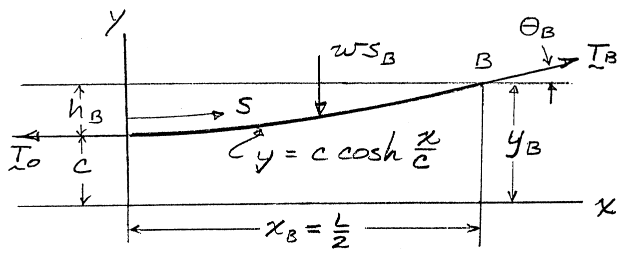
PROBLEM 7.146

A cable of weight w per unit length is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m .

SOLUTION



(a)

$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\quad \min T_{\max}, \quad \frac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$\frac{h}{L} = 0.338$ ◀

(b)

$$T_0 = wc \qquad T_{\max} = wc\cosh\frac{L}{2c} \qquad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

But $\quad T_0 = T_{\max}\cos\theta_B \qquad \frac{T_{\max}}{T_0} = \sec\theta_B$

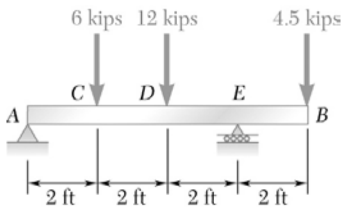
So $\quad \theta_B = \sec^{-1}\left(\frac{y_B}{c}\right) = \sec^{-1}(1.8102)$

$$= 56.46°$$

$\theta_B = 56.5°$ ◀

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$T_{\max} = 0.755wL$ ◀

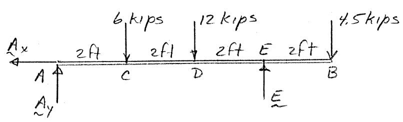


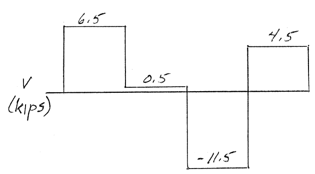
PROBLEM 7.147

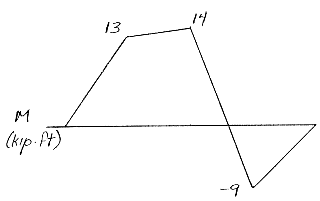
For the beam and loading shown, (*a*) draw the shear and bending moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:







(*a*)

$$\circlearrowleft \Sigma M_A = 0: (6\text{ ft})E - (8\text{ ft})(4.5\text{ kips}) - (4\text{ ft})(12\text{ kips}) - (2\text{ ft})(6\text{ kips}) = 0$$

$$\mathbf{E} = 16\text{ kips}\uparrow$$

$$\circlearrowleft \Sigma M_E = 0: -(6\text{ ft})A_y + (4\text{ ft})(6\text{ kips}) + (2\text{ ft})(12\text{ kips}) - (2\text{ ft})(4.5\text{ kips}) = 0$$

$$\mathbf{A}_y = 6.5\text{ kips}\uparrow$$

Shear Diag: *V* is piece wise constant with discontinuities equal to the forces at *A*, *C*, *D*, *E*, *B*

Moment Diag: *M* is piecewise linear with slope changes at *C*, *D*, *E*

$$M_A = 0$$

$$M_C = (6.5\text{ kips})(2\text{ ft}) = 13\text{ kip}\cdot\text{ft}$$

$$M_C = 13\text{ kip}\cdot\text{ft} + (0.5\text{ kips})(2\text{ ft}) = 14\text{ kip}\cdot\text{ft}$$

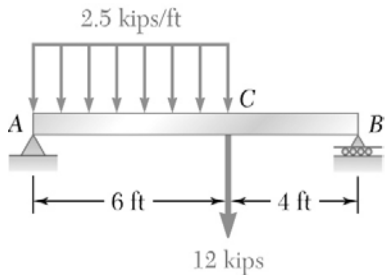
$$M_D = 14\text{ kip}\cdot\text{ft} - (11.5\text{ kips})(2\text{ ft}) = -9\text{ kip}\cdot\text{ft}$$

