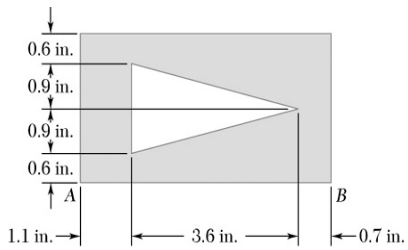
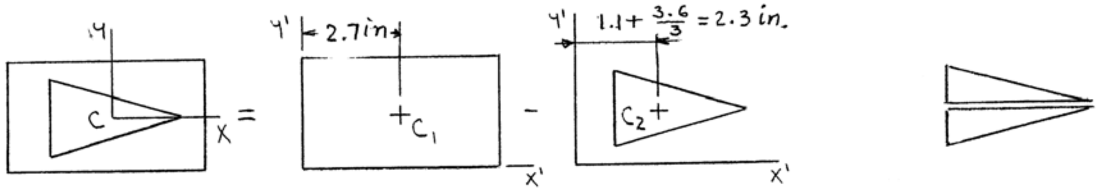



## PROBLEM 9.41

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

## SOLUTION




First calculate the centroid *C* of the area

From symmetry $$\bar{Y} = 0.6\text{ in.} + 0.9\text{ in.} = 1.5\text{ in.}$$

To compute $\bar{X}$ use the equation $$\bar{X}A = \Sigma Ax$$

or
$$\bar{X} = \frac{\left[(3\times 5.4)\text{ in}^2\right]\times(2.7\text{ in.}) - \left[\frac{1}{2}(1.8\times 3.6)\text{ in}^2\right]\times(2.3\text{ in.})}{(3\times 5.4)\text{ in}^2 - \frac{1}{2}(1.8\times 3.6)\text{ in}^2}$$
$$= 2.8\text{ in.}$$

The moment of inertia of the composite area is obtained by subtracting the moment of inertia of the triangle from the moment of inertia of the rectangle

$$\bar{I}_x = (I_x)_1 - (I_x)_2$$

where
$$(I_x)_1 = \frac{1}{12}(5.4\text{ in.})(3\text{ in.})^3 = 12.15\text{ in}^4$$

and
$$(I_x)_2 = 2\left[\frac{1}{12}(3.6\text{ in.})(0.9\text{ in.})^3\right] = 0.4374\text{ in}^4$$

Then
$$\bar{I}_x = (12.15 - 0.4374)\text{ in}^4 = 11.7126\text{ in}^4$$

or $\bar{I}_x = 11.71\text{ in}^4$ ◀

Similarly,
$$\bar{I}_y = (I_y)_1 - (I_y)_2$$

where
$$(I_y)_1 = \frac{1}{12}(3\text{ in.})(5.4\text{ in.})^3 + \left[(3\times 5.4)\text{ in}^2\right](2.8\text{ in.} - 2.7\text{ in.})^2$$
$$= 39.582\text{ in}^4$$

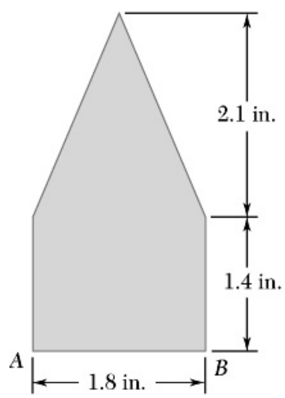
## PROBLEM 9.41 CONTINUED

and $$\left(I_y\right)_2 = \frac{1}{36}(1.8\text{ in.})(3.6\text{ in.})^3 + \left[\frac{1}{2}(1.8)(3.6)\text{ in}^2\right](2.8\text{ in.} - 2.3\text{ in.})^2$$

$$= 3.1428\text{ in}^4$$

Then $$\bar{I}_y = (39.582 - 3.1428)\text{ in}^4 = 36.4392\text{ in}^4$$

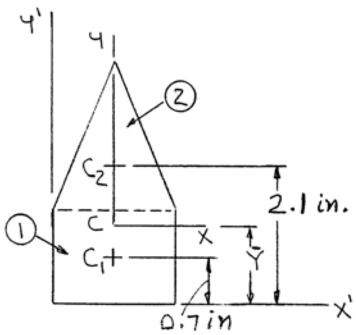
or $\bar{I}_y = 36.4\text{ in}^4$ ◀




## PROBLEM 9.42

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

## SOLUTION




By symmetry $\bar{X} = 0.9 \text{ in.}$

Have $A\bar{Y} = \Sigma\bar{y}A$

Where $A = (1.8 \text{ in.})(1.4 \text{ in.}) + \frac{1}{2}(1.8 \text{ in.})(2.1 \text{ in.})$

