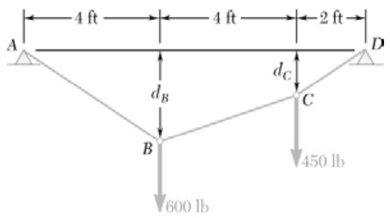
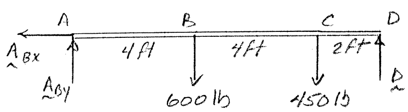


PROBLEM 7.115

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.89*a*.

SOLUTION

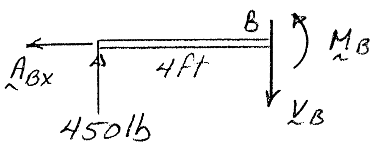
FBD Beam:



$$\circlearrowleft \Sigma M_D = 0: (2\text{ ft})(450\text{ lb}) + (6\text{ ft})(600\text{ lb}) - (10\text{ ft})\mathbf{A}_{By} = 0$$

$$\mathbf{A}_{By} = 450\text{ lb} \uparrow$$

Section AB:



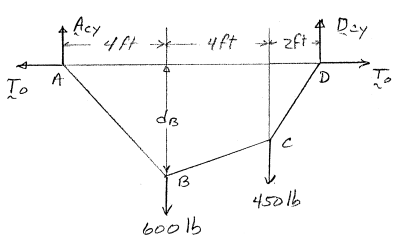
$$\circlearrowleft \Sigma M_B = 0: M_B - (4\text{ ft})(450\text{ lb}) = 0$$

$$M_B = 1800\text{ lb}\cdot\text{ft}$$

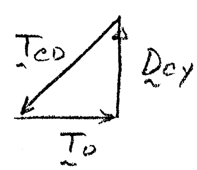
$$\circlearrowleft \Sigma M_A = 0: (10\text{ ft})D_{Cy} - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

$$D_{Cy} = 600\text{ lb}$$

Cable:



(Note: $D_y > A_y$ so $T_{max} = T_{CD}$)



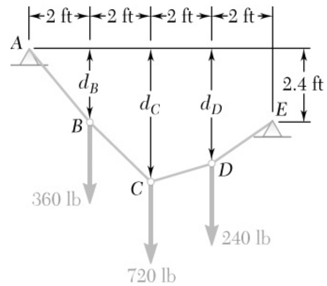
$$T_0 = \sqrt{T_{max}^2 - D_{Cy}^2}$$

$$T_0 = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2}$$

$$T_0 = 398\text{ lb}$$

$$d_B = \frac{M_B}{T_0} = \frac{1800\text{ lb}\cdot\text{ft}}{398\text{ lb}} = 4.523\text{ ft}$$

$$d_B = 4.52\text{ ft} \blacktriangleleft$$

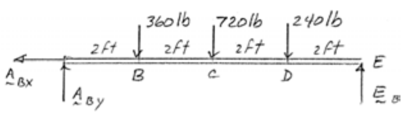


PROBLEM 7.116

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.92*b*.

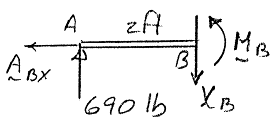
SOLUTION

FBD Beam:



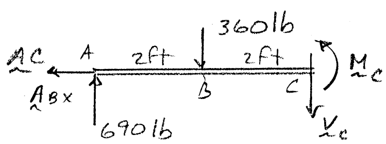
$$\circlearrowleft \Sigma M_E = 0: (2\text{ ft})(240\text{ lb}) + (4\text{ ft})(720\text{ lb}) + (6\text{ ft})(360\text{ lb}) - (8\text{ ft})A_{By} = 0$$

$$\mathbf{A}_{By} = 690\text{ lb}\uparrow$$



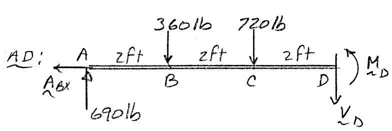
$$\circlearrowleft \Sigma M_B = 0: M_B - (2\text{ ft})(690\text{ lb}) = 0$$

$$M_B = 1380\text{ lb}\cdot\text{ft}$$



$$\circlearrowleft \Sigma M_B = 0: M_C + (2\text{ ft})(360\text{ lb}) - (4\text{ ft})(690\text{ lb}) = 0$$

$$M_C = 2040\text{ lb}\cdot\text{ft}$$



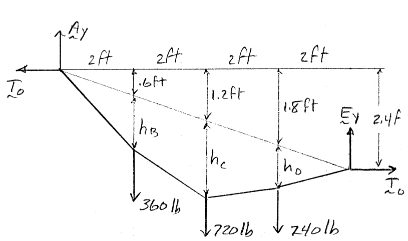
$$\circlearrowleft \Sigma M_D = 0: M_D + (2\text{ ft})(720\text{ lb}) + (4\text{ ft})(360\text{ lb}) - (6\text{ ft})(690\text{ lb}) = 0$$

$$M_D = 1260\text{ lb}\cdot\text{ft}$$

$$h_C = d_C - 1.2\text{ ft} = 3.6\text{ ft} - 1.2\text{ ft} = 2.4\text{ ft}$$

$$T_0 = \frac{M_C}{h_C} = \frac{2040\text{ lb}\cdot\text{ft}}{2.4\text{ ft}} = 850\text{ lb}$$

Cable:



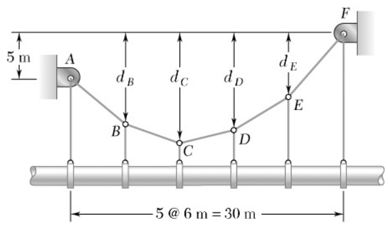
$$h_B = \frac{M_B}{T_0} = \frac{1380\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.6235\text{ ft}$$

$$d_B = h_B + 0.6\text{ ft} \qquad d_B = 2.22\text{ ft} \blacktriangleleft$$

$$h_0 = \frac{M_D}{T_0} = \frac{1260\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.482\text{ ft}$$

$$d_B = h_0 + 1.8\text{ ft} \qquad d_D = 3.28\text{ ft} \blacktriangleleft$$

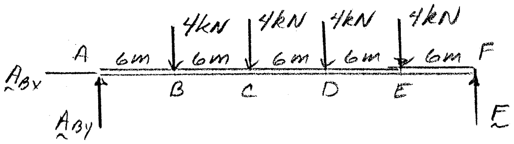
PROBLEM 7.117



Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.94*b*.

