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M4: Finding the factors affecting the period of oscillation 'T' of a spring pendulum and determining the spring constant

▼ 1. Introduction

In this experiment, by using spring pendulum and different masses we can find the factors which are affecting the period of oscillation of a spring pendulum with the help of stop watch. Accordingly, we are also able to determine the spring constant. Here, at first we'll measure the amount of time for 10 oscillations when mass of an object is fixed but we'll change the deflection 3 times. After that, we'll again measure the time for 10 oscillations with fixed deflection including we'll change the amount of mass which are 100gm, 150gm, 200gm, 250 gm and 300 gm. Consequently, we will calculate the period of oscillation for both approaches. Finally, by drawing graph for both approaches we'll calculate the spring constant and we'll establish the relationship between mass and time period as well as deflection and time period.

▼ 2.Theory

In a physical system, where a piece of mass is connected to a spring as if the resulting motion can contain elements of a simple pendulum as well as a spring that is called a spring pendulum.

If we want to stretch a spring or compress a spring of pendulum than the amount of force is essential for this purpose, we can measure this force by following Hooke's law- which is:

$F \propto -y$; y =Distance (stretched or compressed).

$F = -ky$; k =spring constant.

We know that,

$$d^2x/dt^2 = -(kx)/m \dots (i)$$

$$\text{and, } d^2\theta/dt^2 = -(g\theta)/l \dots (ii)$$

If we combine both equations, then we'll get

$$k/m = g/l$$

$\Rightarrow k = mg/l \dots (iii) \rightarrow$ (k is the spring constant measured using linear motion).

The equation of a straight line is $y = ax + b$; where a is the gradient and b is the y intercept. If we compare the third equation (iii) with the equation of straight line then, we'll get the spring constant - $k = g/a \dots (iv)$

By using newton's 2nd law, there is a normal method to analyze the motion of a mass of a string, which is- $T = 2\pi\sqrt{m/k}$

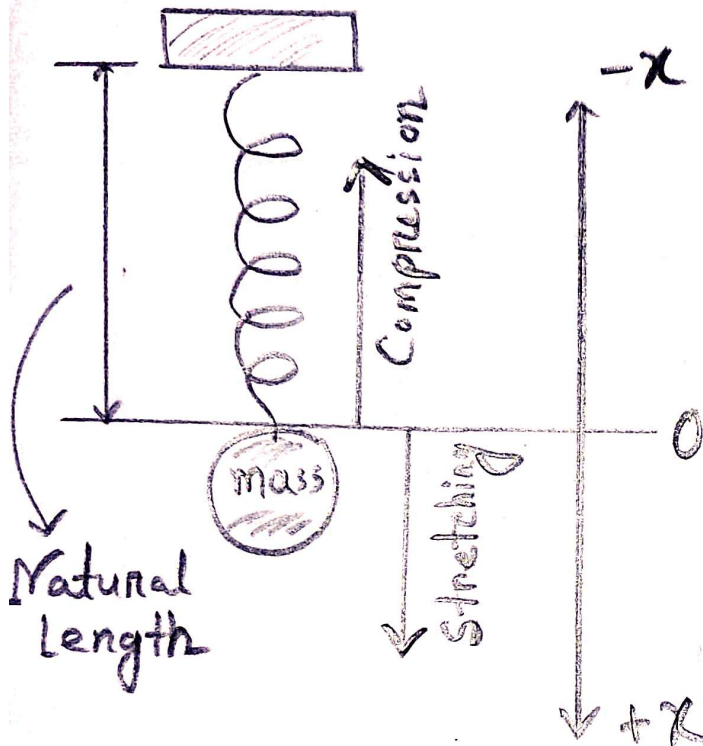
$$\Rightarrow k = 4\pi^2(m/T^2) \dots (v)$$

If we again take a equation of a straight line just like $y = mx + c \dots (vi)$ And then, by comparing equation number (v) and (vi), we'll get the spring constant for spring-

$$k = 4\pi^2/a \dots (vii)$$

When we release the spring pendulum after attaching a mass of an object with it, the time for one complete cycle is called the period. We'll see what will happen after attaching different kinds of mass with the spring pendulum and how its characteristics will change for different masses along with deflection.