$$= (2.52 + 1.89)\text{in}^2 = 4.41 \text{ in}^2$$

Then $(4.41 \text{ in}^2)\bar{Y} = (0.7 \text{ in.})(2.52 \text{ in}^2) + (2.1 \text{ in.})(1.89 \text{ in}^2)$

$$= 5.733 \text{ in}^3$$

or $\bar{Y} = 1.3 \text{ in.}$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(1.8 \text{ in.})(1.4 \text{ in.})^3$

$$+ (2.52 \text{ in}^2)(1.3 \text{ in.} - 0.7 \text{ in.})^2 = 1.3188 \text{ in}^4$$

And $(I_x)_2 = \frac{1}{36}(1.8 \text{ in.})(2.1 \text{ in.})^3$

$$+ (1.89 \text{ in}^2)(2.1 \text{ in.} - 1.3 \text{ in.})^2 = 1.67265 \text{ in}^4$$

Then $\bar{I}_x = (1.3188 + 1.67265)\text{ in}^4 = 2.99145 \text{ in}^4$

or $\bar{I}_x = 2.99 \text{ in}^4$ ◀

Also $\bar{I}_y = (I_y)_1 + (I_y)_2$

where $(I_y)_1 = \frac{1}{12}(1.4 \text{ in.})(1.8 \text{ in.})^3 = 0.6804 \text{ in}^4$

## PROBLEM 9.42 CONTINUED

and

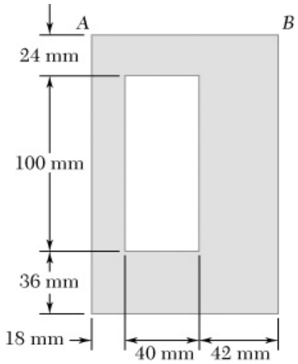
$$\left(I_y\right)_2 = 2\left[\frac{1}{36}(2.1\text{ in.})(0.9\text{ in.})^3 + \left(\frac{1}{2}\times 1.89\text{ in}^2\right)(0.3\text{ in.})^2\right] = 0.25515\text{ in}^4$$

Then

$$\bar{I}_y = (0.6804 + 0.25515)\text{ in}^4$$

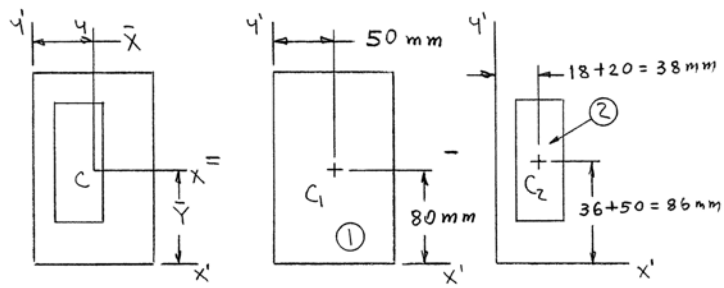
$$\text{or } \bar{I}_y = 0.936\text{ in}^4 \blacktriangleleft$$

## PROBLEM 9.43




Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

## SOLUTION




$$A = A_1 - A_2$$

$$= (100\text{ mm})(160\text{ mm}) - (40\text{ mm})(100\text{ mm})$$

$$= (16\ 000 - 4000)\text{mm}^2$$

$$= 12\ 000\text{ mm}^2$$

First locate the centroid:

Have $\qquad A\bar{X} = \Sigma A\bar{x}$

or $\qquad (12\ 000\text{ mm}^2)\bar{X} = [(16\ 000)(50) - (4000)(38)]\text{mm}^3 = 648\ 000\text{ mm}^3$

or $\qquad \bar{X} = \dfrac{648\ 000\text{ mm}^3}{12\ 000\text{ mm}^2} = 54\text{ mm}$

