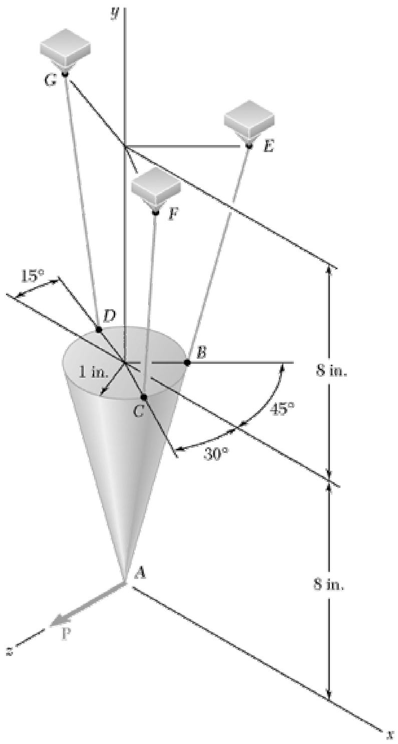


PROBLEM 2.119

A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 2.4 lb and that $P = 0$, determine the tension in each cord.

SOLUTION

Note that because the line of action of each of the cords passes through the vertex *A* of the cone, the cords all have the same length, and the unit vectors lying along the cords are parallel to the unit vectors lying along the generators of the cone.

Thus, for example, the unit vector along *BE* is identical to the unit vector along the generator *AB*.

Hence:

$$\boldsymbol{\lambda}_{AB} = \lambda_{BE} = \frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}$$

It follows that:

$$\mathbf{T}_{BE} = T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\left(\frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{CF} = T_{CF}\boldsymbol{\lambda}_{CF} = T_{CF}\left(\frac{\cos 30°\mathbf{i} + 8\mathbf{j} + \sin 30°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{DG} = T_{DG}\boldsymbol{\lambda}_{DG} = T_{DG}\left(\frac{-\cos 15°\mathbf{i} + 8\mathbf{j} - \sin 15°\mathbf{k}}{\sqrt{65}}\right)$$

At *A*: $\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{BE} + \mathbf{T}_{CF} + \mathbf{T}_{DG} + \mathbf{W} + \mathbf{P} = 0$

PROBLEM 2.119 CONTINUED

Then, isolating the factors if **i**, **j**, and **k** we obtain three algebraic equations:

$$\mathbf{i}: \quad \frac{T_{BE}}{\sqrt{65}}\cos 45^\circ + \frac{T_{CF}}{\sqrt{65}}\cos 30^\circ - \frac{T_{DG}}{\sqrt{65}}\cos 15^\circ = 0$$

or

$$T_{BE}\cos 45^\circ + T_{CF}\cos 30^\circ - T_{DG}\cos 15^\circ = 0 \qquad (1)$$

$$\mathbf{j}: \quad T_{BE}\frac{8}{\sqrt{65}} + T_{CF}\frac{8}{\sqrt{65}} + T_{DG}\frac{8}{\sqrt{65}} - W = 0$$

or

$$T_{BE} + T_{CF} + T_{DG} = \frac{2.4}{8}\sqrt{65} = 0.3\sqrt{65} \qquad (2)$$

$$\mathbf{k}: \quad -\frac{T_{BE}}{\sqrt{65}}\sin 45^\circ + \frac{T_{CF}}{\sqrt{65}}\sin 30^\circ - \frac{T_{DG}}{\sqrt{65}}\sin 15^\circ - P = 0$$

or

$$-T_{BE}\sin 45^\circ + T_{CF}\sin 30^\circ - T_{DG}\sin 15^\circ = P\sqrt{65} \qquad (3)$$

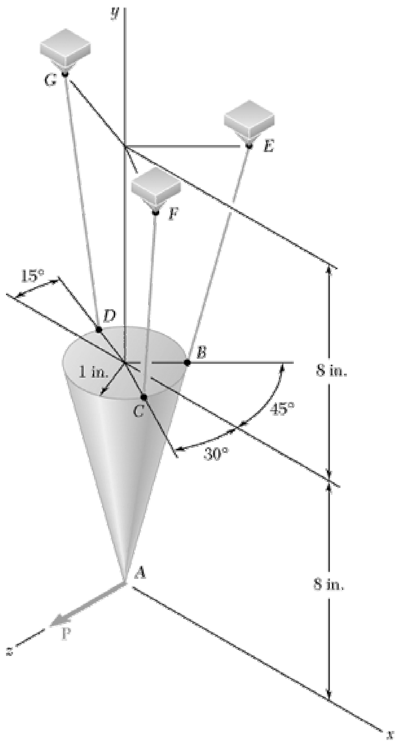
With $P = 0$, the tension in the cords can be found by solving the resulting Equations (1), (2), and (3) using conventional methods in Linear Algebra (elimination, matrix methods or iteration–with MATLAB or Maple, for example). We obtain

