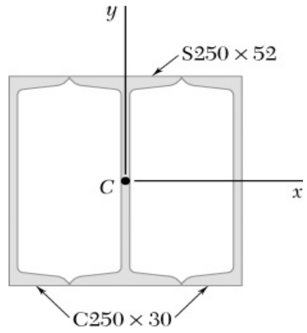


PROBLEM 9.51

Two C250 × 30 channels are welded to a 250 × 52 rolled S section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION

Use Figure 9.13B (textbook) properties of rolled-steel shapes (SI units) to get the values for C250 and S250 shapes

S250 × 52 section: $A = 6670 \text{ mm}^2$

$I_x = 61.2 \times 10^6 \text{ mm}^4$

$I_y = 3.59 \times 10^6 \text{ mm}^4$

C250 × 30 section: $A = 3780 \text{ mm}^2$

$I_x = 32.6 \times 10^6 \text{ mm}^4$

$I_y = 1.14 \times 10^6 \text{ mm}^4$

How, for the combined section:

$$A = A_S + 2A_C$$
$$= [6670 + 2(3780)] \text{mm}^2$$
$$= 14\ 230 \text{ mm}^2$$

$$\bar{I}_x = (I_x)_S + 2(I_x)_C$$
$$= [61.2 \times 10^6 + 2(32.6 \times 10^6)] \text{mm}^4$$

or $\bar{I}_x = 126.4 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_y = (I_y)_S + 2[(I_y)_C + A_C d^2]$$

where *d* is the distance from the centroid of the C section to the centroid *C* of the combined section

Now $$\bar{I}_y = 3.59 \times 10^6 \text{ mm}^4 + 2\left[(1.14 \times 10^6 \text{ mm}^4) + (3780 \text{ mm}^2)\left(\frac{126}{2} + 69 - 15.3\right)^2 \text{ mm}^2\right]$$

$$= (3.59 + 2.28 + 102.9588) \times 10^6 \text{ mm}^4$$

or $\bar{I}_y = 108.8 \times 10^6 \text{ mm}^4$ ◀

Also $$\bar{k}_x = \sqrt{\frac{\bar{I}_x}{A}}$$

$$= \sqrt{\frac{126.4 \times 10^6 \text{ mm}^4}{14\ 230 \text{ mm}^2}}$$

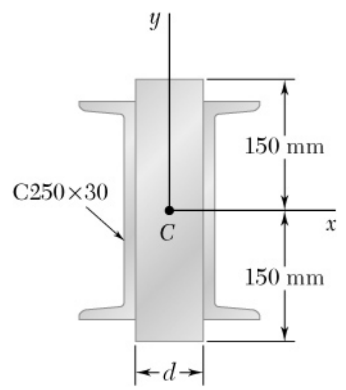
or $\bar{k}_x = 94.2 \text{ mm}$ ◀

PROBLEM 9.51 CONTINUED

And $$\bar{k}_y = \sqrt{\frac{\bar{I}_y}{A}}$$

$$= \sqrt{\frac{108.8 \times 10^6 \text{ mm}^4}{14\ 230 \text{ mm}^2}}$$

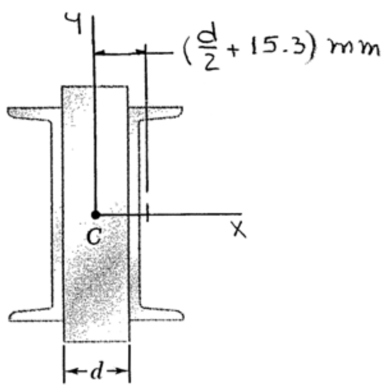
or $\bar{k}_y = 87.5 \text{ mm}$ ◀



PROBLEM 9.52

Two channels are welded to a $d \times$ 300-mm steel plate as shown. Determine the width d for which the ratio $\bar{I}_x / \bar{I}_y$ of the centroidal moments of inertia of the section is 16.

