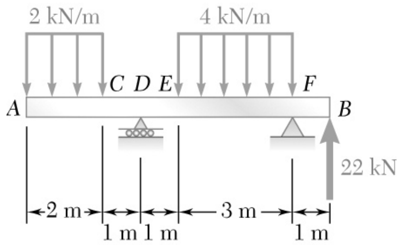
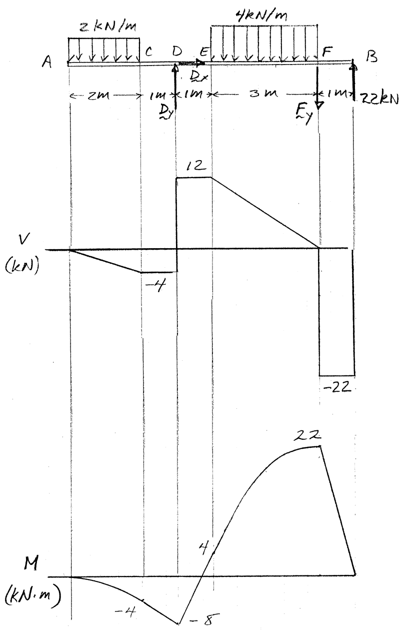


PROBLEM 7.69

Using the method of Sec. 7.6, solve Prob. 7.40.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_F = 0\text{: } (1\text{ m})(22\text{ kN}) + (1.5\text{ m})(4\text{ kN/m})(3\text{ m})$$

$$- (4\text{ m})D_y + (6\text{ m})(2\text{ kN/m})(2\text{ m}) = 0$$

$$\mathbf{D}_y = 16\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0\text{: } 16\text{ kN} + 22\text{ kN} - F_y - (2\text{ kN/m})(2\text{ m})$$

$$-(4\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{F}_y = 22\text{ kN} \downarrow$$

Shear Diag:

$V_A = 0$, then V is linear $\left(\dfrac{dV}{dx} = -2\text{ kN/m}\right)$ to C;

$$V_C = -2\text{ kN/m}(4\text{ m}) = -4\text{ kN}$$

V is constant to D, jumps 16 kN to 12 kN and is constant to E.

Then V is linear $\left(\dfrac{dV}{dx} = -4\text{ kN/m}\right)$ to F.

$$V_F = 12\text{ kN} - (4\text{ kN/m})(3\text{ m}) = 0.$$

V jumps to -22 kN at F, is constant to B, and returns to zero.

$$|V|_{\max} = 22.0\text{ kN} \blacktriangleleft$$

Moment Diag:

$M_A = 0$, M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$ to C.

$$M_C = -\frac{1}{2}(4\text{ kN})(2\text{ m}) = -4\text{ kN}\cdot\text{m}.$$

PROBLEM 7.69 CONTINUED

Then M is linear $\left(\dfrac{dM}{dx} = -4\text{ kN}\right)$ to D.

$$M_D = -4\text{ kN}\cdot\text{m} - (4\text{ kN})(1\text{ m}) = -8\text{ kN}\cdot\text{m}$$

From D to E, M is linear $\left(\dfrac{dM}{dx} = 12\text{ kN}\right)$, and

$$M_E = -8\text{ kN}\cdot\text{m} + (12\text{ kN})(1\text{m})$$

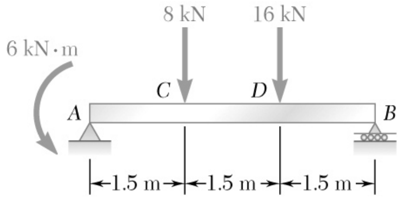
$$M_E = 4\text{ kN}\cdot\text{m}$$

M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with }V\right)$ to F.

$$M_F = 4\text{ kN}\cdot\text{m} + \frac{1}{2}(12\text{ kN})(3\text{ m}) = 22\text{ kN}\cdot\text{m}.$$

Finally, M is linear $\left(\dfrac{dM}{dx} = -22\text{ kN}\right)$, back to zero at B.

$$|M|_{\max} = 22.0\text{ kN}\cdot\text{m} ◀$$

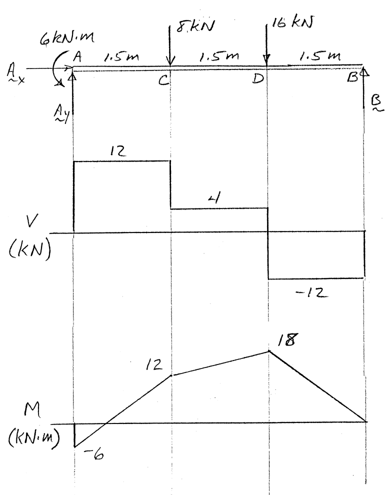


PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (1.5\text{ m})(16\text{ kN})$$

$$+ (3\text{ m})(8\text{ kN}) + 6\text{ kN}\cdot\text{m} - (4.5\text{ m})A_y = 0$$

$$\mathbf{A}_y = 12\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, D, B

$$|V|_{\max} = 12.00\text{ kN} \blacktriangleleft$$

Moment Diag:

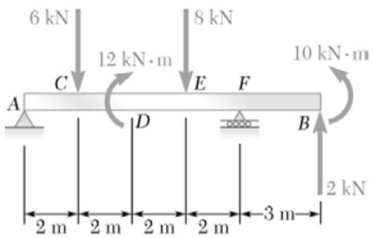
After a jump of $-6\text{ kN}\cdot\text{m}$ at A, M is piecewise linear $\left(\dfrac{dM}{dx} = V\right)$

So $$M_C = -6\text{ kN}\cdot\text{m} + (12\text{ kN})(1.5\text{ m}) = 12\text{ kN}\cdot\text{m}$$

$$M_D = 12\text{ kN}\cdot\text{m} + (4\text{ kN})(1.5\text{ m}) = 18\text{ kN}\cdot\text{m}$$

$$M_B = 18\text{ kN}\cdot\text{m} - (12\text{ kN})(1.5\text{ m}) = 0$$

$$|M|_{\max} = 18.00\text{ kN}\cdot\text{m} \blacktriangleleft$$

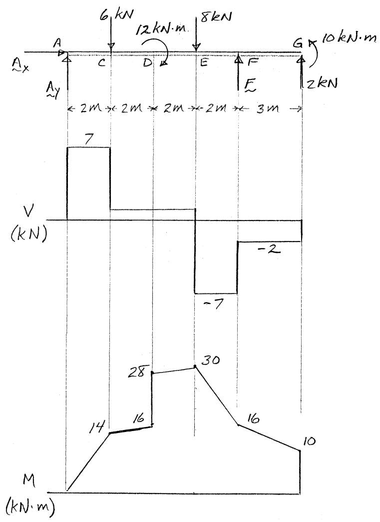


PROBLEM 7.71

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (8\text{ m})F + (11\text{ m})(2\text{ kN}) + 10\text{ kN}\cdot\text{m} - (6\text{ m})(8\text{ kN})$$

$$-12\text{ kN}\cdot\text{m} - (2\text{ m})(6\text{ kN}) = 0 \quad \mathbf{F} = 5\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 8\text{ kN} + 5\text{ kN} + 2\text{ kN} = 0$$

$$\mathbf{A}_y = 7\text{ kN}\uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, E, F, G

Moment Diag:

M is piecewise linear with a discontinuity equal to the couple at D.

$$M_C = (7\text{ kN})(2\text{ m}) = 14\text{ kN}\cdot\text{m}$$

$$M_{D^-} = 14\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_{D^+} = 16\text{ kN}\cdot\text{m} + 12\text{ kN}\cdot\text{m} = 28\text{ kN}\cdot\text{m}$$

