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OLIY MATEMATIKA MISOL VA MASALALARDA

1-qism

ANALITIK GEOMETRIYA, CHIZIQLI ALGEBRA ASOSLARI VA ANALIZGA KIRISH

O'zbekiston faylasuflari milliy jamiyati nashriyoti

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Ushbu qo'llanma 1-qismining mazmuni dasturning analitik geometriya, chiziqli algebra asoslari va analizga kirish. bo'limlarini o'z ichiga oladi. Har bir paragrafda kerakli nazariy ma'lumotlar keltirilgan. Tipik masalalar to'liq yechimlari bilan bayon qilingan. Mustaqil yechish uchun yetarli darajada misol va masalalar keltirilgan.

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MUNDARIJA

I BOB
TEKISLIKDAGI ANALITIK GEOMETRIYA

1-§. DEKART VA QUTB KOORDINATALARI

I. To‘g‘ri chiziqdagi koordinatalar. Kesmani berilgan nisbatda bo‘lish.

x absissaga ega bo‘lgan OX koordinata o‘qining M nuqtasi $M(x)$ bilan belgilanadi. $M_1(x_1)$ va $M_2(x_2)$ nuqtalar orasidagi masofa

$$d = |x_2 - x_1| \tag{1}$$

formula bilan aniqlanadi.

Ixtiyoriy to‘g‘ri chiziqda AB (A—kesmaning boshi, B—oxiri) kesma berilgan bo‘lsin; u holda bu to‘g‘ri chiziqning ixtiyoriy C nuqtasi AB kesmani qandaydir λ nisbatda bo‘ladi, bu yerda $\lambda = \pm |AC|:|CB|$. Agar AC, CB kesmalar bir tomonga qarab yo‘nalgan bo‘lsa "+" ishora, qarama-qarshi tomonga yo‘nalgan bo‘lsa "−" ishora olinadi. Boshqacha qilib aytganda, agar C nuqta A va B nuqtalar orasida yotsa, λ musbat, tashqarida yotsa manfiy bo‘ladi.

Agar A va B nuqtalar OX o‘qida yotsa, $A(x_1)$ va $B(x_2)$ nuqtalarni l nisbatda bo‘luvchi $C(x)$ nuqtaning koordinatalari

$$\bar{x} = \frac{x_1 + \lambda x_2}{1+\lambda} \tag{2}$$

formula bilan aniqlanadi. Xususiy holda, agar $\lambda=1$ bo‘lsa, kesma o‘rtasining koordinatalari uchun

$$\bar{x} = \frac{x_1 + x_2}{2} \tag{3}$$

formula kelib chiqadi.

1. To‘g‘ri chiziqda $A(3)$, $B(-2)$, $C(0)$, $D(2)$, E$(-3{,}5)$ nuqtalarni yasang.

2. AB kesma to‘rtta nuqta bilan beshta teng bo‘lakka bo‘lingan. Agar $A(-3)$, $B(7)$ bo‘lsa, A ga yaqin turgan bo‘linish nuqtasining koordinatasini toping.

Yechish:

$C(x)$ izlangan nuqta; $\lambda=\pm|AC|:|CB| = 1/2$. Demak, (2) formuladan quyidagi ifodani topamiz:

$$\bar{x}=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{-3+\frac{1}{4}\cdot 7}{1+\frac{1}{4}}=-1, \text{ ya'ni } C(-1).$$

3. AB kesma uchlarining koordinatalari berilgan, ya'ni $A(1)$, $B(5)$, C nuqta bu kesmadan tashqarida yotadi, bu nuqtadan A gacha bo'lgan masofa undan B nuqtagacha masofadan 3 marta ko'p. C nuqtaning koordinatasi topilsin.

Yechish:

$\lambda = -|AC| : |CB|$ ligini oson ko'rish mumkin.

$$\bar{x}=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{1-3\cdot 5}{1-3}=7, \text{ ya'ni } C(7).$$

4. 1) $M(3)$ va $N(-5)$; 2) $P(-11/2)$ va $Q(-5/2)$ nuqtalar orasidagi masofani aniqlang.

5. Agar kesma oxirlarining koordinatalari:

1) $A(-6)$ va $B(7)$; 2) $C(-5)$ va $D(1/2)$ bo'lsa, uning o'rtasi koordinatalarini toping.

6. $P(2)$ nuqtaga nisbatan $N(-3)$ nuqtaga simmetrik bo'lgan M nuqtani toping.

7. AB kesma 2 nuqta orqali uchta teng qismga bo'lingan. Agar $A(-1)$, $B(5)$ bo'lsa, bo'linish nuqtalarining kordinatalari aniqlansin.

8. $A(-7)$, $B(-3)$ nuqtalar berilgan. AB kesma tashqarisida C, D nuqtalar yotadi, bunda $|AC| = |BD| = 0{,}5\ |AB|$. C va D nuqtalarning koordinatalari aniqlansin.

2. Tekislikdagi to'g'ri burchakli koordinatalar. Eng sodda masalalar.

Agar berilgan tekislikda XOY dekart koordinata sistemasi berilgan bo'lsa, x, y kordinataga ega bo'lgan M nuqtani $M(x;\ y)$ bilan belgilaymiz.

$M_1(x_1;\ y_1)$, $M_2(x_2;\ y_2)$ nuqtalar orasidagi masofa

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \tag{1}$$

formula bilan hisoblanadi. Xususiy holda *koordinata boshidan* $M(x;\ y)$ nuqtagacha bo‘lgan masofa

$$d=\sqrt{x^2+y^2} \tag{2}$$

formula bilan aniqlanadi.

$A(x_1;\ y_1)$, $B(x_2;\ y_2)$ nuqtalar orasidagi kesmani berilgan λ nisbatda bo‘luvchi $C(\bar{x},\ \bar{y})$ nuqtaning koordinatalari

$$\bar{x}=\frac{x_1+\lambda x_2}{1+\lambda}; \qquad \bar{y}=\frac{y_1+\lambda y_2}{1+\lambda} \tag{3}$$

formulalar bilan aniqlanadi.

Xususiy holda, $\lambda=1$ bo‘lganda *kesma o‘rtasining koordinatalari* quyidagi ifodalar bilan aniqlanadi:

$$\bar{x}=\frac{x_1+x_2}{2}; \qquad \bar{y}=\frac{y_1+y_2}{2}. \tag{4}$$

Uchlarining koordinatalari $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$ bo‘lgan *uchburchak yuzasi*

$$\begin{aligned} S&=\frac{1}{2}\cdot\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|= \\ &=\frac{1}{2}\cdot\left|(x_2-x_1)(y_3-y_1)+(x_3-x_1)(y_2-y_1)\right| \end{aligned} \tag{5}$$

formula yordamida topiladi.

Uchburchak yuzasi

$$S=\frac{1}{2}\Delta, \tag{6}$$

bu yerda $$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

formula bilan hisoblanadi (uchinchi tartibli determinant haqida ushbu bobning 5-§ ida berilgan).

9. Koordinata tekisligida $A(4;\ 3)$, $B(-2;\ 5)$, $C(5;\ -2)$, $D(-4;\ 3)$, $E(-6;\ 0)$ va $F(0;\ 4)$ nuqtalarni yasang.

