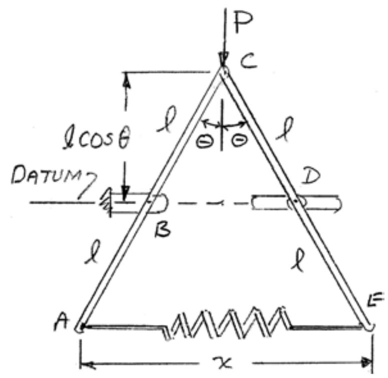
PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.31.

SOLUTION



Spring: $$AE = x = 2(2l\sin\theta) = 4l\sin\theta$$

Unstretched length: $$x_0 = 4l\sin 30^\circ = 2l$$

Deflection of spring $$s = x - x_0$$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_C$$

$$= \frac{1}{2}k\left[2l(2\sin\theta - 1)\right]^2 + P(l\cos\theta)$$

$$V = 2kl^2(2\sin\theta - 1)^2 + Pl\cos\theta$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta - Pl\sin\theta = 0$$

$$(1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{2\sin\theta - 1}{\tan\theta}$$

PROBLEM 10.61 CONTINUED

With $P = 160\text{ N}, l = 200\text{ mm}, \quad \text{and} \quad k = 300\text{ N/m}$

Have
$$\frac{(160\text{ N})}{8(300\text{ N/m})(0.2\text{ m})} = \frac{2\sin\theta - 1}{\tan\theta}$$

or
$$\frac{2\sin\theta - 1}{\tan\theta} = \frac{1}{3}$$

Solving numerically, $\theta = 39.65° \quad \text{and} \quad 68.96°$

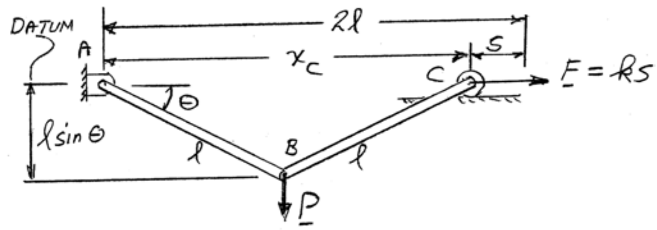
$\theta = 39.7°$ ◀
$\theta = 69.0°$

PROBLEMS 10.62 AND 10.63

10.62: Using the method of Section 10.8, solve Problem 10.33.

10.63: Using the method of Section 10.8, solve Problem 10.34.

SOLUTION



Problem 10.62

Have $$P = 150 \text{ lb}, \ l = 15 \text{ in.}, \text{ and } k = 12.5 \text{ lb/in.}$$

Then $$(1 - \cos\theta)\tan\theta = \frac{150 \text{ lb}}{4(12.5 \text{ lb/in.})(15 \text{ in.})}$$

$$= 0.2$$

Solving numerically, $$\theta = 40.2° \blacktriangleleft$$

Problem 10.63 $$V = \frac{1}{2}ks^2 + Py_B$$

$$V = \frac{1}{2}k(2l - x_C)^2 + Py_B$$

$$x_C = 2l\cos\theta \qquad \text{and} \qquad y_B = -l\sin\theta$$

Thus, $$V = \frac{1}{2}k(2l - 2l\cos\theta)^2 - Pl\sin\theta$$

$$= 2kl^2(1 - \cos\theta)^2 - Pl\sin\theta$$

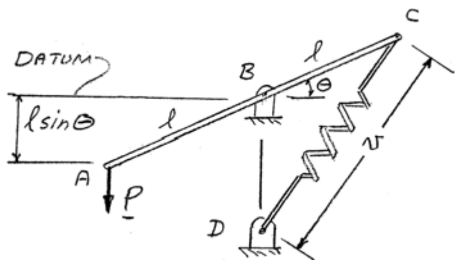
$$\frac{dV}{d\theta} = 2kl^2 2(1 - \cos\theta)\sin\theta - Pl\cos\theta = 0$$

or $$(1 - \cos\theta)\tan\theta = \frac{P}{4kl} \blacktriangleleft$$

PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.35.

SOLUTION



Spring

$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$v = 2l\sin\left(45° + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$

$$v_0 = 2l\sin 45° = \sqrt{2}l$$

Deflection of spring

$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45° + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by $\cos\theta$

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

PROBLEM 10.64 CONTINUED

Then with $P = 150$ lb, $l = 30$ in. and $k = 40$ lb/in.

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{150 \text{ lb}}{(40 \text{ lb/in.})(30 \text{ in.})}$$

$$= 0.125$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.618718$$

Solving numerically, $\theta = 17.83°$ ◀

PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.36.

