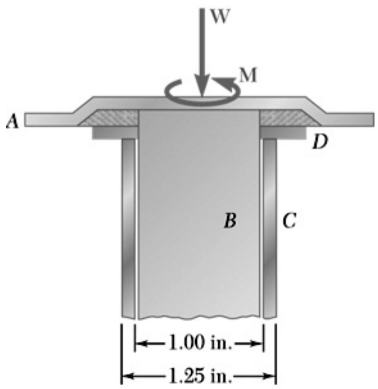


PROBLEM 8.91

The pivot for the seat of a desk chair consists of the steel plate *A*, which supports the seat, the solid steel shaft *B* which is welded to *A* and which turns freely in the tubular member *C*, and the nylon bearing *D*. If a person of weight $W = 180$ lb is seated directly above the pivot, determine the magnitude of the couple **M** for which rotation of the seat is impending knowing that the coefficient of static friction is 0.15 between the tubular member and the bearing.

SOLUTION

For an annular bearing area
$$M = \frac{2}{3}\mu_s P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \quad \text{(Equation 8.8)}$$

Since $R = \dfrac{D}{2}$
$$M = \frac{1}{3}\mu_s P \frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}$$

Now
$$\mu_s = 0.15, \; P = W = 180 \text{ lb}, \; D_1 = 1.00 \text{ in.}, \; D_2 = 1.25 \text{ in.}$$

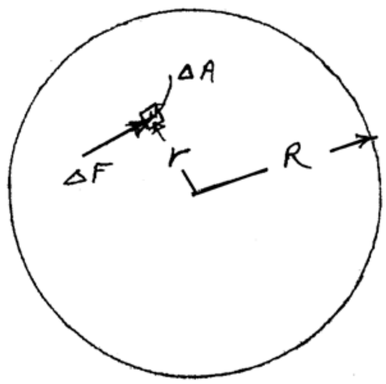
$$M = \frac{0.15}{3}(180 \text{ lb})\frac{(1.25 \text{ in.})^3 - (4 \text{ in.})^3}{(1.25 \text{ in.})^2 - (1 \text{ in.})^2}$$

$$M = 15.25 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

PROBLEM 8.92

As the surfaces of a shaft and a bearing wear out, the frictional resistance of a thrust bearing decreases. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by formula (8.9) for a new bearing.

SOLUTION



Let the normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\Delta F = \mu \Delta N, \ \Delta M = r\Delta F$

The total normal force

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi} \left(\int_0^R \frac{k}{r} r dr \right) d\theta$$

$$P = 2\pi \left(\int_0^R k dr \right) = 2\pi k R \qquad \text{or} \qquad k = \frac{P}{2\pi R}$$

The total couple $M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi} \left(\int_0^R r \mu \frac{k}{r} r dr \right) d\theta$

$$M_{\text{worn}} = 2\pi \mu k \int_0^R r dr = 2\pi \mu k \frac{R^2}{2} = 2\pi \mu \frac{P}{2\pi R} \frac{R^2}{2}$$

or $M_{\text{worn}} = \dfrac{1}{2} \mu P R$

Now $M_{\text{new}} = \dfrac{2}{3} \mu P R$ [Eq. (8.9)]

Thus $\dfrac{M_{\text{worn}}}{M_{\text{new}}} = \dfrac{\frac{1}{2}\mu P R}{\frac{2}{3}\mu P R} = \dfrac{3}{4} = 75\%$ ◀

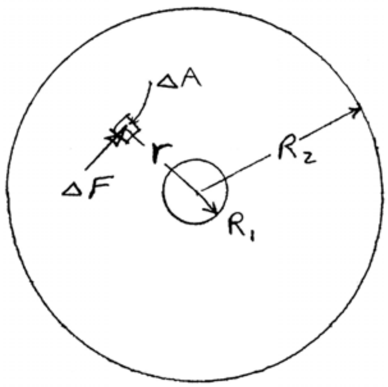
PROBLEM 8.93

Assuming that bearings wear out as indicated in Problem 8.92, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is $M = \frac{1}{2}\mu_k P(R_1 + R_2)$

where $P =$ magnitude of the total axial force

$R_1,\ R_2 =$ inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\quad \Delta F = \mu \Delta N,\ \Delta M = r\Delta F$