10. $A(3;\ 8)$, $B(-5;\ 14)$ nuqtalar orasidagi masofani aniqlang.

Yechish:

(1) formuladan foydalanib, $d=\sqrt{(-5-3)^2+(14-8)^2}=10$ ni topamiz.

11. Uchlari $A(-3;\ -3)$, $B(-1;\ 3)$, $C(11;\ -1)$ bo‘lgan uchburchakning to‘g‘ri burchakli ekanligini ko‘rsating.

Yechish:

Berilgan uchburchakning tomonlari uzunliklarini topamiz:

$$|AB|=\sqrt{(-1+3)^2+(3+3)^2}=\sqrt{40},$$

$$|BC|=\sqrt{(-1+3)^2+(3+3)^2}=\sqrt{160},$$

$$|AC|=\sqrt{(11+3)^2+(-1+3)^2}=\sqrt{200}.$$

$|AB|^2=40$, $|BC|^2=160$, $|AC|^2=200$ bo‘lgani uchun $|AB|^2+|BC|^2=|AC|^2$ bo‘ladi. Shunday qilib, uchburchakning 2 tomoni kvadratlari yig‘indisi uchinchi tomoni kvadratiga teng bo‘lgani uchun ABC uchburchak to‘g‘ri burchakli.

12. AB kesma oxirlarining koordinatalari $A(-2;\ 5)$, $B(4;\ 17)$ berilgan. Bu kesmada C nuqta yotadi, bu nuqtadan A nuqtagacha bo‘lgan masofadan 2 marta katta. C nuqtaning koordinatalari topilsin.

Yechish:

$|AC| = 2|CB|$ bo‘lgani uchun, $\lambda = |AC| : |CB| = 2$. Bu yerda $x_1=-2$, $y_1=5$, $x_2=4$, $y_2=17$. Demak:

$$\bar{x}=\frac{-2+2\cdot 4}{1+2}=2,\qquad \bar{y}=\frac{5+2\cdot 17}{1+2}=13,\quad \text{ya'ni } C(2,13).$$

13. $C(2;\ 3)$ nuqta AB kesma o‘rtasi. Agar $B(7;\ 5)$ bo‘lsa, $+A$ nuqtaning koordinatalarini aniqlang.

Yechish:

$\bar{x}=2$, $\bar{y}=3$, $x_2=7$, $y_2=5$, bundan $2=(x_1+7):2$, $3=(y_1+5):2$. Demak, $x_1=-3$, $y_1=1$, ya’ni $A(-3;\ 1)$.

14. Uchburchak uchlarining koordinatalari berilgan: $A(x_1;\ y_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$. Uchburchak medianalari kesishgan nuqtasining koordinatalarini aniqlang.

Yechish:

AB kesmaning o'rtasi bo'lgan *D* nuqtaning koordinatalarini topamiz;

$$x_D = \frac{x_1 + x_2}{2}, \qquad y_D = \frac{y_1 + y_2}{2}.$$

Medianalar kesishgan *M* nuqta *CD* kesmani 2:1 nisbatda bo'ladi (C nuqtadan hisoblanganda). Demak, *M* nuqtaning koordinatalari

$$\bar{x} = \frac{x_3 + 2x_D}{1+2}, \qquad \bar{y} = \frac{y_3 + 2y_D}{1+2},$$

ya'ni

$$\bar{x} = \frac{x_3 + 2 \cdot \frac{x_1 + x_2}{2}}{3}, \qquad \bar{y} = \frac{y_3 + 2 \cdot \frac{y_1 + y_2}{2}}{3}$$

formulalar bilan aniqlanadi. Shunday qilib:

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3}, \qquad \bar{y} = \frac{y_3 + y_2 + y_3}{3}.$$

15. Uchlari $A(-2; -4)$, $B(2; 8)$, $C(10; 2)$ bo'lgan uchburchak yuzasini aniqlang.

Yechish:

(5) formulani qo'llab topamiz:

$$S = \frac{1}{2} \cdot \left|(2+2) \cdot (2+4) - (10+2) \cdot (8+4)\right| = \frac{1}{2} \cdot \left|24 - 144\right| = 60\, kv.\ b.$$

16. 1) $A(2; 3)$ va $B(-10; -2)$; 2) $C(\sqrt{2}; -\sqrt{7})$ va $D(2\sqrt{2}; 0)$ nuqtalar orasidagi masofani aniqlang.

17. Uchlari $A(4; 3)$, $B(7; 6)$, $C(2; 1)$ bo'lgan uchsburchakning to'g'ri burchakli ekanligini ko'rsating.

18. Uchlari $A(2; -1)$, $B(0; -6)$, $C(-10; -2)$ bo'lgan uchburchakning teng yonli ekanligini ko'rsating.

19. Uchlari $A(-1; -1)$, $B(0; -6)$, $C(-10; -2)$ bo'lgan uchburchak berilgan. *A* uchidan tushurilgan mediana uzunligini toping.

20. *AB* kesmaning oxirlari $A(-3, 7)$, $B(5, 11)$ berilgan. Bu kesma uchta nuqta bilan teng to'rtta bo'lakka ajratilgan. Bo'linish nuqtalarining koordinatalari topilsin.

21. Uchlari $A(1; 5)$, $B(2; 7)$, $C(4; 11)$ bo'lgan uchburchak yuzasini toping.

22. Parallelogramning uchta ketma-ket joylashgan uchlarining koordinatalari berilgan: $A(11;\ 4)$, $B(-1;\ -1)$, $C(5;\ 7)$. To'rtinchi uchining koordinatlari topilsin.

23. Uchburchak ikki uchi: $A(3;\ 8)$, $B(10;\ 2)$ va medianalarning kesishgan nuqtasi $M(1;\ 1)$ berilgan. Uchburchak uchinchi uchining koordinatlarini toping.

24. Uchlari $A(7;\ 2)$, $B(1;\ 9)$, $C(-8;\ -11)$ bo'lgan uchburchak berilgan. Medianalari kesishgan nuqtasini toping.

25. L(0; 0), $M(3;\ 0)$, $N(0;\ 4)$ nuqtalar uchsburchak tomonlari o'rtalarining koordinatalari. Uchburchak yuzasini hisoblang.

3. Qutb koordinatalari.

Qutb koordinatalarida M nuqtaning o'rni uning O qutbidan masofasi $|OM| = \rho$ ($\rho-$ *nuqtaning qutb radius-vektori*) va OM kesmaning qutb o'qi OX bilan tashkil qilgan burchagi θ ($\theta-$ nuqtaning qutb burchagi) bilan aniqlanadi. Qutb o'qidan soat strelkasiga qarama-qarshi olingan θ burchak musbat hisoblanadi.

Agar M nuqta qutb koordinatalariga ega bo'lsa ($\rho > 0,\ 0 \le \theta\, 2\pi$), u holda unga cheksiz ko'p $(\rho,\ \theta + 2k\pi)$, qutb koordinatlari jufti to'g'ri keladi, bunda $k \in Z$.

Agar dekart koordinat sistemasining koordinat boshini qutbga, OX o'qini qutb o'qi bo'yicha yo'naltirsak, u holda M nuqtaning to'g'ri burchakli $(x;\ y)$ koordanatalari bilan$(\rho,\ \theta)$ qutb koordinatalari o'rtasida bog'lanish quyidagi:

$$x = \rho\cos\theta, \qquad y = \rho\sin\theta; \tag{1}$$

$$\rho = \sqrt{x^2 + y^2}, \qquad tg\theta = \frac{y}{x} \tag{2}$$

formulalar bilan ainqlanadi.

