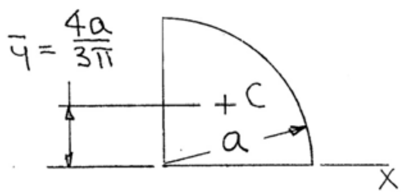
PROBLEM 5.52

Verify that the expressions for the volumes of the first four shapes in Figure 5.21 on page 261 are correct.

SOLUTION

Following the second theorem of Pappus-Guldinus, in each case a specific generating area A will be rotated about the x axis to produce the given shape. Values of $\bar{y}$ are from Fig. 5.8A.

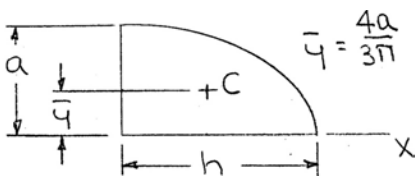
(1) Hemisphere: the generating area is a quarter circle



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}a^2\right)$$

or $V = \frac{2}{3}\pi a^3$ ◀

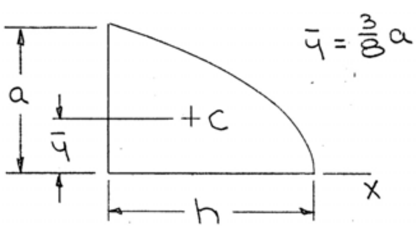
(2) Semiellipsoid of revolution: the generating area is a quarter ellipse



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}ha\right)$$

or $V = \frac{2}{3}\pi a^2 h$ ◀

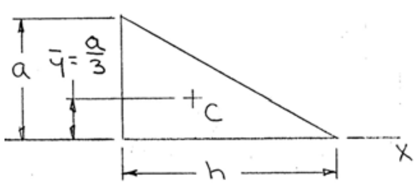
(3) Paraboloid of revolution: the generating area is a quarter parabola



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{3}{8}a\right)\left(\frac{2}{3}ah\right)$$

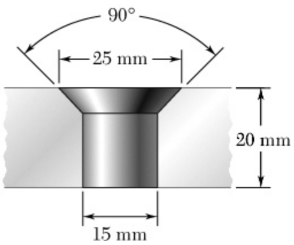
or $V = \frac{1}{2}\pi a^2 h$ ◀

(4) Cone: the generating area is a triangle



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{a}{3}\right)\left(\frac{1}{2}ha\right)$$

or $V = \frac{1}{3}\pi a^2 h$ ◀

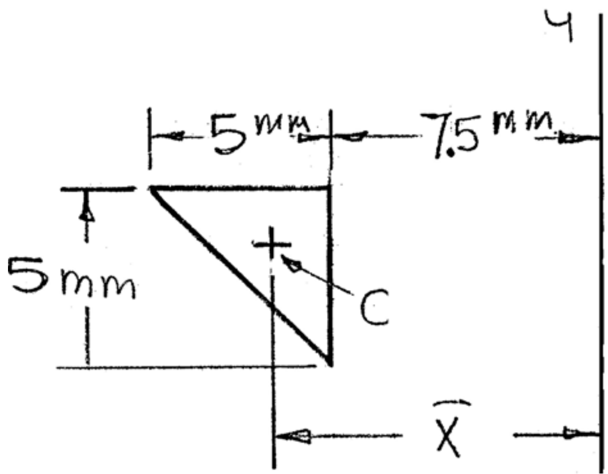


PROBLEM 5.53

A 15-mm-diameter hole is drilled in a piece of 20-mm-thick steel; the hole is then countersunk as shown. Determine the volume of steel removed during the countersinking process.

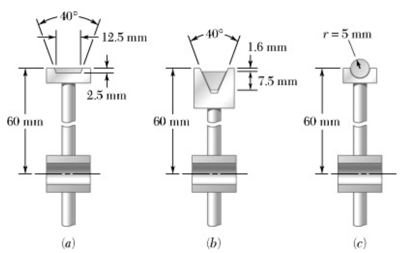
SOLUTION

The required volume can be generated by rotating the area shown about the *y* axis. Applying the second theorem of Pappus-Guldinus, we have



$$V = 2\pi\bar{x}A = 2\pi\left[\left(\frac{5}{3} + 7.5\right)\text{mm}\right] \times \left[\frac{1}{2} \times 5\text{ mm} \times 5\text{ mm}\right]$$

or $V = 720\text{ mm}^3$ ◀



PROBLEM 5.54

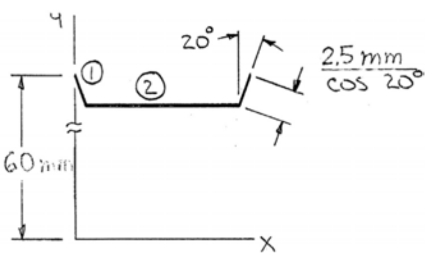
Three different drive belt profiles are to be studied. If at any given time each belt makes contact with one-half of the circumference of its pulley, determine the *contact area* between the belt and the pulley for each design.