SOLUTION

Using the results of Problem 10.64 with $P = 600$ N, $l = 800$ mm, and $k = 4$ kN/m , have

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

$$= \frac{600 \text{ N}}{(4000 \text{ N/m})(0.8 \text{ m})}$$

$$= 0.1875$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.57452$$

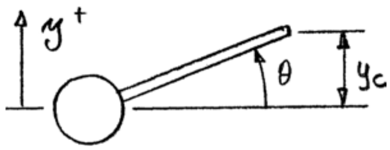
Solving numerically, $\theta = 30.985°$

$\theta = 31.0°$ ◀

PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

SOLUTION



Spring $$V_{SP} = \frac{1}{2}ky_C^2$$

where $$y_C = d_{AC}\tan\theta \qquad d_{AC} = 15\text{ in.}$$

$$\therefore \quad V_{SP} = \frac{1}{2}kd_{AC}^2\tan^2\theta$$

Force P: $$V_P = -Py_P$$

where $$y_P = r\theta \qquad r = 3\text{ in.}$$

$$\therefore \quad V_P = -Pr\theta$$

Then $$V = V_{SP} + V_P$$

$$= \frac{1}{2}kd_{AC}^2\tan^2\theta - Pr\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0: \; kd_{AC}^2\tan\theta\sec^2\theta - Pr = 0$$

or $$(4\text{ lb/in.})(15\text{ in.})^2\tan\theta\sec^2\theta - (96\text{ lb})(3\text{ in.}) = 0$$

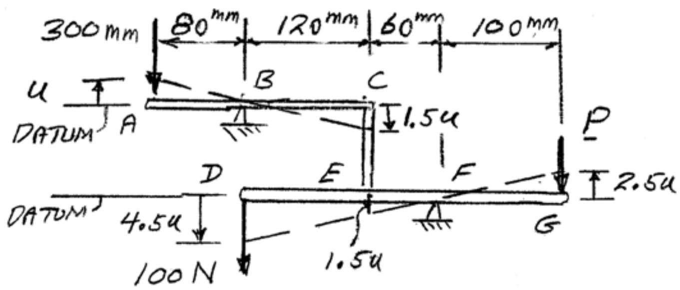
or $$3.125\tan\theta\sec^2\theta - 1 = 0$$

Solving numerically, $\theta = 16.4079°$ $\qquad$ $\theta = 16.41°$ ◀

PROBLEM 10.67

Show that the equilibrium is neutral in Problem 10.1.

SOLUTION



We have $y_A = u$

$$y_D = -4.5u$$

$$y_G = 2.5u$$

Have $V = (300\text{ N})y_A + (100\text{ N})y_D + P(y_E) = 0$

$$V = 300u + 100(-4.5u) + P(2.5u) = 0$$

$$V = (-150 + 2.5P)u$$

$$\frac{dV}{du} = -150 + 2.5P = 0 \text{ so that} \qquad P = 60\text{ N}$$

Substitute $P = 60\text{ N}$ in expression for V: $V = [-150 + 2.5(60)]u$

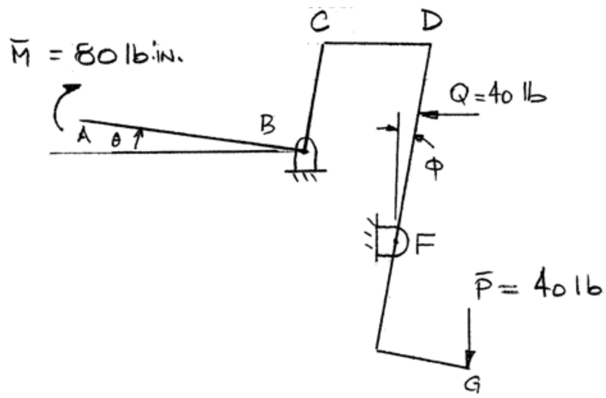
$$= 0$$

$\therefore$ V is constant and equilibrium is neutral ◀

PROBLEM 10.68

Show that the equilibrium is neutral in Problem 10.2.

SOLUTION



Consider a small disturbance of the system so that $\theta \ll 1$

Have $$x_C = x_D, \qquad 5\theta \simeq 15\phi$$

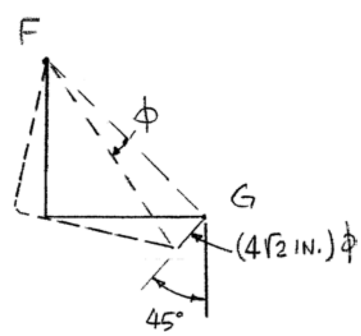
or $$\phi = \frac{\theta}{3}$$

Potential energy $$V = M\theta - Qx_E + Py_G$$

where $$x_E = (10 \text{ in.})\phi$$

$$= \left(\frac{10}{3}\theta\right) \text{ in.}$$

and $$y_G = \left[\left(4\sqrt{2} \text{ in.}\right)\phi\right]\cos 45°$$



PROBLEM 10.68 CONTINUED

Then
$$V = M\theta - \frac{10}{3}Q\theta + \frac{4}{3}P\theta$$

$$= \left(M + \frac{10}{3}Q + \frac{4}{3}P\right)\theta$$

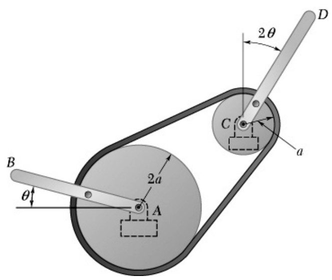
and
$$\frac{dV}{d\theta} = M - \frac{10}{3}Q + \frac{4}{3}P$$

For equilibrium
$$\frac{dV}{d\theta} = 0: \quad M - \frac{10}{3}Q + \frac{4}{3}P = 0$$

$\therefore$ At equilibrium, $V = 0$, a constant, for all values of θ.

