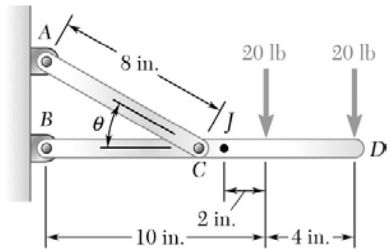
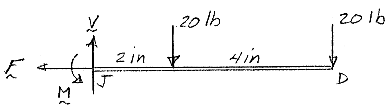


PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at point J of the structure indicated:
Frame and loading of Prob. 6.77.

SOLUTION

FBD JD:



$$\xrightarrow{} \Sigma F_x = 0: -F = 0$$

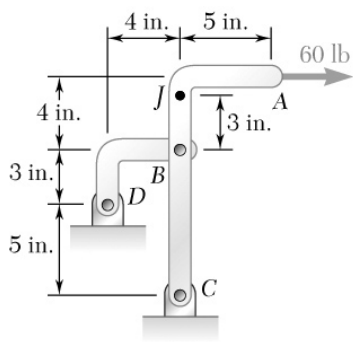
$$\mathbf{F} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: V - 20\text{ lb} - 20\text{ lb} = 0$$

$$\mathbf{V} = 40.0\text{ lb} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (2\text{ in.})(20\text{ lb}) - (6\text{ in.})(20\text{ lb}) = 0$$

$$\mathbf{M} = 160.0\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

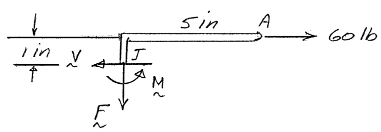


PROBLEM 7.2

Determine the internal forces (axial force, shearing force, and bending moment) at point *J* of the structure indicated:
Frame and loading of Prob. 6.76.

SOLUTION

FBD AJ:



$$\Sigma F_x = 0: 60 \text{ lb} - V = 0$$

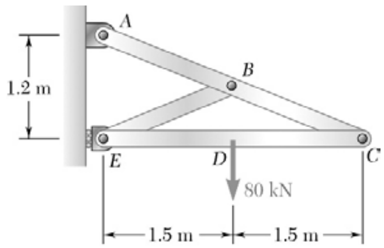
$$\mathbf{V} = 60.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\Sigma F_y = 0: -F = 0$$

$$\mathbf{F} = 0 \blacktriangleleft$$

$$\Sigma M_J = 0: M - (1 \text{ in.})(60 \text{ lb}) = 0$$

$$\mathbf{M} = 60.0 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

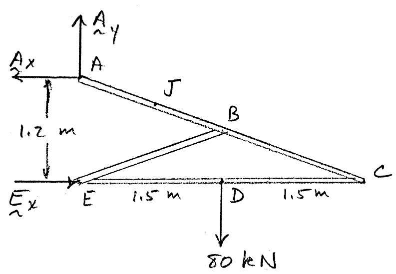


PROBLEM 7.3

For the frame and loading of Prob. 6.80, determine the internal forces at a point *J* located halfway between points *A* and *B*.

SOLUTION

FBD Frame:



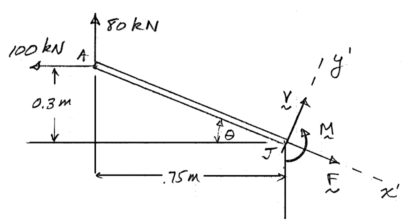
$$\longrightarrow \Sigma F_y = 0\colon\ A_y - 80\text{ kN} = 0 \qquad \mathbf{A}_y = 80\text{ kN} \uparrow$$

$$\circlearrowleft \Sigma M_E = 0\colon\ (1.2\text{ m})A_x - (1.5\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{A}_x = 100\text{ kN} \longleftarrow$$

$$\theta = \tan^{-1}\left(\frac{0.3\text{ m}}{0.75\text{ m}}\right) = 21.801°$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0\colon\ F - (80\text{ kN})\sin 21.801° - (100\text{ kN})\cos 21.801° = 0$$

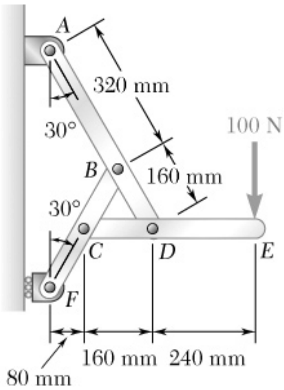
$$\mathbf{F} = 122.6\text{ kN} \searrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0\colon\ V + (80\text{ kN})\cos 21.801° - (100\text{ kN})\sin 21.801° = 0$$

$$\mathbf{V} = 37.1\text{ kN} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0\colon\ M + (.3\text{ m})(100\text{ kN}) - (.75\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{M} = 30.0\text{ kN}\cdot\text{m} \circlearrowright \blacktriangleleft$$

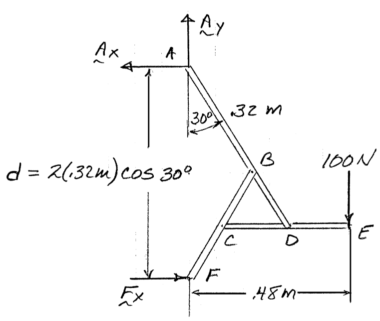


PROBLEM 7.4

For the frame and loading of Prob. 6.101, determine the internal forces at a point J located halfway between points A and B.

SOLUTION

FBD Frame:

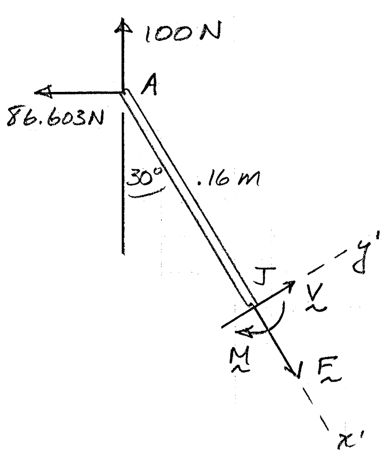


$$\uparrow \Sigma F_y = 0: A_y - 100\text{ N} = 0 \qquad \mathbf{A}_y = 100\text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_F = 0: \left[2(0.32\text{ m})\cos 30°\right]A_x - (0.48\text{ m})(100\text{ N}) = 0$$

$$\mathbf{A}_x = 86.603\text{ N} \longleftarrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: F - (100\text{ N})\cos 30° - (86.603\text{ N})\sin 30° = 0$$

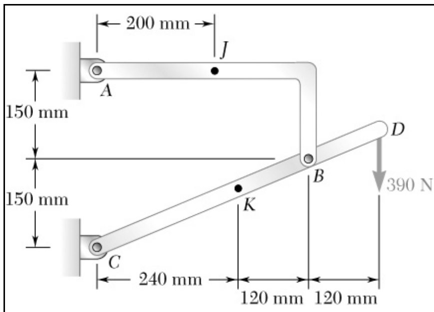
$$\mathbf{F} = 129.9\text{ N} \searrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V + (100\text{ N})\sin 30° - (86.603\text{ N})\cos 30° = 0$$

$$\mathbf{V} = 25.0\text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \left[(0.16\text{ m})\cos 30°\right](86.603\text{ N})$$
$$- \left[(0.16\text{ m})\sin 30°\right](100\text{ N}) - M = 0$$

$$\mathbf{M} = 4.00\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

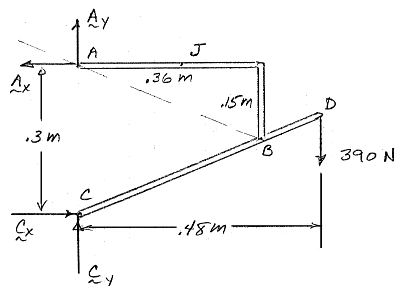


PROBLEM 7.5

Determine the internal forces at point J of the structure shown.

