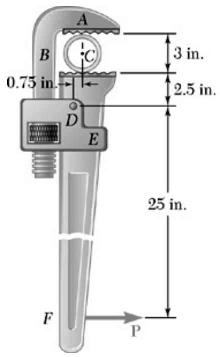
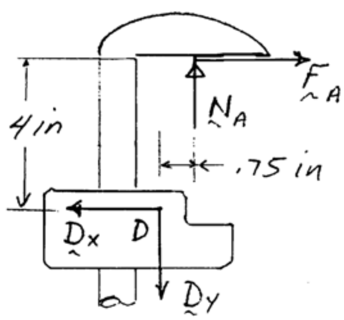
PROBLEM 8.31



Solve Problem 8.30 assuming that the diameter of the pipe is 1.5 in.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75\ \text{in.})N_A - (4\ \text{in.})F_A = 0$$

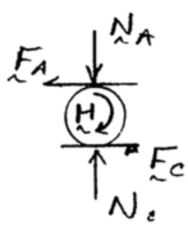
Impending motion: $F_A = \mu_A N_A$

Then $0.75\ \text{in.} - (4\ \text{in.})\mu_A = 0$ $\qquad \mu_A = 0.1875$ ◀

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0$$

so that $D_x = F_A = 0.1875 N_A$

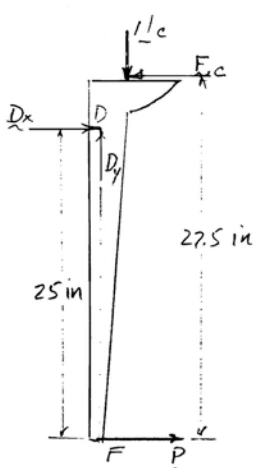
FBD Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



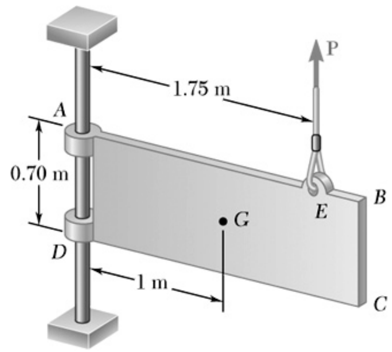
$$\circlearrowleft \Sigma M_F = 0: \quad (27.5\ \text{in.})F_C - (0.75\ \text{in.})N_C - (25\ \text{in.})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

$$27.5\mu_C - 0.75 = 25(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\dfrac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

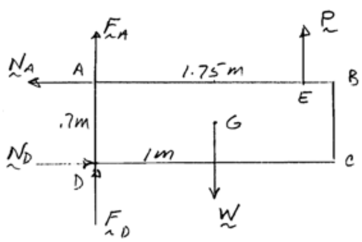


PROBLEM 8.32

The 25-kg plate *ABCD* is attached at *A* and *D* to collars which can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 80$ N.

SOLUTION

FBD plate:



$$W = 25\text{ kg}(9.81\text{ N/kg})$$

$$= 245.25\text{ N}$$

(*a*) $P = 0$; assume equilibrium:

$$\Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W = 0 \qquad N_D = \frac{10W}{7}$$

$$\Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_A = N_D = \frac{10W}{7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

So $\quad (F_A + F_D)_{\max} = \mu_s(N_A + N_D) = \dfrac{20\mu_s W}{7} = 1.143W$

$$\Sigma F_y = 0: \quad F_A + F_D - W = 0$$

$$\therefore \quad F_A + F_D = W < (F_A + F_D)_{\max} \qquad \text{OK.}$$

Plate is in equilibrium ◀

(*b*) $P = 80$ N; assume equilibrium:

$$\Sigma M_A = 0: \quad (1.75\text{ m})P + (0.7\text{ m})N_D - (1\text{ m})W = 0$$

or $\quad N_D = \dfrac{W - 1.75P}{0.7}$

$$\Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_D = N_A = \frac{W - 1.75P}{0.7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_B)_{\max} = \mu_s N_B$$

