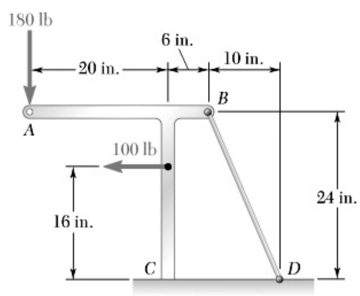
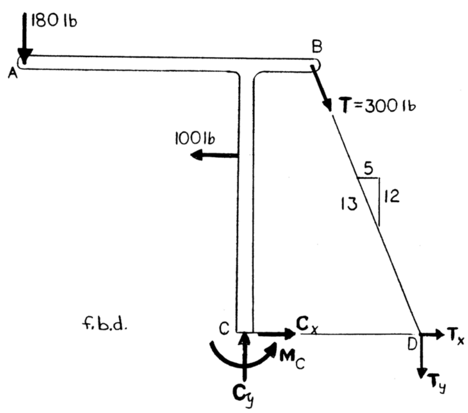
PROBLEM 4.51



Knowing that the tension in wire *BD* is 300 lb, determine the reaction at fixed support *C* for the frame shown.

SOLUTION



From f.b.d. of frame with $T = 300$ lb

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 100 \text{ lb} + \left(\frac{5}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_x = -15.3846 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 15.3846 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 180 \text{ lb} - \left(\frac{12}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_y = 456.92 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 456.92 \text{ lb} \uparrow$$

Then
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(15.3846)^2 + (456.92)^2} = 457.18 \text{ lb}$$

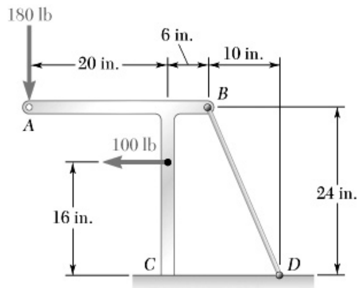
and
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{456.92}{-15.3846}\right) = -88.072°$$

or $\mathbf{C} = 457$ lb $\measuredangle$ 88.1° ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (180 \text{ lb})(20 \text{ in.}) + (100 \text{ lb})(16 \text{ in.}) - \left[\left(\frac{12}{13}\right)300 \text{ lb}\right](16 \text{ in.}) = 0$$

$$\therefore \quad M_C = -769.23 \text{ lb} \cdot \text{in.}$$

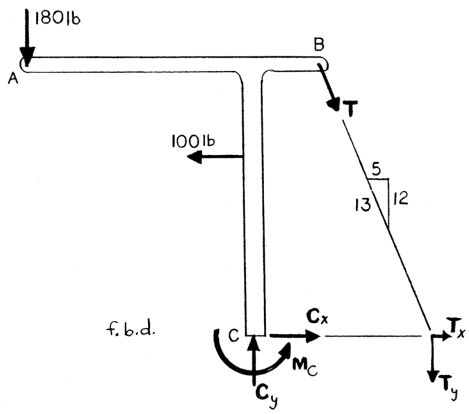
or $\mathbf{M}_C = 769$ lb·in. $\circlearrowright$ ◀



PROBLEM 4.52

Determine the range of allowable values of the tension in wire BD if the magnitude of the couple at the fixed support C is not to exceed $75\ \text{lb}\cdot\text{ft}$.

SOLUTION



$T_{\max}$ From f.b.d. of frame with $\mathbf{M}_C = 75\ \text{lb}\cdot\text{ft} = 900\ \text{lb}\cdot\text{in.}$

$$+\circlearrowleft\ \Sigma M_C = 0:\quad 900\ \text{lb}\cdot\text{in.} + (180\ \text{lb})(20\ \text{in.}) + (100\ \text{lb})(16\ \text{in.}) - \left[\left(\frac{12}{13}\right)T_{\max}\right](16\ \text{in.}) = 0$$

$$\therefore\quad T_{\max} = 413.02\ \text{lb}$$

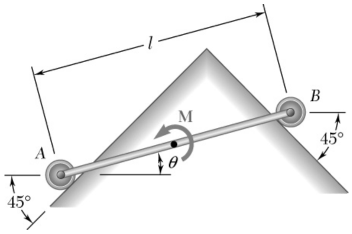
$T_{\min}$ From f.b.d. of frame with $\mathbf{M}_C = 75\ \text{lb}\cdot\text{ft} = 900\ \text{lb}\cdot\text{in.}$

