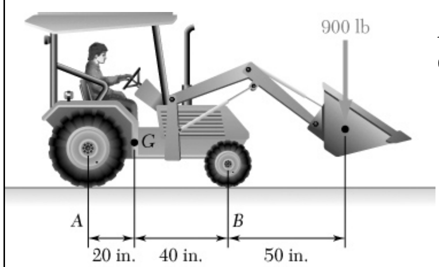
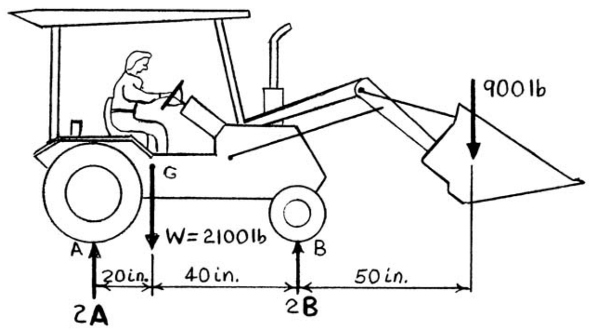
PROBLEM 4.156



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) From f.b.d. of tractor

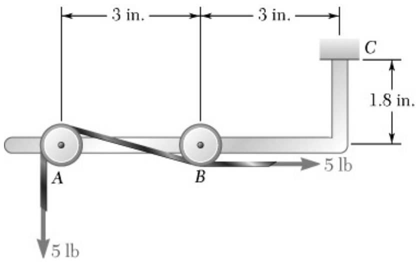
$$+\circlearrowleft \Sigma M_B = 0: \quad (2100 \text{ lb})(40 \text{ in.}) - (2A)(60 \text{ in.}) - (900 \text{ lb})(50 \text{ in.}) = 0$$

$\therefore\ A = 325 \text{ lb}$ or $\mathbf{A} = 325 \text{ lb} \uparrow$ ◀

(*b*) From f.b.d. of tractor

$$+\circlearrowleft \Sigma M_A = 0: \quad (2B)(60 \text{ in.}) - (2100 \text{ lb})(20 \text{ in.}) - (900 \text{ lb})(110 \text{ in.}) = 0$$

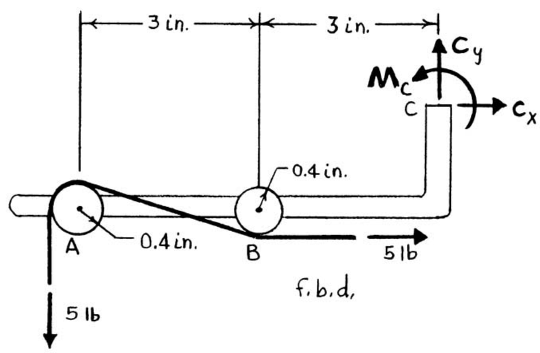
$\therefore\ B = 1175 \text{ lb}$ or $\mathbf{B} = 1175 \text{ lb} \uparrow$ ◀



PROBLEM 4.157

A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION



From f.b.d. of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$\therefore\ C_x = -5\text{ lb}$$

$$+\!\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$\therefore\ C_y = 5\text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{+5}{-5}\right) = -45°$$

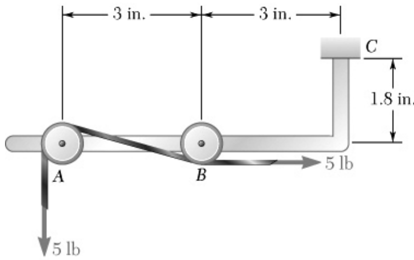
or $\mathbf{C} = 7.07\text{ lb} \measuredangle\ 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.4\text{ in.}) + (5\text{ lb})(2.2\text{ in.}) = 0$$

$$\therefore\ M_C = -43.0\text{ lb}\cdot\text{in}$$

or $\mathbf{M}_C = 43.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

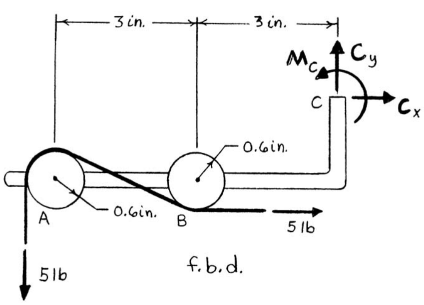
PROBLEM 4.158



Solve Problem 4.157 assuming that 0.6-in.-radius pulleys are used.

