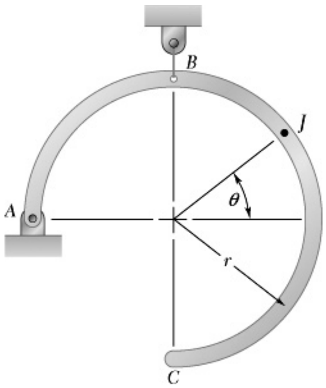
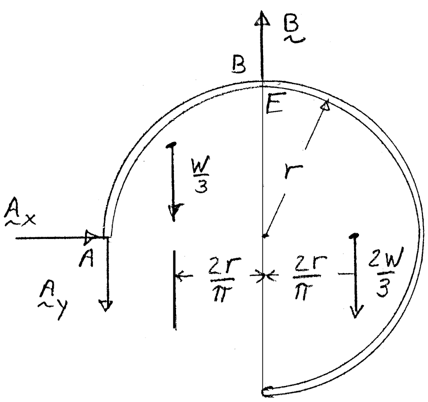
PROBLEM 7.24



A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 120°$.

SOLUTION

(*a*) **FBD Rod:**

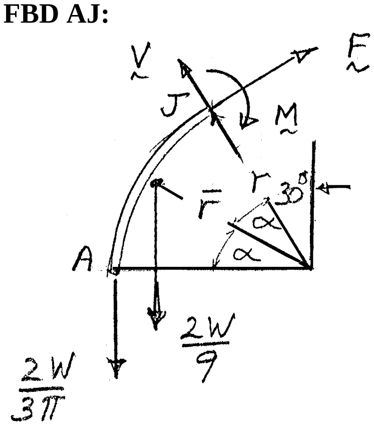


$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: rA_y + \frac{2r}{\pi}\frac{W}{3} - \frac{2r}{\pi}\frac{2W}{3} = 0$$

$$A_y = \frac{2W}{3\pi}$$

FBD AJ:



Note: $$\alpha = \frac{60°}{2} = 30° = \frac{\pi}{6}$$

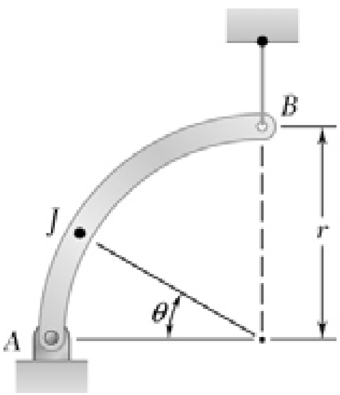
$$\text{Weight of segment} = W\frac{60}{270} = \frac{2W}{9}$$

$$F = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/6}\sin 30° = \frac{3r}{\pi}$$

$$\circlearrowleft \Sigma M_J = 0: \left(\bar{r}\cos\alpha - r\sin 30°\right)\frac{2W}{9} + \left(r - r\sin 30°\right)\frac{2W}{3\pi} - M = 0$$

$$M = \frac{2W}{9}\left(\frac{3r}{\pi}\frac{\sqrt{3}}{2} - \frac{r}{2} + \frac{3r}{2\pi}\right) = Wr\left(\frac{\sqrt{3}}{3\pi} - \frac{1}{9} + \frac{1}{3\pi}\right)$$

$$\mathbf{M} = 0.1788Wr \circlearrowright \blacktriangleleft$$

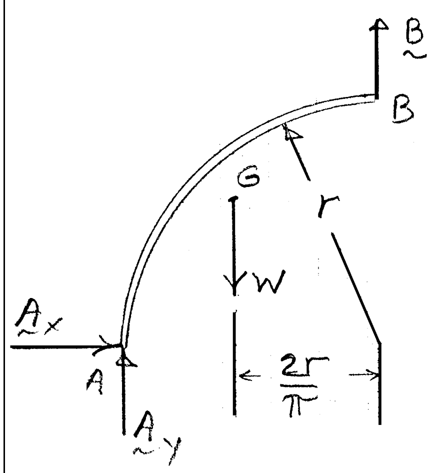


PROBLEM 7.25

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^{\circ}$.

SOLUTION

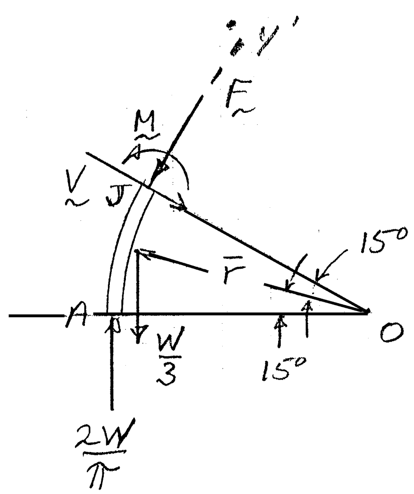
FBD Rod:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi} \uparrow$$

FBD AJ:



$$\alpha = 15^{\circ}, \text{ weight of segment } = W\frac{30^{\circ}}{90^{\circ}} = \frac{W}{3}$$

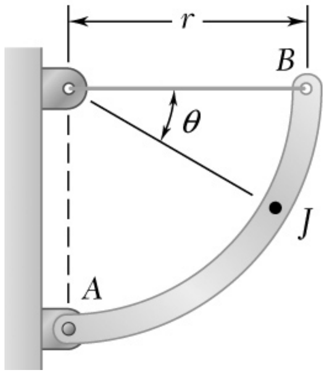
$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/12}\sin 15^{\circ} = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \frac{2W}{\pi}\cos 30^{\circ} - \frac{W}{3}\cos 30^{\circ} - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15^{\circ}\frac{W}{3} = 0$$

$$\mathbf{M} = 0.0557Wr \blacktriangleleft$$

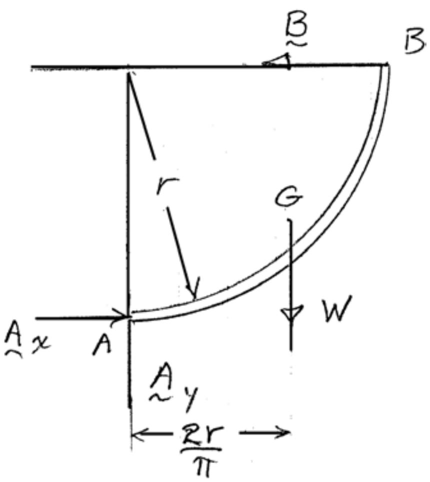


PROBLEM 7.26

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^\circ$.

SOLUTION

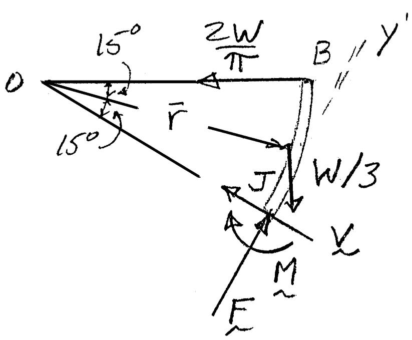
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0:\ rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15^\circ = \frac{\pi}{12}$$

$$\overline{r} = \frac{r}{\pi/12}\sin 15^\circ = 0.98862r$$

$$\text{Weight of segment} = W\frac{30^\circ}{90^\circ} = \frac{W}{3}$$

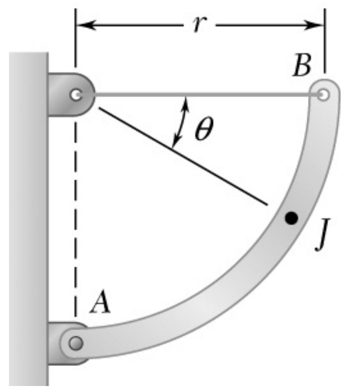
$$\nearrow \Sigma F_{y'} = 0:\ F - \frac{W}{3}\cos 30^\circ - \frac{2W}{\pi}\sin 30^\circ = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0:\ rF - \left(\overline{r}\cos 15^\circ\right)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15^\circ}{3}\right)Wr$$

