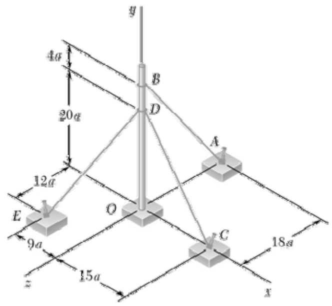
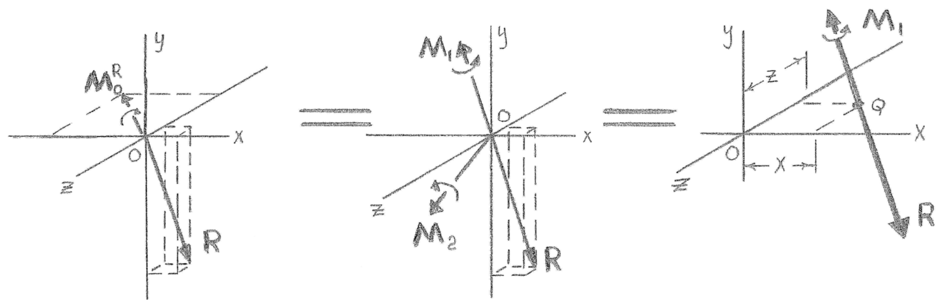


PROBLEM 3.135

A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (*a*) the resultant force $\mathbf{R}$, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



(*a*) First reduce the given force system to a force-couple at the origin.

Have $$\Sigma\mathbf{F}: \quad P\lambda_{BA} + P\lambda_{DC} + P\lambda_{DE} = \mathbf{R}$$

$$\mathbf{R} = P\left[\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right) + \left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right) + \left(\frac{-9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right)\right]$$

$$\therefore \quad \mathbf{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

Have $$\Sigma\mathbf{M}: \quad \Sigma(\mathbf{r}_O \times P) = \mathbf{M}_O^R$$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k})$$

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R$$

where $$\lambda_R = \frac{\mathbf{R}}{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})\frac{25}{27\sqrt{5}P} = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

PROBLEM 3.135 CONTINUED

Then $$M_1 = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \frac{-8Pa}{15\sqrt{5}}$$

and pitch $$p = \frac{M_1}{R} = \frac{-8Pa}{15\sqrt{5}}\left(\frac{25}{27\sqrt{5}P}\right) = \frac{-8a}{81}$$ or $p = -0.0988a$ ◀

(*c*) $$\mathbf{M}_1 = M_1\lambda_R = \frac{-8Pa}{15\sqrt{5}}\left(\frac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$$

Then $$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \frac{8Pa}{675}(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

$$= \left(\frac{3P}{25}\right)\left[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}\right]$$

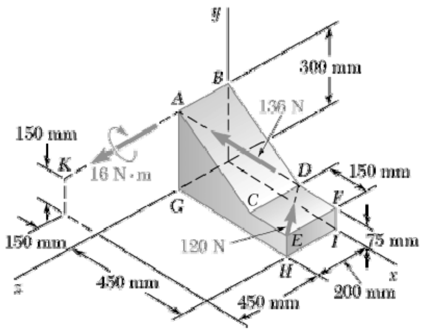
From **i**: $$8(-403)\frac{Pa}{675} = 20z\left(\frac{3P}{25}\right) \qquad \therefore\ z = -1.99012a$$

From **k**: $$8(-406)\frac{Pa}{675} = -20x\left(\frac{3P}{25}\right) \qquad \therefore\ x = 2.0049a$$

∴ The axis of the wrench intersects the *xz*-plane at

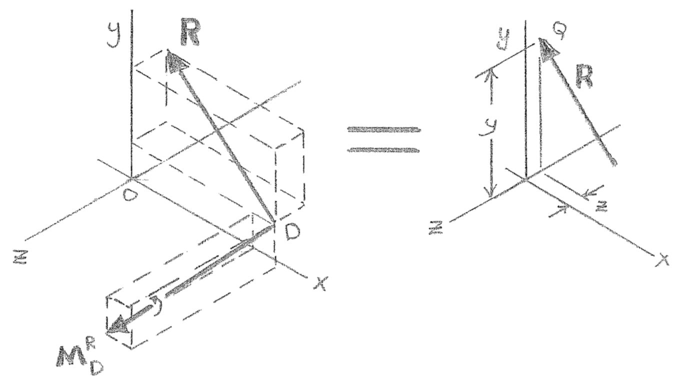
$x = 2.00a,\ z = -1.990a$ ◀

PROBLEM 3.136



Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple at *D*.

