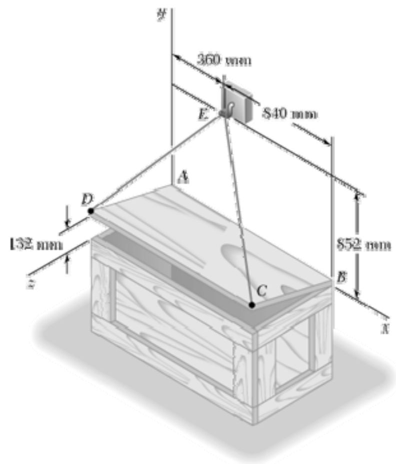
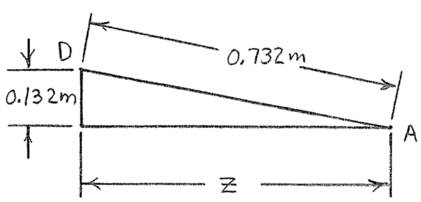


PROBLEM 3.46

The 0.732×1.2-m -m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 54 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *C*.

SOLUTION



First note $$z = \sqrt{(0.732)^2 - (0.132)^2}\ \text{m}$$

$$= 0.720\ \text{m}$$

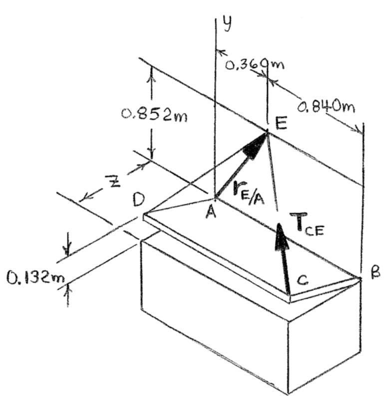
Then $$d_{CE} = \sqrt{(0.840)^2 + (0.720)^2 + (0.720)^2}\ \text{m}$$

$$= 1.32\ \text{m}$$

and $$\mathbf{T}_{CE} = \frac{\mathbf{r}_{E/C}}{d_{CE}}(T_{CE})$$

$$= \frac{-(0.840\ \text{m})\mathbf{i} + (0.720\ \text{m})\mathbf{j} - (0.720\ \text{m})\mathbf{k}}{1.32\ \text{m}}(54\ \text{N})$$

$$= -(36.363\ \text{N})\mathbf{i} + (29.454\ \text{N})\mathbf{j} - (29.454\ \text{N})\mathbf{k}$$



Now $$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{CE}$$

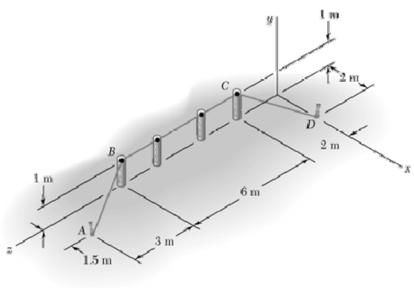
where $$\mathbf{r}_{E/A} = (0.360\ \text{m})\mathbf{i} + (0.852\ \text{m})\mathbf{j}$$

Then $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.360 & 0.852 & 0 \\ -34.363 & 29.454 & -29.454 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(25.095\ \text{N}\cdot\text{m})\mathbf{i} + (10.6034\ \text{N}\cdot\text{m})\mathbf{j} + (39.881\ \text{N}\cdot\text{m})\mathbf{k}$$

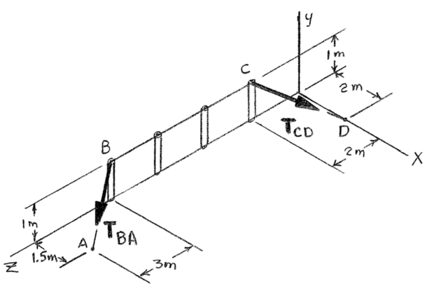
$$\therefore\ M_x = -25.1\ \text{N}\cdot\text{m},\ M_y = 10.60\ \text{N}\cdot\text{m},\ M_z = 39.9\ \text{N}\cdot\text{m}\ ◀$$

PROBLEM 3.47



A fence consists of wooden posts and a steel cable fastened to each post and anchored in the ground at A and D. Knowing that the sum of the moments about the z axis of the forces exerted by the cable on the posts at B and C is $-66\ \text{N}\cdot\text{m}$, determine the magnitude $\mathbf{T}_{CD}$ when $T_{BA} = 56$ N.

