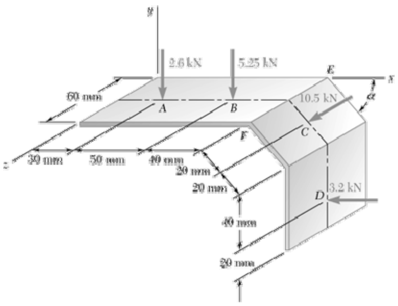
PROBLEM 3.108

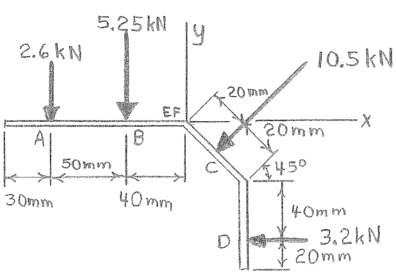


As four holes are punched simultaneously in a piece of aluminum sheet metal, the punches exert on the piece the forces shown. Knowing that the forces are perpendicular to the surfaces of the piece, determine (*a*) the resultant of the applied forces when $\alpha = 45°$ and the point of intersection of the line of action of that resultant with a line drawn through points *A* and *B*, (*b*) the value of α so that the line of action of the resultant passes through fold *EF*.

SOLUTION

Position the origin for the coordinate system along the centerline of the sheet metal at the intersection with line *EF*.

(*a*) Have $\Sigma\mathbf{F} = \mathbf{R}$



$$\mathbf{R} = -\left[2.6\mathbf{j} - 5.25\mathbf{j} - 10.5\left(\cos 45°\mathbf{i} + \sin 45°\mathbf{j}\right) - 3.2\mathbf{i}\right] \text{ kN}$$

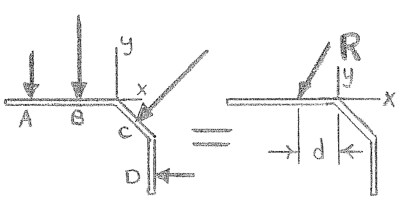
$$\therefore\ \mathbf{R} = -(10.6246 \text{ kN})\mathbf{i} - (15.2746 \text{ kN})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(10.6246)^2 + (15.2746)^2}$$

$$= 18.6064 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-15.2746}{-10.6246}\right) = 55.179°$$

or $\mathbf{R} = 18.61 \text{ kN} \nearrow 55.2°$ ◀



Have $M_{EF} = \Sigma M_{EF}$

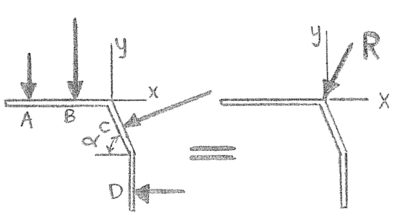
where

$$M_{EF} = (2.6 \text{ kN})(90 \text{ mm}) + (5.25 \text{ kN})(40 \text{ mm})$$

$$- (10.5 \text{ kN})(20 \text{ mm}) - (3.2 \text{ kN})\left[(40 \text{ mm})\sin 45° + 40 \text{ mm}\right]$$

$$\therefore\ M_{EF} = 15.4903 \text{ N}\cdot\text{m}$$

To obtain distance *d* left of *EF*,



Have $M_{EF} = dR_y = d(-15.2746 \text{ kN})$

$$\therefore\ d = \frac{15.4903 \text{ N}\cdot\text{m}}{-15.2746 \times 10^{-3} \text{ N}} = -1.01412 \times 10^{-3} \text{ m}$$

or $d = 1.014$ mm left of *EF* ◀

PROBLEM 3.108 CONTINUED

(*b*) Have $\qquad M_{EF} = 0$

$$M_{EF} = 0 = (2.6 \text{ kN})(90 \text{ mm}) + (5.25 \text{ kN})(40 \text{ mm})$$

$$-(10.5 \text{ kN})(20 \text{ mm})$$

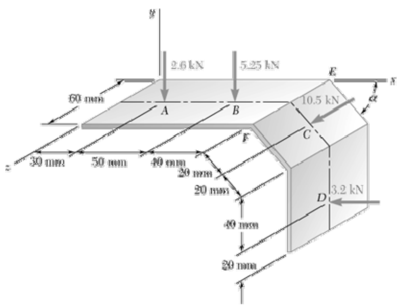
$$-(3.2 \text{ kN})\left[(40 \text{ mm})\sin\alpha + 40 \text{ mm}\right]$$

$$\therefore\ (128 \text{ N}\cdot\text{m})\sin\alpha = 106 \text{ N}\cdot\text{m}$$

