FINAL PROJECT PRESENTATION
BIOSTAT 696

Map Mad Scientists
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MOTIVATION

Light Pollution

Winter night in Kunming, Yunan, China, Jan 16, 2023

MOTIVATION

- **Magnitude of Celestial Body**
  - Range $[-26.73, 30]$
  - Sun -26.73
  - Full moon -12.6
  - Faintest stars observable with naked eye +6
- **Naked Eye Limiting Magnitude (`NELM`)**
- **Magnitude Per Square Arcsecond (`MPSAS`)**

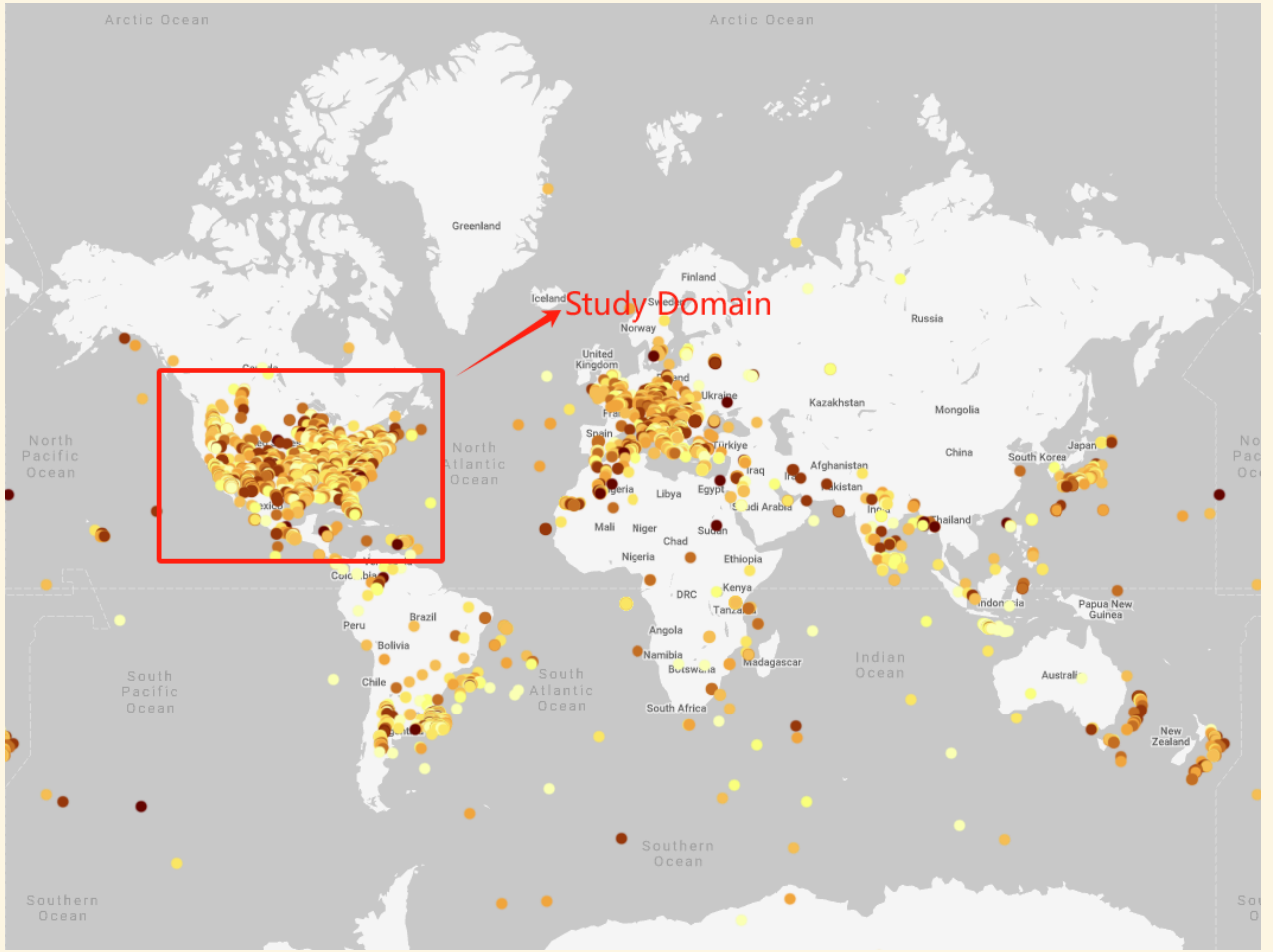
$$NELM = \text{some complex function}(MPSAS)$$

  - Range $[16(brightest) - 22(darkest)]$
  - Measured by equipment (Sky Quality Meter)

DATA SOURCE

Globe at Night Project (Link)

- Light pollution report at 29404 locations (2020)
- Sky brightness measurement (MPSAS) at 2030 locations
  - Around 800 inside continental US
- Darker means higher NELM (less light pollution)



NELM at All Report Locations in 2020

DATA SOURCE

Explanatory Variables

- **Population Density**[1]
- **Land Price**[2]
- **Electricity Consumption**[3]

All are raster datasets and are sampled at the same locations as `MPSAS` measurements.

OUR GOAL

- Apply different spatial Bayesian models and compare performance;
- Impute missing or damaged data in the original dataset (due to unofficial reports);
- Predict light pollution in Michigan.

OUR MODEL

$$y(s) = \beta^T x(s) + w(s) + \epsilon(s), s \in D$$

Response Variable:

$$y(s)$$

- MPSAS measurement at location s (Univariate)

Explanatory Variables

$$x(s) = [x_1(s), x_2(s), x_3(s), x_4(s)]^T$$

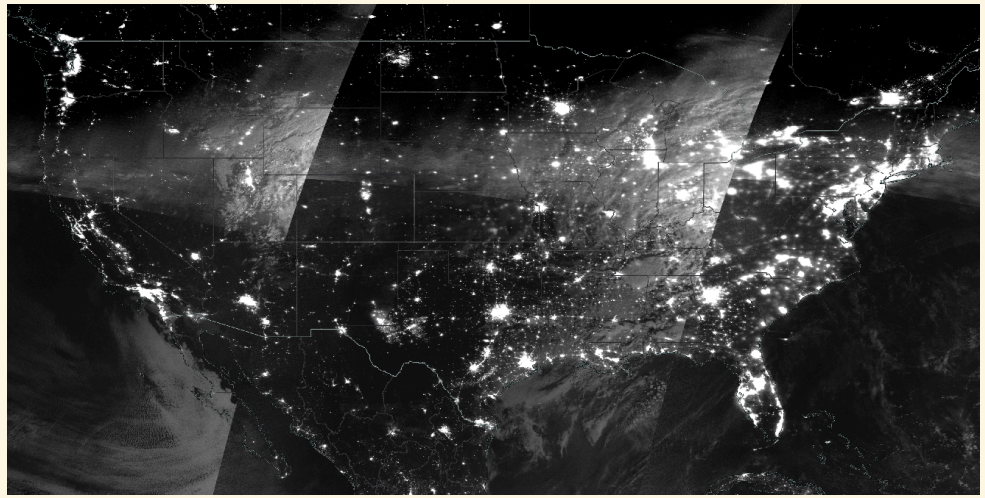
- x_1: Elevation
- x_2: Population Density
- x_3: Land Price
- x_4: Electricity Consumption

