Problem Set 2

Michael Fryer

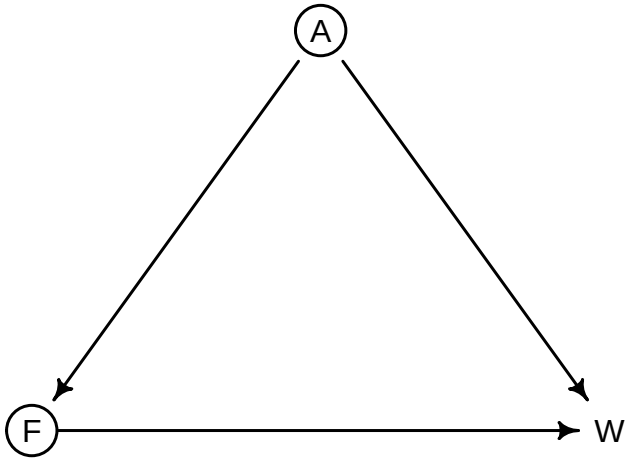
Collaborators: Florian Robrecht

1 Multiple Regression & Causal Models

The `foxes` dataset contains data on urban fox populations.

```
# First, load the foxes dataset
data(foxes)
d <- foxes
# You must set random seed to 390
rseed <- 390
set.seed(rseed)
```

Consider the following hypothesized causal relationship between **territory size** and **body weight** in foxes.



where A, F and W represent random variables `area` (territory size), `avgfood`, and `weight`, respectively.

If this DAG correctly describes the causal relationships, it makes specific predictions about what we should observe in the data. Your task is to test whether the observed patterns match these predictions.

- Territory size (A) has a **direct** effect on weight $(W) : A \rightarrow W$
- Food availability (F) has a **direct** effect on weight $(W) : F \rightarrow W$
- Territory size (A) has an **indirect** effect on weight (W) through food $(F) : A \rightarrow F \rightarrow W$

1.1 Standardize the Values

```
d$A <- standardize(d$area)
d$F <- standardize(d$avgfood)
d$W <- standardize(d$weight)
```

1.2 Part A

a) According to the DAG, territory size effects weight through two paths:

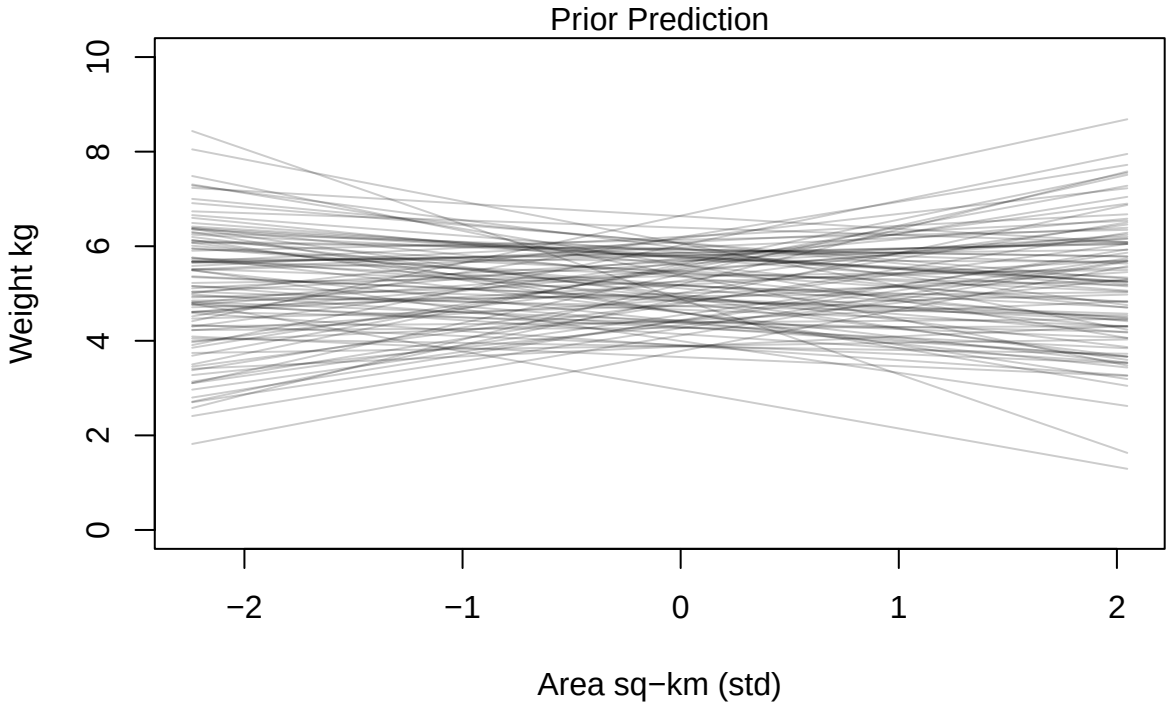
- Direct path: $A \rightarrow W$
- Indirect path: $A \rightarrow F \rightarrow W$

