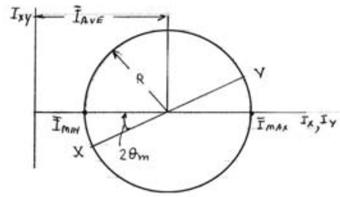
PROBLEM 9.101

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.74

SOLUTION



From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166\times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 0.453\times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = -0.1596\times 10^6 \text{ mm}^4$$

Define points $\quad X\left(0.166, -0.1596\right)\times 10^6 \text{ mm}^4 \quad$ and $\quad Y\left(0.453, -0.1596\right)\times 10^6 \text{ mm}^4$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(0.166 + 0.453\right)\times 10^6 \text{ mm}^4$$

$$= 0.3095\times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{(0.166 - 0.453)10^6}{2}\right]^2 + \left(-0.1596\times 10^6\right)^2}$$

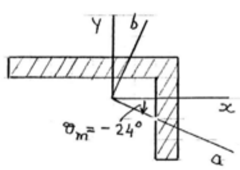
$$= 0.21463\times 10^6 \text{ mm}^4$$

Also
$$2\theta_m = \tan^{-1}\left(\frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}\right) = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04^\circ$$

$$\theta_m = -24.02^\circ$$

or $\theta = -24.0^\circ$ clockwise ◀

Then
$$\bar{I}_{\max,\ \min} = \bar{I}_{\text{ave}} \pm R = \left(0.3095 \pm 0.21463\right)\times 10^6 \text{ mm}^4$$



or $\bar{I}_{\max} = 0.524\times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.0949\times 10^6 \text{ mm}^4$ ◀

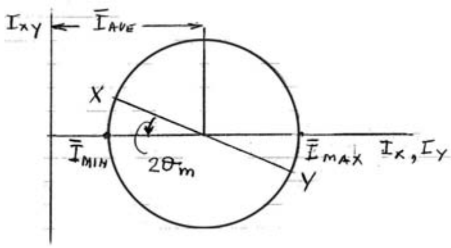
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

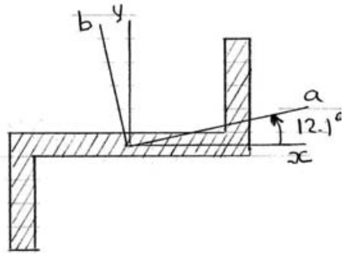
PROBLEM 9.102

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.75

SOLUTION





From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 7.728 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.70134 + 7.728) \times 10^6 \text{ mm}^4 = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{(0.70134 - 7.728) \times 10^6}{2}\right]^2 + \left(1.5732 \times 10^6\right)^2}$$

$$= 3.8495 \times 10^6 \text{ mm}^4$$

Define points $$X\,(0.70134,\ 15732) \times 10^6 \text{ mm}$$

$$Y\,(7.728,\ -1.5732) \times 10^6 \text{ mm}$$

Also $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.122°,\ \theta_m = 12.06°$$

or $\theta_m = 12.06°$ counterclockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (4.2147 \pm 3.8495) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 8.06 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.365 \times 10^6 \text{ mm}^4$ ◀

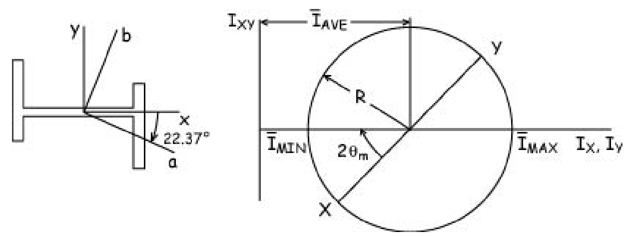
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