26. Qutb koordinatlarida berilgan quyidagi:

$$A(4;\ \frac{\pi}{4}), \qquad B(2;\ \frac{4\pi}{3}), \qquad C(3;\ -\frac{\pi}{6}), \qquad D(-3;\ \frac{\pi}{3}),$$

$$E(0;\ \alpha), \qquad F(-1;\ -\frac{3\pi}{4})$$

nuqtalarini chizing.

27. Agar qutb koordinat boshi bilan, qutb o‘qi musbat abssissa o‘qi bilan ustma-ust tushsa, $M(1;\ -\sqrt{3})$ nuqtaning qutb koordinatlarini toping.

Yechish:

(2) formulaga asosan $\rho=\sqrt{1^2+(-\sqrt{3})^2}=2, tg\theta=-\sqrt{3}$. M nuqta to‘rtinchi chorakda joylashgani uchun $\theta=\frac{5\pi}{3}$. Demak, $M(2;\ \frac{5\pi}{3})$.

28. Agar qutb koordinat boshi bilan, qutb o‘qi absissa o‘qi bilan ustma-ust tushsa, $A(2\sqrt{2},\ \frac{3\pi}{4})$ nuqtaning to‘g‘ri burchakli koordinatasini toping.

Yechish:

(1) formuladan foydalanib topamiz:

$$x=2\sqrt{2}\cdot\cos(\frac{3\pi}{4})=-2$$

$$y=2\sqrt{2}\cdot\sin(\frac{3\pi}{4})=2.$$

Demak, $A(-2;\ 2)$.

29. $A(2\sqrt{3};\ 2)$, $B(0;\ -3)$, $C(-4;\ 4)$, $D(\sqrt{2};\ -\sqrt{2})$, $E(-\sqrt{2};-\sqrt{6})$, $F(-7;\ 0)$ nuqtalarning qutb koordinatalarini toping.

30. $$A(10;\ \frac{\pi}{2}),\quad B(2;\ \frac{5\pi}{4}),\quad C(0;\ \frac{\pi}{10}),$$
$$D(1;\ -\frac{\pi}{4}),\quad E(-1;\ \frac{\pi}{4}),\quad F(1;\ -\frac{\pi}{4})$$

nuqtalarning to‘g‘ri burchakli koordinatalarini toping.

31. $M_1(\rho_1;\ \theta_1)$ va $M_2(\rho_2;\ \theta_2)$ nuqtalar orasidagi masofani toping.

Ko‘rsatma: OM_1M_2 uchburchakka kosinuslar teoremasini qo‘llang.

32. $M(3;\frac{\pi}{4})$ va $N(4;\frac{3\pi}{4})$ nuqtalar orasidagi masofani toping.

33. Qutb o'qiga nisbatan $M(\rho,\theta)$ ga simmetrik bo'lgan nuqtaning qutb koordinatalarini toping.

34. Qutbga nisbatan $M(\rho,\theta)$ ga simmetrik bo'lgan nuqtaning qutb koordinatalarini toping.

35. $(3,\frac{\pi}{6})$; $(5,\frac{2\pi}{3})$; $(2,-\frac{\pi}{6})$ nuqtalarga 1) qutbga; 2) qutb o'qiga simmetrik bo'lgan nuqtalarning qutb koordintalarini toping.

36. Qutbdan o'tib, qutb o'qiga perpendikulyar bo'lgan to'g'ri chiziqqa nisbatan $M(\rho,\theta)$ nuqtaga simmetrik bo'lgan nuqtaning qutb koordinatalarini toping.

4. Chiziq tenglamasi.

xOy tekisligida biror chiziqni nuqtalar to'plami deb qarasak, unga bu chiziqda yotgan ixtiyoriy $M(x,y)$ nuqta koordinatalarini bog'lovchi tenglama to'g'ri keladi. Bunday tenglama *berilgan chiziqning tenglamasi* deb ataladi.

Agar berilgan chiziqning tenglamasiga bu chiziqda yotgan ixtiyoriy nuqtaning koordinatalarini qo'ysak, uni ayniyatga aylantiradi. Chiziqdan tashqaridagi nuqtaning koordinatalari bu chiziq tenglamasini qanoatlantirmaydi.

37. Kesmaning bir oxiri abssissa o'qi, ikkinchi oxiri ordinata o'qi bo'yicha harakatlanadi. Agar kesma uzunligi C - ga teng bo'lsa, bu kesma o'rtasi orqali chiziladigan chiziq tenglamasini toping.

Yechish:

$M(x; y)$ kesmaning o'rtasi bo'lsin. OM kesma (mediana uzunligi) uchburchakgipotenuzasining yarmiga, ya'ni OM kesmaning uzunligi c/2 ga teng. Boshqa tomondan, $|OM|=\sqrt{x^2+y^2}$ (koordinata boshi bilan M nuqta orasidagi masofa). Shunday qilib, $\sqrt{x^2+y^2}=\frac{c}{2}$, yoki $x^2+y^2=\frac{c^2}{4}$. Bu izlangan egri chiziq tenglamasidir. Geometrik jihatdan bu markazi koordinata boshida radiusi $c/2$ bo'lgan aylanadir.

38. Har bir nuqtasidan F(0; 1/4) nuqtagacha, bu nuqtadan $y=-1/4$ to'g'ri chiziqqacha bo'lgan masofalari teng bo'lgan chiziq tenglamasini tuzing.

Yechish:

Izlangan chiziqda ixtiyoriy $M(x, y)$ nuqtani olamiz. M va F nuqtalar orasidagi masofa:

$$|MF|=\sqrt{(x-0)^2+(y-1/4)^2}$$

formula yordamida topiladi. M nuqtadan $y=-1/4$ to'g'ri chiziqqacha bo'lgan masofani oddiy geometrik fikrlashdan topamiz (1-chizma):

$$|MN| = |MK| + |KN| = y+1/4.$$

$|MF| = |MN|$ tenglik izlangan chiziqning ixtiyoriy M nuqtasi uchun o'rinli bo'lgani uchun bu chiziq tenglamasini ushbu

$$\sqrt{x^2+(y-1/4)^2}=y+1/4$$

ko'rinishda yozish mumkin yoki

$$x^2+y^2-\frac{1}{2}y+\frac{1}{16}=y^2+\frac{1}{2}y+\frac{1}{16},$$

ya'ni, $y=x^2$. $y=x^2$ bilan aniqlanadigan chiziq *parabola* deb ataladi.

39. $F_1(a; 0)$, $F_2(-a; 0)$ nuqtalargacha bo'lgan masofalar ko'paytmasi o'zgarmas son a^2/ga teng bo'lgan nuqtalar to'plami tenglamasini tuzing.

