

Problem Set 2

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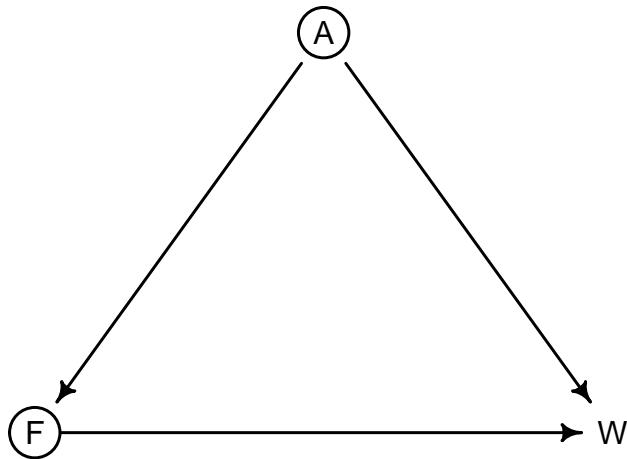
2025-11-16

1 Multiple Regression & Causal Models

The `foxes` dataset contains data on urban fox populations.

```
# First, load the foxes dataset
data(foxes)
d <- foxes
# You must set random seed to 390
rseed <- 390
set.seed(rseed)
```

Consider the following hypothesized causal relationship between **territory size** and **body weight** in foxes.



where A , F and W represent random variables `area` (territory size), `avgfood`, and `weight`, respectively.

If this DAG correctly describes the causal relationships, it makes specific predictions about what we should observe in the data. Your task is to test whether the observed patterns match these predictions.

- Territory size (A) has a **direct** effect on weight (W) : $A \rightarrow W$
- Food availability (F) has a **direct** effect on weight (W) : $F \rightarrow W$
- Territory size (A) has an **indirect** effect on weight (W) through food (F) : $A \rightarrow F \rightarrow W$

1.1 Standardize the Values

```
d$A <- standardize(d$area)
d$F <- standardize(d$avgfood)
d$W <- standardize(d$weight)
```

1.2 Part a

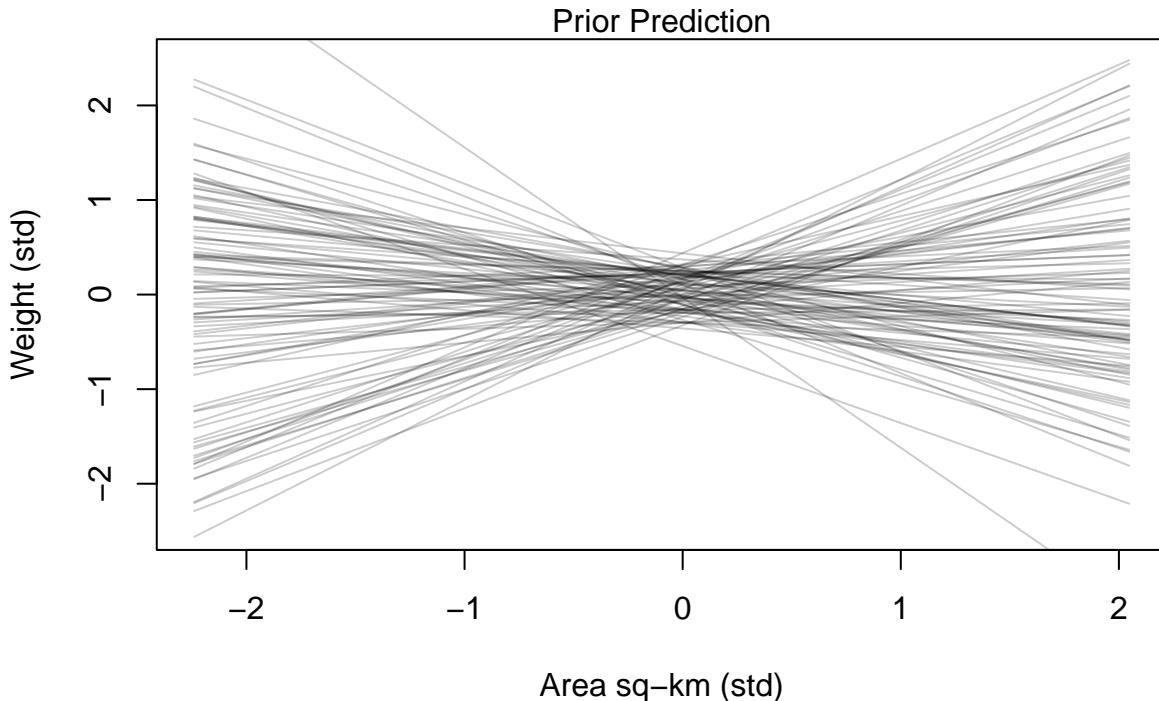
a) According to the DAG, territory size effects weight through two paths:

- Direct path: $A \rightarrow W$
- Indirect path: $A \rightarrow F \rightarrow W$

If we regress weight on territory size without including food, the coefficient should capture both pathways, the “total association” between A and W . Construct a linear regression (`m1a`) using `quap`. Urban foxes in this population have an average weight of 5kg. Use prior predictive simulation to assess the implications of your priors. Standardize the prediction variable.

1.2.1 Prior Predictive Simulation

```
N <- 100
# As our data is standardized, we would expect our intercept, a, to be very
# close to 0
a <- rnorm(N, 0, 0.2)
# b represents the rate of change between our observed and unobserved variables.
# A value of 1 implies that for every one sd of change in our observed var.
# there is 1 sd of change in our unobserved variables. We will choose a sd of
# 0.5 since that means we expect most of these slopes to be in -1 < b < 1
b <- rnorm(N, 0, 0.5)
```



1.2.2 Linear Regression

```
mA <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    # data is standardized so no need to subtract xbar
    mu <- a + b*A,
    # Priors from earlier
```

```

    a ~ dnorm(0, 0.2),
    b ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
),
data=d
)

##           mean        sd      5.5%     94.5%
## a     -3.353656e-07 0.08360863 -0.1336231 0.1336224
## b      1.883325e-02 0.09089577 -0.1264357 0.1641022
## sigma 9.912655e-01 0.06466638  0.8879161 1.0946148

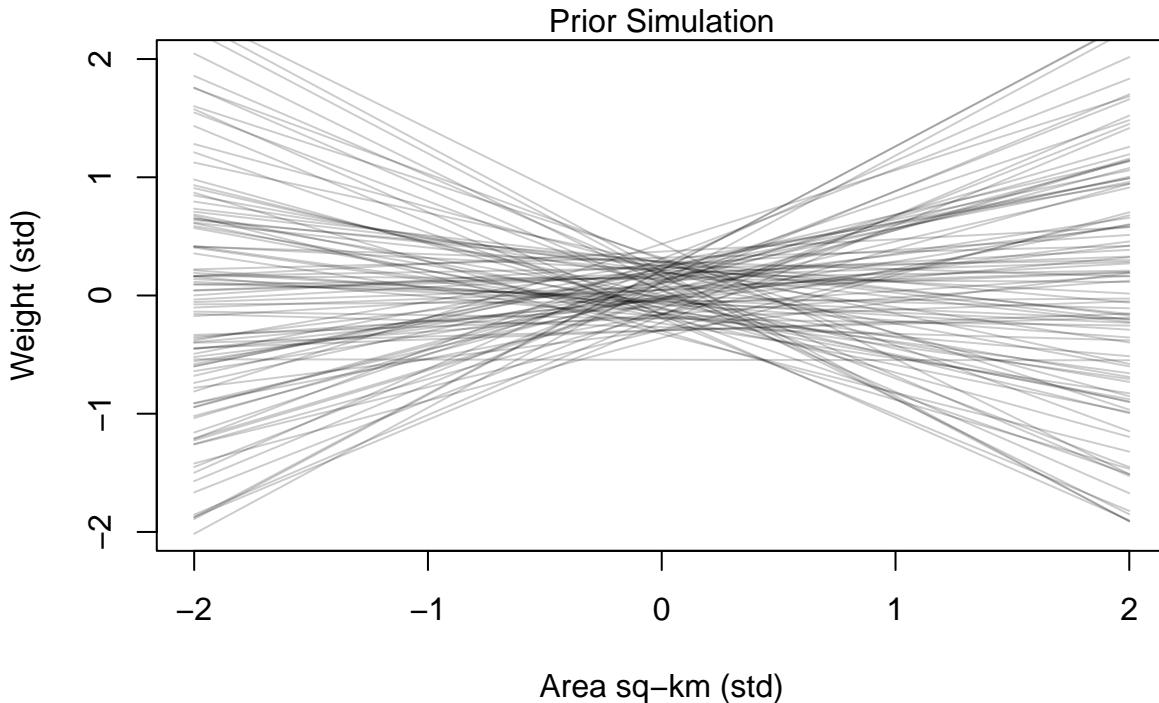
```