$T_{BE} = 0.299\text{ lb}$ ◀

$T_{CF} = 1.002\text{ lb}$ ◀

$T_{DG} = 1.117\text{ lb}$ ◀

PROBLEM 2.120



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 2.4 lb and that $P = 0.1$ lb, determine the tension in each cord.

SOLUTION

See Problem 2.121 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° = 0 \quad (1)$$

$$T_{BE} + T_{CF} + T_{DG} = 0.3\sqrt{65} \quad (2)$$

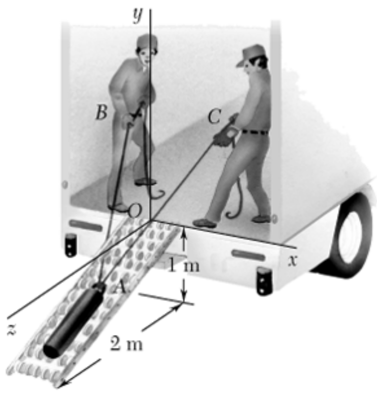
$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = P\sqrt{65} \quad (3)$$

With $P = 0.1\,\text{lb}$, solving (1), (2), and (3), using conventional methods in Linear Algebra (elimination, matrix methods or iteration–with MATLAB or Maple, for example), we obtain

$T_{BE} = 1.006\,\text{lb}$ ◀

$T_{CF} = 0.357\,\text{lb}$ ◀

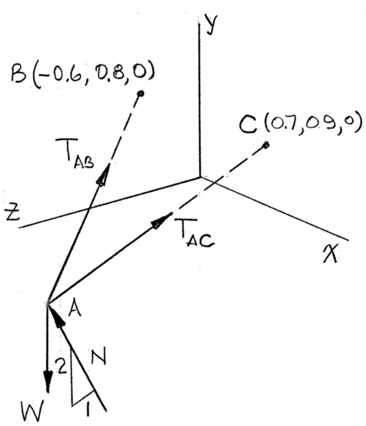
$T_{DG} = 1.056\,\text{lb}$ ◀



PROBLEM 2.121

Using two ropes and a roller chute, two workers are unloading a 200-kg cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of points *A*, *B*, and *C* are, respectively, *A*(0, –0.5 m, 1 m), *B*(–0.6 m, 0.8 m, 0), and *C*(0.7 m, 0.9 m, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION



From the geometry of the chute:

$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k}) = N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

As in Problem 2.11, for example, the force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = -(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.6\text{ m})^2 + (1.3\text{ m})^2 + (1\text{ m})^2} = 1.764\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{1.764\text{ m}}\left[-(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.3436\mathbf{i} + 0.7444\mathbf{j} + 0.5726\mathbf{k})$$

and $\overrightarrow{AC} = (0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$

$$AC = \sqrt{(0.7\text{ m})^2 + (1.4\text{ m})^2 + (-1\text{ m})^2} = 1.8574\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{1.764\text{ m}}\left[(0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.3769\mathbf{i} + 0.7537\mathbf{j} - 0.5384\mathbf{k})$$

Then: $\Sigma\mathbf{F} = 0\text{:}\quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{W} = 0$

PROBLEM 2.121 CONTINUED

With $W = (200 \text{ kg})(9.81 \text{ m/s}) = 1962 \text{ N}$, and equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -0.3436T_{AB} + 0.3769T_{AC} = 0 \qquad (1)$$

$$\mathbf{j}: \quad 0.7444T_{AB} + 0.7537T_{AC} + 0.8944N - 1962 = 0 \qquad (2)$$

