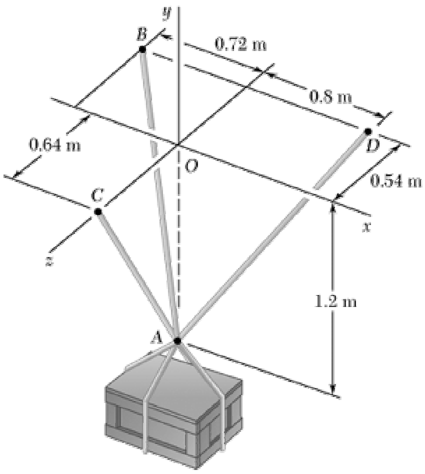
PROBLEM 2.108

A 750-kg crate is supported by three cables as shown. Determine the tension in each cable.



SOLUTION

See Problem 2.105 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

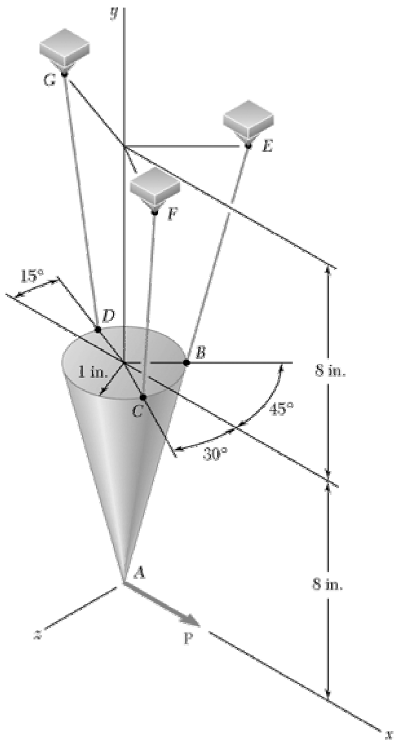
Substituting $W = (750 \text{ kg})(9.81 \text{ m/s}^2) = 7.36 \text{ kN}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$T_{AB} = 2.63 \text{ kN}$ ◀

$T_{AC} = 3.82 \text{ kN}$ ◀

$T_{AD} = 2.43 \text{ kN}$ ◀

PROBLEM 2.109



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex A of the cone. Knowing that $P = 0$ and that the tension in cord BE is 0.2 lb, determine the weight W of the cone.

SOLUTION

Note that because the line of action of each of the cords passes through the vertex A of the cone, the cords all have the same length, and the unit vectors lying along the cords are parallel to the unit vectors lying along the generators of the cone.

Thus, for example, the unit vector along BE is identical to the unit vector along the generator AB.

Hence:
$$\boldsymbol{\lambda}_{AB} = \boldsymbol{\lambda}_{BE} = \frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}$$

It follows that:
$$\mathbf{T}_{BE} = T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\left(\frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{CF} = T_{CF}\boldsymbol{\lambda}_{CF} = T_{CF}\left(\frac{\cos 30°\mathbf{i} + 8\mathbf{j} + \sin 30°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{DG} = T_{DG}\boldsymbol{\lambda}_{DG} = T_{DG}\left(\frac{-\cos 15°\mathbf{i} + 8\mathbf{j} - \sin 15°\mathbf{k}}{\sqrt{65}}\right)$$

PROBLEM 2.109 CONTINUED

At A: $\Sigma\mathbf{F} = 0$: $\mathbf{T}_{BE} + \mathbf{T}_{CF} + \mathbf{T}_{DG} + \mathbf{W} + \mathbf{P} = 0$

Then, isolating the factors of **i**, **j**, and **k**, we obtain three algebraic equations:

$$\mathbf{i}: \quad \frac{T_{BE}}{\sqrt{65}}\cos 45° + \frac{T_{CF}}{\sqrt{65}}\cos 30° - \frac{T_{DG}}{\sqrt{65}}\cos 15° + P = 0$$

or

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° + P\sqrt{65} = 0 \qquad (1)$$

$$\mathbf{j}: \quad T_{BE}\frac{8}{\sqrt{65}} + T_{CF}\frac{8}{\sqrt{65}} + T_{DG}\frac{8}{\sqrt{65}} - W = 0$$

or

$$T_{BE} + T_{CF} + T_{DG} - W\frac{\sqrt{65}}{8} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -\frac{T_{BE}}{\sqrt{65}}\sin 45° + \frac{T_{CF}}{\sqrt{65}}\sin 30° - \frac{T_{DG}}{\sqrt{65}}\sin 15° = 0$$

or

$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = 0 \qquad (3)$$

With $P = 0$ and the tension in cord $BE = 0.2$ lb:

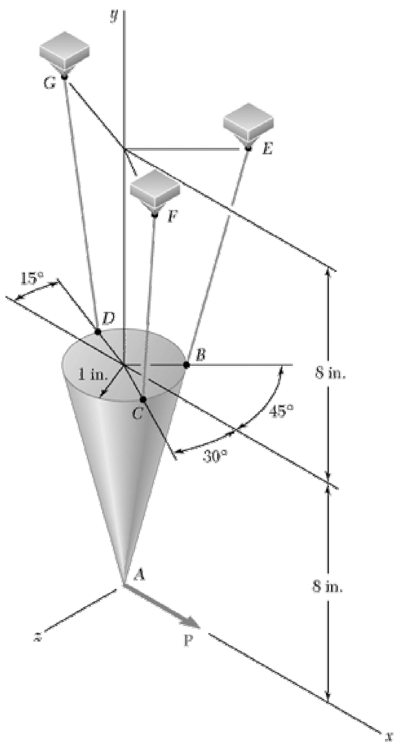
Solving the resulting Equations (1), (2), and (3) using conventional methods in Linear Algebra (elimination, matrix methods or iteration – with MATLAB or Maple, for example), we obtain:

$$T_{CF} = 0.669 \text{ lb}$$

$$T_{DG} = 0.746 \text{ lb}$$

$W = 1.603$ lb ◀

PROBLEM 2.110



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 1.6 lb, determine the range of values of *P* for which cord *CF* is taut.

SOLUTION

See Problem 2.109 for the Figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$\mathbf{i}: \quad T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° + \sqrt{65}P = 0 \tag{1}$$

$$\mathbf{j}: \quad T_{BE} + T_{CF} + T_{DG} - W\frac{\sqrt{65}}{8} = 0 \tag{2}$$

$$\mathbf{k}: \quad -T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = 0 \tag{3}$$

With $W = 1.6$ lb, the range of values of *P* for which the cord *CF* is taut can found by solving Equations (1), (2), and (3) for the tension T_{CF} as a function of *P* and requiring it to be positive (>0).

Solving (1), (2), and (3) with unknown *P*, using conventional methods in Linear Algebra (elimination, matrix methods or iteration – with MATLAB or Maple, for example), we obtain:

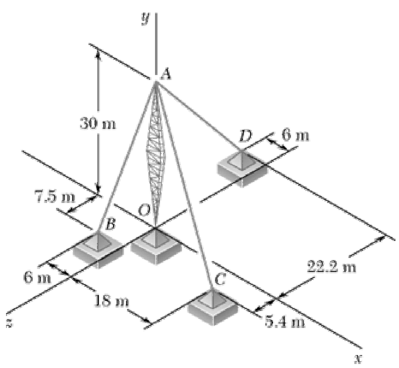
$$T_{CF} = (-1.729P + 0.668)\text{lb}$$

Hence, for $T_{CF} > 0$ $\qquad -1.729P + 0.668 > 0$

or $\qquad P < 0.386$ lb

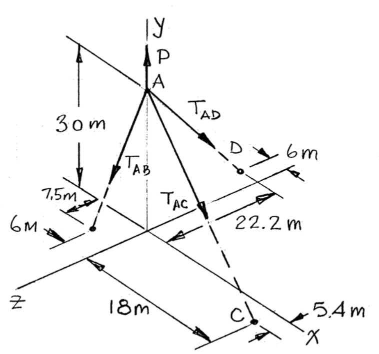
$$\therefore \quad 0 < P < 0.386 \text{ lb} \blacktriangleleft$$

PROBLEM 2.111



A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 3.6 kN, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION



The force in each cable can be written as the product of the magnitude of the force and the unit vector along the cable. That is, with

$$\overrightarrow{AC} = (18\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}$$

$$AC = \sqrt{(18\text{ m})^2 + (-30\text{ m})^2 + (5.4\text{ m})^2} = 35.4\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{35.4\text{ m}}\left[(18\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.5085\mathbf{i} - 0.8475\mathbf{j} + 0.1525\mathbf{k})$$

and
$$\overrightarrow{AB} = -(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (7.5\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-6\text{ m})^2 + (-30\text{ m})^2 + (7.5\text{ m})^2} = 31.5\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{31.5\text{ m}}\left[-(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (7.5\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.1905\mathbf{i} - 0.9524\mathbf{j} + 0.2381\mathbf{k})$$

Finally
$$\overrightarrow{AD} = -(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} - (22.2\text{ m})\mathbf{k}$$

$$AD = \sqrt{(-6\text{ m})^2 + (-30\text{ m})^2 + (-22.2\text{ m})^2} = 37.8\text{ m}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{37.8\text{ m}}\left[-(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} - (22.2\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}(-0.1587\mathbf{i} - 0.7937\mathbf{j} - 0.5873\mathbf{k})$$

PROBLEM 2.111 CONTINUED

With $\mathbf{P} = P\mathbf{j}$, at A:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -0.1905T_{AB} + 0.5085T_{AC} - 0.1587T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -0.9524T_{AB} - 0.8475T_{AC} - 0.7937T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad 0.2381T_{AB} + 0.1525T_{AC} - 0.5873T_{AD} = 0 \tag{3}$$

In Equations (1), (2) and (3), set $T_{AB} = 3.6$ kN, and, using conventional methods for solving Linear Algebraic Equations (MATLAB or Maple, for example), we obtain:

$$T_{AC} = 1.963 \text{ kN}$$

$$T_{AD} = 1.969 \text{ kN}$$

$$\mathbf{P} = 6.66 \text{ kN} \uparrow \blacktriangleleft$$