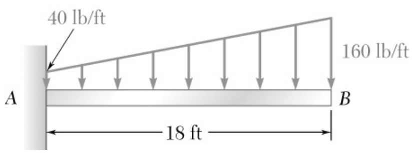
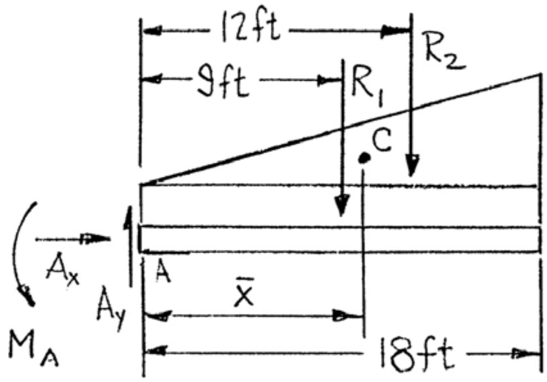


PROBLEM 5.61

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



Resultant $\qquad R = R_1 + R_2$

(*a*) Have $\qquad R_1 = (40 \text{ lb/ft})(18 \text{ ft}) = 720 \text{ lb}$

$$R_2 = \frac{1}{2}(120 \text{ lb/ft})(18 \text{ ft}) = 1080 \text{ lb}$$

or $\qquad R = 1800 \text{ lb}$

The resultant is located at the centroid C of the distributed load $\overline{x}$

Have $\qquad +\circlearrowleft \Sigma M_A:\quad (1800 \text{ lb})\overline{x} = (40 \text{ lb/ft})(18 \text{ ft})(9 \text{ ft}) + \frac{1}{2}(120 \text{ lb/ft})(18 \text{ ft})(12 \text{ ft})$

or $\qquad \overline{x} = 10.80 \text{ ft}$

$$R = 1800 \text{ lb} \blacktriangleleft$$

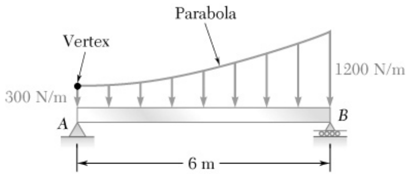
$$\overline{x} = 10.80 \text{ ft}$$

(*b*) $\qquad \xrightarrow{+} \Sigma F_x = 0:\quad A_x = 0$

$+\uparrow \Sigma F_y = 0:\quad A_y - 1800 \text{ lb} = 0,\ A_y = 1800 \text{ lb} \qquad \therefore\ \mathbf{A} = 1800 \text{ lb} \uparrow \blacktriangleleft$

$+\circlearrowleft \Sigma M_A = 0:\quad M_A - (1800 \text{ lb})(10.8 \text{ ft}) = 0$

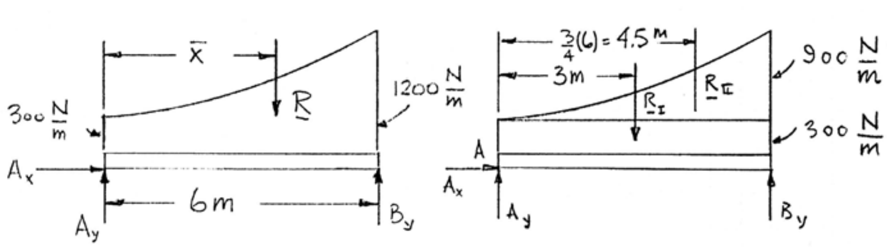
$M_A = 19.444 \text{ lb}\cdot\text{ft} \qquad$ or $\ \mathbf{M}_A = 19.44 \text{ kip}\cdot\text{ft} \circlearrowleft \blacktriangleleft$



PROBLEM 5.62

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) Have

$$R_{\mathrm{I}} = (300\text{ N/m})(6\text{ m}) = 1800\text{ N}$$

$$R_{\mathrm{II}} = \frac{1}{3}(6\text{ m})(900\text{ N/m}) = 1800\text{ N}$$

Then

$$+\uparrow \Sigma F_y: \quad -R = -R_{\mathrm{I}} - R_{\mathrm{II}}$$

or

$$R = 1800\text{ N} + 1800\text{ N} = 3600\text{ N}$$

$$+\circlearrowleft \Sigma M_A: \quad -\bar{x}(3600\text{ N}) = -(3\text{ m})(1800\text{ N}) - (4.5\text{ m})(1800\text{ N})$$

or

$$\bar{x} = 3.75\text{ m}$$

$R = 3600\text{ N}$ ◀

$\bar{x} = 3.75\text{ m}$

(*b*) Reactions

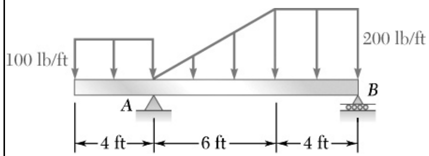
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (6\text{ m})B_y - (3600\text{ N})(3.75\text{ m}) = 0$$

or $B_y = 2250\text{ N}$ $\qquad$ $\mathbf{B} = 2250\text{ N}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y + 2250\text{ N} = 3600\text{ N}$$

