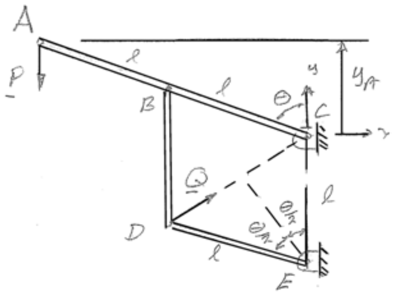
PROBLEM 10.11

Solve Problem 10.10 assuming that the force **P** applied at point *A* acts horizontally to the left.

SOLUTION



Have $$x_A = 2l\sin\theta; \qquad \delta x_A = 2l\cos\theta\delta\theta$$

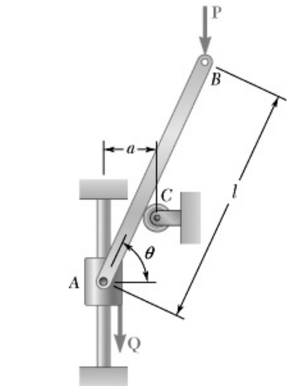
$$CD = 2l\sin\frac{\theta}{2}; \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work: $$\delta U = 0: \quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

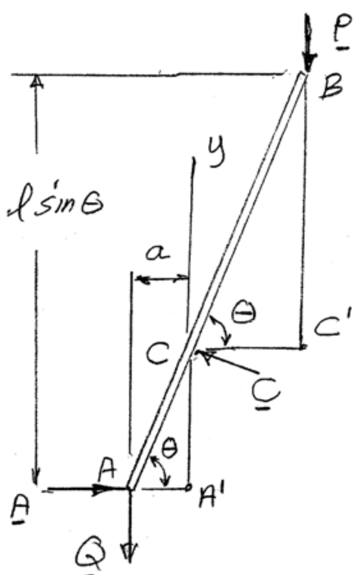
$$Q = 2P\frac{\cos\theta}{\cos(\theta/2)} \blacktriangleleft$$

PROBLEM 10.12



The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work:

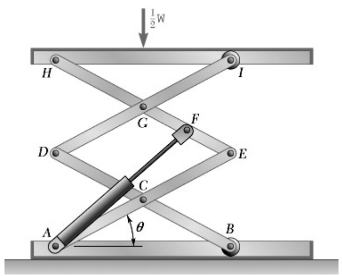
$$\delta U = 0: \quad Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

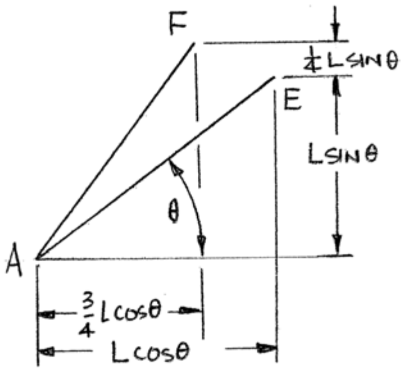
$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right) \blacktriangleleft$$

PROBLEM 10.13



A double scissor lift table is used to raise a 1000-lb machine component. The table consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Each member of the linkage is of length 24 in., and pins C and G are at the midpoints of their respective members. The hydraulic cylinder is pinned at A to the base of the table and at F which is 6 in. from E. If the component is placed on the table so that half of its weight is supported by the system shown, determine the force exerted by each cylinder when $\theta = 30°$.

SOLUTION



First note $\quad y_H = 2L\sin\theta \qquad L = 24$ in. (length of link)

Then $$\delta y_H = 2L\cos\theta\delta\theta$$

Now $$d_{AF} = \sqrt{\left(\frac{3}{4}L\cos\theta\right)^2 + \left(\frac{5}{4}L\sin\theta\right)^2}$$

$$= \frac{1}{4}L\sqrt{9 + 16\sin^2\theta}$$

Then $$\delta d_{AF} = \frac{L}{4}\frac{2(16\sin\theta\cos\theta)}{2\sqrt{9+16\sin^2\theta}}\delta\theta$$

$$= 4L\frac{\sin\theta\cos\theta}{\sqrt{9+16\sin^2\theta}}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad F_{\text{cyl}}\delta d_{AF} - \left(\frac{1}{2}W\right)\delta y_H = 0$$