SOLUTION

FBD Frame:



AB is two-force member, so

$$\frac{A_x}{0.36\text{ m}} = \frac{A_y}{0.15\text{ m}} \qquad A_y = \frac{5}{12}A_x$$

$$\circlearrowleft \Sigma M_C = 0\text{: } (0.3\text{ m})A_x - (0.48\text{ m})(390\text{ N}) = 0$$

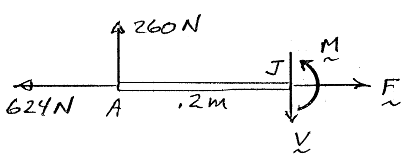
$$\mathbf{A}_x = 624\text{ N} \longleftarrow$$

$$A_y = \frac{5}{12}A_x = 260\text{ N} \text{ or } \mathbf{A}_y = 260\text{ N} \uparrow$$

$$\longrightarrow \Sigma F_x = 0\text{: } F - 624\text{ N} = 0$$

$$\mathbf{F} = 624\text{ N} \longrightarrow \blacktriangleleft$$

FBD AJ:

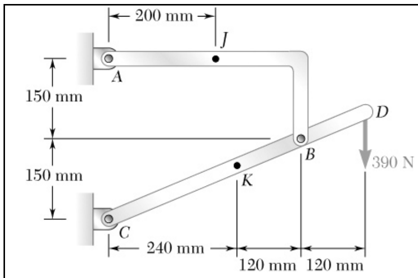


$$\uparrow \Sigma F_y = 0\text{: } 260\text{ N} - V = 0$$

$$\mathbf{V} = 260\text{ N} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0\text{: } M - (0.2\text{ m})(260\text{ N}) = 0$$

$$\mathbf{M} = 52.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

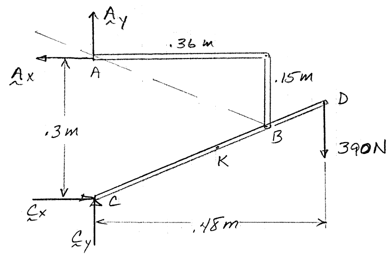


PROBLEM 7.6

Determine the internal forces at point K of the structure shown.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_C = 0: (0.3\text{ m})A_x - (0.48\text{ m})(390\text{ N}) = 0$$

$$\mathbf{A}_x = 624\text{ N} \longleftarrow$$

AB is two-force member, so

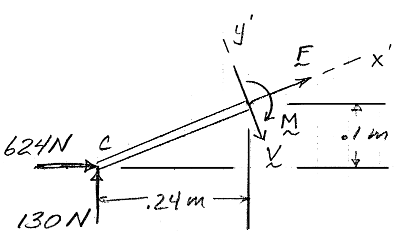
$$\frac{A_x}{0.36\text{ m}} = \frac{A_y}{0.15\text{ m}} \rightarrow A_y = \frac{5}{12}A_x \qquad \mathbf{A}_y = 260\text{ N}\uparrow$$

$$\longrightarrow \Sigma F_x = 0: -A_x + C_x = 0 \qquad \mathbf{C}_x = \mathbf{A}_x = 624\text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: \ A_y + C_y - 390\text{ N} = 0$$

$$C_y = 390\text{ N} - 260\text{ N} = 130\text{ N or } \mathbf{C}_y = 130\text{ N}\uparrow$$

FBD CK:



$$\nearrow \Sigma F_{x'} = 0: F + \frac{12}{13}(624\text{ N}) + \frac{5}{13}(130\text{ N}) = 0$$

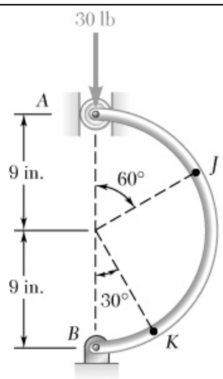
$$F = -626\text{ N} \qquad \mathbf{F} = 626\text{ N} \swarrow \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \frac{12}{13}(130\text{ N}) - \frac{5}{13}(624\text{ N}) - V = 0$$

$$V = -120\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.1\text{ m})(624\text{ N}) - (0.24\text{ m})(130\text{ N}) - M = 0$$

$$\mathbf{M} = 31.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

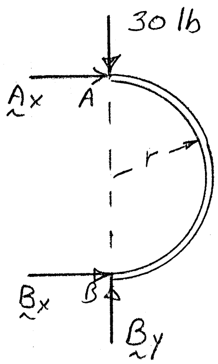


PROBLEM 7.7

A semicircular rod is loaded as shown. Determine the internal forces at point J.

SOLUTION

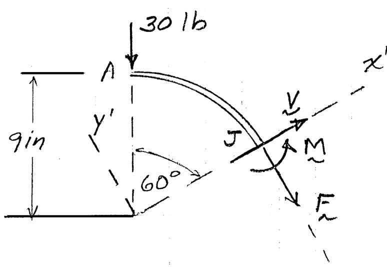
FBD Rod:



$$\Sigma M_B = 0:\ A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

FBD AJ:



$$\Sigma F_{x'} = 0:\ V - (30\text{ lb})\cos 60° = 0$$

$$\mathbf{V} = 15.00\text{ lb} \blacktriangleleft$$

$$\Sigma F_{y'} = 0:\ F + (30\text{ lb})\sin 60° = 0$$

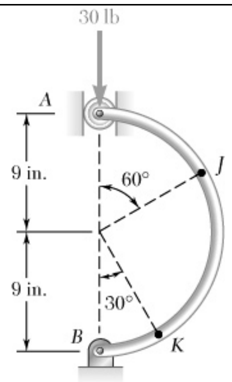
$$F = -25.98\text{ lb}$$

$$\mathbf{F} = 26.0\text{ lb} \blacktriangleleft$$

$$\Sigma M_J = 0:\ M - [(9\text{ in.})\sin 60°](30\text{ lb}) = 0$$

$$M = -233.8\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 234\text{ lb}\cdot\text{in.} \blacktriangleleft$$

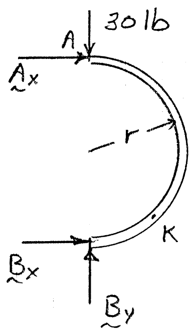


PROBLEM 7.8

A semicircular rod is loaded as shown. Determine the internal forces at point *K*.

SOLUTION

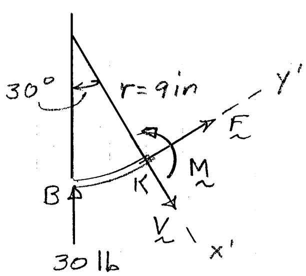
FBD Rod:



$$\uparrow \Sigma F_y = 0: B_y - 30 \text{ lb} = 0 \qquad \mathbf{B}_y = 30 \text{ lb} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: 2rB_x = 0 \qquad \mathbf{B}_x = 0$$

FBD BK:



$$\searrow \Sigma F_{x'} = 0: V - (30 \text{ lb})\cos 30° = 0$$

$$V = 25.98 \text{ lb}$$

$$\mathbf{V} = 26.0 \text{ lb} \searrow \blacktriangleleft$$

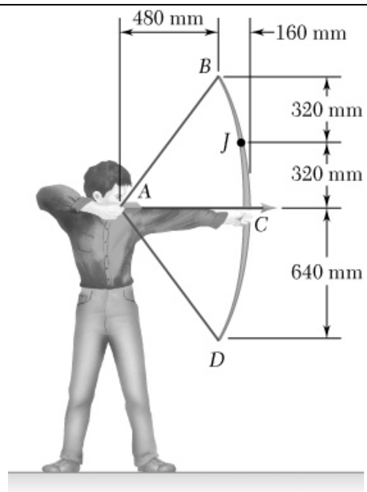
$$\nearrow \Sigma F_{y'} = 0: F + (30 \text{ lb})\sin 30° = 0$$

$$F = -15 \text{ lb}$$

$$\mathbf{F} = 15.00 \text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - \left[(9 \text{ in.})\sin 30°\right](30 \text{ lb}) = 0$$

$$\mathbf{M} = 135.0 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

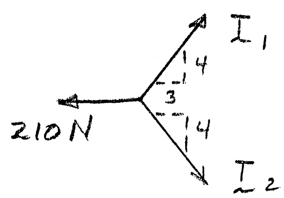


PROBLEM 7.9

An archer aiming at a target is pulling with a 210-N force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at point J.

