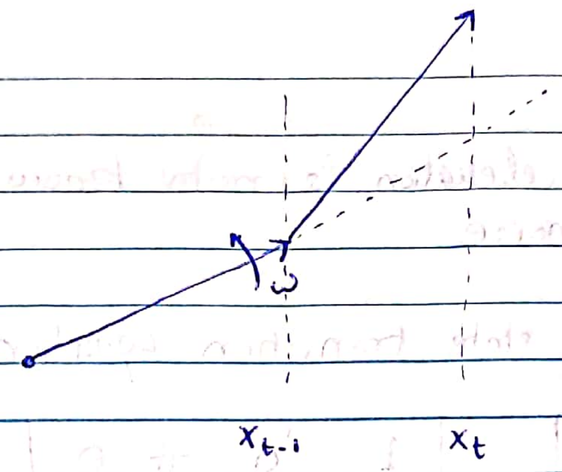
Ans - 2



Let the position x-cordinate at time step t $\rightarrow$ x_t
Let the position y-cordinate at time step t $\rightarrow$ y_t

Similarly V_{x_t} - velocity x-cordinate at time step t
V_{y_t} $\rightarrow$ velocity y-cordinate at time step t

- Prediction Model - (Lidar)/(Radar)

According to the motion equations

$$x_t = x_{t-1} + V_{x_t} dt + \frac{1}{2} a_x dt^2$$

$$y_t = y_{t-1} + V_{y_t} dt + \frac{1}{2} a_y dt^2$$

$$V_{x_t} = V_{x_{t-1}} + a_x dt$$

$$V_{y_t} = V_{y_{t-1}} + a_y dt$$

where dt - is the time step, a_x & a_y is the acceleration of the vehicle which is not known.

As the acceleration is not known we will consider it in noise

so state transition equation is

$$\begin{bmatrix} x_t \\ y_t \\ V_{x_t} \\ V_{y_t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ V_{x_{t-1}} \\ V_{y_{t-1}} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} a_x dt^2 \\ \frac{1}{2} a_y dt^2 \\ a_x dt \\ a_y dt \end{bmatrix}$$

so $F = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\nu = \begin{bmatrix} \frac{1}{2} a_x dt^2 \\ \frac{1}{2} a_y dt^2 \\ a_x dt \\ a_y dt \end{bmatrix}$

to find the covariance 'Q' from ν

$$\nu = \begin{bmatrix} \frac{1}{2} a_x dt^2 \\ \frac{1}{2} a_y dt^2 \\ a_x dt \\ a_y dt \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{2} dt^2 & 0 \\ 0 & \frac{1}{2} dt^2 \\ dt & 0 \\ 0 & dt \end{bmatrix}}_{G} \underbrace{\begin{pmatrix} a_x \\ a_y \end{pmatrix}}_{a}$$

$$Q = G\,E[aa^T]\,G^T = G Q_v G^T \quad (Q = E[\nu\nu^T])$$

$$Q_v = \begin{pmatrix} \sigma_{ax}^2 & \sigma_{axy} \\ \sigma_{axy} & \sigma_{ay}^2 \end{pmatrix}$$

$$Q = G Q_v G^T = \begin{pmatrix} \frac{dt^4}{4}\sigma_{ax}^2 & 0 & \frac{dt^3}{2}\sigma_{ax}^2 & 0 \\ 0 & \frac{dt^4}{4}\sigma_{ay}^2 & 0 & \frac{dt^3}{2}\sigma_{ay}^2 \\ \frac{dt^3}{2}\sigma_{ax}^2 & 0 & dt^2\sigma_{ax}^2 & 0 \\ 0 & \frac{dt^3}{2}\sigma_{ay}^2 & 0 & dt^2\sigma_{ay}^2 \end{pmatrix}$$

Measurement Model (Lidar)

Now we only get measurement of x-position & y-position from Lidar & get three different measurement of distance, bearing & rdot from radar, thus the measurement model will be different for both

As lidar gives direct measurement of x-position & y-position normal kalman filter can be used. (as all relations remains linear)

$$Z = \begin{bmatrix} Z_{x_t} \\ Z_{y_t} \end{bmatrix}$$

$$H_{lidar} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} Z_{x_t} \\ Z_{y_t} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{H_{lidar}} \begin{bmatrix} x_t \\ y_t \\ V_{x_t} \\ V_{y_t} \end{bmatrix} + \xi$$

$$\xi \sim N(0, R)$$

where $R \Rightarrow$ Covariance of the measurement noise.