If we regress weight on territory size without including food, the coefficient should capture both pathways, the "total association" between A and W. Construct a linear regression (`m1a`) using `quap`. Urban foxes in this population have an average weight of 5kg. Use prior predictive simulation to assess the implications of your priors. Standardize the prediction variable.

Based on above, I am assuming that only the prediction variable, area, should be standardized for this model.

1.2.1 Prior Predictive Simulation

```
N <- 100
# If our prediction variable is not a factor, we would expect our intercept
#   to be near the average weight of 5kg. 0.75 represents 15% of the average.
a <- rnorm(N, 5, 0.75)
# b represents the rate of change between our observed and unobserved variables.
#   A value of 1 implies that for every one sd of change in our observed
#   variable there is 1kg of change in our predicted variable
b <- rnorm(N, 0, 0.5)
```



1.2.2 Linear Regression

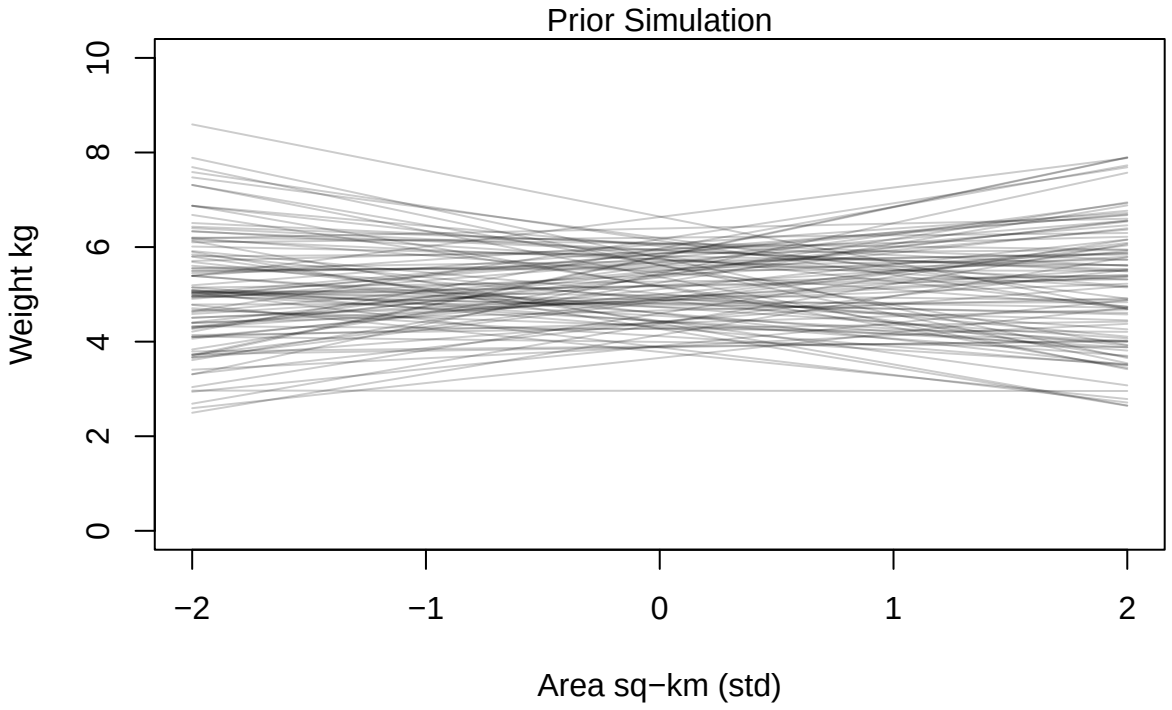
```
m <- quap(
  alist(
    weight ~ dnorm(mu, sigma),
    # No need to subtract mean as our predictor is standardized
    mu <- a + b*A,
    # Priors from earlier
```

```
    a ~ dnorm(5, 0.75),
    b ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data=d
)
```

```
##              mean         sd      5.5%     94.5%
## a     4.53936466 0.10776998  4.367127 4.7116019
## b     0.02200797 0.10683937 -0.148742 0.1927579
## sigma 1.17281090 0.07642636  1.050667 1.2949550
```