SOLUTION

Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by

$$A_C = \pi \bar{y} L = \pi \Sigma \bar{y} L$$

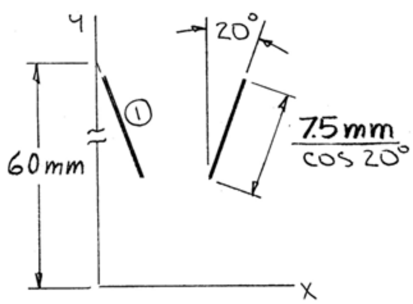
Where the individual lengths are the “Lengths” of the belt cross section that are in contact with the pulley



Have $$A_C = \pi\left[2\left(\bar{y}_1 L_1\right) + \bar{y}_2 L_2\right]$$

$$= \pi\left\{2\left[\left(60 - \frac{2.5}{2}\right)\text{mm}\right]\left[\frac{2.5\text{ mm}}{\cos 20^\circ}\right] + \left[(60 - 2.5)\text{ mm}\right](12.5\text{ mm})\right\}$$

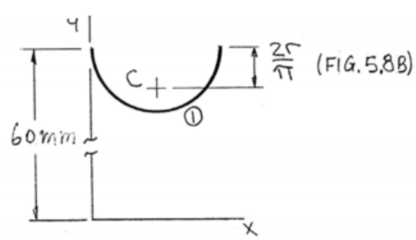
or $A_C = 3.24 \times 10^3\text{ mm}^2$ ◀



Have $$A_C = \pi\left[2\left(\bar{y}_1 L_1\right)\right]$$

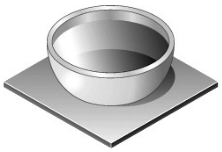
$$= 2\pi\left[\left(60 - 1.6 - \frac{7.5}{2}\right)\text{mm}\right] \times \left(\frac{7.5\text{ mm}}{\cos 20^\circ}\right)$$

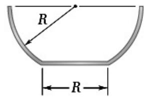
or $A_C = 2.74 \times 10^3\text{ mm}^2$ ◀



Have $$A_C = \pi\left(\bar{y}_1 L_1\right) = \pi\left[\left(60 - \frac{2 \times 5}{\pi}\right)\text{mm}\right](\pi \times 5\text{ mm})$$

or $A_C = 2.80 \times 10^3\text{ mm}^2$ ◀



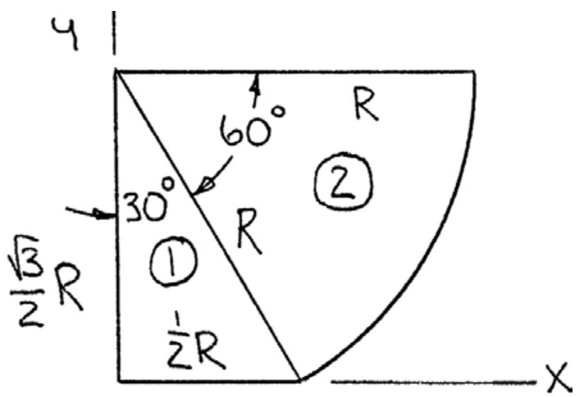


PROBLEM 5.55

Determine the capacity, in gallons, of the punch bowl shown if $R = 12$ in.

SOLUTION

The volume can be generated by rotating the triangle and circular sector shown about the *y* axis. Applying the second theorem of Pappus-Guldinus and using Fig. 5.8A, we have



$$V = 2\pi\bar{x}A = 2\pi\Sigma\bar{x}A = 2\pi\left(\bar{x}_1A_1 + \bar{x}_2A_2\right)$$

$$= 2\pi\left[\left(\frac{1}{3}\times\frac{1}{2}R\right)\left(\frac{1}{2}\times\frac{1}{2}R\times\frac{\sqrt{3}}{2}R\right) + \left(\frac{2R\sin 30^\circ}{3\times\frac{\pi}{6}}\cos 30^\circ\right)\left(\frac{\pi}{6}R^2\right)\right]$$

$$= 2\pi\left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}\right) = \frac{3\sqrt{3}}{8}\pi R^3$$

