Problem Statement: We have an object that is moving in two directions (x and y) with constant velocity (vx, vy) and with a constant turn rate (rate is constant).

We have two sources of measurement:

1. A LIDAR sensor that measures the position of the object in (x, y) co-ordinates with some noise

2. A RADAR that measures the position, relative velocity and heading angle (r, rdot, heading) with some noise

Objective: Develop an extended Kalman filter to predict the position (x, y) velocity (vx, vy), yaw and yaw rate of the object and to output the following time, x_state, y_state, vx_state, y_state, yaw_angle_state, yaw_rate_state, sensor_type x_measured, y_measured, x_ground_truth, y_ground_truth, x_ground_truth, y_ground_truth as well as plot the estimated position (x, y) vs ground truth position (x, y).

Approach for the problem solution:

It involves following steps:

The Extended Kalman filter will be used as few of the models are non-linear in nature.

Motion Model:

1. Defining the prediction model using the physics for basic 2-D motion of a robot with constant turn rate and constant velocity.
2. As the motion model is non-linear, the extended kalam filter will be used. Taylor expansion will be used to linearize the motion model function to obtain the Jacobian matrix which maps the state transition from time step 't-1' to 't'.
3. The Process noise covariance matrix Q will be calculated using the assumption 1.

Measurement Model:

1. The value of R_{lidar}, R_{radar}, σ_a^2, and σ_α^2 will be computed using the provided data of the readings from the lidar and radar and the ground truth values provided.
2. As the lidar readings obtained are linear in nature thus basic Kalman filter can be used for this model and thus linearization is not required.
3. For the measurement model from the radar, the readings obtained are non-linear in nature thus extended Kalman filter will be used. The Jacobian matrix H_t after linearization will also be calculated.

Assumption:

1. As the motion of the robot is constant velocity and constant turn rate model, the acceleration of the robot between consecutive time step is considered as noise and is incorporated in the process noise.

2. As the value of the $\boldsymbol{V_t}$ and $\dot{\boldsymbol{\theta}}_t$ is very uncertain during the initial time steps, thus the initial covariance matrix is initialized as

$$P_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \infty & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \infty \end{bmatrix}$$

3. The initial value of the Lidar data is used to initialize the robot's initial pose.

Given Information:

The data for the lidar and radar reading is given.

The data can be used to calculate the R_{lidar}, R_{lidar}, σ_a^2, and σ_α^2

1. R_{lidar}

$$\frac{1}{N}\sum_{i}^{n}(X_{position} - grount\ truth\ _{X_{position}})^2$$

$Similarly\ for\ Y_{position} can\ be\ done$

lidar_sensor	x_position	y_position	time	gt_x_position	gt_y_position	gt_vx	gt_vy	gt_yaw	gt_yaw_rate		Diff X	Diff Y	diff2 X	diff2 Y
L	3.12E-01	5.80E-01	1.48E+15	6.00E-01	6.00E-01	5.20E+00	0	0	6.91E-03		=B2-E2	-1.97E-02	8.28E-02	3.87E-04
L	1.17E+00	4.81E-01	1.48E+15	1.12E+00	6.00E-01	5.20E+00	5.39E-03	1.04E-03	2.07E-02		5.39E-02	-1.19E-01	2.90E-03	1.42E-02
L	1.65E+00	6.25E-01	1.48E+15	1.64E+00	6.01E-01	5.20E+00	1.80E-02	3.45E-03	3.45E-02		1.07E-02	2.33E-02	1.15E-04	5.45E-04
L	2.19E+00	6.49E-01	1.48E+15	2.16E+00	6.04E-01	5.20E+00	3.77E-02	7.25E-03	4.83E-02		2.91E-02	4.47E-02	8.48E-04	1.99E-03
L	2.66E+00	6.66E-01	1.48E+15	2.68E+00	6.09E-01	5.19E+00	6.46E-02	1.24E-02	6.21E-02		-2.41E-02	5.68E-02	5.79E-04	3.23E-03
L	3.01E+00	6.37E-01	1.48E+15	3.20E+00	6.17E-01	5.19E+00	9.85E-02	1.90E-02	7.58E-02		-1.86E-01	1.98E-02	3.48E-02	3.91E-04
L	3.89E+00	3.12E-01	1.48E+15	3.72E+00	6.29E-01	5.19E+00	1.40E-01	2.69E-02	8.95E-02		1.76E-01	-3.17E-01	3.10E-02	1.01E-01
L	4.31E+00	5.79E-01	1.48E+15	4.24E+00	6.45E-01	5.18E+00	1.88E-01	3.62E-02	1.03E-01		7.30E-02	-6.69E-02	5.33E-03	4.47E-03

$$R_{lidar} = \begin{bmatrix} 0.0228 & 0 \\ 0 & 0.0212 \end{bmatrix}$$

2.

