Note that as the post weight k approaches 0, the perimeter of the hamster pen approaches 1 meter and the polygon approaches a circle. The upper limit for the area is therefore :

$$A = \pi r^2$$
$$= \pi \left(\frac{1}{2\pi}\right)^2$$
$$= \frac{1}{4\pi} \approx 0.079577 m^2$$

As k increases, the number of sides and area decrease.

The area of a regular polygon is:

$$A = \frac{apothem * Perimeter}{2}$$

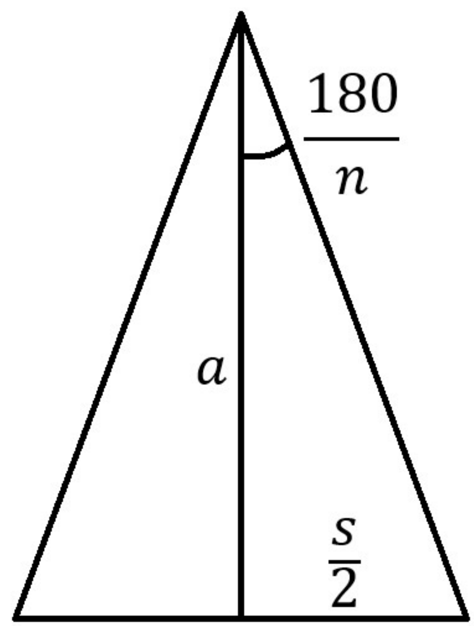
The side length in meters is the same as the side weight in kilograms. Given that the total weight is 1 kilogram and each of the n posts weigh k kilograms, the length of a side is:

$$s = \frac{1 - kn}{n}$$

The apothem can be found from one of the n triangles in the regular polygon :

$$\tan\frac{180}{n} = \frac{\frac{s}{2}}{a}$$

$$a = \frac{\frac{1-kn}{n}}{2\tan\frac{180}{n}}$$



Substituting yields the following:

$$A = \frac{1}{2} * \frac{\frac{1-nk}{n}}{2\tan\frac{180}{n}} * n * \frac{1-nk}{n}$$

$$= \frac{(nk-1)^2}{4n\tan\frac{180}{n}}$$

There is a weight w_n for each n such that the area of a regular *n*-gon and regular *(n+1)*-gon will have the same area.

This yields :

$$\frac{(w_n n - 1)^2}{4n\tan\frac{180}{n}} = \frac{(w_n(n+1) - 1)^2}{4(n+1)\tan\frac{180}{n+1}}$$

Substituting $\tan\frac{180}{n} = t_n$ and $\tan\frac{180}{n+1} = t_{n+1}$ yields:

$$\frac{{w_n}^2n^2 - 2w_nn + 1}{4nt_n} = \frac{w_n^2(n+1)^2 - 2w_n(n+1) + 1}{4(n+1)t_{n+1}}$$

$$w_n^2n^2(n+1)t_{n+1} - 2w_nn(n+1)t_{n+1} + (n+1)t_{n+1} = w_n^2n(n+1)^2t_n - 2w_nn(n+1)t_n + nt_n$$

$$w_n^2[n(n+1)(nt_{n+1} - (n+1)t_n)] + w_n[2n(n+1)(-t_{n+1} + t_n)] + [(n+1)t_{n+1} - nt_n] = 0$$

The discriminant, $b^2 - 4ac$, yields :

$$[2n(n+1)(-t_{n+1} + t_n)]^2 - 4[n(n+1)(nt_{n+1} - (n+1)t_n)][(n+1)t_{n+1} - nt_n]$$

$$= 4n^2(n+1)^2[(t_{n+1}^2 - 2t_{n+1}t_n + t_n^2)] - 4n(n+1)\left[\left((n^2+n)t_{n+1}^2 - (2n^2+1)t_{n+1}t_n + (n^2+n)t_n^2\right)\right]$$

$$= 4n(n+1)\left[\left((n^2+n)t_{n+1}^2 - 2(n^2+n)t_{n+1}t_n + (n^2+n)t_n^2\right) - \left((n^2+n)t_{n+1}^2 - (2n^2+2n+1)t_{n+1}t_n + (n^2+n)t_n^2\right)\right]$$

