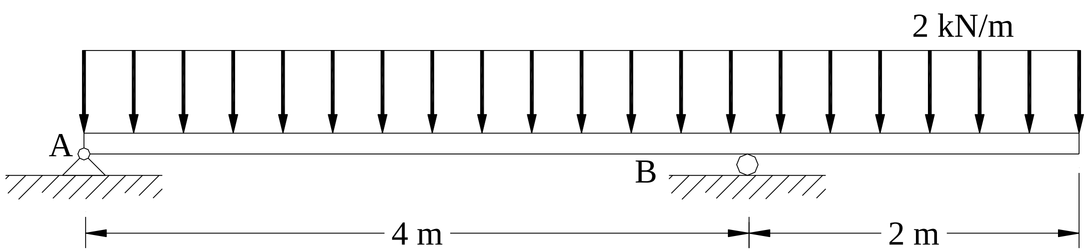
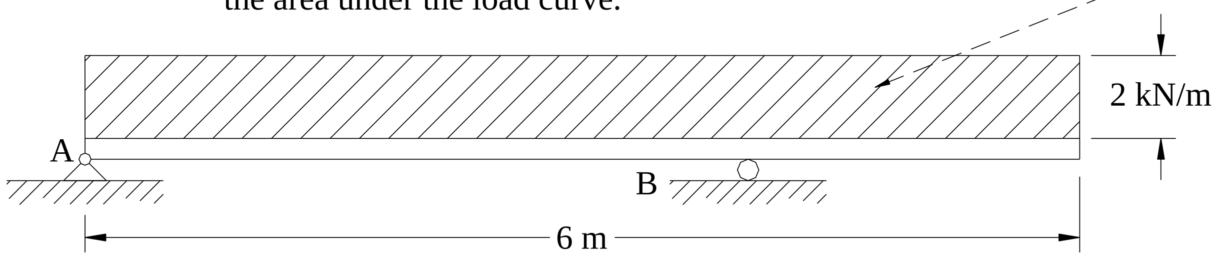
4.6 Distributed Loads on Beams

4.6 Distributed Loads on Beams Example 1, page 1 of 3

1. Determine a) the magnitude and location of the resultant of the distributed load, and b) the reactions at the supports.



1 Distributed load diagram. The magnitude R of the resultant of the distributed load equals the area under the load curve.

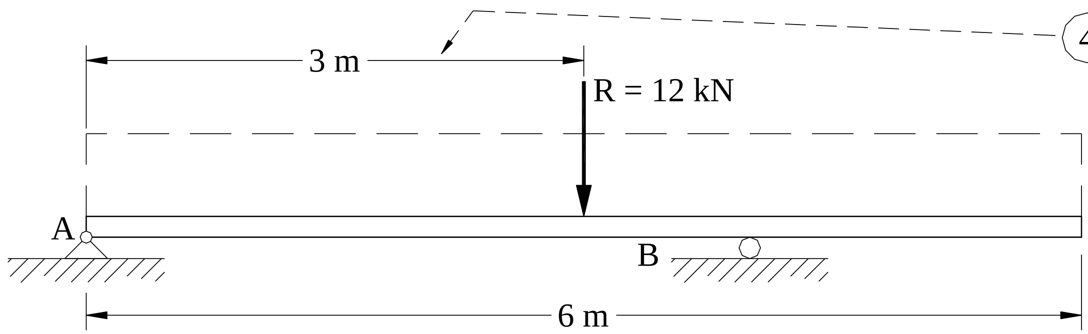


2 R = Area

= (2 kN/m)(6 m)

= 12 kN ←Ans.

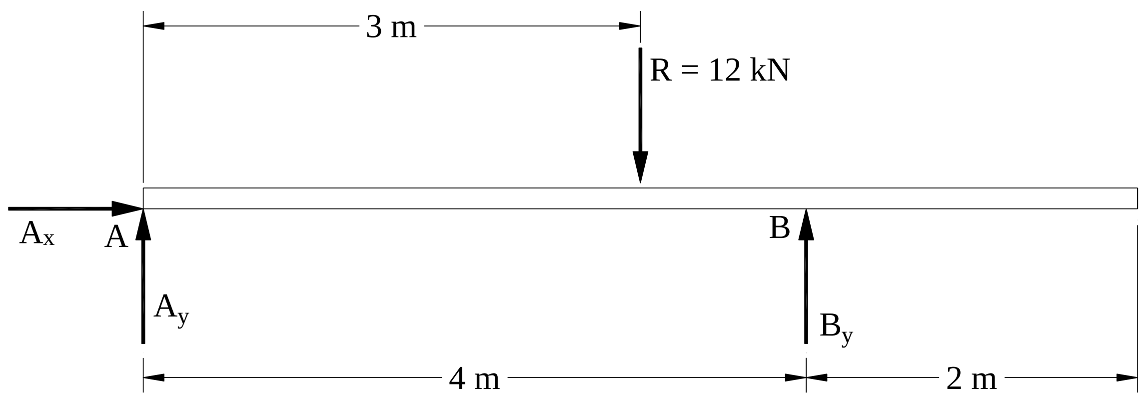
3 The line of action of the resultant, R, passes through the centroid of the load area.



4 Centroid of rectangle lies

$$\frac{6\text{ m}}{2} = 3\text{ m from A} \qquad \leftarrow\text{Ans}$$

5 Free-body diagram



4.6 Distributed Loads on Beams Example 1, page 3 of 3

6 Equations of equilibrium,

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + B_y - 12 \text{ kN} = 0$

$\circlearrowleft + \Sigma M_A = 0$: $(4 \text{ m})(B_y) - (3 \text{ m})(12 \text{ kN}) = 0$

Solving gives

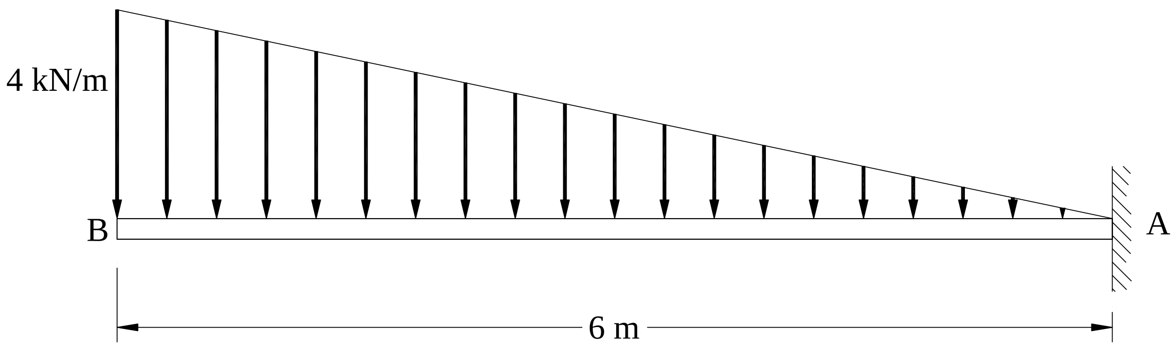
$A_x = 0$ ←Ans.

$A_y = 3 \text{ kN}$ ←Ans.

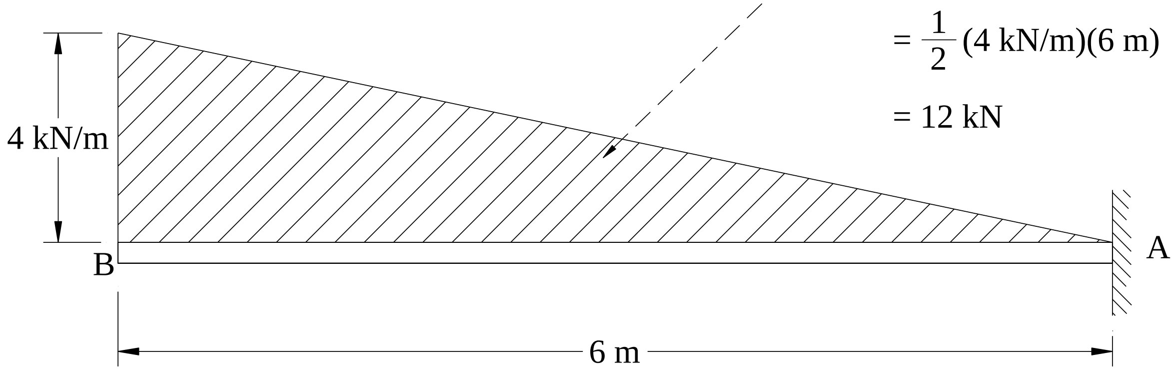
$B_y = 9 \text{ kN}$ ←Ans.

4.6 Distributed Loads on Beams Example 2, page 1 of 3

2. Determine a) the magnitude and location of the resultant of the distributed load, and b) the reactions at the support.



(1) Distributed load diagram. The magnitude R of the resultant of the distributed load equals the area under the load curve.



(2) R = Area

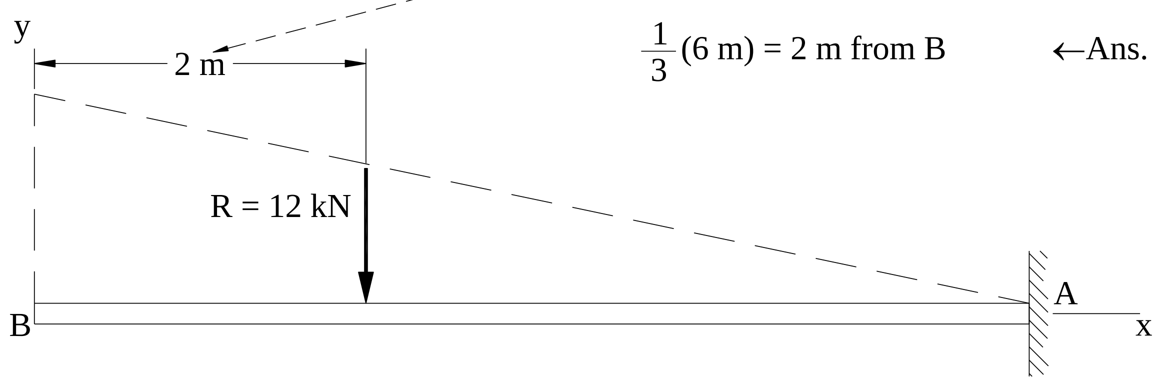
$$= \frac{1}{2}(4 \text{ kN/m})(6 \text{ m})$$

$$= 12 \text{ kN} \qquad \leftarrow \text{Ans.}$$

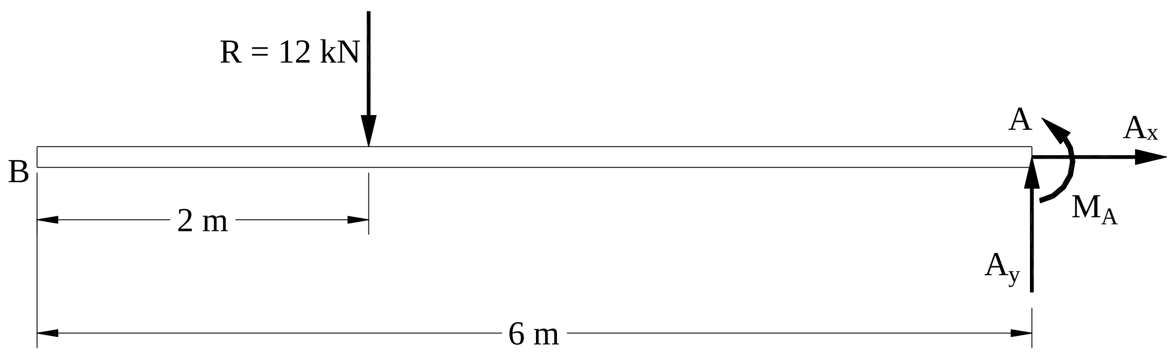
3 The line of action of the resultant, R, passes through the centroid of the triangle.