Yechish:

Izlangan egri chiziqda ixtiyoriy $M(x; y)$ nuqtani olamiz. Bu nuqtadan F_1 va F_2 nuqtalargacha bo'lgan masofalar

$$r_1=\sqrt{(x-a)^2+y^2}, \quad r_2=\sqrt{(x+a)^2+y^2} \text{ ga teng.}$$

Masalaning shartidan $r\cdot r_2=a^2$ ligi kelib chiqadi. Shunday qilib, izlangan egri chiziq tenglamasi:

$$\sqrt{(x-a)^2+y^2}\cdot\sqrt{(x+a)^2+y^2}=a^2,$$

$$[(x-a)^2+y^2][(x+a)^2+y^2]=a^4,$$

$$(x^2-2ax+a^2+y^2)(x^2+2ax+a^2+y^2)=a^4,$$

$$(x^2+y^2+a^2-2ax)(x^2+y^2+a^2-2ax)=a^4,$$

$$(x^2+y^2+a^2)^2-4a^2x^2=a^4,$$

$$(x^2+y^2)^2=2a^2(x^2-y^2).$$

Topilgan egri chiziq *lemniskata* deb ataladi.

40. Lemniskata tenglamasini qutb koordinatalarida yozing va uni chizing.

Yechish:

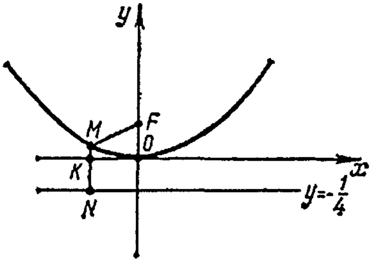
$(x^2+y^2)^2=2a^2(x^2-y^2)$ tenglikda (yuqoridagi masalaga qarang) $x=\rho\cos\theta$, $y=\rho\sin\theta$ formulalari bo'yicha qutb koordinatalariga o'tamiz. U holda

$$(\rho^2\cos^2\theta+\rho^2\sin^2\theta)^2=2a^2(\rho^2\cos^2\theta-\rho^2\sin^2\theta)$$

yoki $\rho^2=2a^2\cos 2\theta$

Bu — lemniskataning qutb koordinatalaridagi formulasidir.

Egri chiziqni chizamiz. Tenglamani ρ ga nisbatan yechamiz, natijada $\rho=\pm\sqrt{2\cos 2\theta}$, o'ng tomonida "$\pm$" turgani uchun va tenglamadan θ ni $-\theta$ bilan almashtirsak, tenglama o'zgarmagani uchun lemniskata *OX* va *OY* o'qlariga nisbatan simmetrik bo'ladi. I-chorak uchun lemniskata shaklini tekshiramiz, ya'ni $\rho\geq 0$, $0\leq\theta<\frac{\pi}{2}$. ρ va θ ning bu qiymatlari uchun $\rho=a\cdot\sqrt{2}\cdot\sqrt{\cos 2\theta}$. θ faqat 0 dan $\frac{\pi}{4}$ gacha o'zgarishini ko'rish mumkin. Shunday qilib, egri chiziqning bu qismi qutb o'qi va nur $\theta=\frac{\pi}{4}$ orasida bo'ladi. Agar $\theta=0$ bo'lsa, $\rho=a\sqrt{2}$ bo'ladi. θ noldan



1-chizma

$\frac{\pi}{4}$ gacha o'sganda ρ ning qiymati 0 gacha kamayadi. Simmetrikligini hisobga olib, lemniskatani yasash mumkin (2-chizma).

41. $A(1,1)$ va $B(3,3)$ nuqtalardan teng uzoqlikda turgan nuqtalar to'plamining tenglamasini tuzing.

Yechish:

M nuqta izlangan egri chiziq nuqtasi bo'lsin, u holda $|MA|=|MB|$ bo'ladi. Ikki nuqta orasidagi masofa formulasidan foydalanib yozamiz:

$$|MA|=\sqrt{(x-1)^2+(y-1)^2}, \qquad |MB|=\sqrt{(x-3)^2+(y-3)^2}.$$

Egri chiziq tenglamasini

$$\sqrt{(x-1)^2+(y-1)^2}=\sqrt{(x-3)^2+(y-3)^2}$$

ko'rinishda yozish mumkin. Oxirgi tenglikning ikki tomonini kvadratga oshirib topamiz:

$$x^2-2x+1+y^2-2x+1=x^2-6x+9+y^2-6y+9$$

O'xshash hadlarni ixchamlab, $x+y-4=0$ tenglamaga ega bo'lamiz.Izlangan nuqtalar to'plami to'g'ri chiziq bo'lib, AB kesmaning o'rtasiga o'tkazilgan perpendikular bo'ladi.

42. M nuqta qutb atrofida tekis aylanadigan nur bo'yicha tekis harakatlanadi. Agar harakat boshida aylanadigan nur qutb o'qi bilan, M nuqta qutb bilan ustma-ust tushsa, M nuqta chizadigan egri chiziq tenglamasini tuzing. Nurni $\theta=1$ (bir radian) ga burganda M nuqta qutbdan A masofaga siljiydi.

Yechish:

Boshlang'ich holatda ρ va θ lar nolga teng. So'ngra vaqtga proporsional holda o'sadi. Ular $\frac{\rho}{\theta}=\text{const}$ to'g'ri proporsional bog'lanishda. $\theta=1$ da $\rho=a$, demak, $\frac{\rho}{\theta}=\frac{a}{1}$, ya'ni $\rho=a\cdot\theta$. $\rho=a\cdot\theta$. egri chiziq *Arximed spirali* deb ataladi.

43. Bir xil diametrli ikkita aylananing biri sirpanmasdan ikkinchisining tashqi tomoni bo'yicha dumalaydi. Ularning diametri A ga teng. Dumalayotgan aylananing aniq bitta nuqtasi chizgan chizig'ining tenglamasini qutb koordinatalarida yozing.

Yechish:

C_1 –dumalayotgan aylana markazining boshlang'ich holati; A–izlangan chiziqni chizuvchi nuqtaning boshlang'ich vaziyati (boshlang'ich onda aylanalar urinadilar; A nuqta B nuqtaga nisbatan diametral joylashgan) (3-chizma). C_2–qo'zg'almas aylananing markazi; C_3–yangi holatdagi dumalayotgan aylananing markazi. M–izlangan chiziqni chizuvchi A nuqtaning yangi holati (C_1 aylana C_3 vaziyatga ko'chganda P nuqta Q vaziyatni oladi. B nuqta D vaziyatni egallaydi, ko'chirish sirpanmasdan bajariladi:

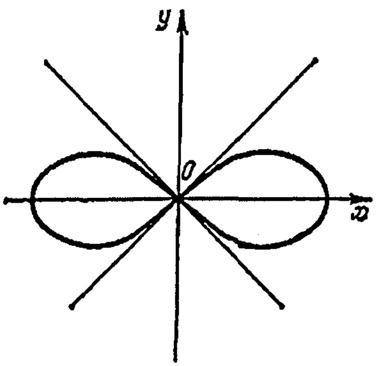
$$\breve{BQ} = \breve{DQ}, \quad Q\hat{C_2}D = Q\hat{C_3}D.)$$

Rasmda qutb va qutb o'qi OX ko'rsatilgan. Izlangan egri chiziqning ixtiyoriy $M(\rho,\theta)$ nuqtasining koordinatalari qanoatlantiradigan tenglamani tuzishimiz kerak. $M\hat{C_3}Q = O\hat{C_2}Q$ ligini ko'rsatish mumkin. OC_2C_3M to'rtburchakning kichik asosi $|C_2 \quad C_3| = a$ bo'lgan teng yonli trapetsiya: C_2C_2' va C_3C_3' $-C_2, C_3$ nuqtadan OM ga tushirilgan perpendikularlar.

