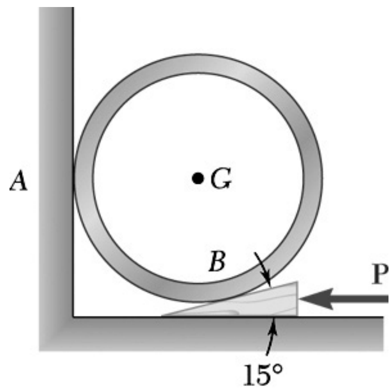
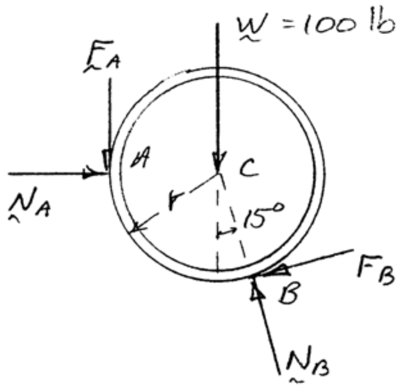
PROBLEM 8.61



A 15° wedge is forced under a 100-lb pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping is impending at *A*.

SOLUTION

FBD pipe:



$$\circlearrowleft \Sigma M_C = 0: \quad rF_A - rF_B = 0$$

or $$F_A = F_B$$

It is apparent that $N_B > N_A$, so if $(\mu_s)_A = (\mu_s)_B$, motion must impend first at *A*. As $(\mu_s)_A$ is increased to some $(\mu_s^*)_A$, motion will impend simultaneously at *A* and *B*.

Then $$F_A = F_B = \mu_{sB} N_B = 0.2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B \cos 15° - F_B \sin 15° - F_A - 100 \text{ lb} = 0$$

$$N_B \cos 15° - 0.2 N_B \sin 15° - 0.2 N_B = 100 \text{ lb}$$

or $$N_B = 140.024 \text{ lb}$$

So $$F_A = F_B = 0.2 N_B = 28.005 \text{ lb}$$

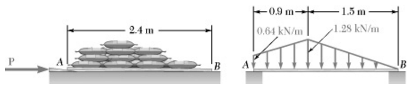
$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B \sin 15° - F_B \cos 15° = 0$$

$$N_A = 140.024 \sin 15° + 28.005 \cos 15° = 63.29 \text{ lb}$$

Then $$(\mu_s^*)_A = \frac{F_A}{N_A} = \frac{28.005 \text{ lb}}{63.29 \text{ lb}}$$

or $$(\mu_s^*)_A = 0.442 \blacktriangleleft$$

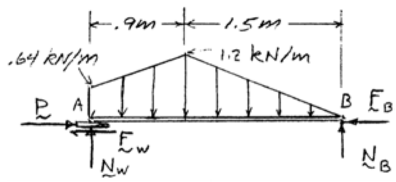
PROBLEM 8.62



Bags of grass seed are stored on a wooden plank as shown. To move the plank, a 9° wedge is driven under end A. Knowing that the weight of the grass seed can be represented by the distributed load shown and that the coefficient of static friction is 0.45 between all surfaces of contact, (a) determine the force **P** for which motion of the wedge is impending, (b) indicate whether the plank will slide on the floor.

SOLUTION

FBD plank + wedge:



(a) $\circlearrowleft \Sigma M_A = 0$: $(2.4\text{ m})N_B - (0.45\text{ m})(0.64\text{ kN/m})(0.9\text{ m})$

$$-(0.6\text{ m})\frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$$

$$-(1.4\text{ m})\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $$N_B = 0.740\text{ kN} = 740\text{ N}$$

$\uparrow \Sigma F_y = 0$: $N_W - (0.64\text{ kN/m})(0.9\text{ m}) - \frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$

$$-\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $$N_W = 1.084\text{ kN} = 1084\text{ N}$$

Assume impending motion of the wedge on the floor and the plank on the floor at B.

