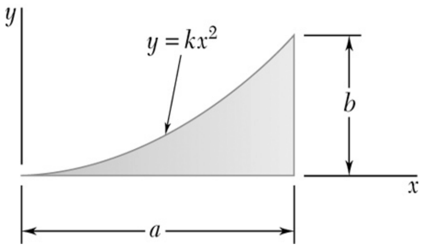
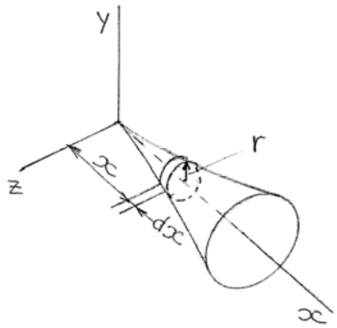



## PROBLEM 9.121

The parabolic spandrel shown is revolved about the $x$ axis to form a homogeneous solid of revolution of mass $m$. Using direct integration, express the moment of inertia of the solid with respect to the $x$ axis in terms of $m$ and $b$.

## SOLUTION




At $\quad x = a,\ y = b: \quad b = ka^2 \quad$ or $\quad k = \dfrac{b}{a^2}$

Then $\quad y = \dfrac{b}{a^2}x^2$

Now $\quad dm = \rho\left(\pi r^2\right)dx$

$$= \pi\rho\left(\frac{b}{a^2}x^2\right)^2 dx$$

Then $\quad m = \pi\rho\dfrac{b^2}{a^4}\int_0^a x^4 dx$

$$= \frac{1}{5}\pi\rho\frac{b^2}{a^4}x^5\Big|_0^a$$

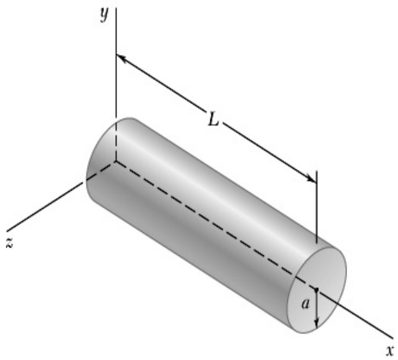
$$= \frac{1}{5}\pi\rho ab^2 \qquad \text{or} \qquad \pi\rho = \frac{5m}{ab^2}$$

Now $\quad d\bar{I}_x = \left(\dfrac{1}{2}r^2\right)dm = \dfrac{1}{2}\left(\dfrac{b}{a^2}x^2\right)^2\left[\pi\rho\left(\dfrac{b}{a^2}x^2\right)^2 dx\right]$

$$= \frac{5m}{ab^2} \times \frac{1}{2}\frac{b^2}{a^4}x^4 \times \frac{b^2}{a^4}x^4 dx = \frac{5}{2}m\frac{b^2}{a^9}x^8 dx$$

Then.. $\quad \bar{I}_x = \dfrac{5}{2}m\dfrac{b^2}{a^9}\int_0^a x^8 dx = \dfrac{5}{2}m\dfrac{b^2}{a^9} \times \dfrac{1}{9}x^9\Big|_0^a$

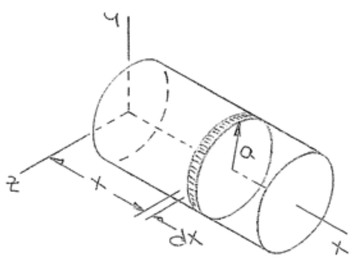
or $\bar{I}_x = \dfrac{5}{18}mb^2$ ◀




## PROBLEM 9.122

Determine by direct integration the moment of inertia with respect to the $z$ axis of the right circular cylinder shown assuming that it has a uniform density and a mass $m$.