By using both spring constant formula we'll draw graphs and get the spring constant as well as we'll compare those graphs to understand the attributes of a spring pendulum.



x = the distance the spring is stretched or compressed

Figure: Spring pendulum

▼ 3. Data and Method

▼ Apparatus

- i) Spring pendulum
- ii) Stop Watch
- iii) Mass (100,150,200,250 and 300 gm)

▼ Procedure

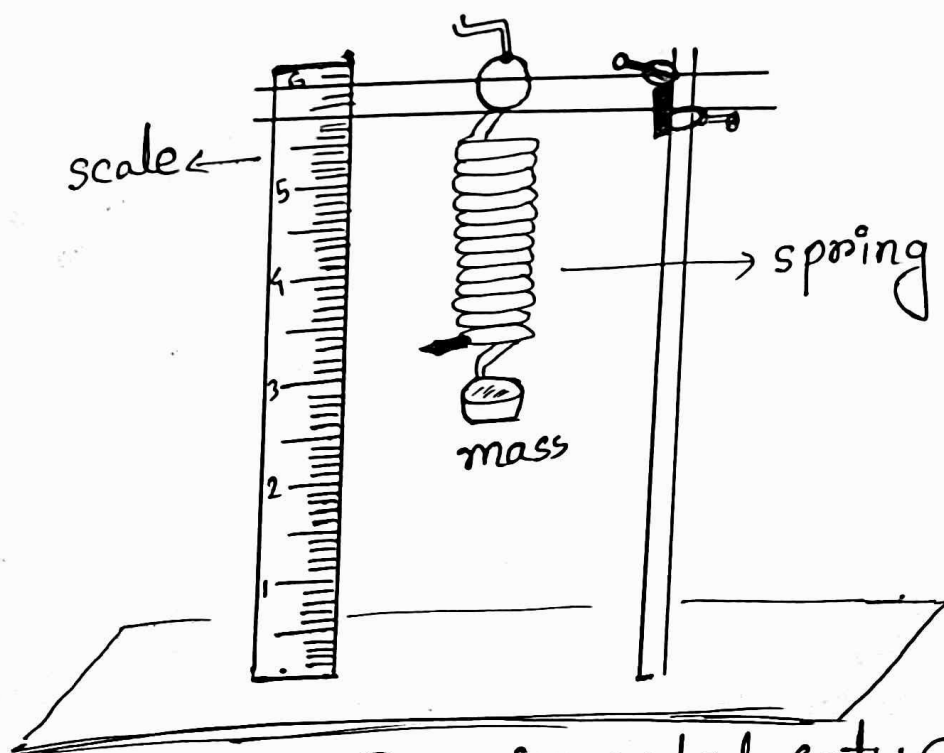
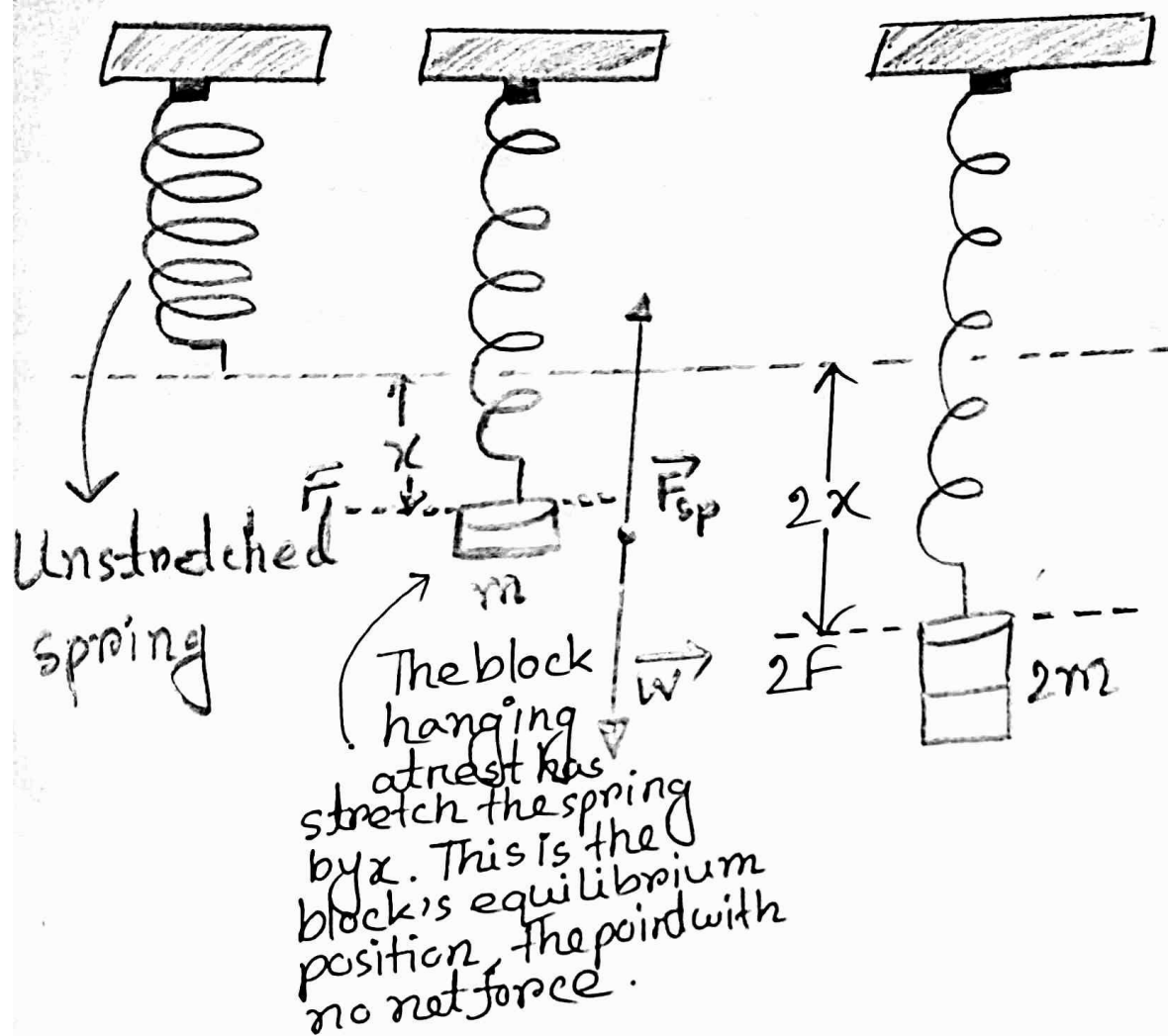