4 Centroid of triangle lies

$\frac{1}{3}$(6 m) = 2 m from B ←Ans.



5 Free-body diagram. Since the support at A is fixed, three reactions are present: A_x, A_y, and M_A.



6 Equations of equilibrium:

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y - 12\text{ kN} = 0$

$\circlearrowleft + \Sigma \overline{M}_A = 0$: $(12\text{ kN})(4\text{ m}) + M_A = 0$

Solving gives

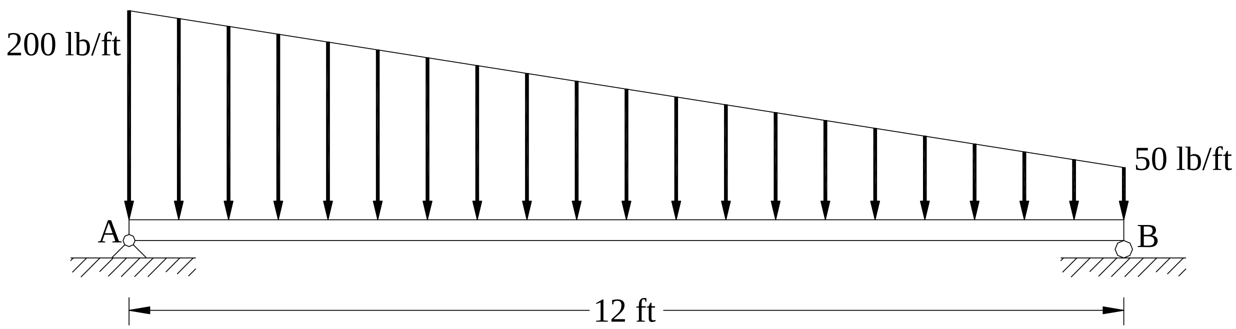
$A_x = 0$ ←Ans.

$A_y = 12\text{ kN}$ ←Ans.

$M_A = -48\text{ kN}\cdot\text{m}$ ←Ans.

4.6 Distributed Loads on Beams Example 3, page 1 of 3

3. Determine a) the magnitude and location of the resultant of the distributed load, and b) the reactions at the supports.



1 Distributed load diagram. The trapezoid loading area can be divided into rectangular and triangular loadings.

2 $F_2 = \text{Area}_2$

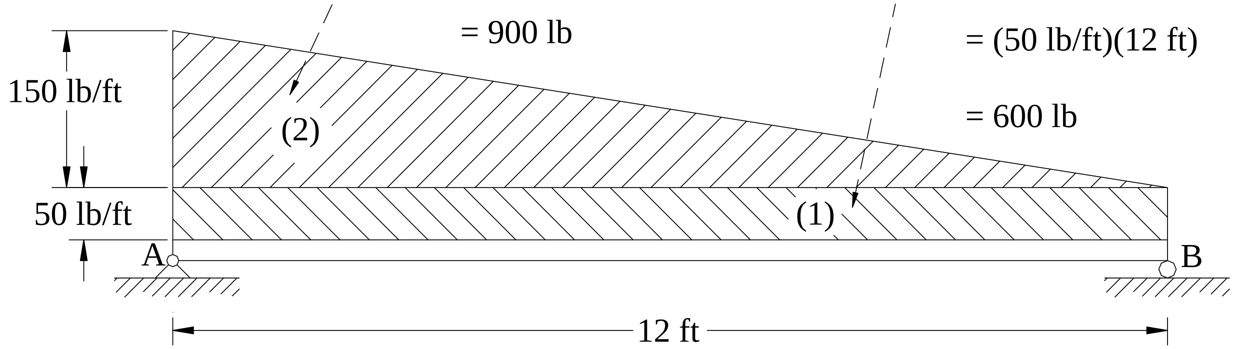
$= \frac{1}{2}(150 \text{ lb/ft})(12 \text{ ft})$

$= 900 \text{ lb}$

3 $F_1 = \text{Area}_1$

$= (50 \text{ lb/ft})(12 \text{ ft})$

$= 600 \text{ lb}$



4 A single resultant, R, can be calculated as:

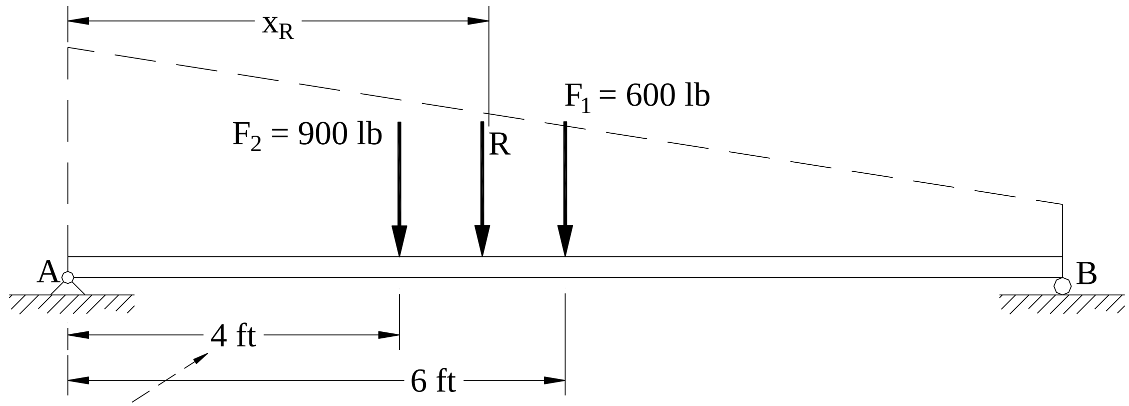
$+\downarrow R = \Sigma F_y$

$= F_1 + F_2$

$= 600 \text{ lb} + 900 \text{ lb}$

$= 1500 \text{ lb}$ ←Ans.

5 The lines of action of F_1 and F_2 pass through the centroids of the rectangular and triangular loading areas respectively.



6 Centroid of triangle lies

$\frac{1}{3}(12 \text{ ft}) = 4 \text{ ft from A}$

7 Centroid of rectangle lies $\frac{1}{2}(12 \text{ ft}) = 6 \text{ ft from A}$

8 To be equivalent, the moment about A produced by R must equal the sum of the moments about A produced by F_1 and F_2.

$\circlearrowleft + \; -x_R R = -(6 \text{ ft})F_1 - (4 \text{ ft})F_2$

where x_R is the distance of R from the support A.

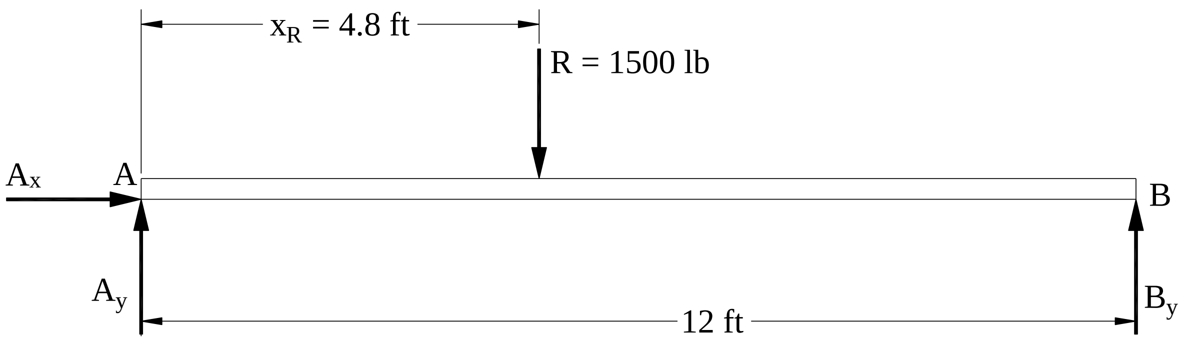
Hence,

$-x_R(1500 \text{ lb}) = -(6 \text{ ft})(600 \text{ lb}) - (4 \text{ ft})(900 \text{ lb})$

$x_R = 4.8 \text{ ft}$ ←Ans.

4.6 Distributed Loads on Beams Example 3, page 3 of 3

9 Free- body diagram



10 Equations of equilibrium

$\xrightarrow{+}$ $\Sigma F_x = 0$: $A_x = 0$

$+\uparrow$ $\Sigma F_y = 0$: $A_y + B_y - 1500 \text{ lb} = 0$

(+) $\Sigma M_A = 0$: $(12 \text{ ft})(B_y) - (1500 \text{ lb})(4.8 \text{ ft}) = 0$

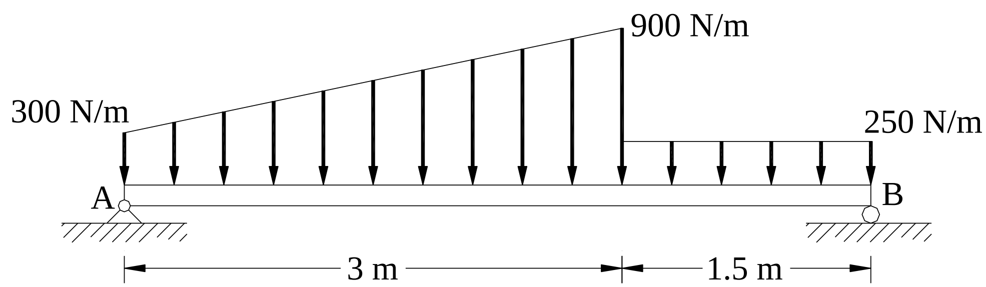
Solving gives

$A_x = 0$ ←Ans.