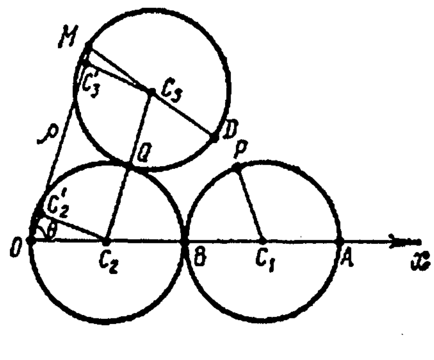
Demak,

$$\rho = \left|OC_2'\right| + \left|C_2'C_3'\right| + \left|C_3'M\right| = \frac{a}{2}\cos\theta + a + \frac{a}{2}\cos\theta = a(1+\cos\theta)$$

Shunday qilib, izlangan chiziqning qutb koordinatalaridagi



2-chizma



3-chizma

tenglamasi $\rho = a(1+\cos\theta)$; bu egri chiziq kardioida deb ataladi. θ ni $-\theta$ ga almashtirganda *kardioida* tenglamasi o'zgarmagani uchun u qutb o'qiga nisbatan simmetrikdir. Agar θ ning qiymati 0 dan π gacha o'zgarsa, u holda ρ ning qiymati $2a$ dan 0 gacha kamayadi.

44. $A(2;\ 0)$ va $B(0;\ 1)$ nuqtalar teng uzoqlikda turgan to'g'ri chiziq tenglamasini toping.

45. $x = y$ tenglama qanday chiziqni aniqlaydi?

46. $x = -y$ tenglama qanday chiziqni aniqlaydi?

47. $A(2;\ 0)$ va $B(0;\ 2)$ nuqtalardan masofalar kvadratlarining yig'indisi A va B nuqtalar orasidagi masofa kvadratiga teng bo'lgan nuqtalar to'plamining tenglamasini tuzing.

48. $A(1;\ 0)$ va $B(0;\ 1)$ nuqtalar orasidagi masofalar yig'indisi 2 ga teng bo'lgan nuqtalar to'plamining tenglamasini tuzing.

49. Markazi qutbda bo'lgan aylana tenglamasini qutb koordinata sistemasida tuzing.

50. Qutbdan o'tadigan va qutb o'qi bilan α burchak tashkil qilgan yarim to'g'ri chiziq tenglamasini qutb koordinat sistemasida yozing.

51. Agar qutb aylanada yotib, qutb o'qi aylana markazidan o'tsa, diametri A ga teng bo'lgan aylana tenglamasini qutb koordinatalarida yozing.

5. Chiziqning parametrik tenglamasi.

Ba'zida nuqtalar to'plamining tenglamasini tuzishda x, y koordinatalarni qandaydir yordamchi parametr t orqali ifodalash qulay keladi (u *parametr* deb ataladi), ya'ni $x = \varphi(t)$, $y = \psi(t)$ tenglamalar sistemasi qaraladi. Izlangan chiziqni bunday tasvirlash *parametrik ko'rinish* deyilib, tenglamalar sistemasi berilgan chiziqning *parametrik tenglamalari* deyiladi. Tenglamalar sistemasidan parametr t ni yuqotib, x, y ni bog'lovchi, ya'ni oddiy $f(x,\ y) = 0$ ko'rinishdagi tenglamaga keltiriladi.

52. Aylananing parametrik tenglamasini tuzing.

Yechish:

Markazi koordinatalar boshida yotgan, radiusi A ga teng aylanani qaraymiz (4-chizma). Unda ixtiyoriy $M(x,\ y)$ nuqtani olamiz. OM radiusi bilan abssissa o'qi orasidagi burchak t ni parametr deb qaraymiz. OMN uchburchakdan $x = a\cos t$, $y = a\sin t$. Shunday qilib, $x = a\cos t$, $y = a\sin t$ aylananing parametrik tenglamasi hisoblanadi. Bu tenglamalardan parametr t ni yo'qotib, aylananing oddiy tenglamasiga kelamiz. Parametr t ni yo'qotish uchun tenglamalarning ikki tomonini kvadratga oshirib, ularning yig'indisini olsak:

$$x^2 + y^2 = a^2\cos^2 t + a^2\sin^2 t = a^2(\cos^2 t + \sin^2 t) = a^2,$$ ya'ni

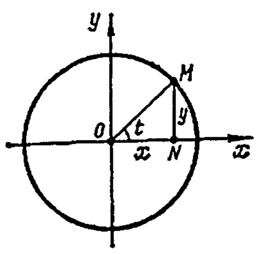
$x^2 + y^2 = a^2$ tenglikni hosil qilamiz. Bu tenglama markazi koordinatalar boshida, radiusi A ga teng bo'lgan aylanadir.

53. Aylananing tayinlangan nuqtasining qo'zgalmas to'gri chiziq bo'ylab sirpanmasdan dumalashidan hosil bo'lgan chiziqning parametrik tenglamasini tuzing.

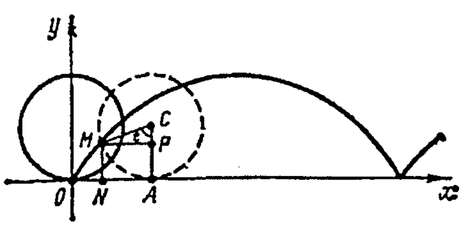
Yechish:

Radiusi A ga teng bo'lgan aylana gorizontal o'q bo'ylab o'ngga sirpanmasdan dumalasin (5-chizma). Bu to'g'ri chiziqni OX o'qi deb, koordinatalar boshini bu o'qdagi biror O nuqtada olamiz. Aylananing tayinlangan nuqtasi deb shunday nuqtani olamizki, aylananing mos holatida O nuqta bilan ustma-ust tushsin. Tayinlangan nuqtadan o'tadigan aylana radiusining burash burchagini t parametr sifatida olamiz.

Ma'lum vaqtda aylana A nuqtada o'qqa urinadi. Aylananing fiksirlangan $M(x; y)$ nuqtasi CM radiusni t burchakka burashga mos keladigan holatni oladi ($t = A\hat{C}M$). Sirpanmasdan dumalagani uchun $|OA| = \overset{\smile}{MA} = at$ bo'ladi. Bundan foydalanib M nuqtaning koordinatalarini t orqali ifodalaymiz:



4-chizma



5-chizma

$$x=|ON|=|OA|-|NA|=\overset{\smile}{MA}-|NA|=at-a\sin t=a(t-\sin t),$$

$$y=|NM|=|AP|=|AC|-|PC|=a-a\cos t=a(1-\cos t).$$

Shunday qilib, izlangan chiziqning parametrik tenglamasi quyidagi $x=a\cdot(t-\sin t)$, $y=a\cdot(1-\cos t)$ ko'rinishda bo'ladi. Bu chiziq sikloida deb ataladi (5-chizma).