## SOLUTION




For the cylinder $m = \rho V = \rho\pi a^2 L$

For the element shown $dm = \rho\pi a^2 dx$

$$= \frac{m}{L}dx$$

and $dI_z = d\bar{I}_z + x^2 dm$

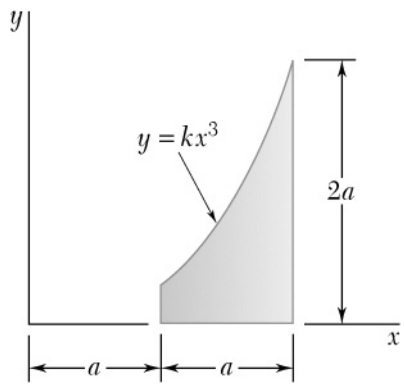
$$= \frac{1}{4}a^2 dm + x^2 dm$$

Then $I_z = \int dI_z = \int_0^L \left(\frac{1}{4}a^2 + x^2\right)\left(\frac{m}{L}dx\right) = \frac{m}{L}\left[\frac{1}{4}a^2 x + \frac{1}{3}x^3\right]_0^L$

$$= \frac{m}{L}\left(\frac{1}{4}a^2 L + \frac{1}{3}L^3\right)$$

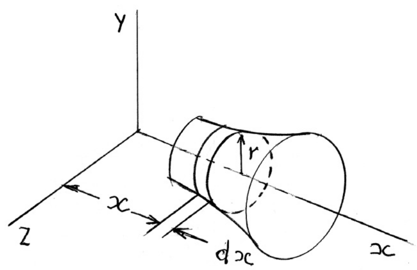
or $I_z = \frac{1}{12}m\left(3a^2 + 4L^2\right)$ ◀

## PROBLEM 9.123




The area shown is revolved about the $x$ axis to form a homogeneous solid of revolution of mass $m$. Determine by direct integration the moment of inertia of the solid with respect to ($a$) the $x$ axis, ($b$) the $y$ axis. Express your answers in terms of $m$ and $a$.

## SOLUTION




At $\quad x = 2a \qquad 2a = k(2a)^3 \qquad$ or $\qquad k = \dfrac{1}{4a^2}$

Then $$y = \frac{1}{4a^2}x^3$$

Now $$dm = \rho\left(\pi r^2 dx\right)$$

$$= \pi\rho\left(\frac{1}{4a^2}x^3\right)^2 dx = \frac{\pi\rho}{16a^4}x^6 dx$$

Then $$m = \frac{\pi\rho}{16a^4}\int_a^{2a} x^6 dx$$

$$= \frac{\pi\rho}{16a^4}\frac{1}{7}x^7\Big|_a^{2a} = \frac{\pi\rho}{112a^4}\left[(2a)^7 - (a)^7\right] = \frac{127}{112}\pi\rho a^3$$

or $\pi\rho = \dfrac{112m}{127a^3}$

($a$) Now $$d\bar{I}_x = \left(\frac{1}{2}r^2\right)dm = \frac{1}{2}\left(\frac{1}{4a^2}x^3\right)^2\left(\frac{\pi\rho}{16a^4}x^6 dx\right)$$

$$= \frac{1}{32a^4}x^6 \times \frac{112m}{127a^3} \times \frac{x^6}{16a^4}dx = \frac{7m}{4064a^{11}}x^{12}dx$$

Then

$$\bar{I}_x = \frac{7m}{4064a^{11}}\int_a^{2a} x^{12}dx = \frac{7m}{4064a^{11}}\frac{1}{13}x^{13}\Big|_a^{2a}$$

$$= \frac{7m}{52832a^{11}}\left[(2a)^{13} - (a)^{13}\right] = \frac{57337}{52832}ma^2 = 1.0853ma^2$$

or $\bar{I}_x = 1.085ma^2$ ◀

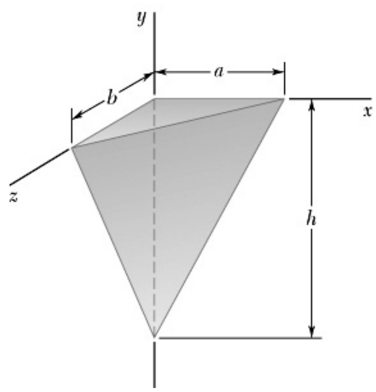
## PROBLEM 9.123 CONTINUED

Have
$$d\bar{I}_y = \left(\frac{1}{4}r^2 + x^2\right)dm = \left[\frac{1}{4}\left(\frac{1}{4a^2}x^3\right)^2 + x^2\right]\frac{\pi\rho}{16a^4}x^6dx$$
$$= \frac{1}{16a^4} \times \frac{112m}{127a^3}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx$$

