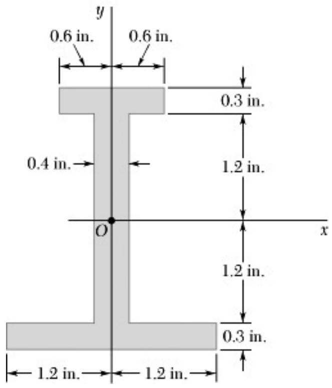
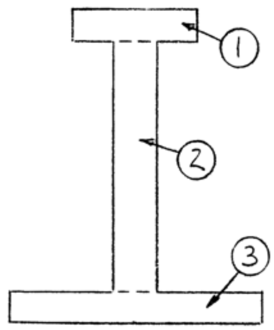
PROBLEM 9.31



Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION



First note that

$$A = A_1 + A_2 + A_3$$
$$= (1.2 \text{ in.})(0.3 \text{ in.}) + (2.4 \text{ in.})(0.4 \text{ in.}) + (2.4 \text{ in.})(0.3 \text{ in.})$$
$$= (0.36 + 0.96 + 0.72)\text{in}^2$$
$$= 2.04 \text{ in}^2$$

Now

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$(I_x)_1 = \frac{1}{12}(1.2 \text{ in.})(0.3 \text{ in.})^3 + (0.36 \text{ in}^2)(1.36 \text{ in.})^2 = 0.6588 \text{ in}^4$$

$$(I_x)_2 = \frac{1}{12}(0.4 \text{ in.})(2.4 \text{ in.})^3 = 0.4608 \text{ in}^4$$

$$(I_x)_3 = \frac{1}{12}(2.4 \text{ in.})(0.3 \text{ in.})^3 + (0.72 \text{ in}^2)(1.35 \text{ in.})^2 = 1.3176 \text{ in}^4$$

Then

$$I_x = 0.6588 \text{ in}^4 + 0.4608 \text{ in}^4 + 1.3176 \text{ in}^4 = 2.4372 \text{ in}^4$$

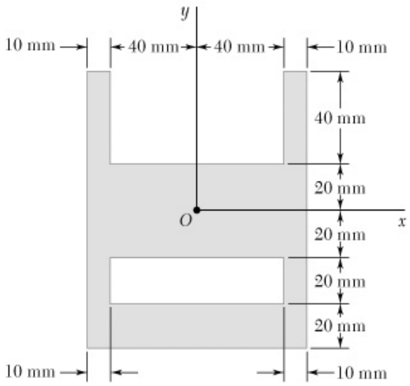
or $I_x = 2.44 \text{ in}^4$ ◀

and

$$k_x^2 = \frac{I_x}{A} = \frac{2.4372 \text{ in}^4}{2.04 \text{ in}^2} = 1.1947 \text{ in}^2$$

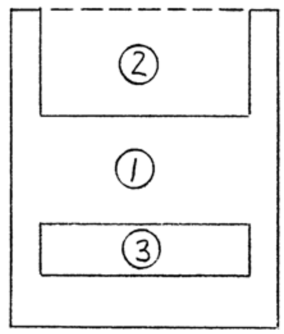
or $k_x = 1.093 \text{ in.}$ ◀

PROBLEM 9.32



Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *x* axis.

SOLUTION



First note that $A = A_1 - A_2 - A_3$

$$= (100 \text{ mm})(120 \text{ mm}) - (80 \text{ mm})(40 \text{ mm}) - (80 \text{ mm})(20 \text{ mm}) = 7200 \text{ mm}^2$$

$$= (12\ 000 - 3200 - 1600)\text{mm}^2 = 7200 \text{ mm}$$

Now $$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(100 \text{ mm})(120 \text{ mm})^3 = 14.4 \times 10^6 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(80 \text{ mm})(40 \text{ mm})^3 + \left(3200 \text{ mm}^2\right)(40 \text{ mm})^2 = 5.5467 \times 10^6 \text{ mm}^4$$

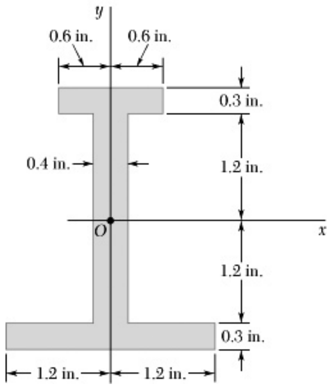
$$(I_x)_3 = \frac{1}{12}(80 \text{ mm})(20 \text{ mm})^2 + \left(1600 \text{ mm}^2\right)(30 \text{ mm})^2 = 1.4933 \times 10^6 \text{ mm}^4$$