And $\qquad A\bar{Y} = \Sigma A\bar{y}$

or $\qquad (12\ 000\text{ mm}^2)\bar{Y} = [(16\ 000)(86) - (4000)(86)]\text{mm}^3 = 936\ 000\text{ mm}^3$

or $\qquad \bar{Y} = \dfrac{936\ 000\text{ mm}^3}{12\ 000\text{ mm}^2} = 78\text{ mm}$

## PROBLEM 9.43 CONTINUED

Now $$\bar{I}_x = (I_x)_1 - (I_x)_2$$

where $$(I_x)_1 = \frac{1}{12}(40\text{ mm})(100\text{ mm})^3 + (16\ 000\text{ mm}^2)(80\text{ mm} - 78\text{ mm})^2$$

$$= 34.197 \times 10^6\text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(40\text{ mm})(100\text{ mm})^3 + (4000\text{ mm}^2)(80\text{ mm} - 78\text{ mm})^2$$

$$= 3.5893 \times 10^6\text{ mm}^4$$

Then $$\bar{I}_x = (34.197 - 3.5893) \times 10^6\text{ mm}^4$$

or $\bar{I}_x = 30.6 \times 10^6\text{ mm}^4$ ◀

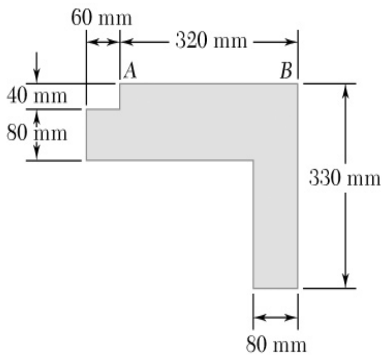
Also $$\bar{I}_y = (I_y)_1 - (I_y)_2$$

where $$(I_y)_1 = \frac{1}{12}(160\text{ mm})(100\text{ mm})^3 + (16\ 000\text{ mm}^2)(54\text{ mm} - 50\text{ mm})^2 = 13.589 \times 10^6\text{ mm}^4$$

$$(I_y)_2 = \frac{1}{12}(100\text{ mm})(40\text{ mm})^3 + (4000\text{ mm}^2)(54\text{ mm} - 38\text{ mm})^2 = 1.5573 \times 10^6\text{ mm}^4$$

Then $$\bar{I}_y = (13.589 - 1.5573) \times 10^6\text{ mm}^4$$

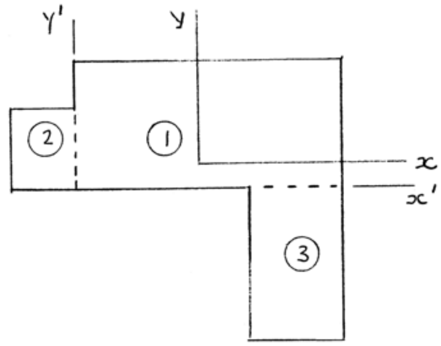
or $\bar{I}_y = 12.03 \times 10^6\text{ mm}^4$ ◀




## PROBLEM 9.44

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

## SOLUTION




First locate centroid

$$\bar{x}_1 = 160 \text{ mm} \qquad \bar{y}_1 = 60 \text{ mm}$$

$$A_1 = 320 \text{ mm} \times 120 \text{ mm} = 38\ 400 \text{ mm}^2$$

$$\bar{x}_2 = -30 \text{ mm} \qquad \bar{y}_2 = 40 \text{ mm}$$

$$A_2 = 60 \text{ mm} \times 80 \text{ mm} = 4800 \text{ mm}^2$$

$$\bar{x}_3 = 280 \text{ mm} \qquad \bar{y}_3 = -105 \text{ mm}$$

$$A_3 = 80 \text{ mm} \times 210 \text{ mm} = 16\ 800 \text{ mm}^2$$

Then

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A}$$

$$= \frac{\left[160(38\ 400) - 30(4800) + 280(16\ 800)\right] \text{mm}^3}{(38\ 400 + 4800 + 16\ 800) \text{ mm}^2}$$

$$= 178.4 \text{ mm}$$

And

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{\left[60(38\ 400) + 40(4800) - 105(16\ 800)\right] \text{mm}^3}{(38\ 400 + 4800 + 16\ 800) \text{ mm}^2}$$

$$= 12.20 \text{ mm}$$

## PROBLEM 9.44 CONTINUED

Then $\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

$$= \left[\frac{1}{12}(320\text{ mm})(120\text{ mm})^3 + \left(38\ 400\text{ mm}^2\right)(60\text{ mm} - 12.2\text{ mm})^2\right]$$