Then
$$\bar{I}_y = \frac{7m}{127a^7}\int_a^{2a}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx = \frac{7m}{127a^7}\left(\frac{1}{832a^4}x^{13} + \frac{1}{9}x^8\right)\Bigg|_a^{2a}$$
$$= \frac{7m}{127a^7}\left[\frac{1}{832a^4}(2a)^{13} + \frac{1}{9}(2a)^9 - \frac{1}{832a^4}(a)^{13} - \frac{1}{9}(a)^9\right]$$
$$= \frac{7m}{127a^7}\left(\frac{8191}{832}a^9 + \frac{511}{9}a^9\right) = 3.67211ma^2$$

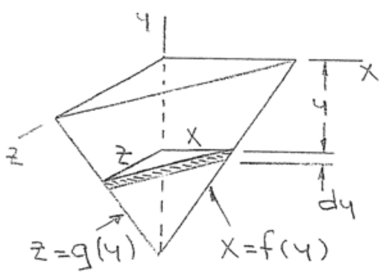
or $\bar{I}_y = 3.67ma^2$ ◀

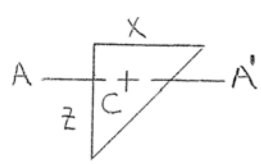
## PROBLEM 9.124




Determine by direct integration the moment of inertia with respect to the $x$ axis of the tetrahedron shown assuming that it has a uniform density and a mass $m$.

## SOLUTION




Have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$

For the element shown $$dm = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

Then $$m = \int dm = \int_{-h}^{0}\frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$

Now, for the element $$I_{AA',\text{area}} = \frac{1}{36}xz^3 = \frac{1}{36}ab^3\left(1 + \frac{y}{h}\right)^4$$

Then $$dI_{AA',\text{ mass}} = \rho t I_{AA',\text{ area}} = \rho(dy)\left[\frac{1}{3}ab^3\left(1 + \frac{y}{h}\right)^4\right]$$

## PROBLEM 9.124 CONTINUED

Now

$$dI_x = dI_{AA',\text{mass}} + \left[y^2 + \left(\frac{1}{3}z\right)^2\right]dm$$

$$= \frac{1}{36}\rho ab^3\left(1+\frac{y}{h}\right)^4 dy$$

$$+\left\{y^2 + \left[\frac{1}{3}b\left(1+\frac{y}{h}\right)\right]^2\right\}\left[\frac{1}{2}\rho ab\left(1+\frac{y}{h}\right)^2 dy\right]$$

$$= \frac{1}{12}\rho ab^3\left(1+\frac{y}{h}\right)^4 dy + \frac{1}{2}\rho ab\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)dy$$

Now $$m = \frac{1}{6}\rho abh$$

Then $$dI_x = \left[\frac{1}{2}m\frac{b^2}{h}\left(1+\frac{y}{h}\right)^4 + \frac{3m}{h}\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

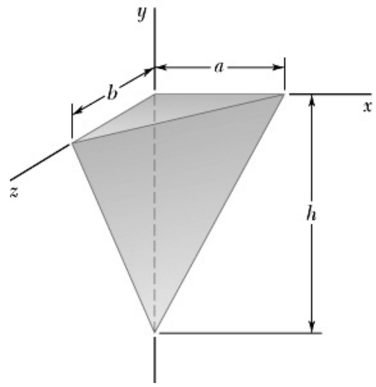
and

$$I_x = \int dI_x = \int_{-h}^{0}\frac{m}{2h}\left[b^2\left(1+\frac{y}{h}\right)^4 + 6\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

$$= \frac{m}{2h}\left[b^2 \times \frac{h}{5}\left(1+\frac{y}{h}\right)^5 + 6\left(\frac{1}{3}y^3 + \frac{1}{2}\frac{y^4}{h} + \frac{y^5}{5h^2}\right)\right]_{-h}^{0}$$

$$= \frac{m}{2h}\left\{\frac{1}{5}b^2h(1)^5 - 6\left[\frac{1}{3}(-h)^3 + \frac{1}{2h}(-h)^4 + \frac{1}{5h^2}(-h)^5\right]\right\}$$