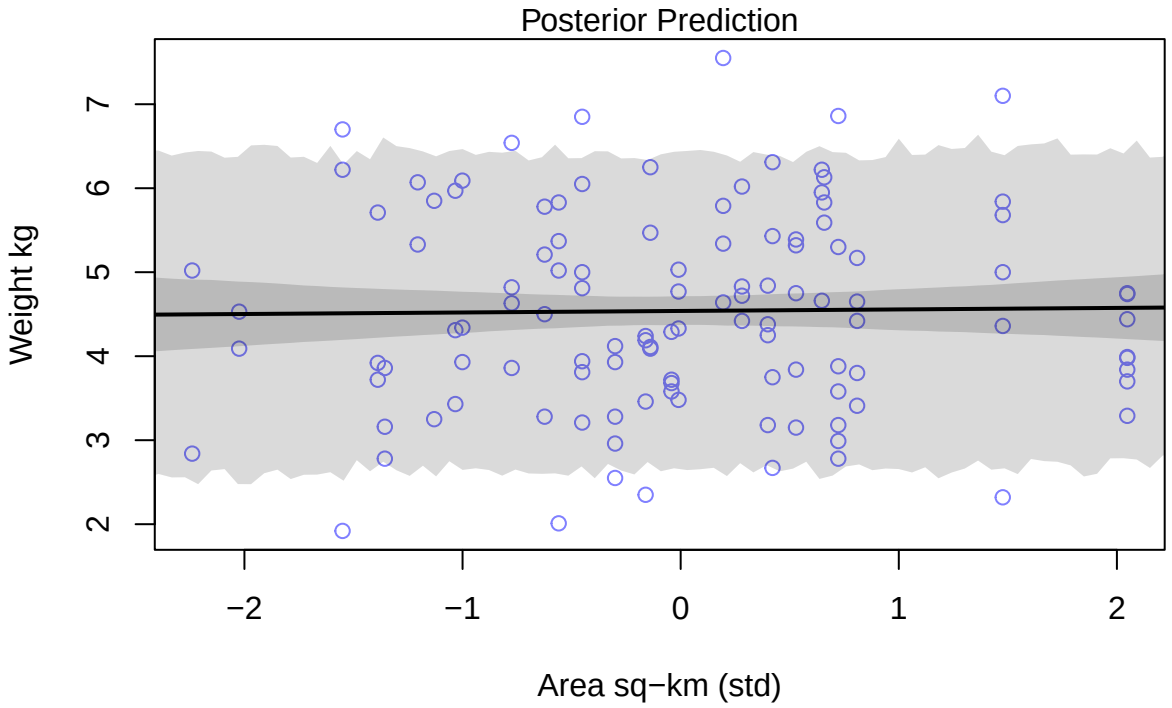
1.2.3 Simulate the Priors

```
set.seed(rseed)
prior <- extract.prior(m)
mu <- link(m, post=prior, data=list(A=c(-2, 2)))
```



1.2.4 Posterior Predictions

```
A.seq <- seq(from=-3, to=3, length.out=N)
mu <- link(m, data=list(A=A.seq))
mu.mean <- apply(mu, 2, mean)
mu.PI <- apply(mu, 2, PI)
sim.weight <- sim(m, data=list(A=A.seq))
weight.PI <- apply(sim.weight, 2, PI, prob=0.89)
```



Question: What association do you observe? What does your analysis suggest about how territory size relates to weight?