$$M_B = -9\text{ kip}\cdot\text{ft} + (4.5\text{ kips})(2\text{ ft}) = 0$$

(*b*) $|V|_{\text{max}} = 11.50$ kips on *DE* ◀

$|M|_{\text{max}} = 14.00$ kip·ft at *D* ◀

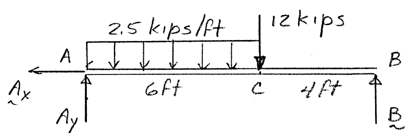
PROBLEM 7.148

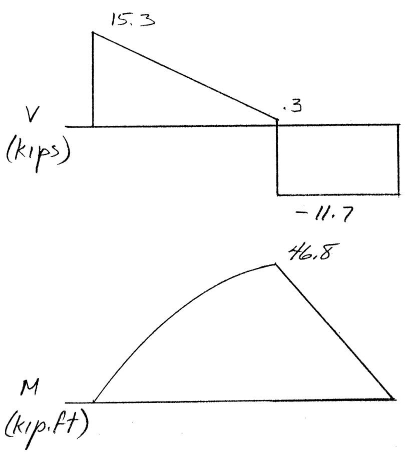


For the beam and loading shown, (*a*) draw the shear and bending moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:





(*a*) $\circlearrowleft \Sigma M_B = 0\text{: } (4 \text{ ft})(12 \text{ kips}) + (7 \text{ ft})(2.5 \text{ kips/ft})(6 \text{ ft})$

$$- (10 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 15.3 \text{ kips} \uparrow$$

Shear Diag: $V_A = A_y = 15.3$ kips, then V is linear

$$\left(\frac{dV}{dx} = -2.5 \text{ kips/ft}\right) \text{to } C.$$

$$V_C = 15.3 \text{ kips} - (2.5 \text{ kips/ft})(6 \text{ ft}) = 0.3 \text{ kips}$$

At C, V decreases by 12 kips to -11.7 kips and is constant to B.

Moment Diag: $M_A = 0$ and M is parabolic

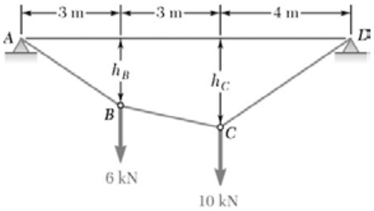
$$\left(\frac{dM}{dx} \text{ decreasing with } V\right) \text{ to } C$$

$$M_C = \frac{1}{2}(15.3 \text{ kips} + 0.3 \text{ kip})(6 \text{ ft}) = 46.8 \text{ kip}\cdot\text{ft}$$

$$M_B = 46.8 \text{ kip}\cdot\text{ft} - (11.7 \text{ kips})(4 \text{ ft}) = 0$$

(*b*) $|V|_{max} = 15.3 \text{ kips}$ ◀

$|M|_{max} = 46.8 \text{ kip}\cdot\text{ft}$ ◀

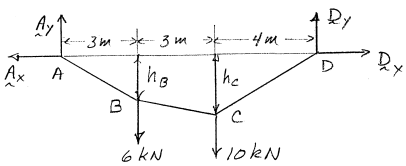


PROBLEM 7.149

Two loads are suspended as shown from the cable *ABCD*. Knowing that $h_B = 1.8$ m, determine (*a*) the distance h_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

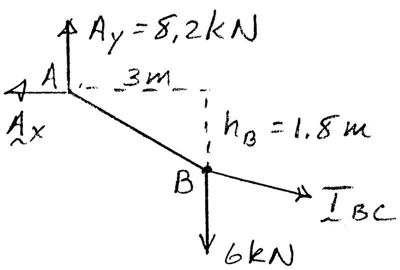
$$\Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN} \uparrow$$

FBD AB:

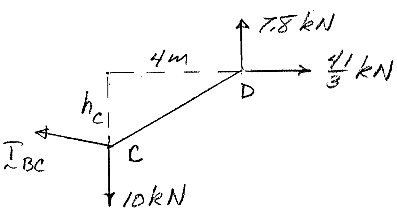


$$\Sigma M_B = 0: (1.8\text{ m})A_x - (3\text{ m})(8.2\text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3}\text{ kN} \leftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:



$$\Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C\left(\frac{41}{3}\text{ kN}\right) = 0$$

$$h_C = 2.283\text{ m}$$

(*a*) $\qquad h_C = 2.28\text{ m}$ ◀

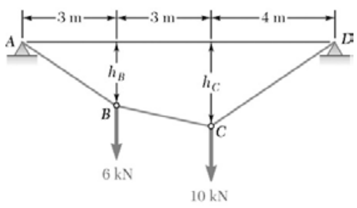
(*b*) $\qquad D_x = 13.67\text{ kN} \rightarrow$ ◀