SOLUTION



Based on $$|\mathbf{M}_z| = \mathbf{k}\cdot\left[(\mathbf{r}_B)_y \times \mathbf{T}_{BA}\right] + \mathbf{k}\cdot\left[(\mathbf{r}_C)_y \times \mathbf{T}_{CD}\right]$$

where

$$\mathbf{M}_z = -(66\ \text{N}\cdot\text{m})\mathbf{k}$$

$$(\mathbf{r}_B)_y = (\mathbf{r}_C)_y = (1\ \text{m})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA}$$

$$= \frac{(1.5\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} + (3\ \text{m})\mathbf{k}}{3.5\ \text{m}}(56\ \text{N})$$

$$= (24\ \text{N})\mathbf{i} - (16\ \text{N})\mathbf{j} + (48\ \text{N})\mathbf{k}$$

$$\mathbf{T}_{CD} = \lambda_{CD}T_{CD}$$

$$= \frac{(2\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} - (2\ \text{m})\mathbf{k}}{3.0\ \text{m}}T_{CD}$$

$$= \frac{1}{3}T_{CD}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

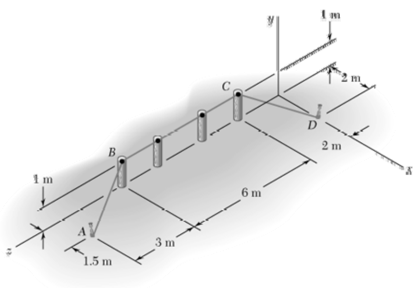
$$\therefore\ -(66\ \text{N}\cdot\text{m}) = \mathbf{k}\cdot\left\{(1\ \text{m})\mathbf{j}\times\left[(24\ \text{N})\mathbf{i} - (16\ \text{N})\mathbf{j} + (48\ \text{N})\mathbf{k}\right]\right\}$$

$$+\ \mathbf{k}\cdot\left\{(1\ \text{m})\mathbf{j}\times\left[\frac{1}{3}T_{CD}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})\right]\right\}$$

or $$-66 = -24 - \frac{2}{3}T_{CD}$$

$$\therefore\ T_{CD} = \frac{3}{2}(66 - 24)\ \text{N}$$

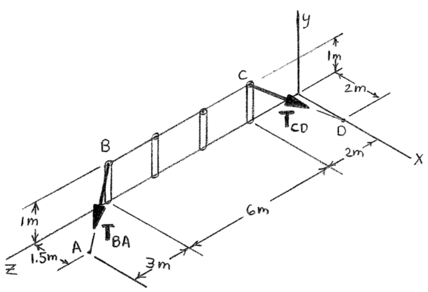
or $T_{CD} = 63.0$ N ◀



PROBLEM 3.48

A fence consists of wooden posts and a steel cable fastened to each post and anchored in the ground at A and D. Knowing that the sum of the moments about the y axis of the forces exerted by the cable on the posts at B and C is 212 N · m, determine the magnitude of $\mathbf{T}_{BA}$ when $T_{CD} = 33$ N.

