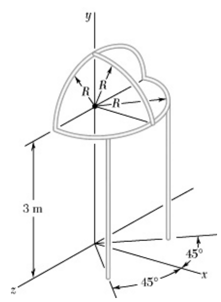
*

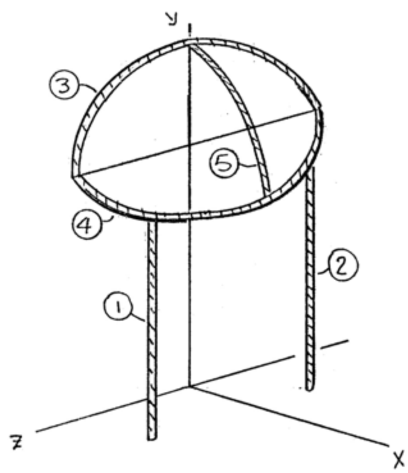



## PROBLEM 5.111

The decorative metalwork at the entrance of a store is fabricated from uniform steel structural tubing. Knowing that $R = 1.2$ m, locate the center of gravity of the metalwork.

## SOLUTION

First, assume that the tubes are homogeneous so that the center of gravity of the metalwork coincides with the centroid of the corresponding line.

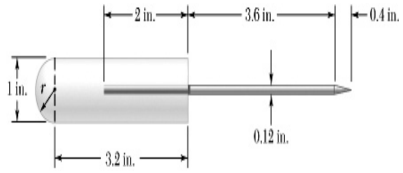



Note that symmetry implies $\bar{Z} = 0$ ◀

| | $L$, m | $\bar{x}$, m | $\bar{y}$, m | $\bar{x}L$, m$^2$ | $\bar{y}L$, m$^2$ |
|---|---|---|---|---|---|
| 1 | 3 | $(1.2)\cos 45° = 0.8485$ | 1.5 | 2.5456 | 4.5 |
| 2 | 3 | $(1.2)\cos 45° = 0.8485$ | 1.5 | 2.5456 | 4.5 |
| 3 | $1.2\pi$ | 0 | 3.7639 | 0 | 14.1897 |
| 4 | $1.2\pi$ | $\frac{(2)(1.2)}{\pi} = 0.7639$ | 3 | 2.88 | 11.3097 |
| 5 | $0.6\pi$ | $\frac{(2)(1.2)}{\pi} = 0.7639$ | 3.7639 | 1.44 | 7.0949 |
| $\Sigma$ | 15.425 | | | 9.4112 | 41.594 |

Have $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(15.425 \text{ m}) = 9.4112 \text{ m}^2$ or $\bar{X} = 0.610$ m ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(15.425 \text{ m}) = 41.594 \text{ m}^2$ or $\bar{Y} = 2.70$ m ◀

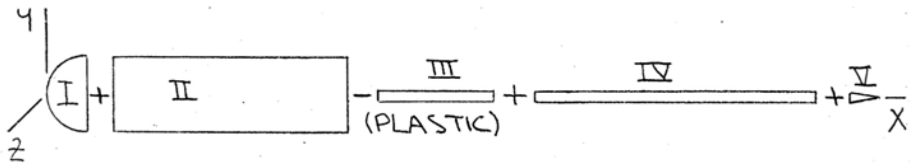



## PROBLEM 5.112

A scratch awl has a plastic handle and a steel blade and shank. Knowing that the specific weight of plastic is $0.0374\ \text{lb/in}^3$ and of steel is $0.284\ \text{lb/in}^3$, locate the center of gravity of the awl.

## SOLUTION

First, note that symmetry implies $\bar{Y} = \bar{Z} = 0$ ◀




$$\bar{x}_{\text{I}} = \frac{5}{8}(0.5\text{ in.}) = 0.3125\text{ in.},\ W_{\text{I}} = \left(0.0374\text{ lb/in}^3\right)\left(\frac{2\pi}{3}\right)(0.5\text{ in.})^3 = 0.009791\text{ lb}$$

$$\bar{x}_{\text{II}} = 1.6\text{ in.} + 0.5\text{ in.} = 2.1\text{ in. } W_{\text{II}} = \left(0.0374\text{ lb/in}^3\right)(\pi)(0.5\text{ in.})^2(3.2\text{ in.}) = 0.093996\text{ lb}$$

$$\bar{x}_{\text{III}} = 3.7\text{ in.} - 1\text{ in.} = 2.7\text{ in.},\ W_{\text{III}} = -\left(0.0374\text{ lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\text{ in.})^2(2\text{ in.}) = -0.000846\text{ lb}$$

$$\bar{x}_{\text{IV}} = 7.3\text{ in.} - 2.8\text{ in.} = 4.5\text{ in.},\ W_{\text{IV}} = \left(0.284\text{ lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\text{ in.})^2(5.6\text{ in.})^2 = 0.017987\text{ lb}$$

