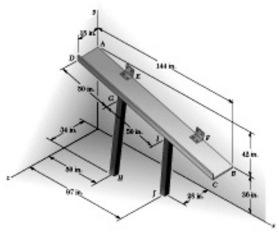
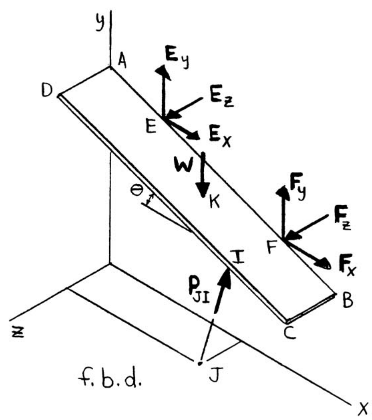
PROBLEM 4.143



While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *IJ* if prop *GH* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42 \text{ in.}}{144 \text{ in.}}\right) = 16.2602^\circ$$

$$x_I = (100 \text{ in.})\cos 16.2602^\circ = 96 \text{ in.}$$

$$y_I = 78 \text{ in.} - (100 \text{ in.})\sin 16.2602^\circ = 50 \text{ in.}$$

$$\lambda_{BA} = \frac{-(144 \text{ in.})\mathbf{i} + (42 \text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2} \text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{I/A} = (96 \text{ in.})\mathbf{i} - (78 \text{ in.} - 50 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k} = (96 \text{ in.})\mathbf{i} - (28 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56 \text{ lb})\mathbf{j}$$

$$\mathbf{P}_{JI} = \lambda_{JI} P_{JI}$$

$$= \frac{-(1 \text{ in.})\mathbf{i} + (50 \text{ in.})\mathbf{j} - (10 \text{ in.})\mathbf{k}}{\sqrt{(1)^2 + (50)^2 + (10)^2} \text{ in.}} P_{JI}$$

$$= \frac{P_{JI}}{51}(-\mathbf{i} + 50\mathbf{j} - 10\mathbf{k})$$

PROBLEM 4.143 CONTINUED

From the f.b.d. of the chute

$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{I/A} \times \mathbf{P}_{JI}\right) = 0$$

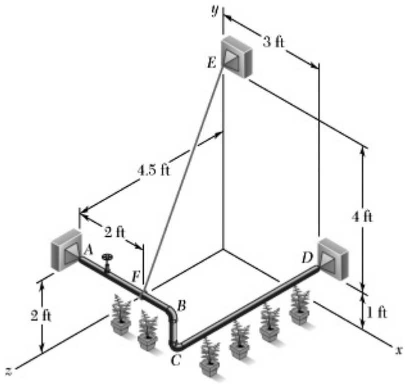
$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 96 & -28 & 18 \\ -1 & 50 & -10 \end{vmatrix} \left[\frac{P_{JI}}{51(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-28)(-10) - (-24)(18)(50) + 7(18)(-1) - (7)(96)(-10)\right]\frac{P_{JI}}{51(25)} = 0$$

$$\therefore \quad P_{JI} = 28.728 \text{ lb}$$

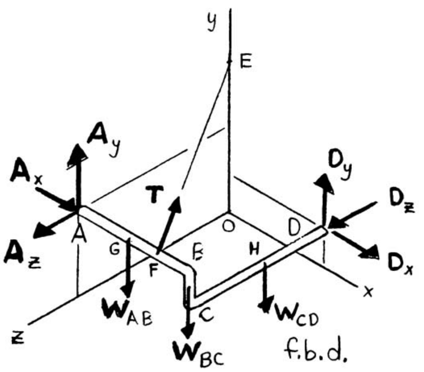
or $P_{JI} = 28.7$ lb ◀

PROBLEM 4.144



To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



First note $\mathbf{r}_{G/A} = (1.5 \text{ ft})\mathbf{i}$

$$\mathbf{W}_{AB} = -(0.85 \text{ lb/ft})(3 \text{ ft})\mathbf{j} = -(2.55 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{F/A} = (2 \text{ ft})\mathbf{i}$$

$$\mathbf{T} = \lambda_{FE}T = \frac{-(2 \text{ ft})\mathbf{i} + (3 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2 + (4.5)^2} \text{ ft}}T$$

$$= \left(\frac{T}{\sqrt{33.25}}\right)(-2\mathbf{i} + 3\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3 \text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(0.85 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3 \text{ ft})\mathbf{i} - (2.25 \text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85 \text{ lb/ft})(4.5 \text{ ft})\mathbf{j} = -(3.825 \text{ lb})\mathbf{j}$$

$$\lambda_{AD} = \frac{(3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2} \text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.144 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{F/A} \times \mathbf{T}\right)$$

$$+ \; \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \quad \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 2 & 0 & 0 \\ -2 & 3 & -4.5 \end{vmatrix} \left(\frac{T}{5.5\sqrt{33.25}}\right)$$