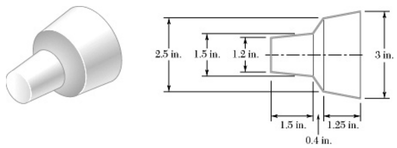
$$= \frac{3\sqrt{3}}{8}\pi(12\text{ in.})^3 = 3526.03\text{ in}^3$$

Since $1\text{ gal} = 231\text{ in}^3$

$$V = \frac{3526.03\text{ in}^3}{231\text{ in}^3/\text{gal}} = 15.26\text{ gal}$$

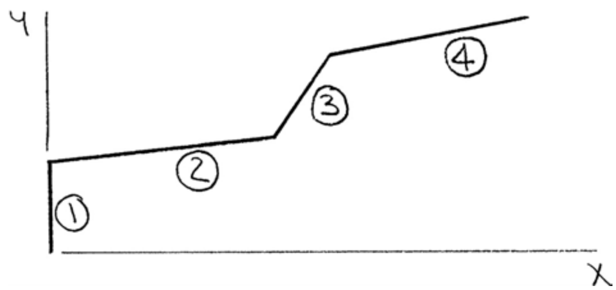
$V = 15.26$ gal ◀

PROBLEM 5.56



The aluminum shade for a small high-intensity lamp has a uniform thickness of 3/32 in. Knowing that the specific weight of aluminum is 0.101 lb/in^3, determine the weight of the shade.

SOLUTION



The weight of the lamp shade is given by

$$W = \gamma V = \gamma A t$$

where A is the surface area of the shade. This area can be generated by rotating the line shown about the x axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \overline{y} L = 2\pi \Sigma \overline{y} L = 2\pi \left(\overline{y}_1 L_1 + \overline{y}_2 L_2 + \overline{y}_3 L_3 + \overline{y}_4 L_4 \right)$$

$$= 2\pi \Bigg[\frac{0.6 \text{ mm}}{2} (0.6 \text{ mm}) + \left(\frac{0.60 + 0.75}{2} \right) \text{mm} \times \sqrt{(0.15 \text{ mm})^2 + (1.5 \text{ mm})^2}$$

$$+ \left(\frac{0.75 + 1.25}{2} \right) \text{mm} \times \sqrt{(0.50 \text{ mm})^2 + (0.40 \text{ mm})^2}$$

$$+ \left(\frac{1.25 + 1.5}{2} \right) \text{mm} \times \sqrt{(0.25 \text{ mm})^2 + (1.25 \text{ mm})^2} \Bigg]$$

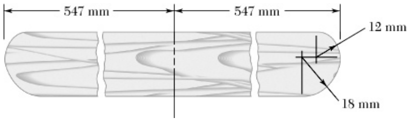
$$= 22.5607 \text{ in}^2$$

Then

$$W = 0.101 \text{ lb/in}^3 \times 22.5607 \text{ in}^2 \times \frac{3}{32} \text{in.} = 0.21362 \text{ lb}$$

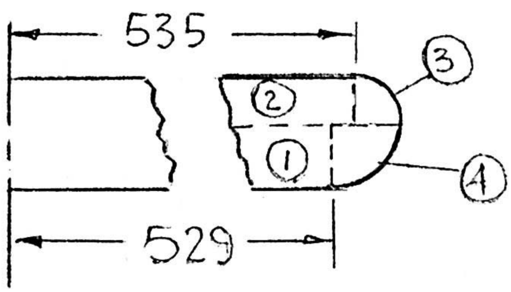
$$W = 0.214 \text{ lb} \blacktriangleleft$$

PROBLEM 5.57



The top of a round wooden table has the edge profile shown. Knowing that the diameter of the top is 1100 mm before shaping and that the density of the wood is $690\,\text{kg/m}^3$, determine the weight of the waste wood resulting from the production of 5000 tops.

SOLUTION



All dimensions are in mm

Have $$V_{\text{waste}} = V_{\text{blank}} - V_{\text{top}}$$

$$V_{\text{blank}} = \pi(550\text{ mm})^2 \times (30\text{ mm}) = 9.075\pi \times 10^6\text{ mm}^3$$

$$V_{\text{top}} = V_1 + V_2 + V_3 + V_4$$

Applying the second theorem of Pappus-Guldinus to parts 3 and 4

$$V_{\text{top}} = \left[\pi(529\text{ mm})^2 \times (18\text{ mm})\right] + \left[\pi(535\text{ mm})^2 \times (12\text{ mm})\right]$$

$$+ 2\pi\left\{\left[\left(535 + \frac{4\times 12}{3\pi}\right)\text{mm}\right] \times \frac{\pi}{4}(12\text{ mm})^2\right\}$$