Then $$I_x = (14.4 - 5.5467 - 1.4933) \times 10^6 \text{ mm}^4 = 7.36 \times 10^6 \text{ mm}^4$$

or $I_x = 7.36 \times 10^6 \text{ mm}^4$ ◀

and $$k_x^2 = \frac{I_x}{A} = \frac{7.36 \times 10^6}{7200} = 1022.2 \text{ mm}^2$$

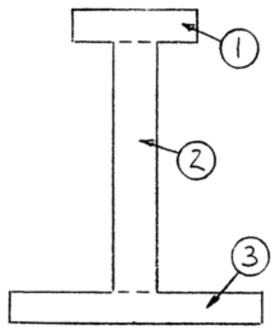
or $k_x = 32.0 \text{ mm}$ ◀



PROBLEM 9.33

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

SOLUTION



First note that

$$A = A_1 + A_2 + A_3$$

$$= (1.2\text{ in.})(0.3\text{ in.}) + (2.4\text{ in.})(0.4\text{ in.}) + (2.4\text{ in.})(0.3\text{ in.})$$

$$= (0.36 + 0.96 + 0.72)\text{ in}^2 = 2.04\text{ in}^2$$

Now

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

Where:

$$(I_y)_1 = \frac{1}{12}(0.3\text{ in.})(1.2\text{ in.})^3 = 0.0432\text{ in}^4$$

$$(I_y)_2 = \frac{1}{12}(2.4\text{ in.})(0.4\text{ in.})^3 = 0.0128\text{ in}^4$$

$$(I_y)_3 = \frac{1}{12}(0.3\text{ in.})(2.4\text{ in.})^3 = 0.3456\text{ in}^4$$

Then

$$I_y = (0.0432 + 0.0128 + 0.3456)\text{ in}^4 = 0.4016\text{ in}^4$$

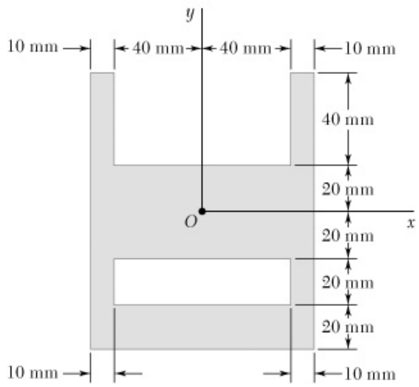
or $I_y = 0.402\text{ in}^4$ ◀

And

$$k_y^2 = \frac{I_y}{A} = \frac{0.4016}{2.04\text{ in}^2} = 0.19686\text{ in}^2$$

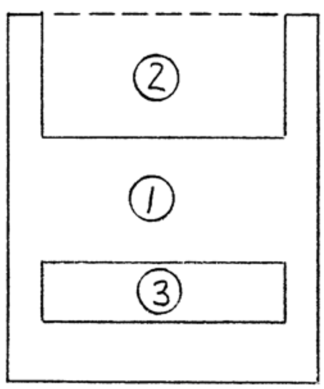
or $k_y = 0.444\text{ in.}$ ◀

PROBLEM 9.34



Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *y* axis.

SOLUTION



First note that $A = A_1 - A_2 - A_3$

$$= (100\text{ mm})(120\text{ mm}) - (80\text{ mm})(40\text{ mm})$$

$$-(80\text{ mm})(20\text{ mm}) = 7200\text{ mm}^2$$

$$= (12\ 000 - 3200 - 1600)\text{mm}^2 = 7200\text{ mm}^2$$

Now $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(120\text{ mm})(100\text{ mm})^3 = 10 \times 10^6\text{ mm}^4$

$$(I_y)_2 = \frac{1}{12}(40\text{ mm})(80\text{ mm})^3 = 1.7067 \times 10^6\text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12}(20\text{ mm})(80\text{ mm})^3 = 0.8533 \times 10^6\text{ mm}^4$$