$$\sin\alpha = 0.828125$$

$$\alpha = 55.907°$$

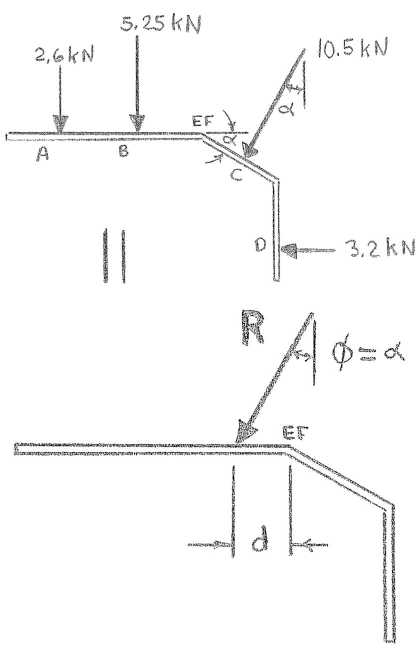
or $\alpha = 55.9°$ ◀



PROBLEM 3.109

As four holes are punched simultaneously in a piece of aluminum sheet metal, the punches exert on the piece the forces shown. Knowing that the forces are perpendicular to the surfaces of the piece, determine (*a*) the value of α so that the resultant of the applied forces is parallel to the 10.5 N force, (*b*) the corresponding resultant of the applied forces and the point of intersection of its line of action with a line drawn through points *A* and *B*.

SOLUTION



(*a*) For the resultant force, **R**, to be parallel to the 10.5 kN force,

$$\alpha = \phi$$

$$\therefore \tan\alpha = \tan\phi = \frac{R_y}{R_x}$$

where

$$R_x = -3.2 \text{ kN} - (10.5 \text{ kN})\sin\alpha$$

$$R_y = -2.6 \text{ kN} - 5.25 \text{ kN} - (10.5 \text{ kN})\cos\alpha$$

$$\therefore \tan\alpha = \frac{3.2 + 10.5\sin\alpha}{7.85 + 10.5\cos\alpha}$$

and $\quad \tan\alpha = \dfrac{3.2}{7.85} = 0.40764$

$$\alpha = 22.178° \qquad \text{or } \alpha = 22.2° \blacktriangleleft$$

(*b*) From $\quad \alpha = 22.178°$

$$R_x = -3.2 \text{ kN} - (10.5 \text{ kN})\sin 22.178° = -7.1636 \text{ kN}$$

$$R_y = -7.85 \text{ kN} - (10.5 \text{ kN})\cos 22.178° = -17.5732 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(7.1636)^2 + (17.5732)^2} = 18.9770 \text{ kN}$$

or $\quad \mathbf{R} = 18.98 \text{ kN} \measuredangle\ 67.8° \blacktriangleleft$

Then

$$M_{EF} = \Sigma M_{EF}$$

where

$$M_{EF} = (2.6 \text{ kN})(90 \text{ mm}) + (5.25 \text{ kN})(40 \text{ mm}) - (10.5 \text{ kN})(20 \text{ mm})$$
$$- (3.2 \text{ kN})\left[(40 \text{ mm})\sin 22.178° + 40 \text{ mm}\right]$$
$$= 57.682 \text{ N}\cdot\text{m}$$

PROBLEM 3.109 CONTINUED

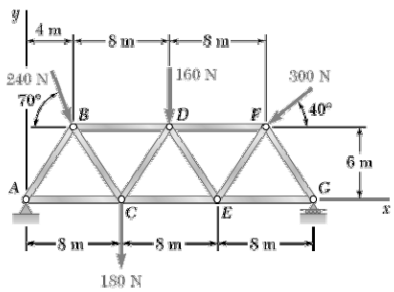
To obtain distance d left of EF,

Have $$M_{EF} = dR_y = d(-17.5732)$$

$$\therefore\ d = \frac{57.682\ \text{N}\cdot\text{m}}{-17.5732 \times 10^3\ \text{N}} = -3.2824 \times 10^{-3}\ \text{m}$$

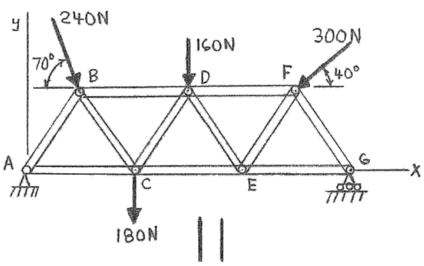
or $d = 3.28$ mm left of EF ◀

PROBLEM 3.110



A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line through points *A* and *G*.

