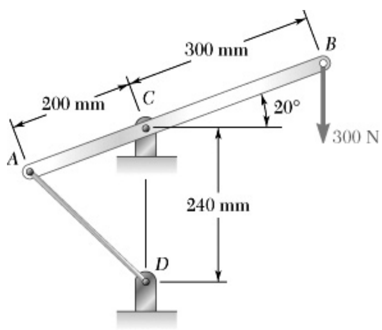
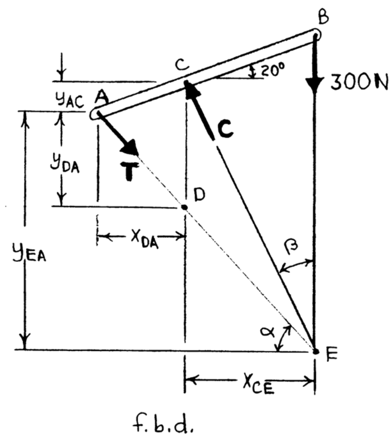
PROBLEM 4.84



Using the method of Section 4.7, solve Problem 4.28.

P4.28 A lever is hinged at *C* and is attached to a control cable at *A*. If the lever is subjected to a 300-N vertical force at *B*, determine
(a) the tension in the cable, (b) the reaction at *C*.

SOLUTION



From geometry of forces acting on lever

$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{DA}}\right)$$

where

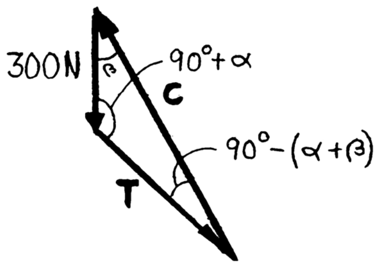
$$y_{DA} = 0.24 \text{ m} - y_{AC} = 0.24 \text{ m} - (0.2 \text{ m})\sin 20°$$

$$= 0.171596 \text{ m}$$

$$x_{DA} = (0.2 \text{ m})\cos 20°$$

$$= 0.187939 \text{ m}$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{0.171596}{0.187939}\right) = 42.397°$$



$$\beta = 90° - \tan^{-1}\left(\frac{y_{AC} + y_{EA}}{x_{CE}}\right)$$

where $\quad x_{CE} = (0.3 \text{ m})\cos 20° = 0.28191 \text{ m}$

$$y_{AC} = (0.2 \text{ m})\sin 20° = 0.068404 \text{ m}$$

$$y_{EA} = (x_{DA} + x_{CE})\tan\alpha$$

$$= (0.187939 + 0.28191)\tan 42.397°$$

$$= 0.42898 \text{ m}$$

$$\therefore \quad \beta = 90° - \tan^{-1}\left(\frac{0.49739}{0.28191}\right) = 29.544°$$

Also, $\quad 90° - (\alpha + \beta) = 90° - 71.941° = 18.0593°$

$$90° + \alpha = 90° + 42.397° = 132.397°$$

PROBLEM 4.84 CONTINUED

Applying the law of sines to the force triangle,

$$\frac{300\text{ N}}{\sin\left[90° - (\alpha + \beta)\right]} = \frac{T}{\sin\beta} = \frac{C}{\sin(90° + \alpha)}$$

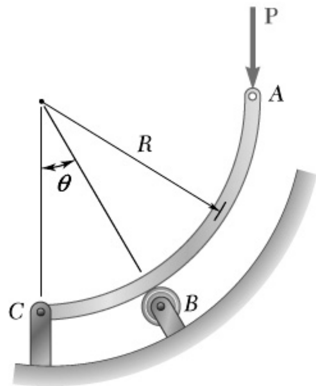
or

$$\frac{300\text{ N}}{\sin 18.0593°} = \frac{T}{\sin 29.544°} = \frac{C}{\sin 132.397°}$$

