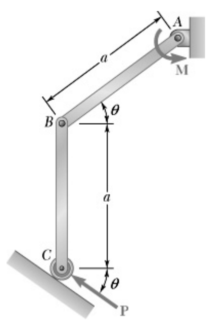
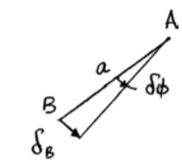
PROBLEM 10.21



For the linkage shown, determine the force **P** required for equilibrium when $a = 18\text{ in.}$, $M = 240\text{ lb}\cdot\text{in.}$, and $\theta = 30°$.

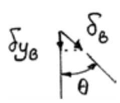
SOLUTION

Consider a virtual counterclockwise rotation $\delta\phi$ of link AB.



Then $$\delta_B = a\delta\phi$$

Note that



$$\delta y_B = \delta_B \cos\theta$$

$$= a\cos\theta\,\delta\phi$$

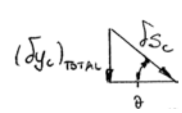
If the incline were removed, point C would move down δy_C as a result of the virtual rotation, where

$$\delta y_C = \delta y_B = a\cos\theta\,\delta\phi$$

For the roller to remain on the incline, the vertical link BC would then have to rotate counterclockwise. Thus, to first order:

$$(\delta y_C)_{\text{total}} \approx \delta y_C$$

Then $$\delta S_C = \frac{(\delta y_C)_{\text{total}}}{\sin\theta}$$



$$= \frac{a\cos\theta\,\delta\phi}{\sin\theta}$$

$$= \frac{a}{\tan\theta}\delta\phi$$

Now, by Virtual Work: $$\delta U = 0: \quad M\delta\phi - P\delta S_C = 0$$

or $$M\delta\phi - P\left(\frac{a}{\tan\theta}\delta\phi\right) = 0$$

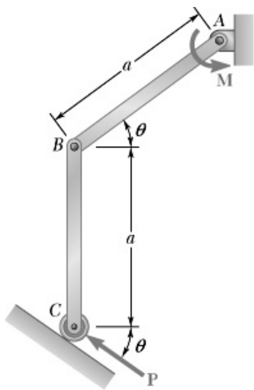
or $$M\tan\theta = Pa$$

With $$M = 240\text{ lb}\cdot\text{in.},\ a = 18\text{ in., and } \theta = 30°$$

$$(240\text{ lb}\cdot\text{in.})\tan 30° = P(18\text{ in.})$$

or $\mathbf{P} = 7.70\text{ lb} \searrow 30.0°$ ◀

PROBLEM 10.22



For the linkage shown, determine the couple **M** required for equilibrium when $a = 2$ ft, $P = 30$ lb, and $\theta = 40°$.

SOLUTION

From the analysis of Problem 10.21, $M \tan\theta = Pa$

Now, with $P = 30$ lb, $a = 2$ ft, and $\theta = 40°$, we have

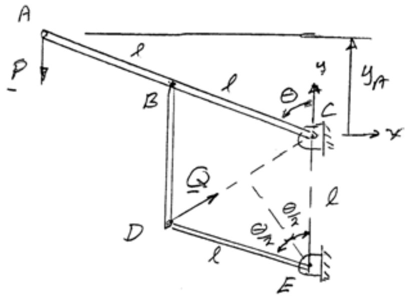
$$M \tan 40° = (30 \text{ lb})(2 \text{ ft})$$

or $\mathbf{M} = 71.5 \text{ lb}\cdot\text{ft}$ ◀

PROBLEM 10.23

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.10 when $P = 60$ lb and $Q = 75$ lb.

SOLUTION



From geometry

$$y_A = 2l\cos\theta, \qquad \delta y_A = -2l\sin\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or