Then $I_y = (10 - 1.7067 - 0.8533) \times 10^6\text{ mm}^4 = 7.44 \times 10^6\text{ mm}^4$

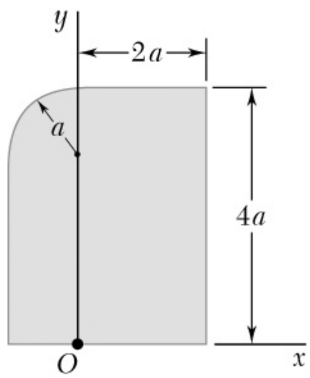
or $I_y = 7.44 \times 10^6\text{ mm}^4$ ◀

And $k_y^2 = \frac{I_y}{A} = \frac{7.44 \times 10^6\text{mm}^4}{7200\text{ mm}^2} = 1033.33\text{ mm}^2$

$$k = 32.14550\text{ mm}$$

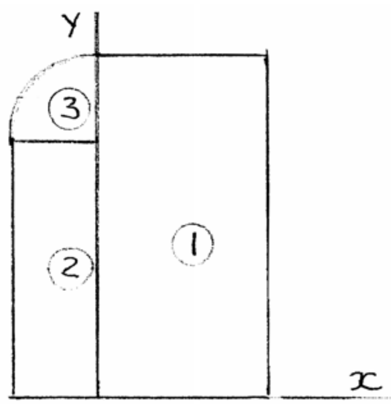
or $k_y = 32.1\text{ mm}$ ◀

PROBLEM 9.35



Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION



Have $I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

$$= \left[\frac{1}{3}(2a)(4a)^3\right] + \left[\frac{1}{3}(a)(3a)^3\right]$$

$$+ \left\{\left[\frac{\pi}{16}a^4 - \frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{4}a^2\left(3a + \frac{4a}{3\pi}\right)^2\right\}$$

$$= \left(\frac{128}{3}a^4\right) + \left(\frac{27}{3}a^4\right) + \left(\frac{\pi}{16} - \frac{4}{9\pi} + \frac{9\pi}{4} + 2 + \frac{4}{9\pi}\right)a^4$$

$$= \left(\frac{161}{3} + \frac{37}{\pi}\right)a^4 = 60.9316a^4$$

or $I_x = 60.9a^4$ ◀

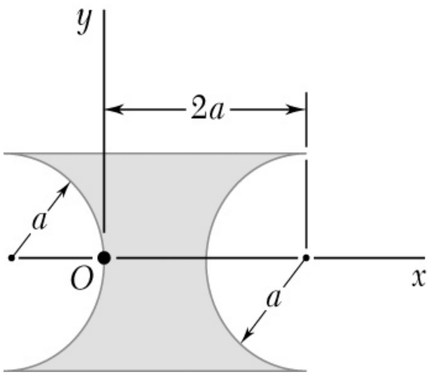
Also $I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

$$= \left[\frac{1}{3}(4a)(2a)^3\right] + \left[\frac{1}{3}(3a)(a)^3\right] + \left[\frac{\pi}{16}a^4\right]$$

$$= \left(\frac{32}{3} + 1 + \frac{\pi}{16}\right)a^4 = 11.8630a^4$$

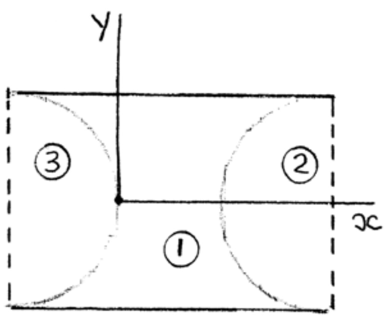
or $I_y = 11.86a^4$ ◀

PROBLEM 9.36



Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION



Have $I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

$$= \left[\frac{1}{12}(3a)(2a)^3\right] - \left[\frac{\pi}{8}a^4\right] - \left[\frac{\pi}{8}a^4\right]$$

$$= \left(2 - \frac{\pi}{8} - \frac{\pi}{8}\right)a^4 = \left(2 - \frac{\pi}{4}\right)a^4$$

or $I_x = 1.215a^4$ ◀

Also $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left[\frac{1}{12}(2a)(3a)^3 + (3a)(2a)\left(\frac{a}{2}\right)^2\right]$$

$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(2a - \frac{4a}{3\pi}\right)^2\right\}$$

