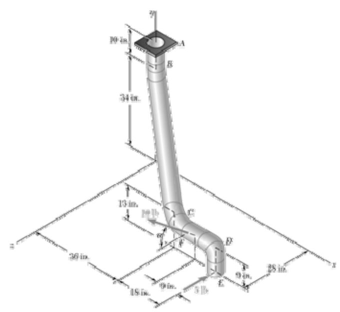


PROBLEM 3.120

A portion of the flue for a furnace is attached to the ceiling at A. While supporting the free end of the flue at F, a worker pushes in at E and pulls out at F to align end E with the furnace. Knowing that the 10-lb force at F lies in a plane parallel to the yz plane and that $\alpha = 60°$, (a) replace the given force system with an equivalent force-couple system at C, (b) determine whether duct CD will tend to rotate clockwise or counterclockwise relative to elbow C, as viewed from D to C.

SOLUTION

(a) Have

$$\mathbf{R} = \Sigma\mathbf{F} = \mathbf{F}_F + \mathbf{F}_E$$

where

$$\mathbf{F}_F = 10\text{ lb}\left[(\sin 60°)\,\mathbf{j} + (\cos 60°)\,\mathbf{k}\right] = (8.6603\text{ lb})\,\mathbf{j} + (5.0\text{ lb})\,\mathbf{k}$$

$$\mathbf{F}_E = -(5\text{ lb})\,\mathbf{k}$$

$$\therefore\quad \mathbf{R} = (8.6603\text{ lb})\,\mathbf{j} \qquad\qquad \text{or } \mathbf{R} = (8.66\text{ lb})\,\mathbf{j} \blacktriangleleft$$

Have

$$\mathbf{M}_C^R = \Sigma(\mathbf{r}\times\mathbf{F}) = \mathbf{r}_{F/C}\times\mathbf{F}_F + \mathbf{r}_{E/C}\times\mathbf{F}_E$$

where

$$\mathbf{r}_{F/C} = (9\text{ in.})\,\mathbf{i} - (2\text{ in.})\,\mathbf{j}$$

$$\mathbf{r}_{E/C} = (18\text{ in.})\,\mathbf{i} - (13\text{ in.})\,\mathbf{j}$$

$$\therefore\quad \mathbf{M}_C^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & -2 & 0 \\ 0 & 8.6603 & 5.0 \end{vmatrix}\text{lb}\cdot\text{in.} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 18 & -13 & 0 \\ 0 & 0 & -5 \end{vmatrix}\text{lb}\cdot\text{in.}$$

$$= (55\text{ lb}\cdot\text{in.})\,\mathbf{i} + (45\text{ lb}\cdot\text{in.})\,\mathbf{j} + (77.942\text{ lb}\cdot\text{in.})\,\mathbf{k}$$

$$\text{or } \mathbf{M}_C^R = (55.0\text{ lb}\cdot\text{in.})\,\mathbf{i} + (45.0\text{ lb}\cdot\text{in.})\,\mathbf{j} + (77.9\text{ lb}\cdot\text{in.})\,\mathbf{k} \blacktriangleleft$$

(b) To determine which direction duct section CD has a tendency to turn, have

$$M_{CD}^R = \boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R$$

where

$$\boldsymbol{\lambda}_{DC} = \frac{-(18\text{ in.})\,\mathbf{i} + (4\text{ in.})\,\mathbf{j}}{2\sqrt{85}\text{ in.}} = \frac{1}{\sqrt{85}}(-9\mathbf{i} + 2\mathbf{j})$$