$$+\left[\frac{1}{12}(60\text{ mm})(80\text{ mm})^3 + \left(4800\text{ mm}^2\right)(40\text{ mm} - 12.2\text{ mm})^2\right]$$

$$+\left[\frac{1}{12}(80\text{ mm})(210\text{ mm})^3 + \left(16\ 800\text{ mm}^2\right)(105\text{ mm} + 12.2\text{ mm})^2\right]$$

$$= \left[(46.080 + 87.7379) + (2.5600 + 3.7096) + (61.7400 + 230.7621)\right] \times 10^6\text{ mm}^4$$

$$= 432.5896 \times 10^6\text{ mm}^4$$

or $\bar{I}_x = 433 \times 10^6\text{ mm}^4$ ◀

And $\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

$$= \left[\frac{1}{12}(120\text{ mm})(320\text{ mm})^3 + \left(38\ 400\text{ mm}^2\right)(178.4\text{ mm} - 160\text{ mm})^2\right]$$

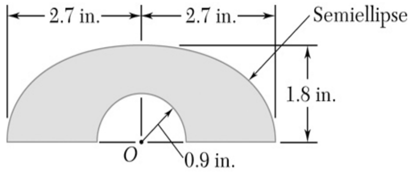
$$+\left[\frac{1}{12}(80\text{ mm})(60\text{ mm})^3 + \left(4800\text{ mm}^2\right)(30\text{ mm} + 178.4\text{ mm})^2\right]$$

$$+\left[\frac{1}{12}(210\text{ mm})(80\text{ mm})^3 + \left(16\ 800\text{ mm}^2\right)(280\text{ mm} - 178.4\text{ mm})^2\right]$$

$$= \left[(327.6800 + 13.0007) + (1.4400 + 208.4667) + (8.9600 + 173.4190)\right] \times 10^6\text{ mm}^4$$

$$= 732.9664 \times 10^6\text{ mm}^4$$

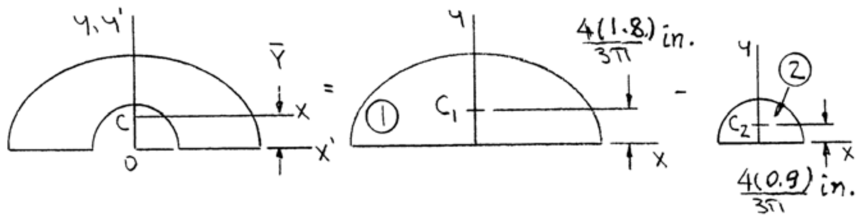
or $\bar{I}_y = 733 \times 10^6\text{ mm}^4$ ◀




## PROBLEM 9.45

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

## SOLUTION




First locate centroid *C* of the area

| | $A$, in$^2$ | $\bar{y}$, in. | $\bar{y}A$, in$^3$ |
|---|---|---|---|
| 1 | $\frac{\pi}{2}(2.7)(1.8) = 7.6341$ | 0.76394 | 5.8319 |
| 2 | $-\frac{\pi}{2}(0.9)^2 = -1.2723$ | 0.38197 | 0.4860 |
| $\Sigma$ | 6.3618 | | 5.3460 |

Then $\qquad \bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y} = \dfrac{5.3460\text{ in}^2}{6.3618\text{ in}^2} = 0.84033\text{ in.}$

(*a*) $$J_O = (J_O)_1 - (J_O)_2 = \frac{\pi}{8}(2.7\text{ in.})(1.8\text{ in.})\left[(2.7)^2 + (1.8)^2\right]\text{in}^2 - \frac{\pi}{4}(0.9\text{ in.})^4$$

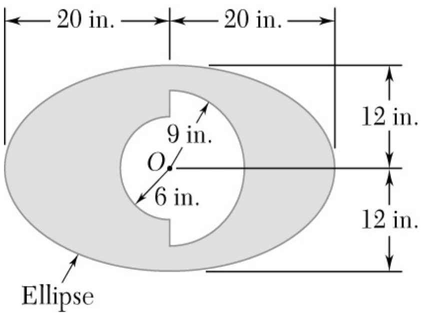
$$= 19.5814\text{ in}^4$$

or $J_O = 19.58\text{ in}^4$ ◀

(*b*) $$J_O = \bar{J}_C + A(\bar{y})^2$$

or $$\bar{J}_C = 19.5814\text{ in}^4 - (6.3618\text{ in}^2)(0.84033\text{ in.})^2 = 15.0890\text{ in}^4$$

