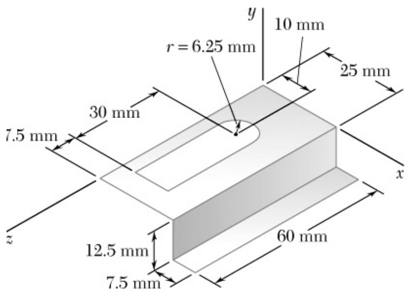
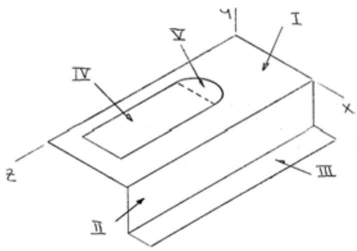
PROBLEM 5.101



A mounting bracket for electronic components is formed from sheet metal of uniform thickness. Locate the center of gravity of the bracket.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)



$$\bar{z}_{\text{V}} = 22.5 - \frac{4(6.25)}{3\pi}$$

$$= 19.85 \text{ mm}$$

$$A_{\text{V}} = -\frac{\pi}{2}(6.25)^2$$

$$= -61.36 \text{ mm}^2$$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	$(25)(60) = 1500$	12.5	0	30	18 750	0	45 000
II	$(12.5)(60) = 750$	25	–6.25	30	18 750	–4687.5	22 500
III	$(7.5)(60) = 450$	28.75	–12.5	30	12 937.5	–5625	13 500
IV	$-(12.5)(30) = -375$	10	0	37.5	–3750	0	–14 062.5
V	–61.36	10	0	19.85	–613.6	0	–1218.0
Σ	2263.64				46 074	–10 313	65 720

Have

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}\left(2263.64 \text{ mm}^2\right) = 46\ 074 \text{ mm}^3 \qquad \text{or } \bar{X} = 20.4 \text{ mm} \blacktriangleleft$$

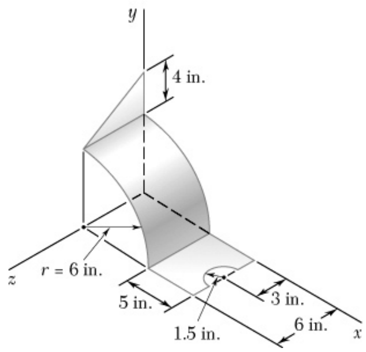
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}\left(2263.64 \text{ mm}^2\right) = -10\ 313 \text{ mm}^3 \qquad \text{or } \bar{Y} = -4.55 \text{ mm} \blacktriangleleft$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A$$

$$\bar{Z}\left(2263.64 \text{ mm}^2\right) = 65\ 720 \text{ mm}^3 \qquad \text{or } \bar{Z} = 29.0 \text{ mm} \blacktriangleleft$$

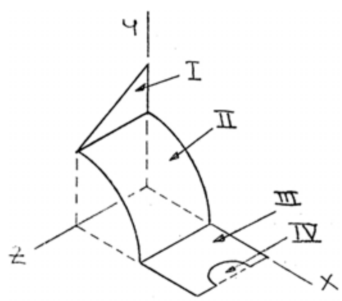
PROBLEM 5.102



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form coincides with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{I}} = 6 + \frac{4}{3} = 7.333 \text{ in.}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(6) = 2 \text{ in.}$$

$$\bar{x}_{\mathrm{II}} = \bar{y}_{\mathrm{II}} = \frac{1}{\pi}(2)(6) = 3.8197 \text{ in.}$$

$$\bar{x}_{\mathrm{IV}} = 11 - \frac{1}{3\pi}(4)(1.5) = 10.363 \text{ in.}$$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3	$\bar{z}A$, in^3
I	12	0	7.333	2	0	88	24
II	56.55	3.8197	3.8197	3	216	216	169.65
III	30	8.5	0	3	255	0	90
IV	–3.534	10.363	0	3	–36.62	0	–10.603
Σ	95.01				434.4	304	273.0

Have

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}\left(95.01 \text{ in}^2\right) = 434 \text{ in}^3 \qquad \text{or } \bar{X} = 4.57 \text{ in.} \blacktriangleleft$$

