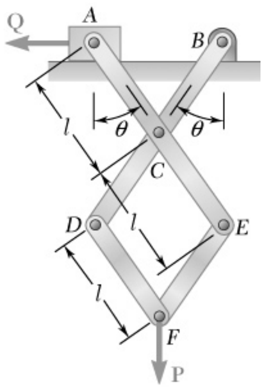
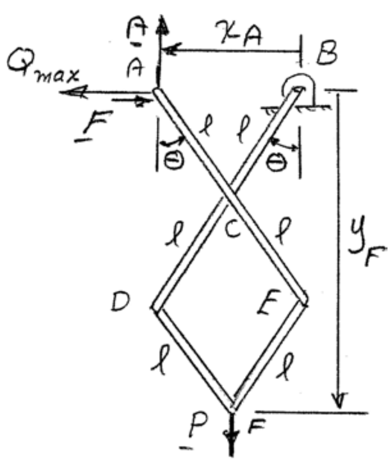


PROBLEM 10.51

Denoting by μ_s the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface, derive expressions in terms of P, μ_s, and θ for the largest and smallest magnitudes of the force **Q** for which equilibrium is maintained.

SOLUTION



For the linkage:

$$+\circlearrowright \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \qquad \text{or} \qquad \mathbf{A} = \frac{P}{2}\uparrow$$

Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $$y_F = 3l\cos\theta$$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

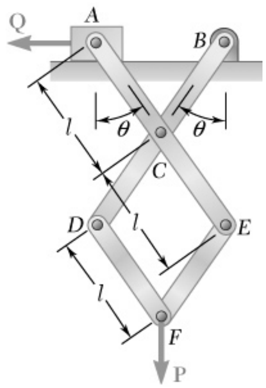
$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of *A* impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$



PROBLEM 10.52

Knowing that the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface is 0.15, determine the magnitudes of the largest and smallest force **Q** for which equilibrium is maintained when $\theta = 30^{\circ}$, $l = 8$ in., and $P = 160$ lb.

SOLUTION

Using the results of Problem 10.52 with

$$\theta = 30°,\ l = 8 \text{ in.}, P = 160 \text{ lb, and } \mu_s = 0.15$$

We have

$$Q_{\max} = \frac{(160 \text{ lb})}{2}(3\tan 30° + 0.15) = 150.56 \text{ lb}$$

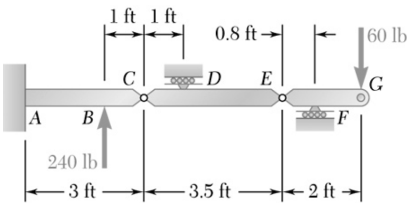
$$Q_{\max} = 150.6 \text{ lb} \blacktriangleleft$$

and

$$Q_{\min} = \frac{(160 \text{ lb})}{2}(3\tan 30° - 0.15) = 126.56 \text{ lb}$$

$$Q_{\min} = 126.6 \text{ lb} \blacktriangleleft$$

PROBLEM 10.53



Using the method of virtual work, determine separately the force and the couple representing the reaction at A.

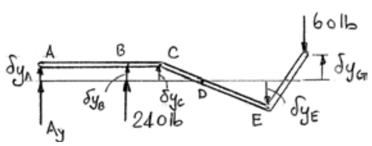
SOLUTION

A_y: Consider an upward displacement δy_A of ABC

ABC: $$\delta y_A = \delta y_B = \delta y_C$$

CDE: $$\frac{\delta y_C}{1\text{ ft}} = \frac{\delta y_E}{2.5\text{ ft}}$$

or $$\delta y_E = 2.5\delta y_A$$



EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

or $$\delta y_G = \frac{1.2\text{ ft}}{0.8\text{ ft}}(2.5\delta y_A)$$

$$= 3.75\delta y_A$$

Virtual Work:

$$\delta U = 0:\quad A_y\delta y_A + (240\text{ lb})\delta y_B - (60\text{ lb})\delta y_G = 0$$

or $$A_y\delta y_A + (240\text{ lb})\delta y_A - (60\text{ lb})3.75\delta y_A = 0$$

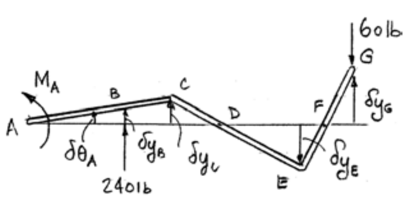
or $$\mathbf{A}_y = 15\text{ lb}\downarrow$$

A_x: Consider a horizontal displacement δx_A:

Virtual Work: $$\delta U = 0:\quad A_x\delta x_A = 0$$

or $$A_x = 0 \qquad \therefore\ \mathbf{A} = 15.00\text{ lb}\downarrow \blacktriangleleft$$

M_A: Consider a counterclockwise rotation about A:



ABC: $$\delta y_B = 2\delta\theta_A, \qquad \delta y_C = 3\delta\theta_A$$

CDE: $$\frac{\delta y_C}{1\text{ ft}} = \frac{\delta y_E}{2.5\text{ ft}}$$

or $$\delta y_E = 2.5(3\delta\theta_A)$$

$$= 7.5\delta\theta_A$$

EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

PROBLEM 10.53 CONTINUED

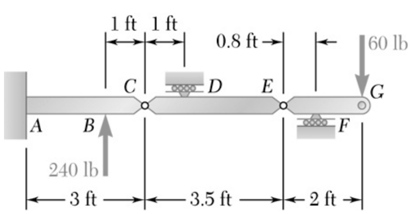
or
$$\delta y_G = \frac{(1.2\text{ ft})}{(0.8\text{ ft})}(7.5\delta\theta_A)$$

$$= 11.25\delta\theta_A$$

Virtual Work: $\delta U = 0$: $M_A\delta\theta_A + (240\text{ lb})\delta y_B - (60\text{ lb})\delta y_G = 0$

or
$$M_A\delta\theta_A + (240\text{ lb})(2\delta\theta_A) - (60\text{ lb})(11.25\delta\theta_A) = 0$$

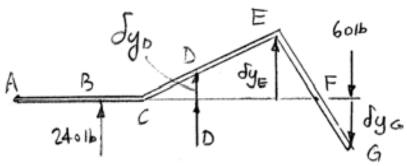
or $\mathbf{M}_A = 195.0\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 10.54

Using the method of virtual work, determine the reaction at *D*.

SOLUTION



Consider an upward displacement δy_E of pin *E*.

CDE: $$\frac{\delta y_D}{1\text{ ft}} = \frac{\delta y_E}{3.5\text{ ft}}$$

or $$\delta y_D = \frac{1}{3.5}\delta E$$

EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

or $$\delta y_G = 1.5\delta y_E$$

Virtual Work:

$$\delta U = 0: \quad D\delta y_D + 60\delta y_G = 0$$

or $$D\left(\frac{1}{3.5}\delta y_E\right) + (60\text{ lb})(1.5\delta y_E) = 0$$

or $$\mathbf{D} = 315\text{ lb} \downarrow \blacktriangleleft$$

PROBLEM 10.55

Referring to Problem 10.41 and using the value found for the force exerted by the hydraulic cylinder *AB*, determine the change in the length of *AB* required to raise the 480-N load 18 mm.

SOLUTION

From the solution to Problem 10.41

$$F_{\text{cyl}} = 397.08 \text{ N}$$

And, Virtual Work: $\delta U = 0\text{:} \quad F_{\text{cyl}}\delta S_{AB} - P\delta y_D = 0$

where $\delta S_{AB} < 0$ for $\delta y_D > 0$

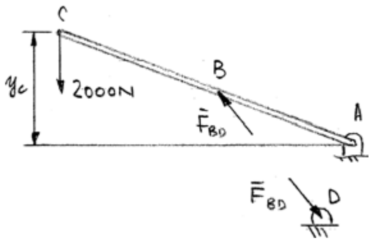
Then $(397.08 \text{ N})\delta S_{AB} - (480 \text{ N})(18 \text{ mm}) = 0$

or $\delta S_{AB} = 21.8 \text{ mm (shortened)}$ ◀

PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder *BD*, determine the change in the length of *BD* required to raise the platform attached at *C* by 50 mm.

SOLUTION



Virtual Work: Assume that both δy_C and δ_{BD} increase

$$\delta U = 0: -(2000 \text{ N})\delta y_C + F_{BD}\delta_{BD} = 0$$

$$\delta y_C = 0.05 \text{ m} \qquad \text{and} \qquad F_{BD} = 9473 \text{ N (from Problem 10.45)}$$

$$-2000(0.05 \text{ m}) + 9473\delta_{BD} = 0$$

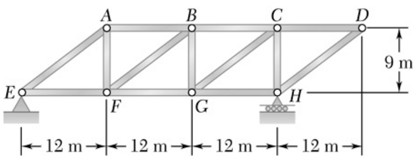
$$\delta_{BD} = 0.010556 \text{ m}$$

$$= 10.556 \text{ mm}$$

The positive sign indicates that *BD* gets longer.

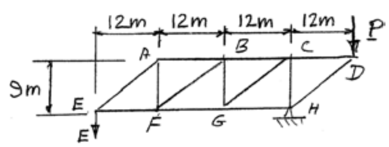
$$\delta_{BD} = 10.56 \text{ mm} \blacktriangleleft$$

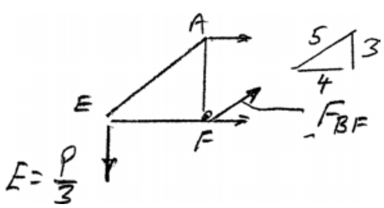
PROBLEM 10.57



Determine the vertical movement of joint D if the length of member BF is increased by 75 mm. (*Hint:* Apply a vertical load at joint D, and, using the methods of Chapter 6, compute the force exerted by member BF on joints B and F. Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member BF. This method should be used only for small changes in the lengths of members.)

