Computational Statistics & Probability

Lab 3 - Causal Models

Fall 2025

Learning Objectives

By the end of this lab, you will be able to:

- Read and interpret causal DAGs (Directed Acyclic Graphs)
- Use the backdoor criterion to identify which variables to condition on
- Distinguish between total causal effects and direct effects
- Recognize masked relationships between variables
- Make counterfactual predictions using causal models
- Understand the difference between causal inference and prediction
- Connect "small world" models to "large world" questions

Introduction: The Mystery of Urban Foxes

Urban foxes are like street gangs. Groups vary in size between two and eight foxes and each group maintains its own (almost exclusive) urban territory. Some territories are larger than others. The data set `foxes` in the `rethinking` package consists of data for 116 foxes from 30 different urban groups in England.

Here's a puzzle that emerged from studying 116 foxes across 30 territories in England:

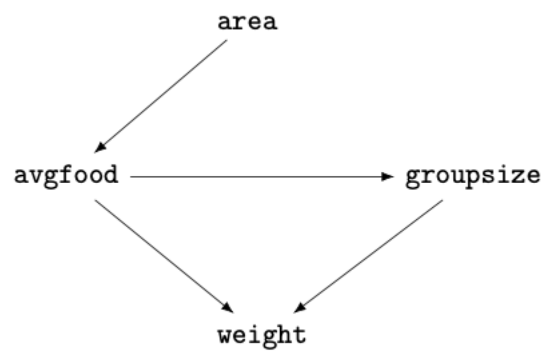
The Puzzle:

- Larger territories have more food
- More food attracts larger groups
- BUT: fox weight shows NO correlation with territory size OR food availability!

How can this be? Today we'll solve this mystery using causal inference.

```r
library(rethinking)
data(foxes)
d <- foxes
```

The variable `area` encodes that some territories are larger than others. The variable `avgfood` encodes that some territories have more food on average than others. And the variable `groupsize` encodes the size of each skulk. Suppose we want to model the `weight` of each fox as a function of the `area` of its territory, the `groupsize` of the skulk it belongs to, and the food (`avgfood`) available in its territory. For our purposes here, assume the following causal DAG:



What the arrows mean:

- `area -> avgfood`: Larger territories have more food
- `avgfood -> groupsize`: More food attracts more foxes
- `avgfood -> weight`: More food makes individual foxes heavier
- `groupsize -> weight`: Larger groups mean competition, making foxes lighter

The causal story: Territory size affects food, which has TWO downstream effects: attracting more group members (bad for individual weight due to competition) and providing more food per fox (good for weight). These effects might cancel out!

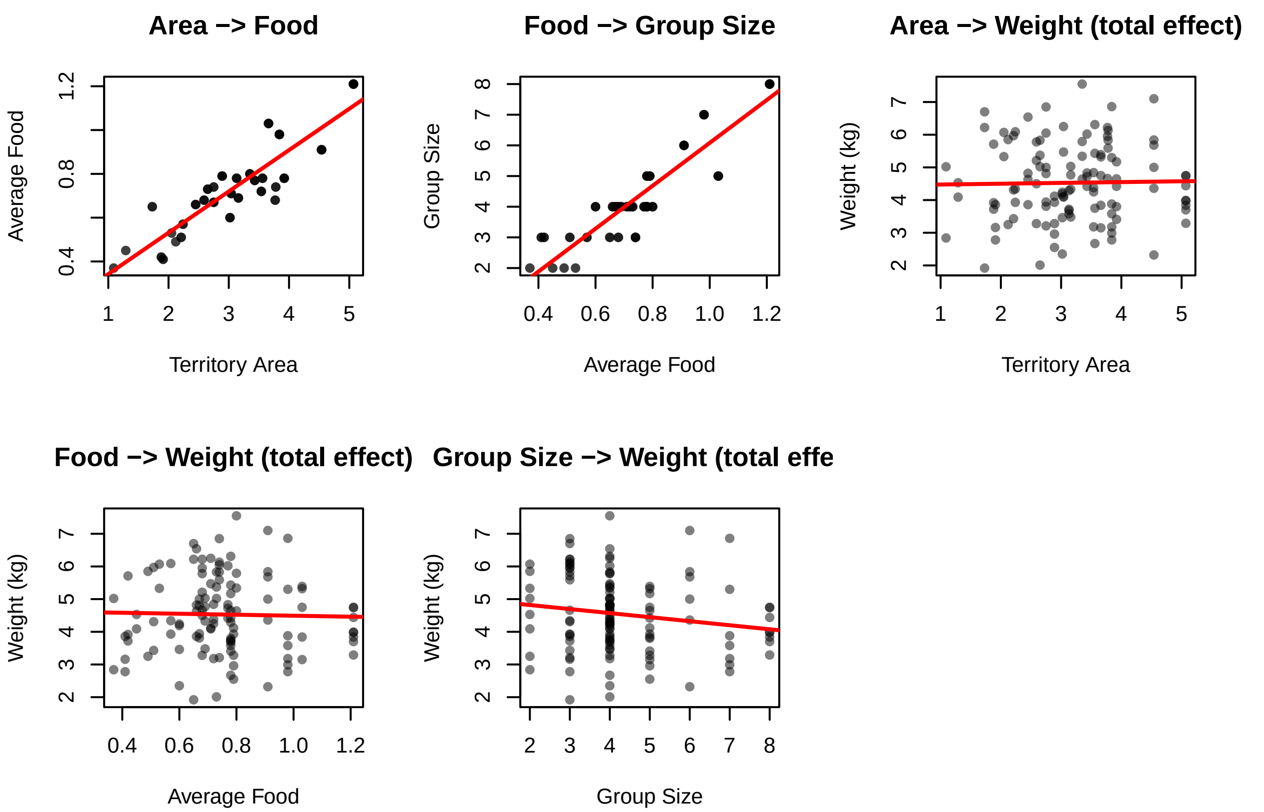
Load and Explore the Data

```
## Number of foxes: 116

## Number of territories: 30

##
## Variable ranges:

##   area: 1.09 5.07

##   avgfood: 0.37 1.21

##   groupsize: 2 8

##   weight: 1.92 7.55 kg
```

Visualize the Puzzle

```r
par(mfrow = c(2, 3))

# Area vs. Food (should be positive)
plot(d$area, d$avgfood,
     xlab = "Territory Area", ylab = "Average Food",
     main = "Area -> Food", pch = 16, col = col.alpha("black", 0.5))
abline(lm(avgfood ~ area, data = d), col = "red", lwd = 2)

# Food vs. Group Size (should be positive)
plot(d$avgfood, d$groupsize,
     xlab = "Average Food", ylab = "Group Size",
     main = "Food -> Group Size", pch = 16, col = col.alpha("black", 0.5))
abline(lm(groupsize ~ avgfood, data = d), col = "red", lwd = 2)

# Area vs. Weight (THE PUZZLE: ~zero!)
```

```r
plot(d$area, d$weight,
     xlab = "Territory Area", ylab = "Weight (kg)",
     main = "Area -> Weight (total effect)", pch = 16, col = col.alpha("black", 0.5))
abline(lm(weight ~ area, data = d), col = "red", lwd = 2)

# Food vs. Weight (ALSO ~zero!)
plot(d$avgfood, d$weight,
     xlab = "Average Food", ylab = "Weight (kg)",
     main = "Food -> Weight (total effect)", pch = 16, col = col.alpha("black", 0.5))
abline(lm(weight ~ avgfood, data = d), col = "red", lwd = 2)

# Group Size vs. Weight (ALSO ~zero!)
plot(d$groupsize, d$weight,
     xlab = "Group Size", ylab = "Weight (kg)",
     main = "Group Size -> Weight (total effect)", pch = 16, col = col.alpha("black", 0.5))
abline(lm(weight ~ groupsize, data = d), col = "red", lwd = 2)

par(mfrow = c(1, 1))
```



The Mystery: Area and Food show clear relationships with each other and with group size, but NOTHING has a visible relationship with weight! We'll solve this mystery using causal inference.