We observe a MAP with a slope of about 0.02. This tells us that territory size gives very little information about weight.

1.3 Part B

b) Regress weight on food availability. That is, construct a `quap` linear regression (`m1b`) to estimate the association of food availability and fox weight. *Before fitting the model*, standardize both `avgfood` and `weight` to have mean 0 and standard deviation 1.

Hint: *With standardized variables, regression slopes represent standardized effect sizes. A slope of 1.0 would indicate a perfect positive relationship, while slopes >2 would be implausibly large for most ecological relationships.*

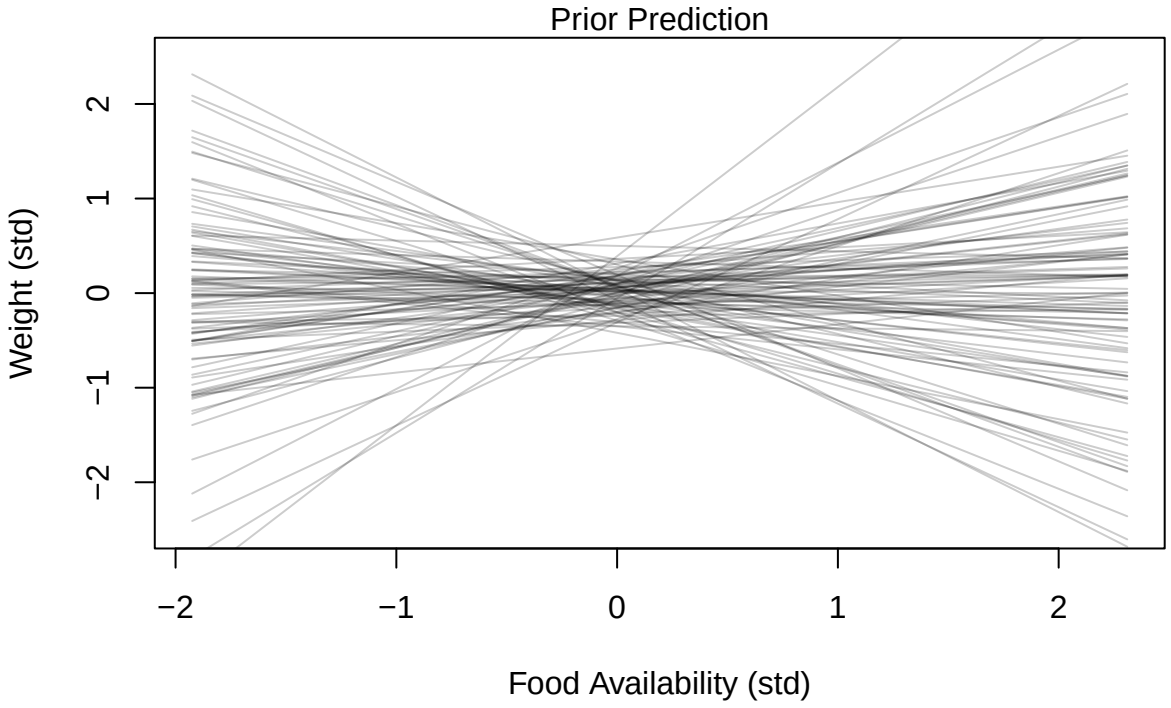
Use prior predictive simulation to assess the implication of your priors. Write 1-2 sentences to justify your priors.

Since both our predictor and prediction variable are standardized, we will simulate priors with mean 0 and a small standard deviation. For our slope pior, b, a value of 1 represents a change of one standard deviation in our predicted variable for 1 standard deviation of change in our predictor variable.

