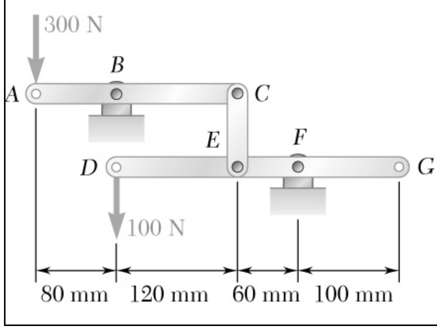
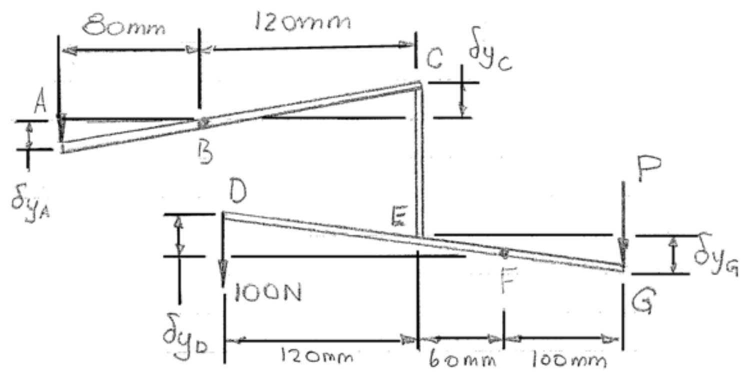


PROBLEM 10.1

Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assuming

$$\delta y_A \downarrow$$

it follows

$$\delta y_C = \frac{120}{80}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{180}{60}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta y_G = \frac{100}{60}\delta y_A = \frac{100}{60}(1.5\delta y_A) = 2.5\delta y_A \downarrow$$

Then, by Virtual Work

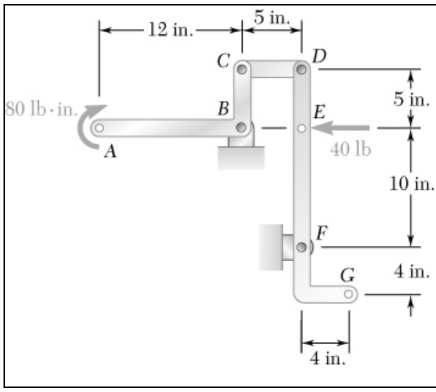
$$\delta U = 0: \quad (300\text{ N})\delta y_A - (100\text{ N})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

$$P = +60\text{ N}$$

$\mathbf{P} = 60\text{ N} \downarrow$ ◀

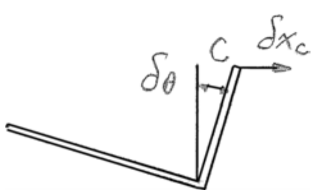


PROBLEM 10.2

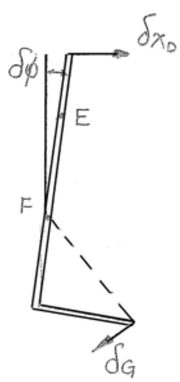
Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.

SOLUTION

Link ABC



Link DEFG



Assume

$$\delta\theta \text{ clockwise}$$

Then for point C

$$\delta x_C = (5\delta\theta)\text{ in.} \longrightarrow$$

and for point D

$$\delta x_D = \delta x_C = (5\delta\theta)\text{ in.} \longrightarrow$$

And for link $DEFG$

$$\delta x_D = 15\delta\phi$$

$$\therefore\quad 5\delta\theta = 15\delta\phi$$

or
$$\delta\phi = \frac{1}{3}\delta\theta$$

Then
$$\delta_G = 4\sqrt{2}\delta\phi = \left(\frac{4}{3}\sqrt{2}\delta\theta\right)\text{ in.}$$