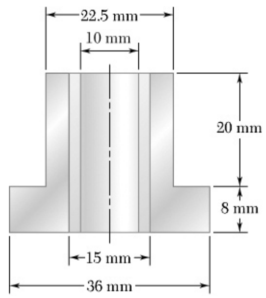
$$\bar{x}_{\text{V}} = 7.3\text{ in.} + \frac{1}{4}(0.4\text{ in.}) = 7.4\text{ in.},\ W_{\text{V}} = \left(0.284\text{ lb/in}^3\right)\left(\frac{\pi}{3}\right)(0.06\text{ in.})^2(0.4\text{ in.}) = 0.000428\text{ lb}$$

| | $W$, lb | $\bar{x}$, in. | $\bar{x}W$, in·lb |
|---|---|---|---|
| I | 0.009791 | 0.3125 | 0.003060 |
| II | 0.093996 | 2.1 | 0.197393 |
| III | –0.000846 | 2.7 | –0.002284 |
| IV | 0.017987 | 4.5 | 0.080942 |
| V | 0.000428 | 7.4 | 0.003169 |
| Σ | 0.12136 | | 0.28228 |

Have $\bar{X}\Sigma W = \Sigma\bar{x}W$: $\bar{X}(0.12136\text{ lb}) = 0.28228\text{ in.}\cdot\text{lb}$

or $\bar{X} = 2.33\text{ in.}$ ◀

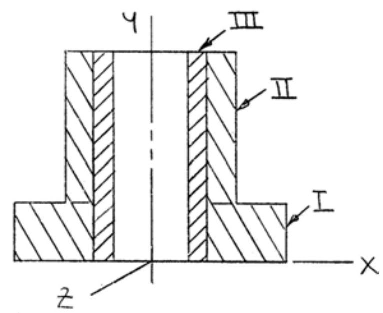
## PROBLEM 5.113




A bronze bushing is mounted inside a steel sleeve. Knowing that the density of bronze is 8800 kg/m$^3$ and of steel is 7860 kg/m$^3$, determine the center of gravity of the assembly.

## SOLUTION

First, note that symmetry implies $\qquad \bar{X} = \bar{Z} = 0$ ◀




Now $\qquad W = (\rho g)V$

$$\bar{y}_{\text{I}} = 4\text{ mm}, \quad W_{\text{I}} = \left(7860\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left\{\left(\frac{\pi}{4}\right)\left[\left(0.036^2 - 0.015^2\right)\text{m}^2\right]\left(0.008\text{ m}\right)\right\}$$

$$= 0.51887\text{ N}$$

$$\bar{y}_{\text{II}} = 18\text{ mm}, \quad W_{\text{II}} = \left(7860\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left\{\left(\frac{\pi}{4}\right)\left[\left(0.0225^2 - 0.05^2\right)\text{m}^2\right]\left(0.02\text{ m}\right)\right\}$$

$$= 0.34065\text{ N}$$

$$\bar{y}_{\text{III}} = 14\text{ mm}, \quad W_{\text{III}} = \left(8800\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left\{\left(\frac{\pi}{4}\right)\left[\left(0.15^2 - 0.10^2\right)\text{m}^2\right]\left(0.028\text{ m}\right)\right\}$$

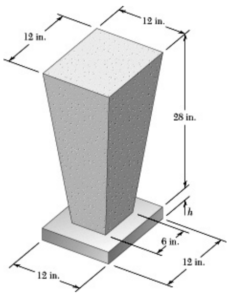
$$= 0.23731\text{ N}$$

Have $\qquad \bar{Y}\Sigma W = \Sigma \bar{y} W$

$$\bar{Y} = \frac{(4\text{ mm})(0.5189\text{ N}) + (18\text{ mm})(0.3406\text{ N}) + (14\text{ mm})(0.2373\text{ N})}{0.5189\text{ N} + 0.3406\text{ N} + 0.2373\text{ N}}$$

or $\bar{Y} = 10.51\text{ mm}$ ◀

(above base)

