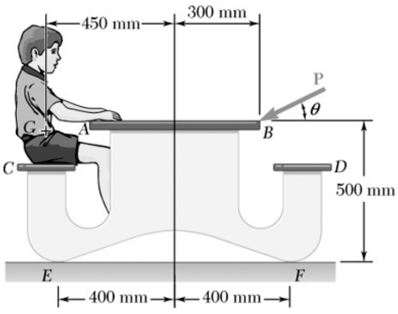
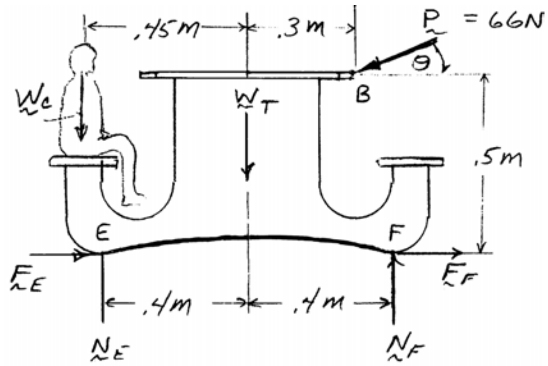
PROBLEM 8.29



A child having a mass of 18 kg is seated halfway between the ends of a small, 16-kg table as shown. The coefficient of static friction is 0.20 between the ends of the table and the floor. If a second child pushes on edge B of the table top at a point directly opposite to the first child with a force $\mathbf{P}$ lying in a vertical plane parallel to the ends of the table and having a magnitude of 66 N, determine the range of values of θ for which the table will (*a*) tip, (*b*) slide.

SOLUTION

FBD table + child:



$$W_C = 18\text{ kg}\left(9.81\text{ m/s}^2\right) = 176.58\text{ N}$$

$$W_T = 16\text{ kg}\left(9.81\text{ m/s}^2\right) = 156.96\text{ N}$$

(*a*) Impending tipping about E, $N_F = F_F = 0$, and

$$\circlearrowleft \Sigma M_E = 0: \quad (0.05\text{ m})(176.58\text{ N}) - (0.4\text{ m})(156.96\text{ N}) + (0.5\text{ m})P\cos\theta - (0.7\text{ m})P\sin\theta = 0$$

$$33\cos\theta - 46.2\sin\theta = 53.955$$

Solving numerically $\qquad \theta = -36.3° \quad$ and $\quad \theta = -72.6°$

Therefore $\qquad -72.6° \le \theta \le -36.3°$ ◀

Impending tipping about F is not possible

(*b*) For impending slip: $\qquad F_E = \mu_s N_E = 0.2N_E \qquad F_F = \mu_s N_F = 0.2N_F$

$$\rightarrow \Sigma F_x = 0: \quad F_E + F_F - P\cos\theta = 0 \quad \text{or} \quad 0.2\left(N_E + N_F\right) = (66\text{ N})\cos\theta$$

$$\uparrow \Sigma F_y = 0: \quad N_E + N_F - 176.58\text{ N} - 156.96\text{ N} - P\sin\theta = 0$$

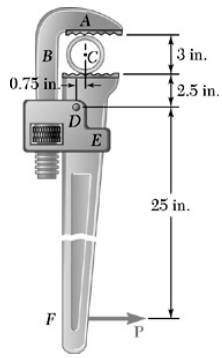
$$N_E + N_F = \left(66\sin\theta + 333.54\right)\text{ N}$$

So $\qquad 330\cos\theta = 66\sin\theta + 333.54$

Solving numerically, $\qquad \theta = -3.66° \quad$ and $\quad \theta = -18.96°$

Therefore, $\qquad -18.96° \le \theta \le -3.66°$ ◀

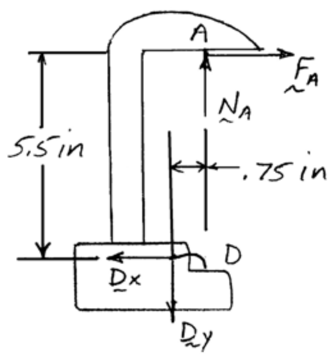
PROBLEM 8.30



A pipe of diameter 3 in. is gripped by the stillson wrench shown. Portions AB and DE of the wrench are rigidly attached to each other, and portion CF is connected by a pin at D. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at A and C.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75\text{ in.})N_A - (5.5\text{ in.})F_A = 0$$

Impending motion: $\qquad F_A = \mu_A N_A$

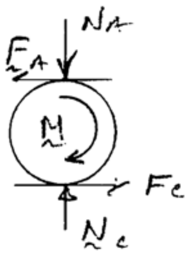
Then $\qquad 0.75 - 5.5\mu_A = 0$

or $\qquad \mu_A = 0.13636$

$$\mu_A = 0.1364 \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \qquad D_x = F_A$$

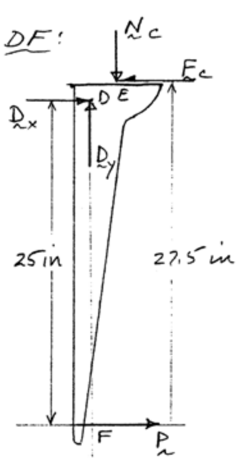
Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5\text{ in.})F_C - (0.75\text{ in.})N_C - (25\text{ in.})D_x = 0$$

Impending motion: $\qquad F_C = \mu_C N_C$

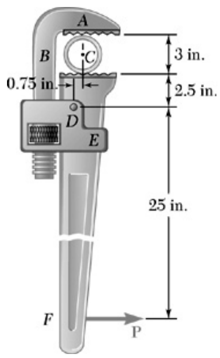
Then $\qquad 27.5\mu_C - 0.75 = 25\dfrac{F_A}{N_C}$

But $\qquad N_C = N_A \quad$ and $\quad \dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $\qquad 27.5\mu_C = 0.75 + 25(0.13636)$

$$\mu_C = 0.1512 \blacktriangleleft$$

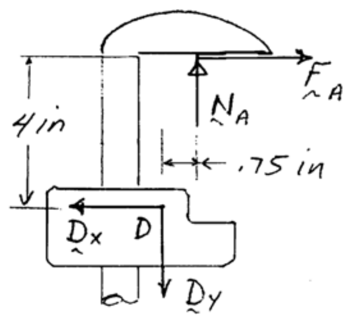
PROBLEM 8.31



Solve Problem 8.30 assuming that the diameter of the pipe is 1.5 in.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75 \text{ in.}) N_A - (4 \text{ in.}) F_A = 0$$

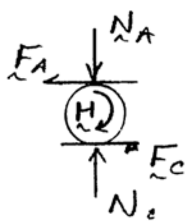
Impending motion: $\qquad F_A = \mu_A N_A$

Then $\qquad 0.75 \text{ in.} - (4 \text{ in.})\mu_A = 0 \qquad\qquad \mu_A = 0.1875$ ◀

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0$$

so that $\qquad D_x = F_A = 0.1875 N_A$

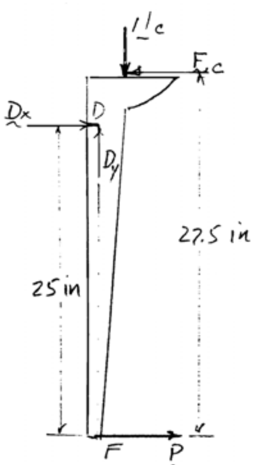
FBD Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



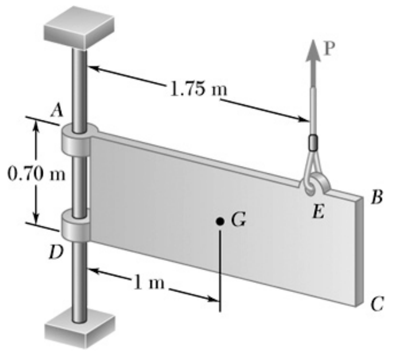
$$\circlearrowleft \Sigma M_F = 0: \quad (27.5 \text{ in.}) F_C - (0.75 \text{ in.}) N_C - (25 \text{ in.}) D_x = 0$$

Impending motion: $\qquad F_C = \mu_C N_C$

$$27.5\mu_C - 0.75 = 25(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\qquad \dfrac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

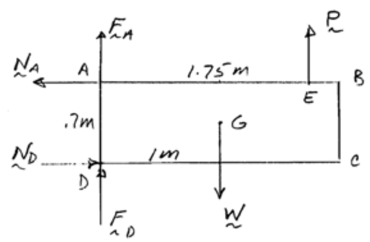


PROBLEM 8.32

The 25-kg plate $ABCD$ is attached at A and D to collars which can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (a) $P = 0$, (b) $P = 80$ N.