SOLUTION

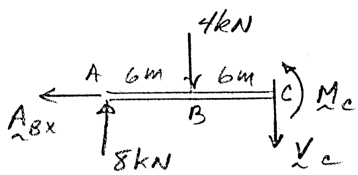
FBD Beam:



By symmetry: $\mathbf{A}_{By} = \mathbf{F} = 8\,kN$

$$M_B = M_E; \qquad M_C = M_D$$

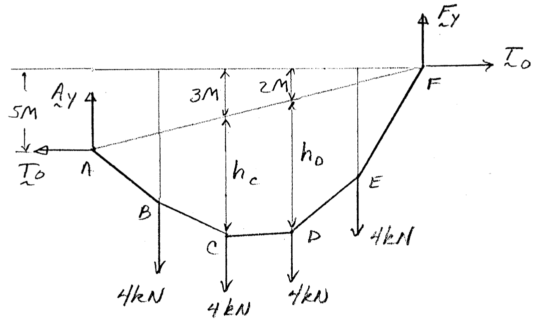
AC:



$$\circlearrowleft \Sigma M_C = 0\text{: } M_C + (6\text{ m})(4\text{ kN}) - (12\text{ m})(8\text{ kN}) = 0$$

$$M_C = 72\text{ kN}\cdot\text{m} \qquad \text{so} \qquad M_D = 72\text{ kN}\cdot\text{m}$$

Cable:



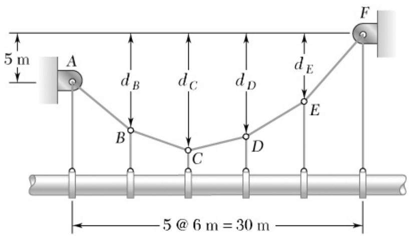
Since $M_D = M_C$

$$h_D = h_C = 12\text{ m} - 3\text{ m} = 9\text{ m}$$

$$d_D = h_D + 2\text{ m} = 11\text{ m}$$

$d_D = 11.00\text{ m}$ ◀

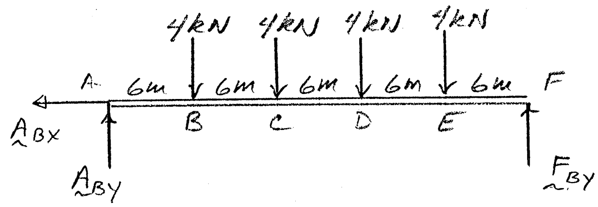
PROBLEM 7.118



Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.95*b*.

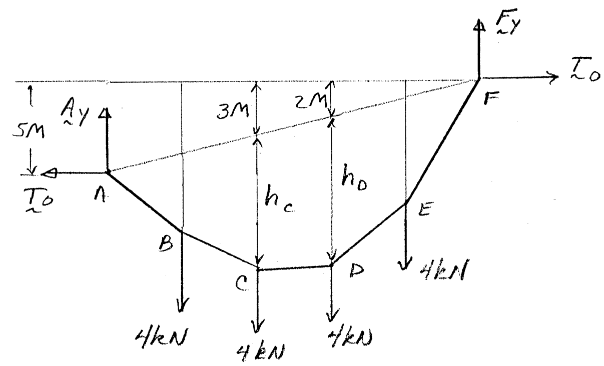
SOLUTION

FBD Beam:



By symmetry: $M_B = M_E$ and $M_C = M_D$

Cable:



Since $M_D = M_C$, $h_D = h_C$

$$h_D = h_C = d_C - 3\text{ m} = 9\text{ m} - 3\text{ m} = 6\text{ m}$$

Then $d_D = h_D + 2\text{ m} = 6\text{ m} + 2\text{ m} = 8\text{ m}$

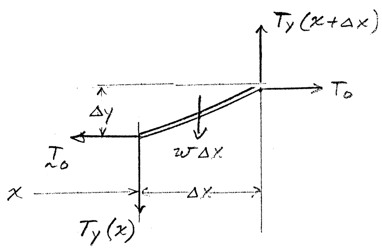
$d_D = 8.00\text{ m}$ ◀

PROBLEM 7.119

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So $$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But $$\frac{T_y}{T_0} = \frac{dy}{dx}$$

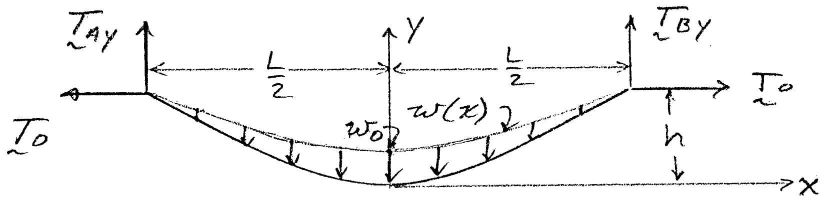
So $$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim\limits_{\Delta x \to 0}$: $$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$ Q.E.D.

PROBLEM 7.120

Using the property indicated in Prob. 7.119, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x / L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

SOLUTION



$$w(x) = w_0 \cos \frac{\pi x}{L}$$

From Problem 7.119

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0} \cos \frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi} \sin \frac{\pi x}{L} \qquad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0 \right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2} \left(1 - \cos \frac{\pi x}{L} \right) \quad \left[\text{using } y(0) = 0 \right] \blacktriangleleft$$

But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2} \left(1 - \cos \frac{\pi}{2} \right) \qquad \text{so} \qquad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

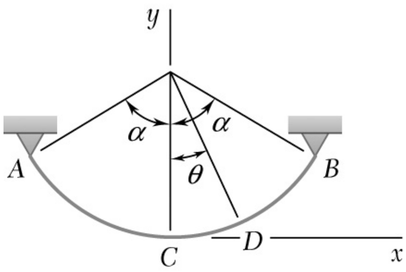
And $$T_0 = T_{\min} \qquad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=\frac{L}{2}} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi}$$

$$T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi} \sqrt{1 + \left(\frac{L}{\pi h} \right)^2} \blacktriangleleft$$

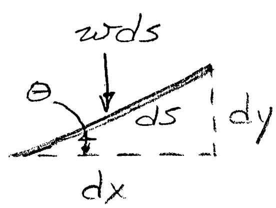
PROBLEM 7.121



If the weight per unit length of the cable *AB* is $w_0 / \cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Prob. 7.119.)

SOLUTION

Elemental Segment:



Load on segment* $\quad w(x)dx = \dfrac{w_0}{\cos^2\theta} ds$

But $\quad dx = \cos\theta ds, \quad$ so $\quad w(x) = \dfrac{w_0}{\cos^3\theta}$

From Problem 7.119 $\quad \dfrac{d^2y}{dx^2} = \dfrac{w(x)}{T_0} = \dfrac{w_0}{T_0\cos^3\theta}$

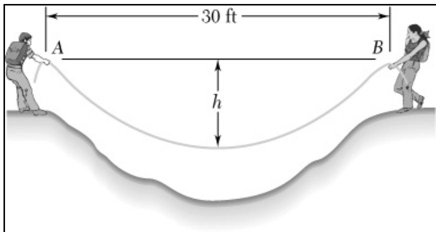
In general $\quad \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}(\tan\theta) = \sec^2\theta \dfrac{d\theta}{dx}$

So $\quad \dfrac{d\theta}{dx} = \dfrac{w_0}{T_0 \cos^3\theta \sec^2\theta} = \dfrac{w_0}{T_0\cos\theta}$

or $\quad \dfrac{T_0}{w_0}\cos\theta d\theta = dx = r d\theta \cos\theta$

Giving $r = \dfrac{T_0}{w_0}$ = constant. So curve is circular arc Q.E.D.

*For large sag, it is not appropriate to approximate *ds* by *dx*.

