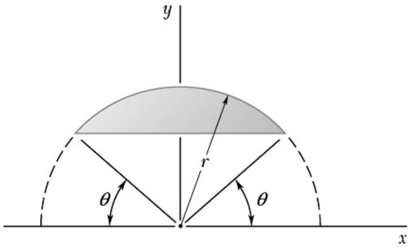
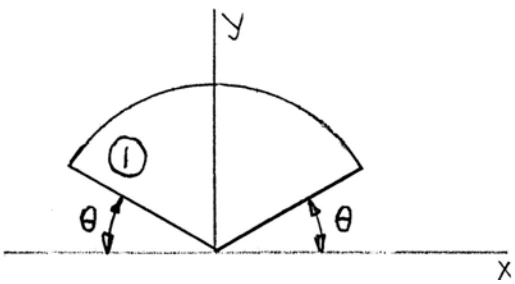
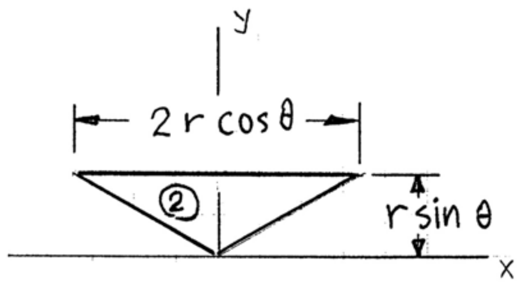
PROBLEM 5.19



The first moment of the shaded area with respect to the x axis is denoted by Q_x. (a) Express Q_x in terms of r and θ. (b) For what value of θ is Q_x maximum, and what is the maximum value?

SOLUTION





(a) With $Q_x = \Sigma \bar{y} A$ and using Fig. 5.8 A,

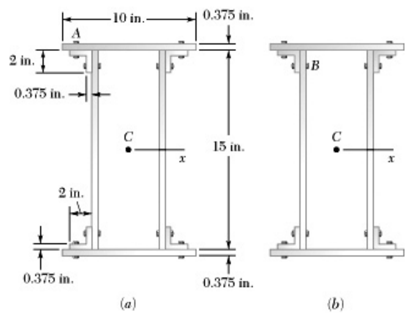
$$Q_x = \left[\frac{\frac{2}{3} r \sin\left(\frac{\pi}{2} - \theta\right)}{\frac{\pi}{2} - \theta}\right]\left[r^2\left(\frac{\pi}{2} - \theta\right)\right] - \left(\frac{2}{3} r \sin\theta\right)\left(\frac{1}{2} \times 2r\cos\theta \times r\sin\theta\right)$$

$$= \frac{2}{3} r^3 \left(\cos\theta - \cos\theta \sin^2\theta\right)$$

or $Q_x = \frac{2}{3} r^3 \cos^3\theta$ ◀

(b) By observation, Q_x is maximum when $\theta = 0$ ◀

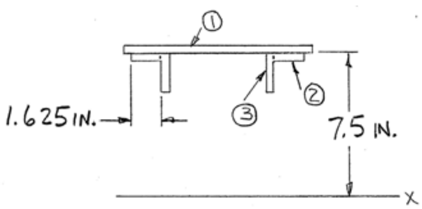
and then $Q_x = \frac{2}{3} r^3$ ◀



PROBLEM 5.20

A composite beam is constructed by bolting four plates to four $2 \times 2 \times 3/8$-in. angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at *A* and *B* are proportional to the first moments with respect to the centroidal *x* axis of the red shaded areas shown, respectively, in parts ***a*** and ***b*** of the figure. Knowing that the force exerted on the bolt at *A* is 70 lb, determine the force exerted on the bolt at *B*.

