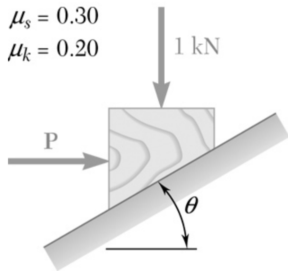
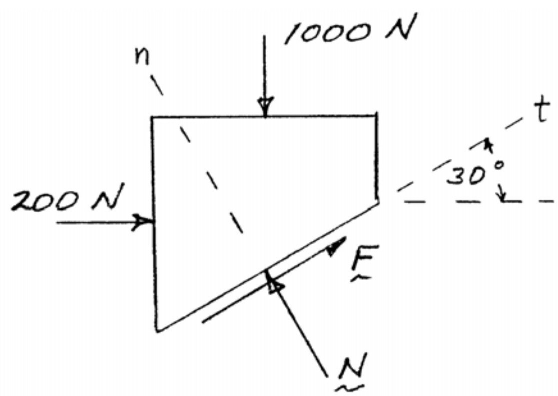


PROBLEM 8.1

Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30^o$ and $P = 200\,\text{N}$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 30° - (200\text{ N})\sin 30° = 0$$

$$N = 966.03\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F + (200\text{ N})\cos 30° - (1000\text{ N})\sin 30° = 0$$

$$F = 326.8\text{ N} = F_{\text{eq.}}$$

But $$F_{\max} = \mu_s N = (0.3)966\text{ N} = 290\text{ N}$$

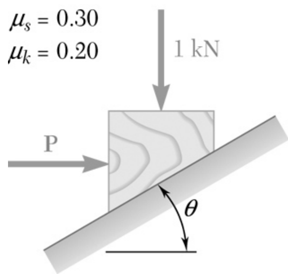
$$F_{\text{eq.}} > F_{\max} \qquad \text{impossible} \Rightarrow \text{Block moves} \blacktriangleleft$$

and $$F = \mu_k N$$

$$= (0.2)(966.03\text{ N})$$

Block slides down $$\mathbf{F} = 193.2\text{ N} \nearrow \blacktriangleleft$$

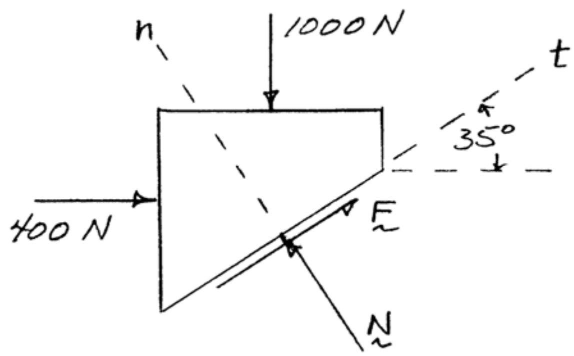
PROBLEM 8.2



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 35^{\text{o}}$ and $P = 400$ N.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 35° - (400\text{ N})\sin 35° = 0$$

$$N = 1048.6\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F - (1000\text{ N})\sin 35° + (400\text{ N})\cos 35° = 0$$

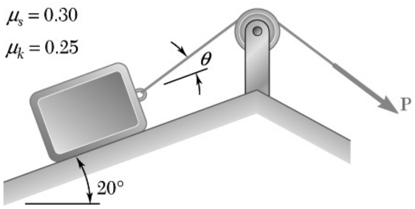
$$F = 246\text{ N} = F_{\text{eq.}}$$

$$F_{\max} = \mu_s N = (0.3)(1048.6\text{ N}) = 314\text{ N}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

$\therefore$ **F** $= 246$ N $\nearrow$ ◀

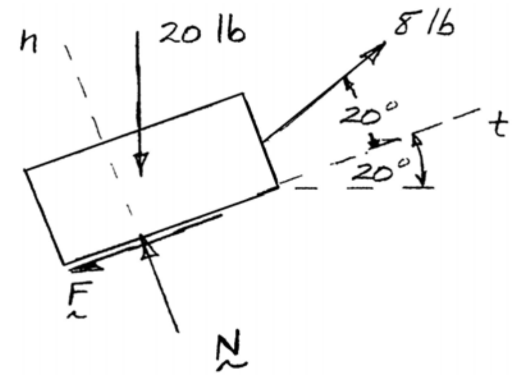
PROBLEM 8.3



Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 8$ lb and $\theta = 20°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (8 \text{ lb})\sin 20° = 0$$

$$N = 16.0577 \text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(16.0577 \text{ lb}) = 4.817 \text{ lb}$$