Then

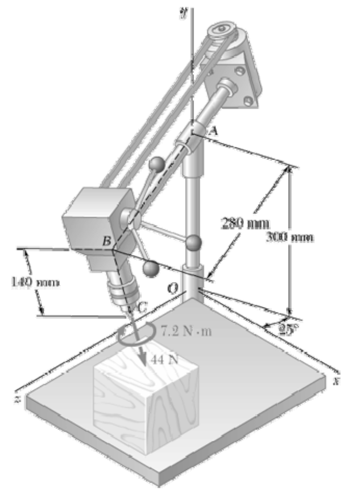
$$M_{CD}^R = \frac{1}{\sqrt{85}}(-9\mathbf{i} + 2\mathbf{j})\cdot(55\mathbf{i} + 45\mathbf{j} + 77.942\mathbf{k})\text{lb}\cdot\text{in.}$$

$$= (-53.690 + 9.7619)\text{lb}\cdot\text{in.}$$

$$= -43.928\text{ lb}\cdot\text{in.}$$

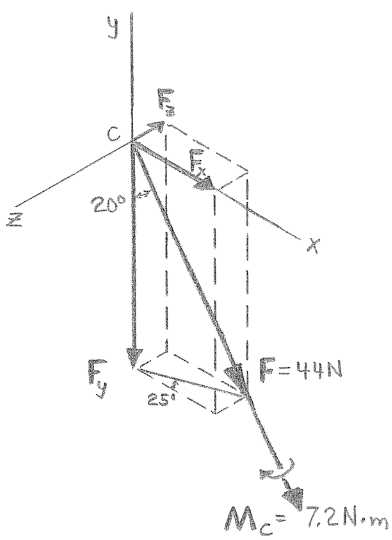
Since $\boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R < 0$, duct DC tends to rotate *clockwise* relative to elbow C as viewed from D to C. $\blacktriangleleft$

PROBLEM 3.121



The head-and-motor assembly of a radial drill press was originally positioned with arm *AB* parallel to the *z* axis and the axis of the chuck and bit parallel to the *y* axis. The assembly was then rotated 25° about the *y* axis and 20° about the centerline of the horizontal arm *AB*, bringing it into the position shown. The drilling process was started by switching on the motor and rotating the handle to bring the bit into contact with the workpiece. Replace the force and couple exerted by the drill press with an equivalent force-couple system at the center *O* of the base of the vertical column.

SOLUTION



Have $\mathbf{R} = \mathbf{F}$

$$= (44\text{ N})\left[(\sin 20^{\circ}\cos 25^{\circ})\mathbf{i} - (\cos 20^{\circ})\mathbf{j} - (\sin 20^{\circ}\sin 25^{\circ})\mathbf{k}\right]$$

$$= (13.6389\text{ N})\mathbf{i} - (41.346\text{ N})\mathbf{j} - (6.3599\text{ N})\mathbf{k}$$

$$\text{or } \mathbf{R} = (13.64\text{ N})\mathbf{i} - (41.3\text{ N})\mathbf{j} - (6.36\text{ N})\mathbf{k} \blacktriangleleft$$

Have $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F} + \mathbf{M}_C$

where

$$\mathbf{r}_{B/O} = \left[(0.280\text{ m})\sin 25^{\circ}\right]\mathbf{i} + (0.300\text{ m})\mathbf{j} + \left[(0.280\text{ m})\cos 25^{\circ}\right]\mathbf{k}$$

$$= (0.118333\text{ m})\mathbf{i} + (0.300\text{ m})\mathbf{j} + (0.25377\text{ m})\mathbf{k}$$

$$\mathbf{M}_C = (7.2\text{ N}\cdot\text{m})\left[(\sin 20^{\circ}\cos 25^{\circ})\mathbf{i} - (\cos 20^{\circ})\mathbf{j} - (\sin 20^{\circ}\sin 25^{\circ})\mathbf{k}\right]$$

$$= (2.2318\text{ N}\cdot\text{m})\mathbf{i} - (6.7658\text{ N}\cdot\text{m})\mathbf{j} - (1.04072\text{ N}\cdot\text{m})\mathbf{k}$$

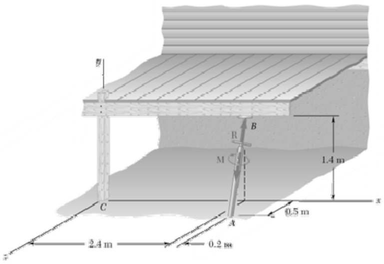
$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.118333 & 0.300 & 0.25377 \\ 13.6389 & -41.346 & -6.3599 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ (2.2318\mathbf{i} - 6.7658\mathbf{j} - 1.04072\mathbf{k})\text{N}\cdot\text{m}$$

$$= (10.8162\text{ N}\cdot\text{m})\mathbf{i} - (2.5521\text{ N}\cdot\text{m})\mathbf{j} - (10.0250\text{ N}\cdot\text{m})\mathbf{k}$$

$$\text{or } \mathbf{M}_O = (10.82\text{ N}\cdot\text{m})\mathbf{i} - (2.55\text{ N}\cdot\text{m})\mathbf{j} - (10.03\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.122