SOLUTION





Apply vertical load P at D.

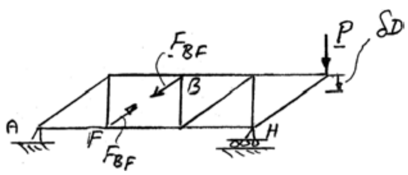
$$+\circlearrowleft \Sigma M_H = 0: \quad -P(12\text{ m}) + E(36\text{ m}) = 0$$

$$\mathbf{E} = \frac{P}{3}\downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

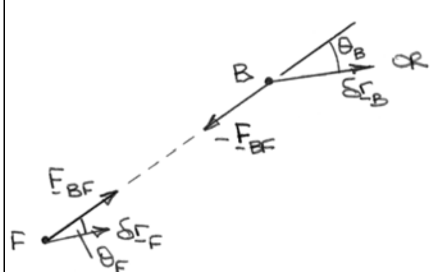


We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

Virtual Work: $\quad \delta U = 0: \quad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}\left(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F\right) = 0$$



where $\left(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F\right) = \delta_{BF}$, which is the change in length of member BF. Thus,

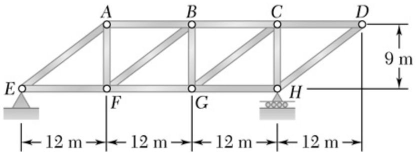
$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(75\text{ mm}) = 0$$

$$\delta D = +41.67\text{ mm}$$

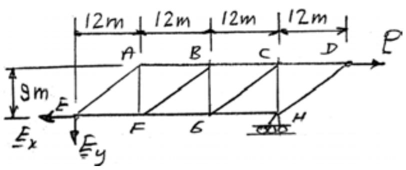
$$\delta D = 41.7\text{ mm}\downarrow \blacktriangleleft$$

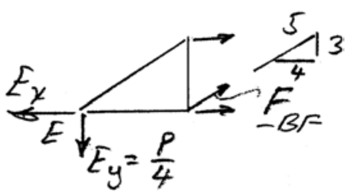
PROBLEM 10.58



Determine the horizontal movement of joint D if the length of member BF is increased by 75 mm. (See the hint for Problem 10.57.)

SOLUTION





Apply horizontal load P at D.

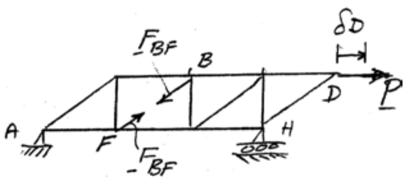
$$+\circlearrowleft \Sigma M_H = 0: \quad P(9\text{ m}) - E_y(36\text{ m}) = 0$$

$$E_y = \frac{P}{4} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

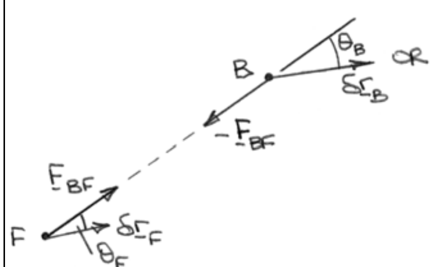
We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have



Virtual Work: $\quad \delta U = 0: \quad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$



where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

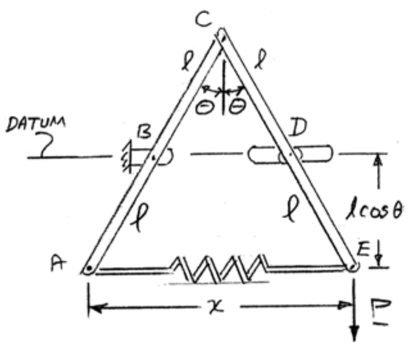
$$P\delta D - \left(\frac{5}{9}P\right)(75\text{ mm}) = 0$$

$$\delta D = 31.25\text{ mm} \qquad\qquad \delta D = 31.3\text{ mm} \longrightarrow \blacktriangleleft$$

PROBLEM 10.59

Using the method of Section 10.8, solve Problem 10.29.

SOLUTION



Spring:

$$AE = x = 2(2l\sin\theta) = 4l\sin\theta$$

Unstretched length:

$$x_0 = 4l\sin 30° = 2l$$

Deflection of spring

$$s = x - x_0$$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_E$$

$$V = \frac{1}{2}k\left[2l(2\sin\theta - 1)\right]^2 + P(-l\cos\theta)$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta + Pl\sin\theta = 0$$

$$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

PROBLEM 10.60

Using the method of Section 10.8, solve Problem 10.30.

SOLUTION

Using the result of Problem 10.59, with

$$P = 160\text{ N},\ l = 200\text{ mm, and } k = 300\text{ N/m}$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{160\text{ N}}{8(300\text{ N/m})(0.2\text{ m})}$$

$$= \frac{1}{3}$$

Solving numerically, $\theta = 25.0°$ ◀