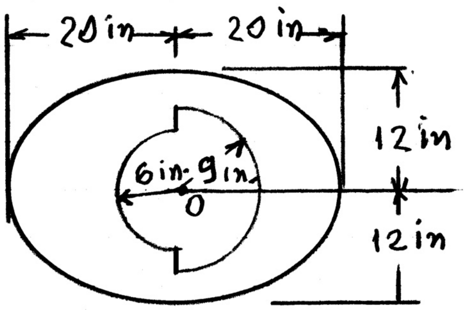
$\bar{J}_C = 15.09\text{ in}^4$ ◀




## PROBLEM 9.46

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

## SOLUTION




First locate centroid

Symmetry implies

$$\bar{Y} = 0$$

$$\bar{x}_1 = 0 \quad A_1 = \pi(20 \text{ in.})(12 \text{ in.})$$

$$= (240\pi) \text{ in}^2$$

$$\bar{x}_2 = \frac{4(9 \text{ in.})}{3\pi} = (12\pi) \text{ in.}$$

$$A_2 = -\frac{\pi}{2}(9 \text{ in.})^2 = -(40.5\pi) \text{ in}^2$$

$$\bar{x}_3 = -\frac{4(6 \text{ in.})}{3\pi} = -\frac{8}{\pi} \text{ in.}$$

$$A_3 = -\frac{\pi}{2}(6 \text{ in.})^2 = -(18\pi) \text{ in}^2$$

Then

$$\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{(0)\left(240\pi \text{ in}^2\right) + (12\pi \text{ in.})\left(-40.5\pi \text{ in}^2\right) - \frac{8 \text{ in.}}{\pi}\left(-18\pi \text{ in}^2\right)}{240\pi \text{ in}^2 - 40.5\pi \text{ in}^2 - 18\pi \text{ in}^2}$$

$$= \frac{-486 \text{ in}^3 + 144 \text{ in}^3}{181.5\pi \text{ in}^2} = \frac{-342 \text{ in}^3}{181.5\pi \text{ in}^2} = -0.59979 \text{ in.}$$

## PROBLEM 9.46 CONTINUED

(*a*) Have

$$J_O = (J_O)_1 - (J_O)_2 - (J_O)_3$$

$$= \frac{\pi}{4}(20\text{ in.})(12\text{ in.})\left[(20\text{ in.})^2 + (12\text{ in.})^2\right] - \left[\frac{\pi}{4}(9\text{ in.})^4\right] - \left[\frac{\pi}{4}(6\text{ in.})^4\right]$$

$$= \pi(32640 - 1640.25 - 324.00)\text{ in}^4 = 96,371\text{ in}^4$$

or $J_O = 96.4 \times 10^3\text{ in}^4$ ◀

(*b*) Have

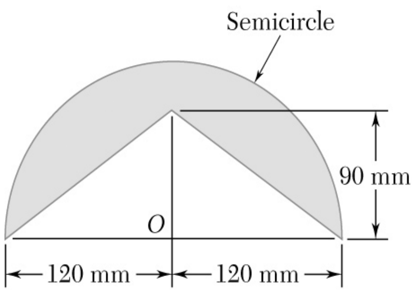
$$J_O = \bar{J}_C + A\bar{x}^2$$

Then

$$\bar{J}_C = 96,371\text{ in}^4 - \left(181.5\pi\text{ in}^2\right)(-0.59979\text{ in.})^2$$

$$= 96,371\text{ in}^4 - 204.5629\text{ in}^4 = 96,166.4379\text{ in}^4$$

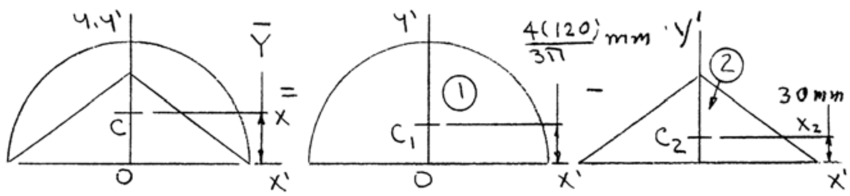
or $\bar{J}_C = 96.2 \times 10^3\text{ in}^4$ ◀