SOLUTION

FBD plate:



$$W = 25\text{ kg}(9.81\text{ N/kg})$$

$$= 245.25\text{ N}$$

(a) $P = 0$; assume equilibrium:

$$\Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W = 0 \qquad N_D = \frac{10W}{7}$$

$$\Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_A = N_D = \frac{10W}{7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

So $$(F_A + F_D)_{\max} = \mu_s(N_A + N_D) = \frac{20\mu_s W}{7} = 1.143W$$

$$\Sigma F_y = 0: \quad F_A + F_D - W = 0$$

$$\therefore \quad F_A + F_D = W < (F_A + F_D)_{\max} \qquad \text{OK.}$$

Plate is in equilibrium ◀

(b) $P = 80$ N; assume equilibrium:

$$\Sigma M_A = 0: \quad (1.75\text{ m})P + (0.7\text{ m})N_D - (1\text{ m})W = 0$$

or $$N_D = \frac{W - 1.75P}{0.7}$$

$$\Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_D = N_A = \frac{W - 1.75P}{0.7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_B)_{\max} = \mu_s N_B$$

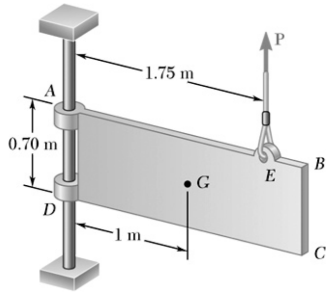
So $$(F_A + F_B)_{\max} = 0.4\frac{2W - 3.5P}{0.7} = 120.29\text{ N}$$

$$\Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$F_A + F_D = W - P = 165.25\text{ N}$$

$$(F_A + F_D)_{\text{equil}} > (F_A + F_D)_{\max}$$

Impossible, so plate slides downward ◀

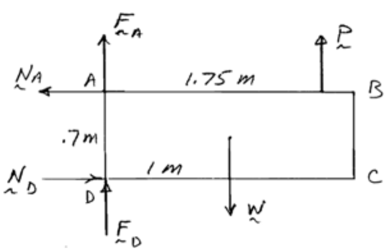


PROBLEM 8.33

In Problem 8.32, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

SOLUTION

FBD plate:



$$W = (25 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 245.25 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7 \text{ m})N_D - (1 \text{ m})W + (1.75 \text{ m})P = 0$$

$$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \quad \text{so that} \quad N_A = N_D = \frac{W - 1.75P}{0.7}$$

Note: N_A and N_D will be > 0 if $P < \frac{4}{7}W$ and < 0 if $P > \frac{4}{7}W$.

Impending motion downward: F_A and F_B are both > 0, so

$$F_A = \mu_s |N_A| = \frac{0.4}{0.7}|W - 1.75P| = \left|\frac{4}{7}W - P\right|$$

$$F_D = \mu_S |N_D| = \left|\frac{4}{7}W - P\right|$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$2\left|\frac{4}{7}W - P\right| - W + P = 0$$

For $P < \frac{4}{7}W$; $\qquad P = \frac{W}{7} = 35.04 \text{ N}$

For $P > \frac{4}{7}W$; $\qquad P = \frac{5W}{7} = 175.2 \text{ N}$

Downward motion for $35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

Alternative Solution

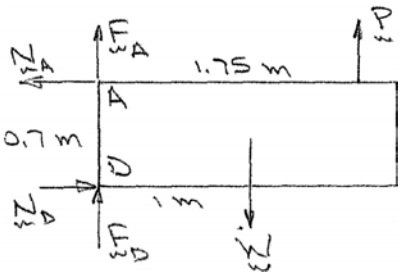
We first observe that for smaller values of the magnitude of **P** that (Case 1) the inner left-hand and right-hand surfaces of collars A and D, respectively, will contact the rod, whereas for larger values of the magnitude of **P** that (Case 2) the inner right-hand and left-hand surfaces of collars A and D, respectively, will contact the rod.

First note: $\qquad W = (25 \text{ kg})(9.81 \text{ m/s}^2)$

$$= 245.25 \text{ N}$$

PROBLEM 8.33 CONTINUED

Case 1



$$\circlearrowleft \Sigma M_D = 0: \quad (0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\text{ N}$$

$$\xrightarrow{} \Sigma F_x = 0: \quad -N_A + N_D = 0$$

or $$N_D = N_A$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25\text{ N} = 0$$

or $$F_A + F_D = (245.25 - P)\text{ N}$$

Now $$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

so that $$(F_A)_{\max} + (F_D)_{\max} = \mu_s(N_A + N_D)$$

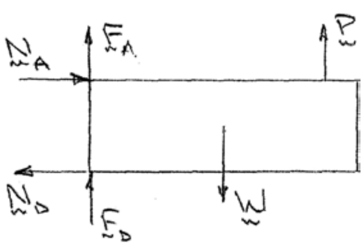
$$= 2(0.4)\left[\frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\right]$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting $$245.25 - P > \frac{8}{7}\left(245.25 - \frac{7}{4}P\right)$$

or $$P > 35.0\text{ N}$$

Case 2



From Case 1: $$N_D = N_A$$

$$F_A + F_D = (245.25 - P)\text{ N}$$

$$(F_A)_{\max} + (F_D)_{\max} = 2\mu_s N_A$$

$$\circlearrowleft \Sigma M_D = 0: \quad -(0.7\text{ m})N_A - (1\text{ m})(245.25\text{ N}) + (1.75\text{ m})P = 0$$

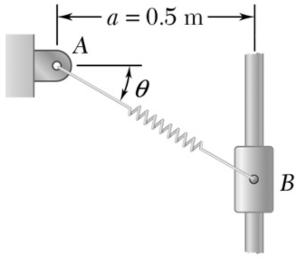
or $$N_A = \frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\text{ N}$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting: $$245.25 - P > 2(0.4)\left[\frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\right]$$

or $$P < 175.2\text{ N}$$

Therefore, have downward motion for $35.0\text{ N} < P < 175.2\text{ N}$ ◀



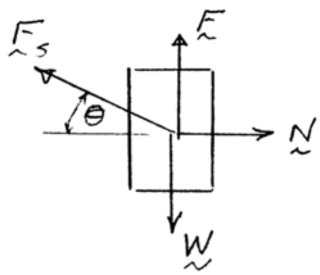
PROBLEM 8.34

A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

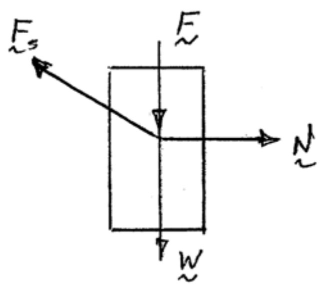
SOLUTION

FBD collar:

Impending motion down:



Impending motion up:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = kx = k\left(\frac{a}{\cos\theta} - a\right) = (1.5 \text{ kN/m})(0.5 \text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75 \text{ kN})\left(\frac{1}{\cos\theta} - 1\right)$$

$\longrightarrow \Sigma F_x = 0: \quad N - F_s\cos\theta = 0$

$$N = F_s\cos\theta = (0.75 \text{ kN})(1 - \cos\theta)$$

Impending slip: $\quad F = \mu_s N = (0.4)(0.75 \text{ kN})(1 - \cos\theta)$

$$= (0.3 \text{ kN})(1 - \cos\theta)$$

$+$ down, $-$ up

$\uparrow \Sigma F_y = 0: \quad F_s\sin\theta \pm F - W = 0$

$$(0.75 \text{ kN})(\tan\theta - \sin\theta) \pm (0.3 \text{ kN})(1 - \cos\theta) - W = 0$$

or $\quad W = (0.3 \text{ kN})[2.5(\tan\theta - \sin\theta) \pm (1 - \cos\theta)]$

