PROBLEM 8.101

A hawser is wrapped two full turns around a bollard. By exerting a 320-N force on the free end of the hawser, a dockworker can resist a force of 20 kN on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 80-kN force is to be resisted by the same 320-N force.

SOLUTION

Two full turns of rope $\rightarrow$ $\quad \beta = 4\pi$ rad

(*a*)

$$\mu_s \beta = \ln \frac{T_2}{T_1} \qquad \text{or} \qquad \mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{4\pi} \ln \frac{20\ 000\ \text{N}}{320\ \text{N}} = 0.329066$$

$$\mu_s = 0.329 \blacktriangleleft$$

(*b*)

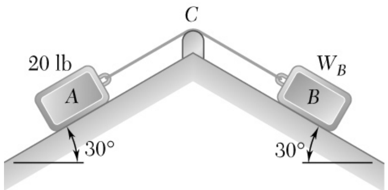
$$\beta = \frac{1}{\mu_s} \ln \frac{T_2}{T_1}$$

$$= \frac{1}{0.329066} \ln \frac{80\ 000\ \text{N}}{320\ \text{N}}$$

$$= 16.799 \text{ rad}$$

$$\beta = 2.67 \text{ turns} \blacktriangleleft$$

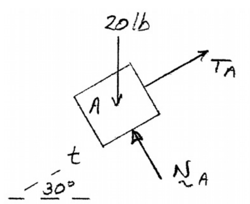
PROBLEM 8.102

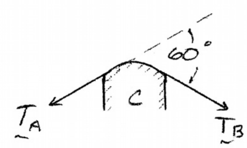


Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that motion of block B up the incline is impending when $W_B = 16$ lb, determine (a) the coefficient of static friction between the rope and the support, (b) the largest value of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

FBD A:

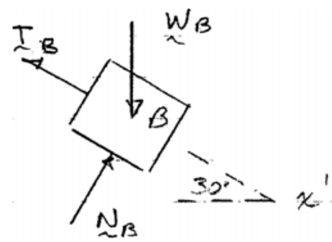




$$\nearrow \Sigma F_t = 0: \quad T_A - 20 \text{ lb} \sin 30° = 0$$

$$T_A = 10 \text{ lb}$$

FBD B:



$$\searrow \Sigma F_{x'} = 0: \quad W_B \sin 30° - T_B = 0$$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \dfrac{\pi}{3}$ rad regardless of shape of support C

(a) For motion of B up incline when $W_B = 16$ lb, $\qquad T_B = \dfrac{W_B}{2} = 8$ lb

and $\qquad \mu_s \beta = \ln \dfrac{T_A}{T_B} \qquad$ or $\qquad \mu_s = \dfrac{1}{\beta} \ln \dfrac{T_A}{T_B} = \dfrac{3}{\pi} \ln \dfrac{10 \text{ lb}}{8 \text{ lb}} = 0.213086$

$$\mu_s = 0.213 \blacktriangleleft$$

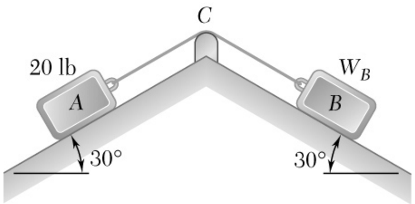
(b) For maximum W_B, motion of B impends down and $T_B > T_A$

So $\qquad T_B = T_A e^{\mu_s \beta} = (10 \text{ lb}) e^{0.213086\pi/3} = 12.500$ lb

Now $\qquad W_B = 2T_B$

So that $\qquad W_B = 25.0 \text{ lb} \blacktriangleleft$

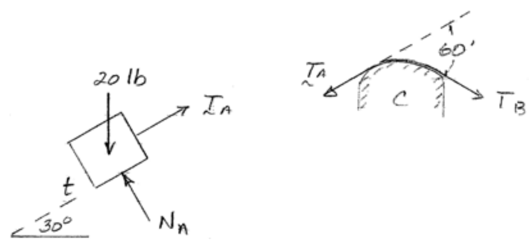
PROBLEM 8.103



Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that the coefficient of static friction between the rope and the support is 0.50, determine the range of values of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

