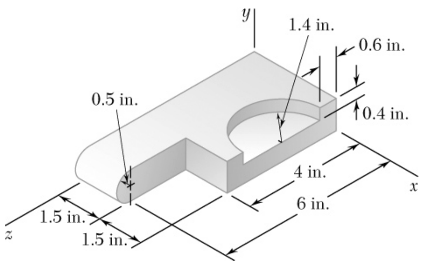
PROBLEM 9.151



Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION

From the solution to Problem 9.147

$$m_1 = 105.676 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_3 = 5.1874 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = 26.419 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_4 = 10.8451 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each component

Now $$I_{uv} = \cancelto{0}{\bar{I}_{u'v'}} + m\bar{u}\,\bar{v} = m\bar{u}\,\bar{v}$$

so that $$(I_{uv})_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$$

Then $$I_{xy} = \Sigma m\bar{x}\,\bar{y} = (105.676 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{1.5}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right) + (26.419 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{0.75}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right)$$

$$+(5.1874 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{0.75}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right)$$

$$-(10.8451 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(3\text{ in.} - \frac{4 \times 1.4 \text{ in.}}{3\pi}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]\left(\frac{0.8}{12}\text{ft}\right)$$

$$= (550.40 + 68.799 + 13.5089 - 144.952) \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 487.76 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{xy} = 0.488 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.151 CONTINUED

$$I_{yz} = \Sigma m\bar{y}\,\bar{z} = \left(105.676\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left(\frac{2}{12}\text{ ft}\right) + \left(26.419\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left(\frac{5}{12}\text{ ft}\right)$$

$$+\left(5.1874\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left[\left(6\text{ in.} + \frac{4\times0.5\text{ in.}}{3\pi}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]$$

$$-\left(10.8451\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.8}{12}\text{ ft}\right)\left(\frac{2}{12}\text{ ft}\right)$$

$$= \left(733.86 + 458.66 + 111.893 - 120.501\right)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 1183.91\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or } I_{yz} = 1.184\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

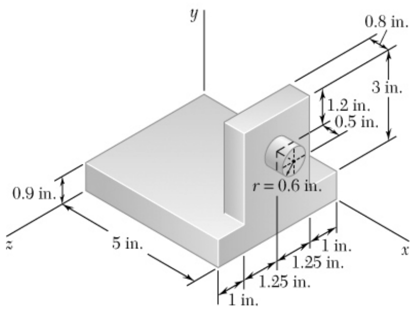
$$I_{zx} = \Sigma m\bar{z}\,\bar{x} = \left(105.676\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{2}{12}\text{ ft}\right)\left(\frac{1.5}{12}\text{ ft}\right) + \left(26.419\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{5}{12}\text{ ft}\right)\left(\frac{0.75}{12}\text{ ft}\right)$$

$$+\left(5.1874\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(6 + \frac{4\times0.5}{3\pi}\right)\text{in.}\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]\left(\frac{0.75}{12}\text{ ft}\right)$$

$$-\left(10.8451\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{2}{12}\text{ ft}\right)\left[\left(3 - \frac{4\times1.4}{3\pi}\right)\text{in.}\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]$$

$$= \left(2201.6 + 687.99 + 167.840 - 362.38\right)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

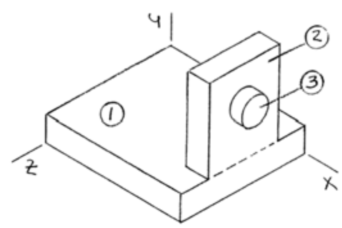
$$= 2695.1\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or } I_{zx} = 2.70\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$



PROBLEM 9.152

Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The specific weight of steel is $0.284\ \text{lb/in}^3$.)

