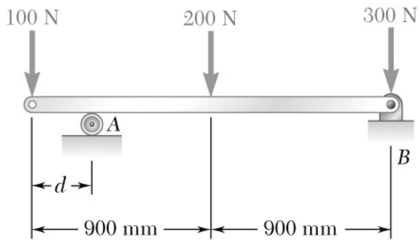
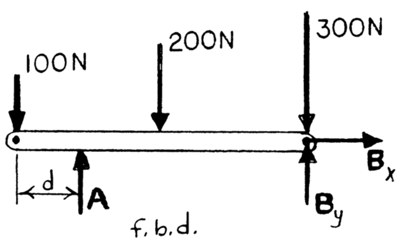
PROBLEM 4.11



The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance d for which the beam is safe.

SOLUTION



From f.b.d. of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (100 + 200 + 300)\,\text{N} = 0$$

or
$$A + B = 600\text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360\text{ N}\,(600\text{ N} - 360\text{ N} = 240\text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad (100\text{ N})(d) - (200\text{ N})(0.9 - d) - (300\text{ N})(1.8 - d)$$
$$+ B(1.8 - d) = 0$$

or
$$d = \frac{720 - 1.8B}{600 - B}$$

Since $B \le 360$ N,

$$d = \frac{720 - 1.8(360)}{600 - 360} = 0.300\text{ m} \qquad \text{or} \qquad d \ge 300\text{ mm}$$

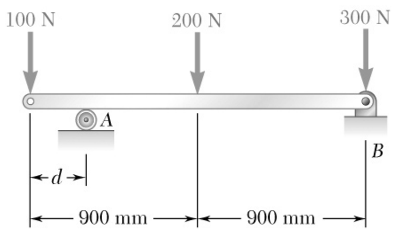
$$+\circlearrowleft \Sigma M_B = 0: \quad (100\text{ N})(1.8) - A(1.8 - d) + (200\text{ N})(0.9) = 0$$

or
$$d = \frac{1.8A - 360}{A}$$

Since $A \le 360$ N,

$$d = \frac{1.8(360) - 360}{360} = 0.800\text{ m} \qquad \text{or} \qquad d \le 800\text{ mm}$$

or $300\text{ mm} \le d \le 800\text{ mm}$ ◀

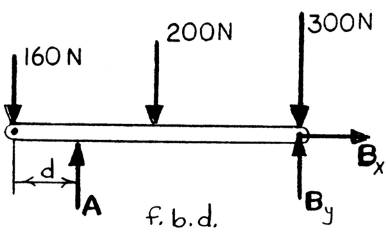


PROBLEM 4.12

Solve Problem 4.11 assuming that the 100-N load is replaced by a 160-N load.

P4.11 The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance d for which the beam is safe.

SOLUTION



From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (160 + 200 + 300)\text{ N} = 0$$

or

$$A + B = 660 \text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360 \text{ N}(660 - 360 = 300 \text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad 160 \text{ N}(d) - 200 \text{ N}(0.9 - d) - 300 \text{ N}(1.8 - d)$$
$$+ B(1.8 - d) = 0$$

or

$$d = \frac{720 - 1.8B}{660 - B}$$

Since $B \le 360$ N,

$$d = \frac{720 - 1.8(360)}{660 - 360} = 0.240 \text{ m} \qquad \text{or} \qquad d \ge 240 \text{ mm}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 160 \text{ N}(1.8) - A(1.8 - d) + 200 \text{ N}(0.9) = 0$$

or

$$d = \frac{1.8A - 468}{A}$$

Since $A \le 360$ N,

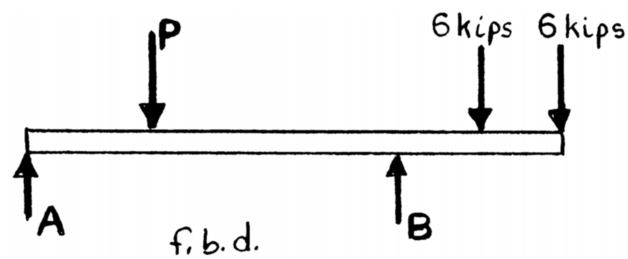
$$d = \frac{1.8(360) - 468}{360} = 0.500 \text{ m} \qquad \text{or} \qquad d \ge 500 \text{ mm}$$

