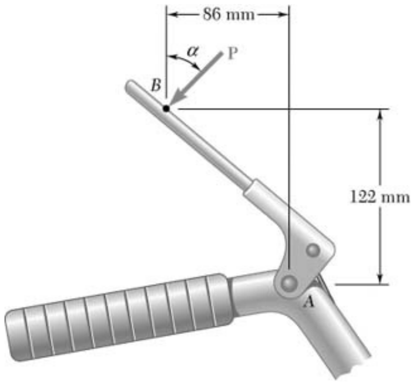
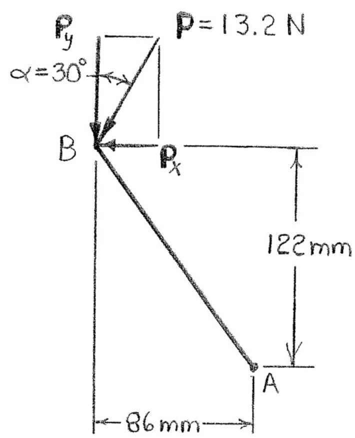
PROBLEM 3.1



A 13.2-N force **P** is applied to the lever which controls the auger of a snowblower. Determine the moment of **P** about *A* when α is equal to 30°.

SOLUTION



First note

$$P_x = P\sin\alpha = (13.2\text{ N})\sin 30^\circ = 6.60\text{ N}$$

$$P_y = P\cos\alpha = (13.2\text{ N})\cos 30^\circ = 11.4315\text{ N}$$

Noting that the direction of the moment of each force component about *A* is counterclockwise,

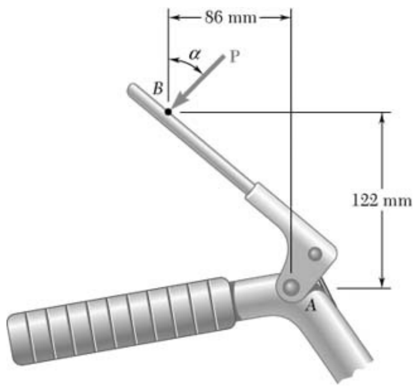
$$M_A = x_{B/A}P_y + y_{B/A}P_x$$

$$= (0.086\text{ m})(11.4315\text{ N}) + (0.122\text{ m})(6.60\text{ N})$$

$$= 1.78831\text{ N}\cdot\text{m}$$

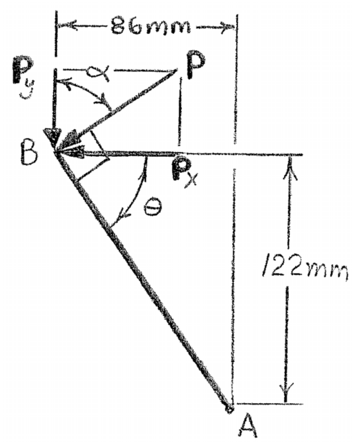
or $\mathbf{M}_A = 1.788\text{ N}\cdot\text{m}$ ↺ ◀

PROBLEM 3.2



The force **P** is applied to the lever which controls the auger of a snowblower. Determine the magnitude and the direction of the smallest force **P** which has a 2.20- N·m counterclockwise moment about *A*.

SOLUTION



For *P* to be a minimum, it must be perpendicular to the line joining points *A* and *B*.

$$r_{AB} = \sqrt{(86\text{ mm})^2 + (122\text{ mm})^2} = 149.265\text{ mm}$$

$$\alpha = \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{122\text{ mm}}{86\text{ mm}}\right) = 54.819°$$

Then $$M_A = r_{AB}P_{\min}$$

or $$P_{\min} = \frac{M_A}{r_{AB}}$$

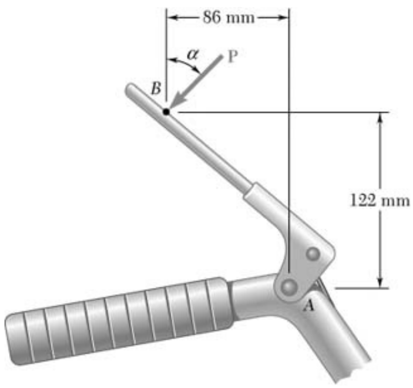
$$= \frac{2.20\text{ N}\cdot\text{m}}{149.265\text{ mm}}\left(\frac{1000\text{ mm}}{1\text{ m}}\right)$$

$$= 14.7389\text{ N}$$

$$\therefore\ \mathbf{P}_{\min} = 14.74\text{ N} \measuredangle\ 54.8°$$

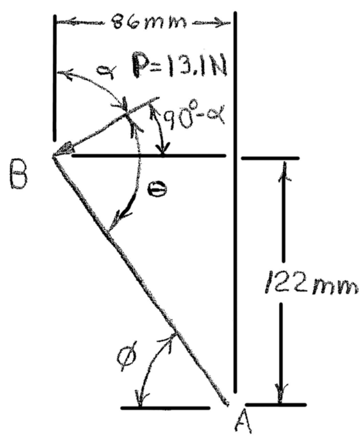
or $\mathbf{P}_{\min} = 14.74\text{ N} \measuredangle\ 35.2°$ ◀