SOLUTION



First compute the mass of each component $\qquad m = \dfrac{\gamma}{g}V$

Then

$$m_1 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}(5\ \text{in.} \times 4.5\ \text{in.} \times 0.9\ \text{in.}) = 0.1786\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}(3\ \text{in.} \times 2.5\ \text{in.} \times 0.8\ \text{in.}) = 0.05292\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left[\pi(0.6\ \text{in.})^2 \times 0.5\ \text{in.}\right] = 0.0049875\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now $I_{uv} = \cancel{\bar{I}_{u'v'}}^{\,0} + m\bar{u}\bar{v}$ so that $(I_{uv})_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$.

	m, $\text{lb}\cdot\text{s}^2/\text{ft}$	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{y}\,\bar{z}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{z}\,\bar{x}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$
1	0.1786	2.5 0.20833	0.45 0.0375	2.25 0.1875	0.20093 $1.39531\cdot10^{-3}$	0.18083 $1.25578\cdot10^{-3}$	1.0046 $6.97656\cdot10^{-3}$
2	0.05292	4.6 0.38333	2.40 0.20	2.25 0.1875	0.58424 $4.0572\cdot10^{-3}$	0.28577 $1.98451\cdot10^{-3}$	0.54772 $3.80362\cdot10^{-3}$
3	0.0049875	5.25 0.4375	2.70 0.225	2.25 0.1875	0.07069 $0.49095\cdot10^{-3}$	0.03030 $0.21041\cdot10^{-3}$	0.05891 $0.40913\cdot10^{-3}$
Σ					0.85586 $5.94347\cdot10^{-3}$	0.4969 $3.45069\cdot10^{-3}$	1.61123 $11.18909\cdot10^{-3}$

PROBLEM 9.152 CONTINUED

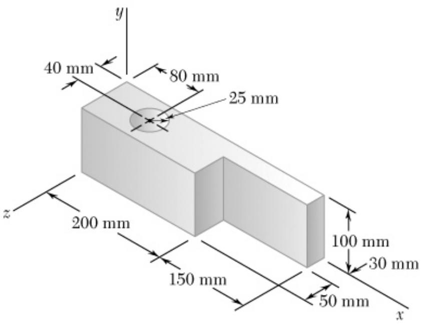
Then

or $I_{xy} = 5.94 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

or $I_{yz} = 3.45 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

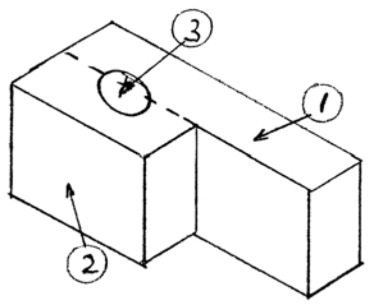
or $I_{zx} = 11.19 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

PROBLEM 9.153



Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The density of aluminum is 2700 kg/m^3.)

SOLUTION



Have $$m = \rho_{\text{al}} V$$

Then $$m_1 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)(0.350 \times 0.100 \times 0.030)\,\text{m}^3$$

$$= 2.8350\,\text{kg}$$

$$m_2 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)(0.200 \times 0.100 \times 0.050)\,\text{m}^3$$

$$= 2.7000\,\text{kg}$$

$$m_3 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)\left[\pi(0.025)^2 \times 0.100\right]\text{m}^3$$

$$= 0.53014\,\text{kg}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = I_{z'x'} = 0$ for each component

Now $$I_{uv} = I_{u'v'} + m\bar{u}\,\bar{v}$$

where $$\bar{I}_{u'v'} = 0$$

$$I_{xy} = \Sigma m\bar{x}\,\bar{y} = (2.8350\,\text{kg})(0.175\,\text{m})(0.050\,\text{m})$$

$$+(2.7000\,\text{kg})(0.100\,\text{m})(0.050\,\text{m}) - (0.53014\,\text{kg})(0.080\,\text{m})(0.050\,\text{m})$$

$$= (24.806 + 13.500 - 2.1206) \times 10^{-3}\,\text{kg}\cdot\text{m}^2 = 36.1854 \times 10^{3}\,\text{kg}\,\text{m}^2$$

or $I_{xy} = 36.2 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \Sigma m\bar{y}\,\bar{z} = (2.8350\,\text{kg})(0.050\,\text{m})(0.015\,\text{m})$$

$$+(2.7000\,\text{kg})(0.050\,\text{m})(0.055\,\text{m}) - (0.53014\,\text{kg})(0.050\,\text{m})(0.040\,\text{m})$$

$$= (2.1263 + 7.4250 - 1.06028) \times 10^{-3}\,\text{kg}\cdot\text{m}^2 = 8.49102 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

or $I_{yz} = 8.49 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.153 CONTINUED