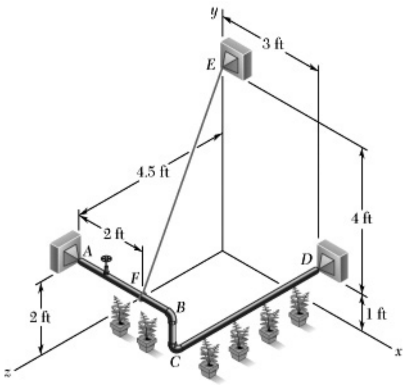
$$+ \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) = 0$$

$$(17.2125) + (-36)\left(\frac{T}{\sqrt{33.25}}\right) + (11.475) + (25.819) = 0$$

$$\therefore \quad T = 8.7306 \text{ lb}$$

or $T = 8.73$ lb ◀

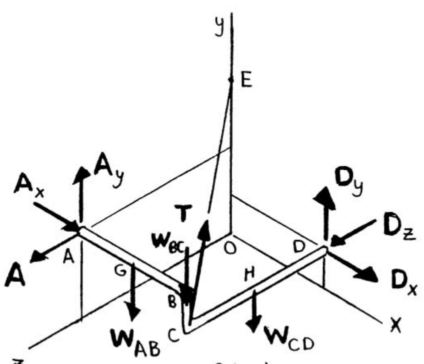
PROBLEM 4.145



Solve Problem 4.144 assuming that cable *EF* is replaced by a cable connecting *E* and *C*.

P4.144 To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



f.b.d.

First note

$$\mathbf{r}_{G/A} = (1.5\text{ ft})\mathbf{i}$$

$$\mathbf{W}_{AB} = -(0.85\text{ lb/ft})(3\text{ ft})\mathbf{j} = -(2.55\text{ lb})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (3\text{ ft})\mathbf{i} - (1\text{ ft})\mathbf{j}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{CE}T = \frac{-(3\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j} - (4.5\text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (4)^2 + (4.5)^2}\text{ ft}}T$$

$$= \left(\frac{T}{\sqrt{45.25}}\right)(-3\mathbf{i} + 4\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3\text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85\text{ lb/ft})(1\text{ ft})\mathbf{j} = -(0.85\text{ lb})\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3\text{ ft})\mathbf{i} - (2.25\text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85\text{ lb/ft})(1\text{ ft})\mathbf{j} = -(3.825\text{ lb})\mathbf{j}$$

$$\boldsymbol{\lambda}_{AD} = \frac{(3\text{ ft})\mathbf{i} - (1\text{ ft})\mathbf{j} - (4.5\text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2}\text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.145 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{T}\right)$$

$$+ \ \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \quad \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & -1 & 0 \\ -3 & 4 & -4.5 \end{vmatrix} \left(\frac{T}{5.5\sqrt{45.25}}\right)$$

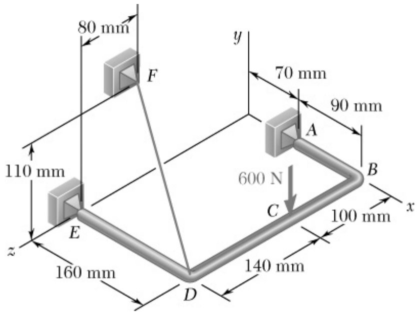
$$+ \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) = 0$$

$$(17.2125) + (-40.5)\left(\frac{T}{\sqrt{45.25}}\right) + (11.475) + (25.819) = 0$$

$$\therefore \quad T = 9.0536 \text{ lb}$$

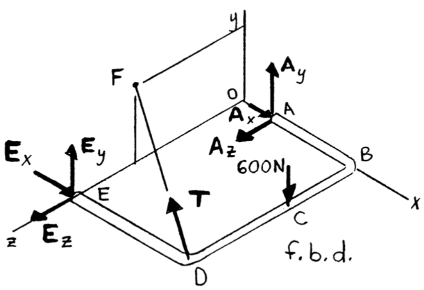
or $T = 9.05$ lb ◀

PROBLEM 4.146



The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\lambda_{AE} = \frac{-(70\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2}\text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90\text{ mm})\mathbf{i} + (100\text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600\text{ N})\mathbf{j}$$

