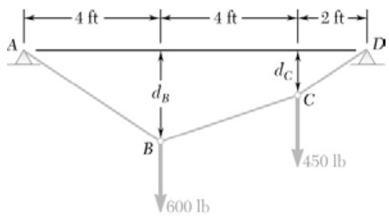
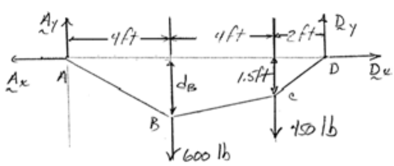


PROBLEM 7.88

Two loads are suspended as shown from cable *ABCD*. Knowing that $d_C = 1.5$ ft, determine (*a*) the distance d_B, (*b*) the components of the reaction at *A*, (*c*) the maximum tension in the cable.

SOLUTION

FBD cable:



$$\circlearrowleft \Sigma M_A = 0: (10\text{ ft})D_y - 8\text{ ft}(450\text{ lb}) - 4\text{ ft}(600\text{ lb}) = 0$$

$$D_y = 600\text{ lb} \uparrow$$

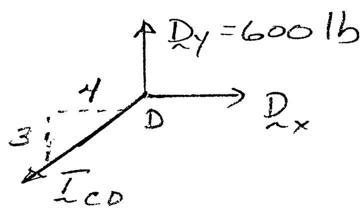
$$\uparrow \Sigma F_y = 0: A_y + 600\text{ lb} - 600\text{ lb} - 450\text{ lb} = 0$$

$$A_y = 450\text{ lb}$$

$$\longleftarrow \Sigma F_x = 0: \quad A_x - D_x = 0 \qquad (1)$$

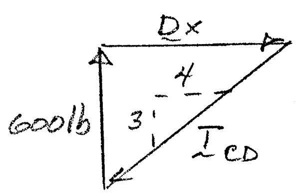
FBD pt D:

$$\frac{600\text{ lb}}{3} = \frac{D_x}{4} = \frac{T_{CD}}{5} : D_x = 800\text{ lb} \longrightarrow = A_x$$



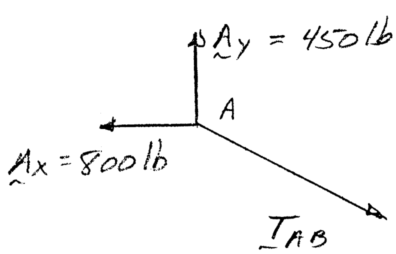
So

And $\qquad T_{CD} = 1000\text{ lb}$



FBD pt A:

$$\frac{800\text{ lb}}{4\text{ ft}} = \frac{450\text{ lb}}{d_B}$$



(*a*) $\qquad d_B = 2.25\text{ ft}$ ◀

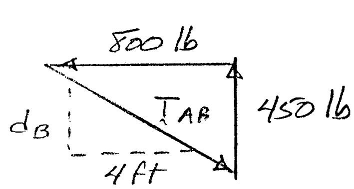
(b) $\qquad \mathbf{A}_x = 800\text{ lb} \longleftarrow$ ◀

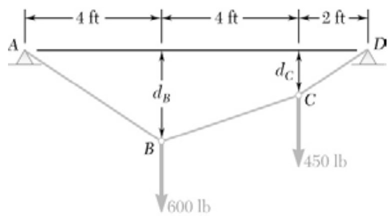
$\qquad \mathbf{A}_y = 450\text{ lb} \uparrow$ ◀

$$T_{AB} = \sqrt{(800\text{ lb})^2 + (450\text{ lb})^2} = 918\text{ lb}$$

So $\qquad$ (*c*) $\qquad T_{\max} = T_{CD} = 1000\text{ lb}$ ◀

Note: T_{CD} is $T_{\max}$ as cable slope is largest in section *CD*.



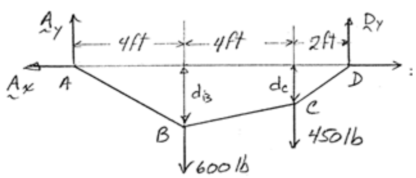


PROBLEM 7.89

Two loads are suspended as shown from cable *ABCD*. Knowing that the maximum tension in the cable is 720 lb, determine (*a*) the distance d_B, (*b*) the distance d_C.

SOLUTION

FBD cable:



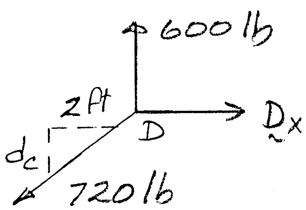
$$\circlearrowleft \Sigma M_A = 0\colon (10\text{ ft})D_y - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

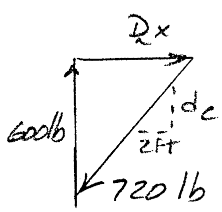
$$\mathbf{D}_y = 600\text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0\colon A_y + 600\text{ lb} - 600\text{ lb} - 450\text{ lb} = 0$$

$$\mathbf{A}_y = 450\text{ lb} \uparrow$$

FBD pt D:





$$\longleftarrow \Sigma F_x = 0\colon A_x - B_x = 0$$

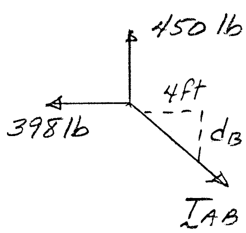
Since $A_x = B_x$; And $D_y > A_y$, Tension $T_{CD} > T_{AB}$

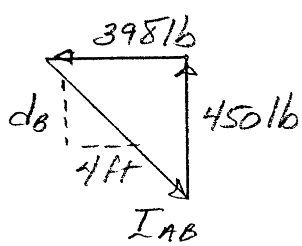
So $T_{CD} = T_{\max} = 720\text{ lb}$

$$D_x = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2} = 398\text{ lb} = A_x$$

$$\frac{d_C}{600\text{ lb}} = \frac{2\text{ ft}}{398\text{ lb}} \qquad d_C = 3.015\text{ ft}$$

FBD pt. A:

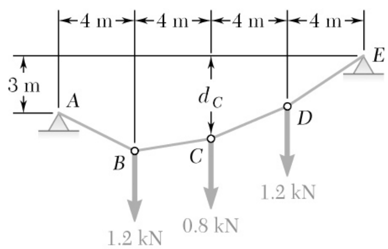




$$\frac{d_B}{450\text{ lb}} = \frac{4\text{ ft}}{398\text{ lb}} \qquad (a) \qquad d_B = 4.52\text{ ft} \blacktriangleleft$$

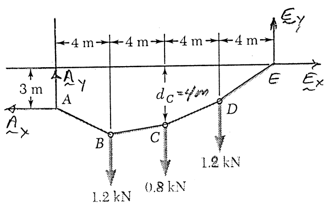
$$(b) \qquad d_C = 3.02\text{ ft} \blacktriangleleft$$

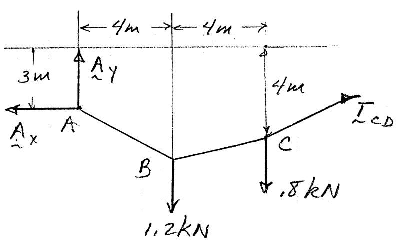
PROBLEM 7.90



Knowing that $d_C = 4$ m, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION





(*a*) **FBD cable:**

$$\Sigma M_E = 0: \ (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN}) - (3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

FBD ABC:

$$\Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) + (1\text{ m})A_x - (8\text{ m})A_y = 0$$

$$A_x - 8A_y = -4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad A_x = 3.2\text{ kN} \qquad A_y = 1\text{ kN}$

