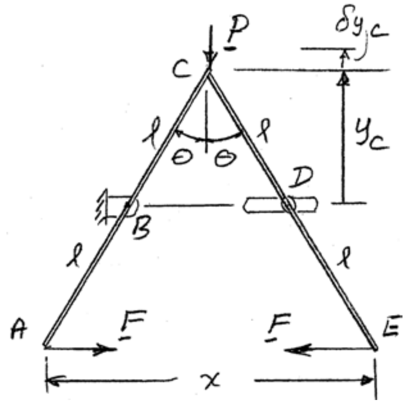
PROBLEM 10.31

Solve Problem 10.30 assuming that force **P** is moved to *C* and acts vertically downward.

SOLUTION



$$y_C = l\cos\theta, \qquad \delta y_C = -l\sin\theta\delta\theta$$

Spring:

$$\text{Unstretched length} = 2l$$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_C - F\delta x$$

$$-P(-l\sin\theta\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\delta\theta) = 0$$

$$P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or

$$\frac{P}{8kl} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

With $l = 200$ mm, $k = 300$ N/m, and $P = 160$ N

$$\frac{(160\text{ N})}{8(300\text{ N/m})(0.2)} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

or

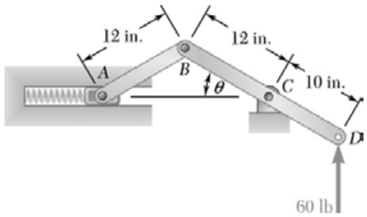
$$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} = \frac{1}{3}$$

Solving numerically, $\theta = 39.65°$

and $\theta = 68.96°$

$\theta = 39.7°$ ◀

and $\theta = 69.0°$ ◀



PROBLEM 10.32

For the mechanism shown, block A can move freely in its guide and rests against a spring of constant 15 lb/in. that is undeformed when $\theta = 45°$. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION

First note $$y_D = 10\sin\theta \text{ (in.)}$$

Then $$\delta y_D = 10\cos\theta\delta\theta \text{ (in.)}$$

Also $$x_A = 2(12\cos\theta) \text{ in.}$$

Then $$(x_A)_0 = (24 \text{ in.})\cos 45°$$

and $$\delta x_A = -24\sin\theta\delta\theta \text{ (in.)}$$

With $\delta\theta < 0$: Virtual Work: $$\delta U = 0: \quad (60 \text{ lb})\delta y_D - F_{SP}|\delta x_A| = 0$$

where $$F_{SP} = k\left[x_A - (x_A)_0\right]$$

$$= (15 \text{ lb/in.})(24\cos\theta - 24\cos 45°)(\text{in.})$$

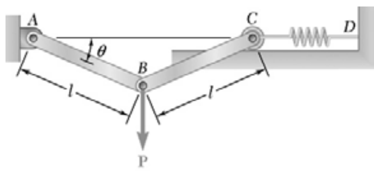
$$= (360 \text{ lb})(\cos\theta - \cos 45°)$$

Then $$(60)(10\cos\theta\delta\theta) - \left[360(\cos\theta - \cos 45°)\right](24\sin\theta\delta\theta) = 0$$

or $$5 - 72\tan\theta(\cos\theta - \cos 45°) = 0$$

Solving numerically, $\theta = 15.03°$ and $\theta = 36.9°$ ◀

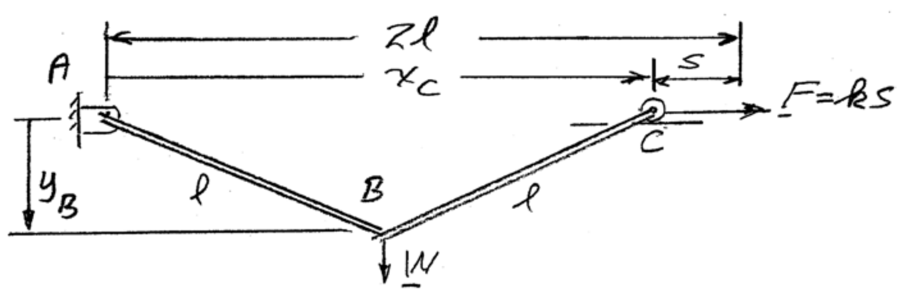
PROBLEM 10.33 AND 10.34



10.33: A force **P** of magnitude 150 lb is applied to the linkage at B. The constant of the spring is 12.5 lb/in., and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that $l = 15$ in., determine the value of θ corresponding to equilibrium.