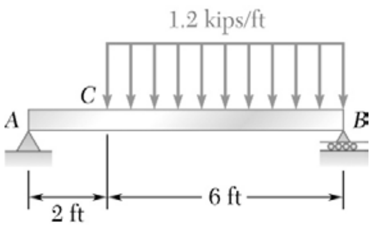
$$M_E = 28\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 30\text{ kN}\cdot\text{m}$$

$$M_F = 30\text{ kN}\cdot\text{m} - (7\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_G = 16\text{ kN}\cdot\text{m} - (2\text{ kN})(3\text{ m}) = 10\text{ kN}\cdot\text{m}$$

(*b*) $|V|_{max} = 7.00\text{ kN}$ ◀

$|M|_{max} = 30.0\text{ kN}$ ◀

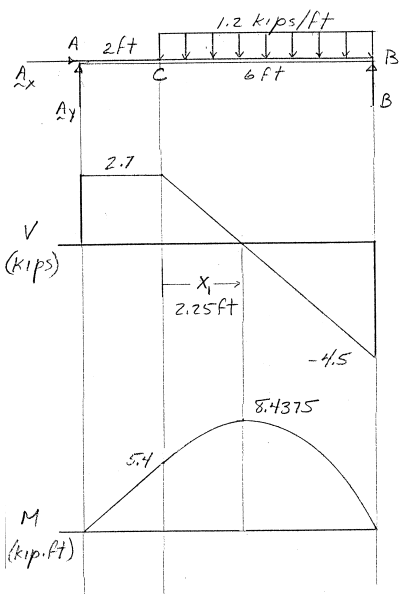


PROBLEM 7.72

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum bending moment.

SOLUTION

(a)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3 \text{ ft})(1.2 \text{ kips/ft})(6 \text{ ft}) - (8 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 2.7 \text{ kips} \uparrow$$

Shear Diag:

$$V = A_y = 2.7 \text{ kips at } A, \text{ is constant to } C, \text{ then linear}$$

$$\left(\frac{dV}{dx} = -1.2 \text{ kips/ft}\right) \text{ to B.} \qquad V_B = 2.7 \text{ kips} - (1.2 \text{ kips/ft})(6 \text{ ft})$$

$$V_B = -4.5 \text{ kips}$$

$$V = 0 = 2.7 \text{ kips} - (1.2 \text{ kips/ft})x_1 \quad \text{at} \quad x_1 = 2.25 \text{ ft}$$

Moment Diag:

$$M_A = 0, \quad M \text{ is linear } \left(\frac{dM}{dx} = 2.7 \text{ kips}\right) \text{ to } C.$$

$$M_C = (2.7 \text{ kips})(2 \text{ ft}) = 5.4 \text{ kip}\cdot\text{ft}$$

Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$

(b)

$$M_{\max} \text{ occurs where } \frac{dM}{dx} = V = 0$$

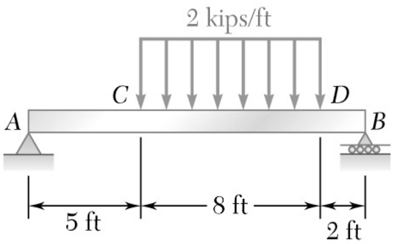
$$M_{\max} = 5.4 \text{ kip}\cdot\text{ft} + \frac{1}{2}(2.7 \text{ kips})x_1; \qquad x_1 = 2.25 \text{ m}$$

$$M_{\max} = 8.4375 \text{ kip}\cdot\text{ft}$$

$$M_{\max} = 8.44 \text{ kip}\cdot\text{ft, 2.25 m right of } C \blacktriangleleft$$

Check: $$M_B = 8.4375 \text{ kip}\cdot\text{ft} - \frac{1}{2}(4.5 \text{ kips})(3.75 \text{ ft}) = 0$$

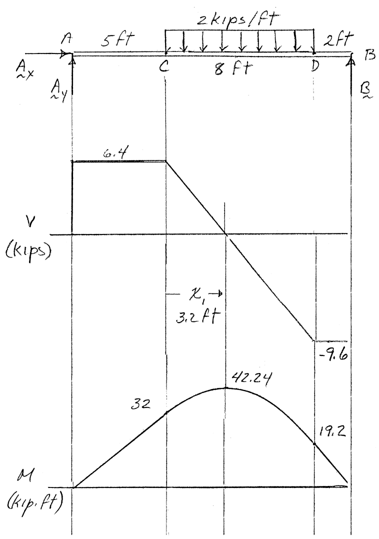
PROBLEM 7.73



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (6 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) - (15 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 6.4 \text{ kips} \uparrow$$

Shear Diag:

$V = A_y = 6.4$ kips at *A*, and is constant to *C*, then linear

$\left(\dfrac{dV}{dx} = -2 \text{ kips/ft}\right)$ to *D*,

$$V_D = 6.4 \text{ kips} - (2 \text{ kips/ft})(8 \text{ ft}) = -9.6 \text{ kips}$$

$$V = 0 = 6.4 \text{ kips} - (2 \text{ kips/ft})x_1 \text{ at } x_1 = 3.2 \text{ ft}$$

Moment Diag:

$M_A = 0$, then M is linear $\left(\dfrac{dM}{dx} = 6.4 \text{ kips}\right)$ to $M_C = (6.4 \text{ kips})(5 \text{ ft})$.

$M_C = 32 \text{ kip}\cdot\text{ft}$. M is then parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$.

(*b*)

$$M_{\max} \text{ occurs where } \frac{dM}{dx} = V = 0.$$

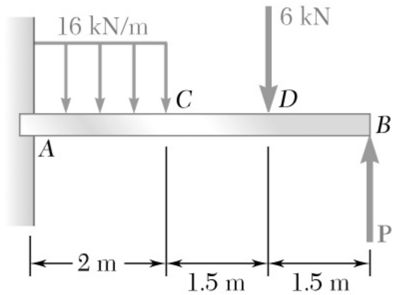
$$M_{\max} = 32 \text{ kip}\cdot\text{ft} + \frac{1}{2}(6.4 \text{ kips})x_1; \qquad x_1 = 3.2 \text{ ft}$$

$$M_{\max} = 42.24 \text{ kip}\cdot\text{ft}$$

$$M_{\max} = 42.2 \text{ kip}\cdot\text{ft, 3.2 ft right of } C \blacktriangleleft$$

$$M_D = 42.24 \text{ kip}\cdot\text{ft} - \frac{1}{2}(9.6 \text{ kips})(4.8 \text{ ft}) = 19.2 \text{ kip}\cdot\text{ft}$$

M is linear from *D* to zero at *B*.

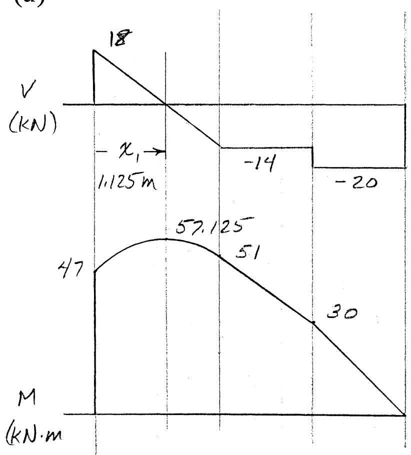


PROBLEM 7.74

For the beam shown, draw the shear and bending-moment diagrams and determine the maximum absolute value of the bending moment knowing that (*a*) $P = 14$ kN, (*b*) $P = 20$ kN.