So $\mathbf{A} = 3.35\text{ kN} \measuredangle 17.35°$ ◀

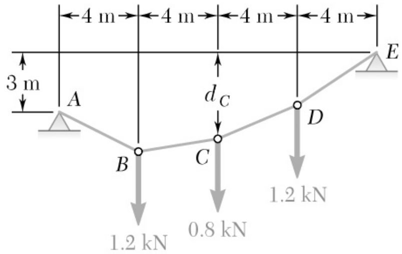
(*b*) cable: $\rightarrow \Sigma F_x = 0: -A_x + E_x = 0$

$$E_x = A_x = 3.2\text{ kN}$$

$$\uparrow \Sigma F_y = 0: \ A_y - (1.2 + 0.8 + 1.2)\text{ kN} + E_y = 0$$

$$E_y = 3.2\text{ kN} - A_y = (3.2 - 1)\text{ kN} = 2.2\text{ kN}$$

So $\mathbf{E} = 3.88\text{ kN} \measuredangle 34.5°$ ◀

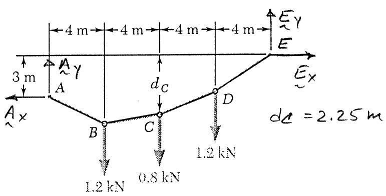


PROBLEM 7.91

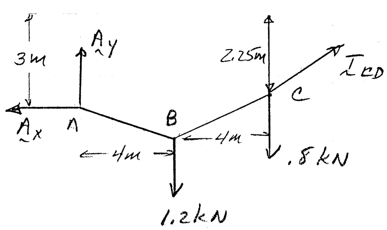
Knowing that $d_C = 2.25$ m, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

FBD Cable:



FBD ABC:



(*a*) $$\circlearrowleft \Sigma M_E = 0: (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN}) - (3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) - (0.75\text{ m})A_x - (8\text{ m})A_y = 0$$

$$0.75A_x + 8A_y = 4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad A_x = \dfrac{32}{3}\text{ kN}, \qquad A_y = -0.4\text{ kN}$

So $\qquad \mathbf{A} = 10.67\text{ kN} \measuredangle\, 2.15°$ ◀

Note: this implies $d_B < 3\text{ m}$ (in fact $d_B = 2.85\text{ m}$)

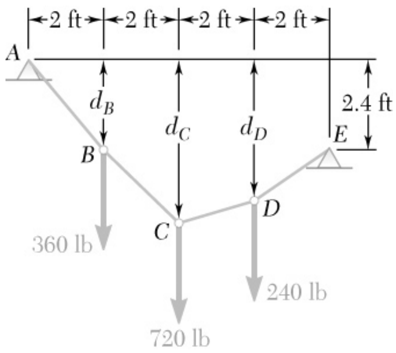
(*b*) FBD cable: $\qquad \xrightarrow{} \Sigma F_x = 0: -\dfrac{32}{3}\text{ kN} + E_x = 0 \qquad E_x = \dfrac{32}{3}\text{ kN}$

$$\uparrow \Sigma F_y = 0: \quad -0.4\text{ kN} - 1.2\text{ kN} - 0.8\text{ kN} - 1.2\text{ kN} + E_y = 0$$

$$E_y = 3.6\text{ kN}$$

$\mathbf{E} = 11.26\text{ kN} \measuredangle\, 18.65°$ ◀

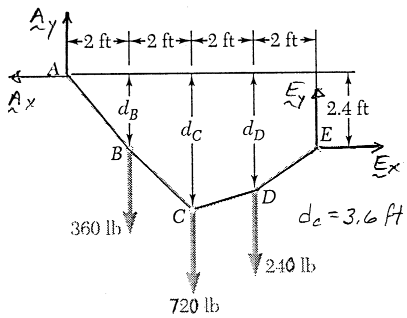
PROBLEM 7.92



Cable *ABCDE* supports three loads as shown. Knowing that $d_C = 3.6$ ft, determine (*a*) the reaction at *E*, (*b*) the distances d_B and d_D.

SOLUTION

FBD Cable:

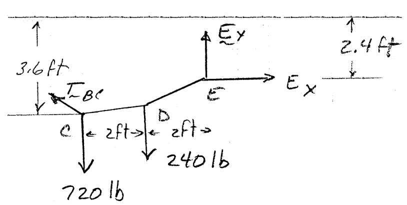


(*a*)

$$\circlearrowleft \Sigma M_A = 0: (2.4 \text{ ft})E_x + (8 \text{ ft})E_y - (2 \text{ ft})(360) - (4 \text{ ft})(720 \text{ lb}) - (6 \text{ ft})(240 \text{ lb}) = 0$$

$$0.3E_x + E_y = 630 \text{ lb} \qquad (1)$$

FBD CDE:



$$\circlearrowleft \Sigma M_C = 0: -(1.2 \text{ ft})E_x + (4 \text{ ft})E_y - (2 \text{ ft})(240 \text{ lb}) = 0$$

$$-0.3E_x + E_y = +120 \text{ lb} \qquad (2)$$

Solving (1) and (2) $\qquad E_x = 850 \text{ lb} \qquad E_y = 375 \text{ lb}$

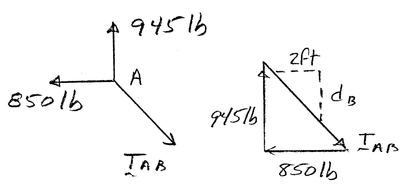
(*a*) $\qquad \mathbf{E} = 929 \text{ lb} \measuredangle 23.8° \blacktriangleleft$

(*b*) cable: $\qquad \rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \qquad A_x = E_x = 850 \text{ lb}$

$$\uparrow \Sigma F_y = 0: A_y - 360 \text{ lb} - 720 \text{ lb} - 240 \text{ lb} + 375 \text{ lb} = 0$$

$$A_y = 945 \text{ lb}$$

Point A:



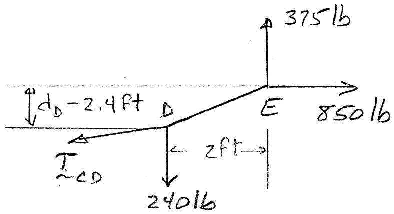
$$\frac{d_B}{2 \text{ ft}} = \frac{945 \text{ lb}}{850 \text{ lb}} \qquad d_B = 2.22 \text{ ft} \blacktriangleleft$$

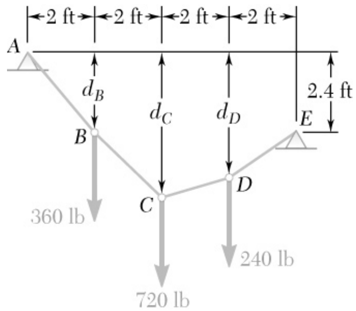
PROBLEM 7.92 CONTINUED

$$\circlearrowleft \Sigma M_D = 0: (2\text{ ft})(375\text{ lb}) - (d_D - 2.4\text{ ft})(850\text{ lb}) = 0$$

Segment DE:

$$d_D = 3.28\text{ ft} \blacktriangleleft$$



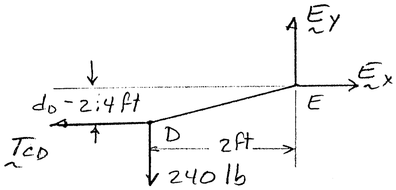


PROBLEM 7.93

Cable *ABCDE* supports three loads as shown. Determine (*a*) the distance d_C for which portion *CD* of the cable is horizontal, (*b*) the corresponding reactions at the supports.

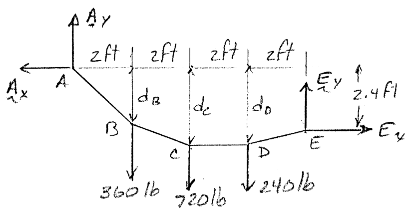
SOLUTION

Segment DE:



$$\uparrow \Sigma F_y = 0: E_y - 240 \text{ lb} = 0 \qquad \mathbf{E}_y = 240 \text{ lb} \uparrow$$

FBD Cable:



$$\circlearrowleft \Sigma M_A = (2.4 \text{ ft})E_x + (8 \text{ ft})(240 \text{ lb}) - (6 \text{ ft})(240 \text{ lb})$$
$$- (4 \text{ ft})(720 \text{ lb}) - (2 \text{ ft})(360 \text{ lb}) = 0$$

$$\mathbf{E}_x = 1300 \text{ lb} \longrightarrow$$

So

From Segment DE:

$$\circlearrowleft \Sigma M_D = 0: (2 \text{ ft})E_y - (d_C - 2.4 \text{ ft})E_x = 0$$

$$d_C = 2.4 \text{ ft} + \frac{E_y}{E_x}(2 \text{ ft}) = (2.4 \text{ ft}) + \frac{240 \text{ lb}}{1300 \text{ lb}}(2 \text{ ft}) = 2.7692 \text{ ft}$$