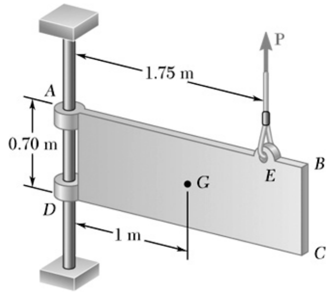
So $\quad (F_A + F_B)_{\max} = 0.4\dfrac{2W - 3.5P}{0.7} = 120.29\text{ N}$

$$\Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$F_A + F_D = W - P = 165.25\text{ N}$$

$$(F_A + F_D)_{\text{equil}} > (F_A + F_D)_{\max}$$

Impossible, so plate slides downward ◀

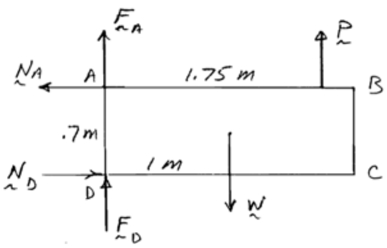


PROBLEM 8.33

In Problem 8.32, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

SOLUTION

FBD plate:



$$W = (25 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 245.25 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7 \text{ m})N_D - (1 \text{ m})W + (1.75 \text{ m})P = 0$$

$$N_D = \frac{W - 1.75P}{0.7}$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_D - N_A = 0 \quad \text{so that} \quad N_A = N_D = \frac{W - 1.75P}{0.7}$$

Note: N_A and N_D will be > 0 if $P < \frac{4}{7}W$ and < 0 if $P > \frac{4}{7}W$.

Impending motion downward: F_A and F_B are both > 0, so

$$F_A = \mu_s |N_A| = \frac{0.4}{0.7}|W - 1.75P| = \left|\frac{4}{7}W - P\right|$$

$$F_D = \mu_S |N_D| = \left|\frac{4}{7}W - P\right|$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$2\left|\frac{4}{7}W - P\right| - W + P = 0$$

For $P < \frac{4}{7}W$; $\qquad P = \frac{W}{7} = 35.04 \text{ N}$

For $P > \frac{4}{7}W$; $\qquad P = \frac{5W}{7} = 175.2 \text{ N}$

Downward motion for $35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

Alternative Solution

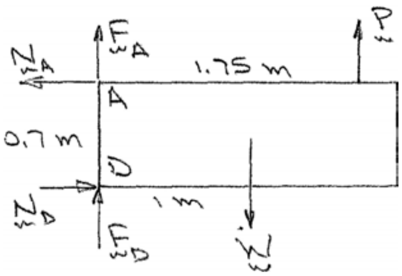
We first observe that for smaller values of the magnitude of **P** that (Case 1) the inner left-hand and right-hand surfaces of collars A and D, respectively, will contact the rod, whereas for larger values of the magnitude of **P** that (Case 2) the inner right-hand and left-hand surfaces of collars A and D, respectively, will contact the rod.

First note: $\qquad W = (25 \text{ kg})(9.81 \text{ m/s}^2)$

$$= 245.25 \text{ N}$$

PROBLEM 8.33 CONTINUED

Case 1



$$\circlearrowleft \Sigma M_D = 0: \quad (0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\text{ N}$$

$$\rightarrow \Sigma F_x = 0: \quad -N_A + N_D = 0$$

or $$N_D = N_A$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25\text{ N} = 0$$

or $$F_A + F_D = (245.25 - P)\text{ N}$$

Now $$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

so that $$(F_A)_{\max} + (F_D)_{\max} = \mu_s(N_A + N_D)$$

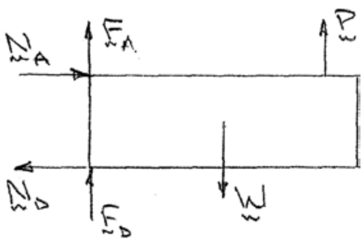
$$= 2(0.4)\left[\frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\right]$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting $$245.25 - P > \frac{8}{7}\left(245.25 - \frac{7}{4}P\right)$$

or $$P > 35.0\text{ N}$$

Case 2



From Case 1: $$N_D = N_A$$

$$F_A + F_D = (245.25 - P)\text{ N}$$

$$(F_A)_{\max} + (F_D)_{\max} = 2\mu_s N_A$$

$$\circlearrowleft \Sigma M_D = 0: \quad -(0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\text{ N}$$

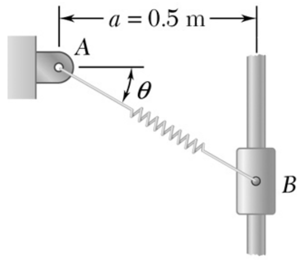
For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting: $$245.25 - P > 2(0.4)\left[\frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\right]$$

or $$P < 175.2\text{ N}$$

Therefore, have downward motion for $35.0\text{ N} < P < 175.2\text{ N}$ ◀

PROBLEM 8.34

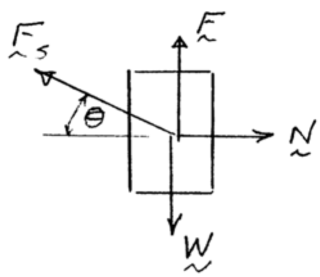


A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^{\circ}$, (*b*) $\theta = 30^{\circ}$.