SOLUTION



From the problem statement: $F \propto Q_x$

so that
$$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}$$

and
$$F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

Now
$$Q_x = \sum \bar{y} A$$

So
$$(Q_x)_A = \left(7.5 \text{ in.} + \frac{0.375}{2} \text{ in.}\right)\left[10 \text{ in.} \times (0.375 \text{ in.})\right] = 28.82 \text{ in}^3$$

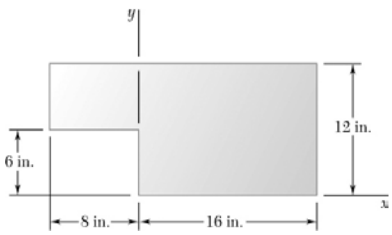
and
$$(Q_x)_B = (Q_x)_A + 2\left(7.5 \text{ in.} - \frac{0.375}{2} \text{ in.}\right)\left[(1.625 \text{ in.})(0.375 \text{ in.})\right] + 2(7.5 \text{ in.} - 1 \text{ in.})\left[(2 \text{ in.})(0.375 \text{ in.})\right]$$

$$= 28.82 \text{ in}^3 + 8.921 \text{ in}^3 + 9.75 \text{ in}^3$$

$$= 47.49 \text{ in}^3$$

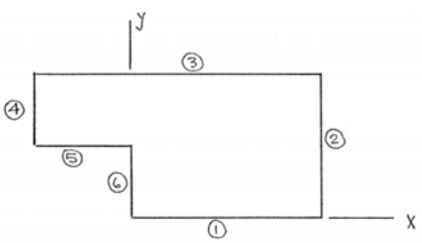
Then
$$F_B = \frac{47.49 \text{ in}^3}{28.82 \text{ in}^3}(70 \text{ lb}) = 115.3 \text{ lb} \blacktriangleleft$$

PROBLEM 5.21



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

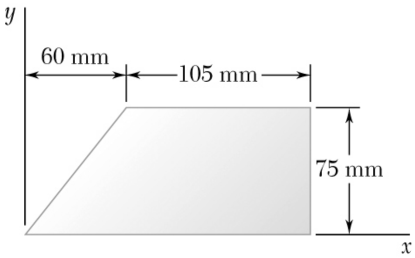
	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
1	16	8	0	128	0
2	12	16	6	102	72
3	24	4	12	96	288
4	6	–8	9	–48	54
5	8	–4	6	–32	48
6	6	0	3	0	18
Σ	72			336	480

Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(72 \text{ in.}) = 336 \text{ in}^2 \qquad \text{or} \quad \bar{X} = 4.67 \text{ in.} \blacktriangleleft$$

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

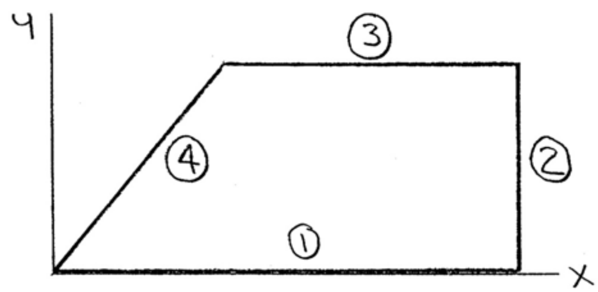
$$\bar{Y}(72 \text{ in.}) = 480 \text{ in}^2 \qquad \text{or} \quad \bar{Y} = 6.67 \text{ in.} \blacktriangleleft$$



PROBLEM 5.22

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

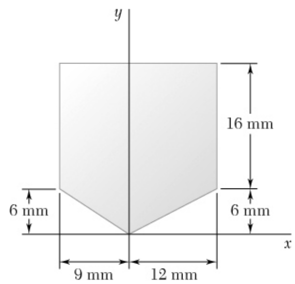
	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	165	82.5	0	13 612	0
2	75	165	37.5	12 375	2812
3	105	112.5	75	11 812	7875
4	$\sqrt{60^2 + 75^2} = 96.05$	30	37.5	2881	3602
Σ	441.05			40 680	14 289

Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(441.05 \text{ mm}) = 40\,680 \text{ mm}^2$$ or $\bar{X} = 92.2$ mm ◀

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

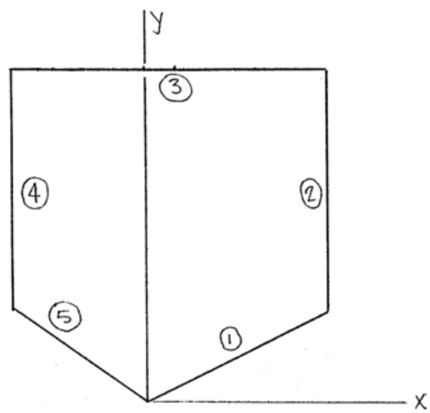
$$\bar{Y}(441.05 \text{ mm}) = 14\,289 \text{ mm}^2$$ $\bar{Y} = 32.4$ mm ◀