SOLUTION

FBD Point A:

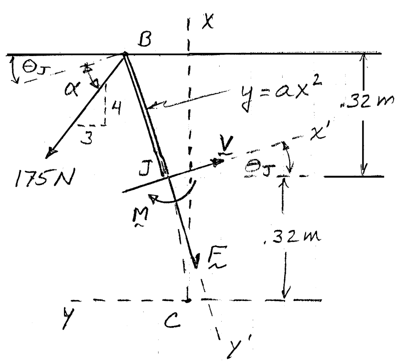


By symmetry $T_1 = T_2$

$$\xrightarrow{} \Sigma F_x = 0:\ 2\left(\frac{3}{5}T_1\right) - 210\text{ N} = 0 \qquad T_1 = T_2 = 175\text{ N}$$

Curve CJB is parabolic: $y = ax^2$

FBD BJ:



At B: $\quad x = 0.64\text{ m}, \qquad y = 0.16\text{ m} \qquad a = \dfrac{0.16\text{ m}}{(0.64\text{ m})^2} = \dfrac{1}{2.56\text{ m}}$

So, at J: $\quad y_J = \dfrac{1}{2.56\text{ m}}(0.32\text{ m})^2 = 0.04\text{ m}$

$$\text{Slope of parabola} = \tan\theta = \frac{dy}{dx} = 2ax$$

At J: $\quad \theta_J = \tan^{-1}\left[\dfrac{2}{2.56\text{ m}}(0.32\text{ m})\right] = 14.036°$

So $\qquad \alpha = \tan^{-1}\dfrac{4}{3} - 14.036° = 39.094°$

$$\nearrow \Sigma F_{x'} = 0:\ V - (175\text{ N})\cos(39.094°) = 0$$

$$\mathbf{V} = 135.8\text{ N} \nearrow \blacktriangleleft$$

$$\searrow \Sigma F_{y'} = 0:\ F + (175\text{ N})\sin(39.094°) = 0$$

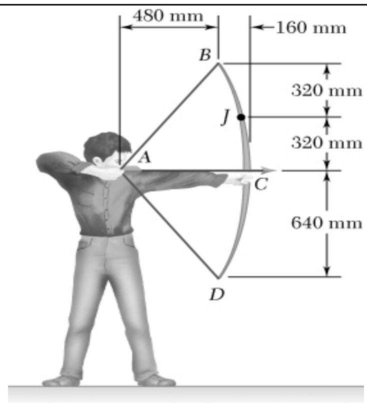
$$F = -110.35\text{ N} \qquad \mathbf{F} = 110.4\text{ N} \nwarrow \blacktriangleleft$$

PROBLEM 7.9 CONTINUED

$$\circlearrowleft \Sigma M_J = 0: M + (0.32\text{ m})\left[\frac{3}{5}(175\text{ N})\right]$$

$$+\left[(0.16 - 0.04)\text{m}\right]\left[\frac{4}{5}(175\text{ N})\right] = 0$$

$$\mathbf{M} = 50.4\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



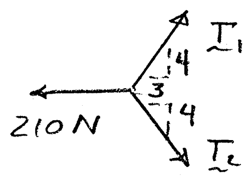
PROBLEM 7.10

For the bow of Prob. 7.9, determine the magnitude and location of the maximum (*a*) axial force, (*b*) shearing force, (*c*) bending moment.

SOLUTION

By symmetry $T_1 = T_2 = T$

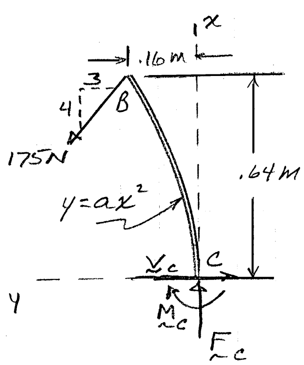
FBD Point A:



$$\longrightarrow \Sigma F_x = 0: 2T_1\left(\frac{3}{5}\right) - 210\text{ N} = 0 \qquad T_1 = 175\text{ N}$$

$$\uparrow \Sigma F_y = 0: F_C - \frac{4}{5}(175\text{ N}) = 0 \qquad F_C = 140\text{ N}\uparrow$$

FBD BC:



$$\longleftarrow \Sigma F_x = 0: \frac{3}{5}(175\text{ N}) - V_C = 0 \qquad V_C = 105\text{ N} \longrightarrow$$

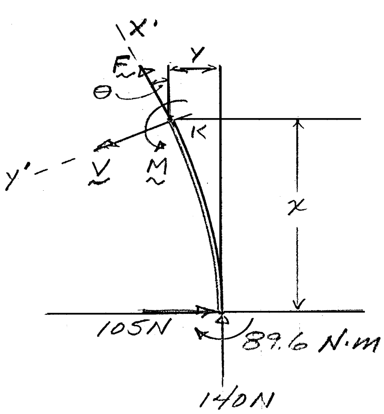
$$\circlearrowleft \Sigma M_C = 0: M_C - (0.64\text{ m})\left[\frac{3}{5}(175\text{ N})\right] - (0.16\text{ m})\left[\frac{4}{5}(175\text{ N})\right] = 0$$

$$M_C = 89.6\text{ N}\cdot\text{m} \circlearrowright$$

Also: if $y = ax^2$ and, at B, $y = 0.16$ m, $x = 0.64$ m

Then $$a = \frac{0.16\text{ m}}{(0.64\text{ m})^2} = \frac{1}{2.56\text{ m}};$$

FBD CK:



And $$\theta = \tan^{-1}\frac{dy}{dx} = \tan^{-1}2ax$$

$$\nwarrow \Sigma F_{x'} = 0: (140\text{ N})\cos\theta - (105\text{ N})\sin\theta + F = 0$$

So $$F = (105\text{ N})\sin\theta - (140\text{ N})\cos\theta$$

$$\frac{dF}{d\theta} = (105\text{ N})\cos\theta + (140\text{ N})\sin\theta$$

$$\swarrow \Sigma F_{y'} = 0: V - (105\text{ N})\cos\theta - (140\text{ N})\sin\theta = 0$$

So $$V = (105\text{ N})\cos\theta + (140\text{ N})\sin\theta$$

PROBLEM 7.10 CONTINUED

And $$\frac{dV}{d\theta} = -(105\text{ N})\sin\theta + (140\text{ N})\cos\theta$$

$$\circlearrowleft \Sigma M_K = 0\text{: } M + x(105\text{ N}) + y(140\text{ N}) - 89.6\text{ N}\cdot\text{m} = 0$$

$$M = -(105\text{ N})x - \frac{(140\text{ N})x^2}{(2.56\text{ m})} + 89.6\text{ N}\cdot\text{m}$$

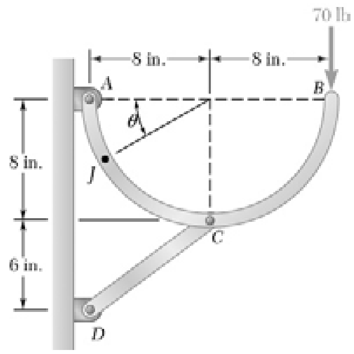
$$\frac{dM}{dx} = -(105\text{ N}) - (109.4\text{ N/m})x + 89.6\text{ N}\cdot\text{m}$$

Since none of the functions, F, V, or M has a vanishing derivative in the valid range of $0 \le x \le 0.64\text{ m}(0 \le \theta \le 26.6°)$, the maxima are at the limits $(x = 0,\ \text{or } x = 0.64\text{ m})$.