SOLUTION



Channel: $\qquad A = 3780 \text{ mm}^2$

$$\bar{I}_x = 32.6 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.14 \times 10^6 \text{ mm}^4$$

Now $\qquad \bar{I}_x = 2(\bar{I}_x)_C + (\bar{I}_x)_{\text{plate}}$

$$= 2(32.6 \times 10^6 \text{ mm}^4) + \frac{d}{12}(300 \text{ mm})^3$$

$$= (65.2 \times 10^6 + 2.25d \times 10^6) \text{ mm}^4$$

And

$$\bar{I}_y = 2(I_y)_{\text{channel}} + (\bar{I}_y)_{\text{plate}}$$

$$= 2\left[1.14 \times 10^6 \text{ mm}^4 + (3780 \text{ mm}^2)\left(\frac{d}{2} + 15.3 \text{ mm}\right)^2\right] + \frac{(300 \text{ mm})d^3}{12}$$

$$= \left[(2.28 \times 10^6 + 1890d + 115.668 \times 10^3 d + 1.7697 \times 10^6) + 25d^3\right] \text{mm}^4$$

$$= (25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6) \text{ mm}^4$$

Given $\qquad \bar{I}_x = 16\bar{I}_y$

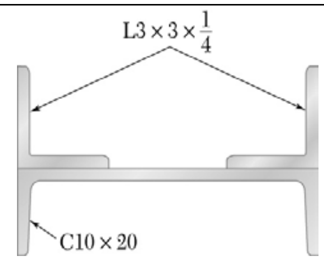
Then $\quad 65.2 \times 10^6 + 2.25d \times 10^6$

$$= 16(25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6)$$

or $\qquad 25d^3 + 1890d^2 - 24.955d - 25300 = 0$

Solving numerically $\qquad d = 12.2935 \text{ mm}$

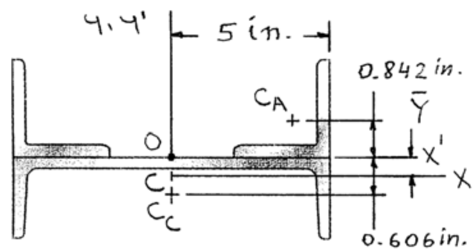
or $\quad d = 12.29 \text{ mm}$ ◀



PROBLEM 9.53

Two L3 × 3 × $\frac{1}{4}$-in. angles are welded to a C10 × 20 channel. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the web of the channel.

SOLUTION



Angle: $A = 1.44\text{ in}^2$

$$\bar{I}_x = \bar{I}_y = 1.24\text{ in}^4$$

Channel: $A = 5.88\text{ in}^2$

$$\bar{I}_x = 2.81\text{ in}^4 \qquad \bar{I}_y = 78.9\text{ in}^4$$

Locate the centroid

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2\left[\left(1.44\text{ in}^2\right)\left(0.842\text{ in.}\right)\right] + \left(5.88\text{ in}^2\right)\left(-0.606\text{ in.}\right)}{2\left(1.44\text{ in}^2\right) + 5.88\text{ in}^2}$$

$$= \frac{\left(2.42496 - 3.5638\right)\text{in}^3}{8.765\text{ in}^4} = -0.12995\text{ in.}$$

Now $$\left(\bar{I}_x\right) = 2\left(I_x\right)_L + \left(I_x\right)_C = 2\left[1.24\text{ in}^4 + \left(1.44\text{ in}^2\right)\left(0.842\text{ in.} + 0.12995\text{ in.}\right)^2\right]$$

$$+\left[2.81\text{ in}^4 + \left(5.88\text{ in}^2\right)\left(0.606\text{ in.} - 0.12995\text{ in.}\right)^2\right]$$

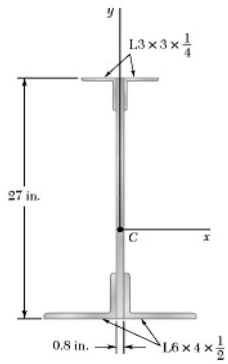
$$= 2\left(2.6003\right)\text{in}^4 + 4.1425\text{ in}^4 = 9.3431\text{ in}^4$$

or $\bar{I}_x = 9.34\text{ in}^4$ ◀

Also $$\left(\bar{I}_y\right) = 2\left(I_y\right)_L + \left(\bar{I}_y\right)_C = 2\left[2.14\text{ in}^4 + 1.44\text{ in}^2\left(5\text{ in.} - 0.842\text{ in.}\right)^2\right] + 7.89\text{ in}^4$$

$$= 2\left(26.136\right)\text{in}^4 + 78.9\text{ in}^4 = 131.17\text{ in}^4$$

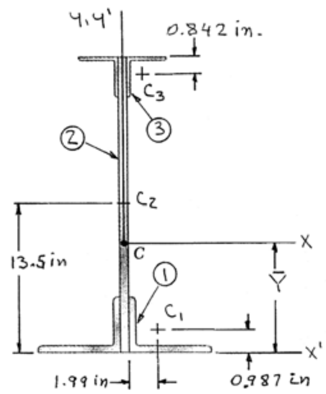
or $\bar{I}_y = 131.2\text{ in}^4$ ◀



PROBLEM 9.54

To form an unsymmetrical girder, two L3 × 3 × $\frac{1}{4}$ -in. angles and two L6 × 4 × $\frac{1}{2}$ -in. angles are welded to a 0.8-in. steel plate as shown. Determine the moments of inertia of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION



Angle:

$$L3 \times 3 \times \frac{1}{4}:$$

$$A = 1.44\ \text{in}^2 \qquad \bar{I}_x = \bar{I}_y = 1.24\ \text{in}^4$$

$$L6 \times 4 \times \frac{1}{2}:$$

$$A = 4.75\ \text{in}^2 \qquad \bar{I}_x = 6.27\ \text{in}^4$$

$$\bar{I}_y = 17.4\ \text{in}^4$$

Plate:

$$A = (27\ \text{in.})(0.8\ \text{in.}) = 21.6\ \text{in}^2$$

$$\bar{I}_x = \frac{1}{12}(0.8\ \text{in.})(27\ \text{in.})^3 = 1312.2\ \text{in}^4$$

$$\bar{I}_y = \frac{1}{12}(27\ \text{in.})(0.8\ \text{in.})^3 = 1.152\ \text{in}^4$$

PROBLEM 9.54 CONTINUED

Centroid: $$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or $$\bar{Y} = \frac{2\left[\left(1.44\text{ in}^2\right)\left(27\text{ in.} - 0.84\text{ in.}\right)\right] + 2\left[\left(4.75\text{ in}^2\right)\left(0.987\text{ in.}\right)\right] + \left(21.6\text{ in}^2\right)\left(13.5\text{ in.}\right)^2}{2\left(1.44\text{ in}^2 + 4.75\text{ in}^2\right) + 21.6\text{ in}^2}$$

$$= \frac{376.31\text{ in}^3}{33.98\text{ in}^2} = 11.0745\text{ in.}$$

Now $$\bar{I}_x = 2\left(I_x\right)_1 + 2\left(I_x\right)_3 + \left(I_x\right)_2$$

$$= 2\left[6.25 + 4.75\left(11.075 - 0.987\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(27 - 0.842 - 11.075\right)^2\right]\text{in}^4$$

$$+\left[1312.2 + 21.6\left(13.5 - 11.075\right)^2\right]\text{in}^4$$

$$= 2\left(489.67\right)\text{in}^4 + 2\left(328.84\right)\text{in}^4 + 1439.22\text{ in}^4 = 3076.24\text{ in}^4$$

or $\bar{I}_x = 3076\text{ in}^4$ ◀

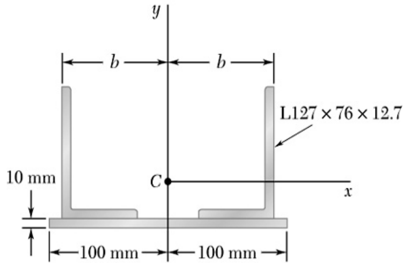
Also $$\left(\bar{I}_y\right) = 2\left(I_y\right)_1 + 2\left(I_y\right)_3 + \left(I_y\right)_2$$

$$= 2\left[17.4 + 4.75\left(0.4 + 1.99\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(0.4 + 0.842\right)^2\right]\text{in}^4 + 1.152\text{ in}^4$$

$$= 2\left(44.532\right)\text{in}^4 + 2\left(3.4613\right)\text{in}^4 + 1.152\text{ in}^4$$

$$= 97.139\text{ in}^4$$

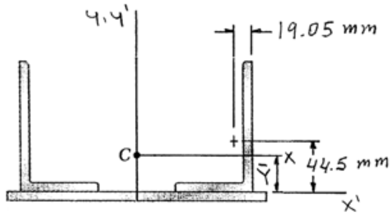
or $\bar{I}_y = 97.1\text{ in}^4$ ◀



PROBLEM 9.55

Two L127 × 76 × 12.7-mm angles are welded to a 10-mm steel plate. Determine the distance *b* and the centroidal moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the combined section knowing that $\bar{I}_y = 3\bar{I}_x$.