or $240 \text{ mm} \le d \le 500 \text{ mm}$ ◀

PROBLEM 4.13

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe knowing that the maximum allowable value of each of the reactions is 45 kips and that the reaction at A must be directed upward.

SOLUTION



For the force of **P** to be a minimum, **A** = 0.

With $A = 0$,

$$+\circlearrowleft \Sigma M_B = 0: \quad P_{\text{min}}(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$\therefore\ P_{\text{min}} = 6.00\text{ kips}$$

For the force **P** to be a maximum, $\mathbf{A} = \mathbf{A}_{\text{max}} = 45\text{ kips}\uparrow$

With $A = 45$ kips,

$$+\circlearrowleft \Sigma M_B = 0: \quad -(45\text{ kips})(9\text{ ft}) + P_{\text{max}}(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$\therefore\ P_{\text{max}} = 73.5\text{ kips}$$

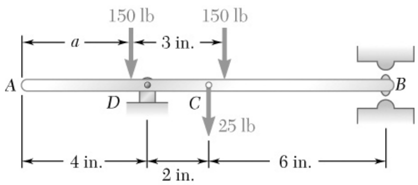
A check must be made to verify the assumption that the maximum value of **P** is based on the reaction force at A. This is done by making sure the corresponding value of B is < 45 kips.

$$+\uparrow \Sigma F_y = 0: \quad 45\text{ kips} - 73.5\text{ kips} + B - 6\text{ kips} - 6\text{ kips} = 0$$

$$\therefore\ B = 40.5\text{ kips} < 45\text{ kips} \qquad \therefore\ \text{ok} \qquad \text{or } P_{\text{max}} = 73.5\text{ kips}$$

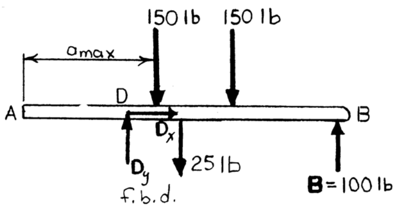
and $6.00\text{ kips} \le P \le 73.5\text{ kips}$ ◀

PROBLEM 4.14



For the beam and loading shown, determine the range of values of the distance a for which the reaction at B does not exceed 50 lb downward or 100 lb upward.

SOLUTION



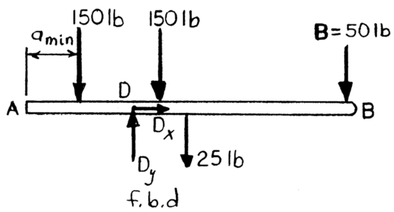
To determine a_{max} the two 150-lb forces need to be as close to B without having the vertical upward force at B exceed 100 lb.

From f.b.d. of beam with $\mathbf{B} = 100 \text{ lb} \uparrow$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(150 \text{ lb})(a_{max} - 4 \text{ in.}) - (150 \text{ lb})(a_{max} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) + (100 \text{ lb})(8 \text{ in.}) = 0$$

or $$a_{max} = 5.00 \text{ in.}$$



To determine a_{min} the two 150-lb forces need to be as close to A without having the vertical downward force at B exceed 50 lb.