The total normal force P is

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r} r dr\right) d\theta$$

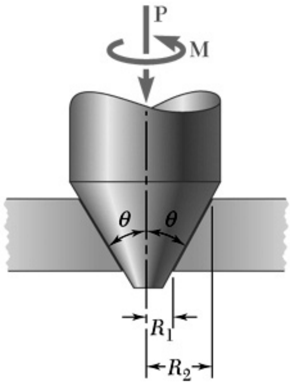
$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k(R_2 - R_1) \qquad \text{or} \qquad k = \frac{P}{2\pi(R_2 - R_1)}$$

The total couple is $\quad M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr\right) d\theta$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} (r dr) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi(R_2 - R_1)}$$

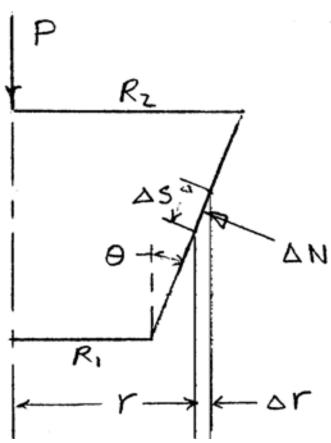
$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$

PROBLEM 8.94



Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is $M = \dfrac{2}{3}\dfrac{\mu_k P}{\sin\theta}\dfrac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = k$,

so $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force $\qquad P = \lim_{\Delta A \to 0} \Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \qquad \text{or} \qquad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple $\qquad M = \lim_{\Delta A \to 0} \Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{\mu k}{\sin\theta} r^2 dr\right)d\phi$

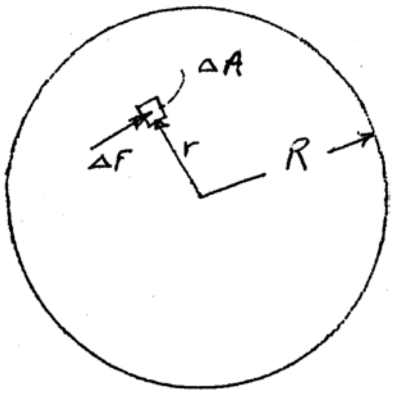
$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2 dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

PROBLEM 8.95

Solve Problem 8.90 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = k\left(1 - \dfrac{r}{R}\right)$

$$\Delta F = \mu \Delta N = \mu k\left(1 - \frac{r}{R}\right)\Delta A = \mu k\left(1 - \frac{r}{R}\right) r\Delta r\Delta\theta$$

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1 - \frac{r}{R}\right) rdr\right] d\theta$$

$$P = 2\pi k\int_0^R \left(1 - \frac{r}{R}\right) rdr = 2\pi k\left(\frac{R^2}{2} - \frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi k R^2 \qquad \text{or} \qquad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A \to 0} \Sigma r\Delta F = \int_0^{2\pi}\left[\int_0^R r\mu k\left(1 - \frac{r}{R}\right) rdr\right] d\theta$$

$$= 2\pi\mu k\int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 2\pi\mu k\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = \frac{1}{6}\pi\mu k R^3$$

$$= \frac{\pi\mu}{6}\frac{3P}{\pi R^2} R^3 = \frac{1}{2}\mu PR$$

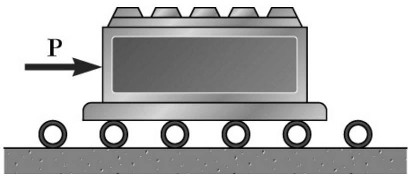
where $\qquad \mu = \mu_k = 0.25 \qquad R = 0.18 \text{ m}$

$$P = W = (25 \text{ kg})(9.81 \text{ m/s}^2) = 245.25 \text{ N}$$

Then $\qquad M = \dfrac{1}{2}(0.25)(245.25 \text{ N})(0.18 \text{ m}) = 5.5181 \text{ N}\cdot\text{m}$

