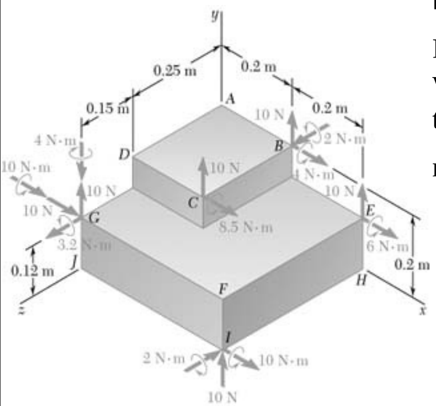
PROBLEM 3.101

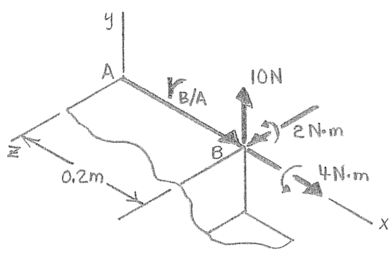


Five separate force-couple systems act at the corners of a metal block, which has been machined into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10\text{ N})\mathbf{j}$ and a couple of moment $\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$ located at point A.

SOLUTION

The equivalent force-couple system at A for each of the five force-couple systems will be determined. Each will then be compared to the given force-couple system to determine if they are equivalent.

Force-couple system at B



Have $\qquad \Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\qquad \mathbf{F} = (10\text{ N})\mathbf{j}$

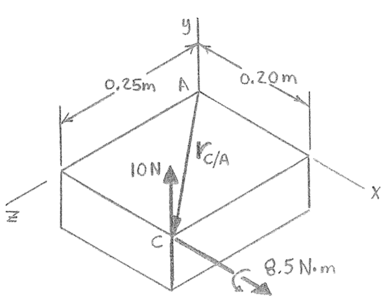
and $\qquad \Sigma\mathbf{M}_A: \quad \Sigma\mathbf{M}_B + (\mathbf{r}_{B/A} \times \mathbf{F}) = \mathbf{M}$

$$(4\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{k} + (0.2\text{ m})\mathbf{i} \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (4\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at A,
Is Not Equivalent ◀

Force-couple system at C



Have $\qquad \Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\qquad \mathbf{F} = (10\text{ N})\mathbf{j}$

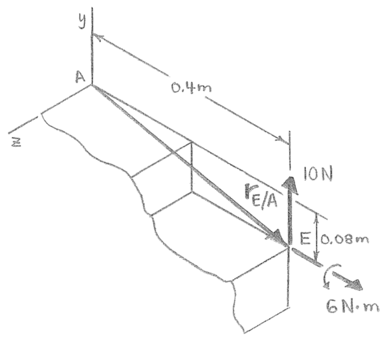
and $\qquad \Sigma\mathbf{M}_A: \quad \mathbf{M}_C + (\mathbf{r}_{C/A} \times \mathbf{F}) = \mathbf{M}$

$$(8.5\text{ N}\cdot\text{m})\mathbf{i} + \left[(0.2\text{ m})\mathbf{i} + (0.25\text{ m})\mathbf{k}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (2.0\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at A,
Is Not Equivalent ◀

PROBLEM 3.101 CONTINUED



Force-couple system at E

Have $\Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

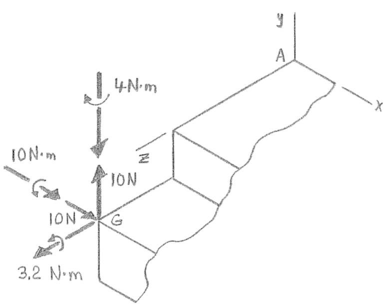
or $\mathbf{F} = (10\text{ N})\mathbf{j}$

and $\Sigma M_A: \quad \mathbf{M}_E + (\mathbf{r}_{E/A} \times \mathbf{F}) = \mathbf{M}$

$$(6\text{ N}\cdot\text{m})\mathbf{i} + \left[(0.4\text{ m})\mathbf{i} - (0.08\text{ m})\mathbf{j}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at *A*,
Is Equivalent ◀



