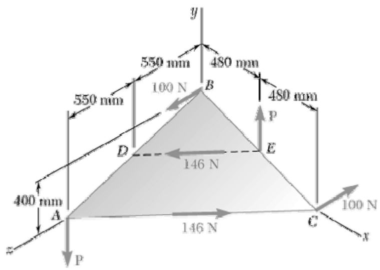


PROBLEM 3.76

Knowing that $P = 210$ N, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Have
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_P$$

where
$$\mathbf{M}_1 = \mathbf{r}_{C/B} \times \mathbf{P}_{1C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.40 & 0 \\ 0 & 0 & -100 \end{vmatrix} = (40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}$$

$$\mathbf{M}_2 = \mathbf{r}_{D/A} \times \mathbf{P}_{2E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.20 & -0.55 \\ -96 & 0 & 110 \end{vmatrix} = (22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

(See Solution to Problem 3.73.)

$$\mathbf{M}_P = \mathbf{r}_{E/A} \times \mathbf{P}_E = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.48 & 0.20 & -1.10 \\ 0 & 210 & 0 \end{vmatrix} = (231\text{ N}\cdot\text{m})\mathbf{i} + (100.8\text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore\ \mathbf{M} = \left[(40 + 22 + 231)\mathbf{i} + (96 + 52.8)\mathbf{j} + (19.2 + 100.8)\mathbf{k}\right]\text{ N}\cdot\text{m}$$

$$= (293\text{ N}\cdot\text{m})\mathbf{i} + (148.8\text{ N}\cdot\text{m})\mathbf{j} + (120\text{ N}\cdot\text{m})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(293)^2 + (148.8)^2 + (120)^2} = 349.84\text{ N}\cdot\text{m}$$

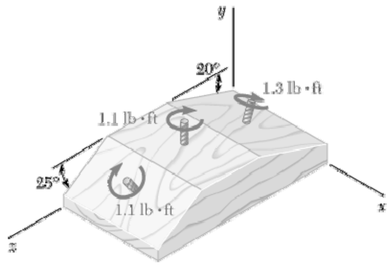
or $M = 350\text{ N}\cdot\text{m}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{293\mathbf{i} + 148.8\mathbf{j} + 120\mathbf{k}}{349.84} = 0.83752\mathbf{i} + 0.42533\mathbf{j} + 0.34301\mathbf{k}$$

$\cos\theta_x = 0.83752 \qquad \therefore\ \theta_x = 33.121°$ or $\theta_x = 33.1°$ ◀

$\cos\theta_y = 0.42533 \qquad \therefore\ \theta_y = 64.828°$ or $\theta_y = 64.8°$ ◀

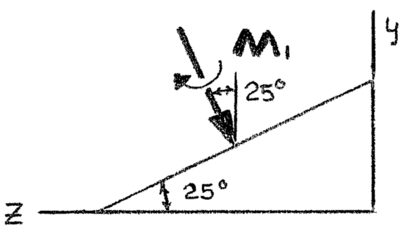
$\cos\theta_z = 0.34301 \qquad \therefore\ \theta_z = 69.940°$ or $\theta_z = 69.9°$ ◀

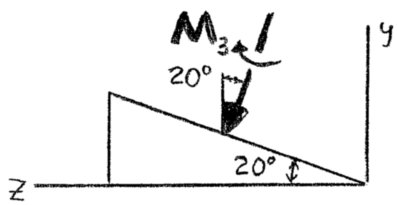


PROBLEM 3.77

In a manufacturing operation, three holes are drilled simultaneously in a workpiece. Knowing that the holes are perpendicular to the surfaces of the workpiece, replace the couples applied to the drills with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION





Have $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$

where $$\mathbf{M}_1 = -(1.1\text{ lb}\cdot\text{ft})(\cos 25°\mathbf{j} + \sin 25°\mathbf{k})$$

$$\mathbf{M}_2 = -(1.1\text{ lb}\cdot\text{ft})\mathbf{j}$$

$$\mathbf{M}_3 = -(1.3\text{ lb}\cdot\text{ft})(\cos 20°\mathbf{j} - \sin 20°\mathbf{k})$$

$$\therefore \quad \mathbf{M} = (-0.99694 - 1.1 - 1.22160)\mathbf{j} + (-0.46488 + 0.44463)\mathbf{k}$$

$$= -(3.3185\text{ lb}\cdot\text{ft})\mathbf{j} - (0.020254\text{ lb}\cdot\text{ft})\mathbf{k}$$

and $$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(0)^2 + (3.3185)^2 + (0.020254)^2}$$

$$= 3.3186\text{ lb}\cdot\text{ft}$$

or $M = 3.32\text{ lb}\cdot\text{ft}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{(0)\mathbf{i} - 3.3185\mathbf{j} - 0.020254\mathbf{k}}{3.3186}$$

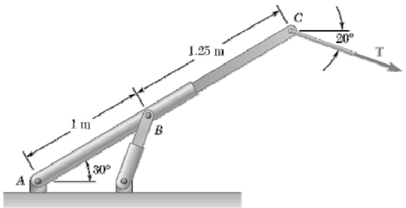
$$= -0.99997\mathbf{j} - 0.0061032\mathbf{k}$$

$\cos\theta_x = 0 \qquad \therefore \quad \theta_x = 90°$ or $\theta_x = 90.0°$ ◀

$\cos\theta_y = -0.99997 \qquad \therefore \quad \theta_y = 179.555°$ or $\theta_y = 179.6°$ ◀

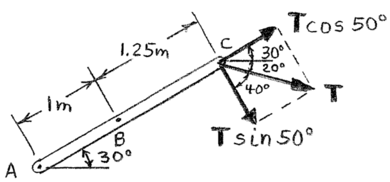
$\cos\theta_z = -0.0061032 \qquad \therefore \quad \theta_z = 90.349°$ or $\theta_z = 90.3°$ ◀

PROBLEM 3.78



The tension in the cable attached to the end C of an adjustable boom ABC is 1000 N. Replace the force exerted by the cable at C with an equivalent force-couple system (a) at A, (b) at B.

SOLUTION



(a) Based on ΣF: $F_A = T = 1000\text{ N}$

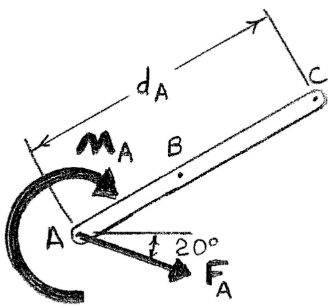
or $\mathbf{F}_A = 1000\text{ N}$ 20° ◀

ΣM_A: $M_A = (T\sin 50°)(d_A)$

$= (1000\text{ N})\sin 50°(2.25\text{ m})$

$= 1723.60\text{ N}\cdot\text{m}$

or $\mathbf{M}_A = 1724\text{ N}\cdot\text{m}$ ◀



(b) Based on ΣF: $F_B = T = 1000\text{ N}$

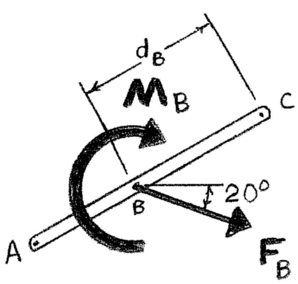
or $\mathbf{F}_B = 1000\text{ N}$ 20° ◀

ΣM_B: $M_B = (T\sin 50°)(d_B)$

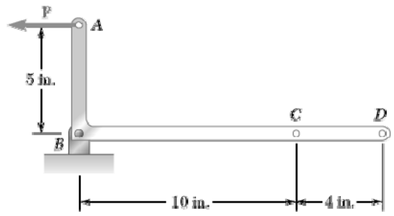
$= (1000\text{ N})\sin 50°(1.25\text{ m})$