SOLUTION



Angle:
$$A = 2420 \text{ mm}^2$$
$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = 1.074 \times 10^6 \text{ mm}^4$$

Plate:
$$A = (200 \text{ mm})(10 \text{ mm}) = 2000 \text{ mm}^2$$
$$\bar{I}_x = \frac{1}{12}(200 \text{ mm})(10 \text{ mm})^3 = 0.01667 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = \frac{1}{12}(10 \text{ mm})(200 \text{ mm})^3 = 6.6667 \times 10^6 \text{ mm}^4$$

Centroid
$$\bar{X} = 0$$
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or
$$\bar{Y} = \frac{2(2420 \text{ mm}^2)(44.5 \text{ mm}) + 2000 \text{ mm}^2(-5 \text{ mm})}{[2(2420) + 2000] \text{ mm}^2} = \frac{205.380 \text{ mm}^3}{6840 \text{ mm}^2}$$
$$= 30.026 \text{ mm}$$

Now
$$\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$$
$$= 2\left[3.93 \times 10^6 + (2420)(44.5 - 30.026)^2\right] \text{mm}^4$$
$$+\left[0.01667 \times 10^6 + (2000)(30.026 + 5)^2\right] \text{mm}^4$$
$$= 2(4.43698 \times 10^6) \text{ mm}^4 + 2.4703 \times 10^6 \text{ mm}^4$$
$$= 11.344 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 11.34 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.55 CONTINUED

Also $$\bar{I}_y = 2\left(I_y\right)_{\text{angle}} + \left(\bar{I}_y\right)_{\text{plate}}$$

Where $$\left(\bar{I}_y\right)_{\text{angle}} = 1.074 \times 10^6 \text{ mm}^4 + \left(2420 \text{ mm}^2\right)\left(b - 19.05 \text{ mm}\right)^2$$

$$= \left[1.074 \times 10^6 + (2420)\left(b^2 - 38.1b + 362.9\right)\right] \text{mm}^4$$

$$= \left[2420b^2 - 92202b + 1.9522 \times 10^6\right] \text{mm}^4$$

and $$\left(\bar{I}_y\right)_{\text{plate}} = 6.6667 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_y = 3\left(\bar{I}_x\right)$$

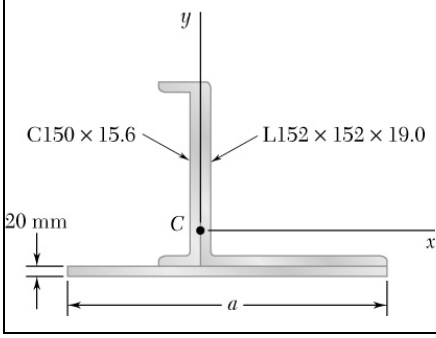
Then $$2\left[2420b^2 - 92202b + 1.9522 \times 10^6\right] \text{mm}^4 + 6.6667 \times 10^6 \text{ mm}^4 = 3\left[\left(11.34 \times 10^6\right) \text{mm}^4\right]$$

or $$2420b^2 - 9.2202b + 1.9522 \times 10^6 - 13.6767 \times 10^6 = 0$$

$$b^2 - 38.1b - 4844.8 = 0$$

$$b = 91.2144 \text{ mm}$$

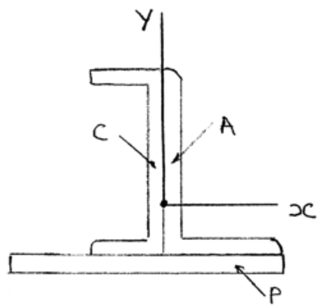
or $b = 91.2$ mm ◀



PROBLEM 9.56

A channel and an angle are welded to an $a \times 20$-mm steel plate. Knowing that the centroidal y axis is located as shown, determine (a) the width a, (b) the moments of inertia with respect to the centroidal x and y axes.

SOLUTION



(a) Using Figure 9.13B

From the geometry of $L152 \times 152 \times 19$, $C150 \times 15.6$, plate $a \times 20$ mm and how they are welded

$$x_A = 44.9 \text{ mm} \quad A_A = 5420 \text{ mm}^2$$

$$x_C = -12.5 \text{ mm} \quad A_C = 1980 \text{ mm}^2$$

$$x_P = -\left(\frac{a}{2} - 152\right) \text{mm} \quad A_P = (20a) \text{ mm}^2$$