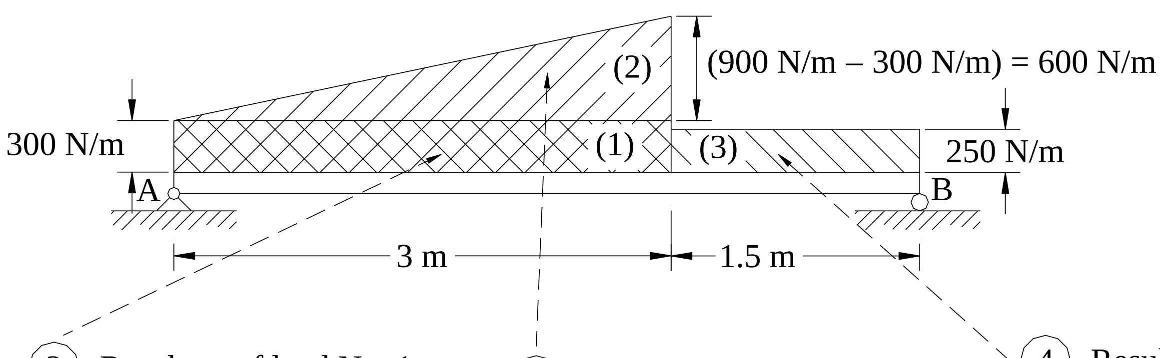
$A_y = 900$ lb ←Ans.

$B_y = 600$ lb ←Ans.

4. Determine the reactions at the supports.



1 Distributed load diagram.



2 Resultant of load No. 1

$F_1 = \text{Area}_1$

$= (300\text{ N/m})(3\text{ m})$

$= 900\text{ N}$

3 Resultant of load No. 2

$F_2 = \text{Area}_2$

$= \dfrac{1}{2}(600\text{ N/m})(3\text{ m})$

$= 900\text{ N}$

4 Resultant of load No. 3

$F_3 = \text{Area}_3$

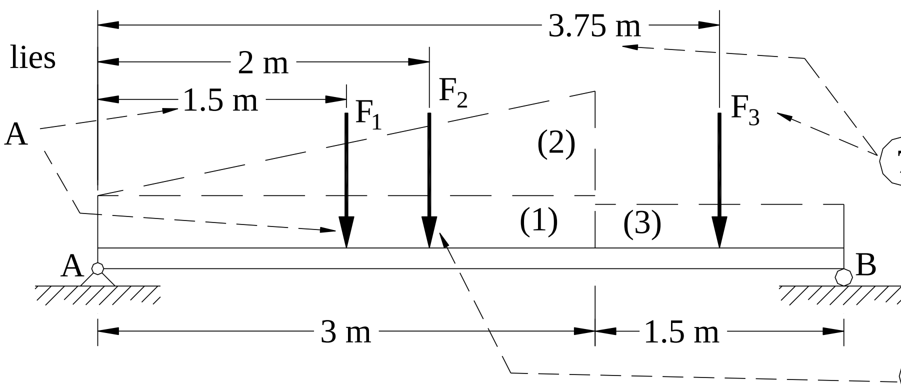
$= (250\text{ N/m})(1.5\text{ m})$

$= 375\text{ N}$

5 The lines of action of resultants F_1, F_2, and F_3 pass through the centroid of their loading areas.

6 Centroid of rectangle 1 lies

$$\frac{3\text{ m}}{2} = 1.5\text{ m from A}$$



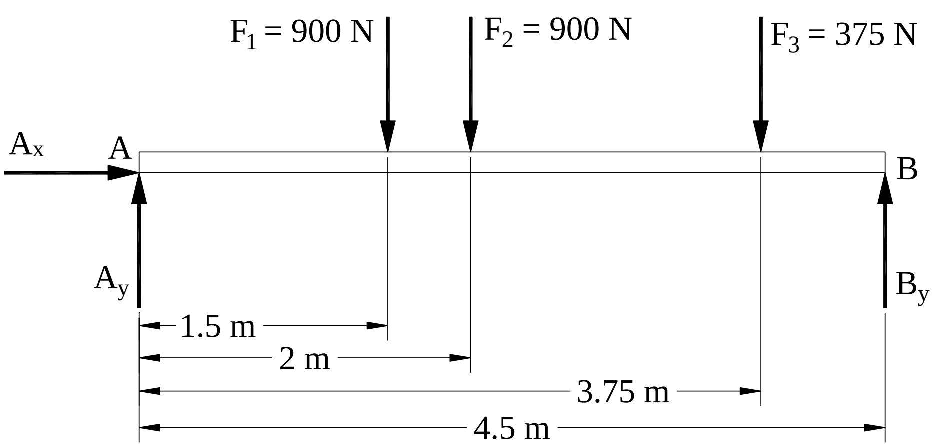
7 Centroid of rectangle 3 lies

$$3\text{ m} + \frac{1.5\text{ m}}{2} = 3.75\text{ m from A}$$

8 Centroid of triangle 2 lies

$$\frac{2}{3}(3\text{ m}) = 2\text{ m from A}$$

9 Free body diagram



10 Equations of equilibrium

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y + B_y - 900\text{ N} - 900\text{ N} - 375\text{ N} = 0$

$+\Sigma M_A = -(900\text{ N})(1.5\text{ m}) - (900\text{ N})(2\text{ m}) - (375\text{ N})(3.75\text{ m}) + B_y(4.5\text{ m}) = 0$

Solving gives

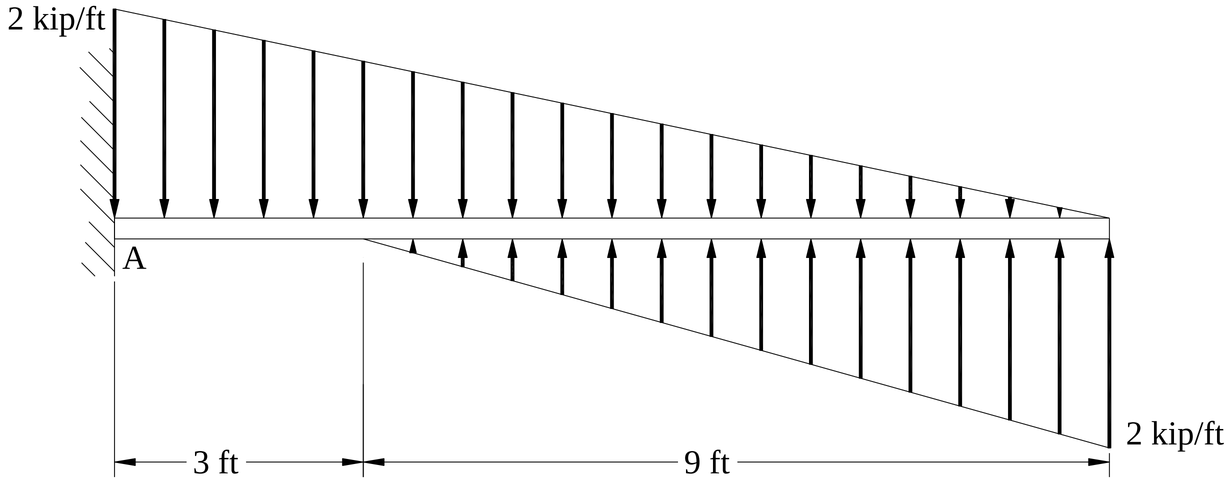
$A_x = 0$ ←Ans.

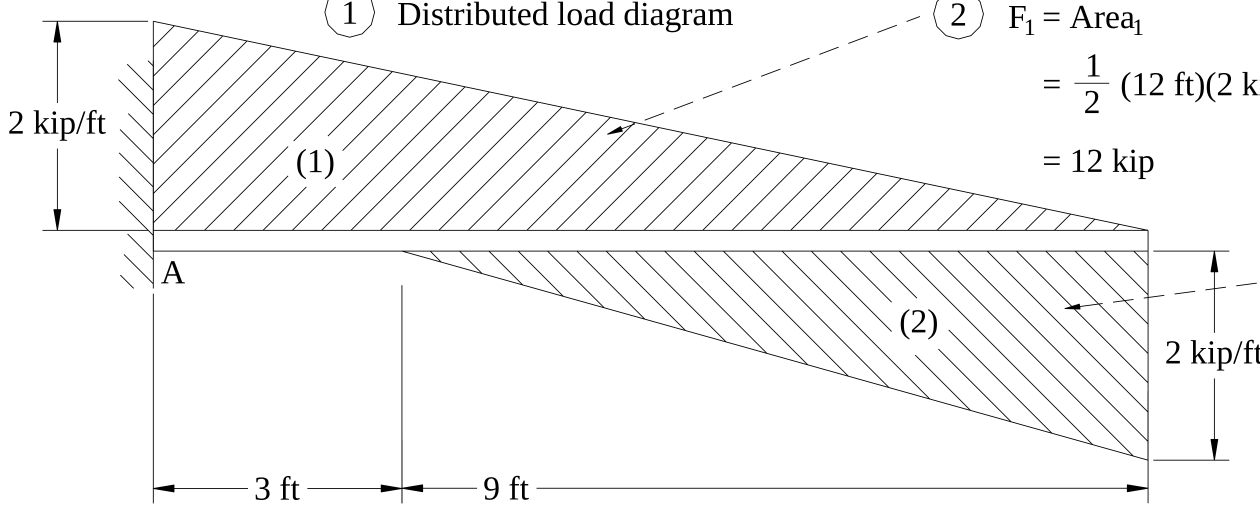
$A_y = 1162\text{ N}$ ←Ans.

$B_y = 1012\text{ N}$ ←Ans.