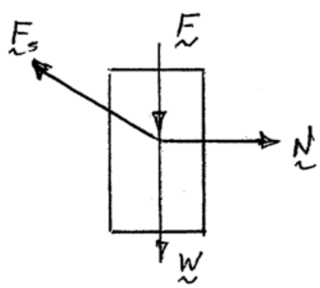
SOLUTION

FBD collar:

Impending motion down:



Impending motion up:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = kx = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right)$$

$$\longrightarrow \Sigma F_x = 0: \quad N - F_s\cos\theta = 0$$

$$N = F_s\cos\theta = (0.75\text{ kN})(1 - \cos\theta)$$

Impending slip: $\quad F = \mu_s N = (0.4)(0.75\text{ kN})(1 - \cos\theta)$

$$= (0.3\text{ kN})(1 - \cos\theta)$$

$+$ down, $-$ up

$$\uparrow \Sigma F_y = 0: \quad F_s\sin\theta \pm F - W = 0$$

$$(0.75\text{ kN})(\tan\theta - \sin\theta) \pm (0.3\text{ kN})(1 - \cos\theta) - W = 0$$

or $\quad W = (0.3\text{ kN})[2.5(\tan\theta - \sin\theta) \pm (1 - \cos\theta)]$

(*a*) $\theta = 20°$: $\quad W_{\text{up}} = -0.00163\text{ kN} \quad (\text{impossible})$

$$W_{\text{down}} = 0.03455\text{ kN} \quad (\text{OK})$$

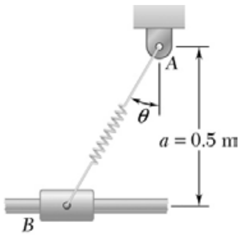
Equilibrium if $0 \le W \le 34.6$ N ◀

(*b*) $\theta = 30°$: $\quad W_{\text{up}} = 0.01782\text{ kN} \quad (\text{OK})$

$$W_{\text{down}} = 0.0982\text{ kN} \quad (\text{OK})$$

Equilibrium if $17.82\text{ N} \le W \le 98.2$ N ◀

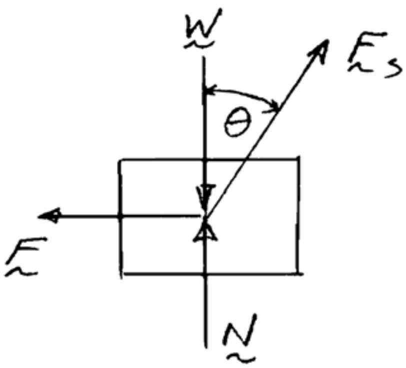
PROBLEM 8.35



A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^{\circ}$, (*b*) $\theta = 30^{\circ}$.

SOLUTION

FBD collar:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = k\left(\frac{a}{\cos\theta} - a\right) = (1.5 \text{ kN/m})(0.5 \text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75 \text{ kN})\left(\frac{1}{\cos\theta} - 1\right) = (750 \text{ N})(\sec\theta - 1)$$

$$\uparrow \Sigma F_y = 0: \quad F_s\cos\theta - W + N = 0$$

or $$W = N + (750 \text{ N})(1 - \cos\theta)$$

Impending slip:

$$F = \mu_s|N| \quad (F \text{ must be } +, \text{ but } N \text{ may be positive or negative})$$

$$\longrightarrow \Sigma F_x = 0: \quad F_s\sin\theta - F = 0$$

or $$F = F_s\sin\theta = (750 \text{ N})(\tan\theta - \sin\theta)$$

(*a*) $\theta = 20^{\circ}$: $\quad F = (750 \text{ N})(\tan 20^{\circ} - \sin 20^{\circ}) = 16.4626 \text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{16.4626 \text{ N}}{0.4} = 41.156 \text{ N}$

(Note: for $|N| < 41.156$ N, motion will occur, equilibrium for $|N| > 41.156$)

But $\quad W = N + (750 \text{ N})(1 - \cos 20^{\circ}) = N + 45.231 \text{ N}$

So equilibrium for $W \leq 4.07$ N and $W \geq 86.4$ N ◀

(*b*) $\theta = 30^{\circ}$: $\quad F = (750 \text{ N})(\tan 30^{\circ} - \sin 30^{\circ}) = 58.013 \text{ N}$