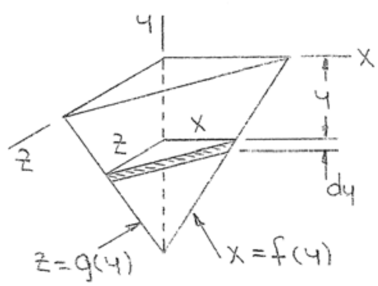
or $$I_x = \frac{1}{10}m\left(b^2 + h^2\right) \blacktriangleleft$$

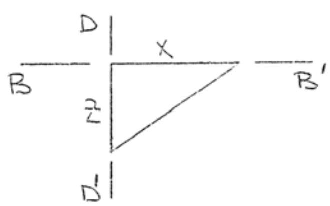



## PROBLEM 9.125

Determine by direct integration the moment of inertia with respect to the *y* axis of the tetrahedron shown assuming that it has a uniform density and a mass *m*.

## SOLUTION




Have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$

For the element shown $$dm = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

Then $$m = \int dm = \int_{-h}^{0} \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$

Also $$I_{BB',\text{area}} = \frac{1}{12}xz^3 \qquad I_{DD',\text{area}} = \frac{1}{12}zx^3$$

Then, using $I_{\text{mass}} = \rho t I_{\text{area}}$ have

$$dI_{BB',\text{mass}} = \rho(dy)\left(\frac{1}{12}xz^3\right) \qquad dI_{DD',\text{mass}} = \rho(dy)\left(\frac{1}{12}zx^3\right)$$

## PROBLEM 9.125 CONTINUED

Now

$$dI_y = dI_{BB',\text{mass}} + dI_{DD',\text{mass}}$$

$$= \frac{1}{12}\rho xz\left(x^2 + z^2\right)dy$$

$$= \frac{1}{12}\rho ab\left(1 + \frac{y}{h}\right)^2\left[\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^2\right]dy$$

Have $$m = \frac{1}{6}\rho abh \Rightarrow dI_y = \frac{m}{2h}\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^4 dy$$

Then

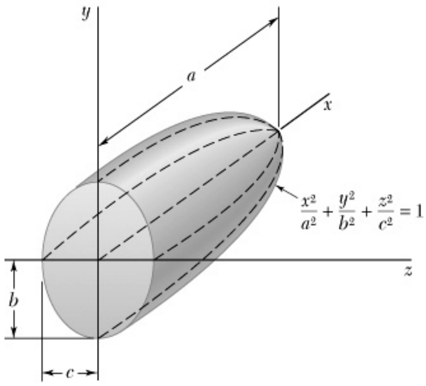
$$I_y = \int dI_y = \int_{-h}^{0}\frac{m}{2h}\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^4 dy$$

$$= \frac{m}{2h}\left(a^2 + b^2\right)\times\frac{h}{5}\left[\left(1 + \frac{y}{h}\right)^5\right]_{-h}^{0}$$

$$= \frac{m}{10}\left(a^2 + b^2\right)\left[(1)^5 - (1-1)^5\right]$$

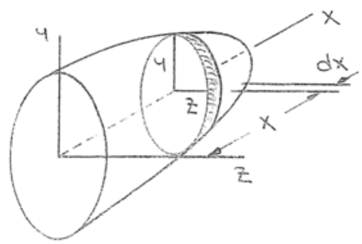
or $I_y = \frac{1}{10}m\left(a^2 + b^2\right)$ ◀

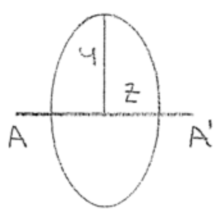
## PROBLEM 9.126




Determine by direct integration the moment of inertia with respect to the $z$ axis of the semiellipsoid shown assuming that it has a uniform density and a mass $m$.

