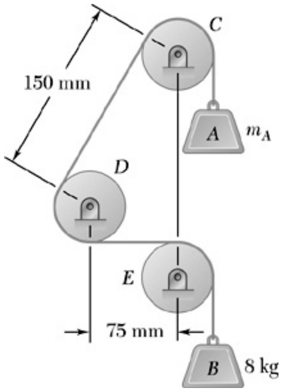
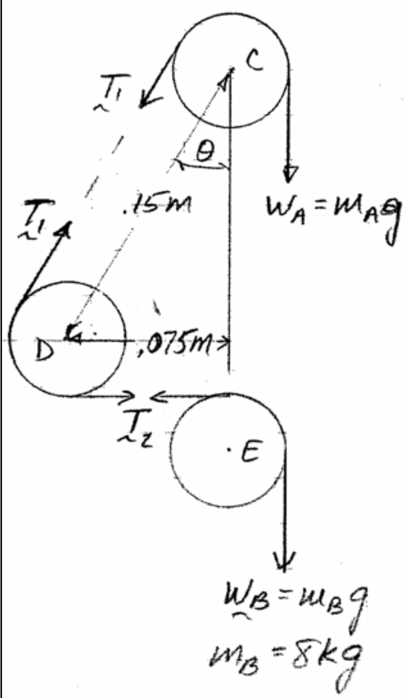


PROBLEM 8.121

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{0.075\text{ m}}{0.15\text{ m}} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \frac{5}{6}\pi,\ \beta_D = \frac{2}{3}\pi,\ \beta_E = \frac{1}{2}\pi$$

(*a*) To raise maximum m_A, with C rotating $W_A = T_1 e^{\mu_s \beta_C}$. If D and E are fixed, cable must slip there, so $T_2 = T_1 e^{\mu_k \beta_D}$

and $$W_B = T_2 e^{\mu_k \beta_E} = T_1 e^{\mu_k(\beta_D + \beta_E)}$$

$$= W_A e^{-\mu_s \beta_C} e^{\mu_k(\beta_D + \beta_E)}$$

$$(8\text{ kg})g = m_A g e^{-0.2\left(\frac{5}{6}\pi\right)} e^{0.15\left(\frac{2}{3}+\frac{1}{2}\right)\pi}$$

$$m_A = 7.79\text{ kg} \blacktriangleleft$$

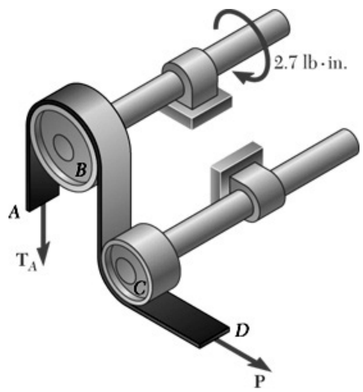
(*b*) With E rotating, $T_2 = W_B e^{\mu_s \beta_E}$. With C and D fixed.

$$T_1 = W_A e^{\mu_k \beta_C} \quad \text{and} \quad T_2 = T_1 e^{\mu_k \beta_D} = W_A e^{\mu_k(\beta_C + \beta_D)}$$

so $$W_B = W_A e^{\mu_k(\beta_C + \beta_D)} e^{-\mu_s \beta_E}$$

$$(8\text{ kg})g = m_A g e^{0.15\left(\frac{5}{6}+\frac{2}{3}\right)\pi} e^{-0.2\left(\frac{1}{2}\pi\right)}$$

$$m_A = 5.40\text{ kg} \blacktriangleleft$$

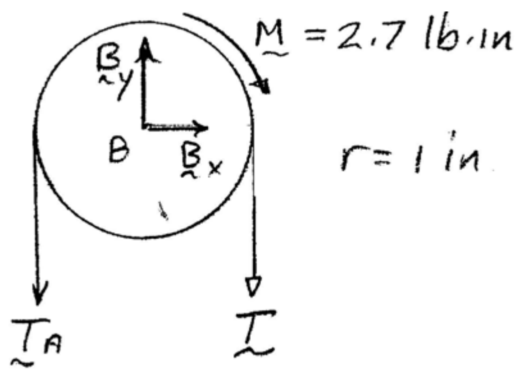


PROBLEM 8.122

A recording tape passes over the 1-in.-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