MODEL ASSUMPTIONS - GP

- $w(s) \sim GP(0, \tau^2 C(\cdot, \cdot))$
- Features
  - Rapid decay in covariance
  - Stationary, isotropic (?)
- Exponential kernel

$$C(s, s') = \sigma^2 \exp -\phi ||s - s'||$$

  - For simplicity
  - Interpretability

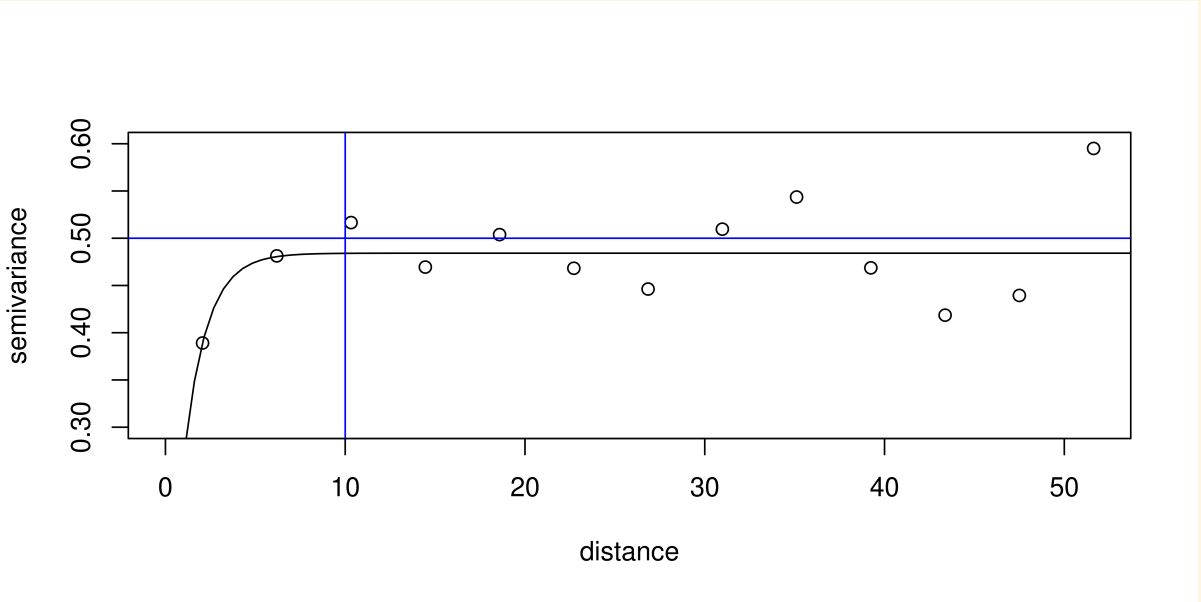


Nighttime Light Image Captured in NASA Worldview[1]

Why not use night time light data as an explanatory variable?

- Only measuring differences
- Does not explain anything

MODEL ASSUMPTIONS - PRIORS



The residual plot of semivariance from nonspatial model could help with our decision of the prior and starting value settings (Banerjee, Carlin, and Gelfand (2014))

- $\beta \sim N(0, 1000 I_p)$
  - We assume independence between explanatory variables

MODEL ASSUMPTIONS - PRIORS

- $\phi \sim U(3/20, 3/0.1)$
  - Since we plan to use exponential model to specify the covariance function, where the spatial correlation is given by

$$p(d) = exp(-d/\phi)$$

    we define the distance, d_0 , at which this correlation drops to 0.05 as the "effective spatial range" ,then

$$\phi = log(0.5)/d_0 \approx 3/d_0$$

  - We assume large decay factor
  - Starting value: 3/10, since the range of the semi-variogram is close to 10

MODEL ASSUMPTIONS - PRIORS

- $\sigma^2 \sim Inv.G(2, 2)$
  - Chose because the sill value of residuals plot is close to 0.5
  - Starting value: same as 0.5
- $\tau^2 \sim Inv.G(2, 0.1)$
  - Chose because we think the mean value of τ^2 should be a small number
  - Starting value 0.1, since the nugget value of the plot is close to 0.
- Tuning: $\phi = 0.05, \tau^2 = 0.01, \sigma^2 = 0.01$

MCMC TRACE - FULL GP

- MCMC samples: 50, 000
- Acceptance rate: 51.7%
- Running time: 157.3s
- Convergence: σ^2 and τ^2 seem to reach convergence, since their trace plot don't have many flat areas, and the density distribution appear to be unimodal and symmetric, indicating precise estimation
- Convergence: trace plot of ϕ tend to have a skewed distribution that does not look ideal and there are a lot of flat sections in its trace plot as well.

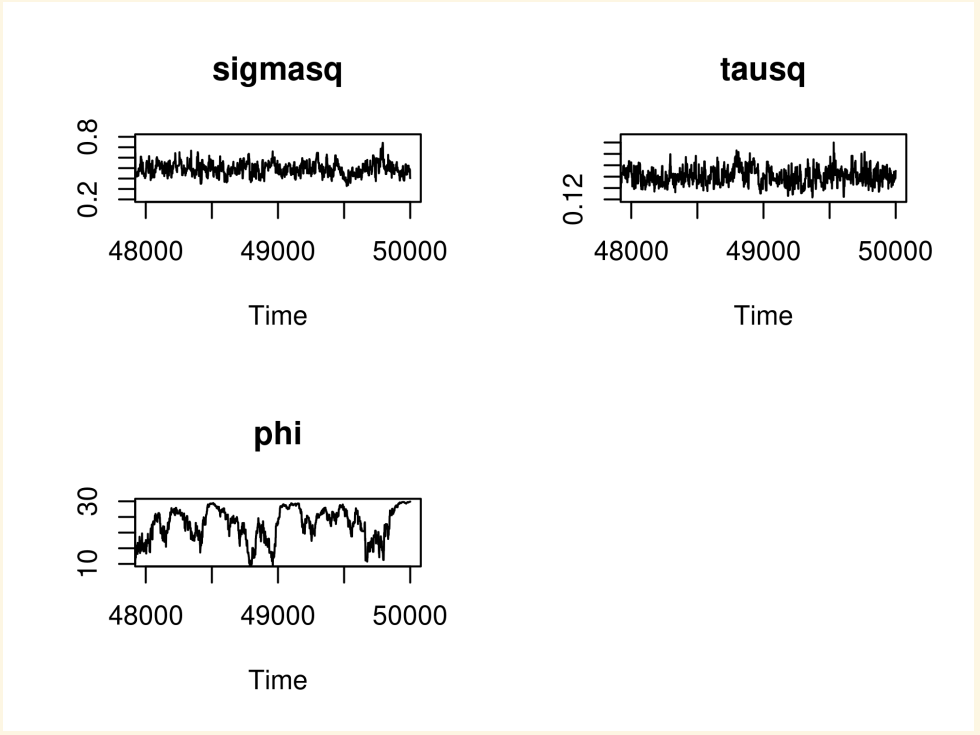
Empirical mean, standard deviation and standard error

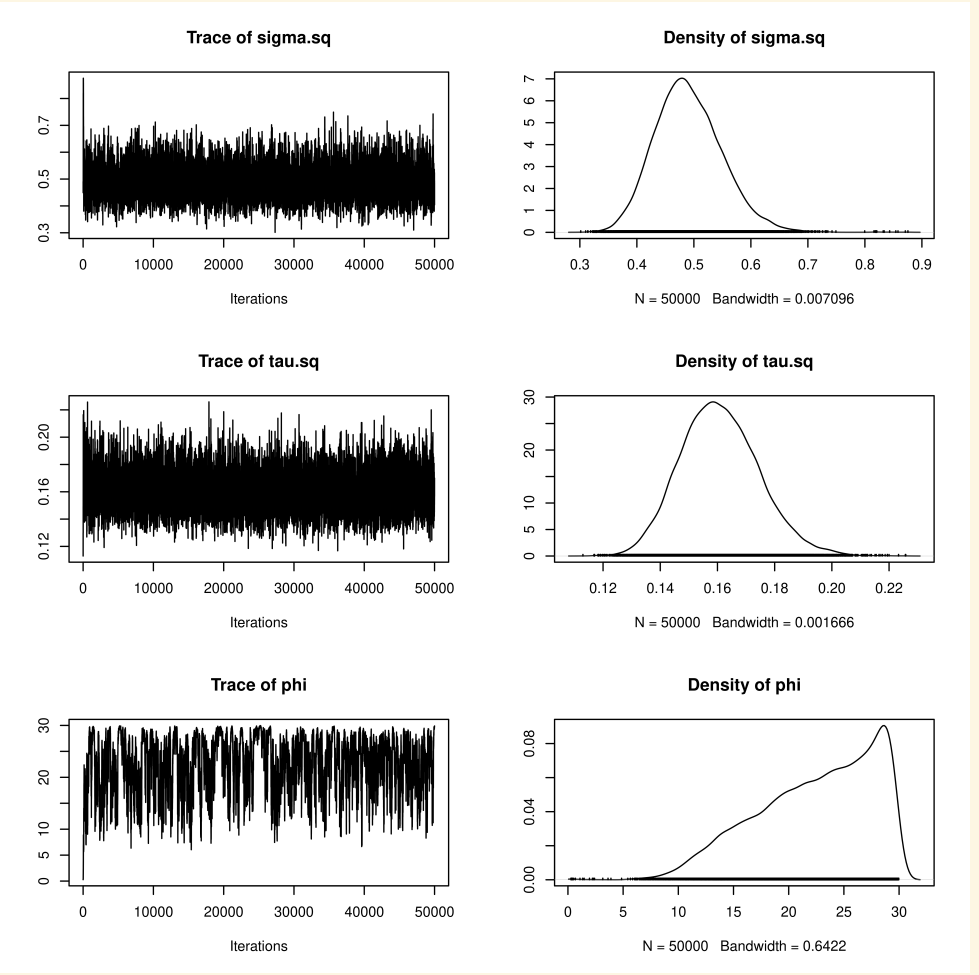
	Mean	SD	Naive SE	Time-series SE
σ^2	0.4683	0.05980	2.674e-04	0.0010711
τ^2	0.1730	0.01491	6.668e-05	0.0002933
ϕ	22.4706	5.42120	2.424e-02	0.3252332