SOLUTION



Based on $$|\mathbf{M}_y| = \mathbf{j}\cdot\left[(\mathbf{r}_B)_z \times \mathbf{T}_{BA} + (\mathbf{r}_C)_z \times \mathbf{T}_{CD}\right]$$

where

$$\mathbf{M}_y = (212\ \text{N}\cdot\text{m})\,\mathbf{j}$$

$$(\mathbf{r}_B)_z = (8\ \text{m})\mathbf{k}$$

$$(\mathbf{r}_C)_z = (2\ \text{m})\mathbf{k}$$

$$\mathbf{T}_{BA} = \lambda_{BA} T_{BA}$$

$$= \frac{(1.5\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} - (3\ \text{m})\mathbf{k}}{3.5\ \text{m}} T_{BA}$$

$$= \frac{T_{BA}}{3.5}(1.5\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

$$\mathbf{T}_{CD} = \lambda_{CD} T_{CD}$$

$$= \frac{(2\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} - (2\ \text{m})\mathbf{k}}{3.0\ \text{m}}(33\ \text{N})$$

$$= (22\mathbf{i} - 11\mathbf{j} - 22\mathbf{k})\ \text{N}$$

$$\therefore\ (212\ \text{N}\cdot\text{m}) = \mathbf{j}\cdot\left\{(8\ \text{m})\mathbf{k} \times \left[\frac{T_{BA}}{3.5}(1.5\mathbf{i} - \mathbf{j} + 3\mathbf{k})\right]\right\}$$

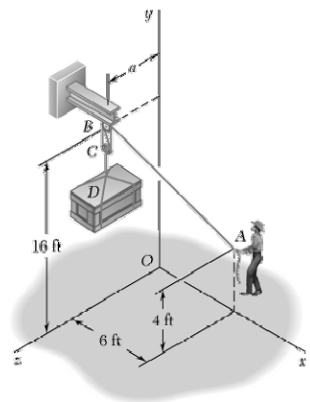
$$+\ \mathbf{j}\cdot\left[(2\ \text{m})\mathbf{k} \times (22\,\mathbf{i} - 11\mathbf{j} - 22\mathbf{k})\ \text{N}\right]$$

or $$212 = \frac{8(1.5)}{3.5} T_{BA} + 2(22)$$

$$\therefore\ T_{BA} = \frac{168}{18.6667}$$

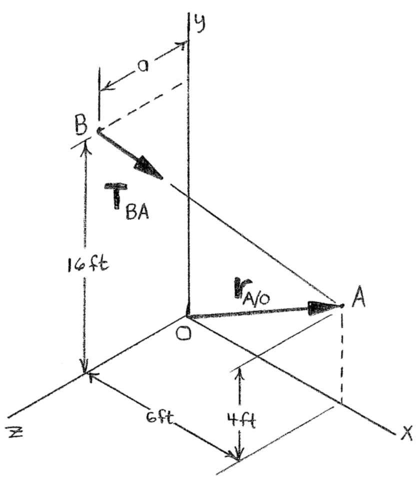
or $T_{BA} = 49.0$ N ◀

PROBLEM 3.49



To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and z axes of the force exerted at B by portion AB of the rope are, respectively, $100\ \text{lb}\cdot\text{ft}$ and $-400\ \text{lb}\cdot\text{ft}$, determine the distance a.

SOLUTION



Based on $$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{BA}$$

where

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= M_x\mathbf{i} + (100\ \text{lb}\cdot\text{ft})\mathbf{j} - (400\ \text{lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{r}_{A/O} = (6\ \text{ft})\mathbf{i} + (4\ \text{ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA}$$

$$= \frac{(6\ \text{ft})\mathbf{i} - (12\ \text{ft})\mathbf{j} - (a)\mathbf{k}}{d_{BA}}T_{BA}$$

$$\therefore\ M_x\mathbf{i} + 100\mathbf{j} - 400\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 4 & 0 \\ 6 & -12 & -a \end{vmatrix}\frac{T_{BA}}{d_{BA}}$$

$$= \frac{T_{BA}}{d_{BA}}\left[-(4a)\mathbf{i} + (6a)\mathbf{j} - (96)\mathbf{k}\right]$$

