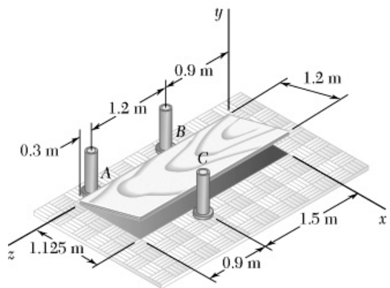
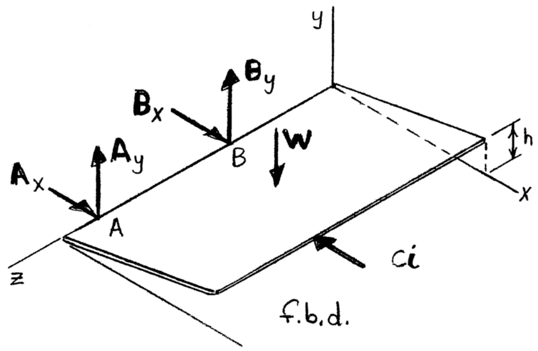
PROBLEM 4.101



A 1.2×2.4-m sheet of plywood having a mass of 17 kg has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction at all surfaces, determine the reactions at *A*, *B*, and *C*.

SOLUTION



First note
$$W = mg = (17\text{ kg})\left(9.81\text{ m/s}^2\right) = 166.77\text{ N}$$

$$h = \sqrt{(1.2)^2 - (1.125)^2} = 0.41758\text{ m}$$

From f.b.d. of plywood sheet

$$\Sigma M_z = 0: \quad C(h) - W\left[\frac{(1.125\text{ m})}{2}\right] = 0$$

$$C(0.41758\text{ m}) - (166.77\text{ N})(0.5625\text{ m}) = 0$$

$$\therefore\ C = 224.65\text{ N} \qquad \text{or} \qquad \mathbf{C} = -(225\text{ N})\mathbf{i}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad -(224.65\text{ N})(0.6\text{ m}) + A_x(1.2\text{ m}) = 0$$

$$\therefore\ A_x = 112.324\text{ N} \qquad \text{or} \qquad \mathbf{A}_x = (112.3\text{ N})\mathbf{i}$$

$$\Sigma M_{B(x\text{-axis})} = 0: \quad (166.77\text{ N})(0.3\text{ m}) - A_y(1.2\text{ m}) = 0$$

$$\therefore\ A_y = 41.693\text{ N} \qquad \text{or} \qquad \mathbf{A}_y = (41.7\text{ N})\mathbf{j}$$

$$\Sigma M_{A(y\text{-axis})} = 0: \quad (224.65\text{ N})(0.6\text{ m}) - B_x(1.2\text{ m}) = 0$$

$$\therefore\ B_x = 112.325\text{ N} \qquad \text{or} \qquad \mathbf{B}_x = (112.3\text{ N})\mathbf{i}$$

PROBLEM 4.101 CONTINUED

$$\Sigma M_{A(x\text{-axis})} = 0: \quad B_y(1.2 \text{ m}) - (166.77 \text{ N})(0.9 \text{ m}) = 0$$

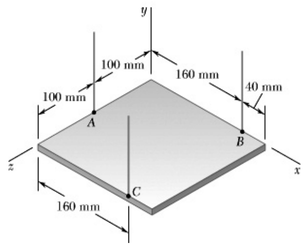
$$\therefore \quad B_y = 125.078 \text{ N} \qquad \text{or} \qquad \mathbf{B}_y = (125.1 \text{ N})\mathbf{j}$$

$$\therefore \quad \mathbf{A} = (112.3 \text{ N})\mathbf{i} + (41.7 \text{ N})\mathbf{j} \blacktriangleleft$$

$$\mathbf{B} = (112.3 \text{ N})\mathbf{i} + (125.1 \text{ N})\mathbf{j} \blacktriangleleft$$

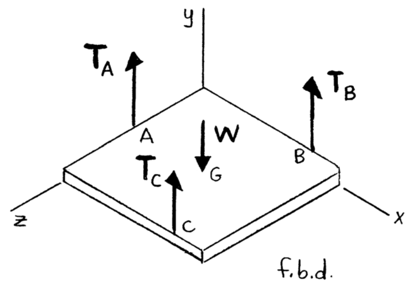
$$\mathbf{C} = -(225 \text{ N})\mathbf{i} \blacktriangleleft$$