Now
$$\delta y_G = \delta_G \cos 45°$$

$$= \left(\frac{4}{3}\sqrt{2}\delta\theta\right)\cos 45°$$

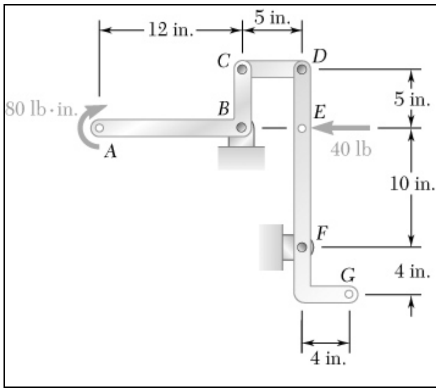
$$= \left(\frac{4}{3}\delta\theta\right)\text{ in.}$$

Then, by Virtual Work.

$$\delta U = 0\colon\quad (80\text{ lb}\cdot\text{in.})\delta\theta - (40\text{ lb})\delta x_E(\text{in.}) + P\delta y_G(\text{in.}) = 0$$

$$80\delta\theta - 40\left(\frac{10}{3}\delta\theta\right) + P\left(\frac{4}{3}\delta\theta\right) = 0$$

or $\mathbf{P} = 40\text{ lb}\downarrow$ ◀

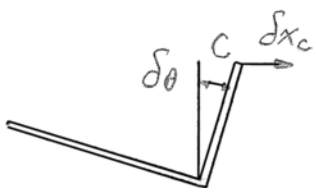


PROBLEM 10.3

Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION

Link ABC



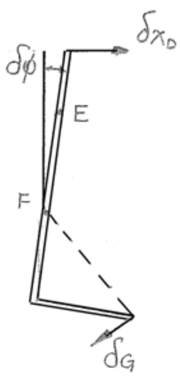
Following the kinematic analysis of Problem 10.2, we have $U = 0$:

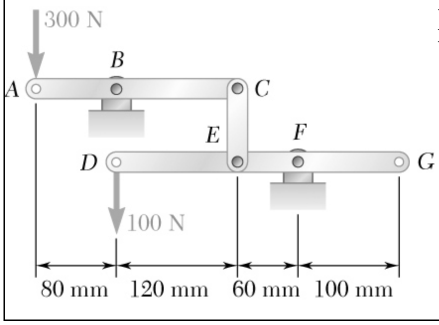
$$\delta U = 0\colon \quad (80 \text{ lb}\cdot\text{in.})\delta\theta - (40 \text{ lb})\delta x_E(\text{in.}) + M\delta\phi = 0$$

$$80\delta\theta - 40\left(\frac{10}{3}\delta\theta\right) + M\left(\frac{1}{3}\delta\theta\right) = 0$$

or $\mathbf{M} = 160 \text{ lb}\cdot\text{in.}$ ◀

Link DEFG

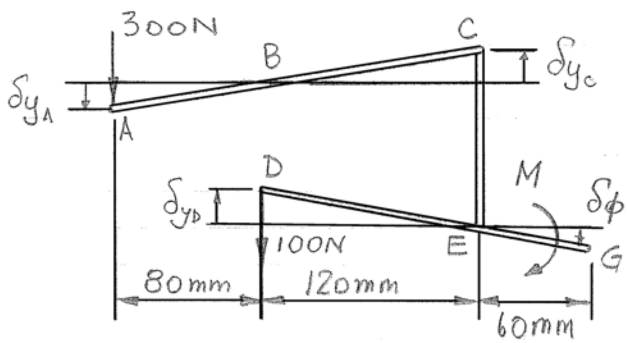




PROBLEM 10.4

Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assuming

$$\delta y_A \downarrow$$

it follows

$$\delta y_C = \frac{120}{80}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{180}{60}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta\phi = \frac{\delta y_E}{60} = \frac{1.5\delta y_A}{60} = \frac{1}{40}\delta y_A$$