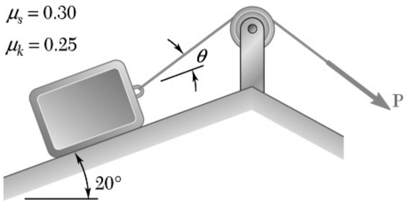
Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (8 \text{ lb})\cos 20° - (20 \text{ lb})\sin 20° - F = 0$$

$$F = 0.6771 \text{ lb} = F_{\text{eq.}}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

and $\mathbf{F} = 0.677$ lb $\swarrow$ ◀

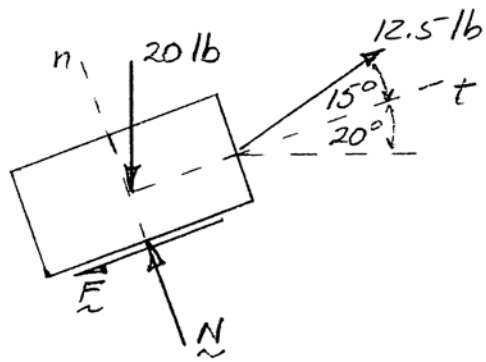


PROBLEM 8.4

Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 12.5$ lb and $\theta = 15°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + (12.5\text{ lb})\sin 15° = 0$$

$$N = 15.559\text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(15.559\text{ lb}) = 4.668\text{ lb}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (12.5\text{ lb})\cos 15° - (20\text{ lb})\sin 20° - F = 0$$

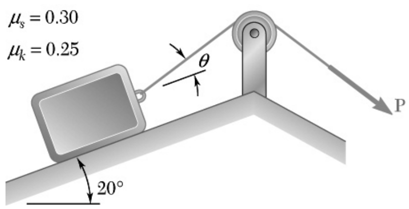
$$F = 5.23\text{ lb} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and $$F = \mu_k N = (0.25)(15.559\text{ lb})$$

$$\mathbf{F} = 3.89\text{ lb} \swarrow$$ ◀

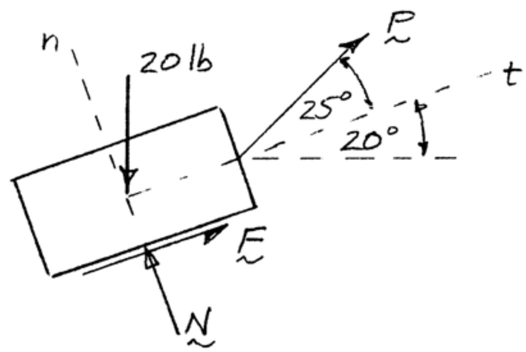
PROBLEM 8.5



Knowing that $\theta = 25°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBD block:



Block is in equilibrium:

$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + P\sin 25° = 0$$

$$N = 18.794\text{ lb} - P\sin 25°$$

$$\nearrow \Sigma F_t = 0: \quad F - (20\text{ lb})\sin 20° + P\cos 25° = 0$$

or

$$F = 6.840\text{ lb} - P\cos 25°$$

Impending motion up: $F = \mu_s N$; Impending motion down: $F = -\mu_s N$

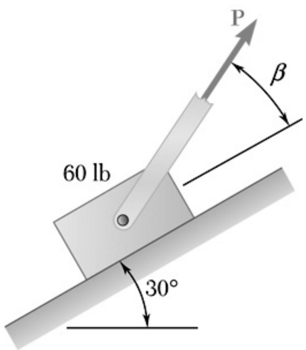
Therefore,

$$6.840\text{ lb} - P\cos 25° = \pm(0.3)(18.794\text{ lb} - P\sin 25°)$$

$$P_{\text{up}} = 12.08\text{ lb} \qquad P_{\text{down}} = 1.542\text{ lb}$$

$$1.542\text{ lb} \le P_{\text{eq.}} \le 12.08\text{ lb} \blacktriangleleft$$

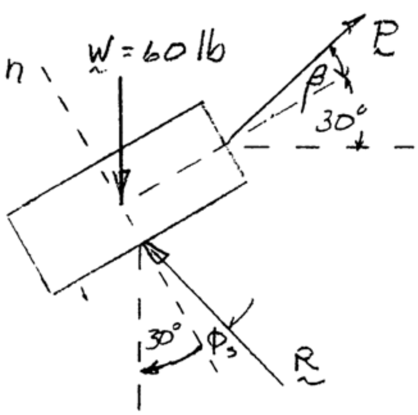
PROBLEM 8.6



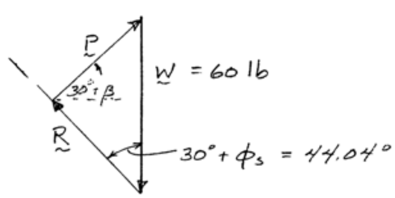
Knowing that the coefficient of friction between the 60-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P for which motion of the block up the incline is impending, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion up)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.04°$$