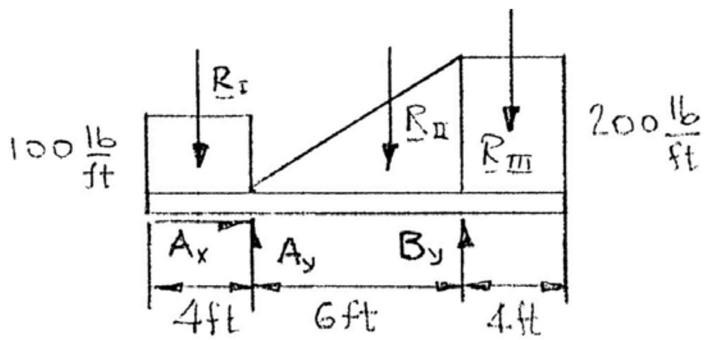
or $A_y = 1350\text{ N}$ $\qquad$ $\mathbf{A} = 1350\text{ N}\uparrow$ ◀



PROBLEM 5.63

Determine the reactions at the beam supports for the given loading.

SOLUTION



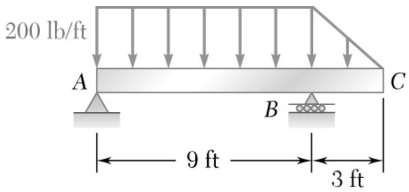
Have
$$R_{\mathrm{I}} = (100 \text{ lb/ft})(4 \text{ ft}) = 400 \text{ lb}$$
$$R_{\mathrm{II}} = \frac{1}{2}(200 \text{ lb/ft})(6 \text{ ft}) = 600 \text{ lb}$$
$$R_{\mathrm{III}} = (200 \text{ lb/ft})(4 \text{ ft}) = 800 \text{ lb}$$

Then
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$
$$+\circlearrowleft \Sigma M_A = 0: \quad (2 \text{ ft})(400 \text{ lb}) - (4 \text{ ft})(600 \text{ lb}) - (12 \text{ ft})(800 \text{ lb}) + (10 \text{ ft})B_y = 0$$

or
$$B_y = 800 \text{ lb} \qquad \mathbf{B} = 800 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800 \text{ lb} - 400 \text{ lb} - 600 \text{ lb} - 800 = 0$$

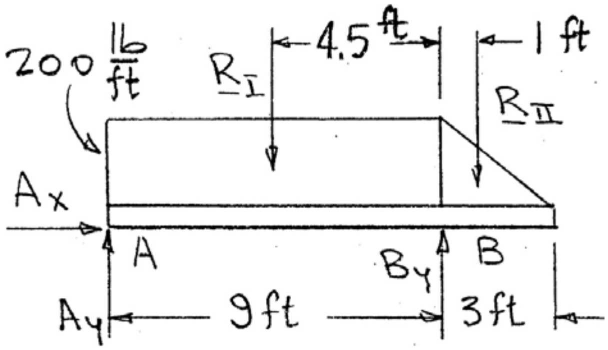
or
$$A_y = 1000 \text{ lb} \qquad \mathbf{A} = 1000 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 5.64

Determine the reactions at the beam supports for the given loading.

SOLUTION



Have $$R_{\mathrm{I}} = (9\text{ ft})(200\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(3\text{ ft})(200\text{ lb/ft}) = 300\text{ lb}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

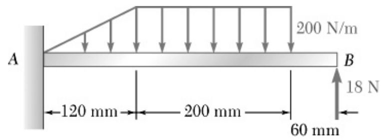
$$+\circlearrowleft \Sigma M_A = 0: \quad -(4.5\text{ ft})(1800\text{ lb}) - (10\text{ ft})(300\text{ lb}) + (9\text{ ft})B_y = 0$$

or $$B_y = 1233.3\text{ lb}$$ **B** $= 1233\text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1800\text{ lb} - 300\text{ lb} + 1233.3\text{ lb} = 0$$