PROBLEM 4.102



The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION



First note $$W = mg = (25 \text{ kg})(9.81 \text{ m/s}^2) = 245.25 \text{ N}$$

From f.b.d. of plate

$$\Sigma M_x = 0: \quad (245.25 \text{ N})(100 \text{ mm}) - T_A(100 \text{ mm}) - T_C(200 \text{ mm}) = 0$$

$$\therefore \quad T_A + 2T_C = 245.25 \text{ N} \tag{1}$$

$$\Sigma M_z = 0: \quad T_B(160 \text{ mm}) + T_C(160 \text{ mm}) - (245.25 \text{ N})(100 \text{ mm}) = 0$$

$$\therefore \quad T_B + T_C = 153.281 \text{ N} \tag{2}$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 245.25 \text{ N} = 0$$

$$\therefore \quad T_B + T_C = 245.25 - T_A \tag{3}$$

Equating Equations (2) and (3) yields

$$T_A = 245.25 \text{ N} - 153.281 \text{ N} = 91.969 \text{ N} \tag{4}$$

or $$T_A = 92.0 \text{ N}$$

Substituting the value of T_A into Equation (1)

$$T_C = \frac{(245.25 \text{ N} - 91.969 \text{ N})}{2} = 76.641 \text{ N} \tag{5}$$

or $$T_C = 76.6 \text{ N}$$

Substituting the value of T_C into Equation (2)

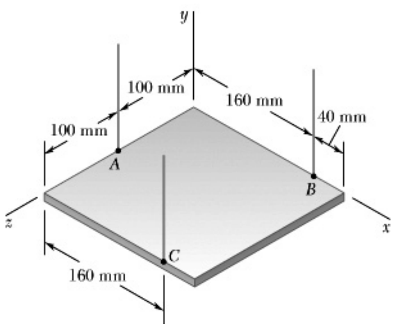
$$T_B = 153.281 \text{ N} - 76.641 \text{ N} = 76.639 \text{ N} \qquad \text{or} \qquad T_B = 76.6 \text{ N}$$

$T_A = 92.0 \text{ N}$ ◀

$T_B = 76.6 \text{ N}$ ◀

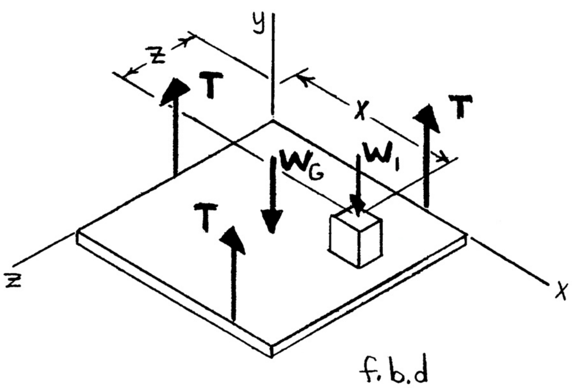
$T_C = 76.6 \text{ N}$ ◀

PROBLEM 4.103



The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the mass and location of the lightest block which should be placed on the plate if the tensions in the three cables are to be equal.

SOLUTION



First note
$$W_G = m_{p1}g = (25\text{ kg})\left(9.81\text{ m/s}^2\right) = 245.25\text{ N}$$

$$W_1 = mg = m\left(9.81\text{ m/s}^2\right) = (9.81m)\text{ N}$$

From f.b.d. of plate

$$\Sigma F_y = 0\text{:}\quad 3T - W_G - W_1 = 0\,(1)$$

$$\Sigma M_x = 0\text{:}\quad W_G(100\text{ mm}) + W_1(z) - T(100\text{ mm}) - T(200\text{ mm}) = 0$$

$$\text{or}\quad -300T + 100W_G + W_1 z = 0 \qquad (2)$$

$$\Sigma M_z = 0\text{:}\quad 2T(160\text{ mm}) - W_G(100\text{ mm}) - W_1(x) = 0$$

$$\text{or}\quad 320T - 100W_G - W_1 x = 0 \qquad (3)$$

Eliminate T by forming $100 \times [\text{Eq. }(1) + \text{Eq. }(2)]$

$$-100W_1 + W_1 z = 0$$

$$\therefore\quad z = 100\text{ mm} \qquad 0 \le z \le 200\text{ mm},\quad \therefore\quad \text{okay}$$