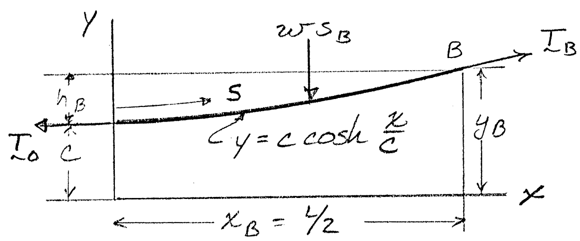


PROBLEM 7.122

Two hikers are standing 30-ft apart and are holding the ends of a 35-ft length of rope as shown. Knowing that the weight per unit length of the rope is 0.05 lb/ft, determine (*a*) the sag *h*, (*b*) the magnitude of the force exerted on the hand of a hiker.

SOLUTION

Half-span:



$$w = 0.05 \text{ lb/ft}, \qquad L = 30 \text{ ft}, \qquad s_B = \frac{35}{2} \text{ ft}$$

$$s_B = c \sinh \frac{y_B}{x_B}$$

$$17.5 \text{ ft} = c \sinh\left(\frac{15 \text{ ft}}{c}\right)$$

Solving numerically, $c = 15.36 \text{ ft}$

Then $y_B = c \cosh \dfrac{x_B}{c} = (15.36 \text{ ft}) \cosh \dfrac{15 \text{ ft}}{15.36 \text{ ft}} = 23.28 \text{ ft}$

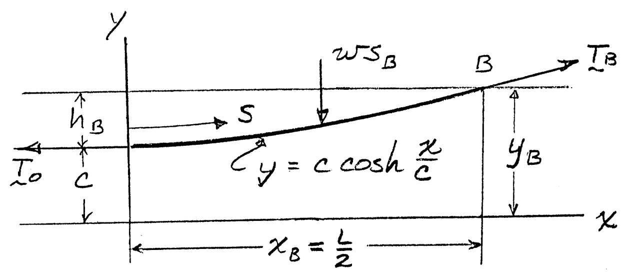
(*a*) $h_B = y_B - c = 23.28 \text{ ft} - 15.36 \text{ ft} = 7.92 \text{ ft}$ ◀

(*b*) $T_B = w y_B = (0.05 \text{ lb/ft})(23.28 \text{ ft}) = 1.164 \text{ lb}$ ◀

PROBLEM 7.123

A 60-ft chain weighing 120 lb is suspended between two points at the same elevation. Knowing that the sag is 24 ft, determine (*a*) the distance between the supports, (*b*) the maximum tension in the chain.

SOLUTION



$$s_B = 30 \text{ ft}, \qquad w = \frac{120 \text{ lb}}{60 \text{ ft}} = 2 \text{ lb/ft}$$

$$h_B = 24 \text{ ft}, \qquad x_B = \frac{L}{2}$$

$$y_B^2 = c^2 + s_B^2 = (h_B + c)^2$$
$$= h_B^2 + 2ch_B + c^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(30 \text{ ft})^2 - (24 \text{ ft})^2}{2(24 \text{ ft})}$$

$$c = 6.75 \text{ ft}$$

Then
$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c}$$

$$x_B = (6.75 \text{ ft}) \sinh^{-1}\left(\frac{30 \text{ ft}}{6.75 \text{ ft}}\right) = 14.83 \text{ ft}$$

(*a*) $L = 2x_B = 29.7 \text{ ft}$ ◀

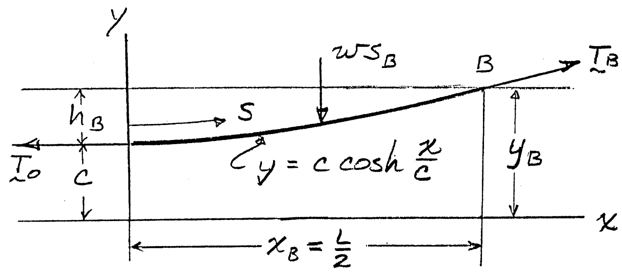
$$T_{\max} = T_B = wy_B = w(c + h_B) = (2 \text{ lb/ft})(6.75 \text{ ft} + 24 \text{ ft}) = 61.5 \text{ lb}$$

(*b*) $T_{\max} = 61.5 \text{ lb}$ ◀

PROBLEM 7.124

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}, \qquad w = \frac{4 \text{ lb}}{200 \text{ ft}} = 0.02 \text{ lb/ft}$$

$$T_{\max} = 16 \text{ lb}$$

$$T_{\max} = T_B = wy_B$$

$$y_B = \frac{T_B}{w} = \frac{16 \text{ lb}}{0.02 \text{ lb/ft}} = 800 \text{ ft}$$

$$c^2 = y_B^2 - s_B^2$$

$$c = \sqrt{(800 \text{ ft})^2 - (100 \text{ ft})^2} = 793.73 \text{ ft}$$

But

$$y_B = x_B \cosh \frac{x_B}{c} \rightarrow x_B = c \cosh^{-1} \frac{y_B}{c}$$

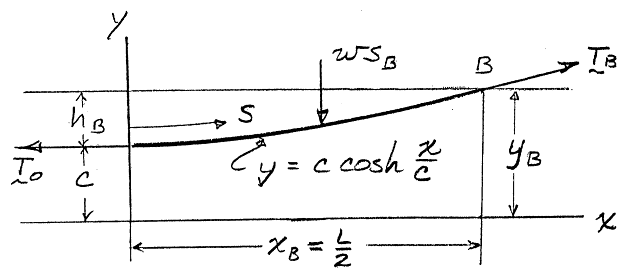
$$= (793.73 \text{ ft}) \ \cosh^{-1}\left(\frac{800 \text{ ft}}{793.73 \text{ ft}}\right) = 99.74 \text{ ft}$$

$$L = 2x_B = 2(99.74 \text{ ft}) = 199.5 \text{ ft} \blacktriangleleft$$

PROBLEM 7.125

An electric transmission cable of length 130 m and mass per unit length of 3.4 kg/m is suspended between two points at the same elevation. Knowing that the sag is 30 m, determine the horizontal distance between the supports and the maximum tension.