$$\mathbf{k}: \quad -0.5726T_{AB} - 0.5384T_{AC} + 0.4472N = 0 \qquad (3)$$

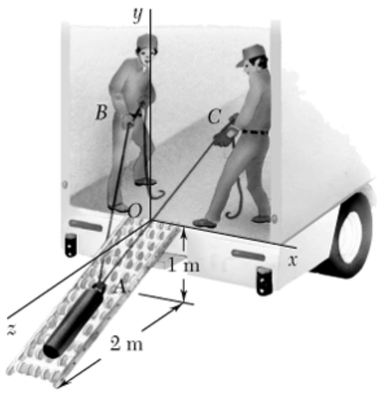
Using conventional methods for solving Linear Algebraic Equations (elimination, MATLAB or Maple, for example), we obtain

$$N = 1311 \text{ N}$$

$$T_{AB} = 551 \text{ N} \blacktriangleleft$$

$$T_{AC} = 503 \text{ N} \blacktriangleleft$$

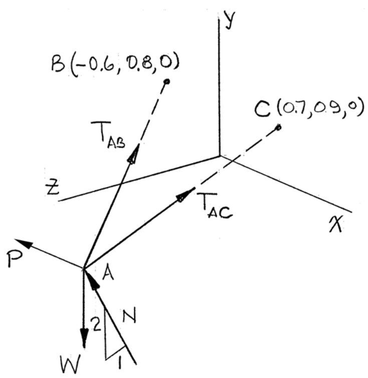
PROBLEM 2.122



Solve Problem 2.121 assuming that a third worker is exerting a force $\mathbf{P} = -(180\text{ N})\mathbf{i}$ on the counterweight.

Problem 2.121: Using two ropes and a roller chute, two workers are unloading a 200-kg cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of points *A*, *B*, and *C* are, respectively, *A*(0, –0.5 m, 1 m), *B*(–0.6 m, 0.8 m, 0), and *C*(0.7 m, 0.9 m, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION



From the geometry of the chute:

$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k}) = N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

As in Problem 2.11, for example, the force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = -(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.6\text{ m})^2 + (1.3\text{ m})^2 + (1\text{ m})^2} = 1.764\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{1.764\text{ m}}\left[-(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.3436\mathbf{i} + 0.7444\mathbf{j} + 0.5726\mathbf{k})$$

and $$\overrightarrow{AC} = (0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0.7\text{ m})^2 + (1.4\text{ m})^2 + (-1\text{ m})^2} = 1.8574\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{1.764\text{ m}}\left[(0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.3769\mathbf{i} + 0.7537\mathbf{j} - 0.5384\mathbf{k})$$

Then: $$\Sigma\mathbf{F} = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

PROBLEM 2.122 CONTINUED

Where $\mathbf{P} = -(180\text{ N})\mathbf{i}$

and $\mathbf{W} = -\left[(200\text{ kg})(9.81\text{ m/s}^2)\right]\mathbf{j}$

$$= -(1962\text{ N})\mathbf{j}$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear equations:

$$\mathbf{i}:\quad -0.3436T_{AB} + 0.3769T_{AC} - 180 = 0$$

$$\mathbf{j}:\quad 0.8944N + 0.7444T_{AB} + 0.7537T_{AC} - 1962 = 0$$

$$\mathbf{k}:\quad 0.4472N - 0.5726T_{AB} - 0.5384T_{AC} = 0$$

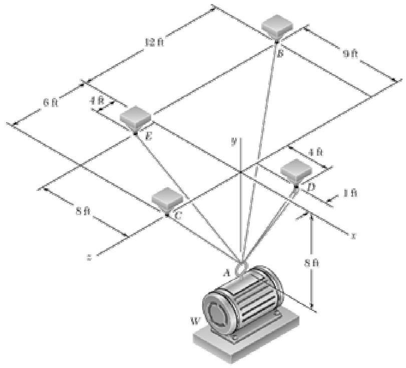
Using conventional methods for solving Linear Algebraic Equations (elimination, MATLAB or Maple, for example), we obtain

$$N = 1302\text{ N}$$

$$T_{AB} = 306\text{ N} \blacktriangleleft$$

$$T_{AC} = 756\text{ N} \blacktriangleleft$$

PROBLEM 2.123



A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that $W = 320$ lb, determine the tension in each cable. (*Hint:* The tension is the same in all portions of cable ADE.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(9\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}$$

$$AB = \sqrt{(-9\text{ ft})^2 + (8\text{ ft})^2 + (-12\text{ ft})^2} = 17\text{ ft}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{17\text{ ft}}\left[-(9\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.5294\mathbf{i} + 0.4706\mathbf{j} - 0.7059\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (8\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}$$