$$\mathbf{r}_{D/A} = (90\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}$$

$$\mathbf{T} = \lambda_{DF}T = \frac{-(160\text{ mm})\mathbf{i} + (110\text{ mm})\mathbf{j} - (80\text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (80)^2}\text{ mm}}T$$

$$= \frac{T}{21}(-16\,\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

From the f.b.d. of the bend rod

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \lambda_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}) = 0$$

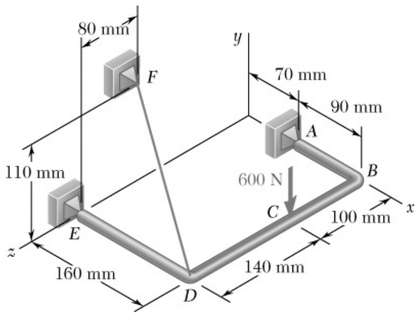
$$\therefore \quad \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 240 \\ -16 & 11 & -8 \end{vmatrix}\left[\frac{T}{25(21)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (18\,480 + 23\,760)\left[\frac{T}{25(21)}\right] = 0$$

$$\therefore \quad T = 853.13\text{ N}$$

or $T = 853$ N ◀

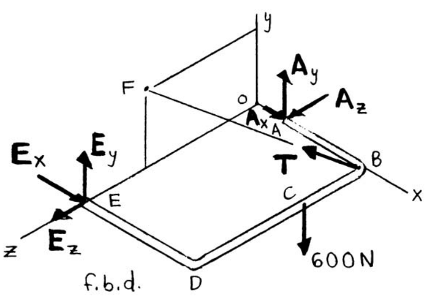
PROBLEM 4.147



Solve Problem 4.146 assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

P4.146 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\boldsymbol{\lambda}_{AE} = \frac{-(70\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2}\text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90\text{ mm})\mathbf{i} + (100\text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600\text{ N})\mathbf{j}$$

$$\mathbf{r}_{B/A} = (90\text{ mm})\mathbf{i}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{BF}T = \frac{-(160\text{ mm})\mathbf{i} + (110\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (160)^2}\text{ mm}}T$$

$$= \frac{1}{251.59}(-160\,\mathbf{i} + 110\mathbf{j} + 160\mathbf{k})$$

From the f.b.d. of the bend rod

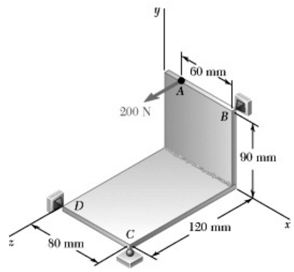
$$\Sigma M_{AE} = 0\text{:}\quad \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\therefore \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 0 \\ -160 & 110 & 160 \end{vmatrix}\left[\frac{T}{25(251.59)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (237\ 600)\left[\frac{T}{25(251.59)}\right] = 0$$

$$\therefore\quad T = 1817.04\text{ N}$$

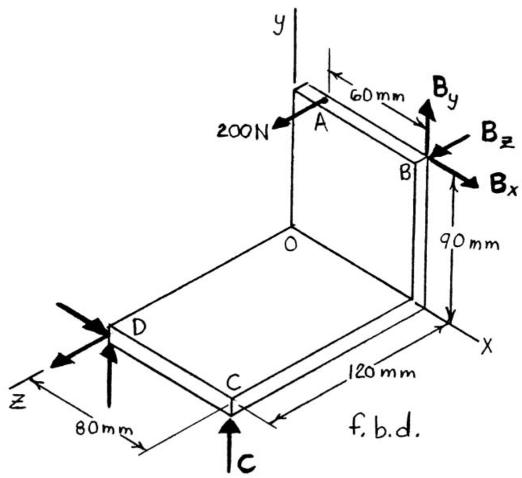
or $T = 1817$ N ◀



PROBLEM 4.148

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at *B* and *D* and by a ball on a horizontal surface at *C*. For the loading shown, determine the reaction at *C*.

