

# Degree of anonymity

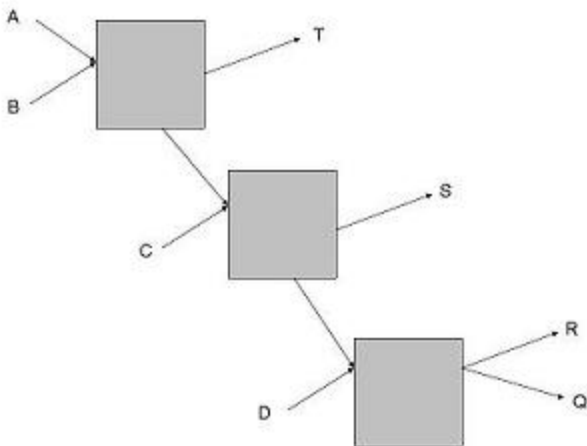
In [anonymity networks](#) (e.g., [Tor](#), [Crowds](#), [Mixmaster](#), I2P, etc.), it is important to be able to measure quantitatively the guarantee that is given to the system. The **degree of anonymity  $d$**  is a device that was proposed at the 2002 Privacy Enhancing Technology (PET) conference. Two papers put forth the idea of using [entropy](#) as the basis for formally measuring anonymity: "Towards an Information Theoretic Metric for Anonymity", and "Towards Measuring Anonymity". The ideas presented are very similar with minor differences in the final definition of  $d$ .

## Background

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Anonymity networks have been developed and many have introduced methods of proving the anonymity guarantees that are possible, originally with simple [Chaum Mixes](#) and Pool Mixes the size of the set of users was seen as the security that the system could provide to a user. This had a number of problems; intuitively if the network is international then it is unlikely that a message that contains only Urdu came from the United States, and vice versa. Information like this and via methods like the [predecessor attack](#) and [intersection attack](#) helps an attacker increase the probability that a user sent the message.

### Example With Pool Mixes



As an example consider the network shown

above, in here  $A, B, C$  and  $D$  are users (senders),  $Q, R, S$ , and  $T$  are servers (receivers), the boxes are mixes, and  $\{A, B\} \in T$ ,  $\{A, B, C\} \in S$  and  $\{A, B, C, D\} \in Q, R$  where  $\in$  denotes the anonymity set. Now as there are **pool mixes** let the cap on the number of incoming messages to wait before sending be **2**; as such if  $A, B$ , or  $C$  is communicating with  $R$  and  $S$  receives a

message then  $\mathcal{S}$  knows that it must have come from  $\mathcal{E}$  (as the links between the mixes can only have 1 message at a time). This is in no way reflected in  $\mathcal{S}$ 's anonymity set, but should be taken into account in the analysis of the network.

## Degree of Anonymity

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The degree of anonymity takes into account the probability associated with each user, it begins by defining the **entropy** of the system (here is where the papers differ slightly but only with notation, we will use the notation from [1] ([https://en.wikipedia.org/wiki/Degree\\_of\\_anonymity#endnote\\_TMA](https://en.wikipedia.org/wiki/Degree_of_anonymity#endnote_TMA))).):

$H(X) := \sum_{i=1}^N \left[ p_i \cdot \lg\left(\frac{1}{p_i}\right) \right]$ , where  $H(X)$  is the entropy of the network,  $N$  is the number of nodes in the network, and  $p_i$  is the probability associated with node  $i$ . Now the maximal **entropy** of a network occurs when there is uniform probability associated with each node  $\left(\frac{1}{N}\right)$  and this yields  $H_M := H(X) \leftarrow \lg(N)$ . The degree of anonymity (now the papers differ slightly in the definition here, [2] ([https://en.wikipedia.org/wiki/Degree\\_of\\_anonymity#endnote\\_TMA](https://en.wikipedia.org/wiki/Degree_of_anonymity#endnote_TMA)) defines a bounded degree where it is compared to  $H_M$  and [3] ([https://en.wikipedia.org/wiki/Degree\\_of\\_anonymity#endnote\\_TIT](https://en.wikipedia.org/wiki/Degree_of_anonymity#endnote_TIT)) gives an unbounded definition—using the entropy directly, we will consider only the bounded case here) is defined as

$d := 1 - \frac{H_M - H(X)}{H_M} = \frac{H(X)}{H_M}$ . Using this anonymity systems can be compared and evaluated using a quantitatively analysis.

## Definition of Attacker

These papers also served to give concise definitions of an attacker:

### Internal/External

an **internal** attacker controls nodes in the network, whereas an **external** can only compromise communication channels between nodes.

### Passive/Active

an **active** attacker can add, remove, and modify any messages, whereas a **passive** attacker can only listen to the messages.

### Local/Global

a **local** attacker has access to only part of the network, whereas a **global** can access the entire network.

## Example $d$

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In the papers there are a number of example calculations of  $d$ ; we will walk through some of them here.

