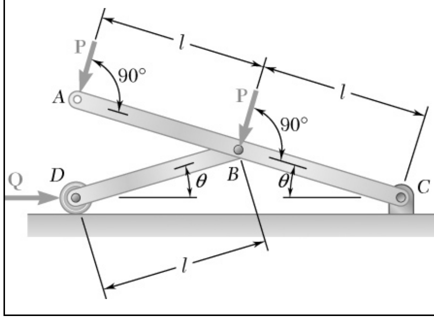
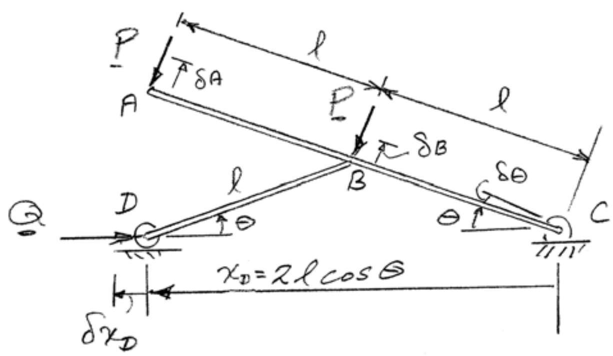
PROBLEM 10.101



Derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the mechanism shown.

SOLUTION



Have $\quad x_D = 2l\cos\theta \quad$ so that $\quad \delta x_D = -2l\sin\theta\delta\theta$

$$\delta A = 2l\delta\theta$$

$$\delta B = l\delta\theta$$

Virtual Work:

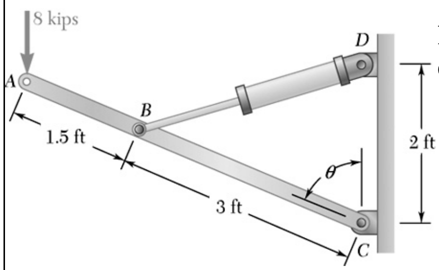
$$\delta U = 0: \quad -Q\delta x_D - P\delta A - P\delta B = 0$$

$$-Q(-2l\sin\theta\delta\theta) - P(2l\delta\theta) - P(l\delta\theta) = 0$$

$$2Ql\sin\theta - 3Pl = 0$$

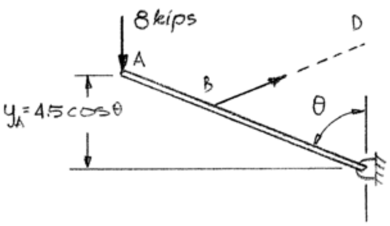
$$Q = \frac{3}{2}\frac{P}{\sin\theta} \blacktriangleleft$$

PROBLEM 10.102



The position of boom ABC is controlled by the hydraulic cylinder BD. For the loading shown, determine the force exerted by the hydraulic cylinder on pin B when $\theta = 70°$.

SOLUTION



First note, by the Law of Cosines

$$DB^2 = (3\text{ ft})^2 + (2\text{ ft})^2 - 2(3\text{ ft})(2\text{ ft})\cos\theta$$

$$= [13 - 12\cos\theta](\text{ft}^2)$$

$$DB = \sqrt{13 - 12\cos\theta}$$

Then $$\delta_B = \delta DB = \frac{1}{2}\frac{(-12)(\sin\theta)}{\sqrt{13 - 12\cos\theta}}\delta\theta$$

or $$\delta_B = \frac{6\sin\theta}{\sqrt{13 - 12\cos\theta}}\delta\theta$$

Also $$y_A = 4.5\cos\theta$$

Then $$\delta y_A = -4.5\sin\theta\delta\theta$$

Virtual Work

$$\delta U = 0: \quad -(8\text{ kips})\delta y_A - F_{DB}\delta_B = 0$$

Then $$-8(-4.5\sin\theta)\delta\theta - F_{DB}\left(\frac{6\sin\theta}{\sqrt{13 - 12\cos\theta}}\right)\delta\theta = 0$$

or $$F_{DB} = \frac{(8)(4.5\sin\theta)}{6\sin\theta}\sqrt{13 - 12\cos\theta}$$

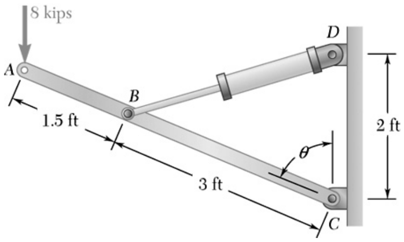
or $$F_{DB} = 6\sqrt{13 - 12\cos\theta}$$

