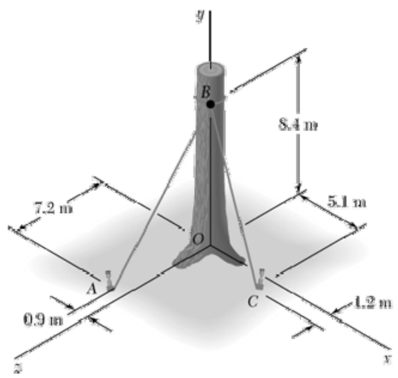
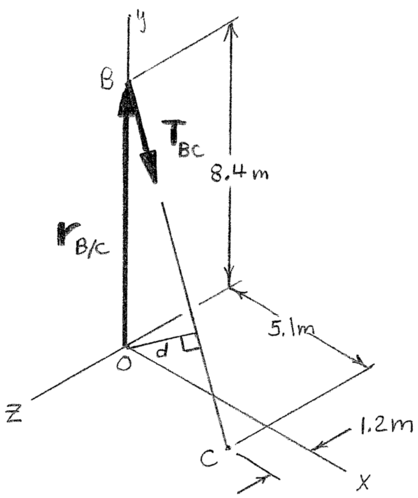
PROBLEM 3.28



In Problem 3.21, determine the perpendicular distance from point *O* to cable *BC*.

Problem 3.21: Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Knowing that the tension in cables *AB* and *BC* are 777 N and 990 N, respectively, determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

SOLUTION



Have $$|\mathbf{M}_O| = T_{BC}d$$

where d = perpendicular distance from O to line BC.

$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BC}$$

$$\mathbf{r}_{B/O} = 8.4\text{ m}\,\mathbf{j}$$

$$\mathbf{T}_{BC} = \lambda_{BC}T_{BC} = \frac{(5.1\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}}{\sqrt{(5.1)^2 + (8.4)^2 + (1.2)^2}\text{ m}}(990\text{ N})$$

$$= (510\text{ N})\mathbf{i} - (840\text{ N})\mathbf{j} + (120\text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ 510 & -840 & 120 \end{vmatrix} = (1008\text{ N}\cdot\text{m})\mathbf{i} - (4284\text{ N}\cdot\text{m})\mathbf{k}$$

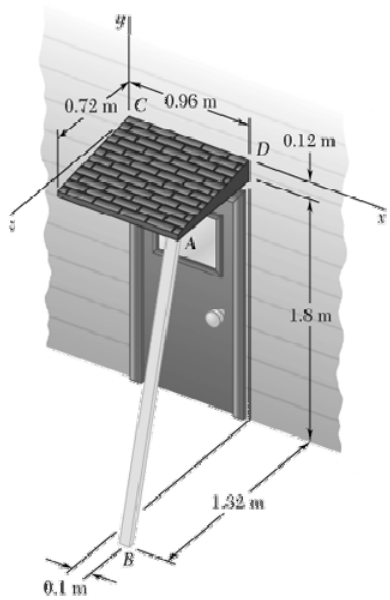
and $$|\mathbf{M}_O| = \sqrt{(1008)^2 + (4284)^2} = 4401.0\text{ N}\cdot\text{m}$$

$$\therefore\ 4401.0\text{ N}\cdot\text{m} = (990\text{ N})d$$

$$d = 4.4454\text{ m}$$

or $d = 4.45$ m ◀

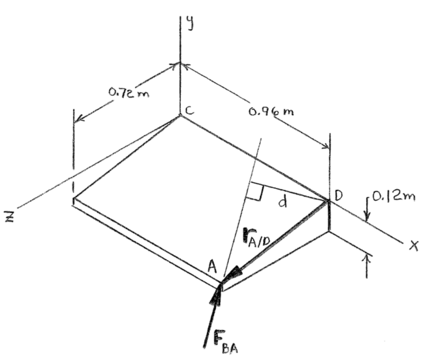
PROBLEM 3.29



In Problem 3.24, determine the perpendicular distance from point D to a line drawn through points A and B.