Figure: Experimental setup of spring pendulum

- 1) Firstly, we have set up the apparatus properly as shown in the above figure. Additionally, we were careful about the position of spring pendulum, in view of that fact that we get accurate measurements.
 - 2) Secondly, we have attached a mass of 150 gm in the bottom part of the spring (as we have seen in the above figure). Also, we have taken the measurement of where the bottom part of the spring pendulum has reached (in the scale which is attached with the pendulum). From that point we have taken the deflection of 1 cm.
 - 3) Thirdly, we have collected a stopwatch which works properly so that we can collect the data related to time accurately. After attaching a mass of 150 gm and deflection of 1 cm, we have released the spring.
 - 4) Subsequently, when we have released the spring, we immediately has started the stop watch as well as collected the time taken for 10 oscillations (to reduce the amount of error we've neglected the first oscillation and started the stopwatch from the 2nd oscillation and stopped it when there was actually 11 oscillations in total, though we have taken the time for 10 oscillations). Just like this, we have collected the time one more time so that we could get average time for 10 oscillations when the deflection is 1 cm.
 - 5) Furthermore, again we have repeated the process with same amount of mass (150 gm) but with deflection of 2 cm and collected the time for 10 oscillations twice just as before.
- Besides, one more time we have repeated the process with deflection of 3 cm but the mass is same which is 150 gm and collected the time twice for 10 oscillations.
- 6) Eventually, we have calculated the average time taken for 10 oscillations and period of oscillation.
 - 7) Additionally, we have repeated the process of taking the time for 10 oscillations but this time we have changed the amount of mass 5 times but did not change the deflection which is fixed now (2 cm). We have attached the mass of 100 gm and taken the measurement of where the bottom part of the spring pendulum has reached (in the scale which is attached with the spring pendulum). From this point, we have stretched the spring 2 cm as well as released the spring pendulum eventually and collected the time for 10 oscillations twice to get more accurate value. (we've started the stop watch when the second oscillation has started to decrease the amount of error though there was exactly 11 oscillations). We have calculated the extension(l) also which is specially the measurement of the main scale division after attaching the mass-the main scale division before attaching the mass.
 - 8) Moreover, after doing this process (no. 7), we have changed the mass from 100 gm to 150 gm but with same deflection (which is 2 cm) and collected the time for 10 oscillations just as before.

And then, we have attached a mass of 200 gm and repeated the process of taking 10 oscillations. Basically, the process has reoccured for 250 gm and 300 gm as well with deflection of 2 cm along with calculated the extension.