$$\mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$

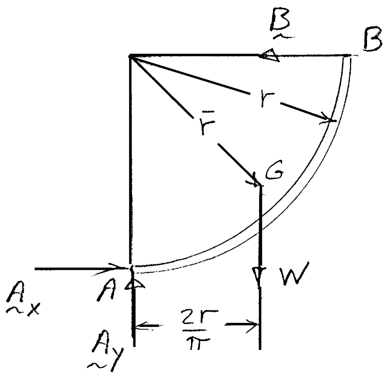


PROBLEM 7.27

For the rod of Prob.7.26, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha,$$

$$\text{Weight of segment } = W\frac{2\alpha}{\pi/2}$$

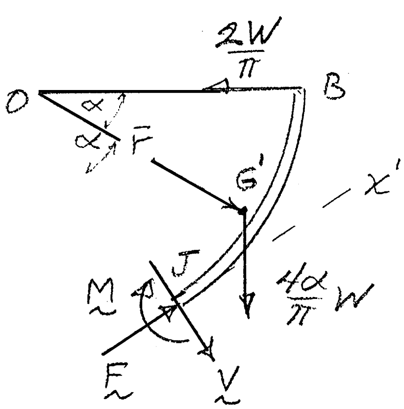
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}\left(\sin 2\alpha + 2\alpha\cos 2\alpha\right)$$

$$= \frac{2W}{\pi}\left(\sin\theta + \theta\cos\theta\right)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: rF - \left(\bar{r}\cos\alpha\right)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr\left(\sin\theta + \theta\cos\theta\right) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $M = \frac{2Wr}{\pi}\left(\sin\theta + \theta\cos\theta - \sin\theta\right)$

or $M = \frac{2}{\pi}Wr\theta\cos\theta$

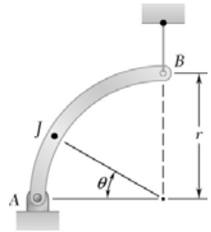
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr\left(\cos\theta - \theta\sin\theta\right) = 0 \quad \text{at } \theta\tan\theta = 1$$

PROBLEM 7.27 CONTINUED

Solving numerically $\theta = 0.8603\text{ rad}$ and $\mathbf{M} = 0.357Wr$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)

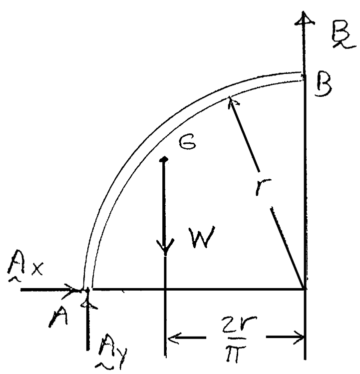


PROBLEM 7.28

For the rod of Prob.7.25, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0:\ A_x = 0$$

$$\Sigma M_B = 0:\ \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

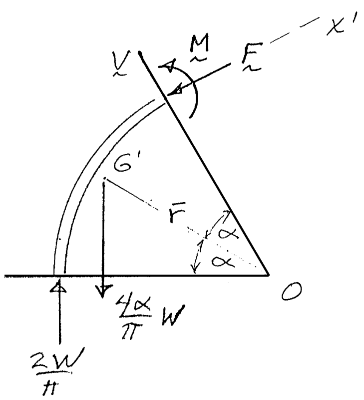
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\pi/2} = \frac{4\alpha}{\pi}W$$

$$\Sigma F_{x'} = 0:\ -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ:



$$\Sigma M_0 = 0:\ M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$$

$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

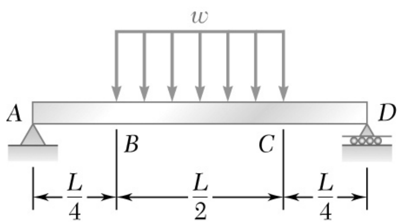
for $(1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \qquad \text{for} \qquad \theta = 0, 1, n\pi \ (n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\theta = 0$ $M = 0$, at $\theta = 1\text{ rad}$

at $\theta = 57.3°$ $M = M_{max} = 0.1009\,Wr$ ◀

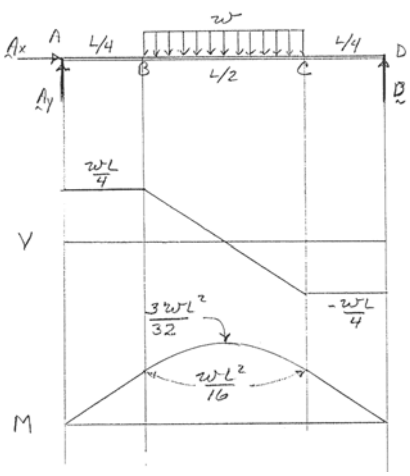


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

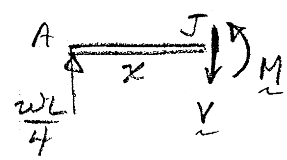
SOLUTION

FBD beam:



(*a*) By symmetry: $A_y = D = \frac{1}{2}(w)\frac{L}{2}$ $\qquad \mathbf{A}_y = \mathbf{D} = \frac{wL}{4} \uparrow$

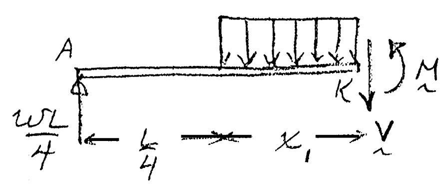
Along AB:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - V = 0 \qquad V = \frac{wL}{4}$$

$$\circlearrowleft \Sigma M_J = 0: M - x\frac{wL}{4} = 0 \qquad M = \frac{wL}{4}x \text{ (straight)}$$

Along BC:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - wx_1 - V = 0$$

$$V = \frac{wL}{4} - wx_1$$

straight with $\qquad V = 0 \quad$ at $\quad x_1 = \frac{L}{4}$

$$\circlearrowleft \Sigma M_k = 0: M + \frac{x_1}{2}wx_1 - \left(\frac{L}{4} + x_1\right)\frac{wL}{4} = 0$$

$$M = \frac{w}{2}\left(\frac{L^2}{8} + \frac{L}{2}x_1 - x_1^2\right)$$

PROBLEM 7.29 CONTINUED

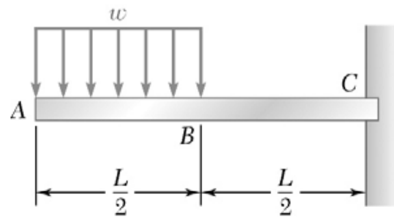
Parabola with $M = \frac{3}{32}wL^2$ at $x_1 = \frac{L}{4}$

Section CD by symmetry

(b) From diagrams:

$$|V|_{max} = \frac{wL}{4} \text{ on } AB \text{ and } CD \blacktriangleleft$$

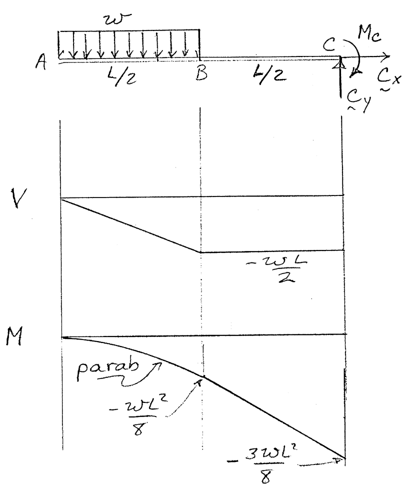
$$|M|_{max} = \frac{3wL^2}{32} \text{ at center } \blacktriangleleft$$



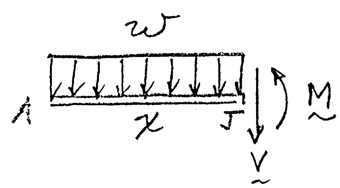
PROBLEM 7.30

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**



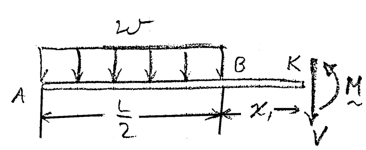
$$\uparrow \Sigma F_y = 0: -wx - V = 0 \qquad V = -wx$$

straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}wx = 0 \qquad M = -\frac{1}{2}wx^2$$

parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$

Along BC:



$$\uparrow \Sigma F_y = 0: -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

$$\circlearrowleft \Sigma M_k = 0: M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

