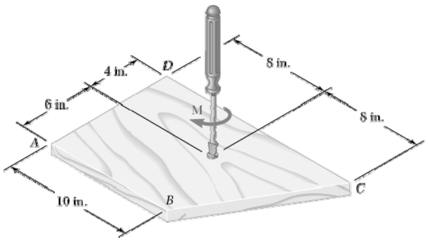
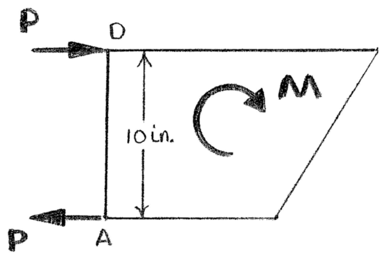
PROBLEM 3.69



A couple M of magnitude $10\ \text{lb}\cdot\text{ft}$ is applied to the handle of a screwdriver to tighten a screw into a block of wood. Determine the magnitudes of the two smallest horizontal forces that are equivalent to M if they are applied (*a*) at corners *A* and *D*, (*b*) at corners *B* and *C*, (*c*) anywhere on the block.

SOLUTION



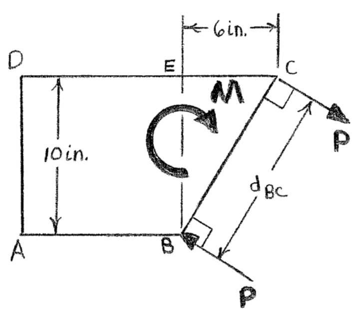
(*a*) Have

$$M = Pd$$

or

$$10\ \text{lb}\cdot\text{ft} = P(10\ \text{in.})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)$$

$$\therefore \quad P = 12\ \text{lb} \qquad \text{or } P_{\text{min}} = 12.00\ \text{lb} \blacktriangleleft$$



(*b*)

$$d_{BC} = \sqrt{(BE)^2 + (EC)^2}$$

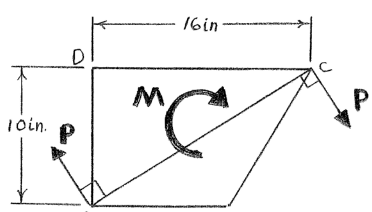
$$= \sqrt{(10\ \text{in.})^2 + (6\ \text{in.})^2} = 11.6619\ \text{in.}$$

Have

$$M = Pd$$

$$10\ \text{lb}\cdot\text{ft} = P(11.6619\ \text{in.})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)$$

$$P = 10.2899\ \text{lb} \qquad \text{or } P = 10.29\ \text{lb} \blacktriangleleft$$



(*c*)

$$d_{AC} = \sqrt{(AD)^2 + (DC)^2}$$

$$= \sqrt{(10\ \text{in.})^2 + (16\ \text{in.})^2} = 2\sqrt{89}\ \text{in.}$$

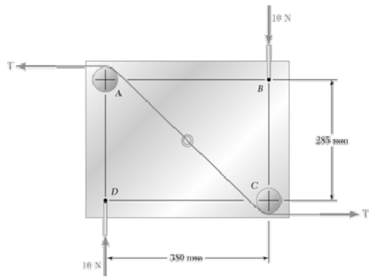
Have

$$M = Pd_{AC}$$

$$10\ \text{lb}\cdot\text{ft} = P\left(2\sqrt{89}\ \text{in.}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)$$

$$P = 6.3600\ \text{lb} \qquad \text{or } P = 6.36\ \text{lb} \blacktriangleleft$$

PROBLEM 3.70



Two 60-mm-diameter pegs are mounted on a steel plate at *A* and *C*, and two rods are attached to the plate at *B* and *D*. A cord is passed around the pegs and pulled as shown, while the rods exert on the plate 10-N forces as indicated. (*a*) Determine the resulting couple acting on the plate when *T* = 36 N. (*b*) If only the cord is used, in what direction should it be pulled to create the same couple with the minimum tension in the cord? (*c*) Determine the value of that minimum tension.