$= 957.56\text{ N}\cdot\text{m}$

or $\mathbf{M}_B = 958\text{ N}\cdot\text{m}$ ◀

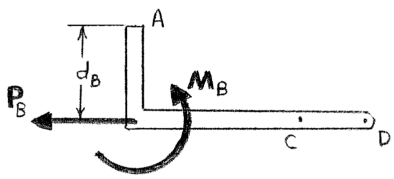


PROBLEM 3.79



The 20-lb horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* which are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \quad P_B = P = 20 \text{ lb}$

or $\mathbf{P}_B = 20 \text{ lb} \longleftarrow$ ◀

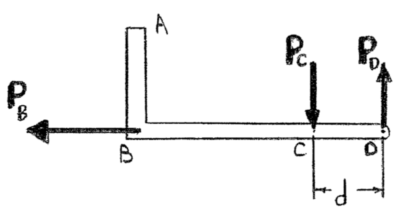
$$\Sigma M: \quad M_B = Pd_B$$

$$= 20 \text{ lb}(5 \text{ in.})$$

$$= 100 \text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_B = 100 \text{ lb}\cdot\text{in.}$ ↺ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then, with P_C and P_D acting as shown,

$$\Sigma M: \quad M_D = P_C d$$

$$100 \text{ lb}\cdot\text{in.} = P_C(4 \text{ in.})$$

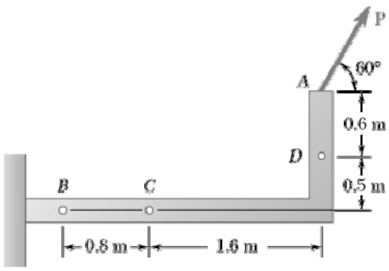
$$\therefore \quad P_C = 25 \text{ lb}$$

or $\mathbf{P}_C = 25 \text{ lb} \downarrow$ ◀

$$\Sigma F_y: \quad 0 = P_D - P_C$$

$$\therefore \quad P_D = 25 \text{ lb}$$

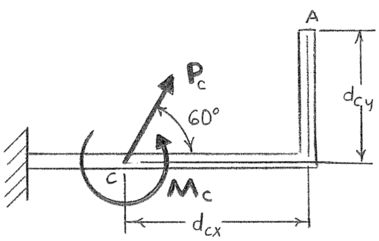
or $\mathbf{P}_D = 25 \text{ lb} \uparrow$ ◀



PROBLEM 3.80

A 700-N force **P** is applied at point *A* of a structural member. Replace **P** with (*a*) an equivalent force-couple system at *C*, (*b*) an equivalent system consisting of a vertical force at *B* and a second force at *D*.

SOLUTION



(*a*) Based on $\Sigma F: \quad P_C = P = 700\text{ N}$

or $\mathbf{P}_C = 700\text{ N} \measuredangle 60°$ ◀

$$\Sigma M_C: \quad M_C = -P_x d_{Cy} + P_y d_{Cx}$$

where $P_x = (700\text{ N})\cos 60° = 350\text{ N}$

$$P_y = (700\text{ N})\sin 60° = 606.22\text{ N}$$

$$d_{Cx} = 1.6\text{ m}$$

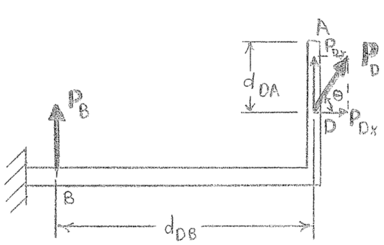
$$d_{Cy} = 1.1\text{ m}$$

$$\therefore \quad M_C = -(350\text{ N})(1.1\text{ m}) + (606.22\text{ N})(1.6\text{ m})$$

$$= -385\text{ N}\cdot\text{m} + 969.95\text{ N}\cdot\text{m}$$

$$= 584.95\text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 585\text{ N}\cdot\text{m}$ ↺ ◀