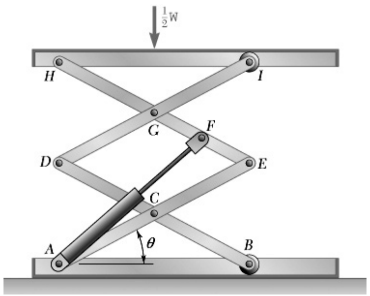
or $$F_{\text{cyl}}\left(4L\frac{\sin\theta\cos\theta}{\sqrt{9+16\sin^2\theta}}\right)\delta\theta - (500\text{ lb})(2L\cos\theta\delta\theta) = 0$$

and $$F_{\text{cyl}}\frac{\sin\theta}{\sqrt{9+16\sin^2\theta}} = 250\text{ lb}$$

Finally, $$F_{\text{cyl}}\frac{\sin 30°}{\sqrt{9+16\sin^2 30°}} = 250\text{ lb}$$

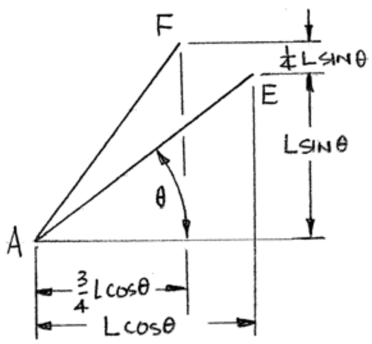
or $F_{\text{cyl}} = 1803$ lb ◀

PROBLEM 10.14



A double scissor lift table is used to raise a 1000-lb machine component. The table consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Each member of the linkage is of length 24 in., and pins C and G are at the midpoints of their respective members. The hydraulic cylinder is pinned at A to the base of the table and at F which is 6 in. from E. If the component is placed on the table so that half of its weight is supported by the system shown, determine the smallest allowable value of θ knowing that the maximum force each cylinder can exert is 8 kips.

SOLUTION



From the results of the Problem 10.13

$$F_{\text{cyl}} \frac{\sin\theta}{\sqrt{9+16\sin^2\theta}} = 250 \text{ lb}$$

Then $$(8000 \text{ lb})\frac{\sin\theta}{\sqrt{9+16\sin^2\theta}} = 250 \text{ lb}$$

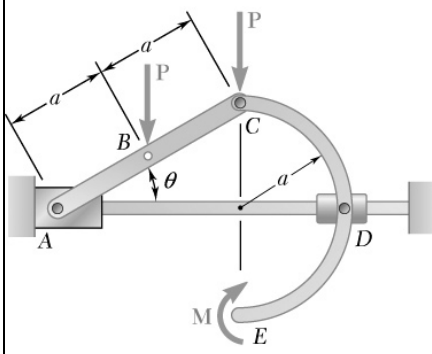
or $$(32\sin\theta)^2 = 9+16\sin^2\theta$$

Thus, $$\sin^2\theta = \frac{9}{1008}$$

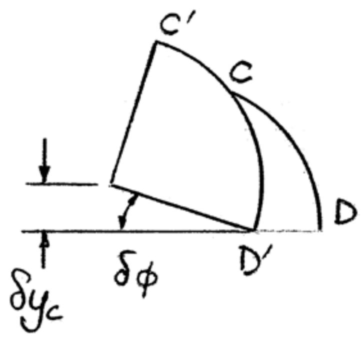
$\theta = 5.42°$ ◀

PROBLEM 10.15

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.



SOLUTION



ABC: $$y_B = a\sin\theta \Rightarrow \delta y_B = a\cos\theta\delta\theta$$

$$y_C = 2a\sin\theta \Rightarrow \delta y_C = 2a\cos\theta\delta\theta$$

CDE: Note that as *ABC* rotates counterclockwise, *CDE* rotates clockwise while it moves to the left.

Then $$\delta y_C = a\delta\phi$$

or $$2a\cos\theta\delta\theta = a\delta\phi$$

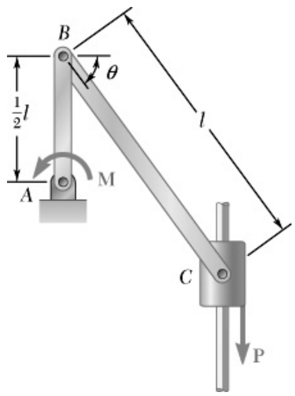
or $$\delta\phi = 2\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_B - P\delta y_C + M\delta\phi = 0$$

$$-P\left(a\cos\theta\delta\theta\right) - P\left(2a\cos\theta\delta\theta\right) + M\left(2\cos\theta\delta\theta\right) = 0$$