1.2.3 Simulate the Priors

```

set.seed(rseed)
prior <- extract.prior(mA)
mu <- link(mA, post=prior, data=list(A=c(-2, 2)))
plot(NULL,
      xlim=c(-2, 2), ylim=c(-2, 2),
      xlab="Area sq-km (std)", ylab="Weight (std")
)
mtext("Prior Simulation")
for (i in 1:N) lines(c(-2, 2), mu[i,], col=col.alpha("black", 0.2))

```



1.2.4 Posterior Predictions

```

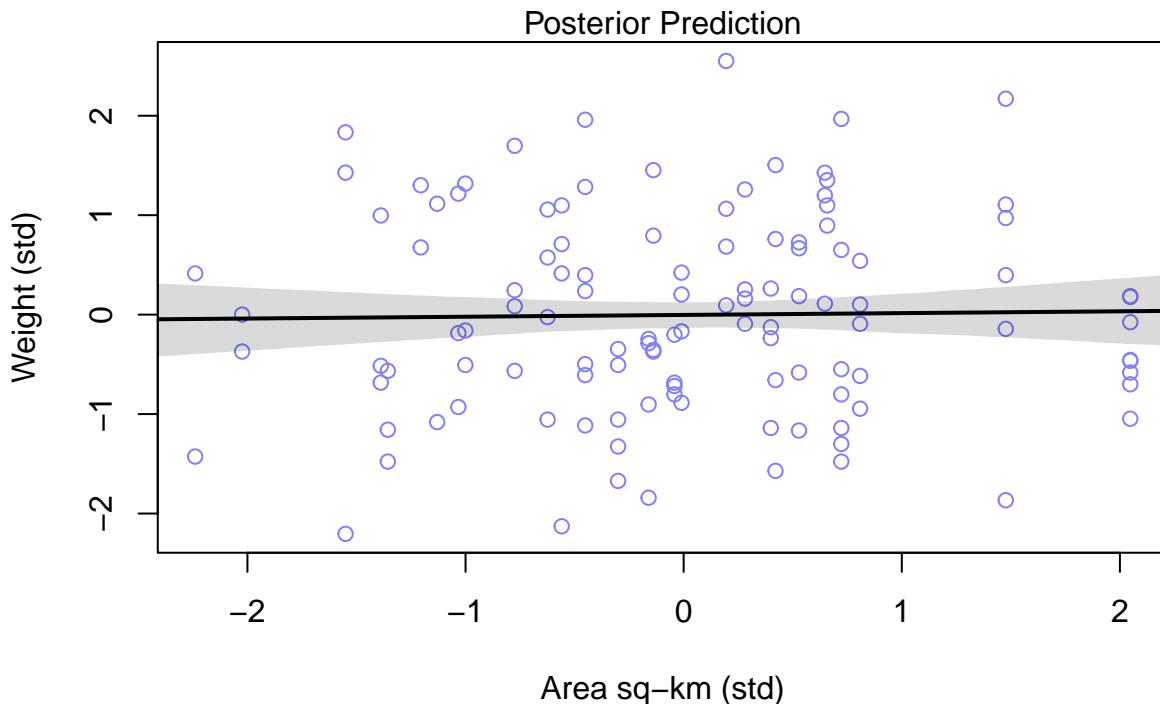
A.seq <- seq(from=-3, to=3, length.out=N)
mu <- link(mA, data=list(A=A.seq))
mu.mean <- apply(mu, 2, mean)
mu.PI <- apply(mu, 2, PI)
# Plot raw data