SOLUTION



$$s_B = 65 \text{ m}, \qquad h_B = 30 \text{ m}$$

$$w = (3.4 \text{ kg/m})(9.81 \text{ m/s}^2) = 33.35 \text{ N/m}$$

$$y_B^2 = c^2 + s_B^2$$

$$(c + h_B)^2 = c^2 + s_B^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(65 \text{ m})^2 - (30 \text{ m})^2}{2(30 \text{ m})}$$

$$= 55.417 \text{ m}$$

Now

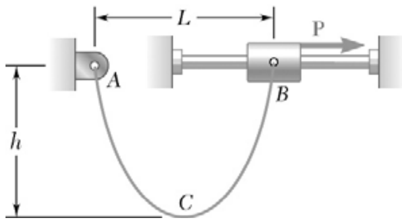
$$s_B = c\sinh\frac{x_B}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (55.417 \text{ m})\sinh^{-1}\left(\frac{65 \text{ m}}{55.417 \text{ m}}\right)$$

$$= 55.335 \text{ m}$$

$$L = 2x_B = 2(55.335 \text{ m}) = 110.7 \text{ m} \blacktriangleleft$$

$$T_{\max} = wy_B = w(c + h_B) = (33.35 \text{ N/m})(55.417 \text{ m} + 30 \text{ m}) = 2846 \text{ N}$$

$$T_{\max} = 2.85 \text{ kN} \blacktriangleleft$$

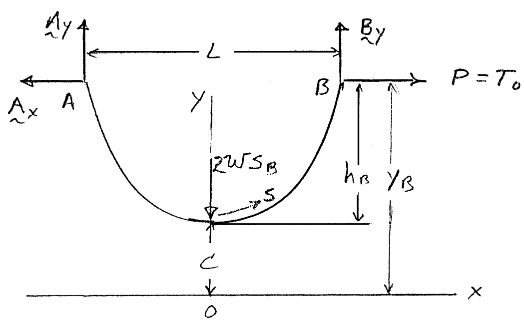


PROBLEM 7.126

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the force **P** for which $h = 12$ m, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s = 30 \text{ m} \qquad \left(\text{so } s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}\right)$$

$$w = (0.3 \text{ kg/m})\left(9.81 \text{ m/s}^2\right) = 2.943 \text{ N/m}$$

$$h_B = 12 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

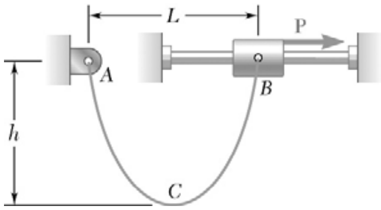
$$c = \frac{(15 \text{ m})^2 - (12 \text{ m})^2}{2(12 \text{ m})} = 3.375 \text{ m}$$

Now
$$s_B = c\sinh\frac{x_B}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (3.375 \text{ m})\sinh^{-1}\left(\frac{15 \text{ m}}{3.375 \text{ m}}\right)$$

$$x_B = 7.4156 \text{ m}$$

$$P = T_0 = wc = (2.943 \text{ N/m})(3.375 \text{ m}) \qquad (a) \qquad \mathbf{P} = 9.93 \text{ N} \longrightarrow \blacktriangleleft$$

$$L = 2x_B = 2(7.4156 \text{ m}) \qquad (b) \qquad L = 14.83 \text{ m} \blacktriangleleft$$

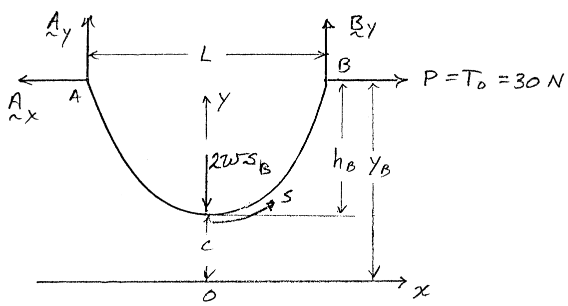


PROBLEM 7.127

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at A and to a collar at B. Knowing that the magnitude of the horizontal force applied to the collar is $P = 30\ \text{N}$, determine (a) the sag h, (b) the corresponding span L.

SOLUTION

FBD Cable:



$$s_T = 30\text{ m}, \qquad w = (0.3\text{ kg/m})(9.81\text{ m/s}^2) = 2.943\text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w}$$

$$c = \frac{30\text{ N}}{2.943\text{ N/m}} = 10.1937\text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \qquad s_B = \frac{30\text{ m}}{2} = 15\text{ m}$$

$$h^2 + 2(10.1937\text{ m})h - 225\text{ m}^2 = 0$$

$$h = 7.9422\text{ m}$$

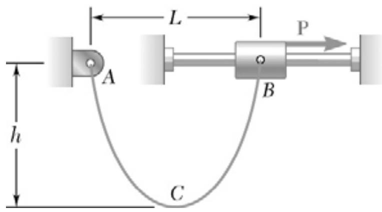
(a) $h = 7.94\text{ m}$ ◀

$$s_B = c\sinh\frac{x_A}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c} = (10.1937\text{ m})\sinh^{-1}\left(\frac{15\text{ m}}{10.1937\text{ m}}\right)$$

$$= 12.017\text{ m}$$

$$L = 2x_B = 2(12.017\text{ m})$$

(b) $L = 24.0\text{ m}$ ◀

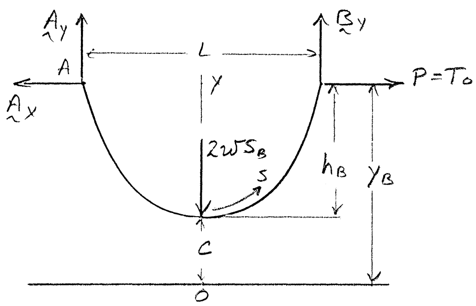


PROBLEM 7.128

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the sag *h* for which $L = 22.5$ m, (*b*) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 30\text{ m} \rightarrow s_B = \frac{30\text{ m}}{2} = 15\text{ m}$$

$$w = (0.3\text{ kg/m})(9.81\text{ m/s}^2) = 2.943\text{ N/m}$$

$$L = 22.5\text{ m}$$

$$s_B = c\sinh\frac{x_B}{c} = c\sinh\frac{L/2}{c}$$

$$15\text{ m} = c\sinh\frac{11.25\text{ m}}{c}$$

Solving numerically: $c = 8.328$ m

$$y_B^2 = c^2 + s_B^2 = (8.328\text{ m})^2 + (15\text{ m})^2 = 294.36\text{ m}^2 \qquad y_B = 17.157\text{ m}$$

$$h_B = y_B - c = 17.157\text{ m} - 8.328\text{ m}$$

(*a*) $h_B = 8.83$ m ◀

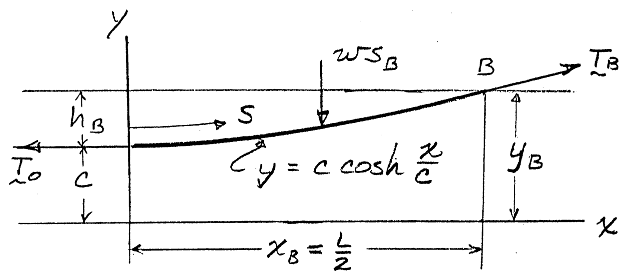
$$P = wc = (2.943\text{ N/m})(8.328\text{ m})$$

(*b*) $\mathbf{P} = 24.5\text{ N} \longrightarrow$ ◀

PROBLEM 7.129

A 30-ft wire is suspended from two points at the same elevation that are 20 ft apart. Knowing that the maximum tension is 80 lb, determine (*a*) the sag of the wire, (*b*) the total weight of the wire.