---

1. Does Territory Size Cause Heavier Foxes?

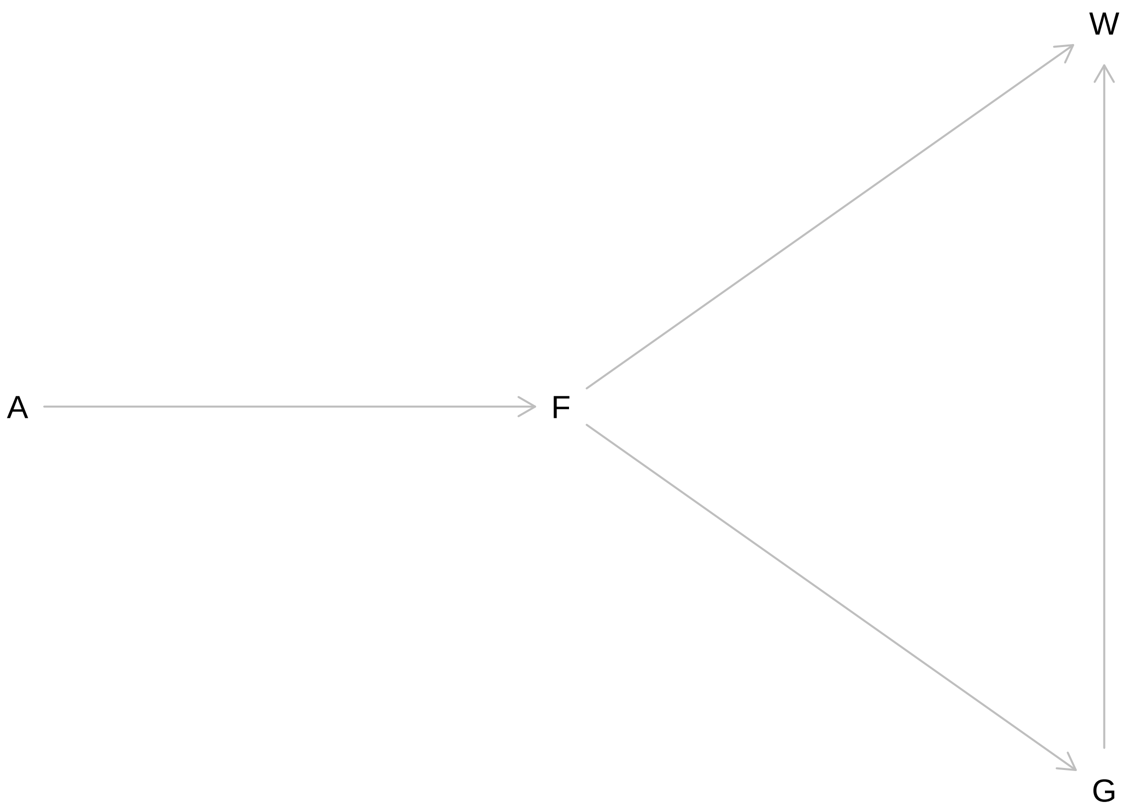
Research Question: If we gave foxes more territory, would they become heavier?

This is asking about the **total causal effect** of `area` on `weight`.

1a. Identify the Causal Path and Adjustment Set

First, let's encode the DAG and use `dagitty` to find what we need to condition on:

```
library(dagitty)

# Define the DAG with coordinates for plotting
dag_foxes <- dagitty("dag{
  A -> F
  F -> G -> W
  F -> W
}")

# Set coordinates for a clean layout
coordinates(dag_foxes) <- list(
  x = c(A = 1, F = 2, G = 3, W = 3),
  y = c(A = 1, F = 1, G = 1.5, W = 0.5)
)

# Plot the DAG
plot(dag_foxes)
```




```
# What do we need to condition on to get A -> W total effect?
adjustmentSets(dag_foxes, exposure = "A", outcome = "W")
```

```
##  {}
```

Question: What does the empty set `{}` mean? Why don't we need to condition on anything?

```
# YOUR ANSWER:
```

```r
# ANSWER:
# The empty set means there are NO backdoor paths from A to W.
# All paths from A to W are causal (forward-flowing through the arrows).
# Therefore, we don't need to condition on anything to estimate the
# total causal effect of A on W.
#
# The causal paths are:
# 1. A -> F -> W (through food)
# 2. A -> F -> G -> W (through food and group size)
```

1b. Prior Predictive Simulation

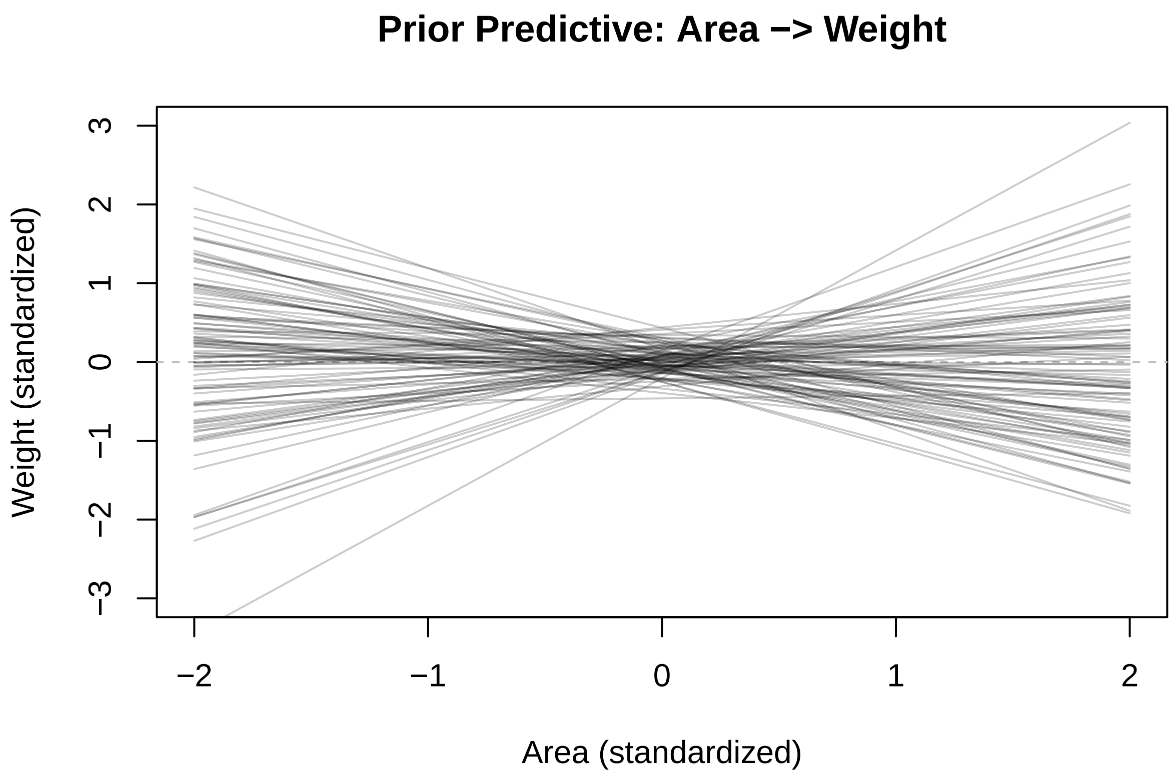
Before fitting, check if our priors make sense. We'll use standardized variables (mean 0, SD 1).

Why standardize? - Makes priors easier to specify (centered at 0) - Makes coefficients comparable - Improves computational efficiency

```r
# Standardize variables
d$A <- standardize(d$area)
d$W <- standardize(d$weight)

# Prior predictive simulation
set.seed(123)
N <- 100

# Sample from priors
# alpha <- # TODO: Normal(0, 0.2)
# beta_A <- # TODO: Normal(0, 0.5)
# sigma <- # TODO: Exponential(1)

# Plot prior predictions
# plot(NULL, xlim = c(-2, 2), ylim = c(-3, 3),
#      xlab = "Area (standardized)", ylab = "Weight (standardized)",
#      main = "Prior Predictive: Area -> Weight")
#
# for (i in 1:N) {
#   curve(alpha[i] + beta_A[i] * x,
#         from = -2, to = 2, add = TRUE,
#         col = col.alpha("black", 0.2))
# }

# Are these priors reasonable?
```

```r
# Standardize variables
d$A <- standardize(d$area)
d$W <- standardize(d$weight)

# Prior predictive simulation
set.seed(123)
N <- 100

# Sample from priors
alpha <- rnorm(N, 0, 0.2)
beta_A <- rnorm(N, 0, 0.5)
sigma <- rexp(N, 1)
```

```r
# Plot prior predictions
plot(NULL, xlim = c(-2, 2), ylim = c(-3, 3),
     xlab = "Area (standardized)", ylab = "Weight (standardized)",
     main = "Prior Predictive: Area -> Weight")

abline(h = 0, lty = 2, col = "gray")

for (i in 1:N) {
  curve(alpha[i] + beta_A[i] * x,
        from = -2, to = 2, add = TRUE,
        col = col.alpha("black", 0.2))
}
```




```r
# INTERPRETATION:
# Most slopes are between -1 and 1 (reasonable for standardized variables)
# Most intercepts near 0 (reasonable - data is centered)
# Some extreme slopes exist, but that's OK with vague priors
# Prior allows both positive and negative relationships
```

1c. Fit the Model

```r
# m1a <- quap(
#   alist(
#     W ~ dnorm(mu, sigma),
#     mu <- a + bA * A,
#     a ~ dnorm(0, 0.2),
#     bA ~ dnorm(0, 0.5),
#     sigma ~ dexp(1)
```

```r
#   ),
#   data = d
# )
#
# precis(m1a)
```

```r
m1a <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    mu <- a + bA * A,
    a ~ dnorm(0, 0.2),
    bA ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data = d
)

precis(m1a)
```

```
##                mean         sd       5.5%     94.5%
## a     1.665819e-07 0.08360976 -0.1336244 0.1336247
## bA    1.883347e-02 0.09089721 -0.1264378 0.1641048
## sigma 9.912818e-01 0.06466904  0.8879282 1.0946354
```