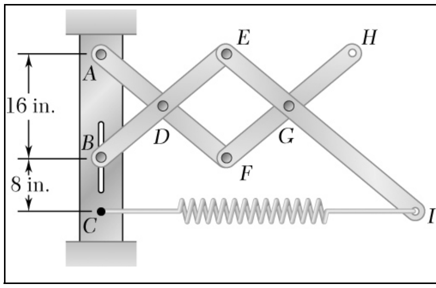
Then, by Virtual Work:

$$\delta U = 0: \quad (300 \text{ N})\delta y_A - (100 \text{ N})\delta y_D + M\delta\phi = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + M\left(\frac{1}{40}\delta y_A\right) = 0$$

$$300 - 450 + \frac{1}{40}M = 0$$

$M = +6000 \text{ N}\cdot\text{mm}$ ↻ $\qquad$ $\mathbf{M} = 6.00 \text{ N}\cdot\text{m}$ ↻ ◀



PROBLEM 10.5

An unstretched spring of constant 4 lb/in. is attached to pins at points C and I as shown. The pin at B is attached to member BDE and can slide freely along the slot in the fixed plate. Determine the force in the spring and the horizontal displacement of point H when a 20-lb horizontal force directed to the right is applied (*a*) at point G, (*b*) at points G and H.

SOLUTION

First note:

$$x_G = 3x_D \Rightarrow \delta x_G = 3\delta x_D$$

$$x_H = 4x_D \Rightarrow \delta x_H = 4\delta x_D$$

$$x_I = 5x_D \Rightarrow \delta x_I = 5\delta x_D$$

(*a*) Virtual Work $\quad \delta U = 0: \quad F_G \delta x_G - F_{SP} \delta x_I = 0$

or $\quad (20 \text{ lb})(3\delta x_D) - F_{SP}(5\delta x_D) = 0$

thus, $F_{SP} = 12.00 \text{ lb } T$ ◀

Now $\quad F_{SP} = k\Delta x_I$

or $\quad 12.00 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$

Thus, $\quad \Delta x_I = 3 \text{ in.}$

and $\quad \delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$

$$\therefore \quad \Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(3 \text{ in.})$$

or $\Delta x_H = 2.40 \text{ in.} \longrightarrow$ ◀

(*b*) Virtual Work: $\quad \delta U = 0: \quad F_G \delta x_G + F_H \delta x_H - F_{SP} \delta x_I = 0$

or $\quad (20 \text{ lb})(3\delta x_D) + (20 \text{ lb})(4\delta x_D) - F_{SP}(5\delta x_D) = 0$

thus, $F_{SP} = 28.0 \text{ lb } T$ ◀

Now $\quad F_{SP} = k\Delta x_I$

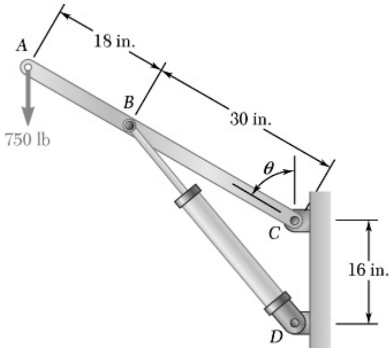
or $\quad 28.0 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$

Thus, $\quad \Delta x_I = 7 \text{ in.}$

From part (*a*) $\quad \Delta x_H = \frac{4}{5}\Delta x_I$

$$= \frac{4}{5}(7 \text{ in.})$$

or $\Delta x_H = 5.60 \text{ in.} \longrightarrow$ ◀



PROBLEM 10.6

An unstretched spring of constant 4 lb/in. is attached to pins at points *C* and *I* as shown. The pin at *B* is attached to member *BDE* and can slide freely along the slot in the fixed plate. Determine the force in the spring and the horizontal displacement of point *H* when a 20-lb horizontal force directed to the right is applied (*a*) at point *E*, (*b*) at points *D* and *E*.