PROBLEM 3.3



A 13.1-N force **P** is applied to the lever which controls the auger of a snowblower. Determine the value of α knowing that the moment of **P** about A is counterclockwise and has a magnitude of $1.95\ \text{N}\cdot\text{m}$.

SOLUTION



By definition $$M_A = r_{B/A} P \sin\theta$$

where $$\theta = \phi + (90° - \alpha)$$

and $$\phi = \tan^{-1}\left(\frac{122 \text{ mm}}{86 \text{ mm}}\right) = 54.819°$$

Also $$r_{B/A} = \sqrt{(86 \text{ mm})^2 + (122 \text{ mm})^2} = 149.265 \text{ mm}$$

Then $$1.95 \text{ N}\cdot\text{m} = (0.149265 \text{ m})(13.1 \text{ N})\sin(54.819° + 90° - \alpha)$$

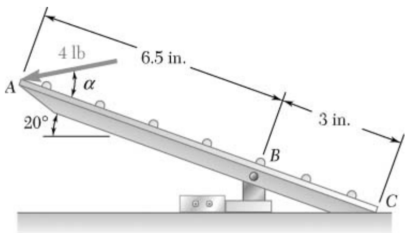
or $$\sin(144.819° - \alpha) = 0.99725$$

or $$144.819° - \alpha = 85.752°$$

and $$144.819° - \alpha = 94.248°$$

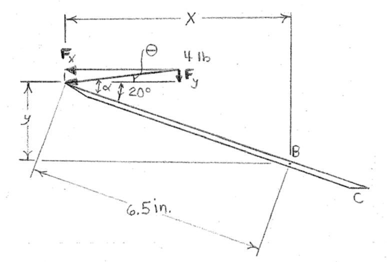
$$\therefore\ \alpha = 50.6°,\ 59.1° \blacktriangleleft$$

PROBLEM 3.4



A foot valve for a pneumatic system is hinged at B. Knowing that $\alpha = 28°$, determine the moment of the 4-lb force about point B by resolving the force into horizontal and vertical components.

SOLUTION



Note that $\theta = \alpha - 20° = 28° - 20° = 8°$

and $F_x = (4\text{ lb})\cos 8° = 3.9611\text{ lb}$

$$F_y = (4\text{ lb})\sin 8° = 0.55669\text{ lb}$$

Also $x = (6.5\text{ in.})\cos 20° = 6.1080\text{ in.}$

$$y = (6.5\text{ in.})\sin 20° = 2.2231\text{ in.}$$

Noting that the direction of the moment of each force component about B is counterclockwise,

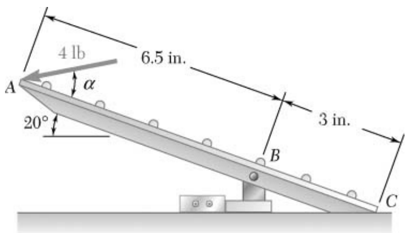
$$M_B = xF_y + yF_x$$

$$= (6.1080\text{ in.})(0.55669\text{ lb}) + (2.2231\text{ in.})(3.9611\text{ lb})$$

$$= 12.2062\text{ lb}\cdot\text{in.}$$

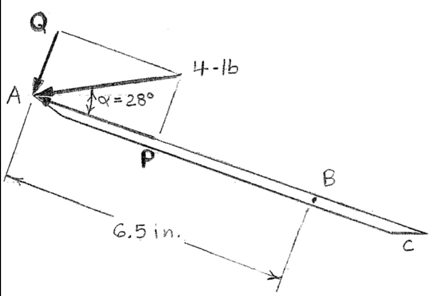
or $\mathbf{M}_B = 12.21\text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.5



A foot valve for a pneumatic system is hinged at B. Knowing that $\alpha = 28°$, determine the moment of the 4-lb force about point B by resolving the force into components along ABC and in a direction perpendicular to ABC.