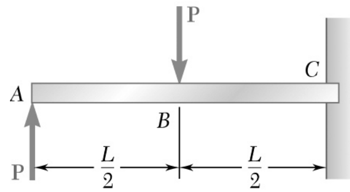
straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams:

$$|V|_{\max} = \frac{wL}{2} \text{ on } BC \blacktriangleleft$$

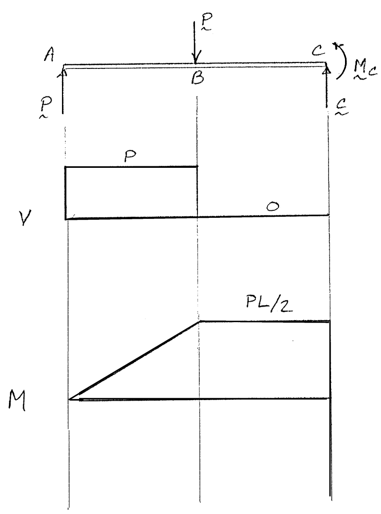
$$|M|_{\max} = \frac{3wL^2}{8} \text{ at } C \blacktriangleleft$$

PROBLEM 7.31

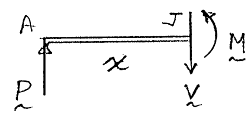


For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**

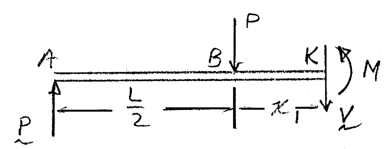


$$\uparrow \Sigma F_y = 0: \; P - V = 0 \qquad V = P$$

$$\Sigma M_J = 0: \; M - Px = 0 \qquad M = Px$$

straight with $M = \dfrac{PL}{2}$ at B

Along BC:



$$\uparrow \Sigma F_y = 0: P - P - V = 0 \qquad V = 0$$

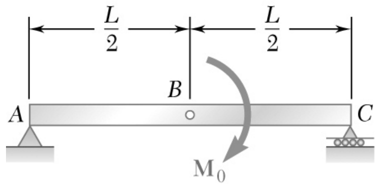
$$\Sigma M_K = 0: M + Px_1 - P\left(\frac{L}{2} + x_1\right) = 0$$

$$M = \frac{PL}{2} \quad \text{(constant)}$$

(*b*) From diagrams: $|V|_{max} = P$ along AB ◀

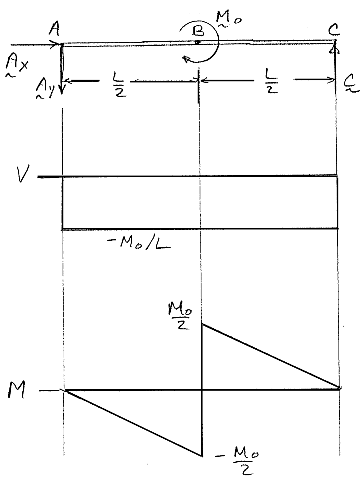
$|M|_{max} = \dfrac{PL}{2}$ along BC ◀

PROBLEM 7.32



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



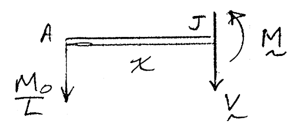
(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0: \; LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: -A_y + C = 0 \qquad \mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:

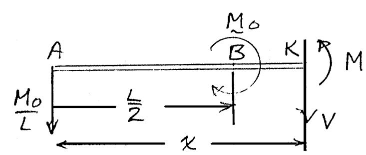


$$\uparrow \Sigma F_y = 0: -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \; x\frac{M_0}{L} + M = 0 \qquad M = -\frac{M_0}{L}x$$

straight with $M = -\dfrac{M_0}{2}$ at B

Along BC:



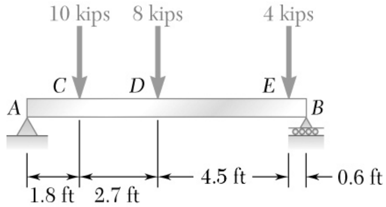
$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_K = 0: \; M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

straight with $M = \dfrac{M_0}{2}$ at B $\qquad M = 0$ at C

(*b*) From diagrams: $\qquad |V|_{max} = P$ everywhere ◀

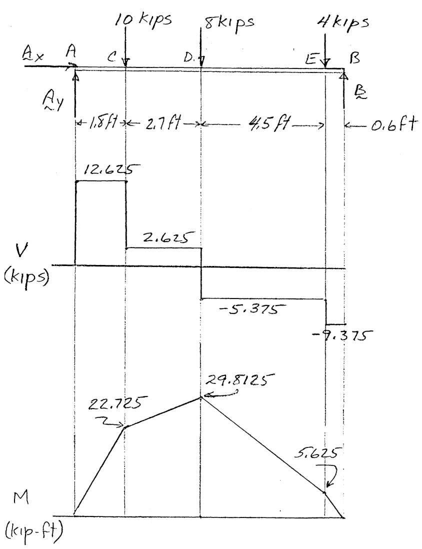
$$|M|_{max} = \frac{M_0}{2} \text{ at } B \blacktriangleleft$$



PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$$\Sigma M_B = 0:$$

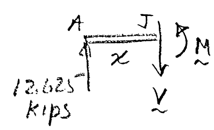
$$(.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips}\uparrow$$

$$\uparrow \Sigma F_y = 0:\ 12.625\text{ kips} - 10\text{ kips} - 8\text{ kips} - 4\text{ kips} + B = 0$$

$$\mathbf{B} = 9.375\text{ kips}\uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0:\ 12.625\text{ kips} - V = 0$$

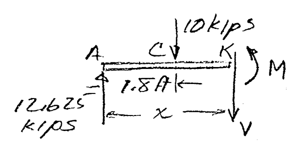
$$V = 12.625\text{ kips}$$

$$\Sigma M_J = 0:\ M - x(12.625\text{ kips}) = 0$$

$$M = (12.625\text{ kips})x$$

$$M = 22.725\text{ kip}\cdot\text{ft at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0:\ 12.625\text{ kips} - 10\text{ kips} - V = 0$$

$$V = 2.625\text{ kips}$$

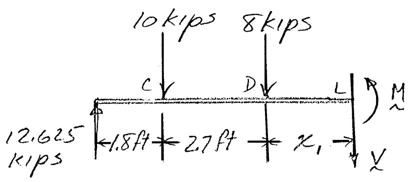
$$\Sigma M_K = 0:\ M + (x - 1.8\text{ ft})(10\text{ kips}) - x(12.625\text{ kips}) = 0$$

$$M = 18\text{ kip}\cdot\text{ft} + (2.625\text{ kips})\,x$$

$$M = 29.8125\text{ kip}\cdot\text{ft at } D\ (x = 4.5\text{ ft})$$

PROBLEM 7.33 CONTINUED

Along DE:



Along DE:

$$\uparrow \Sigma F_y = 0: (12.625 - 10 - 8) \text{ kips} - V = 0 \qquad V = -5.375 \text{ kips}$$

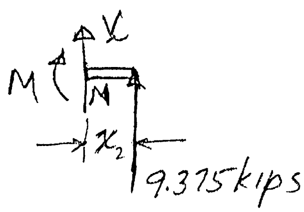
$$\circlearrowleft \Sigma M_L = 0: \; M + x_1(8 \text{ kips}) + (2.7 \text{ ft} + x_1)(10 \text{ kips})$$

$$- (4.5 \text{ ft} + x_1)(12.625 \text{ kips}) = 0$$

$$M = 29.8125 \text{ kip}\cdot\text{ft} - (5.375 \text{ kips})\, x_1$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E \quad (x_1 = 4.5 \text{ ft})$$

Along EB:



Along EB:

$$\uparrow \Sigma F_y = 0: V + 9.375 \text{ kips} = 0 \qquad V = 9.375 \text{ kips}$$