(*a*) $d_C = 2.77$ ft ◀

From FBD Cable:

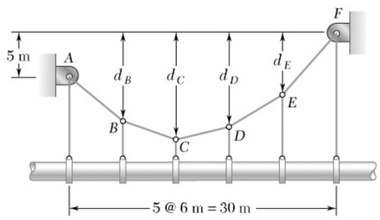
$$\longrightarrow \Sigma F_x = 0: \; -A_x + E_x = 0 \qquad \mathbf{A}_x = 1300 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \; A_y - 360 \text{ lb} - 720 \text{ lb} - 240 \text{ lb} + E_y = 0$$

$$\mathbf{A}_y = 1080 \text{ lb} \uparrow$$

(*b*) $\mathbf{A} = 1.690$ kips $\searrow$ 39.7° ◀

$\mathbf{E} = 1.322$ kips $\nearrow$ 10.46° ◀

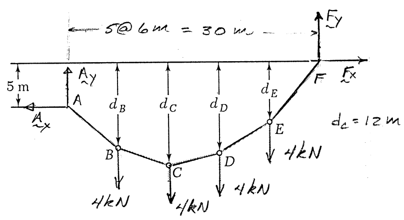


PROBLEM 7.94

An oil pipeline is supported at 6-m intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents, the tension in each hanger is 4 kN. Knowing that $d_C = 12$ m, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable:

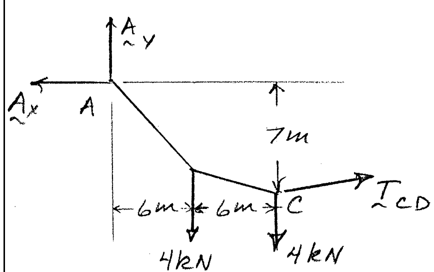


Note: $\mathbf{A}_y$ and $\mathbf{F}_y$ shown are forces on cable, assuming the 4 kN loads at A and F act on supports.

$$\circlearrowleft \Sigma M_F = 0: (6\text{ m})\left[1(4\text{ kN}) + 2(4\text{ kN}) + 3(4\text{ kN}) + 4(4\text{ kN})\right]$$
$$-(30\text{ m})A_y - (5\text{ m})A_x = 0$$

$$A_x + 6A_y = 48\text{ kN} \qquad (1)$$

FBD ABC:



$$\circlearrowleft \Sigma M_C = 0: (6\text{ m})(4\text{ kN}) + (7\text{ m})A_x - (12\text{ m})A_y = 0$$

$$7A_x - 12A_y = -24\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{A}_x = 8\text{ kN} \longrightarrow \qquad \mathbf{A}_y = \dfrac{20}{3}\text{ kN} \uparrow$

From FBD Cable:

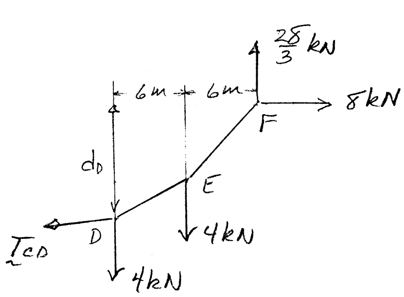
$$\longrightarrow \Sigma F_x = 0: -A_x + F_x = 0 \qquad F_x = A_x = 8\text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - 4(4\text{ kN}) + F_y = 0$$

$$F_y = 16\text{ kN} - A_y = \left(16 - \frac{20}{3}\right)\text{kN} = \frac{28}{3}\text{ kN} > A_y$$

So $\qquad T_{EF} > T_{AB} \qquad T_{\max} = T_{EF} = \sqrt{F_x^2 + F_y^2}$

FBD DEF:

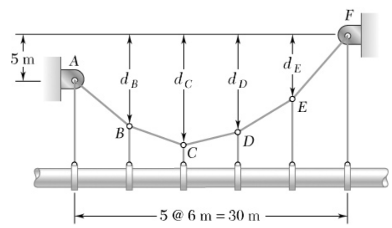


(*a*) $\qquad T_{\max} = \sqrt{(18\text{ kN})^2 + \left(\dfrac{28}{3}\text{ kN}\right)^2} = 12.29\text{ kN}$ ◀

$$\circlearrowleft \Sigma M_D = 0: (12\text{ m})\left(\frac{28}{3}\text{ kN}\right) - d_D(8\text{ kN}) - (6\text{ m})(4\text{ kN}) = 0$$

(*b*) $\qquad d_D = 11.00\text{ m}$ ◀

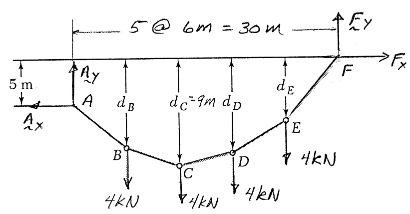
PROBLEM 7.95



Solve Prob. 7.94 assuming that $d_C = 9$ m.

SOLUTION

FBD Cable:



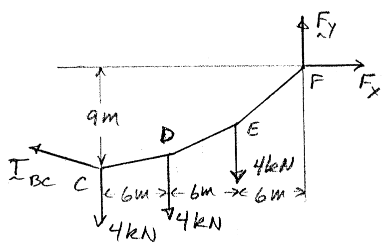
Note: 4 kN loads at A and F act directly on supports, not on cable.

$$\circlearrowleft \Sigma M_A = 0: (30 \text{ m})F_y - (5 \text{ m})F_x$$

$$-(6 \text{ m})\left[1(4 \text{ kN}) + 2(4 \text{ kN}) + 3(4 \text{ kN}) + 4(4 \text{ kN})\right] = 0$$

$$F_x - 6F_y = -48 \text{ kN} \qquad (1)$$

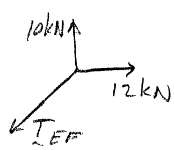
FBD CDEF:



$$\circlearrowleft \Sigma M_C = 0: (18)F_y - (9 \text{ m})F_x - (12 \text{ m})(4 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$$

$$F_x - 2F_y = -8 \text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{F}_x = 12 \text{ kN} \longrightarrow \qquad \mathbf{F}_y = 10 \text{ kN} \uparrow$



$$T_{EF} = \sqrt{(10 \text{ kN})^2 + (12 \text{ kN})^2} = 15.62 \text{ kN}$$

Since slope $EF >$ slope AB this is $T_{\max}$

(a) $\qquad T_{\max} = 15.62 \text{ kN}$ ◀

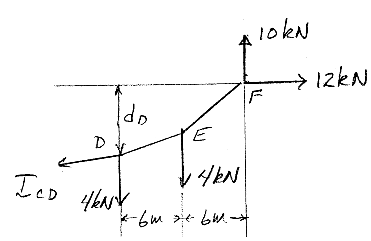
Also could note from FBD cable

$$\uparrow \Sigma F_y = 0: A_y + F_y - 4(4 \text{ kN}) = 0$$

$$A_y = 16 \text{ kN} - 12 \text{ kN} = 4 \text{ kN}$$

Thus $\quad A_y < F_y \quad$ and $\quad T_{AB} < T_{EF}$

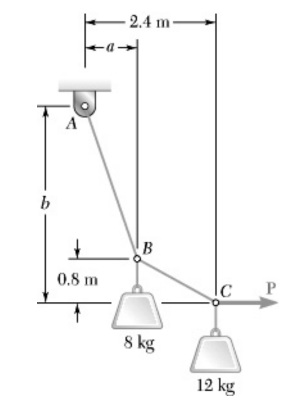
FBD DEF:



(b) $\qquad \circlearrowleft \Sigma M_D = 0: (12 \text{ m})(10 \text{ kN}) - d_D(12 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$

$d_D = 8.00 \text{ m}$ ◀

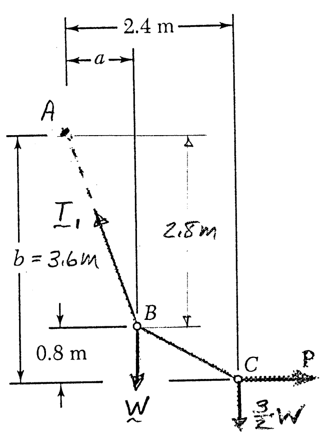
PROBLEM 7.96



Cable *ABC* supports two boxes as shown. Knowing that $b = 3.6$ m, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