SOLUTION



First note

$$\lambda_{BD} = \frac{-(80 \text{ mm})\mathbf{i} - (90 \text{ mm})\mathbf{j} + (120 \text{ mm})\mathbf{k}}{\sqrt{(80)^2 + (90)^2 + (120)^2} \text{ mm}}$$

$$= \frac{1}{17}(-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(60 \text{ mm})\mathbf{i}$$

$$\mathbf{P} = (200 \text{ N})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (80 \text{ mm})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$

From the f.b.d. of the plates

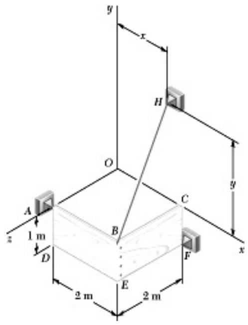
$$\Sigma M_{BD} = 0\text{:} \quad \lambda_{BD} \cdot (\mathbf{r}_{A/B} \times \mathbf{P}) + \lambda_{BD} \cdot (\mathbf{r}_{C/D} \times \mathbf{C}) = 0$$

$$\therefore \quad \begin{vmatrix} -8 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{60(200)}{17}\right] + \begin{vmatrix} -8 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(80)}{17}\right] = 0$$

$$(-9)(60)(200) + (12)(80)C = 0$$

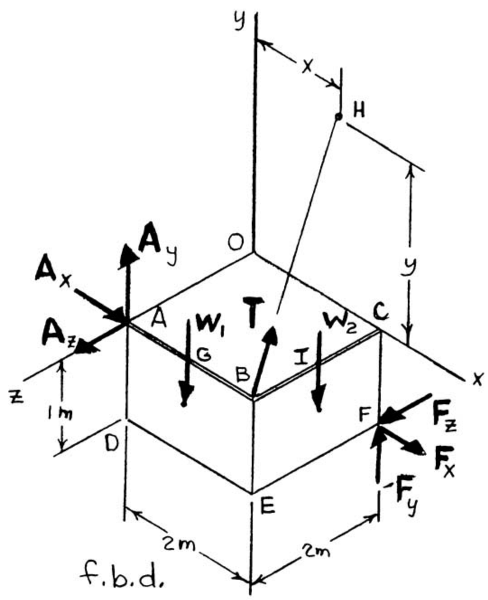
$$\therefore \quad C = 112.5 \text{ N} \qquad \text{or } \mathbf{C} = (112.5 \text{ N})\mathbf{j} \blacktriangleleft$$

PROBLEM 4.149



Two 1×2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at *A* and *F* and by the wire *BH*. Determine (*a*) the location of *H* in the *xy* plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION



Let
$$\mathbf{W}_1 = \mathbf{W}_2 = -(mg)\,\mathbf{j} = -(15\text{ kg})\left(9.81\text{ m/s}^2\right)\mathbf{j}$$
$$= -(147.15\text{ N})\,\mathbf{j}$$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{G/A}\times\mathbf{W}_1\right) + \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{B/A}\times\mathbf{T}\right) + \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{T/A}\times\mathbf{W}_2\right) = 0$$

where
$$\boldsymbol{\lambda}_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

PROBLEM 4.149 CONTINUED

$$\lambda_{BH} = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ x-2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$\frac{2(147.15)}{3} + (-4 - 4y)\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}} + (-2 + 4)\frac{147.15}{3} = 0$$

or
$$T = \frac{147.15}{1 + y}\sqrt{(x-2)^2 + y^2 + (2)^2}$$

For $x - 2$ m, $T = T_{\min}$
$$\therefore \quad T_{\min} = \frac{147.15}{(1+y)}\left(y^2 + 4\right)^{\frac{1}{2}}$$

The y-value for $T_{\min}$ is found from
$$\left(\frac{dT}{dy}\right) = 0: \quad \frac{(1+y)\frac{1}{2}\left(y^2 + 4\right)^{-\frac{1}{2}}(2y) - \left(y^2 + 4\right)^{\frac{1}{2}}(1)}{(1+y)^2} = 0$$

Setting the numerator equal to zero,
$$(1 + y)y = y^2 + 4$$

$$y = 4 \text{ m}$$

Then
$$T\min = \frac{147.15}{(1+4)}\sqrt{(2-2)^2 + (4)^2 + (2)^2} = 131.615 \text{ N}$$

$\therefore$ (*a*) $\quad x = 2.00$ m, $y = 4.00$ m ◀

(*b*) $\quad T_{\min} = 131.6$ N ◀