(*b*) Based on $\Sigma F_x: \quad P_{Dx} = P\cos 60°$

$$= (700\text{ N})\cos 60°$$

$$= 350\text{ N}$$

$$\Sigma M_D: \quad (P\cos 60°)(d_{DA}) = P_B(d_{DB})$$

$$[(700\text{ N})\cos 60°](0.6\text{ m}) = P_B(2.4\text{ m})$$

$$P_B = 87.5\text{ N}$$

or $\mathbf{P}_B = 87.5\text{ N}\uparrow$ ◀

PROBLEM 3.80 CONTINUED

$$\Sigma F_y: \quad P\sin 60° = P_B + P_{Dy}$$

$$(700\text{ N})\sin 60° = 87.5\text{ N} + P_{Dy}$$

$$P_{Dy} = 518.72\text{ N}$$

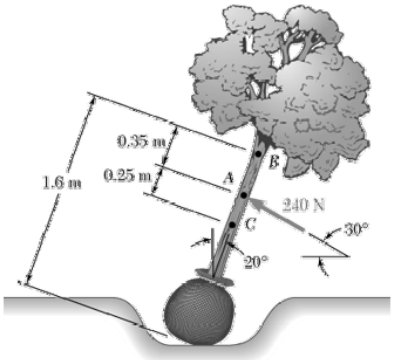
$$P_D = \sqrt{(P_{Dx})^2 + (P_{Dy})^2}$$

$$= \sqrt{(350)^2 + (518.72)^2} = 625.76\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{P_{Dy}}{P_{Dx}}\right) = \tan^{-1}\left(\frac{518.72}{350}\right) = 55.991°$$

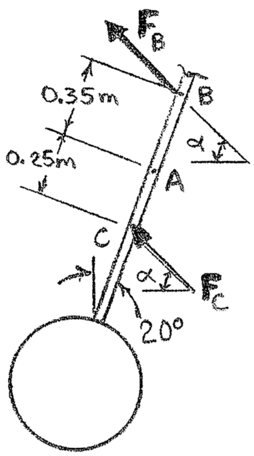
or $P_D = 626\text{ N} \measuredangle\ 56.0°$ ◀

PROBLEM 3.81



A landscaper tries to plumb a tree by applying a 240-N force as shown. Two helpers then attempt to plumb the same tree, with one pulling at B and the other pushing with a parallel force at C. Determine these two forces so that they are equivalent to the single 240-N force shown in the figure.

SOLUTION



Based on

$$\Sigma F_x: \quad -(240 \text{ N})\cos 30° = -F_B \cos\alpha - F_C \cos\alpha$$

or

$$-(F_B + F_C)\cos\alpha = -(240 \text{ N})\cos 30° \qquad (1)$$

$$\Sigma F_y: \quad (240 \text{ N})\sin 30° = F_B \sin\alpha + F_C \sin\alpha$$

or

$$(F_B + F_C)\sin\alpha = (240 \text{ N})\sin 30° \qquad (2)$$

From

$$\frac{\text{Equation (2)}}{\text{Equation (1)}}: \quad \tan\alpha = \tan 30°$$

$$\therefore \quad \alpha = 30°$$

Based on

$$\Sigma M_C: \quad \left[(240 \text{ N})\cos(30° - 20°)\right](0.25 \text{ m}) = (F_B \cos 10°)(0.60 \text{ m})$$

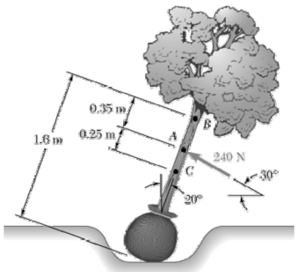
$$\therefore \quad F_B = 100 \text{ N}$$

or $\mathbf{F}_B = 100.0 \text{ N} \measuredangle 30°$ ◀

From Equation (1), $$-(100 \text{ N} + F_C)\cos 30° = -240 \cos 30°$$

$$F_C = 140 \text{ N}$$

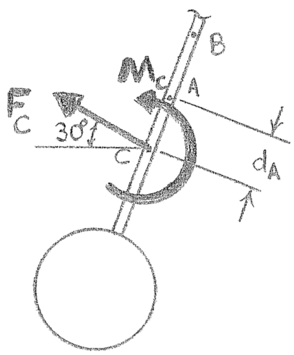
or $\mathbf{F}_C = 140.0 \text{ N} \measuredangle 30°$ ◀



PROBLEM 3.82

A landscaper tries to plumb a tree by applying a 240-N force as shown. (*a*) Replace that force with an equivalent force-couple system at *C*. (*b*) Two helpers attempt to plumb the same tree, with one applying a horizontal force at *C* and the other pulling at *B*. Determine these two forces if they are to be equivalent to the single force of part *a*.