Have $$\Sigma\mathbf{F}: \quad \mathbf{F}_{DA} + \mathbf{F}_{ED} = F_{DA}\boldsymbol{\lambda}_{DA} + F_{ED}\boldsymbol{\lambda}_{ED} = \mathbf{R}$$

where $$\mathbf{F}_{DA} = 136\text{ N}\left[\frac{-(0.300\text{ m})\mathbf{i} + (0.225\text{ m})\mathbf{j} + (0.200\text{ m})\mathbf{k}}{0.425\text{ m}}\right]$$

$$= -(96\text{ N})\mathbf{i} + (72\text{ N})\mathbf{j} + (64\text{ N})\mathbf{k}$$

$$\mathbf{F}_{ED} = 120\text{ N}\left[\frac{-(0.150\text{ m})\mathbf{i} - (0.200\text{ m})\mathbf{k}}{0.250\text{ m}}\right] = -(72\text{ N})\mathbf{i} - (96\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{R} = -(168\text{ N})\mathbf{i} + (72\text{ N})\mathbf{j} - (32\text{ N})\mathbf{k} \ \blacktriangleleft$$

Have $$\Sigma\mathbf{M}_D: \quad \mathbf{M}_A = \mathbf{M}_D^R$$

or $$\mathbf{M}_D^R = (16\text{ N}\cdot\text{m})\left[\frac{-(0.150\text{ m})\mathbf{i} - (0.150\text{ m})\mathbf{j} + (0.450\text{ m})\mathbf{k}}{0.150\sqrt{11}\text{ m}}\right] = \frac{16\text{ N}\cdot\text{m}}{\sqrt{11}}(-\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

PROBLEM 3.136 CONTINUED

The force-couple at D can be replaced by a single force if $\mathbf{R}$ is perpendicular to $\mathbf{M}_D^R$. To be perpendicular, $\mathbf{R}\cdot\mathbf{M}_D^R = 0.$

Have
$$\mathbf{R}\cdot\mathbf{M}_D^R = \left(-168\mathbf{i} + 72\mathbf{j} - 32\mathbf{k}\right)\cdot\frac{16}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right)$$
$$= \frac{128}{\sqrt{11}}\left(21 - 9 - 12\right)$$
$$= 0$$

$\therefore$ Force-couple can be reduced to a single equivalent force. ◀

To determine the coordinates where the equivalent single force intersects the yz-plane, $\mathbf{M}_D^R = \mathbf{r}_{Q/D}\times\mathbf{R}$

where
$$\mathbf{r}_{Q/D} = \left[(0 - 0.300)\text{m}\right]\mathbf{i} + \left[(y - 0.075)\text{m}\right]\mathbf{j} + \left[(z - 0)\text{m}\right]\mathbf{k}$$

$$\therefore \quad \frac{16\text{ N}\cdot\text{m}}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right) = (8\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & (y - 0.075) & z \\ -21 & 9 & -4 \end{vmatrix}\text{m}$$

or

$$\frac{16\text{ N}\cdot\text{m}}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right) = (8\text{ N})\left\{\left[-4(y - 0.075) - 9z\right]\mathbf{i} + \left(-21z - 1.2\right)\mathbf{j} + \left[-2.7 + 21(y - 0.075)\right]\mathbf{k}\right\}\text{m}$$

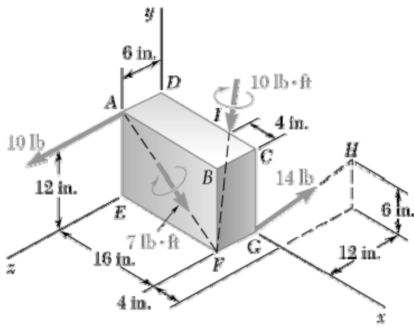
From $\mathbf{j}$: $\quad \frac{-16}{\sqrt{11}} = 8\left(-21z - 1.2\right) \qquad \therefore\ z = -0.028427\text{ m} = -28.4\text{ mm}$

From $\mathbf{k}$: $\quad \frac{48}{\sqrt{11}} = 8\left[-2.7 + 21(y - 0.075)\right] \qquad \therefore\ y = 0.28972\text{ m} = 290\text{ mm}$