Quantiles for each variable

	2.5%	25%	50%	75%	97.5%
σ^2	0.3611	0.4258	0.4648	0.5058	0.5928
τ^2	0.1459	0.1626	0.1724	0.1824	0.2040
ϕ	10.6424	18.8733	23.2973	26.9326	29.8832

MCMC TRACE - FULL GP





MCMC TRACE - LOW RANK GP

- MCMC samples: 50, 000
- Acceptance rate: 7.71%
- knots grid: 6×6
- Running time: 29.8s
- Convergence: Like the fullGP model, the trace plot and density plot of σ^2 and τ^2 suggest convergence, but may not as good as fullGP model since there are more fluctuation in the plots
- The density plot of ϕ is still skewed distributed, but according to the trace plot the convergence performance is better than fullGP model.

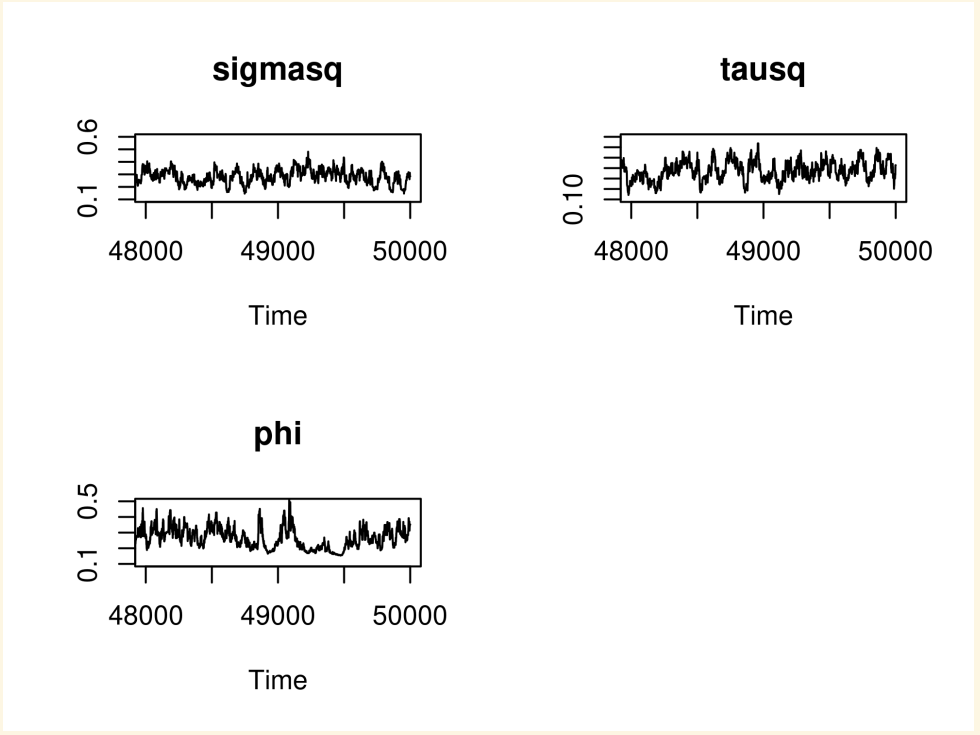
Empirical mean, standard deviation and standard error

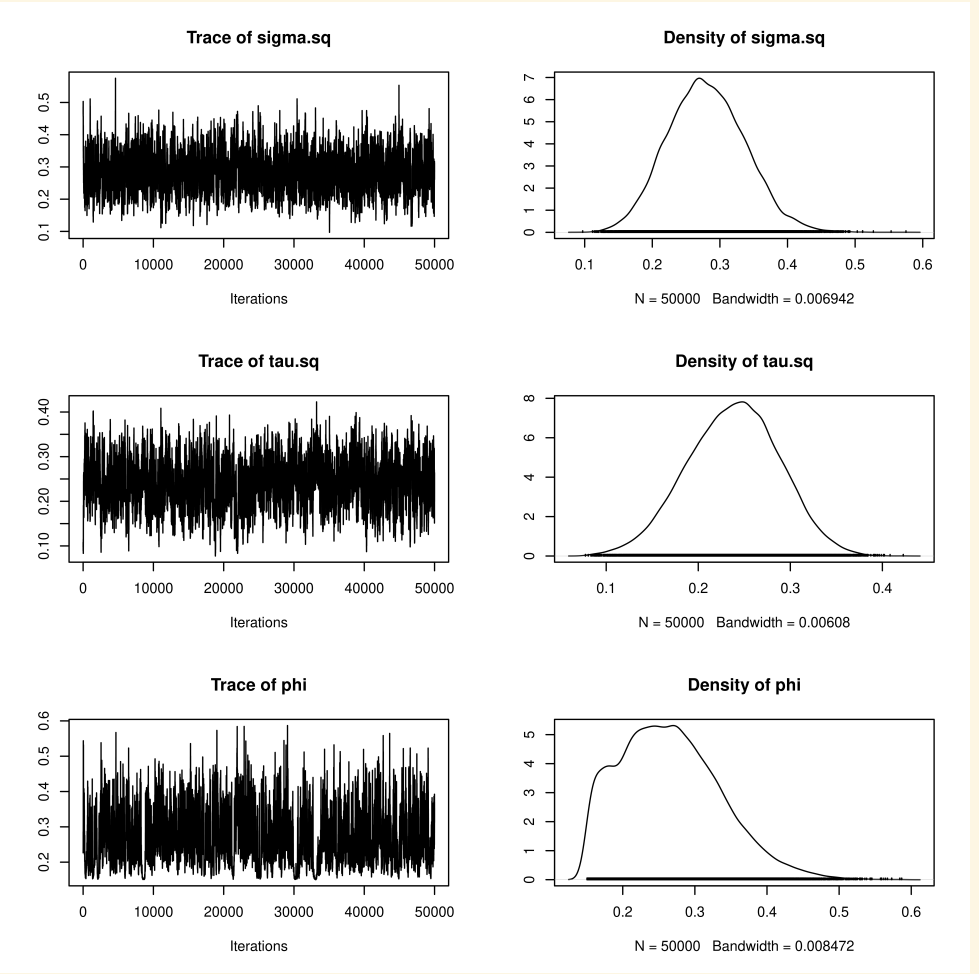
	Mean	SD	Naive SE	Time-series SE
σ^2	0.2024	0.05776	0.0002583	0.002145
τ^2	0.3579	0.05400	0.0002415	0.002185
ϕ	0.2537	0.07618	0.0003407	0.003548

Quantiles for each variable

	2.5%	25%	50%	75%	97.5%
σ^2	0.1067	0.1608	0.1968	0.2381	0.3300
τ^2	0.2444	0.3236	0.3608	0.3959	0.4547
ϕ	0.1533	0.1917	0.2422	0.3010	0.4288

MCMC TRACE - LOW RANK GP





MCMC TRACE - NEAREST NEIGHBOUR GP

- We explored the trace plot of NNGP model but found that the previous prior settings did not fit to the NNGP latent and response model.
- We changed the priors here as
- $\beta \sim N(0, 1000 I_p)$
- $\phi \sim U(15, 60)$
  - Starting value: 30
- $\sigma^2 \sim Inv.G(2, 0.5)$
  - Starting value: 1
- $\tau^2 \sim Inv.G(2, 0.5)$
  - Starting value: 0.5
- Turning: $\phi = 2, \tau^2 = 0.2, \sigma^2 = 0.2;$
- Nearest Neighbor: 10

MCMC TRACE - NEAREST NEIGHBOUR GP

- Acceptance Rate: 17.92%
- Running time: 92.4s
- MCMC Results for Both Intent and Response NNGP Models
  - The trace plot of σ^2, τ^2 show convergence
  - The trace plot of ϕ does not settle around a particular value and instead shows considerable movement across a wide range of values, indicating convergence of ϕ here is not ideal.

