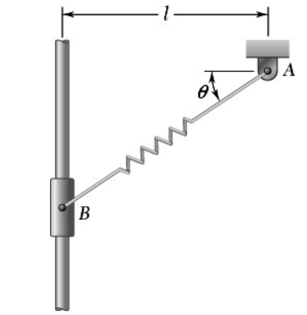
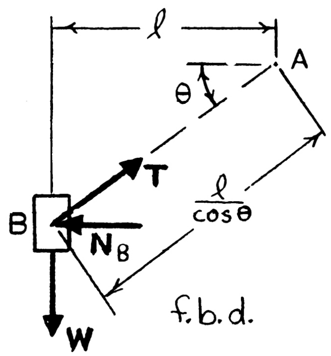
PROBLEM 4.59



A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Derive an equation in θ, W, k, and l which must be satisfied when the collar is in equilibrium. (b) Knowing that $W = 3\text{ lb}$, $l = 6\text{ in.}$, and $k = 8\text{ lb/ft}$, determine the value of θ corresponding to equilibrium.

SOLUTION



First note $\qquad T = ks$

where $\qquad k = \text{spring constant}$

$$s = \text{elongation of spring}$$

$$= \frac{l}{\cos\theta} - l = \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$\therefore \quad T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(a) From f.b.d. of collar B

$$+\uparrow \Sigma F_y = 0: \quad T\sin\theta - W = 0$$

or

$$\frac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0$$

or $\tan\theta - \sin\theta = \dfrac{W}{kl}$ ◀

(b) For $W = 3\text{ lb}$, $l = 6\text{ in.}$, $k = 8\text{ lb/ft}$

$$l = \frac{6\text{ in.}}{12\text{ in./ft}} = 0.5\text{ ft}$$

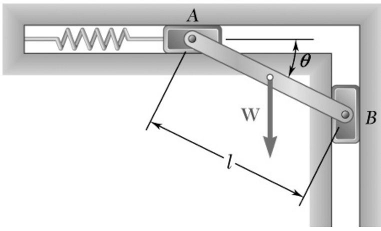
$$\tan\theta - \sin\theta = \frac{3\text{ lb}}{(8\text{ lb/ft})(0.5\text{ ft})} = 0.75$$

Solving Numerically,

$$\theta = 57.957°$$

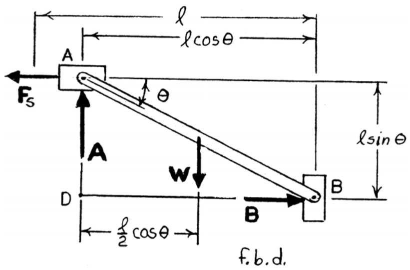
or $\theta = 58.0°$ ◀

PROBLEM 4.60



A slender rod *AB*, of mass *m*, is attached to blocks *A* and *B* which move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when $\theta = 0$. (*a*) Neglecting the mass of the blocks, derive an equation in *m*, *g*, *k*, *l*, and θ which must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ when $m = 2$ kg, $l = 750$ mm, and $k = 30$ N/m.

SOLUTION



First note

$$F_s = \text{spring force} = ks$$

where

$$k = \text{spring constant}$$

$$s = \text{spring deformation}$$

$$= l - l\cos\theta$$

$$= l(1 - \cos\theta)$$

$$\therefore \quad F_s = kl(1 - \cos\theta)$$

(*a*) From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_D = 0: \quad F_s(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(1 - \cos\theta)(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(\sin\theta - \cos\theta\sin\theta) - \left(\frac{W}{2}\right)\cos\theta = 0$$

Dividing by $\cos\theta$

$$kl(\tan\theta - \sin\theta) = \frac{W}{2}$$

$$\therefore \quad \tan\theta - \sin\theta = \frac{W}{2kl}$$

$$\text{or} \quad \tan\theta - \sin\theta = \frac{mg}{2kl} \blacktriangleleft$$

(*b*) For $m = 2$ kg, $l = 750$ mm, $k = 30$ N/m

$$l = 750 \text{ mm} = 0.750 \text{ m}$$

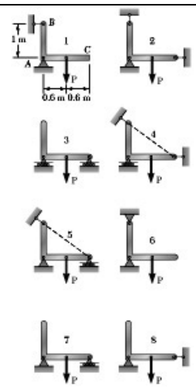
PROBLEM 4.60 CONTINUED

Then $$\tan\theta - \sin\theta = \frac{(2\text{ kg})(9.81\text{ m/s}^2)}{2(30\text{ N/m})(0.750\text{ m})} = 0.436$$