PROBLEM 9.103

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.71

SOLUTION



From Problem 9.71 $\quad \bar{I}_{xy} = -11.0 \text{ in}^4$

Compute $\bar{I}_x$ and $\bar{I}_y$ for area of Problem 9.71

$$\bar{I}_x = \frac{5 \text{ in.} \times (0.5 \text{ in.})^3}{12} + 2\left[\frac{(0.5 \text{ in.})(4 \text{ in.})^3}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(1.0 \text{ in.})^2\right]$$

$$= 9.38542 \text{ in}^4$$

$$\bar{I}_y = 2\left[\frac{(0.5 \text{ in.})^3(4 \text{ in.})}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(2.75 \text{ in.})^2\right] + \frac{0.5 \text{ in.} \times (5 \text{ in.})^3}{12}$$

$$= 35.54167 \text{ in}^4$$

Define points $\quad X(9.38542, \; -11), \quad$ and $\quad Y(35.54167, \; 11)$

Now
$$I_{\text{ave}} = \frac{\bar{I}_x + \bar{I}_y}{2} = \frac{9.38542 \text{ in}^4 + 35.54167 \text{ in}^4}{2} = 22.46354 \text{ in}^4$$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2} = \sqrt{\left(\frac{9.38542 - 35.54167}{2}\right)^2 + (11.0)^2}$$

$$= 17.08910 \text{ in}^4$$

Also
$$2\theta_m = \tan^{-1}\left[\frac{-2(-11.0)}{9.38542 - 35.54167}\right] = -40.067$$

or $\theta_m = -20.033°$ clockwise ◀

Then
$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 22.46354 \pm 17.08910$$

$$= 39.55264, \; 5.37444$$

or $\bar{I}_{\text{max}} = 39.55 \text{ in}^4$ ◀

$\bar{I}_{\text{min}} = 5.37 \text{ in}^4$ ◀

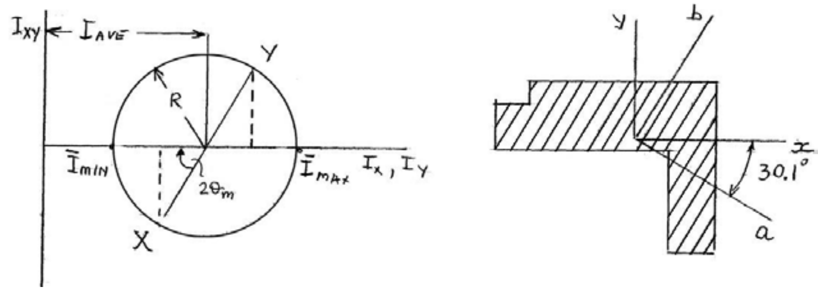
Note: The *a* axis corresponds to $\bar{I}_{\text{min}}$ and *b* axis corresponds to $\bar{I}_{\text{max}}$.

PROBLEM 9.104

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.77

SOLUTION



From Problems 9.44 and 9.77

$$\bar{I}_x = 432.59 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 732.97 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = -261.63 \times 10^6 \text{ mm}^4$$

Define points $$X(432.59, \ -261.63) \times 10^6 \text{ mm}^4$$

$$Y(732.97, \ 261.63) \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(432.59 + 732.97) \times 10^6 = 582.78 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \frac{1}{2}\sqrt{\left(\frac{432.59 - 732.97}{2} \times 10^6\right)^2 + \left(-261.63 \times 10^6\right)^2}$$

$$= 301.67 \times 10^6 \text{ mm}^4$$

Also $$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = \frac{-2(-261.63) \times 10^6}{(432.59 - 732.97) \times 10^6} = -60.14°$$

or $$\theta_m = -30.1° \text{ clockwise} \blacktriangleleft$$

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (582.78 \pm 301.67) \times 10^6 \text{ mm}^4$$

or $$\bar{I}_{\text{max}} = 884 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

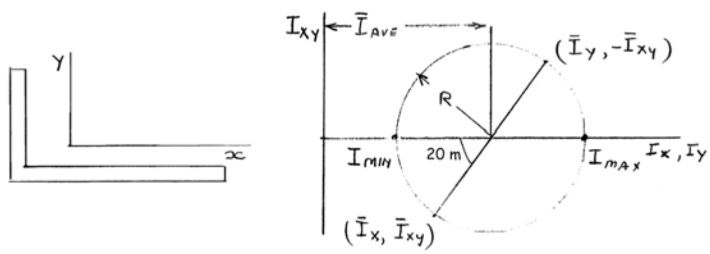
and $$\bar{I}_{\text{min}} = 281 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

PROBLEM 9.105

The moments and product of inertia for an L102 × 76 × 6.4-mm angle cross section with respect to two rectangular axes x and y through C are, respectively, $\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$, $\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$, and $\bar{I}_{xy} < 0$, with the minimum value of the moment of inertia of the area with respect to any axis through C being $\bar{I}_{\min} = 0.051 \times 10^6 \text{ mm}^4$. Using Mohr's circle, determine (*a*) the product of inertia $\bar{I}_{xy}$ of the area, (*b*) the orientation of the principal axes, (*c*) the value of $\bar{I}_{\max}$.