For $$\theta = 70°$$

$$F_{DB} = 17.895\text{ kips}$$

$\mathbf{F}_{DB} = 17.90$ kips ◀

PROBLEM 10.103



The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the largest allowable value of the angle θ if the maximum force that the cylinder can exert on pin *B* is 25 kips.

SOLUTION

From the analysis of Problem 10.102, we have

$$F_{AB} = 6\sqrt{13 - 12\cos\theta}$$

For

$$F_{AB} = 25 \text{ kips}$$

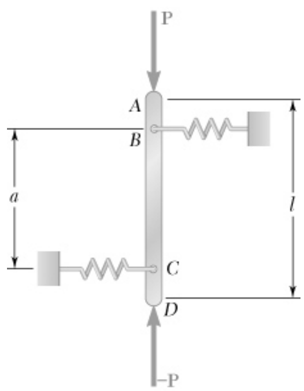
$$6\sqrt{13 - 12\cos\theta} = 25$$

or

$$\cos\theta = \frac{-17.36 + 13}{12} = -0.3633$$

$$\theta = 111.31°$$

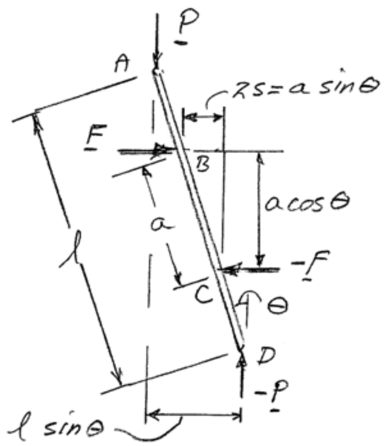
$$\theta = 111.3° \blacktriangleleft$$



PROBLEM 10.104

A vertical bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Determine the range of values of the magnitude P of two equal and opposite vertical forces **P** and $-$**P** for which the equilibrium position is stable if (a) $AB = CD$, (b) $AB = 2CD$.

SOLUTION



For both (a) and (b): Since **P** and $-$**P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and $-$**F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$

We have

$$dU = M_P d\theta + M_F d\theta$$

$$= \left(Pl\sin\theta - Fa\cos\theta\right)d\theta$$

but $$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus, $$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or $$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and $$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

PROBLEM 10.104 CONTINUED

For $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

For Stability: $$\frac{d^2V}{d\theta^2} > 0, \qquad -Pl + \frac{1}{2}ka^2 > 0$$

or (for parts *a* and *b*) $$P < \frac{ka^2}{2l} \blacktriangleleft$$

Note: To check that equilibrium is unstable for $P = \dfrac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \qquad \text{for} \qquad \theta = 0,$$

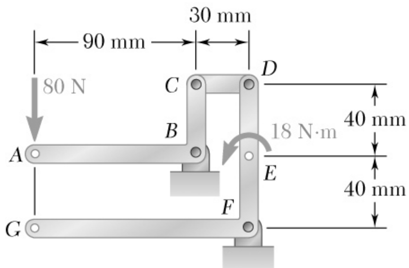
$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $$\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$$

