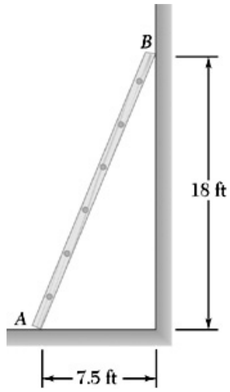
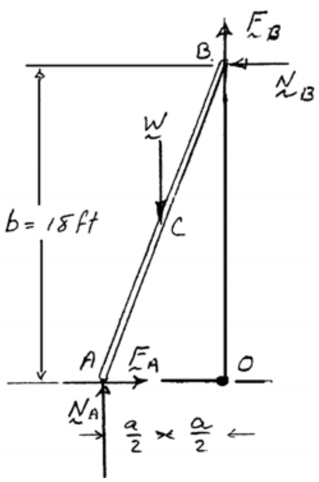


PROBLEM 8.21

A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5$ ft

$b = 18$ ft

Motion impends at both A and B.

$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = \mu_s N_A$$

Then $$F_B = \mu_s N_B = \mu_s^2 N_A$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + F_B = 0 \quad \text{or} \quad N_A\left(1 + \mu_s^2\right) = W$$

$$\Sigma M_O = 0: \quad bN_B + \frac{a}{2}W - aN_A = 0$$

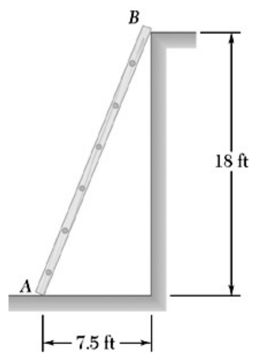
or $$aN_A - b\mu_s N_A = \frac{a}{2}W = \frac{a}{2}N_A\left(1 + \mu_s^2\right)$$

$$\mu_s^2 + \frac{2b}{a}\mu_s - 1 = 0$$

$$\mu_s = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 + 1} = -2.4 \pm 2.6$$

The positive root is physically possible. Therefore, $\mu_s = 0.200$ ◀

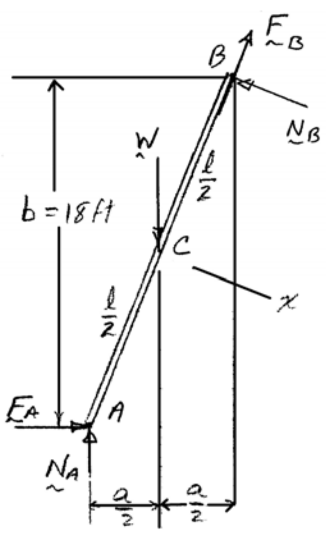
PROBLEM 8.22



A 19.5-ft ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$$a = 7.5\text{ ft}$$

$$l = 19.5\text{ ft}$$

$$\frac{a}{l} = \frac{5}{13}$$

$$\frac{b}{l} = \frac{12}{13}$$

Motion impends at both *A* and *B*, so

$$F_A = \mu_s N_A \qquad \text{and} \qquad F_B = \mu_s N_B$$

$$\circlearrowleft \Sigma M_A = 0: \quad lN_B - \frac{a}{2}W = 0 \qquad \text{or} \qquad N_B = \frac{a}{2l}W = \frac{7.5\text{ ft}}{39\text{ ft}}W$$

or $$N_B = \frac{2.5}{13}W$$

Then $$F_B = \mu_s N_B = \mu_s \frac{2.5W}{13}$$

$$\rightarrow \Sigma F_x = 0: \quad F_A + \frac{5}{13}F_B - \frac{12}{13}N_B = 0$$

$$\mu_s N_A + \frac{12.5}{(13)^2}\mu_s W - \frac{30}{(13)^2}W = 0$$

$$N_A - \frac{W}{(13)^2}\frac{(30 - 12.5\mu_s)}{\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + \frac{12}{13}F_B + \frac{5}{13}N_B = 0$$

$$\left(\frac{30 - 12.5\mu_s}{\mu_s} + 30\mu_s + 12.5\right)\frac{W}{(13)^2} = W$$

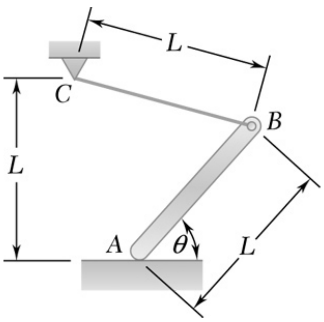
or $$\mu_s^2 - 5.6333\mu_s + 1 = 0$$

$$\mu_s = 2.8167 \pm 2.6332$$

or $$\mu_s = 0.1835 \qquad \text{and} \qquad \mu_s = 5.45$$

The larger value is very unlikely unless the surface is treated with some "non-skid" material.