While a sagging porch is leveled and repaired, a screw jack is used to support the front of the porch. As the jack is expanded, it exerts on the porch the force-couple system shown, where $R = 300\text{ N}$ and $M = 37.5\text{ N}\cdot\text{m}$. Replace this force-couple system with an equivalent force-couple system at C.

SOLUTION

From
$$\mathbf{R}_C = \mathbf{R} = (300\text{ N})\,\boldsymbol{\lambda}_{AB} = 300\text{ N}\left[\frac{-(0.2\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (0.5\text{ m})\mathbf{k}}{1.50\text{ m}}\right]$$

$$\mathbf{R}_C = -(40.0\text{ N})\mathbf{i} + (280\text{ N})\mathbf{j} - (100\text{ N})\mathbf{k} \blacktriangleleft$$

From
$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{R} + \mathbf{M}$$

where

$$\mathbf{r}_{A/C} = (2.6\text{ m})\mathbf{i} + (0.5\text{ m})\mathbf{k}$$

$$\mathbf{M} = (37.5\text{ N}\cdot\text{m})\,\boldsymbol{\lambda}_{BA} = (37.5\text{ N}\cdot\text{m})\left[\frac{(0.2\text{ m})\mathbf{i} - (1.4\text{ m})\mathbf{j} + (0.5\text{ m})\mathbf{k}}{1.50\text{ m}}\right]$$

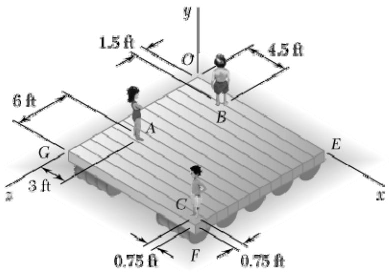
$$= (5.0\text{ N}\cdot\text{m})\mathbf{i} - (35.0\text{ N}\cdot\text{m})\mathbf{j} + (12.5\text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_C = (10\text{ N}\cdot\text{m})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.6 & 0 & 0.5 \\ -4 & 28 & -10 \end{vmatrix} + (5.0\text{ N}\cdot\text{m})\mathbf{i} - (35.0\text{ N}\cdot\text{m})\mathbf{j} + (12.5\text{ N}\cdot\text{m})\mathbf{k}$$

$$= \big[(-140 + 5)\text{N}\cdot\text{m}\big]\mathbf{i} + \big[(-20 + 260 - 35)\text{N}\cdot\text{m}\big]\mathbf{j} + \big[(728 + 12.5)\text{N}\cdot\text{m}\big]\mathbf{k}$$

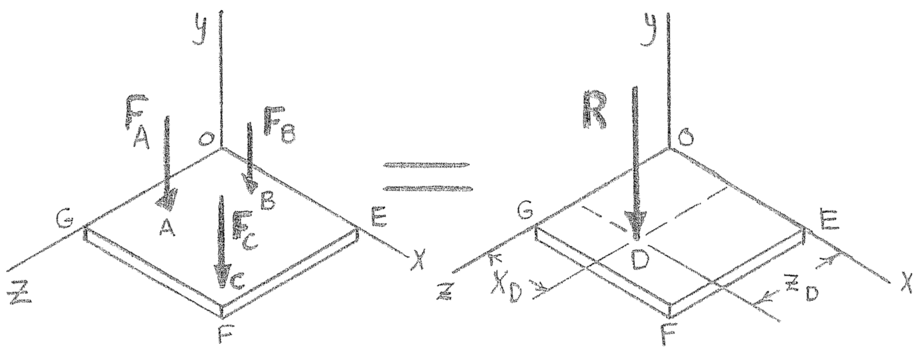
$$\text{or}\quad \mathbf{M}_C = -(135.0\text{ N}\cdot\text{m})\mathbf{i} + (205\text{ N}\cdot\text{m})\mathbf{j} + (741\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.123