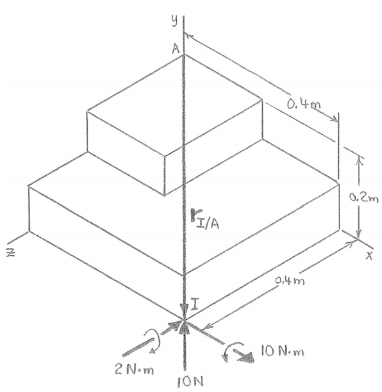
Force-couple system at G

Have $\Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{i} + (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\mathbf{F} = (10\text{ N})\mathbf{i} + (10\text{ N})\mathbf{j}$

$\mathbf{F}$ has two force components

$\therefore$ force-couple system at *G*
Is Not Equivalent ◀



Force-couple system at I

Have $\Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\mathbf{F} = (10\text{ N})\mathbf{j}$

and $\Sigma\mathbf{M}_A: \quad \Sigma\mathbf{M}_I + (\mathbf{r}_{I/A} \times \mathbf{F}) = \mathbf{M}$

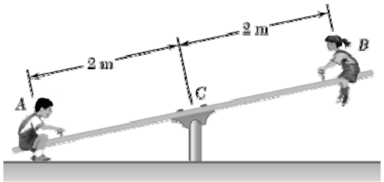
$$(10\text{ N}\cdot\text{m})\mathbf{i} - (2\text{ N}\cdot\text{m})\mathbf{k}$$

$$+\left[(0.4\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

or $\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{k}$

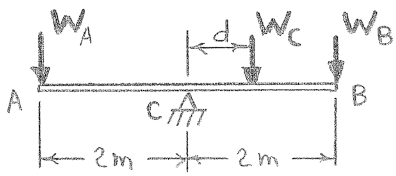
Comparing to given force-couple system at *A*,
Is Not Equivalent ◀

PROBLEM 3.102



The masses of two children sitting at ends A and B of a seesaw are 38 kg and 29 kg, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through C if she has a mass of (a) 27 kg, (b) 24 kg.

SOLUTION



First $\qquad W_A = m_A g = (38 \text{ kg})g$

$$W_B = m_B g = (29 \text{ kg})g$$

(a) $\qquad W_C = m_C g = (27 \text{ kg})g$

For resultant weight to act at C, $\qquad \Sigma M_C = 0$

Then $\quad [(38 \text{ kg})g](2 \text{ m}) - [(27 \text{ kg})g](d) - [(29 \text{ kg})g](2 \text{ m}) = 0$

$$\therefore \quad d = \frac{76 - 58}{27} = 0.66667 \text{ m}$$

or $d = 0.667$ m ◀

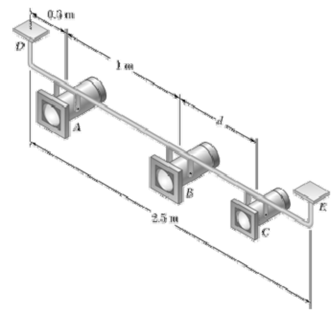
(b) $\qquad W_C = m_C g = (24 \text{ kg})g$

For resultant weight to act at C, $\qquad \Sigma M_C = 0$

Then $\quad [(38 \text{ kg})g](2 \text{ m}) - [(24 \text{ kg})g](d) - [(29 \text{ kg})g](2 \text{ m}) = 0$

$$\therefore \quad d = \frac{76 - 58}{24} = 0.75 \text{ m}$$

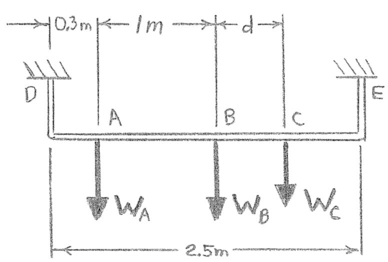
or $d = 0.750$ m ◀



PROBLEM 3.103

Three stage lights are mounted on a pipe as shown. The mass of each light is $m_A = m_B = 1.8$ kg and $m_C = 1.6$ kg. (*a*) If $d = 0.75$ m, determine the distance from D to the line of action of the resultant of the weights of the three lights. (*b*) Determine the value of d so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