(*a*) $\theta = 20^\circ$: $\quad W_{\text{up}} = -0.00163 \text{ kN} \quad (\text{impossible})$

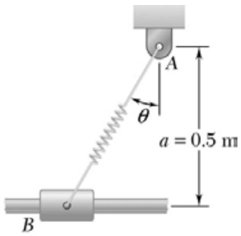
$$W_{\text{down}} = 0.03455 \text{ kN} \quad (\text{OK})$$

Equilibrium if $0 \le W \le 34.6$ N ◀

(*b*) $\theta = 30^\circ$: $\quad W_{\text{up}} = 0.01782 \text{ kN} \quad (\text{OK})$

$$W_{\text{down}} = 0.0982 \text{ kN} \quad (\text{OK})$$

Equilibrium if $17.82 \text{ N} \le W \le 98.2$ N ◀

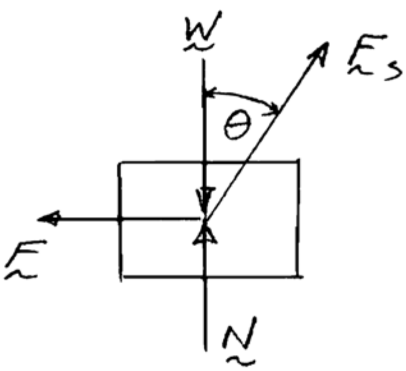


PROBLEM 8.35

A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

SOLUTION

FBD collar:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right) = (750\text{ N})(\sec\theta - 1)$$

$$\uparrow \Sigma F_y = 0: \quad F_s\cos\theta - W + N = 0$$

or $$W = N + (750\text{ N})(1 - \cos\theta)$$

Impending slip:

$$F = \mu_s |N| \quad (F \text{ must be } +, \text{ but } N \text{ may be positive or negative})$$

$$\longrightarrow \Sigma F_x = 0: \quad F_s \sin\theta - F = 0$$

or $$F = F_s \sin\theta = (750\text{ N})(\tan\theta - \sin\theta)$$

(*a*) $\theta = 20^\circ$: $\quad F = (750\text{ N})(\tan 20^\circ - \sin 20^\circ) = 16.4626\text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{16.4626\text{ N}}{0.4} = 41.156\text{ N}$

(Note: for $|N| < 41.156$ N, motion will occur, equilibrium for $|N| > 41.156$)

But $\quad W = N + (750\text{ N})(1 - \cos 20^\circ) = N + 45.231\text{ N}$

So equilibrium for $W \le 4.07$ N and $W \ge 86.4$ N ◀

(*b*) $\theta = 30^\circ$: $\quad F = (750\text{ N})(\tan 30^\circ - \sin 30^\circ) = 58.013\text{ N}$

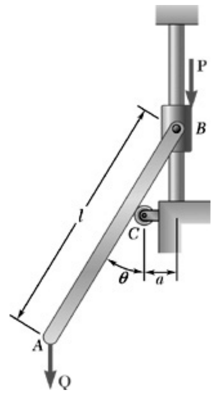
Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{58.013}{0.4} = 145.032\text{ N}$

$$W = N + (750\text{ N})(1 - \cos 30^\circ) = N \pm 145.03\text{ N}$$

$$= -44.55\text{ N (impossible)}, \; 245.51\text{ N}$$

Equilibrium for $W \ge 246$ N ◀

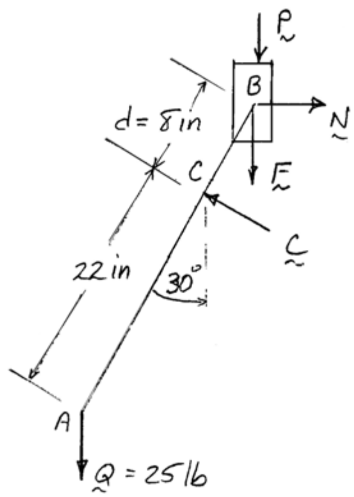
PROBLEM 8.36



The slender rod AB of length $l = 30$ in. is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 4$ in. from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 25$ lb and $\theta = 30^{\circ}$.

SOLUTION

FBD rod + collar:



Note: $d = \dfrac{a}{\sin\theta} = \dfrac{4 \text{ in.}}{\sin 30°} = 8$ in., so $AC = 22$ in.

Neglect weights of rod and collar.

$$\circlearrowleft \Sigma M_B = 0: \quad (30 \text{ in.})(\sin 30°)(25 \text{ lb}) - (8 \text{ in.})C = 0$$

$$C = 46.875 \text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: \quad N - C\cos 30° = 0$$

$$N = (46.875 \text{ lb})\cos 30° = 40.595 \text{ lb}$$

Impending motion up: $\quad F = \mu_s N = 0.25(40.595 \text{ lb})$

$$= 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30° - P - 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} - 10.149 \text{ lb} = -11.71 \text{ lb}$

Impending motion down: Direction of **F** is now upward, but still have

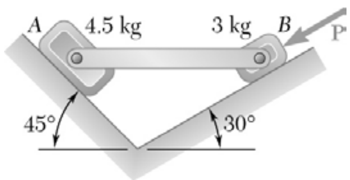
$$|F| = \mu_s N = 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30° - P + 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} + 10.149 \text{ lb} = 8.59 \text{ lb}$

$\therefore$ Equilibrium for $-11.71 \text{ lb} \le P \le 8.59 \text{ lb}$ ◀

PROBLEM 8.37

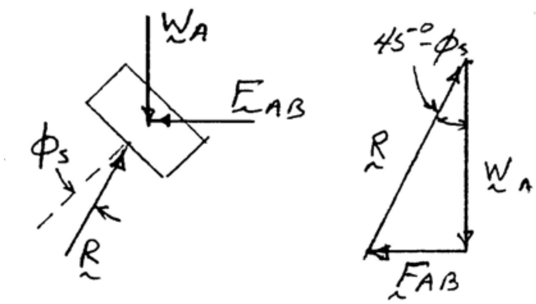


The 4.5-kg block A and the 3-kg block B are connected by a slender rod of negligible mass. The coefficient of static friction is 0.40 between all surfaces of contact. Knowing that for the position shown the rod is horizontal, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBDs: Note: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$

(a) Block A impending slip ↘

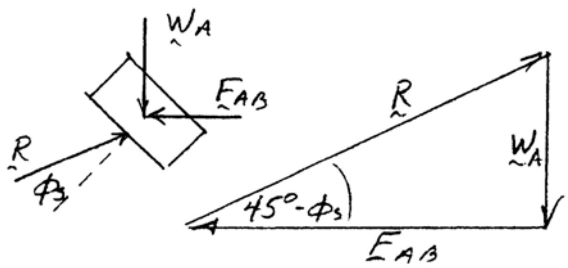


$$F_{AB} = W_A \tan\left(45° - \phi_s\right)$$

$$= \left(4.5 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\tan\left(23.199°\right)$$

$$= 18.9193 \text{ N}$$

(b) Block A impending slip ↖

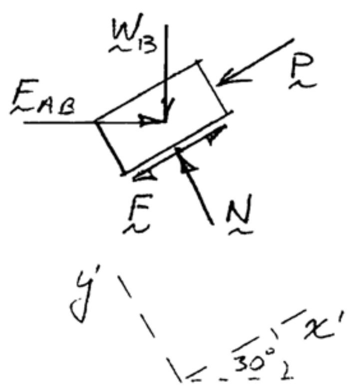


$$F_{AB} = W_A \operatorname{ctn}\left(45° - \phi_s\right)$$

$$= \left(4.5 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\operatorname{ctn}\left(23.199°\right)$$

$$= 103.005 \text{ N}$$

Block B:



$$W_B = \left(3 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)$$

$$= 29.43 \text{ N}$$

From Block B: $\nwarrow \Sigma F_{y'} = 0\text{:} \quad N - W_B \cos 30° - F_{AB} \sin 30° = 0$