Three children are standing on a $15 \times 15\text{-ft}$ raft. If the weights of the children at points *A*, *B*, and *C* are 85 lb, 60 lb, and 90 lb, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



Have $$\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$$

$$-(85 \text{ lb})\mathbf{j} - (60 \text{ lb})\mathbf{j} - (90 \text{ lb})\mathbf{j} = \mathbf{R}$$

$$-(235 \text{ lb})\mathbf{j} = \mathbf{R}$$

or $R = 235$ lb ◀

Have $$\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$$

$$(85 \text{ lb})(9 \text{ ft}) + (60 \text{ lb})(1.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) = (235 \text{ lb})(z_D)$$

$$\therefore \quad z_D = 9.0957 \text{ ft}$$

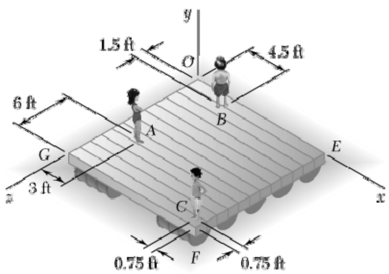
or $z_D = 9.10$ ft ◀

Have $$\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$$

$$(85 \text{ lb})(3 \text{ ft}) + (60 \text{ lb})(4.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) = (235 \text{ lb})(x_D)$$

$$\therefore \quad x_D = 7.6915 \text{ ft}$$

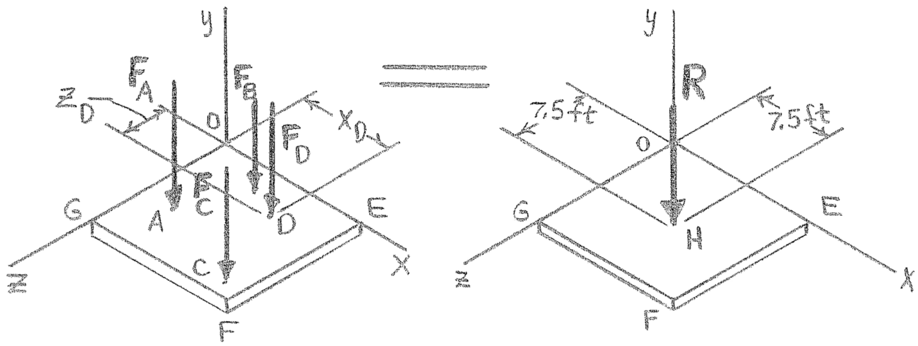
or $x_D = 7.69$ ft ◀



PROBLEM 3.124

Three children are standing on a 15×15-ft raft. The weights of the children at points *A*, *B*, and *C* are 85 lb, 60 lb, and 90 lb, respectively. If a fourth child of weight 95 lb climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



Have $$\Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$$

$$-(85 \text{ lb})\mathbf{j} - (60 \text{ lb})\mathbf{j} - (90 \text{ lb})\mathbf{j} - (95 \text{ lb})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(330 \text{ lb})\mathbf{j}$$

Have $$\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$$

$$(85 \text{ lb})(9 \text{ ft}) + (60 \text{ lb})(1.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) + (95 \text{ lb})(z_D) = (330 \text{ lb})(7.5 \text{ ft})$$

$$\therefore \quad z_D = 3.5523 \text{ ft}$$

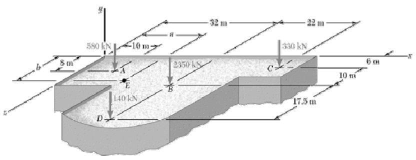
or $z_D = 3.55$ ft ◀

Have $$\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$$

$$(85 \text{ lb})(3 \text{ ft}) + (60 \text{ lb})(4.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) + (95 \text{ lb})(x_D) = (330 \text{ lb})(7.5 \text{ ft})$$