FBD BC:



$$W = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: (3.6\text{ m})P - (2.4\text{ m})\frac{3W}{2} - aW = 0$$

$$P = W\left(1 + \frac{a}{3.6\text{ m}}\right) \qquad (1)$$

$$\longrightarrow \Sigma F_x = 0: -T_{1x} + P = 0 \qquad T_{1x} = P$$

$$\uparrow \Sigma F_y = 0: T_{1y} - W - \frac{3}{2}W = 0 \qquad T_{1y} = \frac{5W}{2}$$

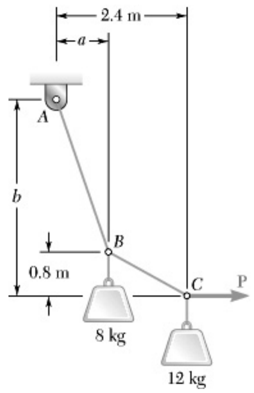
But $\dfrac{T_{1y}}{T_{1x}} = \dfrac{2.8\text{ m}}{a}$ so $\dfrac{5W}{2P} = \dfrac{2.8\text{ m}}{a}$

$$P = \frac{5Wa}{5.6\text{ m}} \qquad (2)$$

Solving (1) and (2): $a = 1.6258$ m, $P = 1.4516W$

So (*a*) $P = 1.4516(78.48) = 113.9\text{ N}$ ◀

(*b*) $a = 1.626\text{ m}$ ◀

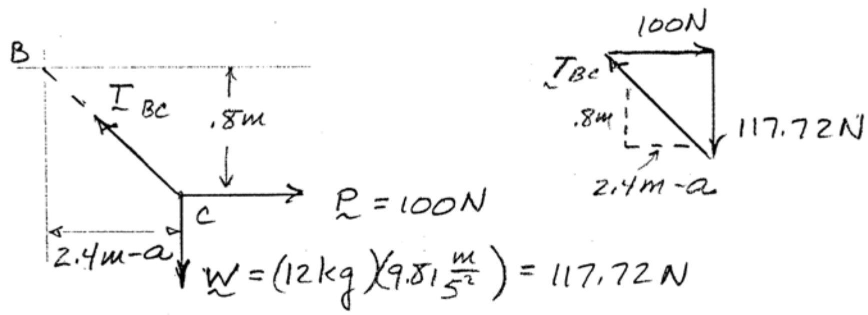


PROBLEM 7.97

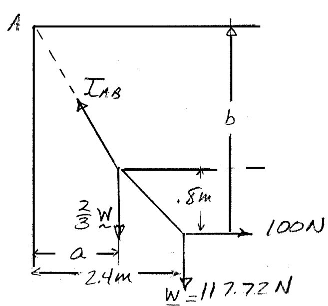
Cable *ABC* supports two boxes as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 100 N is applied at *C*.

SOLUTION

FBD pt C:



Segment BC:



$$\frac{2.4\text{ m} - a}{100\text{ N}} = \frac{0.8\text{ m}}{117.72\text{ N}}$$

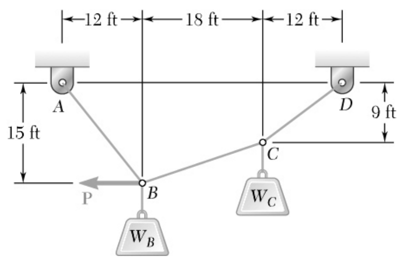
$$a = 1.7204\text{ m}$$

$$a = 1.720\text{ m} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_A = 0: b(100\text{ N}) - (2.4\text{ m})(117.72\text{ N}) - (1.7204\text{ m})\left(\frac{2}{3}117.72\text{ N}\right) = 0$$

$$b = 4.1754\text{ m}$$

$$b = 4.18\text{ m} \blacktriangleleft$$

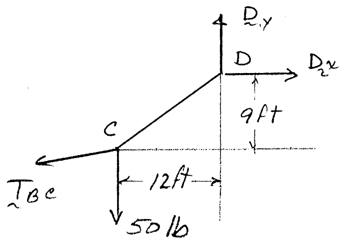


PROBLEM 7.98

Knowing that $W_B = 150$ lb and $W_C = 50$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD CD:

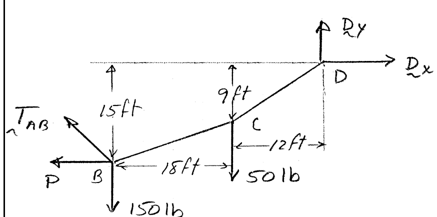


$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$3D_x = 4D_y \qquad (1)$$

$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(50 \text{ lb}) = 0$$

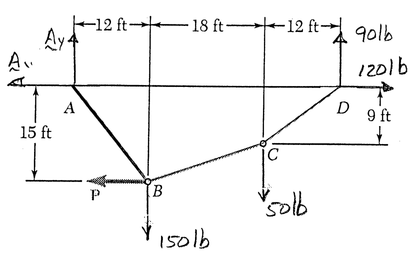
FBD BCD:



$$2D_y - D_x = 60 \text{ lb} \qquad (2)$$

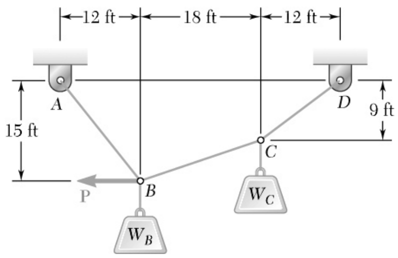
Solving (1) and (2) $\quad \mathbf{D}_x = 120 \text{ lb} \longrightarrow \quad \mathbf{D}_y = 90 \text{ lb} \uparrow$

FBD Cable:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(90 \text{ lb}) - (30 \text{ ft})(50 \text{ lb}) - (12 \text{ ft})(150 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 32.0 \text{ lb} \blacktriangleleft$$

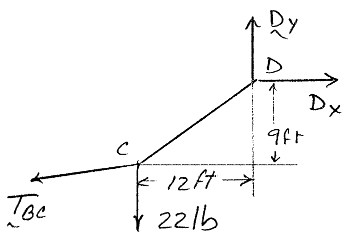


PROBLEM 7.99

Knowing that $W_B = 40$ lb and $W_C = 22$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

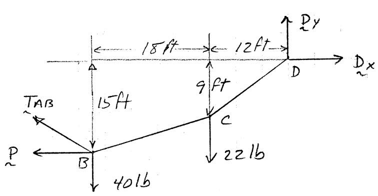
FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$4D_y = 3D_x \qquad (1)$$

FBD BCD:

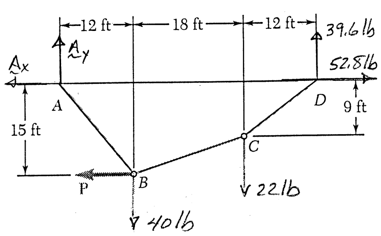


$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(22 \text{ lb}) = 0$$

$$10D_y - 5D_x = 132 \text{ lb} \qquad (2)$$

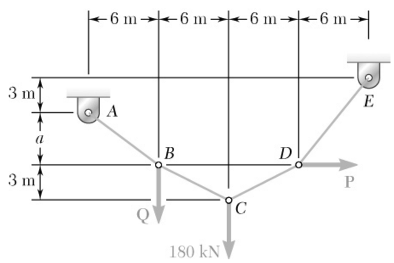
Solving (1) and (2) $\quad \mathbf{D}_x = 52.8 \text{ lb} \longrightarrow \quad \mathbf{D}_y = 39.6 \text{ lb} \uparrow$

FBD Whole:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(39.6 \text{ lb}) - (30 \text{ ft})(22 \text{ lb}) - (12 \text{ ft})(40 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 34.9 \text{ lb} \blacktriangleleft$$