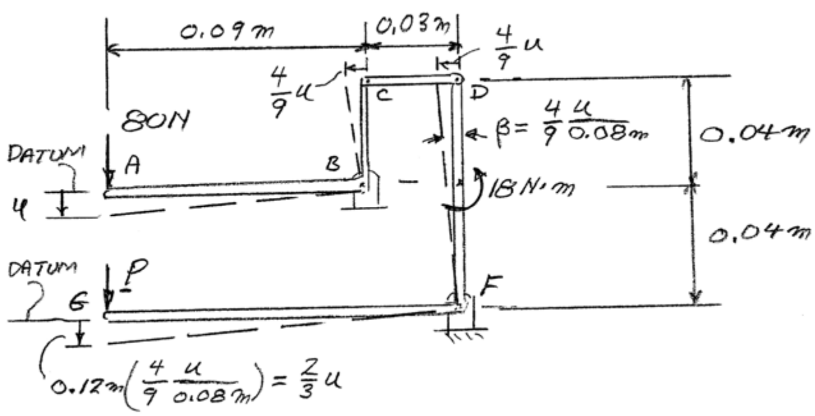
Thus, equilibrium is unstable when $$P = \frac{ka^2}{2l}$$

PROBLEM 10.105

Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.



SOLUTION



$$y_A = -u, \qquad y_G = -\frac{2}{3}u, \qquad \beta = \frac{4u}{0.72}$$

$$V = (80\text{ N})y_A + P(y_G) - (18\text{ N}\cdot\text{m})\beta$$

$$= 80(-u) + P\left(-\frac{2}{3}u\right) - (18)\frac{4u}{0.72}$$

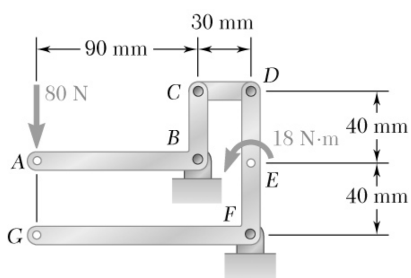
$$\frac{dV}{du} = -80 - \frac{2}{3}P - 100 = 0$$

$$P = -270\text{ N}$$

$\mathbf{P} = 270\text{ N}\uparrow$ ◄

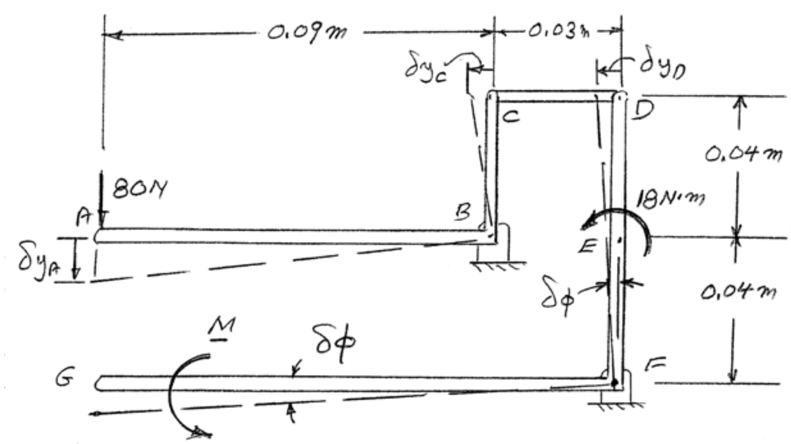
Substituting $P = -270$ N into the expression for V, we have $V = 0$. Thus V is constant and *equilibrium is neutral.*

PROBLEM 10.106



Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\quad \delta y_A \downarrow: \delta y_C = \dfrac{0.04}{0.09}\delta y_A = \dfrac{4}{9}\delta y_A \longleftarrow, \qquad \delta y_D = \delta y_C = \dfrac{4}{9}\delta y_A \longleftarrow$

$$\delta\phi = \frac{\delta y_C}{0.08} = \frac{4}{9}\frac{\delta y_A}{0.08} = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowright$$

Virtual Work:

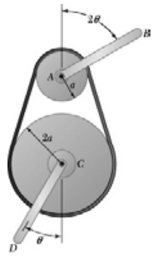
$$\delta U = 0: \quad (80\text{ N})\delta y_A + (18\text{ N}\cdot\text{m})\delta\phi + M\delta\phi = 0$$