First $\qquad W_A = W_B = m_A g = (1.8 \text{ kg})g$

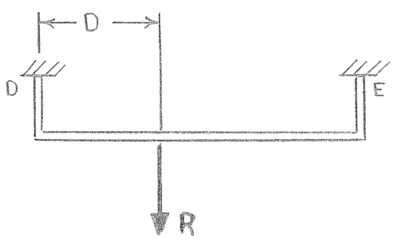
$$W_C = m_C g = (1.6 \text{ kg})g$$

(*a*) $\qquad d = 0.75$ m

Have $\qquad R = W_A + W_B + W_C$

$$R = \left[(1.8 + 1.8 + 1.6)\text{kg}\right]g$$

or $\qquad \mathbf{R} = (5.2g) \text{ N}\downarrow$

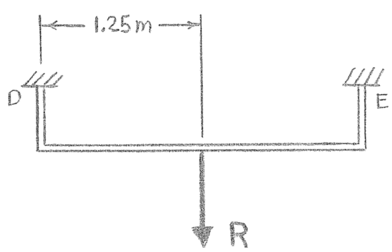


Have

$$\Sigma M_D: \quad -1.8g(0.3 \text{ m}) - 1.8g(1.3 \text{ m}) - 1.6g(2.05 \text{ m}) = -5.2g(D)$$

$$\therefore \quad D = 1.18462 \text{ m}$$

or $D = 1.185$ m ◀



(*b*) $\qquad D = \dfrac{L}{2} = 1.25$ m

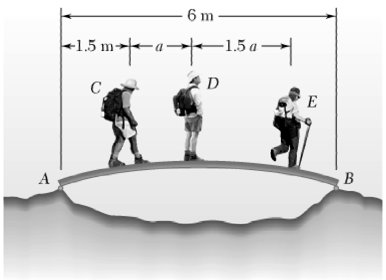
Have

$$\Sigma M_D: \quad -(1.8g)(0.3 \text{ m}) - (1.8g)(1.3 \text{ m}) - (1.6g)(1.3 \text{ m} + d)$$

$$= -(5.2g)(1.25 \text{ m})$$

$$\therefore \quad d = 0.9625 \text{ m}$$

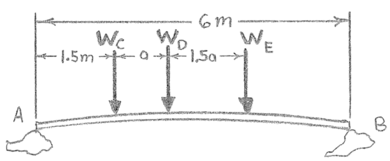
or $d = 0.963$ m ◀



PROBLEM 3.104

Three hikers are shown crossing a footbridge. Knowing that the weights of the hikers at points C, D, and E are 800 N, 700 N, and 540 N, respectively, determine (a) the horizontal distance from A to the line of action of the resultant of the three weights when $a = 1.1$ m, (b) the value of a so that the loads on the bridge supports at A and B are equal.

SOLUTION



(a) $$a = 1.1 \text{ m}$$

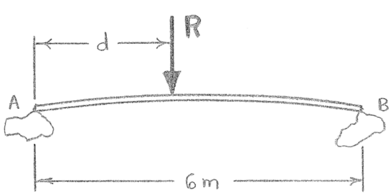
Have $$\Sigma F: \quad -W_C - W_D - W_E = R$$

$$\therefore \quad R = -800 \text{ N} - 700 \text{ N} - 540 \text{ N}$$

$$R = 2040 \text{ N}$$

or $$\mathbf{R} = 2040 \text{ N} \downarrow$$

(a)



