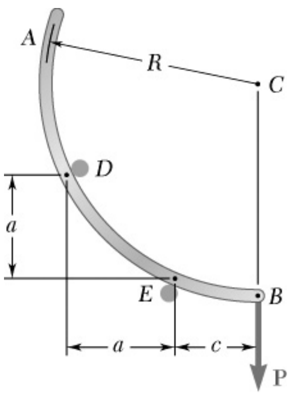
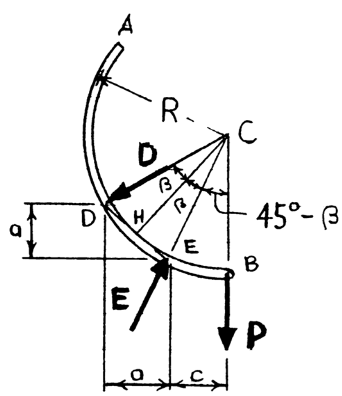
PROBLEM 4.92



Rod *AB* is bent into the shape of a circular arc and is lodged between two pegs *D* and *E*. It supports a load **P** at end *B*. Neglecting friction and the weight of the rod, determine the distance *c* corresponding to equilibrium when $a = 1$ in. and $R = 5$ in.

SOLUTION



Since $$y_{ED} = x_{ED} = a,$$

Slope of *ED* is ⦥ 45°

∴ slope of *HC* is ∡ 45°

Also $$DE = \sqrt{2}a$$

and $$DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$$

For triangles *DHC* and *EHC*

$$\sin\beta = \frac{a/\sqrt{2}}{R} = \frac{a}{\sqrt{2}R}$$

Now $$c = R\sin(45° - \beta)$$

For $$a = 1\,\text{in.} \quad \text{and} \quad R = 5\,\text{in.}$$

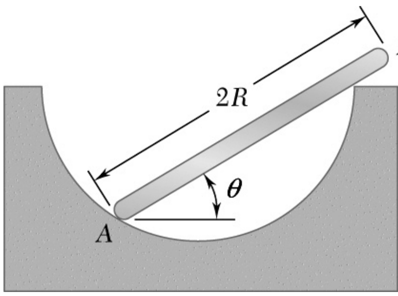
$$\sin\beta = \frac{1\,\text{in.}}{\sqrt{2}(5\,\text{in.})} = 0.141421$$

$$\therefore\ \beta = 8.1301° \qquad \text{or } \beta = 8.13° \blacktriangleleft$$

and $$c = (5\,\text{in.})\sin(45° - 8.1301°) = 3.00\,\text{in.}$$

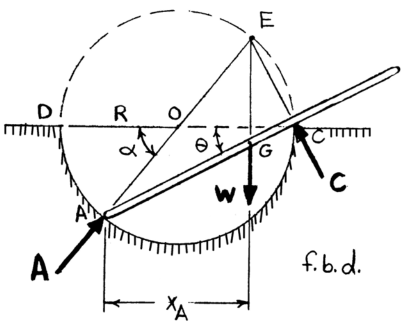
$$\text{or } c = 3.00\,\text{in.} \blacktriangleleft$$

PROBLEM 4.93



A uniform rod AB of weight W and length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction determine the angle θ corresponding to equilibrium.

SOLUTION



Based on the f.b.d., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.

$$\therefore\ \alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE}),$ and $AG, (x_{AG}),$ are equal.

$$\therefore\ x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

or $$4\cos^2\theta - \cos\theta - 2 = 0$$

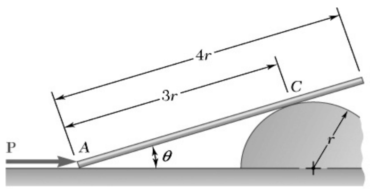
Applying the quadratic equation

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\therefore\ \theta = 32.534° \quad \text{and} \quad \theta = 126.375° \text{(Discard)}$$