Solving Numerically,

$$\theta = 50.328°$$

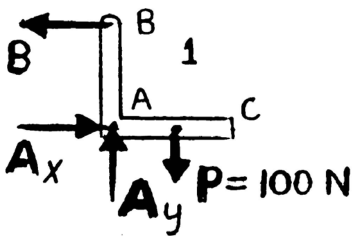
or $\theta = 50.3°$ ◀



PROBLEM 4.61

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket:

$$+\circlearrowleft \Sigma M_A = 0: \quad B(1\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

$$\therefore \quad \mathbf{B} = 60.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 60\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_x = 60.0\text{ N} \longrightarrow$$

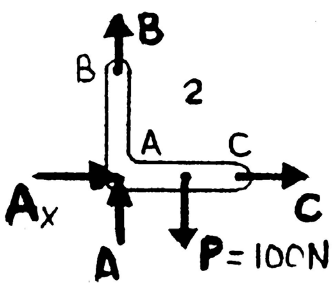
$$+\uparrow \Sigma F_y = 0: \quad A_y - 100\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_y = 100\text{ N} \uparrow$$

Then $\quad A = \sqrt{(60.0)^2 + (100)^2} = 116.619\text{ N}$

and $\quad \theta = \tan^{-1}\left(\dfrac{100}{60.0}\right) = 59.036°$

$$\therefore \quad \mathbf{A} = 116.6\text{ N} \measuredangle 59.0° \blacktriangleleft$$

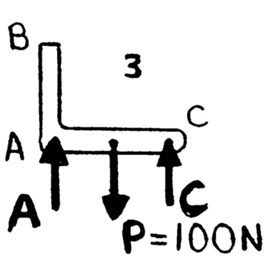


2. Four concurrent reactions through *A*

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀



3. Two reactions

(*a*) Partially constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

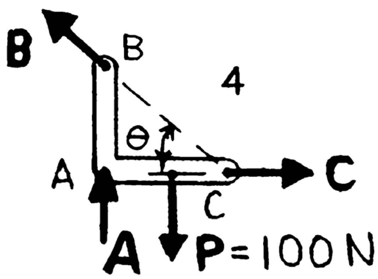
From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad C(1.2 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

$$\therefore \quad \mathbf{C} = 50.0 \text{ N} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A - 100 \text{ N} + 50 \text{ N} = 0$$

$$\therefore \quad \mathbf{A} = 50.0 \text{ N} \uparrow \blacktriangleleft$$



4. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$\theta = \tan^{-1}\left(\frac{1.0}{1.2}\right) = 39.8°$$

$$\overline{BC} = \sqrt{(1.2)^2 + (1.0)^2} = 1.56205 \text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\left(\frac{1.2}{1.56205}\right)B\right](1 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

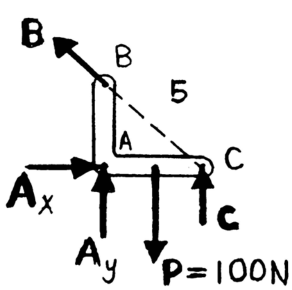
$$\therefore \quad \mathbf{B} = 78.1 \text{ N} \searrow 39.8° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C - (78.102 \text{ N})\cos 39.806° = 0$$

$$\therefore \quad \mathbf{C} = 60.0 \text{ N} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + (78.102 \text{ N})\sin 39.806° - 100 \text{ N} = 0$$

$$\therefore \quad \mathbf{A} = 50.0 \text{ N} \uparrow \blacktriangleleft$$



5. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

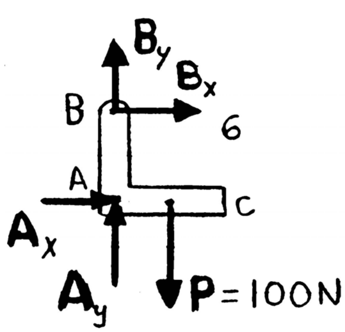
(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_C = 0: \quad (100 \text{ N})(0.6 \text{ m}) - A_y(1.2 \text{ m}) = 0$$

$$\therefore \quad A_y = 50 \text{ N} \qquad \text{or} \quad \mathbf{A}_y = 50.0 \text{ N} \uparrow \blacktriangleleft$$