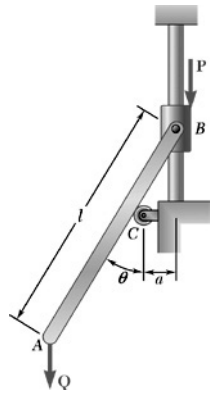
Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{58.013}{0.4} = 145.032 \text{ N}$

$$W = N + (750 \text{ N})(1 - \cos 30^{\circ}) = N \pm 145.03 \text{ N}$$

$$= -44.55 \text{ N (impossible)}, \ 245.51 \text{ N}$$

Equilibrium for $W \geq 246$ N ◀

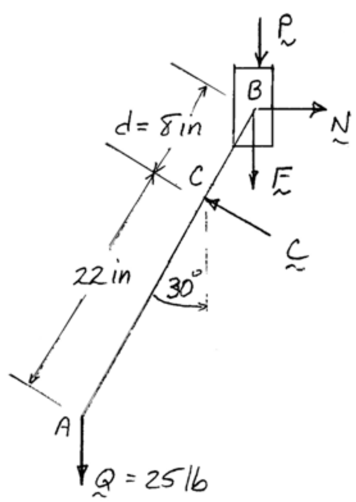
PROBLEM 8.36



The slender rod AB of length $l = 30$ in. is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 4$ in. from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 25$ lb and $\theta = 30^{\circ}$.

SOLUTION

FBD rod + collar:



Note: $d = \dfrac{a}{\sin\theta} = \dfrac{4 \text{ in.}}{\sin 30°} = 8 \text{ in.},$ so $AC = 22$ in.

Neglect weights of rod and collar.

$$\circlearrowleft \Sigma M_B = 0: \quad (30 \text{ in.})(\sin 30°)(25 \text{ lb}) - (8 \text{ in.})C = 0$$

$$C = 46.875 \text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: \quad N - C\cos 30° = 0$$

$$N = (46.875 \text{ lb})\cos 30° = 40.595 \text{ lb}$$

Impending motion up: $\quad F = \mu_s N = 0.25(40.595 \text{ lb})$

$$= 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30° - P - 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} - 10.149 \text{ lb} = -11.71 \text{ lb}$

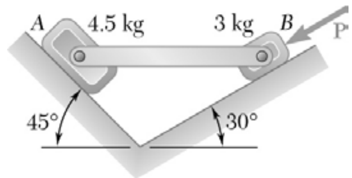
Impending motion down: Direction of **F** is now upward, but still have

$$|F| = \mu_s N = 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30° - P + 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} + 10.149 \text{ lb} = 8.59 \text{ lb}$

$\therefore$ Equilibrium for $-11.71 \text{ lb} \le P \le 8.59 \text{ lb}$ ◀



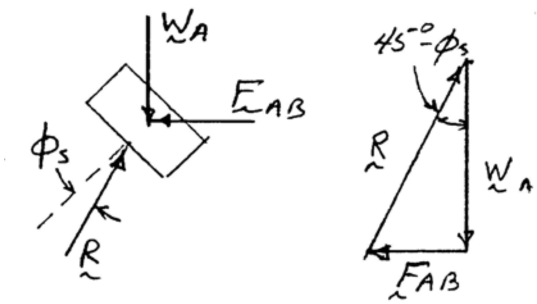
PROBLEM 8.37

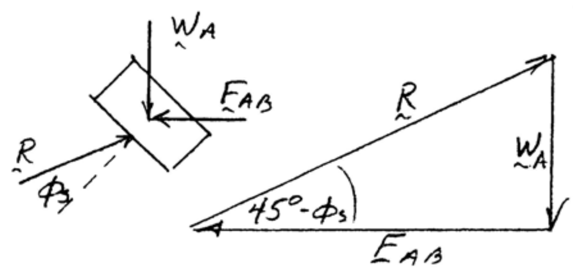
The 4.5-kg block A and the 3-kg block B are connected by a slender rod of negligible mass. The coefficient of static friction is 0.40 between all surfaces of contact. Knowing that for the position shown the rod is horizontal, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBDs: Note: $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.4 = 21.801°$