In any event, the smallest value for equilibrium is $\mu_s = 0.1835$ ◀

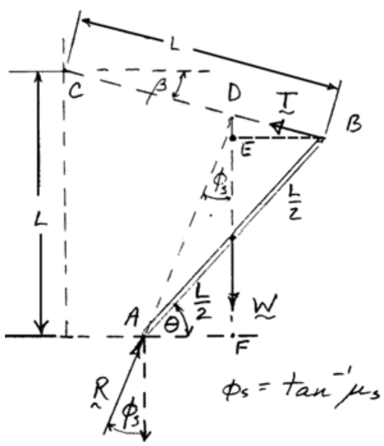


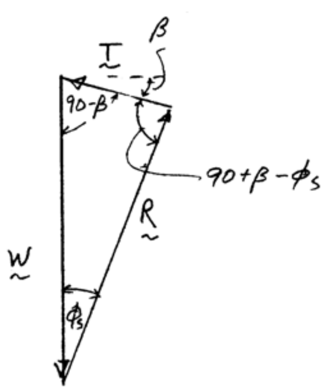
PROBLEM 8.23

End A of a slender, uniform rod of weight W and length L bears on a horizontal surface as shown, while end B is supported by a cord BC of length L. Knowing that the coefficient of static friction is 0.40, determine (*a*) the value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

SOLUTION

FBD rod:





(*a*) Geometry: $BE = \dfrac{L}{2}\cos\theta \qquad DE = \left(\dfrac{L}{2}\cos\theta\right)\tan\beta$

$$EF = L\sin\theta \qquad DF = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

So
$$L\left(\frac{1}{2}\cos\theta\tan\beta + \sin\theta\right) = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

or
$$\tan\beta + 2\tan\theta = \frac{1}{\tan\phi_s} = \frac{1}{\mu_s} = \frac{1}{0.4} = 2.5 \qquad (1)$$

Also,
$$L\sin\theta + L\sin\beta = L$$

or
$$\sin\theta + \sin\beta = 1 \qquad (2)$$

Solving Eqs. (1) and (2) numerically $\theta_1 = 4.62° \qquad \beta_1 = 66.85°$

$$\theta_2 = 48.20° \qquad \beta_2 = 14.75°$$

Therefore, $\theta = 4.62°$ and $\theta = 48.2°$ ◀

(*b*) Now
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$$

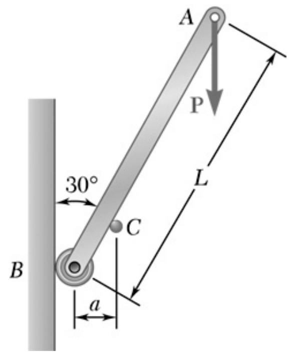
and
$$\frac{T}{\sin\phi_s} = \frac{W}{\sin(90 + \beta - \phi_s)}$$

or
$$T = W\frac{\sin\phi_s}{\sin(90 + \beta - \phi_s)}$$

For $\theta = 4.62° \qquad T = 0.526W$ ◀

$\theta = 48.2° \qquad T = 0.374W$ ◀

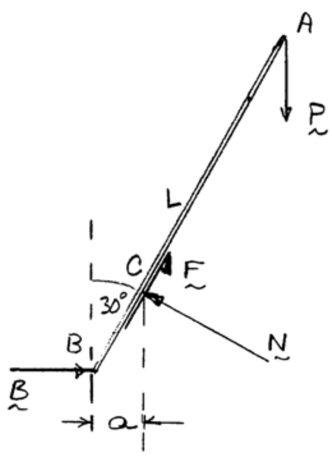
PROBLEM 8.24



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.25 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_B = 0: \quad \frac{a}{\sin 30°} N - L \sin 30° P = 0$$