Problem 3.24: A wooden board AB, which is used as a temporary prop to support a small roof, exerts at point A of the roof a 228 N force directed along BA. Determine the moment about C of that force.

SOLUTION



Have $$|\mathbf{M}_D| = F_{BA}d$$

where d = perpendicular distance from D to line AB.

$$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/D} = -(0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k})}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} - (8.64\text{ N}\cdot\text{m})\mathbf{j} - (1.44\text{ N}\cdot\text{m})\mathbf{k}$$

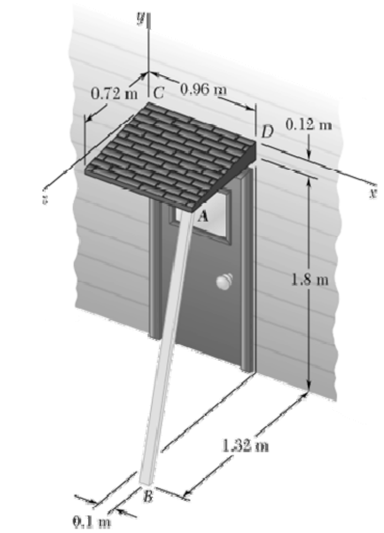
and $$|\mathbf{M}_D| = \sqrt{(146.88)^2 + (8.64)^2 + (1.44)^2} = 147.141\text{ N}\cdot\text{m}$$

$$\therefore\ 147.141\text{ N}\cdot\text{m} = (228\text{ N})d$$

$$d = 0.64536\text{ m}$$

or $d = 0.645\text{ m}$ ◀

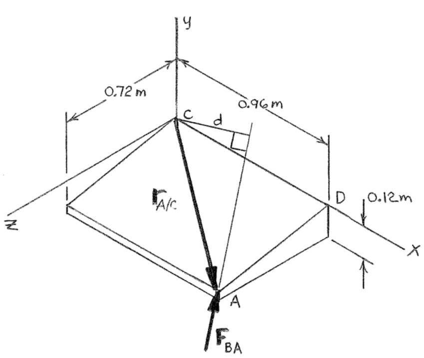
PROBLEM 3.30



In Problem 3.24, determine the perpendicular distance from point C to a line drawn through points A and B.

Problem 3.24: A wooden board AB, which is used as a temporary prop to support a small roof, exerts at point A of the roof a 228 N force directed along BA. Determine the moment about C of that force.

SOLUTION



Have $$|\mathbf{M}_C| = F_{BA}d$$

where d = perpendicular distance from C to line AB.

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/C} = (0.96\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{\left(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6)\mathbf{k}\right)}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} - (60.48\text{ N}\cdot\text{m})\mathbf{j} + (205.92\text{ N}\cdot\text{m})\mathbf{k}$$

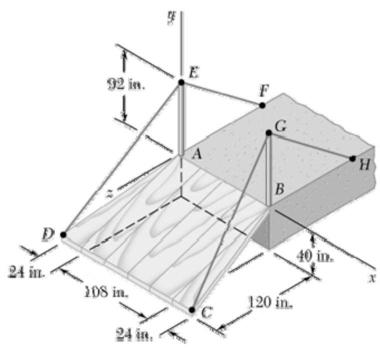
and $$|\mathbf{M}_C| = \sqrt{(146.88)^2 + (60.48)^2 + (205.92)^2} = 260.07\text{ N}\cdot\text{m}$$