Therefore, (*a*) $\mathbf{F}_{max} = 140.0\text{ N}\uparrow$ at C ◀

(*b*) $\mathbf{V}_{max} = 156.5\text{ N}\swarrow$ at B ◀

(*c*) $\mathbf{M}_{max} = 89.6\text{ N}\cdot\text{m}\circlearrowright$ at C ◀

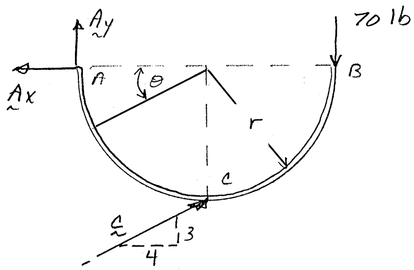


PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at point J knowing that $\theta = 30^{\circ}$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70\text{ lb}) = 0$$

$$\mathbf{C} = 100\text{ lb}$$

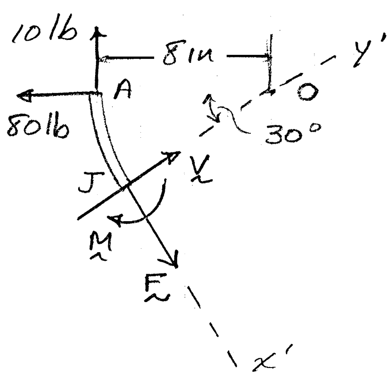
$$\longrightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100\text{ lb}) = 0$$

$$\mathbf{A}_x = 80\text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100\text{ lb}) - 70\text{ lb} = 0$$

$$\mathbf{A}_y = 10\text{ lb} \uparrow$$

FBD AJ:



$$\Sigma F_{x'} = 0: F - (80\text{ lb})\sin 30^{\circ} - (10\text{ lb})\cos 30^{\circ} = 0$$

$$F = 48.66\text{ lb}$$

$$\mathbf{F} = 48.7\text{ lb} \measuredangle 60^{\circ} \blacktriangleleft$$

$$\Sigma F_{y'} = 0: V - (80\text{ lb})\cos 30^{\circ} + (10\text{ lb})\sin 30^{\circ} = 0$$

$$V = 64.28\text{ lb}$$

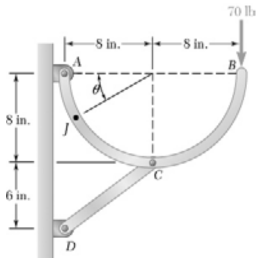
$$\mathbf{V} = 64.3\text{ lb} \measuredangle 30^{\circ} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (8\text{ in.})(48.66\text{ lb}) - (8\text{ in.})(10\text{ lb}) - M = 0$$

$$M = 309.28\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 309\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

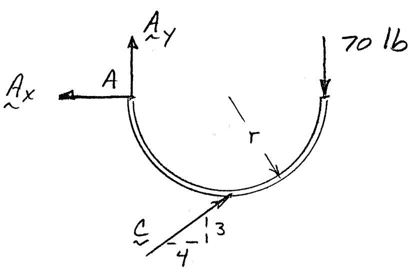
PROBLEM 7.12



A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70 \text{ lb}) = 0$$

$$\mathbf{C} = 100 \text{ lb} \nearrow$$

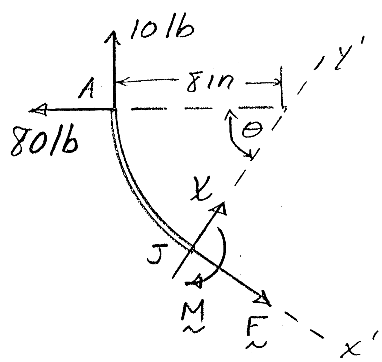
$$\longrightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100 \text{ lb}) = 0$$

$$\mathbf{A}_x = 80 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100 \text{ lb}) - 70 \text{ lb} = 0$$

$$\mathbf{A}_y = 10 \text{ lb} \uparrow$$

FBD AJ:



$$\circlearrowleft \Sigma M_J = 0: M - (8 \text{ in.})(1 - \cos\theta)(10 \text{ lb}) - (8 \text{ in.})(\sin\theta)(80 \text{ lb}) = 0$$

$$M = (640 \text{ lb}\cdot\text{in.})\sin\theta + (80 \text{ lb}\cdot\text{in.})(\cos\theta - 1)$$

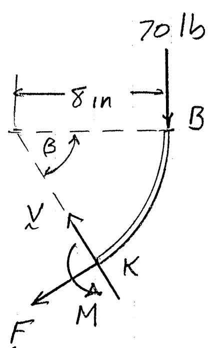
$$\frac{dM}{d\theta} = (640 \text{ lb}\cdot\text{in.})\cos\theta - (80 \text{ lb}\cdot\text{in.})\sin\theta = 0$$

for $\theta = \tan^{-1} 8 = 82.87°$,

where $\dfrac{d^2M}{d\theta^2} = -(640 \text{ lb}\cdot\text{in.})\sin\theta - (80 \text{ lb}\cdot\text{in.})\cos\theta < 0$

So $M = 565 \text{ lb}\cdot\text{in.}$ at $\theta = 82.9°$ is a *max* for *AC*

FBD BK:



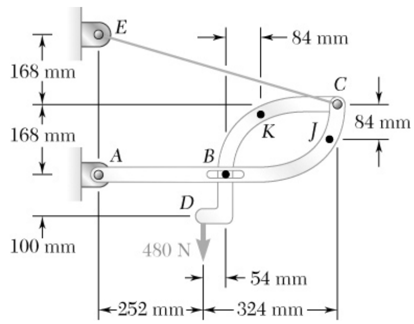
$$\circlearrowleft \Sigma M_K = 0: M - (8 \text{ in.})(1 - \cos\beta)(70 \text{ lb}) = 0$$

$$M = (560 \text{ lb}\cdot\text{in.})(1 - \cos\beta)$$

$$\frac{dM}{d\beta} = (560 \text{ lb}\cdot\text{in.})\sin\beta = 0 \quad \text{for } \beta = 0, \text{ where } M = 0$$

So, for $\beta = \dfrac{\pi}{2}$, $M = 560 \text{ lb}\cdot\text{in.}$ is max for *BC*

$$\therefore \; \mathbf{M}_{max} = 565 \text{ lb}\cdot\text{in. at } \theta = 82.9° \blacktriangleleft$$

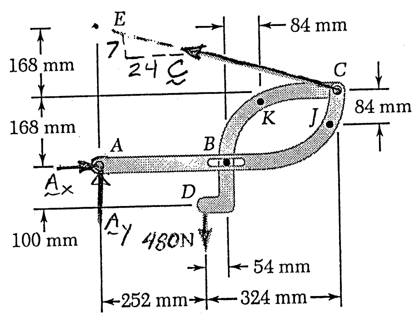


PROBLEM 7.13

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at *D*. Determine the internal forces at point *J*.

SOLUTION

FBD Frame:



$$\Sigma M_A = 0: (0.336 \text{ m})\left(\frac{24}{25}C\right) - (0.252 \text{ m})(480 \text{ N}) = 0$$

$$C = 375 \text{ N}$$

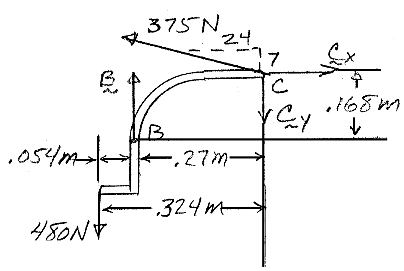
$$\longrightarrow \Sigma F_y = 0: A_x - \frac{24}{25}C = 0 \qquad A_x = \frac{24}{25}(375 \text{ N}) = 360 \text{ N}$$

$$\mathbf{A}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 480 \text{ N} + \frac{7}{24}(375 \text{ N}) = 0$$

$$\mathbf{A}_y = 375 \text{ N} \uparrow$$

FBD CD:



$$\Sigma M_C = 0: (0.324 \text{ m})(480 \text{ N}) - (0.27 \text{ m})B = 0$$

$$B = 576 \text{ N}$$

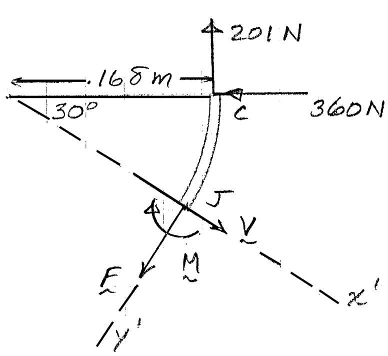
$$\longrightarrow \Sigma F_x = 0: C_x - \frac{24}{25}(375 \text{ N}) = 0$$

$$\mathbf{C}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: -480 \text{ N} + \frac{7}{25}(375 \text{ N}) + (576 \text{ N}) - C_y = 0$$

$$\mathbf{C}_y = 201 \text{ N} \downarrow$$

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: V - (360 \text{ N})\cos 30^\circ - (201 \text{ N})\sin 30^\circ = 0$$