6. Four non-concurrent non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

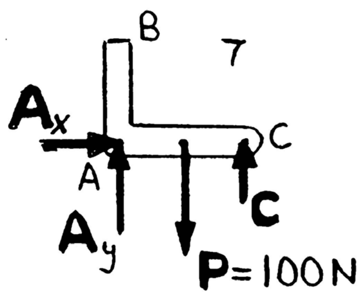
$$\therefore \quad B_x = -60.0\text{ N}$$

or $\mathbf{B}_x = 60.0\text{ N} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -60 + A_x = 0$$

$$\therefore \quad A_x = 60.0\text{ N}$$

or $\mathbf{A}_x = 60.0\text{ N} \longrightarrow$ ◀



7. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(1.2\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

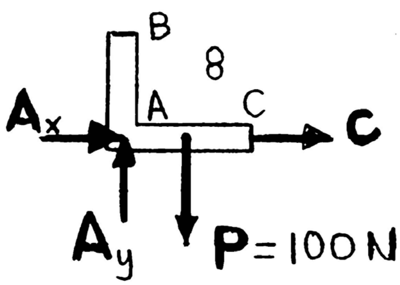
$$\therefore \quad C = 50.0\text{ N}$$

or $\mathbf{C} = 50.0\text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 100\text{ N} + 50.0\text{ N} = 0$$

$$\therefore \quad A_y = 50.0\text{ N}$$

$\therefore \quad \mathbf{A} = 50.0\text{ N} \uparrow$ ◀



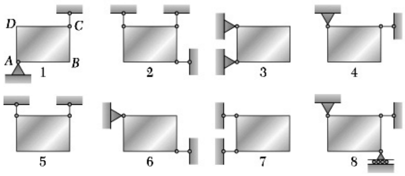
8. Three concurrent, non-parallel reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀

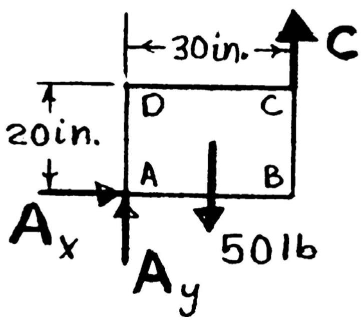
PROBLEM 4.62



Eight identical 20 × 30-in. rectangular plates, each weighing 50 lb, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. For each case, answer the questions listed in Problem 4.61, and, wherever possible, compute the reactions.

P6.1 The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

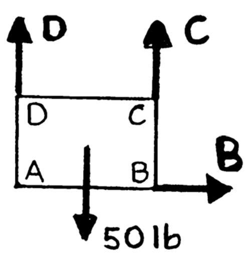
$$+\circlearrowleft \Sigma M_A = 0: \quad C(30\text{ in.}) - 50\text{ lb}(15\text{ in.}) = 0$$

$$\mathbf{C} = 25.0\text{ lb} \uparrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 50\text{ lb} + 25\text{ lb} = 0$$

$$A_y = 25\text{ lb} \qquad \mathbf{A} = 25.0\text{ lb} \uparrow$$ ◀



2. Three non-current, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$\xrightarrow{+} \Sigma F_x = 0: \qquad \mathbf{B} = 0$$ ◀

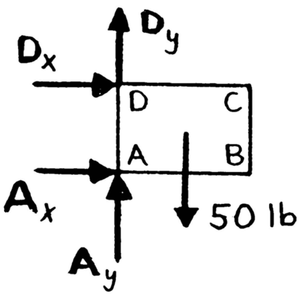
$$+\circlearrowleft \Sigma M_B = 0: \quad (50\text{ lb})(15\text{ in.}) - D(30\text{ in.}) = 0$$

$$\mathbf{D} = 25.0\text{ lb} \uparrow$$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 25.0\text{ lb} - 50\text{ lb} + C = 0$$

$$\mathbf{C} = 25.0\text{ lb} \uparrow$$ ◀

PROBLEM 4.62 CONTINUED



3. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

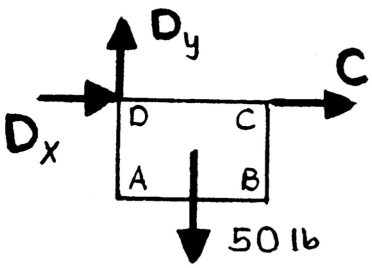
From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad A_x(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.})$$

$$\therefore \quad \mathbf{A}_x = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0$$

$$\therefore \quad \mathbf{D}_x = 37.5 \text{ lb} \longleftarrow \blacktriangleleft$$