## SOLUTION




First note that when

$$z = 0: \qquad y = b\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}$$

$$y = 0: \qquad z = c\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}$$

For the element shown $\qquad dm = \rho(\pi yzdx) = \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$

Then $\qquad m = \int dm = \int_0^a \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$

$$= \pi\rho bc\left[x - \frac{1}{3a^2}x^3\right]_0^a = \frac{2}{3}\pi\rho abc$$

For the element $\qquad I_{AA',\text{area}} = \frac{\pi}{4}zy^3$

Then $\qquad dI_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho(dx)\left(\frac{\pi}{4}zy^3\right)$

Now $\qquad dI_z = dI_{AA',\text{mass}} + x^2dm$

$$= \frac{\pi}{4}\rho b^3c\left(1 - \frac{x^2}{a^2}\right)^2 dx + x^2\left[\pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx\right]$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

## PROBLEM 9.126 CONTINUED

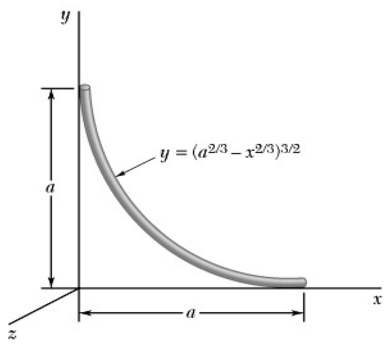
Finally $\quad I_z = \int dI_z$

$$= \frac{3m}{2a}\int_0^a \left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(x - \frac{2}{3}\frac{x^3}{a^2} + \frac{1}{5}\frac{x^5}{a^4}\right) + \left(\frac{1}{3}x^3 - \frac{1}{5}\frac{x^5}{a^2}\right)\right]_0^a$$

$$= \frac{3}{2}m\left[\frac{b^2}{4}\left(1 - \frac{2}{3} + \frac{1}{5}\right) + a^2\left(\frac{1}{3} - \frac{1}{5}\right)\right]$$

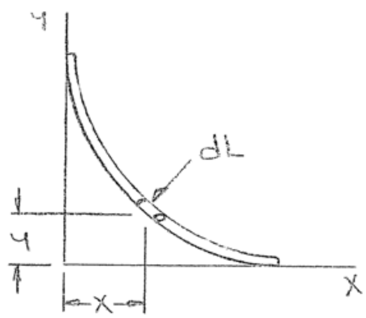
or $I_z = \frac{1}{5}m\left(a^2 + b^2\right)$ ◀




## PROBLEM 9.127

A thin steel wire is bent into the shape shown. Denoting the mass per unit length of the wire by $m'$, determine by direct integration the moment of inertia of the wire with respect to each of the coordinate axes.

## SOLUTION




First note $$\frac{dy}{dx} = -x^{-\frac{1}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

Then $$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-\frac{2}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)$$

$$= \left(\frac{a}{x}\right)^{\frac{2}{3}}$$

For the element shown $$dm = m'dL = m'\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$= m'\left(\frac{a}{x}\right)^{\frac{1}{3}} dx$$

Then $$m = \int dm = \int_0^a m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx = \frac{3}{2}m'a^{\frac{1}{3}}\left[x^{\frac{2}{3}}\right]_0^a = \frac{3}{2}m'a$$

Now $$I_x = \int y^2 dm = \int_0^a\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^3\left(m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx\right)$$

$$= m'a^{\frac{1}{3}}\int_0^a\left(\frac{a^2}{x^{\frac{1}{3}}} - 3a^{\frac{4}{3}}x^{\frac{1}{3}} + 3a^{\frac{2}{3}}x - x^{\frac{5}{3}}\right)dx$$

$$= m'a^{\frac{1}{3}}\left[\frac{3}{2}a^2x^{\frac{2}{3}} - \frac{9}{4}a^{\frac{4}{3}}x^{\frac{4}{3}} + \frac{3}{2}a^{\frac{2}{3}}x^2 - \frac{3}{8}x^{\frac{8}{3}}\right]_0^a$$

$$= m'a^3\left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8}\right) = \frac{3}{8}m'a^3$$

or $I_x = \frac{1}{4}ma^2$ ◀

Symmetry implies $I_y = \frac{1}{4}ma^2$ ◀

## PROBLEM 9.127 CONTINUED

Alternative Solution

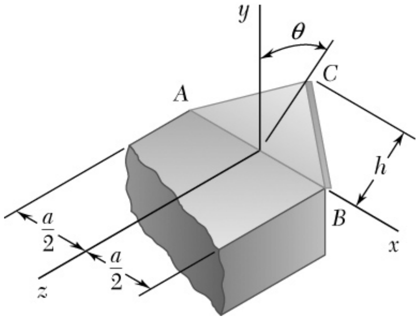
$$I_y = \int x^2 dm = \int_0^a x^2 \left( m' \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} dx \right) = m'a^{\frac{1}{3}} \int_0^a x^{\frac{5}{3}} dx$$