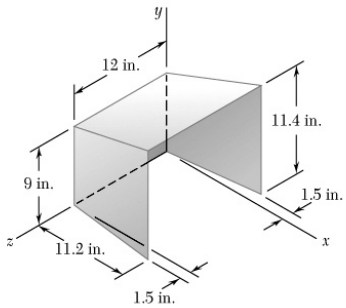
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}\left(95.01 \text{ in}^2\right) = 304.0 \text{ in}^3 \qquad \text{or } \bar{Y} = 3.20 \text{ in.} \blacktriangleleft$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A$$

$$Z\left(95.01 \text{ in}^2\right) = 273.0 \text{ in}^3 \qquad \text{or } \bar{Z} = 2.87 \text{ in.} \blacktriangleleft$$

PROBLEM 5.103

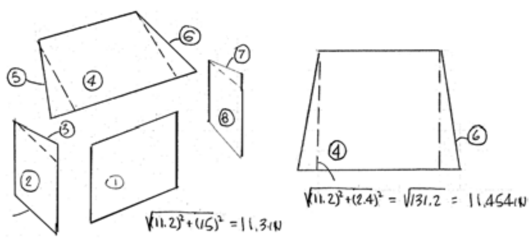


An enclosure for an electronic device is formed from sheet metal of uniform thickness. Locate the center of gravity of the enclosure.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form coincides with the centroid of the corresponding area.

Consider the division of the back, sides, and top into eight segments according to the sketch.



Note that symmetry implies $\bar{Z} = 6.00 \text{ in.}$ ◀

and

$$A_8 = A_2$$
$$A_7 = A_3$$
$$A_6 = A_5$$

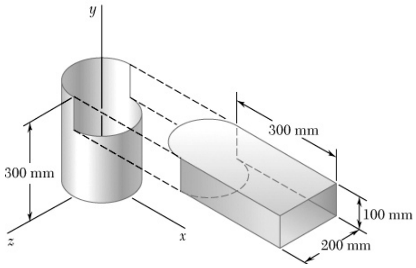
Thus

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$(12)(9) = 108$	0	4.5	0	486
2	$(11.3)(9) = 101.7$	5.6	4.5	569.5	457.6
3	$\frac{1}{2}(11.3)(2.4) = 13.56$	7.467	9.8	101.25	132.89
4	$(12)(11.454) = 137.45$	5.6	10.2	769.72	1402.0
5	$\frac{1}{2}(1.5)(11.454) = 8.591$	7.467	10.6	64.15	91.06
6	8.591	7.467	10.6	64.15	91.06
7	13.56	7.467	9.8	101.25	132.89
8	101.7	5.6	4.5	569.5	457.6
Σ	493.2			2239.5	3251.1

Have $\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(493.2 \text{ in}^2) = 2239.5 \text{ in}^3$ or $\bar{X} = 4.54 \text{ in.}$ ◀

$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(493.2 \text{ in}^2) = 3251.1 \text{ in}^3$ or $\bar{Y} = 6.59 \text{ in.}$ ◀

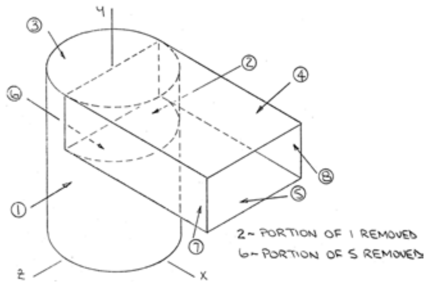
PROBLEM 5.104



A 200-mm-diameter cylindrical duct and a 100 × 200-mm rectangular duct are to be joined as indicated. Knowing that the ducts are fabricated from the same sheet metal, which is of uniform thickness, locate the center of gravity of the assembly.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also note that symmetry implies $\bar{Z} = 0$ ◀



