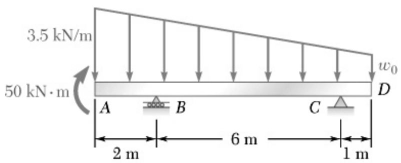
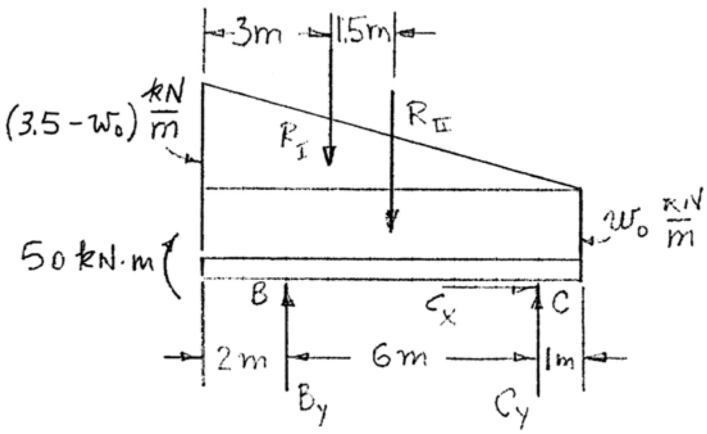


PROBLEM 5.72

Determine (a) the distributed load w_0 at the end D of the beam $ABCD$ for which the reaction at B is zero, (b) the corresponding reactions at C.

SOLUTION



Have $$R_{\mathrm{I}} = \frac{1}{2}(9\text{ m})\left[(3.5 - w_0)\text{ kN/m}\right] = 4.5(3.5 - w_0)\text{ kN}$$

$$R_{\mathrm{II}} = (9\text{ m})(w_0\text{ kN/m}) = 9w_0\text{ kN}$$

(a) Then $$+\circlearrowleft \Sigma M_C = 0: \quad -50\text{ kN}\cdot\text{m} + (5\text{ m})\left[4.5(3.5 - w_0)\text{ kN}\right] + (3.5\text{ m})(9w_0\text{ kN}) = 0$$

or $$9w_0 + 28.75 = 0$$

so $$w_0 = -3.1944\text{ kN/m}$$ $$w_0 = 3.19\text{ kN/m} \uparrow \blacktriangleleft$$

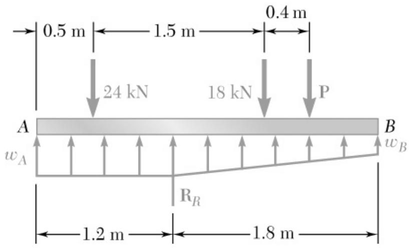
Note: the negative sign means that the distributed force w_0 is upward.

(b) $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad -4.5(3.5 + 3.19)\text{ kN} + 9(3.19)\text{ kN} + C_y = 0$$

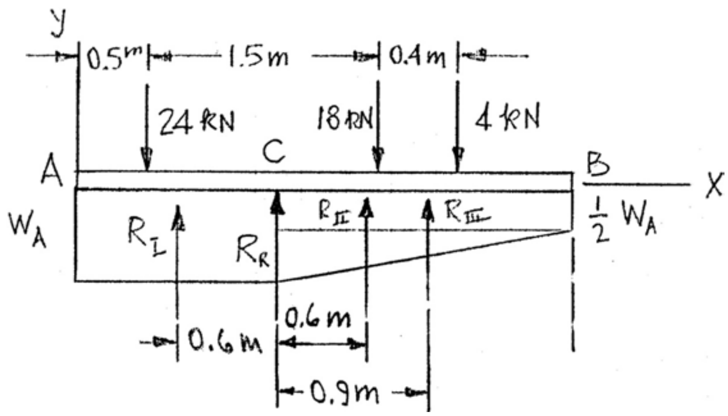
or $$C_y = 1.375\text{ kN}$$ $$\mathbf{C} = 1.375\text{ kN} \uparrow \blacktriangleleft$$

PROBLEM 5.73



A grade beam AB supports three concentrated loads and rests on soil and the top of a large rock. The soil exerts an upward distributed load, and the rock exerts a concentrated load $\mathbf{R}_R$ as shown. Knowing that $P = 4$ kN and $w_B = \frac{1}{2}w_A$, determine the values of w_A and R_R corresponding to equilibrium.