$$\therefore\ 260.07\text{ N}\cdot\text{m} = (228\text{ N})d$$

$$d = 1.14064\text{ m}$$

or $d = 1.141\text{ m}$ ◀

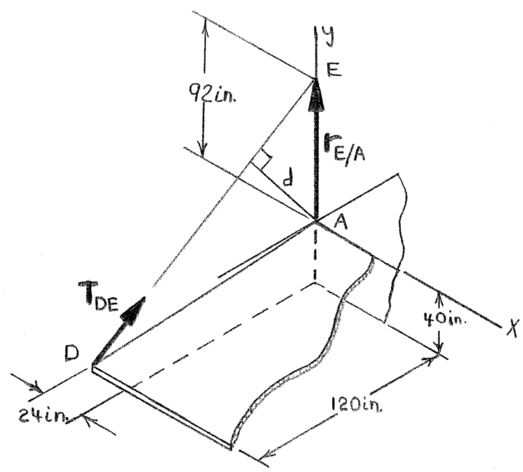
PROBLEM 3.31



In Problem 3.25, determine the perpendicular distance from point *A* to portion *DE* of cable *DEF*.

Problem 3.25: The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



Have $$|\mathbf{M}_A| = T_{DE}d$$

where d = perpendicular distance from A to line DE.

$$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

$$\mathbf{r}_{E/A} = (92 \text{ in.})\,\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE}T_{DE} = \frac{(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= (48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= -(22{,}080 \text{ lb} \cdot \text{in.})\mathbf{i} - (4416 \text{ lb} \cdot \text{in.})\mathbf{k}$$

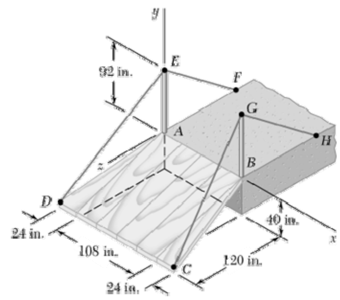
PROBLEM 3.31 CONTINUED

and $$|\mathbf{M}_A| = \sqrt{(22,080)^2 + (4416)^2} = 22,517 \text{ lb}\cdot\text{in.}$$

$$\therefore\ 22,517 \text{ lb}\cdot\text{in.} = (360 \text{ lb})d$$

$$d = 62.548 \text{ in.}$$

or $d = 5.21 \text{ ft}$ ◀

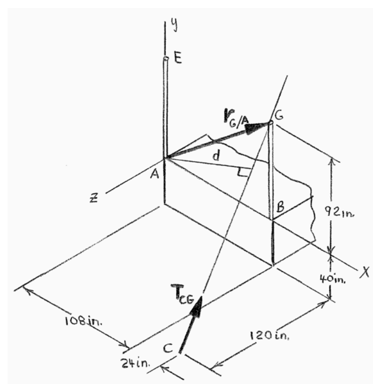


PROBLEM 3.32

In Problem 3.25, determine the perpendicular distance from point *A* to a line drawn through points *C* and *G*.

Problem 3.25: The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



Have $$|\mathbf{M}_A| = T_{CG}d$$

where d = perpendicular distance from A to line CG.

$$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

$$\mathbf{r}_{G/A} = (108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{CG} = \lambda_{CG}T_{CG}$$

$$= \frac{-(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= -(48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 108 & 92 & 0 \\ -48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= -(22,080 \text{ lb}\cdot\text{in.})\mathbf{i} + (25,920 \text{ lb}\cdot\text{in.})\mathbf{j} + (32,928 \text{ lb}\cdot\text{in.})\mathbf{k}$$

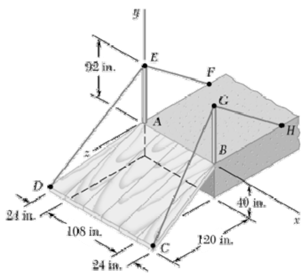
and $$|\mathbf{M}_A| = \sqrt{(22,080)^2 + (25,920)^2 + (32,928)^2} = 47,367 \text{ lb}\cdot\text{in.}$$

$$\therefore 47,367 \text{ lb}\cdot\text{in.} = (360 \text{ lb})d$$

$$d = 131.575 \text{ in.}$$

or $d = 10.96$ ft ◀

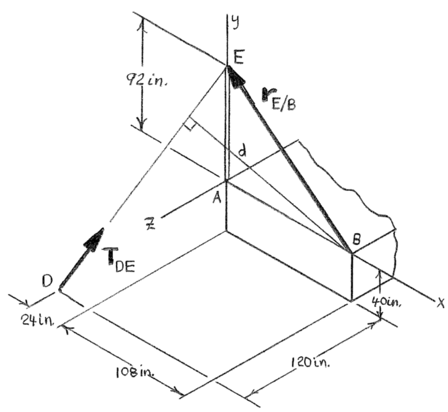
PROBLEM 3.33



In Problem 3.25, determine the perpendicular distance from point B to a line drawn through points D and E.