R_{radar}

$$\frac{1}{N}\sum_{i}^{n} Distance - ground\ truth\ _{distance})^2)$$

radar_sensor	distance	h	relative_vel	time	gt_x_position	gt_y_position	gt_vx	gt_vy	gt_yaw	gt_yaw_rate		gt_distanc	gt_heading	gt_rel_vel		diff2 Dist	diff2 headi	diff2 rel vel	
R	1.01E+00	5.54E-01	4.89E+00	1.47701E+15	8.60E-01	6.00E-01	5.20E+00	1.80E-03	3.46E-04	1.38E-02		1.05E+00	0.6092	4.27E+00		=(B2-M2)^2		3.94E-01	1.00E-01
R	1.05E+00	3.89E-01	4.51E+00	1.47701E+15	1.38E+00	6.01E-01	5.20E+00	1.08E-02	2.07E-03	2.76E-02		1.51E+00	0.410522	4.77E+00		2.09E-01	4.53E-04	6.76E-02	1.00E-01
R	1.70E+00	2.98E-01	5.21E+00	1.47701E+15	1.90E+00	6.02E-01	5.20E+00	2.69E-02	5.18E-03	4.14E-02		1.99E+00	0.307087	4.96E+00		8.69E-02	7.76E-05	6.12E-02	1.00E-01
R	2.04E+00	2.76E-01	5.04E+00	1.47701E+15	2.42E+00	6.06E-01	5.20E+00	5.02E-02	9.67E-03	5.52E-02		2.49E+00	0.245523	5.05E+00		2.02E-01	9.29E-04	6.81E-05	1.00E-01
R	2.99E+00	2.18E-01	5.19E+00	1.47701E+15	2.94E+00	6.13E-01	5.19E+00	8.07E-02	1.55E-02	6.89E-02		3.00E+00	0.205554	5.10E+00		1.28E-04	1.47E-04	8.39E-03	1.00E-01
R	3.59E+00	1.35E-01	5.16E+00	1.47701E+15	3.46E+00	6.23E-01	5.19E+00	1.18E-01	2.28E-02	8.26E-02		3.51E+00	0.178149	5.13E+00		6.40E-03	1.82E-03	1.11E-03	1.00E-01
R	4.26E+00	1.65E-01	5.43E+00	1.47701E+15	3.98E+00	6.37E-01	5.19E+00	1.63E-01	3.14E-02	9.63E-02		4.03E+00	0.158743	5.15E+00		5.19E-02	3.72E-05	8.27E-02	1.00E-01

Similarly, variance can be found for the heading and r_dot

$$R_{radar} = \begin{bmatrix} 0.0928 & 0 & 0 \\ 0 & 5.58 & 0 \\ 0 & 0 & 0.0831 \end{bmatrix}$$

3. σ_a^2

$$\sigma_a^2 = \frac{1}{N}\sum_{i}^{n}\left(\frac{(V_t - V_{t-1})}{\Delta t}\right)^2$$

gt_vy	gt_yaw	gt_yaw_rate		gt_distanc	gt_heading	gt_rel_vel		diff2 Dist	diff2 headi	diff2 rel vel						v	acc	acc^2
1.80E-03	3.46E-04	1.38E-02		1.05E+00	0.6092	4.27E+00		1.14E-03	3.01E-03	3.94E-01	1.00E-01		5.90E-05	8.07E-03		5.20E+00		
1.08E-02	2.07E-03	2.76E-02		1.51E+00	0.410522	4.77E+00		2.09E-01	4.53E-04	6.76E-02	1.00E-01		1.74E-04	2.61E-02		5.20E+00	=(Y3-Y2)/T3	
2.69E-02	5.18E-03	4.14E-02		1.99E+00	0.307087	4.96E+00		8.69E-02	7.76E-05	6.12E-02	1.00E-01		3.74E-04	5.43E-02		5.20E+00	-1.26E-02	1.59E-04
5.02E-02	9.67E-03	5.52E-02		2.49E+00	0.245523	5.05E+00		2.02E-01	9.29E-04	6.81E-05	1.00E-01		6.96E-04	9.26E-02		5.20E+00	-1.76E-02	3.10E-04
8.07E-02	1.55E-02	6.89E-02		3.00E+00	0.205554	5.10E+00		1.28E-04	1.47E-04	8.39E-03	1.00E-01		1.20E-03	1.41E-01		5.19E+00	-2.25E-02	5.08E-04

$$\sigma_a^2 = 0.005072$$

4. σ_α^2

$$\sigma_\alpha^2 = \frac{1}{N}\sum_{i}^{n}\left(\frac{Yaw\ rate_t - Yaw\ rate_{t-1}}{\Delta t}\right)^2$$

gt_vy	gt_yaw	gt_yaw_rate		gt_distanc	gt_heading	gt_rel_vel		diff2 Dist	diff2 headi	diff2 rel vel						v	acc	acc^2
1.80E-03	3.46E-04	1.38E-02		1.05E+00	0.6092	4.27E+00		1.14E-03	3.01E-03	3.94E-01	1.00E-01		5.90E-05	8.07E-03		5.20E+00		
1.08E-02	2.07E-03	2.76E-02		1.51E+00	0.410522	4.77E+00		2.09E-01	4.53E-04	6.76E-02	1.00E-01		1.74E-04	2.61E-02		5.20E+00	=(Y3-Y2)/T3	
2.69E-02	5.18E-03	4.14E-02		1.99E+00	0.307087	4.96E+00		8.69E-02	7.76E-05	6.12E-02	1.00E-01		3.74E-04	5.43E-02		5.20E+00	-1.26E-02	1.59E-04
5.02E-02	9.67E-03	5.52E-02		2.49E+00	0.245523	5.05E+00		2.02E-01	9.29E-04	6.81E-05	1.00E-01		6.96E-04	9.26E-02		5.20E+00	-1.76E-02	3.10E-04
8.07E-02	1.55E-02	6.89E-02		3.00E+00	0.205554	5.10E+00		1.28E-04	1.47E-04	8.39E-03	1.00E-01		1.20E-03	1.41E-01		5.19E+00	-2.25E-02	5.08E-04

$$\sigma_\alpha^2 = 0.009514$$

Defining Parameters:

X_t – *the position X – coordinate of the robot at time step 't'*

Y_t – *the position Y – coordinate of the robot at time step 't'*

V_t – *the velocity of the robot at time step 't'*

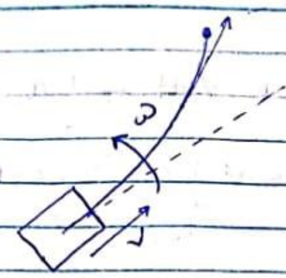
θ_t – *the yaw angle of the robot at time step 't'*

$\dot{\theta}_t$ – *the yaw rate of the robot at time step 't'*

a_t – *the linear acceleration of the robot at time step 't'*

$\boldsymbol{\alpha_t - the\ angular\ acceleration\ of\ the\ robot\ at\ time\ step\ 't'}$

Ans - 2



$\rightarrow$ $V_t \cos(\theta + \omega \Delta t) = V_x$

$V_t \sin(\theta + \omega \Delta t) = V_y$

Let the position x - cordinate at time step t $\rightarrow x_t$
Let the position y - cordinate at time step t $\rightarrow y_t$

Similarly V_{x_t} — velocity x - cordinate at time step t

V_{y_t} — velocity y - cordinate at time step t

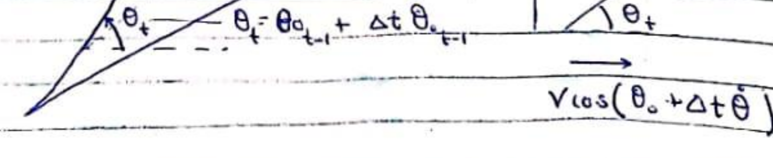
$V \rightarrow$ overall velocity of the robot

$\theta \rightarrow$ yaw angle of the robot

$\dot{\theta} \rightarrow$ yaw rate of the robot

Motion Model / Prediction Model

As the velocity is changing every instance



we know

$$X_t = x_{t-1} + V_x dt + \frac{1}{2} a_x dt^2$$

$$X_t = X_{t-1} + V \int_{t_{t-1}}^{t_t} \cos(\theta + \Delta t\,\dot{\theta})\,dt + \frac{1}{2} a_x \cos\theta\, dt^2$$

Similarly

$$Y_t = Y_{t-1} + V \int_{t_{t-1}}^{t_t} \sin(\theta + (t - t_{t-1})\,\dot{\theta})\,dt + \frac{1}{2} a \sin\theta\, dt^2$$

$$V_t = V_{t-1} + a\Delta t$$

$$\theta_t = \theta_{t-1} + \Delta t\,\dot{\theta}_{t-1} + \frac{1}{2} \Delta t^2 \alpha$$

$$\dot{\theta}_t = \Delta t\, \alpha$$

where $V \rightarrow$ velocity of the robot
$a \rightarrow$ acceleration of the robot
$\alpha \rightarrow$ angular acceleration

$dt \rightarrow$ is the time step

a, α is the linear and angular acceleration of the robot which is considered in the process noise

Considering the acceleration in the process noise

State transition equation is

$$\begin{bmatrix} X_t \\ Y_t \\ V_t \\ \theta_t \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} X_{t-1} \\ Y_{t-1} \\ V_{t-1} \\ \theta_{t-1} \\ \ddot{\theta}_{t-1} \end{bmatrix} + \begin{bmatrix} m \\ n \\ 0 \\ \Delta t \dot{\theta}_{t-1} \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{2}\Delta t^2 \cos\theta_{t-1}\, a \\ \frac{1}{2}\Delta t^2 \sin\theta_{t-1}\, a \\ \Delta t\, a \\ \frac{1}{2}\Delta t^2 \alpha \\ \Delta t\, \alpha \end{bmatrix}}_{\text{process noise}}$$

where

$$m = V \int_{t_{t-1}}^{t_t} \cos(\theta_{t-1} + \Delta t\, \theta_{t-1})\, dt$$

$$n = V \int_{t_{t-1}}^{t_t} \sin(\theta_{t-1} + \Delta t\, \theta_{t-1})\, dt$$

As the state transition equation is not linear so we need to linearize it.

We know the Jacobian F (state transition matrix is)

$$F = \begin{bmatrix} \frac{\partial X_t}{\partial X_{t-1}} & \frac{\partial X_t}{\partial Y_{t-1}} & \frac{\partial X_t}{\partial V_{t-1}} & \frac{\partial X_t}{\partial \theta_{t-1}} & \frac{\partial X_t}{\partial \dot{\theta}_{t-1}} \\ \frac{\partial Y_t}{\partial X_{t-1}} & \frac{\partial Y_t}{\partial Y_{t-1}} & \frac{\partial Y_t}{\partial V_{t-1}} & \frac{\partial Y_t}{\partial \theta_{t-1}} & \frac{\partial Y_t}{\partial \dot{\theta}_{t-1}} \\ \frac{\partial V_t}{\partial X_{t-1}} & \frac{\partial V_t}{\partial Y_{t-1}} & \frac{\partial V_t}{\partial V_{t-1}} & \frac{\partial V_t}{\partial \theta_{t-1}} & \frac{\partial V_t}{\partial \theta_{t-1}} \\ \frac{\partial \theta_t}{\partial X_{t-1}} & \frac{\partial \theta_t}{\partial Y_{t-1}} & \frac{\partial \theta_t}{\partial V_{t-1}} & \frac{\partial \theta_t}{\partial \theta_{t-1}} & \frac{\partial \theta_t}{\partial \dot{\theta}_{t-1}} \\ \frac{\partial \dot{\theta}_t}{\partial X_{t-1}} & \frac{\partial \ddot{\theta}_t}{\partial Y_{t-1}} & \frac{\partial \dot{\theta}_t}{\partial V_{t-1}} & \frac{\partial \dot{\theta}_t}{\partial \theta_{t-1}} & \frac{\partial \dot{\theta}_t}{\partial \dot{\theta}_{t-1}} \end{bmatrix}$$

$$X_t = X_{t-1} + V_{t-1}\int_{t-1}^{t_1} \cos\left(\theta_{t-1} + (t - t_{t-1})\dot{\theta}_{t-1}\right) dt$$

$$X_t - X_{t-1} = \frac{V_{t-1}}{\dot{\theta}_{t-1}} \sin\left[\theta_{t-1} + (t - t_{t-1})\dot{\theta}_{t-1}\right]\Big|_{t_{t-1}}^{t_t}$$

$$X_t - X_{t-1} = \frac{V_{t-1}}{\dot{\theta}_{t-1}}\left(\sin(\theta_{t-1} + \Delta t\,\dot{\theta}_{t-1}) - \sin\theta_{t-1}\right)$$