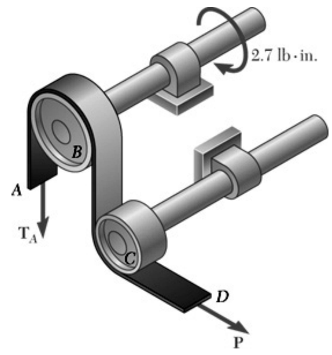
$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb}\cdot\text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $$T_A = Te^{\mu_S \beta} = Te^{0.4\pi}$$

So $$T\left(e^{0.4\pi} - 1\right) = 2.7 \text{ lb}$$

or $$T = 1.0742 \text{ lb}$$

If C is free to rotate, $P = T$ $\quad P = 1.074 \text{ lb}$ ◀

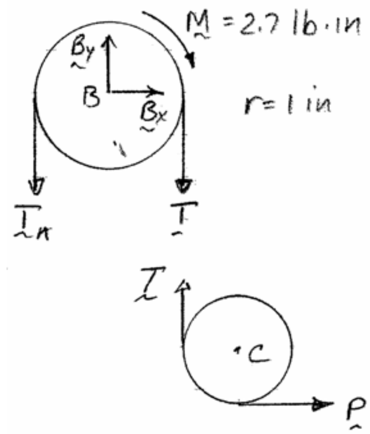


PROBLEM 8.123

Solve Problem 8.122 assuming that the idler drum *C* is frozen and cannot rotate.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb}\cdot\text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $$T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$$

So $$\left(e^{0.4\pi} - 1\right)T = 2.7 \text{ lb}$$

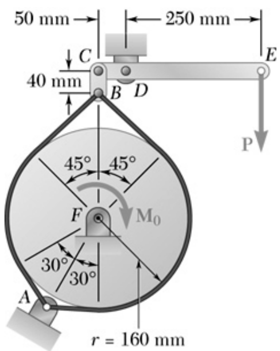
or $$T = 1.07416 \text{ lb}$$

If C is fixed, the tape must slip $\circlearrowleft$

So $$P = Te^{\mu_k \beta_C} = 1.07416 \text{ lb } e^{0.3\frac{\pi}{2}} = 1.7208 \text{ lb}$$

$$P = 1.721 \text{ lb} \blacktriangleleft$$

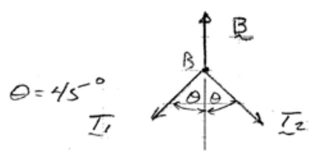
PROBLEM 8.124



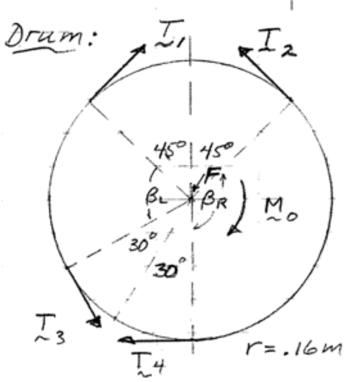
For the band brake shown, the maximum allowed tension in either belt is 5.6 kN. Knowing that the coefficient of static friction between the belt and the 160-mm-radius drum is 0.25, determine (*a*) the largest clockwise moment $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force **P** exerted on end *E* of the lever.