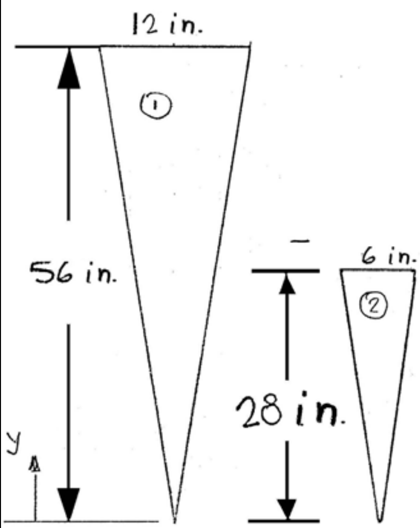



## PROBLEM 5.114

A marker for a garden path consists of a truncated regular pyramid carved from stone of specific weight 160 lb/ft$^3$. The pyramid is mounted on a steel base of thickness $h$. Knowing that the specific weight of steel is 490 lb/ft$^3$ and that steel plate is available in $\frac{1}{4}$ in. increments, specify the minimum thickness $h$ for which the center of gravity of the marker is approximately 12 in. above the top of the base.

## SOLUTION

First, locate the center of gravity of the stone. Assume that the stone is homogeneous so that the center of gravity coincides with the centroid of the corresponding volume.




Have
$$\bar{y}_1 = \frac{3}{4}(56\text{ in.}) = 42\text{ in.},\qquad V_1 = \frac{1}{3}(12\text{ in.})(12\text{ in.})(56\text{ in.})$$
$$= 2688\text{ in}^3$$
$$\bar{y}_2 = \frac{3}{4}(28\text{ in.}) = 21\text{ in.},\qquad V_2 = -\frac{1}{3}(6\text{ in.})(6\text{ in.})(28\text{ in.})$$
$$= -366\text{ in}^3$$

Then
$$V_{\text{stone}} = 2688\text{ in}^3 - 366\text{ in}^3$$
$$= 2352\text{ in}^3$$

and
$$\bar{Y} = \frac{\Sigma\bar{y}V}{\Sigma V}$$
$$= \frac{(42\text{ in.})(2688\text{ in}^3) + (21\text{ in.})(-366\text{ in}^3)}{2352\text{ in}^3}$$
$$= 45\text{ in.}$$

Therefore, the center of gravity of the stone is $(45 - 28)$ in. = 17 in. above the base.

Now
$$W_{\text{stone}} = \gamma_{\text{stone}}V_{\text{stone}} = (160\text{ lb/ft}^3)(2352\text{ in}^3)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$
$$= 217.78\text{ lb}$$
$$W_{\text{steel}} = \gamma_{\text{steel}}V_{\text{steel}}$$
$$= (490\text{ lb/ft}^3)\left[(12\text{ in.})(12\text{ in.})h\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$
$$= (40.833h)\,1\text{b}$$

## PROBLEM 5.114 CONTINUED

Then $$\bar{Y}_{\text{marker}} = \frac{\Sigma yW}{\Sigma W} = 12 \text{ in.}$$

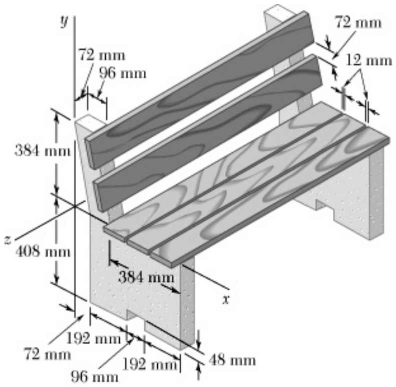
$$= \frac{(17 \text{ in.})(217.78 \text{ lb}) + \left(-\frac{h}{2} \text{ in.}\right)(40.833 \text{ h}) \text{ lb}}{(217.78 + 40.833h)\text{lb}}$$

or $$h^2 + 24h - 53.334 = 0$$

With positive solution $h = 2.0476 \text{ in.}$

$\therefore$ specify $h = 2 \text{ in.}$ ◀

## PROBLEM 5.115




The ends of the park bench shown are made of concrete, while the seat and back are wooden boards. Each piece of wood is $36 \times 120 \times 1180$ mm. Knowing that the density of concrete is $2320\ \text{kg/m}^3$ and of wood is $470\ \text{kg/m}^3$, determine the $x$ and $y$ coordinates of the center of gravity of the bench.

