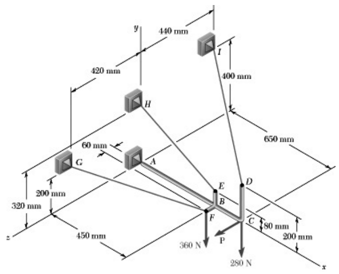
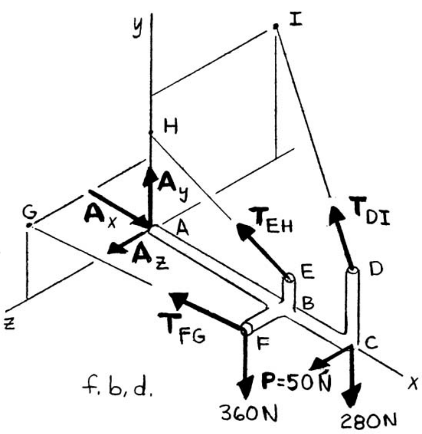
PROBLEM 4.133



The frame shown is supported by three cables and a ball-and-socket joint at A. For $P = 50$ N, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI} T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}} T_{DI}$$

$$= \frac{T_{DI}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH} T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}} T_{EH}$$

$$= \frac{T_{EH}}{17}(-15\mathbf{i} + 8\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG} T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45)^2 + (0.2)^2 + (0.36)^2}\text{ m}} T_{FG}$$

$$= \frac{T_{FG}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times \left[-(280\text{ N})\mathbf{j} + (50\text{ N})\mathbf{k}\right]$$

$$+ \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j}$$

or $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -65 & 20 & -44 \end{vmatrix}\left(\frac{T_{DI}}{81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -280 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -15 & 8 & 0 \end{vmatrix}\left(\frac{T_{EH}}{17}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -45 & 20 & 36 \end{vmatrix}\left(\frac{T_{FG}}{61}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix}(360\text{ N}) = 0$$

and $$(-8.8\mathbf{i} + 28.6\mathbf{j} + 26\mathbf{k})\left(\frac{T_{DI}}{81}\right) + (-32.5\mathbf{j} - 182\mathbf{k}) + (4.8\mathbf{k})\left(\frac{T_{EH}}{17}\right)$$

$$+(-1.2\mathbf{i} - 18.9\mathbf{j} + 9.0\mathbf{k})\left(\frac{T_{FG}}{61}\right) + (0.06\mathbf{i} - 0.45\mathbf{k})(360) = 0$$

PROBLEM 4.133 CONTINUED

From **i**-coefficient $-8.8\left(\frac{T_{DI}}{81}\right) - 1.2\left(\frac{T_{FG}}{61}\right) + 0.06(360) = 0$

$$\therefore \quad 0.108642T_{DI} + 0.0196721T_{FG} = 21.6 \tag{1}$$

From **j**-coefficient $28.6\left(\frac{T_{DI}}{81}\right) - 32.5 - 18.9\left(\frac{T_{FG}}{61}\right) = 0$

$$\therefore \quad 0.35309T_{DI} - 0.30984T_{FG} = 32.5 \tag{2}$$

From **k**-coefficient

$$26\left(\frac{T_{DI}}{81}\right) - 182 + 4.8\left(\frac{T_{EH}}{17}\right) + 9.0\left(\frac{T_{FG}}{61}\right) - 0.45(360) = 0$$

$$\therefore \quad 0.32099T_{DI} + 0.28235T_{EH} + 0.147541T_{FG} = 344 \tag{3}$$

$-3.25 \times$ Equation (1) $\quad -0.35309T_{DI} - 0.063935T_{FG} = -70.201$

Add Equation (2) $\quad \underline{0.35309T_{DI} - 0.30984T_{FG} = 32.5}$

$$-0.37378T_{FG} = -37.701$$

$$\therefore \quad T_{FG} = 100.864 \text{ N}$$

or $\quad T_{FG} = 100.9 \text{ N}$ ◀

Then from Equation (1)

$$0.108642T_{DI} + 0.0196721(100.864) = 21.6$$

$$\therefore \quad T_{DI} = 180.554 \text{ N}$$

or $\quad T_{DI} = 180.6 \text{ N}$ ◀

and from Equation (3)

$$0.32099(180.554) + 0.28235T_{EH} + 0.147541(100.864) = 344$$

$$\therefore \quad T_{EH} = 960.38 \text{ N}$$

or $\quad T_{EH} = 960 \text{ N}$ ◀

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{180.554 \text{ N}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$= -(144.889 \text{ N})\mathbf{i} + (44.581 \text{ N})\mathbf{j} - (98.079 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{960.38 \text{ N}}{17}(-15\mathbf{i} + 8\mathbf{j}) = -(847.39 \text{ N})\mathbf{i} + (451.94 \text{ N})\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{100.864 \text{ N}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

$$= -(74.409 \text{ N})\mathbf{i} + (33.070 \text{ N})\mathbf{j} + (59.527 \text{ N})\mathbf{k}$$