$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + M\left(\frac{50}{9}\delta y_A\right) = 0$$

$$80 + 100 + \frac{50}{9}M = 0$$

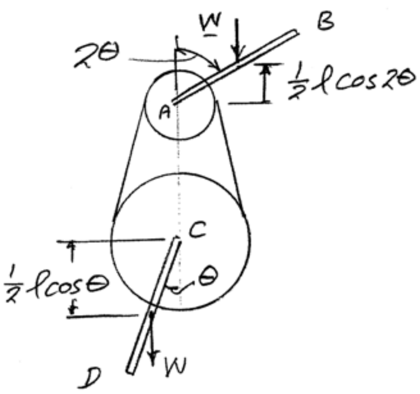
$M = -32.4\text{ N}\cdot\text{m} \qquad\qquad \mathbf{M} = 32.4\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 10.107



Two uniform rods, each of mass m and length l, are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



$$W = mg$$

$$V = W\left(\frac{l}{2}\cos 2\theta\right) - W\left(\frac{l}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = W\frac{l}{2}(-2\sin 2 + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \frac{Wl}{2}(-2\sin 2\theta + \sin\theta) = 0$$

or $$\sin\theta(-4\cos\theta + 1) = 0$$

Solving, $$\theta = 0, 75.5°,\ 180°,\ \text{and}\ \ 284.5°$$

Stability:

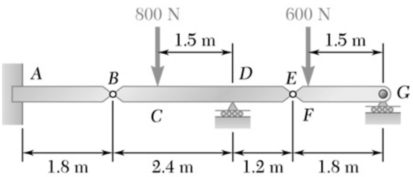
$$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$$

At $\theta = 0$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 - 1) < 0$ $\quad\therefore\quad \theta = 0,$ Unstable ◀

At $\theta = 75.5°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\quad\therefore\quad \theta = 75.5°,$ Stable ◀

At $\theta = 180°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 + 1) < 0$ $\quad\therefore\quad \theta = 180.0°,$ Unstable ◀

At $\theta = 284.5°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\quad\therefore\quad \theta = 285°,$ Stable ◀

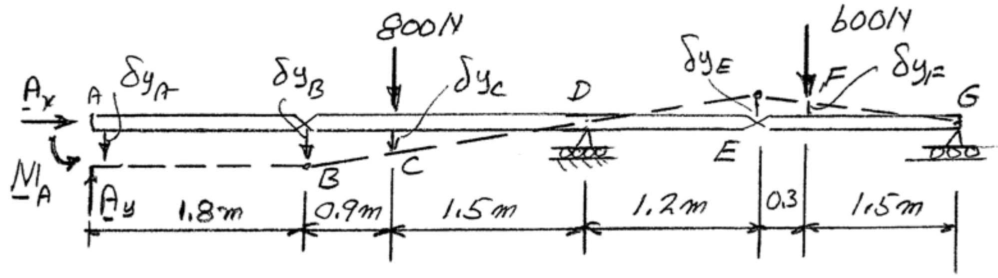


PROBLEM 10.108

Using the method of virtual work, determine separately the force and the couple representing the reaction at *A*.

SOLUTION

Vertical component at A. Move point *A* downward *without* rotation.



Since *AB* remains horizontal, $\delta y_A = \delta y_B$

$$\delta y_C = \frac{5}{8}\delta y_B; \qquad \delta y_E = \frac{1}{2}\delta y_B; \qquad \delta y_E = \frac{5}{6}\delta y_E = \frac{5}{6}\left(\frac{1}{2}\delta y_B\right) = \frac{5}{12}\delta y_B$$

Virtual Work: $\delta U = 0: \; -A\delta y_A + (800 \text{ N})\delta y_C - (600)\delta y_F = 0$

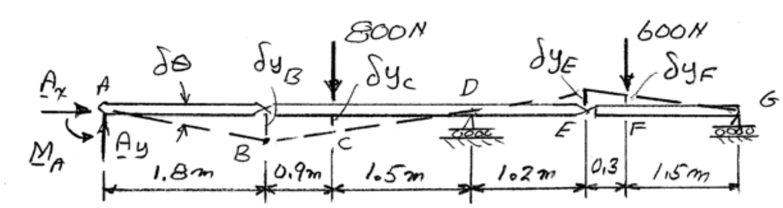
$$-A_y\delta y_B + 800\left(\frac{5}{8}\delta y_B\right) - 600\left(\frac{5}{12}\delta y_B\right) = 0$$