## SOLUTION

First, note that we will account for the two concrete ends by counting twice the weights of components 1, 2, and 3.

$$W_1 = (\rho_c g)V_1 = \left(2320\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(0.480\ \text{m})(0.408\ \text{m})(0.072\ \text{m})\right]$$

$$= 320.9\ \text{N}$$

$$W_2 = -(\rho_c g)V_2 = -\left(2320\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(0.096\ \text{m})(0.048\ \text{m})(0.072\ \text{m})\right]$$

$$= -7.551\ \text{N}$$

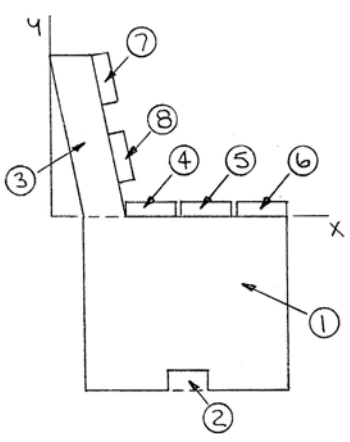
$$W_3 = (\rho_c g)V_3 = \left(2320\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(0.096\ \text{m})(0.384\ \text{m})(0.072\ \text{m})\right]$$

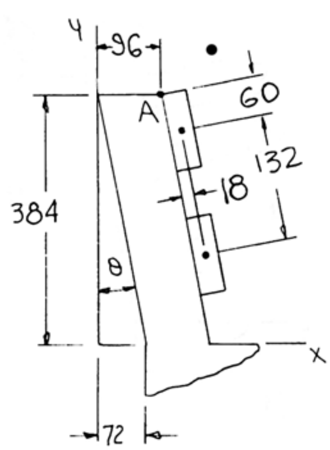
$$= 60.41\ \text{N}$$

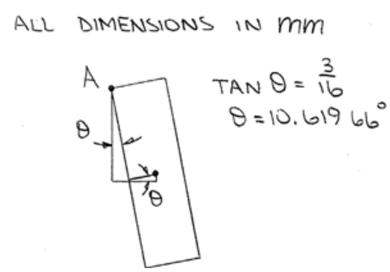
$$W_4 = W_5 = W_6 = W_7 = \rho_w V_{\text{board}}$$

$$= \left(470\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[(0.120\ \text{m})(0.036\ \text{m})(1.180\ \text{m})\right]$$

$$= 23.504\ \text{N}$$



## PROBLEM 5.115 CONTINUED

| | $W$, N | $\bar{x}$, mm | $\bar{y}$, mm | $\bar{x}W$, mm·N | $\bar{y}W$, mm·N |
|---|---|---|---|---|---|
| 1 | $2(320.4) = 641.83$ | 312 | –204 | 200 251.4 | –130 933.6 |
| 2 | $2(-7.551) = -15.10$ | 312 | –384 | –4711.8 | 5799.1 |
| 3 | $2(60.41) = 120.82$ | 84 | 192 | 10 148.5 | 23 196.5 |
| 4 | 23.504 | 228 | 18 | 5358.8 | 423.1 |
| 5 | 23.504 | 360 | 18 | 8461.3 | 423.1 |
| 6 | 23.504 | 442 | 18 | 10 388.5 | 423.1 |
| 7 | 23.504 | 124.7 | 328.3 | 2930.9 | 7716.2 |
| 8 | 23.504 | 160.1 | 139.6 | 3762.9 | 3281.1 |
| Σ | 865.06 | | | 236 590 | –89 671 |

Have $$\bar{X}\Sigma W = \Sigma\bar{x}W: \quad \bar{X}(865.06 \text{ N}) = 236\,590 \text{ mm}\cdot\text{N}$$

or $\bar{X} = 274 \text{ mm}$ ◀

$$\bar{Y}\Sigma W = \Sigma\bar{y}W: \quad \bar{Y}(865.06 \text{ N}) = -89\,671 \text{ mm}\cdot\text{N}$$

or $\bar{Y} = -103.6 \text{ mm}$ ◀

**PROBLEM 5.116**

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Fig. 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A hemisphere.