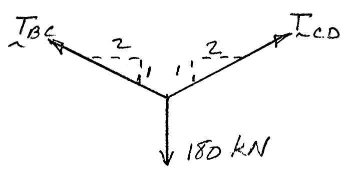
PROBLEM 7.100

If $a = 4.5$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

By symmetry: $T_{BC} = T_{CD} = T$

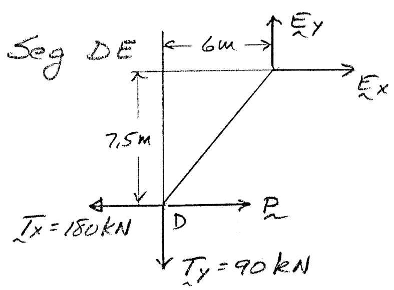
FBD pt C:



$$\uparrow \Sigma F_y = 0: 2\left(\frac{1}{\sqrt{5}}T\right) - 180 \text{ kN} = 0 \qquad T = 90\sqrt{5} \text{ kN}$$

$$T_x = 180 \text{ kN} \qquad T_y = 90 \text{ kN}$$

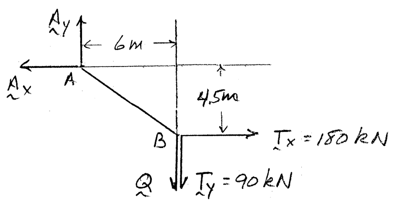
Segment DE:



$$\Sigma M_E = 0: (7.5 \text{ m})(P - 180 \text{ kN}) + (6 \text{ m})(90 \text{ kN}) = 0$$

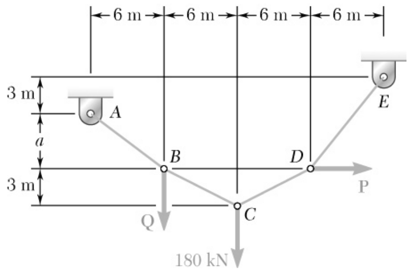
$$P = 108.0 \text{ kN} ◀$$

Segment AB:



$$\Sigma M_A = 0: (4.5 \text{ m})(180 \text{ kN}) - (6 \text{ m})(Q + 90 \text{ kN}) = 0$$

$$Q = 45.0 \text{ kN} ◀$$

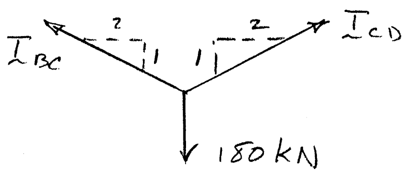


PROBLEM 7.101

If $a = 6$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

FBD pt C:

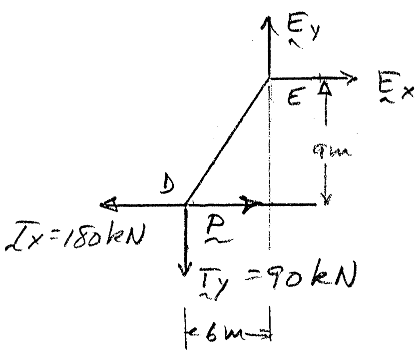


By symmetry: $T_{BC} = T_{CD} = T$

$$\uparrow \Sigma F_y = 0\colon\ 2\left(\frac{1}{\sqrt{5}}T\right) - 180\text{ kN} = 0 \qquad T = 90\sqrt{5}\text{ kN}$$

$$T_x = 180\text{ kN} \qquad T_y = 90\text{ kN}$$

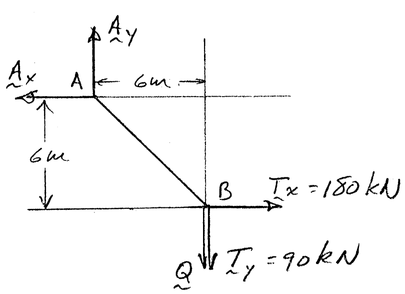
FBD DE:



$$\circlearrowleft \Sigma M_E = 0\colon\ (9\text{ m})(P - 180\text{ kN}) + (6\text{ m})(90\text{ kN}) = 0$$

$$P = 120.0\text{ kN} \blacktriangleleft$$

FBD AB:



$$\circlearrowleft \Sigma M_A = 0\colon\ (6\text{ m})(180\text{ kN}) - (6\text{ m})(Q + 90\text{ kN}) = 0$$

$$Q = 90.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.102

A transmission cable having a mass per unit length of 1 kg/m is strung between two insulators at the same elevation that are 60 m apart. Knowing that the sag of the cable is 1.2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION

(*a*) Since $h = 1.2\text{ m} \ll L = 30\text{ m}$ we can approximate the load as evenly distributed in the horizontal direction.

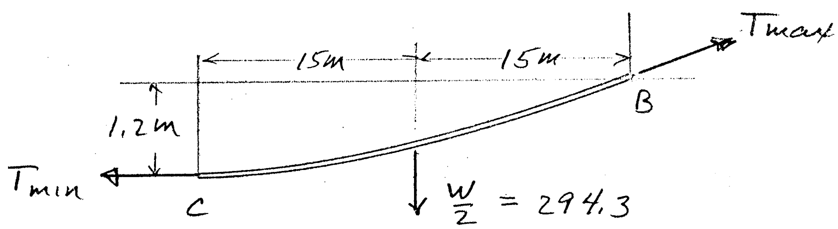
$$w = 1\text{ kg/m}\left(9.81\text{ m/s}^2\right) = 9.81\text{ N/m.}$$

$$w = (60\text{ m})(9.81\text{ N/m})$$

$$w = 588.6\text{ N}$$

Also we can assume that the weight of half the cable acts at the $\frac{1}{4}$ chord point.

FBD half-cable:



$$\circlearrowleft \Sigma M_B = 0: (15\text{ m})(294.3\text{ N}) - (1.2\text{ m})T_{\min} = 0$$

$$T_{\min} = 3678.75\text{ N} = T_{\max\ x}$$

$$\uparrow \Sigma F_y = 0: T_{\max\ y} - 294.3\text{ N} = 0$$

$$T_{\max\ y} = 294.3\text{ N}$$

$$T_{\max} = 3690.5\text{ N}$$

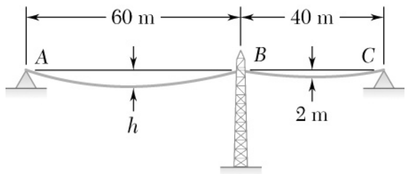
$T_{\max} = 3.69\text{ kN}$ ◀

(*b*)
$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

$$= (30\text{ m})\left[1 + \frac{2}{3}\left(\frac{1.2}{30}\right)^2 - \frac{2}{5}\left(\frac{1.2}{30}\right)^4 + \cdots\right] = 30.048\text{ m} \qquad \text{so} \qquad s = 2s_B = 60.096\text{ m}$$

$s = 60.1\text{ m}$ ◀

Note: The more accurate methods of section 7.10, which assume the load is evenly distributed along the length instead of horizontally, yield $T_{\max} = 3690.5\text{ N}$ and $s = 60.06$ m. Answers agree to 3 digits at least.

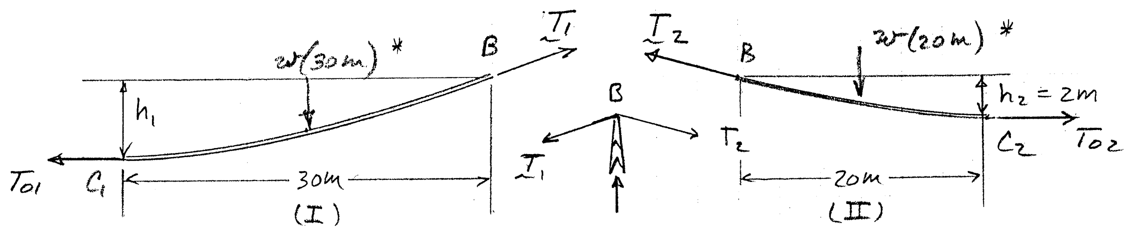


PROBLEM 7.103

Two cables of the same gauge are attached to a transmission tower at *B*. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at *B* is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag *h*, (*b*) the maximum tension in each cable.