$$= m'a^{\frac{1}{3}} \times \frac{3}{8}\left[ x^{\frac{8}{3}} \right]_0^a = \frac{3}{8} m'a^3$$

$$= \frac{1}{4} ma^2$$

Also $$I_z = \int \left( x^2 + y^2 \right) dm = I_y + I_x$$

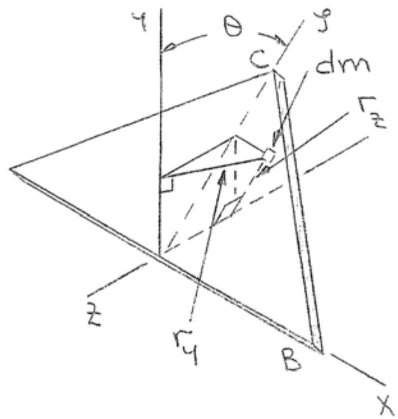
or $I_z = \frac{1}{2} ma^2$ ◀




## PROBLEM 9.128

A thin triangular plate of mass $m$ is welded along its base $AB$ to a block as shown. Knowing that the plate forms an angle $\theta$ with the $y$ axis, determine by direct integration the mass moment of inertia of the plate with respect to ($a$) the $x$ axis, ($b$) the $y$ axis, ($c$) the $z$ axis.

## SOLUTION




For line $BC$

$$\zeta = -\frac{h}{\frac{a}{2}}x + h$$

$$= \frac{h}{a}(a - 2x)$$

Also

$$m = \rho V = \rho t\left(\frac{1}{2}ah\right)$$

$$= \frac{1}{2}\rho tah$$

($a$) Have

$$dI_x = \frac{1}{12}\zeta^2 dm' + \left(\frac{\zeta}{2}\right)^2 dm'$$

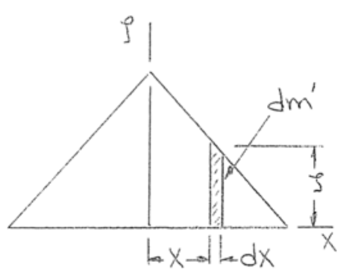
$$= \frac{1}{3}\zeta^2 dm'$$

where

$$dm' = \rho t\zeta\, dx$$

Then

$$I_x = \int dI_x = 2\int_0^{\frac{a}{2}} \frac{1}{3}\zeta^2 (\rho t\zeta\, dx)$$




$$= \frac{2}{3}\rho t\int_0^{\frac{a}{2}} \left[\frac{h}{a}(a - 2x)\right]^3 dx$$

$$= \frac{2}{3}\rho t\frac{h^3}{a^3} \times \frac{1}{4}\left(-\frac{1}{2}\right)\left[(a - 2x)^4\right]_0^{\frac{a}{2}}$$

$$= -\frac{1}{12}\rho t\frac{h^3}{a^3}\left[(a - a)^4 - (a)^4\right]$$

$$= \frac{1}{12}\rho tah^3$$

or $I_x = \dfrac{1}{6}mh^2$ ◀

## PROBLEM 9.128 CONTINUED

Now $$I_\zeta = \int x^2 dm$$

and $$I_\zeta = \int x^2 dm' = 2\int_0^{\frac{a}{2}} x^2 \left(\rho t \zeta\, dx\right)$$

$$= 2\rho t \int_0^{\frac{a}{2}} x^2 \left[\frac{h}{a}\left(a - 2x\right)\right] dx$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^{\frac{a}{2}}$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{4}\left(\frac{a}{2}\right)^4\right]$$

$$= \frac{1}{48}\rho t a^3 h = \frac{1}{24} m a^2$$

(*b*) Have $$I_y = \int r_y^2 dm = \int\left[x^2 + \left(\zeta \sin\theta\right)^2\right] dm$$

$$= \int x^2 dm + \sin^2\theta \int \zeta^2 dm$$

Now $$I_x = \int \zeta^2 dm \Rightarrow I_y = I_\zeta + I_x \sin^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \sin^2\theta$$

$$\text{or } I_y = \frac{m}{24}\left(a^2 + 4h^2 \sin^2\theta\right) \blacktriangleleft$$