SOLUTION

(*a*)



FBD Beam:

$$+\uparrow \Sigma F_y = 0: \; A_y - (16 \text{ kN/m})(2 \text{ m}) - 6 \text{ kN} + P = 0$$

$$A_y = 38 \text{ kN} - P$$

(*a*) $\mathbf{A}_y = 24 \text{ kN} \uparrow$

(*b*) $\mathbf{A}_y = 18 \text{ kN} \uparrow$

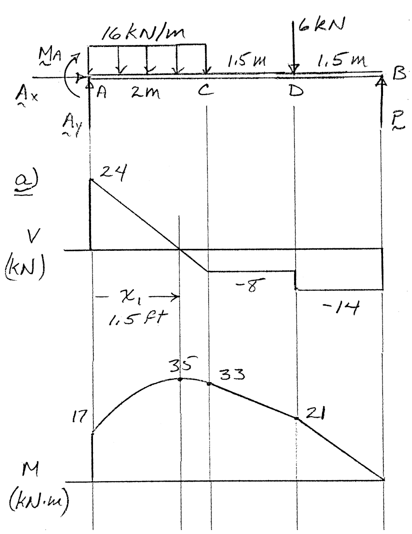
$$\circlearrowleft \Sigma M_A = 0: \; (5 \text{ m})P - (3.5 \text{ m})(6 \text{ kN}) - 1 \text{ m}(16 \text{ kN/m})(2 \text{ m}) - M_A = 0$$

$$M_A = (5 \text{ m})P - 53 \text{ kN}\cdot\text{m}$$

(*a*) $\mathbf{M}_A = 17 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*) $\mathbf{M}_A = 47 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*)



Shear Diags:

$V_A = A_y$. Then V is linear $\left(\dfrac{dV}{dx} = -16 \text{ kN/m}\right)$ to C.

$$V_C = V_A - (16 \text{ kN/m})(2 \text{ m}) = V_A - 32 \text{ kN}$$

(*a*) $V_C = -8$ kN

(*b*) $V_C = -14$ kN

$$V = 0 = V_A - (16 \text{ kN/m})x_1$$

(*a*) $x_1 = 1.5$ m

(*b*) $x_1 = 1.125$ m

V is constant from C to D, decreases by 6 kN at D and is constant to B (at $-P$)

PROBLEM 7.74 CONTINUED

Moment Diags:

$M_A = M_A$ reaction. Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$.

The maximum occurs where $V = 0$. $M_{\max} = M_A + \frac{1}{2}V_A x_1$.

(*a*) $$M_{\max} = 17 \text{ kN}\cdot\text{m} + \frac{1}{2}(24 \text{ kN})(1.5 \text{ m}) = 35.0 \text{ kN}\cdot\text{m} \blacktriangleleft$$

1.5 ft from A ◀

(*b*) $$M_{\max} = 47 \text{ kN}\cdot\text{m} + \frac{1}{2}(18 \text{ kN})(1.125 \text{ m}) = 57.125 \text{ kN}\cdot\text{m}$$

$M_{\max} = 57.1 \text{ kN}\cdot\text{m}$ 1.125 ft from A ◀

$$M_C = M_{\max} + \frac{1}{2}V_C(2 \text{ m} - x_1)$$

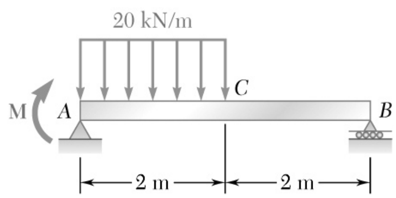
(*a*) $$M_C = 35 \text{ kN}\cdot\text{m} - \frac{1}{2}(8 \text{ kN})(0.5 \text{ m}) = 33 \text{ kN}\cdot\text{m}$$

(*b*) $$M_C = 57.125 \text{ kN}\cdot\text{m} - \frac{1}{2}(14 \text{ kN})(0.875 \text{ m}) = 51 \text{ kN}\cdot\text{m}$$

M is piecewise linear along C, D, B, with $M_B = 0$ and $M_D = (1.5 \text{ m})P$

(*a*) $$M_D = 21 \text{ kN}\cdot\text{m}$$

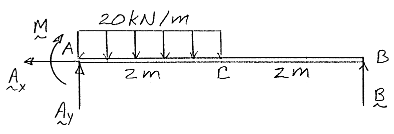
(*b*) $$M_D = 30 \text{ kN}\cdot\text{m}$$



PROBLEM 7.75

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment knowing that (*a*) $M = 0$,
(*b*) $M = 12\ \text{kN}\cdot\text{m}$.

SOLUTION



FBD Beam:

$$\circlearrowleft\ \Sigma M_A = 0: (4\text{ m})B - (1\text{ m})(20\text{ kN/m})(2\text{ m}) - M = 0$$

$$B = 10\text{ kN} + \frac{M}{4\text{ m}}$$

(*a*) $\mathbf{B} = 10\text{ kN}\uparrow$

(*b*) $\mathbf{B} = 13\text{ kN}\uparrow$

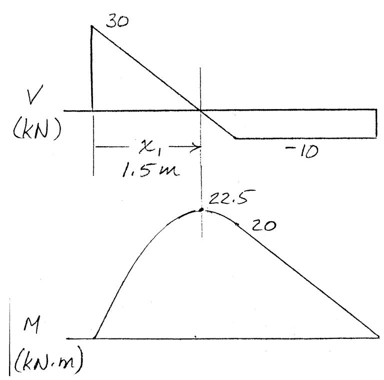
$$\uparrow \Sigma F_y = 0: A_y - (20\text{ kN/m})(2\text{ m}) + B = 0$$

$$A_y = 40\text{ kN} - B$$

(*a*) $\mathbf{A}_y = 30\text{ kN}\uparrow$

(*b*) $\mathbf{A}_y = 27\text{ kN}\uparrow$

(*a*)



Shear Diags:

$V_A = A_y$, then V is linear $\left(\dfrac{dV}{dx} = -20\text{ kN/m}\right)$ to C.

$$V_C = A_y - (20\text{ kN/m})(2\text{ m}) = A_y - 40\text{ kN}$$

(*a*) $V_C = -10\text{ kN}$

(*b*) $V_C = -13\text{ kN}$

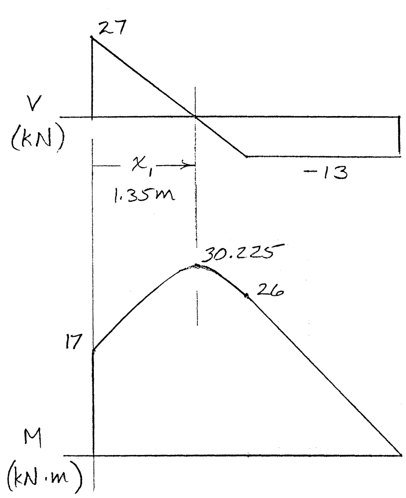
$$V = 0 = A_y - (20\text{ kN/m})x_1 \text{ at } x_1 = \frac{A_y\text{ m}}{20\text{ kN}}$$

(*a*) $x_1 = 1.5\text{ m}$

(*b*) $x_1 = 1.35\text{ m}$

V is constant from C to B.

(*b*)



PROBLEM 7.75 CONTINUED

Moment Diags:

M_A = applied M. Then M is parabolic $\left(\frac{dM}{dx}\text{ decreases with } V\right)$

M is max where $V = 0$. $M_{\max} = M + \frac{1}{2}A_y x_1$.