54. Qanday chiziq $x=t^2$, $y=t^2$ parametrik tenglamalar bilan aniqlanadi.

Yechish:

Parametr t ni yo'qotib $y = x$ tenglamaga kelamiz. Parametrik tenglamalardan $x\geq 0$, $y\geq 0$ ligi kelib chiqadi. Demak, berilgan parametrik tenglamalar birinchi chorak bissektrisasini aniqlaydi.

55. Qanday chiziq $x=\cos t$, $y=\cos^2 t$ parametrik tenglamalar bilan aniqlanadi.

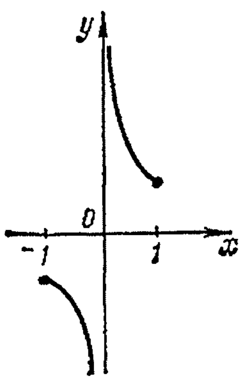
Yechish:

Ikkinchi tenglamadagi cos t o'rniga x ni qo'yib, $y=x^2$ ga ega bo'lamiz. Parametrik tenglamalardan $|x|\leq 1$, $0\leq y\leq 1$ ga ega bo'lamiz. Shunday qilib, parametrik tenglamalar $y=x^2$ parabolaning AOB yoyini aniqlaydi, bu yerda $A(-1;\ 1)$; $B(1;\ 1)$.

56. Qanday chiziq $x=\sin t$, $y=\operatorname{cosec} t$ tenglamalar bilan beriladi.

Yechish:

$y=1/\sin t$ bo'lgani uchun x, y miqdorlar o'rtasidagi teskari proporsional bog'lanishni ifodalaydigan $y=1/x$ tenglamaga ega bo'lamiz. $|x|\leq 1$, $|y|\geq 1$ larni hisobga olib, 6-rasmda tasvirlangan chiziq ko'rinishiga ega bo'lamiz.



6-chizma

57. Qanday chiziq $x = 2t$, $y = 4t$ tenglamalar bilan beriladi.

58. Egri chiziq $x = a\cos\ t$, $y = b\sin\ t$ parametrik tenglamalar bilan beriladi. Uning to'g'ri burchakli koordinatalar sistemasidagi tenglamasini yozing.

Ko'rsatma: Birinchi tenglamani A ga, ikkinchisini B ga bo'lib, t ni yo'qotamiz.

59. Egri chiziq $x = a\sec\ t$, $y = \text{b}\operatorname{tg}\ t$

parametrik tenglamalar bilan berilgan. Uning tenglamasini to‘g‘ri burchakli koordinatalar sistemasida yozing.

60. Qanday chiziq $x = \cos^2 t$, $y = \sin^2 t$ tenglama bilan aniqlanadi?

61. $x = a\cos^3 t$, $y = a \sin^3 t$ tenglamalar bilan beriladigan chiziq astroida deb ataladi. t ni yo‘qotib, astroida tenglamasini to‘g‘ri burchakli koordinatalarda yozing.

62. Markazi O nuqtada, radiusi A ga teng bo‘lgan doiraga soat mili bo‘yicha ip o‘ralgan; ipning oxiri $A(a;\ 0)$ nuqtada bo‘lsin. Ipni (soat miliga qarshi yo‘nalishda) doiradan bo‘shatamiz va har doim oxiriga ipni tarang tortib turamiz. Agar parametr t sifatida OA radius bilan tortilgan ip bilan aylanaga urinish nuktasiga o‘tkazilgan OB radius orasidagi burchakni olsak, ipning oxiri chizgan egri chiziqning parametrik tenglamasini yozing.

2-§. TO‘G‘RI CHIZIQ

1. To‘g‘ri chiziqning umumiy tenglamasi.

x, y larga nisbatan har qanday birinchi darajali tenglama, ya’ni $Ax+By+C=0$ (1) (A, B, C – o‘zgarmas koeffisientlar, $A^2+B^2\neq 0$) tenglama tekislikda qandaydir to‘g‘ri chiziqni aniqlaydi. Bu tenglama *to‘g‘ri chiziqning umumiy tenglamasi* deb ataladi.

Xususiy hollar:

1. $C=0$; $A\neq 0$; $B\neq 0$. $Ax+By=0$ tenglama bilan aniqlanadigan to‘g‘ri chiziq koordinatalar boshidan o‘tadi.

2. $A=0$; $B\neq 0$; $C\neq 0$. $By+C=0$ tenglama bilan aniqlanadigan ($y=-C/B$) to‘g‘ri chiziq OX o‘qiga parallel.

3. $B=0$; $A\neq 0$; $C\neq 0$. $Ax+C=0$ tenglama bilan aniqlanadigan ($x=-C/A$) to‘g‘ri chiziq OY o‘qiga parallel.

4. $B = C = 0$; $A \neq 0$; $Ax = 0$ yoki $x=0$ tenglama bilan aniqlanadigan to‘g‘ri chiziq OY o‘qi bilan ustma-ust tushadi.

5. $A=C=0$; $B\neq 0$, $By=0$ yoki $y=0$ tenglama bilan aniqlanadigan to‘g‘ri chiziq OX o‘qi bilan ustma-ust tushadi.

2. To‘g‘ri chiziqning burchak koeffitsientli tenglamasi.

Agar umumiy tenglamada $B\neq 0$ bo‘lsa, uni y ga nisbatan yechib $y=kx+b$ (2) tenglamani hosil qilamiz (bu yerda $k=-A/B$, $b=-C/B$). Uni *to‘g‘ri chiziqning burchak koeffisientli tenglamasi* deb atashadi, bu yerda $k=\operatorname{tg}\alpha$, α – to‘g‘ri chiziq bilan OX o‘qining musbat yo'nalishi orasidagi burchak. Tenglamaning ozod hadi B to'g'ri chiziqning OY o'qi bilan kesishgan nuqtasi.

3. To'g'ri chiziqning kesmalarga nisbatan tenglamasi.

Agar to'g'ri chiziqning umumiy tenglamasida $C \neq 0$ bo'lsa, tenglamani $-C$ ga bo'lib,

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (3)$$

tenglikga ega bo'lamiz (bu yerda $a=-C/A$, $b=-C/B$). Uni *to'g'ri chiziqning kesmalarga nisbatan tenglamasi* deb atashadi; bunda A – to'g'ri chiziqni OX o'qi, B – esa OY o'qi bilan kesishish nuqtasi. Shuning uchun, a va b to'g'ri chiziqning o'qlardagi kesmalari deyiladi.

4. To'g'ri chiziqning normal tenglamasi.

Agar to'g'ri chiziq umumiy tenglamasining ikki tomonini $\mu=1/\pm\sqrt{A^2+B^2}$ songa ko'paytirsak (μ–normallashtiruvchi ko'paytuvchi, ildiz oldidagi ishorani shunday tanlaymizki $\mu C<0$ bo'lsin),

$$x\cos\varphi+y\sin\varphi-p=0 \qquad (4)$$

ga ega bo'lamiz. Bu tenglikka *to'g'ri chiziqning normal tenglamasi* deyiladi. Bu yerda P koordinatalar boshidan to'g'ri chiziqqa tushirilgan perpendikularning uzunligi, φ– perpendikular bilan OX o'qining musbat yo'nalishi orasidagi burchak.

63. Ordinata o'qidan $b=-3$ kesma ajratuvchi va abssissa o'qining musbat yo'nalishi bilan $\alpha=\pi/6$ burchak tashkil etuvchi chiziq tenglamasini tuzing.