$$N = \frac{L}{a} \sin^2 30° P = \frac{L}{a} \frac{P}{4}$$

$$\text{Impending motion at } C: \left.\begin{array}{r} \text{down} \to F = \mu_s N \\ \text{up} \to F = -\mu_s N \end{array}\right\} F = \pm \frac{N}{4}$$

$$\uparrow \Sigma F_y = 0: \quad F \cos 30° + N \sin 30° - P = 0$$

$$\pm \frac{L}{a} \frac{P}{16} \frac{\sqrt{3}}{2} + \frac{L}{a} \frac{P}{4} \frac{1}{2} = P$$

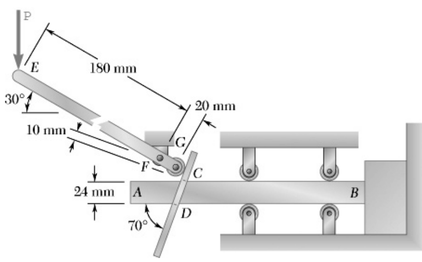
$$\frac{L}{a} \left[\frac{1}{8} \pm \frac{\sqrt{3}}{32} \right] = 1$$

$$\frac{L}{a} = \frac{32}{4 \pm \sqrt{3}}$$

or $\quad \dfrac{L}{a} = 5.583 \quad$ and $\quad \dfrac{L}{a} = 14.110$

For equilibrium: $\quad 5.58 \le \dfrac{L}{a} \le 14.11$ ◀

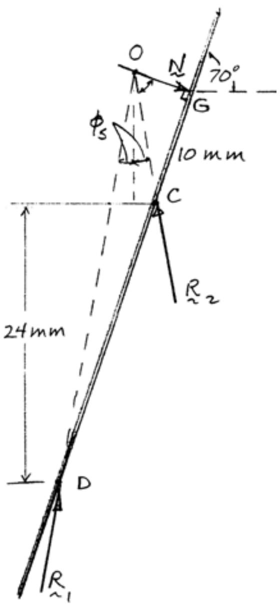
PROBLEM 8.25



The basic components of a clamping device are bar *AB*, locking plate *CD*, and lever *EFG*; the dimensions of the slot in *CD* are slightly larger than those of the cross section of *AB*. To engage the clamp, *AB* is pushed against the workpiece, and then force **P** is applied. Knowing that $P = 160$ N and neglecting the friction force between the lever and the plate, determine the smallest allowable value of the static coefficient of friction between the bar and the plate.

SOLUTION

FBD Plate:



DC is three-force member and motion impends at *C* and *D* (for minimum μ_s).

$$\measuredangle\, OCG = 20° + \phi_s \qquad \measuredangle\, ODG = 20° - \phi_s$$

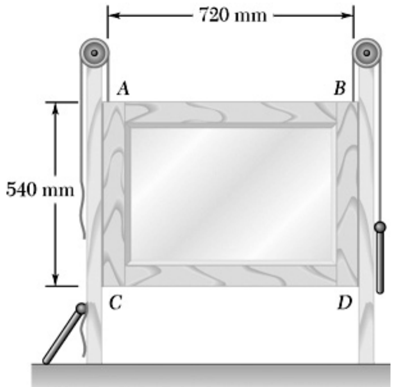
$$\overline{OG} = (10 \text{ mm})\tan(20° + \phi_s) = \left(\frac{24 \text{ mm}}{\sin 70°} + 10 \text{ mm}\right)\tan(20° - \phi_s)$$

or $$\tan(20° + \phi_s) = 3.5540\tan(20° - \phi_s)$$

Solving numerically $$\phi_s = 10.565°$$

Now $$\mu_s = \tan\phi_s$$

so that $$\mu_s = 0.1865 \blacktriangleleft$$

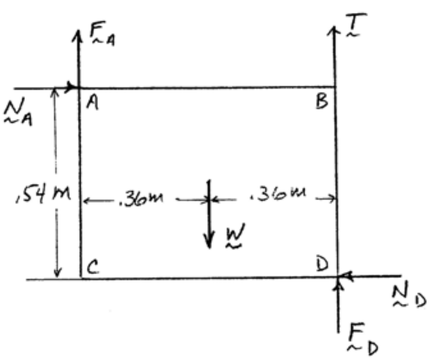


PROBLEM 8.26

A window sash having a mass of 4 kg is normally supported by two 2-kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller that the frame and will bind only at points A and D.)