SOLUTION

Half-cable FBDs:



$T_{1x} = T_{2x}$ to create zero horizontal force on tower $\rightarrow$ thus $T_{01} = T_{02}$

FBD I: $\quad \circlearrowleft \Sigma M_B = 0\colon (15\text{ m})\left[w(30\text{ m})\right] - h_1 T_0 = 0$

$$h_1 = \frac{\left(450\text{ m}^2\right)w}{T_0}$$

FBD II: $\quad \circlearrowleft \Sigma M_B = 0\colon (2\text{ m})T_0 - (10\text{ m})\left[w(20\text{ m})\right] = 0$

$$T_0 = (100\text{ m})w$$

(*a*) $$h_1 = \frac{\left(450\text{ m}^2\right)w}{(100\text{ m})w} = 4.50\text{ m}$$

FBD I: $\quad \rightarrow \Sigma F_x = 0\colon T_{1x} - T_0 = 0$

$$T_{1x} = (100\text{ m})w$$

$$\uparrow \Sigma F_y = 0\colon T_{1y} - (30\text{ m})w = 0$$

$$T_{1y} = (30\text{ m})w$$

$$T_1 = \sqrt{(100\text{ m})^2 + (30\text{ m})^2}\,w$$

$$= (104.4\text{ m})(0.4\text{ kg/m})\left(9.81\text{ m/s}^2\right)$$

$$= 409.7\text{ N}$$

PROBLEM 7.103 CONTINUED

FBD II:

$$\uparrow \Sigma F_y = 0: T_{2y} - (20\text{ m})w = 0$$

$$T_{2y} = (20\text{ m})w$$

$$T_{2x} = T_{1x} = (100\text{ m})w$$

$$T_2 = \sqrt{(100\text{ m})^2 + (20\text{ m})^2}\,w = 400.17\text{ N}$$

(*b*) $T_1 = 410\text{ N}$ ◀

$T_2 = 400\text{ N}$ ◀

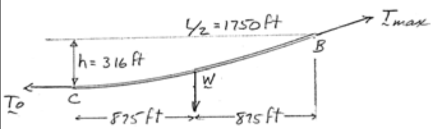
* Since $h \ll L$ it is reasonable to approximate the cable weight as being distributed uniformly along the horizontal. The methods of section 7.10 are more accurate for cables sagging under their own weight.

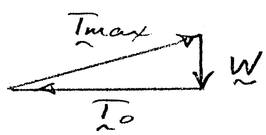
PROBLEM 7.104

The center span of the George Washington Bridge, as originally constructed, consisted of a uniform roadway suspended from four cables. The uniform load supported by each cable was $w = 9.75$ kips/ft along the horizontal. Knowing that the span L is 3500 ft and that the sag h is 316 ft, determine for the original configuration (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

FBD half-span:





$$W = (9.75 \text{ kips/ft})(1750 \text{ ft}) = 17{,}062.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_B = 0\text{:}\ (875 \text{ ft})(17{,}065 \text{ kips}) - (316 \text{ ft})T_0 = 0$$

$$T_0 = 47{,}246 \text{ kips}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}246 \text{ kips})^2 + (17{,}063 \text{ kips})^2}$$

(*a*) $T_{\max} = 50{,}200 \text{ kips}$ ◀

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (1750 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^4 + \cdots\right]$$

$$= 1787.3 \text{ ft}$$

(*b*) $l = 2s_B = 3575 \text{ ft}$ ◀

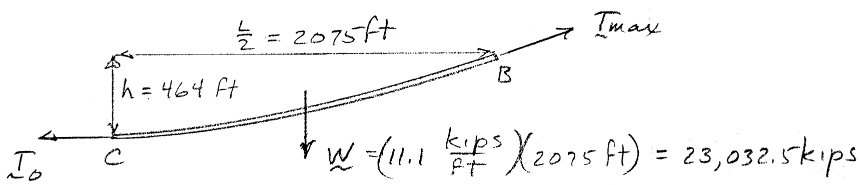
* To get 3-digit accuracy, only two terms are needed.

PROBLEM 7.105

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

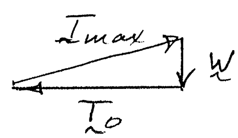
FBD half-span:



(*a*)

$$\circlearrowleft \Sigma M_B = 0: \left(\frac{2075 \text{ ft}}{2}\right)(23032.5 \text{ kips}) - (464 \text{ ft})T_0 = 0$$

$$T_0 = 47,246 \text{ kips}$$



$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47,246 \text{ kips})^2 + (23,033 \text{ kips})^2} = 56,400 \text{ kips} \blacktriangleleft$$

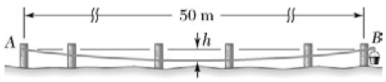
(*b*)

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (2075 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^4 + \cdots\right]$$

$$s_B = 2142 \text{ ft} \qquad l = 2s_B \qquad\qquad l = 4284 \text{ ft} \blacktriangleleft$$

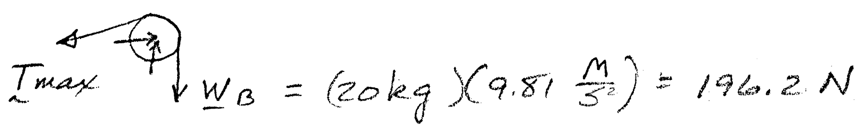
PROBLEM 7.106



To mark the positions of the rails on the posts of a fence, a homeowner ties a cord to the post at *A*, passes the cord over a short piece of pipe attached to the post at *B*, and ties the free end of the cord to a bucket filled with bricks having a total mass of 20 kg. Knowing that the mass per unit length of the rope is 0.02 kg/m and assuming that *A* and *B* are at the same elevation, determine (*a*) the sag *h*, (*b*) the slope of the cable at *B*. Neglect the effect of friction.

SOLUTION

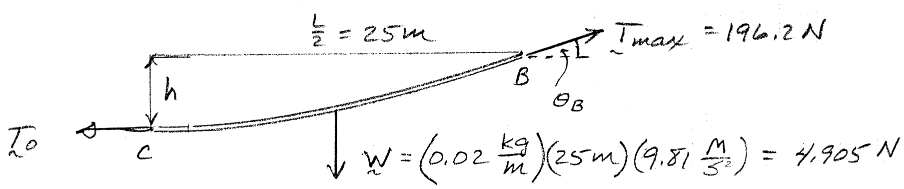
FBD pulley:

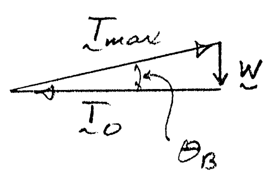


$$\circlearrowleft \Sigma M_P = 0: \left(T_{\max} - W_B\right)r = 0$$

$$T_{\max} = W_B = 196.2 \text{ N}$$

FBD half-span:*





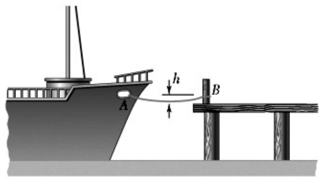
$$T_0 = \sqrt{T_{\max}^2 - W^2} = \sqrt{(196.2 \text{ N})^2 - (4.91 \text{ N})^2} = 196.139 \text{ N}$$

$$\circlearrowleft \Sigma M_B = 0: \left(\frac{25 \text{ m}}{2}\right)(4.905 \text{ N}) - h(196.139 \text{ N}) = 0$$

(*a*) $h = 0.3126 \text{ m} = 313 \text{ mm}$ ◀

(*b*) $\theta_B = \sin^{-1}\dfrac{W}{T_{\max}} = \sin^{-1}\left(\dfrac{4.905 \text{ N}}{196.2 \text{ N}}\right) = 1.433°$ ◀

*See note Prob. 7.103

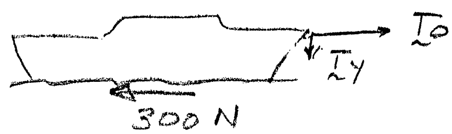


PROBLEM 7.107

A small ship is tied to a pier with a 5-m length of rope as shown. Knowing that the current exerts on the hull of the ship a 300-N force directed from the bow to the stern and that the mass per unit length of the rope is 2.2 kg/m, determine (*a*) the maximum tension in the rope, (*b*) the sag *h*. [*Hint:* Use only the first two terms of Eq. (7.10).]