1d. Interpret the Results

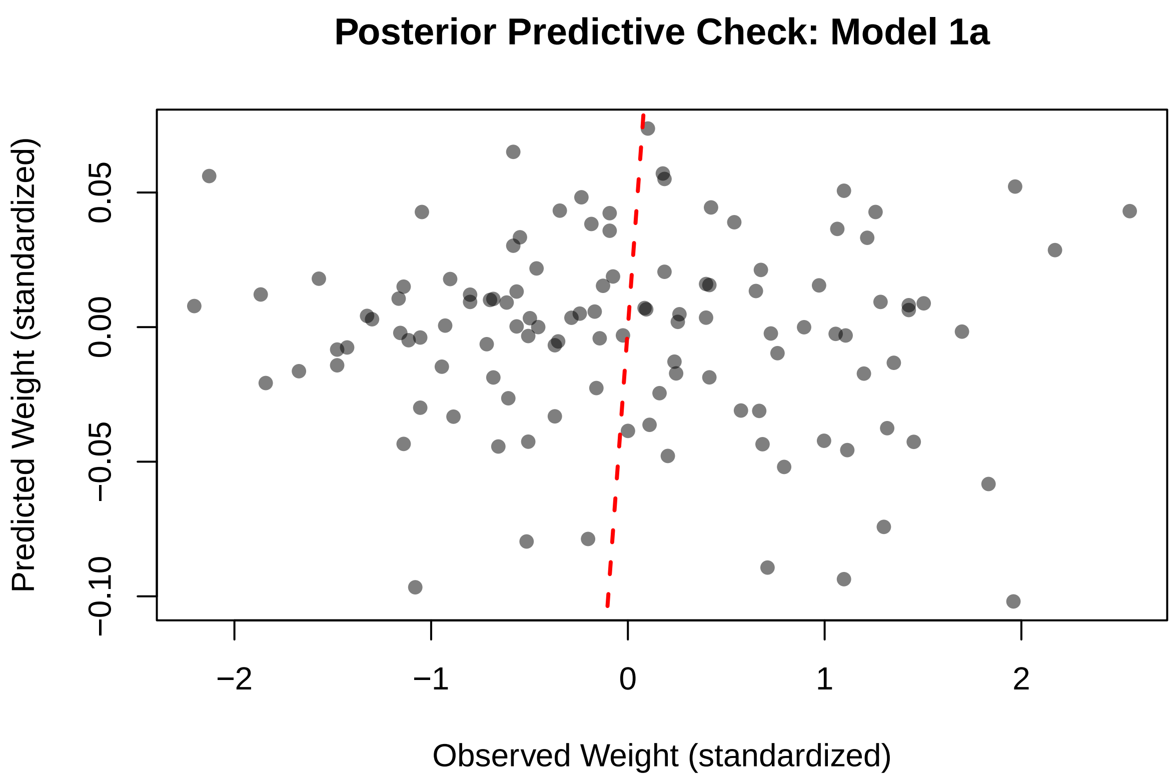
```r
# 1. What is the estimated total causal effect of area on weight?
#
# YOUR ANSWER:

# 2. Is this effect credibly different from zero?
#    (Check if 0 is in the 89% interval)
#
# YOUR ANSWER:

# 3. What does this mean for our research question?
#
# YOUR ANSWER:
```

```r
# ANSWERS:

# 1. Estimated effect: bA approximately 0.019 (essentially zero)
#    In standardized units: 1 SD increase in area -> 0.02 SD increase in weight
#    This is negligible!

# 2. The 89% interval includes 0, so the effect is NOT credibly different from zero
#    We have no evidence that area causes changes in weight

# 3. Research question: "Does more territory make foxes heavier?"
#    Answer: NO - in this data, territory size has no detectable total causal effect
#    on weight. This is surprising! We'll see why when we look at the full story.

# Why might this be?
# The total effect of A on W goes through TWO paths:
```

```r
# A -> F -> W (positive: more food helps)
# A -> F -> G -> W (negative: more food -> more foxes -> more competition)
# These might cancel out!
```

1e. Posterior Predictive Check

```r
# Simulate predictions
weight_pred <- sim(m1a, data = d)
weight_mean <- apply(weight_pred, 2, mean)
weight_PI <- apply(weight_pred, 2, PI, prob = 0.89)

# Observed vs. Predicted
plot(d$W, weight_mean,
     xlab = "Observed Weight (standardized)",
     ylab = "Predicted Weight (standardized)",
     main = "Posterior Predictive Check: Model 1a",
     pch = 16, col = col.alpha("black", 0.5))
abline(0, 1, col = "red", lwd = 2, lty = 2)
```




```r
# Check residuals
residuals <- d$W - weight_mean
cat("Mean residual:", round(mean(residuals), 4), "\n")
```

```
## Mean residual: 0.0024
```

```r
cat("SD residual:", round(sd(residuals), 3), "\n")
```

```
## SD residual: 1.003
```

```
# Model captures very little variation (as expected from near-zero coefficient)
```

---

2. Does Food Availability Cause Heavier Foxes?

Research Question: If we added food to territories, would foxes become heavier?

2a. Identify the Adjustment Set

```
# What do we need to condition on for F -> W total effect?
# adjustmentSets(dag_foxes, exposure = "F", outcome = "W")

# YOUR INTERPRETATION:
```

```
adjustmentSets(dag_foxes, exposure = "F", outcome = "W")
```

```
##  {}
```

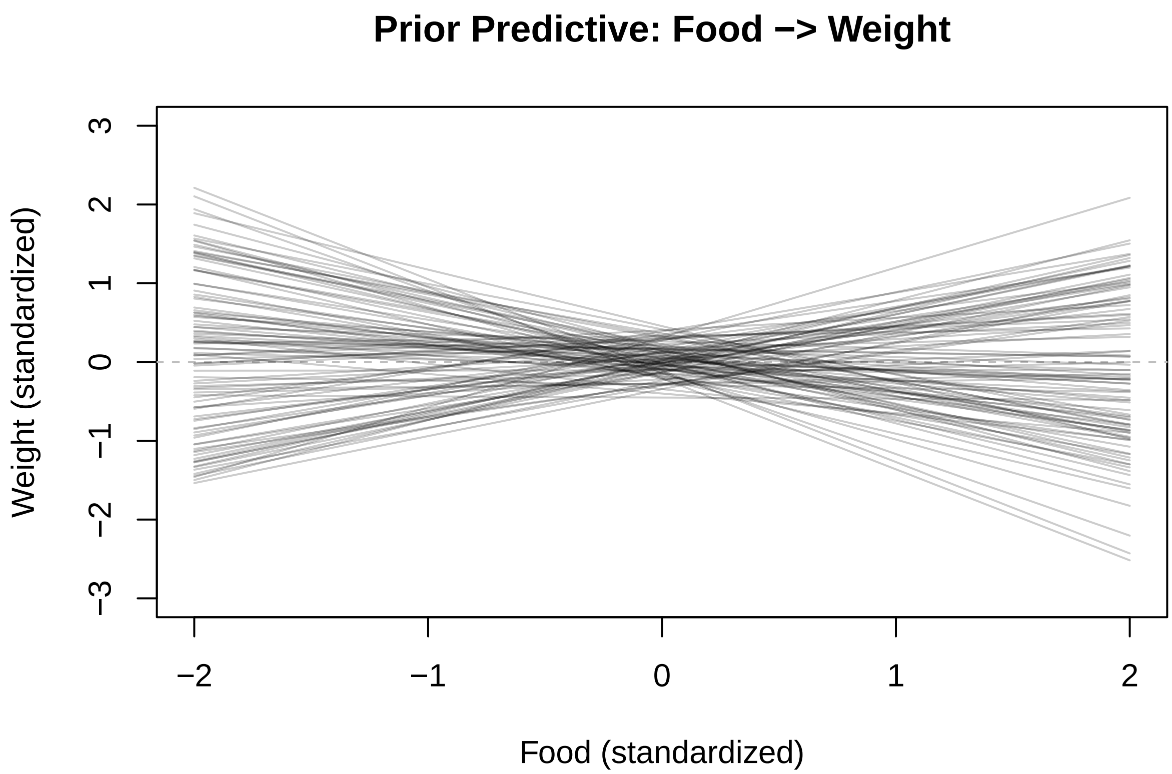
```
# ANSWER: Empty set {}
# No backdoor paths from F to W
# All paths are causal:
# 1. F -> W (direct)
# 2. F -> G -> W (through group size)
# We don't need to condition on anything for the total effect
```

2b. Prior Predictive Simulation

```
# Standardize food
d$F <- standardize(d$avgfood)

# Prior simulation
# set.seed(456)
# N <- 100
# alpha <- # TODO
# beta_F <- # TODO

# Plot
# YOUR CODE HERE
```

```
d$F <- standardize(d$avgfood)

set.seed(456)
N <- 100
alpha <- rnorm(N, 0, 0.2)
beta_F <- rnorm(N, 0, 0.5)

plot(NULL, xlim = c(-2, 2), ylim = c(-3, 3),
     xlab = "Food (standardized)", ylab = "Weight (standardized)",
     main = "Prior Predictive: Food -> Weight")

abline(h = 0, lty = 2, col = "gray")

for (i in 1:N) {
```

```r
  curve(alpha[i] + beta_F[i] * x,
        from = -2, to = 2, add = TRUE,
        col = col.alpha("black", 0.2))
}
```