From the condition
$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = 0$$

$$(44.9 \text{ mm})(5420 \text{ mm}^2) - (12.5 \text{ mm})(1980 \text{ mm}^2) - \left[\left(\frac{a}{2} - 152\right)\text{mm}\right](20a \text{ mm}^2) = 0$$

or
$$a^2 - 304a - 21860.8 = 0 \qquad a = 364.05 \text{ mm}$$

or $a = 364$ mm ◀

And
$$A_P = (20 \text{ mm})(364 \text{ mm})$$

$$= 7280 \text{ mm}^2$$

PROBLEM 9.56 CONTINUED

(*b*) Locate the centroid

$$Y = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{\left(5420\text{ mm}^2\right)\left(44.9\text{ mm}\right) + \left(1980\text{ mm}^2\right)\left(\frac{152}{2}\text{ mm}\right) + \left(7280\text{ mm}^2\right)\left(-10\text{ mm}\right)}{\left(5420 + 1980 + 7280\right)\text{ mm}^2}$$

$$= 21.867\text{ mm}$$

Now $\bar{I}_x = \left(I_x\right)_A + \left(I_x\right)_C + \left(I_x\right)_P$

$$= \left[11.6\times 10^6\text{ mm}^4 + \left(5420\text{ mm}^2\right)\left(44.9\text{ mm} - 21.867\text{ mm}\right)^2\right]$$

$$+\left[6.21\times 10^6\text{ mm}^4 + \left(1980\text{ mm}^2\right)\left(76\text{ mm} - 21.867\text{ mm}\right)^2\right]$$

$$+\left[\frac{1}{12}\left(364.05\text{ mm}\right)\left(20\text{ mm}\right)^3 + \left(7281\text{ mm}^2\right)\left(10\text{ mm} + 21.867\text{ mm}\right)^2\right]$$

$$= \left[\left(11.6 + 2.8754\right) + \left(6.21 + 5.8022\right) + \left(0.2427 + 7.3939\right)\right]\times 10^6\text{ mm}^4$$

$$= \left(14.4754 + 12.0122 + 7.6366\right)\times 10^6\text{ mm}^4$$

$$= 34.1242\times 10^6\text{ mm}^4$$

or $\bar{I}_x = 34.1\times 10^6\text{ mm}^4$ ◀

And $\bar{I}_y = \left(I_y\right)_A + \left(I_y\right)_C + \left(I_y\right)_P$

$$= \left[11.6\times 10^6\text{ mm}^4 + \left(5420\text{ mm}^2\right)\left(44.9\text{ mm}\right)^2\right]$$

$$+\left[0.347\times 10^6\text{ mm}^4 + \left(1980\text{ mm}^2\right)\left(12.5\text{ mm}\right)^2\right]$$

$$+\left[\frac{1}{12}\left(20\text{ mm}\right)\left(364.05\text{ mm}\right)^3 + \left(7821\text{ mm}^2\right)\left(\frac{364.05\text{ mm}}{2} - 152\text{ mm}\right)^2\right]$$

$$= \left(11.6 + 10.9268\right)\times 10^6\text{ mm}^4 + \left(0.347 + 0.3094\right)\times 10^6\text{ mm}^4$$

$$+\left(80.4140 + 6.5638\right)\times 10^6\text{ mm}^4$$

$$= \left(22.5268 + 0.6564 + 86.9778\right)\times 10^6\text{ mm}^4$$

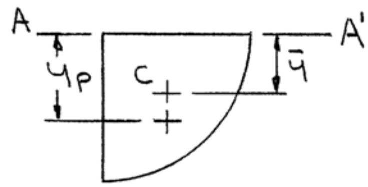
$$= 110.161\times 10^{-6}\text{ mm}^4$$

or $\bar{I}_y = 110.2\times 10^6\text{ mm}^4$ ◀

PROBLEM 9.57

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text have

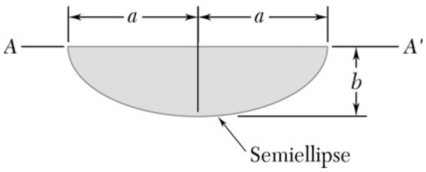
$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a quarter circle $$I_{AA'} = \frac{\pi}{16}r^4$$

and $$\bar{y} = \frac{4r}{3\pi}, \quad A = \frac{\pi}{4}r^2$$

Then $$y_P = \frac{\frac{\pi}{16}r^4}{\left(\frac{4r}{3\pi}\right)\left(\frac{\pi}{4}r^2\right)}$$

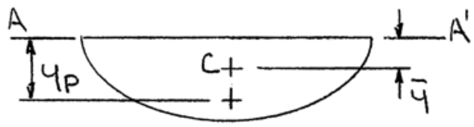
or $$y_P = \frac{3\pi}{16}r \blacktriangleleft$$