(*c*) Have $$I_z = \int r_z^2 dm = \int\left(x^2 + y^2\right) dm$$

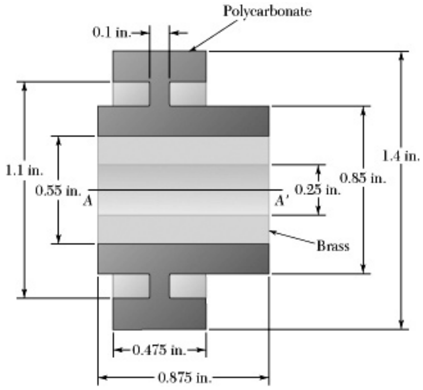
$$= \int\left[x^2 + \left(\zeta \cos\theta\right)^2\right] dm$$

$$= \int x^2 dm + \cos^2\theta \int \zeta^2 dm$$

$$= I_\zeta + I_x \cos^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \cos^2\theta$$

$$\text{or } I_z = \frac{m}{24}\left(a^2 + 4h^2 \cos^2\theta\right) \blacktriangleleft$$




## PROBLEM 9.129

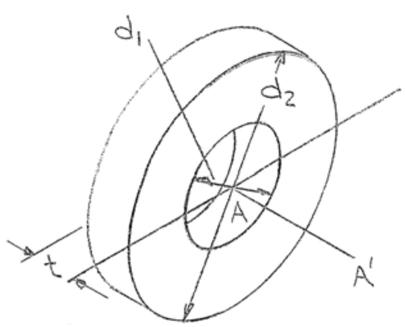
Shown is the cross section of a molded flat-belt pulley. Determine its mass moment of inertia and its radius of gyration with respect to the axis $AA'$. (The specific weight of brass is 0.306 lb/in$^3$ and the specific weight of the fiber-reinforced polycarbonate used is 0.0433 lb/in$^3$.)

## SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that




$$I_{AA'} = \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2$$

$$= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right]$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right)$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right)$$

$$= \frac{1}{8}m\left(d_1^2 + d_2^2\right)$$

Now treat the pulley as four concentric rings and, working from the brass outward, have

$$m = \frac{\pi}{4}\left\{\frac{0.306\,\text{lb/in}^3}{32.2\ \text{ft/s}^2}(0.875\ \text{in.})\left[(0.55\ \text{in.})^2 - (0.25\ \text{in.})^2\right]\right\}$$

$$+ \frac{\pi}{4}\frac{1.0433\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left\{(0.875\ \text{in.})\left[(0.85\ \text{in.})^2 - (0.55\ \text{in.})^2\right]\right.$$

$$+ (0.10\ \text{in.})\left[(1.1\ \text{in.})^2 - (0.85\ \text{in.})^2\right]$$

$$\left. + (0.475\ \text{in.})\left[(1.4\ \text{in.})^2 - (1.1\ \text{in.})^2\right]\right\}$$

$$= \frac{\pi}{128.8}(0.06426 + 0.01593 + 0.00211 + 0.015426)\,\text{lb}\cdot\text{s}^2/\text{ft}$$

## PROBLEM 9.129 CONTINUED

Now $$m = \left(1567.38\cdot 10^{-6} + 388.553\cdot 10^{-6} + 51.465\cdot 10^{-6} + 376.259\cdot 10^{-6}\right)\ \text{lb}\cdot\text{s}^2/\text{ft} = 2383.657\cdot 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Then $$I_{AA'} = \frac{1}{8}\left\{1567.38\cdot 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.25}{12}\ \text{ft}\right)^2 + \left(\frac{0.55}{12}\ \text{ft}\right)^2\right]\right.$$

$$+ 388.553\cdot 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.55}{12}\ \text{ft}\right)^2 + \left(\frac{0.85}{12}\ \text{ft}\right)^2\right]$$

$$+ 51.465\cdot 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.85}{12}\ \text{ft}\right)^2 + \left(\frac{1.1}{12}\ \text{ft}\right)^2\right]$$

$$\left. + 376.259\cdot 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{1.1}{12}\ \text{ft}\right)^2 + \left(\frac{1.4}{12}\ \text{ft}\right)^2\right]\right\}$$

$$= \frac{1}{8}\left(3.9728 + 2.7657 + 0.69067 + 8.2829\right)\cdot 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 1.96401\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{AA'} = 1.964\cdot 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