(a) Block A impending slip ↘ (b) Block A impending slip ↖

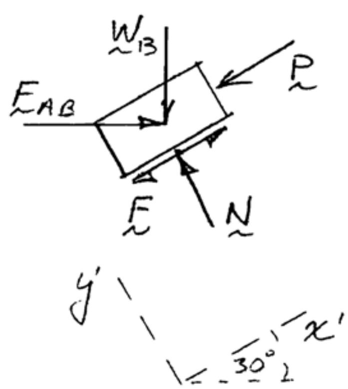




$$F_{AB} = W_A \tan(45° - \phi_s)$$
$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\tan(23.199°)$$
$$= 18.9193\text{ N}$$

$$F_{AB} = W_A \operatorname{ctn}(45° - \phi_s)$$
$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\operatorname{ctn}(23.199°)$$
$$= 103.005\text{ N}$$

Block B:



$$W_B = (3\text{ kg})(9.81\text{ m/s}^2)$$
$$= 29.43\text{ N}$$

From Block B: ↖ $\Sigma F_{y'} = 0$: $N - W_B\cos 30° - F_{AB}\sin 30° = 0$

PROBLEM 8.37 CONTINUED

Case (a) $$N = (29.43\text{ N})\cos 30° + (18.9193\text{ N})\sin 30° = 34.947\text{ N}$$

Impending motion: $$F = \mu_s N = 0.4(34.947\text{ N}) = 13.979\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB}\cos 30° - W_B \sin 30° - 13.979\text{ N} - P = 0$$

$$P = (18.9193\text{ N})\cos 30° - (29.43\text{ N})\sin 30° - 13.979\text{ N}$$

$$= -12.31\text{ N}$$

Case (b) $$N = (29.43\text{ N})\cos 30° + (103.005\text{ N})\sin 30° = 76.9896\text{ N}$$

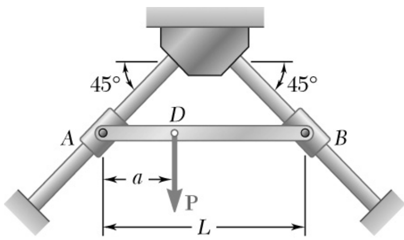
Impending motion: $$F = 0.4(76.9896\text{ N}) = 30.7958\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad (103.005\text{ N})\cos 30° - (29.43\text{ N})\sin 30° + 30.7958\text{ N} - P = 0$$

$$P = 105.3\text{ N}$$

For equilibrium $-12.31\text{ N} \le P \le 105.3\text{ N}$ ◀

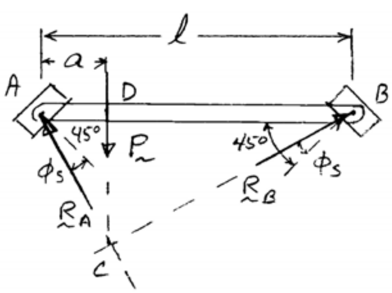
PROBLEM 8.38



Bar AB is attached to collars which can slide on the inclined rods shown. A force **P** is applied at point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a/L* for which equilibrium is maintained.

SOLUTION

FBD bar + collars:



Impending motion

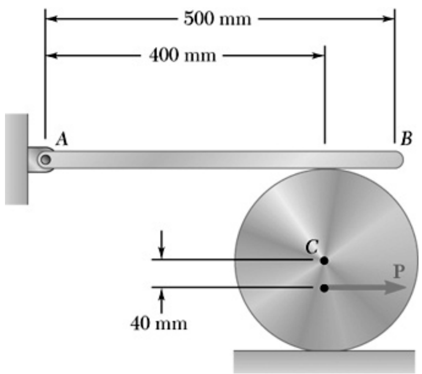
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

Neglect weights: 3-force *FBD* and $\measuredangle ACB = 90°$

So
$$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

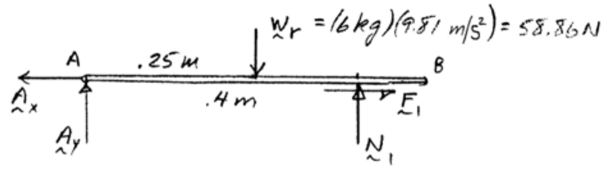


PROBLEM 8.39

The 6-kg slender rod AB is pinned at A and rests on the 18-kg cylinder C. Knowing that the diameter of the cylinder is 250 mm and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained.