(*a*) Note: For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$

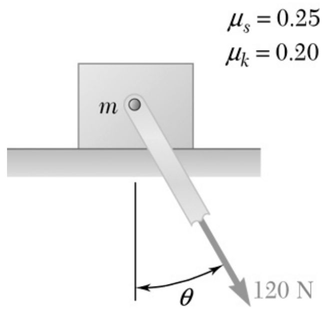
Then $$P = W\sin(30° + \phi_s)$$

$$= (60\text{ lb})\sin 44.04° = 41.71\text{ lb}$$

$P_{\min} = 41.7\text{ lb}$ ◀

(*b*) Have $\beta = \phi_s$ $\qquad\beta = 14.04°$ ◀

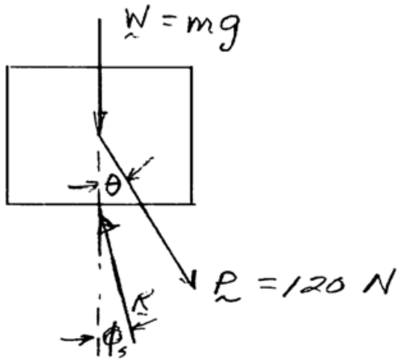
PROBLEM 8.7



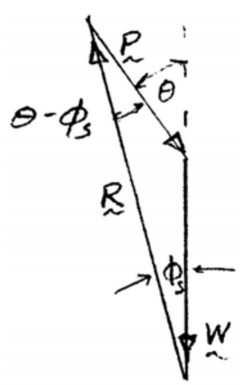
Considering only values of θ less than $90°$, determine the smallest value of θ for which motion of the block to the right is impending when $(a)\, m = 30$ kg, $(b)\, m = 40$ kg.

SOLUTION

FBD block (impending motion to the right)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



$$\frac{P}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W}{P}\sin\phi_s \qquad W = mg$$

(a)

$$m = 30 \text{ kg:} \quad \theta - \phi_s = \sin^{-1}\left[\frac{(30\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 36.499°$$

$$\therefore \quad \theta = 36.499° + 14.036° \qquad \text{or} \quad \theta = 50.5° \blacktriangleleft$$

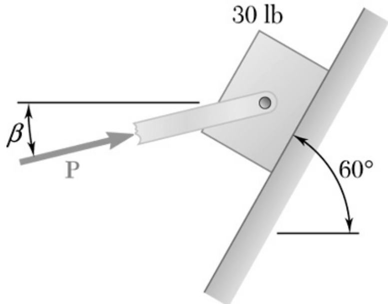
(b)

$$m = 40 \text{ kg:} \quad \theta - \phi_s = \sin^{-1}\left[\frac{(40\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 52.474°$$

$$\therefore \quad \theta = 52.474° + 14.036° \qquad \text{or} \quad \theta = 66.5° \blacktriangleleft$$

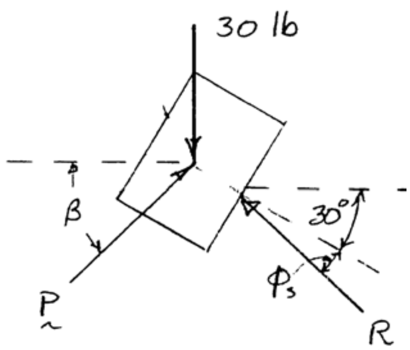
PROBLEM 8.8



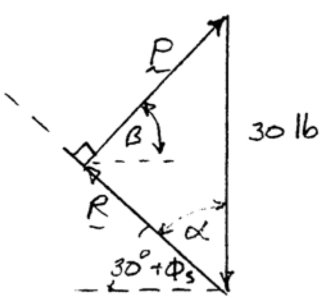
Knowing that the coefficient of friction between the 30-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to maintain the block in equilibrium, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion downward)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



(*a*) Note: For minimum *P*, $\mathbf{P} \perp \mathbf{R}$

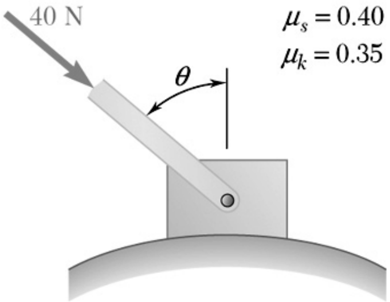
So $\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$

and $P = (30\text{ lb})\sin\alpha = (30\text{ lb})\sin(45.964°) = 21.567\text{ lb}$