$$A_y = +250 \text{ N} \qquad \mathbf{A}_y = 250 \text{ N} \uparrow$$

For horizontal motion $\delta x_A,\; \delta U = 0 = A_x\delta x_A; \qquad A_x = 0$

$$\therefore \; \mathbf{A} = 250 \text{ N} \uparrow \blacktriangleleft$$

For couple $\mathbf{M}_A$, we rotate *AB* about *A* through $\delta\theta$



PROBLEM 10.108 CONTINUED

$$\delta y_B = 1.8\delta\theta; \qquad \delta y_E = \frac{1}{2}\delta y_B = \frac{1}{2}(1.8\delta\theta) = 0.9\delta\theta$$

$$\delta y_C = \frac{5}{8}\delta y_B = \frac{5}{8}(1.8\delta\theta) = 1.25\delta\theta$$

$$\delta y_F = \frac{5}{6}\delta y_E = \frac{5}{6}(0.98\theta) = 0.75\delta\theta$$

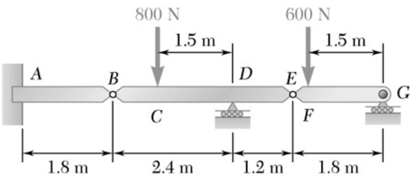
Virtual Work:

$$\delta U = 0: \quad -M_A\delta\theta + (800\text{ N})\delta y_C - (600\text{ N})\delta y_F = 0$$

$$-M_A\delta\theta + 800(1.125\delta\theta) - 600(0.75\delta\theta) = 0$$

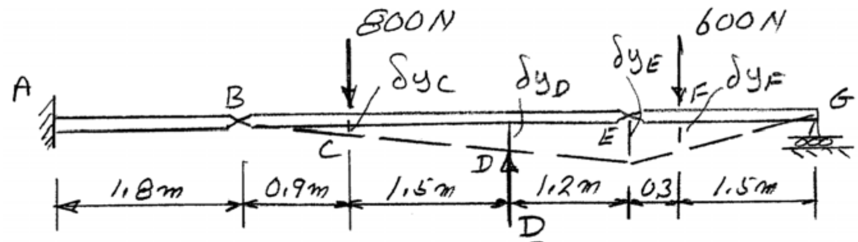
$$M_A = +450\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = 450\text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 10.109



Using the method of virtual work, determine the reaction at *D*.

SOLUTION



We move point *D* downward a distance δy_D

$$\delta y_C = \frac{3}{8}\delta y_D \qquad \delta y_E = \frac{3}{2}\delta y_D$$

$$\delta y_F = \frac{5}{6}\delta y_E = \frac{5}{6}\left(\frac{3}{2}\delta y_D\right) = \frac{5}{4}\delta y_D$$

Virtual Work:

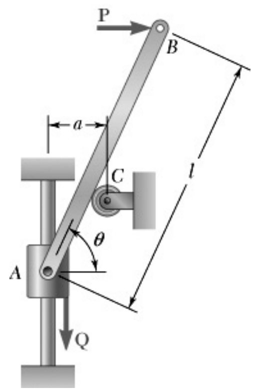
$$\delta U = 0: \quad -D\delta y_D + (800 \text{ N})\delta y_C + (600 \text{ N})\delta y_F = 0$$

$$-D\delta y_D + 800\left(\frac{3}{8}\delta y_D\right) + 600\left(\frac{5}{4}\delta y_D\right) = 0$$

$$D = +1050 \text{ N}$$

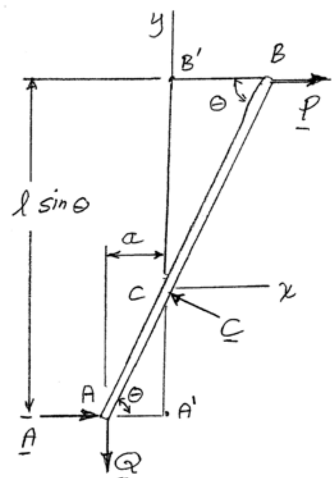
D = 1050 N ↑ ◀

PROBLEM 10.110



The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$: $$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