SOLUTION



Have $$\mathbf{R} = \Sigma\mathbf{F}$$

$$\mathbf{R} = (240\text{ N})(\cos 70°\mathbf{i} - \sin 70°\mathbf{j}) - (160\text{ N})\mathbf{j}$$

$$+ (300\text{ N})(-\cos 40°\mathbf{i} - \sin 40°\mathbf{j}) - (180\text{ N})\mathbf{j}$$

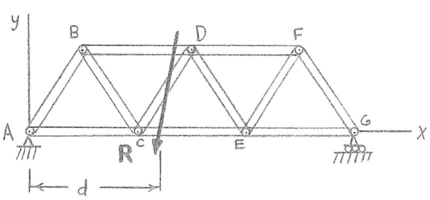
$$\therefore\ \mathbf{R} = -(147.728\text{ N})\mathbf{i} - (758.36\text{ N})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(147.728)^2 + (758.36)^2}$$

$$= 772.62\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-758.36}{-147.728}\right) = 78.977°$$

or $\mathbf{R} = 773\text{ N} \swarrow 79.0°$ ◀



Have $$\Sigma M_A = dR_y$$

where

$$\Sigma M_A = -[240\text{ N}\cos 70°](6\text{ m}) - [240\text{ N}\sin 70°](4\text{ m})$$

$$-(160\text{ N})(12\text{ m}) + [300\text{ N}\cos 40°](6\text{ m})$$

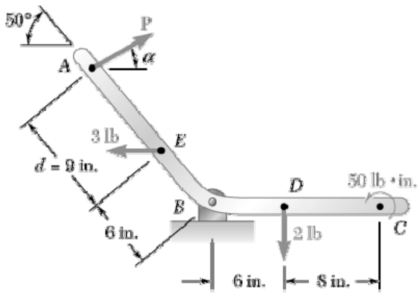
$$-[300\text{ N}\sin 40°](20\text{ m}) - (180\text{ N})(8\text{ m})$$

$$= -7232.5\text{ N}\cdot\text{m}$$

$$\therefore\ d = \frac{-7232.5\text{ N}\cdot\text{m}}{-758.36\text{ N}} = 9.5370\text{ m}$$

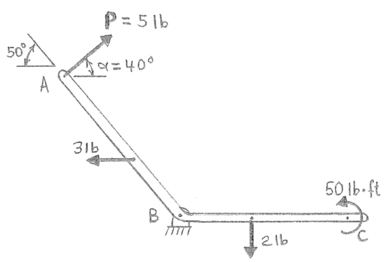
or $d = 9.54$ m to the right of A ◀

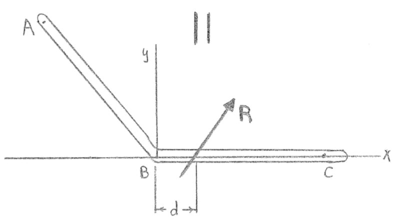
PROBLEM 3.111



Three forces and a couple act on crank *ABC*. For $P = 5$ lb and $\alpha = 40°$, (*a*) determine the resultant of the given system of forces, (*b*) locate the point where the line of action of the resultant intersects a line drawn through points *B* and *C*, (*c*) locate the point where the line of action of the resultant intersects a line drawn through points *A* and *B*.

SOLUTION





(*a*) $$P = 5 \text{ lb}, \qquad \alpha = 40°$$

Have $$\mathbf{R} = \Sigma\mathbf{F}$$

$$= (5 \text{ lb})(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) - (3 \text{ lb})\mathbf{i} - (2 \text{ lb})\mathbf{j}$$

$$\therefore\ \mathbf{R} = (0.83022 \text{ lb})\mathbf{i} + (1.21394 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.83022)^2 + (1.21394)^2}$$

$$= 1.47069 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{1.21394}{0.83022}\right) = 55.632°$$

or $\mathbf{R} = 1.471 \text{ lb} \measuredangle\ 55.6°$ ◀

(*b*) From $$M_B = \Sigma M_B = dR_y$$

where

$$M_B = -\left[(5 \text{ lb})\cos 40°\right]\left[(15 \text{ in.})\sin 50°\right] - \left[(5 \text{ lb})\sin 40°\right]$$

$$\times\left[(15 \text{ in.})\sin 50°\right] + (3 \text{ lb})\left[(6 \text{ in.})\sin 50°\right]$$