SOLUTION



$$L = 20 \text{ ft} \qquad x_B = \frac{20 \text{ ft}}{2} = 10 \text{ ft}$$

$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically: $\quad c = 6.1647 \text{ ft}$

$$y_B = c \cosh \frac{x_B}{c} = (6.1647 \text{ ft}) \cosh\left(\frac{10 \text{ ft}}{1.1647 \text{ ft}}\right)$$

$$y_B = 16.217 \text{ ft}$$

$$h_B = y_B - c = 16.217 \text{ ft} - 6.165 \text{ ft}$$

(*a*) $\qquad h_B = 10.05 \text{ ft}$ ◀

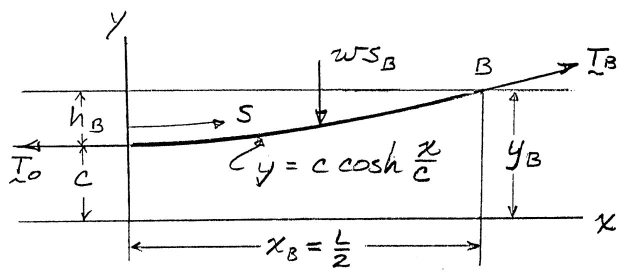
$$T_{\max} = w y_B \qquad \text{and} \qquad W = w(2s_B)$$

So $\qquad W = \frac{T_{\max}}{y_B}(2s_B) = \frac{80 \text{ lb}}{16.217 \text{ ft}}(30 \text{ ft})$ $\qquad$ (*b*) $\qquad \mathbf{W}_m = 148.0 \text{ lb}$ ◀

PROBLEM 7.130

Determine the sag of a 45-ft chain which is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{45 \text{ ft}}{2} = 22.5 \text{ ft} \qquad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$22.5 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

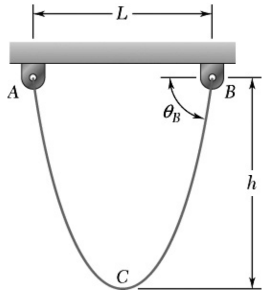
Solving numerically: $c = 4.2023$ ft

$$y_B = c \cosh \frac{x_B}{c}$$

$$= (4.2023 \text{ ft}) \cosh \frac{10 \text{ ft}}{4.2023 \text{ ft}} = 22.889 \text{ ft}$$

$$h_B = y_B - c = 22.889 \text{ ft} - 4.202 \text{ ft}$$

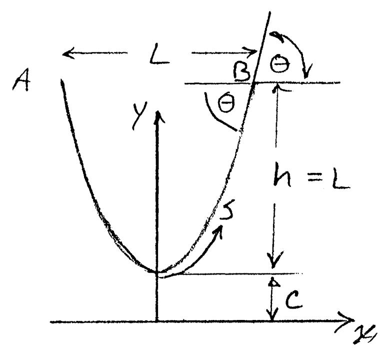
$h_B = 18.69$ ft ◀



PROBLEM 7.131

A 10-m rope is attached to two supports A and B as shown. Determine (a) the span of the rope for which the span is equal to the sag, (b) the corresponding angle θ_B.

SOLUTION



We know $$y = c\cosh\frac{x}{c}$$

At B, $$y_B = c + h = c\cosh\frac{h}{2c}$$

or $$1 = \cosh\frac{h}{2c} - \frac{h}{c}$$

Solving numerically $$\frac{h}{c} = 4.933$$

$$s_B = c\sinh\frac{x_B}{c} \rightarrow \frac{s_T}{2} = c\sinh\frac{h}{2c}$$

So $$c = \frac{s_T}{2\sinh\left(\frac{h}{2c}\right)} = \frac{10\text{ m}}{2\sinh\left(\frac{4.933}{2}\right)} = 0.8550\text{ m}$$

$$h = 4.933c = 4.933(0.8550)\text{ m} = 4.218\text{ m} \qquad h = 4.22\text{ m}$$

(a) $L = h = 4.22\text{ m}$ ◀

From $$y = c\cosh\frac{x}{c}, \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

At B, $$\tan\theta = \left.\frac{dy}{dx}\right|_B = \sinh\frac{L}{2c} = \sinh\frac{4.933}{2} = 5.848$$

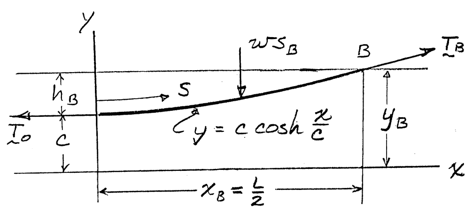
$$\theta = \tan^{-1}5.848$$

(b) $\theta = 80.3°$ ◀

PROBLEM 7.132

A cable having a mass per unit length of 3 kg/m is suspended between two points at the same elevation that are 48 m apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 1800 N.

SOLUTION



$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$L = 48\text{ m}, \qquad T_{\max} \le 1800\text{ N}$$

$$T_{\max} = wy_B \rightarrow y_B = \frac{T_{\max}}{w}$$

$$y_B \le \frac{1800\text{ N}}{29.43\text{ N/m}} = 61.162\text{ m}$$

$$y_B = c\cosh\frac{x_B}{c} \qquad 61.162\text{ m} = c\cosh\frac{48\text{ m}/2}{c} \; *$$

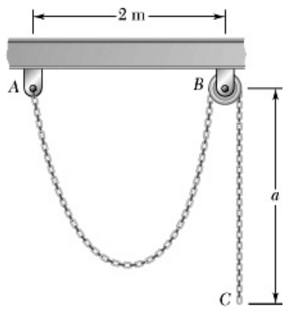
Solving numerically $\quad c = 55.935\text{ m}$

$$h = y_B - c = 61.162\text{ m} - 55.935\text{ m}$$

$$h = 5.23\text{ m} \blacktriangleleft$$

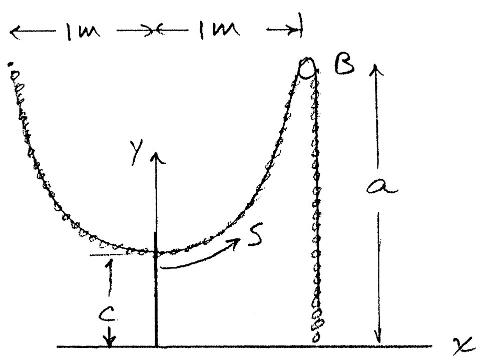
*Note: There is another value of c which will satisfy this equation. It is much smaller, thus corresponding to a much larger h.

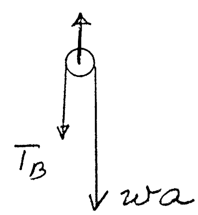
PROBLEM 7.133



An 8-m length of chain having a mass per unit length of 3.72 kg/m is attached to a beam at A and passes over a small pulley at B as shown. Neglecting the effect of friction, determine the values of distance a for which the chain is in equilibrium.