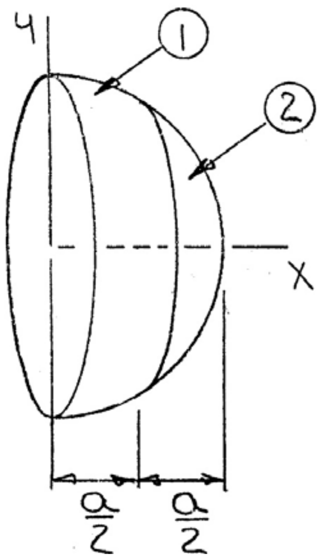
**SOLUTION**

Choose as the element of volume a disk of radius $r$ and thickness $dx$. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi\left(a^2 - x^2\right)dx$$




*Component 1*

$$V_1 = \int_0^{a/2} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_0^{a/2}$$

$$= \frac{11}{24}\pi a^3$$

and

$$\int_1 \bar{x}_{EL} dV = \int_0^{a/2} x\left[\pi\left(a^2 - x^2\right)dx\right]$$

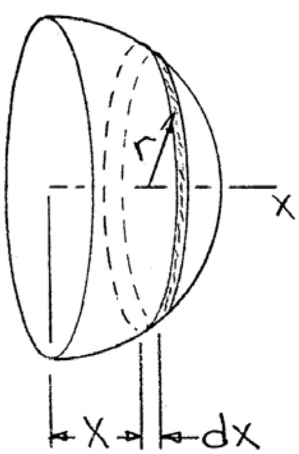
$$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$$

$$= \frac{7}{64}\pi a^4$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4$$

or $\bar{x}_1 = \dfrac{21}{88}a$ ◀




*Component 2*

$$V_2 = \int_{a/2}^{a} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^3$$

## PROBLEM 5.116 CONTINUED

and $$\int_2 \overline{x}_{EL} dV = \int_{a/2}^{a} x\left[\pi\left(a^2 - x^2\right)dx\right] = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^4$$

Now $$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL} dV: \quad \overline{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$$

or $\overline{x}_2 = \dfrac{27}{40}a$ ◀

## PROBLEM 5.117

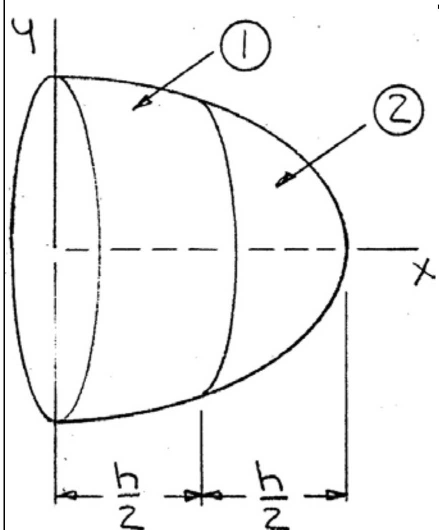
Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Fig. 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A semiellipsoid of revolution.

## SOLUTION

Choose as the element of volume a disk of radius $r$ and thickness $dx$. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\mathrm{EL}} = x$$




The equation of the generating curve is $\dfrac{x^2}{h^2} + \dfrac{y^2}{a^2} = 1$ so that

$r^2 = \dfrac{a^2}{h^2}\left(h^2 - x^2\right)$ and then

$$dV = \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx$$

*Component 1*

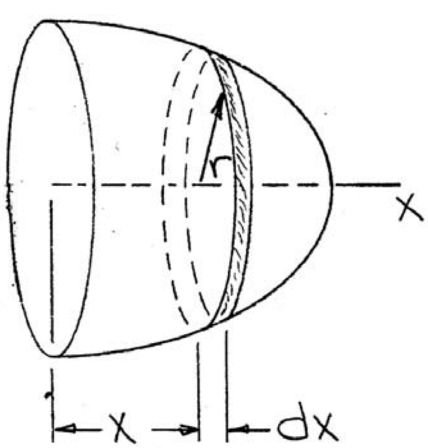
$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{11}{24}\pi a^2 h$$

and $$\int_1 \bar{x}_{EL}\, dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$$

$$= \frac{7}{64}\pi a^2 h^2$$




Now $$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}\, dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^2 h\right) = \frac{7}{64}\pi a^2 h^2$$

or $\bar{x}_1 = \dfrac{21}{88}h$ ◀

## PROBLEM 5.117 CONTINUED

*Component 2*

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi\frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^2 h$$

and $$\int_2 \overline{x}_{EL} dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h^2}\left\{\left[h^2\frac{(h)^2}{2} - \frac{(h)^4}{4}\right] - \left[h^2\frac{\left(\frac{h}{2}\right)^2}{2} - \frac{\left(\frac{h}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^2 h^2$$

Now $$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL} dV: \quad \overline{x}_2\left(\frac{5}{24}\pi a^2 h\right) = \frac{9}{64}\pi a^2 h^2$$

or $\overline{x}_2 = \dfrac{27}{40}h$ ◀