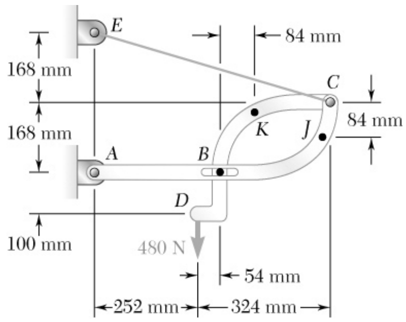
$$\mathbf{V} = 412 \text{ N} \searrow \blacktriangleleft$$

$$\swarrow \Sigma F_{y'} = 0: F + (360 \text{ N})\sin 30^\circ - (201 \text{ N})\cos 30^\circ = 0$$

$$F = -5.93 \text{ N} \qquad \mathbf{F} = 5.93 \text{ N} \nearrow \blacktriangleleft$$

$$\Sigma M_0 = 0: (0.168 \text{ m})(201 \text{ N} + 5.93 \text{ N}) - M = 0$$

$$M = 34.76 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 34.8 \text{ N}\cdot\text{m} \blacktriangleleft$$

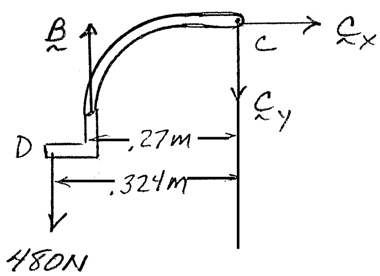


PROBLEM 7.14

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at *D*. Determine the internal forces at point *K*.

SOLUTION

FBD CD:



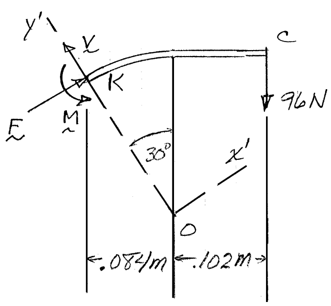
$$\longrightarrow \Sigma F_x = 0: \quad \mathbf{C}_x = 0$$

$$\Sigma M_B = 0: (0.054 \text{ m})(480 \text{ N}) - (0.27 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 96 \text{ N} \downarrow$$

$$\uparrow \Sigma F_y = 0: B - C_y = 0 \qquad \mathbf{B} = 96 \text{ N} \uparrow$$

FBD CK:



$$\nwarrow \Sigma F_{y'} = 0: V - (96 \text{ N})\cos 30° = 0$$

$$\mathbf{V} = 83.1 \text{ N} \nwarrow \blacktriangleleft$$

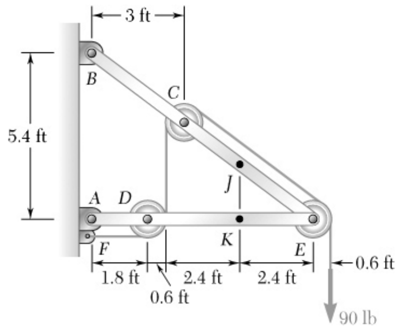
$$\nearrow \Sigma F_{x'} = 0: F - (96 \text{ N})\sin 30° = 0$$

$$\mathbf{F} = 48.0 \text{ N} \nearrow \blacktriangleleft$$

$$\Sigma M_K = 0: M - (0.186 \text{ m})(96 \text{ N}) = 0$$

$$\mathbf{M} = 17.86 \text{ N} \cdot \text{m} \blacktriangleleft$$

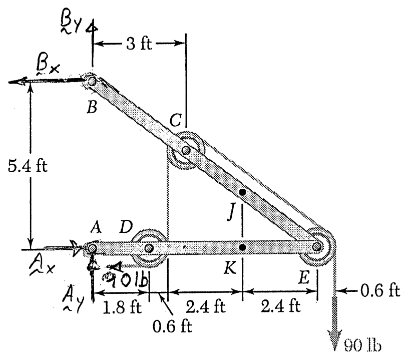
PROBLEM 7.15



Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

FBD Frame:

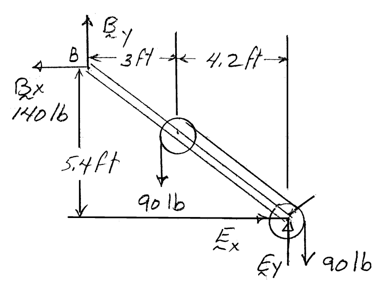


Note: Tension T in cord is 90 lb at any cut. All radii = 0.6 ft

$$\circlearrowleft \Sigma M_A = 0: (5.4\text{ ft})B_x - (7.8\text{ ft})(90\text{ lb}) - (0.6\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{B}_x = 140\text{ lb} \longleftarrow$$

FBD BCE with pulleys and cord:



$$\circlearrowleft \Sigma M_E = 0: (5.4\text{ ft})(140\text{ lb}) - (7.2\text{ ft})B_y$$

$$+ (4.8\text{ ft})90\text{ lb} - (0.6\text{ ft})90\text{ lb} = 0$$

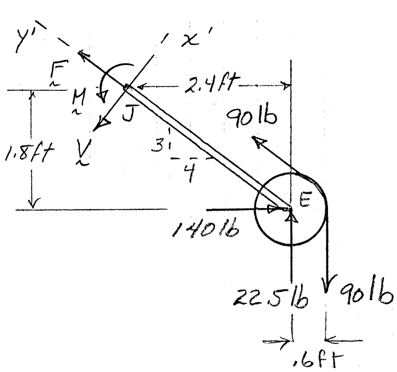
$$\mathbf{B}_y = 157.5\text{ lb} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: E_x - 140\text{ lb} = 0 \qquad \mathbf{E}_x = 140\text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: 157.5\text{ lb} - 90\text{ lb} - 90\text{ lb} + E_y = 0$$

$$\mathbf{E}_y = 22.5\text{ lb} \uparrow$$

FBD EJ:



$$\nearrow \Sigma F_{x'} = 0: -V + \frac{3}{5}(140\text{ lb}) + \frac{4}{5}(22.5\text{ lb} - 90\text{ lb}) = 0$$

$V = 30$ lb $\qquad$ $\mathbf{V} = 30.0\text{ lb} \swarrow$ ◀

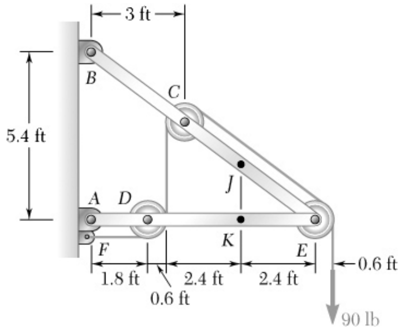
$$\nwarrow \Sigma F_{y'} = 0: F + 90\text{ lb} - \frac{4}{5}(140\text{ lb}) - \frac{3}{5}(90\text{ lb} - 22.5\text{ lb}) = 0$$

$F = 62.5$ lb $\qquad$ $\mathbf{F} = 62.5\text{ lb} \nwarrow$ ◀

$$\circlearrowleft \Sigma M_J = 0: M + (1.8\text{ ft})(140\text{ lb}) + (0.6\text{ ft})(90\text{ lb})$$

$$+ (2.4\text{ ft})(22.5\text{ lb}) - (3.0\text{ ft})(90\text{ lb}) = 0$$

$M = -90$ lb·ft $\qquad$ $\mathbf{M} = 90.0\text{ lb}\cdot\text{ft} \circlearrowright$ ◀

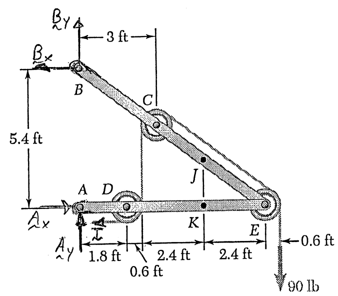


PROBLEM 7.16

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point K of the frame shown.