PROBLEM 4.133 CONTINUED

Then from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 144.889 - 847.39 - 74.409 = 0$$

$$\therefore \quad A_x = 1066.69 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + 44.581 + 451.94 + 33.070 - 360 - 280 = 0$$

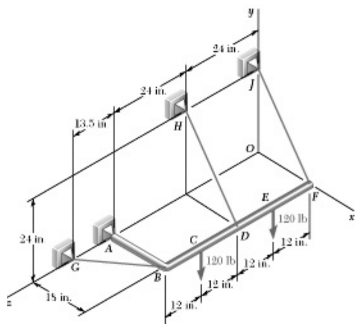
$$\therefore \quad A_y = 110.409 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 98.079 + 59.527 + 50 = 0$$

$$\therefore \quad A_z = -11.448 \text{ N}$$

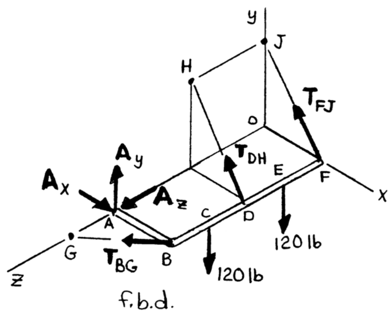
Therefore, $\mathbf{A} = (1067 \text{ N})\mathbf{i} + (110.4 \text{ N})\mathbf{j} - (11.45 \text{ N})\mathbf{k}$ ◀

PROBLEM 4.134



The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{BG} = \lambda_{BG} T_{BG} = \frac{-(18\text{ in.})\mathbf{i} + (13.5\text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2}\text{ in.}} T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH} T_{DH} = \frac{-(18\text{ in.})\mathbf{i} + (24\text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2}\text{ in.}} T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$,

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member *ABF*

$$\Sigma M_{A(x\text{-}axis)} = 0: \quad (0.8T_{FJ})(48\text{ in.}) + (0.8T_{DH})(24\text{ in.}) - (120\text{ lb})(36\text{ in.}) - (120\text{ lb})(12\text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 480 \qquad (1)$$

$$\Sigma M_{A(z\text{-}axis)} = 0: \quad (0.8T_{FJ})(18\text{ in.}) + (0.8T_{DH})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -960 \qquad (2)$$

Equation (1) + Equation (2) $\qquad T_{DH} = 300\text{ lb}$ ◀

Substituting in Equation (1) $\qquad T_{FJ} = 0$ ◀

$$\Sigma M_{A(y\text{-}axis)} = 0: \quad (0.6T_{FJ})(48\text{ in.}) + [0.6(300\text{ lb})](24\text{ in.}) - (0.6T_{BG})(18\text{ in.}) = 0$$

$$\therefore \quad T_{BG} = 400\text{ lb} \blacktriangleleft$$

PROBLEM 4.134 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(300 \text{ lb}) - 0.8(400 \text{ lb}) + A_x = 0$$

$$\therefore \quad A_x = 500 \text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 240 \text{ lb} + A_y = 0$$

$$0.8(300 \text{ lb}) - 240 + A_y = 0$$

$$\therefore \quad A_y = 0$$

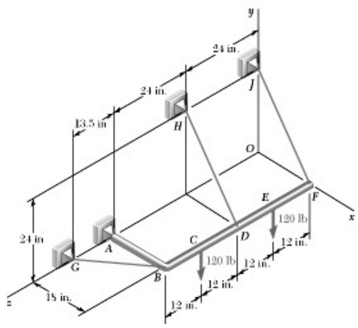
$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(400 \text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -240 \text{ lb}$$

Therefore, $\mathbf{A} = (500 \text{ lb})\mathbf{i} - (240 \text{ lb})\mathbf{k}$ ◀

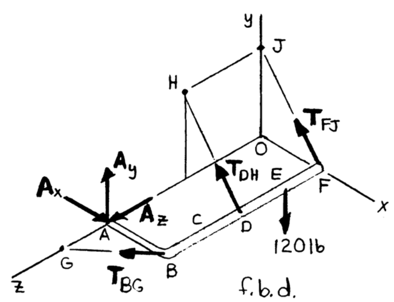
PROBLEM 4.135



Solve Problem 4.134 assuming that the load at C has been removed.