SOLUTION



(*a*) Based on $\Sigma F_x: \quad -(240 \text{ N})\cos 30° = -F_C \cos 30°$

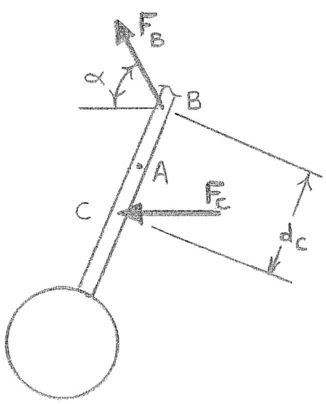
$$\therefore \quad F_C = 240 \text{ N}$$

or $\mathbf{F}_C = 240 \text{ N} \searrow 30°$ ◀

$$\Sigma M_C: \quad \left[(240 \text{ N})\cos 10°\right](d_A) = M_C \qquad d_A = 0.25 \text{ m}$$

$$\therefore \quad M_C = 59.088 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 59.1 \text{ N}\cdot\text{m} \circlearrowleft$ ◀



(*b*) Based on $\Sigma F_y: \quad (240 \text{ N})\sin 30° = F_B \sin\alpha$

or $\qquad F_B \sin\alpha = 120 \qquad (1)$

$$\Sigma M_B: \quad 59.088 \text{ N}\cdot\text{m} - \left[(240 \text{ N})\cos 10°\right](d_C) = -F_C\left(d_C \cos 20°\right)$$

$$59.088 \text{ N}\cdot\text{m} - \left[(240 \text{ N})\cos 10°\right](0.60 \text{ m}) = -F_C\left[(0.60 \text{ m})\cos 20°\right]$$

$$0.56382 F_C = 82.724$$

$$F_C = 146.722 \text{ N}$$

or $\mathbf{F}_C = 146.7 \text{ N} \longleftarrow$ ◀

and $\qquad \Sigma F_x: \quad -(240 \text{ N})\cos 30° = -146.722 \text{ N} - F_B \cos\alpha$

$$F_B \cos\alpha = 61.124 \qquad (2)$$

From

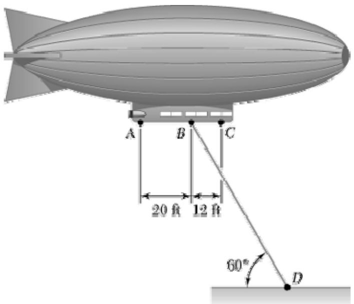
$$\frac{\text{Equation (1)}}{\text{Equation (2)}}: \quad \tan\alpha = \frac{120}{61.124} = 1.96323$$

$\alpha = 63.007°$ $\qquad$ or $\alpha = 63.0°$ ◀

From Equation (1), $\qquad F_B = \dfrac{120}{\sin 63.007°} = 134.670 \text{ N}$

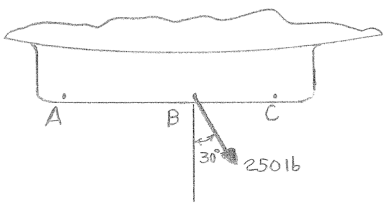
or $\mathbf{F}_B = 134.7 \text{ N} \searrow 63.0°$ ◀

PROBLEM 3.83



A dirigible is tethered by a cable attached to its cabin at B. If the tension in the cable is 250 lb, replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C.

SOLUTION

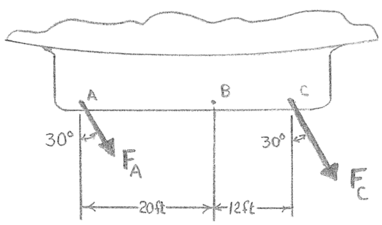


Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (250 \text{ lb})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(250 \text{ lb})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$



Dividing Equation (1) by Equation (2),

$$\frac{(250 \text{ lb})\sin 30°}{-(250 \text{ lb})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$