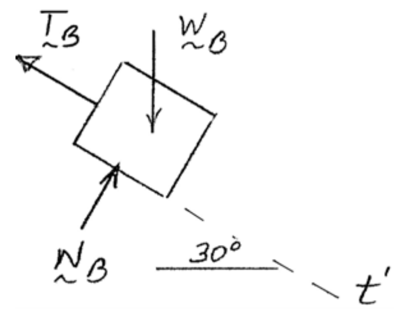
FBD A:



$$\nearrow \Sigma F_t = 0: \quad T_A - 20 \text{ lb} \sin 30° = 0$$

$$T_A = 10 \text{ lb}$$

FBD B:



$$\searrow \Sigma F_{t'} = 0: \quad W_B \sin 30° - T_B = 0$$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \frac{\pi}{3}$ rad, regardless of shape of support C.

For impending motion of B up, $T_A > T_B$, so

$$T_A = T_B e^{\mu_s \beta} \qquad \text{or} \qquad T_B = T_A e^{-\mu_s \beta} = (10 \text{ lb}) e^{-0.5\pi/3} = 5.924 \text{ lb}$$

$$W_B = 2T_B = 11.85 \text{ lb}$$

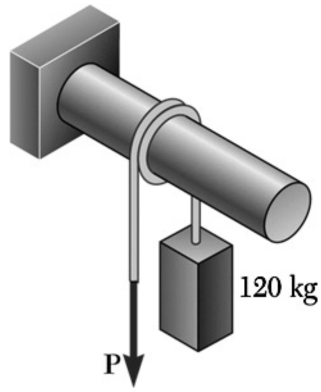
For impending motion of B down, $T_B > T_B$, so

$$T_B = T_A e^{\mu_s \beta} = (10 \text{ lb}) e^{0.5\pi/3} = 16.881 \text{ lb}$$

$$W_B = 2T_B = 33.76 \text{ lb}$$

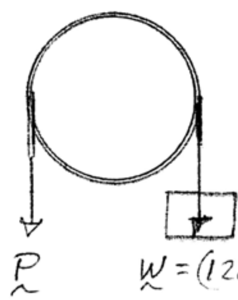
For equilibrium $\qquad 11.85 \text{ lb} \le W_B \le 33.8 \text{ lb}$ ◀

PROBLEM 8.104



A 120-kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$W = (120\text{ kg})(9.81\text{ m/s}^2)$$

$$= 1177.2\text{ N}$$

$$\beta = 1.5\text{ turns} = 3\pi\text{ rad}$$

For impending motion of W up

$$P = We^{\mu_s\beta} = (1177.2\text{ N})e^{(0.15)3\pi}$$

$$= 4839.7\text{ N}$$

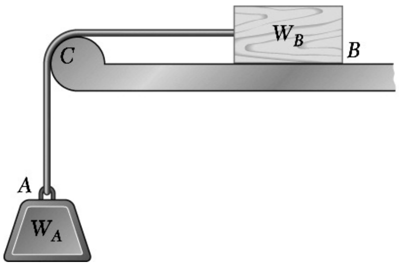
For impending motion of W down

$$P = We^{-\mu_s\beta} = (1177.2\text{ N})e^{-(0.15)3\pi}$$

$$= 286.3\text{ N}$$

For equilibrium

$$286\text{ N} \le P \le 4.84\text{ kN} \blacktriangleleft$$

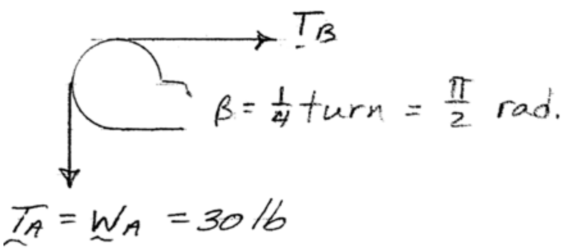


PROBLEM 8.105

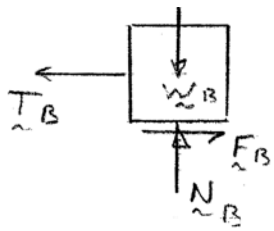
The coefficient of static friction between block B and the horizontal surface and between the rope and support C is 0.40. Knowing that $W_A = 30\text{ lb}$, determine the smallest weight of block B for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \qquad \text{or} \qquad N_B = W_B$$

