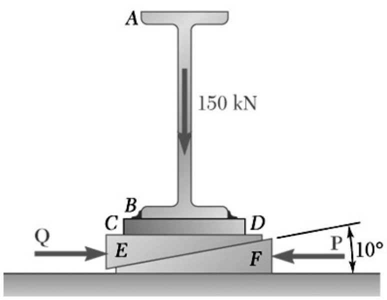
PROBLEM 8.51

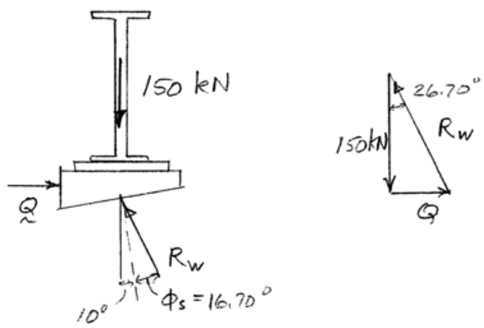


The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

FBD AB + CD + top wedge: Assume top wedge doesn't move

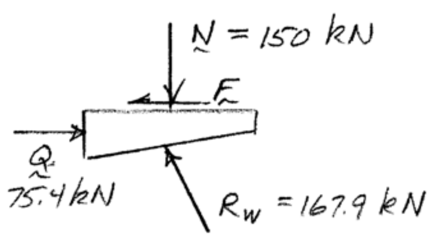


$$R_w = \frac{150 \text{ kN}}{\cos 26.70°} = 167.90 \text{ kN}$$

$$Q = (150 \text{ kN})\tan 26.70° = 75.44 \text{ kN}$$

(*b*) $\mathbf{Q} = 75.4 \text{ kN} \longrightarrow$ ◀

FBD top wedge:

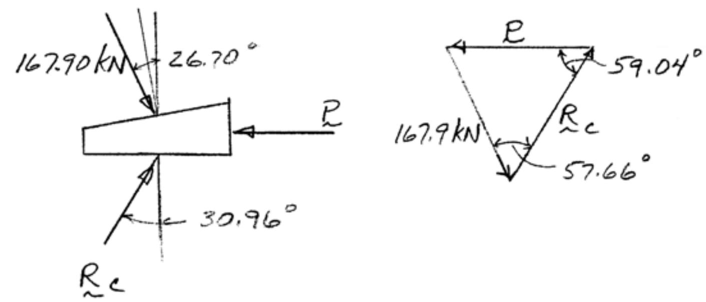


$$\xrightarrow{} \Sigma F_x = 0: \quad 75.44 \text{ kN} - 167.9 \text{ kN} \sin 26.70° - F = 0$$

$$F = 0 \text{ as expected.}$$

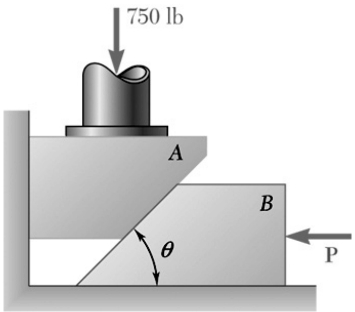
PROBLEM 8.51 CONTINUED

FBD bottom wedge: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 30.96°$ steel on concrete



$$\frac{P}{\sin 57.66°} = \frac{167.90\text{ kN}}{\sin 59.04°}$$

(*a*) $\mathbf{P} = 165.4\text{ kN} \longleftarrow$ ◀



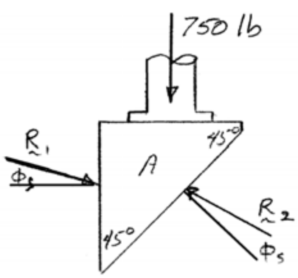
PROBLEM 8.52

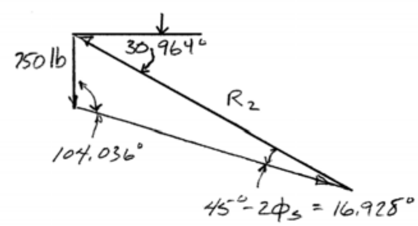
Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