SOLUTION

First note:

$$x_G = 3x_D \Rightarrow \delta x_G = 3\delta x_D$$

$$x_H = 4x_D \Rightarrow \delta x_H = 4\delta x_D$$

$$x_I = 5x_D \Rightarrow \delta x_I = 5\delta x_D$$

(*a*) Virtual Work: $\delta U = 0: \quad F_E \delta x_E - F_{SP} \delta x_I = 0$

or $$(20\text{ lb})(2\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 8.00\text{ lb } T$ ◀

Now $$F_{SP} = k\Delta x_I$$

or $$8.00\text{ lb} = (4\text{ lb/in.})\Delta x_I$$

Thus, $$\Delta x_I = 2\text{ in.}$$

And $$\delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$$

$$\therefore \quad \Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(2\text{ in.})$$

or $\Delta x_H = 1.600\text{ in.} \longrightarrow$ ◀

(*b*) Virtual Work: $\delta U = 0: \quad F_D \delta x_D + F_E \delta x_E - F_{SP} \delta x_I = 0$

or $$(20\text{ lb})\delta x_D + (20\text{ lb})(2\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 12.00\text{ lb } T$ ◀

PROBLEM 10.6 CONTINUED

Now $$F_{SP} = k\Delta x_I$$

or $$12.00\text{ lb} = (4\text{ lb/in.})\Delta x_I$$

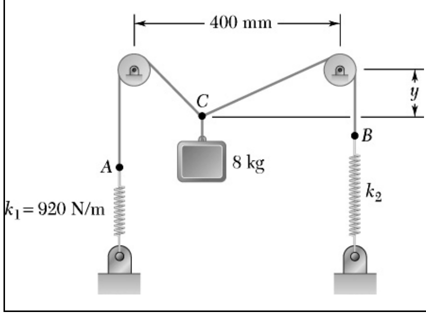
Thus, $$\Delta x_I = 3\text{ in.}$$

From part (*a*) $$\Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(3\text{ in.})$$

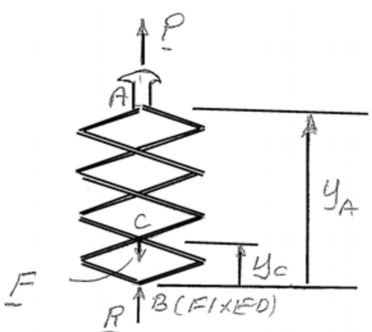
or $\Delta x_H = 2.40\text{ in.} \longrightarrow$ ◀

PROBLEM 10.7



Knowing that the maximum friction force exerted by the bottle on the cork is 300 N, determine (*a*) the force **P** which must be applied to the corkscrew to open the bottle, (*b*) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION



From sketch

$$y_A = 4y_C$$

Thus,

$$\delta y_A = 4\delta y_C$$

(*a*) Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

$$F = 300 \text{ N}: \quad P = \frac{1}{4}(300 \text{ N}) = 75 \text{ N}$$

$$\mathbf{P} = 75.0 \text{ N} \uparrow \blacktriangleleft$$

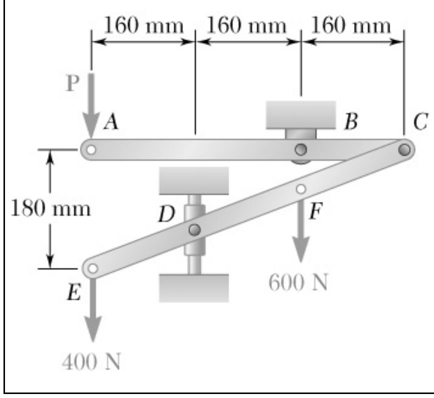
(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0: \quad R + P - F = 0$$