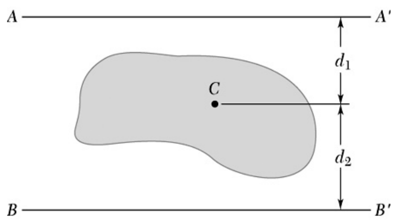
$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(a - \frac{4a}{3\pi}\right)^2\right\}$$

$$= \left(\frac{9}{2} + \frac{3}{2}\right)a^4 - \left(\frac{\pi}{8} - \frac{8}{9\pi} + 2\pi - \frac{8}{3} + \frac{8}{9\pi}\right)a^4$$

$$-\left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} - \frac{4}{3} + \frac{8}{9\pi}\right)a^4 = \left(10 - \frac{11\pi}{4}\right)a^4$$

$$= 1.3606a^4$$

or $I_y = 1.361a^4$ ◀



PROBLEM 9.37

For the 6-in^2 shaded area shown, determine the distance d_2 and the moment of inertia with respect to the centroidal axis parallel to AA' knowing that the moments of inertia with respect to AA' and BB' are 30 in^4 and 58 in^4, respectively, and that $d_1 = 1.25$ in.

SOLUTION

Have
$$I_{AA'} = \bar{I} + Ad_1^2$$

and
$$I_{BB'} = \bar{I} + Ad_2^2$$

subtracting
$$I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$$

or
$$(30 - 58)\text{ in}^4 = \left(6\text{ in}^2\right)\left[(1.25\text{ in.})^2 - d_2^2\right]$$

Solve for d_2

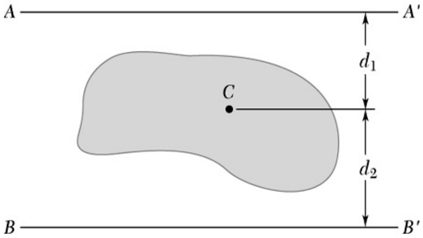
$$d_2^2 = \left(1.25^2 + 4.6667\right)\text{in}^2 = 6.2292\text{ in}^2$$

Then
$$d_2 = 2.4958\text{ in.}$$

or $d_2 = 2.50$ in. ◀

and
$$\bar{I} = I_{AA'} - Ad_1^2 = 30\text{ in}^4 - \left(6\text{ in}^2\right)(1.25\text{ in.})^2 = 20.625\text{ in}^4$$

or $\bar{I} = 20.6\text{ in}^4$ ◀



PROBLEM 9.38

Determine for the shaded region the area and the moment of inertia with respect to the centroidal axis parallel to BB' knowing that $d_1 = 1.25$ in. and $d_2 = 0.75$ in. and that the moments of inertia with respect to AA' and BB' are 20 in^4 and 15 in^4, respectively.

SOLUTION

Have
$$I_{AA'} = \bar{I} + Ad_1^2$$

and
$$I_{BB'} = \bar{I} + Ad_2^2$$

subtracting
$$I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$$

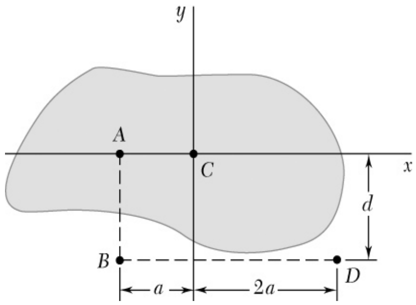
$$20 \text{ in}^4 - 15 \text{ in}^4 = A\left[(1.25)^2 - (0.75)^2\right] \text{in}^2$$

$$5 \text{ in}^4 = A[1.5625 - 0.5625] \text{ in}^2$$

or $A = 5 \text{ in}^2$ ◀

and
$$\bar{I} = I_{AA'} - Ad^2 = 20 \text{ in}^4 - \left(5 \text{ in}^2\right)(1.25 \text{ in.})^2 = 12.1875 \text{ in}^4$$

or $\bar{I} = 12.19 \text{ in}^4$ ◀



PROBLEM 9.39

The centroidal polar moment of inertia $\bar{J}_C$ of the 15.5×10^3 mm^2 shaded region is 250×10^6 mm^4. Determine the polar moments of inertia J_B and J_D of the shaded region knowing that $J_D = 2J_B$ and $d = 100$ mm.