FBD block A:

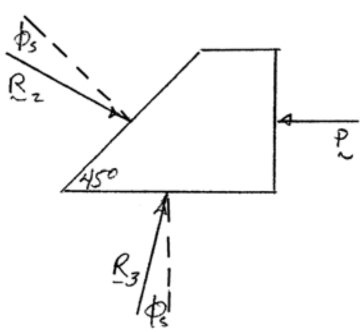


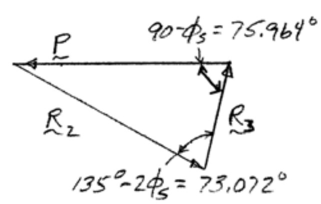


$$\frac{R_2}{\sin 104.036°} = \frac{750 \text{ lb}}{\sin 16.928°}$$

$$R_2 = 2499.0 \text{ lb}$$

FBD wedge B:

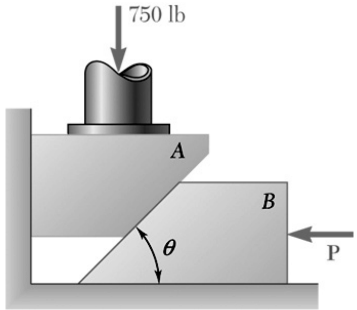




$$\frac{P}{\sin 73.072°} = \frac{2499.0}{\sin 75.964°}$$

$$P = 2464 \text{ lb}$$

$\mathbf{P} = 2.46$ kips ⟵ ◀



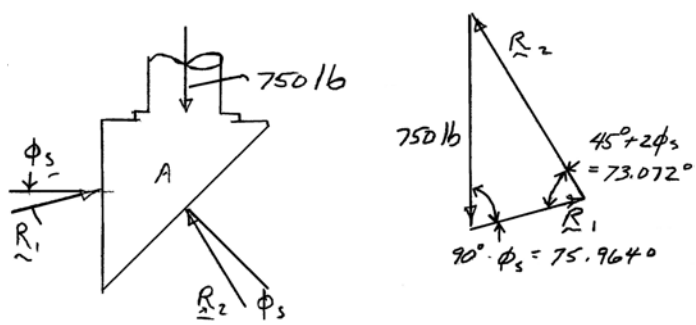
PROBLEM 8.53

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

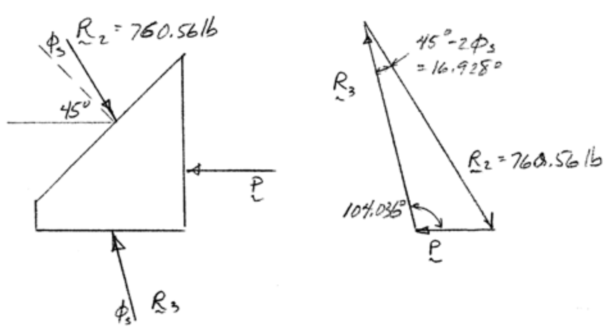
FBD block A:



$$\frac{R_2}{\sin(75.964°)} = \frac{750\text{ lb}}{\sin(73.072°)}$$

$$R_2 = 760.56\text{ lb}$$

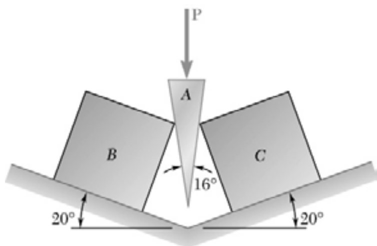
FBD wedge B:



$$\frac{P}{\sin 16.928°} = \frac{760.56}{\sin 104.036°}$$

$$P = 228.3\text{ lb}$$

$\mathbf{P} = 228\text{ lb} \longleftarrow$ ◀



PROBLEM 8.54

A 16° wedge *A* of negligible mass is placed between two 80-kg blocks *B* and *C* which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block *C* and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block *B* and the incline is (*a*) 0.40, (*b*) 0.60.