SOLUTION





Neglect pulley size and friction $\quad T_B = wa$

But $\quad T_B = wy_B \quad$ so $\quad y_B = a$

$$y_B = c\cosh\frac{x_B}{c}$$

$$c\cosh\frac{1\text{ m}}{c} = a$$

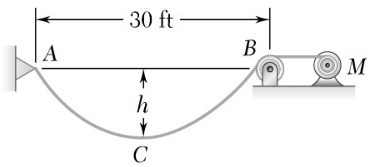
But $\quad s_B = c\sinh\frac{x_B}{c} \qquad \frac{8\text{ m} - a}{2} = c\sinh\frac{1\text{ m}}{c}$

So $$4\text{ m} = c\sinh\frac{1\text{ m}}{c} + \frac{c}{2}\cosh\frac{1\text{ m}}{c}$$

$$16\text{ m} = c\left(3e^{1/c} - e^{-1/c}\right)$$

Solving numerically $\quad c = 0.3773\text{ m},\ 5.906\text{ m}$

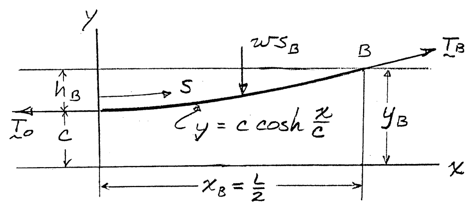
$$a = c\cosh\frac{1\text{ m}}{c} = \begin{cases} (0.3773\text{ m})\cosh\dfrac{1\text{ m}}{0.3773\text{ m}} = 2.68\text{ m} \blacktriangleleft \\ (5.906\text{ m})\cosh\dfrac{1\text{ m}}{5.906\text{ m}} = 5.99\text{ m} \blacktriangleleft \end{cases}$$



PROBLEM 7.134

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 15$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft} \qquad L = 30 \text{ ft} \qquad h_B = 15 \text{ ft}$$

$$y_B = c\cosh\frac{x_B}{c}$$

$$h_B + c = c\cosh\frac{L}{2c}$$

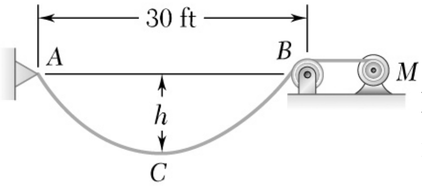
$$15 \text{ ft} = c\left(\cosh\frac{15 \text{ ft}}{c} - 1\right)$$

Solving numerically $\quad c = 9.281 \text{ ft}$

$$y_B = (9.281 \text{ ft})\cosh\frac{15 \text{ ft}}{9.281 \text{ ft}} = 24.281 \text{ ft}$$

$$T_{max} = T_B = wy_B = (0.5 \text{ lb/ft})(24.281 \text{ ft})$$

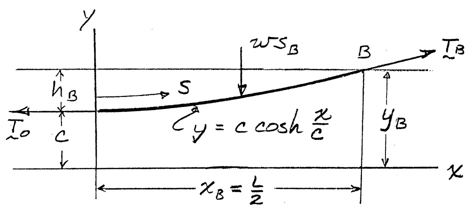
$$T_{max} = 12.14 \text{ lb} \blacktriangleleft$$



PROBLEM 7.135

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 9$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft}, \qquad L = 30 \text{ ft}, \qquad h_B = 9 \text{ ft}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

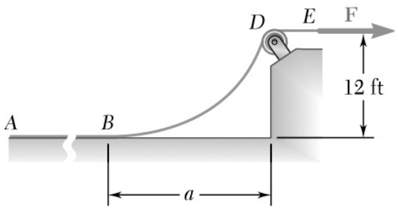
$$9 \text{ ft} = c\left(\cosh\frac{15 \text{ ft}}{c} - 1\right)$$

Solving numerically $c = 13.783$ ft

$$y_B = h_B + c = 9 \text{ ft} + 13.783 \text{ ft} = 21.783 \text{ ft}$$

$$T_{\max} = T_B = wy_B = (0.5 \text{ lb/ft})(21.78 \text{ ft})$$

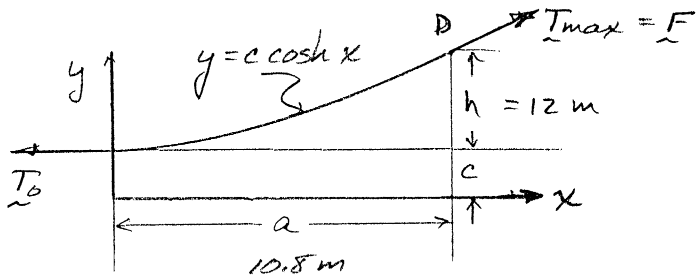
$T_{\max} = 11.39$ lb ◀



PROBLEM 7.136

To the left of point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 10.8$ ft.

SOLUTION



$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$12\text{ m} = c\left(\cosh\frac{10.8\text{ m}}{c} - 1\right)$$

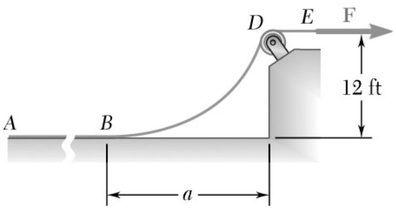
Solving numerically $\qquad c = 6.2136\text{ m}$

Then $\qquad y_B = (6.2136\text{ m})\cosh\dfrac{10.8\text{ m}}{6.2136\text{ m}} = 18.2136\text{ m}$

$$F = T_{\max} = wy_B = (1.5\text{ lb/ft})(18.2136\text{ m})$$

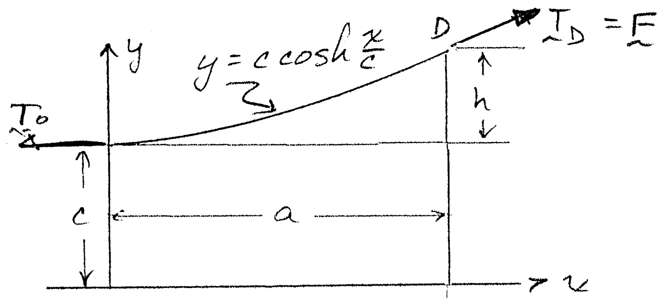
$\mathbf{F} = 27.3\text{ lb} \longrightarrow$ ◀

PROBLEM 7.137



To the left of point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 18$ ft.

SOLUTION



$$y_D = c \cosh \frac{x_D}{c}$$

$$c + h = c \cosh \frac{a}{c}$$

$$h = c\left(\cosh \frac{a}{c} - 1\right)$$

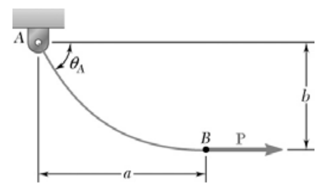
$$12 \text{ ft} = c\left(\cosh \frac{18 \text{ ft}}{c} - 1\right)$$

Solving numerically $\quad c = 15.162$ ft

$$y_B = h + c = 12 \text{ ft} + 15.162 \text{ ft} = 27.162 \text{ ft}$$

$$F = T_D = w y_D = (1.5 \text{ lb/ft})(27.162 \text{ ft}) = 40.74 \text{ lb}$$

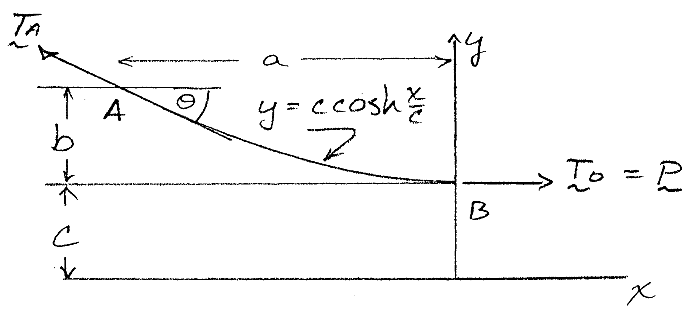
$\mathbf{F} = 40.7 \text{ lb} \longrightarrow$ ◀



PROBLEM 7.138

A uniform cable has a mass per unit length of 4 kg/m and is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 800$ N and $\theta_A = 60°$, determine (*a*) the location of point *B*, (*b*) the length of the cable.