Have

$$\Sigma M_A: \quad -(800 \text{ N})(1.5 \text{ m}) - (700 \text{ N})(2.6 \text{ m}) - (540 \text{ N})(4.25 \text{ m})$$

$$= -R(d)$$

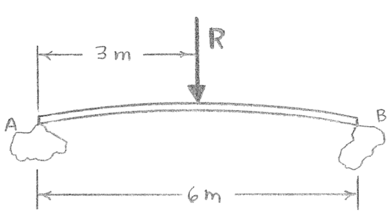
$$\therefore \quad -5315 \text{ N}\cdot\text{m} = -(2040 \text{ N})d$$

and $$d = 2.6054 \text{ m}$$

or $d = 2.61$ m to the right of A ◀

(b) For equal reaction forces at A and B, the resultant, $\mathbf{R}$, must act at the center of the span.

(b)



From $$\Sigma M_A = -R\left(\frac{L}{2}\right)$$

$$\therefore \quad -(800 \text{ N})(1.5 \text{ m}) - (700 \text{ N})(1.5 \text{ m} + a) - (540 \text{ N})(1.5 \text{ m} + 2.5a)$$

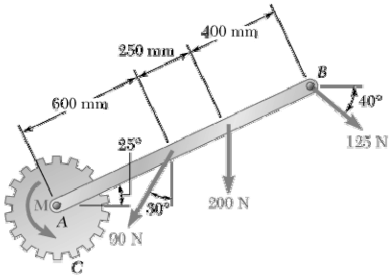
$$= -(2040 \text{ N})(3 \text{ m})$$

$$3060 + 2050a = 6120$$

and $$a = 1.49268 \text{ m}$$

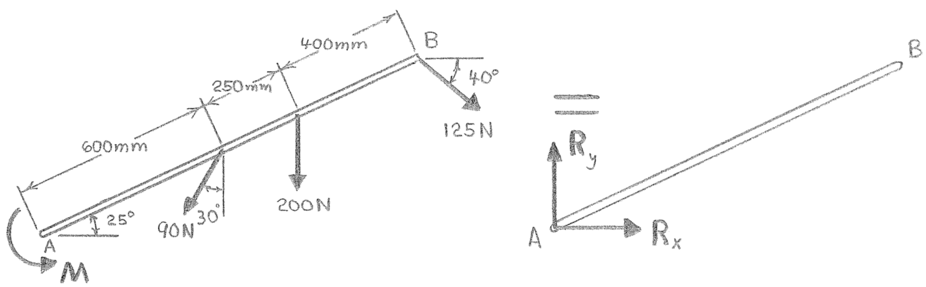
or $a = 1.493$ m ◀

PROBLEM 3.105



Gear *C* is rigidly attached to arm *AB*. If the forces and couple shown can be reduced to a single equivalent force at *A*, determine the equivalent force and the magnitude of the couple **M**.

SOLUTION



For equivalence

$$\Sigma F_x: \quad -(90\text{ N})\sin 30° + (125\text{ N})\cos 40° = R_x$$

$$\text{or } R_x = 50.756\text{ N}$$

$$\Sigma F_y: \quad -(90\text{ N})\cos 30° - 200\text{ N} - (125\text{ N})\sin 40° = R_y$$

$$\text{or } R_y = -358.29\text{ N}$$

Then

$$R = \sqrt{(50.756)^2 + (-358.29)^2} = 361.87\text{ N}$$

and

$$\tan\theta = \frac{R_y}{R_x} = \frac{-358.29}{50.756} = -7.0591 \qquad \therefore \quad \theta = -81.937°$$

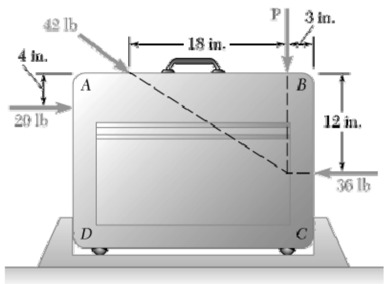
$$\text{or } \mathbf{R} = 362\text{ N} \; 81.9° \blacktriangleleft$$