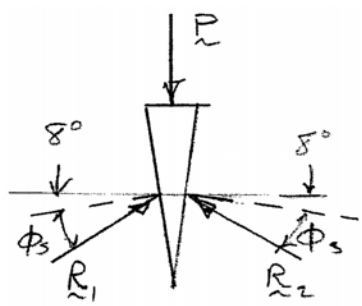
SOLUTION

(*a*)

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.8014°;$$

$$W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$$

FBD wedge:

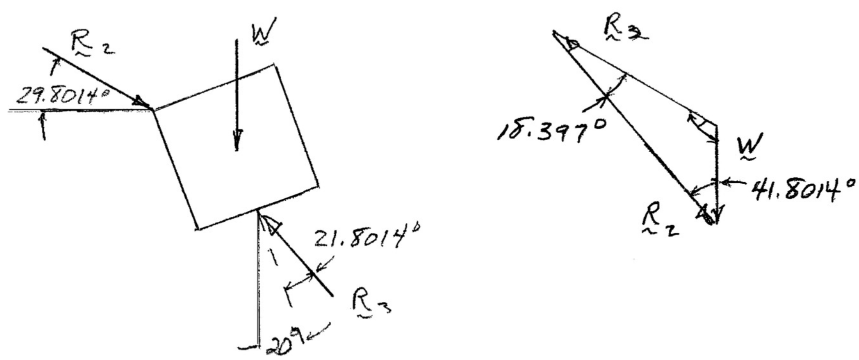


By symmetry: $\mathbf{R}_1 = \mathbf{R}_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_2\sin\left(8° + 21.8014°\right) - P = 0$$

$$P = 0.99400R_2$$

FBD block C:



$$\frac{R_2}{\sin 41.8014°} = \frac{W}{\sin 18.397°}$$

$$R_2 = 2.112\,W$$

PROBLEM 8.54 CONTINUED

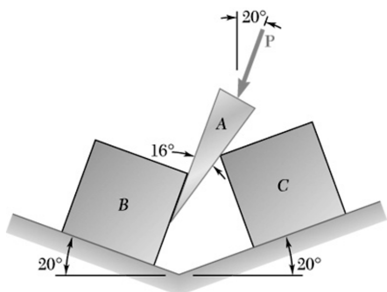
$$P = 0.994R_2 = (0.994)(2.112W)$$

$$P = 2.099(784.8 \text{ N}) = 1647.5 \text{ N}$$

(*a*) $P = 1.648$ kN ◀

(*b*) Note that increasing the friction between block B and the incline has no effect on the above calculations. The physical effect is that slip of B will not impend.

(*b*) $P = 1.648$ kN ◀



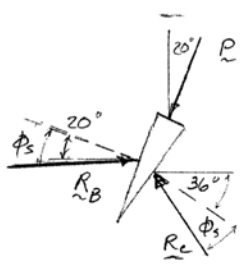
PROBLEM 8.55

A 16° wedge A of negligible mass is placed between two 80-kg blocks B and C which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block C and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block B and the incline is (a) 0.40, (b) 0.60.

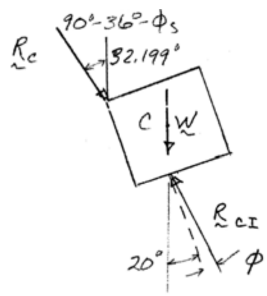
SOLUTION

(a) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$ $\qquad W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

FBD wedge:



FBD block C:

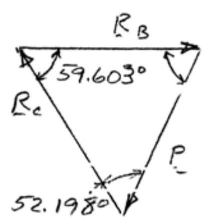


Note that, since $\left(R_{CI}\right)_y > \left(R_C\right)_y$, while the horizontal components are equal,

$$20° + \phi < 32.199°$$

$$\phi < 12.199° < \phi_s$$

Therefore, motion of C is *not* impending; thus, motion of B up the incline is impending.