From f.b.d. of beam with $\mathbf{B} = 50 \text{ lb} \downarrow$

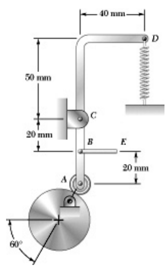
$$+\circlearrowleft \Sigma M_D = 0: \quad (150 \text{ lb})(4 \text{ in.} - a_{min}) - (150 \text{ lb})(a_{min} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) - (50 \text{ lb})(8 \text{ in.}) = 0$$

or $$a_{min} = 1.00 \text{ in.}$$

Therefore, or $1.00 \text{ in.} \le a \le 5.00 \text{ in.}$ ◀

PROBLEM 4.15



A follower *ABCD* is held against a circular cam by a stretched spring, which exerts a force of 21 N for the position shown. Knowing that the tension in rod *BE* is 14 N, determine (*a*) the force exerted on the roller at *A*, (*b*) the reaction at bearing *C*.

SOLUTION

Note: From f.b.d. of *ABCD*

$$A_x = A\cos 60° = \frac{A}{2}$$

$$A_y = A\sin 60° = A\frac{\sqrt{3}}{2}$$

(*a*) From f.b.d. of *ABCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{A}{2}\right)(40 \text{ mm}) - 21 \text{ N}(40 \text{ mm})$$

$$+ 14 \text{ N}(20 \text{ mm}) = 0$$

$$\therefore \ A = 28 \text{ N}$$

or $\mathbf{A} = 28.0 \text{ N} \measuredangle 60°$ ◀

(*b*) From f.b.d. of *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 14 \text{ N} + (28 \text{ N})\cos 60° = 0$$

$$\therefore \ C_x = -28 \text{ N} \qquad \text{or} \qquad \mathbf{C}_x = 28.0 \text{ N} \longleftarrow$$

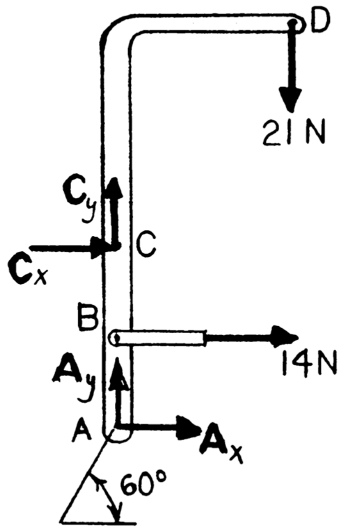
$$+\uparrow \Sigma F_y = 0: \quad C_y - 21 \text{ N} + (28 \text{ N})\sin 60° = 0$$

$$\therefore \ C_y = -3.2487 \text{ N} \qquad \text{or} \qquad \mathbf{C}_y = 3.25 \text{ N} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(28)^2 + (3.2487)^2} = 28.188 \text{ N}$$

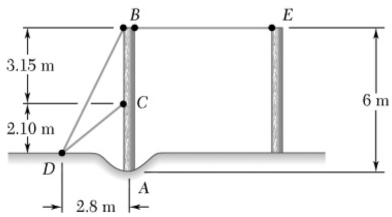
and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-3.2487}{-28}\right) = 6.6182°$$

or $\mathbf{C} = 28.2 \text{ N} \swarrow 6.62°$ ◀



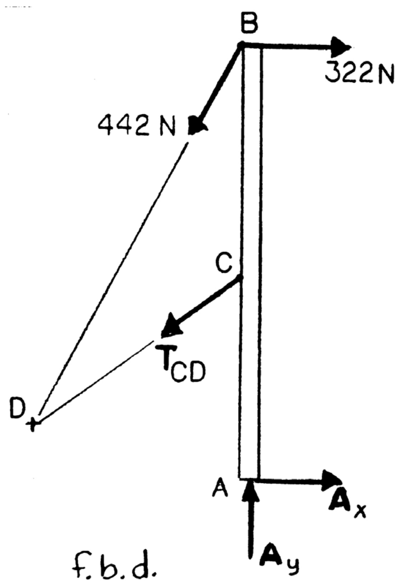
f.b.d.

PROBLEM 4.16



A 6-m-long pole AB is placed in a hole and is guyed by three cables. Knowing that the tensions in cables BD and BE are 442 N and 322 N, respectively, determine (*a*) the tension in cable CD, (*b*) the reaction at A.