1.3.1 Prior Predictive Simulation

```
N <- 100
# As our data is standardized, we would expect our intercept, a, to be very
#   close to 0
a <- rnorm(N, 0, 0.2)
# b represents the rate of change between our observed and unobserved variables.
#   A value of 1 implies that for every one sd of change in our observed var.
#   there is 1 sd of change in our unobserved variables. We will choose a sd of
```

```
#   0.5 since that means we expect most of these slopes to be in -1 < b < 1
b <- rnorm(N, 0, 0.5)
```



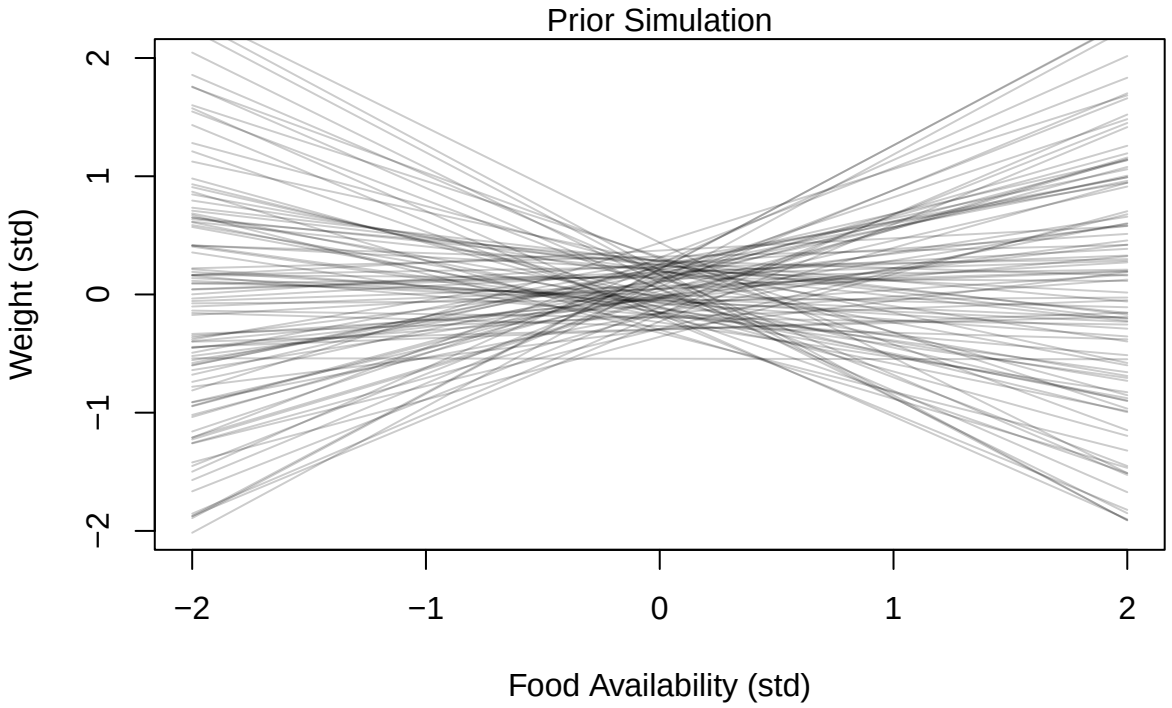
1.3.2 Linear Regression

```
mF <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    # No need to subtract mean as our predictor is standardized
    mu <- a + bF*F,
    # Priors from earlier
    a ~ dnorm(0, 0.2),
    bF ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data=d
)
```

```
##                mean         sd       5.5%     94.5%
## a     -1.073596e-06 0.08360234 -0.1336138 0.1336116
## bF    -2.421160e-02 0.09088778 -0.1694678 0.1210446
## sigma  9.911751e-01 0.06466365  0.8878301 1.0945201
```