(*a*) $T = 477.18$ N or $T = 477$ N ◀

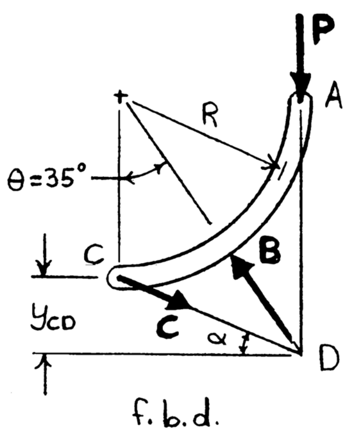
(*b*) $C = 714.67$ N or $\mathbf{C} = 715\text{ N} \measuredangle 60.5°$ ◀

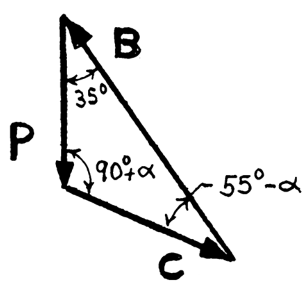
PROBLEM 4.85



Knowing that $\theta = 35^{\circ}$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION





From the geometry of the three forces applied to the member *ABC*

$$\alpha = \tan^{-1}\left(\frac{y_{CD}}{R}\right)$$

where

$$y_{CD} = R\tan 55° - R = 0.42815R$$

$$\therefore \quad \alpha = \tan^{-1}(0.42815) = 23.178°$$

Then $\quad 55° - \alpha = 55° - 23.178° = 31.822°$

$$90° + \alpha = 90° + 23.178° = 113.178°$$

Applying the law of sines to the force triangle,

$$\frac{P}{\sin(55° - \alpha)} = \frac{B}{\sin(90° + \alpha)} = \frac{C}{\sin 35°}$$

or $\quad \dfrac{P}{\sin 31.822°} = \dfrac{B}{\sin 113.178°} = \dfrac{C}{\sin 35°}$

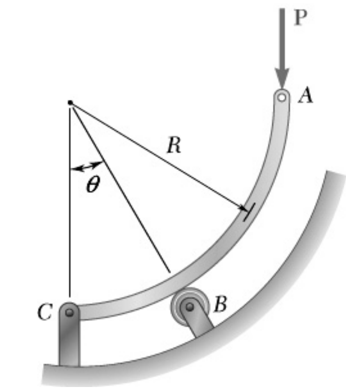
(*a*) $\quad B = 1.74344P$

or $\mathbf{B} = 1.743P$ ⦩ $55.0°$ ◀

(*b*) $\quad C = 1.08780P$

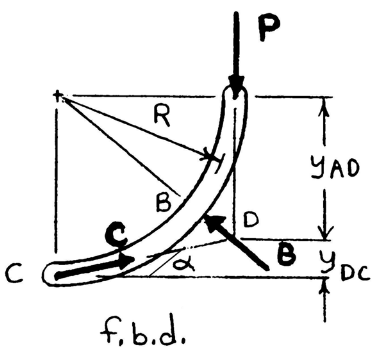
or $\mathbf{C} = 1.088P$ ⦨ $23.2°$ ◀

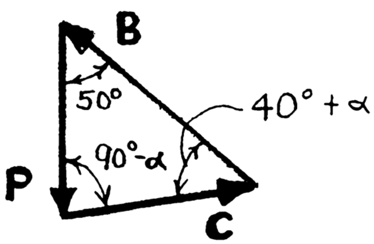
PROBLEM 4.86



Knowing that $\theta = 50^\circ$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION





From the geometry of the three forces acting on member *ABC*

$$\alpha = \tan^{-1}\left(\frac{y_{DC}}{R}\right)$$

where

$$y_{DC} = R - y_{AD} = R\left[1 - \tan\left(90^\circ - 50^\circ\right)\right]$$

$$= 0.160900R$$

$$\therefore \quad \alpha = \tan^{-1}(0.160900) = 9.1406^\circ$$

Then $\qquad 90^\circ - \alpha = 90^\circ - 9.1406^\circ = 80.859^\circ$

$$40^\circ + \alpha = 40^\circ + 9.1406^\circ = 49.141^\circ$$

Applying the law of sines to the force triangle,

$$\frac{P}{\sin(40^\circ + \alpha)} = \frac{B}{\sin(90^\circ - \alpha)} = \frac{C}{\sin 50^\circ}$$

or $\qquad \dfrac{P}{\sin 49.141^\circ} = \dfrac{B}{\sin(80.859^\circ)} = \dfrac{C}{\sin 50^\circ}$