$$\therefore \quad x_D = 7.0263 \text{ ft}$$

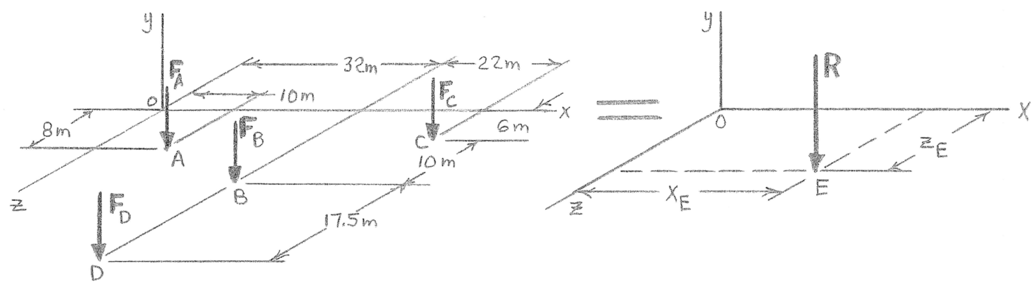
or $x_D = 7.03$ ft ◀



PROBLEM 3.125

The forces shown are the resultant downward loads on sections of the flat roof of a building because of accumulated snow. Determine the magnitude and the point of application of the resultant of these four loads.

SOLUTION



Have $\qquad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$

$$-(580\text{ kN})\mathbf{j} - (2350\text{ kN})\mathbf{j} - (330\text{ kN})\mathbf{j} - (140\text{ kN})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(3400\text{ kN})\mathbf{j} \qquad\qquad R = 3400\text{ kN} \blacktriangleleft$$

Have $\qquad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_E)$

$$(580\text{ kN})(8\text{ m}) + (2350\text{ kN})(16\text{ m}) + (330\text{ kN})(6\text{ m}) + (140\text{ kN})(33.5\text{ m}) = (3400\text{ kN})(z_E)$$

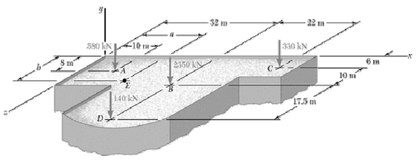
$$\therefore \quad z_E = 14.3853\text{ m} \qquad\qquad \text{or } z_E = 14.39\text{ m} \blacktriangleleft$$

Have $\qquad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_E)$

$$(580\text{ kN})(10\text{ m}) + (2350\text{ kN})(32\text{ m}) + (330\text{ kN})(54\text{ m}) + (140\text{ kN})(32\text{ m}) = (3400\text{ kN})(x_E)$$

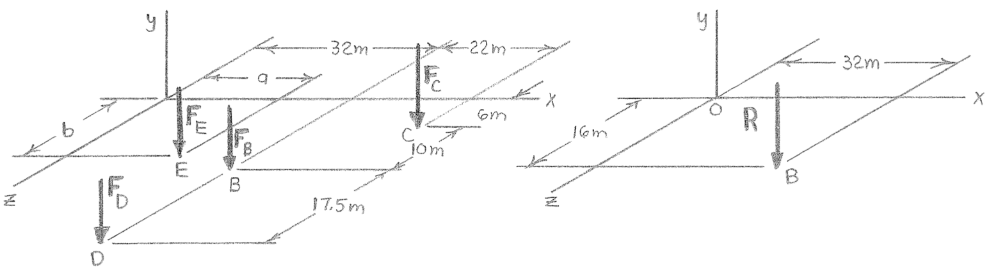
$$\therefore \quad x_E = 30.382\text{ m} \qquad\qquad \text{or } x_E = 30.4\text{ m} \blacktriangleleft$$

PROBLEM 3.126



The forces shown are the resultant downward loads on sections of the flat roof of a building because of accumulated snow. If the snow represented by the 580-kN force is shoveled so that the this load acts at *E*, determine *a* and *b* knowing that the point of application of the resultant of the four loads is then at *B*.