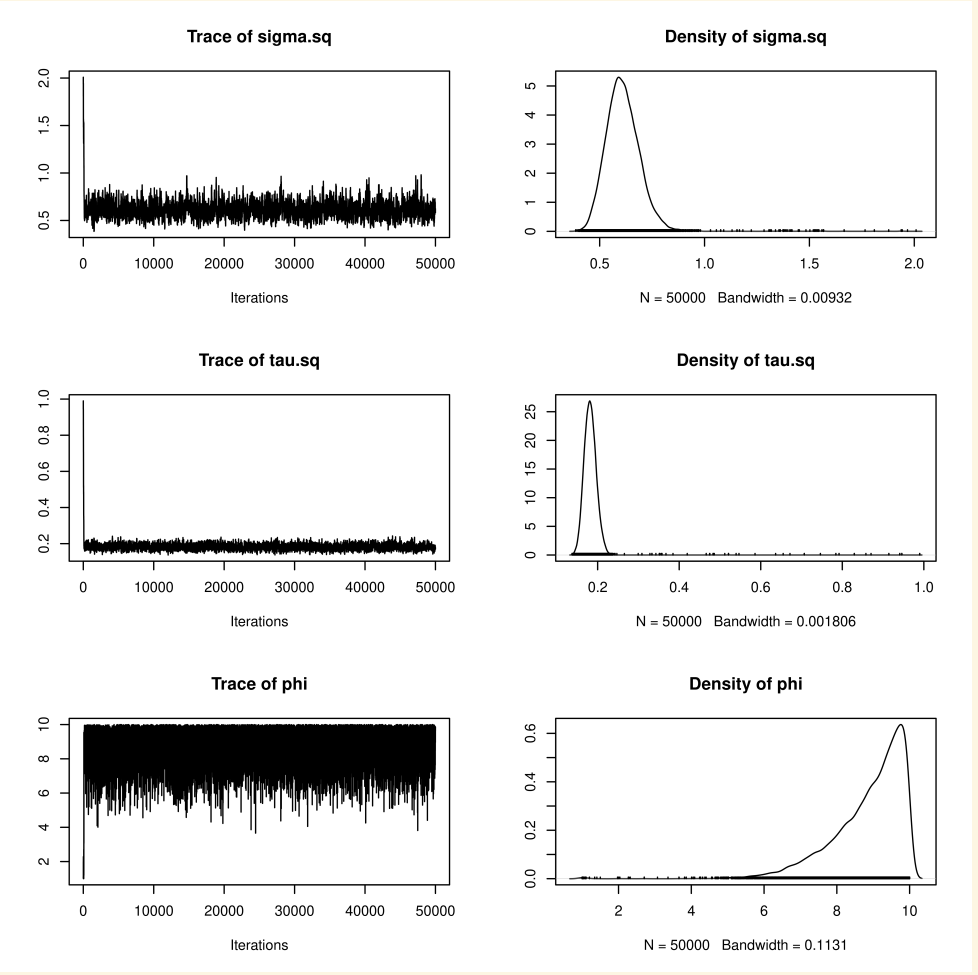
Empirical mean, standard deviation and standard error

	Mean	SD	Naive SE	Time-series SE
σ^2	0.5908	0.06990	3.126e-04	0.001117
τ^2	0.1614	0.01485	6.639e-05	0.000346
ϕ	30.4587	9.31411	4.165e-02	0.532808

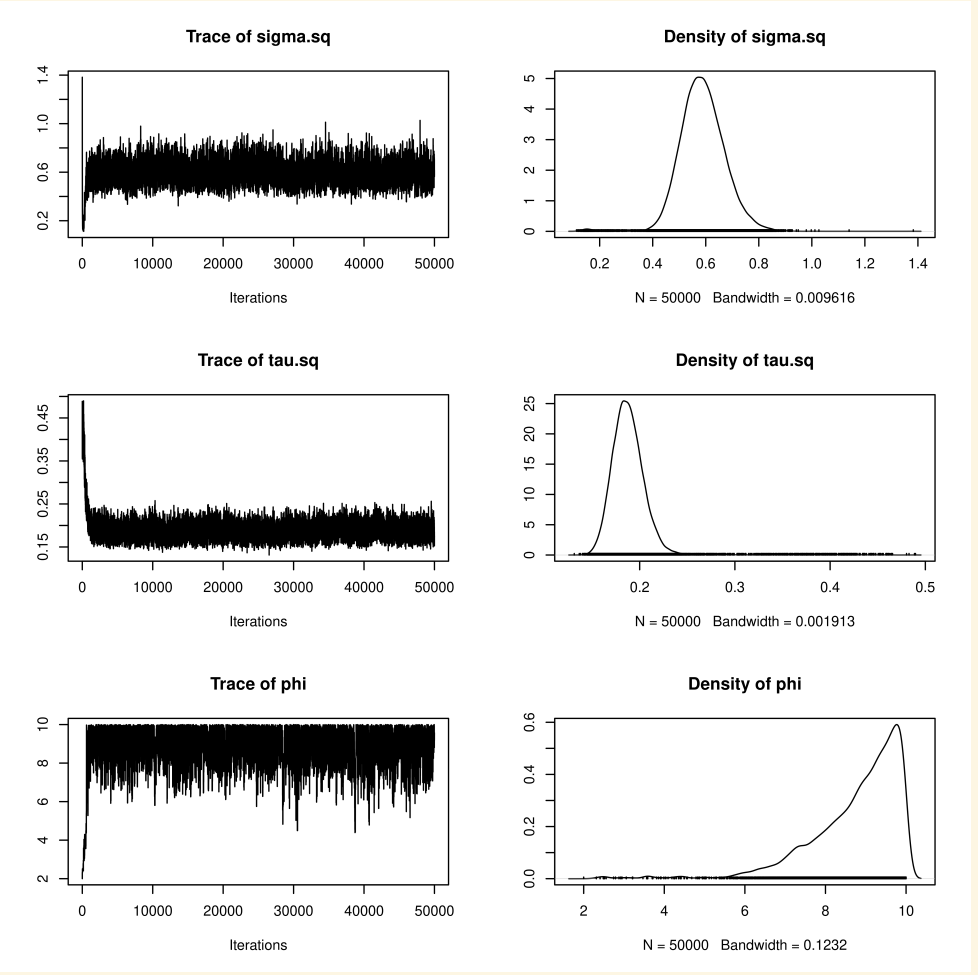
Quantiles for each variable

	2.5%	25%	50%	75%	97.5%
σ^2	0.4650	0.5425	0.5869	0.6347	0.7395
τ^2	0.1352	0.1511	0.1605	0.1707	0.1918
ϕ	16.6526	23.2558	29.1885	36.2258	52.5989