### Crowds

In [Crowds](#) there is a global probability of forwarding ( $p_f$ ), which is the probability a node will forward the message internally instead of routing it to the final destination. Let there be  $C$  corrupt nodes and  $N$  total nodes. In [Crowds](#) the attacker is internal, passive, and local. Trivially

$H_M \leftarrow \lg(N - C)$ , and overall the entropy is

$$H(x) \leftarrow \frac{N - p_f \cdot (N - C - 1)}{N} \cdot \lg \left[ \frac{N}{N - p_f \cdot (N - C - 1)} \right] + p_f \cdot \frac{N - C - 1}{N} \cdot \lg[N/p_f]$$

,  $d$  is this value divided by  $H_M$ .<sup>[4]</sup> ([https://en.wikipedia.org/wiki/Degree\\_of\\_anonymity#endnote\\_TMA](https://en.wikipedia.org/wiki/Degree_of_anonymity#endnote_TMA))

### Onion routing

In [onion routing](#), assuming the attacker can exclude a subset of the nodes from the network, the entropy would easily be  $H(X) \leftarrow \lg(S)$ , where  $S$  is the size of the subset of non-excluded nodes. Under an [attack model](#) where a node can both globally listen to [message passing](#) and is a node on the path this *decreases* to  $H(X) \leftarrow \lg(L)$ , where  $L$  is the length of the onion route (this could be larger or smaller than  $S$ ), as there is no attempt in onion routing to remove the correlation between the incoming and outgoing messages.

### Applications of this metric

In 2004, Diaz, [Sassaman](#), and DeWitte presented an analysis<sup>[5]</sup> ([https://en.wikipedia.org/wiki/Degree\\_of\\_anonymity#endnote\\_CBTMPD](https://en.wikipedia.org/wiki/Degree_of_anonymity#endnote_CBTMPD)) of two anonymous [remailers](#) using the Serjantov and Danezis metric, showing one of them to provide zero anonymity under certain realistic conditions.

## See also

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- [Onion routing](#)
- [Tor \(anonymity network\)](#)

- [Entropy](#)
- [Crowds](#)

## References

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1. ^ See [Towards Measuring Anonymity](http://www.freehaven.net/anonbib/cache/Diaz02.ps.gz) (<http://www.freehaven.net/anonbib/cache/Diaz02.ps.gz>) Claudia Diaz and Stefaan Seys and Joris Claessens and Bart Preneel (April 2002). Roger Dingledine and Paul Syverson (ed.). "Towards measuring anonymity" (<https://web.archive.org/web/20060710023539/http://www.esat.kuleuven.ac.be/~cdiaz/papers/tmAnon.ps.gz>) . *Proceedings of Privacy Enhancing Technologies Workshop (PET 2002)*. Springer-Verlag, LNCS 2482. Archived from [the original](http://www.esat.kuleuven.ac.be/~cdiaz/papers/tmAnon.ps.gz) (<http://www.esat.kuleuven.ac.be/~cdiaz/papers/tmAnon.ps.gz>) on July 10, 2006. Retrieved 2005-11-10.
2. ^ See [Towards an Information Theoretic Metric for Anonymity](https://web.archive.org/web/20040719123728/http://www.cl.cam.ac.uk/~aas23/papers_aas/set.ps) ([https://web.archive.org/web/20040719123728/http://www.cl.cam.ac.uk/~aas23/papers\\_aas/set.ps](https://web.archive.org/web/20040719123728/http://www.cl.cam.ac.uk/~aas23/papers_aas/set.ps)) Andrei Serjantov and George Danezis (April 2002). Roger Dingledine and Paul Syverson (ed.). "Towards an Information Theoretic Metric for Anonymity" ([https://web.archive.org/web/20040719123728/http://www.cl.cam.ac.uk/~aas23/papers\\_aas/set.ps](https://web.archive.org/web/20040719123728/http://www.cl.cam.ac.uk/~aas23/papers_aas/set.ps)) . *Proceedings of Privacy Enhancing Technologies Workshop (PET 2002)*. Springer-Verlag, LNCS 2482. Archived from [the original](http://www.cl.cam.ac.uk/~aas23/papers_aas/set.ps) ([http://www.cl.cam.ac.uk/~aas23/papers\\_aas/set.ps](http://www.cl.cam.ac.uk/~aas23/papers_aas/set.ps)) on July 19, 2004. Retrieved 2005-11-10.
3. ^ See [Comparison Between Two Practical Mix Designs](http://www.cosic.esat.kuleuven.be/publications/article-98.pdf) (<http://www.cosic.esat.kuleuven.be/publications/article-98.pdf>) Claudia Diaz and Len Sassaman and Evelyn Dewitte (September 2004). Dieter Gollmann (ed.). "Comparison Between Two Practical Mix Designs" (<http://www.cosic.esat.kuleuven.be/publications/article-98.pdf>) (PDF). *Proceedings of European Symposium on Research in Computer Security (ESORICS 2004)*. Springer-Verlag, LNCS 3193. Retrieved 2008-06-06.