SOLUTION



Have $$R_{\text{I}} = (1.2 \text{ m})(w_A \text{ kN/m}) = 1.2\, w_A \text{ kN}$$

$$R_{\text{II}} = \frac{1}{2}(1.8 \text{ m})\left(\frac{1}{2}w_A \text{ kN/m}\right) = 0.45\, w_A \text{ kN}$$

$$R_{\text{III}} = (1.8 \text{ m})\left(\frac{1}{2}w_A \text{ kN/m}\right) = 0.9\, w_A \text{ kN}$$

Then $$+\circlearrowleft\ \Sigma M_C = 0:\quad -(0.6 \text{ m})\left[(1.2\, w_A) \text{ kN}\right] + (0.6 \text{ m})\left[(0.45\, w_A)\text{kN/m}\right]$$

$$+ (0.9 \text{ m})\left[(0.9\, w_A)\text{kN/m}\right] - (1.2 \text{ m})(4 \text{ kN/m})$$

$$- (0.8 \text{ m})(18 \text{ kN/m}) + (0.7 \text{ m})(24 \text{ kN/m}) = 0$$

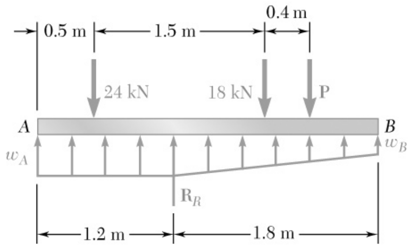
or $$w_A = 6.667 \text{ kN/m}$$ $w_A = 6.67$ kN/m ◀

and $$+\uparrow\ \Sigma F_y = 0:\quad R_R + (1.2 \text{ m})(6.67 \text{ kN/m}) + (0.45 \text{ m})(6.67 \text{ kN/m})$$

$$+ (0.9 \text{ m})(6.67 \text{ kN/m}) - 24 \text{ kN} - 18 \text{ kN} - 4 \text{ kN}$$

or $$R_R = 29.0 \text{ kN}$$ $R_R = 29.0$ kN ◀

PROBLEM 5.74

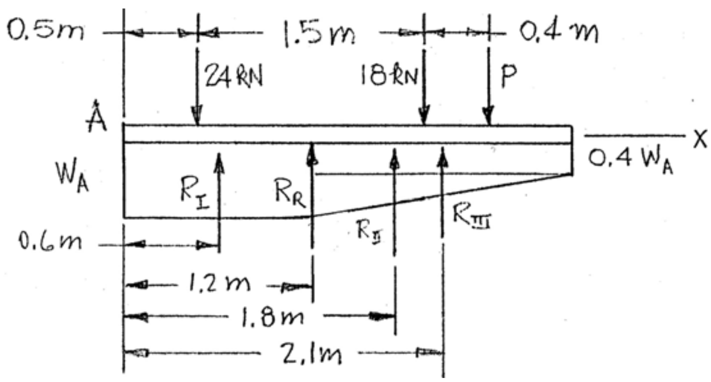


A grade beam *AB* supports three concentrated loads and rests on soil and the top of a large rock. The soil exerts an upward distributed load, and the rock exerts a concentrated load $\mathbf{R}_R$ as shown. Knowing that $w_B = 0.4w_A$, determine (*a*) the largest value of **P** for which the beam is in equilibrium, (*b*) the corresponding value of w_A.

In the following problems, use $\gamma = 62.4\ \text{lb/ft}^3$ for the specific weight of fresh water and $\gamma_C = 150\ \text{lb/ft}^3$ for the specific weight of concrete if U.S. customary units are used. With SI units, use $\rho = 10^3\ \text{kg/m}^3$ for the density of fresh water and $\rho_C = 2.40 \times 10^3\ \text{kg/m}^3$ for the density of concrete. (See the footnote on page 222 for how to determine the specific weight of a material given its density.)