Yechish:

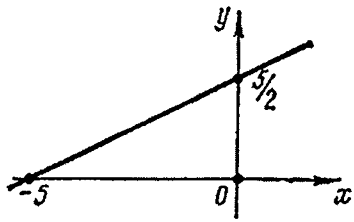
Burchak koeffitsientini topamiz:

$k=tg(\pi/6)=1/\sqrt{3}$

Burchak koeffitsientli tenglamadan foydalanib, $y=(1/\sqrt{3})x-3$ ga ega bo'lamiz. Soddalashtirishlardan so'ng $x-\sqrt{3}y-3\sqrt{3}=0$ ga ega bo'lamiz.

64. Koordinata o'qlaridan $a=2/5$, $b=-1/10$ kesmalar ajratuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: (3) tenglamadan $\frac{x}{2/5}+\frac{y}{(-1/10)}=1$ ni topamiz. Bu tenglamani $(5/2)x - 10y=1$ yoki $5x - 20y - 2 = 0$ ko'rinishga keltirish mumkin.



7-chizma

65. To'g'ri chiziqning $12x-5y-65=0$ umumiy tenglamasi berilgan.

1) burchak koeffitsientli;
2) kesmalarga nisbatan;
3) normal tenglamalarini yozing.

Yechish:

1) Tenglamani y ga nisbatan yechib, burchak koeffitsientli $y = (12/5)x - 13$ tenglamani hosil qilamiz, bu yerda $k=12/5$, $b = -13$.

2) Umumiy tenglamaning ozod hadini o'ng tomonga o'tkazib, ikki tomonni 65 ga bo'lamiz va $(12/65)x-(5/65)y = 1$ ni hosil qilamiz. Oxirgi tenglamani $\frac{x}{65/12}+\frac{y}{(-65/5)}=1$ ko'rinishda yozish mumkin, bu yerda

$a = 65/12$, $b = - 65/5 = -13$.

3) Normallashtiruvchi koeffitsient $\mu=1/\sqrt{12^2+(-5)^2}=1/13$ ni topamiz. Tenglamaning ikki tomonini μ ga ko'paytirib, $(12/13)\,x-(5/13)y-5=0$ normal tenglamani hosil qilamiz, bu yerda $\cos\varphi=12/13$, $\sin\varphi=-5/13$, $p=5$

66. 1) $x-2y+5=0$; 2) $2x+3y=0$; 3) $5x-2=0$; 4) $2y+7=0$ to'g'ri chiziqlarni chizing.

Yechish:

1) Tenglamada $x=0$ desak, $y = 5/2$ ni topamiz. Demak, to'g'ri chiziq ordinata o'qini $B\,(0;\ 5/2)$ nuqtada kesadi; $y = 0$ deb $x = -5$ ni topamiz, ya'ni to'g'ri chiziq abssissa o'qini $A(-5;\ 0)$ nuqtada kesadi. To'g'ri chiziqni A va B nuqtalardan o'tkazish kerak (7-chizma).

2) $2x+3y=0$ to'g'ri chiziq koordinatalar boshidan o'tadi, chunki uning tenglamasida ozod had qatnashmaydi. Tenglamada $x=3$ desak, $6 + 3y = 0$, yoki $y = -2$ bo'ladi. Demak, $M(3;\ -2)$ nuqtani hosil qilamiz. Koordinatalar boshi va M nuqtadan to'g'ri chiziq o'tkazish qoladi.

3) Tenglamani x ga nisbatan yechib, $x = 2/5$ ni hosil qilamiz. Bu to‘g‘ri chiziq ordinata o‘qiga parallel va abssissa o‘qidan 2/5 ga teng kesma ajratadi.

4) Yuqoridagiga o‘xshash $y=-7/2$ ni hosil qilamiz, to‘g‘ri chiziq absissa o‘qiga parallel.

67. To‘g‘ri chiziq tenglamasi $(x+2\sqrt{5})/4+(y-2\sqrt{5})/2=0$ bo‘lsin. Bu to‘g‘ri chiziqning 1) umumiy; 2) burchak koeffitsientli; 3) kesmalarga nisbatan; 4) normal teglamalarni yozing.

68. $2x + 2y - 5 = 0$ tenglama abssissa o‘qining musbat yo‘nalishi bilan qanday burchak hosil qiladi.

69. $4x + 3y - 36 = 0$ to‘g‘ri chiziq bilan koordinata o‘qlaridan hosil bo‘lgan uchburchak yuzasini hisoblang.

70. $20x + 21y = 0$ ni kesmalarga nisbatan yozish mumkinmi?

71. 1) $4x - 5y + 15 = 0$; 2) $2x - y = 0$; 3) $7x - 10 = 0$; 4) $2y + 3 = 0$ to‘g‘ri chiziqlarni chizing.

72. Ordinata o‘qi bilan $b = 1$ va abssissa o‘qining musbat yo‘nalishi bilan $\alpha = 2\pi/3$ burchak hosil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.

73. To‘g‘ri chiziq koordinatalaridan teng musbat kesmalar ajratadi. Agar to‘g‘ri chiziq va koordinata o‘qlari bilan hosil bo‘lgan uchburchak yuzasi 8 kv. birlikka teng bo‘lsa, to‘g‘ri chiziq tenglamasini tuzing.

74. Koordinatalar boshidan va $A(-2; -3)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi tuzilsin.

75. $A(2; 5)$ nuqtadan o‘tuvchi va ordinata o‘qidan $b = 7$ kesma ajratuvchi to‘g‘ri chiziq tenglamasini tuzing.

76. $M(-3; -4)$ nuqtadan o‘tib, koordinata o‘qlariga parallel to‘g‘ri chiziq tenglamasini tuzing.

77. Agar to‘g‘ri chiziqning koordinata o‘qlari orasidagi kesmasining uzunligi $5/\sqrt{2}$ ga teng bo‘lsa, koordinata o‘qlaridan bir xil kesmalar ajratuvchi to‘g‘ri chiziq tenglamasi tuzilsin.

5. To‘g‘ri chiziqlar orasidagi burchak. Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.

$y=k_1x+b_1$, $y=k_2x+b_2$ to‘g‘ri chiziqlar orasidagi burchak

$$tg\alpha=\left|\frac{k_2-k_1}{1+k_1k_2}\right| \qquad (1)$$

formula bilan aniqlanadi.

$k_1 = k_2$ — ikki chiziqning paralellik sharti. $k_1 = -1/k_2$ — ikki to‘g‘ri chiziqning perpendikulyarlik sharti. k burchak koeffisientli va $M(x_1, y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi quyidagi $y - y_1 = k(x - x_1)$ (2) ko‘rinishda yoziladi.

$M_1(x_1; y_1)$ va $M_2(x_2; y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \qquad (3)$$

va to‘g‘ri chiziqning burchak koeffitsienti

$$k = \frac{y_2 - y_1}{x_2 - x_1} \qquad (4)$$

formuladan topiladi.

Agar $x_1 = x_2$ bo‘lsa, M_1, M_2 nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi $x = x_1$, $y_1 = y_2$ bo‘lsa $y = y_1$ bo‘ladi.