SOLUTION

FBD pin B:



Drum:



(*a*) By symmetry: $T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2} T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2} T_1 = \sqrt{2} T_2 \qquad (1)$$

For impending rotation ↻:

$$T_3 > T_1 = T_2 > T_4, \text{ so } T_3 = T_{\max} = 5.6 \text{ kN}$$

Then $$T_1 = T_3 e^{-\mu_s \beta_L} = (5.6 \text{ kN}) e^{-0.25\left(\frac{\pi}{4} + \frac{\pi}{6}\right)}$$

or $$T_1 = 4.03706 \text{ kN} = T_2$$

and $$T_4 = T_2 e^{-\mu_s \beta_R} = (4.03706 \text{ kN}) e^{-0.25\left(\frac{3\pi}{4}\right)}$$

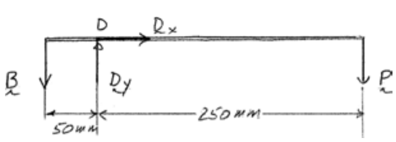
or $$T_4 = 2.23998 \text{ kN}$$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_4 - T_3 + T_2 - T_1) = 0$$

or $$M_0 = (0.16 \text{ m})(5.6 \text{ kN} - 2.23998 \text{ kN}) = 0.5376 \text{ kN} \cdot \text{m}$$

$$\mathbf{M}_0 = 538 \text{ N} \cdot \text{m} \ \circlearrowright \ ◀$$

Lever:



(*b*) Using Equation (1)

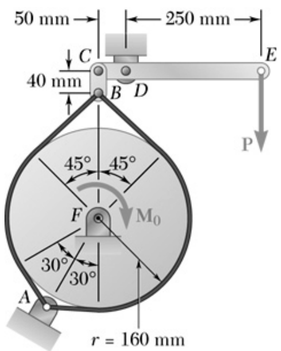
$$B = \sqrt{2} T_1 = \sqrt{2}(4.03706 \text{ kN})$$

$$= 5.70927 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(5.70927 \text{ kN}) - (0.25 \text{ m}) P = 0$$

$$\mathbf{P} = 1.142 \text{ kN} \downarrow ◀$$

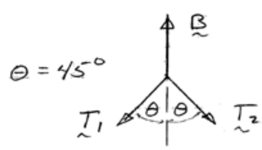
PROBLEM 8.125



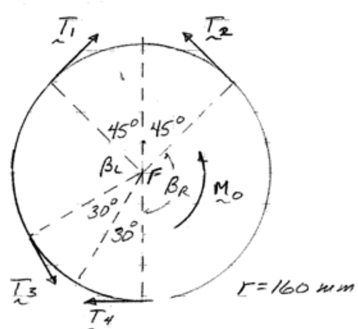
Solve Problem 8.124 assuming that a counterclockwise moment is applied to the drum.

SOLUTION

FBD pin B:



FBD Drum



(*a*) By symmetry: $\quad T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2}T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2}T_1 \qquad (1)$$

For impending rotation ↺:

$$T_4 > T_2 = T_1 > T_3, \text{ so } T_4 = T_{\max} = 5.6 \text{ kN}$$

Then $\quad T_2 = T_4 e^{-\mu_s \beta_R} = (5.6 \text{ kN})e^{-0.25\left(\frac{3\pi}{4}\right)}$

or $\quad T_2 = 3.10719 \text{ kN} = T_1$

and $\quad T_3 = T_1 e^{-\mu_s \beta_L} = (3.10719 \text{ kN})e^{-0.25\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$

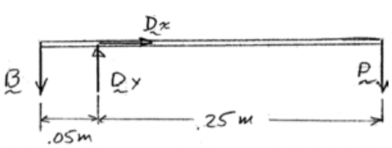
or $\quad T_3 = 2.23999 \text{ kN}$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_2 - T_1 + T_3 - T_4) = 0$$

$$M_0 = (160 \text{ mm})(5.6 \text{ kN} - 2.23999 \text{ kN}) = 537.6 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_0 = 538 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

FBD Lever:



(*b*) Using Equation (1)