SOLUTION

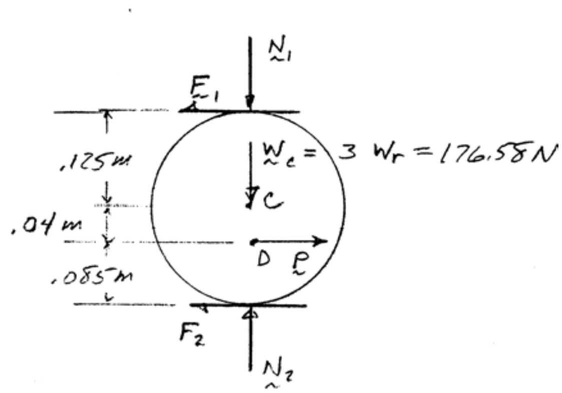
FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.4 \text{ m})N_1 - (0.25 \text{ m})W_r = 0$$

$$N_1 = 0.625W_r = 36.7875 \text{ N}$$

FBD cylinder:



Cylinder:

$$\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_C = 0 \quad \text{or} \quad N_2 = 0.625W_r + 3W_r = 3.625W_r = 5.8N_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.165 \text{ m})F_1 - (0.085 \text{ m})F_2 = 0 \quad \text{or} \quad F_2 = 1.941F_1$$

Since $\mu_{s1} = \mu_{s2}$, motion will impend first at top of the cylinder

So $$F_1 = \mu_s N_1 = 0.35(36.7875 \text{ N}) = 12.8756 \text{ N}$$

and $$F_2 = 1.941(12.8756 \text{ N}) = 24.992 \text{ N}$$

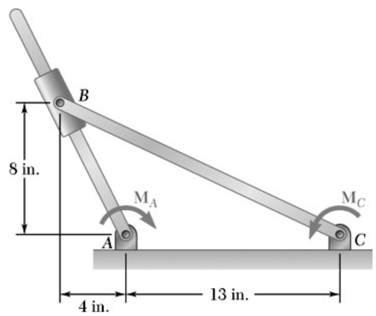
$$\left[\text{Check } F_2 = 25 \text{ N} < \mu_S N_2 = 74.7 \text{ N} \qquad \text{OK}\right]$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_1 - F_2 = 0$$

or $$P = 12.8756 \text{ N} + 24.992 \text{ N}$$

or $P = 37.9 \text{ N}$ ◀

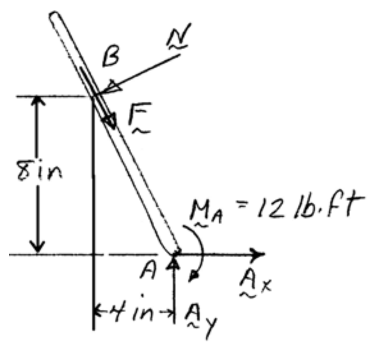
PROBLEM 8.40



Two rods are connected by a collar at B. A couple $\mathbf{M}_A$ of magnitude $12\ \text{lb}\cdot\text{ft}$ is applied to rod AB. Knowing that $\mu_s = 0.30$ between the collar and rod AB, determine the largest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



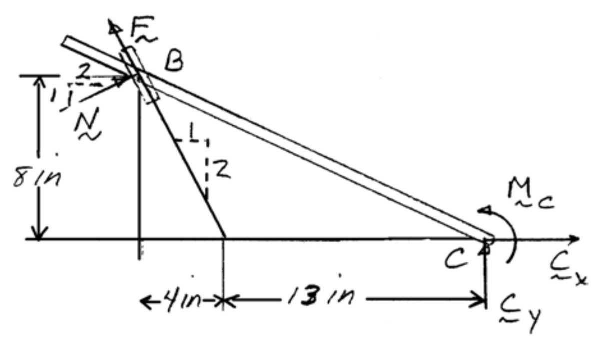
$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{8\ \text{in}^2 + 4\ \text{in}^2}\,(N) - M_A = 0$$

$$N = \frac{(12\ \text{lb}\cdot\text{ft})(12\ \text{in./ft})}{8.9443\ \text{in.}} = 16.100\ \text{lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\ \text{lb}) = 4.83\ \text{lb}$

(Note: For max, M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:



$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\ \text{in.})\frac{1}{\sqrt{5}}N - (8\ \text{in.})\frac{2}{\sqrt{5}}N - (13\ \text{in.})\frac{2}{\sqrt{5}}F = 0$$

or

$$M_C = \frac{17\ \text{in.}}{\sqrt{5}}(16.100\ \text{lb}) + \frac{16\ \text{in.}}{\sqrt{5}}(16.100\ \text{lb}) + \frac{26\ \text{in.}}{\sqrt{5}}(4.830\ \text{lb}) = 293.77\ \text{lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\max} = 24.5\ \text{lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$