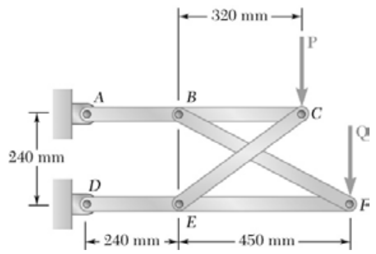
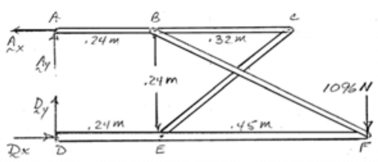


PROBLEM 6.105

Knowing that $P = 0$ and $Q = 1096$ N, determine for the frame and loading shown (*a*) the reaction at *D*, (*b*) the force in member *BF*.

SOLUTION

FBD Frame:



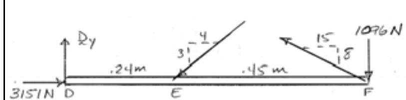
$$\circlearrowleft \Sigma M_A = 0: (0.24\text{ m})D_x - (0.69\text{ m})(1096\text{ N}) = 0 \qquad \mathbf{D}_x = 3151\text{ N} \longrightarrow$$

$$\circlearrowleft \Sigma M_D = 0: (0.69\text{ m})\frac{8}{17}F_{BF} - (0.69\text{ m})(1096\text{ N}) - (0.24\text{ m})\frac{3}{5}F_{EC} = 0$$

FBD member DF:

$$0.3247F_{BF} - 0.144F_{EC} = 756.24\text{ N}$$

$$\circlearrowleft \Sigma F_x = 0: 3151\text{ N} - \frac{4}{5}F_{EC} - \frac{15}{17}F_{BF} = 0$$



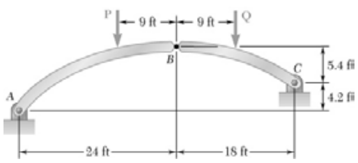
$$0.8824F_{BF} + 0.800F_{EC} = 3151\text{ N}$$

Solving: $F_{BF} = 2737$ N $\qquad F_{BF} = 2.74$ kN T ◀

$$\circlearrowleft \Sigma M_E = 0: (0.45\text{ m})\left[\frac{8}{17}(2737\text{ N}) - 1096\text{ N}\right] - (0.24\text{ m})D_y = 0$$

$\mathbf{D}_y = 360.06$ N ↑ $\qquad \mathbf{D} = 3.17$ kN $\measuredangle$ 6.52° ◀

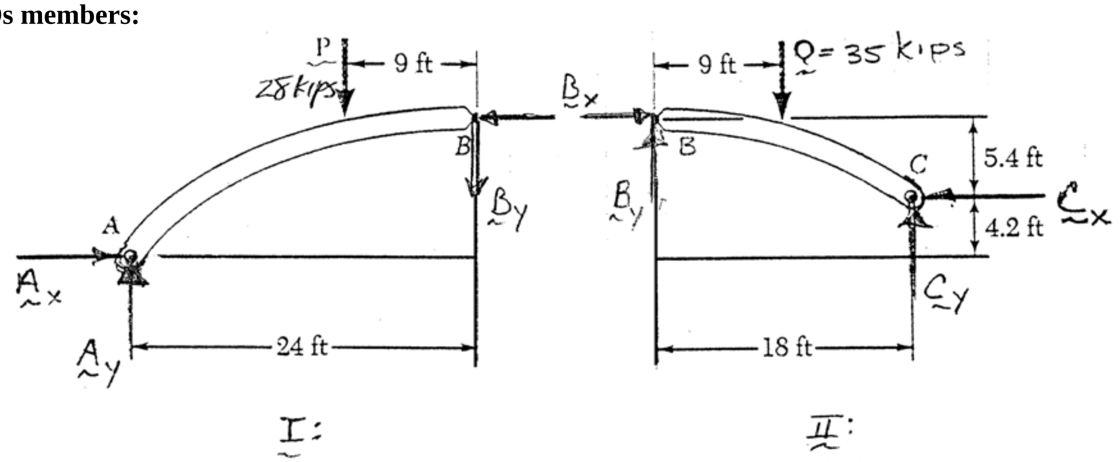
PROBLEM 6.106



The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 28$ kips and $Q = 35$ kips, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION

FBDs members:



From FBD I: $\Sigma M_A = 0$: $(9.6 \text{ ft})B_x - (24 \text{ ft})B_y - (15 \text{ ft})(28 \text{ kips}) = 0$

$$1.2B_x - 3.0B_y = 52.5 \text{ kips}$$

FBD II: $\Sigma M_C = 0$: $(5.4 \text{ ft})B_x + (18 \text{ ft})B_y - (9 \text{ ft})(35 \text{ kips}) = 0$

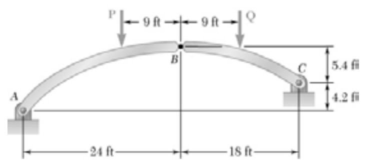
$$0.6B_x - 2B_y = 35 \text{ kips}$$

Solving: $B_x = 50$ kips; $B_y = 2.5$ kips as drawn, so on *AB*: $\mathbf{B}_x = 50.0 \text{ kips} \longleftarrow$ ◀

$\mathbf{B}_y = 2.50 \text{ kips} \downarrow$ ◀

FBD I: $\longrightarrow \Sigma F_x = 0$: $A_x - 50 \text{ kips} = 0$ $\mathbf{A}_x = 50.0 \text{ kips} \longrightarrow$ ◀

$\uparrow \Sigma F_x = 0$: $A_y - 28 \text{ kips} - 2.5 \text{ kips} = 0$ $\mathbf{A}_y = 30.5 \text{ kips} \uparrow$ ◀

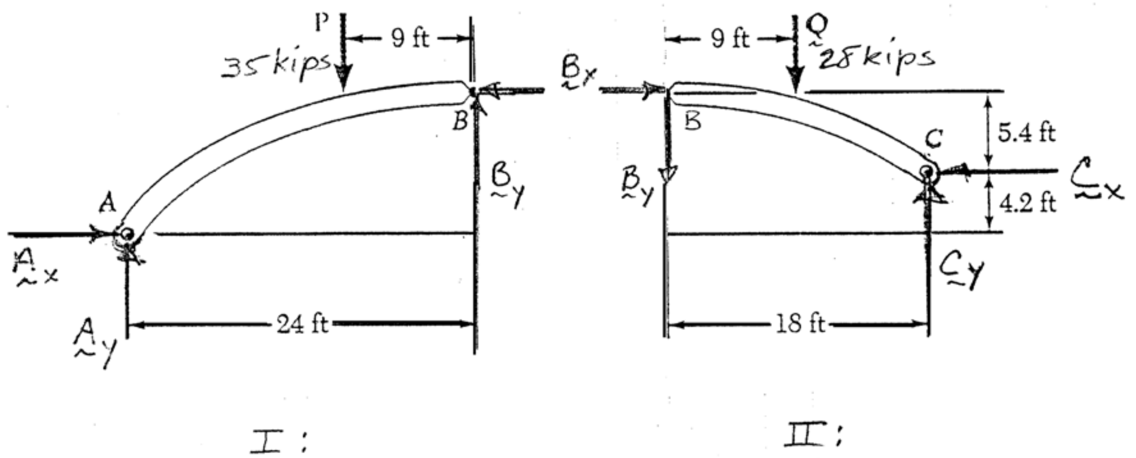


PROBLEM 6.107

The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 35\text{ kips}$ and $Q = 28\text{ kips}$, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION

member FBDs:



From FBD I: $\circlearrowleft\ \Sigma M_A = 0\text{: } (9.6\text{ ft})B_x + (24\text{ ft})B_y - (15\text{ ft})(35\text{ kips}) = 0$

$$3.2B_x + 8B_y = 175\text{ kips}$$

FBD I: $\circlearrowleft\ \Sigma M_C = 0\text{: } (5.4\text{ ft})B_x - (18\text{ ft})B_y - (9\text{ ft})(28\text{ kips}) = 0$