$D_y = 7.80\text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3}\text{ kN}\right)^2 + (8.2\text{ kN})^2}$$

(*c*) $\qquad T_{\max} = 15.94\text{ kN}$ ◀

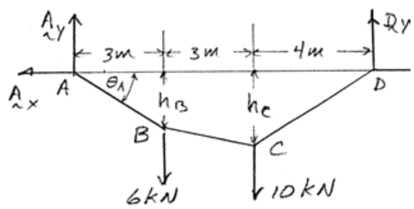


PROBLEM 7.150

Knowing that the maximum tension in cable *ABCD* is 15 kN, determine (*a*) the distance h_B, (*b*) the distance h_C.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN}\uparrow$$

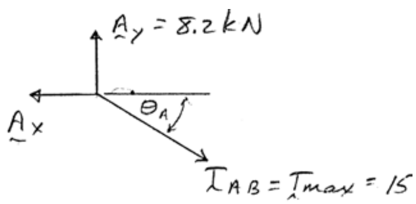
$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN}\uparrow$$

Since $\qquad A_x = D_x \qquad$ and $\qquad A_y > D_y, \qquad T_{max} = T_{AB}$

$$\uparrow \Sigma F_y = 0: 8.2\text{ kN} - (15\text{ kN})\sin\theta_A = 0$$

FBD pt A:

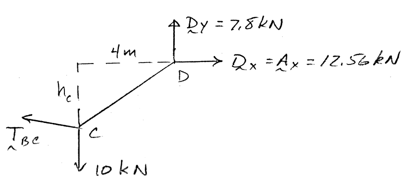


$$\theta_A = \sin^{-1}\frac{8.2\text{ kN}}{15\text{ kN}} = 33.139°$$

$$\rightarrow \Sigma F_x = 0: -A_x + (15\text{ kN})\cos\theta_A = 0$$

$$A_x = (15\text{ kN})\cos(33.139°) = 12.56\text{ kN}$$

FBD CD:

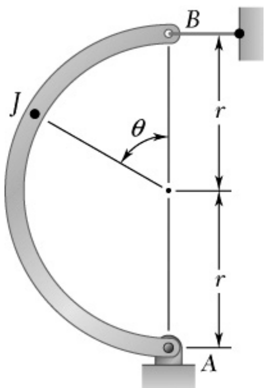


From FBD cable: $\qquad h_B = (3\text{ m})\tan\theta_A = (3\text{ m})\tan(33.139°)$

(*a*) $\qquad h_B = 1.959\text{ m}$ ◀

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C(12.56\text{ kN}) = 0$$

(*b*) $\qquad h_C = 2.48\text{ m}$ ◀

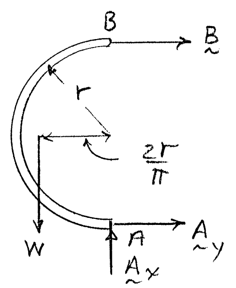


PROBLEM 7.151

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 60°$.

SOLUTION

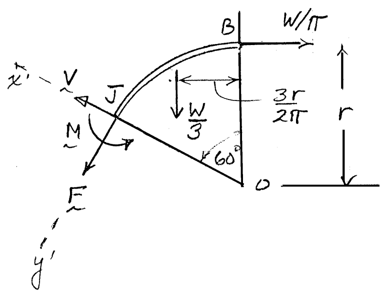
FBD Rod:



$$\Sigma M_A = 0: \frac{2r}{\pi}W - 2rB = 0$$

$$\mathbf{B} = \frac{W}{\pi} \longrightarrow$$

FBD BJ:



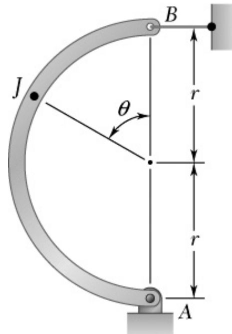
$$\Sigma F_{y'} = 0: F + \frac{W}{3}\sin 60° - \frac{W}{\pi}\cos 60° = 0$$

$$F = -0.12952W$$

$$\Sigma M_0 = 0: r\left(F - \frac{W}{\pi}\right) + \frac{3r}{2\pi}\left(\frac{W}{3}\right) + M = 0$$

$$M = Wr\left(0.12952 + \frac{1}{\pi} - \frac{1}{2\pi}\right) = 0.28868Wr$$

On BJ $\qquad \mathbf{M}_J = 0.289Wr$ ◀

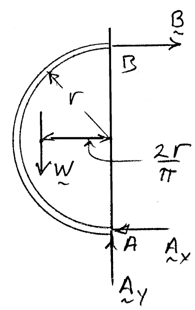


PROBLEM 7.152

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 150°$.