$$+\circlearrowleft\ \Sigma M_C = 0:\quad -900\ \text{lb}\cdot\text{in.} + (180\ \text{lb})(20\ \text{in.}) + (100\ \text{lb})(16\ \text{in.}) - \left[\left(\frac{12}{13}\right)T_{\min}\right](16\ \text{in.}) = 0$$

$$\therefore\quad T_{\min} = 291.15\ \text{lb}$$

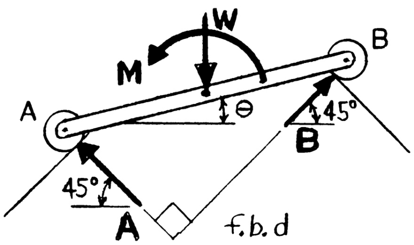
$$\therefore\quad 291\ \text{lb} \le T \le 413\ \text{lb}\ \blacktriangleleft$$

PROBLEM 4.53



Uniform rod AB of length l and weight W lies in a vertical plane and is acted upon by a couple **M**. The ends of the rod are connected to small rollers which rest against frictionless surfaces. (a) Express the angle θ corresponding to equilibrium in terms of M, W, and l. (b) Determine the value of θ corresponding to equilibrium when $M = 1.5\text{ lb}\cdot\text{ft}$, $W = 4\text{ lb}$, and $l = 2\text{ ft}$.

SOLUTION



(a) From f.b.d. of uniform rod AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A\cos 45° + B\cos 45° = 0$$

$$\therefore\ -A + B = 0 \qquad \text{or} \qquad B = A \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad A\sin 45° + B\sin 45° - W = 0$$

$$\therefore\ A + B = \sqrt{2}W \tag{2}$$

From Equations (1) and (2)

$$2A = \sqrt{2}W$$

$$\therefore\ A = \frac{1}{\sqrt{2}}W$$

From f.b.d. of uniform rod AB

$$+\circlearrowleft \Sigma M_B = 0: \quad W\left[\left(\frac{l}{2}\right)\cos\theta\right] + M$$

$$-\left(\frac{1}{\sqrt{2}}W\right)\left[l\cos\left(45° - \theta\right)\right] = 0 \tag{3}$$

From trigonometric identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Equation (3) becomes

$$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\left(\cos\theta + \sin\theta\right) = 0$$

PROBLEM 4.53 CONTINUED

or $$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\cos\theta - \left(\frac{Wl}{2}\right)\sin\theta = 0$$

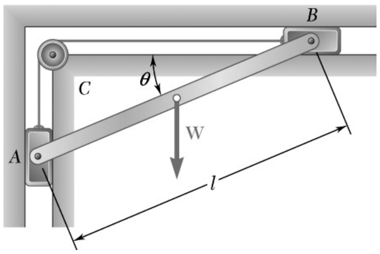
$$\therefore \quad \sin\theta = \frac{2M}{Wl}$$

$$\text{or} \quad \theta = \sin^{-1}\left(\frac{2M}{Wl}\right) \blacktriangleleft$$

(*b*) $$\theta = \sin^{-1}\left[\frac{2(1.5\text{ lb}\cdot\text{ft})}{(4\text{ lb})(2\text{ ft})}\right] = 22.024°$$

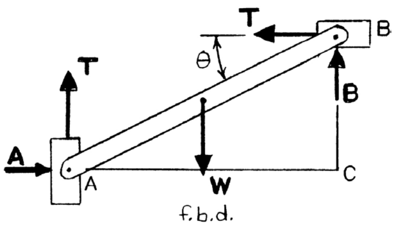
$$\text{or} \quad \theta = 22.0° \blacktriangleleft$$

PROBLEM 4.54



A slender rod *AB*, of weight *W*, is attached to blocks *A* and *B*, which move freely in the guides shown. The blocks are connected by an elastic cord which passes over a pulley at *C*. (*a*) Express the tension in the cord in terms of *W* and θ. (*b*) Determine the value of θ for which the tension in the cord is equal to 3*W*.