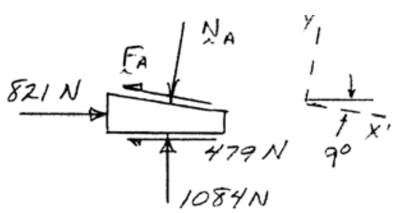
So $$F_W = \mu_s N_W = 0.45(1084\text{ N}) = 478.8\text{ N}$$

and $$F_B = \mu_s N_B = 0.45(740\text{ N}) = 333\text{ N}$$

$\rightarrow \Sigma F_x = 0$: $P - F_W - F_B = 0$

or $$P = 478.8\text{ N} + 333\text{ N} \qquad P = 821\text{ N} \blacktriangleleft$$

Check wedge:



(b) $\uparrow \Sigma F_y = 0$: $(1084\text{ N})\cos 9° + (821\text{ N} - 479\text{ N})\sin 9° - N_A = 0$

or $$N_A = 1124\text{ N}$$

$\searrow \Sigma F_x = 0$: $(821\text{ N} - 479\text{ N})\cos 9° - (1084\text{ N})\sin 9° - F_A = 0$

or $$F_A = 168\text{ N}$$

$$F_A < \mu_s N_A = 0.45(1124\text{ N}) = 506\text{ N}$$

So, no impending motion at wedge/plank

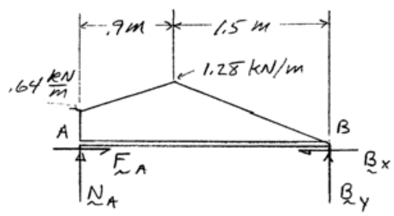
$\therefore$ Impending motion of plank on floor at B $\blacktriangleleft$

PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is driven under the plank at B instead of at A.

SOLUTION

FBD plank:



$$\rightarrow \Sigma F_x = 0: \quad F_A - B_x = 0$$

$$F_A = B_x$$

(a) $$\Sigma M_A = 0: \quad (2.4\text{ m})B_y - (0.45\text{ m})(0.64\text{ kN/m})(0.9\text{ m})$$

$$-(0.6\text{ m})\frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$$

$$-(1.4\text{ m})\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $$B_y = 0.740\text{ kN} = 740\text{ N}$$

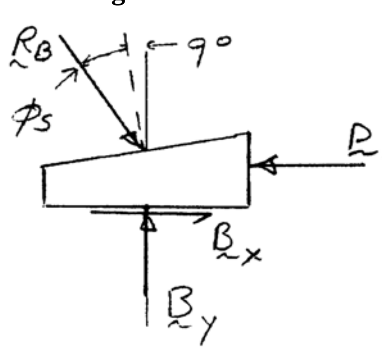
$$\uparrow \Sigma F_y = 0: \quad N_A - (0.64\text{ kN/m})(0.9\text{ m}) - \frac{1}{2}(0.64\text{ kN/m})(0.9\text{ m})$$

$$-\frac{1}{2}(1.28\text{ kN/m})(1.5\text{ m}) = 0$$

or $$N_A = 1.084\text{ kN} = 1084\text{ N}$$

Since $B_y < N_A$, assume impending motion of the wedge under the plank at B.

FBD wedge:



$$(R_B)_y = B_y = 740\text{ N} \quad \text{and} \quad B_x = \mu_s B_y = 0.45(740\text{ N}) = 333\text{ N}$$

$$(R_B)_x = (R_B)_y \tan(9° + \phi_s)$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.45 = 24.228°$$

So $$(R_B)_x = (740\text{ N})\tan(9° + 24.228°) = 485\text{ N}$$

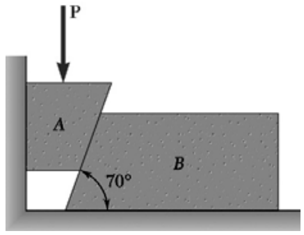
$$\rightarrow \Sigma F_x = 0: \quad 485\text{ N} - 333\text{ N} - P = 0$$

$$\mathbf{P} = 818\text{ N} \leftarrow \blacktriangleleft$$

(b) Check:

$$F_A = B_x = 333\text{ N} \quad \text{and} \quad \frac{F_A}{N_A} = \frac{333}{1084} = 0.307 < \mu_s \qquad \text{OK}$$