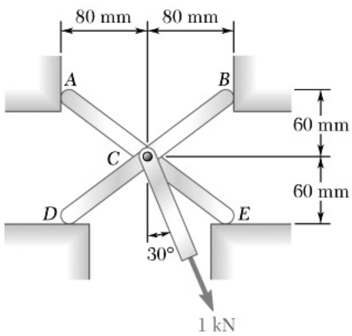
$$0.6B_x - 2B_y = 28\text{ kips}$$

Solving: $B_x = 51.25\text{ kips}$; $B_x = 1.375\text{ kips}$ as drawn, so on *AB*: $\mathbf{B}_x = 51.3\text{ kips} \longleftarrow$ ◀

$\mathbf{B}_y = 1.375\text{ kips} \uparrow$ ◀

FBD I: $\longrightarrow \Sigma F_x = 0\text{: } A_x - 51.25\text{ kips}$ $\mathbf{A}_x = 51.3\text{ kips} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0\text{: } A_y - 35\text{ kips} + 1.375\text{ kips}$ $\mathbf{A}_y = 33.6\text{ kips} \uparrow$ ◀

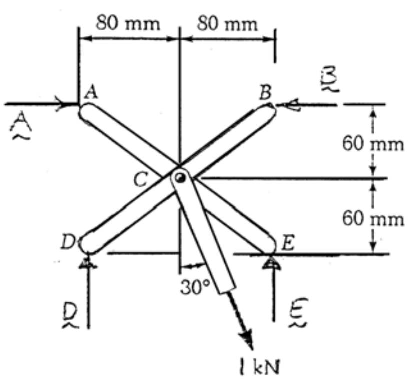


PROBLEM 6.108

For the frame and loading shown, determine the reactions at A, B, D, and E. Assume that the surface at each support is frictionless.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_A = 0: (0.16 \text{ m})E - (0.08 \text{ m})(1 \text{ kN})\cos 30°$$

$$-(0.06 \text{ m})(1 \text{ kN}) \sin 30° = 0$$

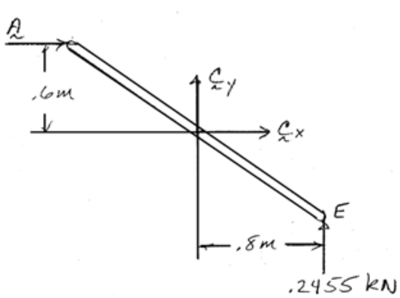
$E = 0.2455 \text{ kN}$ $\qquad$ $\mathbf{E} = 246 \text{ N} \uparrow$ ◀

$\uparrow \Sigma F_y = 0: D + 0.2455 \text{ kN} - (1 \text{ kN})\cos 30° = 0$ $\qquad$ $D = 0.6205 \text{ kN}$

$\mathbf{D} = 621 \text{ N} \uparrow$ ◀

$\longrightarrow \Sigma F_x = 0: A - B + (1 \text{ kN})\sin 30° = 0$ $\qquad$ $B = A + 0.5 \text{ kN}$

FBD member ACE:

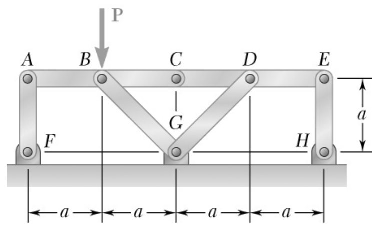


$\circlearrowleft \Sigma M_C = 0: (0.8 \text{ m})(0.2455 \text{ kN}) - (0.6 \text{ m})(A) = 0$ $\qquad$ $A = 0.3274 \text{ kN}$

$\mathbf{A} = 327 \text{ N} \longrightarrow$ ◀

From above $\qquad$ $B = A + 0.05 \text{ kN}$

$B = (0.327 + 0.50) \text{kN} = 0.827 \text{ kN}$ $\qquad$ $\mathbf{B} = 827 \text{ N} \longleftarrow$ ◀