## PROBLEM 9.47

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

## SOLUTION



| | $A$, mm$^2$ | $\bar{y}$, mm | $\bar{y}A$, mm$^3$ |
|---|---|---|---|
| 1 | $\frac{\pi}{2}(120)^2 = 22\ 619.5$ | 50.9296 | $1.1520 \times 10^6$ |
| 2 | $-\frac{1}{2}(240)(90) = -10\ 800$ | 30 | $-0.324 \times 10^6$ |
| $\Sigma$ | 11 819.5 | | $0.828 \times 10^6$ |

Now
$$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A} = \frac{0.828 \times 10^6 \text{ mm}^3}{11819.5 \text{ mm}^2} = 70.054 \text{ mm}$$

(*a*)
$$J_O = (J_O)_1 - (J_O)_2$$

where
$$(J_O)_1 = \frac{\pi}{4}\left(120 \text{ mm}^4\right) = 162.86 \times 10^6 \text{ mm}^4$$

and
$$(J_O)_2 = (I_{x'})_2 + (I_{y'})_2 = \frac{1}{12}(240 \text{ mm})(90 \text{ mm})^3 + 2\left[\frac{1}{12}(90 \text{ mm})(120 \text{ mm})^3\right]$$

$$= 40.5 \times 10^6 \text{ mm}^4$$

Then
$$J_O = (162.86 - 40.5) \times 10^6 \text{ mm}^4 = 122.36 \times 10^6 \text{ mm}^4$$

or $J_O = 122.4 \times 10^6 \text{ mm}^4$ ◀

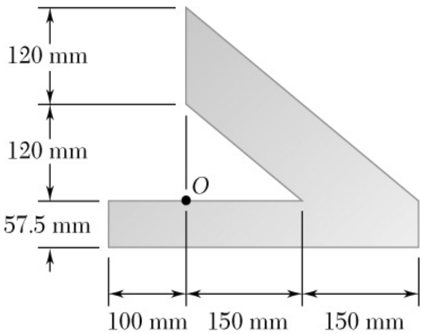
## PROBLEM 9.47 CONTINUED

(*b*)
$$J_O = \bar{J}_C + A\bar{y}^2$$

or
$$\bar{J}_C = 122.36 \times 10^6\ \text{mm}^4 - \left(11\,819.5\ \text{mm}^2\right)\left(70.054\ \text{mm}\right)^2$$

$$= \left(122.36 - 58.005\right)10^6\ \text{mm}^4$$

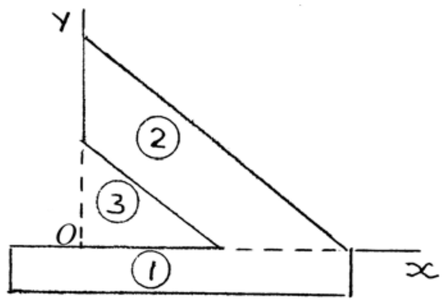
or $\bar{J}_C = 64.4 \times 10^6\ \text{mm}^4$ ◀




## PROBLEM 9.48

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

## SOLUTION




First locate centroid

$$\bar{x}_1 = 100\text{ mm} \qquad \bar{y}_1 = -28.75\text{ mm}$$

$$A_1 = (400\text{ mm})(57.5\text{ mm}) = 23\ 000\text{ mm}^2$$

$$\bar{x}_2 = 100\text{ mm} \qquad \bar{y}_2 = 80\text{ mm}$$

$$A_2 = \frac{1}{2}(300\text{ mm})(240\text{ mm}) = 36\ 000\text{ mm}^2$$

$$\bar{x}_3 = 50\text{ mm} \qquad \bar{y}_3 = 40\text{ mm}$$

$$A_3 = -\frac{1}{2}(150\text{ mm})(120\text{ mm}) = -9000\text{ mm}^2$$

Then $$\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{(100\text{ mm})(23\ 000\text{ mm}^2) + (100\text{ mm})(36\ 000\text{ mm}^2) + (50\text{ mm})(-9000\text{ mm}^2)}{23\ 000\text{ mm}^2 + 36\ 000\text{ mm}^2 - 9000\text{ mm}^2}$$

$$= \frac{(2.3 + 3.6 - 0.45)\times 10^6\text{ mm}^3}{50\times 10^3\text{ mm}^2} = 109.0\text{ mm}$$