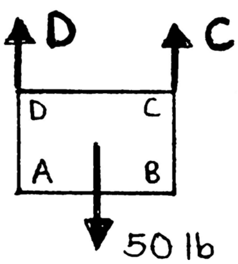


4. Three concurrent reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀



5. Two parallel reactions

(*a*) Partial constraint ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad C(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{C} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D - 50 \text{ lb} + 25 \text{ lb} = 0$$

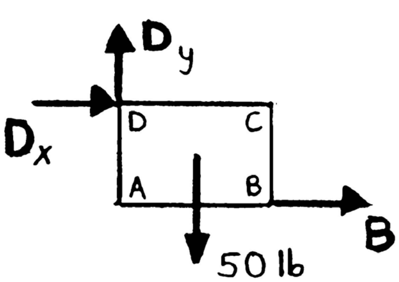
$$\mathbf{D} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

6. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀



From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad B(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

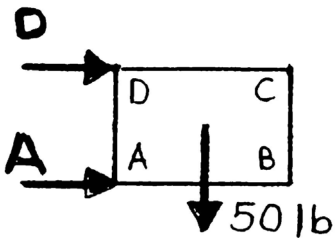
$$\mathbf{B} = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0 \qquad \mathbf{D}_x = 37.5 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} = 0 \qquad \mathbf{D}_y = 50.0 \text{ lb} \uparrow$$

$$\text{or } \mathbf{D} = 62.5 \text{ lb} \searrow 53.1^\circ \blacktriangleleft$$

PROBLEM 4.62 CONTINUED

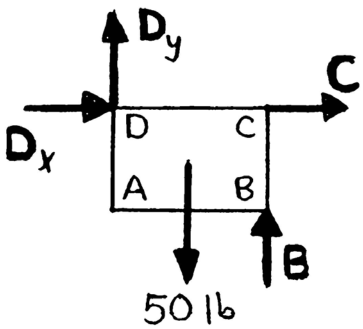


7. Two parallel reactions

(*a*) Improperly constrained ◀

(*b*) Reactions determined by dynamics ◀

(*c*) No equilibrium ◀



8. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$+\circlearrowright \Sigma M_D = 0: \quad B(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{B} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

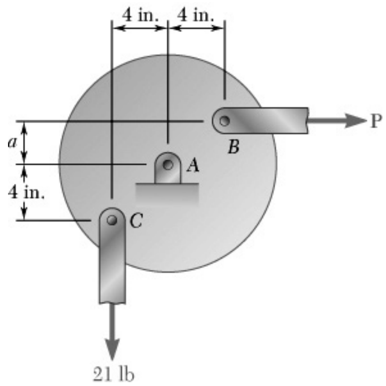
$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} + 25.0 \text{ lb} = 0$$

$$\mathbf{D}_y = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

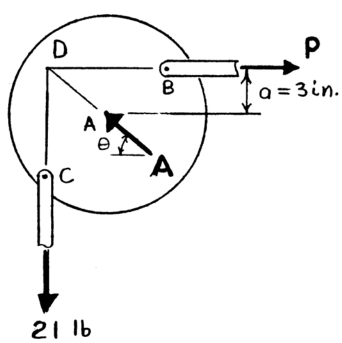
$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + C = 0$$

PROBLEM 4.63

Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Knowing that $a = 3.0$ in., determine the value of P and the reaction at A.



SOLUTION



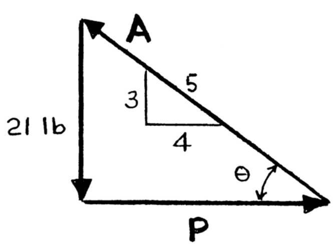
As shown on the f.b.d., the wheel is a three-force body. Let point D be the intersection of the three forces.

From force triangle

$$\frac{A}{5} = \frac{P}{4} = \frac{21\text{ lb}}{3}$$

$$\therefore \quad P = \frac{4}{3}(21\text{ lb}) = 28\text{ lb}$$

or $P = 28.0$ lb ◀



and

$$A = \frac{5}{3}(21\text{ lb}) = 35\text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.870°$$

$\therefore \quad \mathbf{A} = 35.0$ lb $\searrow$ 36.9° ◀