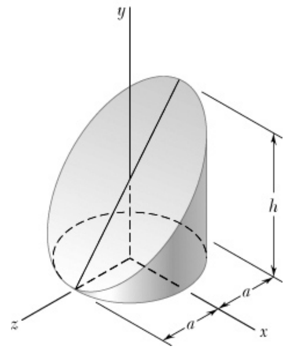
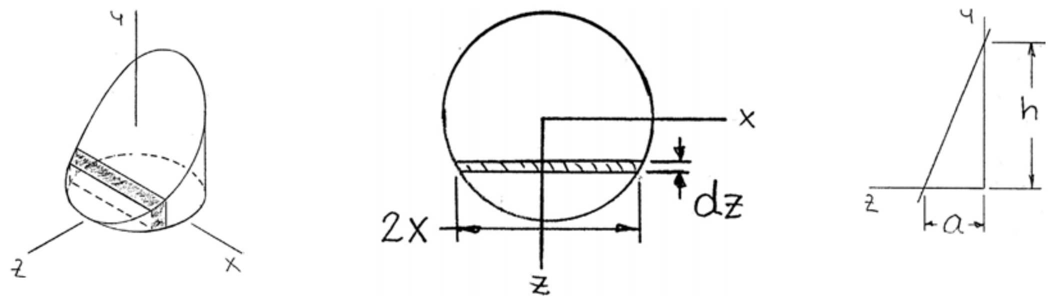
PROBLEM 5.129



Locate the centroid of the section shown, which was cut from a circular cylinder by an inclined plane.

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀



Choose as the element of volume a vertical slice of width $2x$, thickness dz, and height y. Then

$$dV = 2xy\,dz, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad \bar{z}_{EL} = z$$

Now $\quad x = \sqrt{a^2 - z^2} \quad$ and $\quad y = \frac{h}{2} - \frac{h}{2a}z = \frac{h}{2}\left(1 - \frac{z}{a}\right)$

So $\quad dV = h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz$

Then $\quad V = \int_0^a h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz = h\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2\sin^{-1}\left(\frac{z}{a}\right)\right] + \frac{1}{3a}\left(a^2 - z^2\right)^{3/2}\right\}\Bigg|_{-a}^{a}$

$$= \frac{1}{2}a^2h\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= \frac{\pi}{2}a^2h$$

PROBLEM 5.129 CONTINUED

Then

$$\int \overline{y}_{EL} dV = \int_{-a}^{a} \left[\frac{1}{2} \times \frac{h}{2}\left(1 - \frac{z}{a}\right)\right]\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$$

$$= \frac{h^2}{4}\int_{-a}^{a} \sqrt{a^2 - z^2}\left(1 - 2\frac{z}{a} + \frac{z^2}{a^2}\right)dz$$

$$= \frac{h^2}{4}\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2 \sin^{-1}\left(\frac{z}{a}\right)\right] + \left[\frac{2}{3a}\left(a^2 - z^2\right)^{\frac{3}{2}}\right]\right.$$

$$\left. + \frac{1}{a^2}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= \frac{5h^2a^2}{32}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

Then

$$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}\left(\frac{\pi a^2}{2}h\right) = \frac{5h^2a^2}{32}(\pi)$$

$$\text{or } \overline{y} = \frac{5}{16}h \blacktriangleleft$$

and

$$\int z_{EL} dV = \int_{-a}^{a} z\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$$

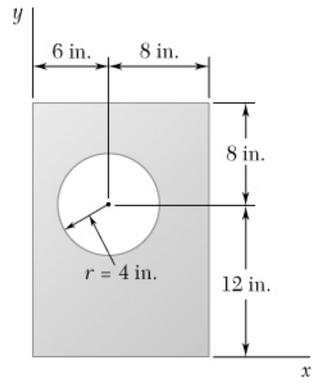
$$= h\left\{-\frac{1}{3}\left(a^2 - z^2\right)^{\frac{3}{2}} - \frac{1}{a}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= -\frac{a^3h}{8}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$\overline{z}V = \int \overline{z}_{EL} dV: \quad \overline{z}\left(\frac{\pi a^2 h}{2}\right) = -\frac{\pi a^3 h}{8}$$