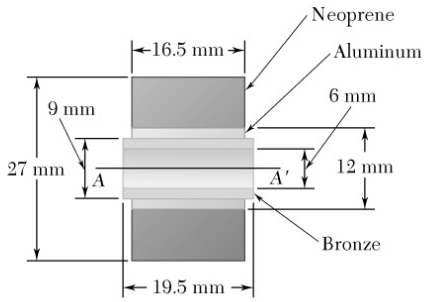
and $$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.96401\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2}{2383.657\times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}}$$

$$= 8.23947\times 10^{-4}\ \text{ft}^2$$

$$k_{AA'} = 2.87044\cdot 10^{-2}\ \text{ft} = 0.34445\ \text{in.}$$

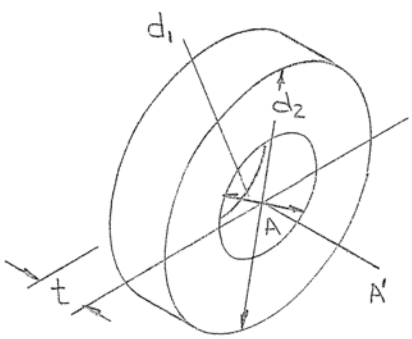
or $k_{AA'} = 0.344$ in. ◀

## PROBLEM 9.130




Shown is the cross section of an idler roller. Determine its moment of inertia and its radius of gyration with respect to the axis $AA'$. (The density of bronze is 8580 kg/m$^3$; of aluminum, 2770 kg/m$^3$; and of neoprene, 1250 kg/m$^3$.)

## SOLUTION




First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\rho t\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that

$$I_{AA'} = \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2$$

$$= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right]$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right)$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right)$$

$$= \frac{1}{8}m\left(d_1^2 + d_2^2\right)$$

Now treat the roller as three concentric rings and, working from the bronze outward, have

Have $$m = \frac{\pi}{4}\Big\{\left(8580\text{ kg/m}^3\right)(0.0195\text{ m})\left[(0.009\text{ m})^2 - (0.006\text{ m})^2\right]$$

$$+ \left(2770\text{ kg/m}^3\right)(0.0165\text{ m})\left[(0.012\text{ m})^2 - (0.009\text{ m})^2\right]$$

$$+ \left(1250\text{ kg/m}^3\right)(0.0165\text{ m})\left[(0.027\text{ m})^2 - (0.012\text{ m})^2\right]\Big\}$$

$$= \frac{\pi}{4}[7.52895 + 2.87942 + 12.06563] \times 10^{-3}\text{ kg}$$

$$= 5.9132 \times 10^{-3}\text{ kg} + 2.26149 \times 10^{-3}\text{ kg}$$

$$+ 9.47632 \times 10^{-3}\text{ kg}$$

$$= 17.6510 \times 10^{-3}\text{ kg}$$

## PROBLEM 9.130 CONTINUED

And $$I_{AA'} = \frac{1}{8}\left\{\left(5.9132\times10^{-3}\text{ kg}\right)\left[(0.006)^2 + (0.009)^2\right]\text{m}^2\right.$$

$$+\left(2.26149\times10^{-3}\text{ kg}\right)\left[(0.009)^2 + (0.012)^2\right]\text{m}^2$$

$$\left.+\left(9.47632\times10^{-3}\text{ kg}\right)\left[(0.012)^2 + (0.027)^2\right]\text{m}^2\right\}$$

$$= \frac{1}{8}\left(691.844 + 508.835 + 8272.827\right)10^{-9}\text{ kg}\cdot\text{m}^2$$

$$= 1.18419\times10^{-6}\text{ kg}\cdot\text{m}^2$$

or $I_{AA'} = 1.184\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

Now $$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.18419\times10^{-6}\text{ kg m}^2}{17.6510\times10^{-3}\text{ kg}}$$

$$= 67.08902\times10^{-6}\text{ m}^2$$

$$k_{AA'} = 8.19079\times10^{-3}\text{ m}$$

or $k_{AA'} = 8.19\text{ mm}$ ◀