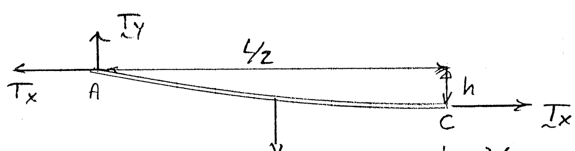
SOLUTION

(*a*) **FBD ship:**



$$\rightarrow \Sigma F_x = 0: T_0 - 300\text{ N} = 0 \qquad T_0 = 300\text{ N}$$

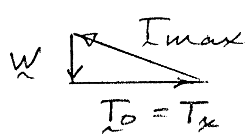
FBD half-span:*



$$\underset{\sim}{W} = \left(2.2\ \frac{\text{kg}}{\text{m}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.5\text{ m}) = 53.955\text{ N}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(300\text{ N})^2 = (54\text{ N})^2} = 305\text{ N} \blacktriangleleft$$

(*b*) $$\circlearrowleft \Sigma M_A = 0: hT_x - \frac{L}{4}W = 0 \qquad h = \frac{LW}{4T_x}$$



$$s = x\left[1 + \frac{2}{3}\left(\frac{4}{x}\right)^2 + \cdots\right] \qquad \text{but} \qquad y_A = h = \frac{LW}{4T_x} \qquad \text{so} \qquad \frac{y_A}{x_A} = \frac{W}{2T_x}$$

$$(2.5\text{ m}) = \frac{L}{2}\left[1 + \frac{2}{3}\left(\frac{53.955\text{ N}}{600\text{ N}}\right)^2 - \cdots\right] \rightarrow L = 4.9732\text{ m}$$

$$\text{So} \quad h = \frac{LW}{4T_x} = 0.2236\text{ m} \qquad\qquad h = 224\text{ mm} \blacktriangleleft$$

*See note Prob. 7.103

PROBLEM 7.108

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allowed for the effect of extreme temperature changes which cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

Knowing

$$l = 2s_{\text{TOT}} = L\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \frac{2}{5}\left(\frac{h}{L/2}\right)^2 + \cdots\right]$$

Winter:

$$l_w = (4260 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{386 \text{ ft}}{2130 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{386 \text{ ft}}{2130 \text{ ft}}\right)^4 + \cdots\right] = 4351.43 \text{ ft}$$

Summer:

$$l_s = (4260 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{394 \text{ ft}}{2130 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{394 \text{ ft}}{2130 \text{ ft}}\right)^4 + \cdots\right] = 4355.18 \text{ ft}$$

$$\Delta l = l_s - l_w = 3.75 \text{ ft} \blacktriangleleft$$

PROBLEM 7.109

A cable of length $L+\Delta$ is suspended between two points which are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 30$ m and $\Delta = 1.2$ m, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

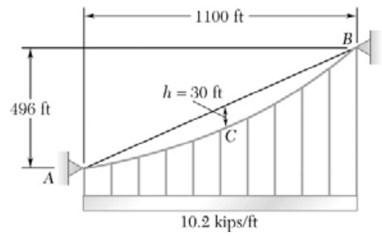
(*a*)

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \cdots\right]$$

$$L + \Delta = 2s_{\text{TOT}} = L\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \cdots\right]$$

$$\frac{\Delta}{L} = \frac{2}{3}\left(\frac{2h}{L}\right)^2 = \frac{8}{3}\left(\frac{h}{L}\right)^2 \rightarrow h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*) For $L = 30$ m, $\Delta = 1.2$ m $\quad h = 3.67$ m $\blacktriangleleft$

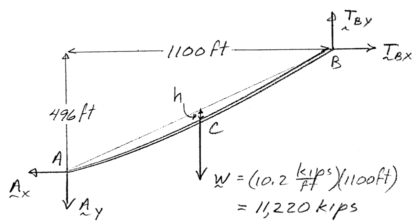


PROBLEM 7.110

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance *h* from each cable to the chord *AB* is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at *B*.

SOLUTION

FBD AB:

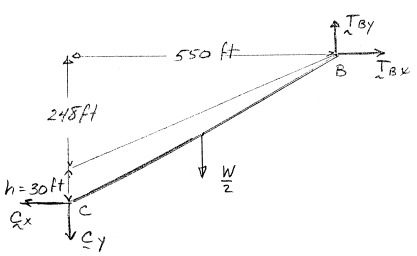


$$\circlearrowleft \Sigma M_A = 0: (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:

$$\circlearrowleft \Sigma M_C = 0: (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$



$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

Solving (1) and (2)

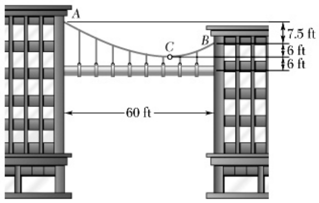
$$T_{By} = 28,798 \text{ kips}$$

$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that (*a*) $T_{max} = 58,900$ kips ◀

(*b*) $\theta_B = 29.2°$ ◀

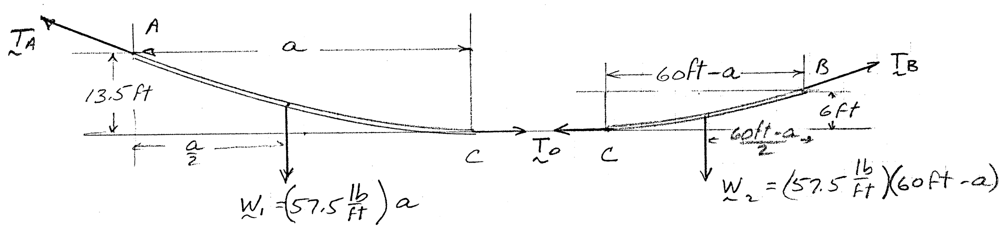


PROBLEM 7.111

A steam pipe weighting 50 lb/ft that passes between two buildings 60 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable is equivalent to a uniformly distributed loading of 7.5 lb/ft, determine (*a*) the location of the lowest point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION

FBD AC: **FBD CB:**



$$\Sigma M_A = 0: (13.5 \text{ ft})T_0 - \frac{a}{2}(57.5 \text{ lb/ft})a = 0$$

$$T_0 = (2.12963 \text{ lb/ft}^2)a^2 \qquad (1)$$

$$\Sigma M_B = 0: \frac{60 \text{ ft} - a}{2}(57.5 \text{ lb/ft})(60 \text{ ft} - a) - (6 \text{ ft})T_0 = 0$$

$$6T_0 = (28.75 \text{ lb/ft}^2)\left[3600 \text{ ft}^2 - (120 \text{ ft})a + a^2\right] \qquad (2)$$

Using (1) in (2) $\quad 0.55a^2 - (120 \text{ ft})a + 3600 \text{ ft}^2 = 0$

Solving: $\quad a = (108 \pm 72)$ ft $\qquad a = 36$ ft (180 ft out of range)

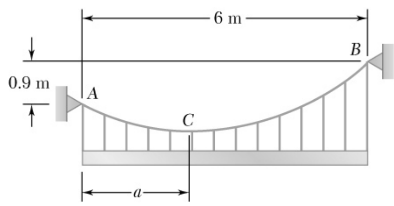
So *C* is 36 ft from *A*

(*a*) *C* is 6 ft below and 24 ft left of *B* ◀

$$T_0 = 2.1296 \text{ lb/ft}^2 (36 \text{ ft})^2 = 2760 \text{ lb}$$

$$W_1 = (57.5 \text{ lb/ft})(36 \text{ ft}) = 2070 \text{ lb}$$

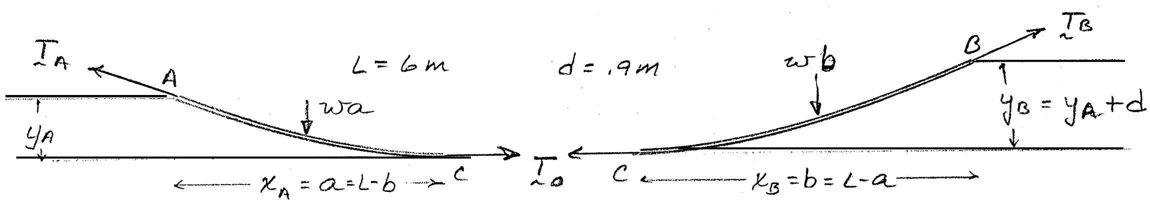
(*b*) $\quad T_{max} = T_A = \sqrt{T_0^2 + W_1^2} = \sqrt{(2760 \text{ lb})^2 + (2070 \text{ lb})^2} = 3450 \text{ lb}$ ◀



PROBLEM 7.112

Chain AB supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. If the maximum tension in the cable is not to exceed 8 kN, determine (a) the horizontal distance a from A to the lowest point C of the chain, (b) the approximate length of the chain.