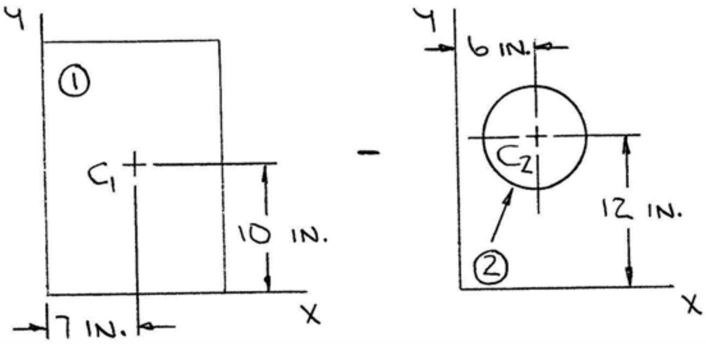
$$\text{or } \overline{z} = -\frac{a}{4} \blacktriangleleft$$

PROBLEM 5.130



Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	–301.59	–603.19
Σ	229.73			1658.41	2196.8

Then

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}\left(229.73\text{ in}^2\right) = 1658.41\text{ in}^3$$

or $\bar{X} = 7.22$ in. ◀

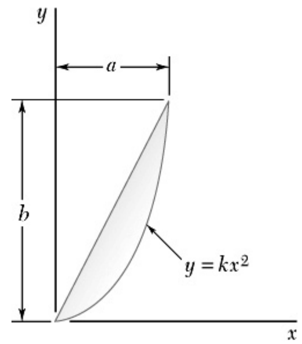
and

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}\left(229.73\text{ in}^2\right) = 2196.8\text{ in}^3$$

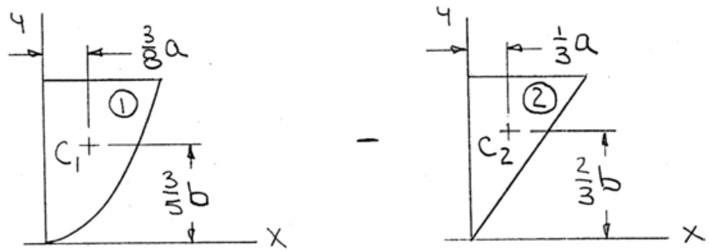
or $\bar{Y} = 9.56$ in. ◀

PROBLEM 5.131



For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$.

SOLUTION



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$\frac{a^2b}{4}$	$\frac{2ab^2}{5}$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-\frac{a^2b}{6}$	$-\frac{ab^2}{3}$
Σ	$\frac{1}{6}ab$			$\frac{a^2b}{12}$	$\frac{ab^2}{15}$

Then
$$\bar{X}\Sigma A = \Sigma\bar{x}A$$
$$\bar{X}\left(\frac{1}{6}ab\right) = \frac{a^2b}{12}$$

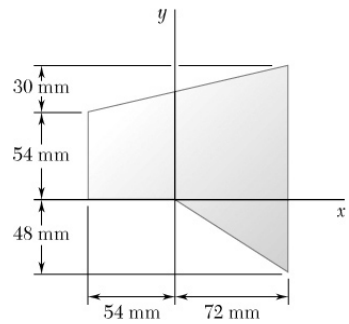
or
$$\bar{X} = \frac{1}{2}a$$
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$
$$\bar{Y}\left(\frac{1}{6}ab\right) = \frac{ab^2}{15}$$