R can be obtained from data.

From Lidar data:

$$R_{lidar} = \begin{bmatrix} 0.0228 & 0 \\ 0 & 0.0212 \end{bmatrix}$$

Similarly a_x noise

$$\sigma_{ax}^2 = 12.6 =$$
$$\sigma_{ay}^2 = 1.88$$

So $\sigma_{ax}^2 = 12.57$, $\sigma_{ay}^2 = 1.875$
(to be more precise)

Measurement Model - (Radar)

we know that

$$Z_{radar} = \begin{bmatrix} Z_\rho \\ Z_\phi \\ Z_{\dot{\rho}} \end{bmatrix}$$

To Calulate the Jacobian H_t

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{bmatrix} \sqrt{x_t^2 + y_t^2} \\ \tan^{-1}(y_t/x_t) \\ \dfrac{V_{x_t} x_t + V_{y_t} y_t}{\sqrt{x_t^2 + y_t^2}} \end{bmatrix}$$

$$H_t = \begin{pmatrix} \dfrac{\partial \rho}{\partial x_t} & \dfrac{\partial \rho}{\partial y_t} & \dfrac{\partial \rho}{\partial V_x} & \dfrac{\partial \rho}{\partial V_y} \\ \dfrac{\partial \phi}{\partial x_t} & \dfrac{\partial \phi}{\partial y_t} & \dfrac{\partial \phi}{\partial V_x} & \dfrac{\partial \phi}{\partial V_y} \\ \dfrac{\partial \dot{\rho}}{\partial x_t} & \dfrac{\partial \dot{\rho}}{\partial y_t} & \dfrac{\partial \dot{\rho}}{\partial V_x} & \dfrac{\partial \dot{\rho}}{\partial V_y} \end{pmatrix}$$

$$\frac{\partial \rho}{\partial x_t} = \frac{\partial}{\partial x_t}\left(\sqrt{x_t^2 + y_t^2}\right)$$

$$\frac{\partial \rho}{\partial x_t} = \frac{2X_t}{2\sqrt{x_t^2 + y_t^2}} = \frac{X_t}{\sqrt{X_t^2 + Y_t^2}}$$

$$\frac{\partial \rho}{\partial y_t} = \frac{Y_t}{\sqrt{X_t^2 + Y_t^2}}$$

$$\frac{\partial \phi}{\partial x_t} = \tan^{-1}\left(y_t / x_t\right)$$

$$= \frac{1}{1 + \left(\frac{y_t}{x_t}\right)^2} \times \left(\frac{-y_t}{x_t^2}\right)$$

$$\frac{\partial \phi}{\partial x_t} = \frac{-y_t}{x_t^2 + y_t^2}$$

$$\frac{\partial \phi}{\partial y_t} = -\frac{x_t}{\left(\sqrt{x_t^2+y_t^2}\right)^2}$$

$$\frac{\partial \dot{\rho}}{\partial x_t} = \frac{\partial}{\partial x_t}\left(\frac{V_x x_t + V_y . y_t}{\sqrt{x_t^2+y_t^2}}\right)$$

$$\frac{\partial \dot{\rho}}{\partial x_t} = \frac{y_t\left(V_x y_t - V_y x_t\right)}{\left(x_t^2+y_t^2\right)^{3/2}}$$

$$\frac{\partial \dot{\rho}}{\partial y_t} = \frac{\partial}{\partial y_t}\left(\frac{V_x x_t + V_y y_t}{\sqrt{x_t^2+y_t^2}}\right.$$

$$\frac{\partial \dot{\rho}}{d y_t} = \frac{x_t\left(x_t V_y - y_t V_x\right)}{\left(\sqrt{x_t^2+y_t^2}\right)^3}$$