j

SOLUTION



Have $R_{\text{I}} = (1.2\text{ m})(w_A\text{ kN/m}) = 1.2\,w_A\text{ kN}$

$$R_{\text{II}} = \frac{1}{2}(1.8\text{ m})(0.6\,w_A\text{ kN/m}) = 0.54\,w_A\text{ kN}$$

$$R_{\text{III}} = (1.8\text{ m})(0.4\,w_A\text{ kN/m}) = 0.72\,w_A\text{ kN}$$

(*a*) Then $+\circlearrowleft\ \Sigma M_A = 0$: $(0.6\text{ m})[(1.2\,w_A)\text{ kN}] + (1.2\text{ m})R_R + (1.8\text{ m})[(0.54\,w_A)\text{ kN}]$

$$+ (2.1\text{ m})[(0.72\,w_A)\text{ kN}] - (0.5\text{ m})(24\text{ kN})$$

$$- (2.0\text{ m})(18\text{ kN}) + (2.4\text{ m})P = 0$$

or $$3.204\,w_A + 1.2\,R_R - 2.4\,P = 48 \qquad (1)$$

and $+\uparrow\ \Sigma F_y = 0$: $R_R + 1.2\,W_A + 0.54\,W_A + 0.72\,W_A - 24 - 18 - P = 0$

or $$R_R + 2.46\,W_A - P = 42 \qquad (2)$$

Now combine Eqs. (1) and (2) to eliminate w_A:

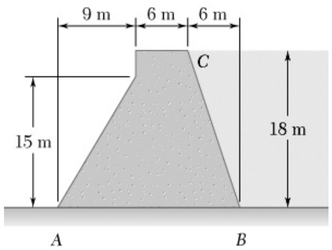
$$(3.204)\text{Eq. }2 - (2.46)\text{Eq. }1 \Rightarrow\ 0.252\,R_R = 16.488 - 2.7\,P$$

Since R_R must be ≥ 0, the maximum acceptable value of P is that for which $R = 0$,

or $P = 6.1067\text{ kN}$ $\qquad$ $P = 6.11\text{ kN}$ ◀

(*b*) Then, from Eq. (2): $2.46\,W_A - 6.1067 = 42$ $\qquad$ or $W_A = 19.56\text{ kN/m}$ ◀

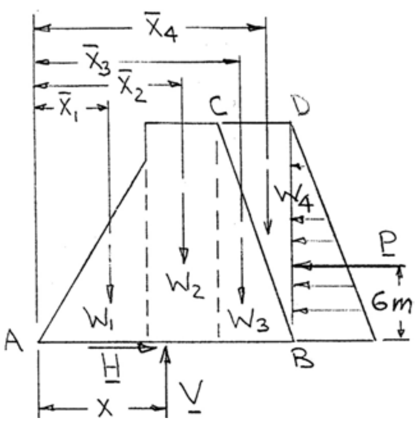
PROBLEM 5.75



The cross section of a concrete dam is as shown. For a dam section of unit width, determine (*a*) the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of the reaction forces of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

In the following problems, use $\gamma = 62.4 \text{ lb/ft}^3$ for the specific weight of fresh water and $\gamma_C = 150 \text{ lb/ft}^3$ for the specific weight of concrete if U.S. customary units are used. With SI units, use $\rho = 10^3 \text{ kg/m}^3$ for the density of fresh water and $\rho_C = 2.40 \times 10^3 \text{ kg/m}^3$ for the density of concrete. (See the footnote on page 222 for how to determine the specific weight of a material given its density.)

SOLUTION



The free body shown consists of a 1-m thick section of the dam and the triangular section *BCD* of the water behind the dam.

Note:

$$\bar{X}_1 = 6 \text{ m}$$

$$\bar{X}_2 = (9+3) \text{ m} = 12 \text{ m}$$

$$\bar{X}_3 = (15+2) \text{ m} = 17 \text{ m}$$

$$\bar{X}_4 = (15+4) \text{ m} = 19 \text{ m}$$

(*a*) Now $\quad W = \rho g V \quad$ so that

$$W_1 = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(9 \text{ m})(15 \text{ m})(1 \text{ m})\right] = 1589 \text{ kN}$$

$$W_2 = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[(6 \text{ m})(18 \text{ m})(1 \text{ m})\right] = 2543 \text{ kN}$$

$$W_3 = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(6 \text{ m})(18 \text{ m})(1 \text{ m})\right] = 1271 \text{ kN}$$

$$W_4 = (2400 \text{ kg/m}^3)(9.81 \text{ m/s}^2)\left[\frac{1}{2}(6 \text{ m})(18 \text{ m})(1 \text{ m})\right] = 529.7 \text{ kN}$$