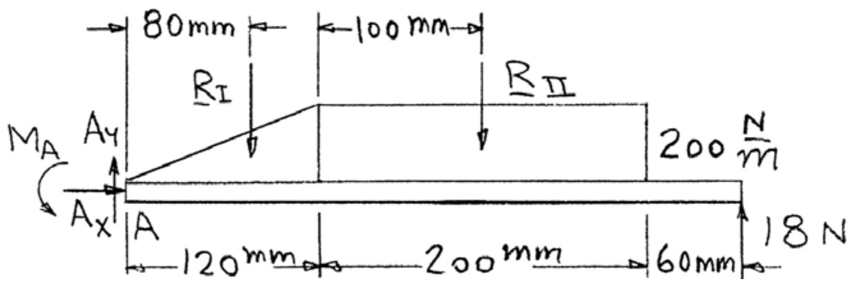
or $$A_y = 866.7\text{ lb}$$ **A** $= 867\text{ lb} \uparrow$ ◀

PROBLEM 5.65



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have $$R_{\mathrm{I}} = \frac{1}{2}(200 \text{ N/m})(0.12 \text{ m}) = 12 \text{ N}$$

$$R_{\mathrm{II}} = (200 \text{ N/m})(0.2 \text{ m}) = 40 \text{ N}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

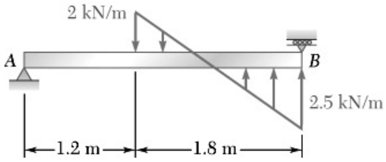
$$+\uparrow \Sigma F_y = 0: \quad A_y + 18 \text{ N} - 12 \text{ N} - 40 \text{ N} = 0$$

or $$A_y = 34 \text{ N}$$ $\mathbf{A} = 34.0 \text{ N} \uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (0.8 \text{ m})(12 \text{ N}) - (0.22 \text{ m})(40 \text{ N}) + (0.38 \text{ m})(18 \text{ N})$$

or $$M_A = 2.92 \text{ N}\cdot\text{m}$$ $\mathbf{M}_A = 2.92 \text{ N}\cdot\text{m} \circlearrowleft$ ◀

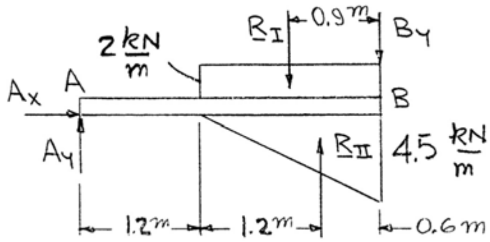
PROBLEM 5.66



Determine the reactions at the beam supports for the given loading.

SOLUTION

First, replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a linear relation between load and distance, and the values at the end points are the same.



Have $$R_I = (1.8\text{ m})(2000\text{ N/m}) = 3600\text{ N}$$

$$R_{II} = \frac{1}{2}(1.8\text{ m})(4500\text{ N/m}) = 4050\text{ N}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

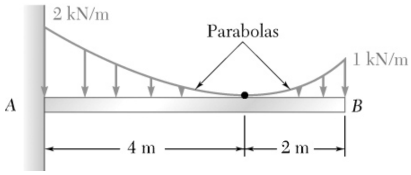
$$+\circlearrowleft \Sigma M_B = 0: \quad -(3\text{ m})A_y - (2.1\text{ m})(3600\text{ N}) + (2.4\text{ m})(4050\text{ N})$$

or $$A_y = 270\text{ N}$$ $\mathbf{A} = 270\text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 270\text{ N} - 3600\text{ N} + 4050\text{ N} - B_y = 0$$

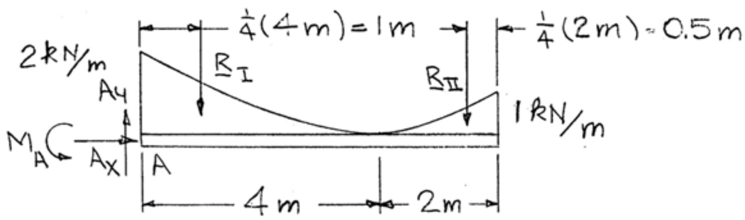
or $$B_y = 720\text{ N}$$ $\mathbf{B} = 720\text{ N} \downarrow$ ◀

PROBLEM 5.67



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have $$R_{\text{I}} = \frac{1}{3}(4\text{ m})(2000\text{ kN/m}) = 2667\text{ N}$$

$$R_{\text{II}} = \frac{1}{3}(2\text{ m})(1000\text{ kN/m}) = 666.7\text{ N}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