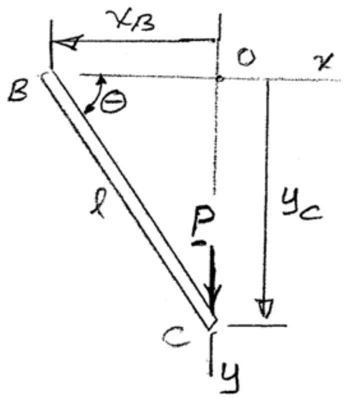
or $M = \dfrac{3}{2}Pa$ ◀

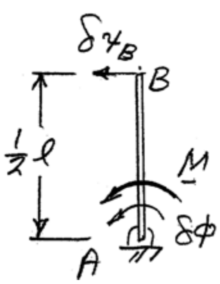


PROBLEM 10.16

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION





Have

$$x_B = l\cos\theta$$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Now $$\delta x_B = \frac{1}{2}l\delta\theta$$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or $$\delta\phi = -2\sin\theta\delta\theta$$

Virtual Work:

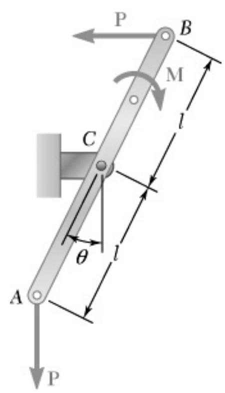
$$\delta U = 0: \quad M\delta\varphi + P\delta y_C = 0$$

$$M(-2\sin\theta\delta\theta) + P(l\cos\theta\delta\theta) = 0$$

or $$M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta}$$

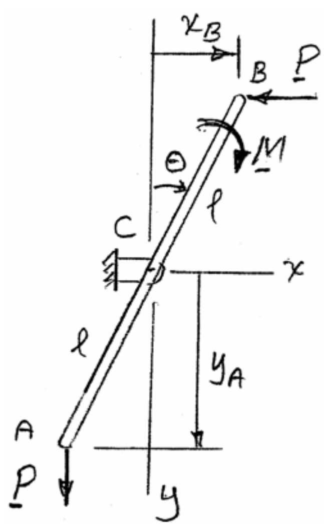
$$M = \frac{Pl}{2\tan\theta} \blacktriangleleft$$

PROBLEM 10.17



Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



Have

$$x_B = l\sin\theta$$

$$\delta x_B = l\cos\theta\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$

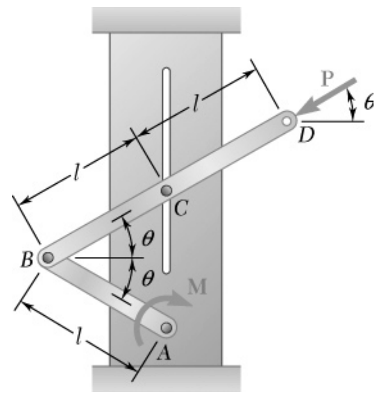
Virtual Work:

$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$

$$M\delta\theta - P(l\cos\theta\delta\theta) + P(-l\sin\theta\delta\theta) = 0$$

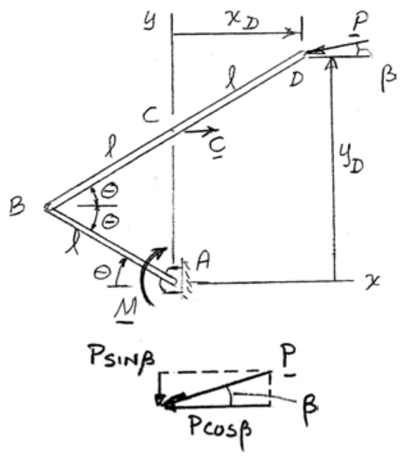
$$M = Pl(\sin\theta + \cos\theta) \blacktriangleleft$$

PROBLEM 10.18



The pin at C is attached to member BCD and can slide along a slot cut in the fixed plate shown. Neglecting the effect of friction, derive an expression for the magnitude of the couple **M** required to maintain equilibrium when the force **P** which acts at D is directed (a) as shown, (b) vertically downward, (c) horizontally to the right.

SOLUTION



Have $$x_D = l\cos\theta$$

$$\delta x_D = -l\sin\theta\delta\theta$$

$$y_D = 3l\sin\theta$$

$$\delta y_D = 3l\cos\theta\delta\theta$$

Virtual Work: $\delta U = 0$: $M\delta\theta - (P\cos\beta)\delta x_D - (P\sin\beta)\delta y_D = 0$

$$M\delta\theta - (P\cos\beta)(-l\sin\theta\delta\theta) - (P\sin\beta)(3l\cos\theta\delta\theta) = 0$$

$$M = Pl(3\sin\beta\cos\theta - \cos\beta\sin\theta) \qquad (1)$$

(a) For **P** directed along BCD, $\beta = \theta$

Equation (1): $M = Pl(3\sin\theta\cos\theta - \cos\theta\sin\theta)$

$M = Pl(2\sin\theta\cos\theta)$ $\qquad M = Pl\sin 2\theta$ ◀

(b) For **P** directed ↓, $\beta = 90°$

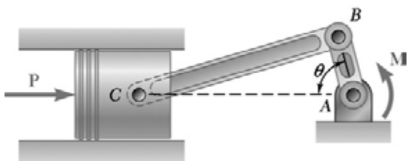
Equation (1): $M = Pl(3\sin 90°\cos\theta - \cos 90°\sin\theta)$