$$+ 2\pi\left\{\left[\left(529 + \frac{4\times 18}{3\pi}\right)\text{mm}\right] \times \frac{\pi}{4}(18\text{ mm})^2\right\}$$

$$= \pi(5.0371 + 3.347 + 0.1222 + 0.2731) \times 10^6\text{ mm}^3$$

$$= 8.8671\pi \times 10^6\text{ mm}^3$$

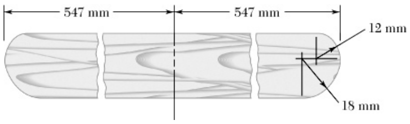
$$\therefore \quad V_{\text{waste}} = (9.0750 - 8.8671)\pi \times 10^6\text{ mm}^3$$

$$= 0.2079\pi \times 10^{-3}\text{ m}^3$$

Finally $$W_{\text{waste}} = \rho_{\text{wood}} V_{\text{waste}}\, g\, N_{\text{tops}}$$

$$= 690\text{ kg/m}^3 \times \left(0.2079\pi \times 10^{-3}\text{ m}^3\right) \times 9.81\text{ m/s}^2 \times 5000\,(\text{tops})$$

or $W_{\text{waste}} = 2.21\text{ kN}$ ◀



PROBLEM 5.58

The top of a round wooden table has the shape shown. Determine how many liters of lacquer are required to finish 5000 tops knowing that each top is given three coats of lacquer and that 1 liter of lacquer covers 12 m^2.

SOLUTION

Referring to the figure in solution of Problem 5.57 and using the first theorem of Pappus-Guldinus, we have

$$A_{\text{surface}} = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{edge}}$$

$$= \left[\pi(535 \text{ mm})^2\right] + \left[\pi(529 \text{ mm})^2\right]$$

$$+ \left\{2\pi\left[\left(535 + \frac{2 \times 12}{\pi}\right) \text{mm}\right] \times \frac{\pi}{2}(12 \text{ mm})\right\}$$

$$+ \left\{2\pi\left[\left(529 + \frac{2 \times 18}{\pi}\right) \text{mm}\right] \times \frac{\pi}{2}(18 \text{ mm})\right\}$$

$$= 617.115\pi \times 10^3 \text{ mm}^2$$

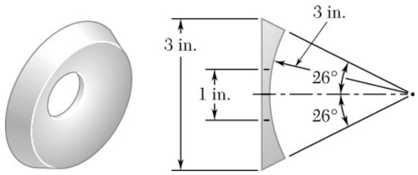
Then

$$\# \text{ liters} = A_{\text{surface}} \times \text{Coverage} \times N_{\text{tops}} \times N_{\text{coats}}$$

$$= 617.115\pi \times 10^{-3} \text{ m}^2 \times \frac{1 \text{ liter}}{12 \text{ m}^2} \times 5000 \times 3$$

or $\# \text{ liters} = 2424 \text{ L}$ ◀

PROBLEM 5.59

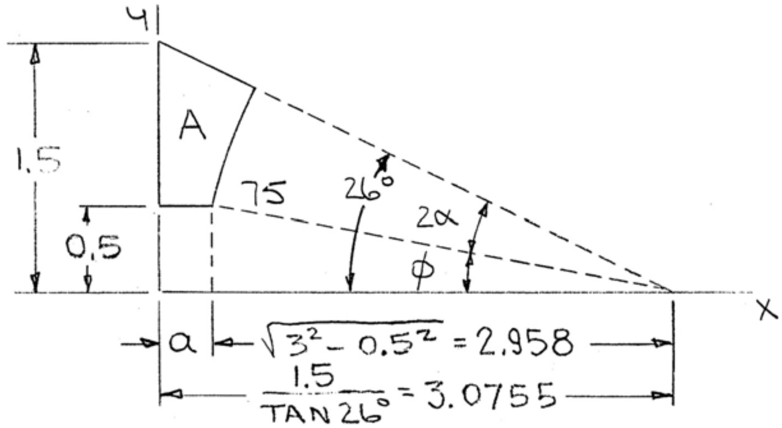


The escutcheon (a decorative plate placed on a pipe where the pipe exits from a wall) shown is cast from yellow brass. Knowing that the specific weight of yellow brass is 0.306 lb/in^3. determine the weight of the escutcheon.

SOLUTION

The weight of the escutcheon is given by $W = (\text{specific weight})V$

where V is the volume of the plate. V can be generated by rotating the area A about the x axis.