10.34: A vertical force **P** is applied to the linkage at B. The constant of the spring is k, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ, P, l, and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \qquad \delta x_C = -2l\sin\theta\delta\theta$$

$$y_B = l\sin\theta \qquad \delta y_B = l\cos\theta\delta\theta$$

$$F = ks = k(2l - x_C) = 2kl(1 - \cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

Problem 10.33: Given: $l = 0.3$ m, $W = 600$ N, $k = 2500$ N/m

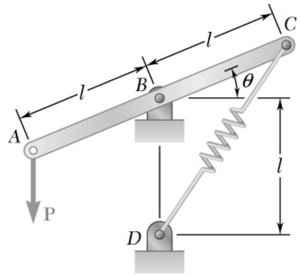
Then $$(1-\cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

or $$(1-\cos\theta)\tan\theta = 0.2$$

Solving numerically, $$\theta = 40.22°$$

$$\theta = 40.2° \blacktriangleleft$$

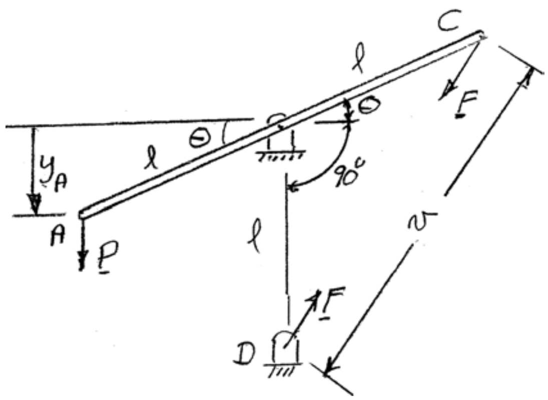
Problem 10.34: From above $$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$



PROBLEM 10.35

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated. $P = 150$ lb, $l = 30$ in., $k = 40$ lb/in.

SOLUTION



$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ:

$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$\delta v = l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta$$

Stretched length:

$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

Then $$F = ks = kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta = 0$$

or $$\frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

PROBLEM 10.35 CONTINUED

Now, with $P = 150$ lb, $l = 30$ in., and $k = 40$ lb/in.

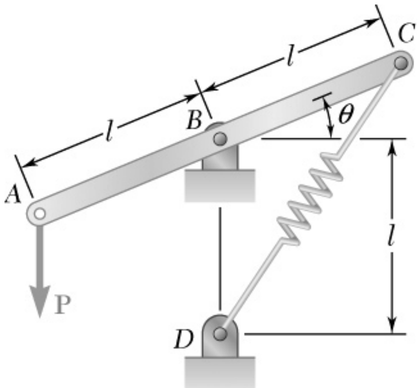
$$\frac{(150\text{ lb})}{(40\text{ lb/in.})(30\text{ in.})} = 1 - \sqrt{2}\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta} = 0.61872$$

Solving numerically, $\theta = 17.825°$

$\theta = 17.83°$ ◀



PROBLEM 10.36

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated. $P = 600$ N, $l = 800$ mm, $k = 4$ kN/m.