(*a*) $$|M|_{\max} = \frac{1}{2}(30\text{ kN})(1.5\text{ m}) = 22.5\text{ kN}\cdot\text{m} \blacktriangleleft$$

1.500 m from A ◀

(*b*) $$M_{\max} = 12\text{ kN}\cdot\text{m} + \frac{1}{2}(27\text{ kN})(1.35\text{ m}) = 30.225\text{ kN}\cdot\text{m} \blacktriangleleft$$

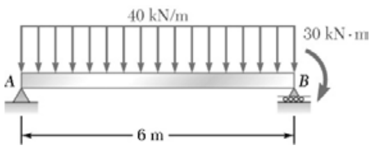
$|M|_{\max} = 30.2$ kN   1.350 m from A ◀

$$M_C = M_{\max} - \frac{1}{2}V_C(2\text{ m} - x_1)$$

(*a*) $$M_C = 20\text{ kN}\cdot\text{m}$$

(*b*) $$M_C = 26\text{ kN}\cdot\text{m}$$

Finally, M is linear $\left(\frac{dM}{dx} = V_C\right)$ to zero at B.

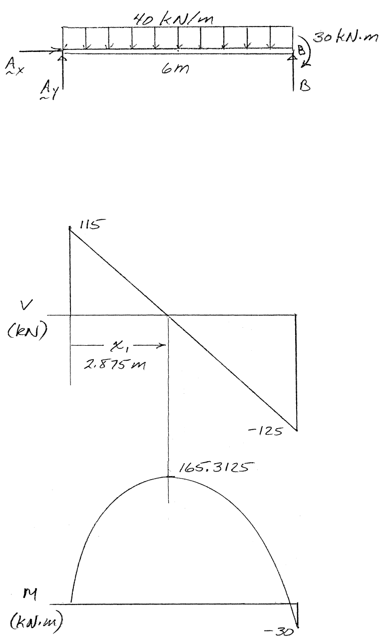


PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3\text{ m})(40\text{ kN/m})(6\text{ m}) - (30\text{ kN}\cdot\text{m}) - (6\text{ m})A_y = 0$$

$$\mathbf{A}_y = 115\text{ kN} \uparrow$$

Shear Diag:

$V_A = A_y = 115\text{ kN}$, then V is linear $\left(\dfrac{dM}{dx} = -40\text{ kN/m}\right)$ to B.

$$V_B = 115\text{ kN} - (40\text{ kN/m})(6\text{ m}) = -125\text{ kN}.$$

$$V = 0 = 115\text{ kN} - (40\text{ kN/m})x_1 \text{ at } x_1 = 2.875\text{ m}$$

Moment Diag:

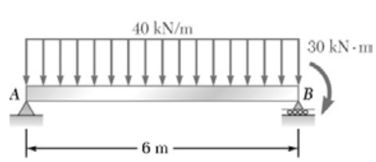
$M_A = 0$. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$,

$$M_{\max} = \frac{1}{2}(115\text{ kN/m})(2.875\text{ m}) = 165.3125\text{ kN}\cdot\text{m}$$

$$\begin{aligned} M_B &= M_{\max} - \frac{1}{2}(125\text{ kN})(6\text{ m} - x_1) \\ &= 165.3125\text{ kN}\cdot\text{m} - \frac{1}{2}(125\text{ kN})(6 - 2.875)\text{m} \\ &= -30\text{ kN}\cdot\text{m as expected.} \end{aligned}$$

(*b*) $\quad |M|_{\max} = 165.3\text{ kN}\cdot\text{m}\,(2.88\text{ m from } A)$ ◀

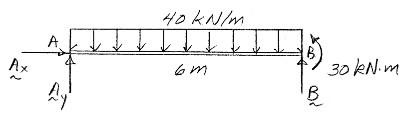
PROBLEM 7.77



Solve Prob. 7.76 assuming that the $30\ \text{kN}\cdot\text{m}$ couple applied at B is counterclockwise

SOLUTION

(a)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0\text{: } 30\ \text{kN}\cdot\text{m} + (3\ \text{m})(40\ \text{kN/m})(6\ \text{m}) - (6\ \text{m})A_y = 0$$

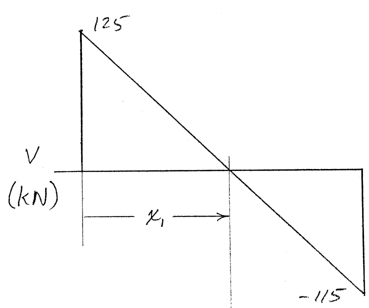
$$\mathbf{A}_y = 125\ \text{kN} \uparrow$$

Shear Diag:

$V_A = A_y = 125\ \text{kN},\ V$ is linear $\left(\dfrac{dV}{dx} = -40\ \text{kN/m}\right)$ to B.

$$V_B = 125\ \text{kN} - (40\ \text{kN/m})(6\ \text{m}) = -115\ \text{kN}$$

$$V = 0 = 115\ \text{kN} - (40\ \text{kN/m})x_1 \text{ at } x_1 = 3.125\ \text{m}$$



Moment Diag:

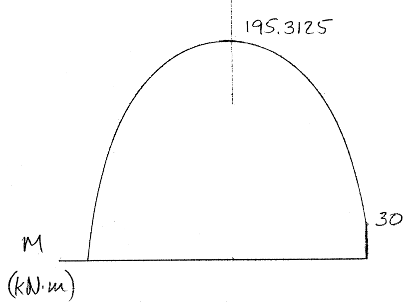
$M_A = 0.$ Then M is parabolic $\left(\dfrac{dM}{dx}\ \text{decreases with}\ V\right)$. Max M occurs where $V = 0$,

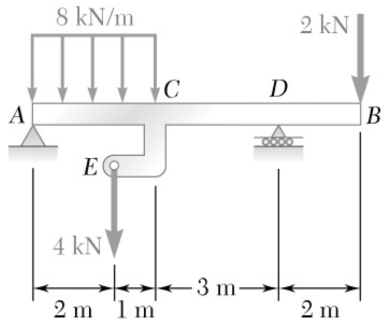
$$M_{\text{max}} = \frac{1}{2}(125\ \text{kN})(3.125\ \text{m}) = 195.3125\ \text{kN}\cdot\text{m}$$

(b) $$|M|_{\text{max}} = 195.3\ \ \text{kN}\cdot\text{m}(3.125\ \text{m from } A) \blacktriangleleft$$

$$M_B = 195.3125\ \text{kN}\cdot\text{m} - \frac{1}{2}(115\ \text{kN})(6 - 3.125)\text{m}$$

$$M_B = 30\ \text{kN}\cdot\text{m} \text{ as expected.}$$



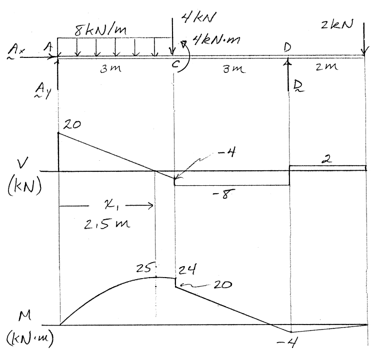


PROBLEM 7.78

For beam AB, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(a)
Replacing the load at E with equivalent force-couple at C:



$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) - (3\text{ m})(4\text{ kN}) - (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) - 4\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{D} = 10\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + 10\text{ kN} - 2\text{ kN} - 4\text{ kN} - (8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{A}_y = 20\text{ kN}\uparrow$$

Shear Diag:

$V_A = A_y = 20\text{ kN}$, then V is linear $\left(\dfrac{dV}{dx} = -8\ \text{kN/m}\right)$ to C.

$$V_C = 20\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -4\text{ kN}$$

$$V = 0 = 20\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = 2.5\text{ m}$$

At C, V decreases by 4 kN to -8 kN.

At D, V increases by 10 kN to 2 kN.