Similarly

$$Y_t - Y_{t-1} = -\frac{V_{t-1}}{\dot{\theta}_{t-1}}\left(\cos(\theta_{t-1} + \Delta t\,\dot{\theta}_{t-1}) - \cos\theta_{t-1}\right)$$

$$F = \begin{bmatrix} 1 & 0 & \frac{\sin(\theta_{t-1} + \Delta t\,\dot{\theta}_{t-1}) - \sin\theta_{t-1}}{\dot{\theta}_{t-1}} & \frac{V_{t-1}}{\dot{\theta}_{t-1}}\left(\cos(\theta_{t-1} + \Delta t\,\dot{\theta}_{t-1}) - \cos\theta\right) & \frac{\Delta t\,v}{\dot{\theta}}\cos(\theta + \Delta t\,\dot{\theta}) - \frac{v}{\dot{\theta}^2}\left(\sin(\theta + \Delta t\,\dot{\theta}) - \sin\theta\right) \\ 0 & 1 & \frac{-\cos(\theta + \Delta t\,\dot{\theta}) - \cos\theta}{\dot{\theta}} & \frac{v}{\dot{\theta}}\left(\sin(\theta + \Delta t\,\dot{\theta}) - \sin\theta\right) & \frac{\Delta t\,v}{\dot{\theta}}\left(\sin(\theta + \Delta t\,\dot{\theta})\right) + \frac{v}{\dot{\theta}^2}\left(\cos(\theta + \Delta t\,\dot{\theta}) - \cos\theta\right) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} \frac{1}{2}\Delta t^2 \cos\theta_{t-1}\, a \\ \frac{1}{2}\Delta t^2 \sin\theta_{t-1}\, a \\ \Delta t a \\ \frac{1}{2}\Delta t^2 \alpha \\ \Delta t\, \alpha \end{bmatrix}$$

$$v = \underbrace{\begin{bmatrix} \frac{1}{2}\Delta t^2 \cos\theta & 0 \\ \frac{1}{2}\Delta t^2 \sin\theta & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ 0 & \Delta t \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} a \\ \alpha \end{bmatrix}}_{a}$$

$$Q = G E[aa^T] G^T = G Q_v G^T \qquad Q = E[vv^T]$$

$$Q_v = \begin{bmatrix} \sigma_a^2 & \sigma_a \sigma_\alpha \\ \sigma_\alpha \sigma_a & \sigma_\alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\Delta t^2 \cos\theta & 0 \\ \frac{1}{2}\Delta t^2 \sin\theta & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} \sigma_a^2 & \sigma_a \sigma_a \\ \sigma_\alpha \sigma_a & \sigma_\alpha^2 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\Delta t^2 \cos\theta & \frac{1}{2}\Delta t^2 \sin\theta & \Delta t & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\Delta t^2 & \Delta t \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\Delta t^2\cos\theta & 0 \\ \frac{1}{2}\Delta t^2\sin\theta & 0 \\ \Delta t & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} \frac{1}{2}\Delta t^2\cos\theta\,\sigma_a^2 & \frac{1}{2}\Delta t^2\sin\theta\,\sigma_a^2 & \sigma_a^2\Delta t & 0 & 0 \\ 0 & 0 & 0 & \sigma_\alpha^2\frac{1}{2}\Delta t^2 & \sigma_\alpha^2\Delta t \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{1}{4}\Delta t^2\cos^2\theta\,\sigma_a^2 & \frac{1}{4}\Delta t^4\sin\theta\cos\theta\,\sigma_a^2 & \frac{1}{2}\Delta t^3\cos\theta\,\sigma_a^2 & 0 & 0 \\ \frac{1}{4}\Delta t^4\cos\theta\sin\theta\,\sigma_a^2 & \frac{1}{4}\Delta t^4\sin^2\theta\,\sigma_a^2 & \frac{1}{2}\Delta t^3\sin\theta\,\sigma_a^2 & 0 & 0 \\ \frac{1}{2}\Delta t^3\cos\theta\,\sigma_a^2 & \frac{1}{2}\Delta t^3\sin\theta\,\sigma_a^2 & \sigma_a^2\Delta t^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4}\Delta t^4\sigma_\alpha^2 & \frac{1}{2}\Delta t^3\sigma_\alpha^2 \\ 0 & 0 & 0 & \sigma_\alpha^2\frac{1}{2}\Delta t^3 & \sigma_\alpha^2\Delta t^2 \end{bmatrix}$$

Measurement Model (Lidar)

Now we only get measurement of x-position & y-position from LiDAR and get three different measurement of distance, bearing and r-dot from radar, thus the measurement model will be different for both