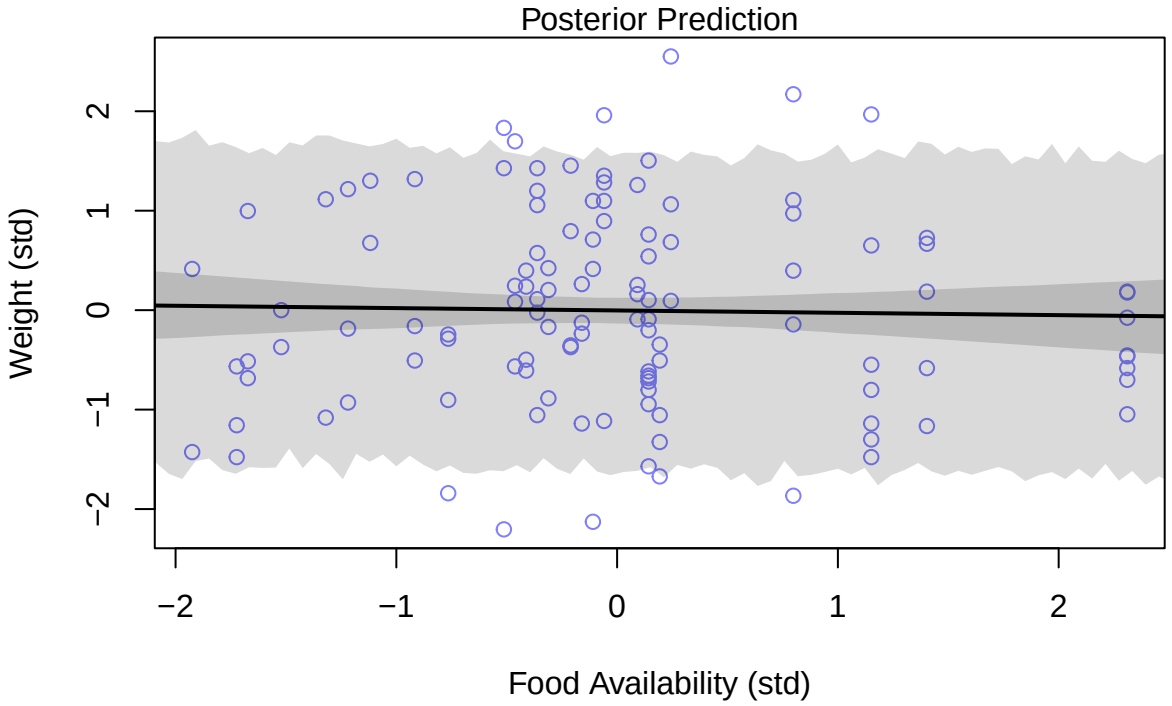
1.3.3 Simulate the Priors

```
set.seed(rseed)
prior <- extract.prior(mF)
mu <- link(mF, post=prior, data=list(F=c(-2, 2)))
```



1.3.4 Posterior Predictions

```
F.seq <- seq(from=-3, to=3, length.out=N)
mu <- link(mF, data=list(F=F.seq))
mu.mean <- apply(mu, 2, mean)
mu.PI <- apply(mu, 2, PI)
sim.W <- sim(mF, data=list(F=F.seq))
W.PI <- apply(sim.W, 2, PI, prob=0.89)
```



As with the previous part, the mean slope is near 0 implying food availability gives little information about weight. It is notable that what little impact food availability does has on weight appears to be negative in our dataset.

1.4 Part C

c) Now regress weight on *both* territory size and food availability Construct a `quap` model (`m1c`) that includes both predictors. Use the standardized variables. Explain your findings with 3-4 sentences and appropriate plots.

In the below analysis, I will standardize both the prediction variables and the predictor variable. This is done so we can accurately compare the models to one another.

1.4.1 Prior Predictive Simulation

```
N <- 100
# As our data is standardized, we would expect our intercept, a, to be very
#   close to 0
a <- rnorm(N, 0, 0.2)
# bA from above
bA <- rnorm(N, 0, 0.5)
# bF from above
bF <- rnorm(N, 0, 0.5)
```