$M = 3Pl\cos\theta$ ◀

(c) For **P** directed ⟶, $\beta = 180°$

Equation (1): $M = Pl(3\sin 180°\cos\theta - \cos 180°\sin\theta)$

$M = Pl\sin\theta$ ◀

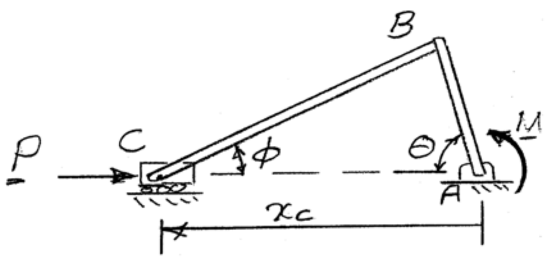


PROBLEM 10.19

A 1-kip force **P** is applied as shown to the piston of the engine system. Knowing that $AB = 2.5$ in. and $BC = 10$ in., determine the couple **M** required to maintain the equilibrium of the system when (*a*) $\theta = 30°$, (*b*) $\theta = 150°$.

SOLUTION

Analysis of the geometry:



Law of Sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \tag{1}$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \tag{2}$$

Now, from Equation (1)

$$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \tag{3}$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}\left(\sin\theta\cos\phi + \sin\phi\cos\theta\right)\delta\theta$$

Then

$$\delta x_C = -\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta$$

PROBLEM 10.19 CONTINUED

Virtual Work: $$\delta U = 0: \quad -P\delta x_C - M\delta\theta = 0$$

$$-P\left[-\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P \qquad (4)$$

For the given conditions: $P = 1.0\text{ kip} = 1000\text{ lb}$, $AB = 2.5\text{ in.}$, and $BC = 10\text{ in.}$:

(a) When $$\theta = 30°: \quad \sin\phi = \frac{2.5}{10}\sin 30°, \qquad \phi = 7.181°$$

$$M = (2.5\text{ in.})\frac{\sin(30° + 7.181°)}{\cos 7.181°}(1.0\text{ kip}) = 1.5228\text{ kip}\cdot\text{in.}$$

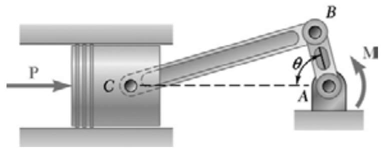
$$= 0.1269\text{ kip}\cdot\text{ft}$$

or $\mathbf{M} = 126.9\text{ lb}\cdot\text{ft}$ ◀

(b) When $$\theta = 150°: \quad \sin\phi = \frac{2.5}{10}\sin 150°, \qquad \phi = 7.181°$$

$$M = (2.5\text{ in.})\frac{\sin(150° + 7.181°)}{\cos 7.181°}(1.0\text{ kip}) = 0.97722\text{ kip}\cdot\text{in.}$$

or $\mathbf{M} = 81.4\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 10.20

A couple **M** of magnitude $75 \text{ lb} \cdot \text{ft}$ is applied as shown to the crank of the engine system. Knowing that $AB = 2.5$ in. and $BC = 10$ in., determine the force **P** required to maintain the equilibrium of the system when (a) $\theta = 60°$, (b) $\theta = 120°$.

SOLUTION

From the analysis of Problem 10.19,

$$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P$$

Now, with $M = 75 \text{ lb} \cdot \text{ft} = 900 \text{ lb} \cdot \text{in.}$

(a) For $\theta = 60°$

$$\sin\phi = \frac{2.5}{10}\sin 60°, \qquad \phi = 12.504°$$

$$(900 \text{ lb} \cdot \text{in.}) = (2.5 \text{ in.})\frac{\sin(60° + 12.504°)}{\cos 12.504°}(P)$$

or

$$P = 368.5 \text{ lb}$$

$$\mathbf{P} = 369 \text{ lb} \longrightarrow \blacktriangleleft$$

(b) For $\theta = 120°$

$$\sin\phi = \frac{2.5}{10}\sin 120°, \qquad \phi = 12.504°$$

$$(900 \text{ lb} \cdot \text{in.}) = (2.5 \text{ in.})\frac{\sin(120° + 12.504°)}{\cos 12.504°}(P)$$

or

$$P = 476.7 \text{ lb}$$

$$\mathbf{P} = 477 \text{ lb} \longrightarrow \blacktriangleleft$$