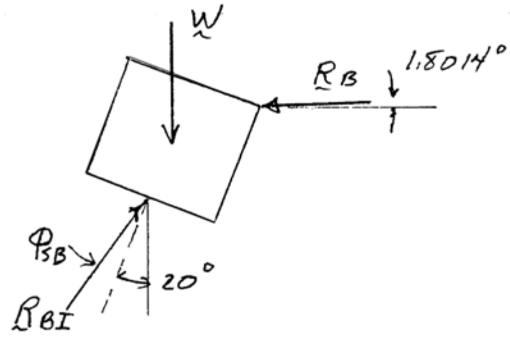


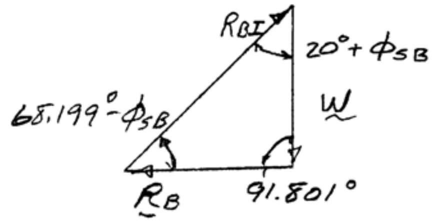
$$\frac{R_B}{\sin 52.198°} = \frac{P}{\sin 59.603°}$$

$$P = 1.0916R_B$$

PROBLEM 8.55 CONTINUED

FBD block B:





$$\frac{R_B}{\sin(20° + \phi_{sB})} = \frac{W}{\sin(68.199° - \phi_{sB})}$$

or
$$R_B = \frac{W\sin(20° + \phi_{sB})}{\sin(68.199° - \phi_{sB})}$$

(*a*) Have $\phi_{sB} = \phi_s = 21.801°$

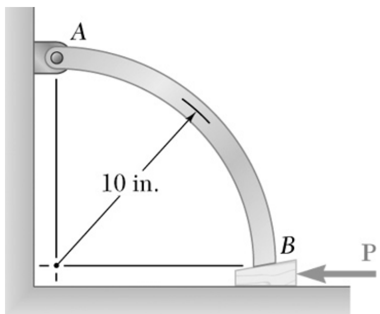
Then
$$R_B = \frac{(784.8 \text{ N})\sin(20° + 21.801°)}{\sin(68.199° - 21.801°)} = 722.37 \text{ N}$$

and
$$P = 1.0916(722.37 \text{ N})$$
or $P = 789 \text{ N}$ ◀

(*b*) Have $\phi_{sB} = \tan^{-1}\mu_{sB} = \tan^{-1}0.6 = 30.964°$

Then
$$R_B = \frac{(784.8 \text{ N})\sin(20° + 30.964°)}{\sin(68.199° - 30.964°)} = 1007.45 \text{ N}$$

and
$$P = 1.0916(1007.45 \text{ N})$$
or $P = 1100 \text{ N}$ ◀

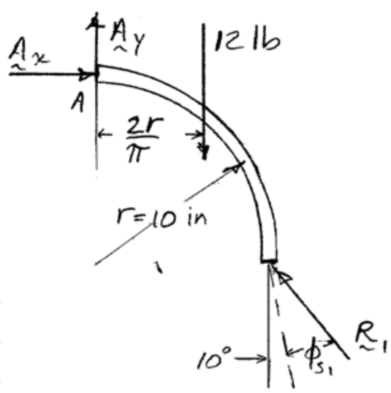


PROBLEM 8.56

A $10°$ wedge is to be forced under end B of the 12-lb rod AB. Knowing that the coefficient of static friction is 0.45 between the wedge and the rod and 0.25 between the wedge and the floor, determine the smallest force **P** required to raise end B of the rod.