2c. Fit the Model

```r
# m1b <- quap(
#   alist(
#     W ~ dnorm(mu, sigma),
#     mu <- a + bF * F,
#     a ~ dnorm(0, 0.2),
#     bF ~ dnorm(0, 0.5),
#     sigma ~ dexp(1)
#   ),
#   data = d
# )
#
# precis(m1b)
```

```r
m1b <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    mu <- a + bF * F,
    a ~ dnorm(0, 0.2),
    bF ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data = d
```

```r
)

precis(m1b)
```

```
##                mean         sd       5.5%     94.5%
## a     -1.677778e-09 0.08360017 -0.1336092 0.1336092
## bF    -2.421163e-02 0.09088502 -0.1694634 0.1210402
## sigma  9.911440e-01 0.06465859  0.8878071 1.0944809
```

2d. Interpret the Results

```r
# 1. What is the total causal effect of food on weight?
#
# YOUR ANSWER:

# 2. Compare this to the area effect. What's going on?
#
# YOUR ANSWER:
```

```r
# ANSWERS:

# 1. Effect: bF approximately -0.024 (essentially zero, possibly slightly negative!)
#    Food appears to have NO total causal effect on weight
#    Even more surprising than area!

# 2. THE PUZZLE DEEPENS:
#    - Area -> Food (we saw this in the data)
#    - Food should help foxes gain weight (biologically obvious)
#    - But food shows NO effect on weight!
#
#    Something is canceling out the effect of food.
#    Hypothesis: Food attracts more foxes -> competition -> offsets benefits
#    We'll test this in the next section!

cat("Mystery: Why do area and food have no effect on weight?\n")
```

```
## Mystery: Why do area and food have no effect on weight?
```

```r
cat("Hint: Look at the two causal paths from F to W:\n")
```

```
## Hint: Look at the two causal paths from F to W:
```

```r
cat("  1. F -> W (direct, should be positive)\n")
```

```
##   1. F -> W (direct, should be positive)
```

```r
cat("  2. F -> G -> W (indirect through competition)\n")
```

```
##   2. F -> G -> W (indirect through competition)
```

```r
cat("These might cancel each other out!\n")
```

```
## These might cancel each other out!
```

---

3. Does Group Size Cause Heavier Foxes?

Research Question: If we removed some foxes from a group, would the remaining foxes become heavier?

3a. Identify the Adjustment Set

Critical difference: Group size has a BACKDOOR path to weight!

```
# What do we need to condition on for G -> W?
# adjustmentSets(dag_foxes, exposure = "G", outcome = "W")

# Draw the backdoor path:
#
# YOUR ANSWER:
```

```
adjustmentSets(dag_foxes, exposure = "G", outcome = "W")
```

```
## { F }
```

```
# ANSWER: We must condition on {F}
#
# The backdoor path: G <- F -> W
# Food is a common cause of both group size and weight
# If we don't condition on F, we'll get a confounded estimate
#
# Intuition: Territories with more food have BOTH more foxes AND heavier
# individual foxes. This creates a spurious correlation unless we control for food.

cat("\nCausal paths from G to W:\n")
```

```
##
## Causal paths from G to W:
```

```
cat("  - G -> W (direct effect we want)\n")
```

```
##   - G -> W (direct effect we want)
```

```
cat("  - G <- F -> W (backdoor, must block by conditioning on F)\n")
```

```
##   - G <- F -> W (backdoor, must block by conditioning on F)
```

3b. Prior Predictive Simulation

Now we have TWO predictors: F and G

```
# Standardize group size
d$G <- standardize(d$groupsize)

# Prior simulation with TWO predictors
# set.seed(789)
# N <- 50
# alpha <- # TODO
# beta_F <- # TODO
# beta_G <- # TODO

# Visualize: harder with 2 predictors!
# Show some example planes in 3D or contour plots
```

```r
d$G <- standardize(d$groupsize)

set.seed(789)
N <- 50
alpha <- rnorm(N, 0, 0.2)
beta_F <- rnorm(N, 0, 0.5)
beta_G <- rnorm(N, 0, 0.5)

# With 2 predictors, let's check predictions at a few combinations
F_test <- c(-1, 0, 1)  # Low, medium, high food
G_test <- c(-1, 0, 1)  # Small, medium, large groups

cat("Prior predictions for different (F, G) combinations:\n")
```

```
## Prior predictions for different (F, G) combinations:
```

```r
for (f_val in F_test) {
  for (g_val in G_test) {
    pred <- alpha + beta_F * f_val + beta_G * g_val
    cat(sprintf("  F=%+.0f, G=%+.0f: mean=%.2f, range=[%.2f, %.2f]\n",
                f_val, g_val, mean(pred), min(pred), max(pred)))
  }
}
```

```
##   F=-1, G=-1: mean=-0.05, range=[-2.06, 1.70]
##   F=-1, G=+0: mean=-0.06, range=[-1.27, 1.56]
##   F=-1, G=+1: mean=-0.08, range=[-1.64, 1.43]
##   F=+0, G=-1: mean=-0.01, range=[-1.39, 1.45]
##   F=+0, G=+0: mean=-0.03, range=[-0.62, 0.35]
##   F=+0, G=+1: mean=-0.04, range=[-1.52, 1.66]
##   F=+1, G=-1: mean=0.02, range=[-1.01, 1.79]
##   F=+1, G=+0: mean=0.00, range=[-1.21, 1.08]
##   F=+1, G=+1: mean=-0.01, range=[-1.67, 2.32]
```

```r
# Priors are vague but reasonable (centered on 0, not too extreme)
```

3c. Fit the Model with Adjustment

This is the key model!

```r
# m1c <- quap(
#   alist(
#     W ~ dnorm(mu, sigma),
#     mu <- a + bF * F + bG * G,
#     a ~ dnorm(0, 0.2),
#     bF ~ dnorm(0, 0.5),
#     bG ~ dnorm(0, 0.5),
#     sigma ~ dexp(1)
#   ),
#   data = d
# )
#
# precis(m1c)
```

```r
m1c <- quap(
  alist(
```

```r
    W ~ dnorm(mu, sigma),
    mu <- a + bF * F + bG * G,
    a ~ dnorm(0, 0.2),
    bF ~ dnorm(0, 0.5),
    bG ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data = d
)

precis(m1c)
```

```
##                mean         sd       5.5%      94.5%
## a     -2.178860e-07 0.08013807 -0.1280763  0.1280759
## bF     4.772528e-01 0.17912323  0.1909793  0.7635263
## bG    -5.735257e-01 0.17914173 -0.8598288 -0.2872226
## sigma  9.420440e-01 0.06175256  0.8433515  1.0407365
```

3d. THE BIG REVEAL: Masked Effects!

```r
# Compare all three models:
# precis(m1a)  # Area only
# precis(m1b)  # Food only
# precis(m1c)  # Food + Group size

# What happened? Why are the effects suddenly non-zero?
#
# YOUR INTERPRETATION:
```

```
## ==========================================

## THE MYSTERY SOLVED: MASKED EFFECTS

## ==========================================

## Model 1a (Area only):

##               mean         sd       5.5%     94.5%
## a     1.665819e-07 0.08360976 -0.1336244 0.1336247
## bA    1.883347e-02 0.09089721 -0.1264378 0.1641048
## sigma 9.912818e-01 0.06466904  0.8879282 1.0946354

## Model 1b (Food only):

##                mean         sd       5.5%     94.5%
## a     -1.677778e-09 0.08360017 -0.1336092 0.1336092
## bF    -2.421163e-02 0.09088502 -0.1694634 0.1210402
## sigma  9.911440e-01 0.06465859  0.8878071 1.0944809

## Model 1c (Food + Group Size):

##                mean         sd       5.5%      94.5%
## a     -2.178860e-07 0.08013807 -0.1280763  0.1280759
## bF     4.772528e-01 0.17912323  0.1909793  0.7635263
## bG    -5.735257e-01 0.17914173 -0.8598288 -0.2872226
## sigma  9.420440e-01 0.06175256  0.8433515  1.0407365

## ==========================================
```