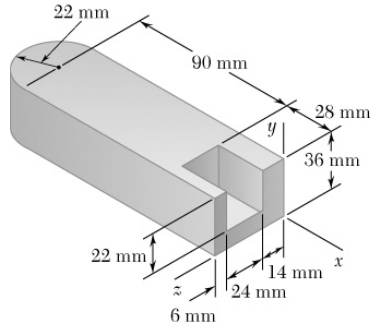
$$I_{zx} = \Sigma m\bar{z}\,\bar{x} = (2.8350\,\text{kg})(0.015\,\text{m})(0.175\,\text{m})$$

$$+(\,2.7000\,\text{kg})(0.055\,\text{m})(0.100\,\text{m}) - (0.53014\,\text{kg})(0.040\,\text{m})(0.080\,\text{m})$$

$$= (7.4419 + 14.850 - 1.69645)10^{-3}\,\text{kg}\cdot\text{m}^2 = 20.59545\times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

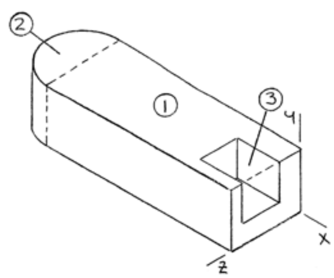
or $I_{zx} = 20.6\times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.154



Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The density of aluminum is 2700 kg/m^3.)

SOLUTION



Have $\quad m = \rho V$

Then
$$m_1 = \left(2700\text{ kg/m}^3\right)\left(0.118 \times 0.036 \times 0.044\right)\text{ m}^3 = 0.50466\text{ kg}$$

$$m_2 = \left(2700\text{ kg/m}^3\right)\left[\frac{\pi}{2}(0.022)^2 \times 0.036\right]\text{m}^3 = 0.07389\text{ kg}$$

$$m_3 = \left(2700\text{ kg/m}^3\right)\left(0.028 \times 0.022 \times 0.024\right)\text{ m}^3 = 0.03992\text{ kg}$$

Now observe that $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$ and $\bar{I}_{z'x'}$ are zero because of symmetry

Now
$$\bar{x}_2 = -\left(0.118 + \frac{4 \times 0.023}{3\pi}\right)\text{m} = -0.12734\text{ m}$$

$$\bar{y}_3 = \left(0.036 - \frac{0.062}{2}\right)\text{m} = 0.025\text{ m}$$

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$ kg·m^2	$m\bar{y}\,\bar{z}$ kg·m^2	$m\bar{z}\,\bar{x}$ kg·m^2
1	0.50466	–0.059	0.018	0.022	-0.53595×10^{-3}	0.19985×10^{-3}	-0.65505×10^{-3}
2	0.07389	–0.12734	0.018	0.022	-0.16932×10^{-3}	0.02926×10^{-3}	-0.20695×10^{-3}
3	0.03992	–0.041	0.025	0.026	-0.01397×10^{-3}	0.02594×10^{-3}	-0.01453×10^{-3}

PROBLEM 9.154 CONTINUED

And

$$I_{xy} = \Sigma\left(\cancel{I_{x'y'}}^{\,0} + m\bar{x}\,\bar{y}\right)$$

$$I_{yz} = \Sigma\left(\cancel{I_{y'z'}}^{\,0} + m\bar{y}\,\bar{z}\right)$$

$$I_{zx} = \Sigma\left(\cancel{I_{z'x'}}^{\,0} + m\bar{x}\,\bar{z}\right)$$

Finally $$I_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2 - \left(I_{xy}\right)_3 = -0.6913\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -0.691\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