$\therefore$ line of action of $\mathbf{R}$ intersects the yz-plane at

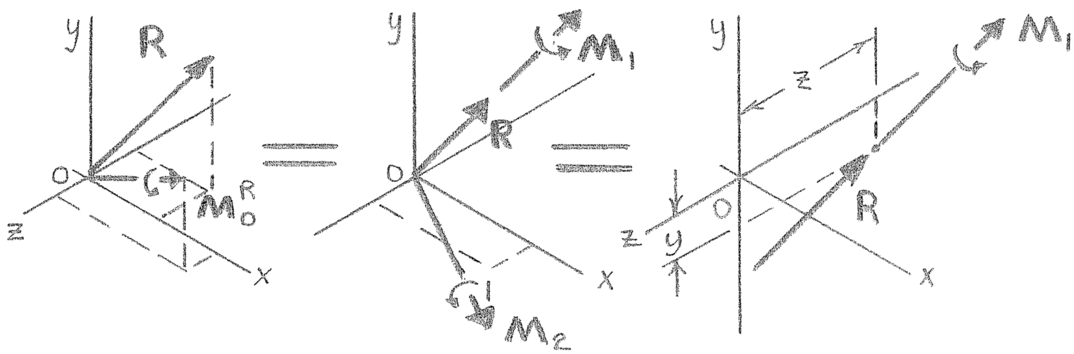
$y = 290\text{ mm},\ z = -28.4\text{ mm}$ ◀

PROBLEM 3.137



Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma\mathbf{F}:\quad \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$$

$$\therefore\quad \mathbf{R} = (10\text{ lb})\mathbf{k} + 14\text{ lb}\left[\frac{(4\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} - (12\text{ in.})\mathbf{k}}{14\text{ in.}}\right] = (4\text{ lb})\mathbf{i} + (6\text{ lb})\mathbf{j} - (2\text{ lb})\mathbf{k} \blacktriangleleft$$

and $$R = \sqrt{56}\text{ lb}$$

Have $$\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \left[(12\text{ in.})\mathbf{j} \times (10\text{ lb})\mathbf{k}\right] + \left\{(16\text{ in.})\mathbf{i} \times \left[(4\text{ lb})\mathbf{i} + (6\text{ lb})\mathbf{j} - (12\text{ lb})\mathbf{k}\right]\right\}$$

$$+ (84\text{ lb}\cdot\text{in.})\left[\frac{(16\text{ in.})\mathbf{i} - (12\text{ in.})\mathbf{j}}{20\text{ in.}}\right] + (\ 120\text{ lb}\cdot\text{in.})\left[\frac{(4\text{ in.})\mathbf{i} - (12\text{ in.})\mathbf{j} + (6\text{ in.})\mathbf{k}}{14\text{ in.}}\right]$$

$$\therefore\quad \mathbf{M}_0^R = (221.49\text{ lb}\cdot\text{in.})\mathbf{i} + (38.743\text{ lb}\cdot\text{in.})\mathbf{j} + (147.429\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$= (18.4572\text{ lb}\cdot\text{ft})\mathbf{i} + (3.2286\text{ lb}\cdot\text{ft})\mathbf{j} + (12.2858\text{ lb}\cdot\text{ft})\mathbf{k}$$

PROBLEM 3.137 CONTINUED

The force-couple at O can be replaced by a single force if the direction of **R** is perpendicular to $\mathbf{M}_O^R$.

To be perpendicular $\mathbf{R}\cdot\mathbf{M}_O^R = 0$

Have
$$\mathbf{R}\cdot\mathbf{M}_O^R = (4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) = 0?$$
$$= 73.829 + 19.3716 - 24.572$$
$$\neq 0$$

$\therefore$ System cannot be reduced to a single equivalent force.

To reduce to an equivalent wrench, the moment component along the line of action of **P** is found.

$$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$
$$= \left[\frac{(4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})}{\sqrt{56}}\right]\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k})$$
$$= 9.1709 \text{ lb}\cdot\text{ft}$$

and
$$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R = (9.1709 \text{ lb}\cdot\text{ft})(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$

And pitch
$$p = \frac{M_1}{R} = \frac{9.1709 \text{ lb}\cdot\text{ft}}{\sqrt{56} \text{ lb}} = 1.22551 \text{ ft}$$

or $p = 1.226$ ft ◀

Have
$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) - (9.1709)(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$
$$= (13.5552 \text{ lb}\cdot\text{ft})\mathbf{i} - (4.1244 \text{ lb}\cdot\text{ft})\mathbf{j} + (14.7368 \text{ lb}\cdot\text{ft})\mathbf{k}$$