Also $\quad P = \frac{1}{2}Ap = \frac{1}{2}\left[(18 \text{ m})(1 \text{ m})\right]\left[(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(18 \text{ m})\right]$

$$= 1589 \text{ kN}$$

Then $\quad \xrightarrow{+} \Sigma F_x = 0: \quad H - 1589 \text{ kN} = 0$

or $\quad H = 1589 \text{ kN} \qquad \mathbf{H} = 1589 \text{ kN} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad V - 1589 \text{ kN} - 2543 \text{ kN} - 1271 \text{ kN} - 529.7 \text{ kN}$

or $\quad V = 5933 \text{ kN} \qquad \mathbf{V} = 5.93 \text{ MN} \uparrow$ ◀

PROBLEM 5.75 CONTINUED

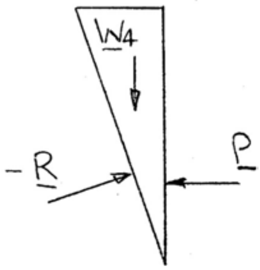
(*b*) Have $\circlearrowleft + \; \Sigma M_A = 0\!: \quad X(5933\text{ kN}) + (6\text{ m})(1589\text{ kN})$

$$- (6\text{ m})(1589\text{ kN}) - (12\text{ m})(2543\text{ kN})$$

$$- (17\text{ m})(1271\text{ kN}) - (19\text{ m})(529.7) = 0$$

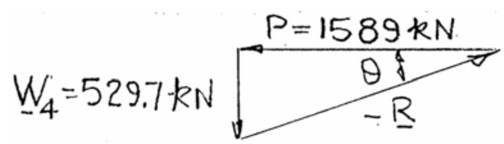
or $\quad X = 10.48\text{ m} \qquad\qquad X = 10.48\text{ m}$ ◀

to the right of A



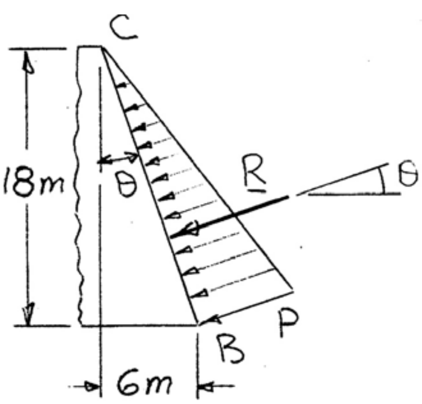
(*c*) Consider water section *BCD* as the free body.

Have $\quad \Sigma\mathbf{F} = 0$



Then $\quad -\mathbf{R} = 1675\text{ kN} \quad 18.43°$

or $\mathbf{R} = 1675\text{ kN} \quad 18.43°$ ◀



Alternative solution to part (c)

Consider the face *BC* of the dam.

Have $\quad BC = \sqrt{6^2 + 18^2} = 18.9737\text{ m}$

$$\tan\theta = \frac{6}{18} \qquad \theta = 18.43°$$

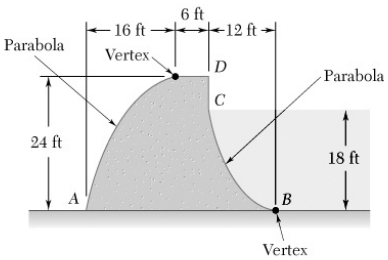
and $\quad p = (\rho g)h = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(18\text{ m})$

$$= 176.6\text{ kN/m}^2$$

Then $\quad R = \frac{1}{2}Ap = \frac{1}{2}\left[(18.97\text{ m})(1\text{ m})\right](176.6\text{ kN/m}^2)$

$$= 1675\text{ kN}$$

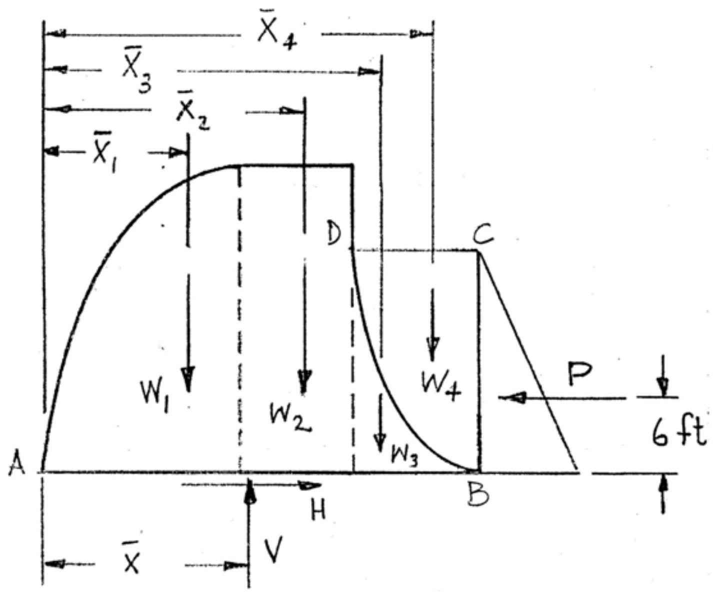
$$\therefore \; \mathbf{R} = 1675\text{ kN} \quad 18.43°$$