No impending slip of plank at A ◀

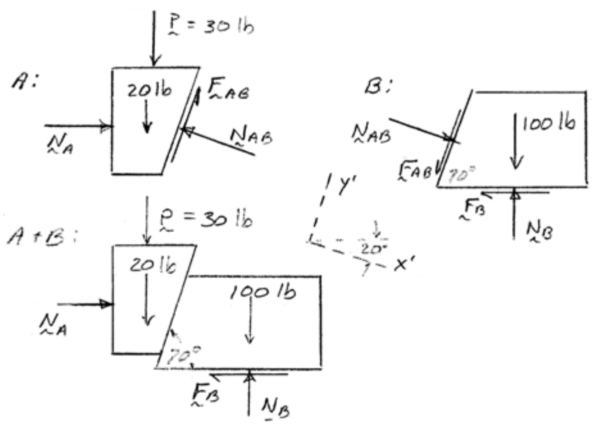


PROBLEM 8.64

The 20-lb block A is at rest against the 100-lb block B as shown. The coefficient of static friction μ_s is the same between blocks A and B and between block B and the floor, while friction between block A and the wall can be neglected. Knowing that $P = 30$ lb, determine the value of μ_s for which motion is impending.

SOLUTION

FBD's:



Impending motion at all surfaces $\qquad F_{AB} = \mu_s N_{AB}$

$$F_B = \mu_s N_B$$

$A + B$: $\quad \uparrow \Sigma F_y = 0{:}\quad N_B - 30\text{ lb} - 20\text{ lb} - 100\text{ lb} = 0$

or $\qquad N_B = 150\text{ lb}$

and $\qquad F_B = \mu_s N_B = (150\text{ lb})\mu_s$

$\longrightarrow \Sigma F_x = 0{:}\quad N_A - F_B = 0 \qquad$ so that $\qquad N_A = (150\text{ lb})\mu_s$

A: $\quad \searrow \Sigma F_{x'} = 0{:}\quad N_A \cos 20° + (30\text{ lb} + 20\text{ lb})\sin 20° - N_{AB} = 0$

or $\qquad N_{AB} = 17.1010\text{ lb} + \mu_s(140.954\text{ lb})$

$\nearrow \Sigma F_{y'} = 0{:}\quad F_{AB} + N_A \sin 20° - (30\text{ lb} + 20\text{ lb})\cos 20° = 0$

or $\qquad F_{AB} = 46.985\text{ lb} - \mu_s(51.303\text{ lb})$

But $\qquad F_{AB} = \mu_s N_{AB}{:}\quad 46.985 - 51.303\mu_s = 17.101\mu_s + 140.954\mu_s^2$

$$\mu_s^2 + 0.4853\mu_s - 0.3333 = 0$$

$$\mu_s = -0.2427 \pm 0.6263$$

$\mu_s > 0 \qquad$ so $\qquad \mu_s = 0.384$ ◀

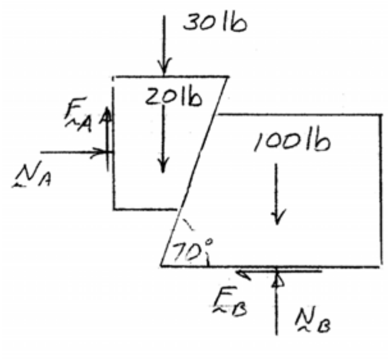
PROBLEM 8.65

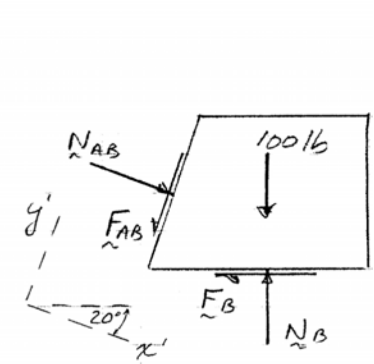
Solve Problem 8.64 assuming that μ_s is the coefficient of static friction between all surfaces of contact.