SOLUTION



Have $$\Sigma \mathbf{F}: \quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{F}_E = \mathbf{R}$$

$$-(2350\text{ kN})\mathbf{j} - (330\text{ kN})\mathbf{j} - (140\text{ kN})\mathbf{j} - (580\text{ kN})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(3400\text{ kN})\mathbf{j}$$

Have $$\Sigma M_x: \quad F_B(z_B) + F_C(z_C) + F_D(z_D) + F_E(z_E) = R(z_B)$$

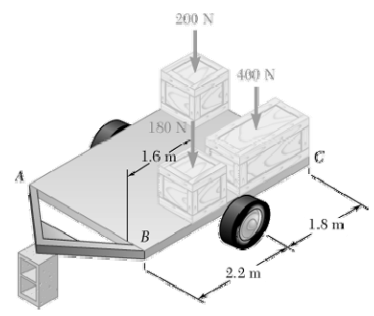
$$(2350\text{ kN})(16\text{ m}) + (330\text{ kN})(6\text{ m}) + (140\text{ kN})(33.5\text{ m}) + (580\text{ kN})(b) = (3400\text{ kN})(16\text{ m})$$

$$\therefore \quad b = 17.4655\text{ m}$$ or $b = 17.47$ m ◀

Have $$\Sigma M_z: \quad F_B(x_B) + F_C(x_C) + F_D(x_D) + F_E(x_E) = R(x_B)$$

$$(2350\text{ kN})(32\text{ m}) + (330\text{ kN})(54\text{ m}) + (140\text{ kN})(32\text{ m}) + (580\text{ kN})(a) = (3400\text{ kN})(32\text{ m})$$

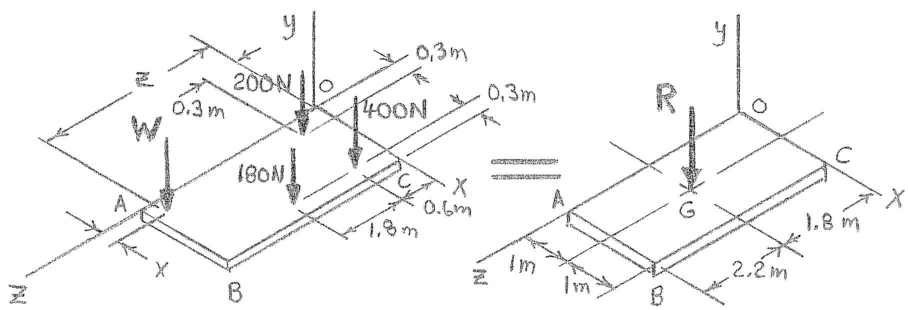
$$\therefore \quad a = 19.4828\text{ m}$$ or $a = 19.48$ m ◀



PROBLEM 3.127

A group of students loads a 2×4-m flatbed trailer with two $0.6 \times 0.6 \times 0.6$-m boxes and one $0.6 \times 0.6 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.6 \times 0.6 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.6 \times 0.6 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.6×0.6-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

Have $$\Sigma \mathbf{F}: \quad -(200 \text{ N})\mathbf{j} - (400 \text{ N})\mathbf{j} - (180 \text{ N})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(780 \text{ N})\mathbf{j}$$

Have $$\Sigma M_z: \quad (200 \text{ N})(0.3 \text{ m}) + (400 \text{ N})(1.7 \text{ m}) + (180 \text{ N})(1.7 \text{ m}) = (780 \text{ N})(x)$$

$$\therefore \quad x = 1.34103 \text{ m}$$

Have $$\Sigma M_x: \quad (200 \text{ N})(0.3 \text{ m}) + (400 \text{ N})(0.6 \text{ m}) + (180 \text{ N})(2.4 \text{ m}) = (780 \text{ N})(z)$$

$$\therefore \quad z = 0.93846 \text{ m}$$

From the statement of the problem, it is known that the resultant of **R** from the original loading and the lightest load **W** passes through G, the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load **W** is to be as small as possible, the fourth box should be placed as far from G as possible without the box overhanging the trailer. These two requirements imply

$$(0.3 \text{ m} \le x \le 1 \text{ m}) \quad (1.8 \text{ m} \le z \le 3.7 \text{ m})$$