SOLUTION

FBD rod:

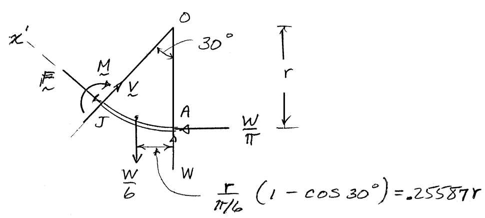


$$\uparrow \Sigma F_y = 0: A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

$$\Sigma M_B = 0: \frac{2r}{\pi}W - 2rA_x = 0$$

$$\mathbf{A}_x = \frac{W}{\pi} \longleftarrow$$

FBD AJ:



$$\nwarrow \Sigma F_{x'} = 0: \frac{W}{\pi}\cos 30° + \frac{5W}{6}\sin 30° - F = 0 \quad F = 0.69233W \searrow$$

$$\circlearrowleft \Sigma M_0 = 0: 0.25587r\left(\frac{W}{6}\right) + r\left(F - \frac{W}{\pi}\right) - M = 0$$

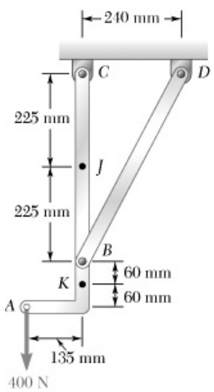
$$M = Wr\left[\frac{0.25587}{6} + 0.69233 - \frac{1}{\pi}\right]$$

$$M = Wr(0.4166)$$

On AJ $\qquad \mathbf{M} = 0.417Wr \circlearrowright$ ◀

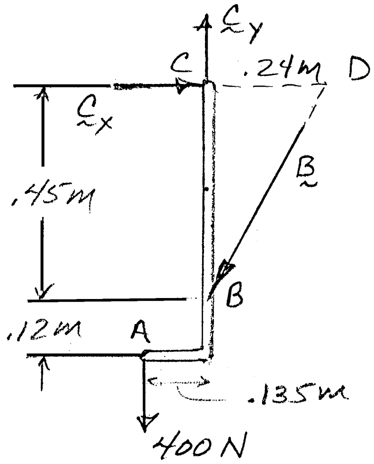
PROBLEM 7.153

Determine the internal forces at point J of the structure shown.



SOLUTION

FBD ABC:



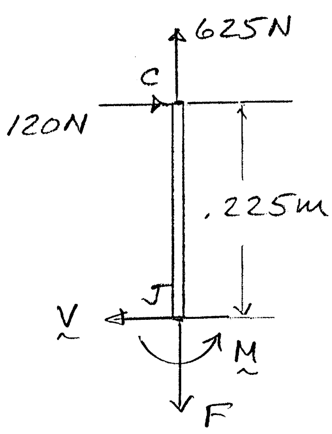
$$\circlearrowleft \Sigma M_D = 0: (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 625 \text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0$$

$$\mathbf{C}_x = 120 \text{ N} \longrightarrow$$

FBD CJ:



$$\uparrow \Sigma F_y = 0: 625 \text{ N} - F = 0$$

$$\mathbf{F} = 625 \text{ N} \downarrow \blacktriangleleft$$

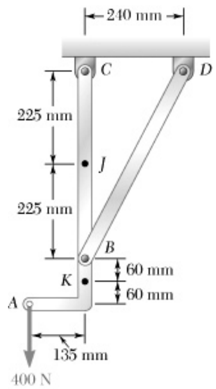
$$\longrightarrow \Sigma F_x = 0: 120 \text{ N} - V = 0$$

$$\mathbf{V} = 120.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (0.225 \text{ m})(120 \text{ N}) = 0$$

$$\mathbf{M} = 27.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

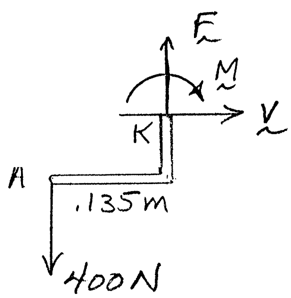
PROBLEM 7.154



Determine the internal forces at point K of the structure shown.