PROBLEM 8.37 CONTINUED

Case (*a*) $N = (29.43\text{ N})\cos 30° + (18.9193\text{ N})\sin 30° = 34.947\text{ N}$

Impending motion: $F = \mu_s N = 0.4(34.947\text{ N}) = 13.979\text{ N}$

$\nearrow \Sigma F_{x'} = 0$: $F_{AB}\cos 30° - W_B \sin 30° - 13.979\text{ N} - P = 0$

$$P = (18.9193\text{ N})\cos 30° - (29.43\text{ N})\sin 30° - 13.979\text{ N}$$

$$= -12.31\text{ N}$$

Case (*b*) $N = (29.43\text{ N})\cos 30° + (103.005\text{ N})\sin 30° = 76.9896\text{ N}$

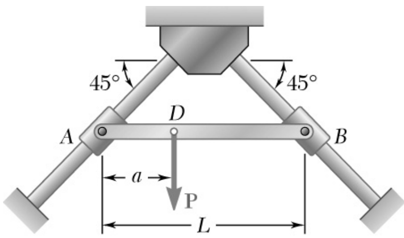
Impending motion: $F = 0.4(76.9896\text{ N}) = 30.7958\text{ N}$

$\nearrow \Sigma F_{x'} = 0$: $(103.005\text{ N})\cos 30° - (29.43\text{ N})\sin 30° + 30.7958\text{ N} - P = 0$

$$P = 105.3\text{ N}$$

For equilibrium $-12.31\text{ N} \le P \le 105.3\text{ N}$ ◀

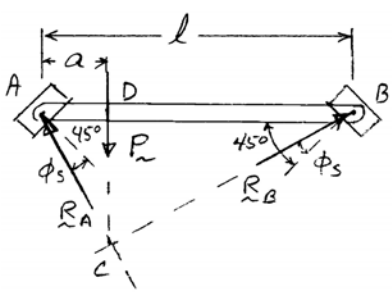
PROBLEM 8.38



Bar AB is attached to collars which can slide on the inclined rods shown. A force **P** is applied at point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a/L* for which equilibrium is maintained.

SOLUTION

FBD bar + collars:



Impending motion

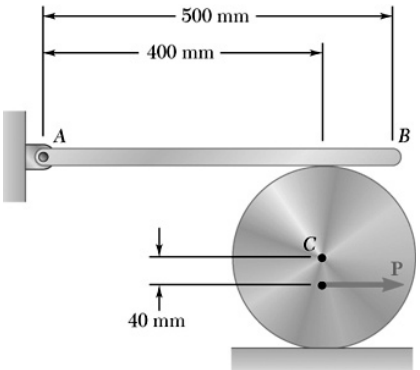
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

Neglect weights: 3-force *FBD* and $\measuredangle\, ACB = 90°$

So $$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

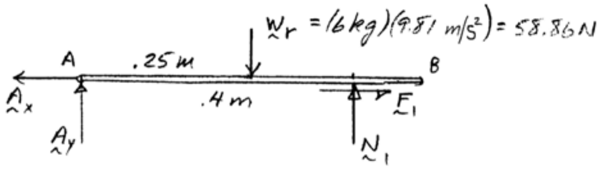


PROBLEM 8.39

The 6-kg slender rod AB is pinned at A and rests on the 18-kg cylinder C. Knowing that the diameter of the cylinder is 250 mm and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained.

SOLUTION

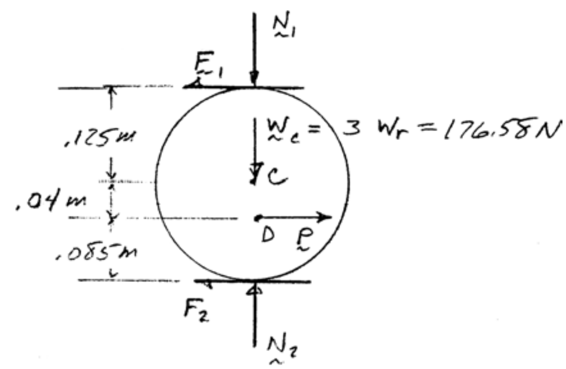
FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.4\text{ m})N_1 - (0.25\text{ m})W_r = 0$$

$$N_1 = 0.625 W_r = 36.7875\text{ N}$$

FBD cylinder:



Cylinder:

$$\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_C = 0 \quad \text{or} \quad N_2 = 0.625 W_r + 3W_r = 3.625 W_r = 5.8 N_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.165\text{ m})F_1 - (0.085\text{ m})F_2 = 0 \quad \text{or} \quad F_2 = 1.941 F_1$$

Since $\mu_{s1} = \mu_{s2}$, motion will impend first at top of the cylinder

So $$F_1 = \mu_s N_1 = 0.35(36.7875\text{ N}) = 12.8756\text{ N}$$

and $$F_2 = 1.941(12.8756\text{ N}) = 24.992\text{ N}$$

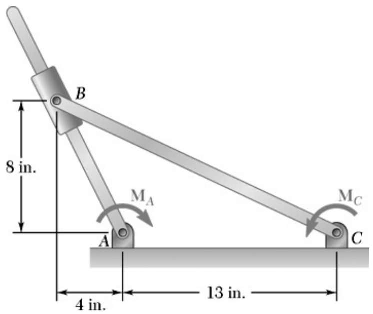
$$\left[\text{Check } F_2 = 25\text{ N} < \mu_S N_2 = 74.7\text{ N} \qquad \text{OK}\right]$$

$$\rightarrow \Sigma F_x = 0: \quad P - F_1 - F_2 = 0$$

or $$P = 12.8756\text{ N} + 24.992\text{ N}$$

or $P = 37.9\text{ N}$ ◀

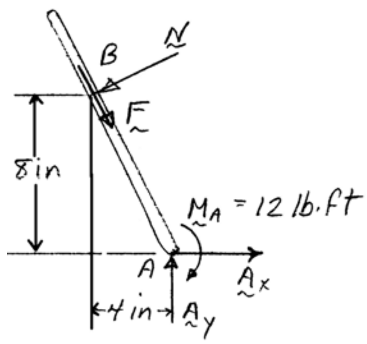
PROBLEM 8.40



Two rods are connected by a collar at B. A couple $\mathbf{M}_A$ of magnitude $12\ \text{lb}\cdot\text{ft}$ is applied to rod AB. Knowing that $\mu_s = 0.30$ between the collar and rod AB, determine the largest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



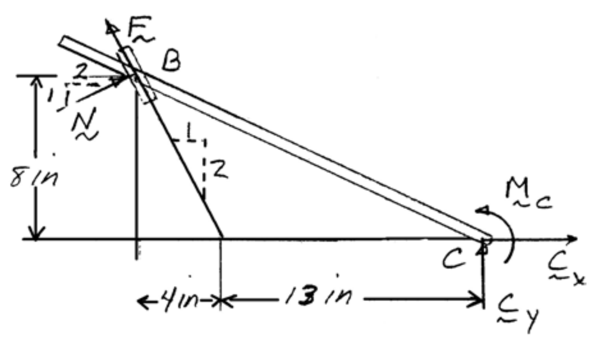
$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{8\ \text{in}^2 + 4\ \text{in}^2}\,(N) - M_A = 0$$

$$N = \frac{(12\ \text{lb}\cdot\text{ft})(12\ \text{in./ft})}{8.9443\ \text{in.}} = 16.100\ \text{lb}$$

Impending motion: $F = \mu_s N = 0.3(16.100\ \text{lb}) = 4.83\ \text{lb}$

(Note: For max, M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:

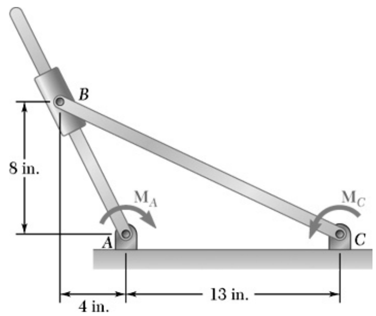


$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\ \text{in.})\frac{1}{\sqrt{5}}N - (8\ \text{in.})\frac{2}{\sqrt{5}}N - (13\ \text{in.})\frac{2}{\sqrt{5}}F = 0$$

or $$M_C = \frac{17\ \text{in.}}{\sqrt{5}}(16.100\ \text{lb}) + \frac{16\ \text{in.}}{\sqrt{5}}(16.100\ \text{lb}) + \frac{26\ \text{in.}}{\sqrt{5}}(4.830\ \text{lb}) = 293.77\ \text{lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\max} = 24.5\ \text{lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

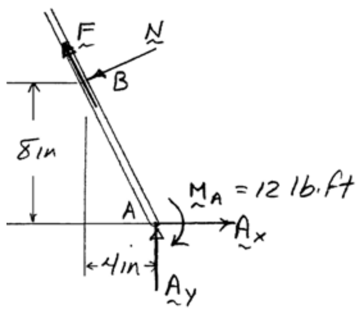
PROBLEM 8.41



In Problem 8.40, determine the smallest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad N\left(\sqrt{8\text{ in}^2 + 4\text{ in}^2}\right) - M_A = 0$$

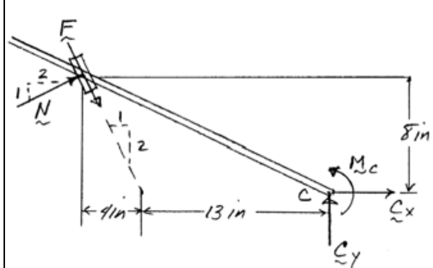
$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\text{ lb})$

$$= 4.830\text{ lb}$$

(Note: For min. M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:



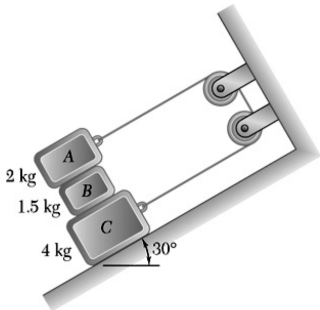
$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N + (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

$$M_C = \frac{1}{\sqrt{5}}\left[(17\text{ in.} + 16\text{ in.})(16.100\text{ lb}) - (26\text{ in.})(4.830\text{ lb})\right]$$

$$= 181.44\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\min} = 15.12\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

PROBLEM 8.42

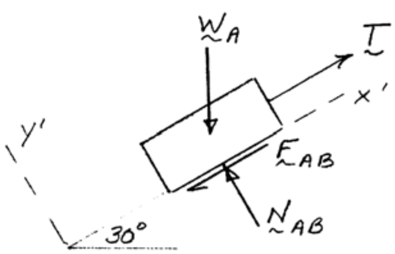


Blocks A, B, and C having the masses shown are at rest on an incline. Denoting by μ_s the coefficient of static friction between all surfaces of contact, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

For impending motion, C will start down and A will start up. Since, the normal force between B and C is larger than that between A and B, the corresponding friction force can be larger as well. Thus we assume that motion impends between A and B.

FBD A:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{AB} - W_A \cos 30° = 0; \; N_{AB} = \frac{\sqrt{3}}{2} W_A$$

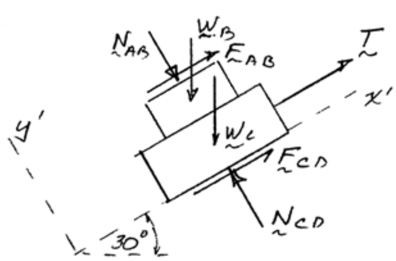
Impending motion: $$F_{AB} = \mu_s N_{AB} = \frac{\sqrt{3}}{2} W_A \mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T - F_{AB} - W_A \sin 30° = 0$$

or $$T = \left(\sqrt{3}\mu_s + 1\right)\frac{W_A}{2}$$

$$\nwarrow \Sigma F_{y'} = 0: \quad N_{CD} - N_{AB} - \left(W_B + W_C\right)\cos 30° = 0$$

FBD B + C:



or $$N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)$$

Impending motion: $$F_{CD} = \mu_s N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)\mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T + F_{AB} + F_{CD} - \left(W_B + W_C\right)\sin 30° = 0$$

$$T = \frac{W_B + W_C}{2} - \frac{\sqrt{3}}{2}\mu_s\left(2W_A + W_B + W_C\right)$$

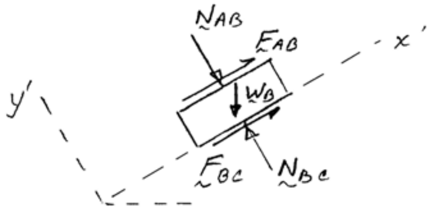
Equating T's: $$\sqrt{3}\mu_s\left(3W_A + W_B + W_C\right) = W_B + W_C - W_A$$

$$\mu_s = \frac{m_B + m_C - m_A}{\left(3m_A + m_B + m_C\right)\sqrt{3}} = \frac{1.5 \text{ kg} + 4 \text{ kg} - 2 \text{ kg}}{\left(6 \text{ kg} + 1.5 \text{ kg} + 4 \text{ kg}\right)\sqrt{3}}$$

$$\mu_s = 0.1757 \blacktriangleleft$$

PROBLEM 8.42 CONTINUED

FBD B:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{BC} - N_{AB} - W_B \cos 30° = 0$$

or
$$N_{BC} = \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$(F_{BC})_{\max} = \mu_s N_{BC} = 0.1757 \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$= 0.1522(m_A + m_B)g = 0.1522(3.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 5.224 \text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB} + F_{BC} - W_B \sin 30° = 0$$

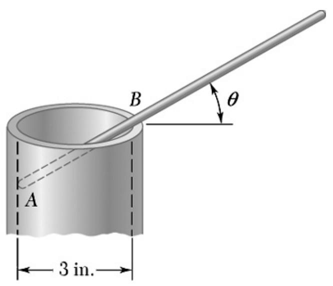
or
$$F_{BC} = -F_{AB} + \frac{1}{2}W_B = -\frac{\sqrt{3}}{2}W_A(0.1757) + \frac{W_B}{2}$$

$$= (-0.1522 m_A + 0.5 m_B)g$$

$$= [-0.1522(2 \text{ kg}) + 0.5(1.5 \text{ kg})](9.81 \text{ m/s}^2)$$

$$= 4.37 \text{ N}$$

$$F_{BC} < F_{BC\max} \qquad \text{OK}$$

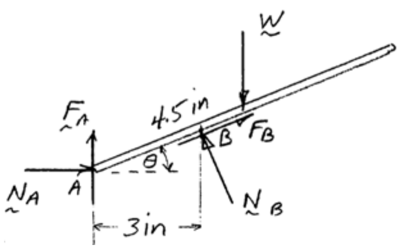


PROBLEM 8.43

A slender steel rod of length 9 in. is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

FBD rod:



$$\Sigma M_A = 0: \quad \frac{3 \text{ in.}}{\cos\theta} N_B - \left[(4.5 \text{ in.})\cos\theta\right] W = 0$$

or $$N_B = \left(1.5\cos^2\theta\right)W$$

Impending motion: $$F_B = \mu_s N_B = \left(1.5\mu_s \cos^2\theta\right)W$$

$$= \left(0.3\cos^2\theta\right)W$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

or $$N_A = \left(1.5\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= \left(0.3\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

$$\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos\theta + F_B \sin\theta - W = 0$$

or $$F_A = W\left(1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta\right)$$