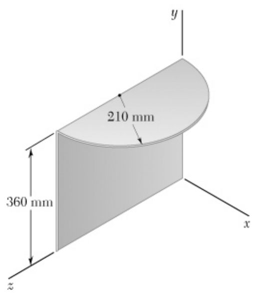
$$I_{yz} = \left(I_{yz}\right)_1 + \left(I_{yz}\right)_2 - \left(I_{yz}\right)_3 = 0.20317\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{yz} = 0.203\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{zx} = \left(I_{zx}\right)_1 + \left(I_{zx}\right)_2 - \left(I_{zx}\right)_3 = -0.84747\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

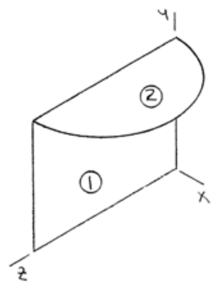
or $I_{zx} = -0.848\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.155



A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is 7860 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



Have

$$m = \rho v = \rho_{st} tA$$

Then

$$m_1 = \left(7860 \text{ kg/m}^3\right)\left(0.003 \times 0.420 \times 0.360\right)\text{m}^3 = 3.5653 \text{ kg}$$

$$m_2 = \left(7860 \text{ kg/m}^3\right)\left(0.003 \text{ m}\right)\left[\frac{\pi}{2}\left(0.210 \text{ m}\right)^2\right] = 1.6334 \text{ kg}$$

$$\overline{x}_2 = \frac{4\left(0.210 \text{ m}\right)}{3\pi} = 0.089127 \text{ m}$$

Now observe that

$$\overline{I}_{x'y'} = \overline{I}_{y'z'} = \overline{I}_{z'x'} = 0$$

	m, kg	$\overline{x}$, m	$\overline{y}$, m	$\overline{z}$, m	$m\overline{x}\,\overline{y}$, $\text{kg}\cdot\text{m}^2$	$m\overline{y}\,\overline{z}$, $\text{kg}\cdot\text{m}^2$	$m\overline{z}\,\overline{x}$, $\text{kg}\cdot\text{m}^2$
1	3.5653	0	0.8	0.21	0	134.768×10^{-3}	0
2	1.6334	0.089127	0.36	0.21	52.409×10^{-3}	123.485×10^{-3}	30.572×10^{-3}
Σ					52.409×10^{-3}	258.253×10^{-3}	30.572×10^{-3}

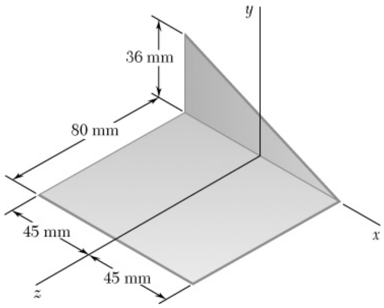
Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\overline{I}_{x'y'}} + m\overline{x}\,\overline{y}\right) \qquad \text{or} \quad I_{xy} = 52.4 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\overline{I}_{y'z'}} + m\overline{y}\,\overline{z}\right) \qquad \text{or} \quad I_{yz} = 258 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

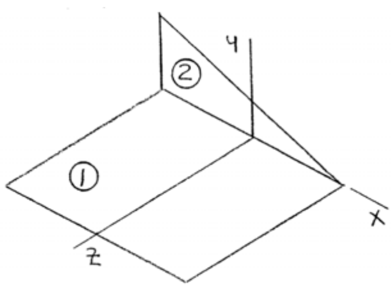
$$I_{zx} = \Sigma\left(\cancelto{0}{\overline{I}_{z'x'}} + m\overline{z}\,\overline{x}\right) \qquad \text{or} \quad I_{zx} = 30.6 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

PROBLEM 9.156



A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is $7860\ \text{kg/m}^3$, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component