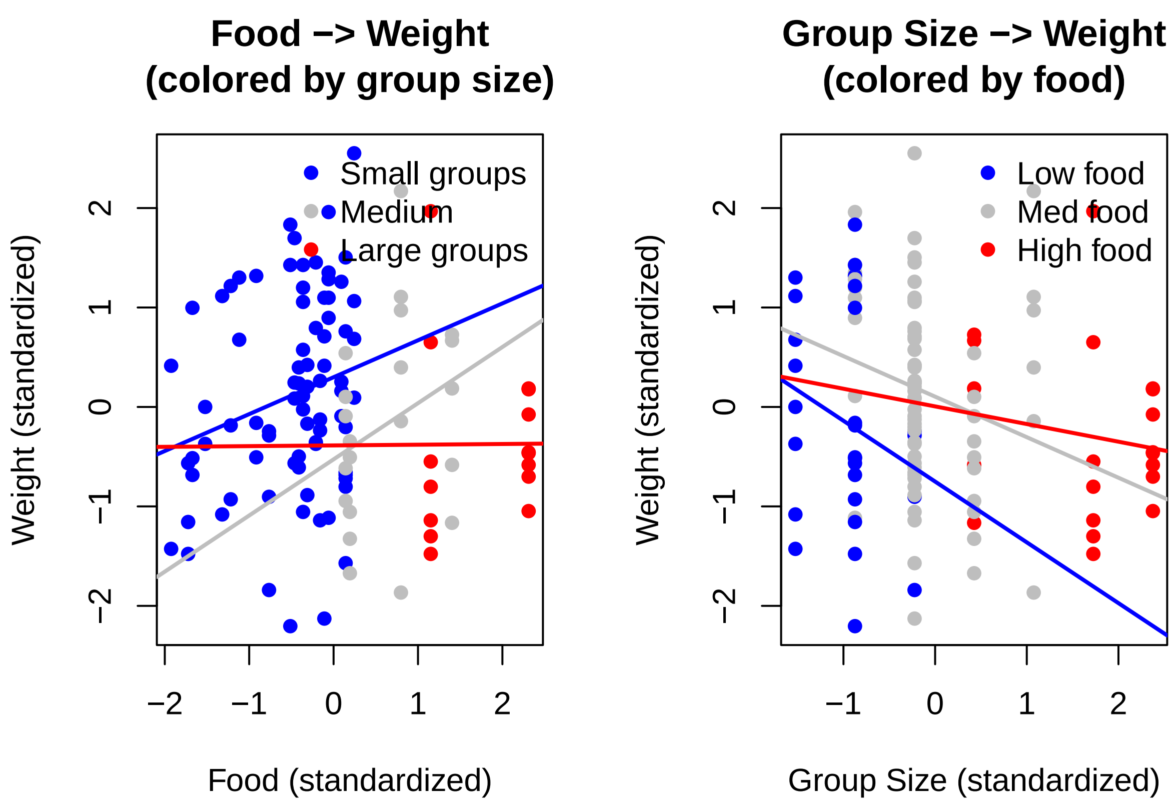
```
## WHAT HAPPENED?
## ============================================
##
## Models 1a & 1b: Effects near ZERO
## Model 1c: STRONG effects for both F and G!
## Food (F):
##   - Unconditional (m1b): approximately 0
##   - Conditional on G (m1c): approximately +0.48 (POSITIVE!)
##   - Interpretation: More food DOES make foxes heavier,
##     but ONLY when controlling for group size
## Group size (G):
##   - Effect (m1c): approximately -0.57 (NEGATIVE!)
##   - Interpretation: Larger groups -> more competition -> lighter foxes
## THE MASKING MECHANISM:
##   More food -> Two effects:
##     1. Direct: F -> W (+0.48) makes foxes HEAVIER
##     2. Indirect: F -> G -> W creates competition
##        F -> G (positive), G -> W (-0.57)
##        Net indirect effect: NEGATIVE
##   These effects CANCEL OUT in the total effect!
## This is why area and food showed no bivariate correlation with weight.
## The effects were there all along - just hidden!
```

3e. Visualize the Masking

Let's see this graphically:

```r
par(mfrow = c(1, 2))

# Left plot: Food vs. Weight, colored by group size
# Categorize group size for coloring
G_cat <- cut(d$G, breaks = 3, labels = c("Small", "Medium", "Large"))
colors <- c("blue", "gray", "red")
col_vec <- colors[as.numeric(G_cat)]

plot(d$F, d$W,
     col = col_vec, pch = 16,
     xlab = "Food (standardized)",
     ylab = "Weight (standardized)",
     main = "Food -> Weight\n(colored by group size)")

# Add regression lines for each group size category
for (i in 1:3) {
  subset_data <- d[G_cat == levels(G_cat)[i], ]
```

```r
  if (nrow(subset_data) > 5) {
    abline(lm(W ~ F, data = subset_data), col = colors[i], lwd = 2)
  }
}

legend("topright", legend = c("Small groups", "Medium", "Large groups"),
       col = colors, pch = 16, bty = "n")

# Right plot: Group size vs. Weight, colored by food
F_cat <- cut(d$F, breaks = 3, labels = c("Low", "Med", "High"))
col_vec2 <- colors[as.numeric(F_cat)]

plot(d$G, d$W,
     col = col_vec2, pch = 16,
     xlab = "Group Size (standardized)",
     ylab = "Weight (standardized)",
     main = "Group Size -> Weight\n(colored by food)")

for (i in 1:3) {
  subset_data <- d[F_cat == levels(F_cat)[i], ]
  if (nrow(subset_data) > 5) {
    abline(lm(W ~ G, data = subset_data), col = colors[i], lwd = 2)
  }
}

legend("topright", legend = c("Low food", "Med food", "High food"),
       col = colors, pch = 16, bty = "n")
```



```r
par(mfrow = c(1, 1))

cat("\nINTERPRETATION:\n")
```

```
##
## INTERPRETATION:
```

```r
cat("Left plot: Within each group size, more food -> heavier foxes (POSITIVE slopes)\n")
```

```
## Left plot: Within each group size, more food -> heavier foxes (POSITIVE slopes)
```

```r
cat("Right plot: Within each food level, larger groups -> lighter foxes (NEGATIVE slopes)\n")
```

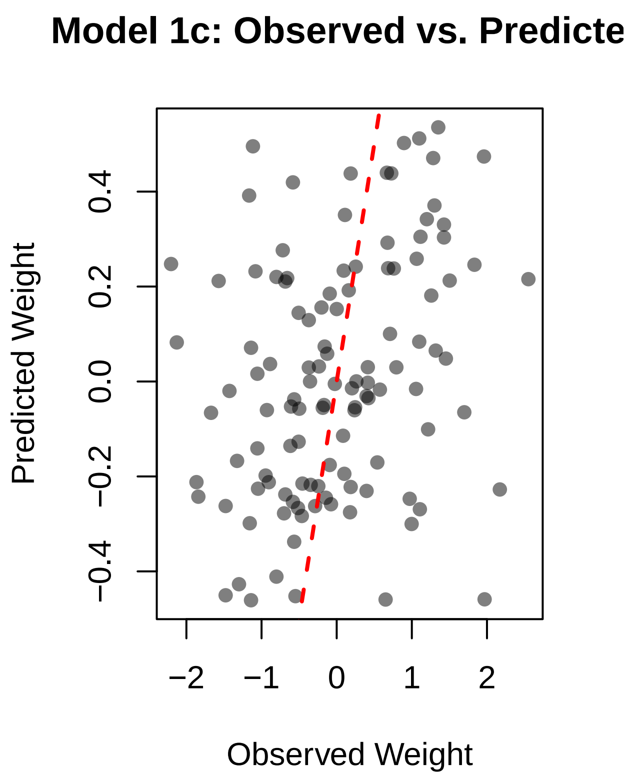
```
## Right plot: Within each food level, larger groups -> lighter foxes (NEGATIVE slopes)
```

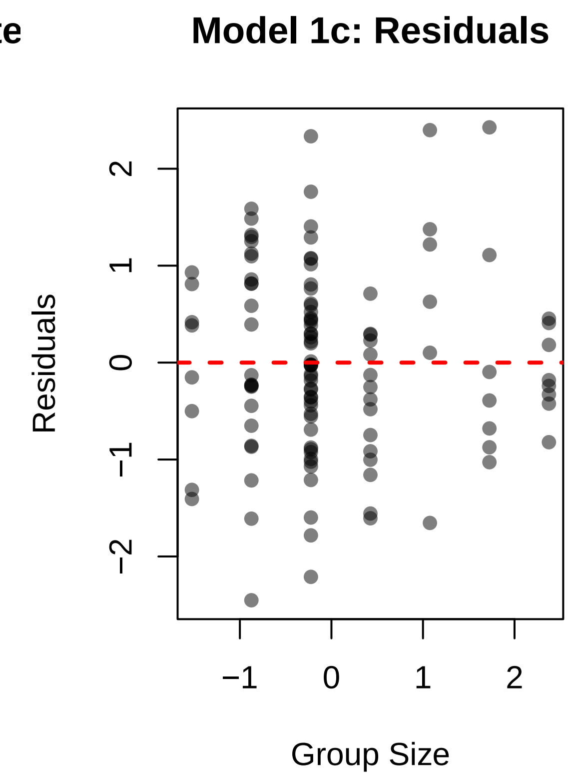
```r
cat("But unconditionally (ignoring colors), both show approximately zero correlation!\n")
```

```
## But unconditionally (ignoring colors), both show approximately zero correlation!
```

3f. Posterior Predictive Check

```r
weight_pred_c <- sim(m1c, data = d)
weight_mean_c <- apply(weight_pred_c, 2, mean)
weight_PI_c <- apply(weight_pred_c, 2, PI, prob = 0.89)

par(mfrow = c(1, 2))

# Observed vs. Predicted
plot(d$W, weight_mean_c,
     xlab = "Observed Weight",
     ylab = "Predicted Weight",
     main = "Model 1c: Observed vs. Predicted",
     pch = 16, col = col.alpha("black", 0.5))
abline(0, 1, col = "red", lwd = 2, lty = 2)

# Residuals
residuals_c <- d$W - weight_mean_c
plot(d$G, residuals_c,
     xlab = "Group Size",
     ylab = "Residuals",
     main = "Model 1c: Residuals",
     pch = 16, col = col.alpha("black", 0.5))
abline(h = 0, col = "red", lwd = 2, lty = 2)
```






```r
par(mfrow = c(1, 1))

cat("Model 1c fits much better than 1a or 1b:\n")
```

```
## Model 1c fits much better than 1a or 1b:
```

```r
cat("   - Residual SD:", round(sd(residuals_c), 3), "\n")
```

```
##    - Residual SD: 0.954
```

```r
cat("   - Compare to m1a:", round(sd(d$W - weight_mean), 3), "\n")
```

```
##    - Compare to m1a: 1.003
```

```r
cat("   - Explains more variation in weight\n")
```

```
##    - Explains more variation in weight
```