SOLUTION

FBD's:

A + *B*: *B*:





Impending motion at all surfaces, so

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$F_{AB} = \mu_s N_{AB}$$

$A+B$: $\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0$ or $N_A = F_B = \mu_s N_B$

$\uparrow \Sigma F_y = 0: \quad F_A - 30\text{ lb} - 20\text{ lb} - 100\text{ lb} + N_B = 0$ or $\mu_s N_A + N_B = 150\text{ lb}$

So

$$N_B = \frac{150\text{ lb}}{1+\mu_s^2} \qquad \text{and} \qquad F_B = \frac{\mu_s}{1+\mu_s^2}(150\text{ lb})$$

B: $\searrow \Sigma F_{x'} = 0: \quad N_{AB} + (100\text{ lb} - N_B)\sin 20° - F_B \cos 20° = 0$

or

$$N_{AB} = N_B \sin 20° + F_B \cos 20° - (100\text{ lb})\sin 20°$$

$\nearrow \Sigma F_{y'} = 0: \quad -F_{AB} + (N_B - 100\text{ lb})\cos 20° - F_B \sin 20° = 0$

or

$$F_{AB} = N_B \cos 20° - F_B \sin 20° - (100\text{ lb})\cos 20°$$

PROBLEM 8.65 CONTINUED

Now $$F_{AB} = \mu_s N_{AB}: \quad \frac{150\text{ lb}}{1+\mu_s^2}\cos 20° - \frac{\mu_s}{1+\mu_s^2}(150\text{ lb})\sin 20° - (100\text{ lb})\cos 20°$$

$$= \frac{\mu_s}{1+\mu_s^2}(150\text{ lb})\sin 20° + \frac{\mu_s^2}{1+\mu_s^2}(150\text{ lb})\cos 20° - \mu_s(100\text{ lb})\sin 20°$$

$$2\mu_s^3 - 5\mu_s^2 \operatorname{ctn} 20° - 4\mu_s + \operatorname{ctn} 20° = 0$$

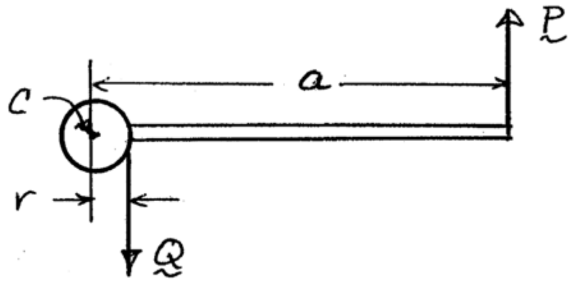
Solving numerically: $\mu_s = 0.330$ ◀

PROBLEM 8.66

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

FBD jack handle:

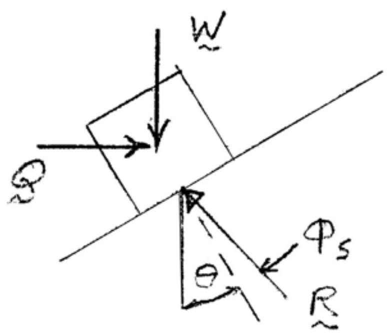


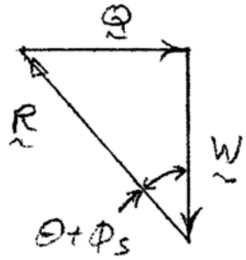
See Section 8.6

$$\circlearrowleft \Sigma M_C = 0: \quad aP - rQ = 0 \text{ or } P = \frac{r}{a}Q$$

FBD block on incline:

(*a*) Raising load



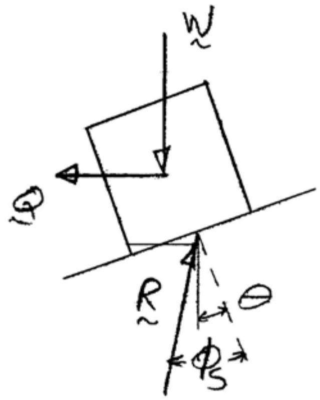


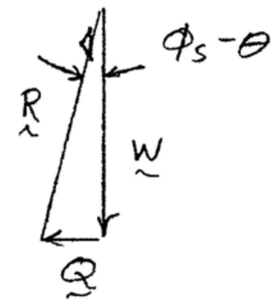
$$Q = W\tan(\theta + \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

PROBLEM 8.66 CONTINUED

(b) Lowering load if screw is self-locking (i.e.: if $\phi_s > \theta$)

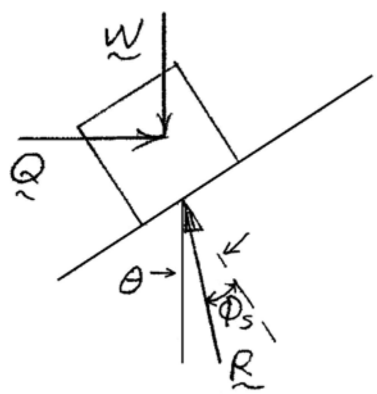


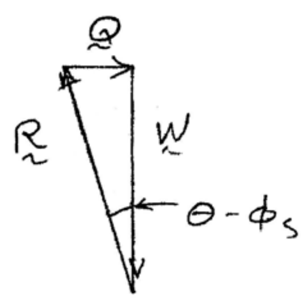


$$Q = W\tan\left(\phi_s - \theta\right)$$

$$P = \frac{r}{a}W\tan\left(\phi_s - \theta\right) \blacktriangleleft$$

(c) Holding load is screw is not self-locking $\left(\text{i.e. if } \phi_s < \theta\right)$

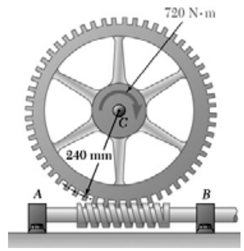




$$Q = W\tan\left(\theta - \phi_s\right)$$

$$P = \frac{r}{a}W\tan\left(\theta - \phi_s\right) \blacktriangleleft$$

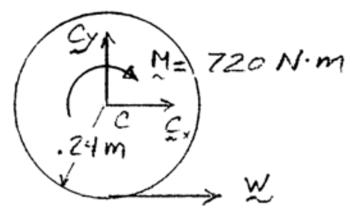
PROBLEM 8.67



The square-threaded worm gear shown has a mean radius of 30 mm and a lead of 7.5 mm. The larger gear is subjected to a constant clockwise couple of 720 N·m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

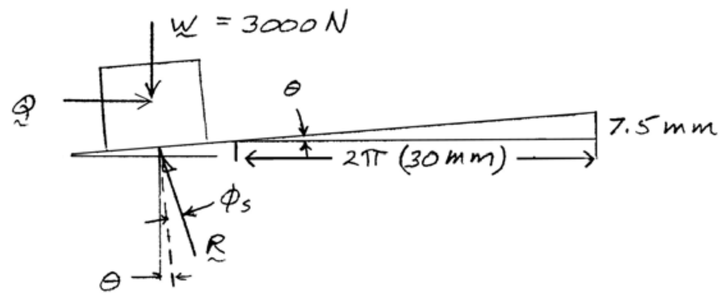
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24\text{ m})W - 720\text{ N}\cdot\text{m} = 0$$

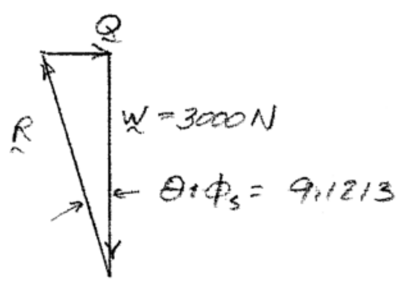
$$W = 3000\text{ N}$$

Block on incline:



$$\theta = \tan^{-1}\frac{7.5\text{ mm}}{2\pi(30\text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$