Also

$$\Sigma M_A: \quad M - [(90\text{ N})\sin 35°](0.6\text{ m}) - [(200\text{ N})\cos 25°](0.85\text{ m}) - [(125\text{ N})\sin 65°](1.25\text{ m}) = 0$$

$$\therefore \quad M = 326.66\text{ N}\cdot\text{m}$$

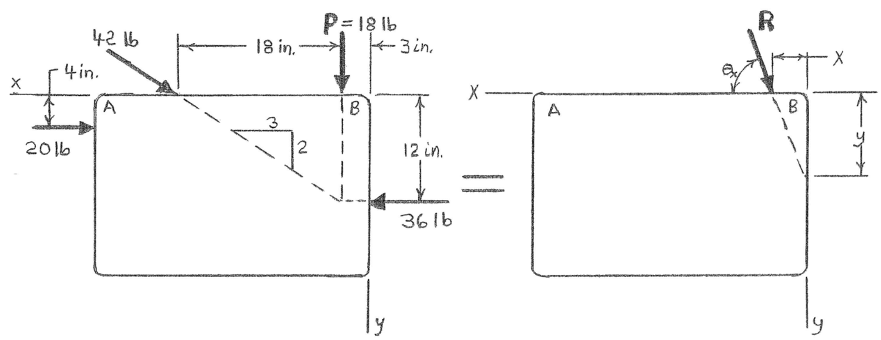
$$\text{or } M = 327\text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.106

To test the strength of a 25×20-in. suitcase, forces are applied as shown. If $P = 18$ lb, (*a*) determine the resultant of the applied forces, (*b*) locate the two points where the line of action of the resultant intersects the edge of the suitcase.

SOLUTION



(*a*) $P = 18$ lb

Have
$$\Sigma \mathbf{F}: \quad -(20 \text{ lb})\mathbf{i} + \frac{42 \text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j}) + (18 \text{ lb})\mathbf{j} + (36 \text{ lb})\mathbf{i} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$\therefore\; -(18.9461 \text{ lb})\mathbf{i} + (41.297 \text{ lb})\mathbf{j} = R_x\mathbf{i} + R_y\mathbf{j}$$

or
$$\mathbf{R} = -(18.95 \text{ lb})\mathbf{i} + (41.3 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(18.9461)^2 + (41.297)^2} = 45.436 \text{ lb}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{41.297}{-18.9461}\right) = -65.355°$$

or $\mathbf{R} = 45.4 \text{ lb} \nwarrow 65.4°$ ◀

(*b*) Have
$$\Sigma \mathbf{M}_B = \mathbf{M}_B$$

$$\mathbf{M}_B = (4 \text{ in.})\mathbf{j} \times (-20 \text{ lb})\mathbf{i} + (21 \text{ in.})\mathbf{i} \times \left[\frac{42 \text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j})\right] + (12 \text{ in.})\mathbf{j} \times (36 \text{ lb})\mathbf{i} + (3 \text{ in.})\mathbf{i} \times (18 \text{ lb})\mathbf{j}$$

$$\therefore\; \mathbf{M}_B = (191.246 \text{ lb} \cdot \text{in.})\mathbf{k}$$

PROBLEM 3.106 CONTINUED

Since $$\mathbf{M}_B = \mathbf{r}_B \times \mathbf{R}$$

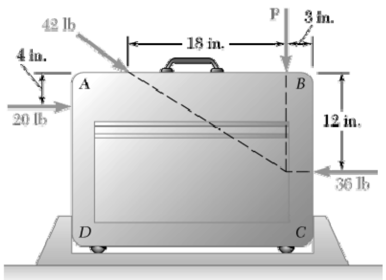
$$\therefore\ (191.246\text{ lb}\cdot\text{in.})\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ -18.9461 & 41.297 & 0 \end{vmatrix} = (41.297x + 18.9461y)\mathbf{k}$$