SOLUTION

(*a*) Have

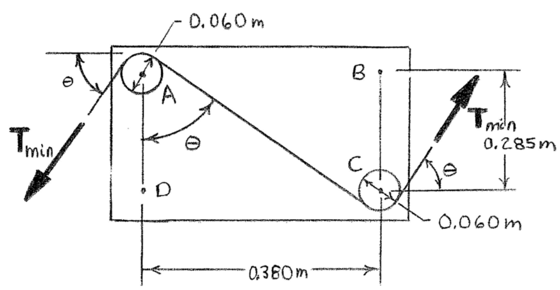
$$M = \Sigma(Fd)$$

$$= (36\text{ N})(0.345\text{ m}) - (10\text{ N})(0.380\text{ m})$$

$$= 8.62\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 8.62\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

(*b*)



Have $M = Td = 8.62\text{ N}\cdot\text{m}$

For *T* to be minimum, *d* must be maximum.

$$\therefore\ T_{\text{min}} \text{ must be perpendicular to line } AC$$

$$\tan\theta = \frac{0.380\text{ m}}{0.285\text{ m}} = 1.33333$$

and $\theta = 53.130°$

$$\text{or } \theta = 53.1° \blacktriangleleft$$

(*c*) Have $M = T_{\text{min}} d_{\text{max}}$

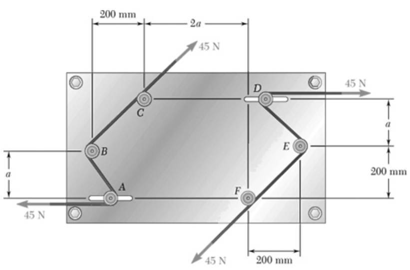
where $M = 8.62\text{ N}\cdot\text{m}$

$$d_{\text{max}} = \left[\sqrt{(0.380)^2 + (0.285)^2} + 2(0.030)\right]\text{ m} = 0.535\text{ m}$$

$$\therefore\ 8.62\text{ N}\cdot\text{m} = T_{\text{min}}(0.535\text{ m})$$

$$T_{\text{min}} = 16.1121\text{ N}$$

$$\text{or } T_{\text{min}} = 16.11\text{ N} \blacktriangleleft$$



PROBLEM 3.71

The steel plate shown will support six 50-mm-diameter idler rollers mounted on the plate as shown. Two flat belts pass around the rollers, and rollers A and D will be adjusted so that the tension in each belt is 45 N. Determine (a) the resultant couple acting on the plate if $a = 0.2$ m, (b) the value of a so that the resultant couple acting on the plate is $54\ \text{N}\cdot\text{m}$ clockwise.

SOLUTION

(a) Note when $a = 0.2$ m, $\mathbf{r}_{C/F}$ is perpendicular to the inclined 45 N forces.

Have

$$M = \Sigma(Fd)$$

$$= -(45\ \text{N})\left[a + 0.2\ \text{m} + 2(0.025\ \text{m})\right]$$

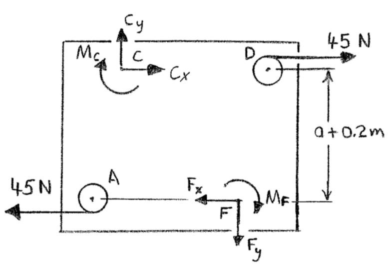
$$-(45\ \text{N})\left[2a\sqrt{2} + 2(0.025\ \text{m})\right]$$

For $a = 0.2$ m,

$$M = -(45\ \text{N})(0.450\ \text{m} + 0.61569\ \text{m})$$

$$= -47.956\ \text{N}\cdot\text{m}$$

or $\mathbf{M} = 48.0\ \text{N}\cdot\text{m}$ ↻ ◀



(b) $\mathbf{M} = 54.0\ \text{N}\cdot\text{m}$ ↻

M = Moment of couple due to horizontal forces at A and D

+ Moment of force-couple systems at C and F about C.

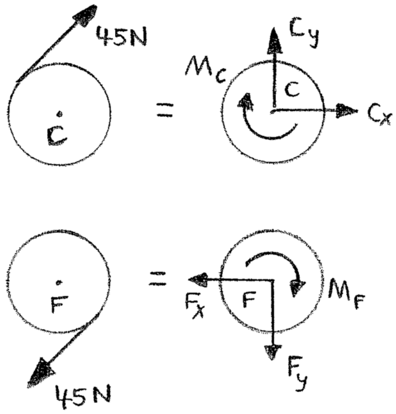
$$-54.0\ \text{N}\cdot\text{m} = -45\ \text{N}\left[a + 0.2\ \text{m} + 2(0.025\ \text{m})\right]$$

$$+\left[M_C + M_F + F_x(a + 0.2\ \text{m}) + F_y(2a)\right]$$

where $M_C = -(45\ \text{N})(0.025\ \text{m}) = -1.125\ \text{N}\cdot\text{m}$

$$M_F = M_C = -1.125\ \text{N}\cdot\text{m}$$

PROBLEM 3.71 CONTINUED



$$F_x = \frac{-45}{\sqrt{2}}\text{ N}$$

$$F_y = \frac{-45}{\sqrt{2}}\text{ N}$$

$$\therefore \quad -54.0\text{ N}\cdot\text{m} = -45\text{ N}\left(a + 0.25\text{ m}\right) - 1.125\text{ N}\cdot\text{m} - 1.125\text{ N}\cdot\text{m}$$

$$\frac{-45\text{ N}}{\sqrt{2}}\left(a + 0.2\text{ m}\right) - \frac{45\text{ N}}{\sqrt{2}}\left(2a\right)$$