Require
$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$
$$(13.5552\mathbf{i} - 4.1244\mathbf{j} + 14.7368\mathbf{k}) = (y\mathbf{j} + z\mathbf{k}) \times (4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$$
$$= -(2y + 6z)\mathbf{i} + (4z)\mathbf{j} - (4y)\mathbf{k}$$

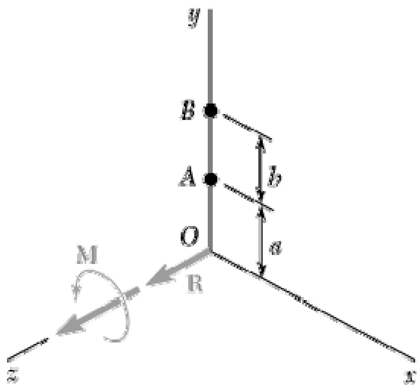
From **j**: $\quad -4.1244 = 4z \quad$ or $\quad z = -1.0311$ ft

From **k**: $\quad 14.7368 = -4y \quad$ or $\quad y = -3.6842$ ft

$\therefore$ line of action of the wrench intersects the yz plane at

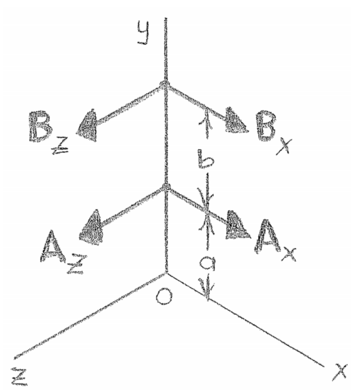
$y = -3.68$ ft, $z = 1.031$ ft ◀

PROBLEM 3.138



Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the y axis and applied respectively at A and B.

SOLUTION



Express the forces at A and B as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system

$$\Sigma F_x: \quad A_x + B_x = 0 \tag{1}$$

$$\Sigma F_z: \quad A_z + B_z = R \tag{2}$$

$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \tag{3}$$

$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \tag{4}$$

From Equation (1), $\quad B_x = -A_x$

Substitute into Equation (4)

$$-A_x(a) + A_x(a+b) = M$$

$$\therefore \quad A_x = \frac{M}{b} \qquad \text{and} \qquad B_x = -\frac{M}{b}$$

From Equation (2), $\quad B_z = R - A_z$

and Equation (3), $\quad A_z a + (R - A_z)(a+b) = 0$

$$\therefore \quad A_z = R\left(1 + \frac{a}{b}\right)$$

PROBLEM 3.138 CONTINUED

and $$B_z = R - R\left(1 + \frac{a}{b}\right)$$

$$\therefore \quad B_z = -\frac{a}{b}R$$

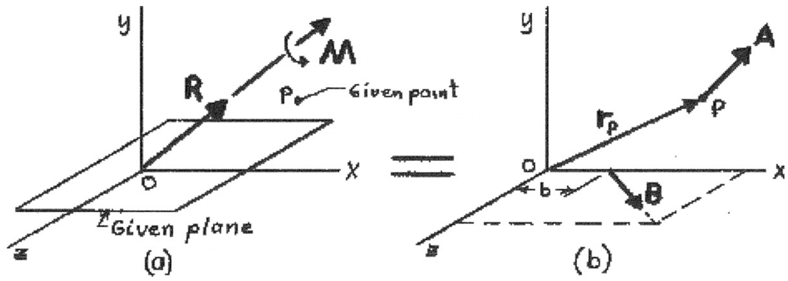
Then $$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.139

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the *xy* plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let A be the force passing through the given point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \qquad \text{and} \qquad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \qquad \text{and} \qquad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of point *P* is given, it follows that the scalar components (*x*, *y*, *z*) of the position vector $\mathbf{r}_P$ are also known.

Then, for equivalence of the two systems

$$\Sigma F_x: \quad R_x = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R_y = A_y \tag{2}$$

$$\Sigma F_z: \quad R_z = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \tag{4}$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \tag{5}$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \tag{6}$$

PROBLEM 3.139 CONTINUED

Based on the above six independent equations for the six unknowns $\left(A_x,\ A_y,\ A_z,\ B_x,\ B_z,\ b\right)$, there exists a unique solution for **A** and **B**.

From Equation (2) $$A_y = R_y \blacktriangleleft$$

Equation (6) $$A_x = \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

Equation (1) $$B_x = R_x - \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

Equation (4) $$A_z = \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

Equation (3) $$B_z = R_z - \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

Equation (5) $$b = \frac{\left(xM_x + yM_y + zM_z\right)}{\left(M_x - yR_z + zR_y\right)} \blacktriangleleft$$