```

```

plot(W ~ A, data=d, col=rangi2, xlab="Area sq-km (std)", ylab="Weight (std)")
mtext("Posterior Prediction")
# Plot MAP line
lines(A.seq, mu.mean, lwd=2)
# Plot HDPI region for line
shade(mu.PI, A.seq)

```



Question: What association do you observe? What does your analysis suggest about how territory size relates to weight?

AHHHHHHHHHHHHHHHHH

1.3 Part b

b) Regress weight on food availability. That is, construct a quap linear regression (`m1b`) to estimate the association of food availability and fox weight. *Before fitting the model*, standardize both `avgfood` and `weight` to have mean 0 and standard deviation 1.

Hint: *With standardized variables, regression slopes represent standardized effect sizes. A slope of 1.0 would indicate a perfect positive relationship, while slopes >2 would be implausibly large for most ecological relationships.*

Use prior predictive simulation to assess the implication of your priors. Write 1-2 sentences to justify your priors.

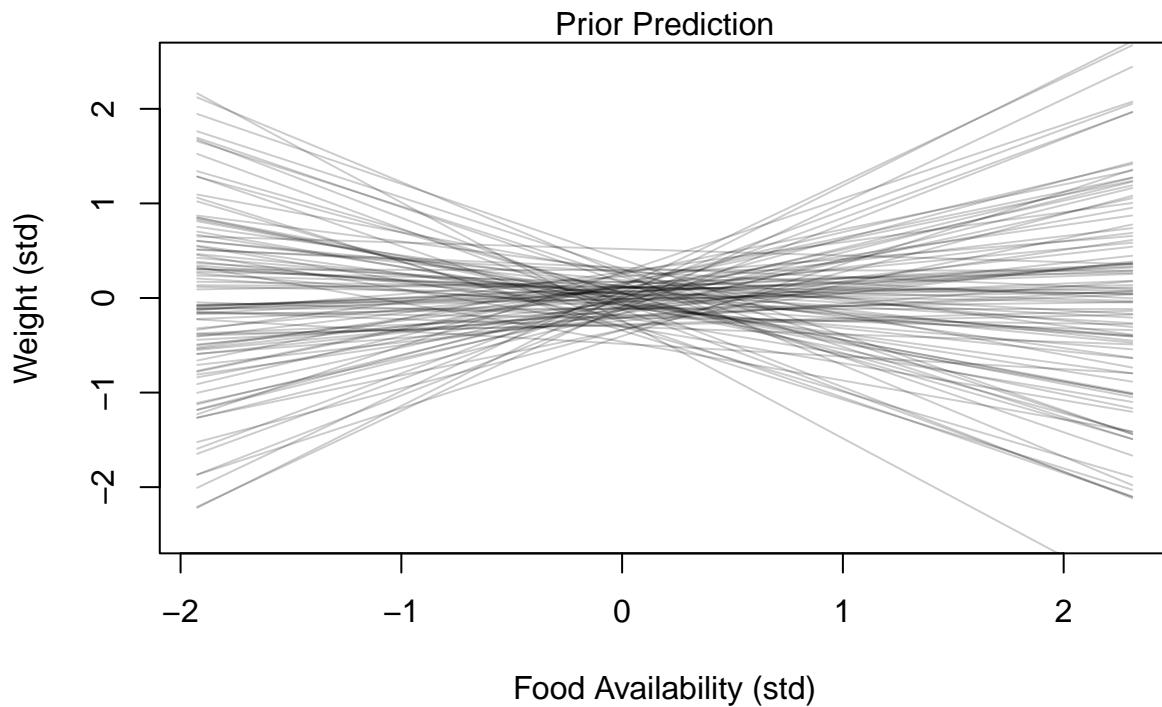
1.3.1 Prior Predictive Simulation

```

N <- 100
# As our data is standardized, we would expect our intercept, a, to be very
# close to 0
a <- rnorm(N, 0, 0.2)
# b represents the rate of change between our observed and unobserved variables.
# A value of 1 implies that for every one sd of change in our observed var.