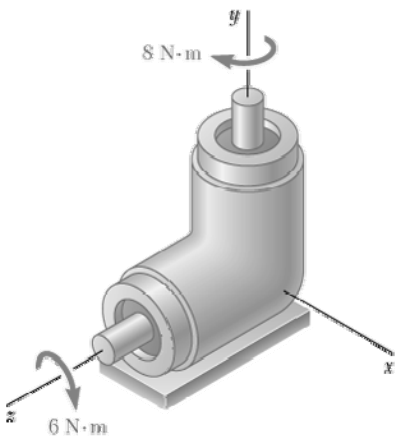
$$1.20 = a + 0.25 + 0.025 + 0.025 + \frac{a}{\sqrt{2}} + \frac{0.20}{\sqrt{2}} + \frac{2a}{\sqrt{2}}$$

$$3.1213a = 0.75858$$

$$a = 0.24303\text{ m}$$

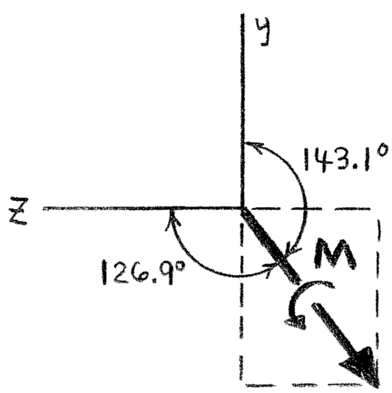
or $a = 243\text{ mm}$ ◀

PROBLEM 3.72



The shafts of an angle drive are acted upon by the two couples shown. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Based on $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

where

$$\mathbf{M}_1 = -(8\ \text{N}\cdot\text{m})\,\mathbf{j}$$

$$\mathbf{M}_2 = -(6\ \text{N}\cdot\text{m})\mathbf{k}$$

$$\therefore\quad \mathbf{M} = -(8\ \text{N}\cdot\text{m})\,\mathbf{j} - (6\ \text{N}\cdot\text{m})\mathbf{k}$$

and $$|\mathbf{M}| = \sqrt{(8)^2 + (6)^2} = 10\ \text{N}\cdot\text{m}$$

or $M = 10.00\ \text{N}\cdot\text{m}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-(8\ \text{N}\cdot\text{m})\,\mathbf{j} - (6\ \text{N}\cdot\text{m})\mathbf{k}}{10\ \text{N}\cdot\text{m}} = -0.8\mathbf{j} - 0.6\mathbf{k}$$

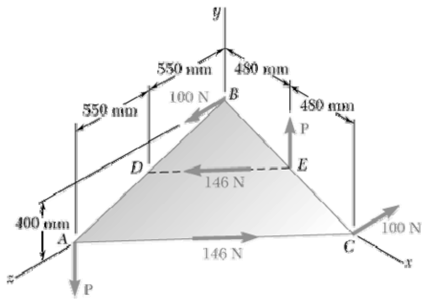
or $$\mathbf{M} = |\mathbf{M}|\lambda = (10\ \text{N}\cdot\text{m})(-0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\cos\theta_x = 0 \qquad \therefore\quad \theta_x = 90°$$

$$\cos\theta_y = -0.8 \qquad \therefore\quad \theta_y = 143.130°$$

$$\cos\theta_z = -0.6 \qquad \therefore\quad \theta_z = 126.870°$$

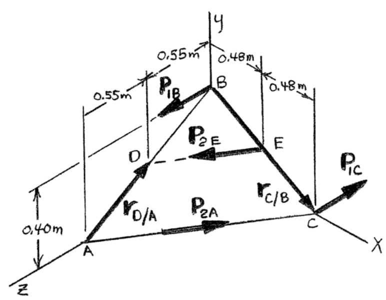
or $\theta_x = 90.0°,\ \ \theta_y = 143.1°,\ \ \theta_z = 126.9°$ ◀