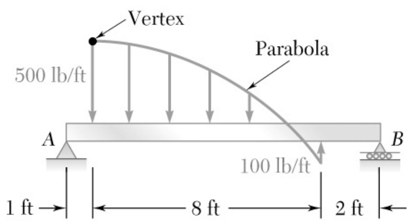
$$+\uparrow \Sigma F_y = 0: \quad A_y - 2667\text{ N} - 666.7\text{ N} = 0$$

or $$A_y = 3334\text{ N}$$ $\mathbf{A} = 3.33\text{ kN}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (1\text{ m})(2667\text{ N}) - (5.5\text{ m})(666.7\text{ N})$$

or $$M_A = 6334\text{ N}\cdot\text{m}$$ $\mathbf{M}_A = 6.33\text{ kN}\cdot\text{m}\circlearrowleft$ ◀

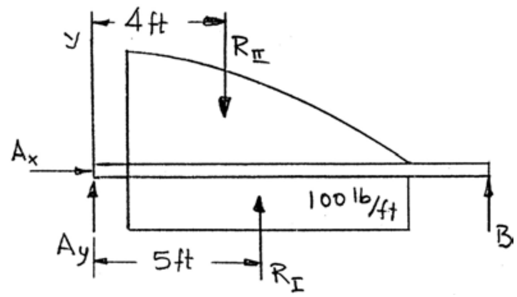
PROBLEM 5.68



Determine the reactions at the beam supports for the given loading.

SOLUTION

First, replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a parabolic relation between load and distance, and the values at end points are the same.



Have $$R_{\mathrm{I}} = (8\text{ ft})(100\text{ lb/ft}) = 800\text{ lb}$$

$$R_{\mathrm{II}} = \frac{2}{3}(8\text{ ft})(600\text{ lb/ft}) = 3200\text{ lb}$$

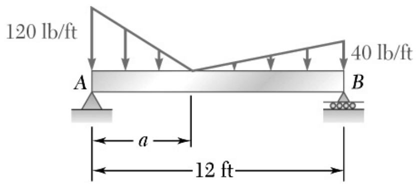
Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 11B + (5\text{ ft})(800\text{ lb}) - (4\text{ ft})(3200)\text{lb} = 0$$

or $\mathbf{B} = 800\text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 3200\text{ lb} + 800\text{ lb} + 800\text{ lb} = 0$$

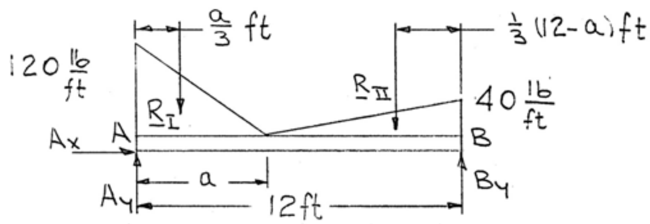
or $\mathbf{A} = 1600\text{ lb} \uparrow$ ◀



PROBLEM 5.69

Determine (*a*) the distance *a* so that the vertical reactions at supports *A* and *B* are equal, (*b*) the corresponding reactions at the supports.

SOLUTION



(*a*) Have

$$R_{\mathrm{I}} = \frac{1}{2}(a \text{ ft})(120 \text{ lb/ft}) = (60a) \text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(12 - a)(40 \text{ lb/ft}) = (240 - 20a) \text{ lb}$$

Then $+\uparrow \Sigma F_y = 0$: $\quad A_y - 60a - (240 - 2a) + B_y = 0$

or $$A_y + B_y = 240 + 40a$$

Now $$A_y = B_y \Rightarrow A_y = B_y = 120 + 20a \qquad (1)$$

Also $+\circlearrowleft \Sigma M_B = 0$: $\quad -(12 \text{ m})A_y + [(60a) \text{ lb}]\left[\left(12 - \frac{a}{3}\right) \text{ft}\right] + \left[\left(\frac{1}{3}(12 - a) \text{ ft}\right)\right][(240 - 20a) \text{ lb}] = 0$

or $$A_y = 80 - \frac{140}{3}a - \frac{10}{9}a^2 \qquad (2)$$

Equating Eqs. (1) and (2)