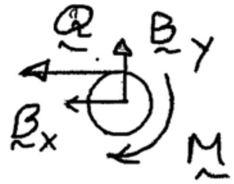


$$Q = (3000\text{ N})\tan 9.1213°$$

$$= 481.7\text{ N}$$

PROBLEM 8.67 CONTINUED

Worm gear:



$$r = 30 \text{ mm}$$

$$= 0.030 \text{ m}$$

$$\circlearrowright \Sigma M_B = 0: \quad rQ - M = 0$$

$$M = rQ = (0.030 \text{ m})(481.7 \text{ N})$$

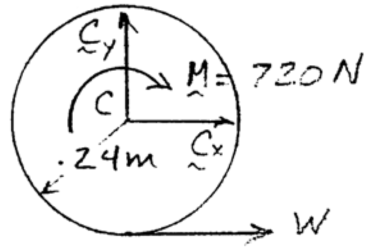
$$M = 14.45 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 8.68

In Problem 8.67, determine the couple that must be applied to shaft *AB* in order to rotate the gear clockwise.

SOLUTION

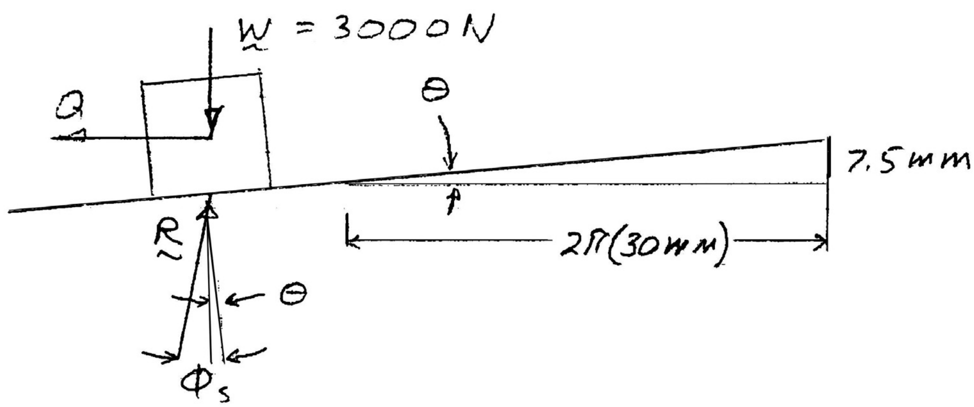
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N}\cdot\text{m} = 0$$

$$W = 3000 \text{ N}$$

Block on incline:



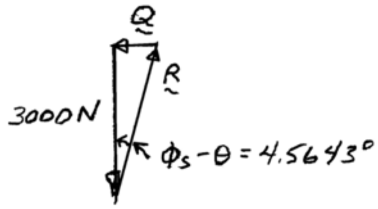
$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu = \tan^{-1}0.12$$

$$\phi_s = 6.8428°$$

$$\phi_s - \theta = 4.5643°$$

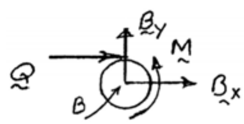
PROBLEM 8.68 CONTINUED



$$Q = (3000\text{ N})\tan 4.5643°$$

$$= 239.5\text{ N}$$

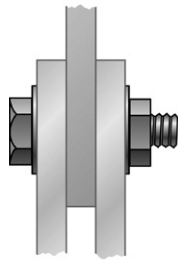
Worm gear:



$r = 30$ mm

$$\circlearrowleft \Sigma M_B = 0: \quad M - rQ = 0$$

$$M = rQ = (0.030\text{ m})(239.5\text{ N}) = 7.18\text{ N}\cdot\text{m} \blacktriangleleft$$