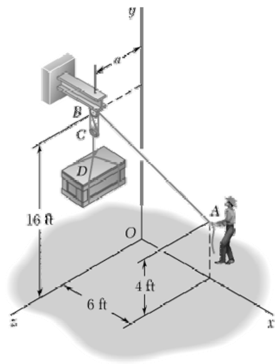
From $\mathbf{j}$-coefficient: $$100d_{AB} = 6aT_{BA} \quad \text{or} \quad T_{BA} = \frac{100}{6a}d_{BA} \qquad (1)$$

From $\mathbf{k}$-coefficient: $$-400d_{AB} = -96T_{BA} \quad \text{or} \quad T_{BA} = \frac{400}{96}d_{BA} \qquad (2)$$

Equating Equations (1) and (2) yields $$a = \frac{100(96)}{6(400)}$$

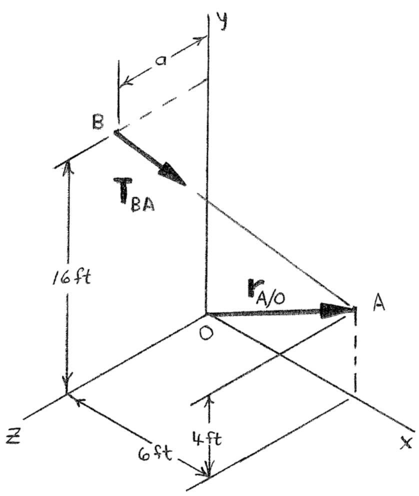
or $a = 4.00\ \text{ft}$ ◀

PROBLEM 3.50



To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the man applies a 200-lb force to end A of the rope and that the moment of that force about the y axis is $175\ \text{lb}\cdot\text{ft}$, determine the distance a.

SOLUTION



Based on $$|\mathbf{M}_y| = \mathbf{j}\cdot\left(\mathbf{r}_{A/O}\times\mathbf{T}_{BA}\right)$$

where $$\mathbf{r}_{A/O} = (6\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA} = \frac{\mathbf{r}_{A/B}}{d_{BA}}T_{BA}$$

$$= \frac{(6\text{ ft})\mathbf{i} - (12\text{ ft})\mathbf{j} - (a)\mathbf{k}}{d_{BA}}(200\text{ lb})$$

$$= \frac{200}{d_{BA}}(6\mathbf{i} - 12\mathbf{j} - a\mathbf{k})$$

$$\therefore\ 175\text{ lb}\cdot\text{ft} = \begin{vmatrix} 0 & 1 & 0 \\ 6 & 4 & 0 \\ 6 & -12 & -a \end{vmatrix}\frac{200}{d_{BA}}$$

$$175 = \left[0 - 6(-a)\right]\frac{200}{d_{BA}}$$

where $$d_{BA} = \sqrt{(6)^2 + (12)^2 + (a)^2}\ \text{ft}$$

$$= \sqrt{180 + a^2}\ \text{ft}$$

$$\therefore\ 175\sqrt{180 + a^2} = 1200a$$

or $$\sqrt{180 + a^2} = 6.8571a$$

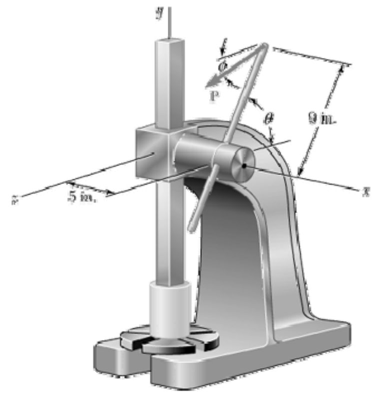
Squaring each side

$$180 + a^2 = 47.020a^2$$

Solving $$a = 1.97771\text{ ft}$$

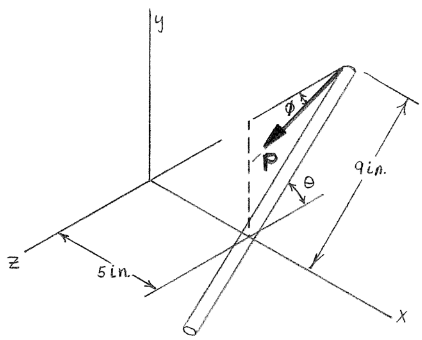
or $a = 1.978\text{ ft}$ ◀

PROBLEM 3.51



A force **P** is applied to the lever of an arbor press. Knowing that **P** lies in a plane parallel to the *yz* plane and that $M_x = 230\ \text{lb}\cdot\text{in.}$, $M_y = -200\ \text{lb}\cdot\text{in.}$, and $M_z = -35\ \text{lb}\cdot\text{in.}$, determine the magnitude of **P** and the values of ϕ and θ.