Finally, $\qquad Q = \dfrac{M}{d} = \dfrac{5.5181 \text{ N}\cdot\text{m}}{0.4 \text{ m}} \qquad Q = 13.80 \text{ N}$ ◀

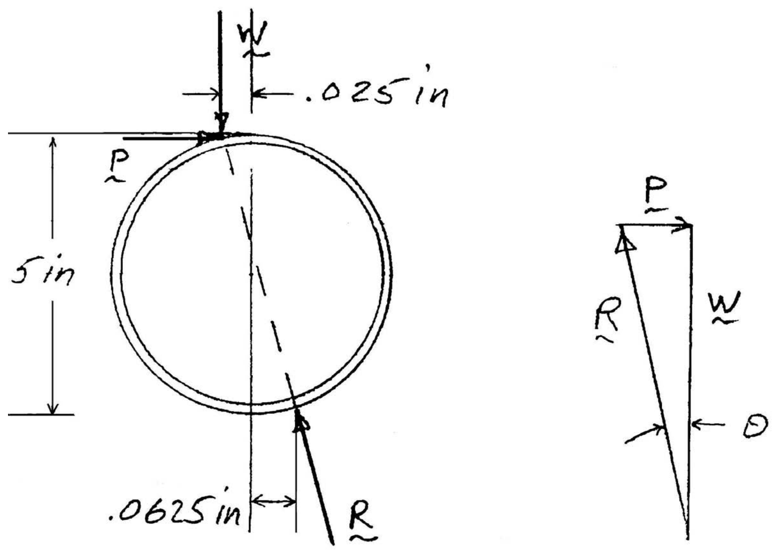
PROBLEM 8.96



A 1-ton machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 5 in. Knowing that the coefficient of rolling resistance is 0.025 in. between the pipes and the base and 0.0625 in. between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$\theta = \sin^{-1}\frac{0.025\text{ in.} + 0.0625\text{ in.}}{5\text{ in.}} = 1.00257°$$

$P = W\tan\theta$ for each pipe, so also for total

$$P = (2000\text{ lb})\tan(1.00257°)$$

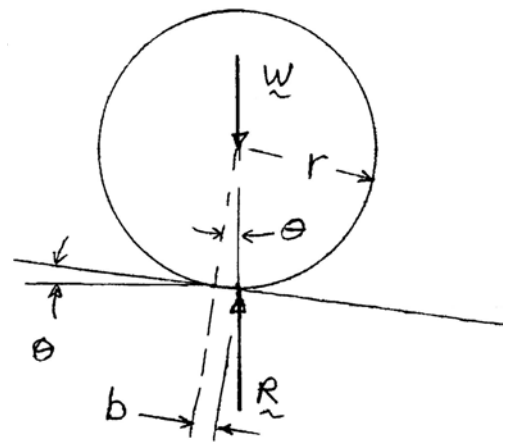
$P = 35.0\text{ lb}$ ◀

PROBLEM 8.97

Knowing that a 120-mm-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (60\text{ mm})(0.02)$$

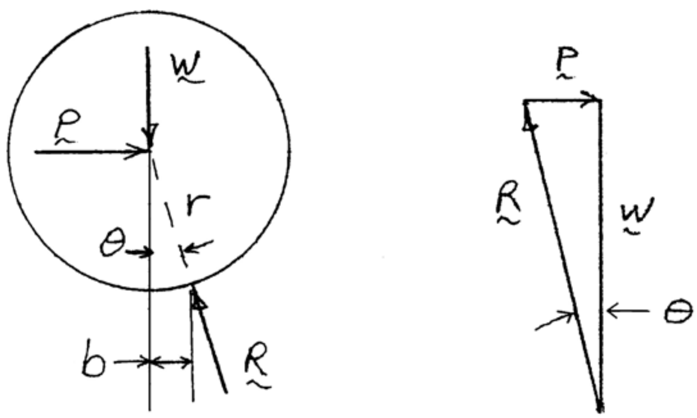
$$b = 1.200\text{ mm} \blacktriangleleft$$

PROBLEM 8.98

Determine the horizontal force required to move a 1-Mg automobile with 460-mm-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 1 mm.