$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)}$$

With

$$P = 60\text{ lb}, \qquad Q = 75\text{ lb}$$

$$(75\text{ lb}) = 2(60\text{ lb})\frac{\sin\theta}{\cos(\theta/2)}$$

$$\frac{\sin\theta}{\cos(\theta/2)} = 0.625$$

or

$$\frac{2\sin(\theta/2)\cos(\theta/2)}{\cos(\theta/2)} = 0.625$$

$$\theta = 36.42°$$

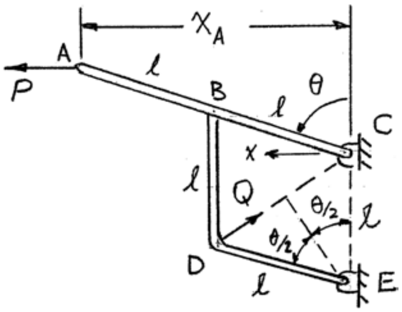
$$\theta = 36.4° \blacktriangleleft$$

(Additional solutions discarded as not applicable are $\theta = \pm 180°$)

PROBLEM 10.24

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.11 when $P = 20$ lb and $Q = 25$ lb.

SOLUTION



$$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}$$

$$\delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or
$$Q = 2P\frac{\cos\theta}{\cos(\theta/2)}$$

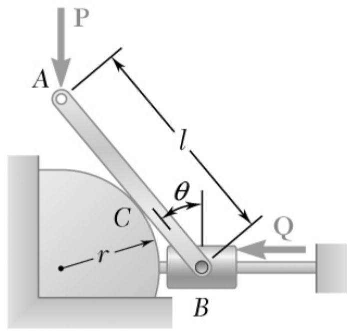
With $\quad P = 20 \text{ lb} \quad$ and $\quad Q = 25 \text{ lb}$

$$(25 \text{ lb}) = 2(20 \text{ lb})\frac{\cos\theta}{\cos(\theta/2)}$$

or
$$\frac{\cos\theta}{\cos(\theta/2)} = 0.625$$

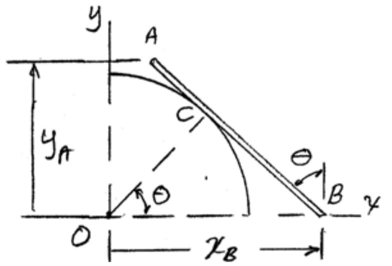
Solving numerically, $\quad \theta = 56.615° \qquad \theta = 56.6°$ ◀

PROBLEM 10.25



A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 300$ mm, $r = 90$ mm, $P = 60$ N, and $Q = 120$ N.

SOLUTION



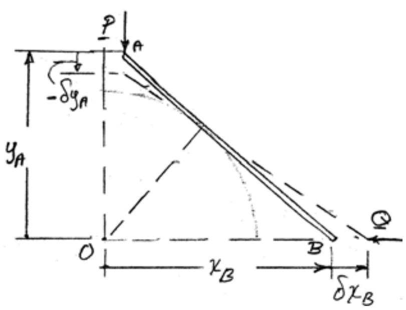
Geometry $\qquad OC = r$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \qquad \delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\,\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \tag{1}$$

Then, with $l = 300$ mm, $r = 90$ mm, $P = 60$ N, and $Q = 120$ N

$$\cos^2\theta = \frac{(120\text{ N})(90\text{ mm})}{(60\text{ N})(300\text{ mm})} = 0.6$$

or $\qquad \theta = 39.231°$ $\qquad \theta = 39.2°$ ◀

PROBLEM 10.26

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 280$ mm, $r = 100$ mm, $P = 300$ N, and $Q = 600$ N.

SOLUTION

From the analysis of Problem 10.25

$$\cos^2\theta = \frac{Qr}{Pl}$$

Then with $l = 280$ mm, $r = 100$ mm, $P = 300$ N, and $Q = 600$ N

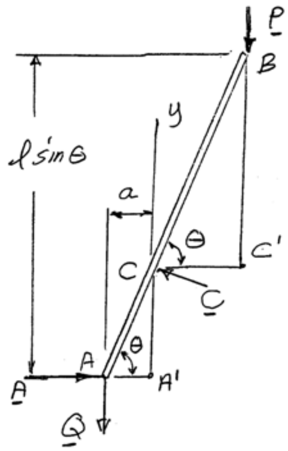
$$\cos^2\theta = \frac{(600\text{ N})(100\text{ mm})}{(300\text{ N})(280\text{ mm})} = 0.71429$$

or $$\theta = 32.311°$$ $\theta = 32.3°$ ◀

PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.12 when $l = 600$ mm, $a = 100$ mm, $P = 100$ N, and $Q = 160$ N.