SOLUTION

FBD AK:



$$\rightarrow \Sigma F_x = 0: V = 0$$

$$\mathbf{V} = 0 \blacktriangleleft$$

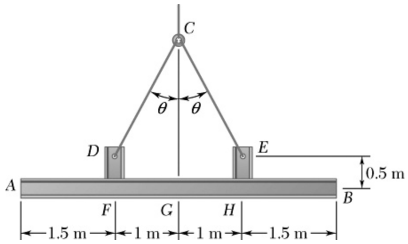
$$\uparrow \Sigma F_y = 0: F - 400 \text{ N} = 0$$

$$\mathbf{F} = 400 \text{ N} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$$\mathbf{M} = 54.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

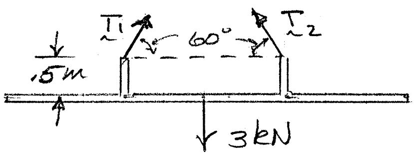
PROBLEM 7.155



Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing the $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

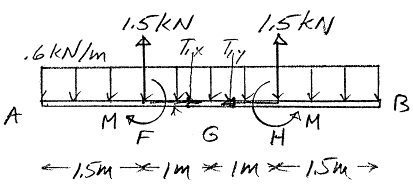
SOLUTION

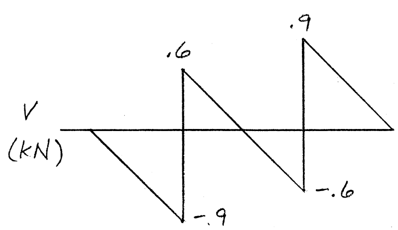
FBD Beam + channels:

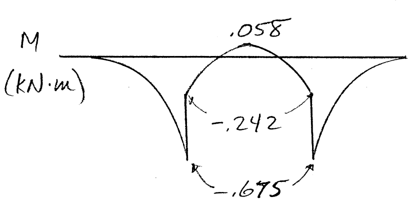


FBD Beam:

With cable force replaced by equivalent force-couple system at *F* and *G*







(*a*) By symmetry: $T_1 = T_2 = T$

$$\uparrow \Sigma F_y = 0:\ 2T\sin 60° - 3\text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}}\text{ kN} \qquad T_{1x} = \frac{3}{2\sqrt{3}} \qquad T_{1y} = \frac{3}{2}\text{ kN}$$

$$M = (0.5\text{ m})\frac{3}{2\sqrt{3}}\text{ kN} = 0.433\text{ kN}\cdot\text{m}$$

Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6\text{ kN/m}\right)\text{ with 1.5 kN}$$

discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6\text{ kN/m})(1.5\text{ m}) = 0.9\text{ kN}$$

V increases by 1.5 kN to $+\ 0.6$ kN at F^+

$$V_G = 0.6\text{ kN} - (0.6\text{ kN/m})(1\text{ m}) = 0$$

Finish by invoking symmetry

Moment Diagram: *M* is piecewise parabolic

$\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ with discontinuities of .433 kN at *F* and *H*.

$$M_{F^-} = -\frac{1}{2}(0.9\text{ kN})(1.5\text{ m}) = -0.675\text{ kN}\cdot\text{m}$$

M increases by $0.433\text{ kN}\cdot\text{m}$ to $-0.242\text{ kN}\cdot\text{m}$ at F^+

$$M_G = -0.242\text{ kN}\cdot\text{m} + \frac{1}{2}(0.6\text{ kN})(1\text{ m}) = 0.058\text{ kN}\cdot\text{m}$$

Finish by invoking symmetry

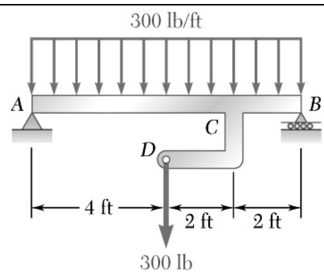
(*b*) $|V|_{\max} = 900\text{ N}$ ◀

at F^- and G^+

$|M|_{\max} = 675\text{ N}\cdot\text{m}$ ◀

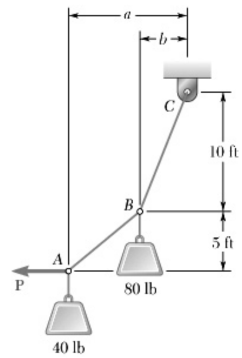
at *F* and *G*

PROBLEM 7.156



(*a*) Draw the shear and bending moment diagrams for beam *AB*,
(*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