SOLUTION

FBD wheel:



$$r = 230 \text{ mm}$$

$$b = 1 \text{ mm}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

$$P = (1000 \text{ kg})(9.81 \text{ m/s}^2)\tan\left(\sin^{-1}\frac{1}{230}\right)$$

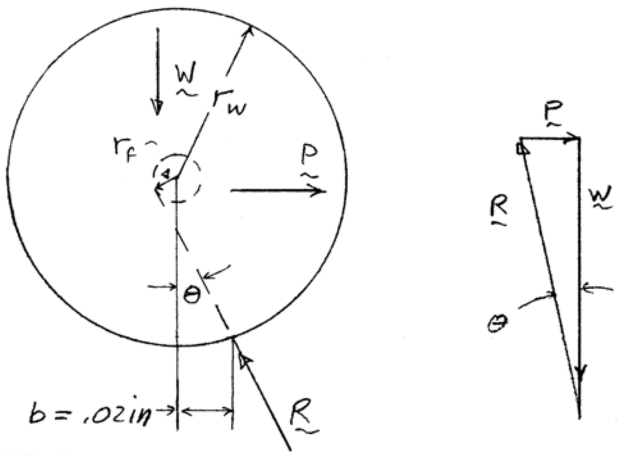
$$P = 42.7 \text{ N} \blacktriangleleft$$

PROBLEM 8.99

Solve Problem 8.88 including the effect of a coefficient of rolling resistance of 0.02 in.

SOLUTION

FBD wheel:



$$r_f = r_a \sin\phi = r_a \sin\left(\tan^{-1}\mu\right)$$

$$= (2.5\text{ in.})\sin\left(\tan^{-1}\mu\right)$$

$P = W\tan\theta$ for each wheel, so also for total $P = W\tan\theta$

$$\tan\theta \approx \frac{b + r_f}{r_w} \text{ for small } \theta$$

So
$$P = (70{,}000\text{ lb})\frac{(0.02\text{ in.}) + r_f}{16\text{ in.}}$$

(*a*) For impending motion, use $\mu_s = 0.02$:

Then $r_f = 0.04999$ in. and $P = 306$ lb ◀

(*b*) For steady motion, use $\mu_k = 0.015$:

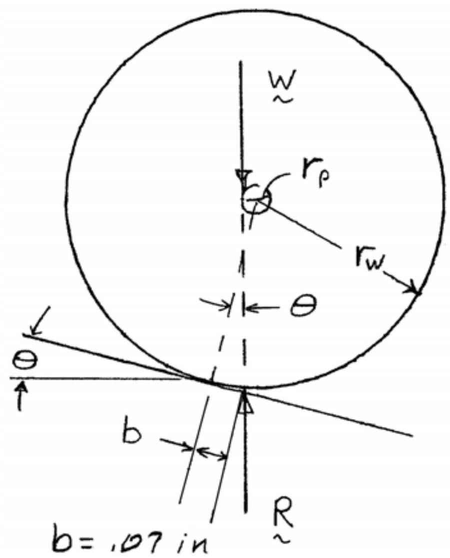
Then $r_f = 0.037496$ in. and $P = 252$ lb ◀

PROBLEM 8.100

Solve Problem 8.89 including the effect of a coefficient of rolling resistance of 0.07 in.

SOLUTION

FBD wheel:



The wheel is a two-force body, so **R** and **W** are colinear and tangent to the friction circle.

$$\tan\theta = \text{slope} = 0.02$$

$$\tan\theta \approx \frac{b + r_f}{r_w} \qquad \text{or} \qquad r_w \approx \frac{b + r_f}{\tan\theta}$$

Now
$$r_f = r_a \sin\phi_k = r_a \sin\left(\tan^{-1}\mu_k\right)$$

$$= (0.5\text{ in.})\sin\left(\tan^{-1}0.1\right)$$

$$= 0.049752$$

$$\therefore \quad r_w \approx \frac{0.07\text{ in.} + 0.049752\text{ in.}}{0.02} = 5.9876\text{ in.}$$

$$d = 2r_w$$

$d = 11.98$ in. ◀