Based on

$$\Sigma M_C: \quad [(250 \text{ lb})\cos 30°](12 \text{ ft}) = (F_A \cos 30°)(32 \text{ ft})$$

$$\therefore \quad F_A = 93.75 \text{ lb}$$

or $\mathbf{F}_A = 93.8 \text{ lb} \searrow 60°$ ◀

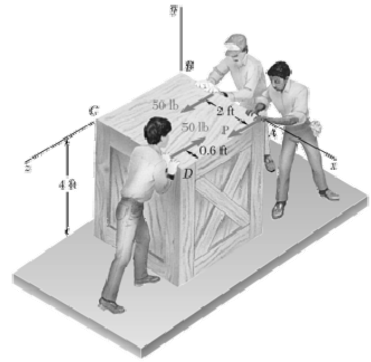
Based on

$$\Sigma M_A: \quad -[(250 \text{ lb})\cos 30°](20 \text{ ft}) = (F_C \cos 30°)(32 \text{ ft})$$

$$\therefore \quad F_C = 156.25 \text{ lb}$$

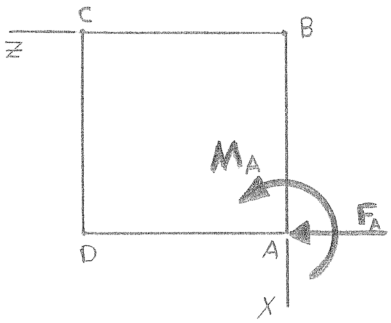
or $\mathbf{F}_C = 156.3 \text{ lb} \searrow 60°$ ◀

PROBLEM 3.84



Three workers trying to move a $3 \times 3 \times 4$-ft crate apply to the crate the three horizontal forces shown. (*a*) If $P = 60$ lb, replace the three forces with an equivalent force-couple system at *A*. (*b*) Replace the force-couple system of part *a* with a single force, and determine where it should be applied to side *AB*. (*c*) Determine the magnitude of **P** so that the three forces can be replaced with a single equivalent force applied at *B*.

SOLUTION



(*a*)

(*a*) Based on

$$\Sigma F_z: \quad -50 \text{ lb} + 50 \text{ lb} + 60 \text{ lb} = F_A$$

$$F_A = 60 \text{ lb}$$

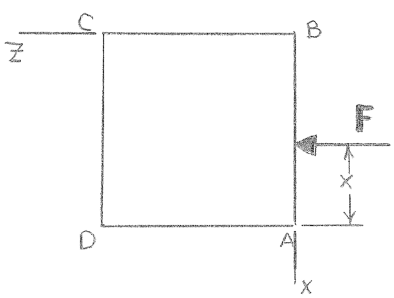
$$\text{or } \mathbf{F}_A = (60.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Based on

$$\Sigma M_A: \quad (50 \text{ lb})(2 \text{ ft}) - (50 \text{ lb})(0.6 \text{ ft}) = M_A$$

$$M_A = 70 \text{ lb}\cdot\text{ft}$$

$$\text{or } \mathbf{M}_A = (70.0 \text{ lb}\cdot\text{ft})\mathbf{j} \blacktriangleleft$$



(*b*)

(*b*) Based on

$$\Sigma F_z: \quad -50 \text{ lb} + 50 \text{ lb} + 60 \text{ lb} = F$$

$$F = 60 \text{ lb}$$

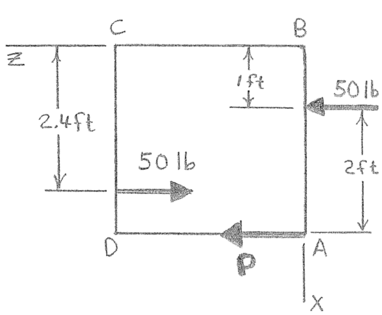
$$\text{or } \mathbf{F} = (60.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Based on

$$\Sigma M_A: \quad 70 \text{ lb}\cdot\text{ft} = 60 \text{ lb}(x)$$

$$x = 1.16667 \text{ ft}$$

$$\text{or } x = 1.167 \text{ ft from } A \text{ along } AB \blacktriangleleft$$



(*c*)

(*c*) Based on

$$\Sigma M_B: \quad -(50 \text{ lb})(1 \text{ ft}) + (50 \text{ lb})(2.4 \text{ ft}) - P(3 \text{ ft}) = R(0)$$

$$P = \frac{70}{3} = 23.333 \text{ lb}$$

$$\text{or } P = 23.3 \text{ lb} \blacktriangleleft$$