Moment Diag:

$M_A = 0$, then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$.

$$M_{\max} = \frac{1}{2}(20\text{ kN})(2.5\text{ m}) = 25\text{ kN}\cdot\text{m}$$

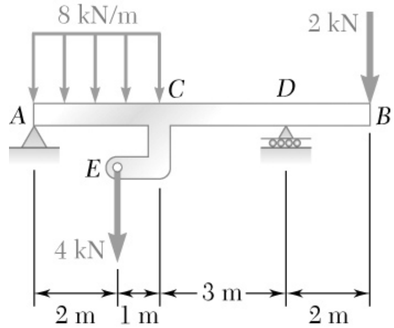
(b) $\quad M_{\max} = 25.0\ \text{kN}\cdot\text{m}, 2.50\text{ m from } A$ ◀

PROBLEM 7.78 CONTINUED

$$M_C = 25 \text{ kN}\cdot\text{m} - \frac{1}{2}(4 \text{ kN})(0.5 \text{ m}) = 24 \text{ kN}\cdot\text{m}.$$

At *C*, *M* decreases by $4 \text{ kN}\cdot\text{m}$ to $20 \text{ kN}\cdot\text{m}$. From *C* to *B*, *M* is piecewise linear with $\frac{dM}{dx} = -8 \text{ kN}$ to *D*, then $\frac{dM}{dx} = +2 \text{ kN}$ to *B*.

$$M_D = 20 \text{ kN}\cdot\text{m} - (8 \text{ kN})(3 \text{ m}) = -4 \text{ kN}\cdot\text{m}. \quad M_B = 0$$

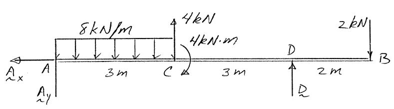


PROBLEM 7.79

Solve Prob. 7.78 assuming that the 4-kN force applied at E is directed upward.

SOLUTION

(a)
Replacing the load at E with equivalent force-couple at C.

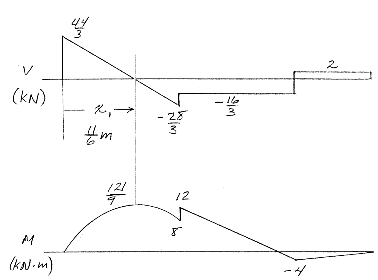


$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) + (3\text{ m})(4\text{ kN})$$
$$- 4\text{ kN}\cdot\text{m} - (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{D} = \frac{22}{3}\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{22}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) + 4\text{ kN} - 2\text{ kN} = 0$$

$$\mathbf{A}_y = \frac{44}{3}\text{ kN} \uparrow$$



Shear Diag:

$V_A = A_y = \dfrac{44}{3}$ kN, then V is linear $\left(\dfrac{dV}{dx} = -8\text{ kN/m}\right)$ to C.

$$V_C = \frac{44}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -\frac{28}{3}\text{ kN}$$

$$V = 0 = \frac{44}{3}\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = \frac{11}{6}\text{ m.}$$

At C, V increases 4 kN to $-\dfrac{16}{3}$ kN.

At D, V increases $\dfrac{22}{3}$ kN to 2 kN.

PROBLEM 7.79 CONTINUED

Moment Diag:

$M_A = 0.$ Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right).$ Max M occurs where $V = 0.$

$$M_{\max} = \frac{1}{2}\left(\frac{44}{3}\text{ kN}\right)\left(\frac{11}{6}\text{ m}\right) = \frac{121}{9}\text{ kN}\cdot\text{m}$$

(b) $M_{\max} = 13.44\ \text{kN}\cdot\text{m}$ at 1.833 m from A ◀

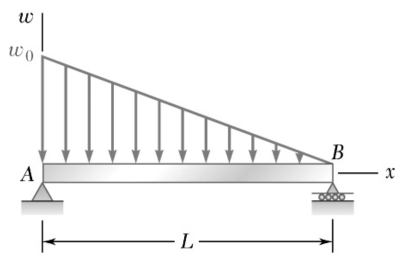
$$M_C = \frac{121}{9}\text{ kN}\cdot\text{m} - \frac{1}{2}\left(\frac{28}{3}\text{ kN}\right)\left(\frac{7}{6}\text{ m}\right) = 8\text{ kN}\cdot\text{m}.$$

At C, M increases by $4\text{ kN}\cdot\text{m}$ to $12\text{ kN}\cdot\text{m}$. Then M is linear $\left(\dfrac{dM}{dx} = -\dfrac{16}{3}\text{ kN}\right)$ to D.

$M_D = 12\text{ kN}\cdot\text{m} - \left(\dfrac{16}{3}\text{ kN}\right)(3\text{ m}) = -4\text{ kN}\cdot\text{m}.$ M is again linear

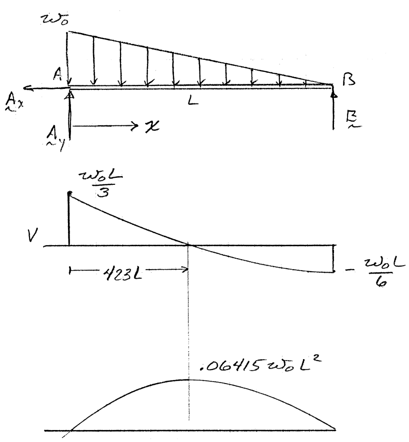
$\left(\dfrac{dM}{dx} = 2\text{ kN}\right)$ to zero at B.

PROBLEM 7.80



For the beam and loading shown, (*a*) derive the equations of the shear and bending-moment curves, (*b*) draw the shear and bending-moment diagrams, (*c*) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\text{Distributed load } w = w_0\left(1 - \frac{x}{L}\right) \qquad \left(\text{total} = \frac{1}{2}w_0L\right)$$

$$\circlearrowleft \Sigma M_A = 0: \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \qquad \mathbf{A}_y = \frac{w_0L}{3} \uparrow$$

Shear:

$$V_A = A_y = \frac{w_0L}{3},$$

Then
$$\frac{dV}{dx} = -w \rightarrow V = V_A - \int_0^x w_0\left(1 - \frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2 = w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $x = L$, $V = -\dfrac{w_0L}{6}$;

$$V = 0 \quad \text{at} \quad \left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3} = 0 \rightarrow \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$$

Moment:

$$M_A = 0,$$

Then
$$\left(\frac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

PROBLEM 7.80 CONTINUED

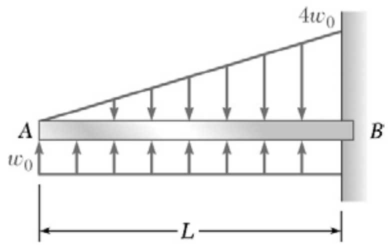
$$M_{max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

(*a*) $$V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

(*c*) $$M_{max} = 0.0642\, w_0 L^2 \blacktriangleleft$$

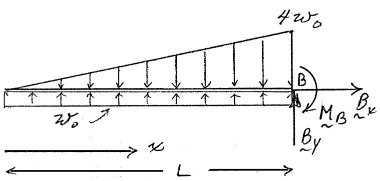
$$\text{at } x = 0.423L \blacktriangleleft$$



PROBLEM 7.81

For the beam and loading shown, (*a*) derive the equations of the shear and bending-moment curves, (*b*) draw the shear and bending-moment diagrams, (*c*) determine the magnitude and location of the maximum bending moment.