P4.157 A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION



From f.b.d of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5 \text{ lb}) = 0$$

$$\therefore \; C_x = -5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5 \text{ lb}) = 0$$

$$\therefore \; C_y = 5 \text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711 \text{ lb}$$

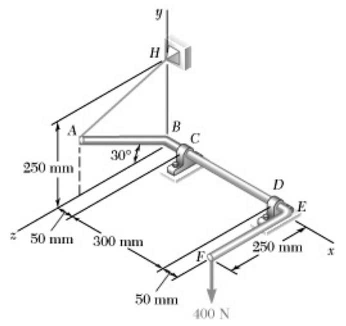
and $$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = -45.0°$$

or $\mathbf{C} = 7.07 \text{ lb} \measuredangle 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5 \text{ lb})(6.6 \text{ in.}) + (5 \text{ lb})(2.4 \text{ in.}) = 0$$

$$\therefore \; M_C = -45.0 \text{ lb}\cdot\text{in.}$$

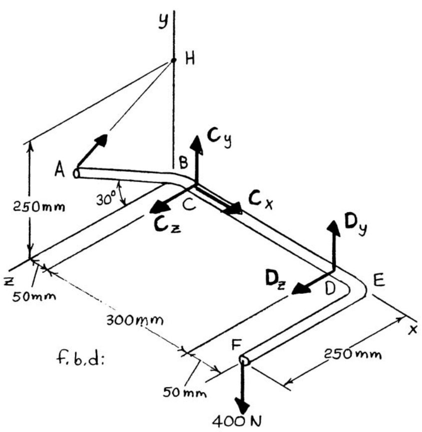
or $\mathbf{M}_C = 45.0 \text{ lb}\cdot\text{in.} \circlearrowright$ ◀



PROBLEM 4.159

The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



(*a*) From f.b.d. of bent rod

$$\Sigma M_{CD} = 0: \quad \lambda_{CD} \cdot \left(\mathbf{r}_{H/B} \times \mathbf{T}\right) + \lambda_{CD} \cdot \left(\mathbf{r}_{F/E} \times \mathbf{F}\right) = 0$$

where $\quad \lambda_{CD} = \mathbf{i}$

$$\mathbf{r}_{H/B} = (0.25\text{ m})\mathbf{j}$$

$$\mathbf{T} = \lambda_{AH} T$$

$$= \frac{(y_{AH})\mathbf{j} - (z_{AH})\mathbf{k}}{\sqrt{(y_{AH})^2 + (z_{AH})^2}} T$$

$$y_{AH} = (0.25\text{ m}) - (0.25\text{ m})\sin 30°$$

$$= 0.125\text{ m}$$

$$z_{AH} = (0.25\text{ m})\cos 30°$$

$$= 0.21651\text{ m}$$

$$\therefore\ \mathbf{T} = \frac{T}{0.25}(0.125\mathbf{j} - 0.21651\mathbf{k})$$

$$\mathbf{r}_{F/E} = (0.25\text{ m})\mathbf{k}$$

$$\mathbf{F} = -400\text{ N }\mathbf{j}$$

$$\therefore \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.125 & -0.21651 \end{vmatrix} (0.25)\left(\frac{T}{0.25}\right) + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} (0.25)(400\text{ N}) = 0$$

$$-0.21651T + 0.25(400\text{ N}) = 0$$

$$\therefore\ T = 461.88\text{ N}$$

or $T = 462$ N ◀

PROBLEM 4.159 CONTINUED

(*b*) From f.b.d. of bent rod

$$\Sigma F_x = 0: \quad C_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -\left[(461.88\text{ N})\sin 30°\right](0.35\text{ m}) - C_y(0.3\text{ m})$$

$$-(400\text{ N})(0.05\text{ m}) = 0$$

$$\therefore\ C_y = -336.10\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(0.3\text{ m}) - \left[(461.88\text{ N})\cos 30°\right](0.35\text{ m}) = 0$$