SOLUTION



Note:

$$\overline{DB} = \sqrt{(2.8)^2 + (5.25)^2} = 5.95 \text{ m}$$

$$\overline{DC} = \sqrt{(2.8)^2 + (2.10)^2} = 3.50 \text{ m}$$

(*a*) From f.b.d. of pole

$$+\circlearrowleft \Sigma M_A = 0: \quad -(322 \text{ N})(6 \text{ m}) + \left[\left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})\right](6 \text{ m})$$

$$+\left[\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)T_{CD}\right](2.85 \text{ m}) = 0$$

$$\therefore \ T_{CD} = 300 \text{ N}$$

or $T_{CD} = 300$ N ◀

(*b*) From f.b.d. of pole

$$\xrightarrow{+} \Sigma F_x = 0: \quad 322 \text{ N} - \left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})$$

$$-\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) + A_x = 0$$

$$\therefore \ A_x = 126 \text{ N} \qquad \text{or} \qquad \mathbf{A}_x = 126 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - \left(\frac{5.25 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N}) - \left(\frac{2.10 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) = 0$$

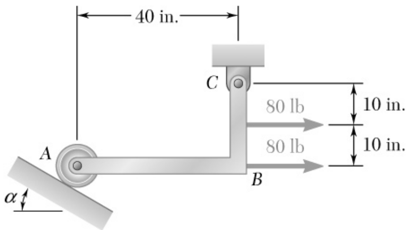
$$\therefore \ A_y = 570 \text{ N} \qquad \text{or} \qquad \mathbf{A}_y = 570 \text{ N} \uparrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(126)^2 + (570)^2} = 583.76 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{570 \text{ N}}{126 \text{ N}}\right) = 77.535°$$

or $\mathbf{A} = 584$ N $\measuredangle$ 77.5° ◀

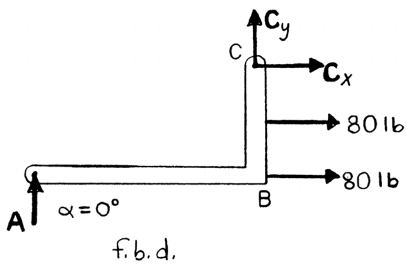
PROBLEM 4.17



Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30^{\circ}$.

SOLUTION

(*a*)



(*a*) $\alpha = 0°$

From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (80 \text{ lb})(10 \text{ in.}) + (80 \text{ lb})(20 \text{ in.}) - A(40 \text{ in.}) = 0$$

$$\therefore\ A = 60 \text{ lb}$$

or $\mathbf{A} = 60.0 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y + 60 \text{ lb} = 0$$

$$\therefore\ C_y = -60 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 60 \text{ lb} \downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80 \text{ lb} + 80 \text{ lb} + C_x = 0$$

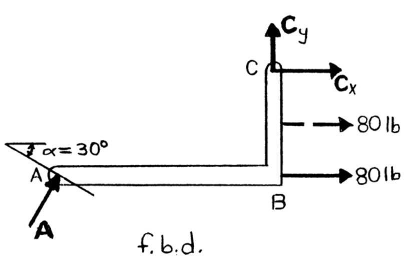
$$\therefore\ C_x = -160 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 160 \text{ lb} \longleftarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(160)^2 + (60)^2} = 170.880 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-60}{-160}\right) = 20.556°$$

or $\mathbf{C} = 170.9 \text{ lb} \measuredangle 20.6°$ ◀

(*b*)



(*b*) $\alpha = 30°$

From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (80 \text{ lb})(10 \text{ in.}) + (80 \text{ lb})(20 \text{ in.}) - (A\cos 30°)(40 \text{ in.})$$

$$+ (A\sin 30°)(20 \text{ in.}) = 0$$

$$\therefore\ A = 97.399 \text{ lb}$$

or $\mathbf{A} = 97.4 \text{ lb} \measuredangle 60°$ ◀

PROBLEM 4.17 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80 \text{ lb} + 80 \text{ lb} + (97.399 \text{ lb})\sin 30° + C_x = 0$$

$$\therefore \; C_x = -208.70 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 209 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + (97.399 \text{ lb})\cos 30° = 0$$