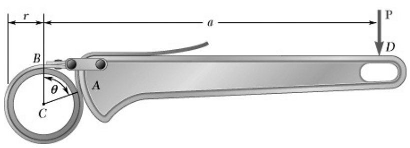
$$B = \sqrt{2}T_1 = \sqrt{2}(3.10719 \text{ kN})$$

$$B = 4.3942 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(4.3942 \text{ kN}) - (0.25 \text{ m})P = 0$$

$$\mathbf{P} = 879 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 8.126

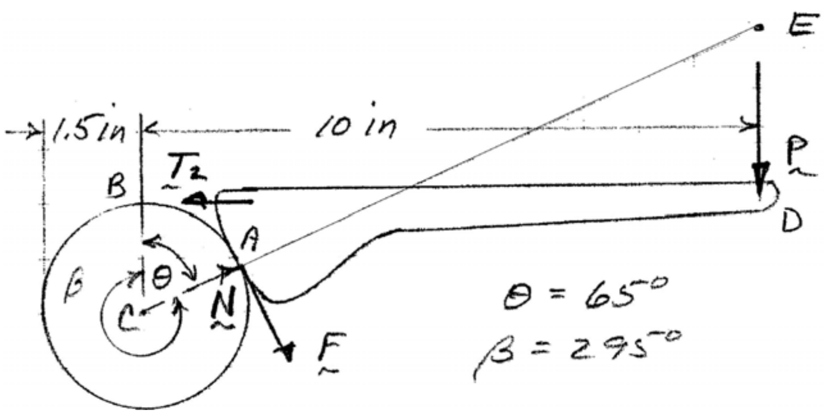


The strap wrench shown is used to grip the pipe firmly without marring the surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 10$ in., $r = 1.5$ in., and $\theta = 65^\circ$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10\text{ in.}}{\sin 65^\circ} - 1.5\text{ in.}\right)F - \left(\frac{10\text{ in.}}{\tan 65^\circ} - 1.5\text{ in.}\right)T_2 = 0$$

$$9.5338F = 3.1631T_2 \qquad \text{or} \qquad 3.01406 = \frac{T_2}{F} \tag{1}$$

$$\longrightarrow \Sigma F_x = 0: \quad -T_2 + N\sin 65^\circ + F\cos 65^\circ = 0$$

Impending slipping:

$$N = F/\mu_s$$

So

$$F\left(\frac{\sin 65^\circ}{\mu_s} + \cos 65^\circ\right) = T_2$$

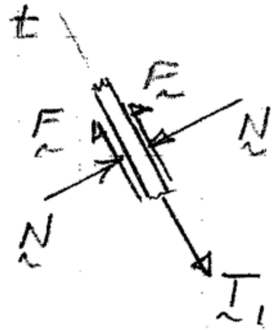
or

$$\frac{0.90631}{\mu_s} + 0.42262 = \frac{T_2}{F} \tag{2}$$

Solving Equations (1) and (2) yields $\mu_s = 0.3497$; must still check belt on pipe.

PROBLEM 8.126 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

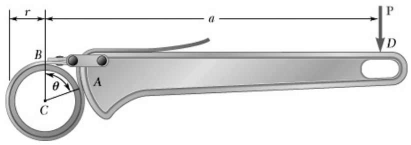
Belt impending slipping: $$\ln \frac{T_2}{T_1} = \mu_s \beta$$

So $$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{\beta} \ln \frac{T_2}{2F}$$