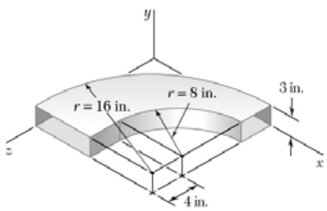
	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3
1	$\pi(0.2)(0.3) = 0.1885$	0	0.15	0	0.028274
2	$-\frac{\pi}{2}(0.2)(0.1) = -0.0314$	$\frac{2(0.1)}{\pi} = 0.06366$	0.25	–0.02000	–0.007854
3	$\frac{\pi}{2}(0.1)^2 = 0.01571$	$\frac{-4(0.1)}{3\pi} = -0.04244$	0.30	–0.000667	0.004712
4	$(0.3)(0.2) = 0.060$	0.15	0.30	0.00900	0.001800
5	$(0.3)(0.2) = 0.060$	0.15	0.20	0.00900	0.001200
6	$-\frac{\pi}{2}(0.1)^2 = -0.1571$	$\frac{4(0.1)}{3\pi}$	0.20	–0.000667	–0.003142
7	$(0.3)(0.1) = 0.030$	0.15	0.25	0.004500	0.007500
8	$(0.3)(0.1) = 0.030$	0.15	0.25	0.004500	0.007500
Σ	0.337080			0.023667	0.066991

Have $\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(0.337080 \text{ mm}^2) = 0.023667 \text{ mm}^3$

or $\bar{X} = 0.0702$ m $\qquad \bar{X} = 70.2$ mm ◀

$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}\left(0.337080 \text{ mm}^2\right) = 0.066991 \text{ mm}^3$

or $\bar{Y} = 0.19874$ m $\qquad \bar{Y} = 198.7$ mm ◀

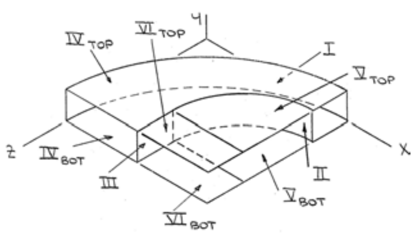


PROBLEM 5.105

An elbow for the duct of a ventilating system is made of sheet metal of uniform thickness. Locate the center of gravity of the elbow.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also, note that the shape of the duct implies $\overline{Y} = 1.5 \text{ in.}$ ◀



Note that $\bar{x}_{\text{I}} = \bar{z}_{\text{I}} = 16 \text{ in.} - \dfrac{2}{\pi}(16 \text{ in.}) = 5.81408 \text{ in.}$

$$\bar{x}_{\text{II}} = 16 \text{ in.} - \frac{2}{\pi}(8 \text{ in.}) = 10.9070 \text{ in.}$$

$$\bar{z}_{\text{II}} = 12 \text{ in.} - \frac{2}{\pi}(8 \text{ in.}) = 6.9070 \text{ in.}$$

$$\bar{x}_{\text{IV}} = \bar{z}_{\text{IV}} = 16 \text{ in.} - \frac{4}{3\pi}(16 \text{ in.}) = 9.2094 \text{ in.}$$

$$\bar{x}_{\text{V}} = 16 \text{ in.} - \frac{4}{3\pi}(8 \text{ in.}) = 12.6047 \text{ in.}$$

$$\bar{z}_{\text{V}} = 12 \text{ in.} - \frac{4}{3\pi}(8 \text{ in.}) = 8.6047 \text{ in.}$$

Also note that the corresponding top and bottom areas will contribute equally when determining $\bar{x}$ and $\bar{z}$.

Thus

	A, in^2	$\bar{x}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{z}A$, in^3
I	$\frac{\pi}{2}(16)(3) = 75.3982$	5.81408	5.81408	438.37	438.37
II	$\frac{\pi}{2}(8)(3) = 37.6991$	10.9070	6.9070	411.18	260.39
III	$4(3) = 12$	8	14	96.0	168.0
IV	$2\left(\frac{\pi}{4}\right)(16)^2 = 402.1239$	9.2094	9.2094	3703.32	3703.32
V	$-2\left(\frac{\pi}{4}\right)(8)^2 = -100.5309$	12.6047	8.6047	–1267.16	–865.04
VI	$-2(4)(8) = -64$	12	14	–768.0	–896.0
Σ	362.69			2613.71	2809.04