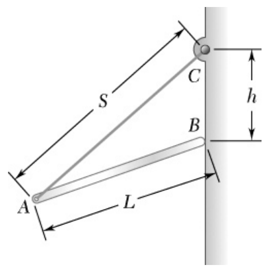
(*a*) $\qquad B = 1.30540P$

or $\mathbf{B} = 1.305P \measuredangle 40.0^\circ$ ◀

(*b*) $\qquad C = 1.01286P$

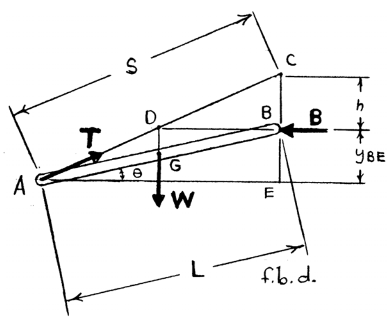
or $\mathbf{C} = 1.013P \measuredangle 9.14^\circ$ ◀

PROBLEM 4.87



A slender rod of length L and weight W is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

SOLUTION



From the f.b.d of the three-force member AB, forces must intersect at D. Since the force T intersects point D, directly above G,

$$y_{BE} = h$$

For triangle ACE:

$$S^2 = (AE)^2 + (2h)^2 \tag{1}$$

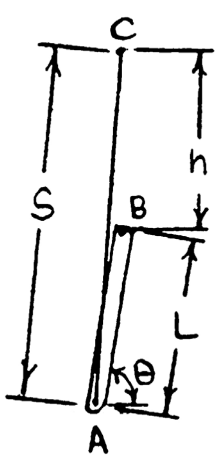
For triangle ABE:

$$L^2 = (AE)^2 + (h)^2 \tag{2}$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2 \tag{3}$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}} \blacktriangleleft$$



As length S increases relative to length L, angle θ increases until rod AB is vertical. At this vertical position:

$$h + L = S \qquad \text{or} \qquad h = S - L$$

Therefore, for all positions of AB $\qquad h \geq S - L \qquad (4)$

or $$\sqrt{\frac{S^2 - L^2}{3}} \geq S - L$$

or $$S^2 - L^2 \geq 3(S - L)^2 = 3(S^2 - 2SL + L^2) = 3S^2 - 6SL + 3L^2$$

or $$0 \geq 2S^2 - 6SL + 4L^2$$

and $$0 \geq S^2 - 3SL + 2L^2 = (S - L)(S - 2L)$$

For $$S - L = 0 \qquad S = L$$

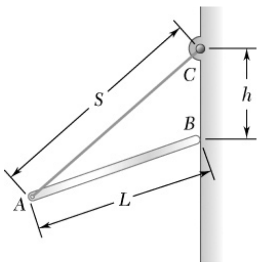
$\therefore$ Minimum value of S is L

For $$S - 2L = 0 \qquad S = 2L$$

$\therefore$ Maximum value of S is $2L$

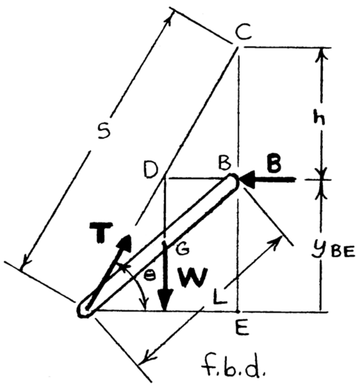
Therefore, equilibrium does not exist if $S > 2L$ ◀

PROBLEM 4.88



A slender rod of length $L = 200$ mm is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 300$ mm. Knowing that the mass of the rod is 1.5 kg, determine (*a*) the distance *h*, (*b*) the tension in the cord, (*c*) the reaction at *B*.