Have $$m = \rho_{sT} V = \rho_{sT} tA$$

Then $$m_1 = \left(7860\ \text{kg/m}^3\right)\left[(0.003)(0.08)(0.09)\right]\text{m}^3$$

$$= 0.169776\ \text{kg}$$

$$m_2 = 7860\ \text{kg/m}^3\left[(0.003)\left(\frac{1}{2} \times 0.09 \times 0.036\right)\right]\text{m}^3$$

$$= 0.03820\ \text{kg}$$

Now observe that

$$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{y'z'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0$$

$$\left(\bar{I}_{y'z'}\right)_2 = \left(\bar{I}_{z'x'}\right)_2 = 0$$

From Sample Problem 9.6 $$\left(\bar{I}_{x'y'}\right)_{2,\text{area}} = -\frac{1}{72}b_2^2h_2^2$$

Then $$\left(\bar{I}_{x'y'}\right)_2 = \rho_{sT} t\left(I_{x'y'}\right)_{2,\text{area}} = \rho_{sT} t\left(-\frac{1}{72}b_2^2h_2^2\right) = -\frac{1}{36}m_2b_2h_2$$

Also $$\bar{x}_1 = \bar{y}_1 = \bar{z}_2 = 0 \qquad \bar{x}_2 = \left(-0.045 + \frac{0.09}{3}\right)\text{m} = -0.015\ \text{m}$$

PROBLEM 9.156 CONTINUED

Finally..

$$I_{xy} = \Sigma\left(\bar{I}_{xy} + m\bar{x}\,\bar{y}\right) = (0+0) + \left[-\frac{1}{36}(0.03820\,\text{kg})(0.09\,\text{m})(0.036\,\text{m})\right.$$

$$\left. + (0.03820\,\text{kg})(-0.015\,\text{m})\left(\frac{0.036\,\text{m}}{3}\right)\right]$$

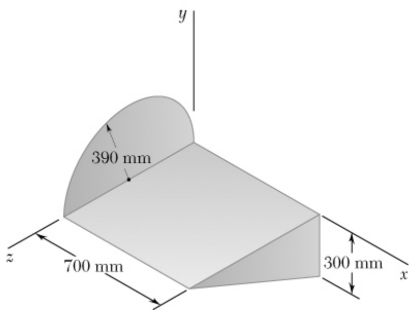
$$= \left(-3.4379\times10^{-6} - 6.876\times10^{-6}\right)\text{kg}\cdot\text{m}^2 = -10.3139\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -10.31\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

And $I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\,\bar{y}\,\bar{z}\right) = (0+0) + (0+0) = 0$ or $I_{yz} = 0$ ◀

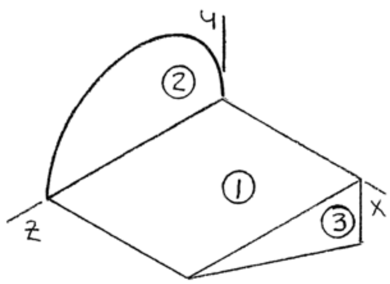
$I_{zx} = \Sigma\left(\bar{I}_{zx'} + m\,\bar{z}\,\bar{x}\right) = (0+0) + (0+0) = 0$ or $I_{zx} = 0$ ◀

PROBLEM 9.157



A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is 7860 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component

Have.. $$m = \rho_{sT}V = \rho_{sT}tA$$

Then $$m_1 = \left(7860 \text{ kg/m}^3\right)\left[(0.003)(0.7)(0.78)\right]\text{m}^3 = 12.785 \text{ kg}$$

$$m_2 = \left(7860 \text{ kg/m}^3\right)\left[(0.003)\left(\frac{\pi}{2}\times 0.39^2\right)\right]\text{m}^3 = 5.6337 \text{ kg}$$

$$m_3 = \left(7860 \text{ kg/m}^3\right)\left[(0.003)\left(\frac{1}{2}\times 0.78\times 0.3\right)\right]\text{m}^3 = 2.7589 \text{ kg}$$

Now observe that because of symmetry the centroidal products of inertia of components 1 and 2 are zero and $\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{z'x'}\right)_3 = 0$