MCMC TRACE - NEAREST NEIGHBOUR GP



Response Model

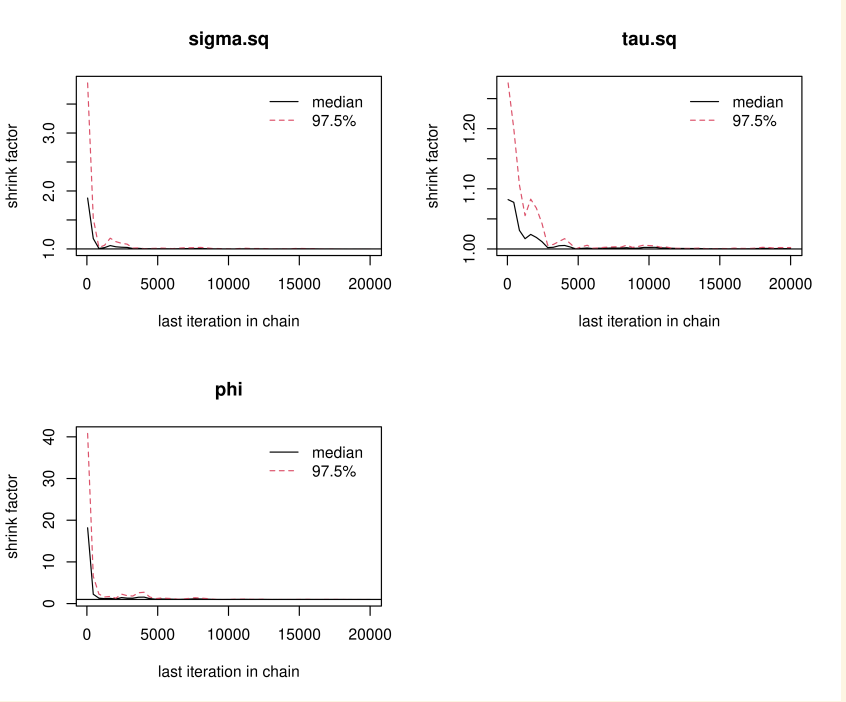


Latent Model

MCMC DIAGNOSES - FULL GP

Gelman Dignostic:

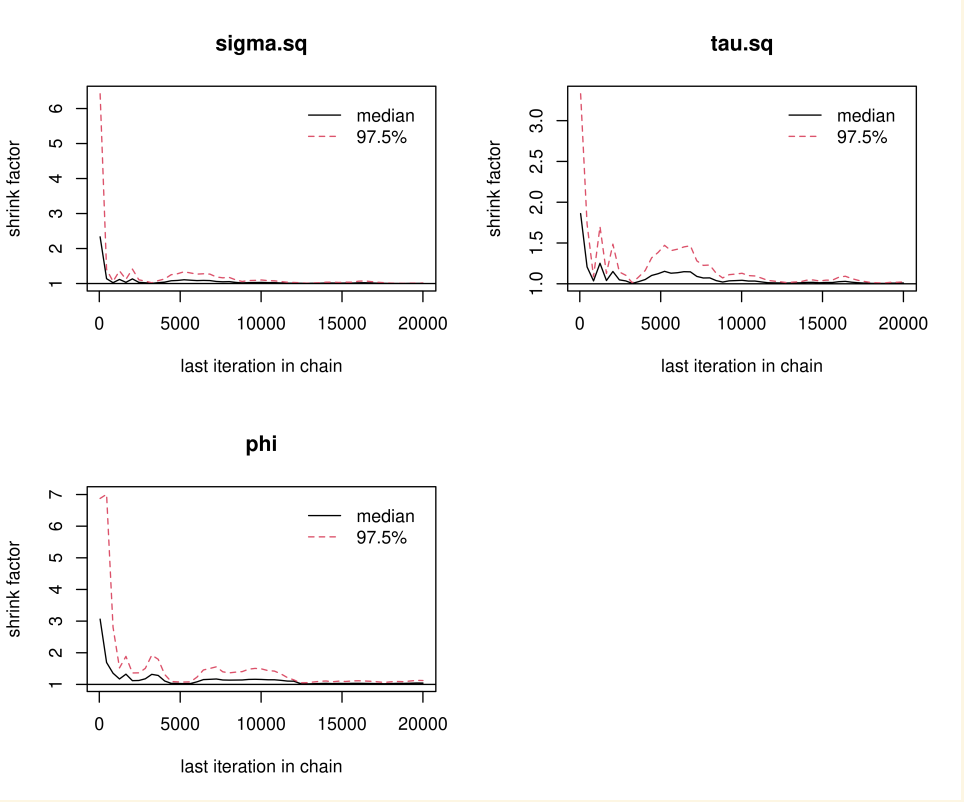
- We changed the starting values and ran MCMC for each model for 3 times to gain the plot of shrink factor that considers within-chain variance and between-chain variance.
- In fullGP model, given multiple runs, the shrink factors of σ^2 and ϕ are quickly drop to 1, that show good convergence, while τ^2 's plot drops a bit slowly.
- Multivariate PSRF(Potential Scale Reduction Factor): 1 (Indicates convergence as well)



MCMC DIAGNOSES - LRGP

Gelman Dignostic

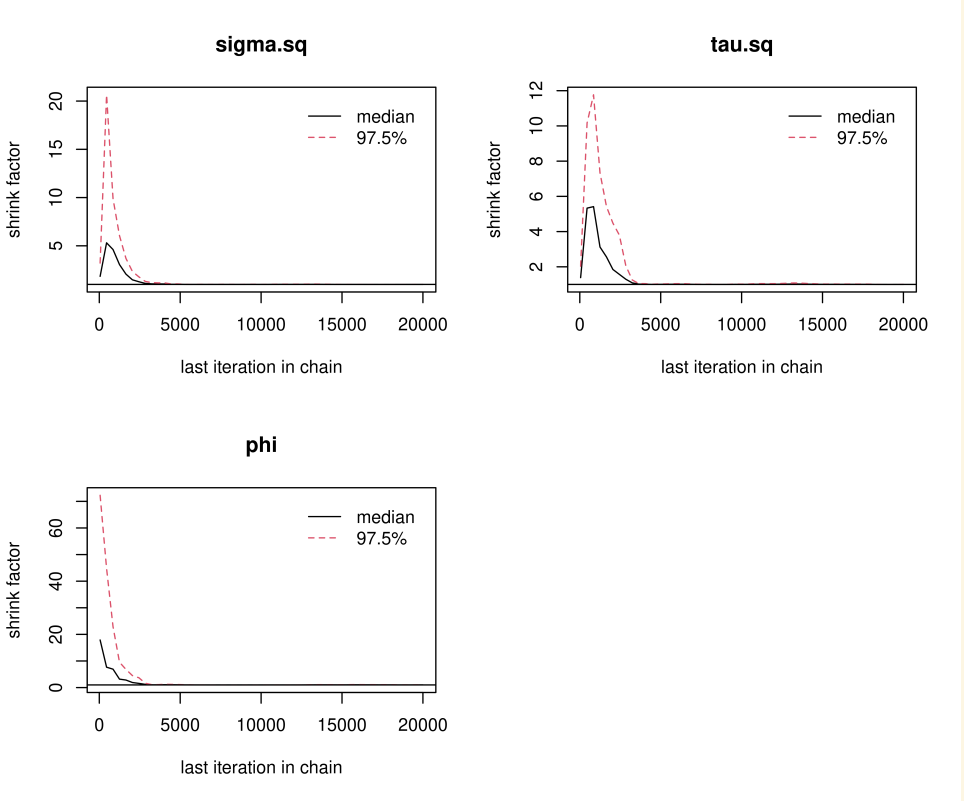
- Lack of convergence according to the fluctuation of the plots of all the parameters
- Multivariate PSRF: 1.03



MCMC DIAGNOSES - NNGP

Gelman Dignostic

- All the plots are dropped quickly to 1 -> Convergence
- σ^2 and τ^2 show peaks at the start of their PSRF plots
- Multivariate PSRF: 1



MCMC DIAGNOSES - SUMMARY

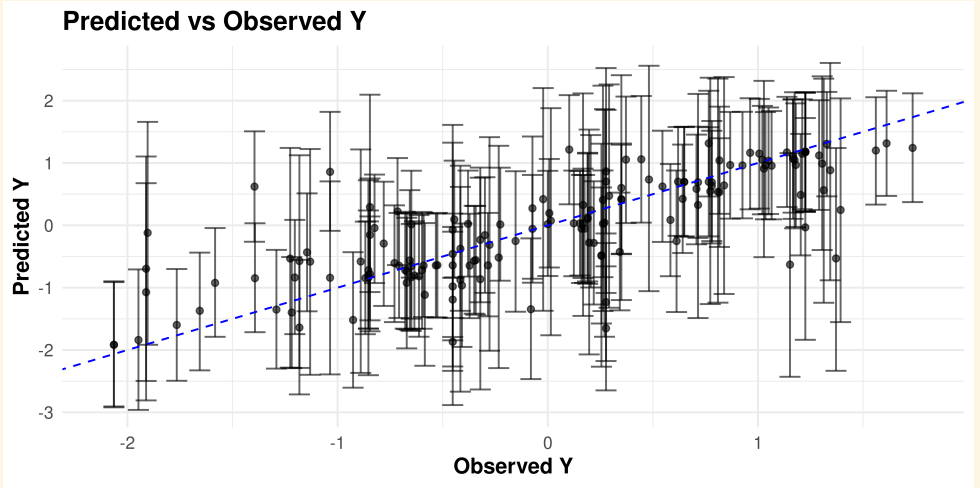
	Full GP	LRGP	NNGP (latent)	NNGP (Response)
$DIC				
bar.D	-532.3148	179.1369	-	-
D.bar.Omega	-775.2592	164.0365	-	-
pD	242.9444	15.1004	245.9763	-
DIC	-289.3703	194.2373	880.0929	-
L	-	-	-194.0702	-
$GP				
G	61.08594	293.8502	61.19716	340.3536
P	138.35194	323.2817	141.61156	479.2075
D	199.43788	617.1320	202.80872	819.5612
$GRS	663.4488	-156.3317	653.3704	-272.9311

