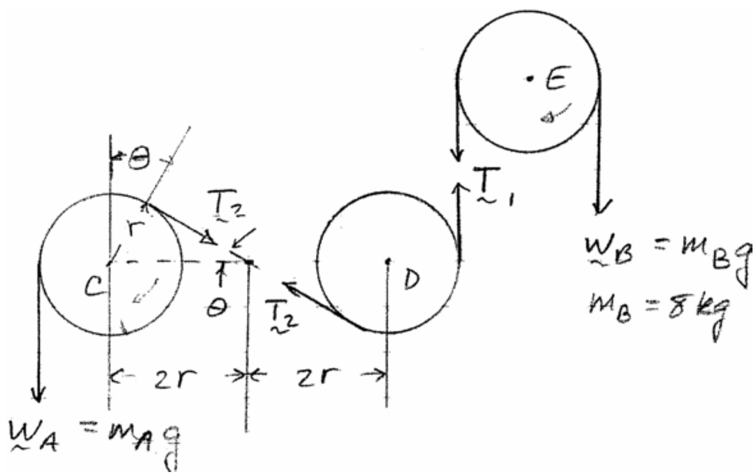
Equilibrium for $2.81\text{ kg} \le m_A \le 22.8\text{ kg}$ ◀



PROBLEM 8.119

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note:
$$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6}\text{ rad}$$

$$\beta_C = \beta_D = \frac{2\pi}{3} \qquad \text{and} \qquad \beta_E = \pi$$

Mass *A* moves up

(*a*) *C* rotates, for maximum W_A have no belt slipping on *C*, so

$$W_A = T_2 e^{\mu_s \beta_C}$$

D and *E* are fixed, so
$$T_1 = T_2 e^{\mu_k \beta_D}$$

and
$$W_B = T_1 e^{\mu_k \beta_E} = T_2 e^{\mu_k(\beta_D + \beta_E)} \Rightarrow T_2 = W_B e^{-\mu_k(\beta_D + \beta_E)}$$

Thus
$$m_A g = m_B g e^{\mu_s \beta_C - \mu_k(\beta_D + \beta_E)} \qquad \text{or} \qquad m_A = (8\text{ kg})e^{\left(\frac{0.4\pi}{3} - 0.1\pi - 0.15\pi\right)}$$

$$m_A = 5.55\text{ kg} \blacktriangleleft$$

PROBLEM 8.119 CONTINUED

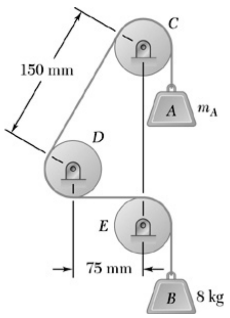
(*b*) E rotates, no belt slip on E, so $$T_1 = W_B e^{\mu_s \beta_E}$$

C and D fixed, so $$T_1 = T_2 e^{\mu_k \beta_D} = W_A e^{\mu_k (\beta_C + \beta_D)}$$

or $$m_A g = T_1 e^{-\mu_k (\beta_C + \beta_D)} = m_B g e^{\mu_s \beta_E - \mu_k (\beta_C + \beta_D)}$$

Then $$m_A = (8 \text{ kg}) e^{(0.2\pi - 0.1\pi - 0.1\pi)} = 8.00 \text{ kg}$$

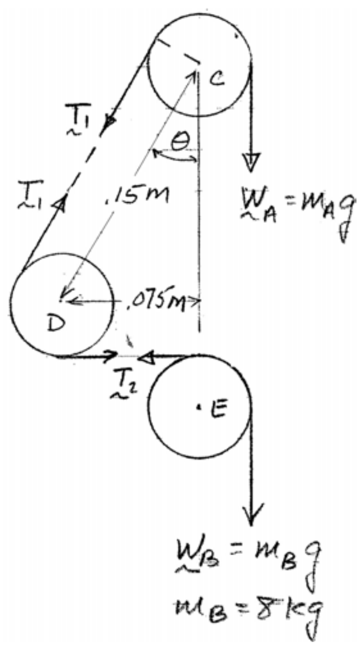
$$m_A = 8.00 \text{ kg} \blacktriangleleft$$



PROBLEM 8.120

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (a) if pulley D is locked, (b) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{0.075\text{ m}}{0.15\text{ m}} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \frac{5}{6}\pi,\ \beta_D = \frac{2}{3}\pi,\ \beta_E = \frac{1}{2}\pi$$

(a) All pulleys locked, slipping at all surfaces.

For m_A impending $\uparrow$, $$T_1 = W_A e^{\mu_s \beta_C},$$

$$T_2 = T_1 e^{\mu_s \beta_D}, \qquad \text{and} \qquad W_B = T_2 e^{\mu_k \beta_E},$$

So $$m_B g = m_A g e^{\mu_s(\beta_C + \beta_D + \beta_E)}$$

$$8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} \qquad \text{or} \qquad m_A = 2.28\text{ kg}$$

For m_A impending down, all tension ratios are inverted, so

$$m_A = (8\text{ kg})e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} = 28.1\text{ kg}$$

Equilibrium for $2.28\text{ kg} \le m_A \le 28.1\text{ kg}$ ◀

(b) Pulleys C and E locked, D free $\Rightarrow T_1 = T_2$, other ratios as in (a)

m_A impending $\uparrow$, $$T_1 = W_A e^{\mu_s \beta_C} = T_2$$

and $$W_B = T_2 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$$

So $$m_B g = m_A g\, e^{\mu(\beta_C + \beta_E)} \qquad \text{or} \qquad 8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$

$$m_A = 3.46\text{ kg}$$

m_A impending $\downarrow$, all tension ratios are inverted, so

$$m_A = 8\text{ kg}\, e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$

$$= 18.49\text{ kg}$$

Equilibrium for $3.46\text{ kg} \le m_A \le 18.49\text{ kg}$ ◀