Also $$\left(\bar{I}_{y'z'}\right)_{3,\text{mass}} = \rho_{sT}t\left(\bar{I}_{y'z'}\right)_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a $90°$ rotation, have

$$\left(\bar{I}_{y'z'}\right)_{3,\text{area}} = \frac{1}{72}b_3^2h_3^2$$

Then

$$\left(\bar{I}_{y'z'}\right)_3 = \rho_{sT}t\left(\frac{1}{72}b_3^2h_3^2\right) = \frac{1}{36}m_3b_3h_3$$

Also $$\bar{y}_1 = \bar{x}_2 = 0 \qquad \bar{y}_2 = \frac{4\times 0.39 \text{ m}}{3\pi} = 0.16552 \text{ m}$$

Finally $$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) = (0+0) + (0+0)$$

$$+\left[0 + (2.7589 \text{ kg})(0.7 \text{ m})\left(\frac{0.3\text{m}}{2}\right)\right] = -0.19312 \text{ kg}\cdot\text{m}^2$$

or $I_{xy} = -0.1931 \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.157 CONTINUED

$$I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\bar{y}\,\bar{z}\right)$$

$$= (0 + 0) + \left[0 + (5.6337\text{ kg})(0.16552\text{ m})(0.39\text{ m})\right]$$

$$+ (2.7589\text{ kg})\left[\frac{1}{36}(0.78\text{ m})(0.3\text{ m}) + \left(\frac{-0.3\text{ m}}{3}\right)\left(\frac{0.78\text{ m}}{3}\right)\right]$$

$$= (0.36367 + 0.017933 - 0.07173)\text{ kg}\cdot\text{m}^2 = 0.30987\text{ kg}\cdot\text{m}^2$$

or $I_{yz} = 0.310\text{ kg}\cdot\text{m}^2$ ◀

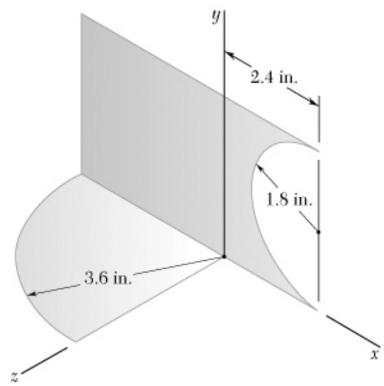
$$I_{zx} = \Sigma\left(\bar{I}_{z'x'} + m\bar{z}\,\bar{x}\right) = \left[0 + (12.875\text{ kg})(0.35\text{ m})(0.39\text{ m})\right]$$

$$+ (0 + 0) + \left[0 + (2.7589\text{ kg})\left(\frac{0.78\text{ m}}{2}\right)(0.7\text{ m})\right]$$

$$= (1.75744 + 0.50212)\text{ kg}\cdot\text{m}^2 = 2.25956\text{ kg}\cdot\text{m}^2$$

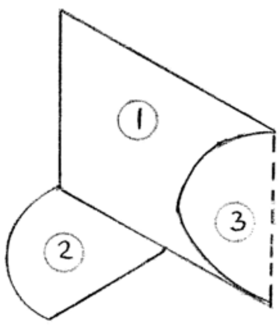
or $I_{zx} = 2.26\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.158



A section of sheet steel 0.08 in. thick is cut and bent into the machine component shown. Knowing that the specific weight of steel is 490 lb/ft^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First note $$m = \rho_{sT} V = \frac{\gamma_{sT}}{g} tA$$

Then $$m_1 = \left(\frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[(6 \times 3.6)\text{in}^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 15.2174 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(\frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[\frac{\pi}{2}(1.8 \text{ in.})^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 3.5855 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(\frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)(0.08 \text{ in.})\left[\frac{\pi}{4}(3.6 \text{ in.})^2\right]\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 7.1710 \times 10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$$