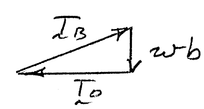
SOLUTION



$$+\circlearrowleft \Sigma M_A = 0:\ y_A T_0 - \frac{a}{2} wa = 0 \qquad +\circlearrowleft \Sigma M_B = 0:\ -y_B T_0 + \frac{b}{2} wb = 0$$

$$y_A = \frac{wa^2}{2T_0} \qquad y_B = \frac{wb^2}{2T_0}$$

$$d = \left(y_B - y_B\right) = \frac{w}{2T_0}\left(b^2 - a^2\right)$$



But $\qquad T_0 = \sqrt{T_B^2 - (wb)^2} = \sqrt{T_{\max}^2 - (wb)^2}$

$$\therefore \quad (2d)^2\left[T_{\max}^2 - (wb)^2\right] = w^2\left(b^2 - a^2\right)^2 = L^2 w^2\left(4b^2 - 4Lb + L^2\right)$$

or $\qquad 4\left(L^2 + d^2\right)b^2 - 4L^3 b + \left(L^4 - 4d^2\frac{T_{\max}^2}{w^2}\right) = 0$

Using $\qquad L = 6$ m, $\quad d = 0.9$ m, $\quad T_{\max} = 8$ kN, $\quad w = (85\text{ kg/m})\left(9.81\text{ m/s}^2\right) = 833.85$ N/m

yields $\qquad b = (2.934 \pm 1.353)$ m $\qquad b = 4.287$ m $\qquad$ (since $b > 3$ m)

(a) $\qquad a = 6\text{ m} - b = 1.713\text{ m}$ ◀

PROBLEM 7.112 CONTINUED

$$T_0 = \sqrt{T_{\max}^2 - (wb)^2} = 7156.9 \text{ N}$$

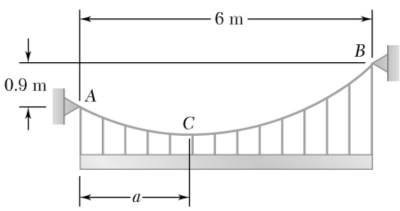
$$\frac{y_A}{x_A} = \frac{wa}{2T_0} = 0.09979 \qquad \frac{y_B}{x_B} = \frac{wb}{2T_0} = 0.24974$$

$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 + \cdots\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= (1.713 \text{ m})\left[1 + \frac{2}{3}(0.09979)^2\right] + (4.287 \text{ m})\left[1 + \frac{2}{3}(0.24974)^2\right] = 6.19 \text{ m}$$

(b) $\qquad l = 6.19 \text{ m}$ ◀

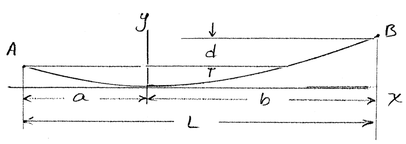
PROBLEM 7.113



Chain *AB* of length 6.4 m supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. Determine (*a*) the horizontal distance *a* from *A* to the lowest point *C* of the chain, (*b*) the maximum tension in the chain.

SOLUTION

Geometry:

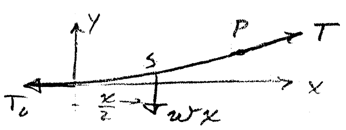


$$\circlearrowleft \Sigma M_P = 0: \frac{x}{2}wx - yT_0 = 0$$

$$y = \frac{wx^2}{2T_0} \qquad \text{so} \qquad \frac{y}{x} = \frac{wx}{2T_0}$$

$$\text{and } d = y_B - y_A = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

FBD Segment:



Also
$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{a}\right)^2\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{b}\right)^2\right]$$

$$l - L = \frac{2}{3}\left[\left(\frac{y_A}{a}\right)^2 + \left(\frac{y_B}{b}\right)^2\right] = \frac{w^2}{6T_0^2}\left(a^3 + b^3\right)$$

$$= \frac{1}{6}\frac{4d^2}{\left(b^2 - a^2\right)^2}\left(a^3 + b^3\right) = \frac{2}{3}\frac{d^2\left(a^3 + b^3\right)}{\left(b^2 - a^2\right)^2}$$

Using $l = 6.4$ m, $L = 6$ m, $d = 0.9$ m, $b = 6\text{ m} - a$, and solving for a, knowing that $a < 3$ ft

$a = 2.2196$ m (*a*) $a = 2.22$ m ◀

Then
$$T_0 = \frac{w}{2d}\left(b^2 - a^2\right)$$

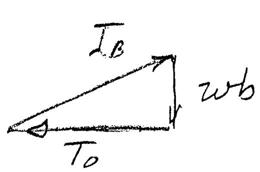
And with
$$w = \left(85\text{ kg/m}\right)\left(9.81\text{ m/s}^2\right) = 833.85\text{ N/m}$$

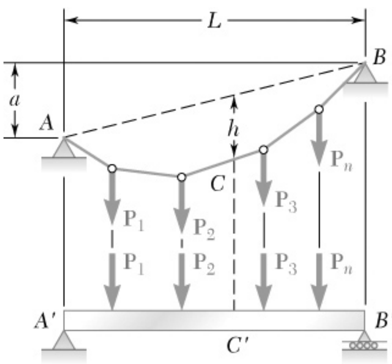
And
$$b = 6\text{ m} - a = 3.7804\text{ m} \qquad T_0 = 4338\text{ N}$$

$$T_{\max} = T_B = \sqrt{T_0^2 + (wb)^2}$$

$$= \sqrt{(4338\text{ N})^2 + (833.85\text{ N/m})^2(3.7804\text{ m})^2}$$

$T_{\max} = 5362$ N (*b*) $T_{\max} = 5.36$ kN ◀



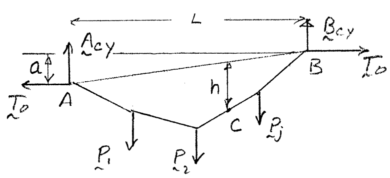


PROBLEM 7.114

A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product , $T_0 h$ where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between point C and the chord joining the points of support A and B.

SOLUTION

FBD Cable:

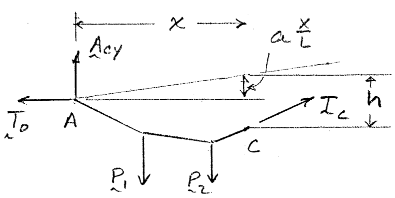


$$\circlearrowleft \Sigma M_B = 0: LA_{Cy} + aT_0 - \Sigma M_{B\text{ loads}} = 0 \qquad (1)$$

(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\text{ left}} = 0 \qquad (2)$$

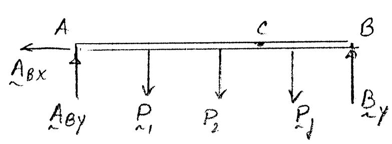
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} = 0 \qquad (3)$$

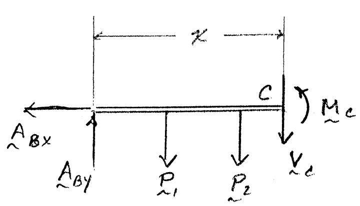
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0: LA_{By} - \Sigma M_{B\text{ loads}} = 0 \qquad (4)$$

$$\circlearrowleft \Sigma M_C = 0: xA_{By} - \Sigma M_{C\text{ left}} - M_C = 0 \qquad (5)$$

FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.