PROBLEM 3.127 CONTINUED

Let $x = 0.3$ m, $\quad \Sigma M_{Gz}: \quad (200 \text{ N})(0.7 \text{ m}) - (400 \text{ N})(0.7 \text{ m}) - (180 \text{ N})(0.7 \text{ m}) + W(0.7 \text{ m}) = 0$

$$\therefore \quad W = 380 \text{ N}$$

$$\Sigma M_{Gx}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + (380 \text{ N})(z - 1.8 \text{ m}) = 0$$

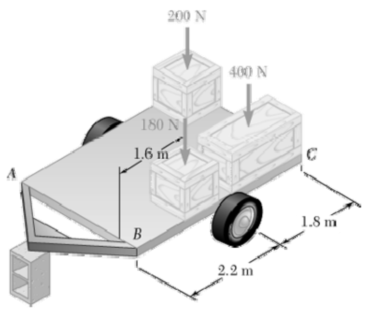
$$\therefore \; z = 3.5684 \text{ m} < 3.7 \text{ m} \qquad \therefore \quad \text{acceptable}$$

Let $z = 3.7$ m, $\quad \Sigma M_{Gx}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + W(1.7 \text{ m}) = 0$

$$\therefore \quad W = 395.29 \text{ N} > 380 \text{ N}$$

Since the weight W found for $x = 0.3$ m is less than W found for $z = 3.7$ m, $x = 0.3$ m results in the smallest weight W.

or $W = 380$ N at $(0.3 \text{ m}, 0, 3.57 \text{ m})$ ◀

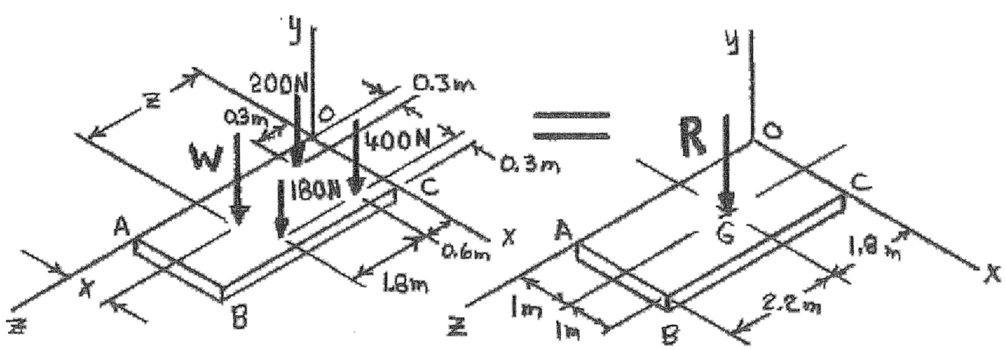


PROBLEM 3.128

Solve Problem 3.127 if the students want to place as much weight as possible in the fourth box and that at least one side of the box must coincide with a side of the trailer.

Problem 3.127: A group of students loads a 2×4-m flatbed trailer with two $0.6 \times 0.6 \times 0.6$-m boxes and one $0.6 \times 0.6 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.6 \times 0.6 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the largest additional weight on the trailer with the box having at least one side coinsiding with the side of the trailer, the box must be as close as possible to point G. For $x = 0.6$ m, with a small side of the box touching the z-axis, satisfies this condition.

Let $x = 0.6$ m, $\quad \Sigma M_{Gz}: \quad (200 \text{ N})(0.7 \text{ m}) - (400 \text{ N})(0.7 \text{ m}) - (180 \text{ N})(0.7 \text{ m}) + W(0.4 \text{ m}) = 0$

$$\therefore \quad W = 665 \text{ N}$$

and $\quad \Sigma M_{GX}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + (665 \text{ N})(z - 1.8 \text{ m}) = 0$

$$\therefore \quad z = 2.8105 \text{ m} \qquad (2 \text{ m} < z < 4 \text{ m}) \qquad \therefore \quad \text{acceptable}$$

or $W = 665$ N at $(0.6 \text{ m}, 0, 2.81 \text{ m})$ ◀