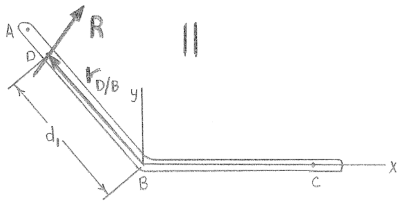
$$-\,(2 \text{ lb})(6 \text{ in.}) + 50 \text{ lb}\cdot\text{in.}$$

$$\therefore\ M_B = -23.211 \text{ lb}\cdot\text{in.}$$

and $$d = \frac{M_B}{R_y} = \frac{-23.211 \text{ lb}\cdot\text{in.}}{1.21394 \text{ lb}} = -19.1205 \text{ in.}$$

or $d = 19.12$ in. to the left of *B* ◀

PROBLEM 3.111 CONTINUED



(c) From $$\mathbf{M}_B = \mathbf{r}_{D/B} \times \mathbf{R}$$

$$-(23.211\text{ lb}\cdot\text{in.})\mathbf{k} = (-d_1\cos 50°\mathbf{i} + d_1\sin 50°\mathbf{j})$$

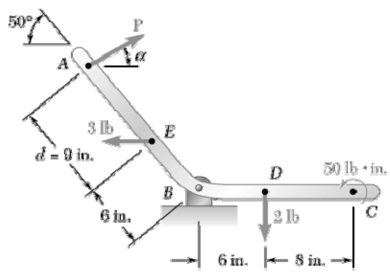
$$\times\left[(-0.83022\text{ lb})\mathbf{i} + (1.21394\text{ lb})\mathbf{j}\right]$$

$$-(23.211\text{ lb}\cdot\text{in.})\mathbf{k} = (-0.78028d_1 - 0.63599d_1)\mathbf{k}$$

$$\therefore\quad d_1 = \frac{23.211}{1.41627} = 16.3889\text{ in.}$$

or $$d_1 = 16.39\text{ in. from } B \text{ along line } AB$$

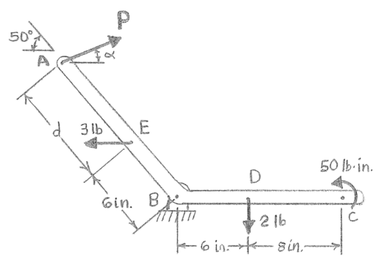
or 1.389 in. above and to the left of A ◀



PROBLEM 3.112

Three forces and a couple act on crank *ABC*. Determine the value of *d* so that the given system of forces is equivalent to zero at (*a*) point *B*, (*b*) point *D*.

SOLUTION



Based on $\Sigma F_x = 0$

$$P\cos\alpha - 3\text{ lb} = 0$$

$$\therefore\quad P\cos\alpha = 3\text{ lb} \qquad (1)$$

and $\Sigma F_y = 0$

$$P\sin\alpha - 2\text{ lb} = 0$$

$$\therefore\quad P\sin\alpha = 2\text{ lb} \qquad (2)$$

Dividing Equation (2) by Equation (1),

$$\tan\alpha = \frac{2}{3}$$

$$\therefore\quad \alpha = 33.690°$$

Substituting into Equation (1),

$$P = \frac{3\text{ lb}}{\cos 33.690°} = 3.6056\text{ lb}$$

or $\mathbf{P} = 3.61\text{ lb} \measuredangle\ 33.7°$

(*a*) Based on $\Sigma M_B = 0$

$$-\left[(3.6056\text{ lb})\cos 33.690°\right]\left[(d + 6\text{ in.})\sin 50°\right]$$

$$-\left[(3.6056\text{ lb})\sin 33.690°\right]\left[(d + 6\text{ in.})\cos 50°\right]$$

$$+(3\text{ lb})\left[(6\text{ in.})\sin 50°\right] - (2\text{ lb})(6\text{ in.}) + 50\text{ lb}\cdot\text{in.} = 0$$

$$-3.5838d = -30.286$$

$$\therefore\ d = 8.4509\text{ in.}$$

or $d = 8.45$ in. ◀

PROBLEM 3.112 CONTINUED

(*b*) Based on $\Sigma M_D = 0$

$$-\left[(3.6056 \text{ lb})\cos 33.690°\right]\left[(d + 6 \text{ in.})\sin 50°\right]$$

$$-\left[(3.6056 \text{ lb})\sin 33.690°\right]\left[(d + 6 \text{ in.})\cos 50° + 6 \text{ in.}\right]$$

$$+(3 \text{ lb})\left[(6 \text{ in.})\sin 50°\right] + 50 \text{ lb}\cdot\text{in.} = 0$$