SOLUTION



From the f.b.d of the three-force member *AB*, forces must intersect at *D*. Since the force *T* intersects point *D*, directly above *G*,

$$y_{BE} = h$$

For triangle *ACE*:

$$S^2 = (AE)^2 + (2h)^2 \qquad (1)$$

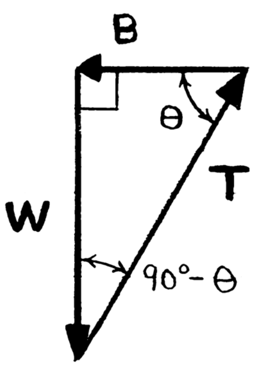
For triangle *ABE*:

$$L^2 = (AE)^2 + (h)^2 \qquad (2)$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}}$$



(*a*) For $L = 200$ mm and $S = 300$ mm

$$h = \sqrt{\frac{(300)^2 - (200)^2}{3}} = 129.099 \text{ mm}$$

or $h = 129.1$ mm ◀

(*b*) Have $W = mg = (1.5\text{ kg})(9.81\text{ m/s}^2) = 14.715\text{ N}$

and $$\theta = \sin^{-1}\left(\frac{2h}{s}\right) = \sin^{-1}\left[\frac{2(129.099)}{300}\right]$$

$$\theta = 59.391°$$

From the force triangle

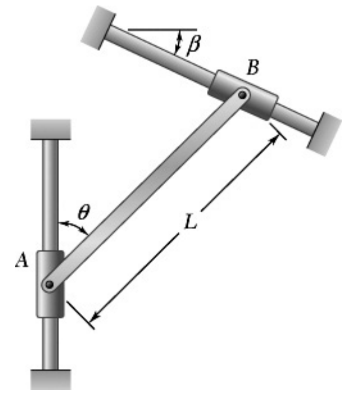
$$T = \frac{W}{\sin\theta} = \frac{14.715\text{ N}}{\sin 59.391°} = 17.0973\text{ N}$$

or $T = 17.10$ N ◀

(*c*) $$B = \frac{W}{\tan\theta} = \frac{14.715\text{ N}}{\tan 59.391°} = 8.7055\text{ N}$$

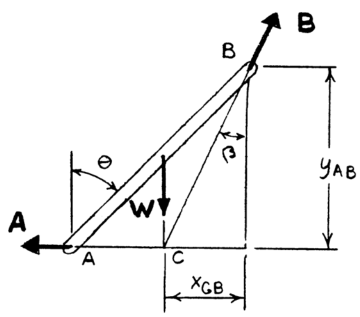
or $\mathbf{B} = 8.71\text{ N} \longleftarrow$ ◀

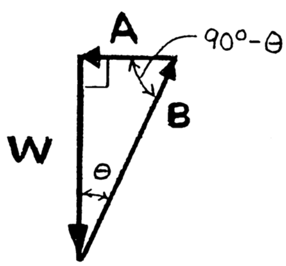
PROBLEM 4.89



A slender rod of length L and weight W is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION





As shown in the f.b.d of the slender rod AB, the three forces intersect at C. From the force geometry

$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

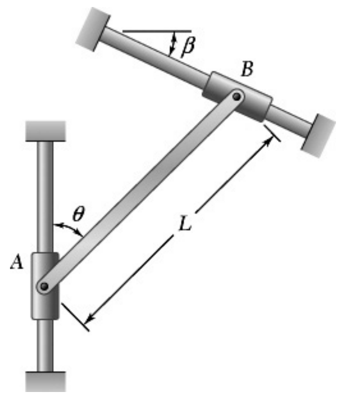
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\therefore \quad \tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta} = \frac{1}{2}\tan\theta$$