or
$$\bar{Y} = \frac{2}{5}b$$

Now
$$\bar{X} = \bar{Y} \Rightarrow \qquad \frac{1}{2}a = \frac{2}{5}b$$

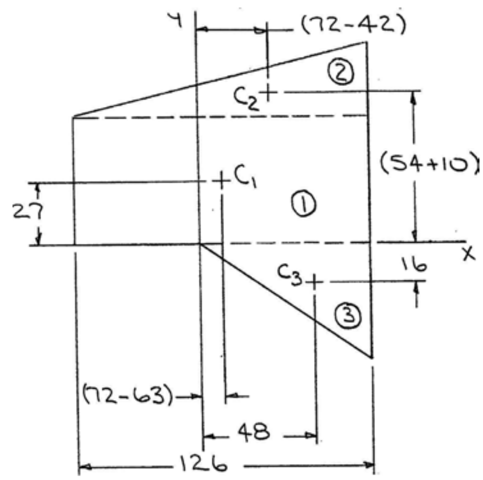
or $\dfrac{a}{b} = \dfrac{4}{5}$ ◀

PROBLEM 5.132



Locate the centroid of the plane area shown.

SOLUTION



Dimensions in mm

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$126 \times 54 = 6804$	9	27	61 236	183 708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56 700	120 960
3	$\frac{1}{2} \times 72 \times = 1728$	48	–16	82 944	–27 648
Σ	10 422			200 880	277 020

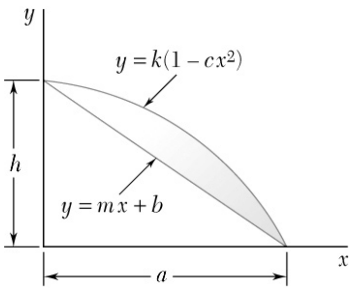
Then
$$\bar{X}\Sigma A = \Sigma \bar{x}A$$
$$\bar{X}\left(10\ 422\ \text{m}^2\right) = 200\ 880\ \text{mm}^2$$

or $\bar{X} = 19.27$ mm ◀

and
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$
$$\bar{Y}\left(10\ 422\ \text{m}^2\right) = 270\ 020\ \text{mm}^3$$

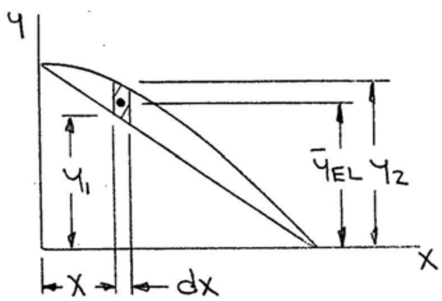
or $\bar{Y} = 26.6$ mm ◀

PROBLEM 5.133



Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



By observation

$$y_1 = -\frac{h}{a}x + h$$

$$= h\left(1 - \frac{x}{a}\right)$$

For y_2: At $x = 0, y = h$: $h = k(1 - 0)$ or $k = h$

At $x = a, y = 0$: $0 = h(1 - ca^2)$ or $C = \frac{1}{a^2}$

Then $$y_2 = h\left(1 - \frac{x^2}{a^2}\right)$$

Now $$dA = (y_2 - y_1)dx = h\left[\left(1 - \frac{x^2}{a^2}\right) - \left(1 - \frac{x}{a}\right)\right]dx$$

$$= h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$\overline{x}_{EL} = x$ $$\overline{y}_{EL} = \frac{1}{2}(y_1 - y_2) = \frac{h}{2}\left[\left(1 - \frac{x}{a}\right) + \left(1 - \frac{x^2}{a^2}\right)\right]$$

$$= \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)$$

Then $$A = \int dA = \int_0^a h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx = h\left[\frac{x^2}{2a} - \frac{x^3}{3a^2}\right]_0^a$$

$$= \frac{1}{6}ah$$

and $$\int \overline{x}_{EL} dA = \int_0^a x\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right] = h\left[\left(\frac{x^3}{3a} - \frac{x^4}{4a^2}\right)\right]_0^a$$

$$= \frac{1}{12}a^2h$$

PROBLEM 5.133 CONTINUED

$$\int \overline{y}_{EL} dA = \int_0^a \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right]$$