4.6 Distributed Loads on Beams Example 5, page 1 of 3

5. Determine the reactions at the support.





1 Distributed load diagram

2 $F_1 = \text{Area}_1$

$= \frac{1}{2}(12 \text{ ft})(2 \text{ kip/ft})$

$= 12 \text{ kip}$

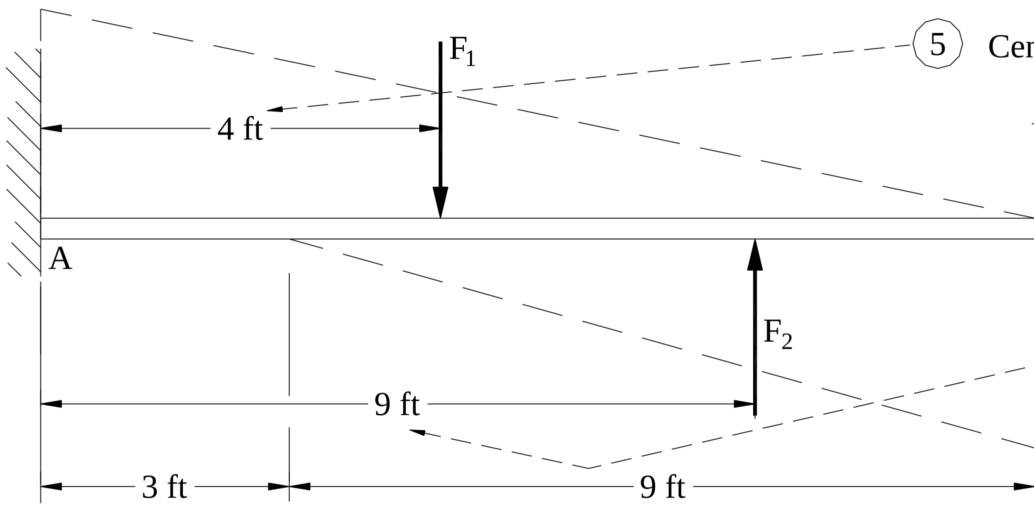
3 $F_2 = \text{Area}_2$

$= \frac{1}{2}(9 \text{ ft})(2 \text{ kip/ft})$

$= 9 \text{ kip}$

4.6 Distributed Loads on Beams Example 5, page 2 of 3

4 The lines of actions of the resultants pass through the centroids of the loading areas.



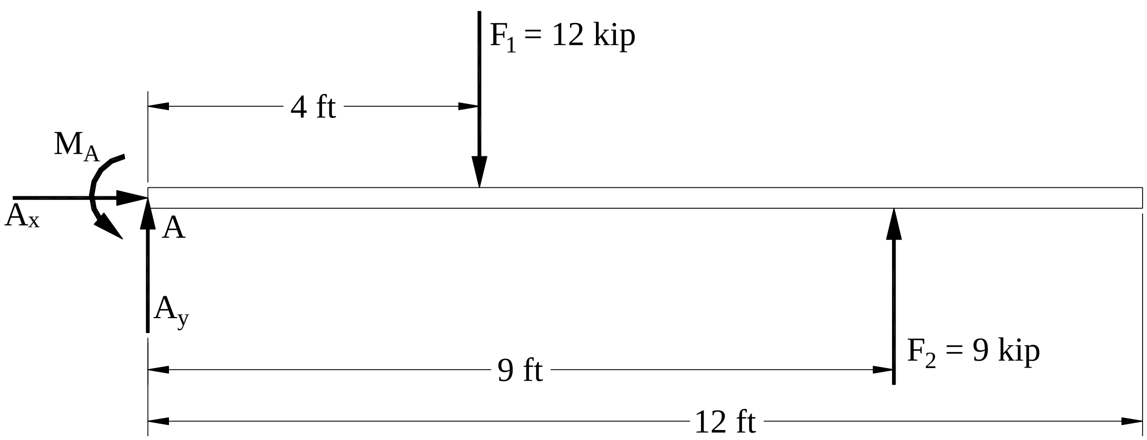
5 Centroid of triangle 1 lies

$$\frac{1}{3}(12\text{ ft}) = 4\text{ ft from A}$$

6 Centroid of triangle 2 lies

$$3\text{ ft} + \frac{2}{3}(9\text{ ft}) = 9\text{ ft from A}$$

7 Free- body diagram



8 Equations of equilibrium

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 12 \text{ kip} + 9 \text{ kip} = 0$

$+ \Sigma \overline{M}_A = 0$: $M_A - 12 \text{ kip}(4 \text{ ft}) + 9 \text{ kip}(9 \text{ ft}) = 0$

Solving gives

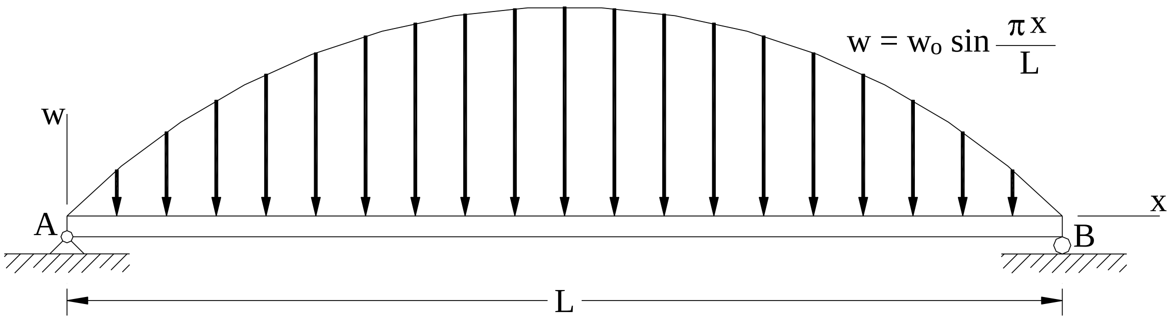
$A_x = 0$ ←Ans.

$A_y = 3$ kip ←Ans.

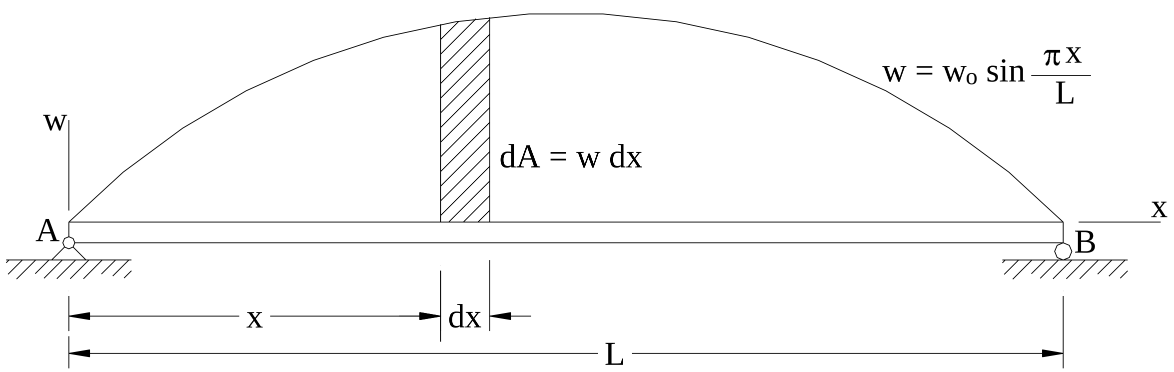
$M_A = -33$ kip·ft ←Ans.

4.6 Distributed Loads on Beams Example 6, page 1 of 3

6. Determine the reactions at the supports.



1 Distributed load diagram. The magnitude and location of the resultant of the given load will be determined by integration.

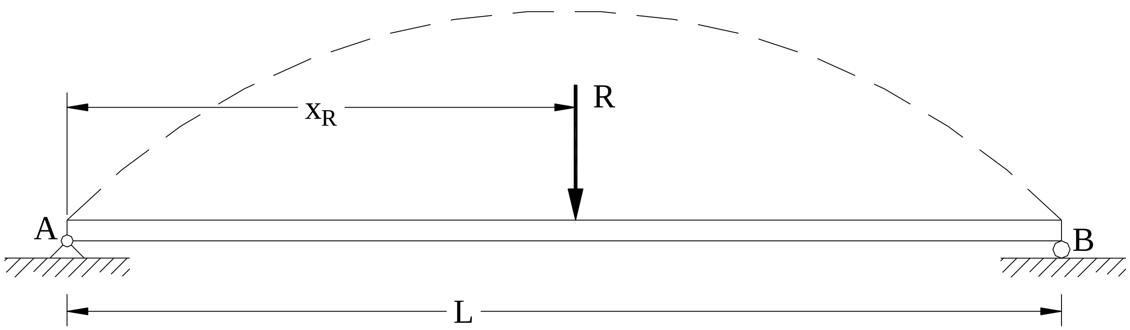


2 The resultant force $R = \int_A dA = \int_0^L w\, dx$

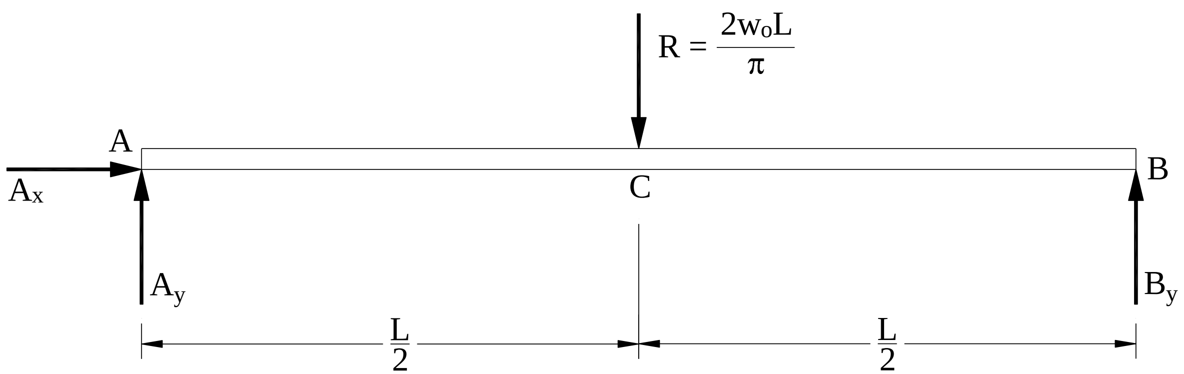
$$R = \int_0^L w_o \sin\frac{\pi x}{L}\, dx = -w_o\left[\frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^L = \frac{2w_oL}{\pi}$$

3 Resultant force diagram. The location x_R of R measured from A is

$$x_R = \frac{\int x\,dA}{\int dA} = \frac{\int_0^L x\, w_o \sin\frac{\pi x}{L}\,dx}{\frac{2w_oL}{\pi}} = \frac{L}{2}$$



4 Free- body diagram



4.6 Distributed Loads on Beams Example 6, page 3 of 3

5 Equations of equilibrium

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + B_y - \dfrac{2w_oL}{\pi} = 0$

$\circlearrowleft + \Sigma M_A = 0$: $B_yL - \left(\dfrac{2w_oL}{\pi}\right)\dfrac{L}{2} = 0$

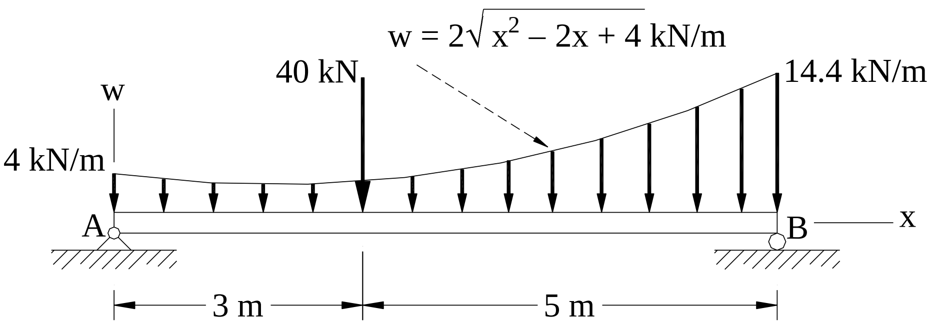
Solving gives

$A_x = 0$ ←Ans.