$P = 21.6\text{ lb}$ ◀

(*b*) $\beta = 46.0°$ ◀

PROBLEM 8.9

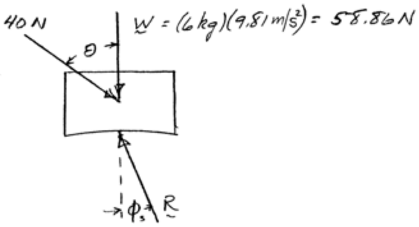


$\mu_s = 0.40$
$\mu_k = 0.35$

A 6-kg block is at rest as shown. Determine the positive range of values of θ for which the block is in equilibrium if (*a*) θ is less than 90°, (*b*) θ is between 90° and 180°.

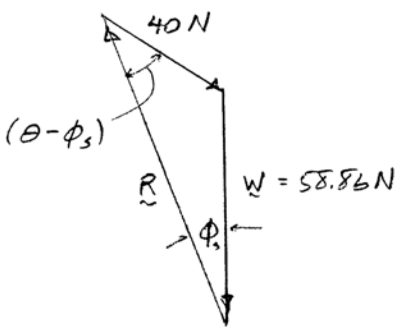
SOLUTION

FBD block (impending motion)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$$

(*a*) $0° \le \theta \le 90°$:



$$\frac{58.86 \text{ N}}{\sin(\theta - \phi_s)} = \frac{40 \text{ N}}{\sin\phi_s}$$

$$\theta - \phi_s = \sin^{-1}\frac{58.86 \text{ N}}{40 \text{ N}}\sin(21.801°)$$

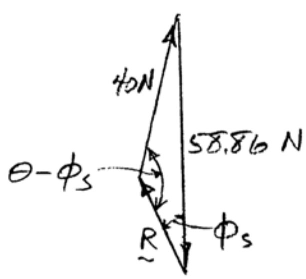
$$= 33.127°,\ 146.873°$$

$$\theta = 54.9° \quad \text{and} \quad \theta = 168.674°$$

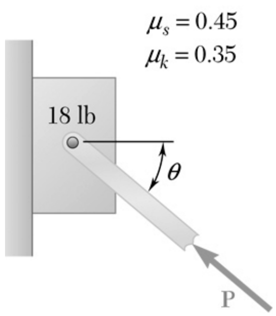
$\therefore$ (*a*) Equilibrium for $0 \le \theta \le 54.9°$ ◀

(*b*) $90° \le \theta \le 180°$:

(*b*) and for $168.7° \le \theta \le 180.0°$ ◀



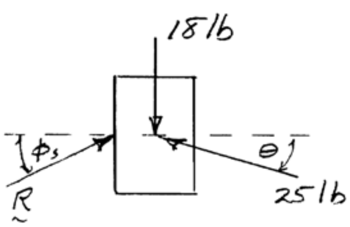
PROBLEM 8.10

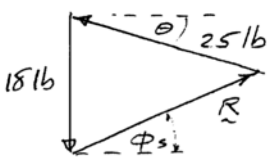


Knowing that $P = 25$ lb, determine the range of values of θ for which equilibrium of the 18-lb block is maintained.

SOLUTION

FBD block (impending motion down)





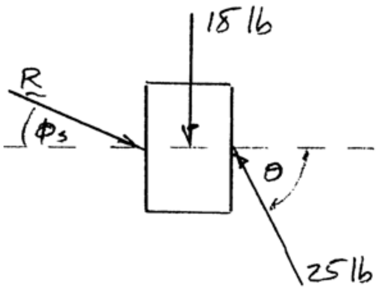
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.228°$$

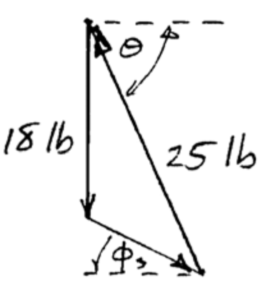
$$\frac{25 \text{ lb}}{\sin(90° - \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta + \phi_s)}$$

$$\theta + \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° - 24.228°)\right] = 41.04°$$

$$\theta = 16.81°$$

Impending motion up:





$$\frac{25 \text{ lb}}{\sin(90° + \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta - \phi_s)}$$

$$\theta - \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° + 24.228°)\right] = 41.04°$$

$$\theta = 65.27°$$

Equilibrium for $16.81° \le \theta \le 65.3°$ ◀