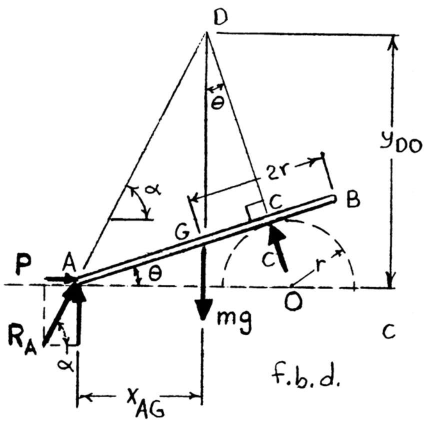
or $\theta = 32.5°$ ◀

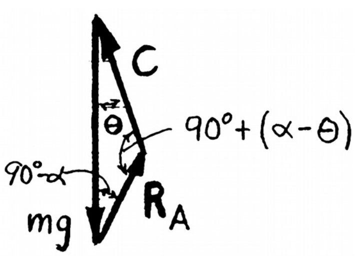
PROBLEM 4.94



A uniform slender rod of mass m and length $4r$ rests on the surface shown and is held in the given equilibrium position by the force **P**. Neglecting the effect of friction at A and C, (a) determine the angle θ, (b) derive an expression for P in terms of m.

SOLUTION





The forces acting on the three-force member intersect at D.

(a) From triangle ACO

$$\theta = \tan^{-1}\left(\frac{r}{3r}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.4349^\circ \qquad \text{or} \quad \theta = 18.43^\circ \blacktriangleleft$$

(b) From triangle DCG

$$\tan\theta = \frac{r}{DC}$$

$$\therefore \quad DC = \frac{r}{\tan\theta} = \frac{r}{\tan 18.4349^\circ} = 3r$$

and

$$DO = DC + r = 3r + r = 4r$$

$$\alpha = \tan^{-1}\left(\frac{y_{DO}}{x_{AG}}\right)$$

where

$$y_{DO} = (DO)\cos\theta = (4r)\cos 18.4349^\circ$$
$$= 3.4947r$$

and

$$x_{AG} = (2r)\cos\theta = (2r)\cos 18.4349^\circ$$
$$= 1.89737r$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{3.4947r}{1.89737r}\right) = 63.435^\circ$$

where

$$90^\circ + (\alpha - \theta) = 90^\circ + 45^\circ = 135.00^\circ$$

Applying the law of sines to the force triangle,

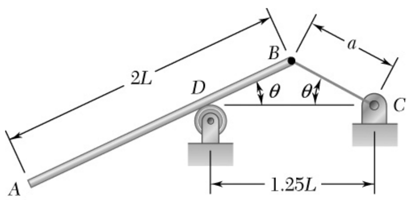
$$\frac{mg}{\sin[90^\circ + (\alpha - \theta)]} = \frac{R_A}{\sin\theta}$$

$$\therefore \quad R_A = (0.44721)mg$$

Finally,

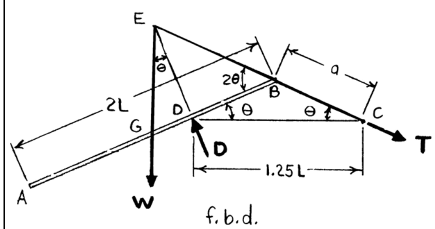
$$P = R_A \cos\alpha$$
$$= (0.44721mg)\cos 63.435^\circ$$
$$= 0.20000mg \qquad \text{or} \quad P = \frac{mg}{5} \blacktriangleleft$$

PROBLEM 4.95



A uniform slender rod of length $2L$ and mass m rests against a roller at D and is held in the equilibrium position shown by a cord of length a. Knowing that $L = 200$ mm, determine (a) the angle θ, (b) the length a.

SOLUTION



(a) The forces acting on the three-force member AB intersect at E. Since triangle DBC is isosceles, $DB = a$.

From triangle BDE

$$ED = DB\tan 2\theta = a\tan 2\theta$$

From triangle GED

$$ED = \frac{(L-a)}{\tan\theta}$$

$$\therefore \quad a\tan 2\theta = \frac{L-a}{\tan\theta} \qquad \text{or} \qquad a(\tan\theta\tan 2\theta + 1) = L \qquad (1)$$

From triangle BCD

$$a = \frac{\frac{1}{2}(1.25L)}{\cos\theta} \qquad \text{or} \qquad \frac{L}{a} = 1.6\cos\theta \qquad (2)$$