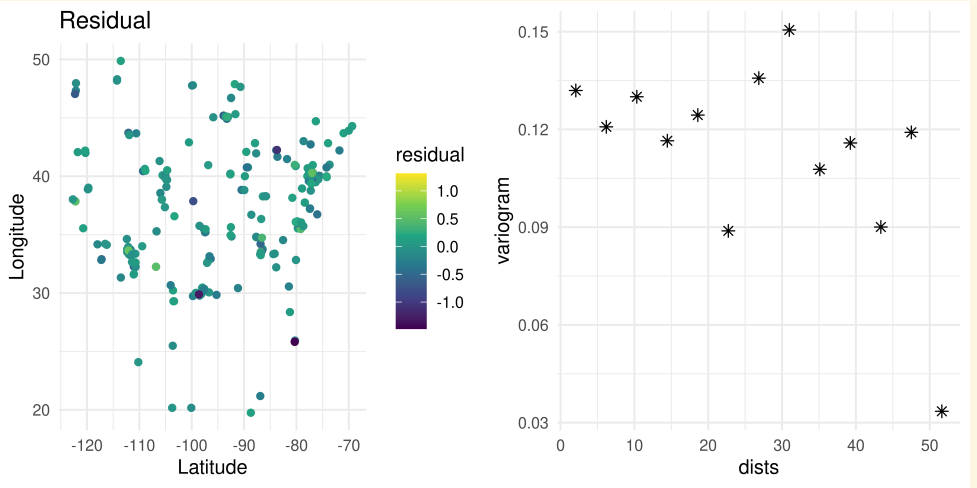
MCMC DIAGNOSES - SUMMARY

Effective Sample Size	Full GP	LRGP	NNGP (latent)	NNGP (Response)
σ^2	3445.5804	725.3773	3912.5011	3444.001
τ^2	2586.3832	610.8421	1840.5902	3590.207
ϕ	293.4634	461.0998	305.5915	2941.707

VALIDATION - FULL GP

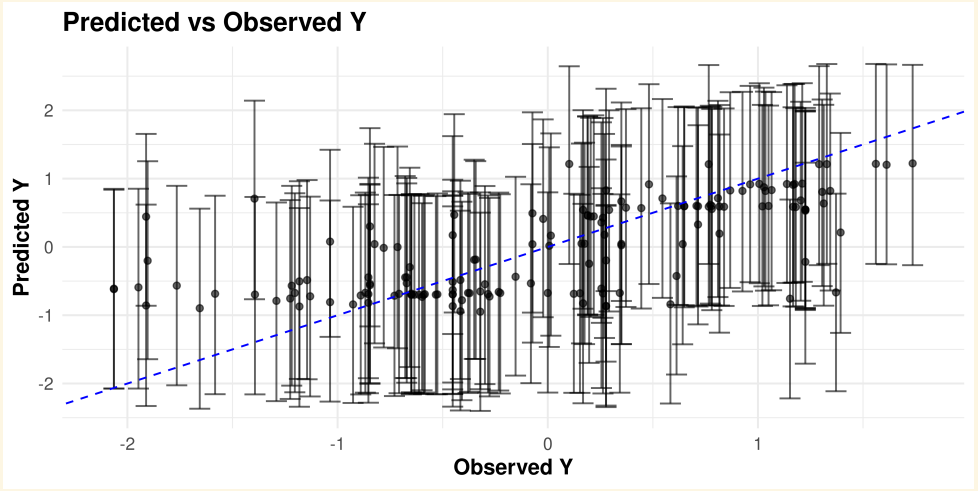
- Testing set: 25% of the data
- Most of the points are close to reference line ($slope = 1$)
- Low confidence intervals among all the models
- Nearly no spatial dependence on the semivariogram
- Low residual values (close to 0.1)

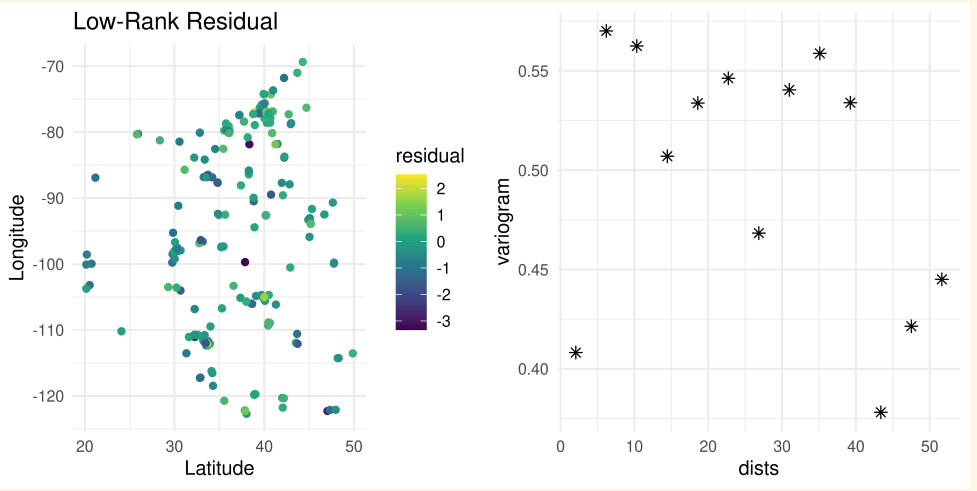




VALIDATION - LOW RANK GP

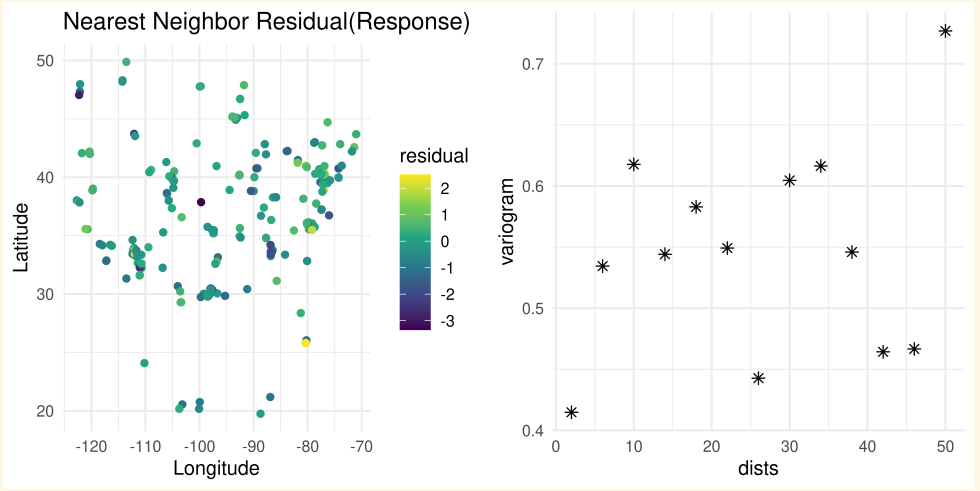
- More deviation points from reference line comparing to full rank GP model
- Higher confidence intervals than full GP model
- Less spatial dependence than LM model