$$120 + 20a = 80 - \frac{140}{3}a - \frac{10}{9}a^2$$

or $$\frac{40}{3}a^2 - 320a + 480 = 0$$

Then $$a = 1.6077 \text{ ft}, \qquad a = 22.392$$

Now $$a \le 12 \text{ ft}$$

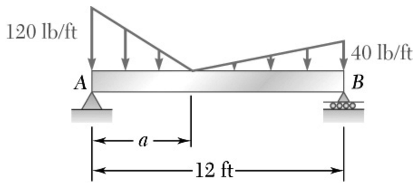
$a = 1.608$ ft ◀

(*b*) Have $\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x = 0$

Eq. (1) $$A_y = B_y = 120 + 20(1.61) = 152.2 \text{ lb}$$

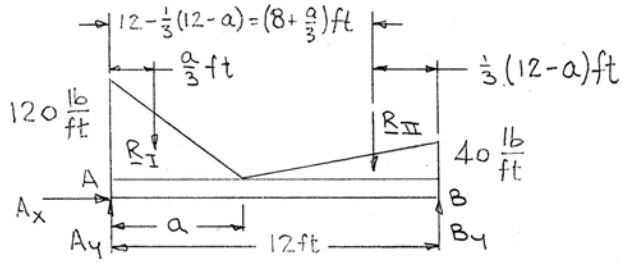
$\mathbf{A} = \mathbf{B} = 152.2 \text{ lb} \uparrow$ ◀

PROBLEM 5.70



Determine (*a*) the distance *a* so that the vertical reaction at support *B* is minimum, (*b*) the corresponding reactions at the supports.

SOLUTION



(*a*) Have $R_{\mathrm{I}} = \frac{1}{2}(a\ \mathrm{ft})(120\ \mathrm{lb/ft}) = 60a\ \mathrm{lb}$

$$R_{\mathrm{II}} = \frac{1}{2}\left[(12-a)\ \mathrm{ft}\right](40\ \mathrm{lb/ft}) = (240-20a)\ \mathrm{lb}$$

Then $+\circlearrowleft \Sigma M_A = 0$: $-\left(\frac{a}{3}\mathrm{ft}\right)(60a\ \mathrm{lb}) - \left[(240-20a)\ \mathrm{lb}\right]\left[\left(8+\frac{a}{3}\right)\mathrm{ft}\right] + (12\ \mathrm{ft})B_y = 0$

or $$B_y = \frac{10}{9}a^2 - \frac{20}{3}a + 160 \qquad (1)$$

Then $$\frac{dB_y}{da} = \frac{20}{9}a - \frac{20}{3} = 0$$ or $a = 3.00$ ft ◀

(*b*) Eq. (1) $$B_y = \frac{10}{9}(3.00)^2 - \frac{20}{3}(3.00) + 160$$

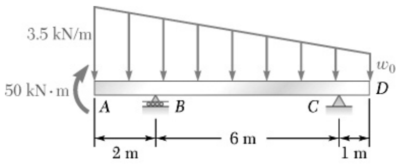
$= 150$ lb **B** $= 150.0$ lb ↑ ◀

and $\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - \left[60(3.00)\right]\mathrm{lb} - \left[240 - 20(3.00)\right]\mathrm{lb} + 150\ \mathrm{lb} = 0$

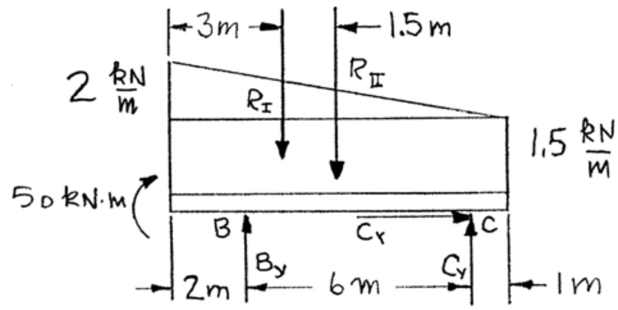
or $A_y = 210$ lb **A** $= 210$ lb ↑ ◀

PROBLEM 5.71



Determine the reactions at the beam supports for the given loading when $w_0 = 1.5$ kN/m.

SOLUTION



Have
$$R_{\mathrm{I}} = \frac{1}{2}(9\text{ m})(2\text{ kN/m}) = 9\text{ kN}$$

$$R_{\mathrm{II}} = (9\text{ m})(1.5\text{ kN/m}) = 13.5\text{ kN}$$

Then
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -50\text{ kN}\cdot\text{m} - (1\text{ m})(9\text{ kN}) - (2.5\text{ m})(13.5\text{ kN}) + (6\text{ m})C_y = 0$$

or
$$C_y = 15.4583\text{ kN}$$
$\mathbf{C} = 15.46$ kN $\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9\text{ kN} - 13.5\text{ kN} + 15.4583 = 0$$

or
$$B_y = 7.0417\text{ kN}$$
$\mathbf{B} = 7.04$ kN $\uparrow$ ◀