PROBLEM 5.23

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	$\sqrt{12^2 + 6^2} = 13.416$	6	3	80.50	40.25
2	16	12	14	192	224
3	21	1.5	22	31.50	462
4	16	–9	14	–144	224
5	$\sqrt{6^2 + 9^2} = 10.817$	–4.5	3	–48.67	32.45
Σ	77.233			111.32	982.7

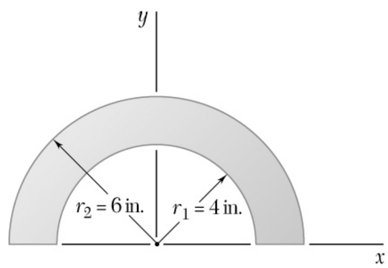
Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(77.233 \text{ mm}) = 111.32 \text{ mm}^2$$ or $\bar{X} = 1.441$ mm ◀

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

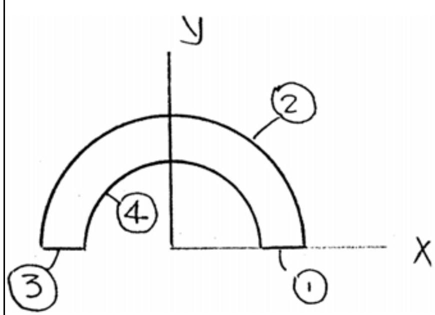
$$\bar{Y}(77.233 \text{ mm}) = 982.7 \text{ mm}^2$$ or $\bar{Y} = 12.72$ mm ◀

PROBLEM 5.24



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

By symmetry $\bar{X} = 0$ ◀

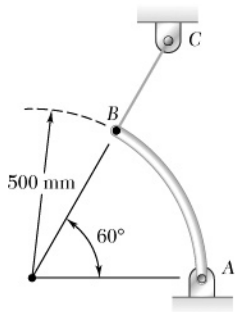
	L, in.	$\bar{y}$, in.	$\bar{y}L$, in^2
1	2	0	0
2	$\pi(6)$	$\dfrac{2(6)}{\pi} = 3.820$	72
3	2	0	0
4	$\pi(4)$	$\dfrac{2(4)}{\pi} = 2.546$	32
Σ	35.416		104

Then $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

$$\bar{Y}(35.416 \text{ in.}) = 104 \text{ in}^2$$

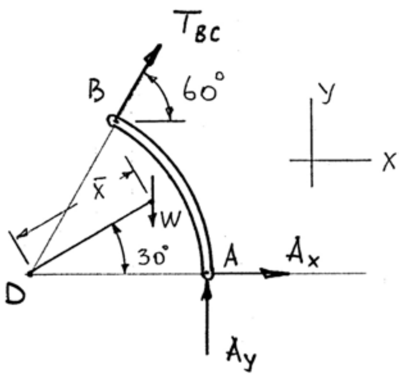
or $\bar{Y} = 2.94$ in. ◀

PROBLEM 5.25



A $750 = \text{g}$ uniform steel rod is bent into a circular arc of radius 500 mm as shown. The rod is supported by a pin at *A* and the cord *BC*. Determine (*a*) the tension in the cord, (*b*) the reaction at *A*.

SOLUTION



First note, from Figure 5.8B: $\bar{X} = \dfrac{(0.5\text{ m})\sin 30°}{\pi/6}$

$$= \frac{1.5}{\pi}\text{ m}$$

Then
$$\begin{aligned} W &= mg \\ &= (0.75\text{ kg})(9.81\text{ m/s}^2) \\ &= 7.358\text{ N} \end{aligned}$$

Also note that ΔABD is an equilateral triangle.
Equilibrium then requires

(*a*) $\Sigma M_A = 0$:

$$\left[0.5\text{ m} - \left(\frac{1.5}{\pi}\text{ m}\right)\cos 30°\right](7.358\text{ N}) - \left[(0.5\text{ m})\sin 60°\right]T_{BC} = 0$$