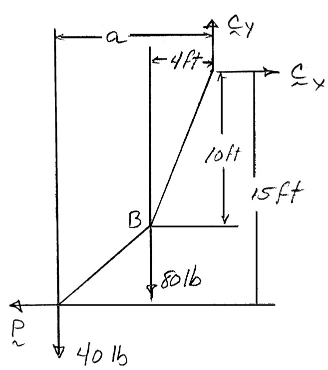
PROBLEM 7.157



Cable *ABC* supports two loads as shown. Knowing that $b = 4$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

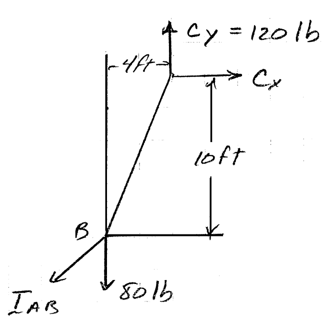
FBD ABC:



$$\uparrow \Sigma F_y = 0: -40 \text{ lb} - 80 \text{ lb} + C_y = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: (4 \text{ ft})(120 \text{ lb}) - (10 \text{ ft})C_x = 0$$

$$\mathbf{C}_x = 48 \text{ lb} \longrightarrow$$

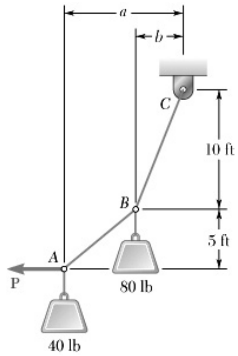
From ABC: $\longrightarrow \Sigma F_x = 0: -P + C_x = 0$

$$P = C_x = 48 \text{ lb}$$

(*a*) $P = 48.0$ lb ◀

$$\circlearrowleft \Sigma M_C = 0: (4 \text{ ft})(80 \text{ lb}) + a(40 \text{ lb}) - (15 \text{ ft})(48 \text{ lb}) = 0$$

(*b*) $a = 10.00$ ft ◀

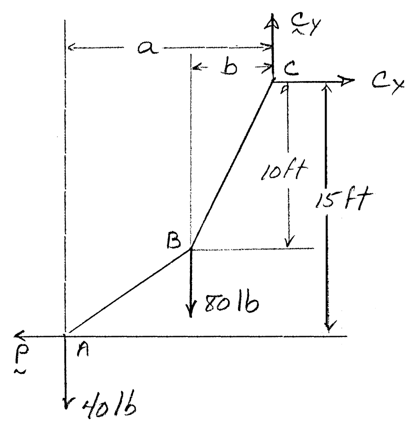


PROBLEM 7.158

Cable *ABC* supports two loads as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 60 lb is applied at *A*.

SOLUTION

FBD ABC:

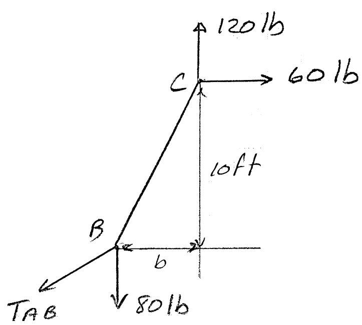


$$\xrightarrow{} \Sigma F_x = 0: C_x - P = 0 \qquad C_x = 60 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: C_y - 40 \text{ lb} - 80 \text{ lb} = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

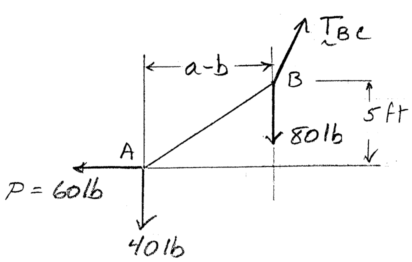
FBD BC:



$$\circlearrowleft \Sigma M_B = 0: b(120 \text{ lb}) - (10 \text{ ft})(60 \text{ lb}) = 0$$

$$b = 5.00 \text{ ft} \blacktriangleleft$$

FBD AB:



$$\Sigma M_B = 0: (a - b)(40 \text{ lb}) - (5 \text{ ft})60 \text{ lb} = 0$$

$$a - b = 7.5 \text{ ft}$$

$$a = b + 7.5 \text{ ft}$$
$$= 5 \text{ ft} + 7.5 \text{ ft}$$

$$a = 12.50 \text{ ft} \blacktriangleleft$$