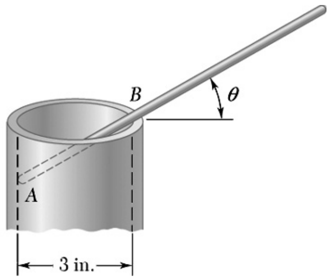
Equating F_A's

$$0.3\cos^2\theta\left(\sin\theta - 0.2\cos\theta\right) = 1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta$$

$$0.6\cos^2\theta\sin\theta + 1.44\cos^3\theta = 1$$

Solving numerically $\theta = 35.8°$ ◀

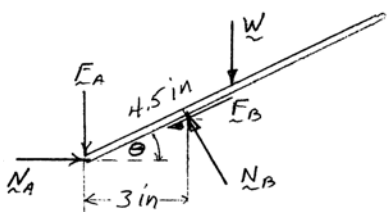
PROBLEM 8.44



In Problem 8.43, determine the smallest value of θ for which the rod will not fall out of the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3 \text{ in.}}{\cos\theta} N_B - \left[(4.5 \text{ in.})\cos\theta\right] W = 0$$

or $$N_B = 1.5W\cos^2\theta$$

Impending motion: $$F_B = \mu_s N_B = 0.2\left(1.5W\cos^2\theta\right)$$

$$= 0.3W\cos^2\theta$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta - F_B\cos\theta = 0$$

or $$N_A = W\cos^2\theta\left(1.5\sin\theta + 0.3\cos\theta\right)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= W\cos^2\theta\left(0.3\sin\theta + 0.06\cos\theta\right)$$

$$\uparrow \Sigma F_y = 0: \quad N_B\cos\theta - F_B\sin\theta - W - F_A = 0$$

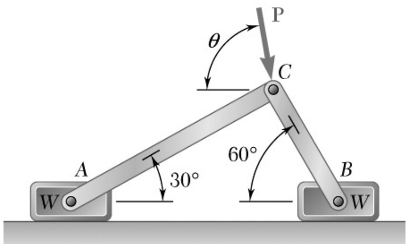
or $$F_A = W\left[\cos^2\theta\left(1.5\cos\theta - 0.3\sin\theta\right) - 1\right]$$

Equating F_A's:

$$\cos^2\theta\left(1.44\cos\theta - 0.6\sin\theta\right) = 1$$

Solving numerically $\theta = 20.5°$ ◀

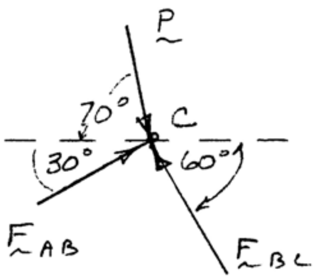
PROBLEM 8.45

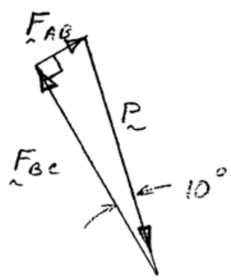


Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 70^{\circ}$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:





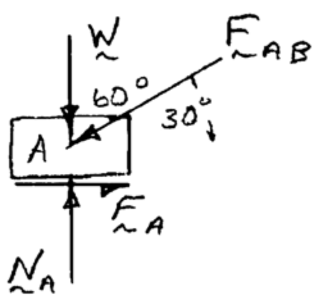
$$F_{AB} = P\sin 10^{\circ} = 0.173648P$$

$$F_{BC} = P\cos 10^{\circ} = 0.98481P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AB}\sin 30^{\circ} = 0$$

or $$N_A = W + 0.173648P\sin 30^{\circ} = W + 0.086824P$$

FBD block A:



$$\longrightarrow \Sigma F_x = 0: \quad F_A - F_{AB}\cos 30^{\circ} = 0$$

or $$F_A = 0.173648P\cos 30^{\circ} = 0.150384P$$

For impending motion at A: $$F_A = \mu_s N_A$$

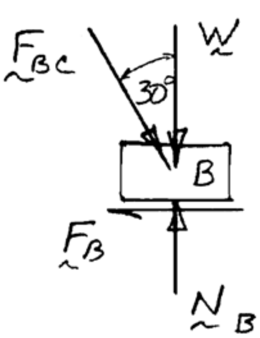
Then $$N_A = \frac{F_A}{\mu_s}: \quad W + 0.086824P = \frac{0.150384}{0.3}P$$

or $$P = 2.413W$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30^{\circ} = 0$$

$$N_B = W + 0.98481P\cos 30^{\circ} = W + 0.85287P$$

FBD block B:



$$\longrightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30^{\circ} - F_B = 0$$

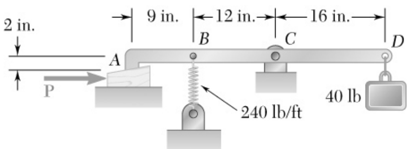
$$F_B = 0.98481P\sin 30^{\circ} = 0.4924P$$

For impending motion at B: $$F_B = \mu_s N_B$$

Then $$N_B = \frac{F_B}{\mu_s}: \quad W + 0.85287P = \frac{0.4924P}{0.3}$$

or $$P = 1.268W$$

Thus, maximum P for equilibrium $$P_{\max} = 1.268W \blacktriangleleft$$



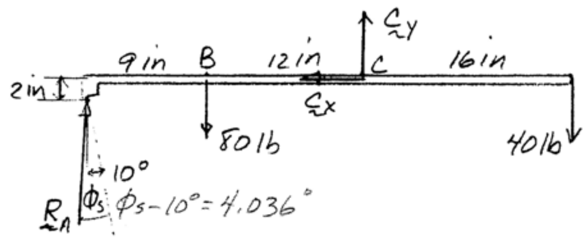
PROBLEM 8.46

A 40-lb weight is hung from a lever which rests against a 10° wedge at *A* and is supported by a frictionless hinge at *C*. Knowing that the coefficient of static friction is 0.25 at both surfaces of the wedge and that for the position shown the spring is stretched 4 in., determine (*a*) the magnitude of the force **P** for which motion of the wedge is impending, (*b*) the components of the corresponding reaction at *C*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos(\phi_s - 10°)$$

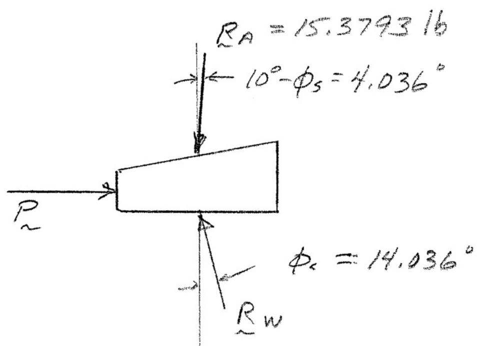
$$+ (2 \text{ in.})R_A\sin(\phi_s - 10°) = 0$$

or $\qquad R_A = 15.3793 \text{ lb}$

(*b*) $\quad \xrightarrow{} \Sigma F_x = 0: \quad (15.379 \text{ lb})\sin(4.036°) - C_x = 0 \qquad \mathbf{C}_x = 1.082 \text{ lb} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad (15.379 \text{ lb})\cos(4.036°) - 80 \text{ lb} - 40 \text{ lb} + C_y = 0 \qquad \mathbf{C}_y = 104.7 \text{ lb} \uparrow$ ◀

FBD wedge:



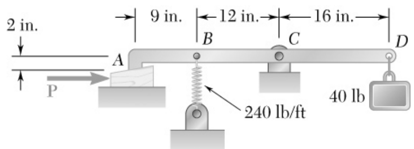
$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (15.3793 \text{ lb})\cos 4.036° = 0$

or $\qquad R_W = 15.8133 \text{ lb}$

(*a*) $\quad \xrightarrow{} \Sigma F_x = 0: \quad P - (15.3793 \text{ lb})\sin 4.036° - (15.8133 \text{ lb})\sin 14.036° = 0$

$P = 4.92 \text{ lb}$ ◀

PROBLEM 8.47

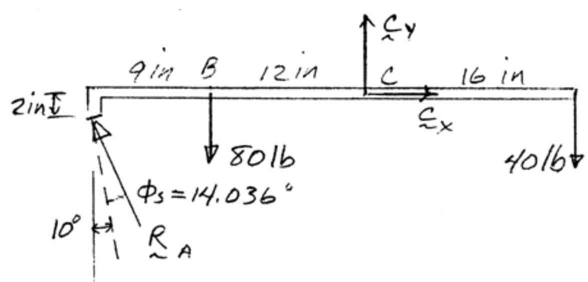


Solve Problem 8.46 assuming that force **P** is directed to the left.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240\text{ lb/ft})\left(\frac{4\text{ in.}}{12\text{ in./ft}}\right) = 80\text{ lb}$$