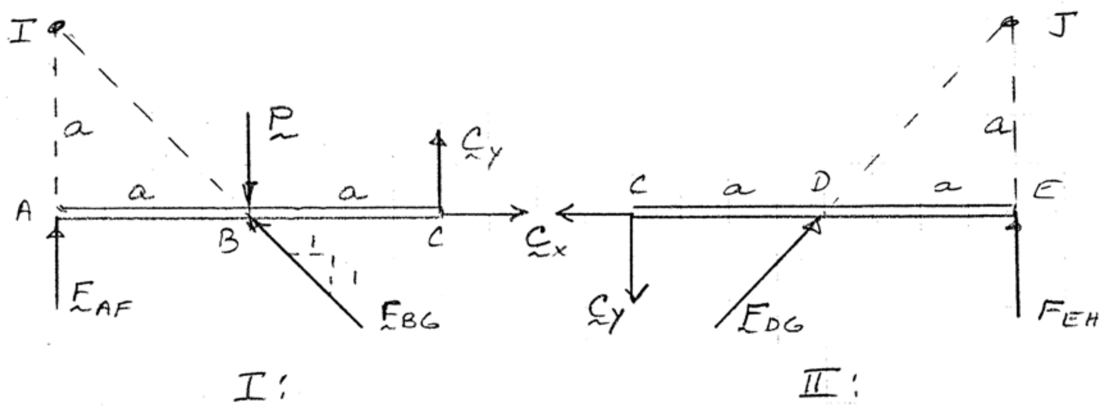


PROBLEM 6.109

Members *ABC* and *CDE* are pin-connected at *C* and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



FBD I: $\Sigma M_I = 0: 2aC_y + aC_x - aP = 0 \qquad 2C_y + C_x = P$

FBD II: $\Sigma M_J = 0: 2aC_y - aC_x = 0 \qquad 2C_y - C_x = 0$

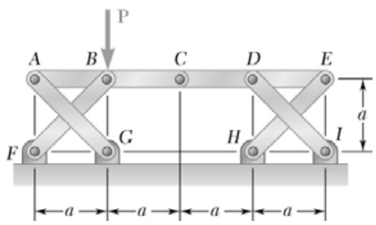
Solving: $C_x = \dfrac{P}{2};\ C_y = \dfrac{P}{4}$ as shown

FBD I: $\longrightarrow \Sigma F_x = 0: -\dfrac{1}{\sqrt{2}}F_{BG} + C_x = 0 \qquad F_{BG} = C_x\sqrt{2}$ $\qquad F_{BG} = \dfrac{\sqrt{2}}{2}P$ C ◀

$\uparrow \Sigma F_y = 0: F_{AF} - P + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + \dfrac{P}{4} = 0$ $\qquad F_{AF} = \dfrac{P}{4}$ C ◀

FBD II: $\longrightarrow \Sigma F_x = 0: -C_x + \dfrac{1}{\sqrt{2}}F_{DG} = 0 \qquad F_{DG} = C_x\sqrt{2}$ $\qquad F_{DG} = \dfrac{\sqrt{2}}{2}P$ C ◀

$\uparrow \Sigma F_y = 0: -C_y + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \qquad F_{EH} = \dfrac{P}{4} - \dfrac{P}{2} = -\dfrac{P}{4}$ $\qquad F_{EH} = \dfrac{P}{4}$ T ◀

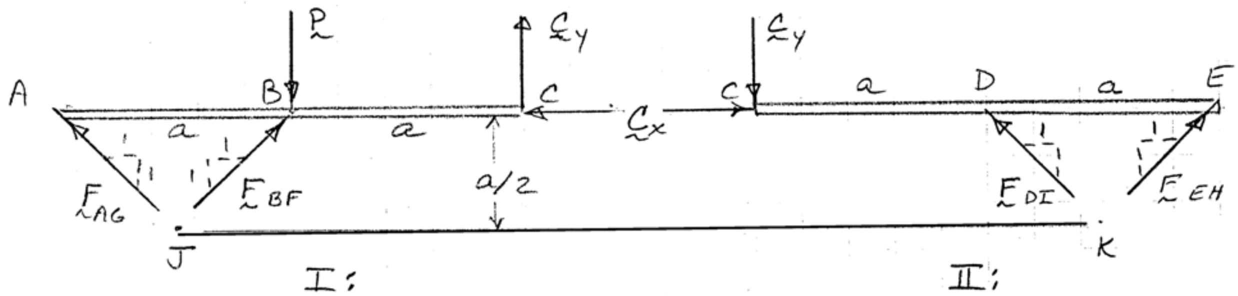


PROBLEM 6.110

Members *ABC* and *CDE* are pin-connected at *C* and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