And $$\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{(-28.75\text{ mm})(23\ 000\text{ mm}^2) + (80\text{ mm})(36\ 000\text{ mm}^2) + (40\text{ mm})(-9000\text{ mm}^2)}{50\times 10^3\text{ mm}^2}$$

$$= \frac{(-661.25 + 2880 - 360)\times 10^3\text{ mm}^3}{50\times 10^3\text{ mm}^2} = 37.175\text{ mm}$$

## PROBLEM 9.48 CONTINUED

(*a*) Now $J_O = I_x + I_y$

where $I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$

$$(I_x)_1 = \frac{1}{3}(400\text{ mm})(57.5\text{ mm})^3 = 25.3479 \times 10^6\text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(300\text{ mm})(240\text{ mm})^3 = 345.6000 \times 10^6\text{ mm}^4$$

$$(I_x)_3 = \frac{1}{12}(150\text{ mm})(120\text{ mm})^3 = 21.6000 \times 10^6\text{ mm}^4$$

Then $I_x = (25.3479 + 345.6000 - 21.6000) \times 10^6\text{ mm}^4$

$$= 349.348 \times 10^6\text{ mm}^4$$

Also $I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(57.5\text{ mm})(400\text{ mm})^3 + (23\ 000\text{ mm}^2)(100\text{ mm})^2$

$$= (306.6667 + 230.0000) \times 10^6\text{ mm}^4 = 536.6667 \times 10^6\text{ mm}^4$$

$$(I_y)_2 = \frac{1}{12}(240\text{ mm})(300\text{ mm})^3 = 540.0000 \times 10^6\text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12}(150\text{ mm})(120\text{ mm})^3 = 33.7500 \times 10^6\text{ mm}^4$$

Then $I_y = (536.6667 + 540 - 33.75) \times 10^6\text{ mm}^4 = 1042.917 \times 10^6\text{ mm}^4$

Finally, $J_O = (349.348 + 1042.917) \times 10^6\text{ mm}^4 = 1392.265 \times 10^6\text{ mm}^4$

or $J_O = 1392 \times 10^6\text{ mm}^4$ ◀

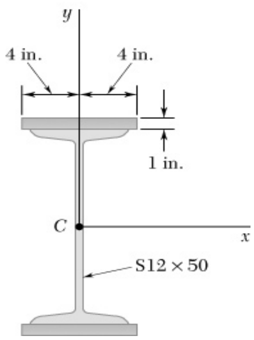
(*b*) Have $J_O = \bar{J}_C + Ad^2$ where $d^2 = \bar{X}^2 + \bar{Y}^2$

Then $\bar{J}_C = 1392.265 \times 10^6\text{ mm}^4 - (50 \times 10^3\text{ mm}^2)\left[(109.0\text{ mm})^2 + (37.175\text{ mm})^2\right]$

$$= (1392.265 - 594.050 - 69.099) \times 10^6\text{ mm}^4$$

$$= 729.1660 \times 10^6\text{ mm}^4$$

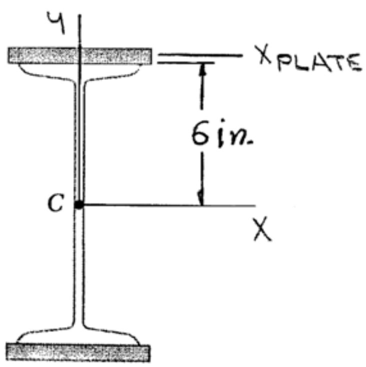
or $\bar{J}_C = 729 \times 10^6\text{ mm}^4$ ◀




## PROBLEM 9.49

Two 1-in. steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the section with respect to the centroidal $x$ and $y$ axes.