SOLUTION



Given: $\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$, $\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$ and $\bar{I}_{xy} < 0$

Note: A review of a table of rolled-steel shapes reveals that the given values of $\bar{I}_x$ and $\bar{I}_y$ are obtained when the 102 mm leg of the angle is parallel to the x axis. For $\bar{I}_{xy} < 0$ the angle must be oriented as shown.

(*a*) Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.166 + 0.453) \times 10^6 \text{ mm}^4$$
$$= 0.3095 \times 10^6 \text{ mm}^4$$

Now
$$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \qquad \text{or} \qquad R = \bar{I}_{\text{ave}} - \bar{I}_{\min}$$

Then
$$R = (0.3095 - 0.051) \times 10^6 \text{ mm}^4$$
$$= 0.2585 \times 10^6 \text{ mm}^4$$

From
$$R^2 = \left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2$$

$$\bar{I}_{xy} = \sqrt{\left[(0.2585)^2 - \left(\frac{0.166 - 0.453}{2}\right)^2\right]} \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = \pm 0.21501 \times 10^6 \text{ mm}^4$$

Since $\bar{I}_{xy} < 0$, $\bar{I}_{xy} = -0.21501 \times 10^6 \text{ mm}^4$ or $\bar{I}_{xy} = -0.215 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.105 CONTINUED

(*b*)

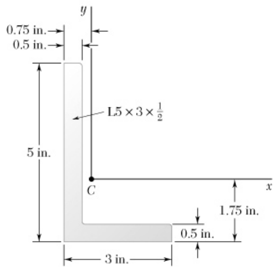
$$2\theta_m = \tan^{-1}\left[\frac{-2(-0.21501)}{0.166 - 0.453}\right] = -56.28°$$

or $\theta_m = -28.1$ clockwise ◀

(*c*)

$$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = (0.3095 + 0.2585) \times 10^6 \text{ mm}^4$$

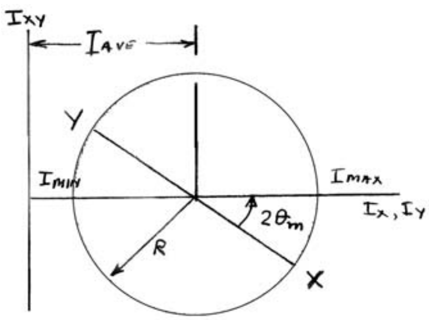
or $\bar{I}_{\max} = 0.568 \times 10^6 \text{ mm}^4$ ◀



PROBLEM 9.106

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Fig. 9.13.)

SOLUTION



From Figure 9.13 $\quad \bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\quad \bar{I}_{xy} = -2.81 \text{ in}^4$

The Mohr's circle is defined by the diameter XY where

$$X\left(9.45,\ -2.81\right) \text{ in}^4$$

$$Y\left(2.58,\ 2.81\right) \text{ in}^4$$

Now $\quad \bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(9.45 \text{ in}^4 + 2.58 \text{ in}^4\right)$

$$= 6.015 \text{ in}^4$$

and $\quad R = \sqrt{\left[\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\right]^2 + \bar{I}_{xy}^2}$

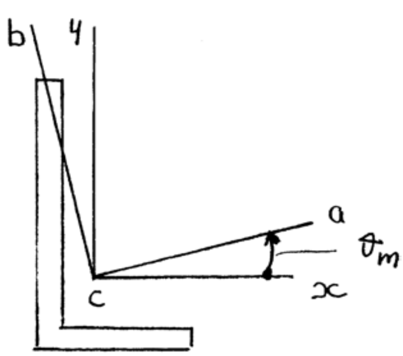
$$= \sqrt{\frac{1}{2}\left(9.45 \text{ in}^4 - 2.58 \text{ in}^4\right)^2 + \left(2.81 \text{ in}^4\right)^2}$$

$$= 5.612 \text{ in}^4$$

$$\tan 2\theta_m = \frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = \frac{-2\left(-2.81 \text{ in}^4\right)}{9.45 \text{ in}^4 - 2.58 \text{ in}^4} = 0.81805$$

or $\quad 2\theta_m = 32.285°$

or $\theta_m = 19.643°$ counterclockwise ◀



About C.