PROBLEM 3.73

Knowing that $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Have $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

where $$\mathbf{M}_1 = \mathbf{r}_{C/B} \times \mathbf{P}_{1C}$$

$$\mathbf{r}_{C/B} = (0.96\text{ m})\mathbf{i} - (0.40\text{ m})\mathbf{j}$$

$$\mathbf{P}_{1C} = -(100\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.40 & 0 \\ 0 & 0 & -100 \end{vmatrix} = (40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}$$

Also, $$\mathbf{M}_2 = \mathbf{r}_{D/A} \times \mathbf{P}_{2E}$$

$$\mathbf{r}_{D/A} = (0.20\text{ m})\mathbf{j} - (0.55\text{ m})\mathbf{k}$$

$$\mathbf{P}_{2E} = \lambda_{ED} P_{2E}$$

$$= \frac{-(0.48\text{ m})\mathbf{i} + (0.55\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.55)^2}\text{ m}}(146\text{ N})$$

$$= -(96\text{ N})\mathbf{i} + (110\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.20 & -0.55 \\ -96 & 0 & 110 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$= (22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

PROBLEM 3.73 CONTINUED

and $$\mathbf{M} = \left[(40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}\right] + \left[(22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}\right]$$

$$= (62.0\text{ N}\cdot\text{m})\mathbf{i} + (148.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(62.0)^2 + (148.8)^2 + (19.2)^2}$$

$$= 162.339\text{ N}\cdot\text{m}$$

or $M = 162.3\text{ N}\cdot\text{m}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{62.0\mathbf{i} + 148.8\mathbf{j} + 19.2\mathbf{k}}{162.339}$$

$$= 0.38192\mathbf{i} + 0.91660\mathbf{j} + 0.118271\mathbf{k}$$

$$\cos\theta_x = 0.38192 \qquad \therefore \quad \theta_x = 67.547°$$

or $\theta_x = 67.5°$ ◀

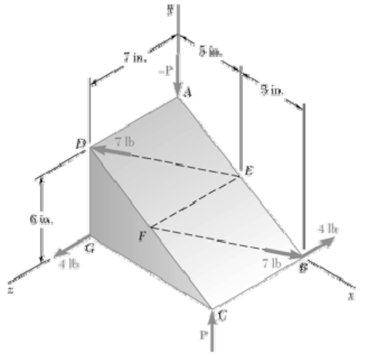
$$\cos\theta_y = 0.91660 \qquad \therefore \quad \theta_y = 23.566°$$

or $\theta_y = 23.6°$ ◀

$$\cos\theta_z = 0.118271 \qquad \therefore \quad \theta_z = 83.208°$$

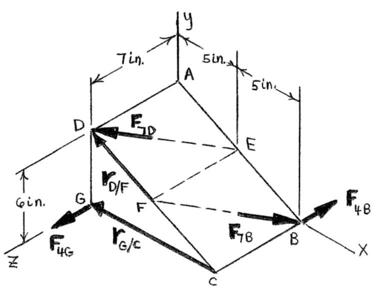
or $\theta_z = 83.2°$ ◀

PROBLEM 3.74



Knowing that $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Have $$\mathbf{M} = \mathbf{M}_4 + \mathbf{M}_7$$

where $$\mathbf{M}_4 = \mathbf{r}_{G/C} \times \mathbf{F}_{4G}$$

$$\mathbf{r}_{G/C} = -(10 \text{ in.})\mathbf{i}$$

$$\mathbf{F}_{4G} = (4 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_4 = -(10 \text{ in.})\mathbf{i} \times (4 \text{ lb})\mathbf{k} = (40 \text{ lb}\cdot\text{in.})\mathbf{j}$$

Also, $$\mathbf{M}_7 = \mathbf{r}_{D/F} \times \mathbf{F}_{7D}$$

$$\mathbf{r}_{D/F} = -(5 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$

$$\mathbf{F}_{7D} = \lambda_{ED} F_{7D}$$

$$= \frac{-(5 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j} + (7 \text{ in.})\mathbf{k}}{\sqrt{(5)^2 + (3)^2 + (7)^2} \text{ in.}}(7 \text{ lb})$$

$$= \frac{7 \text{ lb}}{\sqrt{83}}(-5\mathbf{i} + 3\mathbf{j} + 7\mathbf{k})$$

$$\therefore \quad \mathbf{M}_7 = \frac{7 \text{ lb}\cdot\text{in.}}{\sqrt{83}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 0 \\ -5 & 3 & 7 \end{vmatrix} = \frac{7 \text{ lb}\cdot\text{in.}}{\sqrt{83}}(21\mathbf{i} + 35\mathbf{j} + 0\mathbf{k})$$