$$AC = \sqrt{(0\text{ ft})^2 + (8\text{ ft})^2 + (6\text{ ft})^2} = 10\text{ ft}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{10\text{ ft}}\left[(0\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (4\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (1\text{ ft})\mathbf{k}$$

$$AD = \sqrt{(4\text{ ft})^2 + (8\text{ ft})^2 + (-1\text{ ft})^2} = 9\text{ ft}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{ADE}\frac{\overrightarrow{AD}}{AD} = \frac{T_{ADE}}{9\text{ ft}}\left[(4\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (1\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{ADE}(0.4444\mathbf{i} + 0.8889\mathbf{j} - 0.1111\mathbf{k})$$

PROBLEM 2.123 CONTINUED

Finally,

$$\overrightarrow{AE} = (-8\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} + (4\text{ ft})\mathbf{k}$$

$$AE = \sqrt{(-8\text{ ft})^2 + (8\text{ ft})^2 + (4\text{ ft})^2} = 12\text{ ft}$$

$$\mathbf{T}_{AE} = T\boldsymbol{\lambda}_{AE} = T_{ADE}\frac{\overrightarrow{AE}}{AE} = \frac{T_{ADE}}{12\text{ ft}}\left[(-8\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} + (4\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AE} = T_{ADE}(-0.6667\mathbf{i} + 0.6667\mathbf{j} + 0.3333\mathbf{k})$$

With the weight of the machinery, $\mathbf{W} = -W\mathbf{j}$, at A, we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + 2\mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero, we obtain the following linear algebraic equations:

$$-0.5294T_{AB} + 2(0.4444T_{ADE}) - 0.6667T_{ADE} = 0 \qquad (1)$$

$$0.4706T_{AB} + 0.8T_{AC} + 2(0.8889T_{ADE}) + 0.6667T_{ADE} - W = 0 \qquad (2)$$

$$-0.7059T_{AB} + 0.6T_{AC} - 2(0.1111T_{ADE}) + 0.3333T_{ADE} = 0 \qquad (3)$$

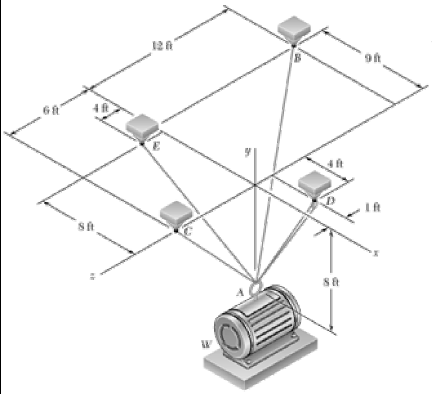
Knowing that $W = 320\text{ lb}$, we can solve Equations (1), (2) and (3) using conventional methods for solving Linear Algebraic Equations (elimination, matrix methods via MATLAB or Maple, for example) to obtain

$T_{AB} = 46.5\text{ lb}$ ◀

$T_{AC} = 34.2\text{ lb}$ ◀

$T_{ADE} = 110.8\text{ lb}$ ◀

PROBLEM 2.124



A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that the tension in cable AB is 68 lb, determine (*a*) the tension in AC, (*b*) the tension in ADE, (*c*) the weight W. (*Hint:* The tension is the same in all portions of cable ADE.)

SOLUTION

See Problem 2.123 for the analysis leading to the linear algebraic Equations (1), (2), and (3), below:

$$-0.5294T_{AB} + 2\left(0.4444T_{ADE}\right) - 0.6667T_{ADE} = 0 \tag{1}$$

$$0.4706T_{AB} + 0.8T_{AC} + 2\left(0.8889T_{ADE}\right) + 0.6667T_{ADE} - W = 0 \tag{2}$$

$$-0.7059T_{AB} + 0.6T_{AC} - 2\left(0.1111T_{ADE}\right) + 0.3333T_{ADE} = 0 \tag{3}$$

Knowing that the tension in cable AB is 68 lb, we can solve Equations (1), (2) and (3) using conventional methods for solving Linear Algebraic Equations (elimination, matrix methods via MATLAB or Maple, for example) to obtain

(*a*) $T_{AC} = 50.0 \text{ lb}$ ◀

(*b*) $T_{AE} = 162.0 \text{ lb}$ ◀

(*c*) $W = 468 \text{ lb}$ ◀