$$\therefore\ C_z = 466.67\text{ N}$$

or $\mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

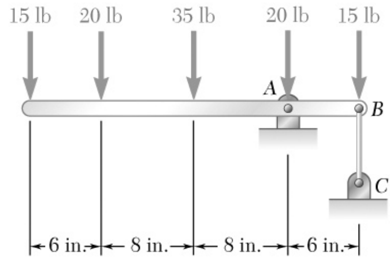
$$\Sigma F_y = 0: \quad D_y - 336.10\text{ N} + (461.88\text{ N})\sin 30° - 400\text{ N} = 0$$

$$\therefore\ D_y = 505.16\text{ N}$$

$$\Sigma F_z = 0: \quad D_z + 466.67\text{ N} - (461.88\text{ N})\cos 30° = 0$$

$$\therefore\ D_z = -66.670\text{ N}$$

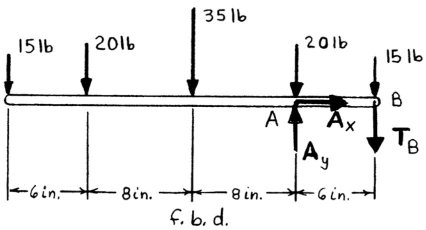
or $\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀



PROBLEM 4.160

For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION



(*a*) From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.})$$

$$+ (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$$

$$\therefore A_y = 245\text{ lb}$$

$$\text{or} \quad \mathbf{A} = 245\text{ lb} \uparrow \blacktriangleleft$$

(*b*) From f.b.d of beam

$$+\circlearrowleft \Sigma M_A = 0: \quad (15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.})$$

$$- (15\text{ lb})(6\text{ in.}) - T_B(6\text{ in.}) = 0$$

$$\therefore T_B = 140.0\text{ lb}$$

$$\text{or} \quad T_B = 140.0\text{ lb} \blacktriangleleft$$

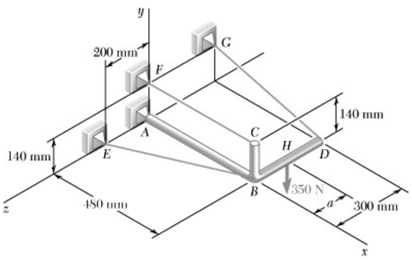
Check:

$$+\uparrow \Sigma F_y = 0: \quad -15\text{ lb} - 20\text{ lb} - 35\text{ lb} - 20\text{ lb}$$

$$- 15\text{ lb} - 140\text{ lb} + 245\text{ lb} = 0?$$

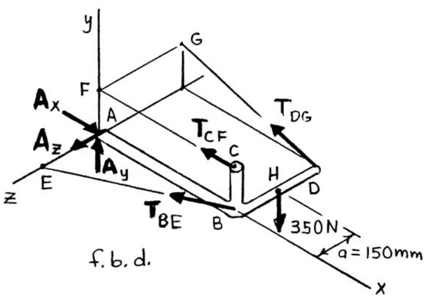
$$245\text{ lb} - 245\text{ lb} = 0 \quad \text{ok}$$

PROBLEM 4.161



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For a = 150 mm, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE} T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.15\text{ m}) = 0$$

or $T_{DG} = 625\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 625\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48\text{ m}) = 0$$

or $T_{BE} = 975\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 625\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

or $T_{CF} = 600\text{ N}$ ◀

PROBLEM 4.161 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600\text{ N} - \left(\frac{12}{13} \times 975\text{ N}\right) - \left(\frac{24}{25} \times 625\text{ N}\right) = 0$$

$$\therefore\ A_x = 2100\text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350\text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625\text{ N}\right) - 350\text{ N} = 0$$

$$\therefore\ A_y = 175.0\text{ N}$$

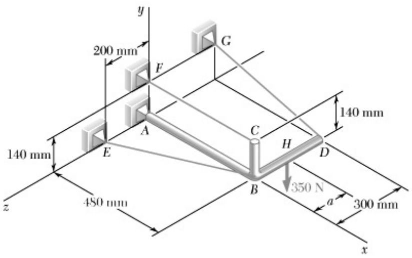
$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975\text{ N}\right) = 0$$

$$\therefore\ A_z = -375\text{ N}$$

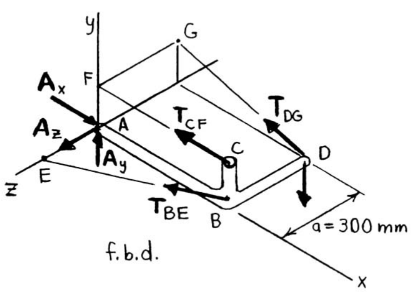
Therefore $\mathbf{A} = (2100\text{ N})\mathbf{i} + (175.0\text{ N})\mathbf{j} - (375\text{ N})\mathbf{k}$ ◀