Now, $3 \times [\text{Eq. }(3)] - 320 \times [\text{Eq. }(1)]$ yields

$$3(320T) - 3(100)W_G - 3W_1 x - 320(3T) + 320W_G + 320W_1 = 0$$

PROBLEM 4.103 CONTINUED

or
$$20W_G + (320 - 3x)W_1 = 0$$

or
$$\frac{W_1}{W_G} = \frac{20}{(3x - 320)}$$

The smallest value of $\frac{W_1}{W_G}$ will result in the smallest value of W_1 since W_G is given.

$$\therefore \quad \text{Use } x = x_{\text{max}} = 200 \text{ mm}$$

and then
$$\frac{W_1}{W_G} = \frac{20}{3(200) - 320} = \frac{1}{14}$$

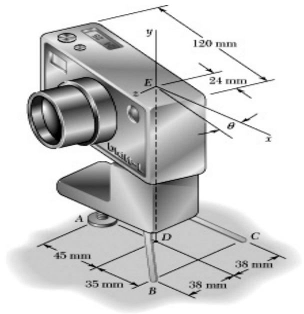
$$\therefore \quad W_1 = \frac{W_G}{14} = \frac{245.25 \text{ N}}{14} = 17.5179 \text{ N}(\text{minimum})$$

and
$$m = \frac{W_1}{g} = \frac{17.5179 \text{ N}}{9.81 \text{ m/s}^2} = 1.78571 \text{ kg}$$

or $m = 1.786$ kg ◀

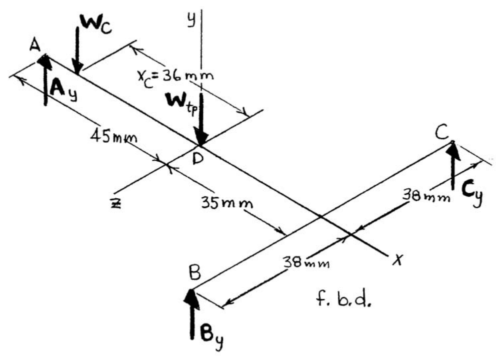
at $x = 200$ mm, $z = 100$ mm ◀

PROBLEM 4.104



A camera of mass 240 g is mounted on a small tripod of mass 200 g. Assuming that the mass of the camera is uniformly distributed and that the line of action of the weight of the tripod passes through *D*, determine (*a*) the vertical components of the reactions at *A*, *B*, and *C* when $\theta = 0$, (*b*) the maximum value of θ if the tripod is not to tip over.

SOLUTION



First note
$$W_C = m_C g = (0.24\text{ kg})(9.81\text{ m/s}^2) = 2.3544\text{ N}$$

$$W_{\text{tp}} = m_{\text{tp}} g = (0.20\text{ kg})(9.81\text{ m/s}^2) = 1.9620\text{ N}$$

For $\theta = 0$
$$x_C = -(60\text{ mm} - 24\text{ mm}) = -36\text{ mm}$$

$$z_C = 0$$

(*a*) From f.b.d. of camera and tripod as projected onto plane *ABCD*

$$\Sigma F_y = 0:\quad A_y + B_y + C_y - W_C - W_{\text{tp}} = 0$$

$$\therefore\quad A_y + B_y + C_y = 2.3544\text{ N} + 1.9620\text{ N} = 4.3164\text{ N} \qquad (1)$$

$$\Sigma M_x = 0:\quad C_y(38\text{ mm}) - B_y(38\text{ mm}) = 0 \qquad \therefore\quad C_y = B_y \qquad (2)$$

$$\Sigma M_z = 0:\quad B_y(35\text{ mm}) + C_y(35\text{ mm}) + (2.3544\text{ N})(36\text{ mm}) - A_y(45\text{ mm}) = 0$$

$$\therefore\quad 9A_y - 7B_y - 7C_y = 16.9517 \qquad (3)$$

Substitute C_y with B_y from Equation (2) into Equations (1) and (3), and solve by elimination

$$7(A_y + 2B_y = 4.3164)$$

$$\underline{9A_y - 14B_y = 16.9517}$$

$$16A_y = 47.166$$

PROBLEM 4.104 CONTINUED

$$\therefore \quad A_y = 2.9479 \text{ N}$$

or $\mathbf{A}_y = 2.95 \text{ N} \uparrow$ ◀

Substituting $A_y = 2.9479 \text{ N}$ into Equation (1)