SOLUTION

FBD AB:

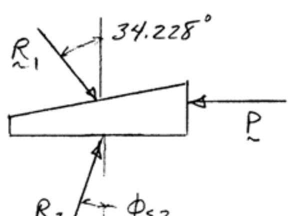


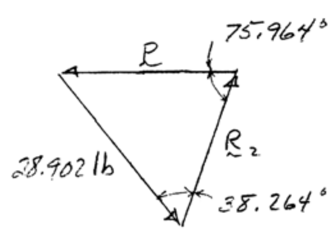
$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.45 = 24.228°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1 \cos(10° + 24.228°) - rR_1 \sin(10° + 24.228°) - \frac{2r}{\pi}(12 \text{ lb}) = 0$$

$$R_1 = 28.902 \text{ lb}$$

FBD wedge:



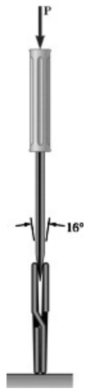


$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.25 = 14.036°$$

$$\frac{P}{\sin(38.264°)} = \frac{28.902 \text{ lb}}{\sin 75.964°};$$

$\mathbf{P} = 22.2 \text{ lb} \longleftarrow$ ◀

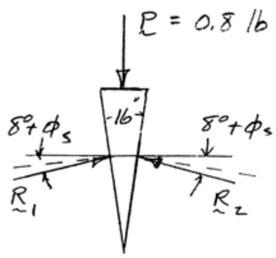
PROBLEM 8.57



A small screwdriver is used to pry apart the two coils of a circular key ring. The wedge angle of the screwdriver blade is 16° and the coefficient of static friction is 0.12 between the coils and the blade. Knowing that a force **P** of magnitude 0.8 lb was required to insert the screwdriver to the equilibrium position shown, determine the magnitude of the forces exerted on the ring by the screwdriver immediately after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(8° + \phi_s) - P = 0$$

Have $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.843°$ $\qquad P = 0.8 \text{ lb}$

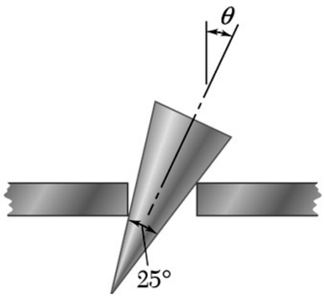
So $R_1 = R_2 = 1.5615 \text{ lb}$

When **P** is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components, $R_1 \cos(14.843°)$, only

Therefore, side forces are 1.509 lb ◀

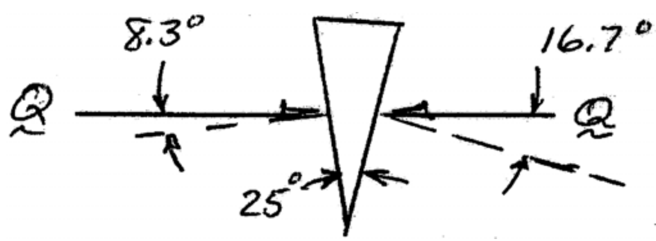
But these will occur only instantaneously as the angle between the force and the wedge normal is $8° > \phi_s = 6.84°$, so the screwdriver will slip out.

PROBLEM 8.58



A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

SOLUTION

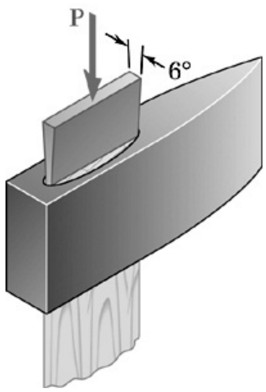


As the plates are moved, the angle θ will decrease.

(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.2 = 11.31°$. As θ decreases, the minimum angle at the contact approaches $12.5° > \phi_s = 11.31°$, so the wedge will slide up and out from the slot. ◀

(*b*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70°$. As θ decreases, the angle at one contact reaches $16.7°$. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$) The wedge binds in the slot. ◀