---

4. Compare All Three Models

4a. Side-by-Side Coefficient Comparison

```r
library(knitr)

# Extract coefficients
coef_1a <- precis(m1a, depth = 2)
coef_1b <- precis(m1b, depth = 2)
coef_1c <- precis(m1c, depth = 2)

comparison <- data.frame(
```

```r
  Model = c("1a: Area only", "1b: Food only", "1c: Food + Group"),
  bA = c(coef_1a["bA", "mean"], NA, NA),
  bF = c(NA, coef_1b["bF", "mean"], coef_1c["bF", "mean"]),
  bG = c(NA, NA, coef_1c["bG", "mean"]),
  sigma = c(coef_1a["sigma", "mean"],
            coef_1b["sigma", "mean"],
            coef_1c["sigma", "mean"])
)

kable(comparison, digits = 3,
      caption = "Model Comparison: Coefficients",
      col.names = c("Model", "beta_area", "beta_food", "beta_group", "sigma"))
```

Table 1: Model Comparison: Coefficients

Model	beta_area	beta_food	beta_group	sigma
1a: Area only	0.019	NA	NA	0.991
1b: Food only	NA	-0.024	NA	0.991
1c: Food + Group	NA	0.477	-0.574	0.942

```r
cat("
Key observations:
1. Area and Food ALONE show approximately zero effects (models 1a, 1b)
2. Food + Group TOGETHER show strong effects (model 1c)
3. Sigma is lower in model 1c (better fit)
4. This is a textbook example of MASKING
")
```

```
##
## Key observations:
## 1. Area and Food ALONE show approximately zero effects (models 1a, 1b)
## 2. Food + Group TOGETHER show strong effects (model 1c)
## 3. Sigma is lower in model 1c (better fit)
## 4. This is a textbook example of MASKING
```

4b. Model Comparison with WAIC

```r
compare(m1a, m1b, m1c)
```

```
##         WAIC       SE     dWAIC      dSE    pWAIC      weight
## m1c 323.6275 16.15847  0.000000       NA 3.644200 0.986730456
## m1b 333.4093 13.79771  9.781771 6.701192 2.392591 0.007415047
## m1a 333.8819 13.75758 10.254373 6.922185 2.718104 0.005854498
```

```r
cat("\nInterpretation:\n")
```

```
##
## Interpretation:
```

```r
cat("Model 1c has the lowest WAIC (best out-of-sample predictions)\n")
```

```
## Model 1c has the lowest WAIC (best out-of-sample predictions)
```

```r
cat("Models 1a and 1b are essentially equivalent (both bad)\n")
```

```
## Models 1a and 1b are essentially equivalent (both bad)
```

```r
cat("The DAG told us we needed to condition on F when estimating G -> W\n")
```

```
## The DAG told us we needed to condition on F when estimating G -> W
```

```r
cat("Without this, we get confounded estimates!\n")
```

```
## Without this, we get confounded estimates!
```

---

5. Counterfactual Predictions

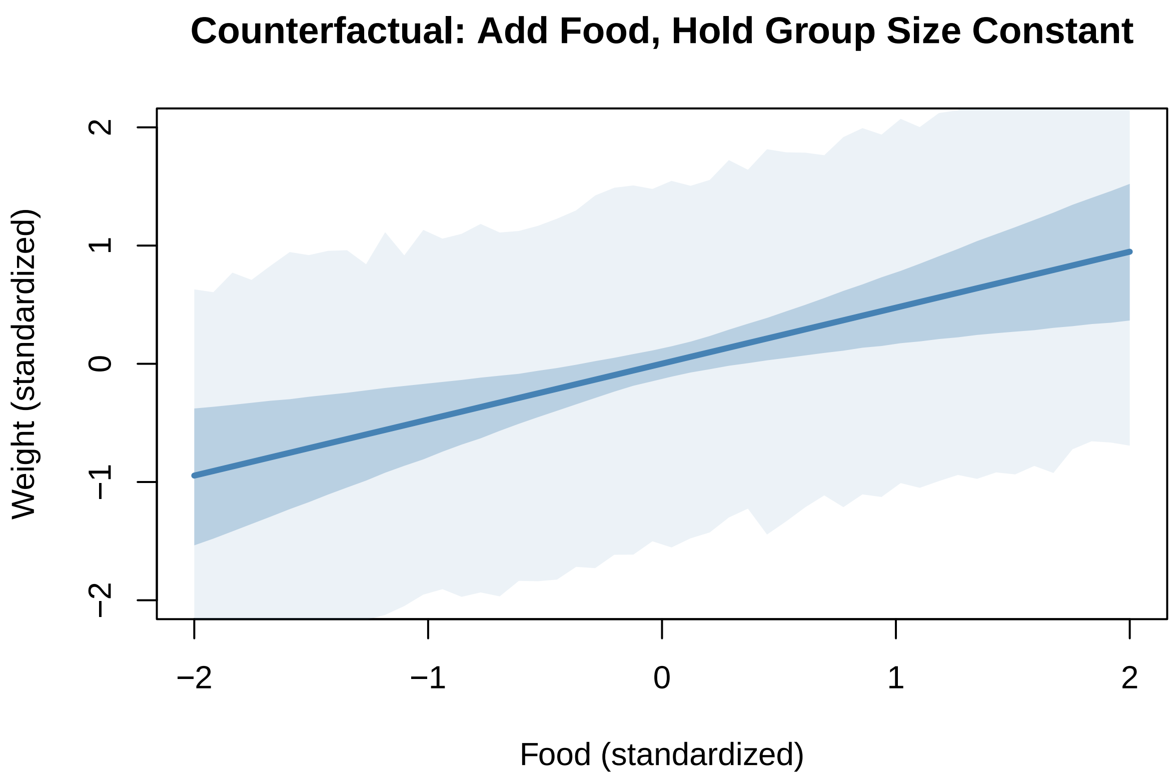
Now let's use our causal model to make predictions about interventions.

5a. Intervention 1: Add Food (Magical Intervention)

Scenario: We magically add food WITHOUT attracting more foxes (do-operator).

```r
# Simulate: Increase F by 1 SD, keep G constant
# What happens to weight?

# new_data <- data.frame(
#   F = seq(-2, 2, length.out = 50),
#   G = 0  # Hold group size at mean
# )

# mu <- # TODO: use link()
# weight_sim <- # TODO: use sim()

# Plot the counterfactual
```

```r
# Simulate: Increase F by 1 SD, keep G constant
new_data <- data.frame(
  F = seq(-2, 2, length.out = 50),
  G = 0  # Hold group size at mean
)

mu <- link(m1c, data = new_data)
mu_mean <- apply(mu, 2, mean)
mu_PI <- apply(mu, 2, PI, prob = 0.89)

weight_sim <- sim(m1c, data = new_data)
weight_PI <- apply(weight_sim, 2, PI, prob = 0.89)

plot(new_data$F, mu_mean, type = "l", lwd = 3, col = "steelblue",
     xlab = "Food (standardized)",
     ylab = "Weight (standardized)",
     main = "Counterfactual: Add Food, Hold Group Size Constant",
     ylim = c(-2, 2))

shade(mu_PI, new_data$F, col = col.alpha("steelblue", 0.3))
shade(weight_PI, new_data$F, col = col.alpha("steelblue", 0.1))
```




```r
cat(
  "\nInterpretation:\n",
  "If we could add food WITHOUT attracting more foxes:\n",
  "  - 1 SD increase in food -> approximately 0.48 SD increase in weight\n",
  "  - This is substantial!\n",
  "  - Effect: bF = +0.48\n\n",
  "But in reality, adding food WILL attract more foxes...\n"
)
```

```
##
## Interpretation:
##  If we could add food WITHOUT attracting more foxes:
##    - 1 SD increase in food -> approximately 0.48 SD increase in weight
##    - This is substantial!
##    - Effect: bF = +0.48
##
##  But in reality, adding food WILL attract more foxes...
```

5b. Intervention 2: Add Food (Realistic Scenario)