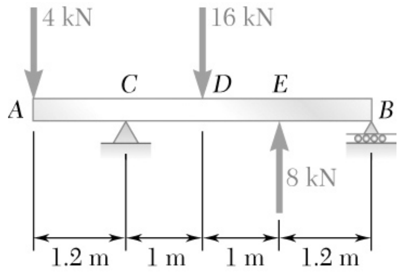
$$\circlearrowleft \Sigma M_N = 0: \; x_2(9.375 \text{ kip}) - M = 0$$

$$M = (9.375 \text{ kips})\, x_2$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E$$

(b) From diagrams: $|V|_{max} = 12.63 \text{ kips on } AC$ ◀

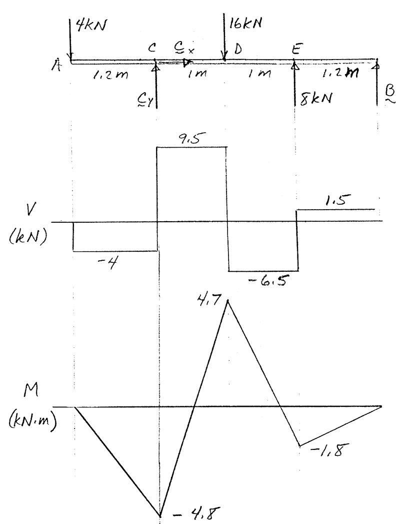
$|M|_{max} = 29.8 \text{ kip}\cdot\text{ft at } D$ ◀



PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$\circlearrowleft \Sigma M_C = 0:$

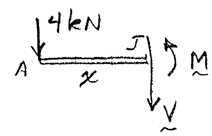
$$(1.2\text{ m})(4\text{ kN}) - (1\text{ m})(16\text{ kN}) + (2\text{ m})(8\text{ kN}) + (3.2\text{ m})B = 0$$

$$\mathbf{B} = -1.5\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -4\text{ kN} + C_y - 16\text{ kN} + 8\text{ kN} - 1.5\text{ kN} = 0$$

$$\mathbf{C}_y = 13.5\text{ kN} \uparrow$$

Along AC:



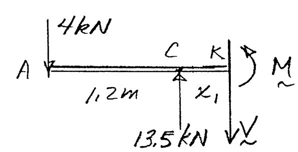
$$\uparrow \Sigma F_y = 0: -4\text{ kN} - V = 0$$

$$V = -4\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(4\text{ kN}) = 0 \qquad M = -4\text{ kN}\,x$$

$$M = -4.8\text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -4\text{ kN} + 13.5\text{ kN} - V = 0$$

$$V = 9.5\text{ kN}$$

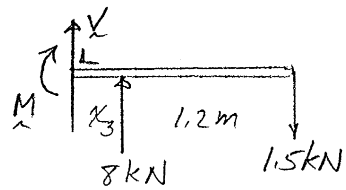
$$\circlearrowleft \Sigma M_K = 0: M + (1.2\text{ m} + x_1)(4\text{ kN}) - x_1(13.5\text{ kN}) = 0$$

$$M = -4.8\text{ kN}\cdot\text{m} + (9.5\text{ kN})x_1$$

$$M = 4.7\text{ kN}\cdot\text{m at } D\ (x_1 = 1\text{ m})$$

PROBLEM 7.34 CONTINUED

Along DE:



$$\uparrow \Sigma F_y = 0: V + 8 \text{ kN} - 1.5 \text{ kN} = 0$$

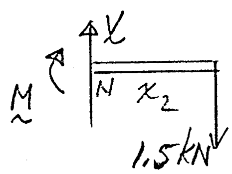
$$V = -6.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_3(8 \text{ kN}) + (x_3 + 1.2 \text{ m})(1.5 \text{ kN}) = 0$$

$$M = -1.8 \text{ kN}\cdot\text{m} + (6.5 \text{ kN})x_3$$

$$M = 4.7 \text{ kN}\cdot\text{m at } D \ (x_3 = 1 \text{ m})$$

Along EB:



$$\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0$$

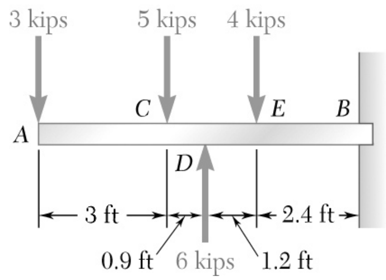
$$V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: \ x_2(1.5 \text{ kN}) + M = 0$$

$$M = -(1.5 \text{ kN})x_2 \qquad M = -1.8 \text{ kN}\cdot\text{m at } E$$

(b) From diagrams: $|V|_{\max} = 9.50 \text{ kN}\cdot$ *on* CD ◀

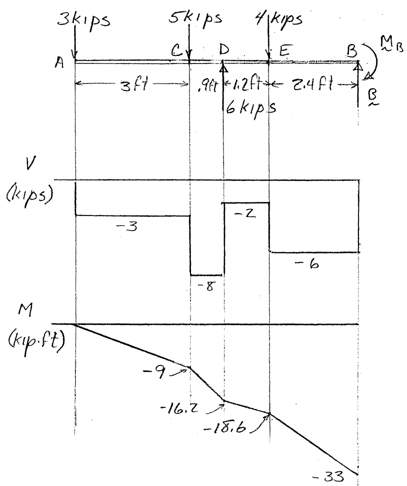
$|M|_{\max} = 4.80 \text{ kN}\cdot\text{m at } C$ ◀



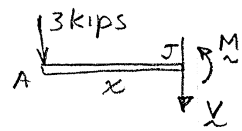
PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AC:**

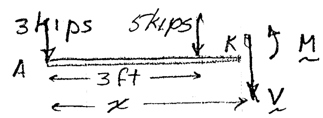


$$+\!\uparrow \Sigma F_y = 0: -3 \text{ kip} - V = 0 \qquad V = -3 \text{ kips}$$

$$\Sigma M_J = 0: \; M + x(3 \text{ kips}) = 0 \qquad M = (3 \text{ kips})x$$

$$M = -9 \text{ kip}\cdot\text{ft at } C$$

Along CD:



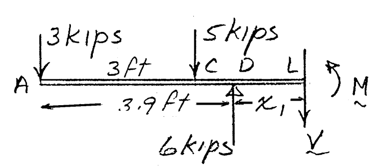
$$+\!\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} - V = 0 \qquad V = -8 \text{ kips}$$

$$\Sigma M_K = 0: \; M + (x - 3 \text{ ft})(5 \text{ kips}) + x(3 \text{ kips}) = 0$$

$$M = +15 \text{ kip}\cdot\text{ft} - (8 \text{ kips})x$$

$$M = -16.2 \text{ kip}\cdot\text{ft at } D \; (x = 3.9 \text{ ft})$$

Along DE:



$$+\!\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - V = 0$$

$$V = -2 \text{ kips}$$

$$\Sigma M_L = 0: \; M - x_1(6 \text{ kips}) + (.9 \text{ ft} + x_1)(5 \text{ kips})$$

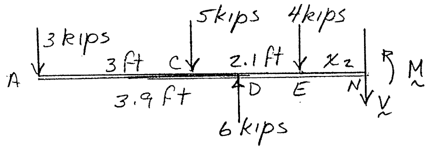
$$+ (3.9 \text{ ft} + x_1)(3 \text{ kips}) = 0$$

$$M = -16.2 \text{ kip}\cdot\text{ft} - (2 \text{ kips})x_1$$

$$M = -18.6 \text{ kip}\cdot\text{ft at } E \; (x_1 = 1.2 \text{ ft})$$

PROBLEM 7.35 CONTINUED

Along EB:



$+\uparrow \Sigma F_y = 0$: $-3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - 4 \text{ kips} - V = 0$ $\qquad V = -6 \text{ kips}$

$$\circlearrowleft \Sigma M_N = 0: \; M + (4 \text{ kips})x_2 + (2.1 \text{ ft} + x_2)(5 \text{ kips})$$