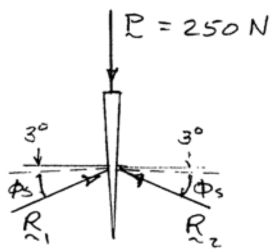
PROBLEM 8.59



A 6° steel wedge is driven into the end of an ax handle to lock the handle to the ax head. The coefficient of static friction between the wedge and the handle is 0.35. Knowing that a force **P** of magnitude 250 N was required to insert the wedge to the equilibrium position shown, determine the magnitude of the forces exerted on the handle by the wedge after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry $R_1 = R_2$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.35 = 19.29°$$

$$\uparrow \Sigma F_y = 0: \quad 2R\sin(19.29° + 3°) - P = 0$$

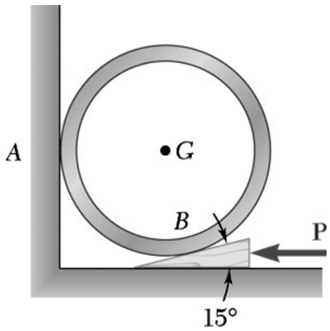
$$R_1 = R_2 = 329.56 \text{ N}$$

When force **P** is removed, the vertical components of R_1 and R_2 vanish, leaving only the horizontal components $H_1 = H_2 = R\cos(22.29°)$

$$H_1 = H_2 = 305 \text{ N} \blacktriangleleft$$

Since the wedge angle $3° < \phi_s = 19.3°,$ the wedge is "self-locking" and will remain seated.

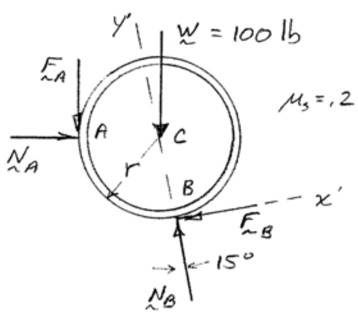
PROBLEM 8.60



A 15° wedge is forced under a 100-lb pipe as shown. The coefficient of static friction at all surfaces is 0.20. Determine (*a*) at which surface slipping of the pipe will first occur, (*b*) the force **P** for which motion of the wedge is impending.

SOLUTION

FBD pipe:



(*a*) $\Sigma M_C = 0: \quad rF_A - rF_B = 0$

or $$F_A = F_B$$

But it is apparent that $N_B > N_A$, so since $(\mu_s)_A = (\mu_s)_B$,
motion must first impend at A ◀

and $$F_B = F_A = \mu_s N_A = 0.2N_A$$

(*b*) $\Sigma M_B = 0: \quad (r\sin 15°)W + r(1+\sin 15°)F_A - (r\cos 15°)N_A = 0$

$$0.2588(100\text{ lb}) + 1.2588(0.2N_A) - 0.9659N_A = 0$$

or $$N_A = 36.24\text{ lb}$$

and $$F_A = 7.25\text{ lb}$$

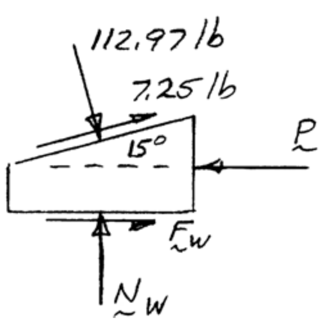
$\Sigma F_{y'} = 0: \quad N_B - N_A\sin 15° - F_A\cos 15° - W\cos 15° = 0$

$$N_B = (36.24\text{ lb})\sin 15° + (7.25\text{ lb} + 100\text{ lb})\cos 15°$$

$$= 112.97\text{ lb}$$

(note $N_B > N_A$ as stated, and $F_B < \mu_s N_B$)

FBD wedge:



$\Sigma F_y = 0: \quad N_W + (7.25\text{ lb})\sin 15° - (112.97\text{ lb})\cos 15° = 0$

$$N_W = 107.24\text{ lb}$$

Impending slip: $\quad F_W = \mu_s N_W = 0.2(107.24) = 21.45\text{ lb}$

$\Sigma F_x = 0: \quad 21.45\text{ lb} + (7.25\text{ lb})\cos 15° + (112.97\text{ lb})\sin 15° - P = 0$

$\mathbf{P} = 57.7\text{ lb} \longleftarrow$ ◀