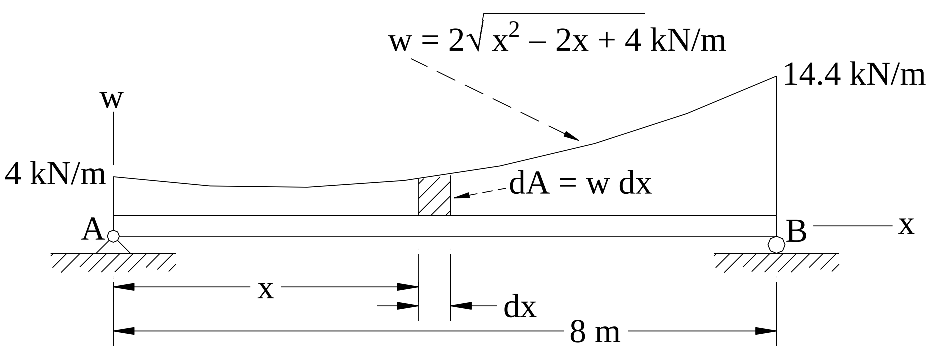
$A_y = \dfrac{w_oL}{\pi}$ ←Ans.

$B_y = \dfrac{w_oL}{\pi}$ ←Ans.

7. Determine the reactions at the supports.



1 Distributed load diagram. The magnitude and location of the force represented by the distributed load will be determined by integration.

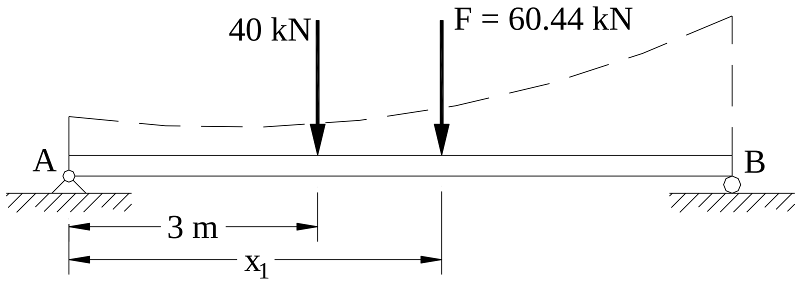


2 The force $F = \int dA = \int w\,dx$

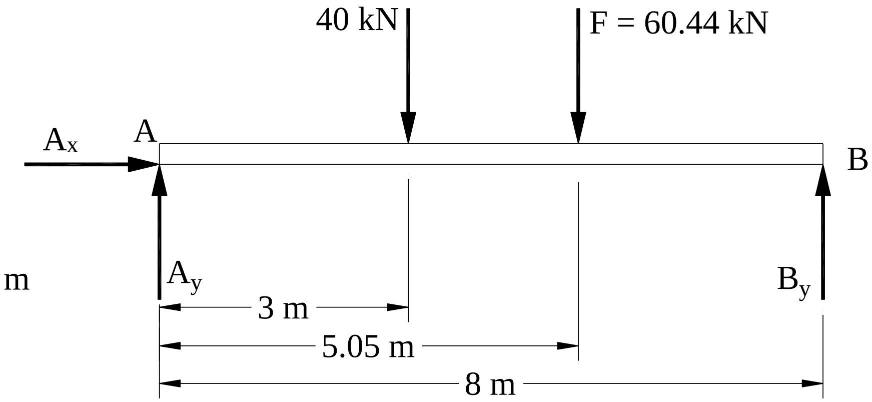
$$F = \int_0^8 2\sqrt{x^2 - 2x + 4}\,dx = 60.44 \text{ kN}$$

and F acts through the point defined by x_1:

$$x_1 = \frac{\int x\,dA}{\int dA} = \frac{\int_0^8 2x\sqrt{x^2 - 2x + 4}\,dx}{60.44} = \frac{305.09}{60.44} = 5.05 \text{ m}$$



3 Free- body diagram



4 Equations of equilibrium

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + B_y - 40 \text{ kN} - 60.44 \text{ kN} = 0$

$\circlearrowleft + \ \Sigma M_A = 0$: $B_y(8 \text{ m}) - 40 \text{ kN}(3 \text{ m}) - 60.44 \text{ kN}(5.05 \text{ m}) = 0$

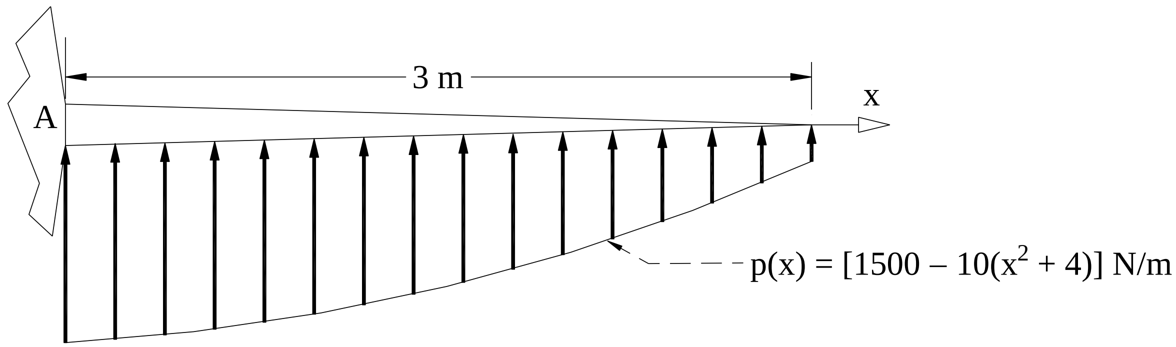
Solving gives

$A_x = 0$ ←Ans.

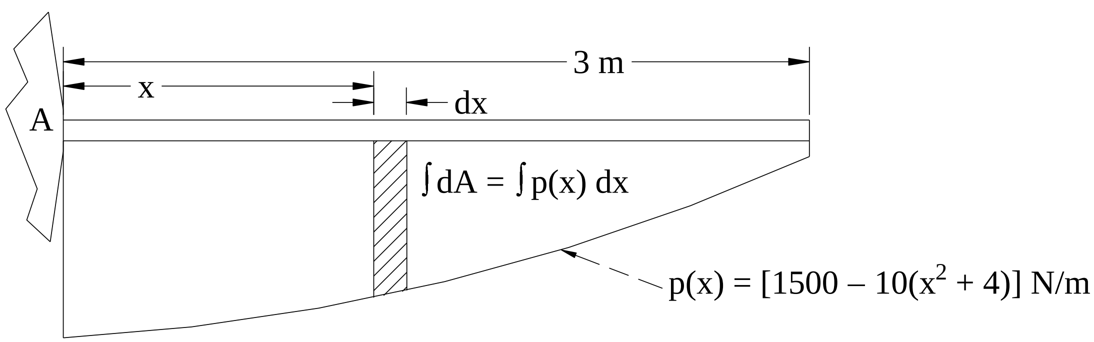
$A_y = 47.3$ kN ←Ans.

$B_y = 53.1$ kN ←Ans.

8. The lift force acting on an airplane wing can be modeled by the equation shown. Determine the force and moment at the point where the wing is attached to the fuselage.



1 Distributed load diagram. The magnitude and location of the resultant force will be determine by integration.

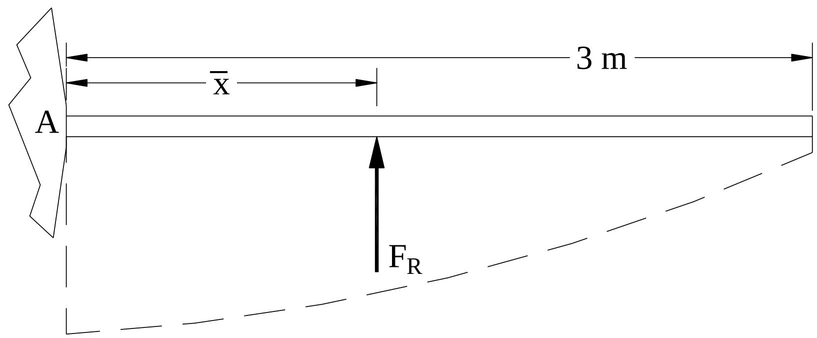


2 The resultant force R is equal to the area under the loading curve.

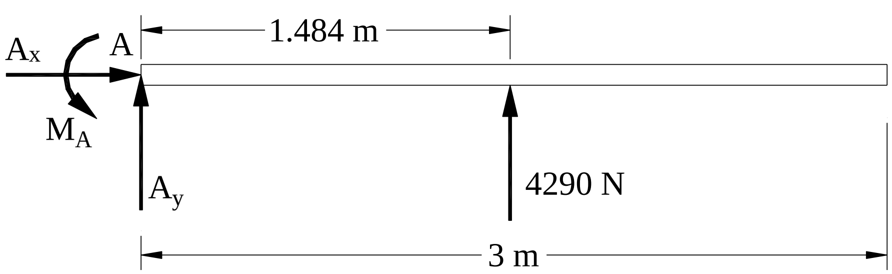
$$R = \int_A dA = \int_0^3 [1500 - 10(x^2 + 4)]\, dx = 4290 \text{ N}$$

The line of action of R passes through the point defined by x_R:

$$x_R = \frac{\int x\, dA}{\int dA} = \frac{\int_0^3 x[1500 - 10(x^2 + 4)]\, dx}{4290} = \frac{6368}{4290} = 1.484 \text{ m}$$



3 Free- body diagram



4.6 Distributed Loads on Beams Example 8, page 3 of 3

4 Equations of equilibrium

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 4290 \text{ N} = 0$

$\circlearrowleft + \Sigma \overline{M}_A = 0$: $M_A + (4290 \text{ N})(1.484 \text{ m}) = 0$