SOLUTION



For $\Delta AA'C$: $\quad A'C = a\tan\theta$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\,\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

or $$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$$

With $\quad l = 600$ mm, $a = 100$ mm, $P = 100$ N, and $Q = 160$ N

$$(160\text{ N}) = (100\text{ N})\left(\frac{600\text{ mm}}{100\text{ mm}}\cos^3\theta - 1\right)$$

or $$\cos^3\theta = 0.4333$$

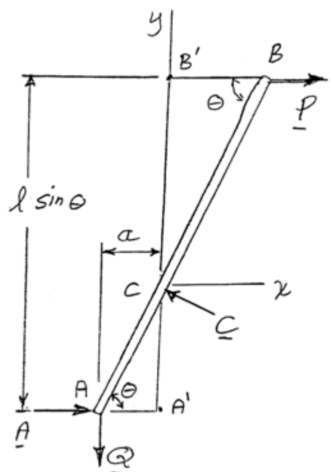
$$\theta = 40.82°$$

$\theta = 40.8°$ ◀

PROBLEM 10.28

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.13 when $l = 900$ mm, $a = 150$ mm $P = 75$ N, and $Q = 135$ N.

SOLUTION



For $\Delta AA'C$: $$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\,\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

$$Pl\sin\theta\cos^2\theta = Qa$$

or $$Q = P\frac{l}{a}\sin\theta\cos^2\theta$$

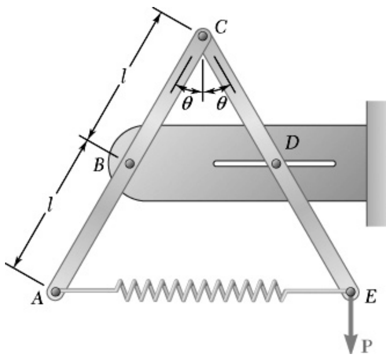
With $l = 900$ mm, $a = 150$ mm, $P = 75$ N, and $Q = 135$ N

$$135\text{ N} = (75\text{ N})\frac{900\text{ mm}}{150\text{ mm}}\sin\theta\cos^2\theta$$

or $$\sin\theta\cos^2\theta = 0.300$$

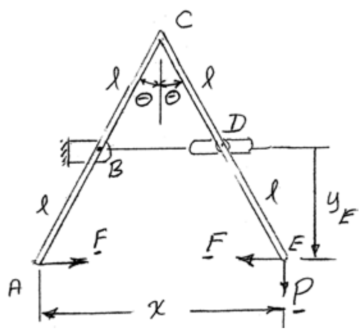
Solving numerically, $\theta = 19.81°$ and $51.9°$ ◀

PROBLEM 10.29



Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is k, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in $P, \theta,\ l$, and k that must be satisfied when the system is in equilibrium.

SOLUTION



$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

Spring:

$$\text{Unstretched length} = 2l$$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\,\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

Virtual Work:

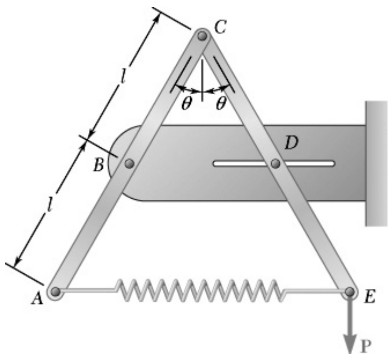
$$\delta U = 0: \quad P\delta y_E - F\delta x = 0$$

$$P(-l\sin\theta\,\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\,\delta\theta) = 0$$

$$-P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or $$\frac{P}{8kl} = (1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} \qquad \frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

PROBLEM 10.30



Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is 300 N/m, and the spring is unstretched when $\theta = 30°$. Knowing that $l = 200\text{ mm}$ and neglecting the mass of the rods, determine the value of θ corresponding to equilibrium when $P = 160\text{ N}$.

SOLUTION

From the analysis of Problem 10.29,

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

Then with

$$P = 160\text{ N},\ l = 0.2\text{ m, and } k = 300\text{ N/m}$$

$$\frac{160\text{ N}}{8(300\text{ N/m})(0.2\text{ m})} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{1}{3} = 0.3333$$

Solving numerically,

$$\theta = 24.98°$$

$\theta = 25.0°$ ◀