▼ Data

▼ Table-1: Period of oscillation 'T' as a function of deflection. (m= 150 gm)

Deflection(L)	Time taken for 10 oscillations	Avg. time taken for 10 oscillations(10T)	Period of oscillation(T)
1 cm	i. 7.20 sec		
	ii. 7.25 sec	7.225 sec	0.7725 sec
2 cm	i. 7.23 sec		
	ii. 7.22 sec	7.225 sec	0.7725 sec
3 cm	i. 7.26 sec		
	ii. 7.18 sec	7.220 sec	0.7720 sec

▼ Table-2: Period of oscillation 'T' as a function of mass. (Deflection= 2 cm)

Mass m	Time taken for 10 oscillations(10T)	Avg. time taken for 10 oscillations(10T)	Period of oscillation(T)	T ²	E
100 gm	i. 5.45 sec				
	ii. 5.60 sec	5.525 sec	0.5525 sec	0.31 sec ²	1.
150 gm	i. 7.13 sec				
	ii. 7.12 sec	7.125 sec	0.7125 sec	0.51 sec ²	4.
200 gm	i. 7.72 sec				
	ii. 7.84 sec	7.780 sec	0.778 sec	0.61 sec ²	7.
250 gm	i. 8.93 sec				
	ii. 8.51 sec	8.72 sec	0.872 sec	0.76 sec ²	1'
300 gm	i. 9.52 sec				
	ii. 9.54 sec	9.53 sec	0.953 sec	0.91 sec ²	14

▼ 4. Analysis and Result

```
import numpy as np
import matplotlib.pyplot as plt
```

```
m = np.arange(100, 301, 50)
```

```

l = [1.6, 4.8, 7.9, 11, 14.1]

t = [[5.45, 5.60], [7.13, 7.12], [7.72, 7.84], [8.93, 8.51], [9.52, 9.54]]

T2 = []
for i in t: T2.append((np.mean(i)/10.0)**2)

print(m) #gm
print(l,)
print(T2) #sec^2

[100 150 200 250 300]
[1.6, 4.8, 7.9, 11, 14.1]
[0.30525625, 0.50765625, 0.6052839999999998, 0.7603839999999998, 0.9082089999999999]

```

Graph: T^2 vs Mass, when spring constant $k = g/a$; $g = 981 \text{ cm/s}^2$

```

x, y = m, l

plt.plot(x, y, '.')

A = np.vstack([x, np.ones(len(x))]).T
a, b = np.linalg.lstsq(A, y, rcond=None)[0]

plt.grid()
plt.xlabel('Mass (m) in gm')
plt.ylabel('T^2 in sec^2')

line = a*x + b
plt.plot(x, line, '-')

k = 981 / a
print("Spring constant %.2f N/cm"%k)
print(b)

```

Spring constant 15721.15 N/cm
-4.599999999999994

Graph: T^2 vs Mass, when spring constant $k = (4\pi^2)/a$.

$x, y = m, T^2$

```
plt.plot(x, y, '.')

```

```
A = np.vstack([x, np.ones(len(x))]).T
a, b = np.linalg.lstsq(A, y, rcond=None)[0]

```

```
plt.grid()
plt.xlabel('Mass (m) in gm')
plt.ylabel('T^2 in sec^2')

```

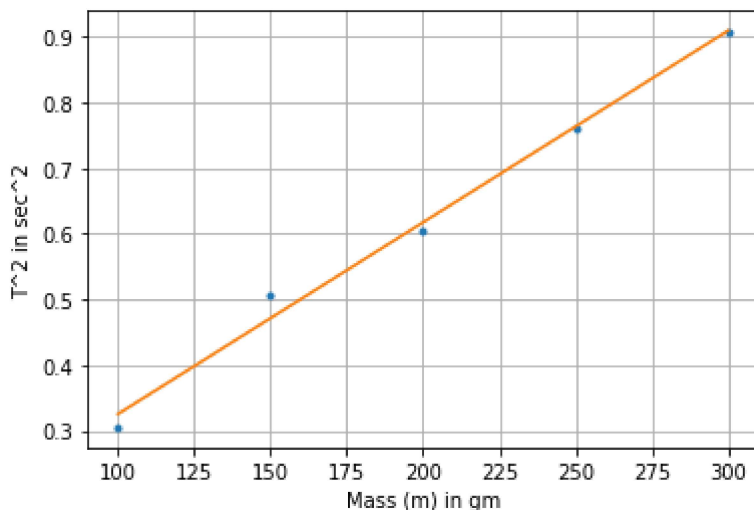
```
line = a*x + b
plt.plot(x, line, '-')