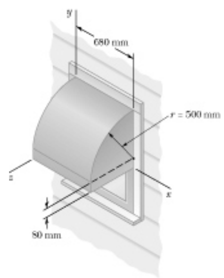
Have $\overline{X}\Sigma A = \Sigma \bar{x}A$: $\overline{X}(362.69 \text{ in}^2) = 2613.71 \text{ in}^3$

or $\overline{X} = 7.21 \text{ in.}$ ◀

$\overline{Z}\Sigma A = \Sigma \bar{z}A$: $\overline{Z}(362.69 \text{ in}^2) = 2809.04 \text{ in}^3$

or $\overline{Z} = 7.74 \text{ in.}$ ◀

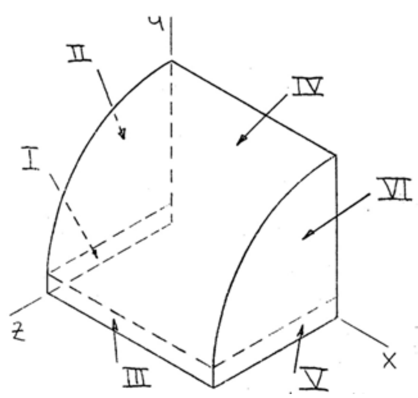
PROBLEM 5.106



A window awning is fabricated from sheet metal of uniform thickness. Locate the center of gravity of the awning.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{II}} = \bar{y}_{\mathrm{VI}} = 80 + \frac{(4)(500)}{3\pi} = 292.2 \text{ mm}$$

$$\bar{z}_{\mathrm{II}} = \bar{z}_{\mathrm{VI}} = \frac{(4)(500)}{3\pi} = 212.2 \text{ mm}$$

$$\bar{y}_{\mathrm{IV}} = 80 + \frac{(2)(500)}{\pi} = 398.3 \text{ mm}$$

$$\bar{z}_{\mathrm{IV}} = \frac{(2)(500)}{\pi} = 318.3 \text{ mm}$$

$$A_{\mathrm{II}} = A_{\mathrm{VI}} = \frac{\pi}{4}(500)^2 = 196\,350 \text{ mm}^2$$

$$A_{\mathrm{IV}} = \frac{\pi}{2}(500)(680) = 534\,071 \text{ mm}^2$$

	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	(80)(500) = 40 000	40	250	1.6×10^6	10×10^6
II	196 350	292.2	212.2	57.4×10^6	41.67×10^6
III	(80)(680) = 54 400	40	500	0.2176×10^6	27.2×10^6
IV	534 071	398.3	318.3	212.7×10^6	170×10^6
V	(80)(500) = 40 000	40	250	1.6×10^6	10×10^6
VI	196 350	292.2	212.2	57.4×10^6	41.67×10^6
Σ	1.061×10^6			332.9×10^6	300.5×10^6

Now, symmetry implies $\bar{X} = 340 \text{ mm}$ ◀

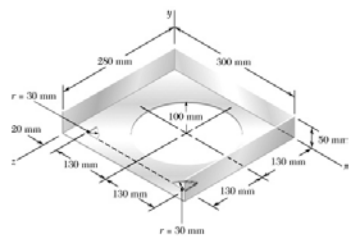
and $\bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}\left(1.061 \times 10^6 \text{ mm}^2\right) = 332.9 \times 10^6 \text{ mm}^3$

or $\bar{Y} = 314 \text{ mm}$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A: \quad \bar{Z}\left(1.061 \times 10^6 \text{ mm}^2\right) = 300.5 \times 10^6 \text{ mm}^3$

or $\bar{Z} = 283 \text{ mm}$ ◀

PROBLEM 5.107

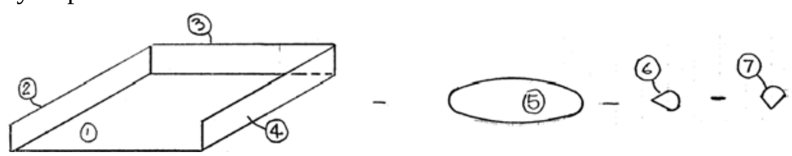


The thin, plastic front cover of a wall clock is of uniform thickness. Locate the center of gravity of the cover.

SOLUTION

First, assume that the plastic is homogeneous so that the center of gravity of the cover coincides with the centroid of the corresponding area.