Using Equation (1) $$\mu_s = \frac{180}{295\pi} \ln 1.50703$$

$$= 0.0797$$

$\therefore$ for self-locking, need $\mu_s = 0.350$ ◀



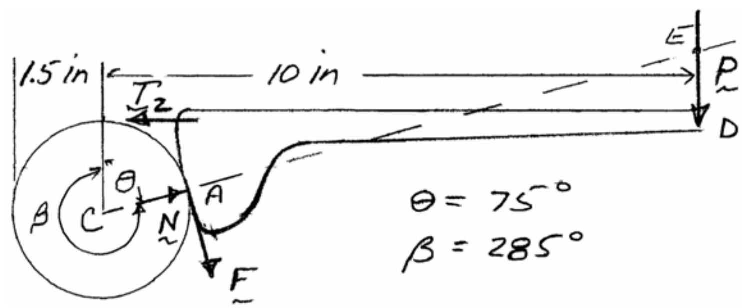
PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75^{\circ}$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10 \text{ in.}}{\sin 75^{\circ}} - 1.5 \text{ in.}\right)F - \left(\frac{10 \text{ in.}}{\tan 75^{\circ}} - 1.5 \text{ in.}\right)T_2 = 0$$

or
$$\frac{T_2}{F} = 7.5056 \qquad (1)$$

$$\longrightarrow \Sigma F_x = 0: \quad -T_2 + N\sin 75^{\circ} + F\cos 75^{\circ} = 0$$

Impending slipping:
$$N = F/\mu_s$$

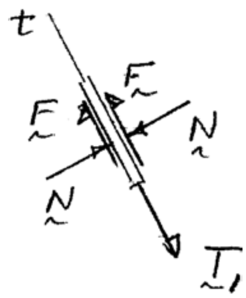
So
$$F\left(\frac{\sin 75^{\circ}}{\mu_s} + \cos 75^{\circ}\right) = T_2$$

or
$$\frac{T_2}{F} = \frac{0.96593}{\mu_s} + 0.25882 \qquad (2)$$

Solving Equations (1) and (2): $\mu_s = 0.13329$; must still check belt on pipe.

PROBLEM 8.127 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

Impending belt slipping: $$\ln \frac{T_2}{T_1} = \mu_s \beta$$

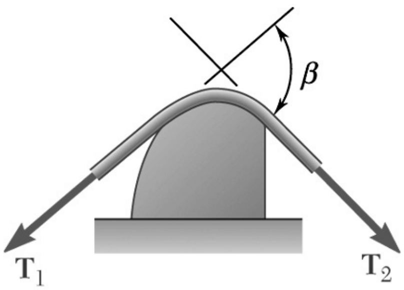
So $$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{\beta} \ln \frac{T_2}{2F}$$

Using Equation (1): $$\mu_s = \frac{180}{285\pi} \ln \frac{7.5056}{2}$$

$$= 0.2659$$

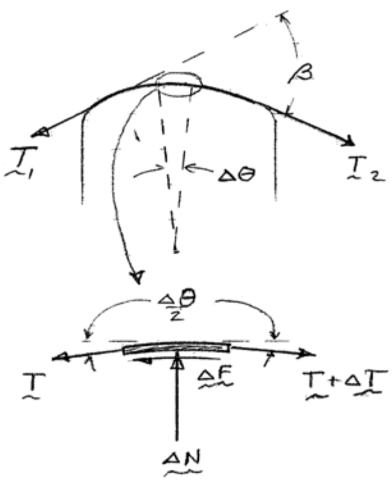
$\therefore$ for self-locking, $\mu_s = 0.266$ ◀

PROBLEM 8.128



Prove that Equatins (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - \left[T + (T + \Delta T)\right]\sin\frac{\Delta\theta}{2} = 0$$

or $$\Delta N = (2T + \Delta T)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad \left[(T + \Delta T) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or $$\Delta F = \Delta T\cos\frac{\Delta\theta}{2}$$

Impending slipping: $$\Delta F = \mu_s \Delta N$$

So $$\Delta T\cos\frac{\Delta\theta}{2} = \mu_s 2T\sin\frac{\Delta\theta}{2} + \mu_s\Delta T\frac{\sin\Delta\theta}{2}$$

In limit as $\Delta\theta \to 0: \quad dT = \mu_s T d\theta,$ or $\dfrac{dT}{T} = \mu_s d\theta$