As the lidar gives direct measurement of x - position & y - position normal kalman filter can be used (as well relations remains linear)

$$Z = \begin{bmatrix} Z_{x_t} \\ Z_{y_t} \end{bmatrix}$$

$$H_{lidar} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} Z_{x_t} \\ Z_{y_t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \\ V_t \\ \theta_t \\ \dot{\theta}_t \end{bmatrix} + \xi$$

$$\xi \sim \mathcal{N}(0, R)$$

where $R \Rightarrow$ Covariance of the measurement noise

R can be obtained from data

From Lidar data :

$$R_{lidar} = \begin{bmatrix} 0.0228 & 0 \\ 0 & 0.0212 \end{bmatrix}$$

Similarly α & a noise

$$\sigma_a^2 = 0.005072$$

$$\sigma_\alpha^2 = 0.009514$$

Measurement Model - Radar

we know that

$$Z_{radar} = \begin{bmatrix} z_\rho \\ z_\phi \\ z_{\dot\rho} \end{bmatrix}$$

To calculate the Jacobian H_t

$$\begin{pmatrix} \rho \\ \phi \\ \dot\rho \end{pmatrix} = \begin{pmatrix} \sqrt{X_t^2 + Y_t^2} \\ \tan^{-1}\left(Y_t / X_t\right) \\ \dfrac{V\cos\theta\, X_t + V\sin\theta\, Y_t}{\sqrt{X_t^2 + Y_t^2}} \end{pmatrix}$$

$$H_t = \begin{bmatrix} \frac{\partial \rho}{\partial x_t} & \frac{\partial \rho}{\partial y_t} & \frac{\partial \rho}{\partial v_t} & \frac{\partial \rho}{\partial \theta_t} & \frac{\partial \rho}{\partial \dot{\theta}_t} \\ \frac{\partial \phi}{\partial x_t} & \frac{\partial \phi}{\partial y_t} & \frac{\partial \phi}{\partial v_t} & \frac{\partial \phi}{\partial \theta_t} & \frac{\partial \phi}{\partial \dot{\theta}_t} \\ \frac{\partial \dot{\rho}}{\partial x_t} & \frac{\partial \dot{\rho}}{\partial y_t} & \frac{\partial \dot{\rho}}{\partial v_t} & \frac{\partial \dot{\rho}}{\partial \theta_t} & \frac{\partial \dot{\rho}}{\partial \dot{\theta}_t} \end{bmatrix}.$$

$$\frac{\partial \rho}{\partial x_t} = \frac{\partial}{\partial x_t}\left(\sqrt{x_t^2 + y_t^2}\right)$$

$$\frac{\partial \rho}{\partial x_t} = \frac{2x_t}{2\sqrt{x_t^2 + y_t^2}} = \frac{x_t}{\sqrt{x_t^2 + y_t^2}}$$

$$\frac{\partial \rho}{\partial y_t} = \frac{y_t}{\sqrt{x_t^2 + y_t^2}}$$

$$\frac{\partial \phi}{\partial x_t} = \tan^{-1}\left(y_t / x_t\right)$$

$$= \frac{1}{1 + \left(\frac{y_t}{x_t}\right)^2} \times \left(-\frac{y_t}{x_t^2}\right)$$

$$\frac{\partial \phi}{\partial x_t} = \frac{-y_t}{x_t^2 + y_t^2}$$

$$\frac{\partial \phi}{\partial y_t} = \frac{-x_t}{\left(\sqrt{x_t^2 + y_t^2}\right)^2}$$

$$\frac{\partial \dot{\rho}}{\partial x_t} = \frac{\partial}{\partial x_t}\left(\frac{V\cos\theta\, x_t + V\sin\theta\, y_t}{\sqrt{x_t^2 + y_t^2}}\right)$$

$$\frac{\partial \dot{\rho}}{\partial x_t} = \frac{y_t\left(V\cos\theta\, y_t - V\sin\theta\, x_t\right)}{\left(x_t^2 + y_t^2\right)^{3/2}}$$

$$\frac{\partial \dot{\rho}}{\partial y_t} = \frac{x_t\left(x_t V_y - y_t V_x\right)}{\left(\sqrt{x_t^2 + y_t^2}\right)^3}$$

$$\frac{\partial \dot{\rho}}{\partial v} = \frac{x_t\cos\theta + y_t\sin\theta}{\sqrt{x_t^2 + y_t^2}}$$

$$\frac{\partial \dot{\rho}}{\partial \theta} = \frac{V y_t \cos\theta - V x_t \sin\theta}{\sqrt{x_t^2 + y_t^2}}$$

$$\frac{\partial \dot{\rho}}{\partial \dot{\theta}} = 0$$

to convert ρ, ϕ, $\dot{\rho}$ to our state variables

$$\rho = \sqrt{x_t^2 + y_t^2}$$

$$\phi = \tan^{-1}\left(y_t / x_t \right)$$

$$\dot{\rho} = \frac{V\cos\theta \, x_t + V\sin\theta \, Y_t}{\sqrt{x_t^2 + Y_t^2}}$$

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{x_t^2 + Y_t^2} \\ \tan^{-1}\left(y_t / x_t \right) \\ \dfrac{V\cos\theta \, X_t + V\sin\theta \, Y_t}{\sqrt{X_t^2 + y_t^2}} \end{pmatrix}$$