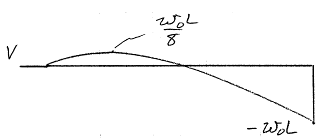
SOLUTION



Distributed load $$w = w_0\left[4\left(\frac{x}{L}\right) - 1\right]$$

Shear: $$\frac{dV}{dx} = -w, \text{ and } V(0) = 0, \text{ so}$$

$$V = \int_0^x -w\,dx = -\int_0^{x/L} Lw\,d\left(\frac{x}{L}\right)$$

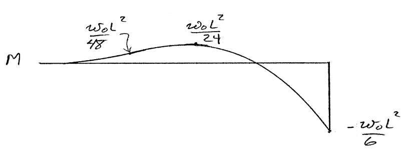


$$V = \int_0^{x/L} w_0L\left[1 - 4\left(\frac{x}{L}\right)\right]d\left(\frac{x}{L}\right) = w_0L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$$

Notes: At $x = L, V = -w_0L$

And $V = 0$ at $$\left(\frac{x}{L}\right) = 2\left(\frac{x}{L}\right)^2 \quad \text{or} \quad \frac{x}{L} = \frac{1}{2}$$

Also V is max where $w = 0$ $\left(\frac{x}{L} = \frac{1}{4}\right)$



$$V_{\max} = \frac{1}{8}w_0L$$

Moment: $$M(0) = 0, \frac{dM}{dx} = V$$

$$M = \int_0^x v\,dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\int_0^{x/L}\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

(*a*) $$V = w_0L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right] ◀$$

$$M = w_0L^2\left[\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{2}{3}\left(\frac{x}{L}\right)^3\right] ◀$$

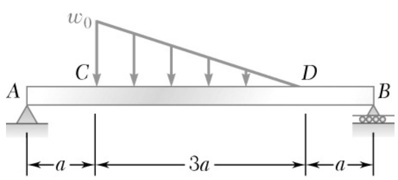
PROBLEM 7.81 CONTINUED

$$M_{\max} = \frac{1}{24} w_0 L^2 \text{ at } \quad x = \frac{L}{2}$$

$$M_{\min} = -\frac{1}{6} w_0 L^2 \text{ at } x = L$$

$$M_{\max} = \frac{w_0 L^2}{24} \text{ at } x = \frac{L}{2}$$

$$(c) \qquad |M|_{\max} = -M_{\min} = \frac{w_0 L^2}{6} \text{ at } B \blacktriangleleft$$

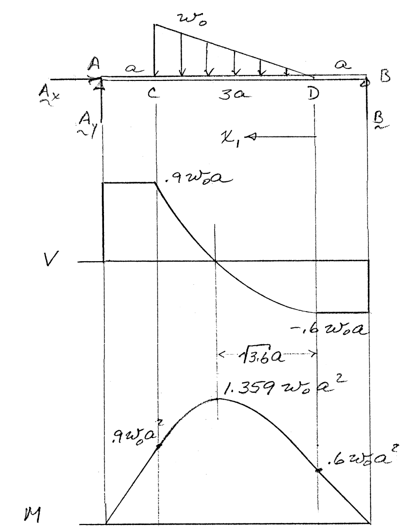


PROBLEM 7.82

For the beam shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment. (*Hint:* Derive the equations of the shear and bending-moment curves for portion *CD* of the beam.)

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0:\ (3a)\left[\frac{1}{2}w_0(3a)\right] - 5aA_y = 0 \qquad \mathbf{A}_y = 0.9w_0a \uparrow$$

$$\uparrow \Sigma F_y = 0:\ 0.9w_0a - \frac{1}{2}w_0(3a) + B = 0$$

$$\mathbf{B} = 0.6w_0a \uparrow$$

Shear Diag:

$V = A_y = 0.9w_0a$ from A to C and $V = B = -0.6w_0a$ from B to D.

Then from D to C, $w = w_0\dfrac{x_1}{3a}$. If x_1 is measured right to left, $\dfrac{dV}{dx_1} = +w$ and $\dfrac{dM}{dx_1} = -V$. So, from D, $V = -0.6w_0a + \int_0^{x_1}\dfrac{w_0}{3a}x_1dx_1$,

$$V = w_0a\left[-0.6 + \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]$$

$$\text{Note: } V = 0 \text{ at } \left(\frac{x_1}{a}\right)^2 = 3.6,\ x_1 = \sqrt{3.6}a$$

Moment Diag:

$M = 0$ at A, increasing linearly $\left(\dfrac{dM}{dx_1} = 0.9w_0a\right)$ to $M_C = 0.9w_0a^2$.

Similarly $M = 0$ at B increasing linearly $\left(\dfrac{dM}{dx} = 0.6w_0a\right)$ to

$M_D = 0.6w_0a^2$. Between C and D

$$M = 0.6w_0a^2 + w_0a\int_0^{x_1}\left[0.6 - \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]dx_1,$$

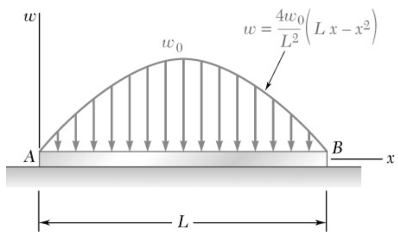
$$M = w_0a^2\left[0.6 + 0.6\left(\frac{x_1}{a}\right) - \frac{1}{18}\left(\frac{x_1}{a}\right)^3\right]$$

PROBLEM 7.82 CONTINUED

(*b*)

At $\dfrac{x_1}{a} = \sqrt{3.6}, \; M = M_{\max} = 1.359 w_0 a^2$ ◀

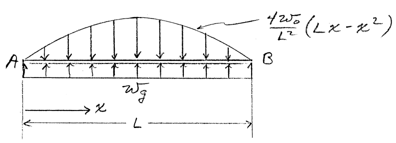
$x_1 = 1.897a$ left of D ◀



PROBLEM 7.83

Beam *AB*, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*) $$+\uparrow \Sigma F_y = 0: \; w_g L - \int_0^L \frac{4w_0}{L^2}\left(Lx - x^2\right)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L \qquad w_g = \frac{2w_0}{3}$$

Define $\xi = \dfrac{x}{L}$ so $d\xi = \dfrac{dx}{L}$ $\rightarrow$ net load $w = 4w_0\left[\dfrac{x}{L} - \left(\dfrac{x}{L}\right)^2\right] - \dfrac{2}{3}w_0$

or $$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi =$$

$$0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\left(\xi - 3\xi^2 + 2\xi^3\right) \blacktriangleleft$$

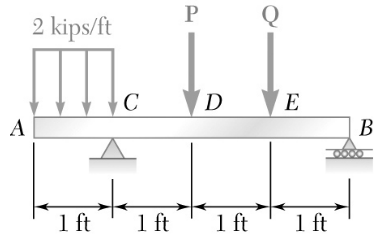
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi \left(\xi - 3\xi^2 + 2\xi^3\right)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2\left(\xi^2 - 2\xi^3 + \xi^4\right) \blacktriangleleft$$

(*b*) Max *M* occurs where $V = 0 \rightarrow 1 - 3\xi + 2\xi^2 = 0 \rightarrow \xi = \dfrac{1}{2}$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

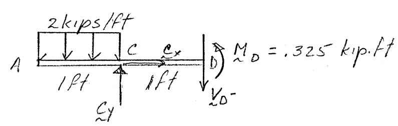


PROBLEM 7.84

The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +325 lb ft at *D* and +800 lb ft at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

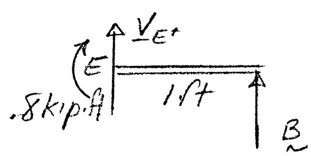
FBD ACD:



$$(a) \curvearrowleft \Sigma M_{D^-} = 0\text{: } 0.325\text{ kip}\cdot\text{ft} - (1\text{ ft})C_y + (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft}) = 0$$