Impending motion $\qquad F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$

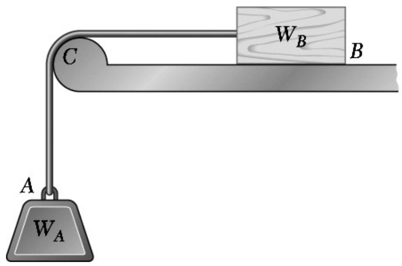
$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = 0.4 W_B$$

At support, for impending motion of W_A down, $\qquad W_A = T_B e^{\mu_s \beta}$

so $\qquad T_B = W_A e^{-\mu_s \beta} = (30\text{ lb}) e^{-(0.4)\pi/2} = 16.005\text{ lb}$

Now $\qquad W_B = \dfrac{T_B}{0.4}$

so that $\qquad W_B = 40.0\text{ lb}$ ◀

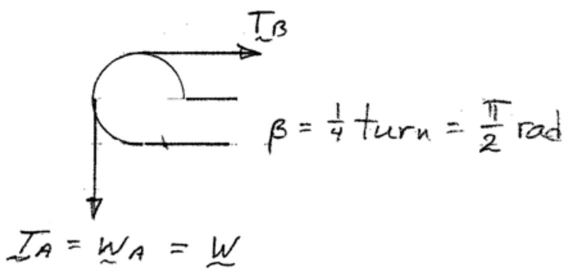


PROBLEM 8.106

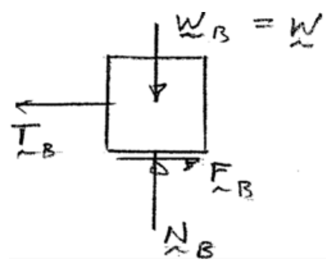
The coefficient of static friction μ_s is the same between block B and the horizontal surface and between the rope and support C. Knowing that $W_A = W_B$, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Support at C



FBD B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W = 0 \qquad \text{or} \qquad N_B = W$$

Impending motion: $$F_B = \mu_s N_B = \mu_s W$$

$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = \mu_s W$$

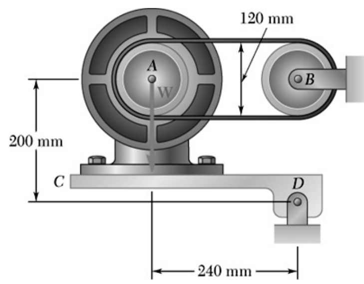
Impending motion of rope on support: $$W = T_B e^{\mu_s \beta} = \mu_s W e^{\mu_s \beta}$$

or $$1 = \mu_s e^{\mu_s \beta}$$

or $$\mu_s e^{\frac{\pi}{2}\mu_s} = 1$$

Solving numerically: $$\mu_s = 0.475 ◀$$

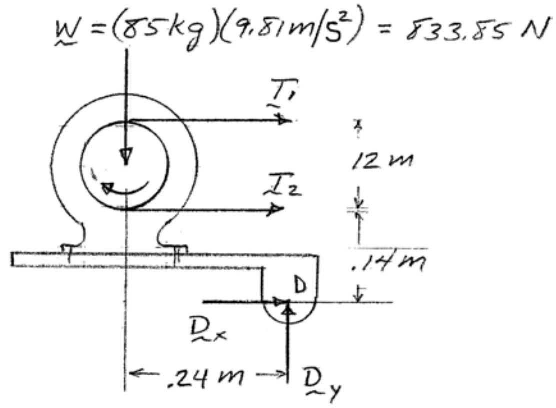
PROBLEM 8.107



In the pivoted motor mount shown, the weight **W** of the 85-kg motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest torque which can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor + mount:



For impending slipping of belt, $$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_1$$