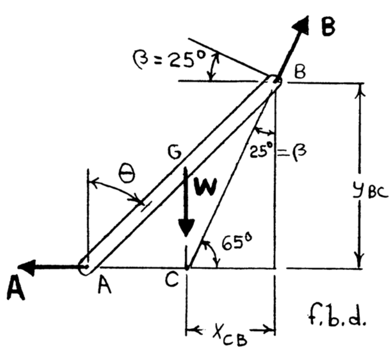
or $\tan\theta = 2\tan\beta$ ◀

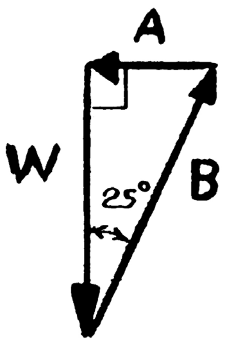
PROBLEM 4.90



A 10-kg slender rod of length L is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 25°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION





(*a*) As shown in the f.b.d. of the slender rod AB, the three forces intersect at C. From the geometry of the forces

$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\therefore \quad \tan\beta = \frac{1}{2}\tan\theta$$

or

$$\tan\theta = 2\tan\beta$$

For

$$\beta = 25°$$

$$\tan\theta = 2\tan 25° = 0.93262$$

$$\therefore \quad \theta = 43.003°$$

or $\theta = 43.0°$ ◀

(*b*)

$$W = mg = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$$

From force triangle

$$A = W\tan\beta$$

$$= (98.1\text{ N})\tan 25°$$

$$= 45.745\text{ N}$$

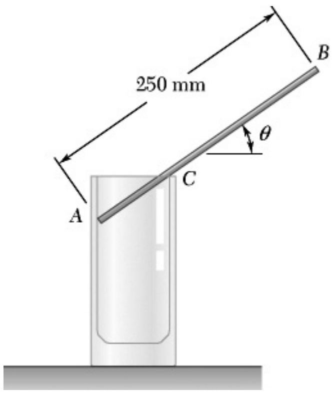
or $\mathbf{A} = 45.7\text{ N} \longleftarrow$ ◀

and

$$B = \frac{W}{\cos\beta} = \frac{98.1\text{ N}}{\cos 25°} = 108.241\text{ N}$$

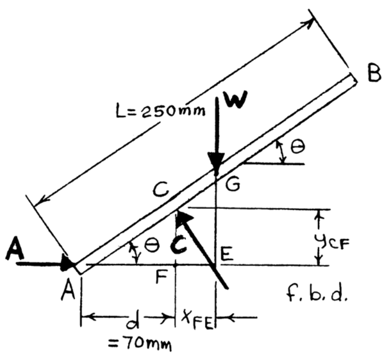
or $\mathbf{B} = 108.2\text{ N} \measuredangle\, 65.0°$ ◀

PROBLEM 4.91



A uniform slender rod of mass 5 g and length 250 mm is balanced on a glass of inner diameter 70 mm. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION



From the geometry of the forces acting on the three-force member *AB*

Triangle *ACF*

$$y_{CF} = d\tan\theta$$

Triangle *CEF* $$x_{FE} = y_{CF}\tan\theta = d\tan^2\theta$$

Triangle *AGE*

$$\cos\theta = \frac{d + x_{FE}}{\left(\frac{L}{2}\right)} = \frac{d + d\tan^2\theta}{\left(\frac{L}{2}\right)}$$

$$= \frac{2d}{L}\left(1 + \tan^2\theta\right)$$

Now $$\left(1 + \tan^2\theta\right) = \sec^2\theta \qquad \text{and} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Then $$\cos\theta = \frac{2d}{L}\sec^2\theta = \frac{2d}{L}\left(\frac{1}{\cos^2\theta}\right)$$

$$\therefore \quad \cos^3\theta = \frac{2d}{L}$$

For $$d = 70 \text{ mm} \quad \text{and} \quad L = 250 \text{ mm}$$

$$\cos^3\theta = \frac{2(70)}{250} = 0.56$$

$$\therefore \quad \cos\theta = 0.82426$$

and $$\theta = 34.487°$$

or $\theta = 34.5°$ ◀