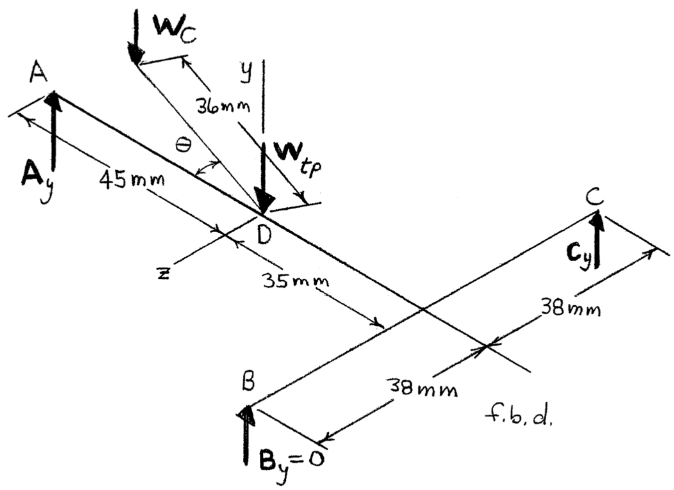
$$2.9479 \text{ N} + 2B_y = 4.3164$$

$$\therefore \quad B_y = 0.68425 \text{ N}$$

$$C_y = 0.68425 \text{ N}$$

or $\mathbf{B}_y = \mathbf{C}_y = 0.684 \text{ N} \uparrow$ ◀

(*b*) $B_y = 0$ for impending tipping



From f.b.d. of camera and tripod as projected onto plane *ABCD*

$$\Sigma F_y = 0: \quad A_y + C_y - W_C - W_{\text{tp}} = 0$$

$$\therefore \quad A_y + C_y = 4.3164 \text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad C_y(38 \text{ mm}) - (2.3544 \text{ N})\left[(36 \text{ mm})\sin\theta\right] = 0$$

$$\therefore \quad C_y = 2.2305\sin\theta \qquad (2)$$

$$\Sigma M_z = 0: \quad C_y(35 \text{ mm}) - A_y(45 \text{ mm}) + (2.3544 \text{ N})\left[(36 \text{ mm})\cos\theta\right] = 0$$

$$\therefore \quad 9A_y - 7C_y = (16.9517 \text{ N})\cos\theta \qquad (3)$$

Forming $7 \times [\text{Eq. } (1)] + [\text{Eq. } (3)]$ yields

$$16A_y = 30.215 \text{ N} + (16.9517 \text{ N})\cos\theta \qquad (4)$$

PROBLEM 4.104 CONTINUED

Substituting Equation (2) into Equation (3)

$$9A_y - (15.6134\text{ N})\sin\theta = (16.9517\text{ N})\cos\theta \qquad (5)$$

Forming $9 \times [\text{Eq. } (4)] - 16 \times [\text{Eq. } (5)]$ yields

$$(249.81\text{ N})\sin\theta = 271.93\text{ N} - (118.662\text{ N})\cos\theta$$

or
$$\cos^2\theta = [2.2916\text{ N} - (2.1053\text{ N})\sin\theta]^2$$

Now
$$\cos^2\theta = 1 - \sin^2\theta$$

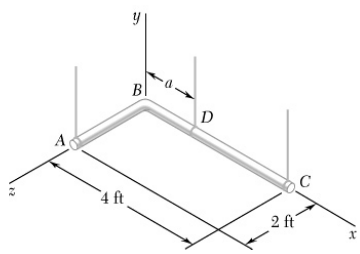
$$\therefore \quad 5.4323\sin^2\theta - 9.6490\sin\theta + 4.2514 = 0$$