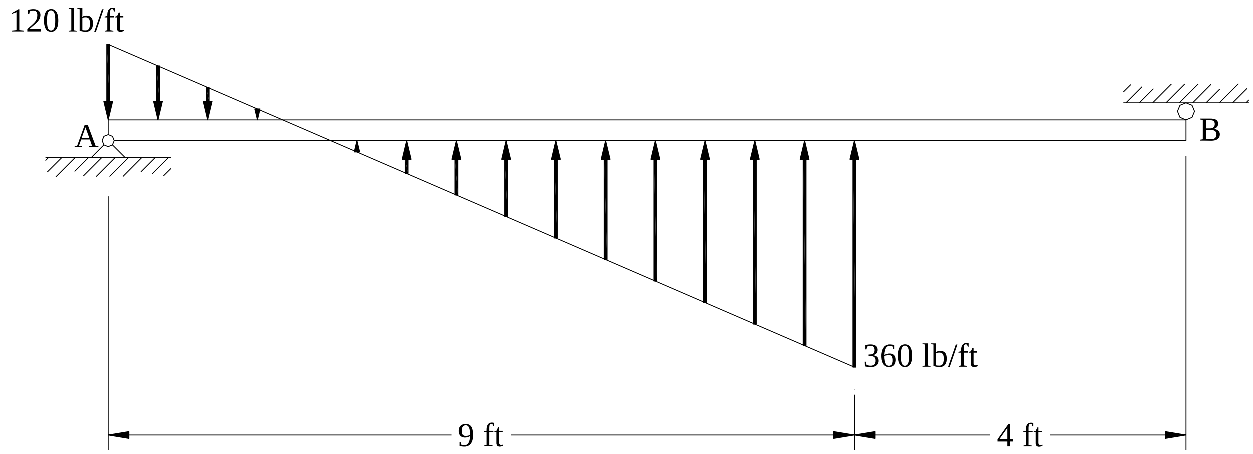
Solving gives

$A_x = 0$ ←Ans.

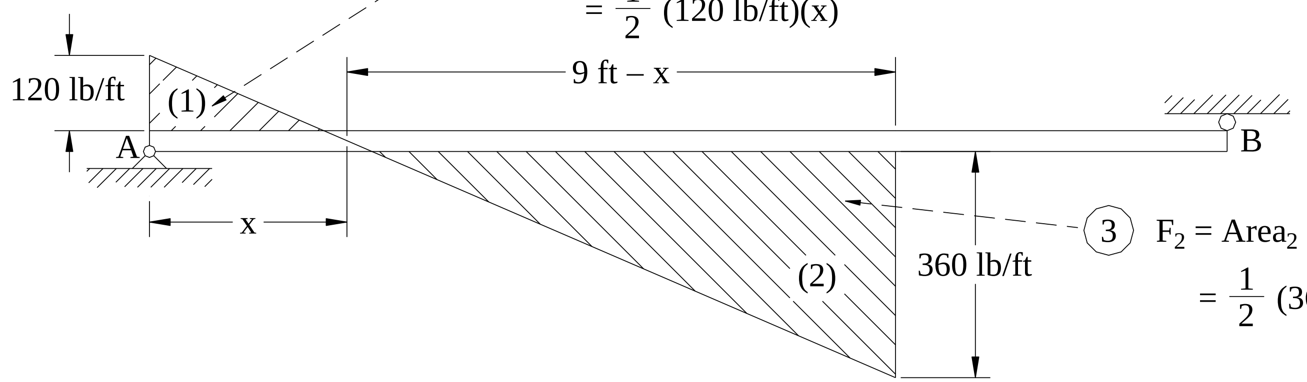
$A_y = -4290 \text{ N} = -4.29 \text{ kN}$ ←Ans.

$M_A = -6366 \text{ N}\cdot\text{m} = -6.37 \text{ kN}\cdot\text{m}$ ←Ans.

9. Determine the reactions at the supports.



1 Distributed load diagram. The distance x will be determined later.



2 $F_1 = \text{Area}_1$

$$= \frac{1}{2}(120 \text{ lb/ft})(x)$$

3 $F_2 = \text{Area}_2$

$$= \frac{1}{2}(360 \text{ lb/ft})(9 \text{ ft} - x)$$

4.6 Distributed Loads on Beams Example 9, page 2 of 3

4 From similar triangles, $\dfrac{x}{120 \text{ lb/ft}} = \dfrac{9 - x}{360 \text{ lb/ft}}$

Solving gives

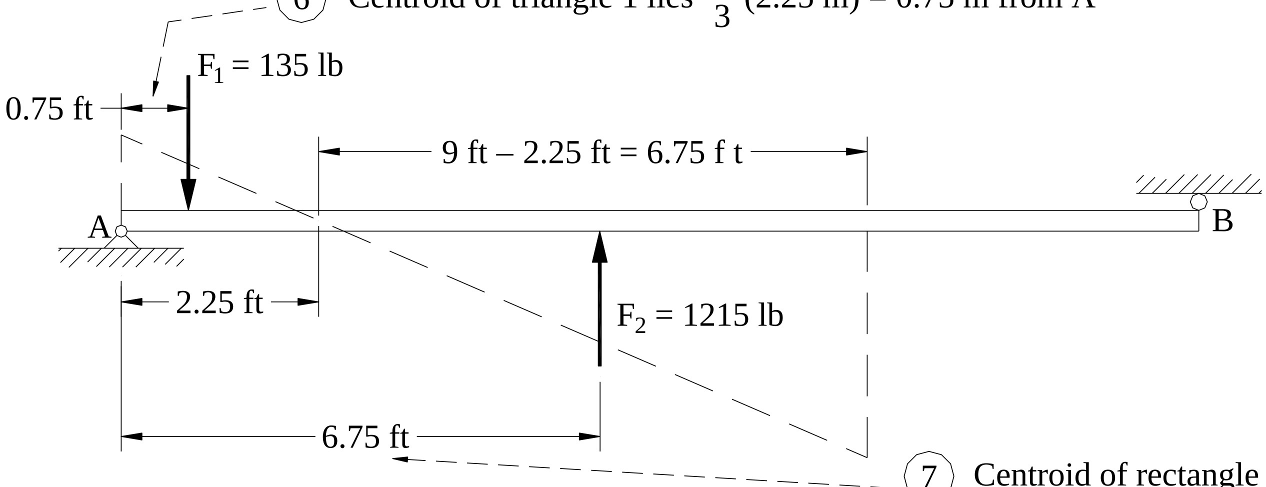
$$x = 2.25 \text{ ft.}$$

The force represented by upper triangle $F_1 = \dfrac{1}{2}(120 \text{ lb/ft})(2.25 \text{ ft}) = 135 \text{ lb.}$

The force represented by lower triangle $F_2 = \dfrac{1}{2}(360 \text{ lb/ft})(9 \text{ ft} - 2.25 \text{ ft}) = 1215 \text{ lb.}$

5 The lines of action of these forces pass through the centroids of the corresponding areas.

6 Centroid of triangle 1 lies $\dfrac{1}{3}(2.25 \text{ m}) = 0.75 \text{ m}$ from A

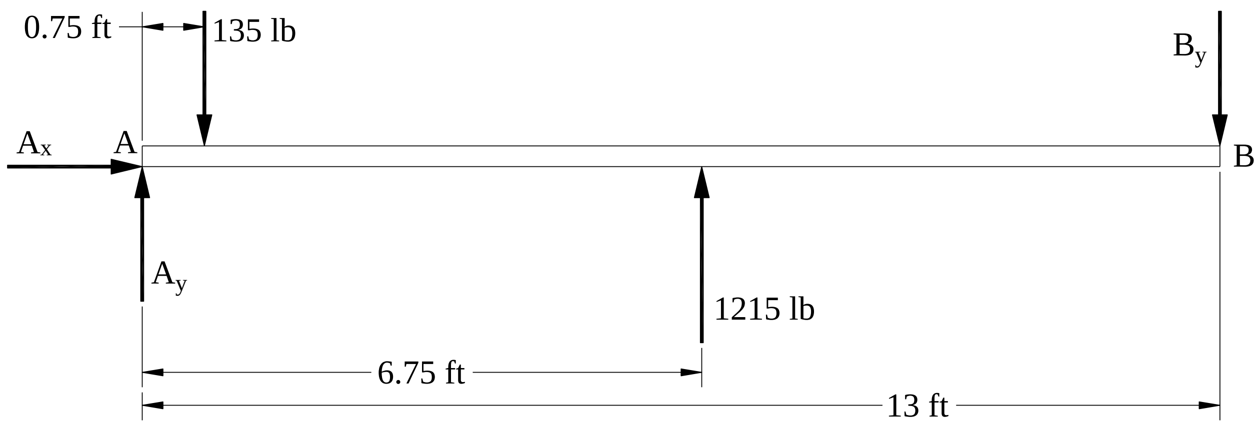


7 Centroid of rectangle 2 lies

$$2.25 \text{ ft} + \frac{2}{3}(6.75 \text{ ft}) = 6.75 \text{ ft from A}$$

4.6 Distributed Loads on Beams Example 9, page 3 of 3

8 Free-body diagram



9 Equations of equilibrium

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y - 135 \text{ lb} + 1215 \text{ lb} - B_y = 0$

$\circlearrowleft + \Sigma M_A = 0$: $-(135 \text{ lb})(0.75 \text{ ft}) + (1215 \text{ lb})(6.75 \text{ ft}) - B_y(13 \text{ ft}) = 0$

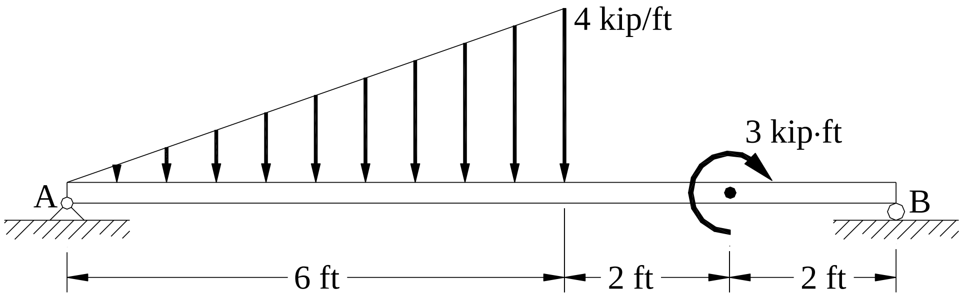
Solving we get

$A_x = 0$ ←Ans.