SOLUTION



Based on $$M_x = (P\cos\phi)\left[(9\text{ in.})\sin\theta\right] - (P\sin\phi)\left[(9\text{ in.})\cos\theta\right] \qquad (1)$$

$$M_y = -(P\cos\phi)(5\text{ in.}) \qquad (2)$$

$$M_z = -(P\sin\phi)(5\text{ in.}) \qquad (3)$$

Then $$\frac{\text{Equation (3)}}{\text{Equation (2)}}: \quad \frac{M_z}{M_y} = \frac{-(P\sin\phi)(5)}{-(P\cos\phi)(5)}$$

or $$\tan\phi = \frac{-35}{-200} = 0.175 \qquad \phi = 9.9262°$$

or $\phi = 9.93°$ ◀

Substituting ϕ into Equation (2)

$$-200\ \text{lb}\cdot\text{in.} = -(P\cos 9.9262°)(5\text{ in.})$$

$$P = 40.608\text{ lb}$$

or $P = 40.6$ lb ◀

Then, from Equation (1)

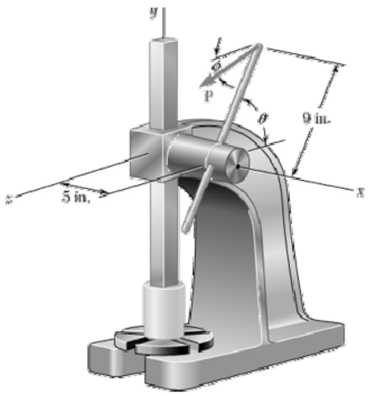
$$230\ \text{lb}\cdot\text{in.} = \left[(40.608\text{ lb})\cos 9.9262°\right]\left[(9\text{ in.})\sin\theta\right]$$

$$-\left[(40.608\text{ lb})\sin 9.9262°\right]\left[(9\text{ in.})\cos\theta\right]$$

or $$0.98503\sin\theta - 0.172380\cos\theta = 0.62932$$

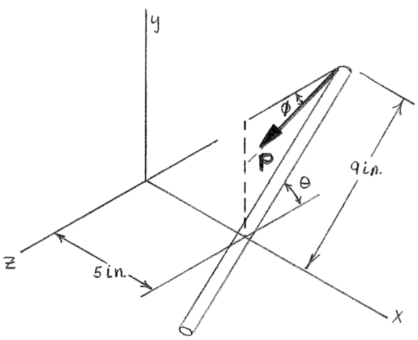
Solving numerically, $\theta = 48.9°$ ◀

PROBLEM 3.52



A force **P** is applied to the lever of an arbor press. Knowing that **P** lies in a plane parallel to the *yz* plane and that $M_y = -180\text{ lb}\cdot\text{in.}$ and $M_z = -30\text{ lb}\cdot\text{in.}$, determine the moment M_x of **P** about the *x* axis when $\theta = 60°$.