SOLUTION



$$w = 4 \text{ kg/m}\left(9.81 \text{ m/s}^2\right) = 39.24 \text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{800 \text{ N}}{39.24 \text{ N/m}}$$

$$c = 20.387 \text{ m}$$

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$$

$$a = c\sinh^{-1}(\tan\theta) = (20.387 \text{ m})\sinh^{-1}(\tan 60°)$$

$$a = 26.849 \text{ m}$$

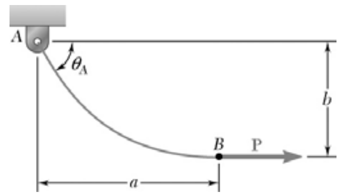
$$y_A = c\cosh\frac{a}{c} = (20.387 \text{ m})\cosh\frac{26.849 \text{ m}}{20.387 \text{ m}} = 40.774 \text{ m}$$

$$b = y_A - c = 40.774 \text{ m} - 20.387 \text{ m} = 20.387 \text{ m}$$

So (*a*) *B* is 26.8 m right and 20.4 m down from *A* ◀

$$s = c\sinh\frac{a}{c} = (20.387 \text{ m})\sinh\frac{26.849 \text{ m}}{20.387 \text{ m}} = 35.31 \text{ m}$$

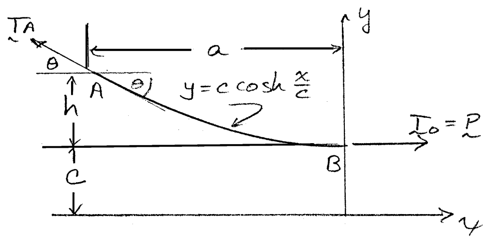
(*b*) $s = 35.3$ m ◀



PROBLEM 7.139

A uniform cable having a mass per unit length of 4 kg/m is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 600$ N and $\theta_A = 60°$, determine (*a*) the location of point *B*, (*b*) the length of the cable.

SOLUTION



$$w = (4\text{ kg/m})(9.81\text{ m/s}^2) = 39.24\text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{600\text{ N}}{39.24\text{ N/m}}$$

$$c = 15.2905\text{ m}$$

$$y = c\cosh\frac{x}{c} \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

At *A*: $\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$

So $a = c\sinh^{-1}(\tan\theta) = (15.2905\text{ m})\sinh^{-1}(\tan 60°) = 20.137\text{ m}$

$$y_B = h + c = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

$$= (15.2905\text{ m})\left(\cosh\frac{20.137\text{ m}}{15.2905\text{ m}} - 1\right)$$

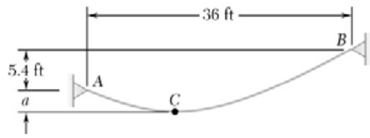
$$= 15.291\text{ m}$$

So (*a*) *B* is 20.1 m right and 15.29 m down from *A* ◀

$$s = c\sinh\frac{a}{c} = (15.291\text{ m})\sinh\frac{20.137\text{ m}}{15.291\text{ m}} = 26.49\text{ m}$$

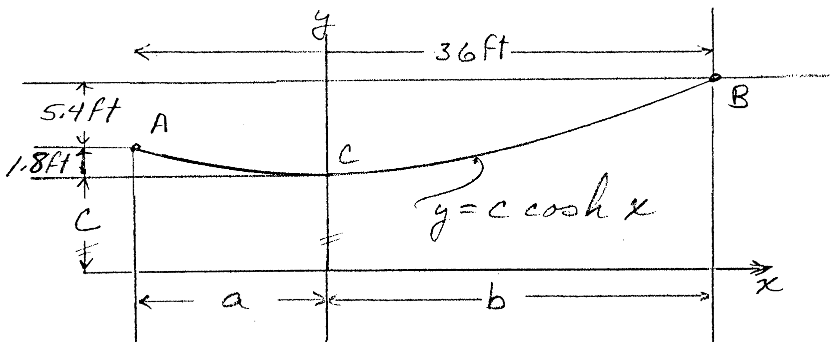
(*b*) $s = 26.5$ m ◀

PROBLEM 7.140



The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance $a = 1.8$ ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 1.8 \text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{1.8 \text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 7.2 \text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{7.2 \text{ ft}}{c}\right)$$

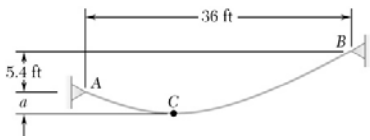
But $\quad a + b = 36 \text{ ft} = c\left[\cosh^{-1}\left(1 + \frac{1.8 \text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{7.2 \text{ ft}}{c}\right)\right]$

Solving numerically $\quad c = 40.864$ ft

Then $\quad b = (40.864 \text{ ft})\cosh^{-1}\left(1 + \frac{7.2 \text{ ft}}{40.864 \text{ ft}}\right) = 23.92 \text{ ft}$

(*a*) $\quad C$ is 23.9 ft left of and 7.20 ft below B ◀

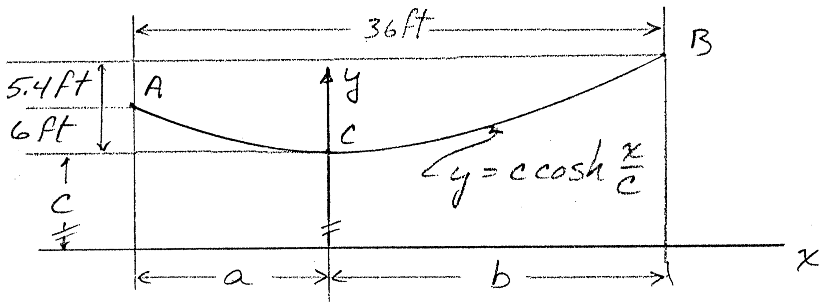
$T_{max} = wy_B = (0.3 \text{ lb/ft})(40.864 \text{ ft} + 7.2 \text{ ft})$ $\quad$ (*b*) $\quad T_{max} = 14.42$ lb ◀



PROBLEM 7.141

The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance $a = 6$ ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 6\text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{6\text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 11.4\text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{11.4\text{ ft}}{c}\right)$$

So $$a + b = c\left[\cosh^{-1}\left(1 + \frac{6\text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{11.4\text{ ft}}{\text{c}}\right)\right] = 36\text{ ft}$$

Solving numerically $$c = 20.446\text{ ft}$$

$$b = (20.446\text{ ft})\cosh^{-1}\left(1 + \frac{11.4\text{ ft}}{20.446\text{ ft}}\right) = 20.696\text{ ft}$$