SOLUTION



First resolve the 4-lb force into components **P** and **Q**, where

$$Q = (4.0 \text{ lb})\sin 28° = 1.87787 \text{ lb}$$

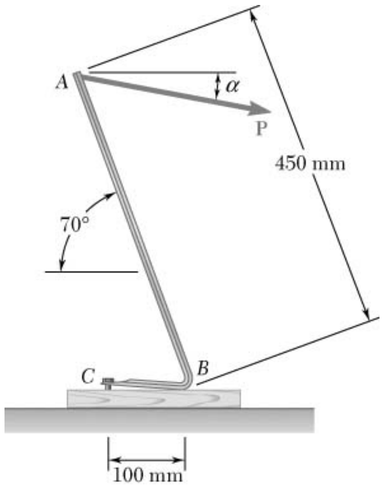
Then

$$M_B = r_{A/B}Q$$

$$= (6.5 \text{ in.})(1.87787 \text{ lb})$$

$$= 12.2063 \text{ lb}\cdot\text{in.}$$

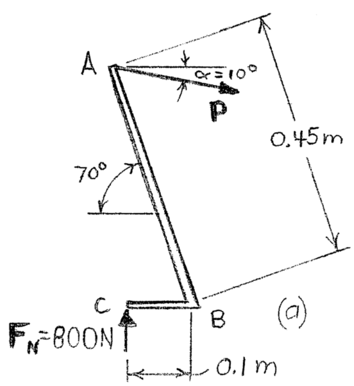
or $\mathbf{M}_B = 12.21 \text{ lb}\cdot\text{in.}$ ↻ ◀

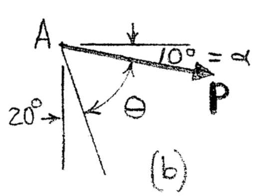


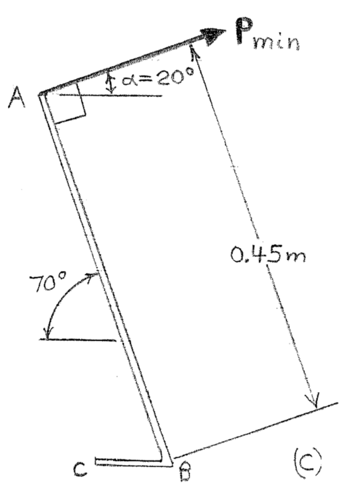
PROBLEM 3.6

It is known that a vertical force of 800 N is required to remove the nail at C from the board. As the nail first starts moving, determine (*a*) the moment about B of the force exerted on the nail, (*b*) the magnitude of the force $\mathbf{P}$ which creates the same moment about B if $\alpha = 10°$, (*c*) the smallest force $\mathbf{P}$ which creates the same moment about B.

SOLUTION







(*a*) Have

$$M_B = r_{C/B}F_N$$

$$= (0.1\text{ m})(800\text{ N})$$

$$= 80.0\text{ N}\cdot\text{m}$$

or $\mathbf{M}_B = 80.0\text{ N}\cdot\text{m}$ ◀

(*b*) By definition

$$M_B = r_{A/B}P\sin\theta$$

where

$$\theta = 90° - (90° - 70°) - \alpha$$

$$= 90° - 20° - 10°$$

$$= 60°$$

$$\therefore\quad 80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P\sin 60°$$

$$P = 205.28\text{ N}$$

or $P = 205\text{ N}$ ◀

(*c*) For $\mathbf{P}$ to be minimum, it must be perpendicular to the line joining points A and B. Thus, $\mathbf{P}$ must be directed as shown.

Thus

$$M_B = dP_{\min} = r_{A/B}P_{\min}$$

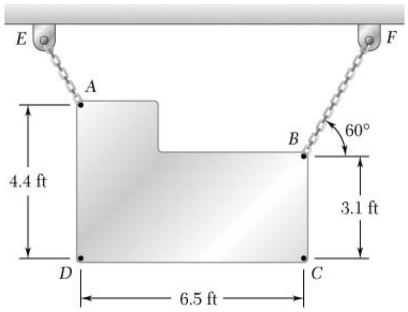
or

$$80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P_{\min}$$

$$\therefore\quad P_{\min} = 177.778\text{ N}$$

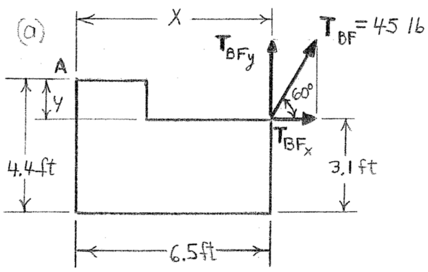
or $\mathbf{P}_{\min} = 177.8\text{ N} \measuredangle 20°$ ◀

PROBLEM 3.7



A sign is suspended from two chains *AE* and *BF*. Knowing that the tension in *BF* is 45 lb, determine (*a*) the moment about *A* of the force exert by the chain at *B*, (*b*) the smallest force applied at *C* which creates the same moment about *A*.