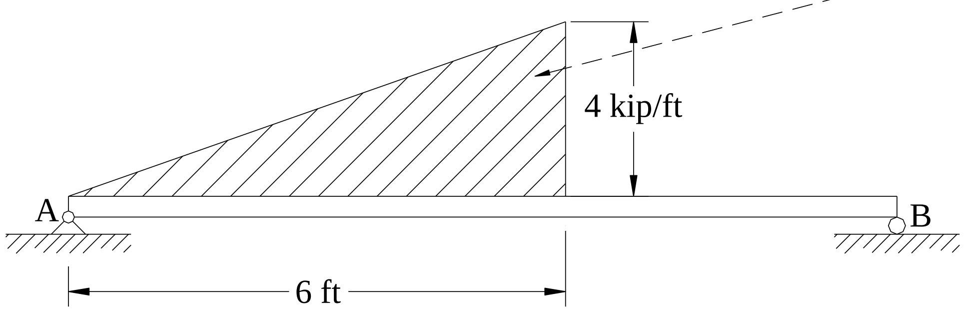
$A_y = -457$ lb ←Ans.

$B_y = 623$ lb ←Ans.

10. Determine the reactions at the supports.



1 Distributed load diagram



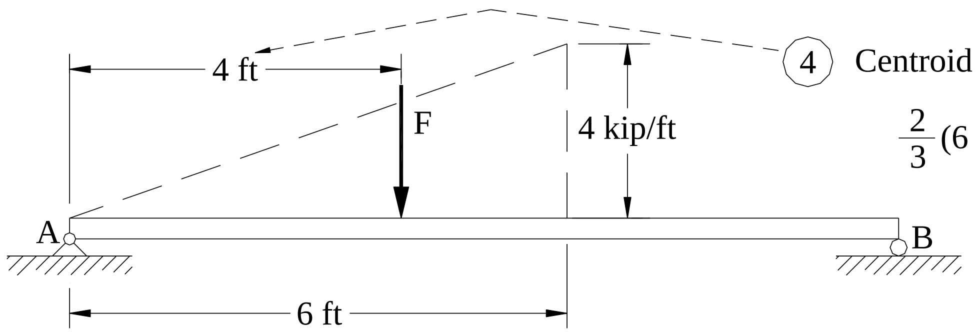
2 Resultant

F = Area

$$= \frac{1}{2}(4\text{ kip/ft})(6\text{ ft})$$

$$= 12\text{ kip}$$

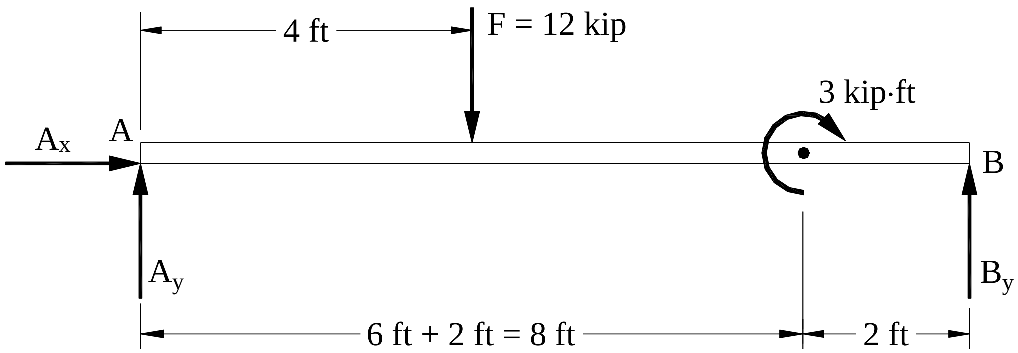
3 The line of action of the resultant F passes through the centroid of the loading area.



4 Centroid of triangle lies

$$\frac{2}{3}(6\text{ ft}) = 4\text{ ft from A}$$

5 Free-body diagram



4.6 Distributed Loads on Beams Example 10, page 3 of 3

6 Equations of equilibrium

$\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y - 12\text{ kip} + B_y = 0$

$\circlearrowleft+\ \Sigma M_A = 0$: $-(12\text{ kip})(4\text{ ft}) - (3\text{ kip}\cdot\text{ft}) + B_y(6\text{ ft} + 2\text{ ft} + 2\text{ ft}) = 0$

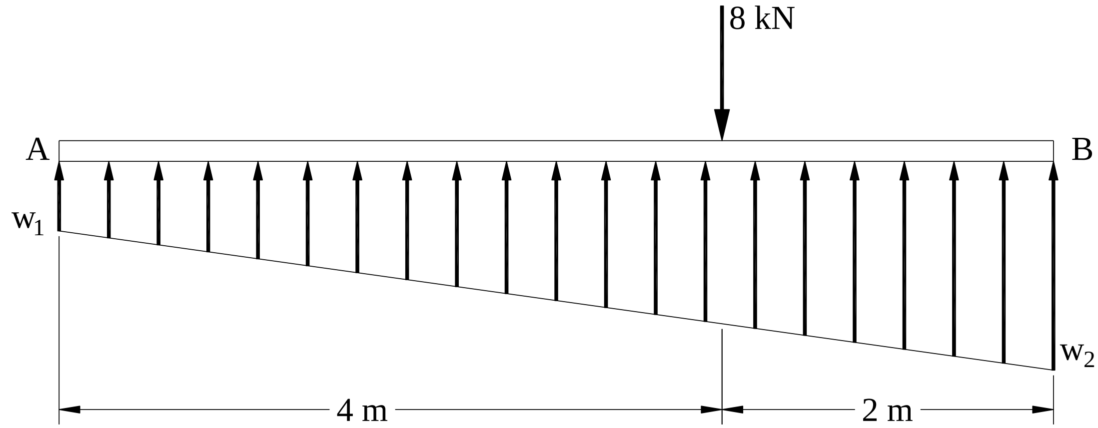
Solving gives

$A_x = 0$ ←Ans.

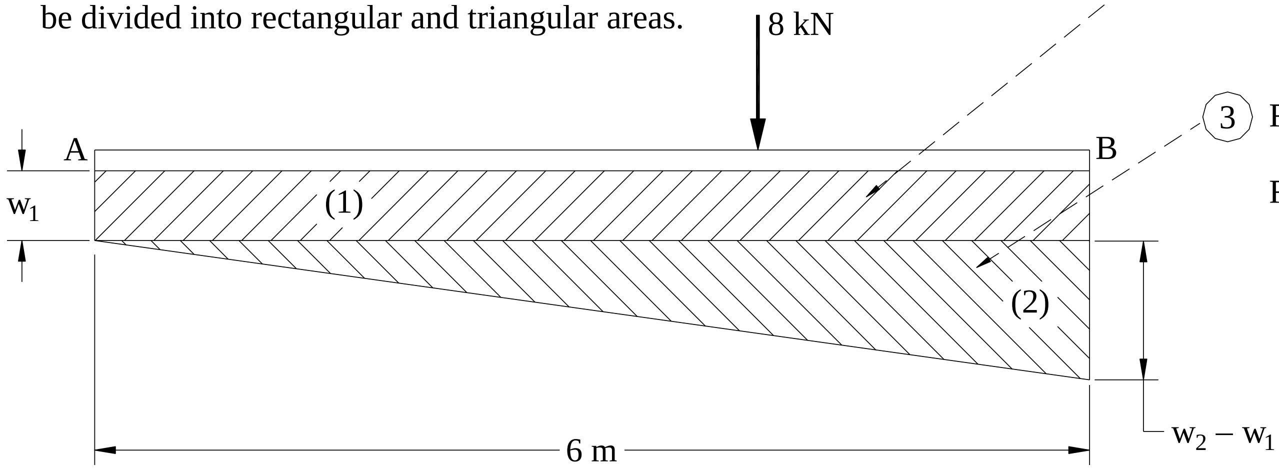
$A_y = 6.90\text{ kip}$ ←Ans.

$B_y = 5.10\text{ kip}$ ←Ans.

11. Determine w_1 and w_2 so that the beam is in equilibrium.



1 Distributed load diagram. The trapezoidal area can be divided into rectangular and triangular areas.



2 Resultant of load No. 1

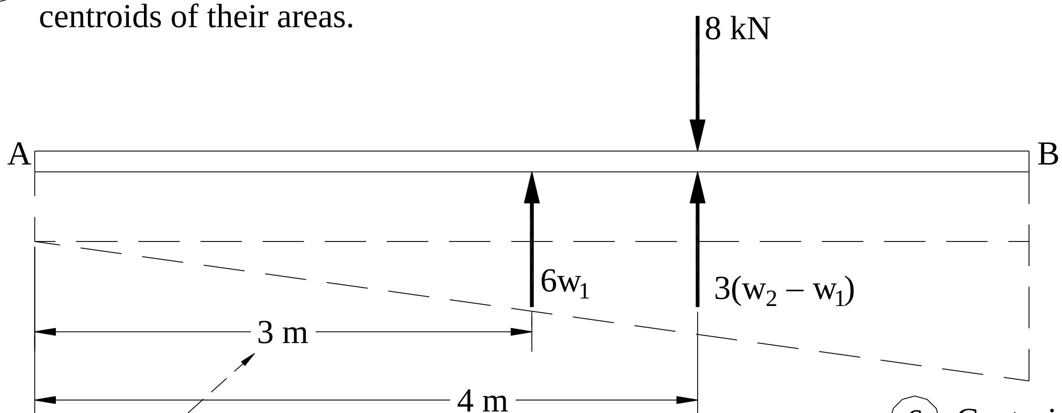
$$F_1 = \text{Area}_1$$
$$= (w_1)(6\text{ m})$$
$$= 6w_1$$

3 Resultant of load No. 2

$$F_2 = \text{Area}_2$$
$$= \frac{1}{2}(w_2 - w_1)(6\text{ m})$$
$$= 3(w_2 - w_1)$$