```

```
k = (4*np.pi**2) / a
print("Spring constant %.2f N/cm"%k)
print(b)

```

Spring constant 13532.67 N/cm
0.033904599999999924



Graph: T vs Deflecting (from Table-1)

```
deflection_in_cm=[1,2,3]
t_DifferentDeflection=[[7.20,7.25],[7.23,7.22],[7.26,7.18]]

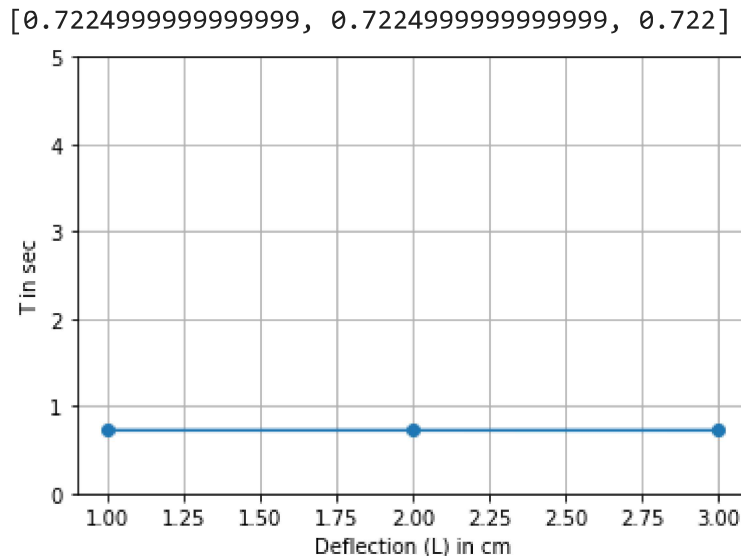
timePeriod_in_second = []
for i in t_DifferentDeflection: timePeriod_in_second.append(np.mean(i)/10)
print(timePeriod_in_second)

plt.plot(deflection_in_cm,timePeriod_in_second,'-o')

```



```
plt.ylim([0,5])
plt.grid()
plt.xlabel('Deflection (L) in cm')
plt.ylabel('T in sec')
plt.show()
```



Here, when we use the spring constant $k=g/a$, we got the Spring constant 15721.15 N/cm and y intercept is -4.5999999999999994.

Additionally, when we use the spring constant $k=4\pi^2/a$, we got Spring constant 13532.67 N/m and y intercept is 0.033904599999999924.

There is some difference because we have done the experiment by us instead of a machine or robot. That is why, there is some error for spring constant.

▼ 5. Discussion

At first, when we set up our apparatus, we were attentive at this moment so that we could get accurate measurements. There was probably systematic error. Systematic errors are errors associated with the particular instruments or techniques used to carry out the measurements. Since, it is almost impossible to get perfect data about the measurements so, there were some random errors also though we were careful. Because, error might be caused by us in this experiment since, everything was handled by us instead of any kind of machines. There also might be error in recording the time for each 10 oscillations, since it has done by us, we might start our stopwatch too immediate or making a small amount of late. To reduce the error related to time we have taken the time twice for each 10 oscillations and then calculated the average for each. From the average time for 10 oscillation we were able to calculate the period of oscillation.

We have understood that, if we don't change the mass then it does not matter how much deflection is happened there, the time taken for 10 oscillations will be same every time. But, if the mass is different and deflection is same for every mass then the time will change for each 10 oscillation. The amount of time will increase if the amount of mass attached with the spring is increased. In addition, the extension(l) will also increase if the amount of mass is increased.

Moreover, we can clearly see the characteristics in the graphs, in T^2 vs Mass graph, we can see that it is increasing, but in T vs Deflection graph, it is a straight line.

▼ 6. Conclusion

The above experiment was all about to find out the factors which are affecting the period of oscillation ' T ' of a spring pendulum and also to determine the spring constant. Additionally, we have also learned about what is a spring pendulum and its characteristics. We have done the experiment in two ways to understand the attributes of a spring pendulum. The first way is- the amount of mass was fixed(150 gm) but we take three different deflections and by doing this we have learned that, the period will be equal every time since, it depends on mass, not on the deflection. The 2nd way is- changing the mass but deflection is fixed, we have acquired the knowledge that, the period of oscillation changes with the change of mass that is the period of oscillation can be reduced by reducing the mass as well as the period of oscillation can be increased by increasing the amount of mass.