SOLUTION

From the analysis of Problem 10.35, we have

$$\frac{P}{kl} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

With $P = 600$ N, $l = 800$ mm, and $k = 4$ kN/m

$$\frac{(600\text{ N})}{(4000\text{ N/m})(0.8\text{ m})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

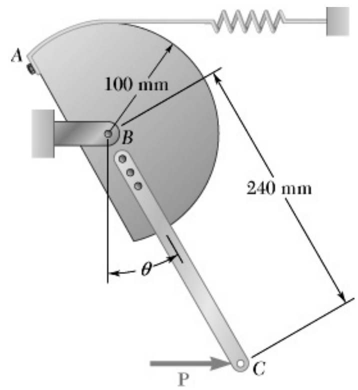
or

$$\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta} = 0.57452$$

Solving numerically, $\theta = 30.98°$

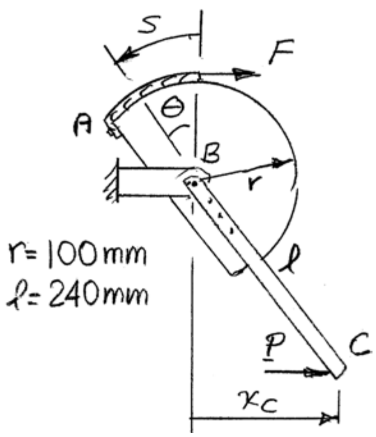
$\theta = 31.0°$ ◀

PROBLEM 10.37



A horizontal force **P** of magnitude 160 N is applied to the mechanism at C. The constant of the spring is $k = 1.8\,\text{kN/m}$, and the spring is unstretched when $\theta = 0$. Neglecting the mass of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION



Have $s = r\theta \qquad \delta s = r\delta\theta$

$$F = ks = kr\theta$$

and $x_C = l\sin\theta$

$$\delta x_C = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_C - F\delta s = 0$$

$$Pl\cos\theta\delta\theta - kr\theta(r\delta\theta) = 0$$

or

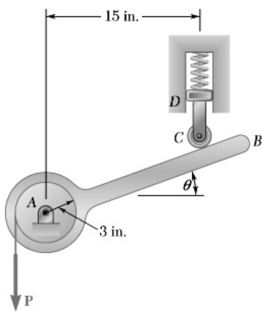
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\theta}$$

$$\frac{(160\text{ N})(0.24\text{ m})}{(1800\text{ N/m})(0.1\text{ m})^2} = \frac{\theta}{\cos\theta}$$

$$2.1333 = \frac{\theta}{\cos\theta}$$

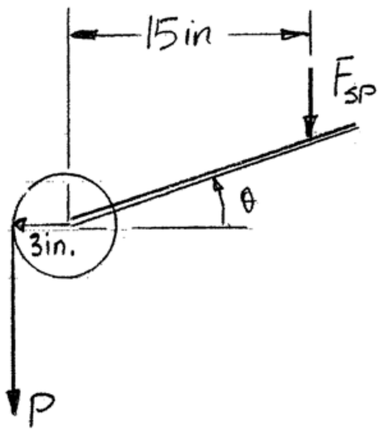
Solving numerically, $\theta = 1.054\text{ rad} = 60.39°$ $\qquad \theta = 60.4°$ ◀

PROBLEM 10.38



A cord is wrapped around drum *A* which is attached to member *AB*. Block *D* can move freely in its guide and is fastened to link *CD*. Neglecting the weight of *AB* and knowing that the spring is of constant 4 lb/in. and is undeformed when $\theta = 0$, determine the value of θ corresponding to equilibrium when a downward force **P** of magnitude 96 lb is applied to the end of the cord.

SOLUTION



Have $$y_C = 15\tan\theta \ (\text{in.})$$

Then $$\delta y_C = 15\sec^2\theta\delta\theta \ (\text{in.})$$

Virtual Work: $$\delta U = 0: \quad P\delta s_P - F_{SP}\delta y_C = 0$$

where $$\delta s_P = (3 \text{ in.})\delta\theta$$

and $$F_{SP} = ky_C$$

$$= (4 \text{ lb/in.})(15 \text{ in.})\tan\theta$$

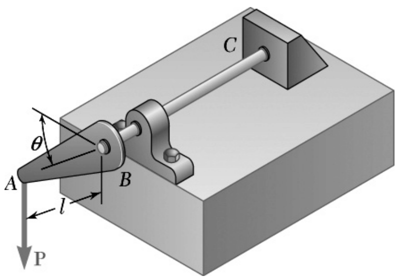
$$= 60\tan\theta \ (\text{lb})$$

Then $$(96 \text{ lb})(3 \text{ in.})\delta\theta - \left[(60\tan\theta) \text{ lb}\right]\left[(15\sec^2\theta\delta\theta) \text{ in.}\right] = 0$$