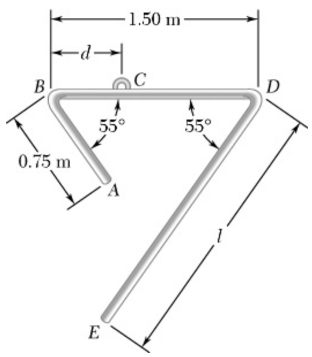
$$= \frac{h^2}{2}\int_0^a \left(2\frac{x}{a} - 3\frac{x^2}{a^2} + \frac{x^4}{a^4}\right)dx$$

$$= \frac{h^2}{2}\left[\frac{x^2}{a} - \frac{x^3}{a^2} + \frac{x^5}{5a^4}\right]_0^a = \frac{1}{10}ah^2$$

$$\overline{x}A = \int \overline{x}_{EL} dA\text{:}\quad x\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2h \qquad \overline{x} = \frac{1}{2}a \blacktriangleleft$$

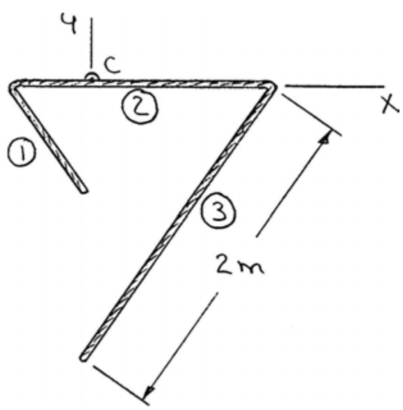
$$\overline{y}A = \int \overline{y}_{EL} dA\text{:}\quad y\left(\frac{1}{6}ah\right) = \frac{1}{10}a^2h \qquad \overline{y} = \frac{3}{5}h \blacktriangleleft$$

PROBLEM 5.134



Member *ABCDE* is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at *C* and that $l = 2$ m, determine the distance d so that portion *BCD* of the member is horizontal.

SOLUTION



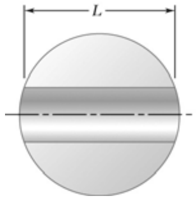
First note that for equilibrium, the center of gravity of the component must lie on a vertical line through C. Further, because the tubing is uniform, the center of gravity of the component will coincide with the centroid of the corresponding line. Thus, $\bar{X} = 0$

So that $\Sigma \bar{x} L = 0$

Then
$$-\left(d - \frac{0.75}{2}\cos 55°\right)\text{m} \times (0.75\text{ m})$$
$$+ (0.75 - d)\text{m} \times (1.5\text{ m})$$
$$+ \left[(1.5 - d)\text{m} - \left(\frac{1}{2} \times 2\text{ m} \times \cos 55°\right)\right] \times (2\text{ m}) = 0$$

or
$$(0.75 + 1.5 + 2)d = \left[\frac{1}{2}(0.75)^2 - 2\right]\cos 55° + (0.75)(1.5) + 3$$

or $d = 0.739$ m ◀

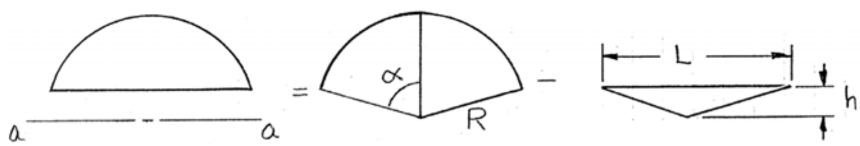


PROBLEM 5.135

A cylindrical hole is drilled through the center of a steel ball bearing shown here in cross section. Denoting the length of the *hole* by L, show that the volume of the steel remaining is equal to the volume of a sphere of diameter L.

SOLUTION

Calculate volumes by rotating cross sections about a line and using Theorem II of Pappus-Guldinus



For the sector: $\bar{y}_{AA} = \dfrac{2R\sin\alpha}{3\alpha}$ $\qquad A = \alpha R^2$