Now the measurement model is

$$\begin{bmatrix} Z_\rho \\ Z_\phi \\ Z_{\dot{\rho}} \end{bmatrix} = \begin{bmatrix} \dfrac{x_t}{\sqrt{x_t^2 + y_t^2}} & \dfrac{y_t}{\sqrt{x_t^2 + Y_t^2}} & 0 & 0 & 0 \\ \dfrac{-y_t}{\left(\sqrt{x_t^2 + Y_t^2}\right)^2} & \dfrac{X_t}{\left(\sqrt{x_t^2 + Y_t^2}\right)^2} & 0 & 0 & 0 \\ \dfrac{Y_t\left(V\cos\theta \, y_t - V\sin\theta \, X_t\right)}{\left(x_t^2 + y_t^2\right)^{3/2}} & \dfrac{X_t\left(X_t V\sin\theta - Y_t V\cos\theta\right)}{\left(X_t^2 + Y_t^2\right)^{3/2}} & a_{33} & a_{34} & a_{35} \end{bmatrix}$$

$$a_{33} = \frac{X_t\cos\theta + Y_t\sin\theta}{\sqrt{X_t^2 + Y_t^2}}, \quad a_{34} = \frac{V y_t\cos\theta - V x_t\sin\theta}{\sqrt{X_t^2 + Y_t^2}}$$

$$a_{35} = 0$$

$$\varepsilon_e \sim \mathcal{N}(0, R_{radar})$$

$$R_{radar} = \begin{bmatrix} 0.0928 & 0 & 0 \\ 0 & 5.58 & 0 \\ 0 & 0 & 0.0831 \end{bmatrix}$$

can be obtained from data.

Fixed values → R_{lidar}, R_{radar}, Q, F, H_{radar}, H_{lidar}

Varying / Updating variables → P, X, K.

Initial Condition

X_0 = first reading of the sensor

$$P_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{bmatrix}$$

As the value of velocity is not known we can initialize its covariance as 1200. And as the reading of X_t & Y_t is received.

USING EXTENDED KALMAN FILTER ALGORITHM, WE HAVE

Algorithm Extended_Kalman_filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

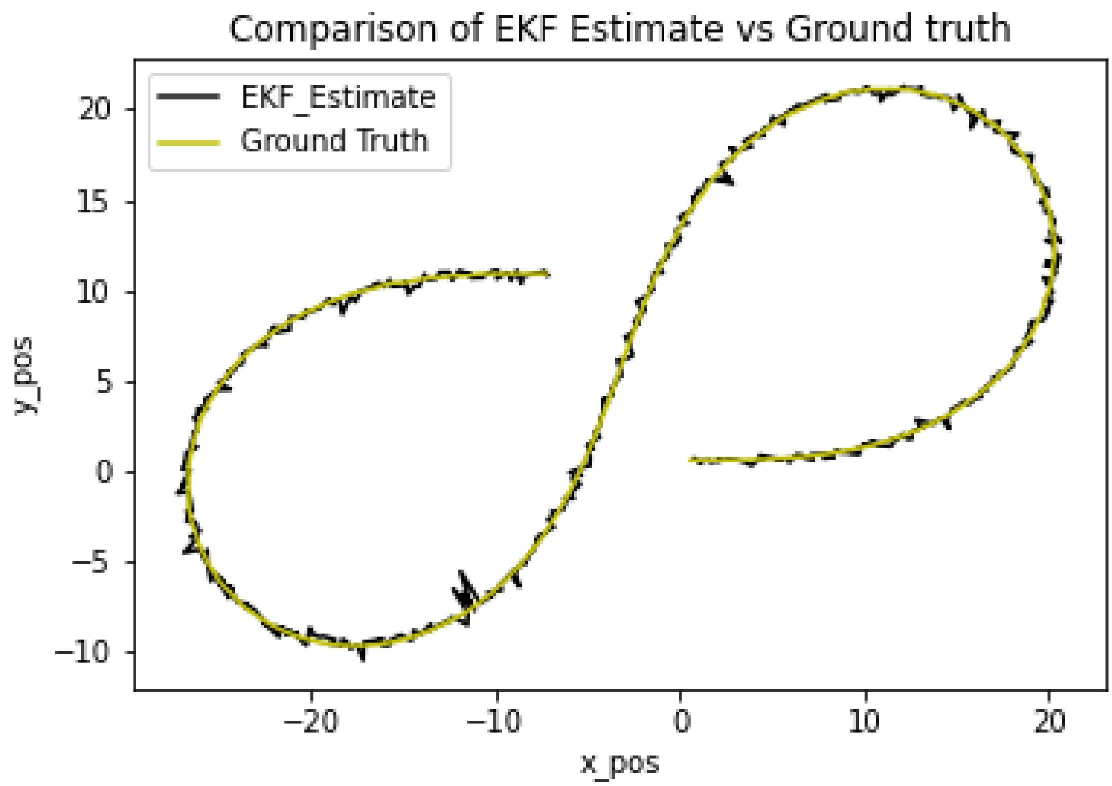
$$\bar{\mu}_t = g(u_t, \mu_{t-1})$$
$$\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^T + R_t$$
$$K_t = \bar{\Sigma}_t\, H_t^T (H_t\, \bar{\Sigma}_t\, H_t^T + Q_t)^{-1}$$
$$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$$
$$\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$$