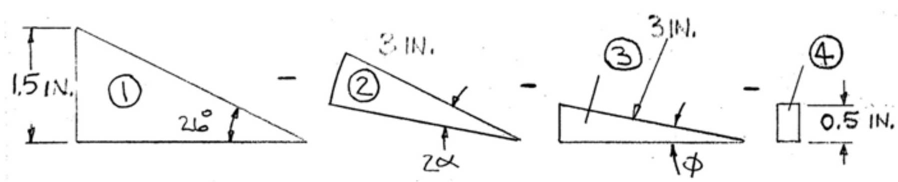


Have $a = 3.0755\text{ in.} - 2.958\text{ in.} = 0.1175\text{ in.}$

and $\sin\phi = \dfrac{0.5}{3} \Rightarrow \phi = 0.16745\text{ R} = 9.5941°$

Then $2\alpha = 26° - 9.5941° = 16.4059°$ or $\alpha = 8.20295° = 0.143169\text{ rad}$

The area A can be obtained by combining the following four areas, as indicated.



Applying the second theorem of Pappus-Guldinus and then using Figure 5.8A, we have

$$V = 2\pi\bar{y}A = 2\pi\Sigma\bar{y}A$$

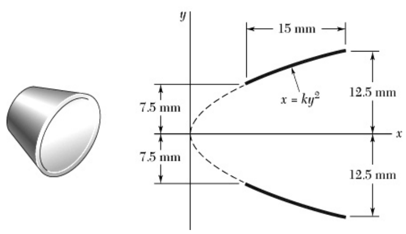
PROBLEM 5.59 CONTINUED

	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$\frac{1}{2}(3.0755)(1.5) = 2.3066$	$\frac{1}{3}(1.5) = 0.5$	1.1533
2	$-\alpha(3)^2 = -1.28851$	$\frac{2(3)\sin\alpha}{3\alpha} \times \sin(\alpha + \phi) = 0.60921$	–0.78497
3	$-\frac{1}{2}(2.958)(0.5) = -0.7395$	$\frac{1}{3}(0.5) = 0.16667$	–0.12325
4	$-(0.1755)(0.5) = -0.05875$	$\frac{1}{2}(0.5) = 0.25$	–0.14688
			$\Sigma\bar{y}A = 0.44296$ in^3

Then $$V = 2\pi\left(0.44296 \text{ in}^3\right) = 1.4476 \text{ in}^3$$

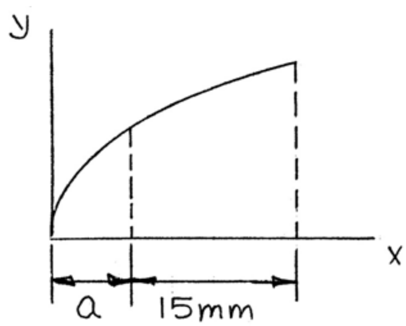
so that $$W = 1.4476 \text{ in}^3\left(0.306 \text{ lb/in}^3\right) = 0.44296 \text{ lb}$$ $W = 0.443$ lb ◀

PROBLEM 5.60



The reflector of a small flashlight has the parabolic shape shown. Determine the surface area of the inside of the reflector.

SOLUTION



First note that the required surface area A can be generated by rotating the parabolic cross section through 2π radians about the x axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \bar{y} L$$

Now, since $\quad x = ky^2, \quad$ at $\quad x = a: a = k(7.5)^2$

or $$a = 56.25\,k \qquad (1)$$

At $\quad x = (a + 15)\,\text{mm}: a + 15 = k(12.5)^2$

or $$a + 15 = 156.25k \qquad (2)$$

Then $$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \frac{a + 15}{a} = \frac{156.25k}{56.25k} \quad \text{or} \quad a = 8.4375\,\text{mm}$$

$$\text{Eq. (1)} \Rightarrow \; k = 0.15\frac{1}{\text{mm}}$$

$$\therefore \; x = 0.15\,y^2 \qquad \text{and} \qquad \frac{dx}{dy} = 0.3y$$

Now $$dL = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}dy = \sqrt{1 + 0.09y^2}\,dy$$

So $$A = 2\pi\bar{y}L \qquad \text{and} \qquad \bar{y}L = \int y\,dL$$

$$\therefore \; A = 2\pi\int_{7.5}^{12.5} y\sqrt{1 + 0.09y^2}\,dy$$

$$= 2\pi\left[\frac{2}{3}\left(\frac{1}{0.18}\right)\left(1 + 0.09y^2\right)^{3/2}\right]_{7.5}^{12.5}$$

$$= 1013\,\text{mm}^2 \qquad \text{or} \quad A = 1013\,\text{mm}^2 \blacktriangleleft$$