```

```
#   there is 1 sd of change in our unobserved variables. We will choose a sd of
#   0.5 since that means we expect most of these slopes to be in -1 < b < 1
b <- rnorm(N, 0, 0.5)
```



1.3.2 Linear Regression

```
mF <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    # data is standardized so no need to subtract xbar
    mu <- a + b*F,
    # Priors from earlier
    a ~ dnorm(0, 0.2),
    b ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data=d
)

##               mean        sd      5.5%     94.5%
## a       3.769415e-09 0.08360017 -0.1336092 0.1336092
## b      -2.421162e-02 0.09088502 -0.1694634 0.1210402
## sigma  9.911440e-01 0.06465859  0.8878071 1.0944809
```

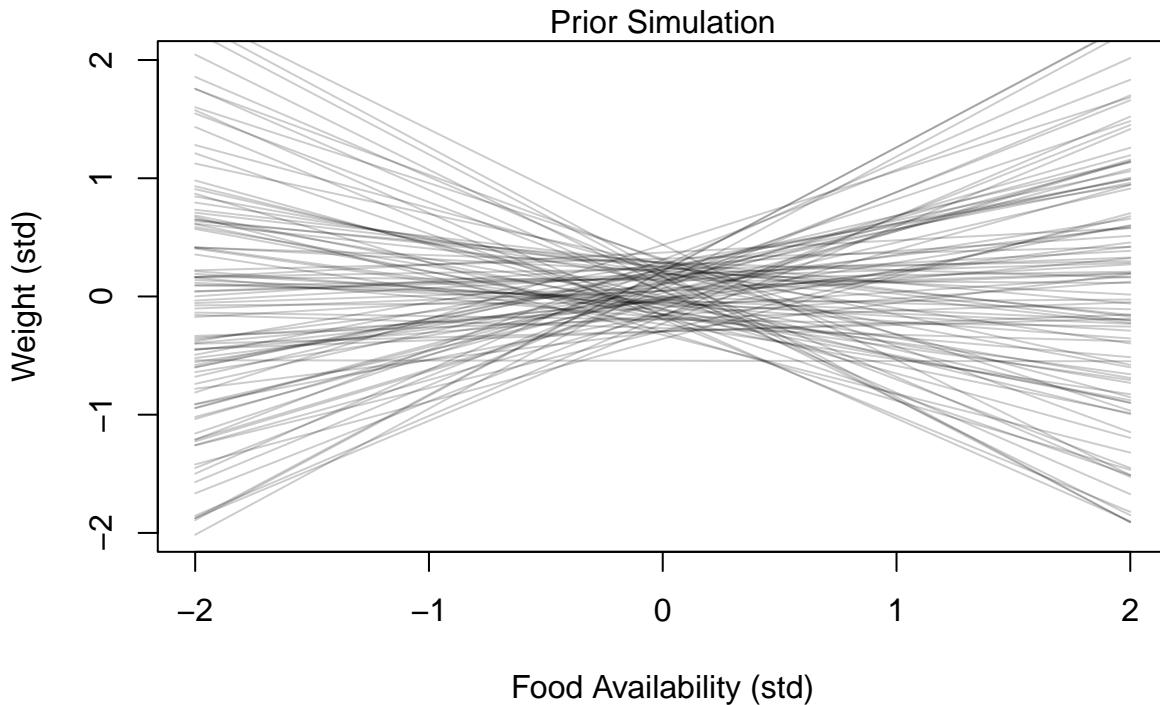
1.3.3 Simulate the Priors

```
set.seed(rseed)
prior <- extract.prior(mF)
mu <- link(mF, post=prior, data=list(F=c(-2, 2)))
plot(NULL,
  xlim=c(-2, 2), ylim=c(-2, 2),
```

```

    xlab="Food Availability (std)", ylab="Weight (std)"
)
mtext("Prior Simulation")
for (i in 1:N) lines(c(-2, 2), mu[i,], col=col.alpha("black", 0.2))