FBD lever:



$$\Sigma M_C = 0: \quad (12\text{ in.})(80\text{ lb}) - (16\text{ in.})(40\text{ lb}) - (21\text{ in.})R_A\cos 24.036°$$

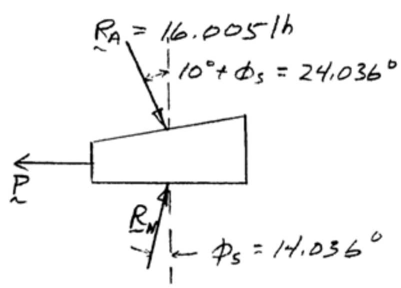
$$- (2\text{ in.})R_A \sin 24.036° = 0$$

or $\qquad R_A = 16.005$ lb

(*b*) $\quad \longrightarrow \Sigma F_x = 0: \quad C_x - (16.005\text{ lb})\sin 24.036° = 0 \qquad\qquad \mathbf{C}_x = 6.52\text{ lb} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad C_y - 80\text{ lb} - 40\text{ lb} + (16.005\text{ lb})\cos(24.036°) = 0 \qquad\qquad \mathbf{C}_y = 105.4\text{ lb} \uparrow$ ◀

FBD wedge:



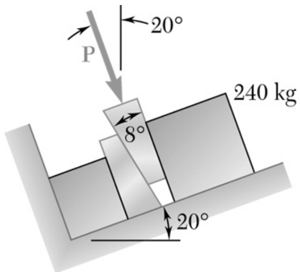
$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (16.005\text{ lb})\cos 24.036° = 0$$

or $\qquad R_W = 15.067$ lb

(*a*) $\quad \longrightarrow \Sigma F_x = 0: \quad (16.005\text{ lb})\sin 24.036° + (15.067\text{ lb})\sin 14.036° - P = 0$

$P = 10.17$ lb ◀

PROBLEM 8.48

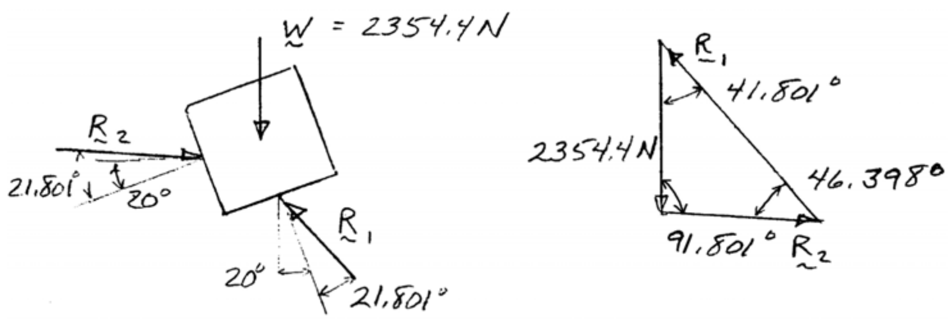


Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

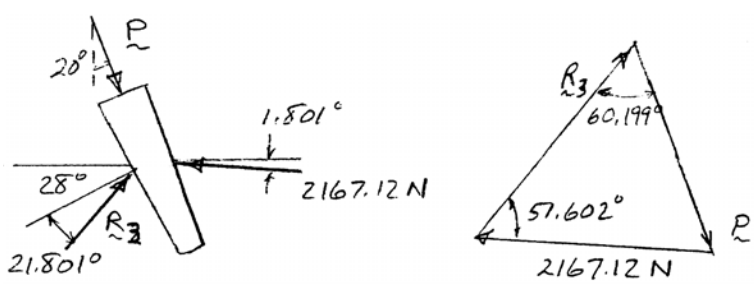
FBD block:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 46.398°}$$

$$R_2 = 2167.12\text{ N}$$

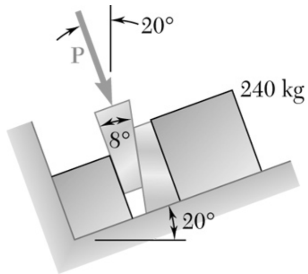
FBD wedge:



$$\frac{P}{\sin 51.602°} = \frac{2167.12\text{ N}}{\sin 60.199°}$$

$P = 1957$ N

$P = 1.957$ kN ◀



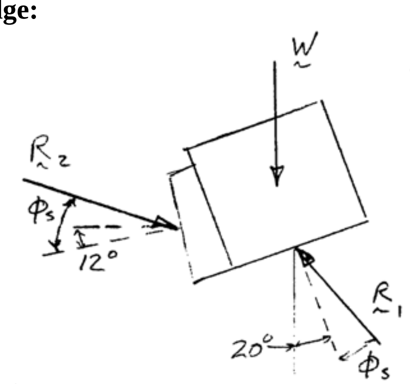
PROBLEM 8.49

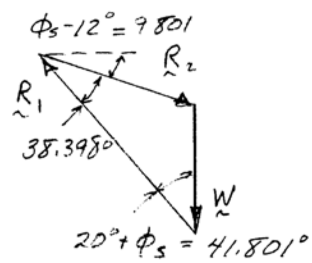
Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

FBD block + wedge:

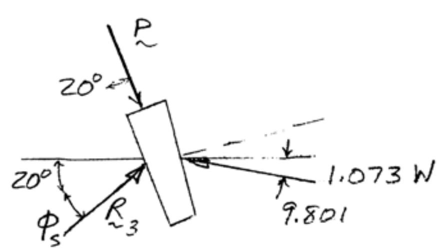


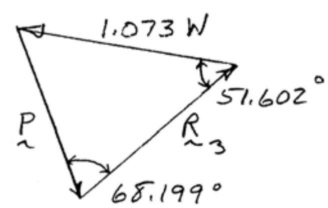


$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 38.398°}$$

$$R_2 = 2526.6\text{ N}$$

FBD wedge:

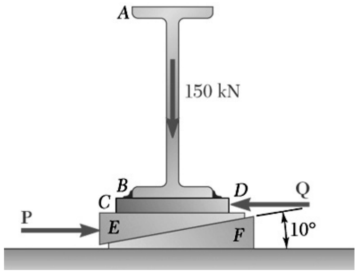




$$\frac{P}{\sin 51.602°} = \frac{2526.6\text{ N}}{\sin 68.199°}$$

$$P = 2132.7\text{ N}$$

$P = 2.13\text{ kN}$ ◀

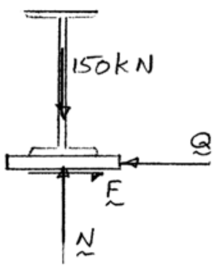


PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges E and F. The base plate CD has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

FBD AB + CD:



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

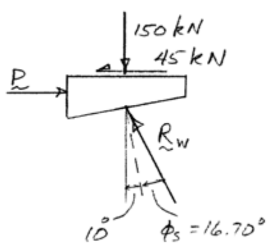
$$\uparrow \Sigma F_y = 0: \quad N - 150 \text{ kN} = 0 \qquad N = 150 \text{ kN}$$

Impending motion: $\quad F = \mu_s N = 0.3(150 \text{ kN}) = 45 \text{ kN}$

$$\longrightarrow \Sigma F_x = 0: \quad F - Q = 0$$

$$(b)\ \mathbf{Q} = 45.0 \text{ kN} \longleftarrow \blacktriangleleft$$

FBD top wedge:



Assume bottom wedge doesn't move:

$$\uparrow \Sigma F_y = 0: \quad R_W \cos(10° + 16.70°) - 150 \text{ kN} = 0$$

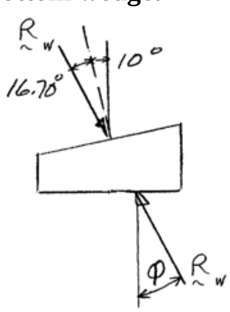
$$R_W = 167.9 \text{ kN}$$

$$\longrightarrow \Sigma F_x = 0: \quad P - 45 \text{ kN} - (167.9 \text{ kN})\sin 26.70° = 0$$

$$P = 120.44 \text{ kN}$$

$$(a)\ \mathbf{P} = 120.4 \text{ kN} \longrightarrow \blacktriangleleft$$

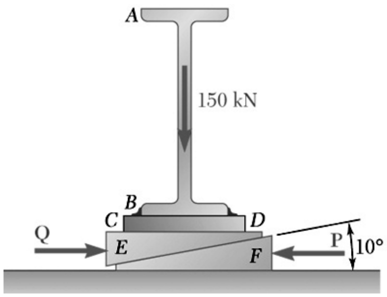
FBD bottom wedge:



Bottom wedge is two-force member, so $\phi = 26.70°$ for equilibrium, but

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 31.0° \ (\text{steel on concrete})$$

So $\qquad \phi < \phi_s \qquad$ OK.



