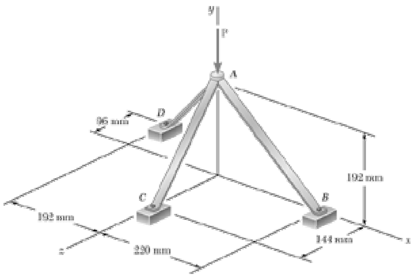
PROBLEM 2.101



The support assembly shown is bolted in place at *B*, *C*, and *D* and supports a downward force **P** at *A*. Knowing that the forces in members *AB*, *AC*, and *AD* are directed along the respective members and that the force in member *AB* is 146 N, determine the magnitude of **P**.

SOLUTION

Note that *AB*, *AC*, and *AD* are in compression.

Have

$$d_{BA} = \sqrt{(-220\text{ mm})^2 + (192\text{ mm})^2 + (0)^2} = 292\text{ mm}$$

$$d_{DA} = \sqrt{(192\text{ mm})^2 + (192\text{ mm})^2 + (96\text{ mm})^2} = 288\text{ mm}$$

$$d_{CA} = \sqrt{(0)^2 + (192\text{ mm})^2 + (-144\text{ mm})^2} = 240\text{ mm}$$

and

$$\mathbf{F}_{BA} = F_{BA}\lambda_{BA} = \frac{146\text{ N}}{292\text{ mm}}\left[(-220\text{ mm})\mathbf{i} + (192\text{ mm})\mathbf{j}\right]$$

$$= -(110\text{ N})\mathbf{i} + (96\text{ N})\mathbf{j}$$

$$\mathbf{F}_{CA} = F_{CA}\lambda_{CA} = \frac{F_{CA}}{240\text{ mm}}\left[(192\text{ mm})\mathbf{j} - (144\text{ mm})\mathbf{k}\right]$$

$$= F_{CA}(0.80\mathbf{j} - 0.60\mathbf{k})$$

$$\mathbf{F}_{DA} = F_{DA}\lambda_{DA} = \frac{F_{DA}}{288\text{ mm}}\left[(192\text{ mm})\mathbf{i} + (192\text{ mm})\mathbf{j} + (96\text{ mm})\mathbf{k}\right]$$

$$= F_{DA}\left[0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}\right]$$

With $\quad \mathbf{P} = -P\mathbf{j}$

At *A*: $\quad \Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{CA} + \mathbf{F}_{DA} + \mathbf{P} = 0$

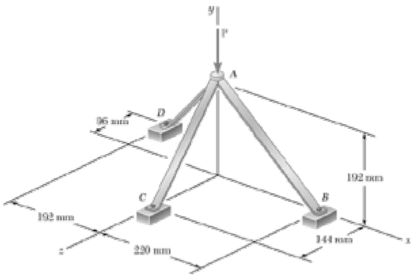
i-component: $\quad -(110\text{ N}) + 0.66667F_{DA} = 0 \quad$ or $\quad F_{DA} = 165\text{ N}$

j-component: $\quad 96\text{ N} + 0.80F_{CA} + 0.66667(165\text{ N}) - P = 0 \qquad (1)$

k-component: $\quad -0.60F_{CA} + 0.33333(165\text{ N}) = 0 \qquad (2)$

Solving (2) for F_{CA} and then using that result in (1), gives $\quad P = 279\text{ N}$ ◀

PROBLEM 2.102



The support assembly shown is bolted in place at *B*, *C*, and *D* and supports a downward force **P** at *A*. Knowing that the forces in members *AB*, *AC*, and *AD* are directed along the respective members and that **P** = 200 N, determine the forces in the members.

SOLUTION

With the results of 2.101:

$$\mathbf{F}_{BA} = F_{BA}\lambda_{BA} = \frac{F_{BA}}{292 \text{ mm}}\left[(-220 \text{ mm})\mathbf{i} + (192 \text{ mm})\mathbf{j}\right]$$

$$= F_{BA}\left[-0.75342\mathbf{i} + 0.65753\mathbf{j}\right]\text{N}$$

$$\mathbf{F}_{CA} = F_{CA}\lambda_{CA} = \frac{F_{CA}}{240 \text{ mm}}\left[(192 \text{ mm})\mathbf{j} - (144 \text{ mm})\mathbf{k}\right]$$

$$= F_{CA}(0.80\mathbf{j} - 0.60\mathbf{k})$$

$$\mathbf{F}_{DA} = F_{DA}\lambda_{DA} = \frac{F_{DA}}{288 \text{ mm}}\left[(192 \text{ mm})\mathbf{i} + (192 \text{ mm})\mathbf{j} + (96 \text{ mm})\mathbf{k}\right]$$

$$= F_{DA}\left[0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}\right]$$

With: $\mathbf{P} = -(200 \text{ N})\mathbf{j}$

At *A*: $\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{CA} + \mathbf{F}_{DA} + \mathbf{P} = 0$