P4.134 The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First

$$\mathbf{T}_{BG} = \lambda_{BG} T_{BG} = \frac{-(18 \text{ in.})\mathbf{i} + (13.5 \text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2} \text{ in.}} T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH} T_{DH} = \frac{-(18 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2} \text{ in.}} T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member ABF

$$\Sigma M_{A(x\text{-}axis)} = 0: \quad (0.8T_{FJ})(48 \text{ in.}) + (0.8T_{DH})(24 \text{ in.}) - (120 \text{ lb})(36 \text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 360 \tag{1}$$

$$\Sigma M_{A(z\text{-}axis)} = 0: \quad (0.8T_{FJ})(18 \text{ in.}) + (0.8T_{DH})(18 \text{ in.}) - (120 \text{ lb})(18 \text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -480 \tag{2}$$

Equation (1) + Equation (2) $\qquad T_{DH} = 75.0 \text{ lb}$ ◀

Substituting into Equation (2) $\qquad T_{FJ} = 75.0 \text{ lb}$ ◀

$$\Sigma M_{A(y\text{-}axis)} = 0: \quad (0.6T_{FJ})(48 \text{ in.}) + (0.6T_{DH})(24 \text{ in.}) - (0.6T_{BG})(18 \text{ in.}) = 0$$

or

$$(75.0 \text{ lb})(48 \text{ in.}) + (75.0 \text{ lb})(24 \text{ in.}) = T_{BG}(18 \text{ in.})$$

$T_{BG} = 300 \text{ lb}$ ◀

PROBLEM 4.135 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(75.0 + 75.0) - 0.8(300) + A_x = 0$$

$$\therefore \quad A_x = 330 \text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 120 \text{ lb} + A_y = 0$$

$$0.8(150 \text{ lb}) - 120 \text{ lb} + A_y = 0$$

$$\therefore \quad A_y = 0$$

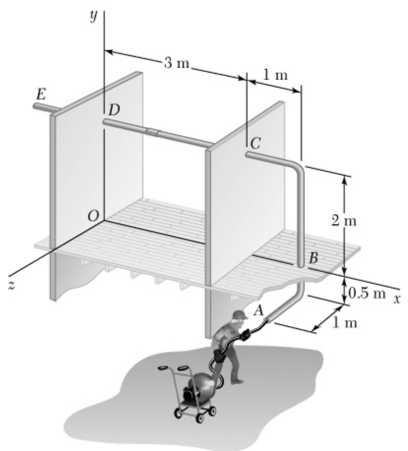
$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(300 \text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -180 \text{ lb}$$

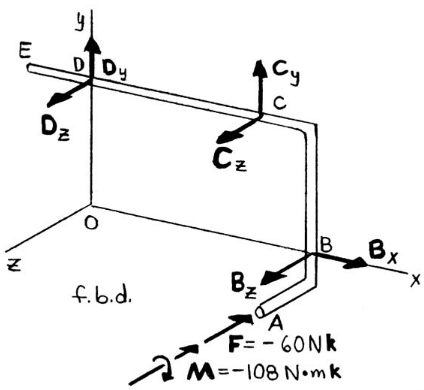
Therefore $\mathbf{A} = (330 \text{ lb})\mathbf{i} - (180 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.136



In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

$$\text{and} \quad \mathbf{B} = (75.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - 108\ \text{N}\cdot\text{m} = 0$$

$$\therefore \quad C_y = 36.0\ \text{N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\ \text{m}) - (75\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20.0\ \text{N}$$

$$\text{and} \quad \mathbf{C} = (36.0\ \text{N})\mathbf{j} - (20.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad D_y + 36.0 = 0$$

$$\therefore \quad D_y = -36.0\ \text{N}$$

$$\Sigma F_z = 0: \quad D_z - 20.0\ \text{N} + 75.0\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

$$\text{and} \quad \mathbf{D} = -(36.0\ \text{N})\mathbf{j} + (5.00\ \text{N})\mathbf{k} \blacktriangleleft$$