For the triangle: $h = \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2} = \dfrac{1}{2}\sqrt{4R^2 - L^2}$ $\qquad \bar{y}_{AA} = \dfrac{2}{3}h = \dfrac{1}{3}\sqrt{4R^2 - L^2},$

$$A = \frac{1}{2}(L)(h)$$

$$= \frac{1}{4}L\sqrt{4\text{R}^2 - L^2}$$

Using Theorem II of Pappus-Guldinus

$$V_{\text{ball}} = 2\pi\left(\bar{y}_{AA}\right)_1 A_1 - 2\pi\left(\bar{y}_{AA}\right)_2 A_2$$

$$= 2\pi\left[\frac{2R\sin\alpha}{3\alpha}\left(\alpha R^2\right) - \left(\frac{1}{3}\sqrt{4R^2 - L^2}\right)\left(\frac{1}{4}L\sqrt{4R^2 - L^2}\right)\right]$$

$$= 2\pi\left[\frac{2}{3}R^3\sin\alpha - \frac{L}{12}\left(4R^2 - L^2\right)\right]$$

Now $\qquad R\sin\alpha = \dfrac{L}{2}$

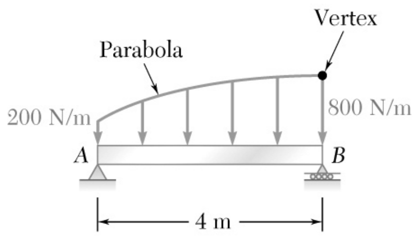
Then $\qquad V = 2\pi\left[\dfrac{2}{3}\left(\dfrac{L}{2}\right)R^2 - \dfrac{1}{3}LR^2 + \dfrac{1}{12}L^3\right]$

$$= \frac{\pi}{6}L^3$$

Note $V_{\text{sphere}} = \dfrac{4}{3}\pi r$ where r is the radius

If $r = \dfrac{L}{2}$, then $\qquad V_{\text{sphere}} = \dfrac{4}{3}\pi\left(\dfrac{L}{2}\right)^3 = \dfrac{\pi}{6}L^3$

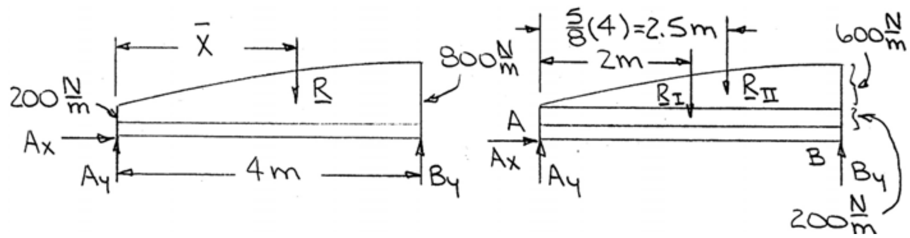
Therefore, $\qquad V_{\text{ball}} = V_{\text{sphere}} = \dfrac{\pi}{6}L^3$ Q.E.D. ◀



PROBLEM 5.136

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) Have $R_{\mathrm{I}} = (4\text{ m})(200\text{ N/m}) = 800\text{ N}$

$$R_{\mathrm{II}} = \frac{2}{3}(4\text{ m})(600\text{ N/m}) = 1600\text{ N}$$

Then ΣF_y: $-R = -R_{\mathrm{I}} - R_{\mathrm{II}}$

or $R = 800 + 1600 = 2400\text{ N}$

and ΣM_A: $-\bar{X}(2400) = -2(800) - 2.5(1600)$

or $\bar{X} = \dfrac{7}{3}\text{ m}$

$$\therefore\ \mathbf{R} = 2400\text{ N}\downarrow,\ \bar{X} = 2.33\text{ m} \blacktriangleleft$$

(*b*) Reactions

$$\xrightarrow{+} \Sigma F_x = 0:\ A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0:\ (4\text{ m})B_y - \left(\frac{7}{3}\text{ m}\right)(2400\text{ N}) = 0$$

or $B_y = 1400\text{ N}$

$$+\uparrow \Sigma F_y = 0:\ A_y + 1400\text{ N} - 2400\text{ N} = 0$$

or $A_y = 1000\text{ N}$

$$\therefore\ \mathbf{A} = 1000\text{ N}\uparrow,\ \mathbf{B} = 1400\text{ N}\uparrow \blacktriangleleft$$