1.4.2 Linear Regression

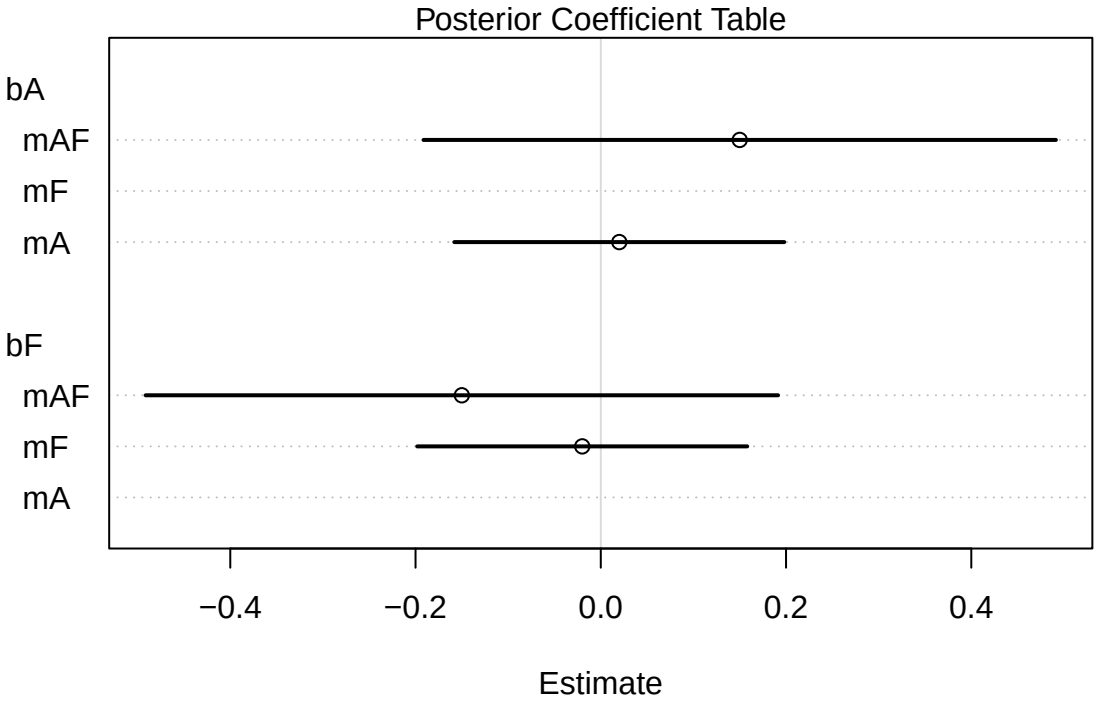
```
# Area predictor with standardized weight
mA <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    # No need to subtract mean as our predictor is standardized
    mu <- a + bA*A,
    # Priors from earlier
    a ~ dnorm(0, 0.2),
    bA ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data=d
)
# Food availability predictor reused from above
# Both predictors with standardized weight
mAF <- quap(
  alist(
    W ~ dnorm(mu, sigma),
    # No need to subtract mean as our predictor is standardized
    mu <- a + bA*A + bF*F,
    # Priors from earlier
    a ~ dnorm(0, 0.2),
    bA ~ dnorm(0, 0.5),
    bF ~ dnorm(0, 0.5),
    sigma ~ dexp(1)
  ),
  data=d
)
```

```
## Standardized Area Predictors:

##                mean         sd       5.5%     94.5%
## a     -1.482298e-06 0.08360918 -0.1336251 0.1336221
## bA     1.883367e-02 0.09089648 -0.1264365 0.1641038
## sigma  9.912735e-01 0.06466768  0.8879220 1.0946249

## Standardized Area and Food Availability Predictors:

##                mean         sd       5.5%     94.5%
## a      6.301500e-07 0.08334404 -0.1331992 0.1332005
## bA     1.461372e-01 0.17418828 -0.1322494 0.4245237
## bF    -1.490384e-01 0.17418845 -0.4274252 0.1293484
## sigma  9.874680e-01 0.06444170  0.8844777 1.0904583
```



In the above figure, we can see that mA and mF alone maintain a mean of approximately 0. Only when adding the other parameter do stray away from the mean and increase in standard deviation. This implies that considering both mA and mF at the same time produces more uncertainty that just considering one at a time.

2 AI Declaration

AI was not used for this assignment.