From FBD I: $\circlearrowleft \Sigma M_J = 0\colon \frac{a}{2}C_x + \frac{3a}{2}C_y - \frac{a}{2}P = 0 \qquad C_x + 3C_y = P$

FBD II: $\circlearrowleft \Sigma M_K = 0\colon \frac{a}{2}C_x - \frac{3a}{2}C_y = 0 \qquad C_x - 3C_y = 0$

Solving : $C_x = \frac{P}{2};\ C_y = \frac{P}{6}$ as drawn

FBD I: $\circlearrowleft \Sigma M_B = 0\colon aC_y - a\frac{1}{\sqrt{2}}F_{AG} = 0 \qquad F_{AG} = \sqrt{2}C_y = \frac{\sqrt{2}}{6}P$

$F_{AG} = \frac{\sqrt{2}}{6}P$ C ◀

$\longrightarrow \Sigma F_x = 0\colon -\frac{1}{\sqrt{2}}F_{AG} + \frac{1}{\sqrt{2}}F_{BF} - C_x = 0 \qquad F_{BF} = F_{AG} + C_x\sqrt{2} = \frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

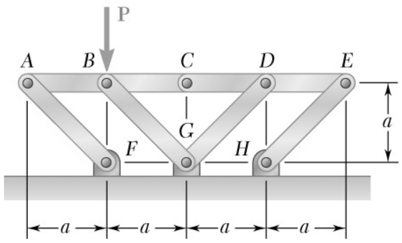
$F_{BF} = \frac{2\sqrt{2}}{3}P$ C ◀

FBD II: $\circlearrowleft \Sigma M_D = 0\colon a\frac{1}{\sqrt{2}}F_{EH} + aC_y = 0 \qquad F_{EH} = -\sqrt{2}C_y = -\frac{\sqrt{2}}{6}P$

$F_{EH} = \frac{\sqrt{2}}{6}P$ T ◀

$\longrightarrow \Sigma F_x = 0\colon C_x - \frac{1}{\sqrt{2}}F_{DI} + \frac{1}{\sqrt{2}}F_{EH} = 0 \qquad F_{DI} = F_{EH} + C_x\sqrt{2} = -\frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

$F_{DI} = \frac{\sqrt{2}}{3}P$ C ◀

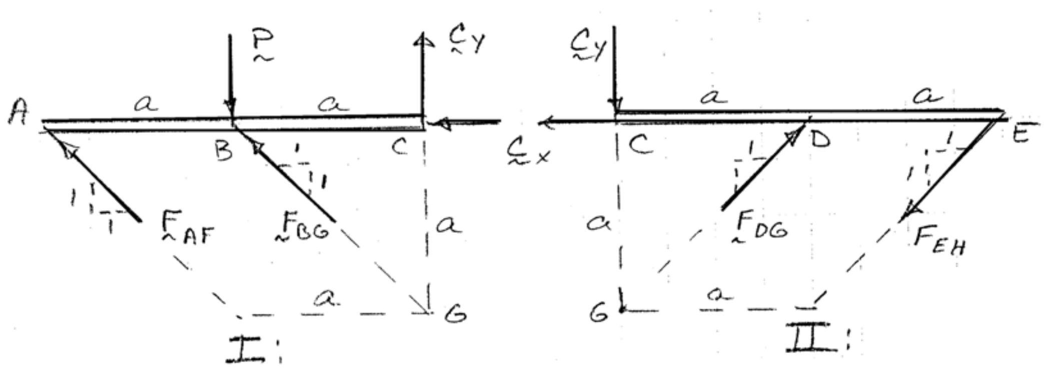


PROBLEM 6.111

Members ABC and CDE are pin-connected at C and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