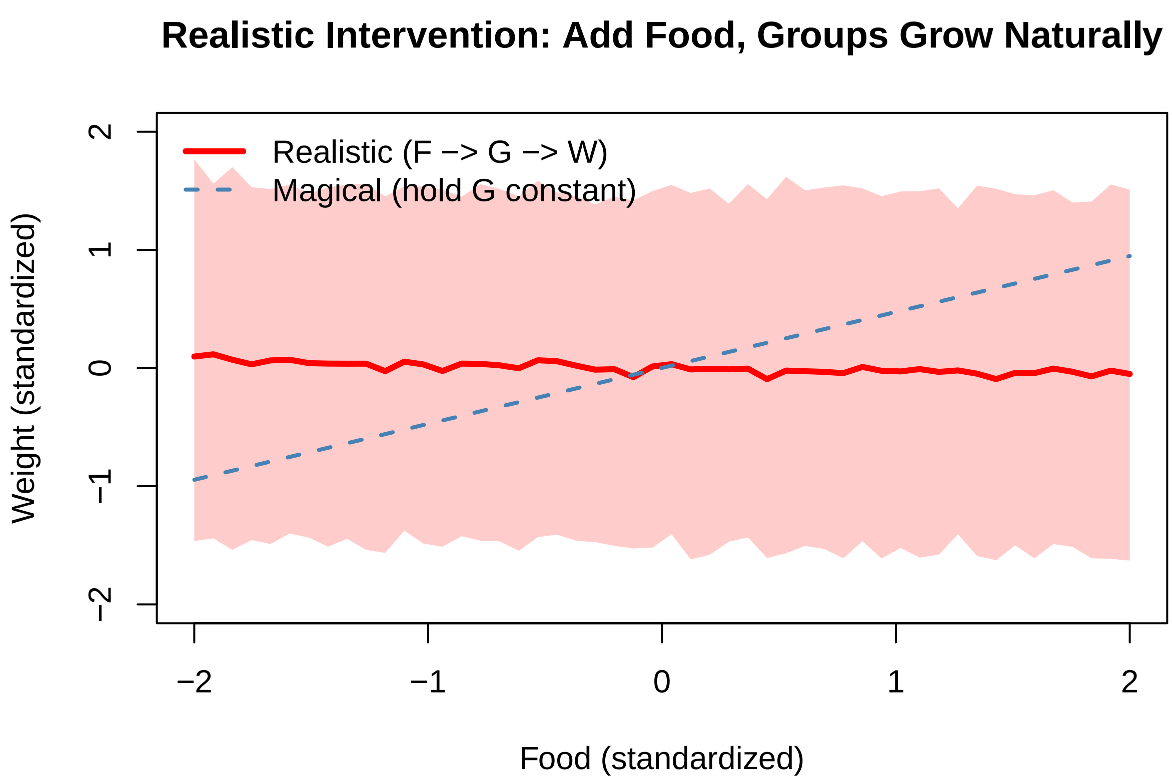
Scenario: Add food, and group size increases naturally.

```r
# Model the F -> G relationship first
m_FG <- quap(
  alist(
    G ~ dnorm(mu, sigma),
    mu <- a + bF * F,
    a ~ dnorm(0, 0.2),
    bF ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
```

```r
  ),
  data = d
)

precis(m_FG)
```

```
##                mean         sd        5.5%      94.5%
## a     -5.779296e-08 0.03916850 -0.06259888 0.06259876
## bF     8.957172e-01 0.03999379  0.83179943 0.95963504
## sigma  4.301880e-01 0.02816943  0.38516785 0.47520824
```

```r
# Now simulate: increase F, let G change naturally
F_seq <- seq(-2, 2, length.out = 50)

# Predict G given F
G_pred <- sim(m_FG, data = data.frame(F = F_seq))
G_mean <- apply(G_pred, 2, mean)

# Now predict W given both F and G
new_data_realistic <- data.frame(F = F_seq, G = G_mean)
weight_realistic <- sim(m1c, data = new_data_realistic)
weight_realistic_mean <- apply(weight_realistic, 2, mean)
weight_realistic_PI <- apply(weight_realistic, 2, PI, prob = 0.89)

plot(F_seq, weight_realistic_mean, type = "l", lwd = 3, col = "red",
     xlab = "Food (standardized)",
     ylab = "Weight (standardized)",
     main = "Realistic Intervention: Add Food, Groups Grow Naturally",
     ylim = c(-2, 2))

shade(weight_realistic_PI, F_seq, col = col.alpha("red", 0.2))

# Compare to magical intervention
lines(new_data$F, mu_mean, lwd = 2, col = "steelblue", lty = 2)

legend("topleft",
       legend = c("Realistic (F -> G -> W)", "Magical (hold G constant)"),
       col = c("red", "steelblue"),
       lwd = c(3, 2), lty = c(1, 2), bty = "n")
```




```r
cat(
  "\nINTERPRETATION:\n",
  "Realistic scenario: Adding food has LITTLE total effect\n",
  "  - Direct effect (F -> W): +0.48\n",
  "  - Indirect effect (F -> G -> W): approximately -0.5\n",
  "  - Net effect: approximately 0.0 (nearly zero!)\n",
  "This explains why we saw no effect in models 1a and 1b!\n",
  "The direct and indirect effects cancel out.\n"
)
```

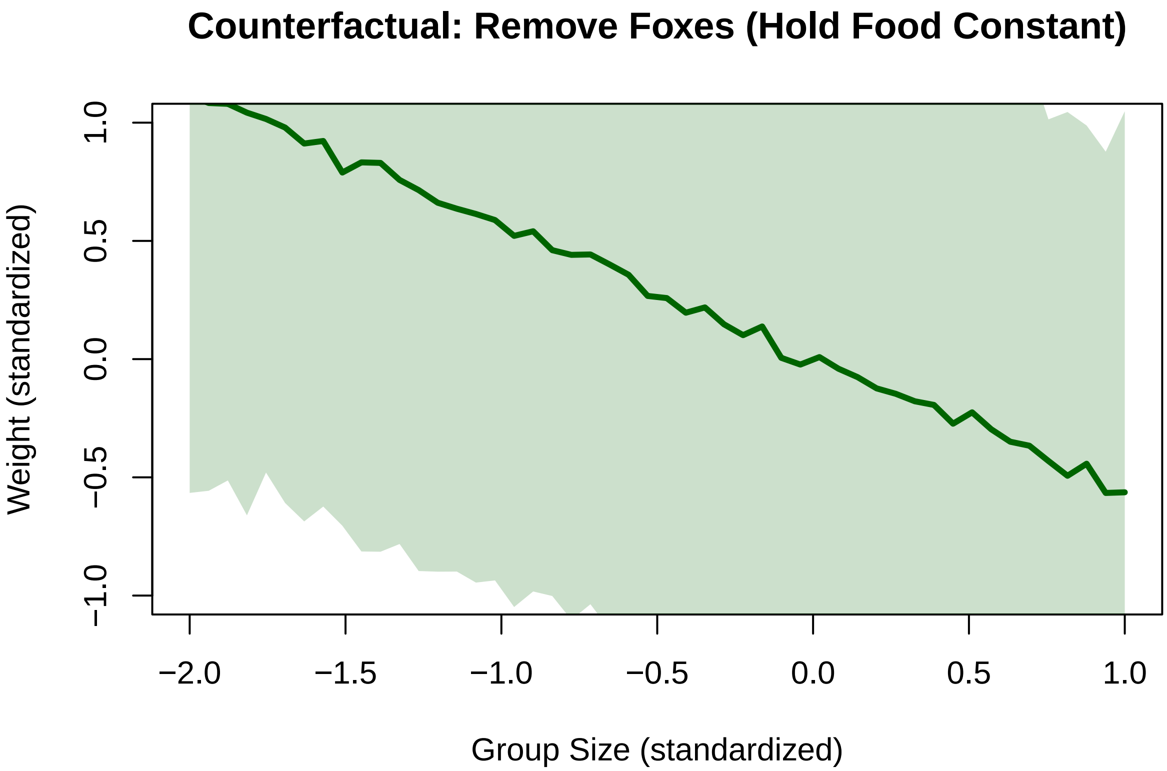
```
##
## INTERPRETATION:
##  Realistic scenario: Adding food has LITTLE total effect
##    - Direct effect (F -> W): +0.48
##    - Indirect effect (F -> G -> W): approximately -0.5
##    - Net effect: approximately 0.0 (nearly zero!)
##  This explains why we saw no effect in models 1a and 1b!
##  The direct and indirect effects cancel out.
```

5c. Intervention 3: Reduce Group Size

Scenario: Remove foxes from a territory.

```r
# What if we removed foxes (reduced G) without changing F?
new_data_remove <- data.frame(
  F = 0,  # Average food
  G = seq(1, -2, length.out = 50)  # Reduce group size
)

weight_remove <- sim(m1c, data = new_data_remove)
```

```r
weight_remove_mean <- apply(weight_remove, 2, mean)
weight_remove_PI <- apply(weight_remove, 2, PI, prob = 0.89)

plot(new_data_remove$G, weight_remove_mean,
     type = "l", lwd = 3, col = "darkgreen",
     xlab = "Group Size (standardized)",
     ylab = "Weight (standardized)",
     main = "Counterfactual: Remove Foxes (Hold Food Constant)",
     ylim = c(-1, 1))

shade(weight_remove_PI, new_data_remove$G, col = col.alpha("darkgreen", 0.2))
```




```r
cat("
Interpretation:
Removing foxes from a territory:
  - Reducing G by 1 SD -> approximately 0.57 SD increase in weight
  - This works because we're reducing competition
  - Effect: bG = -0.57

POLICY IMPLICATION:
If you want heavier foxes, don't add food - manage group sizes!
")
```

```
##
## Interpretation:
## Removing foxes from a territory:
##   - Reducing G by 1 SD -> approximately 0.57 SD increase in weight
##   - This works because we're reducing competition
##   - Effect: bG = -0.57
##
```

```
## POLICY IMPLICATION:
## If you want heavier foxes, don't add food - manage group sizes!
```