SOLUTION



(*a*) From f.b.d. of rod *AB*

$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{l}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$\therefore\ T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by $\cos\theta$,

$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right)$$

$$\text{or}\quad T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

(*b*) For $T = 3W$,

$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

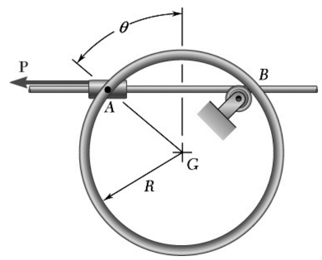
$$\therefore\ 1 - \tan\theta = \frac{1}{6}$$

or

$$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806^\circ$$

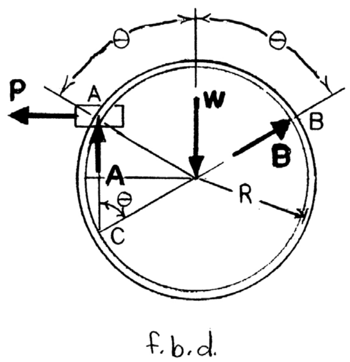
$$\text{or}\quad \theta = 39.8^\circ \blacktriangleleft$$

PROBLEM 4.55



A thin, uniform ring of mass m and radius R is attached by a frictionless pin to a collar at A and rests against a small roller at B. The ring lies in a vertical plane, and the collar can move freely on a horizontal rod and is acted upon by a horizontal force **P**. (a) Express the angle θ corresponding to equilibrium in terms of m and P. (b) Determine the value of θ corresponding to equilibrium when $m = 500$ g and $P = 5$ N.

SOLUTION



(a) From f.b.d. of ring

$$+\circlearrowleft \Sigma M_C = 0: \quad P(R\cos\theta + R\cos\theta) - W(R\sin\theta) = 0$$

$$2P = W\tan\theta \quad \text{where} \quad W = mg$$

$$\therefore \quad \tan\theta = \frac{2P}{mg}$$

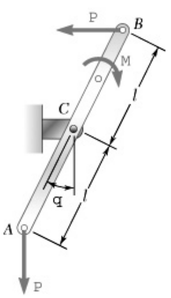
$$\text{or} \quad \theta = \tan^{-1}\left(\frac{2P}{mg}\right) \blacktriangleleft$$

(b) Have $\quad m = 500 \text{ g} = 0.500 \text{ kg} \;\text{ and }\; P = 5 \text{ N}$

$$\therefore \quad \theta = \tan^{-1}\left[\frac{2(5 \text{ N})}{(0.500 \text{ kg})(9.81 \text{ m/s}^2)}\right]$$

$$= 63.872°$$

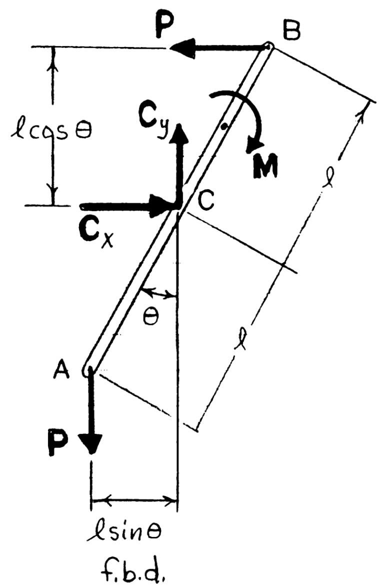
$$\text{or} \quad \theta = 63.9° \blacktriangleleft$$



PROBLEM 4.56

Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150$ lb · in., $P = 20$ lb, and $l = 6$ in.

SOLUTION



(a) From f.b.d. of rod AB

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or} \quad \sin\theta + \cos\theta = \frac{M}{Pl} \blacktriangleleft$$

(b) For $M = 150$ lb·in., $P = 20$ lb, and $l = 6$ in.

$$\sin\theta + \cos\theta = \frac{150 \text{ lb}\cdot\text{in.}}{(20 \text{ lb})(6 \text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\quad \sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + \left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25$$

$$\left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(6.25) - 4(2)(0.5625)}}{2(2)}$$