SOLUTION



(*a*) Have $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BF}$

Noting that the direction of the moment of each force component about *A* is counterclockwise,

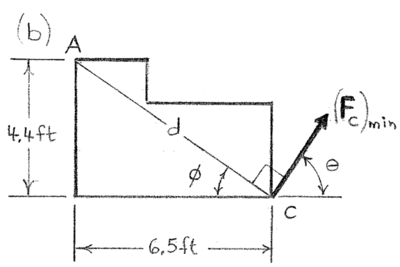
$$M_A = xT_{BFy} + yT_{BFx}$$

$$= (6.5 \text{ ft})(45 \text{ lb})\sin 60° + (4.4 \text{ ft} - 3.1 \text{ ft})(45 \text{ lb})\cos 60°$$

$$= 282.56 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 283 \text{ lb}\cdot\text{ft}$ ◀

(*b*) Have $\mathbf{M}_A = \mathbf{r}_{C/A} \times (\mathbf{F}_C)_{\min}$



For $\mathbf{F}_C$ to be minimum, it must be perpendicular to the line joining points *A* and *C*.

$$\therefore\ M_A = d(F_C)_{\min}$$

where $d = r_{C/A} = \sqrt{(6.5 \text{ ft})^2 + (4.4 \text{ ft})^2} = 7.8492 \text{ ft}$

$$\therefore\quad 282.56 \text{ lb}\cdot\text{ft} = (7.8492 \text{ ft})(F_C)_{\min}$$

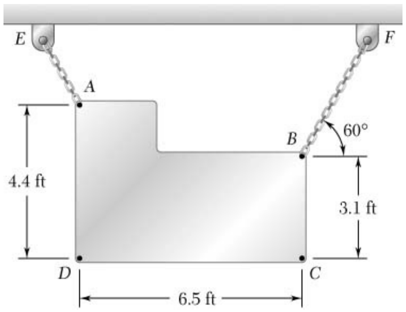
$$(F_C)_{\min} = 35.999 \text{ lb}$$

$$\phi = \tan^{-1}\left(\frac{4.4 \text{ ft}}{6.5 \text{ ft}}\right) = 34.095°$$

$$\theta = 90° - \phi = 90° - 34.095° = 55.905°$$

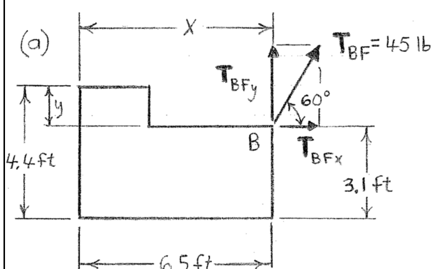
or $(\mathbf{F}_C)_{\min} = 36.0 \text{ lb} \measuredangle\ 55.9°$ ◀

PROBLEM 3.8



A sign is suspended from two chains *AE* and *BF*. Knowing that the tension in *BF* is 45 lb, determine (*a*) the moment about *A* of the force exerted by the chain at *B*, (*b*) the magnitude and sense of the vertical force applied at *C* which creates the same moment about *A*, (*c*) the smallest force applied at *B* which creates the same moment about *A*.

SOLUTION



(*a*) Have $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BF}$

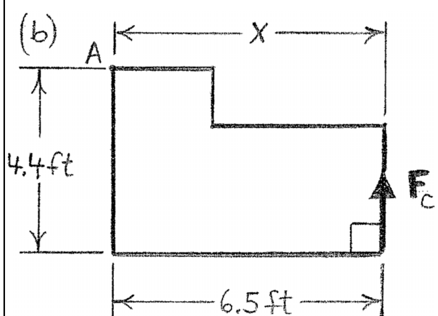
Noting that the direction of the moment of each force component about *A* is counterclockwise,

$$M_A = xT_{BFy} + yT_{BFx}$$

$$= (6.5 \text{ ft})(45 \text{ lb})\sin 60° + (4.4 \text{ ft} - 3.1 \text{ ft})(45 \text{ lb})\cos 60°$$

$$= 282.56 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 283 \text{ lb}\cdot\text{ft}$ ◀