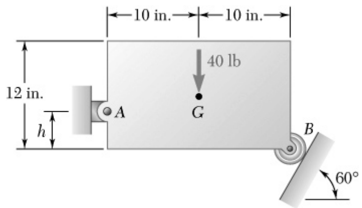
$$\therefore \; C_y = -84.350 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 84.4 \text{ lb} \downarrow$$

Then $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(208.70)^2 + (84.350)^2} = 225.10 \text{ lb}$

and $\quad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-84.350}{-208.70}\right) = 22.007°$

or $\mathbf{C} = 225 \text{ lb} \measuredangle 22.0°$ ◀

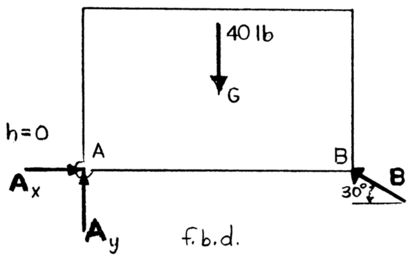
PROBLEM 4.18



Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 8$ in.

SOLUTION

(*a*)



(*a*) $h = 0$

From f.b.d. of plate

$$+\circlearrowleft\ \Sigma M_A = 0:\quad (B\sin 30°)(20\text{ in.}) - (40\text{ lb})(10\text{ in.}) = 0$$

$$\therefore\ B = 40\text{ lb}$$

or $\mathbf{B} = 40.0\text{ lb} \searrow 30°$ ◀

$$\xrightarrow{+}\ \Sigma F_x = 0:\quad A_x - (40\text{ lb})\cos 30° = 0$$

$\therefore\ A_x = 34.641$ lb or $\mathbf{A}_x = 34.6\text{ lb} \longrightarrow$

$$+\uparrow\ \Sigma F_y = 0:\quad A_y - 40\text{ lb} + (40\text{ lb})\sin 30° = 0$$

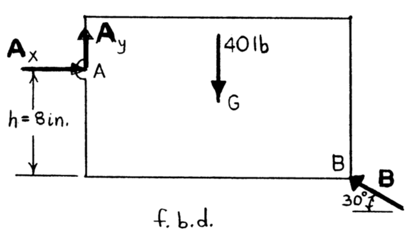
$\therefore\ A_y = 20$ lb or $\mathbf{A}_y = 20.0\text{ lb} \uparrow$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(34.641)^2 + (20)^2} = 39.999\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{20}{34.641}\right) = 30.001°$$

or $\mathbf{A} = 40.0\text{ lb} \measuredangle 30°$ ◀

(*b*)



(*b*) $h = 8$ in.

From f.b.d. of plate

$$+\circlearrowleft\ \Sigma M_A = 0:\quad (B\sin 30°)(20\text{ in.}) - (B\cos 30°)(8\text{ in.})$$

$$-(40\text{ lb})(10\text{ in.}) = 0$$

$$\therefore\ B = 130.217\text{ lb}$$

or $\mathbf{B} = 130.2\text{ lb} \searrow 30.0°$ ◀

PROBLEM 4.18 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (130.217 \text{ lb})\cos 30° = 0$$

$$\therefore A_x = 112.771 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_x = 112.8 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 40 \text{ lb} + (130.217 \text{ lb})\sin 30° = 0$$

$$\therefore A_y = -25.108 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_y = 25.1 \text{ lb} \downarrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(112.771)^2 + (25.108)^2} = 115.532 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-25.108}{112.771}\right) = -12.5519°$$

$$\text{or } \mathbf{A} = 115.5 \text{ lb} \searrow 12.55° \blacktriangleleft$$