Hence, equating the three (**i**, **j**, **k**) components to 0 gives three equations

i-component: $-0.75342F_{BA} + 0.66667F_{DA} = 0$ (1)

j-component: $0.65735F_{BA} + 0.80F_{CA} + 0.66667F_{DA} - 200 \text{ N} = 0$ (2)

k-component: $-0.60F_{CA} + 0.33333F_{DA} = 0$ (3)

Solving (1), (2), and (3), gives

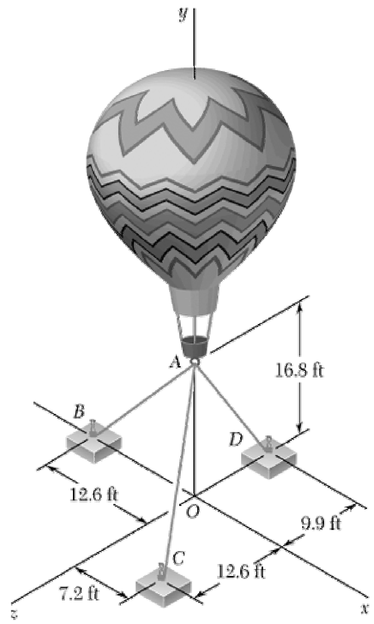
$$F_{BA} = 104.5 \text{ N}, \qquad F_{CA} = 65.6 \text{ N}, \qquad F_{\text{DA}} = 118.1 \text{ N}$$

$F_{BA} = 104.5$ N ◀

$F_{CA} = 65.6$ N ◀

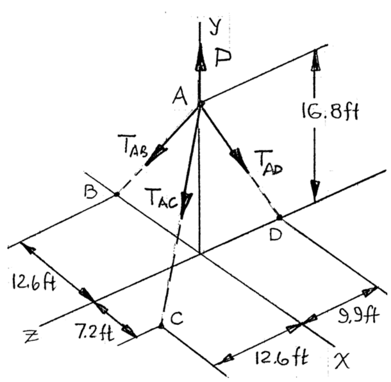
$F_{DA} = 118.1$ N ◀

PROBLEM 2.103



Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AB is 60 lb.

SOLUTION



The forces applied at A are:

$$\mathbf{T}_{AB},\ \mathbf{T}_{AC},\ \mathbf{T}_{AD} \text{ and } \mathbf{P}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(12.6 \text{ ft})\mathbf{i} - (16.8 \text{ ft})\mathbf{j} \qquad AB = 21 \text{ ft}$$

$$\overrightarrow{AC} = (7.2 \text{ ft})\mathbf{i} - (16.8 \text{ ft})\mathbf{j} + (12.6 \text{ ft})\mathbf{k} \qquad AC = 22.2 \text{ ft}$$

$$\overrightarrow{AD} = -(16.8 \text{ ft})\mathbf{j} - (9.9 \text{ ft})\mathbf{k} \qquad AD = 19.5 \text{ ft}$$

and
$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.3242\mathbf{i} - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.8615\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.103 CONTINUED

Equilibrium Condition

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$\left(-0.6T_{AB} + 0.3242T_{AC}\right)\mathbf{i} + \left(-0.8T_{AB} - 0.75676T_{AC} - 0.8615T_{AD} + P\right)\mathbf{j}$$

$$+ \left(0.56757T_{AC} - 0.50769T_{AD}\right)\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.3242T_{AC} = 0 \qquad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.8615T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

Setting $T_{AB} = 60$ lb in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 111 \text{ lb}$$

$$T_{AD} = 124.2 \text{ lb}$$

$\mathbf{P} = 239 \text{ lb} \uparrow$ ◀