(*b*) Have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$

or $M_A = xF_C$

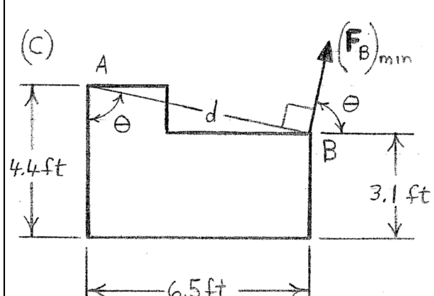
$$\therefore \; F_C = \frac{M_A}{x} = \frac{282.56 \text{ lb}\cdot\text{ft}}{6.5 \text{ ft}} = 43.471 \text{ lb}$$

or $\mathbf{F}_C = 43.5 \text{ lb} \uparrow$ ◀

(*c*) Have $\mathbf{M}_A = \mathbf{r}_{B/A} \times (\mathbf{F}_B)_{\min}$

For $\mathbf{F}_B$ to be minimum, it must be perpendicular to the line joining points *A* and *B*.

$$\therefore \; M_A = d(F_B)_{\min}$$

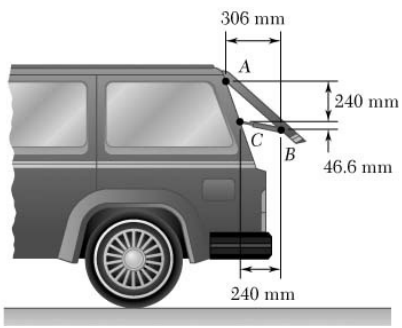


where $d = \sqrt{(6.5 \text{ ft})^2 + (4.4 \text{ ft} - 3.1 \text{ ft})^2} = 6.6287 \text{ ft}$

$$\therefore \; (F_B)_{\min} = \frac{M_A}{d} = \frac{282.56 \text{ lb}\cdot\text{ft}}{6.6287 \text{ ft}} = 42.627 \text{ lb}$$

and $\theta = \tan^{-1}\left(\dfrac{6.5 \text{ ft}}{4.4 \text{ ft} - 3.1 \text{ ft}}\right) = 78.690°$

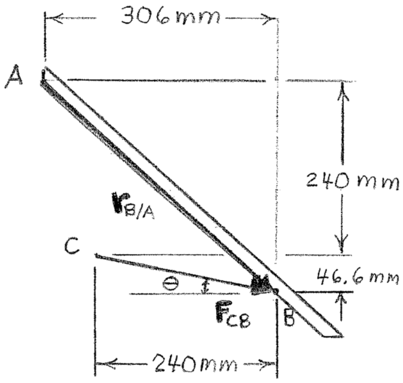
or $(\mathbf{F}_B)_{\min} = 42.6 \text{ lb} \measuredangle \; 78.7°$ ◀



PROBLEM 3.9

The tailgate of a car is supported by the hydraulic lift *BC*. If the lift exerts a 125-N force directed along its center line on the ball and socket at *B*, determine the moment of the force about *A*.

SOLUTION



First note $$d_{CB} = \sqrt{(240 \text{ mm})^2 + (46.6 \text{ mm})^2}$$

$$= 244.48 \text{ mm}$$

Then $$\cos\theta = \frac{240 \text{ mm}}{244.48 \text{ mm}}$$

$$\sin\theta = \frac{46.6 \text{ mm}}{244.48 \text{ mm}}$$

and $$\mathbf{F}_{CB} = F_{CB}\cos\theta\mathbf{i} - F_{CB}\sin\theta\mathbf{j}$$

$$= \frac{125 \text{ N}}{244.48 \text{ mm}}\left[(240 \text{ mm})\mathbf{i} - (46.6 \text{ mm})\mathbf{j}\right]$$

Now $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where $$\mathbf{r}_{B/A} = (306 \text{ mm})\mathbf{i} - (240 \text{ mm} + 46.6 \text{ mm})\mathbf{j}$$

$$= (306 \text{ mm})\mathbf{i} - (286.6 \text{ mm})\mathbf{j}$$

Then $$\mathbf{M}_A = \left[(306 \text{ mm})\mathbf{i} - (286.6 \text{ mm})\mathbf{j}\right] \times \frac{125 \text{ N}}{244.48}(240\mathbf{i} - 46.6\mathbf{j})$$

$$= (27878 \text{ N}\cdot\text{mm})\mathbf{k} = (27.878 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = 27.9 \text{ N}\cdot\text{m}$ ◀