Hence, equilibrium is neutral

Q.E.D.◀



PROBLEM 10.69

Two identical uniform rods, each of weight W and length L, are attached to pulleys that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the pulleys, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Let each rod be of length L and weight W. Then the potential energy V is

$$V = W\left(\frac{L}{2}\sin\theta\right) + W\left(\frac{L}{2}\cos 2\theta\right)$$

Then

$$\frac{dV}{d\theta} = \frac{W}{2}L\cos\theta - WL\sin 2\theta$$

For equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{W}{2}L\cos\theta - WL\sin 2\theta = 0$$

or

$$\cos\theta - 2\sin 2\theta = 0$$

Solving numerically or using a computer algebra system, such as Maple, gives four solutions:

$$\theta = 1.570796327 \text{ rad} = 90.0°$$

$$\theta = -1.570796327 \text{ rad} = 270°$$

$$\theta = 0.2526802551 \text{ rad} = 14.4775°$$

$$\theta = 2.888912399 \text{ rad} = 165.522°$$

Now

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}WL\sin\theta - 2WL\cos 2\theta$$

$$= -WL\left(\frac{1}{2}\sin\theta + 2\cos 2\theta\right)$$

PROBLEM 10.69 CONTINUED

At $\theta = 14.4775°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 14.4775° + 2\cos\left[2(14.4775°)\right]\right\}$$

$$= -1.875WL \ (< 0) \qquad \therefore \quad \theta = 14.48°, \ \text{Unstable} \blacktriangleleft$$

At $\theta = 90°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 90° + 2\cos 180°\right\}$$

$$= 1.5WL \ (> 0) \qquad \therefore \quad \theta = 90°, \ \text{Stable} \blacktriangleleft$$

At $\theta = 165.522°$

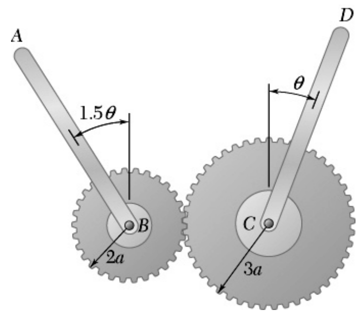
$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 165.522° + 2\cos(2 \times 165.522°)\right\}$$

$$= -1.875WL \ (< 0) \qquad \therefore \quad \theta = 165.5°, \ \text{Unstable} \blacktriangleleft$$

At $\theta = 270°$

$$\frac{d^2V}{d\theta^2} = -WL\left(\frac{1}{2}\sin 270° + 2\cos 540°\right)$$

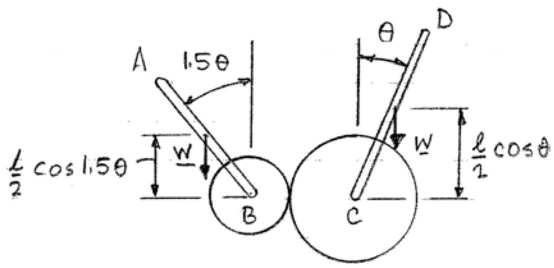
$$= 2.5WL \ (> 0) \qquad \therefore \quad \theta = 270°, \ \text{Stable} \blacktriangleleft$$



PROBLEM 10.70

Two uniform rods, each of mass m and length l, are attached to gears as shown. For the range $0 \le \theta \le 180^\circ$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \qquad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-1.5\sin 1.5\theta) + \frac{Wl}{2}(-\sin\theta)$$

$$= -\frac{Wl}{2}(1.5\sin 1.5\theta + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

For equilibrium $$\frac{dV}{d\theta} = 0\text{: } 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8^\circ$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

PROBLEM 10.70 CONTINUED

At $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left(2.25\cos 0° + \cos 0°\right)$$

$$= -\frac{Wl}{2}\left(3.25\right) \ \left(< 0\right)$$

$\therefore\ \theta = 0,$ Unstable ◀

At $\theta = 137.8°$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left[2.25\cos\left(1.5 \times 137.8°\right) + \cos 137.8°\right]$$

$$= \frac{Wl}{2}\left(2.75\right) \ \left(> 0\right)$$

$\therefore\ \theta = 137.8°,$ Stable ◀