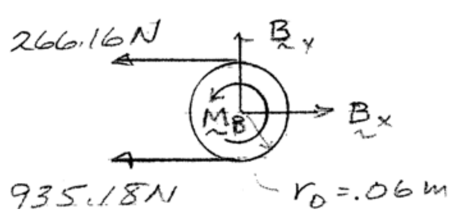
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})(833.85\text{ N}) - (0.14\text{ m})T_2 - (0.26\text{ m})T_1 = 0$$

$$\left[(0.14\text{ m})(3.5136) + 0.26\text{ m}\right]T_1 = 200.124\text{ N}\cdot\text{m}$$

or $$T_1 = 266.16\text{ N}$$

and $$T_2 = 3.5136 T_1 = 935.18\text{ N}$$

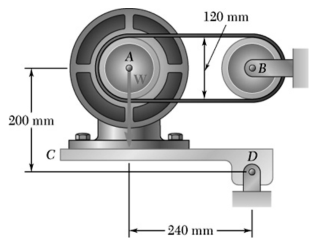
FBD drum:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (0.06\text{m})(266.16\text{ N} - 935.18\text{ N}) = 0$$

$$M_B = 40.1\text{ N}\cdot\text{m} \blacktriangleleft$$

(Compare to $M_B = 81.7\text{ N}\cdot\text{m}$ using V-belt, Problem 8.130.)

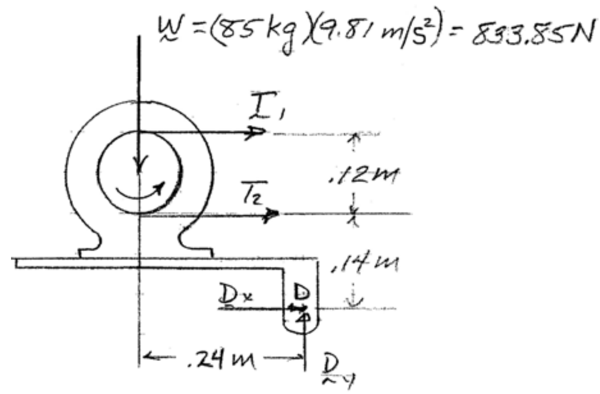


PROBLEM 8.108

Solve Problem 8.107 assuming that the drive drum *A* is rotating counterclockwise.

SOLUTION

FBD motor + mount:



Impending slipping of belt: $$T_1 = T_2 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_2$$

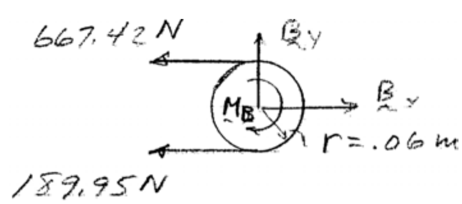
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})W - (0.26\text{ m})T_1 - (0.14\text{ m})T_2 = 0$$

$$\left[(0.26\text{ m})(3.5136) + 0.14\text{ m}\right]T_2 = (0.24\text{ m})(833.85\text{ N})$$

or $$T_2 = 189.95\text{ N}$$

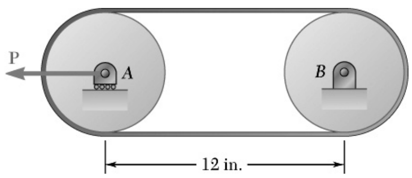
and $$T_1 = 667.42\text{ N}$$

FBD drum:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.06\text{ m})(667.42\text{ N} - 189.95\text{ N}) - M_B = 0$$

$$M_B = 28.6\text{ N}\cdot\text{m} \blacktriangleleft$$

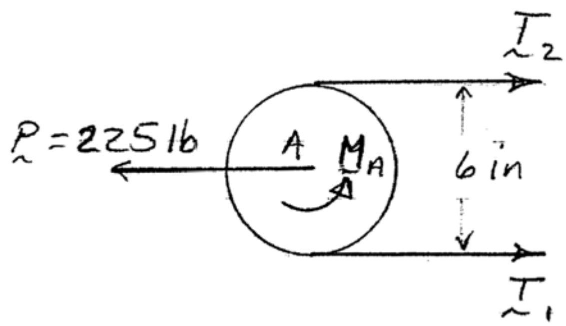


PROBLEM 8.109

A flat belt is used to transmit a torque from pulley *A* to pulley *B*. The radius of each pulley is 3 in., and a force of magnitude $P = 225\text{ lb}$ is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest torque which can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