PROBLEM 4.162



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* (a = 300 mm), determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG}T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}}T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50}T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}}T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52}T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$$

From f.b.d of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.3\text{ m}) = 0$$

$$\text{or} \quad T_{DG} = 1250\text{ N} \blacktriangleleft$$

$$\Sigma M_y = 0: \quad \left(\frac{24}{25}\times 1250\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

$$\text{or} \quad T_{BE} = 1950\text{ N} \blacktriangleleft$$

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25}\times 1250\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

$$\text{or} \quad T_{CF} = 0 \blacktriangleleft$$

PROBLEM 4.162 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x + 0 - \left(\frac{12}{13} \times 1950 \text{ N}\right) - \left(\frac{24}{25} \times 1250 \text{ N}\right) = 0$$

$$\therefore \; A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 1250 \text{ N}\right) - 350 \text{ N} = 0$$

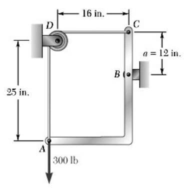
$$\therefore \; A_y = 0$$

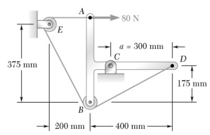
$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 1950 \text{ N}\right) = 0$$

$$\therefore \; A_z = -750 \text{ N}$$

Therefore $\mathbf{A} = (3000 \text{ N})\mathbf{i} - (750 \text{ N})\mathbf{k}$ ◀



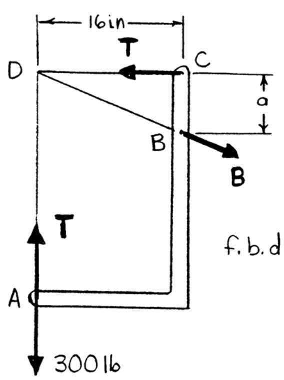


PROBLEM 4.163

In the problems listed below, the rigid bodies considered were completely constrained and the reactions were statically determinate. For each of these rigid bodies it is possible to create an improper set of constraints by changing a dimension of the body. In each of the following problems determine the value of a which results in improper constraints. (*a*) Problem 4.81, (*b*) Problem 4.82.

SOLUTION

(*a*)



(*a*) $$+\circlearrowleft \Sigma M_B = 0: \quad (300\text{ lb})(16\text{ in.}) - T(16\text{ in.}) + T(a) = 0$$

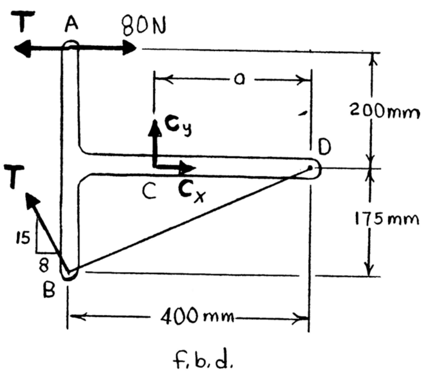
or $$T = \frac{(300\text{ lb})(16\text{ in.})}{(16 - a)\text{in.}}$$

$\therefore$ T becomes infinite when

$$16 - a = 0$$

or $a = 16.00$ in. ◀

(*b*)



(*b*) $$+\circlearrowleft \Sigma M_C = 0: \quad (T - 80\text{ N})(0.2\text{ m}) - \left(\frac{8}{17}T\right)(0.175\text{ m})$$

$$-\left(\frac{15}{17}T\right)(0.4\text{ m} - a) = 0$$

$$0.2T - 16.0 - 0.82353T - 0.35294T + 0.88235Ta = 0$$

or $$T = \frac{16.0}{0.88235a - 0.23529}$$

$\therefore$ T becomes infinite when

$$0.88235a - 0.23529 = 0$$

$$a = 0.26666\text{ m}$$

or $a = 267$ mm ◀