$$R + 75 \text{ N} - 300 \text{ N} = 0$$

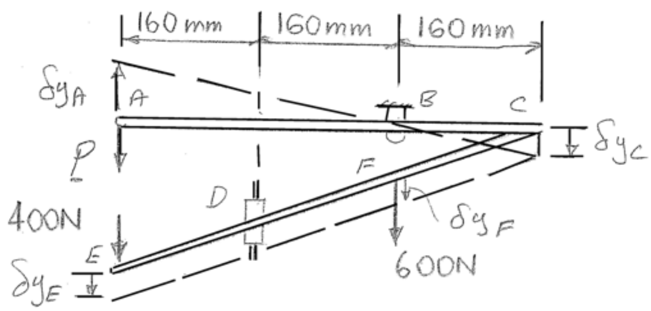
$$\mathbf{R} = 225 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 10.8



The two-bar linkage shown is supported by a pin and bracket at B and a collar at D that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$

Have $\delta y_C = \dfrac{160}{320}\delta y_A$ or $\delta y_C = \dfrac{1}{2}\delta y_A \downarrow$

Since bar CD moves in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2}\delta y_A \downarrow$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A + (400\text{ N})\delta y_E + (600\text{ N})\delta y_F = 0$$

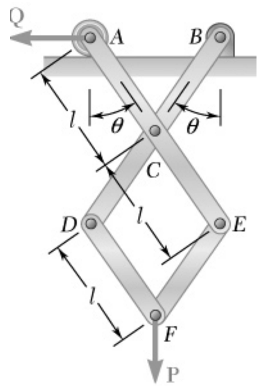
$$-P\delta y_A + (400\text{ N})\left(\frac{1}{2}\delta y_A\right) + (600\text{ N})\left(\frac{1}{2}\delta y_A\right) = 0$$

or

$$P = 500\text{ N}$$

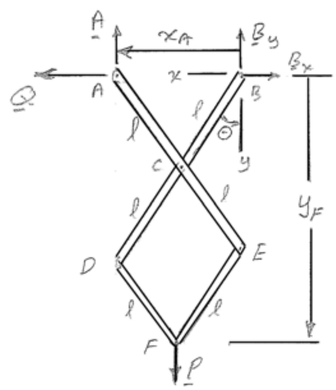
$\mathbf{P} = 500\text{ N} \downarrow$ ◀

PROBLEM 10.9



The mechanism shown is acted upon by the force **P**; derive an expression for the magnitude of the force **Q** required for equilibrium.

SOLUTION



Virtual Work:

Have $x_A = 2l\sin\theta$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

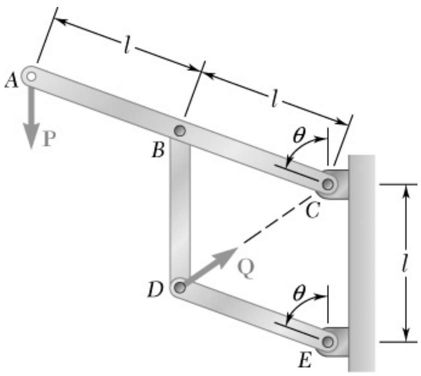
and $y_F = 3l\cos\theta$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work: $\delta U = 0$: $Q\delta x_A + P\delta y_F = 0$

$$Q(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

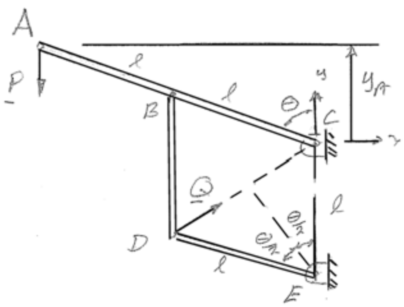
$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$



PROBLEM 10.10

Knowing that the line of action of the force **Q** passes through point C, derive an expression for the magnitude of **Q** required to maintain equilibrium

SOLUTION



Have $\quad y_A = 2l\cos\theta; \qquad \delta y_A = -2l\sin\theta\,\delta\theta$

$$CD = 2l\sin\frac{\theta}{2}; \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)} \blacktriangleleft$$