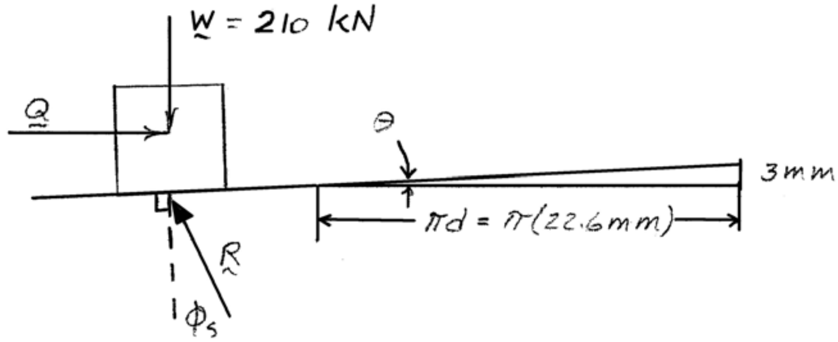


PROBLEM 8.69

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

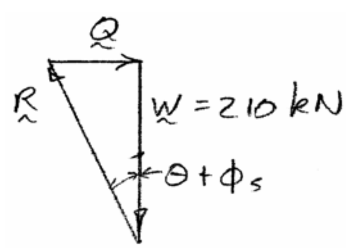
FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi} = 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.8014°$$



$$Q = (210\text{ kN})\tan(21.8014° + 2.4195°)$$

$$Q = 94.47\text{ kN}$$

$$\text{Torque} = \frac{d}{2}Q = \frac{22.6\text{ mm}}{2}(94.47\text{ kN})$$

$$= 1067.5\text{ N}\cdot\text{m}$$

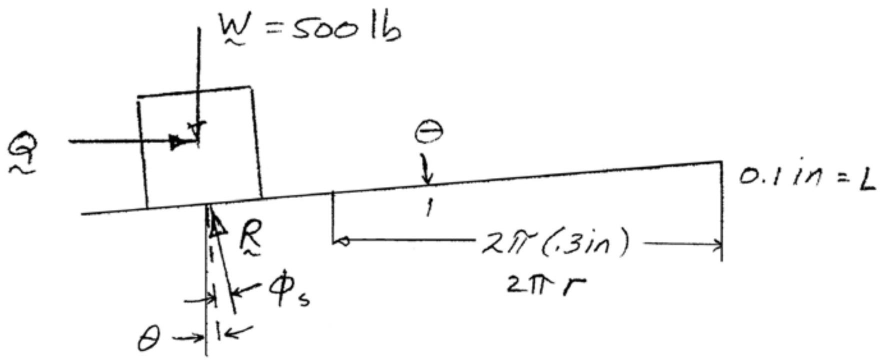
$$\text{Torque} = 1.068\text{ kN}\cdot\text{m} \blacktriangleleft$$

PROBLEM 8.70

The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 0.3 in. and pitch 0.1 in. Rod *A* has a right-handed thread and rod *B* a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

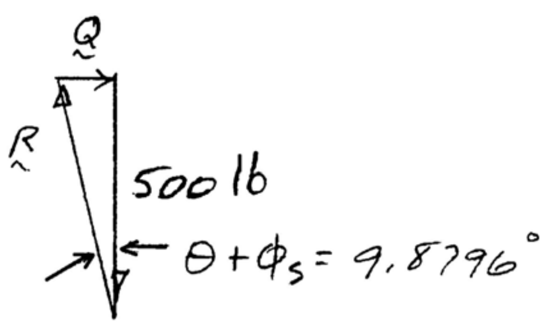
SOLUTION

Block on incline:



$$\theta = \tan^{-1}\frac{0.1\text{ in.}}{2\pi(0.3\text{ in.})} = 3.0368°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$



$$Q = (500\text{ lb})\tan 9.8796° = 87.08\text{ lb}$$

Couple on each side

$$M = rQ = (0.3\text{ in.})(87.08\text{ lb}) = 26.12\text{ lb}\cdot\text{in.}$$

Couple to turn $= 2M = 52.2\text{ lb}\cdot\text{in.}$ ◀