PROBLEM 5.76

The cross section of a concrete dam is as shown. For a dam section of unit width, determine (*a*) the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of the reaction forces of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION



The free body shown consists of a 1-ft thick section of the dam and the parabolic section of water above (and behind) the dam.

Note

$$\bar{x}_1 = \frac{5}{8}(16\text{ ft}) = 10\text{ ft}$$

$$\bar{x}_2 = \left[16 + \frac{1}{2}(6)\right]\text{ft} = 19\text{ ft}$$

$$\bar{x}_3 = \left[22 + \frac{1}{4}(12)\right]\text{ft} = 25\text{ ft}$$

$$\bar{x}_4 = \left[22 + \frac{5}{8}(12)\right]\text{ft} = 29.5\text{ ft}$$

PROBLEM 5.76 CONTINUED

Now $$W = \gamma V$$

$$W_1 = \left(150\ \text{lb/ft}^3\right)\left[\frac{2}{3}(16\ \text{ft})(24\ \text{ft}) \times (1\ \text{ft})\right] = 38,400\ \text{lb}$$

$$W_2 = \left(150\ \text{lb/ft}^3\right)\left[(6\ \text{ft})(24\ \text{ft}) \times (1\ \text{ft})\right] = 21,600\ \text{lb}$$

$$W_3 = \left(150\ \text{lb/ft}^3\right)\left[\frac{1}{3}(12\ \text{ft})(18\ \text{ft}) \times (1\ \text{ft})\right] = 10,800\ \text{lb}$$

$$W_4 = \left(62.4\ \text{lb/ft}^3\right)\left[\frac{2}{3}(12\ \text{ft})(18\ \text{ft}) \times (1\ \text{ft})\right] = 8985.6\ \text{lb}$$

Also $$P = \frac{1}{2}Ap = \frac{1}{2}\left[(18 \times 1)\ \text{ft}^2\right] \times \left(62.4\ \text{lb/ft}^3 \times 18\ \text{ft}\right) = 10,108.8\ \text{lb}$$

(a) Then $$\xrightarrow{+} \Sigma F_x = 0: \quad H - 10,108.8\ \text{lb} = 0$$

or $\mathbf{H} = 10.11\ \text{kips} \longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad V - 38,400\ \text{lb} - 21,600\ \text{lb} - 10,800\ \text{lb} - 8995.6\ \text{lb} = 0$$

or $$V = 79,785.6$$ $\mathbf{V} = 79.8\ \text{kips} \uparrow$ ◀

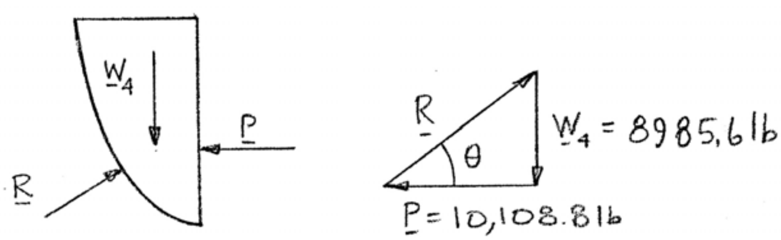
(b) $$+\circlearrowleft \Sigma M_A = 0: \quad \overline{X}(79,785.6\ \text{lb}) - (6\ \text{ft})(38,400\ \text{lb}) - (19\ \text{ft})(21,600\ \text{lb}) - (25\ \text{ft})(10,800\ \text{lb})$$
$$- (29.5\ \text{ft})(8985.6\ \text{lb}) + (6\ \text{ft})(10,108.8\ \text{lb}) = 0$$

or $$\overline{X} = 15.90\ \text{ft}$$

The point of application of the resultant is 15.90 ft to the right of A ◀

(c) Consider the water section BCD as the free body.