So $$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu_s d\theta;$$

and $$\ln\frac{T_2}{T_1} = \mu_s\beta$$

or $T_2 = T_1 e^{\mu_s\beta}$ ◀

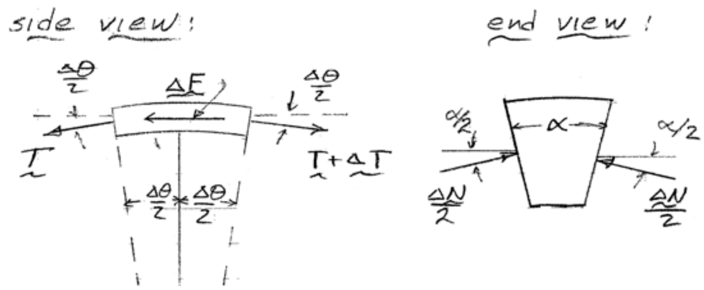
Note: Nothing above depends on the shape of the surface, except it is assumed smooth.

PROBLEM 8.129

Complete the derivation of Equation (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:



$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - \left[T + \left(T + \Delta T\right)\right]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad \left[\left(T + \Delta T\right) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

Impending slipping: $$\Delta F = \mu_s \Delta N \Rightarrow \Delta T\cos\frac{\Delta\theta}{2} = \mu_s \frac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$$

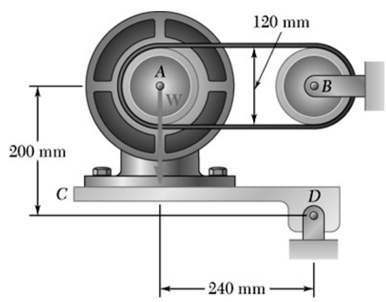
In limit as $\Delta\theta \to 0$: $$dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \qquad \text{or} \qquad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}d\theta$$

So $$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}\int_0^{\beta} d\theta$$

or $$\ln\frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin\frac{\alpha}{2}}$$

or $$T_2 = T_1 e^{\mu_s\beta/\sin\frac{\alpha}{2}} \blacktriangleleft$$

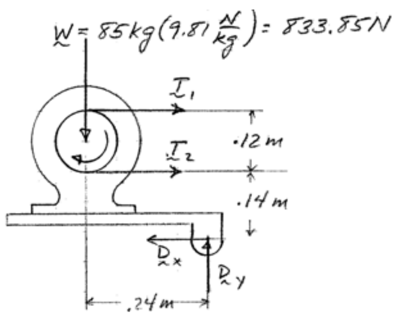
PROBLEM 8.130



Solve Problem 8.107 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^{\circ}$. (The angle α is as shown in Figure 8.15*a*.)

SOLUTION

FBD motor + mount:



$$\Sigma M_D = 0: \quad (0.24\text{ m})W - (0.26\text{ m})T_1 - (0.14\text{ m})T_2 = 0$$

Impending slipping: $$T_2 = T_1 e^{\mu_S \beta/\sin\frac{\alpha}{2}}$$

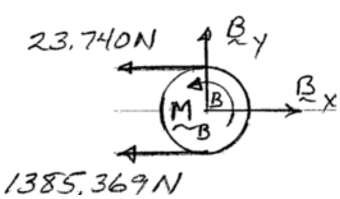
$$T_2 = T_1 e^{\frac{0.4\pi}{\sin 18^{\circ}}} = 58.356 T_1$$

Thus $$(0.24\text{ m})(833.85\text{ N}) - \left[0.26\text{ m} + (0.14\text{ m})(58.356)\right]T_1 = 0$$

$$T_1 = 23.740\text{ N}$$

$$T_2 = 1385.369\text{ N}$$

FBD Drum:



$$\Sigma M_B = 0: \quad M_B + (0.06\text{ m})(23.740\text{ N} - 1385.369\text{ N}) = 0$$

$$M_B = 81.7\text{ N}\cdot\text{m} \blacktriangleleft$$

(Compare to $M_B = 40.1\text{ N}\cdot\text{m}$ using flat belt, Problem 8.107.)