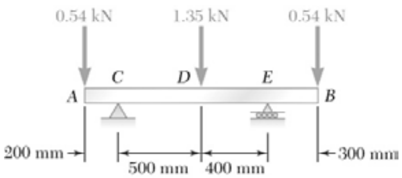
$$+ (5.1 \text{ ft} + x_2)(3 \text{ kips}) - (1.2 \text{ ft} + x_2)(6 \text{ kips}) = 0$$

$$M = -18.6 \text{ kip}\cdot\text{ft} - (6 \text{ kips})x_2$$

$$M = -33 \text{ kip}\cdot\text{ft at } B \quad (x_2 = 2.4 \text{ ft})$$

(b) From diagrams: $\qquad |V|_{\max} = 8.00 \text{ kips on } CD$ ◀

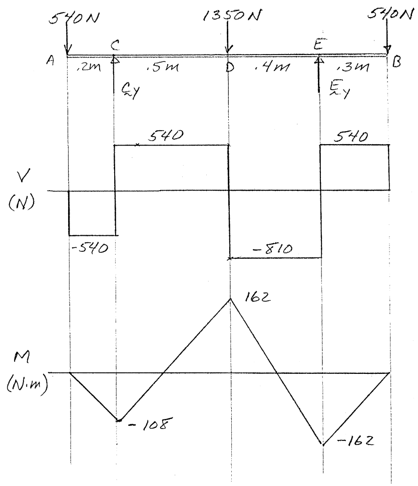
$|M|_{\max} = 33.0 \text{ kip}\cdot\text{ft at } B$ ◀



PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:** $\circlearrowleft \Sigma M_E = 0$:

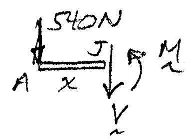
$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\text{ m})(1350\text{ N}) - (0.3\text{ m})(540\text{ N}) = 0$$

$$\mathbf{C}_y = 1080\text{ N} \uparrow$$

$$\uparrow \Sigma F_y = 0\text{: } -540\text{ N} + 1080\text{ N} - 1350\text{ N}$$

$$-540\text{ N} + E_y = 0 \qquad \mathbf{E}_y = 1350\text{ N} \uparrow$$

Along AC:

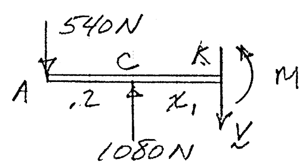


$$\uparrow \Sigma F_y = 0\text{: } -540\text{ N} - V = 0$$

$$V = -540\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0\text{: } x(540\text{ N}) + M = 0 \qquad M = -(540\text{ N})x$$

Along CD:



$$\uparrow \Sigma F_y = 0\text{: } -540\text{ N} + 1080\text{ N} - V = 0 \qquad V = 540\text{ N}$$

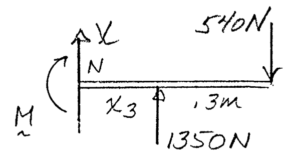
$$\circlearrowleft \Sigma M_K = 0\text{: } M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$$

$$M = -108\text{ N}\cdot\text{m} + (540\text{ N})x_1$$

$$M = 162\text{ N}\cdot\text{m at } D\ (x_1 = 0.5\text{ m})$$

PROBLEM 7.36 CONTINUED

Along DE:



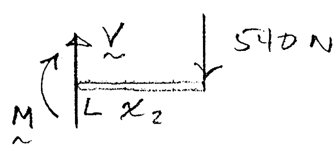
$+\uparrow \Sigma F_y = 0: \quad V + 1350 \text{ N} - 540 \text{ N} = 0 \qquad V = -810 \text{ N}$

$+\circlearrowleft \Sigma M_N = 0: \; M + (x_3 + 0.3 \text{ m})(540 \text{ N}) - x_3(1350 \text{ N}) = 0$

$M = -162 \text{ N}\cdot\text{m} + (810 \text{ N})x_3$

$M = 162 \text{ N}\cdot\text{m} \;$ at $D \; (x_3 = 0.4)$

Along EB:



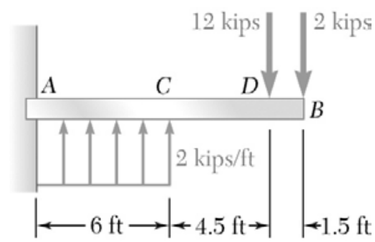
$+\uparrow \Sigma F_y = 0: V - 540 \text{ N} = 0 \qquad V = 540 \text{ N}$

$+\circlearrowleft \Sigma M_L = 0: M + x_2(540 \text{ N}) = 0 \qquad M = -540 \text{ N}\, x_2$

$M = -162 \text{ N}\cdot\text{m at } E \quad (x_2 = 0.3 \text{ m})$

(*b*) From diagrams $\qquad |V|_{max} = 810 \text{ N on } DE$ ◀

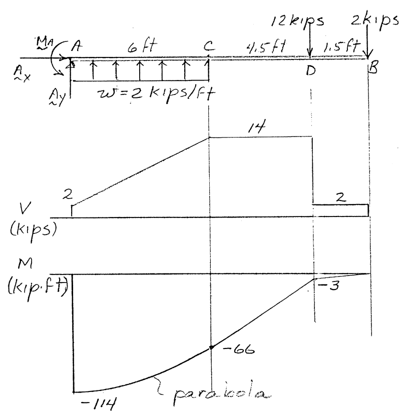
$|M|_{max} = 162.0 \text{ N}\cdot\text{m at } D \text{ and } E$ ◀



PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: A_y + (6\text{ ft})(2\text{ kips/ft}) - 12\text{ kips} - 2\text{ kips} = 0$$

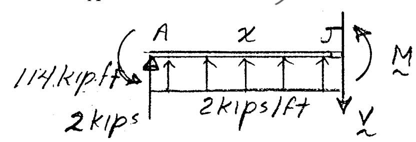
$$\mathbf{A}_y = 2\text{ kips}\uparrow$$

$$\Sigma M_A = 0: M_A + (3\text{ ft})(6\text{ ft})(2\text{ kips/ft})$$

$$-(10.5\text{ ft})(12\text{ kips}) - (12\text{ ft})(2\text{ kips}) = 0$$

$$\mathbf{M}_A = 114\text{ kip}\cdot\text{ft}$$

Along AC:



$$+\uparrow \Sigma F_y = 0: 2\text{ kips} + x(2\text{ kips/ft}) - V = 0$$

$$V = 2\text{ kips} + (2\text{ kips/ft})\,x$$

$$V = 14\text{ kips at } C\ (x = 6\text{ ft})$$

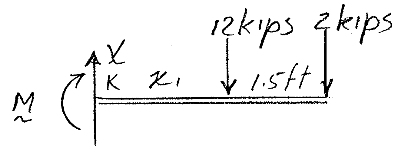
$$\Sigma M_J = 0: 114\text{ kip}\cdot\text{ft} - x(2\text{ kips})$$

$$-\frac{x}{2}x(2\text{ kips/ft}) + M = 0$$

$$M = (1\text{ kip/ft})x^2 + (2\text{ kips})x - 114\text{ kip}\cdot\text{ft}$$

$$M = -66\text{ kip}\cdot\text{ft at } C\ (x = 6\text{ ft})$$

Along CD:



$$+\uparrow \Sigma F_y = 0: V - 12\text{ kips} - 2\text{ kips} = 0 \qquad V = 14\text{ kips}$$

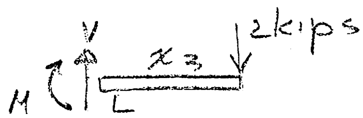
$$\Sigma M_k = 0: -M - x_1(12\text{ kips}) - (1.5\text{ ft} + x_1)(2\text{ kips}) = 0$$

PROBLEM 7.37 CONTINUED

$$M = -3\text{ kip}\cdot\text{ft} - (14\text{ kips})x_1$$

$$M = -66\text{ kip}\cdot\text{ft at } C \quad (x_1 = 4.5\text{ ft})$$

Along DB:



$$\uparrow \Sigma F_y = 0: \quad V - 2\text{ kips} = 0 \qquad V = +2\text{ kips}$$