SOLUTION

FBD Whole:

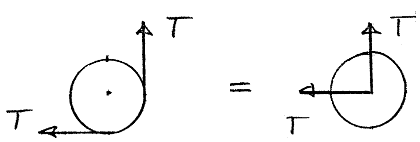


Note: $T = 90$ lb

$$\circlearrowleft \Sigma M_B = 0: (5.4 \text{ ft})A_x - (6 \text{ ft})(90 \text{ lb}) - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{A}_x = 2.30 \text{ lb} \longrightarrow$$

FBD AE:



Note: Cord tensions moved to point D as per Problem 6.91

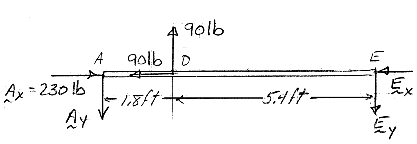
$$\longrightarrow \Sigma F_x = 0: 230 \text{ lb} - 90 \text{ lb} - E_x = 0$$

$$\mathbf{E}_x = 140 \text{ lb} \longleftarrow$$

$$\circlearrowleft \Sigma M_A = 0: (1.8 \text{ ft})(90 \text{ lb}) - (7.2 \text{ ft})E_y = 0$$

$$\mathbf{E}_y = 22.5 \text{ lb} \downarrow$$

FBD KE:



$$\longrightarrow \Sigma F_x = 0: F - 140 \text{ lb} = 0$$

$$\mathbf{F} = 140.0 \text{ lb} \longrightarrow \blacktriangleleft$$

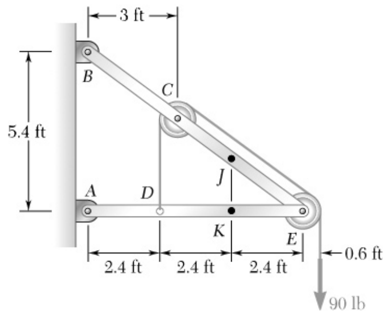
$$\uparrow \Sigma F_y = 0: V - 22.5 \text{ lb} = 0$$

$$\mathbf{V} = 22.5 \text{ lb} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - (2.4 \text{ ft})(22.5 \text{ lb}) = 0$$

$$\mathbf{M} = 54.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

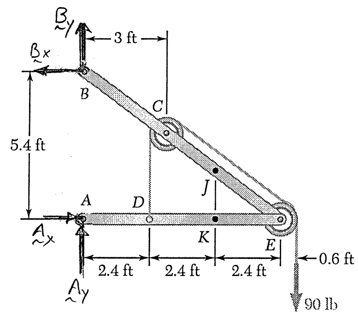
PROBLEM 7.17



Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

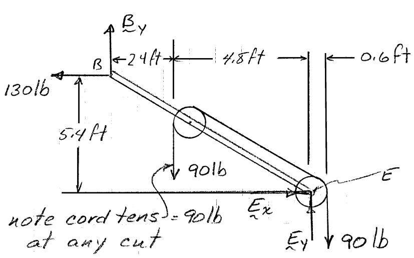
FBD Whole:



$$\circlearrowleft \Sigma M_A = 0: (5.4 \text{ ft})B_x - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{B}_x = 130 \text{ lb} \longleftarrow$$

FBD BE with pulleys and cord:



$$\circlearrowleft \Sigma M_E = 0: (5.4 \text{ ft})(130 \text{ lb}) - (7.2 \text{ ft})B_y$$
$$+ (4.8 \text{ ft})(90 \text{ lb}) - (0.6 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{B}_y = 150 \text{ lb} \uparrow$$

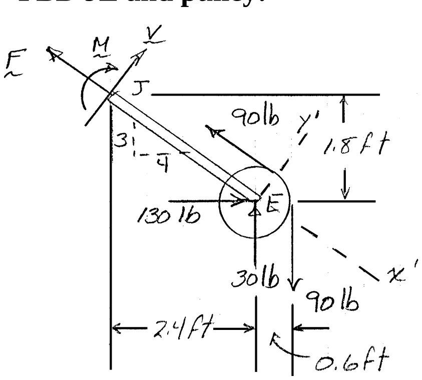
$$\longrightarrow \Sigma F_x = 0: E_x - 130 \text{ lb} = 0$$

$$\mathbf{E}_x = 130 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: E_y + 150 \text{ lb} - 90 \text{ lb} - 90 \text{ lb} = 0$$

$$\mathbf{E}_y = 30 \text{ lb} \uparrow$$

FBD JE and pulley:



$$\searrow \Sigma F_{x'} = 0: -F - 90 \text{ lb} + \frac{4}{5}(130 \text{ lb}) + \frac{3}{5}(90 \text{ lb} - 30 \text{ lb}) = 0$$

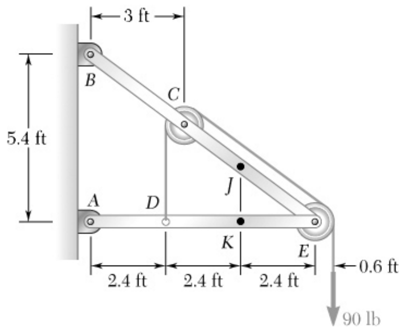
$$\mathbf{F} = 50.0 \text{ lb} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V + \frac{3}{5}(130 \text{ lb}) + \frac{4}{5}(30 \text{ lb} - 90 \text{ lb}) = 0$$

$$V = -30 \text{ lb} \qquad \mathbf{V} = 30.0 \text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: -M + (1.8 \text{ ft})(130 \text{ lb}) + (2.4 \text{ ft})(30 \text{ lb}) + (0.6 \text{ ft})(90 \text{ lb})$$
$$- (3.0 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{M} = 90.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

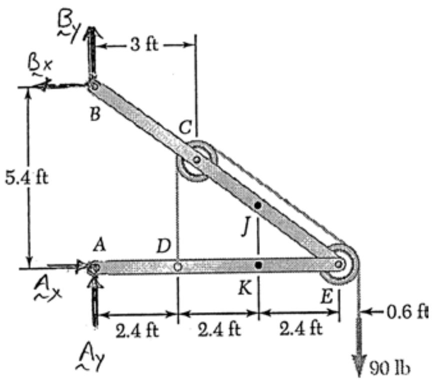


PROBLEM 7.18

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point K of the frame shown.

SOLUTION

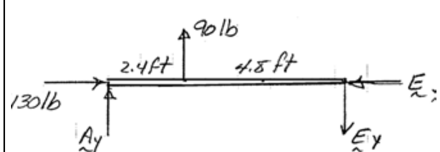
FBD Whole:



$$\circlearrowleft \Sigma M_B = 0: (5.4 \text{ ft})A_x - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{A}_x = 130 \text{ lb} \longrightarrow$$

FBD AE:

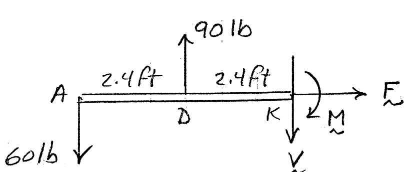


$$\circlearrowleft \Sigma M_E = 0: -(7.2 \text{ ft})A_y - (4.8 \text{ ft})(90 \text{ lb}) = 0$$

$$A_y = -60 \text{ lb} \qquad \mathbf{A}_y = 60 \text{ lb} \downarrow$$

$$\longrightarrow \Sigma F_x = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

FBD AK:

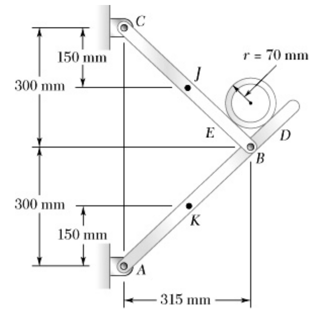


$$\uparrow \Sigma F_y = 0: -60 \text{ lb} + 90 \text{ lb} - V = 0$$

$$\mathbf{V} = 30.0 \text{ lb} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (4.8 \text{ ft})(60 \text{ lb}) - (2.4 \text{ ft})(90 \text{ lb}) - M = 0$$

$$\mathbf{M} = 72.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

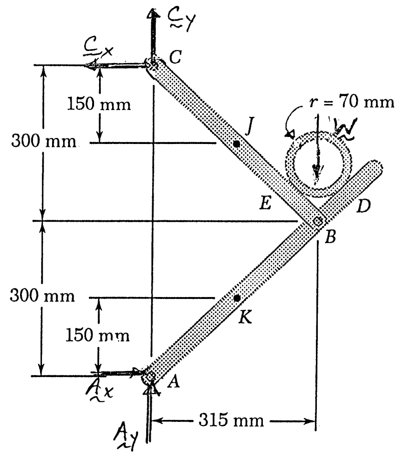


PROBLEM 7.19

A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *J*.