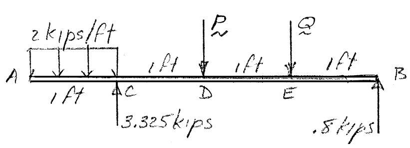
$$\mathbf{C}_y = 3.325\text{ kips} \uparrow$$

FBD EB:



$$\curvearrowleft \Sigma M_E = 0\text{: } (1\text{ ft})B - 0.8\text{ kip}\cdot\text{ft} = 0 \qquad \mathbf{B} = 0.8\text{ kip} \uparrow$$

FBD Beam:



$$\curvearrowleft \Sigma M_D = 0\text{: } (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft}) - (1\text{ ft})(3.325\text{ kips})$$

$$-(1\text{ ft})Q + 2\text{ ft}(0.8\text{ kips}) = 0$$

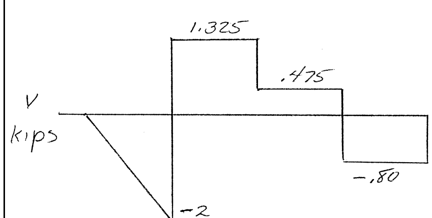
$$Q = 1.275\text{ kips}$$

$$\uparrow \Sigma F_y = 0\text{: } 3.325\text{ kips} + 0.8\text{ kips} - 1.275\text{ kips}$$

$$-(2\text{ kips/ft})(1\text{ft}) - P = 0 \qquad P = 0.85\text{ kip} \downarrow$$

(*a*) $\mathbf{P} = 850\text{ lb} \downarrow$ ◀

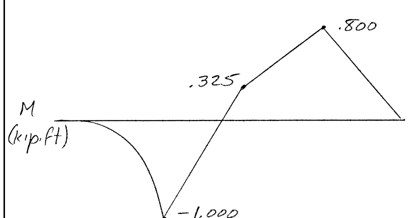
$\mathbf{Q} = 1.275\text{ kips} \downarrow$ ◀



(*b*) **Shear Diag:**

V is linear $\left(\dfrac{dV}{dx} = -2\text{ kips/ft}\right)$ from 0 at *A* to

$-(2\text{ kips/ft})(1\text{ ft}) = -2\text{ kips}$ at *C*. Then *V* is piecewise constant with discontinuities equal to forces at *C*, *D*, *E*, *B*



Moment Diag:

M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$ from 0 at *A* to

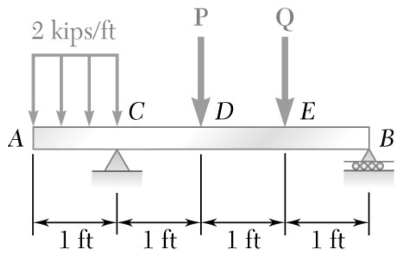
$-\dfrac{1}{2}(2\text{ kips})(1\text{ ft}) = -1\text{ kip}\cdot\text{ft}$ at *C*. Then *M* is piecewise linear with

PROBLEM 7.84 CONTINUED

$$M_D = -1\text{ kip}\cdot\text{ft} + (1.325\text{ kips})(1\text{ ft}) = 0.325\text{ kip}\cdot\text{ft}$$

$$M_E = 0.325\text{ kip}\cdot\text{ft} + (0.475\text{ kips})(1\text{ ft}) = 0.800\text{ kip}\cdot\text{ft}$$

$$M_B = 0.8\text{ kip}\cdot\text{ft} - (0.8\text{ kip})(1\text{ ft}) = 0$$

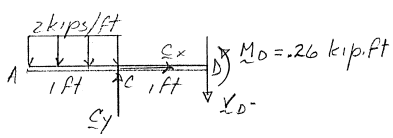


PROBLEM 7.85

Solve Prob. 7.84 assuming that the bending moment was found to be +260 lb ft at D and +860 lb ft at E.

SOLUTION

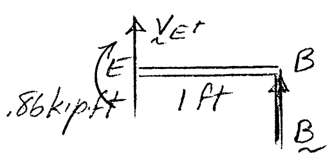
FBD ACD:



(a) $\circlearrowleft \Sigma M_D = 0\text{: } 0.26\,\text{kip}\cdot\text{ft} - (1\,\text{ft})C_y + (1.5\,\text{ft})(2\,\text{kips/ft})(1\,\text{ft}) = 0$

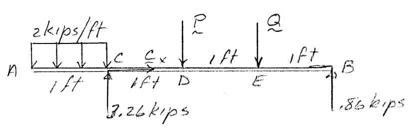
$$\mathbf{C}_y = 3.26\ \text{kips}\ \uparrow$$

FBD DB:



$\circlearrowleft \Sigma M_E = 0\text{: } (1\,\text{ft})B - 0.86\ \text{kip}\cdot\text{ft}$ $\qquad \mathbf{B} = 0.86\ \text{kip}\ \uparrow$

FBD Beam:



$$\circlearrowleft \Sigma M_D = 0\text{: } (1.5\,\text{ft})(2\,\text{kips/ft})(1\,\text{ft}) - (1\,\text{ft})(3.26\,\text{kips}) + (1\,\text{ft})Q + (2\,\text{ft})(0.86\,\text{kips}) = 0$$

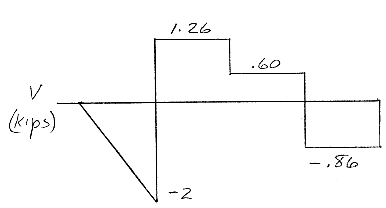
$Q = 1.460\ \text{kips}$ $\qquad \mathbf{Q} = 1.460\ \text{kips}\ \downarrow$ ◀

$$\uparrow \Sigma F_y = 0\text{: } 3.26\,\text{kips} + 0.86\,\text{kips} - 1.460\,\text{kips} - P - (2\,\text{kips/ft})(1\,\text{ft}) = 0$$

$$P = 0.66\ \text{kips}$$

$\mathbf{P} = 660\ \text{lb}\ \downarrow$ ◀

(b) **Shear Diag:**

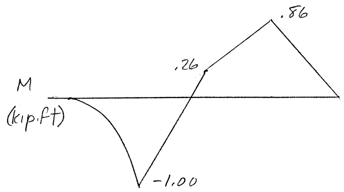


V is linear $\left(\dfrac{dV}{dx} = -2\ \text{kips/ft}\right)$ from 0 at A to $-(2\,\text{kips/ft})(1\,\text{ft}) = -2\ \text{kips}$ at C. Then V is piecewise constant with discontinuities equal to forces at C, D, E, B.