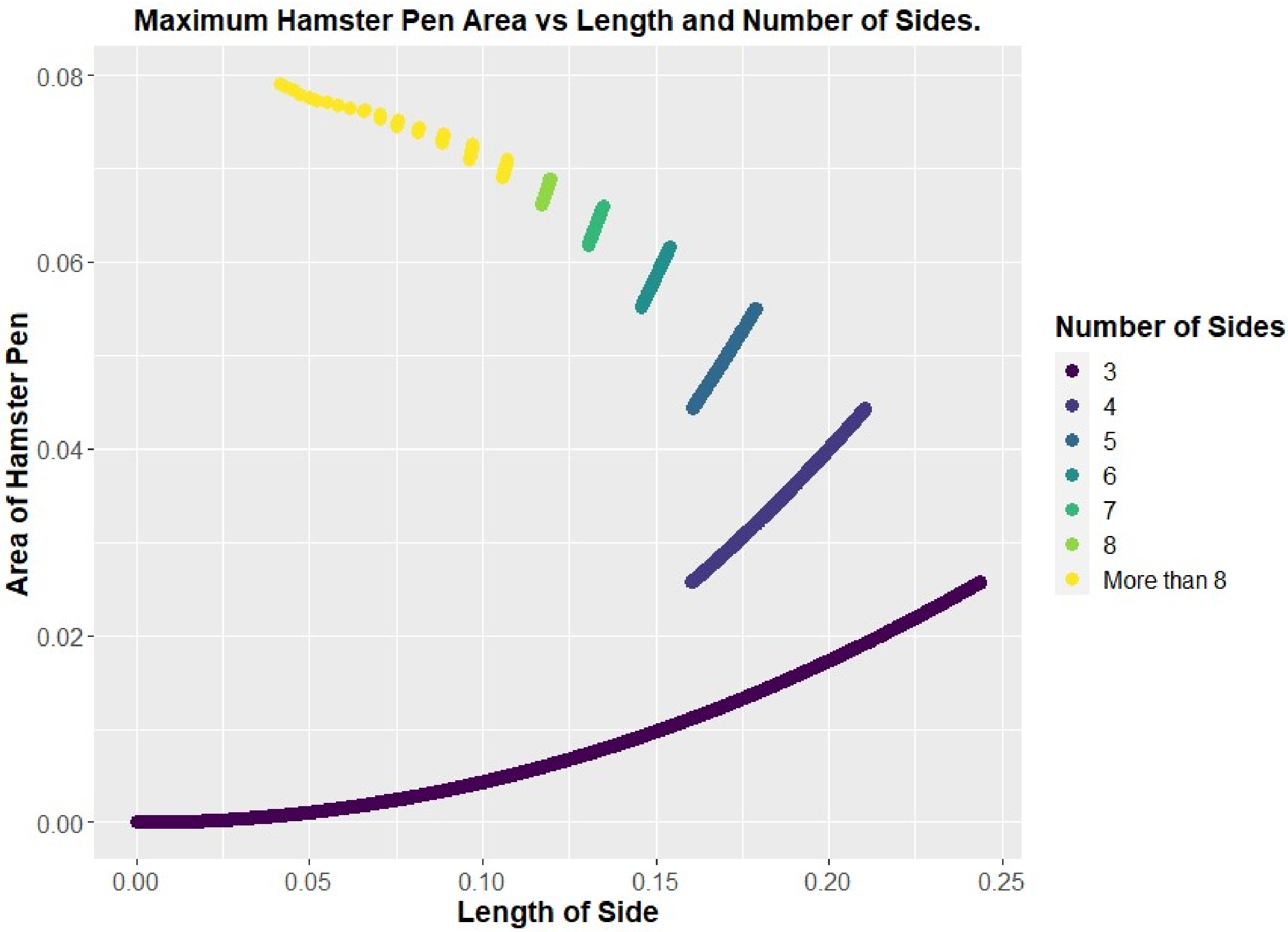
$$= 4n(n+1)t_{n+1}t_n$$

Completing the quadratic formula :

$$\boxed{w_n = \frac{2n(n+1)(t_{n+1} - t_n) + \sqrt{4n(n+1)t_{n+1}t_n}}{2n(n+1)(nt_{n+1} - (n+1)t_n)}}$$

The above equation yields the following table that maximizes the hamster pen area (rounded to 7 decimal places) :

For n-gon	Maximum weight of post (kg)	Minimum weight of post (kg)
3	0.3333333	0.0896422
4	0.0896422	0.0395738
5	0.0395738	0.0210155
6	0.0210155	0.0125110
7	0.0125110	0.0080559
8	0.0080559	0.0054941
9	0.0054941	0.0039154



Rohan Lewis

2020.08.24