```

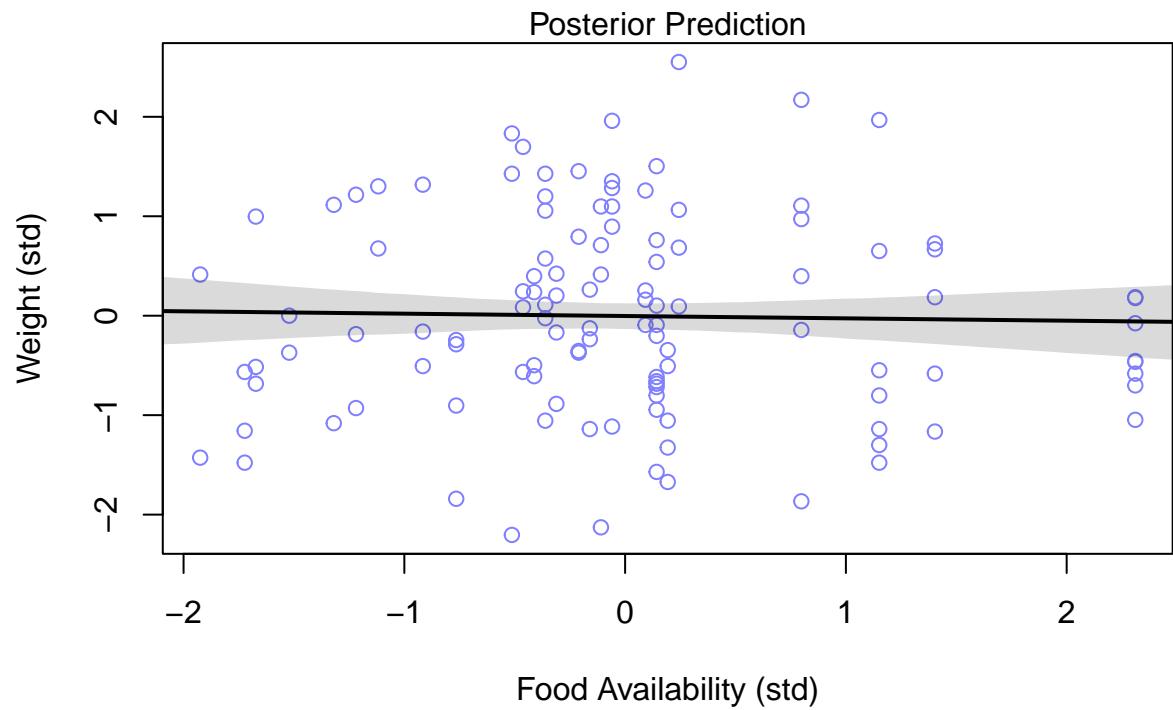


1.3.4 Posterior Predictions

```

F.seq <- seq(from=-3, to=3, length.out=N)
mu <- link(mF, data=list(F=F.seq))
mu.mean <- apply(mu, 2, mean)
mu.PI <- apply(mu, 2, PI)
# Plot raw data
plot(W ~ F, data=d, col=rangi2,
      xlab="Food Availability (std)", ylab="Weight (std)"
)
mtext("Posterior Prediction")
# Plot MAP line
lines(F.seq, mu.mean, lwd=2)
# Plot HDPI region for line
shade(mu.PI, F.seq)

```



c) Now regress weight on *both* territory size and food availability. Construct a `quap` model (`m1c`) that includes both predictors. Use the standardized variables. Explain your findings with 3-4 sentences and appropriate plots.

2 AI Declaration