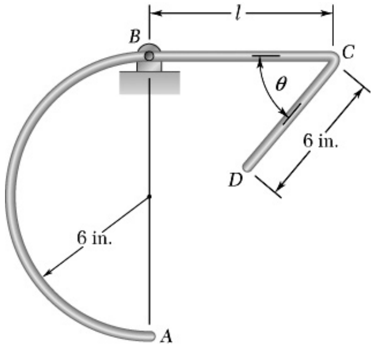
or $T_{BC} = 1.4698\text{ N}$ or $T_{BC} = 1.470\text{ N}$ ◀

(*b*) $\Sigma F_x = 0$: $A_x + (1.4698\text{ N})\cos 60° = 0$

or $A_x = -0.7349\text{ N}$

$\Sigma F_y = 0$: $A_y - 7.358\text{ N} + (1.4698\text{ N})\sin 60° = 0$

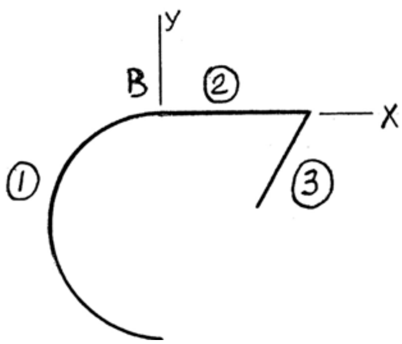
or $A_y = 6.085\text{ N}$ thus $\mathbf{A} = 6.13\text{ N}$ ⦪ $83.1°$ ◀



PROBLEM 5.26

The homogeneous wire *ABCD* is bent as shown and is supported by a pin at *B*. Knowing that $l = 8\text{ in.}$, determine the angle θ for which portion *BC* of the wire is horizontal.

SOLUTION



First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *B*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_B = 0$, which implies that $\bar{x} = 0$ or $\Sigma xL = 0$

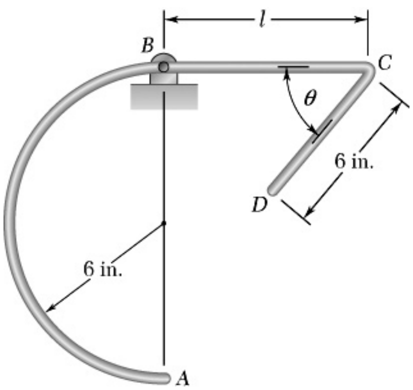
Hence

$$-\frac{2(6\text{ in.})}{\pi}(\pi \times 6\text{ in.}) + \left(\frac{8\text{ in.}}{2}\right)(8\text{ in.})$$

$$+\left(8\text{ in.} - \frac{6\text{ in.}}{2}\cos\theta\right)(6\text{ in.}) = 0$$

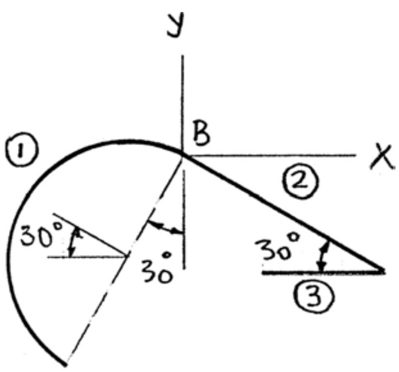
Then $\qquad \cos\theta = \dfrac{4}{9} \qquad$ or $\theta = 63.6°$ ◀

PROBLEM 5.27



The homogeneous wire *ABCD* is bent as shown and is supported by a pin at *B*. Knowing that $\theta = 30°$, determine the length l for which portion *CD* of the wire is horizontal.

SOLUTION



First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *B*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_B = 0$, which implies that $\bar{x} = 0$ or $\Sigma x_i L_i = 0$

Hence
$$-\left[\frac{2(6\text{ in.})}{\pi}\cos 30° + (6\text{ in.})\sin 30°\right](\pi \times 6\text{ in.})$$

$$+\left[\frac{(l\text{ in.})}{2}\cos 30°\right](l\text{ in.})$$

$$+\left[(l\text{ in.})\cos 30° - \frac{6\text{ in.}}{2}\right](6\text{ in.}) = 0$$

or
$$l^2 + 12.0l - 316.16 = 0$$

with roots $l_1 = 12.77$ and -24.77.

Taking the positive root

$l = 12.77$ in. ◀