Note that symmetry implies $$\left(\bar{I}_{x'y'}\right)_{1,2} = \left(\bar{I}_{y'z'}\right)_{1,2} = \left(\bar{I}_{z'x'}\right)_{1,2} = 0$$

$$\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{y'z'}\right)_3 = 0$$

Now $$I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$$

PROBLEM 9.158 CONTINUED

Thus $$I_{xy} = \Sigma m\bar{x}\,\bar{y}$$

$$= m_1\bar{x}_1\bar{y}_1 - m_2\bar{x}_2\bar{y}_2 + m_3 x_3 \cancelto{0}{\bar{y}_3}$$

$$= \left(15.2174 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(-\frac{0.6}{12}\ \text{ft}\right)\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$-\left(3.5855 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(2.4 - \frac{4 \times 1.8}{3\pi}\right)\text{in.} \times \frac{1\ \text{ft}}{12\ \text{in.}}\right]\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$= \left(-114.131 - 73.326\right) \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{xy} = -187.5 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now $$I_{yz} = \Sigma m\bar{y}\,\bar{z} = m_1\bar{y}_1\cancelto{0}{\bar{z}_1} - m_2\bar{y}_2\cancelto{0}{\bar{z}_2} + m_3\bar{y}_3\cancelto{0}{\bar{z}_3}$$

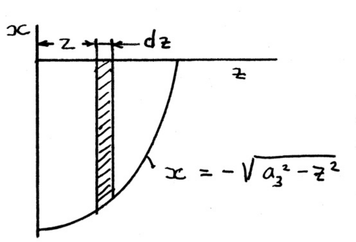
or $I_{yz} = 0$ ◀

Also $$I_{zx} = \left(I_{zx}\right)_1 - \left(I_{zx}\right)_2 + \left(I_{zx}\right)_3$$

$$= m_1\cancelto{0}{\bar{z}_1}\bar{x}_1 - m_2\cancelto{0}{\bar{z}_2}\bar{x}_2 + \left(I_{zx}\right)_3$$

Now determine $\left(I_{zx}\right)_3$

Have $$\left(dI_{zx}\right)_3 = \left(\cancelto{0}{d\bar{I}_{z'x'}}\right)_3 + \bar{z}\,\bar{x}\,dm$$



$$= (z)\left(-\frac{x}{2}\right)\left(\frac{\gamma_{sT}}{g}t\left|x\right|dz\right)$$

$$= -\frac{1}{2}\frac{\gamma_{sT}}{g}tz\left(a_3^2 - z^2\right)dz$$

Now $$m_3 = \frac{\gamma_{sT}}{g}t\left(\frac{\pi}{4}a_3^2\right)$$

or $$\frac{\gamma_{sT}}{g}t = \frac{4m_3}{\pi a_3^2}$$

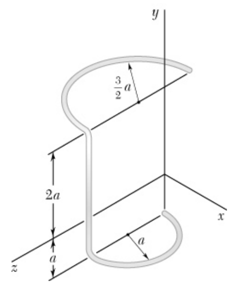
Therefore, $$\left(I_{zx}\right)_3 = \frac{2m_3}{\pi a_3^2}\int_0^a\left(a_3^2 z - z^3\right)dz = -\frac{2m_3}{\pi a_3^2}\left(\frac{1}{2}a_3^2 z^2 - \frac{1}{4}z^4\right)\Bigg|_0^a$$

$$= -\frac{1}{2\pi}m_3 a_3^2$$

Finally $$I_{zx} = -\frac{1}{2\pi}\left(7.1710 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{3.6}{12}\ \text{ft}\right)^2$$

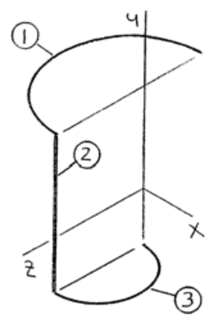
or $I_{zx} = -102.7 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.159



Brass wire with a weight per unit length w is used to form the figure shown. Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. Have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then..

$$m_1 = \frac{w}{g}\left(\pi \times \frac{3}{2}a\right) = \frac{3}{2}\pi \frac{w}{g}a$$

$$m_2 = \frac{w}{g}(3a) = 3\frac{w}{g}a$$

$$m_3 = \frac{w}{g}(\pi \times a) = \pi \frac{w}{g}a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of each component are zero because of symmetry.