SOLUTION

Have $$J_B = \bar{J}_C + Ad_{CB}^2$$

and $$J_D = \bar{J}_C + Ad_{CD}^2$$

Now $$J_D = 2J_B$$

Then $$\bar{J}_C + Ad_{CD}^2 = 2\left(\bar{J}_C + Ad_{CB}^2\right)$$

Now $$d_{CB}^2 = a^2 + d^2 \qquad \text{and} \qquad d_{CD}^2 = (2a)^2 + d^2$$

Substituting $$A\left(4a^2 + d^2\right) = \bar{J}_C + 2A\left(a^2 + d^2\right)$$

or $$a^2 = \frac{1}{2}\left(\frac{\bar{J}_C}{A} + d^2\right)$$

$$= \frac{1}{2}\left[\frac{250 \times 10^6 \text{ mm}^4}{15.5 \times 10^3 \text{ mm}^2} + (100 \text{ mm})^2\right] = 13064.5 \text{ mm}^2$$

or $$a = 114.300 \text{ mm}$$

Then $$J_B = 250 \times 10^6 \text{ mm}^4 + \left(15.5 \times 10^3 \text{ mm}^2\right)\left[(114.300 \text{ mm})^2 + (100 \text{ mm})^2\right]$$

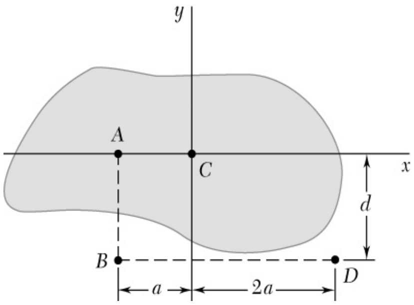
$$= \left(250 \times 10^6 + 357.5 \times 10^6\right) \text{mm}^4 = 607.5 \times 10^6 \text{ mm}^4$$

or $J_B = 608 \times 10^6$ mm^4 ◀

And $$J_D = 250 \times 10^6 \text{ mm}^4 + \left(15.5 \times 10^3 \text{ mm}^2\right)\left[(228.60 \text{ mm})^2 + (100 \text{ mm})^2\right]$$

$$= \left(250 \times 10^6 + 964.99 \times 10\right) \text{mm}^4 = 1214.99 \times 10^6 \text{ mm}^4$$

or $J_D = 1215 \times 10^6$ mm^4 ◀



PROBLEM 9.40

Determine the centroidal polar moment of inertia $\bar{J}_C$ of the 10×10^3 mm^2 shaded area knowing that the polar moments of inertia of the area with respect to points *A*, *B*, and *D* are $J_A = 45 \times 10^6$ mm^4, $J_B = 130 \times 10^6$ mm^4, and $J_D = 252 \times 10^6$ mm^4, respectively.

SOLUTION

Have $\quad J_A = \bar{J}_C + Ad_{CA}^2 \quad$ where $\quad d_{CA}^2 = a^2$

Then $$J_A = \bar{J}_C + Aa^2 \qquad (1)$$

Have $\quad J_B = \bar{J}_C + Ad_{CB}^2 \quad$ where $\quad d_{CB}^2 = a^2 + d^2$

Then $$J_B = \bar{J}_C + A\left(a^2 + d^2\right) \qquad (2)$$

Have $\quad J_D = \bar{J}_C + Ad_{CD}^2 \quad$ where $\quad d_{CD}^2 = 4a^2 + d^2$

Then $$J_D = \bar{J}_C + A\left(4a^2 + d^2\right) \qquad (3)$$

Then Equation (3) – Equation(2): $$J_D - J_B = 3Aa^2 \qquad (4)$$

and Equation(4) – 3[Equation(1)]: $$\left(J_D - J_B\right) - 3J_A = -3\bar{J}_C$$

or $$\bar{J}_C = J_A - \frac{1}{3}\left(J_D - J_B\right)$$

$$= 45 \times 10^6 \text{ mm}^4 - \frac{1}{3}\left(252 \times 10^6 - 130 \times 10^6\right) \text{mm}^4 = 4.3333 \times 10^6 \text{ mm}^4$$

or $\bar{J}_C = 4.33 \times 10^6$ mm^4 ◀

Note $$a = 63.77 \text{ mm}$$

and $$d = 92.195 \text{ mm}$$