Next, note that symmetry implies $\bar{X} = 150.0$ mm



	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}$, mm^3
1	$(300)(280)$ $= 84\ 000$	0	140	0	11 760 000
2	$(280)(50)$ $= 14\ 000$	25	140	350 000	1 960 000
3	$(300)(50)$ $= 15\ 000$	25	0	375 000	0
4	$(280)(50)$ $= 14\ 000$	25	140	350 000	1 960 000
5	$-\pi(100)^2$ $= -31\ 416$	0	130	0	–4 084 070
6	$\frac{-\pi}{4}(30)^2$ $= -706.86$	0	$260 - \frac{(4)(30)}{3\pi} = 247.29$	0	–174 783
7	$\frac{-\pi}{4}(30)^2$ $= -706.86$	0	$260 - \frac{(4)(30)}{3\pi} = 247.29$	0	–174 783
Σ	94 170			1 075 000	11 246 363

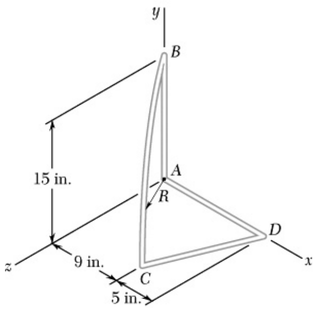
Have $\quad \bar{Y}\Sigma A = \Sigma\bar{y}A$: $\quad \bar{Y}\left(94\ 170\text{ mm}^2\right) = 1\ 075\ 000\text{ mm}^3$

or $\bar{Y} = 11.42$ mm ◀

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\quad \bar{Z}\left(94\ 170\text{ mm}^2\right) = 11\ 246\ 363\text{ mm}^3$

or $\bar{Z} = 119.4$ mm ◀

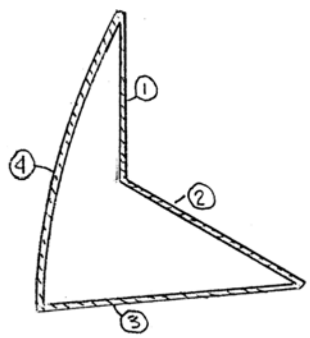
PROBLEM 5.108



A thin steel wire of uniform cross section is bent into the shape shown, where arc BC is a quarter circle of radius R. Locate its center of gravity.

SOLUTION

First, assume that the wire is homogeneous so that its center of gravity coincides with the centroid of the corresponding line.



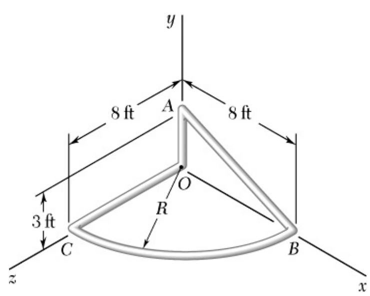
	L, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2	$\bar{z}L$, in^2
1	15	0	7.5	0	0	112.5	0
2	14	7	0	0	98	0	0
3	13	$9\left(\frac{5}{12}\right) = 11.5$	0	6	149.5	0	78
4	$\frac{\pi}{2}(15) = 23.56$	$\frac{3}{5}\left(\frac{2\times 15}{\pi}\right) = 5.73$	$\frac{30}{\pi} = 9.549$	$\frac{24}{\pi} = 7.639$	135.0	225.0	180.0
Σ	65.56				382.5	337.5	258.0

Have $\bar{X}\Sigma L = \Sigma\bar{x}L$: $\bar{X}(65.56\text{ in.}) = 382.5\text{ in}^2$ or $\bar{X} = 5.83\text{ in.}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\bar{Y}(65.56\text{ in.}) = 337.5\text{ in}^2$ or $\bar{Y} = 5.15\text{ in.}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\bar{Z}(65.56\text{ in.}) = 258.0\text{ in}^2$ or $\bar{Z} = 3.94\text{ in.}$ ◀

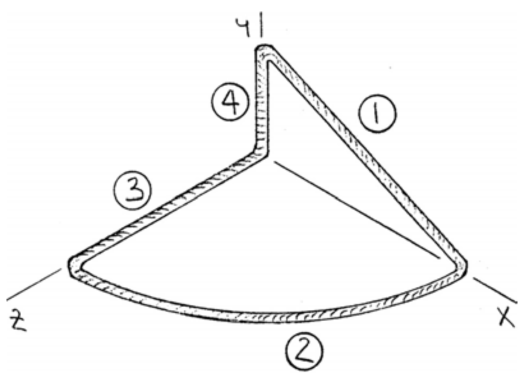
PROBLEM 5.109



A thin steel wire of uniform cross section is bent into the shape shown, where arc *BC* is a quarter circle of radius *R*. Locate its center of gravity.