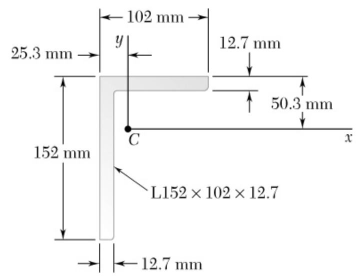
Now $\quad \bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = \left(6.015 \pm 5.612\right)\text{in}^4$

or $\bar{I}_{\text{max}} = 11.63 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 0.403 \text{ in}^4$ ◀

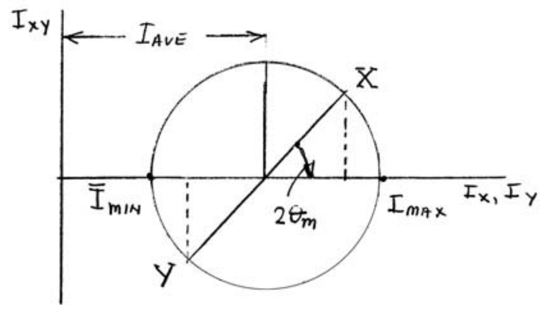
From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{max}}$ and the b axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.107



Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Fig. 9.13.)

SOLUTION



From Figure 9.13B: $\bar{I}_x = 7.20 \times 10^6 \text{ mm}^4, \quad \bar{I}_y = 2.64 \times 10^6 \text{ mm}^4$

Have $\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2,$ where $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$ and $\bar{I}_{x'y'} = 0$

Now $\bar{x}_1 = \dfrac{102}{2} - 25.3 = 25.7 \text{ mm}, \quad \bar{y}_1 = 50.3 - \dfrac{12.7}{2} = 43.95 \text{ mm}$

$$A_1 = 102 \times 12.7 = 1295.4 \text{ mm}^2$$

$$\bar{x}_2 = -25.3 - \frac{12.7}{2} = -18.95 \text{ mm} \qquad \bar{y}_2 = -\left[\frac{1}{2}(152 - 12.7) - (50.3 - 12.7)\right] = 32.05 \text{ mm}$$

$$A_2 = (12.7)(152 - 12.7) = 1769.11 \text{ mm}^2$$

Then $\bar{I}_{xy} = \left\{\left[(25.7 \text{ mm})(43.95 \text{ mm})\left(1295.4 \text{ mm}^2\right)\right] + \left[(-18.95 \text{ mm})(-32.05 \text{ mm})\left(1769.11 \text{ mm}^2\right)\right]\right\} \times 10^6$

$$= (1.46317 + 1.07446) \times 10^6 \text{ mm}^4 = 2.5376 \times 10^6 \text{ mm}^4$$

The Mohr's circle is defined by points X and Y, where

$$X\left(\bar{I}_x, \bar{I}_{xy}\right), \; Y\left(\bar{I}_y, -\bar{I}_{xy}\right)$$

Now $\bar{I}_{\text{ave}} = \dfrac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \dfrac{1}{2}(7.20 + 2.64) \times 10^6 \text{ mm}^4 = 4.92 \times 10^6 \text{ mm}^4$

PROBLEM 9.107 CONTINUED

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right) + \bar{I}_{xy}^2} = \left[\sqrt{\frac{1}{2}(7.20 - 2.64)^2 + 2.5376^2}\right] \times 10^6 \text{ mm}^4$$

$$= 3.4114 \times 10^6 \text{ mm}^4$$

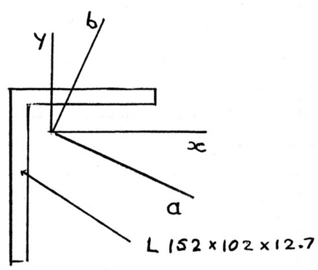
$$\tan\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.5376)}{(7.20 - 2.64)} = -1.11298, \qquad 2\theta = -48.0607°$$

or $\theta = -24.0°$ clockwise ◀

Now $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (4.92 \pm 3.4114) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 8.33 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 1.509 \times 10^6 \text{ mm}^4$ ◀



Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{max}}$ and the *b* axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.108

For a given area the moments of inertia with respect to two rectangular centroidal x and y axes are $\bar{I}_x = 640 \text{ in}^4$ and $\bar{I}_y = 280 \text{ in}^4$, respectively. Knowing that after rotating the x and y axes about the centroid 60° clockwise the product of inertia relative to the rotated axes is -180 in^4, use Mohr's circle to determine (*a*) the orientation of the principal axes, (*b*) the centroidal principal moments of inertia.

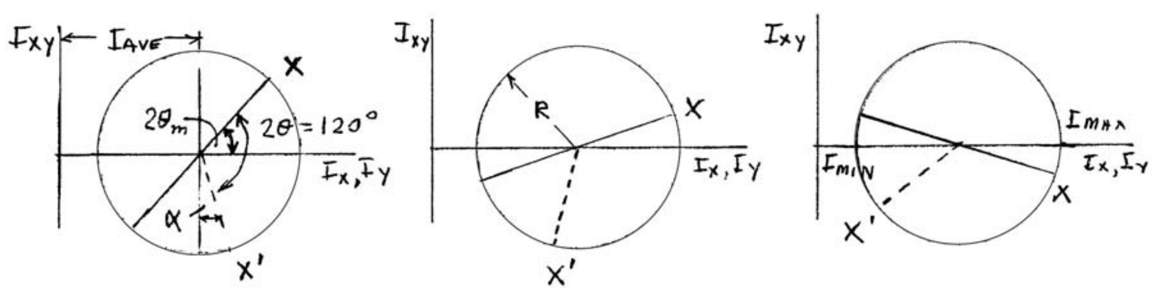
SOLUTION

Have $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(640 \text{ in}^4 + 280 \text{ in}^4\right) = 460 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = \frac{1}{2}\left(640 \text{ in}^4 - 280 \text{ in}^4\right) = 180 \text{ in}^4$$

Also have $$\bar{I}_{x'y'} = -180 \text{ in}^4, \qquad 2\theta = -120°, \qquad I_x > I_y$$

Letting the points $\left(\bar{I}_x, \bar{I}_{xy}\right)$ and $\left(\bar{I}_{x'}, \bar{I}_{x'y'}\right)$ be denoted by X an X', respectively, three possible Mohr's circles can be constructed



Assume the first case applies

Then $$\frac{\bar{I}_x - \bar{I}_y}{2} = R\cos 2\theta_m \qquad \text{or} \qquad R\cos 2\theta_m = 180 \text{ in}^4$$

Also $$\left|\bar{I}_{x'y'}\right| = R\cos\alpha \qquad \text{or} \qquad R\cos\alpha = 180 \text{ in}^4$$

$$\therefore \quad \alpha = \pm 2\theta_m$$

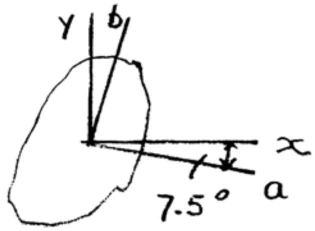
Also have $$120° = 2\theta_m + \left(90° - \alpha\right) \qquad \text{or} \qquad 2\theta_m - \alpha = 30°$$

$$\therefore \quad \alpha = -2\theta_m \qquad \text{and} \qquad 2\left(2\theta_m\right) = 30° \qquad \text{or} \qquad 2\theta_m = |\alpha| = 15°$$

Note $$\left.\begin{aligned} 2\theta_m &> 0 \\ \alpha &< 0 \end{aligned}\right\} \text{ implies case 2 applies}$$