SOLUTION

FBD Whole:

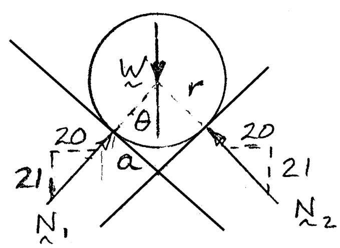


$$W = (3\text{ m})(28\text{ kg/m})(9.81\text{ m/s}^2) = 824.04\text{ N}$$

$$\circlearrowleft \Sigma M_A = (0.6\text{ m})C_x - (0.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{C}_x = 432.62\text{ N} \longleftarrow$$

FBD pipe:



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0:\ 2\frac{21}{29}N_1 - W = 0$$

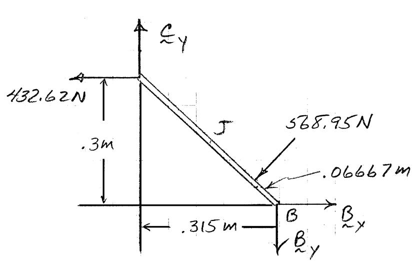
$$N_1 = \frac{29}{42}\,(824.04\text{ N})$$

$$= 568.98\text{ N}$$

Also note: $$a = r\tan\theta = 70\text{ mm}\left(\frac{20}{21}\right)$$

$$a = 66.67\text{ mm}$$

FBD BC:



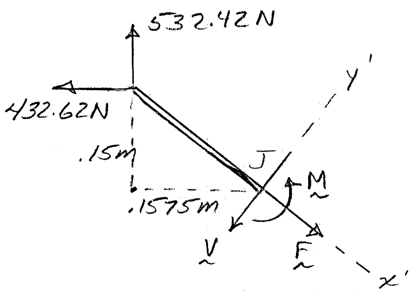
$$\circlearrowleft \Sigma M_B = 0:\ (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})C_y$$

$$+ (0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{C}_y = 532.42\text{ N} \uparrow$$

PROBLEM 7.19 CONTINUED

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: F - \frac{21}{29}(432.62 \text{ N}) - \frac{20}{29}(532.42 \text{ N}) = 0$$

$$\mathbf{F} = 680 \text{ N} \searrow \blacktriangleleft$$

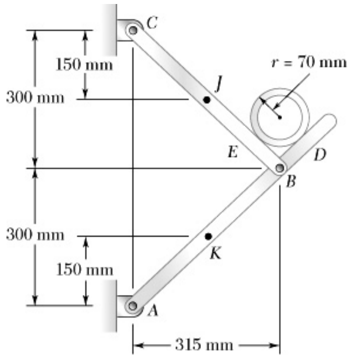
$$\nearrow \Sigma F_{y'} = 0: \frac{21}{29}(532.42 \text{ N}) - \frac{20}{29}(432.62 \text{ N}) - V = 0$$

$$\mathbf{V} = 87.2 \text{ N} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: (0.15 \text{ m})(432.62 \text{ N}) - (0.1575 \text{ m})(532.42 \text{ N}) + M = 0$$

$$\mathbf{M} = 18.96 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

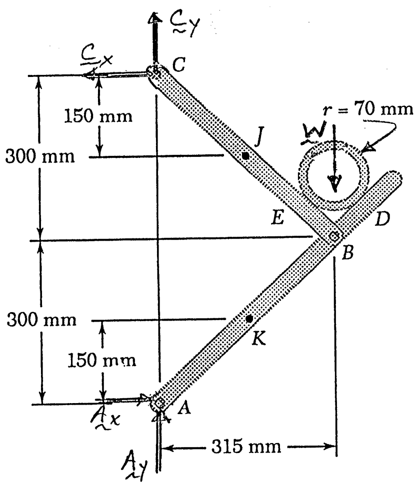
PROBLEM 7.20



A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *K*.

SOLUTION

FBD Whole:

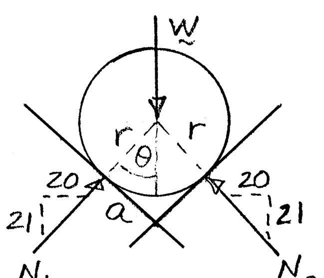


$$W = (3\text{ m})(28\text{ kg/m})(9.81\text{ m/s}^2) = 824.04\text{ N}$$

$$\circlearrowleft \Sigma M_C = 0: (.6\text{ m})A_x - (.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{A}_x = 432.62\text{ N} \longrightarrow$$

FBD pipe



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0: 2\frac{21}{29}N_1 - W = 0$$

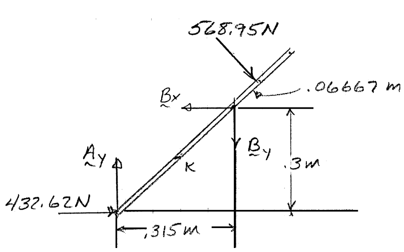
$$N_2 = \frac{29}{42}\,824.04\text{ N}$$

$$= 568.98\text{ N}$$

Also note: $$a = r\tan\theta = (70\text{ mm})\frac{20}{21}$$

$$a = 66.67\text{ mm}$$

FBD AD:



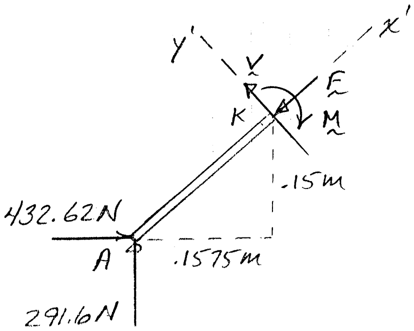
$$\circlearrowleft \Sigma M_B = 0: (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})A_y$$

$$-(0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{A}_y = 291.6\text{ N} \uparrow$$

PROBLEM 7.20 CONTINUED

FBD AK:



$$\nearrow \Sigma F_{x'} = 0: \frac{21}{29}(432.62 \text{ N}) + \frac{20}{29}(291.6 \text{ N}) - F = 0$$

$$\mathbf{F} = 514 \text{ N} \swarrow \blacktriangleleft$$

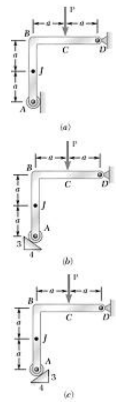
$$\nwarrow \Sigma F_{y'} = 0: \frac{21}{29}(291.6 \text{ N}) - \frac{20}{29}(432.62 \text{ N}) + V = 0$$

$$\mathbf{V} = 87.2 \text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.15 \text{ m})(432.62 \text{ N}) - (0.1575 \text{ m})(291.6 \text{ N}) - M = 0$$

$$\mathbf{M} = 18.97 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

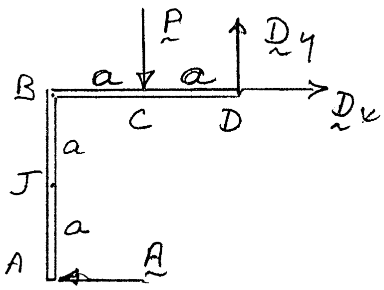
PROBLEM 7.21



A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.