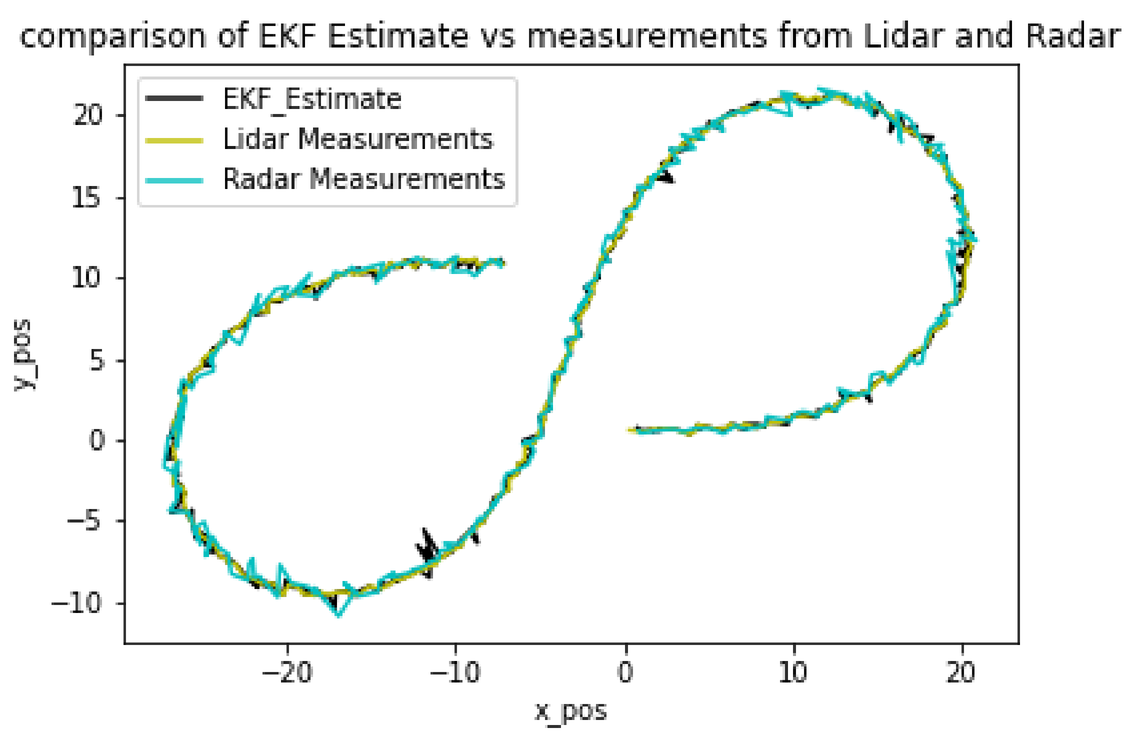
return μ_t, Σ_t

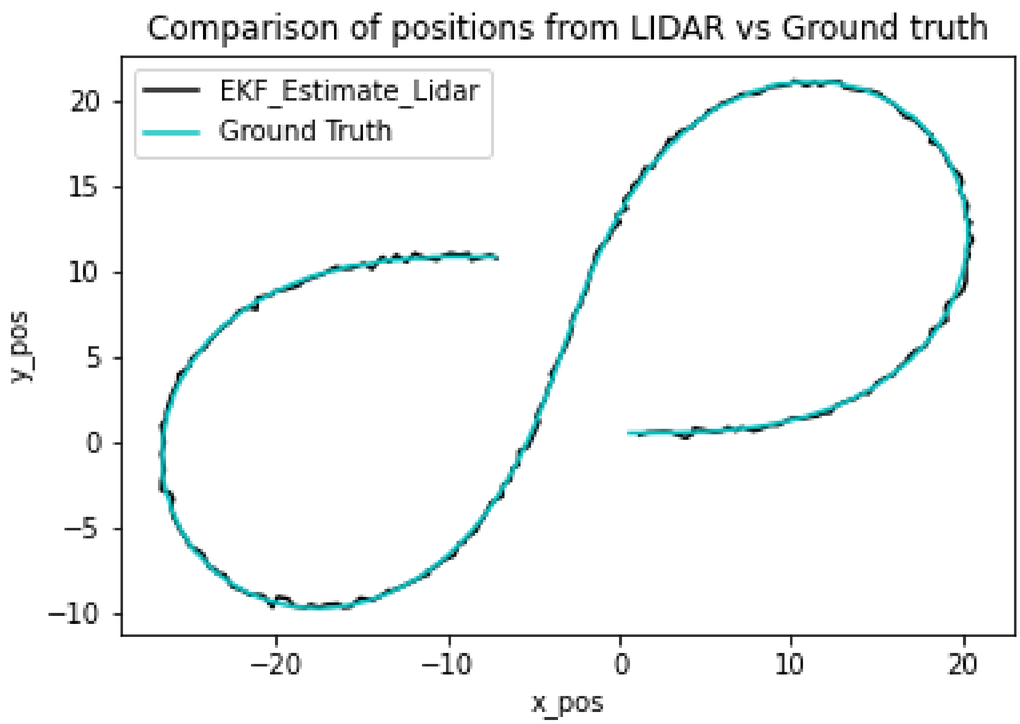
Application of extended Kalman filter is in the python file

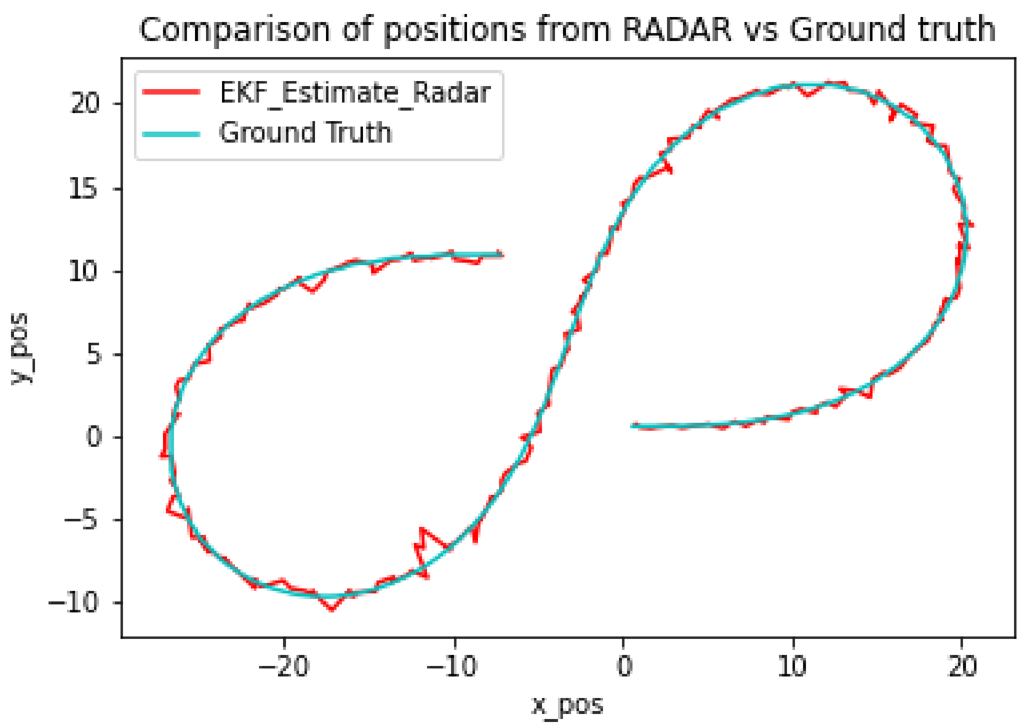
Result:

Following are the graphs of the extended Kalman filter obtained:


Comparison of EKF Estimate vs Ground truth
EKF_Estimate
Ground Truth
y_pos
x_pos
20
15
10
5
0
−5
−10
−20
−10
0
10
20


comparison of EKF Estimate vs measurements from Lidar and Radar
EKF_Estimate
Lidar Measurements
Radar Measurements
y_pos
x_pos
20
15
10
5
0
−5
−10
−20
−10
0
10
20





Output Obtained:

The output obtained after applying the extended Kalman filter is of the form.

	time	x_state	y_state	vx_state	vx_state	yaw_angle_state	yaw_rate_state	sensor_type	x_measured	y_measured	x_ground_truth	y_ground_truth	vx_ground_truth	vy_ground_truth
0	1.47701E+15	0.783673589	0.727186479	15.63249473	0.005408591	0.000345984	0.006928031	R	0.858931228	0.531354068	0.86	0.6	5.2	0.0018
1	1.47701E+15	1.172499163	0.483524244	9.624571863	-2.179512154	-0.222696897	-2.579629952	L	1.17	0.481	1.12	0.6	5.2	0.00539
2	1.47701E+15	1.02329886	0.548566179	6.730807733	-0.094883373	-0.014095944	2.140674157	R	0.971553225	0.398226482	1.38	0.601	5.2	0.0108
3	1.47701E+15	1.648069476	0.625256225	11.00240034	0.806813003	0.073199615	1.906690288	L	1.65	0.625	1.64	0.601	5.2	0.018
4	1.47701E+15	1.678719701	0.484069503	8.537679948	0.311978495	0.036525116	0.628535783	R	1.625073551	0.499135205	1.9	0.602	5.2	0.0269
5	1.47701E+15	2.189542569	0.647394021	9.843280889	1.424375774	0.143707864	1.505049445	L	2.19	0.649	2.16	0.604	5.2	0.0377
6	1.47701E+15	2.025424086	0.550370629	8.059488759	1.069107727	0.131882092	0.654476044	R	1.962792466	0.555918821	2.42	0.606	5.2	0.0502
7	1.47701E+15	2.65831536	0.666723713	11.52386545	1.382068308	0.119360871	0.128963698	L	2.66	0.666	2.68	0.609	5.19	0.0646
8	1.47701E+15	2.94134546	0.646982804	9.675708641	0.22846114	0.023607439	-1.010107191	R	2.91923255	0.646669405	2.94	0.613	5.19	0.0807
9	1.47701E+15	3.012623128	0.637215037	3.62025615	-0.12921991	-0.035678435	-1.111740772	L	3.01	0.637	3.2	0.617	5.19	0.0985
10	1.47701E+15	3.530521978	0.509420126	4.109960888	-0.61507264	-0.148551676	-1.83147582	R	3.557335779	0.483179217	3.46	0.623	5.19	0.118
11	1.47701E+15	3.889221189	0.314073432	6.461250542	-1.900199351	-0.286027446	-2.362978915	L	3.89	0.312	3.72	0.629	5.19	0.14
12	1.47701E+15	4.238984282	0.637522574	6.376666714	-0.278094299	-0.043583616	1.841314318	R	4.202142194	0.69971493	3.98	0.637	5.19	0.163
13	1.47701E+15	4.311473968	0.580284799	2.684677736	0.019097897	0.007113545	1.366122858	L	4.31	0.579	4.24	0.645	5.18	0.188
14	1.47701E+15	4.607182402	0.676960586	3.220166007	0.297978396	0.092272334	1.555991262	R	4.61894745	0.688639567	4.5	0.655	5.18	0.214
15	1.47701E+15	4.353430001	0.894624381	-2.0429621	-0.54391679	0.260203351	2.601979463	L	4.35	0.899	4.75	0.667	5.18	0.243
16	1.47701E+15	5.131686336	0.740933779	-0.366174738	-0.195373655	0.490128385	3.737395973	R	5.207718251	0.66495911	5.01	0.68	5.17	0.273
17	1.47701E+15	5.513654343	0.651869783	2.308899826	1.916065553	0.692685434	3.919502802	L	5.52	0.648	5.27	0.694	5.17	0.304
18	1.47701E+15	5.256492883	0.672859482	2.449536633	3.045803483	0.89347942	3.884291051	R	5.230829281	0.641346267	5.53	0.71	5.16	0.338

Result_EKF.csv is also attached which reflects the required values for all the estimation states.