Have $$\Sigma \mathbf{F} = 0$$

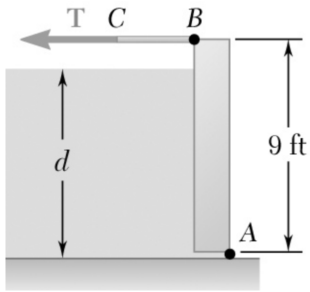


$$\therefore \quad R = 13.53\ \text{kips}$$
$$\theta = 41.6°$$

On the face BD of the dam

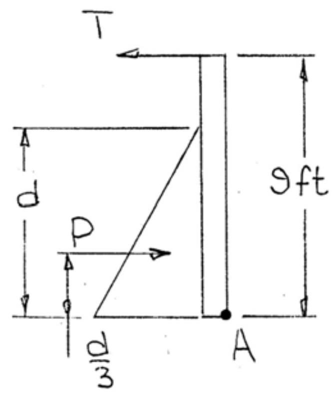
$\mathbf{R} = 13.53\ \text{kips} \nearrow 41.6°$ ◀

PROBLEM 5.77



The 9×12-ft side AB of a tank is hinged at its bottom A and is held in place by a thin rod BC. The maximum tensile force the rod can withstand without breaking is 40 kips, and the design specifications require the force in the rod not exceed 20 percent of this value. If the tank is slowly filled with water, determine the maximum allowable depth of water d in the tank.

SOLUTION



Consider the free-body diagram of the side.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma d)$$

Now $$+\circlearrowleft \Sigma M_A = 0\text{:} \quad (9\text{ ft})T - \frac{d}{3}P = 0$$

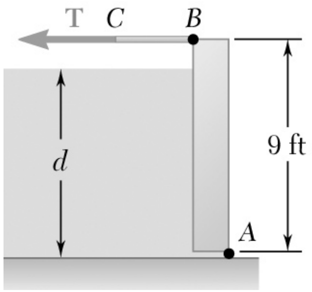
Then, for $d_{\max}$:

$$(9\text{ ft})\left[(0.2)\left(40\times 10^3\text{ lb}\right)\right] - \frac{d_{\max}}{3}\left\{\frac{1}{2}\left[(12\text{ ft})(d_{\max})\right]\left(62.4\text{ lb/ft}^3\right)d_{\max}\right\} = 0$$

or $$216\times 10^3\text{ ft}^3 = 374.4\, d_{\max}^3$$

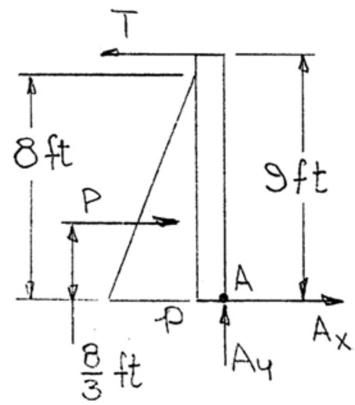
or $$d_{\max}^3 = 576.92\text{ ft}^3$$ $$d_{\max} = 8.32\text{ ft} \blacktriangleleft$$

PROBLEM 5.78



The $9 \times 12\text{-ft}$ side of an open tank is hinged at its bottom A and is held in place by a thin rod. The tank is filled with glycerine, whose specific weight is $80\ \text{lb/ft}^3$. Determine the force **T** in the rod and the reactions at the hinge after the tank is filled to a depth of 8 ft.

SOLUTION



Consider the free-body diagram of the side.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma d)$$

$$= \frac{1}{2}\left[(8\text{ ft})(12\text{ ft})\right]\left(80\text{ lb/ft}^3\right)(8\text{ ft}) = 30{,}720\text{ lb}$$

Then $$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (9\text{ ft})T - \left(\frac{8}{3}\text{ ft}\right)(30{,}720\text{ lb}) = 0$$

or $$T = 9102.22\text{ lb}$$

$$\mathbf{T} = 9.10\text{ kips} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 30{,}720\text{ lb} - 9102.22\text{ lb} = 0$$

or $$A = -21{,}618\text{ lb}$$

$$\mathbf{A} = 21.6\text{ kips} \longleftarrow \blacktriangleleft$$