SOLUTION

First, assume that the wire is homogeneous so that its center of gravity coincides with the centroid of the corresponding line



4

Have $$\bar{x}_2 = \bar{z}_2 = \frac{(2)(8)}{\pi} = \frac{16}{\pi}\text{ ft}$$

$$L_1 = \sqrt{8^2 + 3^2} = 8.5440\text{ ft}$$

$$L_2 = \frac{8\pi}{2} = 4\pi\text{ ft}$$

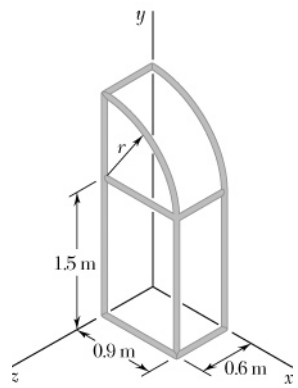
	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2	$\bar{z}L$, ft^2
1	8.5440	4	1.5	0	34.176	12.816	0
2	4π	16π	0	$\frac{16}{\pi}$	64.0	0	64.0
3	8	0	0	4	0	0	32
4	3	0	1.5	0	0	4.5	0
	32.110				98.176	17.316	96.0

Have $\bar{X}\Sigma L = \Sigma\bar{x}L$: $\bar{X}(32.110\text{ ft}) = 98.176\text{ ft}^2$ or $\bar{X} = 3.06\text{ ft}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\bar{Y}(32.110\text{ ft}) = 17.316\text{ ft}^2$ or $\bar{Y} = 0.539\text{ ft}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\bar{Z}(32.110\text{ ft}) = 96.0\text{ ft}^2$ or $\bar{Z} = 2.99\text{ ft}$ ◀

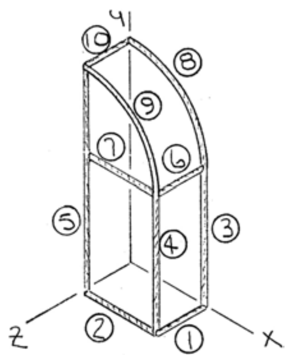
PROBLEM 5.110



The frame of a greenhouse is constructed from uniform aluminum channels. Locate the center of gravity of the portion of the frame shown.

SOLUTION

First, assume that the channels are homogeneous so that the center of gravity of the frame coincides with the centroid of the corresponding line.



Note $\quad \bar{x}_8 = \bar{x}_9 = \dfrac{(2)(0.9)}{\pi} = 0.57296 \text{ m}$

$$\bar{y}_8 = \bar{y}_9 = 1.5 + \frac{(2)(0.9)}{\pi} = 2.073 \text{ m}$$

$$L_7 = L_8 = \frac{\pi}{2}(0.9) = 1.4137 \text{ m}$$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	0.6	0.9	0	0.3	0.540	0	0.18
2	0.9	0.45	0	0.6	0.4050	0	0.54
3	1.5	0.9	0.75	0	1.350	1.125	0
4	1.5	0.9	0.75	0.6	1.350	1.125	0.9
5	2.4	0	1.2	0.6	0	2.880	1.44
6	0.6	0.9	1.5	0.3	0.540	0.9	0.18
7	0.9	0.45	1.5	0.6	0.4050	1.350	0.54
8	1.4137	0.573	2.073	0	0.8100	2.9306	0
9	1.4137	0.573	2.073	0.6	0.8100	2.9306	0.8482
10	0.6	0	2.4	0.3	0	1.440	0.18
Σ	11.827				6.210	14.681	4.8082

Have $\quad \bar{X}\Sigma L = \Sigma \bar{x}L: \quad \bar{X}(11.827 \text{ m}) = 6.210 \text{ m}^2 \quad$ or $\bar{X} = 0.525 \text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L: \quad \bar{Y}(11.827 \text{ m}) = 14.681 \text{ m}^2 \quad$ or $\bar{Y} = 1.241 \text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L: \quad \bar{Z}(11.827 \text{ m}) = 4.8082 \text{ m}^2 \quad$ or $\bar{Z} = 0.406 \text{ m}$ ◀