FBD I: $\circlearrowright\ \Sigma M_B = 0\colon aC_y - a\dfrac{1}{\sqrt{2}}F_{AF} = 0 \qquad F_{AF} = \sqrt{2}C_y$

FBD II: $\circlearrowright\ M_D = 0\colon aC_y - a\dfrac{1}{\sqrt{2}}F_{EH} = 0 \qquad F_{EH} = \sqrt{2}C_y$

FBDs combined: $\circlearrowright\ \Sigma M_G = 0\colon aP - a\dfrac{1}{\sqrt{2}}F_{AF} - a\dfrac{1}{\sqrt{2}}F_{EH} = 0 \qquad P = \dfrac{1}{\sqrt{2}}\sqrt{2}C_y + \dfrac{1}{\sqrt{2}}\sqrt{2}C_y$

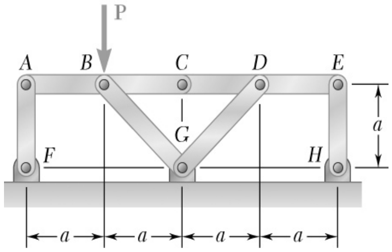
$$C_y = \frac{P}{2}$$

so $F_{AF} = \dfrac{\sqrt{2}}{2}P$ C ◀

$F_{EH} = \dfrac{\sqrt{2}}{2}P$ T ◀

FBD I: $\uparrow\ \Sigma F_y = 0\colon \dfrac{1}{\sqrt{2}}F_{AF} + \dfrac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \qquad \dfrac{P}{2} + \dfrac{1}{\sqrt{2}}F_{BG} - P + \dfrac{P}{2} = 0 \qquad F_{BG} = 0$ ◀

FBD II: $\uparrow\ \Sigma F_y = 0\colon -C_y + \dfrac{1}{\sqrt{2}}F_{DG} - \dfrac{1}{\sqrt{2}}F_{EH} = 0 \qquad -\dfrac{P}{2} + \dfrac{1}{\sqrt{2}}F_{DG} - \dfrac{P}{2} = 0 \qquad F_{DG} = \sqrt{2}\,P$ C ◀

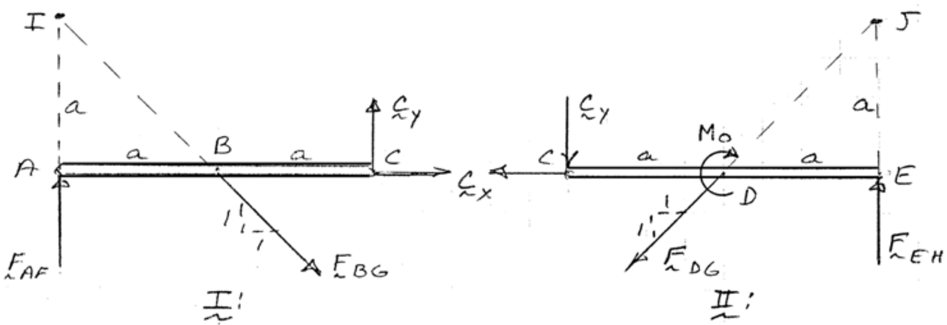


PROBLEM 6.112

Solve Prob. 6.109 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

SOLUTION

FBDs members:



FBD I: $\quad \circlearrowleft \Sigma M_A = 0: 2aC_y - a\dfrac{1}{\sqrt{2}}F_{BG} = 0 \qquad F_{BG} = 2\sqrt{2}C_y$

FBD II: $\quad \circlearrowleft \Sigma M_E = 0: 2aC_y - M_0 + a\dfrac{1}{\sqrt{2}}F_{DG} = 0 \qquad F_{DG} = -2\sqrt{2}C_y + \dfrac{\sqrt{2}}{a}M_0$

FBDs combined: $\quad \xrightarrow{} \Sigma F_x = 0: \dfrac{1}{\sqrt{2}}F_{BG} + C_x - C_x - \dfrac{1}{\sqrt{2}}F_{DG} = 0$

$$F_{BG} = F_{DG}: 2\sqrt{2}C_y = -2\sqrt{2}C_y + \frac{\sqrt{2}}{a}M_0 \qquad C_y = \frac{M_0}{4a}$$