Problem 3.25: The ramp $ABCD$ is supported by cables at corners C and D. The tension in each of the cables is 360 lb. Determine the moment about A of the force exerted by (a) the cable at D, (b) the cable at C.

SOLUTION



Have $$|\mathbf{M}_B| = T_{DE} d$$

where $$d = \text{perpendicular distance from } B \text{ to line } DE.$$

$$\mathbf{M}_B = \mathbf{r}_{E/B} \times \mathbf{T}_{DE}$$

$$\mathbf{r}_{E/B} = -(108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE} = \frac{(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= (48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore\ \mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -108 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{lb} \cdot \text{in.}$$

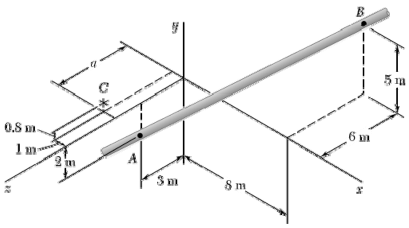
$$= -(22{,}080 \text{ lb} \cdot \text{in.})\mathbf{i} - (25{,}920 \text{ lb} \cdot \text{in.})\mathbf{j} - (32{,}928 \text{ lb} \cdot \text{in.})\mathbf{k}$$

and $$|\mathbf{M}_B| = \sqrt{(22{,}080)^2 + (25{,}920)^2 + (32{,}928)^2} = 47{,}367 \text{ lb} \cdot \text{in.}$$

$$\therefore\ 47{,}367 \text{ lb} \cdot \text{in.} = (360 \text{ lb})d$$

$$d = 131.575 \text{ in.}$$

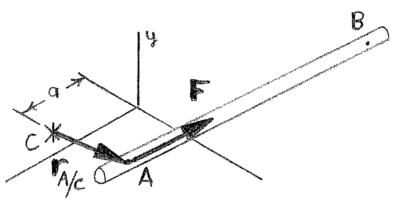
or $d = 10.96$ ft ◀



PROBLEM 3.34

Determine the value of a which minimizes the perpendicular distance from point C to a section of pipeline that passes through points A and B.

SOLUTION



Assuming a force $\mathbf{F}$ acts along AB,

$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

where

$$d = \text{perpendicular distance from } C \text{ to line } AB$$

$$\mathbf{F} = \lambda_{AB} F = \frac{(8\text{ m})\mathbf{i} + (7\text{ m})\mathbf{j} - (9\text{ m})\mathbf{k}}{\sqrt{(8)^2 + (7)^2 + (9)^2}\text{ m}} F$$

$$= F(0.57437)\mathbf{i} + (0.50257)\mathbf{j} - (0.64616)\mathbf{k}$$

$$\mathbf{r}_{A/C} = (1\text{ m})\mathbf{i} - (2.8\text{ m})\mathbf{j} - (a - 3\text{ m})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2.8 & 3 - a \\ 0.57437 & 0.50257 & -0.64616 \end{vmatrix} F$$

$$= \left[(0.30154 + 0.50257a)\mathbf{i} + (2.3693 - 0.57437a)\mathbf{j}\right.$$

$$\left. + 2.1108\mathbf{k}\right]F$$

Since $\quad |\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}^2|} \quad$ or $\quad |\mathbf{r}_{A/C} \times \mathbf{F}^2| = (dF)^2$

$$\therefore\ (0.30154 + 0.50257a)^2 + (2.3693 - 0.57437a)^2 + (2.1108)^2 = d^2$$