(a) *C* is 20.7 ft left of and 11.4 ft below *B* ◀

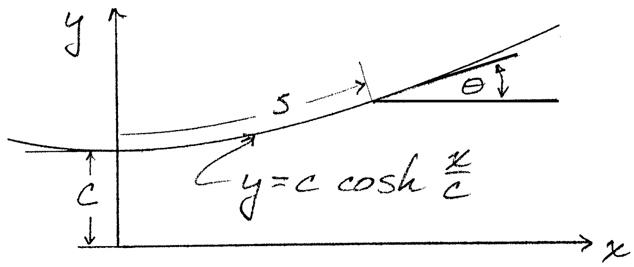
$$T_{\max} = wy_B = (0.3\text{ lb/ft})(20.446\text{ ft})\cosh\left(\frac{20.696\text{ ft}}{20.446\text{ ft}}\right) = 9.554\text{ lb}$$

(*b*) $T_{\max} = 9.55\text{ lb}$ ◀

PROBLEM 7.142

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c\ \tan\theta$, (*b*) $y = c\ \sec\theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \quad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2\left(\cosh^2 x = \sinh^2 x + 1\right)$$

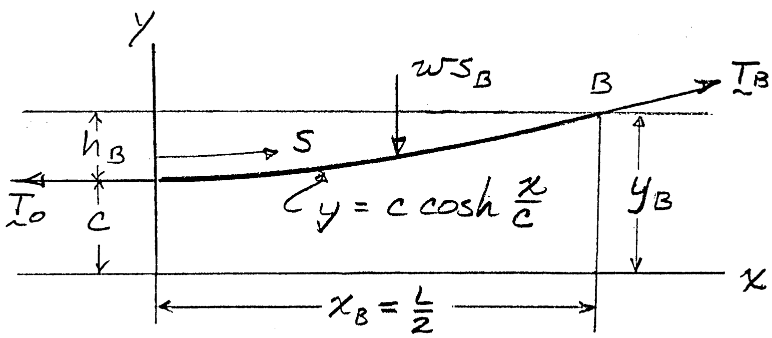
So $$y^2 = c^2\left(\tan^2\theta + 1\right) = c^2\sec^2\theta$$

And $$y = c\sec\theta \quad \text{Q.E.D.}$$

PROBLEM 7.143

(a) Determine the maximum allowable horizontal span for a uniform cable of mass per unit length m' if the tension in the cable is not to exceed a given value T_m. (b) Using the result of part a, determine the maximum span of a steel wire for which $m' = 0.34$ kg/m and $T_m = 32$ kN.

SOLUTION



$$T_B = T_{max} = wy_B$$

$$= wc\cosh\frac{x_B}{c} = w\frac{L}{2}\left(\frac{2c}{L}\right)\cosh\frac{L}{2c}$$

Let $\xi = \dfrac{L}{2c}$ so $T_{max} = \dfrac{wL}{2\xi}\cosh\xi$

$$\frac{dT_{max}}{d\xi} = \frac{wL}{2\xi}\left(\sinh\xi - \frac{1}{\xi}\cosh\xi\right)$$

For $\min T_{max}$, $\tanh\xi - \dfrac{1}{\xi} = 0$

Solving numerically $\xi = 1.1997$

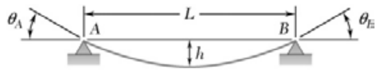
$$(T_{max})_{min} = \frac{wL}{2(1.9997)}\cosh(1.1997) = 0.75444wL$$

(a) $L_{max} = \dfrac{T_{max}}{0.75444w} = 1.3255\dfrac{T_{max}}{w}$ ◀

If $T_{max} = 32$ kN and $w = (0.34 \text{ kg/m})(9.81 \text{ m/s}^2) = 3.3354$ N/m

$$L_{max} = 1.3255\frac{32.000 \text{ N}}{3.3354 \text{ N/m}} = 12\,717 \text{ m}$$

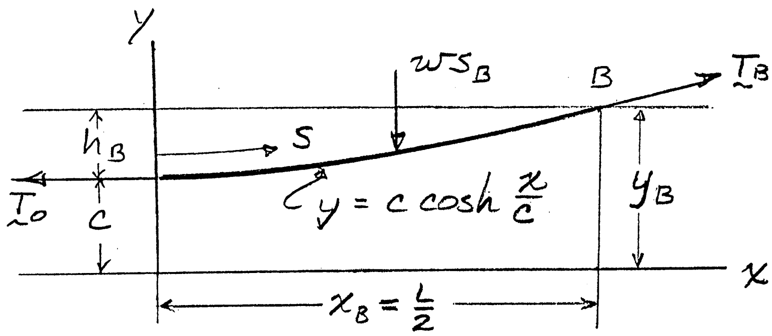
(b) $L_{max} = 12.72$ km ◀



PROBLEM 7.144

A cable has a weight per unit length of 2 lb/ft and is supported as shown. Knowing that the span L is 18 ft, determine the two values of the sag h for which the maximum tension is 80 lb.

SOLUTION



$$y_{max} = c \cosh \frac{L}{2c} = h + c$$

$$T_{max} = w y_{max} \qquad y_{max} = \frac{T_{max}}{w}$$

$$y_{max} = \frac{80 \text{ lb}}{2 \text{ lb/ft}} = 40 \text{ ft}$$

$$c \cosh \frac{9 \text{ ft}}{c} = 40 \text{ ft}$$

Solving numerically $\quad c_1 = 2.6388$ ft

$$c_2 = 38.958 \text{ ft}$$

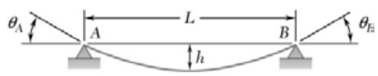
$$h = y_{max} - c$$

$$h_1 = 40 \text{ ft} - 2.6388 \text{ ft}$$

$h_1 = 37.4$ ft ◀

$$h_2 = 40 \text{ ft} - 38.958 \text{ ft}$$

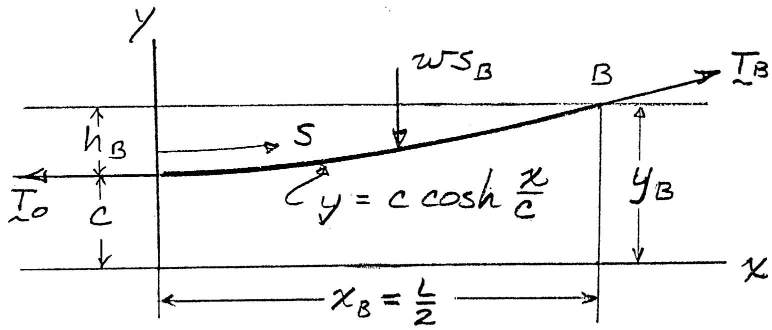
$h_2 = 1.042$ ft ◀



PROBLEM 7.145

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable *AB*.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1$$

$$= 0.154701$$

$$\frac{h_B}{L} = \frac{h_B / c}{2(L/2c)}$$

$$= \frac{0.5(0.154701)}{0.549306} = 0.14081$$

$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$