or $$3.125\tan\theta\sec^2\theta = 1$$

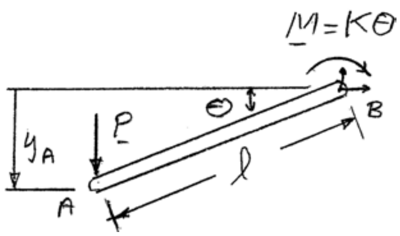
Solving numerically, $$\theta = 16.41° \blacktriangleleft$$

PROBLEM 10.39



The lever AB is attached to the horizontal shaft BC which passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 400$ lb, $l = 10$ in., and $K = 150$ lb · ft/rad.

SOLUTION



Have $$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\delta\theta - K\theta\delta\theta = 0$$

or $$\frac{\theta}{\cos\theta} = \frac{Pl}{K} \qquad (1)$$

With $P = 400$ lb, $l = 10$ in., and $K = 150$ lb·ft/rad

$$\frac{\theta}{\cos\theta} = \frac{(400\text{ lb})\left(\frac{10\text{ in.}}{12\text{ in./ft}}\right)}{150\text{ lb}\cdot\text{ft/rad}}$$

or $$\frac{\theta}{\cos\theta} = 2.2222$$

Solving numerically, $\theta = 61.25°$ $\qquad \theta = 61.2°$ ◀

PROBLEM 10.40

Solve Problem 10.39 assuming that $P = 1.26$ kips, $l = 10$ in., and $K = 150$ lb · ft/rad. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, and $360° < \theta < 450°$.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 1.26 \text{ kip}, \; l = 10 \text{ in., and } K = 150 \text{ lb}\cdot\text{ft/rad}$$

we have

$$\frac{\theta}{\cos\theta} = \frac{(1260 \text{ lb})\left(\dfrac{10 \text{ in.}}{12 \text{ in./ft}}\right)}{150 \text{ lb}\cdot\text{ft/rad}}$$

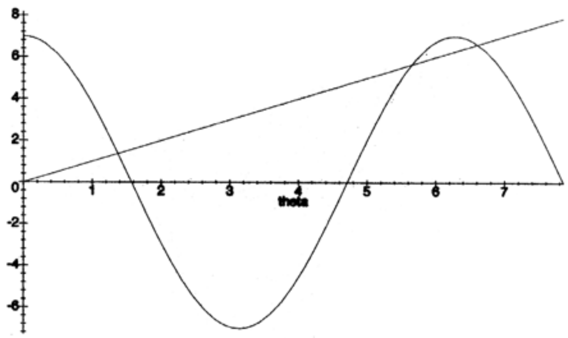
or

$$\frac{\theta}{\cos\theta} = 7 \qquad \text{or} \qquad \theta = 7\cos\theta \qquad (1)$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: plot({theta, 7 * cos(theta)}, t = 0..5 * Pi/2);

Thus, we plot $y = \theta$ and $y = 7\cos\theta$ in the range

$$0 \le \theta \le \frac{5\pi}{2}$$



We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ.

$$0 \le \theta \le 90°\left(\frac{\pi}{2}\right); \qquad \theta = 1.37333 \text{ rad}, \qquad \theta = 78.69° \qquad \theta = 78.7° \blacktriangleleft$$

$$270 \le \theta \le 360°\left(\frac{3\pi}{2} \le \theta \le 2\pi\right); \qquad \theta = 5.65222 \text{ rad}, \qquad \theta = 323.85° \qquad \theta = 324° \blacktriangleleft$$

$$360 \le \theta \le 450°\left(2\pi \le \theta \le \frac{5\pi}{2}\right); \qquad \theta = 6.61597 \text{ rad}, \qquad \theta = 379.07 \qquad \theta = 379° \blacktriangleleft$$