SOLUTION

FBD window:



$$W = (4\text{ kg})(9.81\text{ m/s}^2) = 39.24\text{ N}$$

$$T = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N} = \frac{W}{2}$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - N_D = 0 \qquad N_A = N_D$$

Impending motion: $\quad F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$$\Sigma M_D = 0: \quad (0.36\text{ m})W - (0.54\text{ m})N_A - (0.72\text{ m})F_A = 0$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T$$

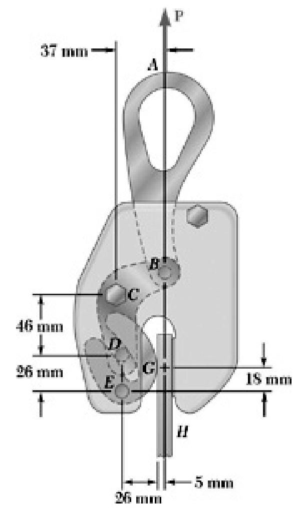
$$= \frac{W}{2}$$

Now $\quad F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$

Then $\quad \dfrac{W}{2} = 2\mu_s \dfrac{2W}{3 + 4\mu_s}$

or $\quad \mu_s = 0.750$ ◀

PROBLEM 8.27



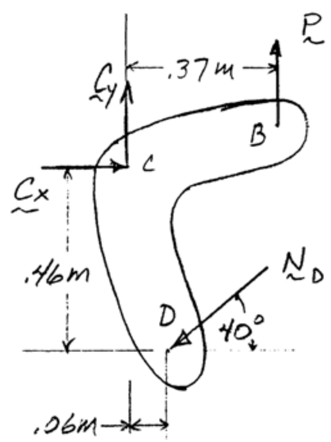
The steel-plate clamp shown is used to lift a steel plate *H* of mass 250 kg. Knowing that the normal force exerted on steel cam *EG* by pin *D* forms an angle of 40° with the horizontal and neglecting the friction force between the cam and the pin, determine the smallest allowable value of the coefficient of static friction.

SOLUTION

FBDs:

(Note: **P** is vertical as *AB* is two force member; also $P = W$ since clamp + plate is a two force *FBD*)

BCD:

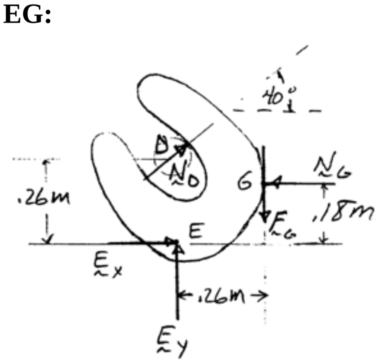


$$\circlearrowleft \Sigma M_C = 0: \quad (0.37 \text{ m})P - (0.46 \text{ m})N_D \cos 40°$$

$$- (0.06 \text{ m})N_D \sin 40° = 0$$

or $$N_D = 0.94642P = 0.94642W$$

EG:



$$\circlearrowleft \Sigma M_E = 0: \quad (0.18 \text{ m})N_G - (0.26 \text{ m})F_G - (0.26 \text{ m})N_D \cos 40° = 0$$

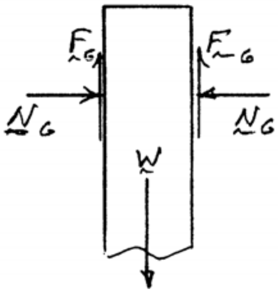
Impending motion: $$F_G = \mu_s N_G$$

Combining $$(18 + 26\mu_s)N_G = 19.9172N_D$$

$$= 18.850W$$

PROBLEM 8.27 CONTINUED

Plate:

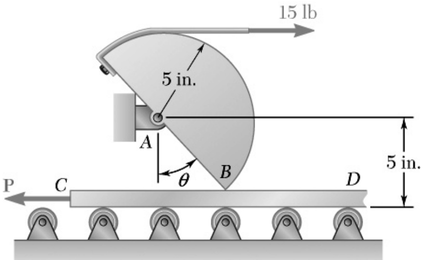


From plate: $F_G = \dfrac{W}{2}$ so that $N_G = \dfrac{W}{2\mu_s}$

Then $(18 + 26\mu_s)\dfrac{W}{2\mu_s} = 18.85W$

$\mu_s = 0.283$ ◀

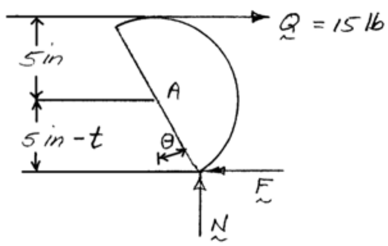
PROBLEM 8.28

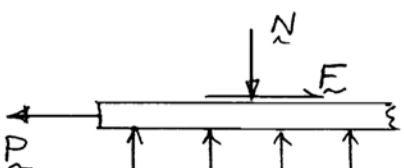


The 5-in.-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** for which motion of the plate is impending knowing that the plate is 1 in. thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking, (that is, for which the plate cannot be moved however large the force **P** may be).

SOLUTION

FBDs:





From plate: $\longrightarrow \Sigma F_x = 0: \quad F - P = 0 \qquad F = P$

From cam geometry: $\cos\theta = \dfrac{5\text{ in.} - t}{5\text{ in.}}$

$$\circlearrowleft \Sigma M_A = 0: \quad \left[(5\text{ in.})\sin\theta\right]N - \left[(5\text{ in.})\cos\theta\right]F - (5\text{ in.})Q = 0$$

Impending motion: $F = \mu_s N$

So $N\sin\theta - \mu_s N\cos\theta = Q = 15\text{ lb}$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta}$$

So $P = F = \mu_s N = \dfrac{\mu_s Q}{\sin\theta - \mu_s\cos\theta}$

(*a*) $t = 1\text{ in.} \Rightarrow \cos\theta = \dfrac{4\text{ in.}}{5\text{ in.}} = 0.8; \ \sin\theta = 0.6$

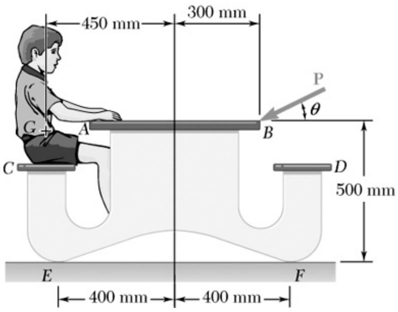
$$P = \frac{(0.45)(15\text{ lb})}{0.6 - (0.45)(0.8)} = 28.125\text{ lb}; \ \mathbf{P} = 28.1\text{ lb} \longleftarrow \blacktriangleleft$$

(*b*) $P \to \infty: \ \sin\theta - \mu_s\cos\theta = \dfrac{\mu_s Q}{P} \longrightarrow 0$

Thus $\tan\theta \to \mu_s = 0.45$ so that $\theta = 24.228°$

But $(5\text{ in.})\cos\theta = 5\text{ in.} - t$ or $t = (5\text{ in.})(1 - \cos\theta)$

$$t = 0.440\text{ in.} \blacktriangleleft$$

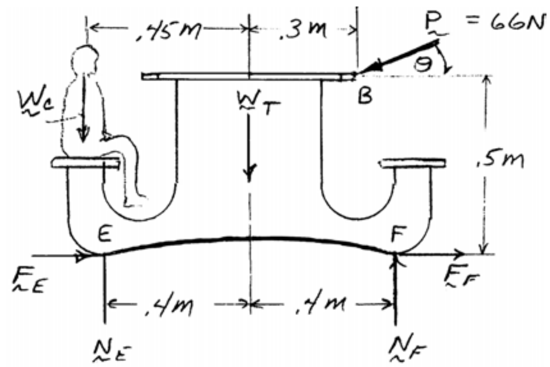


PROBLEM 8.29

A child having a mass of 18 kg is seated halfway between the ends of a small, 16-kg table as shown. The coefficient of static friction is 0.20 between the ends of the table and the floor. If a second child pushes on edge B of the table top at a point directly opposite to the first child with a force $\mathbf{P}$ lying in a vertical plane parallel to the ends of the table and having a magnitude of 66 N, determine the range of values of θ for which the table will (*a*) tip, (*b*) slide.