Substituting Equation (2) into Equation (1) yields

$$1.6\cos\theta = 1 + \tan\theta\tan 2\theta$$

Now

$$\tan\theta\tan 2\theta = \frac{\sin\theta}{\cos\theta}\frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1}$$

$$= \frac{2(1-\cos^2\theta)}{2\cos^2\theta - 1}$$

Then

$$1.6\cos\theta = 1 + \frac{2(1-\cos^2\theta)}{2\cos^2\theta - 1}$$

or

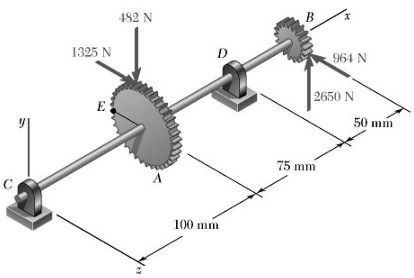
$$3.2\cos^3\theta - 1.6\cos\theta - 1 = 0$$

Solving numerically $\theta = 23.515°$ or $\theta = 23.5°$ ◀

(b) From Equation (2) for $L = 200$ mm and $\theta = 23.5°$

$$a = \frac{5}{8}\frac{(200\text{ mm})}{\cos 23.515°} = 136.321\text{ mm}$$

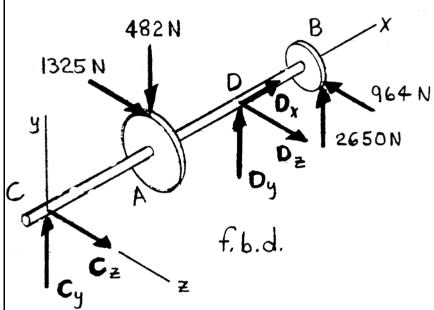
or $a = 136.3$ mm ◀



PROBLEM 4.96

Gears A and B are attached to a shaft supported by bearings at C and D. The diameters of gears A and B are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at C and D. Assume that the bearing at C does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+ (2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 963.71\text{ N}$$

or $$\mathbf{C}_y = (964\text{ N})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (1325\text{ N})(75\text{ mm})$$

$$+ (964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -843.29\text{ N}$$

or $$\mathbf{C}_z = (843\text{ N})\mathbf{k}$$

$$\text{and} \quad \mathbf{C} = (964\text{ N})\mathbf{j} - (843\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+ (2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -3131.7\text{ N}$$

or $$\mathbf{D}_y = -(3130\text{ N})\mathbf{j}$$

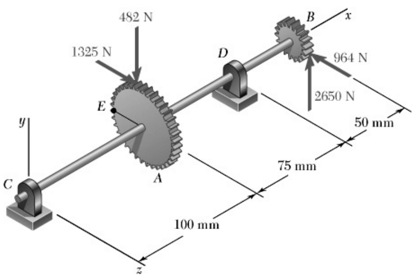
$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(1325\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

$$+ (964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 482.29\text{ N}$$

or $$\mathbf{D}_z = (482\text{ N})\mathbf{k}$$

$$\text{and} \quad \mathbf{D} = -(3130\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k} \blacktriangleleft$$

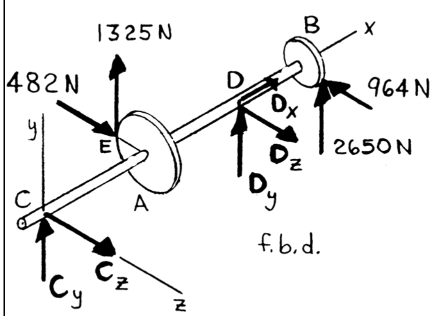


PROBLEM 4.97

Solve Problem 4.96 assuming that for gear A the tangential and radial forces are acting at E, so that $\mathbf{F}_A = (1325\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k}$.

P4.96 Gears A and B are attached to a shaft supported by bearings at C and D. The diameters of gears A and B are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at C and D. Assume that the bearing at C does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-}axis)} = 0: \quad -C_y(175\text{ mm}) - (1325\text{ N})(75\text{ mm})$$

$$+ (2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 189.286\text{ N}$$

or $$\mathbf{C}_y = (189.3\text{ N})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+ (964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -482.00\text{ N}$$

or $$\mathbf{C}_z = -(482\text{ N})\mathbf{k}$$

and $\mathbf{C} = (189.3\text{ N})\mathbf{j} - (482\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{C(z\text{-axis})} = 0: \quad (1325\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+ (2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -4164.3\text{ N}$$

or $$\mathbf{D}_y = -(4160\text{ N})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