$$= 0.76835(21\mathbf{i} + 35\mathbf{j}) \text{ lb}\cdot\text{in.}$$

PROBLEM 3.74 CONTINUED

and $$\mathbf{M} = \left[(40\text{ lb}\cdot\text{in.})\mathbf{j}\right] + \left[0.76835(21\mathbf{i} + 35\mathbf{j})\text{ lb}\cdot\text{in.}\right]$$

$$= (16.1353\text{ lb}\cdot\text{in.})\mathbf{i} + (66.892\text{ lb}\cdot\text{in.})\mathbf{j}$$

$$|\mathbf{M}| = \sqrt{(M_x)^2 + (M_y)^2} = \sqrt{(16.1353)^2 + (66.892)^2}$$

$$= 68.811\text{ lb}\cdot\text{in.}$$

or $M = 68.8\text{ lb}\cdot\text{in.}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{(16.1353\text{ lb}\cdot\text{in.})\mathbf{i} + (66.892\text{ lb}\cdot\text{in.})\mathbf{j}}{68.811\text{ lb}\cdot\text{in.}}$$

$$= 0.23449\mathbf{i} + 0.97212\mathbf{j}$$

$$\cos\theta_x = 0.23449 \qquad \therefore \quad \theta_x = 76.438°$$

or $\theta_x = 76.4°$ ◀

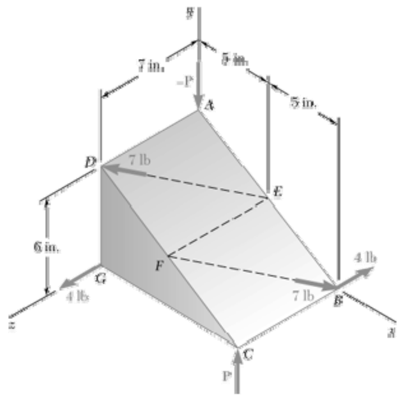
$$\cos\theta_y = 0.97212 \qquad \therefore \quad \theta_y = 13.5615°$$

or $\theta_y = 13.56°$ ◀

$$\cos\theta_z = 0.0 \qquad \therefore \quad \theta_z = 90°$$

or $\theta_z = 90.0°$ ◀

PROBLEM 3.75



Knowing that $P = 5$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Have
$$\mathbf{M} = \mathbf{M}_4 + \mathbf{M}_7 + \mathbf{M}_5$$

where

$$\mathbf{M}_4 = \mathbf{r}_{G/C} \times \mathbf{F}_{4G} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & 0 & 0 \\ 0 & 0 & 4 \end{vmatrix} \text{ lb}\cdot\text{in.} = (40 \text{ lb}\cdot\text{in.})\,\mathbf{j}$$

$$\mathbf{M}_7 = \mathbf{r}_{D/F} \times \mathbf{F}_{7D} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 0 \\ -5 & 3 & 7 \end{vmatrix} \left(\frac{7}{\sqrt{83}}\right) \text{ lb}\cdot\text{in.} = 0.76835(21\mathbf{i} + 35\mathbf{j}) \text{ lb}\cdot\text{in.}$$

(See Solution to Problem 3.74.)

$$\mathbf{M}_5 = \mathbf{r}_{C/A} \times \mathbf{F}_{5C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -6 & 7 \\ 0 & 5 & 0 \end{vmatrix} \text{ lb}\cdot\text{in.} = -(35 \text{ lb}\cdot\text{in.})\mathbf{i} + (50 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\therefore \quad \mathbf{M} = \left[(16.1353 - 35)\mathbf{i} + (40 + 26.892)\,\mathbf{j} + (50)\mathbf{k}\right] \text{ lb}\cdot\text{in.}$$

$$= -(18.8647 \text{ lb}\cdot\text{in.})\mathbf{i} + (66.892 \text{ lb}\cdot\text{in.})\,\mathbf{j} + (50 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(18.8647)^2 + (66.892)^2 + (50)^2} = 85.618 \text{ lb}\cdot\text{in.}$$

or $M = 85.6$ lb·in. ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-18.8647\mathbf{i} + 66.892\mathbf{j} + 50\mathbf{k}}{85.618} = -0.22034\mathbf{i} + 0.78129\mathbf{j} + 0.58399\mathbf{k}$$

$\cos\theta_x = -0.22034 \qquad \therefore \quad \theta_x = 102.729°$ or $\theta_x = 102.7°$ ◀

$\cos\theta_y = 0.78129 \qquad \therefore \quad \theta_y = 38.621°$ or $\theta_y = 38.6°$ ◀

$\cos\theta_z = 0.58399 \qquad \therefore \quad \theta_z = 54.268°$ or $\theta_z = 54.3°$ ◀