Virtual Work:

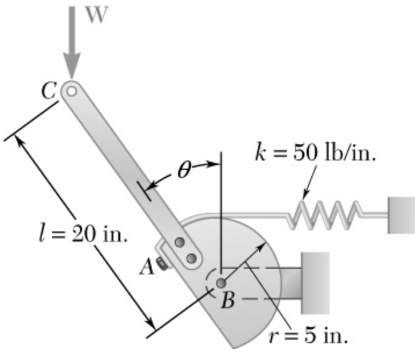
$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\,\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

or $$Pl\sin\theta\cos^2\theta = Qa$$

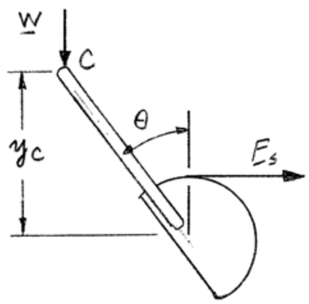
$$Q = P\frac{l}{a}\sin\theta\cos^2\theta \blacktriangleleft$$

PROBLEM 10.111



A load **W** of magnitude 100 lb is applied to the mechanism at C. Knowing that the spring is unstretched when $\theta = 15°$, determine the value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION



Have $\qquad y_C = l\cos\theta$

$$V = \frac{1}{2}k\left[r(\theta - \theta_0)\right]^2 + Wy_C \qquad \theta_0 = 15° = \frac{\pi}{12}\text{ rad}$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl\sin\theta$$

Equilibrium $\qquad \dfrac{dV}{d\theta} = 0: \quad kr^2(\theta - \theta_0) - wl\sin\theta = 0 \qquad (1)$

With $\qquad W = 100$ lb, $R = 50$ lb/in., $l = 20$ in., and $r = 5$ in.

$$(50\text{ lb/in.})(25\text{ in}^2)\left(\theta - \frac{\pi}{12}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or $\qquad 0.625\theta - \sin\theta = 0.16362$

Solving numerically, $\qquad \theta = 1.8145\text{ rad} = 103.97°$

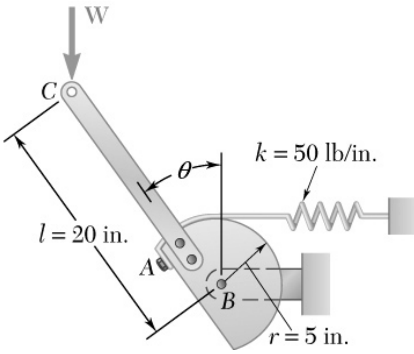
$\theta = 104.0°$ ◀

Stability $\qquad \dfrac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta \qquad (2)$

or $\qquad = 1250 - 2000\cos\theta$

For $\theta = 104.0°$: $\qquad = 1734\text{ in.}\cdot\text{lb} > 0 \qquad \therefore$ Stable ◀

PROBLEM 10.112



A load **W** of magnitude 100 lb is applied to the mechanism at C. Knowing that the spring is unstretched when $\theta = 30°$, determine the value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.111, particularly Equations (1), with 15° replace by $30°\left(\frac{\pi}{6}\text{ rad}\right)$:

For equilibrium
$$kr^2\left(\theta - \frac{\pi}{6}\right) - Wl\sin\theta = 0$$

With
$$k = 50\text{ lb/in.},\ W = 100\text{ lb},\ r = 5\text{ in., and } l = 20\text{ in.}$$

$$(50\text{ lb/in.})(25\text{ in.}^2)\left(\theta - \frac{\pi}{6}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or
$$1250\theta - 654.5 - 2000\sin\theta = 0$$

Solving numerically,
$$\theta = 1.9870\text{ rad} = 113.8°$$

$$\theta = 113.8° \blacktriangleleft$$

Stability: Equation (2), Problem 111:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$$

or
$$= 1250 - 2000\cos\theta$$

For $\theta = 113.8°$:
$$= 2057\text{ in.}\cdot\text{lb} > 0 \qquad \therefore\ \text{Stable} \blacktriangleleft$$