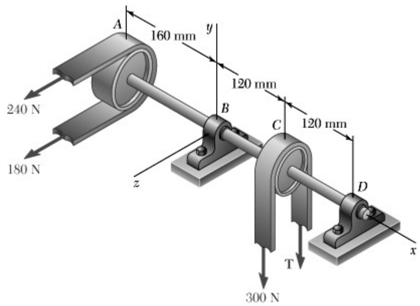
$$+(964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 964.00\text{ N}$$

or $$\mathbf{D}_z = (964\text{ N})\mathbf{k}$$

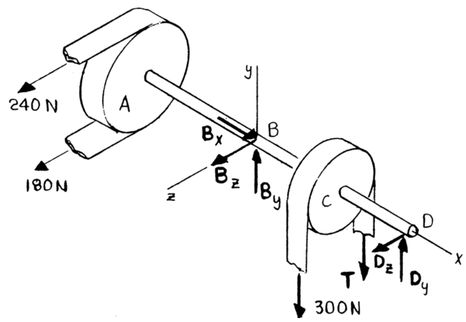
and $\mathbf{D} = -(4160\text{ N})\mathbf{j} + (964\text{ N})\mathbf{k}$ ◀

PROBLEM 4.98



Two transmission belts pass over sheaves welded to an axle supported by bearings at *B* and *D*. The sheave at *A* has a radius of 50 mm, and the sheave at *C* has a radius of 40 mm. Knowing that the system rotates with a constant rate, determine (*a*) the tension *T*, (*b*) the reactions at *B* and *D*. Assume that the bearing at *D* does not exert any axial thrust and neglect the weights of the sheaves and the axle.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

(*a*) $$\Sigma M_{x\text{-axis}} = 0: \quad (240\text{ N} - 180\text{ N})(50\text{ mm}) + (300\text{ N} - T)(40\text{ mm}) = 0$$

$$\therefore\ T = 375\text{ N} \blacktriangleleft$$

(*b*) $$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad (300\text{ N} + 375\text{ N})(120\text{ mm}) - B_y(240\text{ mm}) = 0$$

$$\therefore\ B_y = 337.5\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad (240\text{ N} + 180\text{ N})(400\text{ mm}) + B_z(240\text{ mm}) = 0$$

$$\therefore\ B_z = -700\text{ N}$$

$$\text{or}\quad \mathbf{B} = (338\text{ N})\mathbf{j} - (700\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -(300\text{ N} + 375\text{ N})(120\text{ mm}) + D_y(240\text{ mm}) = 0$$

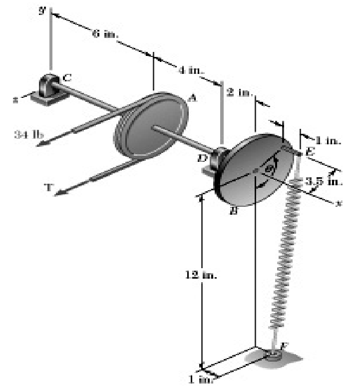
$$\therefore\ D_y = 337.5\text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad (240\text{ N} + 180\text{ N})(160\text{ mm}) + D_z(240\text{ mm}) = 0$$

$$\therefore\ D_z = -280\text{ N}$$

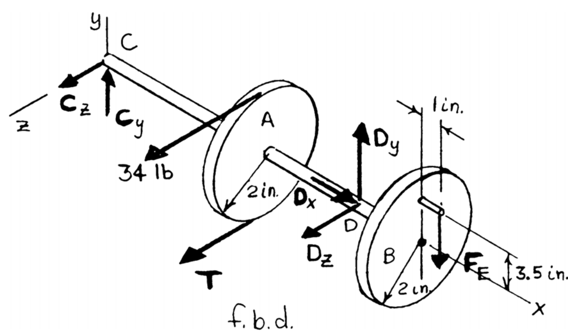
$$\text{or}\quad \mathbf{D} = (338\text{ N})\mathbf{j} - (280\text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.99



For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 180°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

$$x = (y_E)_{\text{final}} - (y_E)_{\text{initial}} = (12 \text{ in.} + 3.5 \text{ in.}) - (12 \text{ in.} - 3.5 \text{ in.}) = 7.0 \text{ in.}$$

$$\therefore \quad F_E = (2 \text{ lb/in.})(7.0 \text{ in.}) = 14.0 \text{ lb}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0: \quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) = 0$$

$$\therefore \quad T = 34 \text{ lb}$$

or $T = 34.0 \text{ lb}$ ◀

(*b*)