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$\frac{3}{2}\pi\frac{w}{g}a$	$-\frac{2}{\pi}\left(\frac{3}{2}a\right)$	$2a$	$\frac{1}{2}a$	$-9\frac{w}{g}a^3$	$\frac{3}{2}\pi\frac{w}{g}a^3$	$-\frac{9}{4}\frac{w}{g}a^3$
2	$3\frac{w}{g}a$	0	$\frac{1}{2}a$	$2a$	0	$3\frac{w}{g}a^3$	0
3	$\pi\frac{w}{g}a$	$\frac{2}{\pi}(a)$	$-a$	a	$-2\frac{w}{g}a^3$	$-\pi\frac{w}{g}a^3$	$2\frac{w}{g}a^3$
Σ					$-11\frac{w}{g}a^3$	$\frac{w}{g}\left(\frac{\pi}{2}+3\right)a^3$	$-\frac{1}{4}\frac{w}{g}a^3$

PROBLEM 9.159 CONTINUED

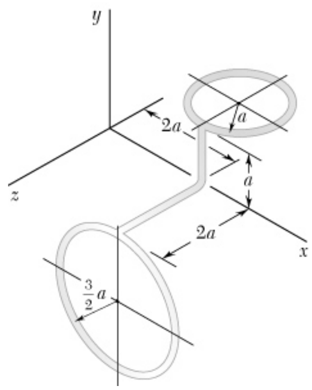
Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) \qquad \text{or} \quad I_{xy} = -11\frac{w}{g}a^3 \blacktriangleleft$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) \qquad \text{or} \quad I_{yz} = \frac{1}{2}\frac{w}{g}a^3(\pi + b) \blacktriangleleft$$

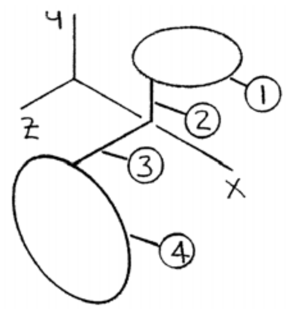
$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) \qquad \text{or} \quad I_{zx} = -\frac{1}{4}\frac{w}{g}a^3 \blacktriangleleft$$

PROBLEM 9.160



Brass wire with a weight per unit length w is used to form the figure shown. Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. Have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then

$$m_1 = \frac{w}{g}(2\pi \times a) = 2\pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g}(a) = \frac{w}{g} a$$

$$m_3 = \frac{w}{g}(2a) = 2\frac{w}{g} a$$

$$m_4 = \frac{w}{g}\left(2\pi \times \frac{3}{2} a\right) = 3\pi \frac{w}{g} a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

PROBLEM 9.160 CONTINUED

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$2\pi \frac{w}{g} a$	$2a$	a	$-a$	$4\pi \frac{w}{g} a^3$	$-2\pi \frac{w}{g} a^3$	$-4\pi \frac{w}{g} a^3$
2	$\frac{w}{g} a$	$2a$	$\frac{1}{2} a$	0	$\frac{w}{g} a^3$	0	0
3	$2\frac{w}{g} a$	$2a$	0	a	0	0	$4\frac{w}{g} a^3$
4	$3\pi \frac{w}{g} a$	$2a$	$-\frac{3}{2} a$	$2a$	$-9\pi \frac{w}{g} a^3$	$-9\pi \frac{w}{g} a^3$	$12\pi \frac{w}{g} a^3$
Σ					$\frac{w}{g}(1-5\pi)a^3$	$-11\pi \frac{w}{g} a^3$	$4\frac{w}{g}(1+2\pi)a^3$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) \quad \text{or} \quad I_{xy} = \frac{w}{g} a^3 (1 - 5\pi) \blacktriangleleft$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) \quad \text{or} \quad I_{yz} = -11\pi \frac{w}{g} a^3 \blacktriangleleft$$

$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) \quad \text{or} \quad I_{zx} = 4\frac{w}{g} a^3 (1 + 2\pi) \blacktriangleleft$$