For Shire *n-shire* in the Riddler Township, the population can be expressed as :

$$P(n) = 10n + 1$$

And the total population is :

$$\sum_{n=1}^{10} (10n + 1) = 10 \sum_{n=1}^{10} n + 10 = 550$$

The number of electoral votes can be expressed as :

$$V(n) = n + 2$$

And the total number of votes is :

$$\sum_{n=1}^{10} (n + 2) = \sum_{n=1}^{10} n + 20 = 75$$

The majority cutoff for electoral votes is 38. The sum of the three largest shires is too small $(10 + 11 + 12 = 33)$ and the sum of the seven smallest shires is too large $(3 + 4 + 5 + 6 + 7 + 8 + 9 = 42)$. In order to achieve exactly 38, either four, five, or six shires is necessary.

The cutoff for majority of the population for each *n-shire* is :

$$M(n) = 5n + 1$$

In order to minimize the percentage of the popular vote, assume exactly the cutoff majority voting population, which means the weight of each constituent for *n-shire* is :

$$V(n) = \frac{V(n)}{M(n)} = \frac{n+2}{5n+1} = \frac{n + \frac{1}{5}}{5n+1} + \frac{\frac{9}{5}}{5n+1} = \frac{1}{5} + \frac{9}{25n+5}$$

It follows that smaller shire constituents have more weight than that of larger shires. Making a coalition of six shires,

$$s_1, s_2, s_3, s_4, s_5, s_6$$

the respective sum of their votes can be expressed as:

$$CoalitionVotes = 38 = \sum_{k=1}^{6} V(s_k) = \sum_{k=1}^{6} (s_k + 2) = 12 + \sum_{k=1}^{6} s_k$$

$$\sum_{k=1}^{6} s_k = 26$$

The population of that coalition can be expressed as :

$$CoalitionPopulation = \sum_{k=1}^{6} M(s_k) = \sum_{k=1}^{6} (5s_k + 1) = 6 + 5 \sum_{k=1}^{6} s_k = 136$$

The following coalitions satisfy both of these conditions.

[3, 4, 5, 6, 8, 12]   [3, 4, 5, 7, 8, 11]   [3, 4, 6, 7, 8, 10]

[3, 4, 5, 6, 9, 11]   [3, 4, 5, 7, 9, 10]   [3, 5, 6, 7, 8, 9]

Each of these coalitions has 38 electoral votes and only 136 out of the 560 constituents, $\approx 24.29\%$, voting for the winner.

Note that for a coalition of five shires,

$$CoalitionPopulation = 5 + 5(38 - 10) = 145$$

And for a coalition of four shires,

$$CoalitionPopulation = 4 + 5(38 - 8) = 154$$

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