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(10 \text{ in.}) - F_E(2 \text{ in.} + 1 \text{ in.}) = 0$$

$$-C_y(10 \text{ in.}) - 14.0 \text{ lb}(3 \text{ in.}) = 0$$

$$\therefore \quad C_y = -4.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(4.20 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(10 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) = 0$$

$$\therefore \quad C_z = -27.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(27.2 \text{ lb})\mathbf{k}$$

and $\mathbf{C} = -(4.20 \text{ lb})\mathbf{j} - (27.2 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.99 CONTINUED

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10\text{ in.}) - F_E(12\text{ in.} + 1\text{ in.}) = 0$$

or

$$D_y(10\text{ in.}) - 14.0(13\text{ in.}) = 0$$

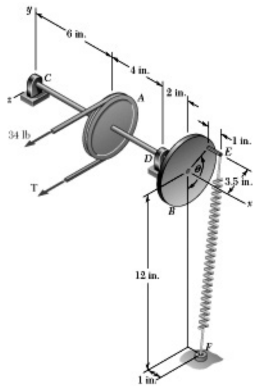
$$\therefore\ D_y = 18.2\text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (18.20\text{ lb})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -2(34\text{ lb})(6\text{ in.}) - D_z(10\text{ in.}) = 0$$

$$\therefore\ D_z = -40.8\text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(40.8\text{ lb})\mathbf{k}$$

and $\mathbf{D} = (18.20\text{ lb})\mathbf{j} - (40.8\text{ lb})\mathbf{k}$ ◀

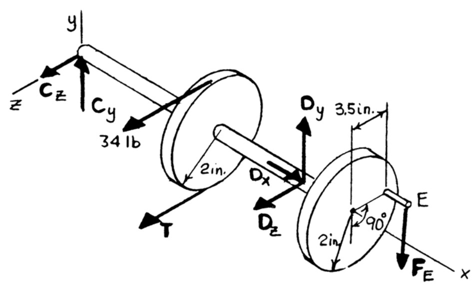
PROBLEM 4.100



Solve Problem 4.99 for $\theta = 90°$.

P4.99 For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 90°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

and $$x = L_{\text{final}} - L_{\text{initial}} = \left(\sqrt{(3.5)^2 + (12)^2}\right) - (12 - 3.5) = 12.5 - 8.5 = 4.0 \text{ in.}$$

$$\therefore\ F_E = (2 \text{ lb/in.})(4.0 \text{ in.}) = 8.0 \text{ lb}$$

Then $$\mathbf{F}_E = \frac{-12.0}{12.5}(8.0 \text{ lb})\mathbf{j} + \frac{3.5}{12.5}(8.0 \text{ lb})\mathbf{k} = -(7.68 \text{ lb})\mathbf{j} + (2.24 \text{ lb})\mathbf{k}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0: \quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) - (7.68 \text{ lb})(3.5 \text{ in.}) = 0$$

$$\therefore \quad T = 20.56 \text{ lb} \qquad \text{or } T = 20.6 \text{ lb} \blacktriangleleft$$

(*b*) $$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(10 \text{ in.}) - (7.68 \text{ lb})(3.0 \text{ in.}) = 0$$

$$\therefore \quad C_y = -2.304 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(2.30 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(10 \text{ in.}) + (34 \text{ lb})(4.0 \text{ in.}) + (20.56 \text{ lb})(4.0 \text{ in.}) - (2.24 \text{ lb})(3 \text{ in.}) = 0$$

$$\therefore \quad C_z = -21.152 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(21.2 \text{ lb})\mathbf{k}$$

and $\mathbf{C} = -(2.30 \text{ lb})\mathbf{j} - (21.2 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.100 CONTINUED

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10 \text{ in.}) - (7.68 \text{ lb})(13 \text{ in.}) = 0$$

$$\therefore \quad D_y = 9.984 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (9.98 \text{ lb})\,\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(34 \text{ lb})(6 \text{ in.}) - (20.56 \text{ lb})(6 \text{ in.}) - D_z(10 \text{ in.}) - (2.24 \text{ lb})(13 \text{ in.}) = 0$$

$$\therefore \quad D_z = -35.648 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(35.6 \text{ lb})\mathbf{k}$$

and $\mathbf{D} = (9.98 \text{ lb})\,\mathbf{j} - (35.6 \text{ lb})\mathbf{k}$ ◀