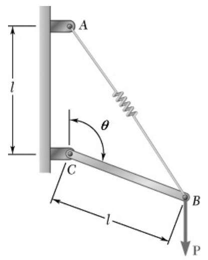
$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\quad \sin\theta = 0.95572 \quad$ and $\quad \sin\theta = 0.29428$

$$\therefore \quad \theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

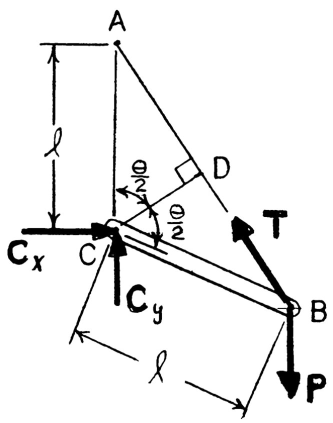
$$\text{or} \quad \theta = 17.11° \text{ and } \theta = 72.9° \blacktriangleleft$$

PROBLEM 4.57



A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 90^{\circ}$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to equilibrium in terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium when $P = \frac{1}{4}kl$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where

$$s = \text{elongation of spring}$$

$$= \left(\overline{AB}\right)_{\theta} - \left(\overline{AB}\right)_{\theta=90^{\circ}}$$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin\left(\frac{90^{\circ}}{2}\right)$$

$$= 2l\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]$$

$$\therefore \quad T = 2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] \qquad (1)$$

(*a*) From f.b.d. of rod *BC*

$$+\circlearrowleft \Sigma M_C = 0: \quad T\left[l\cos\left(\frac{\theta}{2}\right)\right] - P\left(l\sin\theta\right) = 0$$

Substituting *T* From Equation (1)

$$2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\left[l\cos\left(\frac{\theta}{2}\right)\right] - P\left(l\sin\theta\right) = 0$$

$$2kl^2\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\cos\left(\frac{\theta}{2}\right) - Pl\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = 0$$

Factoring out $\quad 2l\cos\left(\frac{\theta}{2}\right)$, leaves

PROBLEM 4.57 CONTINUED

$$kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] - P\sin\left(\frac{\theta}{2}\right) = 0$$

or

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\left(\frac{kl}{kl - P}\right)$$

$$\therefore \quad \theta = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl - P\right)}\right] \blacktriangleleft$$

(*b*) $P = \dfrac{kl}{4}$

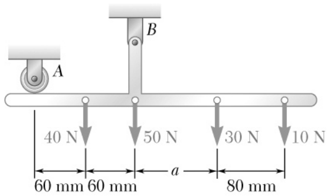
$$\theta = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl - \frac{kl}{4}\right)}\right] = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}}\left(\frac{4}{3\,\mathrm{kl}}\right)\right] = 2\sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$$

$$= 2\sin^{-1}\left(0.94281\right)$$

$$= 141.058°$$

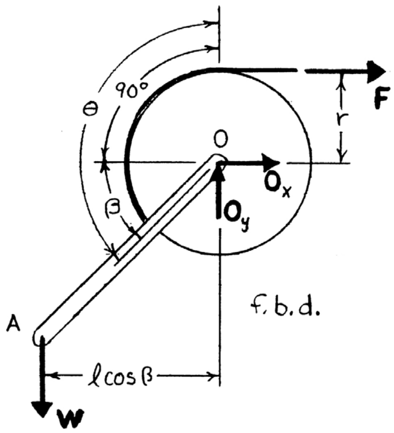
$$\text{or} \quad \theta = 141.1° \blacktriangleleft$$

PROBLEM 4.58



Solve Sample Problem 4.5 assuming that the spring is unstretched when $\theta = 90^\mathrm{o}$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where

$$s = \text{deformation of spring}$$

$$= r\beta$$

$$\therefore \quad F = kr\beta$$

From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_0 = 0: \quad W(l\cos\beta) - F(r) = 0$$

or

$$Wl\cos\beta - kr^2\beta = 0$$

$$\therefore \quad \cos\beta = \frac{kr^2}{Wl}\beta$$

For $\quad k = 250 \text{ lb/in.}, \; r = 3 \text{ in.}, \; l = 8 \text{ in.}, \; W = 400 \text{ lb}$

$$\cos\beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\beta$$

or

$$\cos\beta = 0.703125\beta$$

Solving numerically,

$$\beta = 0.89245 \text{ rad}$$

or

$$\beta = 51.134°$$

Then

$$\theta = 90° + 51.134° = 141.134°$$

or $\theta = 141.1°$ ◀