SOLUTION

FBD table + child:



$$W_C = 18\text{ kg}\left(9.81\text{ m/s}^2\right) = 176.58\text{ N}$$

$$W_T = 16\text{ kg}\left(9.81\text{ m/s}^2\right) = 156.96\text{ N}$$

(*a*) Impending tipping about E, $N_F = F_F = 0$, and

$$\circlearrowleft \Sigma M_E = 0: \quad (0.05\text{ m})(176.58\text{ N}) - (0.4\text{ m})(156.96\text{ N}) + (0.5\text{ m})P\cos\theta - (0.7\text{ m})P\sin\theta = 0$$

$$33\cos\theta - 46.2\sin\theta = 53.955$$

Solving numerically $\qquad \theta = -36.3° \quad$ and $\quad \theta = -72.6°$

Therefore $\qquad -72.6° \le \theta \le -36.3°$ ◀

Impending tipping about F is not possible

(*b*) For impending slip: $\qquad F_E = \mu_s N_E = 0.2N_E \qquad F_F = \mu_s N_F = 0.2N_F$

$$\longrightarrow \Sigma F_x = 0: \quad F_E + F_F - P\cos\theta = 0 \quad \text{or} \quad 0.2\left(N_E + N_F\right) = (66\text{ N})\cos\theta$$

$$\uparrow \Sigma F_y = 0: \quad N_E + N_F - 176.58\text{ N} - 156.96\text{ N} - P\sin\theta = 0$$

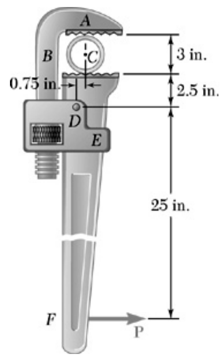
$$N_E + N_F = \left(66\sin\theta + 333.54\right)\text{ N}$$

So $\qquad 330\cos\theta = 66\sin\theta + 333.54$

Solving numerically, $\qquad \theta = -3.66° \quad$ and $\quad \theta = -18.96°$

Therefore, $\qquad -18.96° \le \theta \le -3.66°$ ◀

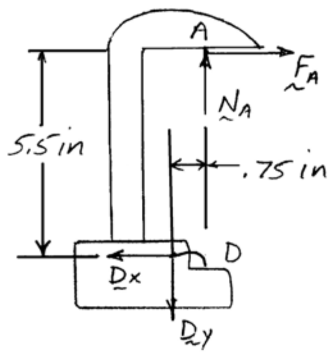
PROBLEM 8.30



A pipe of diameter 3 in. is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75\text{ in.})N_A - (5.5\text{ in.})F_A = 0$$

Impending motion: $\quad F_A = \mu_A N_A$

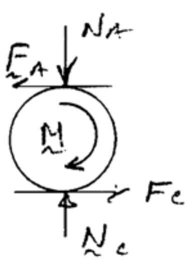
Then $\quad 0.75 - 5.5\mu_A = 0$

or $\quad \mu_A = 0.13636$

$$\mu_A = 0.1364 \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \qquad D_x = F_A$$

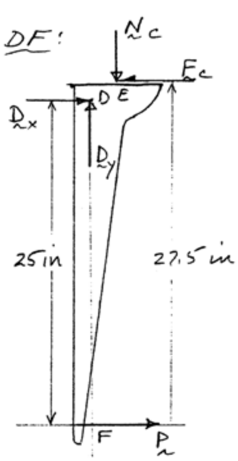
Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5\text{ in.})F_C - (0.75\text{ in.})N_C - (25\text{ in.})D_x = 0$$

Impending motion: $\quad F_C = \mu_C N_C$

Then $\quad 27.5\mu_C - 0.75 = 25\dfrac{F_A}{N_C}$

But $\quad N_C = N_A \quad$ and $\quad \dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $\quad 27.5\mu_C = 0.75 + 25(0.13636)$

$$\mu_C = 0.1512 \blacktriangleleft$$