Moment Diag:

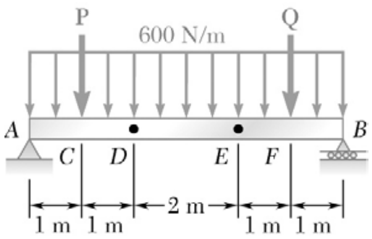
M is parabolic $\left(\dfrac{dM}{dx}\ \text{decreasing with}\ V\right)$ from 0 at A to $-\dfrac{1}{2}(2\,\text{kips/ft})(1\,\text{ft}) = -1\,\text{kip}\cdot\text{ft}$ at C. Then M is piecewise linear with

PROBLEM 7.85 CONTINUED



$M_0 = 0.26 \text{ kip}\cdot\text{ft}$

$M_E = 0.86 \text{ kip}\cdot\text{ft}, \quad M_B = 0$

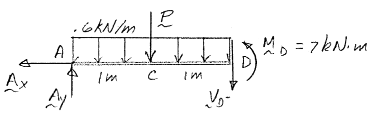


PROBLEM 7.86

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+7\text{ kN}\cdot\text{m}$ at D and $+5\text{ kN}\cdot\text{m}$ at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

FBD AD:

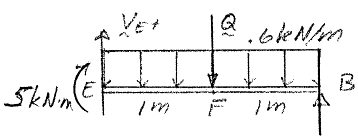


(a)

$$\circlearrowleft \Sigma M_D = 0\text{:}\quad 7\text{ kN}\cdot\text{m} + (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - (2\text{ m})A_y = 0$$

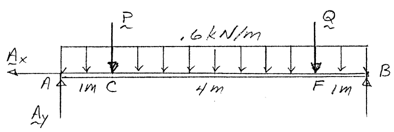
$$2A_y - P = 8.2\text{ kN} \qquad (1)$$

FBD EB:



$$\circlearrowleft \Sigma M_E = 0\text{: } (2\text{ m})B - (1\text{ m})Q - (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - 5\text{ kN}\cdot\text{m} = 0$$

$$2B - Q = 6.2\text{ kN} \qquad (2)$$



$$\circlearrowleft \Sigma M_A = 0\text{:}\quad (6\text{ m})B - (1\text{ m})P - (5\text{ m})Q - (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) = 0$$

$$6B - P - 5Q = 10.8\text{ kN} \qquad (3)$$

$$\circlearrowleft \Sigma M_B = 0\text{: } (1\text{ m})Q + (5\text{ m})P + (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) - (6\text{ m})A = 0$$

$$6A - Q - 5P = 10.8\text{ kN} \qquad (4)$$

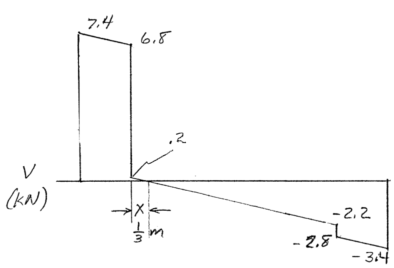
Solving (1)–(4): $\mathbf{P} = 6.60\text{ kN}\downarrow$, $\mathbf{Q} = 600\text{ N}\downarrow$ ◀

$\mathbf{A}_y = 7.4\text{ kN}\uparrow$, $\quad \mathbf{B} = 3.4\text{ kN}\uparrow$

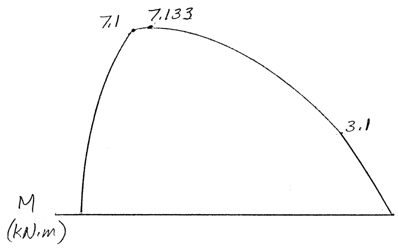
(b) **Shear Diag:**

V is piecewise linear with $\dfrac{dV}{dx} = -0.6\text{ kN/m}$ throughout, and discontinuities equal to forces at A, C, F, B.

Note $\quad V = 0 = 0.2\text{ kN} - (0.6\text{ kN/m})x$ at $x = \dfrac{1}{3}\text{ m}$



PROBLEM 7.86 CONTINUED



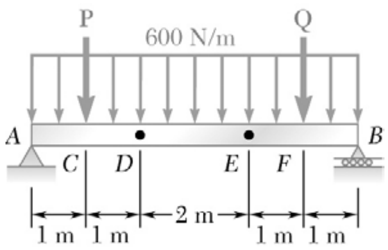
Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with }V\right)$ with “breaks” in slope at C and F.

$$M_C = \frac{1}{2}(7.4 + 6.8)\,\text{kN}(1\,\text{m}) = 7.1\,\text{kN}\cdot\text{m}$$

$$M_{\max} = 7.1\,\text{kN}\cdot\text{m} + \frac{1}{2}(0.2\,\text{kN})\left(\frac{1}{3}\,\text{m}\right) = 7.13\dot{3}\,\text{kN}\cdot\text{m}$$

$$M_F = 7.13\dot{3}\,\text{kN}\cdot\text{m} - \frac{1}{2}(2.2\,\text{kN})\left(3\frac{2}{3}\,\text{m}\right) = 3.1\,\text{kN}\cdot\text{m}$$

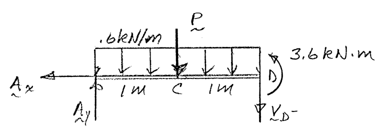


PROBLEM 7.87

Solve Prob. 7.86 assuming that the bending moment was found to be $+3.6\text{ kN}\cdot\text{m}$ at D and $+4.14\text{ kN}\cdot\text{m}$ at E.

SOLUTION

FBD AD:

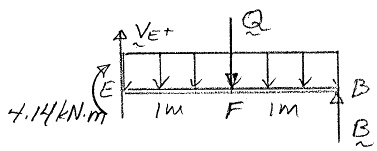


(*a*)

$$\curvearrowleft \Sigma M_D = 0\text{: } 3.6\,\text{kN}\cdot\text{m} + (1\text{ m})P + (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - (2\text{ m})A_y = 0$$

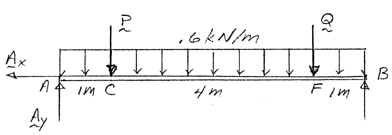
$$2A_y - P = 4.8\text{ kN} \qquad (1)$$

FBD EB:



$$\curvearrowleft \Sigma M_E = 0\text{: } \quad (2\text{ m})B - (1\text{ m})Q - (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - 4.14\text{ kN}\cdot\text{m} = 0$$

$$2B - Q = 5.34\text{ kN} \qquad (2)$$



$$\curvearrowleft \Sigma M_A = 0\text{: } (6\text{ m})B - (5\text{ m})Q - (1\text{ m})P - (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) = 0$$

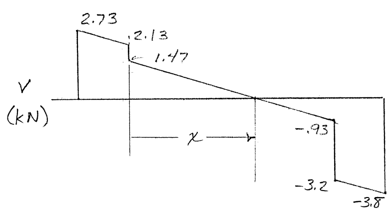
$$6B - P - 5Q = 10.8\text{ kN} \qquad (3)$$

By symmetry:

$$6A - Q - 5P = 10.8\text{ kN} \qquad (4)$$

Solving (1)–(4) $\qquad \mathbf{P} = 660\text{ N}\downarrow, \; \mathbf{Q} = 2.28\text{ kN}\downarrow$ ◀

$$A_y = 2.73\text{ kN}\uparrow, B = 3.81\text{ kN}\uparrow$$



(*b*) **Shear Diag:**

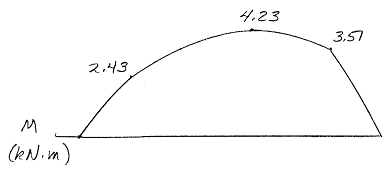
V is piecewise linear with $\left(\dfrac{dV}{dx} = -0.6\text{ kN/m}\right)$ throughout, and discontinuities equal to forces at A, C, F, B.

Note that $V = 0 = 1.47\text{ kN} - (0.6\text{ kN/m})x$ at $x = 2.45\text{ m}$

Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$, with "breaks" in slope at C and F.

PROBLEM 7.87 CONTINUED



$$M_C = \frac{1}{2}(2.73 + 2.13)\,\text{kN}(1\,\text{m}) = 2.43\,\text{kN}\cdot\text{m}$$

$$M_{\text{max}} = 2.43\,\text{kN}\cdot\text{m} + \frac{1}{2}(1.47\,\text{kN})(2.45\,\text{m}) = 4.231\,\text{kN}\cdot\text{m}$$

$$M_F = 4.231\,\text{kN}\cdot\text{m} - \frac{1}{2}(0.93\,\text{kN})(1.55\,\text{m}) = 3.51\,\text{kN}\cdot\text{m}$$