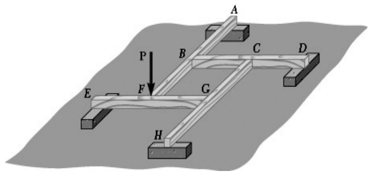
$$F_{BG} = 2\sqrt{2}\frac{M_0}{4a} \qquad\qquad F_{BG} = \frac{\sqrt{2}}{2}\frac{M_0}{a} \text{ T} \blacktriangleleft$$

$$F_{DG} = \frac{\sqrt{2}}{a}M_0 - 2\sqrt{2}\frac{M_0}{4a} \qquad\qquad F_{DG} = \frac{\sqrt{2}}{2}\frac{M_0}{a} \text{ T} \blacktriangleleft$$

FBD I: $\uparrow \Sigma F_y = 0: F_{AF} - \dfrac{1}{\sqrt{2}}F_{BG} + C_y = 0 \qquad F_{AF} = \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{2}\dfrac{M_0}{a} - \dfrac{M_0}{4a} \qquad F_{AF} = \dfrac{M_0}{4a} \text{ C} \blacktriangleleft$

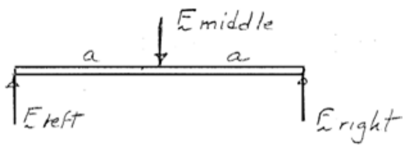
FBD II: $\circlearrowleft \Sigma H_D = 0: aC_y - M_0 + aF_{EH} = 0 \qquad aF_{EH} = M_0 - a\dfrac{M_0}{4a} \qquad F_{EH} = \dfrac{3}{4}\dfrac{M_0}{a} \text{ C} \blacktriangleleft$

PROBLEM 6.113



Four wooden beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.

SOLUTION



Note that, if we assume P is applied to EG, each individual member FBD looks like

so $$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$

Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

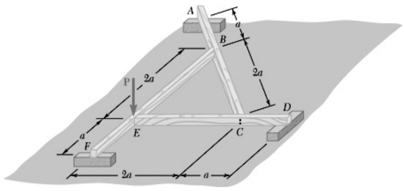
$$P + F = 2G(=4C = 8B = 16F) = 2E$$

From $$P + F = 16F, \quad F = \frac{P}{15}$$ so $$\mathbf{A} = \frac{P}{15}\uparrow \blacktriangleleft$$

$$\mathbf{D} = \frac{2P}{15}\uparrow \blacktriangleleft$$

$$\mathbf{H} = \frac{4P}{15}\uparrow \blacktriangleleft$$

$$\mathbf{E} = \frac{8P}{15}\uparrow \blacktriangleleft$$

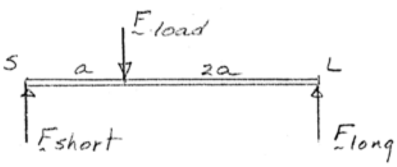


PROBLEM 6.114

Three wooden beams, each of length of $3a$, are nailed together to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, and F.

SOLUTION

Note that, if we assume P is applied to BF, each individual member FBD looks like:



so $$F_{\text{short}} = 2F_{\text{long}} = \frac{2}{3}F_{\text{load}}$$

(by moment equations about S and L).

Labeling each interaction force with the letter corresponding to the joint of application, we have:

$$F = \frac{2(P+E)}{3} = 2B$$

$$E = \frac{C}{3} = \frac{D}{2}$$

$$C = \frac{B}{3} = \frac{A}{2}$$

so $$\frac{2(P+E)}{3} = 2B = 6C = 18E \qquad P + E = 27E \qquad \mathbf{E} = \frac{P}{26}\uparrow$$

so $$\mathbf{D} = 2\mathbf{E} = \frac{P}{13}\uparrow \quad \blacktriangleleft$$

$$A = 2C = 3E \qquad \mathbf{A} = \frac{3P}{13}\uparrow \quad \blacktriangleleft$$

$$F = \frac{2}{3}\left(P + \frac{P}{26}\right) \qquad \mathbf{F} = \frac{9P}{13}\uparrow \quad \blacktriangleleft$$