---

6. Small World vs. Large World

6a. What Our Model Assumes

Our model assumes the DAG is correct:

```
     area
       |
       v
    avgfood -> groupsize
       |          |
       v          v
       ->  weight <-
```

Assumptions: 1. No unmeasured confounders 2. Linear relationships 3. No interactions 4. Food and group size are the ONLY causes of weight variation 5. No measurement error 6. Relationships are stable across all territories

Question: Which assumptions are most likely to be violated?

```r
# YOUR ANSWER:
```

```r
# LIKELY VIOLATIONS:

# 1. Unmeasured confounders:
#    - Sex (males likely heavier)
#    - Age (adults heavier than juveniles)
#    - Season (weight varies seasonally)
#    - Health status
#    - Individual variation in metabolism

# 2. No interactions:
#    - In Assignment 2, you likely found an F x G interaction!
#    - The effect of food might DEPEND on group size
#    - Small groups: lots of food per fox
#    - Large groups: food spread thin even with more total food

# 3. Linear relationships:
#    - Threshold effects? (minimum food needed to survive)
#    - Diminishing returns? (extra food helps less when already well-fed)

# 4. Causal structure:
#    - Could weight CAUSE group size? (Weak foxes expelled?)
#    - Could there be feedback loops?

cat("\nThis is the SMALL WORLD (our model) vs. LARGE WORLD (reality) distinction\n")
```

```
##
## This is the SMALL WORLD (our model) vs. LARGE WORLD (reality) distinction
```

```r
cat("Our model is useful but incomplete!\n")
```

```
## Our model is useful but incomplete!
```

6b. Connection to Assignment 2

In Assignment 2, you likely found evidence for an interaction between food and group size.

What does this mean for our DAG?

```
# How would you modify the DAG to include an interaction?
# What does an interaction mean causally?
#
# YOUR ANSWER:
```

```
cat("
INTERACTION EFFECTS AND DAGS
==============================

In Assignment 2, you probably found that:
  - Food helps MORE when group size is small
  - Food helps LESS (or not at all) when group size is large

Causal interpretation:
  - Small groups: Each fox gets more food -> more weight gain
  - Large groups: Food is diluted -> little weight gain
  - This is an interaction: effect of F DEPENDS on G

How to include in DAG?
  - Interactions are tricky in DAGs
  - One approach: Add a node for F x G
  - Better: Recognize DAGs show qualitative structure
  - Statistical models add quantitative details (like interactions)

The lesson:
  - DAGs guide WHICH variables to include
  - They don't specify EXACT functional forms
  - That's why we check residuals and compare models!
")
```

```
##
## INTERACTION EFFECTS AND DAGS
## ==============================
##
## In Assignment 2, you probably found that:
##   - Food helps MORE when group size is small
##   - Food helps LESS (or not at all) when group size is large
##
## Causal interpretation:
##   - Small groups: Each fox gets more food -> more weight gain
##   - Large groups: Food is diluted -> little weight gain
##   - This is an interaction: effect of F DEPENDS on G
##
## How to include in DAG?
##   - Interactions are tricky in DAGs
##   - One approach: Add a node for F x G
##   - Better: Recognize DAGs show qualitative structure
##   - Statistical models add quantitative details (like interactions)
##
## The lesson:
```

```
##   - DAGs guide WHICH variables to include
##   - They don't specify EXACT functional forms
##   - That's why we check residuals and compare models!
```

---

7. Reflection Questions

a) Causation vs. Prediction

```r
# Consider these three goals:
# 1. Predict fox weight given measurements (area, food, group size)
# 2. Understand what causes fox weight variation
# 3. Decide how to make foxes heavier (policy intervention)
#
# Which model (m1a, m1b, or m1c) is best for each goal? Why?
#
# YOUR ANSWER:
```

```r
cat("
ANSWERS:

Goal 1: PREDICTION (just forecast weight)
  Best model: m1c (lowest WAIC, best out-of-sample)
  Could also use all three variables: area + food + group size
  For prediction, we don't care about causation
  We just want the best forecasts

Goal 2: UNDERSTAND CAUSES
  Best model: m1c (food + group size)
  This reveals the masked effects
  Shows that food helps and competition hurts
  The DAG guides which variables to condition on

Goal 3: POLICY/INTERVENTION
  Best model: m1c (for counterfactuals)
  But must think carefully about interventions:
    - Adding food: realistic effect is approximately zero (attracts foxes)
    - Reducing group size: would increase weight
  Causal model lets us simulate interventions
  This is the VALUE of causal inference!

KEY LESSON:
  - Prediction: Include all useful variables
  - Causation: Follow the DAG
  - These can suggest different models!
")
```

```
##
## ANSWERS:
##
## Goal 1: PREDICTION (just forecast weight)
##   Best model: m1c (lowest WAIC, best out-of-sample)
##   Could also use all three variables: area + food + group size
##   For prediction, we don't care about causation
```

```
##   We just want the best forecasts
##
## Goal 2: UNDERSTAND CAUSES
##   Best model: m1c (food + group size)
##   This reveals the masked effects
##   Shows that food helps and competition hurts
##   The DAG guides which variables to condition on
##
## Goal 3: POLICY/INTERVENTION
##   Best model: m1c (for counterfactuals)
##   But must think carefully about interventions:
##     - Adding food: realistic effect is approximately zero (attracts foxes)
##     - Reducing group size: would increase weight
##   Causal model lets us simulate interventions
##   This is the VALUE of causal inference!
##
## KEY LESSON:
##   - Prediction: Include all useful variables
##   - Causation: Follow the DAG
##   - These can suggest different models!
```

b) Masked Effects in Other Domains

Can you think of other situations where effects might cancel out?

```
# Examples from: economics, medicine, education, etc.
#
# YOUR ANSWER:
```

```
cat(
  "EXAMPLES OF MASKED EFFECTS:\n\n",
  "1. EDUCATION:\n",
  "   Class size -> Test scores\n",
  "   - Small classes: more attention (positive)\n",
  "   - Small classes often used for struggling students (selection)\n",
  "   - Total effect may appear zero or negative!\n\n",
  "2. MEDICINE:\n",
  "   Exercise -> Health outcomes\n",
  "   - Direct effect: positive\n",
  "   - But sick people exercise more (try to improve health)\n",
  "   - Could mask the positive effect\n\n",
  "3. ECONOMICS:\n",
  "   Interest rates -> Investment\n",
  "   - Low rates encourage borrowing (positive)\n",
  "   - But low rates set during recessions (negative)\n",
  "   - Correlation may be zero or opposite\n\n",
  "4. NUTRITION:\n",
  "   Calories -> Weight\n",
  "   - More calories -> heavier (direct)\n",
  "   - But people who eat more may exercise more\n",
  "   - Total effect smaller than direct effect\n\n",
  "LESSON: Bivariate correlations can be VERY misleading!\n",
  "Always think about confounders and mediators.\n"
)
```

```
## EXAMPLES OF MASKED EFFECTS:
##
##  1. EDUCATION:
##     Class size -> Test scores
##     - Small classes: more attention (positive)
##     - Small classes often used for struggling students (selection)
##     - Total effect may appear zero or negative!
##
##  2. MEDICINE:
##     Exercise -> Health outcomes
##     - Direct effect: positive
##     - But sick people exercise more (try to improve health)
##     - Could mask the positive effect
##
##  3. ECONOMICS:
##     Interest rates -> Investment
##     - Low rates encourage borrowing (positive)
##     - But low rates set during recessions (negative)
##     - Correlation may be zero or opposite
##
##  4. NUTRITION:
##     Calories -> Weight
##     - More calories -> heavier (direct)
##     - But people who eat more may exercise more
##     - Total effect smaller than direct effect
##
##  LESSON: Bivariate correlations can be VERY misleading!
##  Always think about confounders and mediators.
```

---

8. Summary

What We Learned

1. **Causal DAGs** guide which variables to condition on
    - Use backdoor criterion to identify adjustment sets
    - No backdoor -> no adjustment needed for total effect
    - Backdoor present -> must condition on blocking set
2. **Masked Effects** occur when:
    - One variable has multiple causal paths to outcome
    - Paths have opposite signs
    - Total effect cancels out
    - Conditioning reveals hidden relationships
3. **Counterfactual Reasoning** requires causal models:
    - "Do-operator" is not the same as observing
    - Can simulate interventions
    - Guides policy decisions
4. **Small World vs. Large World**:
    - Models are useful but incomplete
    - Always check assumptions
    - Use posterior predictive checks
    - Consider unmeasured confounders

The Fox Story

Mystery: Why doesn't food or territory affect fox weight?

Solution: Masking! - More territory -> more food -> two effects: - Direct: heavier foxes (F -> W) - Indirect: attracts more foxes -> competition -> lighter foxes (F -> G -> W) - These cancel out in the total effect

Evidence: When we condition on group size, both effects appear: - Food: +0.48 (positive) - Group size: -0.57 (negative)

Lesson: Correlation is not causation, and LACK of correlation doesn't mean no causation!

Connection to Course Themes

- **Bayesian workflow**: Prior predictive -> Fit -> Posterior predictive
- **Causal inference**: Goes beyond prediction to understanding and intervention
- **Model comparison**: WAIC helps choose between models
- **Uncertainty**: Always present, always quantified
- **Generative thinking**: Build models from causes to effects

---