Using quadratic formula to solve,

$$\sin\theta = 0.80981 \quad \text{and} \quad \sin\theta = 0.96641$$

$$\therefore \quad \theta = 54.078° \quad \text{and} \quad \theta = 75.108°$$

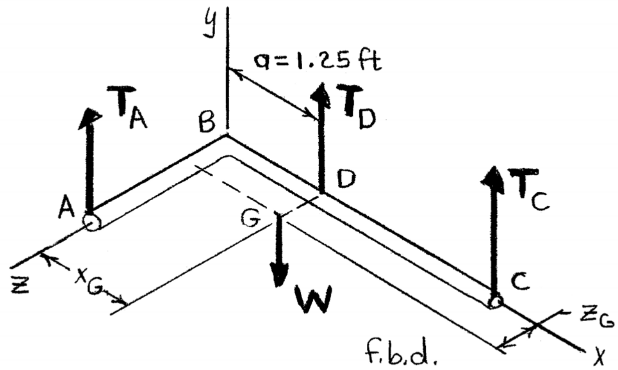
or $\theta_{max} = 54.1°$ before tipping ◀



PROBLEM 4.105

Two steel pipes *AB* and *BC*, each having a weight per unit length of 5 lb/ft, are welded together at *B* and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note

$$W_{AB} = (5 \text{ lb/ft})(2 \text{ ft}) = 10 \text{ lb}$$

$$W_{BC} = (5 \text{ lb/ft})(4 \text{ ft}) = 20 \text{ lb}$$

$$W = W_{AB} + W_{BC} = 30 \text{ lb}$$

To locate the equivalent force of the pipe assembly weight

$$\mathbf{r}_{G/B} \times \mathbf{W} = \Sigma(\mathbf{r_i} \times \mathbf{W_i}) = \mathbf{r}_{G(AB)} \times \mathbf{W}_{AB} + \mathbf{r}_{G(BC)} \times \mathbf{W}_{BC}$$

or

$$(x_G\mathbf{i} + z_G\mathbf{k}) \times (-30 \text{ lb})\mathbf{j} = (1 \text{ ft})\mathbf{k} \times (-10 \text{ lb})\mathbf{j} + (2 \text{ ft})\mathbf{i} \times (-20 \text{ lb})\mathbf{j}$$

$$\therefore \quad -(30 \text{ lb})x_G\mathbf{k} + (30 \text{ lb})z_G\mathbf{i} = (10 \text{ lb}\cdot\text{ft})\mathbf{i} - (40 \text{ lb}\cdot\text{ft})\mathbf{k}$$

From **i**-coefficient

$$z_G = \frac{10 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = \frac{1}{3}\text{ft}$$

k-coefficient

$$x_G = \frac{40 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = 1\frac{1}{3}\text{ft}$$

From f.b.d. of piping

$$\Sigma M_x = 0: \quad W(z_G) - T_A(2 \text{ ft}) = 0$$

$$\therefore \quad T_A = \left(\frac{1}{2} \text{ ft}\right)30 \text{ lb}\left(\frac{1}{3}\text{ft}\right) = 5 \text{ lb} \qquad \text{or} \qquad T_A = 5.00 \text{ lb}$$

$$\Sigma F_y = 0: \quad 5 \text{ lb} + T_D + T_C - 30 \text{ lb} = 0$$

$$\therefore \quad T_D + T_C = 25 \text{ lb} \tag{1}$$

PROBLEM 4.105 CONTINUED

$$\Sigma M_z = 0: \quad T_D(1.25 \text{ ft}) + T_C(4 \text{ ft}) - 30 \text{ lb}\left(\frac{4}{3}\text{ft}\right) = 0$$

$$\therefore \quad 1.25T_D + 4T_C = 40 \text{ lb}\cdot\text{ft} \qquad (2)$$

$-4[\text{Equation } (1)]$

$$-4T_D - 4T_C = -100 \qquad (3)$$

Equation (2) + Equation (3)

$$-2.75T_D = -60$$

$$\therefore \quad T_D = 21.818 \text{ lb} \qquad \text{or} \qquad T_D = 21.8 \text{ lb}$$

From Equation (1)

$$T_C = 25 - 21.818 = 3.1818 \text{ lb} \qquad \text{or} \qquad T_C = 3.18 \text{ lb}$$

Results:

$T_A = 5.00$ lb ◀

$T_C = 3.18$ lb ◀

$T_D = 21.8$ lb ◀