Setting $\dfrac{d}{da}(d^2) = 0$ to find a to minimize d

$$2(0.50257)(0.30154 + 0.50257a)$$

$$+ 2(-0.57437)(2.3693 - 0.57437a) = 0$$

Solving $\qquad a = 2.0761\text{ m}$

or $\quad a = 2.08\text{ m}$ ◀

PROBLEM 3.35

Given the vectors $\mathbf{P} = 7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{Q} = -3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$, and $\mathbf{S} = 8\mathbf{i} + \mathbf{j} - 9\mathbf{k}$, compute the scalar products $\mathbf{P}\cdot\mathbf{Q}$, $\mathbf{P}\cdot\mathbf{S}$, and $\mathbf{Q}\cdot\mathbf{S}$.

SOLUTION

$$\mathbf{P}\cdot\mathbf{Q} = (7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\cdot(-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$$

$$= (7)(-3) + (-2)(-4) + (5)(6)$$

$$= 17$$

or $\mathbf{P}\cdot\mathbf{Q} = 17$ ◀

$$\mathbf{P}\cdot\mathbf{S} = (7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\cdot(8\mathbf{i} + \mathbf{j} - 9\mathbf{k})$$

$$= (7)(8) + (-2)(1) + (5)(-9)$$

$$= 9$$

or $\mathbf{P}\cdot\mathbf{S} = 9$ ◀

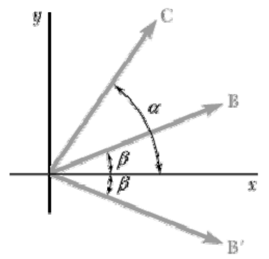
$$\mathbf{Q}\cdot\mathbf{S} = (-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})\cdot(8\mathbf{i} + \mathbf{j} - 9\mathbf{k})$$

$$= (-3)(8) + (-4)(1) + (6)(-9)$$

$$= -82$$

or $\mathbf{Q}\cdot\mathbf{S} = -82$ ◀

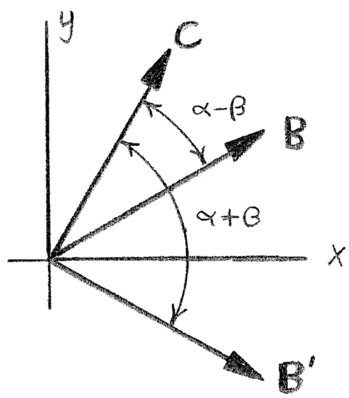
PROBLEM 3.36



Form the scalar products $\mathbf{B} \cdot \mathbf{C}$ and $\mathbf{B}' \cdot \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\cos\alpha \, \cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta).$$

SOLUTION



By definition

$$\mathbf{B} \cdot \mathbf{C} = BC\cos(\alpha - \beta)$$

where $$\mathbf{B} = B[(\cos\beta)\mathbf{i} + (\sin\beta)\mathbf{j}]$$

$$\mathbf{C} = C[(\cos\alpha)\mathbf{i} + (\sin\alpha)\mathbf{j}]$$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (B\sin\beta)(C\sin\alpha) = BC\cos(\alpha-\beta)$$

or $$\cos\beta\cos\alpha + \sin\beta\sin\alpha = \cos(\alpha-\beta) \qquad (1)$$

By definition

$$\mathbf{B}' \cdot \mathbf{C} = BC\cos(\alpha + \beta)$$

where $$\mathbf{B}' = [(\cos\beta)\mathbf{i} - (\sin\beta)\mathbf{j}]$$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (-B\sin\beta)(C\sin\alpha) = BC\cos(\alpha+\beta)$$

or $$\cos\beta\cos\alpha - \sin\beta\sin\alpha = \cos(\alpha+\beta) \qquad (2)$$

Adding Equations (1) and (2),

$$2\cos\beta\cos\alpha = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

or $$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \blacktriangleleft$$