For $$y = 0, \qquad x = \frac{191.246}{41.297} = 4.6310\text{ in.}$$

or $x = 4.63$ in. ◀

For $$x = 0, \qquad y = \frac{191.246}{18.9461} = 10.0942\text{ in.}$$

or $y = 10.09$ in. ◀

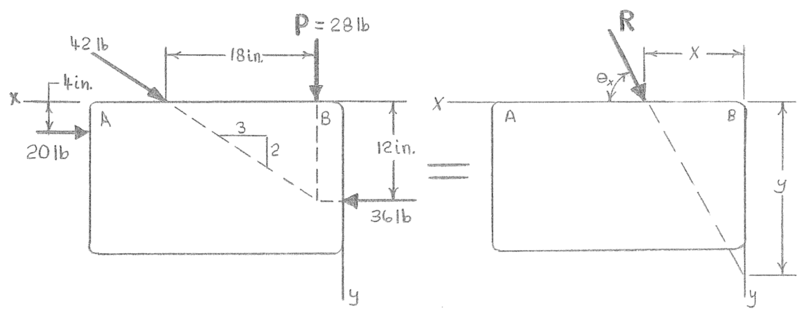


PROBLEM 3.107

Solve Problem 3.106 assuming that $P = 28$ lb.

Problem 3.106: To test the strength of a 25×20-in. suitcase, forces are applied as shown. If $P = 18$ lb, (*a*) determine the resultant of the applied forces, (*b*) locate the two points where the line of action of the resultant intersects the edge of the suitcase.

SOLUTION



(*a*) $P = 28$ lb

Have
$$\Sigma\mathbf{F}: \quad -(20\text{ lb})\mathbf{i} + \frac{42}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j}) + (28\text{ lb})\mathbf{j} + (36\text{ lb})\mathbf{i} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$\therefore\ -(18.9461\text{ lb})\mathbf{i} + (51.297\text{ lb})\mathbf{j} = R_x\mathbf{i} + R_y\mathbf{j}$$

or
$$\mathbf{R} = -(18.95\text{ lb})\mathbf{i} + (51.3\text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(18.9461)^2 + (51.297)^2} = 54.684\text{ lb}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{51.297}{-18.9461}\right) = -69.729°$$

or $\mathbf{R} = 54.7$ lb ⦪ 69.7° ◀

(*b*) Have
$$\Sigma\mathbf{M}_B = \mathbf{M}_B$$

$$\mathbf{M}_B = (4\text{ in.})\mathbf{j} \times (-20\text{ lb})\mathbf{i} + (21\text{ in.})\mathbf{i} \times \left[\frac{42\text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j})\right] + (12\text{ in.})\mathbf{j} \times (36\text{ lb})\mathbf{i} + (3\text{ in.})\mathbf{i} \times (28\text{ lb})\mathbf{j}$$

$$\therefore\quad \mathbf{M}_B = (221.246\text{ lb}\cdot\text{in.})\mathbf{k}$$

PROBLEM 3.107 CONTINUED

Since

$$\mathbf{M}_B = \mathbf{r}_B \times \mathbf{R}$$

$$\therefore \left(221.246 \text{ lb}\cdot\text{in.}\right)\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ -18.9461 & 51.297 & 0 \end{vmatrix} = \left(51.297x + 18.9461y\right)\mathbf{k}$$

For $\quad y = 0, \qquad x = \dfrac{221.246}{51.297} = 4.3130 \text{ in.}$ $\qquad$ or $x = 4.31$ in. ◀

For $\quad x = 0, \qquad y = \dfrac{221.246}{18.9461} = 11.6776 \text{ in.}$ $\qquad$ or $y = 11.68$ in. ◀