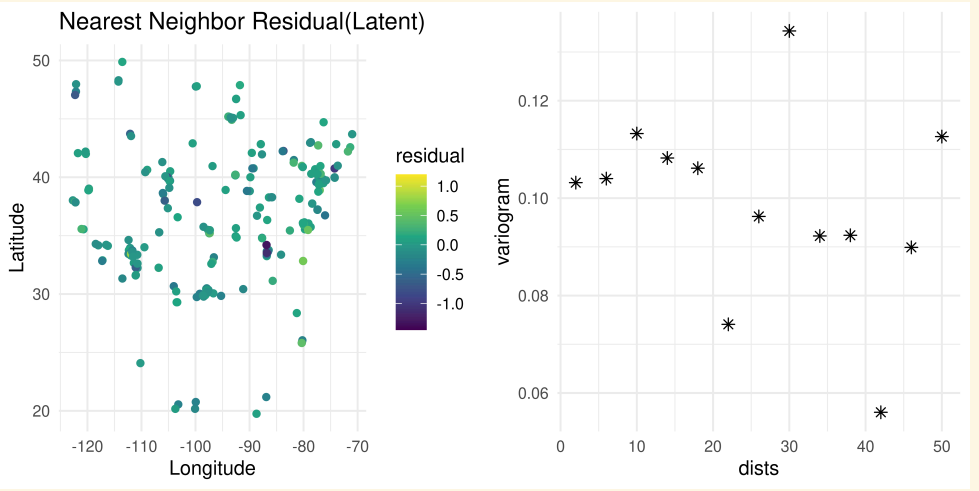




VALIDATION - NNGP

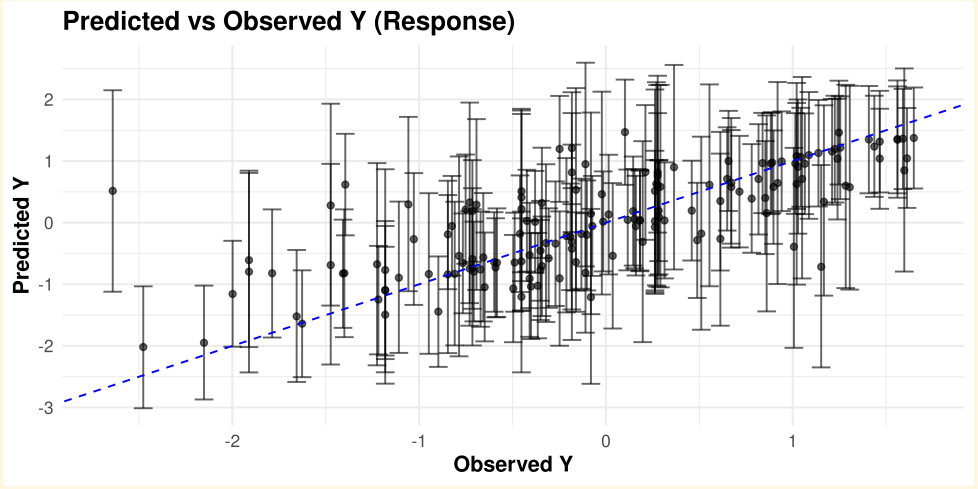
- According to residuals, latent model is significantly better than response model, the latter one seems failing to capture spatial variance that still can notice in the semivariogram.
- In terms of the distribution of residuals, the two models tend to have similar spatial patterns that points with larger residuals are both gathered around the eastern part of the research region.

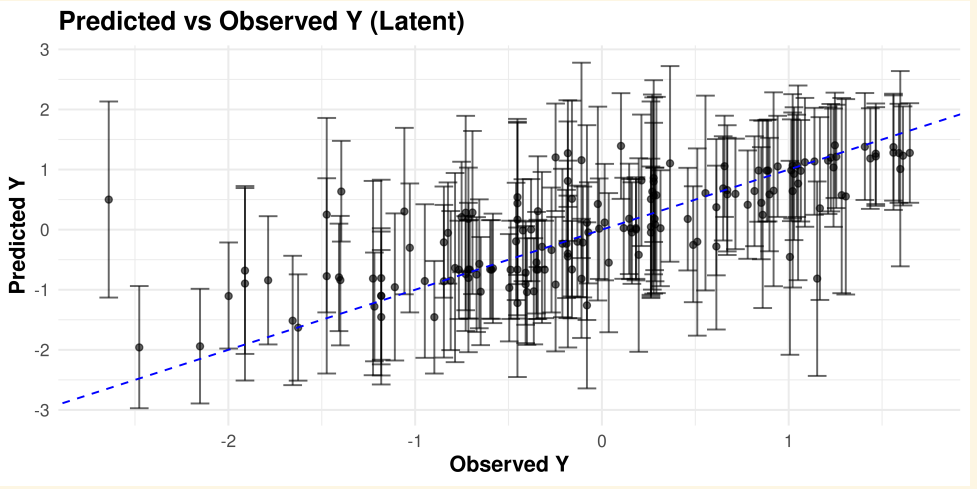




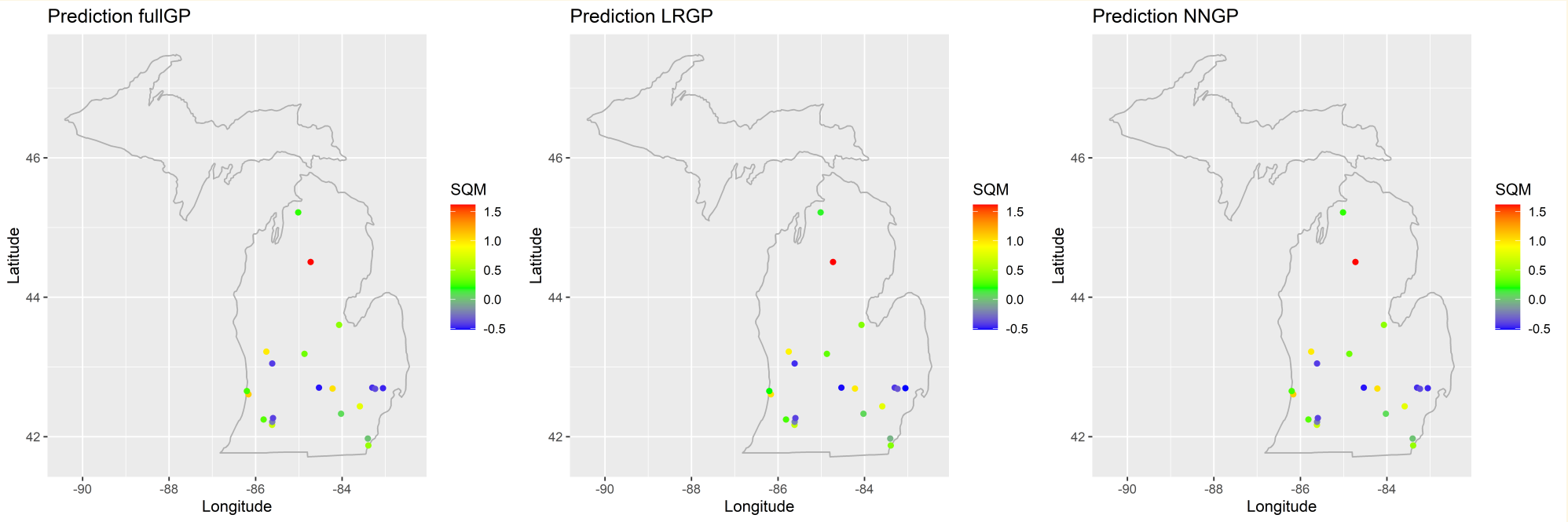
VALIDATION - NNGP

- Comparing two models' prediction on validation data, the scatters from latent model are closer to the reference line and has narrower confidence intervals generally.





