1 Preliminaries

1.1 Resources

- **Introduction to Differential Equations (online textbook) by Trefor Bazett**
  (Note: this is the same as our class textbook but with videos and exercises)
  - YT playlist
- Differential Equations (Khan Academy)
- Linear Algebra (Trefor Bazett, YT playlist)
- Linear Algebra (Khan Academy)

1.2 HW/Assignments

Homework is "required", but not submitted/graded.

MATLAB assignments submitted on independent Canvas (information).

1.3 Prerequisites

- Functions
  - Domain/range, basic classifications
- Limits
  - L'Hopital's rule
- Differentiation/Integration
  - Basic formula sheet; know basic techniques/identities
- Parametric Curves
- Linear Algebra:
  - Matricies, determinants, systems of linear equations, eigenvalues/eigenvectors

2 Differential Equations

2.1 Definition

Differential equations involve the derivative of some unknown function:

$$\frac{dx}{dt} = k \cdot x(t). \tag{1}$$

"Solving" a differential equation involves finding an equation for $x(t)$. There is usually a family of *explicit* solutions and one *general* solutions. The general solution for (1) is found by:

$$\begin{aligned} \int \frac{dx}{x(t)} &= \int k \; dt \\ \ln|x(t)| &= kt + C \\ x(t) &= Ce^{kt}. \end{aligned}$$

We can solve for C when given some initial condition $x(t_0) = x_0$.

2.2 Classifications

- Ordinary vs. partial (the function being differentiated is a function of how many variables?)
- Linear vs. nonlinear (is the function or its derivatives ever raised to a power?)
  - Note that $y^3 y'$ is a nonlinear term.
  - Order: greatest degree of a derivative.

The explicit form of first-order (linear) ODEs is:

$$\boxed{y' = f(t, y)} \tag{2}$$

2.3 Worked Example: Free Fall with Air Resistance

Example 2.1: An object in free fall will experience the forces of gravity and air friction. Newton's second law tells us:

$$\begin{aligned} F_{net} &= F_g - F_{air} \\ m\left(\tfrac{dv}{dt}\right) &= mg - kv \\ &\left(\text{let } b = \tfrac{k}{m}\right) \\ \tfrac{dv}{dt} &= g - bv. \end{aligned}$$

At this point, we define $u := g - bv$ and substitute $\frac{du}{dt} = -b\left(\frac{dv}{dt}\right)$:

$$\begin{aligned} \tfrac{-1}{b}\left(\tfrac{du}{dt}\right) &= u \\ \tfrac{du}{dt} &= -bu, \end{aligned}$$

yielding a differential equation which can be solved to yield the general equation

$$v(t) = \frac{1}{b}\left(g - Ce^{-bt}\right).$$