SOLUTION



Based on $$M_x = (P\cos\phi)[(9\text{ in.})\sin\theta] - (P\sin\phi)[(9\text{ in.})\cos\theta] \quad (1)$$

$$M_y = -(P\cos\phi)(5\text{ in.}) \quad (2)$$

$$M_z = -(P\sin\phi)(5\text{ in.}) \quad (3)$$

Then $$\frac{\text{Equation (3)}}{\text{Equation (2)}}: \quad \frac{M_z}{M_y} = \frac{-(P\sin\phi)(5)}{-(P\cos\phi)(5)}$$

or $$\frac{-30}{-180} = \tan\phi$$

$$\therefore\ \phi = 9.4623°$$

From Equation (3),

$$-30\text{ lb}\cdot\text{in.} = -(P\sin 9.4623°)(5\text{ in.})$$

$$\therefore\ P = 36.497\text{ lb}$$

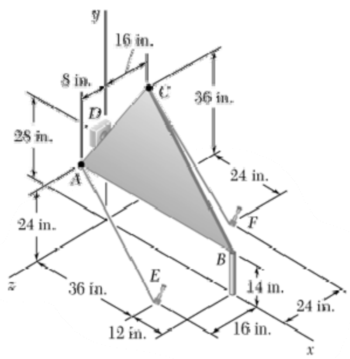
From Equation (1),

$$M_x = (36.497\text{ lb})(9\text{ in.})(\cos 9.4623°\sin 60° - \sin 9.4623°\cos 60°)$$

$$= 253.60\text{ lb}\cdot\text{in.}$$

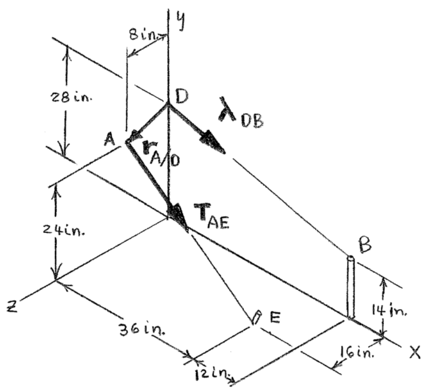
or $M_x = 254\text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.53



The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 220 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \left(\mathbf{r}_{A/D} \times \mathbf{T}_{AE}\right)$$

where $$\boldsymbol{\lambda}_{DB} = \frac{(48\text{ in.})\mathbf{i} - (14\text{ in.})\mathbf{j}}{50\text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{A/D} = -(4\text{ in.})\mathbf{j} + (8\text{ in.})\mathbf{k}$$

$$\mathbf{T}_{AE} = \boldsymbol{\lambda}_{AE} T_{AE} = \frac{\left[(36\text{ in.})\mathbf{i} - (24\text{ in.})\mathbf{j} + (8\text{ in.})\mathbf{k}\right]}{44\text{ in.}}(220\text{ lb})$$

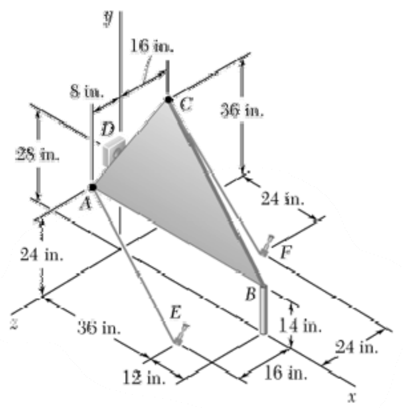
$$= (180\text{ lb})\mathbf{i} - (120\text{ lb})\mathbf{j} + (40\text{ lb})\mathbf{k}$$

$$\therefore\ M_{DB} = \begin{vmatrix} 0.960 & -0.280 & 0 \\ 0 & -4 & 8 \\ 180 & -120 & 40 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= (0.960)\left[(-4)(40) - (8)(-120)\right] + (-0.280)\left[8(180) - 0\right]$$

$$= 364.8\text{ lb}\cdot\text{in.}$$

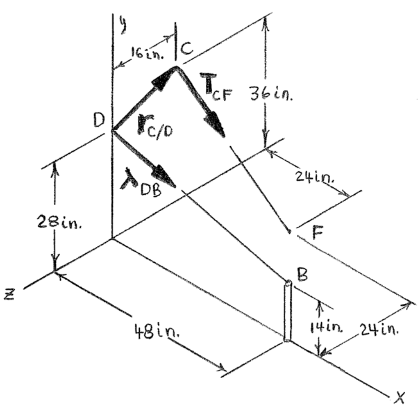
or $M_{DB} = 365\text{ lb}\cdot\text{in.}$ ◀