PREDICTION



Prediction for Points with Missing Values within Michigan

FURTHER ATTEMPTS: NON-STATIONARY MODEL[1]

Model Description

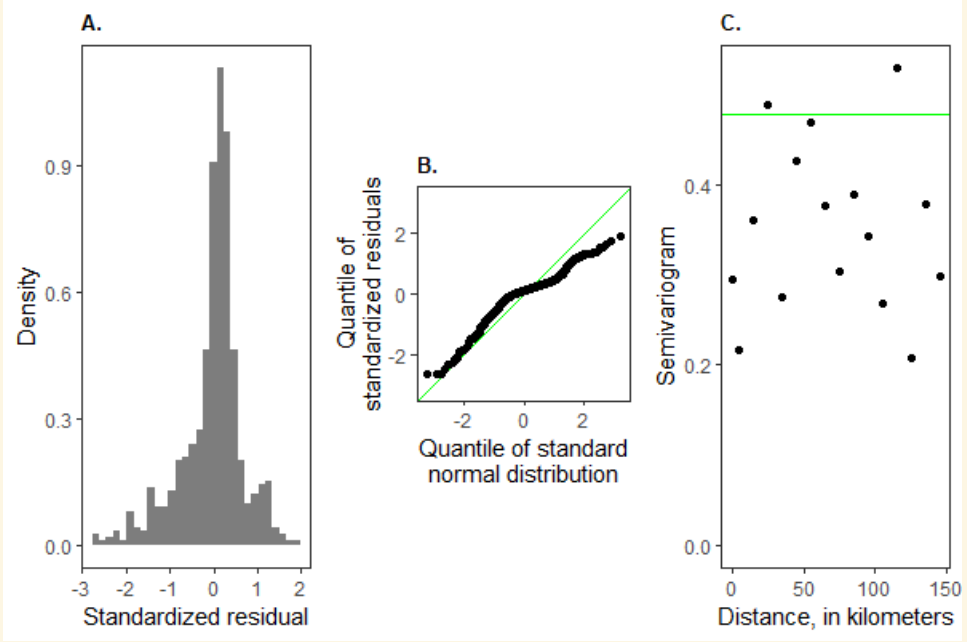
- $Y = X + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$
- $X \sim MVN(\mu 1 + B\phi, diag[(p \exp[B\psi])^2])$
- $\phi \sim MVN(0, \Sigma_\phi)$
- $\Sigma_\phi = (\tau_\phi^2[D_\phi - \alpha_\phi W_\phi]^{-1})$

Priors

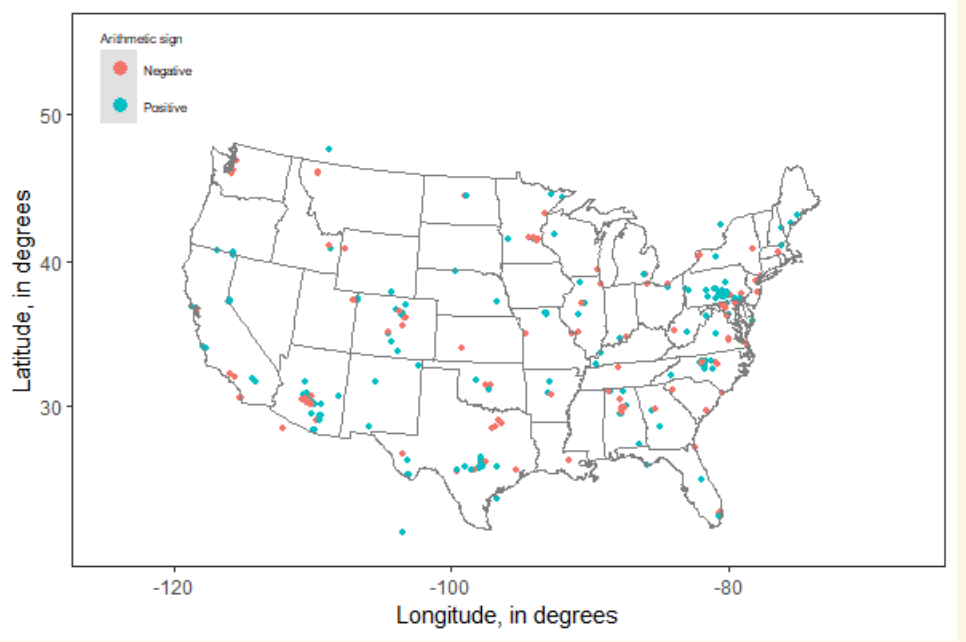
- $\alpha_\phi \sim Beta(p_1, p_2)$
- $\tau_\phi^2 \sim Gamma(q_1, q_2)$
- $\rho \sim \text{Truncated Cauchy}(0, s)$

FURTHER ATTEMPTS: NON-STATIONARY MODEL

Fitting Results



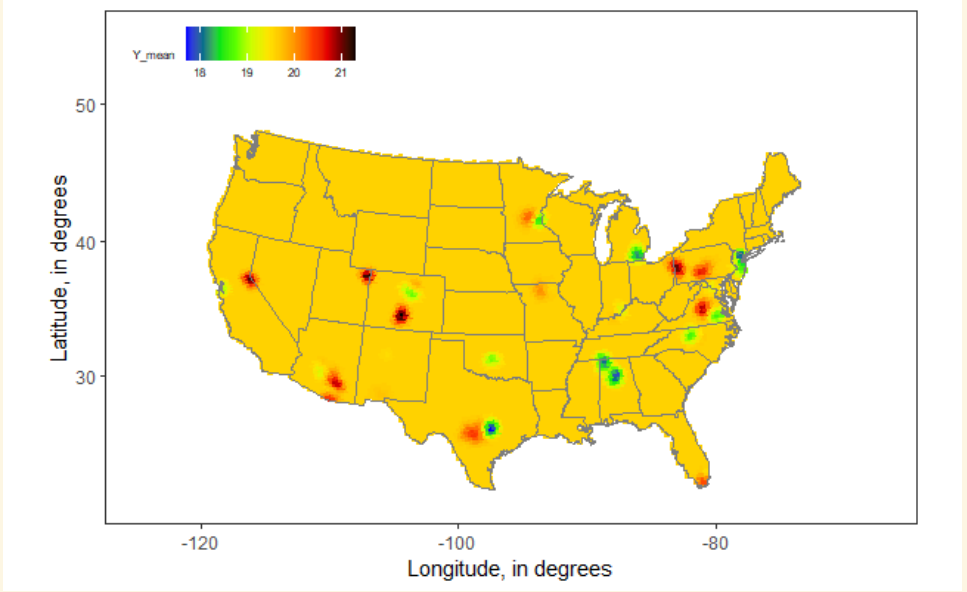
Charts of Residuals



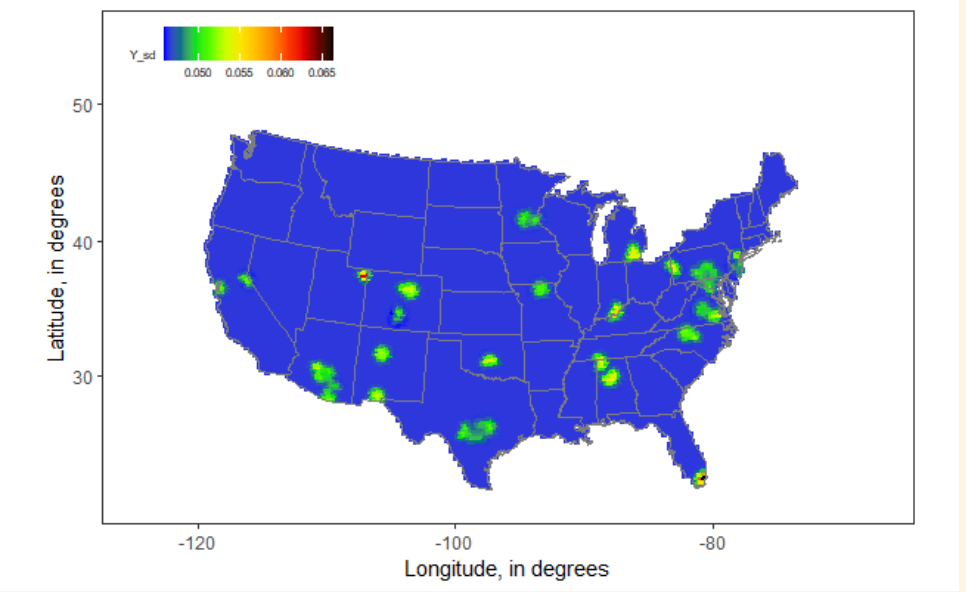
Signs of Residuals on Each Location

FURTHER ATTEMPTS: NON-STATIONARY MODEL

Prediction results



Non-Stationary Model Prediction: Mean



Non-Stationary Model Prediction: Standard Deviation

DISCUSSION AND IMPROVEMENTS

- Scale
- Cloud cover
- Static
- Response model vs Latent model

  Response model's performance on our dataset is not as good as latent model, which is probably caused by the setting of the priors since response model's nugget term is within the covariance, may consider adjust the prior of NNGP model more meticulously.

THANK YOU!