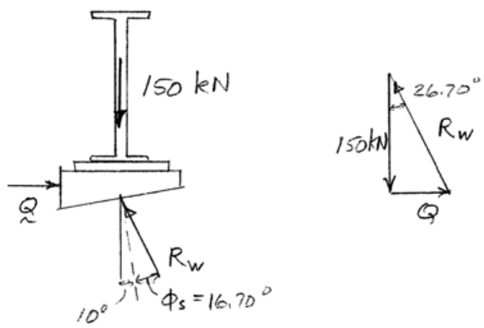
PROBLEM 8.51

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

FBD AB + CD + top wedge: Assume top wedge doesn't move

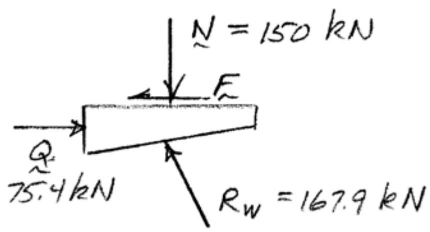


$$R_w = \frac{150 \text{ kN}}{\cos 26.70°} = 167.90 \text{ kN}$$

$$Q = (150 \text{ kN})\tan 26.70° = 75.44 \text{ kN}$$

(*b*) $\mathbf{Q} = 75.4 \text{ kN} \longrightarrow$ ◀

FBD top wedge:

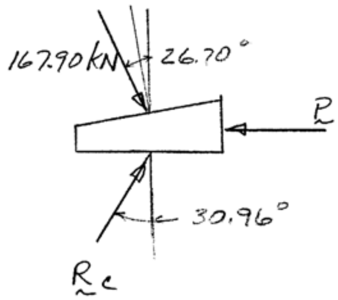


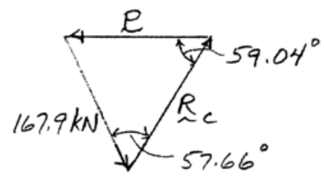
$$\longrightarrow \Sigma F_x = 0: \quad 75.44 \text{ kN} - 167.9 \text{ kN} \sin 26.70° - F = 0$$

$$F = 0 \text{ as expected.}$$

PROBLEM 8.51 CONTINUED

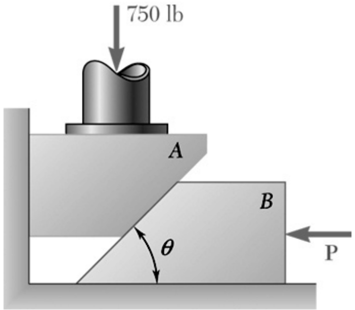
FBD bottom wedge: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 30.96°$ steel on concrete





$$\frac{P}{\sin 57.66°} = \frac{167.90 \text{ kN}}{\sin 59.04°}$$

(*a*) $\mathbf{P} = 165.4 \text{ kN} \longleftarrow$ ◀



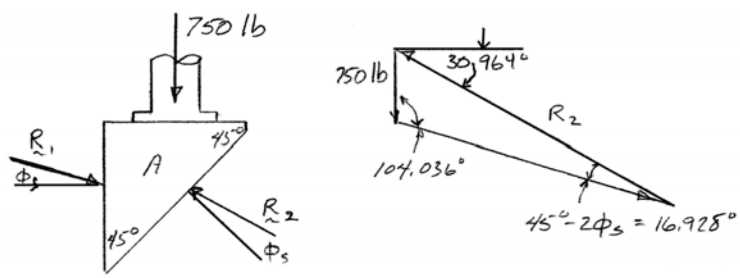
PROBLEM 8.52

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

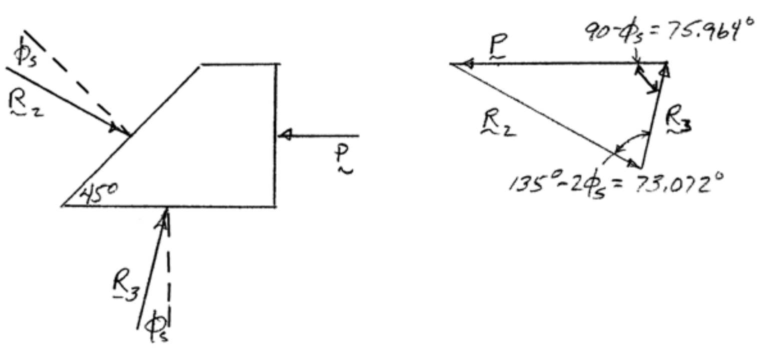
FBD block A:



$$\frac{R_2}{\sin 104.036°} = \frac{750\text{ lb}}{\sin 16.928°}$$

$$R_2 = 2499.0\text{ lb}$$

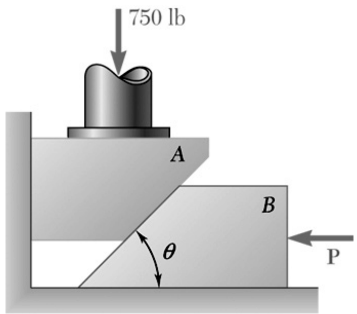
FBD wedge B:



$$\frac{P}{\sin 73.072°} = \frac{2499.0}{\sin 75.964°}$$

$$P = 2464\text{ lb}$$

$\mathbf{P} = 2.46\text{ kips} \longleftarrow$ ◀



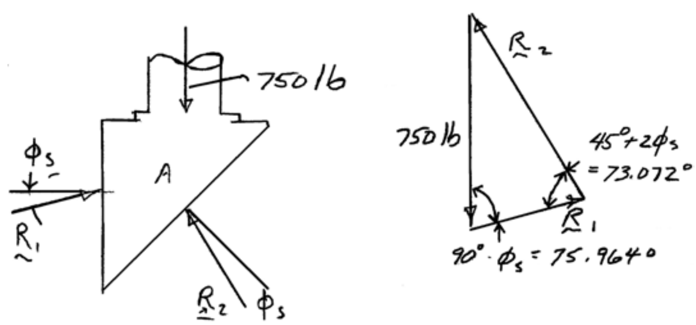
PROBLEM 8.53

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force $\mathbf{P}$ for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

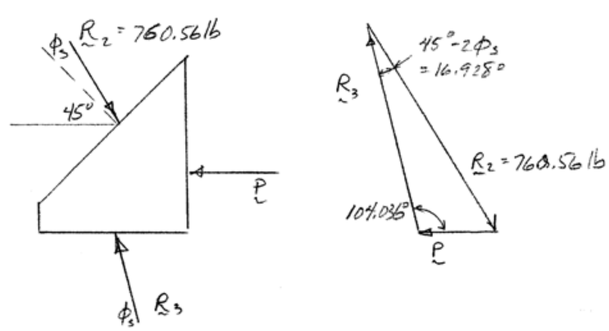
FBD block A:



$$\frac{R_2}{\sin(75.964°)} = \frac{750\text{ lb}}{\sin(73.072°)}$$

$$R_2 = 760.56\text{ lb}$$

FBD wedge B:



$$\frac{P}{\sin 16.928°} = \frac{760.56}{\sin 104.036°}$$

$$P = 228.3\text{ lb}$$

$\mathbf{P} = 228\text{ lb} \longleftarrow$ ◀