PROBLEM 9.108 CONTINUED

(a) Therefore, $\theta_m = 7.5°$ clockwise ◀



(b) Have $R\cos 15° = 180$ or $R = 186.35 \text{ in}^4$

Then $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 460 \pm 186.350$

or $\bar{I}_{\text{max}} = 646 \text{ in}^4$ ◀

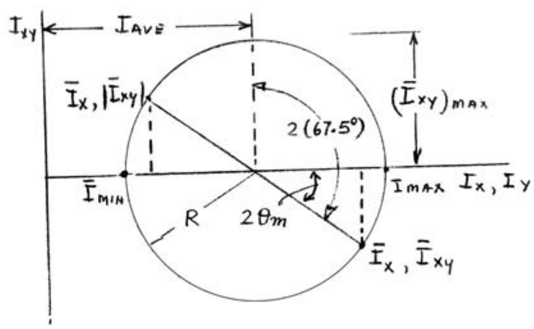
and $\bar{I}_{\text{min}} = 274 \text{ in}^4$ ◀

Note: From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{max}}$ and the b axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.109

It is known that for a given area $\bar{I}_y = 300 \text{ in}^4$ and $\bar{I}_{xy} = -125 \text{ in}^4$, where the x and y axes are rectangular centroidal axes. If the axis corresponding to the maximum product of inertia is obtained by rotating the x axis 67.5° counterclockwise about C, use Mohr's circle to determine (a) the moment of inertia $\bar{I}_x$ of the area, (b) the principal centroidal moments of inertia.

SOLUTION



First assume $\bar{I}_x > \bar{I}_y$

(Note: Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to the $\left(\bar{I}_{xy}\right)_{\max}$ is obtained by rotating the x axis through $67.5°$ counterclockwise)

From Mohr's circle have $$2\theta_m = 2(67.5°) - 90° = 45°$$

(a) From $$\tan 2\theta_m = \frac{2\left|\bar{I}_{xy}\right|}{\bar{I}_x - \bar{I}_y}$$

Have $$\bar{I}_x = \bar{I}_y + 2\frac{\left|\bar{I}_{xy}\right|}{\tan 2\theta_m} = 300 \text{ in}^4 + 2\frac{125 \text{ in}^4}{\tan 45°} = 550 \text{ in}^4$$

or $\bar{I}_x = 550 \text{ in}^4$ ◀

(b) Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{550 + 300}{2} \text{ in}^4 = 425 \text{ in}^4$$

and $$R = \frac{\left|\bar{I}_{xy}\right|}{\sin 2\theta_m} = \frac{125 \text{ in}^4}{\sin 45°} = 176.78 \text{ in}^4$$

Then $$\bar{I}_{\max,\ \min} = \bar{I}_{\text{ave}} \pm R = (425 \pm 176.76)\text{in}^4$$
$$= (601.78,\ 248.22)\text{in}^4$$

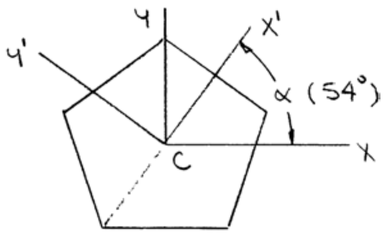
or $\bar{I}_{\max} = 602 \text{ in}^4$ ◀

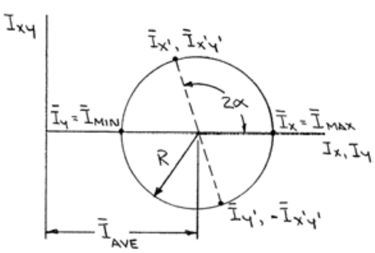
and $\bar{I}_{\min} = 248 \text{ in}^4$ ◀

PROBLEM 9.110

Using Mohr's circle, show that for any regular polygon (such as a pentagon) (*a*) the moment of inertia with respect to every axis through the centroid is the same, (*b*) the product of inertia with respect to every pair of rectangular axes through the centroid is zero.

SOLUTION





Consider the regular pentagon shown, with centroidal axes *x* and *y*.

Because the *y* axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0.$ Since $\bar{I}_{xy} = 0,$ the *x* and *y* axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min},$ the Mohr's circle is then drawn as shown.

Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'},$ and product of inertia, $\bar{I}_{x'y'},$ are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0.$ For this to be possible on the Mohr's circle, the radius *R* must be equal to zero (thus, the circle degenerates into a point). With $R = 0,$ it immediately follows that

(*a*) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through *C*) ◀

(*b*) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at *C*)◀