4.6 Distributed Loads on Beams Example 11, page 2 of 3

4 The lines of action of the resultants pass through the centroids of their areas.



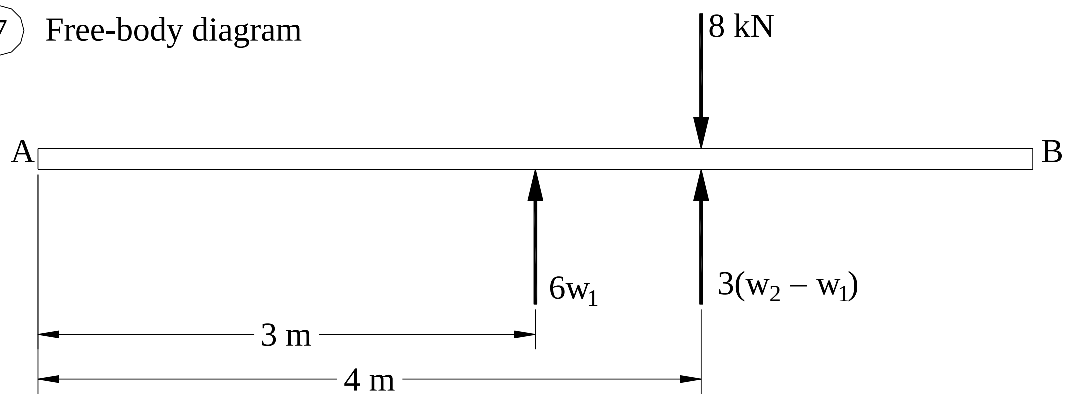
6 Centroid of triangle 2 lies

$$\frac{2}{3}(6\text{ m}) = 4\text{ m from A}$$

5 Centroid of rectangle 1 lies

$$\frac{6\text{ m}}{2} = 3\text{m from A}$$

7 Free-body diagram



8 Equations of equilibrium

$+\uparrow \Sigma F_y = 0$: $6w_1 + 3(w_2 - w_1) - 8\text{ kN} = 0$

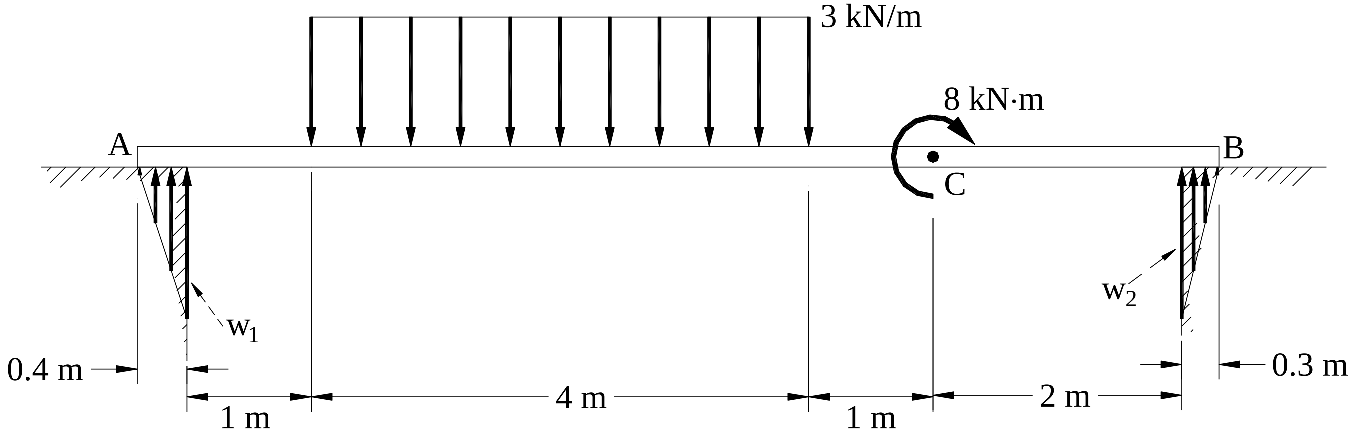
$\circlearrowleft + \Sigma M_A = 0$: $6w_1(3\text{ m}) + 3(w_2 - w_1)(4\text{ m}) - 8\text{ kN}(4\text{ m}) = 0$

Solving gives

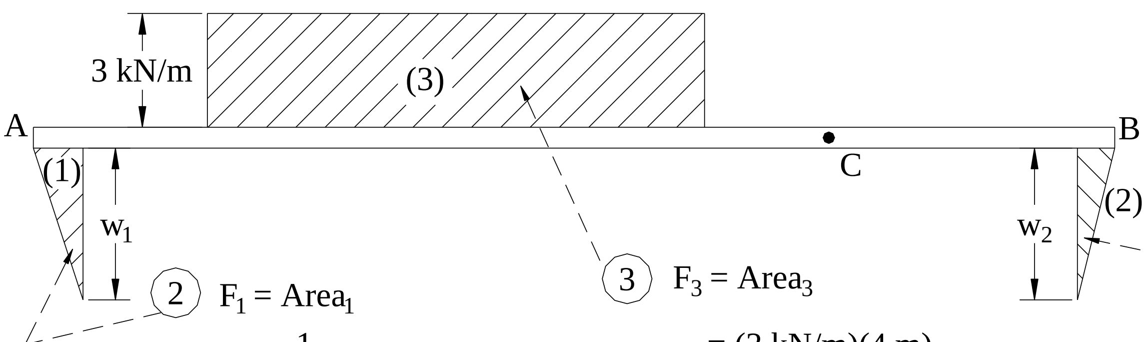
$w_1 = 0$ ←Ans.

$w_2 = \frac{8}{3}\text{ kN/m}$ ←Ans.

12. The forces from the supports are approximately represented as triangular distributed loads. Determine the values of w_1 and w_2.



1 Distributed load diagram.



2 $F_1 = \text{Area}_1$

$= \dfrac{1}{2}\, w_1(0.4\text{ m})$

$= 0.2w_1$

3 $F_3 = \text{Area}_3$

$= (3\text{ kN/m})(4\text{ m})$

$= 12\text{ kN}$

4 $F_2 = \text{Area}_2$

$= \dfrac{1}{2}\, w_2(0.3\text{ m})$

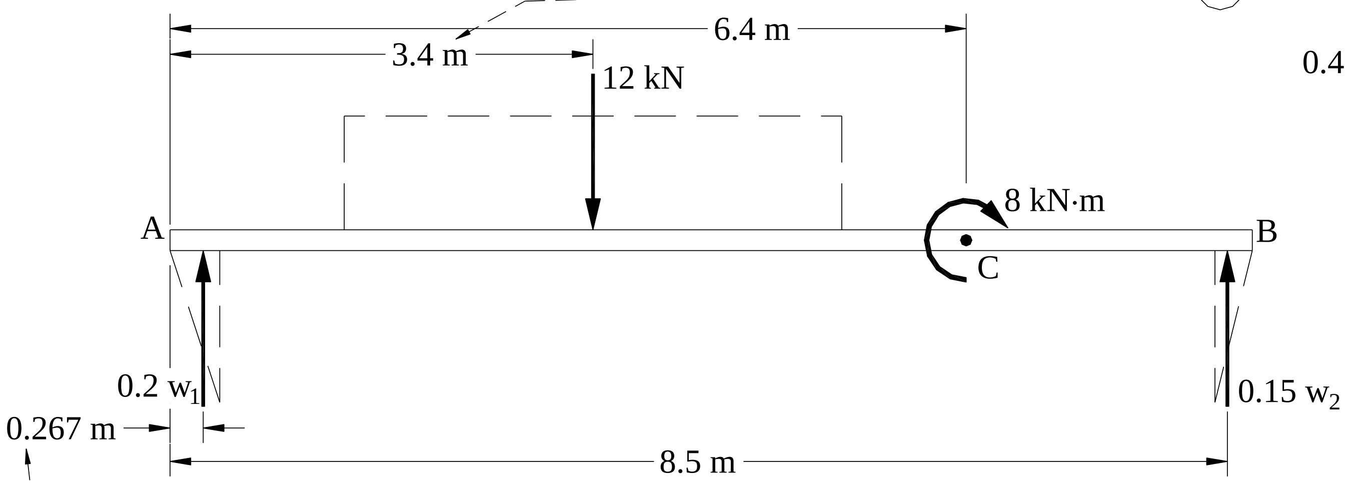
$= 0.15w_2$

4.6 Distributed Loads on Beams Example 12, page 2 of 3

5 The lines of action of the resultant forces pass through the centroids of their loading areas.

6 Centroid of rectangle lies

$$0.4 \text{ m} + 1 \text{ m} + \frac{4 \text{ m}}{2} = 3.4 \text{ m from A}$$



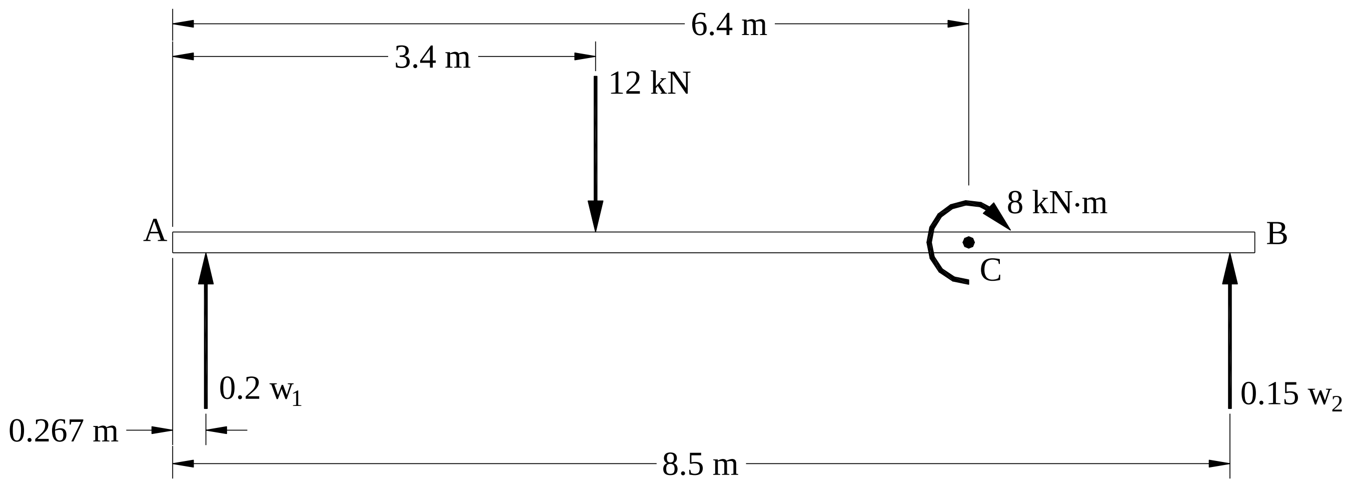
7 Centroid of triangle 1 lies

$$\frac{2}{3}(0.4 \text{ m}) = 0.267 \text{ m from A}$$

8 Centroid of triangle 2 lies

$$0.4 \text{ m} + 1 \text{ m} + 4 \text{ m} + 1 \text{ m} + 2 \text{ m} + \frac{1}{3}(0.3 \text{ m}) = 8.5 \text{ m from A}$$

9 Free- body diagram



10 Equations of equilibrium

$+\uparrow \Sigma F_y = 0$: $0.2w_1 + 0.15w_2 - 12\text{ kN} = 0$

$\circlearrowleft + \Sigma M_A = 0$: $(0.2w_1)(0.267\text{ m}) - (12\text{ kN})(3.4\text{ m}) - 8\text{ kN}\cdot\text{m} + (0.15w_2)(8.5\text{ m}) = 0$

Solving gives

$w_1 = 32.3\text{ kN/m}$ ←Ans.

$w_2 = 36.9\text{ kN/m}$ ←Ans.