PROBLEM 3.54

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 132 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CF}\right)$$

where $$\boldsymbol{\lambda}_{DB} = \frac{(48\text{ in.})\mathbf{i} - (14\text{ in.})\mathbf{j}}{50\text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}$$

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{(24\text{ in.})\mathbf{i} - (36\text{ in.})\mathbf{j} - (8\text{ in.})\mathbf{k}}{44\text{ in.}}(132\text{ lb})$$

$$= (72\text{ lb})\mathbf{i} - (108\text{ lb})\mathbf{j} - (24\text{ lb})\mathbf{k}$$

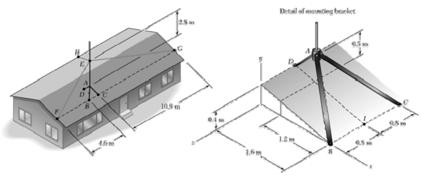
$$\therefore\ M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & 8 & -16 \\ 72 & -108 & -24 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= 0.96\left[(8)(-24) - (-16)(-108)\right] + (-0.28)\left[(-16)(72) - 0\right]$$

$$= -1520.64\text{ lb}\cdot\text{in.}$$

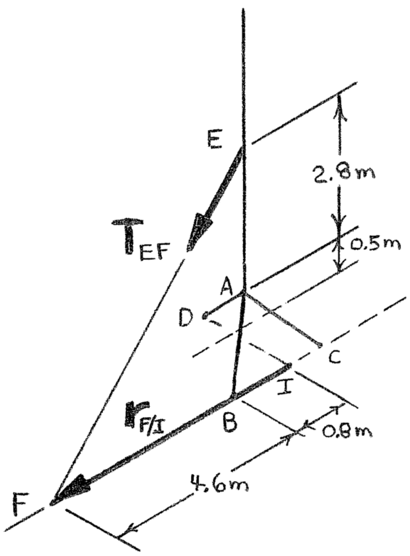
or $M_{DB} = -1521\text{ lb}\cdot\text{in.}$ ◀

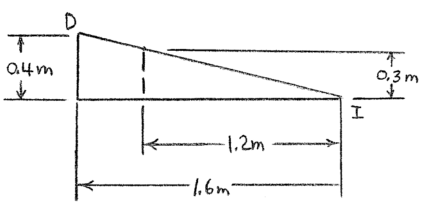
PROBLEM 3.55



A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EF* at *E* is 66 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION





Have $$M_{DI} = \lambda_{DI} \cdot \left[\mathbf{r}_{F/I} \times \mathbf{T}_{EF}\right]$$

where $$\lambda_{DI} = \frac{(1.6\text{ m})\mathbf{i} - (0.4\text{ m})\mathbf{j}}{\sqrt{(1.6)^2 + (0.4)^2}\text{ m}} = \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

$$\mathbf{r}_{F/I} = (4.6\text{ m} + 0.8\text{ m})\mathbf{k} = (5.4\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF}$$

$$= \frac{(1.2\text{ m})\mathbf{i} - (3.6\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}}{6.6\text{ m}}(66\text{ N})$$

$$= (12\text{ N})\mathbf{i} - (36\text{ N})\mathbf{j} + (54\text{ N})\mathbf{k}$$

$$= 6\left[(2\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (9\text{ N})\mathbf{k}\right]$$

$$\therefore\ M_{DI} = \frac{(6\text{ N})(5.4\text{ m})}{\sqrt{17}}\begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & 1 \\ 2 & -6 & 9 \end{vmatrix}$$

$$= 7.8582\left[(0 + 24) + (-2 - 0)\right]$$

$$= 172.879\text{ N}\cdot\text{m}$$

or $M_{DI} = 172.9\text{ N}\cdot\text{m}$ ◀