FBD pulley A:



Impending slipping of belt: $T_2 = T_1 e^{\mu_s \beta}$

$$T_2 = T_1 e^{0.35\pi} = 3.0028T_1$$

$$\longrightarrow \Sigma F_x = 0: \quad T_1 + T_2 - 225\text{ lb} = 0$$

$$T_1(1 + 3.0028) = 225\text{ lb} \qquad \text{or} \qquad T_1 = 56.211\text{ lb}$$

$$T_2 = 3.0028T_1 \qquad \text{or} \qquad T_2 = 168.79\text{ lb}$$

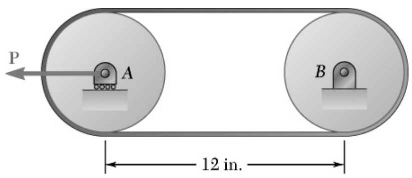
(*a*) $\circlearrowleft \Sigma M_A = 0: \quad M_A + (6\text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3\text{ in.})(168.79\text{ lb} - 56.21\text{ lb})$

$\therefore$ max. torque: $M_A = 338\text{ lb}\cdot\text{in.}$ ◀

(*b*) max. tension: $T_2 = 168.8\text{ lb}$ ◀

(Compare with $M_A = 638\text{ lb}\cdot\text{in.}$ with V-belt, Problem 8.131.)

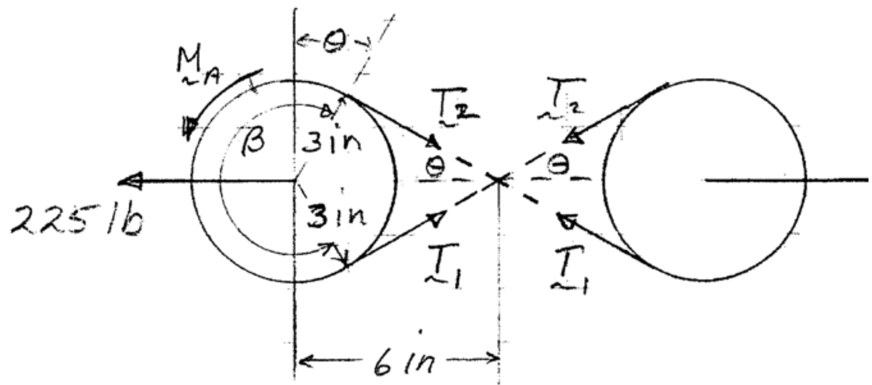
PROBLEM 8.110



Solve Problem 8.109 assuming that the belt is looped around the pulleys in a figure eight.

SOLUTION

FBDs pulleys:



$$\theta = \sin^{-1}\frac{3 \text{ in.}}{6 \text{ in.}} = 30° = \frac{\pi}{6}\text{rad.}$$

$$\beta = \pi + 2\frac{\pi}{6} = \frac{4\pi}{3}$$

Impending belt slipping: $$T_2 = T_1 e^{\mu_s \beta}$$

$$T_2 = T_1 e^{(0.35)4\pi/3} = 4.3322T_1$$

$$\xrightarrow{} \Sigma F_x = 0: \quad T_1 \cos 30° + T_2 \cos 30° - 225 \text{ lb} = 0$$

$$(T_1 + 4.3322T_1)\cos 30° = 225 \text{ lb} \qquad \text{or} \qquad T_1 = 48.7243 \text{ lb}$$

$$T_2 = 4.3322T_1 \qquad \text{so that} \qquad T_2 = 211.083 \text{ lb}$$

(*a*) $$\circlearrowleft \Sigma M_A = 0: \quad M_A + (3 \text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3 \text{ in.})(211.083 \text{ lb} - 48.224 \text{ lb})$$

$$M_{\max} = M_A = 487 \text{ lb}\cdot\text{in.} ◀$$

(*b*) $$T_{\max} = T_2 = 211 \text{ lb} ◀$$