## SOLUTION




S-section

$$A = 14.7\text{ in}^2$$

$$\bar{I}_x = 305\text{ in}^4$$

$$\bar{I}_y = 15.7\text{ in}^4$$

$$A = A_S + 2A_{\text{plate}}$$

$$= 14.7\text{ in}^2 + 2(8\text{ in.})(1\text{ in.}) = 30.7\text{ in}^2$$

$$\bar{I}_x = (\bar{I}_x)_S + 2(\bar{I}_x)_{\text{plate}}$$

$$= 305\text{ in}^4 + 2\left[\frac{(8\text{ in.})(1\text{ in.})^3}{12} + (8\text{ in.})(1\text{ in.})(6.5\text{ in.})^2\right]$$

$$= (305 + 677.33)\text{ in}^4 = 982.33\text{ in}^4$$

or $\bar{I}_x = 9.82\text{ in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A} = \dfrac{982.33\text{ in}^4}{30.7\text{ in}^4} = 31.998\text{ in}^2$

or $\bar{k}_x = 5.66\text{ in.}$ ◀

Also $\bar{I}_y = (\bar{I}_y)_S + 2(\bar{I}_y)_{\text{plate}} = 15.7\text{ in}^4 + 2\left[\dfrac{(1\text{ in.})(8\text{ in.})^3}{12}\right]$

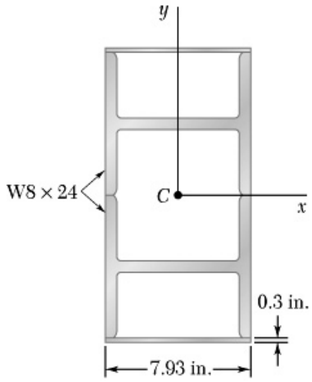
$$= (15.7 + 85.333)\text{ in}^4 = 101.03\text{ in}^4$$

or $\bar{I}_y = 101.0\text{ in}^4$ ◀

and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A} = \dfrac{101.03\text{ in}^4}{30.7\text{ in}^2} = 3.29098\text{ in}^2$

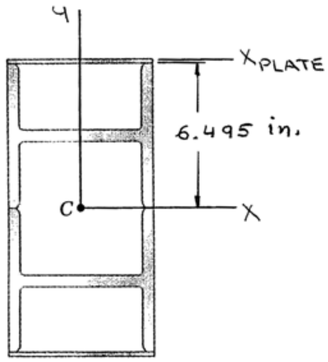
or $\bar{k}_y = 1.814\text{ in.}$ ◀

## PROBLEM 9.50




To form a reinforced box section, two rolled W sections and two plates are welded together. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal axes shown.

## SOLUTION




W-section

$$A = 7.08\text{ in}^2$$

$$\bar{I}_x = 18.3\text{ in}^4$$

$$\bar{I}_y = 82.8\text{ in}^4$$

$$A = 2A_W + 2A_{\text{plate}}$$

$$= 2\left[7.08\text{ in}^2 + (7.93\text{ in.})(0.3\text{ in.})\right]$$

$$= 18.918\text{ in}^2$$

Now

$$\bar{I}_x = 2(\bar{I}_x)_W + 2(\bar{I}_x)_{\text{plate}}$$

$$= 2\left[18.3\text{ in}^4 + (7.08\text{ in}^2)\left(\frac{6.495\text{ in.}}{2}\right)^2\right]$$

$$+ 2\left\{\frac{(7.93\text{ in.})(0.3\text{ in.})^3}{12} + \left[(7.93\text{ in.})(0.3\text{ in.})\right](6.495\text{ in.} + 0.15\text{ in.})^2\right\}$$

$$= 2\left[92.967\text{ in}^4\right] + 2\left[105.07\text{ in}^4\right] = 396.07\text{ in}^4$$

or $\bar{I}_x = 396\text{ in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A} = \dfrac{396.07\text{ in}^4}{18.918\text{ in}^2} = 20.936\text{ in}^2$

or $\bar{k}_x = 4.58\text{ in.}$ ◀

**PROBLEM 9.50 CONTINUED**

Also

$$\bar{I}_y = 2\left(\bar{I}_y\right)_W + 2\left(\bar{I}_y\right)_{\text{plate}}$$

$$= 2\left(82.8\text{ in}^4\right) + 2\left[\frac{(0.3\text{ in.})(7.93\text{ in.})^3}{12}\right] = (165.60 + 24.9339)\text{ in}^4$$

$$= 190.53\text{ in}^4$$

or $\bar{I}_y = 190.5\text{ in}^4$ ◀

and $$\bar{k}_y^2 = \frac{\bar{I}_y}{A} = \frac{190.53\text{ in}^4}{18.918\text{ in}^2} = 10.072$$

or $\bar{k}_x = 3.17$ in. ◀