$$\frac{\partial \dot{\rho}}{\partial V_x} = \frac{\partial}{\partial V_x}\left(\frac{V_x x_t + V_y x_t}{\sqrt{x_t^2+y_t^2}}\right.$$

$$\frac{\partial \dot{\rho}}{d V_x} = \frac{x_t}{\sqrt{x_t^2+y_t^2}}$$

$$\frac{\partial \dot{\rho}}{\partial V_y} = \frac{y_t}{\sqrt{x_t^2+y_t^2}}$$

to convert $\rho, \phi, \dot{\rho}$ to our state variables

$$\rho = \sqrt{x_t^2 + y_t^2}$$

$$\phi = \tan^{-1}\left(y_t / x_t \right)$$

$$\dot{\rho} = \frac{V_{x_t} x_t + V_{y_t} y_t}{\sqrt{x_t^2 + y_t^2}}$$

$$\begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{x_t^2 + y_t^2} \\ \tan^{-1}\left(y_t / x_t \right) \\ \dfrac{V_{x_t} x_t + V_{y_t} y_t}{\sqrt{x_t^2 + y_t^2}} \end{pmatrix}$$

Now the measurement model is

$$\begin{bmatrix} Z_\rho \\ Z_\phi \\ Z_{\dot{\rho}} \end{bmatrix} = \underbrace{\begin{bmatrix} \dfrac{x_t}{\sqrt{x_t^2+y_t^2}} & \dfrac{y_t}{\sqrt{x_t^2+y_t^2}} & 0 & 0 \\ \dfrac{-y_t}{\left(\sqrt{x_t^2+y_t^2}\right)^2} & \dfrac{x_t}{\left(\sqrt{x_t^2+y_t^2}\right)^2} & 0 & 0 \\ \dfrac{y_t(V_x y_t - V_y x_t)}{\left(\sqrt{x_t^2+y_t^2}\right)^3} & \dfrac{x_t(x_t V_y - y_t V_x)}{\left(\sqrt{x_t^2+y_t^2}\right)^3} & \dfrac{x_t}{\sqrt{x_t^2+y_t^2}} & \dfrac{y_t}{\sqrt{x_t^2+y_t^2}} \end{bmatrix}}_{H} \begin{bmatrix} x_t \\ y_t \\ V_{x_t} \\ V_{y_t} \end{bmatrix} + \xi_\varphi$$

So

$$H_{radar} = \begin{bmatrix} \dfrac{x_t}{\sqrt{x_t^2+y_t^2}} & \dfrac{y_t}{\sqrt{x_t^2+y_t^2}} & 0 & 0 \\[2ex] \dfrac{-y_t}{(\sqrt{x_t^2+y_t^2})^2} & \dfrac{x_t}{(\sqrt{x_t^2+y_t^2})^2} & 0 & 0 \\[2ex] \dfrac{y_t(v_x y_t - v_y x_t)}{(x_t^2+y_t^2)^{3/2}} & \dfrac{x_t(x_t v_y - y_t v_x)}{(x_t^2+y_t^2)^{3/2}} & \dfrac{x_t}{\sqrt{x_t^2+y_t^2}} & \dfrac{y_t}{\sqrt{x_t^2+y_t^2}} \end{bmatrix}$$

$$\varepsilon_e \sim N(0, R_{radar})$$

$$R_{radar} = \begin{bmatrix} 0.0928 & 0 & 0 \\ 0 & 5.58 & 0 \\ 0 & 0 & 0.0831 \end{bmatrix}$$

can be obtained from data

Fixed values $\rightarrow$ R_{Lidar}, R_{radar}, Q, F, H_{radar}, H_{lidar}

Varying / Updating variables $\rightarrow$ P, X, K.

Initial Condition

$X_0 =$ first reading of the sensor

$$P_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1200 & 0 \\ 0 & 0 & 0 & 1200 \end{bmatrix}$$

As the value of velocity is not known we can initialize its covariance as 1200.

And as the reading of X_t & Y_t is received