SOLUTION

(*a*) **FBD Rod:**



$\circlearrowleft \Sigma M_D = 0: aP - 2aA = 0$

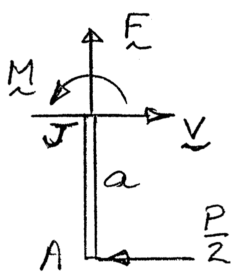
$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$\longrightarrow \Sigma F_x = 0: V - \frac{P}{2} = 0$

$$\mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

$\uparrow \Sigma F_y = 0:$ $\mathbf{F} = 0 \blacktriangleleft$

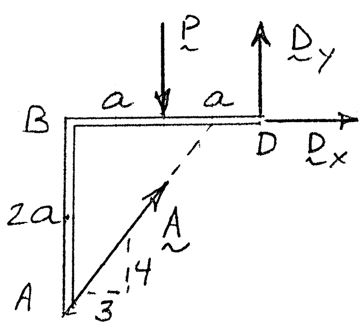
FBD AJ:



$\circlearrowleft \Sigma M_J = 0: M - a\frac{P}{2} = 0$

$$\mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

(*b*) **FBD Rod:**

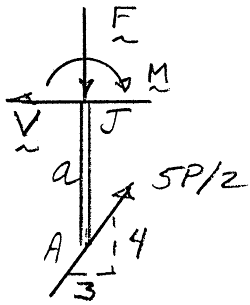


$\circlearrowleft \Sigma M_D = 0: aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

PROBLEM 7.21 CONTINUED

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \frac{3}{5}\frac{5P}{2} - V = 0$$

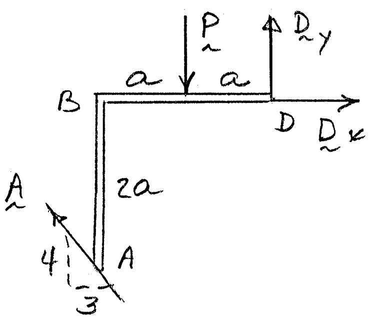
$$\mathbf{V} = \frac{3P}{2} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{2} - F = 0$$

$$\mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \blacktriangleleft$$

(*c*) **FBD Rod:**



$$\Sigma M_D = 0: aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

$$\rightarrow \Sigma F_x = 0: V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{V} = \frac{3P}{14} \rightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{14} - F = 0$$

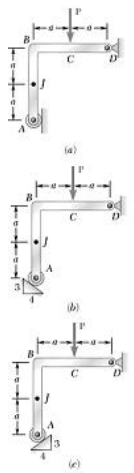
$$\mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\Sigma M_J = 0: M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{M} = \frac{3}{14}aP \blacktriangleleft$$

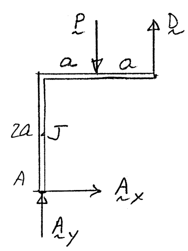
PROBLEM 7.22

A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.



SOLUTION

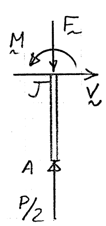
(*a*) **FBD Rod:**



$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\circlearrowleft \Sigma M_D = 0: aP - 2aA_y = 0 \qquad A_y = \frac{P}{2}$$

FBD AJ:



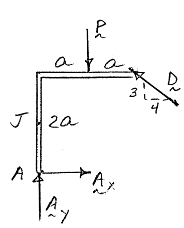
$$\rightarrow \Sigma F_x = 0: \mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{P}{2} - F = 0$$

$$\mathbf{F} = \frac{P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \mathbf{M} = 0 \blacktriangleleft$$

(*b*) **FBD Rod:**



$$\circlearrowleft \Sigma M_A = 0$$

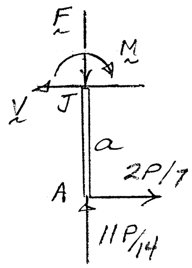
$$2a\left(\frac{4}{5}D\right) + 2a\left(\frac{3}{5}D\right) - aP = 0 \qquad D = \frac{5P}{14}$$

$$\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\frac{5}{14}P = 0 \qquad A_x = \frac{2P}{7}$$

$$\uparrow \Sigma F_y = 0: A_y - P + \frac{3}{5}\frac{5}{14}P = 0 \qquad A_y = \frac{11P}{14}$$

PROBLEM 7.22 CONTINUED

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \frac{2}{7}P - V = 0$$

$$\mathbf{V} = \frac{2P}{7} \leftarrow$$ ◀

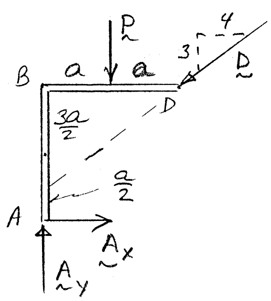
$$\uparrow \Sigma F_y = 0: \frac{11P}{14} - F = 0$$

$$\mathbf{F} = \frac{11P}{14} \downarrow$$ ◀

$$\Sigma M_J = 0: a\frac{2P}{7} - M = 0$$

$$\mathbf{M} = \frac{2}{7}aP$$ ◀

(c) **FBD Rod:**

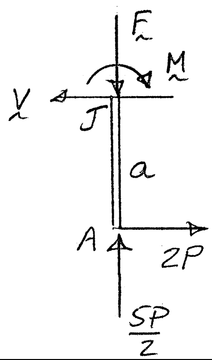


$$\Sigma M_A = 0: \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:



$$\rightarrow \Sigma F_x = 0: 2P - V = 0$$

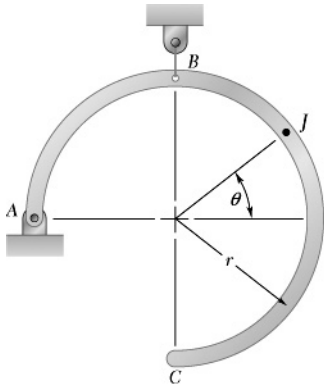
$$\mathbf{V} = 2P \leftarrow$$ ◀

$$\uparrow \Sigma F_y = 0: \frac{5P}{2} - F = 0$$

$$\mathbf{F} = \frac{5P}{2} \downarrow$$ ◀

$$\Sigma M_J = 0: a(2P) - M = 0$$

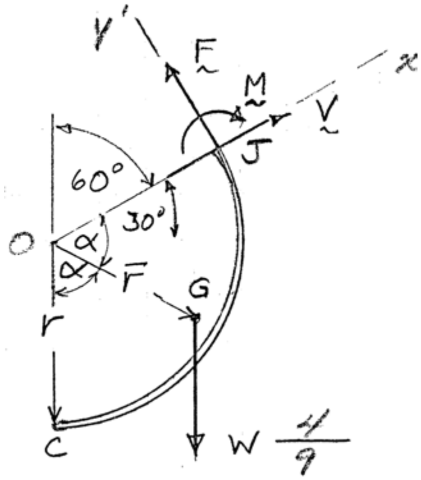
$$\mathbf{M} = 2aP$$ ◀



PROBLEM 7.23

A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 30°$.

SOLUTION



Note $\alpha = \dfrac{180° - 60°}{2} = 60° = \dfrac{\pi}{3}$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{3r}{\pi}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}r$$

$$\text{Weight of section} = W\frac{120}{270} = \frac{4}{9}W$$

$$\nwarrow \Sigma F_{y'} = 0:\ F - \frac{4}{9}W\cos 30° = 0 \qquad F = \frac{2\sqrt{3}}{9}W$$

$$\circlearrowleft \Sigma M_0 = 0:\ rF - (\bar{r}\sin 60°)\frac{4W}{9} - M = 0$$

$$M = r\left[\frac{2\sqrt{3}}{9} - \frac{3\sqrt{3}}{2\pi}\frac{\sqrt{3}}{2}\frac{4}{9}\right]W = \left[\frac{2\sqrt{3}}{9} - \frac{1}{\pi}\right]Wr$$

$$\mathbf{M} = 0.0666Wr \circlearrowright \blacktriangleleft$$