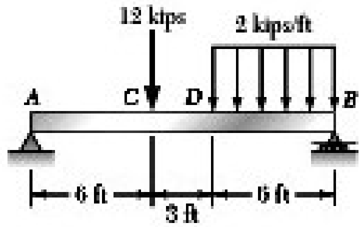
$$\circlearrowleft \Sigma M_L = 0: -M - 2\text{ kip } x_3 = 0$$

$$M = -(2\text{ kips})x_3$$

$$M = -3\text{ kip}\cdot\text{ft at } D \ (x = 1.5\text{ ft})$$

(b) From diagrams: $|V|_{max} = 14.00\text{ kips on } CD$ ◀

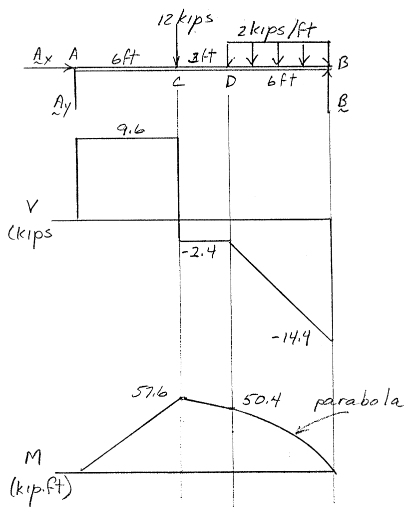
$|M|_{max} = 114.0\text{ kip}\cdot\text{ft at } A$ ◀



PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

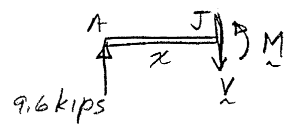
$$\Sigma M_A = (15\text{ ft})B - (12\text{ ft})(2\text{ kips/ft})(6\text{ ft}) - (6\text{ ft})(12\text{ kips}) = 0$$

$$\mathbf{B} = 14.4\text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0:\ A_y - 12\text{ kips} - (2\text{ kips/ft})(6\text{ ft}) + 14.4\text{ kips}$$

$$\mathbf{A}_y = 9.6\text{ kips} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0:\ 9.6\text{ kips} - V = 0$$

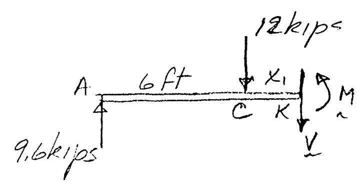
$$V = 9.6\text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0:\ M - x(9.6\text{ kips}) = 0$$

$$M = (9.6\text{ kips})x$$

$$M = 57.6\text{ kip}\cdot\text{ft at } C\ (x = 6\text{ ft})$$

Along CD:



$$\uparrow \Sigma F_y = 0:\ 9.6\text{ kips} - 12\text{ kips} - V = 0$$

$$V = -2.4\text{ kips}$$

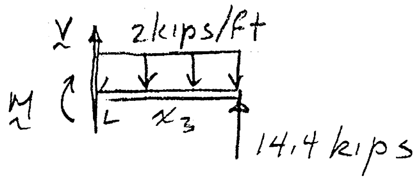
$$\circlearrowleft \Sigma M_K = 0:\ M + x_1(12\text{ kips}) - (6\text{ ft} + x_1)(9.6\text{ kips}) = 0$$

$$M = 57.6\text{ kip}\cdot\text{ft} - (2.4\text{ kips})x_1$$

$$M = 50.4\text{ kip}\cdot\text{ft at } D$$

PROBLEM 7.38 CONTINUED

Along DB:



$$\Sigma F_y = 0: \ V - x_3\left(2 \text{ kips/ft}\right) + 14.4 \text{ kips} = 0$$

$$V = -14.4 \text{ kips} + \left(2 \text{ kips/ft}\right)x_3$$

$$V = -2.4 \text{ kips at } D$$

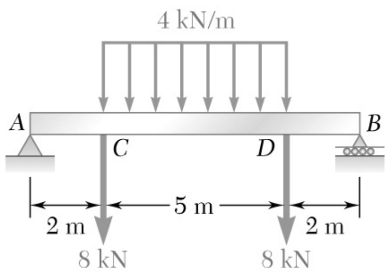
$$\Sigma M_L = 0: M + \frac{x_3}{2}\left(2 \text{ kips/ft}\right)\left(x_3\right) - x_3\left(14.4 \text{ kips}\right) = 0$$

$$M = \left(14.4 \text{ kips}\right)x_3 - \left(1 \text{ kip/ft}\right)x_3^{\ 2}$$

$$M = 50.4 \text{ kip}\cdot\text{ft at } D \ \left(x_3 = 6 \text{ ft}\right)$$

(b) From diagrams: $|V|_{\max} = 14.40$ kips at B ◀

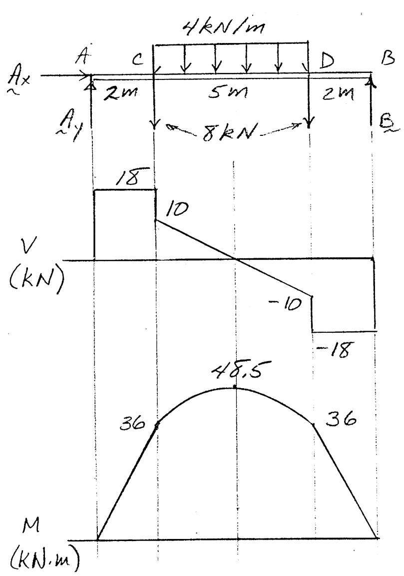
$|M|_{\max} = 57.6$ kip·ft at C ◀



PROBLEM 7.39

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

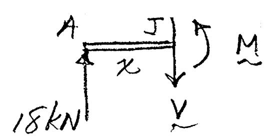
SOLUTION



(a) By symmetry:

$$A_y = B = 8\text{ kN} + \frac{1}{2}(4\text{ kN/m})(5\text{ m}) \qquad \mathbf{A}_y = \mathbf{B} = 18\text{ kN} \uparrow$$

Along AC:

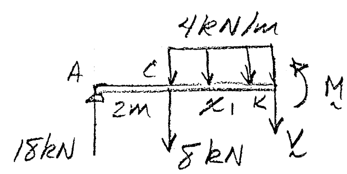


$$+\uparrow \Sigma F_y = 0\text{: } 18\text{ kN} - V = 0 \qquad V = 18\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0\text{: } M - x(18\text{ kN}) \qquad M = (18\text{ kN})x$$

$$M = 36\text{ kN}\cdot\text{m at } C \ (x = 2\text{ m})$$

Along CD:



$$+\uparrow \Sigma F_y = 0\text{: } 18\text{ kN} - 8\text{ kN} - (4\text{ kN/m})x_1 - V = 0$$

$$V = 10\text{ kN} - (4\text{ kN/m})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ m}(\text{at center})$$

$$\circlearrowleft \Sigma M_K = 0\text{: } M + \frac{x_1}{2}(4\text{ kN/m})x_1 + (8\text{ kN})x_1 - (2\text{ m} + x_1)(18\text{ kN}) = 0$$

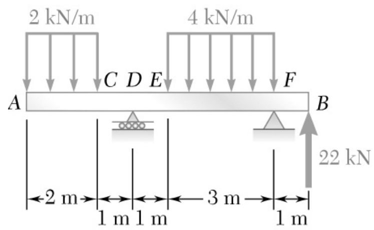
$$M = 36\text{ kN}\cdot\text{m} + (10\text{ kN/m})x_1 - (2\text{ kN/m})x_1^2$$

$$M = 48.5\text{ kN}\cdot\text{m at } x_1 = 2.5\text{ m}$$

Complete diagram by symmetry

(b) From diagrams: $|V|_{max} = 18.00\text{ kN on } AC \text{ and } DB$ ◀

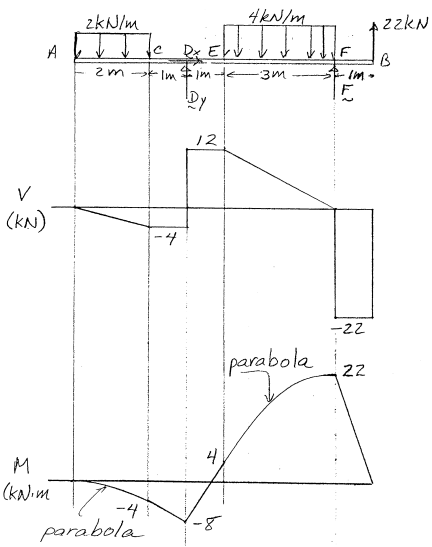
$|M|_{max} = 48.5\text{ kN}\cdot\text{m at center}$ ◀



PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) $$\Sigma M_D = 0: (2\text{ m})(2\text{ kN/m})(2\text{ m}) - (2.5\text{ m})(4\text{ kN/m})(3\text{ m})$$

$$-(4\text{ m})F - (5\text{ m})(22\text{ kN}) = 0$$

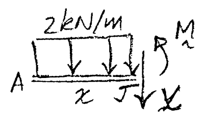
$$\mathbf{F} = 22\text{ kN} \downarrow$$

$$\uparrow \Sigma F_y = 0: -(2\text{ m})(2\text{ kN/m}) + D_y$$

$$-(3\text{ m})(4\text{ kN/m}) - 22\text{ kN} + 22\text{ kN} = 0$$

$$\mathbf{D}_y = 16\text{ kN} \uparrow$$

Along AC:



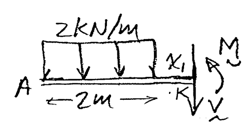
$$\uparrow \Sigma F_y = 0: -x(2\text{ kN/m}) - V = 0$$

$$V = -(2\text{ kN/m})x \qquad V = -4\text{ kN at } C$$

$$\Sigma M_J = 0: M + \frac{x}{2}(2\text{ kN/m})(x) \neq 0$$

$$M = -(1\text{ kN/m})x^2 \qquad M = -4\text{ kN}\cdot\text{m at } C$$

Along CD:



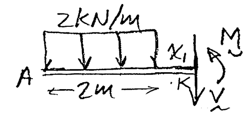
$$\uparrow \Sigma F_y = 0: -(2\text{ m})(2\text{ kN/m}) - V = 0 \qquad V = -4\text{ kN}$$

$$\Sigma M_K = 0: (1\text{ m} + x_1)(2\text{ kN/m})(2\text{ m}) = 0$$

$$M = -4\text{ kN}\cdot\text{m} - (4\text{ kN/m})x_1 \qquad M = -8\text{ kN}\cdot\text{m at } D$$

PROBLEM 7.40 CONTINUED

Along DE:

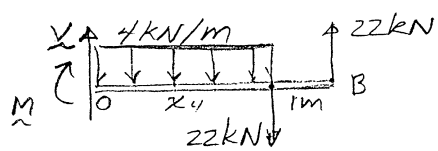


$$\uparrow \Sigma F_y = 0: -(2\ \text{kN/m})(2\ \text{m}) + 16\ \text{kN} - V = 0 \qquad V = 12\ \text{kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_2(16\ \text{kN}) + (x_2 + 2\ \text{m})(2\ \text{kN/m})(2\ \text{m}) = 0$$

$$M = -8\ \text{kN}\cdot\text{m} + (12\ \text{kN})x_2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along EF:



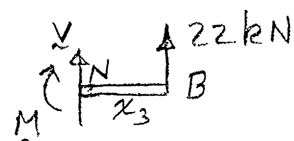
$$\uparrow \Sigma F_y = 0: V - x_4(4\ \text{kN/m}) - 22\ \text{kN} + 22\ \text{kN} = 0$$

$$V = (4\ \text{kN/m})x_4 \qquad V = 12\ \text{kN at } E$$

$$\circlearrowleft \Sigma M_0 = 0: M + \frac{x_4}{2}(4\ \text{kN/m})x_4 - (1\ \text{m})(22\ \text{kN}) = 0$$

$$M = 22\ \text{kN}\cdot\text{m} - (2\ \text{kN/m})x_4^2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along FB:



$$\uparrow \Sigma F_y = 0: V + 22\ \text{kN} = 0 \qquad V = 22\ \text{kN}$$

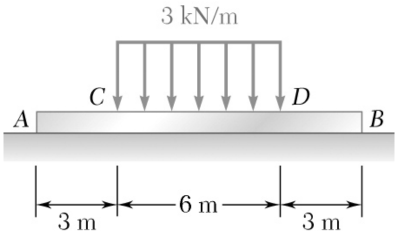
$$\circlearrowleft \Sigma M_N = 0: M - x_3(22\ \text{kN}) = 0$$

$$M = (22\ \text{kN})x_3$$

$$M = 22\ \text{kN}\cdot\text{m at } F$$

(*b*) From diagrams: $|V|_{max} = 22.0\ \text{kN on } FB$ ◀

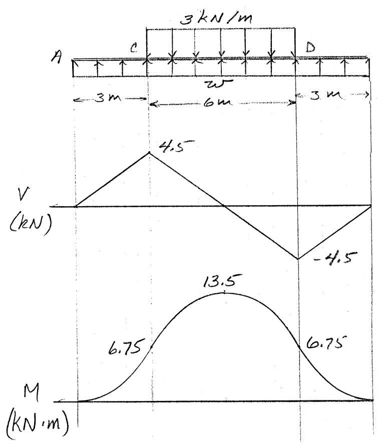
$|M|_{max} = 22.0\ \text{kN}\cdot\text{m at } F$ ◀



PROBLEM 7.41

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

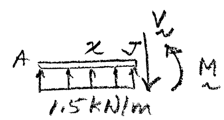
SOLUTION



(a) $\uparrow \Sigma F_y = 0: (12\text{ m})w - (6\text{ m})(3\text{ kN/m}) = 0$

$$w = 1.5\text{ kN/m}$$

Along AC:



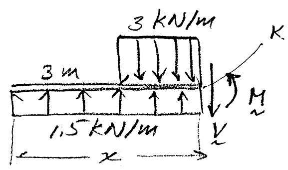
$$\uparrow \Sigma F_y = 0: x(1.5\text{ kN/m}) - V = 0 \qquad V = (1.5\text{ kN/m})x$$

$$V = 4.5\text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(1.5\text{ kN/m})(x) = 0$$

$$M = (0.75\text{ kN/m})x^2 \qquad M = 6.75\text{ N}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: x(1.5\text{ kN/m}) - (x - 3\text{ m})(3\text{ kN/m}) - V = 0$$

$$V = 9\text{ kN} - (1.5\text{ kN/m})x \qquad V = 0 \text{ at } x = 6\text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: M + \left(\frac{x - 3\text{ m}}{2}\right)(3\text{ kN/m})(x - 3\text{ m}) - \frac{x}{2}(1.5\text{ kN/m})x = 0$$

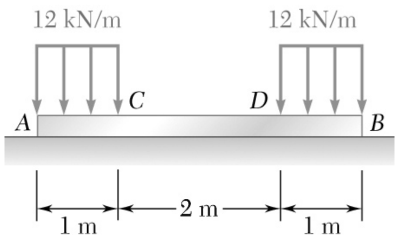
$$M = -13.5\text{ kN}\cdot\text{m} + (9\text{ kN})x - (0.75\text{ kN/m})x^2$$

$$M = 13.5\text{ kN}\cdot\text{m at center } (x = 6\text{ m})$$

Finish by symmetry

(b) From diagrams: $|V|_{\max} = 4.50\text{ kN at } C \text{ and } D$ ◀

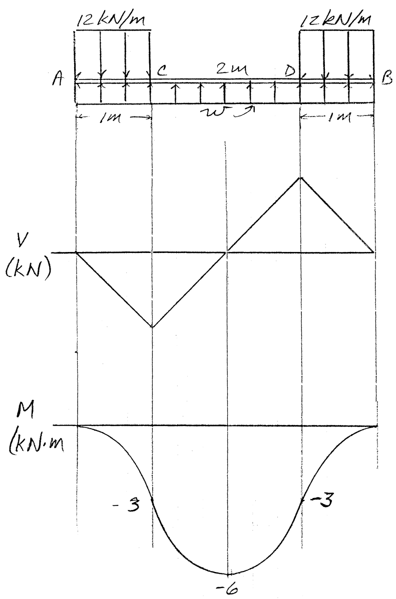
$$|M|_{\max} = 13.50\text{ kN}\cdot\text{m at center}$$ ◀



PROBLEM 7.42

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

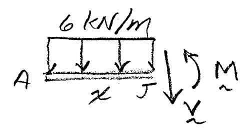


(a) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: (4\text{ m})(w) - (2\text{ m})(12\text{ kN/m}) = 0$$

$$w = 6\text{ kN/m}$$

Along AC:



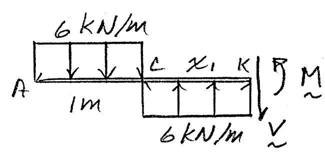
$$\uparrow \Sigma F_y = 0: -x(6\text{ kN/m}) - V = 0 \qquad V = -(6\text{ kN/m})x$$

$$V = -6\text{ kN at } C\,(x = 1\text{ m})$$

$$\Sigma M_J = 0: M + \frac{x}{2}(6\text{ kN/m})(x) = 0$$

$$M = -(3\text{ kN/m})x^2 \qquad M = -3\text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -(1\text{ m})(6\text{ kN/m}) + x_1(6\text{ kN/m}) - v = 0$$

$$V = (6\text{ kN/m})(1\text{ m} - x_1) \qquad V = 0 \text{ at } x_1 = 1\text{ m}$$

$$\Sigma M_K = 0: M + (0.5\text{ m} + x_1)(6\text{ kN/m})(1\text{ m}) - \frac{x_1}{2}(6\text{ kN/m})x_1 = 0$$

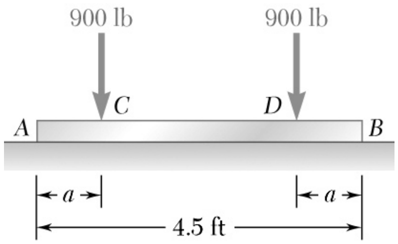
$$M = -3\text{ kN}\cdot\text{m} - (6\text{ kN})x_1 + (3\text{ kN/m})x_1^2$$

$$M = -6\text{ kN}\cdot\text{m at center } (x_1 = 1\text{ m})$$

Finish by symmetry

(b) From diagrams: $|V|_{\max} = 6.00\text{ kN at } C \text{ and } D$ ◀

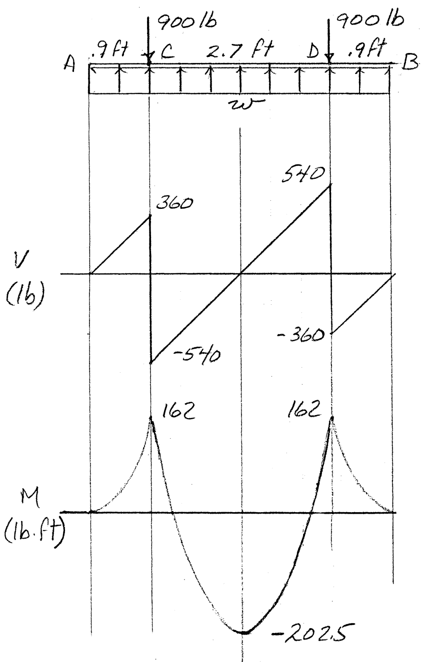
$|M|_{\max} = 6.00\text{ kN at center}$ ◀



PROBLEM 7.43

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.9$ ft, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

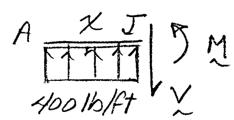


(a) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: (4.5 \text{ ft})w - 900 \text{ lb} - 900 \text{ lb} = 0$$

$$w = 400 \text{ lb/ft}$$

Along AC:



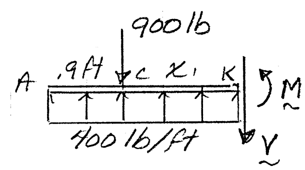
$$+\uparrow \Sigma F_y = 0: x(400 \text{ lb}) - V = 0 \qquad V = (400 \text{ lb})x$$

$$V = 360 \text{ lb at } C \quad (x = 0.9 \text{ ft})$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = (200 \text{ lb/ft})x^2 \qquad M = 162 \text{ lb}\cdot\text{ft at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0: (0.9 \text{ ft} + x_1)(400 \text{ lb/ft}) - 900 \text{ lb} - V = 0$$

$$V = -540 \text{ lb} + (400 \text{ lb/ft})x_1 \qquad V = 0 \text{ at } x_1 = 1.35 \text{ ft}$$

$$\circlearrowleft \Sigma M_K = 0: \; M + x_1(900 \text{ lb}) - \frac{0.9 \text{ ft} + x_1}{2}(400 \text{ lb/ft})(0.9 \text{ ft} + x_1) = 0$$

$$M = 162 \text{ lb}\cdot\text{ft} - (540 \text{ lb})x_1 + (200 \text{ lb/ft})x_1^2$$

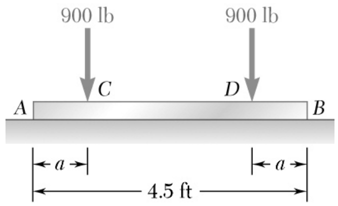
$$M = -202.5 \text{ lb}\cdot\text{ft at center} \quad (x_1 = 1.35 \text{ ft})$$

Finish by symmetry

(b) From diagrams:

$$|V|_{\max} = 540 \text{ lb at } C^+ \text{ and } D^- \blacktriangleleft$$

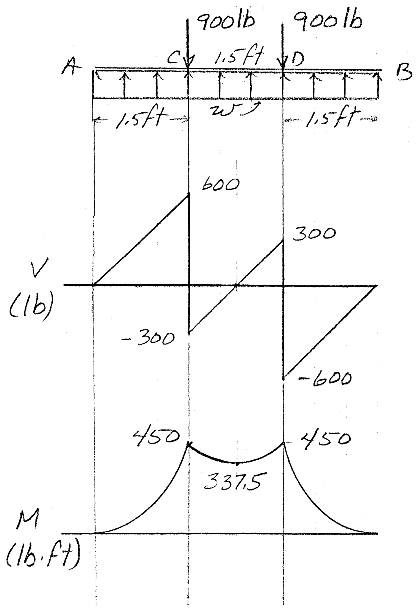
$$|M|_{\max} = 203 \text{ lb}\cdot\text{ft at center} \blacktriangleleft$$



PROBLEM 7.44

Solve Prob. 7.43 assuming that $a = 1.5$ ft.

SOLUTION

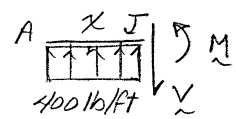


(a) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: (4.5 \text{ ft})w - 900 \text{ lb} - 900 \text{ lb} = 0$$

$$w = 400 \text{ lb/ft}$$

Along AC:



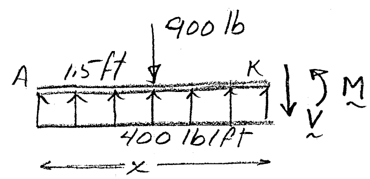
$$+\uparrow \Sigma F_y = 0: \qquad x(400 \text{ lb/ft}) - V = 0$$

$$V = (400 \text{ lb/ft})x \qquad V = 600 \text{ lb at } C \quad (x = 1.5 \text{ ft})$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = (200 \text{ lb/ft})x^2 \qquad M = 450 \text{ lb}\cdot\text{ft at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0: x(400 \text{ lb/ft}) - 900 \text{ lb} - V = 0$$

$$V = -900 \text{ lb} + (400 \text{ lb/ft})x \qquad V = -300 \text{ at } x = 1.5 \text{ ft}$$

$$V = 0 \text{ at } x = 2.25 \text{ ft}$$

$$\circlearrowleft \Sigma M_K = 0: M + (x - 1.5 \text{ ft})(900 \text{ lb}) - \frac{x}{2}(400 \text{ lb/ft})x = 0$$

$$M = 1350 \text{ lb}\cdot\text{ft} - (900 \text{ lb})x + (200 \text{ lb/ft})x^2$$

$$M = 450 \text{ lb}\cdot\text{ft at } x = 1.5 \text{ ft}$$

$$M = 337.5 \text{ lb}\cdot\text{ft at } x = 2.25 \text{ ft (center)}$$

Finish by symmetry

(b) From diagrams:

$$|V|_{\max} = 600 \text{ lb at } C^- \text{ and } D^+ \blacktriangleleft$$

$$|M|_{\max} = 450 \text{ lb}\cdot\text{ft at } C \text{ and } D \blacktriangleleft$$