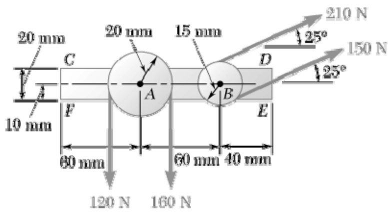
$$-3.5838d = -30.286$$

$$\therefore\ d = 8.4509 \text{ in.}$$

or $d = 8.45$ in. ◀

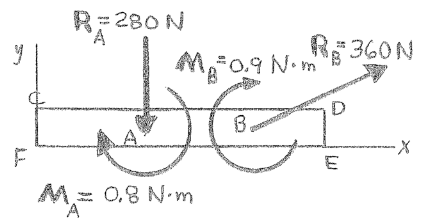
This result is expected, since $\mathbf{R} = 0$ and $\mathbf{M}_B^R = 0$ for $d = 8.45$ in. implies that $\mathbf{R} = 0$ and $\mathbf{M} = 0$ at any other point for the value of *d* found in part *a*.

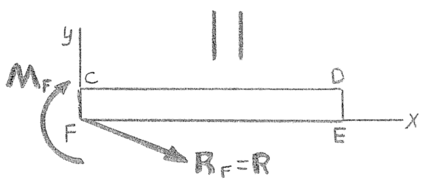
PROBLEM 3.113

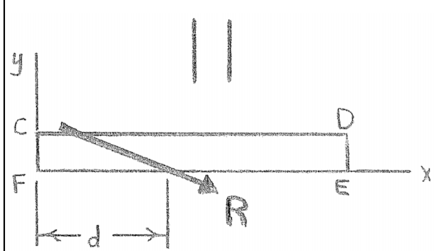


Pulleys *A* and *B* are mounted on bracket *CDEF*. The tension on each side of the two belts is as shown. Replace the four forces with a single equivalent force, and determine where its line of action intersects the bottom edge of the bracket.

SOLUTION







Equivalent force-couple at *A* due to belts on pulley *A*

Have $\qquad \Sigma\mathbf{F}: \quad -120\text{ N} - 160\text{ N} = R_A$

$$\therefore \quad \mathbf{R}_A = 280\text{ N} \downarrow$$

Have $\qquad \Sigma\mathbf{M}_A: \quad -40\text{ N}(0.02\text{ m}) = M_A$

$$\therefore \quad \mathbf{M}_A = 0.8\text{ N}\cdot\text{m} \circlearrowright$$

Equivalent force-couple at *B* due to belts on pulley *B*

Have $\qquad \Sigma\mathbf{F}: \quad (210\text{ N} + 150\text{ N}) \measuredangle\, 25° = \mathbf{R}_B$

$$\therefore \quad \mathbf{R}_B = 360\text{ N} \measuredangle\, 25°$$

Have $\qquad \Sigma\mathbf{M}_B: \quad -60\text{ N}(0.015\text{ m}) = M_B$

$$\therefore \quad \mathbf{M}_B = 0.9\text{ N}\cdot\text{m} \circlearrowright$$

Equivalent force-couple at *F*

Have $\qquad \Sigma\mathbf{F}: \quad \mathbf{R}_F = (-280\text{ N})\mathbf{j} + (360\text{ N})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$

$$= (326.27\text{ N})\mathbf{i} - (127.857\text{ N})\mathbf{j}$$

$$R = R_F = \sqrt{R_{Fx}^2 + R_{Fy}^2} = \sqrt{(326.27)^2 + (127.857)^2} = 350.43\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_{Fy}}{R_{Fx}}\right) = \tan^{-1}\left(\frac{-127.857}{326.27}\right) = -21.399°$$

or $\mathbf{R}_F = \mathbf{R} = 350\text{ N} \searrow 21.4°$ ◀

PROBLEM 3.113 CONTINUED

Have

$$\Sigma \mathbf{M}_F: \quad M_F = -(280\text{ N})(0.06\text{ m}) - 0.80\text{ N}\cdot\text{m}$$

$$-\left[(360\text{ N})\cos 25°\right](0.010\text{ m})$$

$$+\left[(360\text{ N})\sin 25°\right](0.120\text{ m}) - 0.90\text{ N}\cdot\text{m}$$

$$\mathbf{M}_F = -(3.5056\text{ N}\cdot\text{m})\mathbf{k}$$

To determine where a single resultant force will intersect line *FE*,

$$M_F = dR_y$$

$$\therefore \quad d = \frac{M_F}{R_y} = \frac{-3.5056\text{ N}\cdot\text{m}}{-127.857\text{ N}} = 0.027418\text{ m} = 27.418\text{ mm}$$

or $d = 27.4\text{ mm}$ ◀