PROBLEM 9.58

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text have

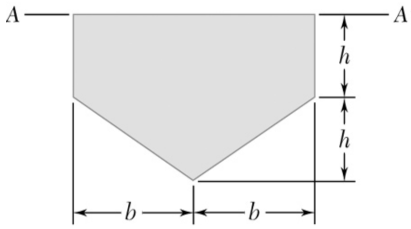
$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a semiellipse $$I_{AA'} = \frac{\pi}{8}ab^3$$

$$\bar{y} = \frac{4b}{3\pi}, \quad A = \frac{\pi}{2}ab$$

Then $$y_P = \frac{\frac{\pi}{8}ab^3}{\left(\frac{4b}{3\pi}\right)\left(\frac{\pi}{2}ab\right)}$$

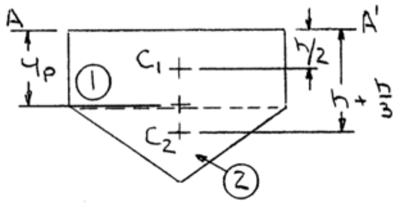
or $y_P = \dfrac{3\pi}{16}b$ ◀



PROBLEM 9.59

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text

Have
$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

Now
$$\bar{Y}A = \Sigma\bar{y}A$$

$$= \frac{h}{2}(2b \times h) + \frac{4}{3}h\left(\frac{1}{2} \times 2b \times h\right)$$

$$= \frac{7}{3}bh^2$$

And
$$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$$

where

$$(I_{AA'})_1 = \frac{1}{3}(2b)(h)^3 = \frac{2}{3}bh^3$$

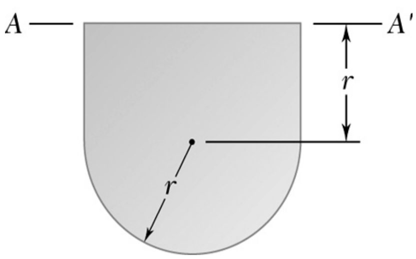
$$(I_{AA'})_2 = \bar{I}_x + Ad^2 = \frac{1}{36}(2b)(h)^3 + \left(\frac{1}{2} \times 2b \times h\right)\left(\frac{4}{3}h\right)^2$$

$$= \frac{11}{6}bh^3$$

Then
$$I_{AA'} = \frac{2}{3}bh^3 + \frac{11}{6}bh^3 = \frac{5}{2}bh^3$$

Finally,
$$y_P = \frac{\frac{5}{2}bh^3}{\frac{7}{3}bh^2}$$

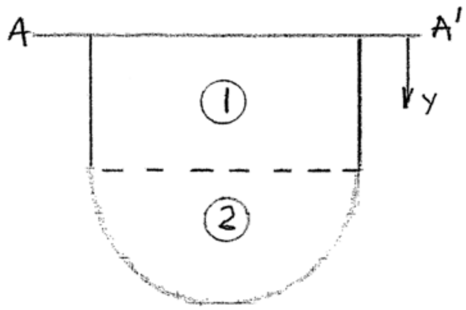
or $y_P = \dfrac{15}{14}h$ ◀



PROBLEM 9.60

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text

Have $$y_P = \frac{I_{AA'}}{\bar{y}A}$$

where $$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$$

$$= \left[\frac{1}{3}(2r)(r)^3\right] + \left\{\left[\frac{\pi}{8}r^4 - \frac{\pi}{2}r^2\left(\frac{4r}{3\pi}\right)^2\right] + \frac{\pi}{2}r^2\left(r + \frac{4r}{3\pi}\right)^2\right\}$$

$$= \frac{2}{3}r^4 + \left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} + \frac{4}{3} + \frac{9}{8\pi}\right)r^4 = \left(2 + \frac{5\pi}{8}\right)r^4$$

And $$\bar{Y}A = \Sigma\bar{y}A = \left[\frac{r}{2}(2r \times r)\right] + \left[\left(r + \frac{4r}{3\pi}\right)\left(\frac{\pi}{2}r^2\right)\right]$$

$$= \left(1 + \frac{\pi}{2} + \frac{2}{3}\right)r^3 = \left(\frac{5}{3} + \frac{\pi}{2}\right)r^3$$

Then $$y_P = \frac{\left(2 + \frac{5\pi}{8}\right)r^4}{\left(\frac{5}{3} + \frac{\pi}{2}\right)r^3} = 1.2242r$$

or $y_P = 1.224r$ ◀