6. To‘g‘ri chiziqlarning kesishuvi. Nuqtadan to‘g‘ri chiziqqacha masofa. To‘g‘ri chiziqlar dastasi.

Agar $A_1/A_2 = B_1/B_2$ bo‘lsa, $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to‘g‘ri chiziqlarning kesishgan nuqtasi ularning tenglamalari birga echib topiladi.

$M(x_0, y_0)$ nuqtadan $Ax + By + C = 0$ to‘g‘ri chiziqqacha masofa quyidagicha topiladi:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}. \qquad (1)$$

$A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to‘g‘ri chiziqlar orasidagi burchak bissektrisasining tenglamasi

$$\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} \pm \frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}} = 0 \qquad (2)$$

bo‘ladi. Agar $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to‘g‘ri chiziqlar kesishsa, u holda $A_1x + B_1y + C_1 + \lambda(A_2x + B_2y + C_2) = 0$ (3) tenglama (λ—sonli ko‘paytuvchi) to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tadigan to‘g‘ri chiziqni aniqlaydi. (3) da λ ga har xil qiymatlar berib, markaz deb ataluvchi kesishgan nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasiga tegishli har xil to‘g‘ri chiziqlarni hosil qilamiz.

78. $y = -3x+7$ va $y = 2x + 1$ to‘g‘ri chiziqlar orasidagi o‘tkir burchakni aniqlang.

Yechish: $k_1=-3$, $k_2=2$ deb 5-banddagi (1) dan topamiz:

$$\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right|, \text{ ya'ni } \varphi = \frac{\pi}{4}.$$

79. $4x - 6y + 7 = 0$ va $20x - 30y - 11 = 0$ to‘g‘ri chiziqlarning parallelligini ko‘rsating.

Yechish: Har bir tenglamani burchak koeffitsientli ko‘rinishga keltiramiz:

$y=(2/3)x+7/6$ va $y=(2/3)x-11/30$. Bu to‘g‘ri chiziqlarning burchak koeffisientlari $k_1 = k_2 = 2/3$ga teng, ya’ni to‘g‘ri chiziqlar parallel.

80. $3x-5y+7=0$ va $10x+6y-3=0$ to‘g‘ri chiziqlarning perpendikularligini ko‘rsating.

Yechish: Tenglamalarni burchak koeffitsientli ko‘rinishga keltiramiz:

$y=(3/5)x+7/5$ va $y=(-5/3)x+1/2$, bu yerda $k_1=3/5$, $k_2=-5/3$, $k_1 = -1/k_2$ bo‘lgani uchun to‘g‘ri chiziqlar perpendikular.

81. $M(-1;\ 3)$ va $N(2;\ 5)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

Yechish: 5-chi banddagi (3) formuladan $x_2=2$, $y_2=5$, $x_1=-1$, $y_1 = 3$ deb olamiz:

$$\frac{y-3}{5-3}=\frac{x+1}{2+1} \quad \text{yoki} \quad \frac{y-3}{2}=\frac{x+1}{3}.$$

Shunday qilib, izlangan tenglama $2x - 3y + 11 = 0$ bo‘ladi.

Tenglamaning to‘g‘ri topilganini tekshirib ko‘rish mumkin. Buning uchun M va N nuqtalarning koordinatalari bu tenglamani qanoatlantirishini ko‘rsatish kerak. Haqiqatan ham:

$2(-1)-3\cdot3+11=0$, $\quad 2\cdot2-3\cdot5+11=0$.

82. $A(-2;\ 4)$ va $B(-2;\ -1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

Yechish: $x_1= x_2= -2$ bo‘lgani uchun to‘g‘ri chiziq tenglamasi $x = -2$ bo‘ladi. (Ordinata o‘qiga parallel).

83. $3x-2y+1=0$ va $2x+5y-12=0$ to‘g‘ri chiziqlarning kesishishini ko‘rsating va kesishgan nuqtani toping.

Yechish: $3/2 \neq (-2)/5$ bo'lgani uchun to'g'ri chiziqlar kesishishadi.

$$\begin{cases} 3x-2y+1=0 \\ 2x+5y-12=0 \end{cases}$$

sistemasini yechib quyidagilarni topamiz: $x = 1$, $y = 2$, ya'ni to'g'ri chiziqlar (1; 2) nuqtada kesishishadi.

84. To'g'ri chiziq tenglamasidan foydalanmasdan $M(x_0;\ y_0)$ nuqtadan $Ax + By + C = 0$ to'g'ri chiziqqacha masofani toping.

Yechish: Masala $M(x_0;\ y_0)$ nuqtadan berilgan to'g'ri chiziqqa tushirilgan perpendikular asosi, N gacha bo'lgan masofani topishga keltiriladi. MN to'gri chiziq tenglamasini tuzamiz. Berilgan to'g'ri chiziq burchak koeffitsienti $-A/B$ bo'lgani uchun MN to'g'ri chiziqning burchak koeffitsienti B/A bo'ladi (perpendikularlik shartidan) va MN to'g'ri chiziq, tenglamasi $y - y_0 = (B/A)(x - x_0)$ bo'ladi. U bunday $(x - x_0)/A = (y - y_0)/B$ ko'rinishda yoziladi.

$$Ax + By + C = 0, \qquad (x - x_0)/A = (y - y_0)/B$$

tenglamalar sistemasini yozib, N nuqtaning koordinatalarini topamiz.

Yordamchi noma'lum t ni kiritamiz: $(x - x_0)/A = (y - y_0)/B = t$. U holda $x = x_0 + At$, $y = y_0 + Bt$ bo'ladi. Bu ifodani berilgan to'g'ri chiziq tenglamasiga qo'yib topamiz: $A(x_0 + At) + B(y_0 + Bt) + C = 0$. Bundan $t = -(Ax_0 + By_0 + C)/(A_2 + B_2)$. t ni $x = x_0 + At$, $y = y_0 + Bt$ larga qo'yib, N ning koordinatalarini topamiz:

$$x = x_0 - A\frac{Ax_0 + By_0 + C}{A^2 + B^2}, \quad y = y_0 - B\frac{Ax_0 + By_0 + C}{A^2 + B^2}.$$

Endi M va N nuqtalar orasidagi masofani topamiz:

$$d = \sqrt{(x - x_0)^2 + (y - y_0)^2} =$$

$$= \sqrt{\left(A\frac{Ax_0 + By_0 + C}{A^2 + B^2}\right)^2 + \left(B\frac{Ax_0 + By_0 + C}{A^2 + B^2}\right)^2} = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

85. $M(1;\ 2)$ nuqtadan $20x - 21y - 58 = 0$ to'g'ri chiziqqacha masofani aniqlang.

Yechish:

$$d = \frac{|20 \cdot 1 - 21 \cdot 2 - 58|}{\sqrt{400 + 44}} = \frac{|20 - 42 - 58|}{29} = \frac{|-80|}{29} = 2\frac{22}{29}.$$

86. ℓ to‘g‘ri chiziq berilgan: $4x-3y-7=0$. $A(5/2;\ 1)$, $B(3;\ 2)$, $C(1;\ -1)$, $D(0;\ -2)$, $E(4;\ 3)$, $F(5;\ 2)$ nuqtalardan qaysi biri ℓ to‘g‘ri chiziqda yotadi.

Yechish: Agar nuqta to‘g‘ri chiziqda yotsa, u holda uning koordinatalari to‘g‘ri chiziq tenglamasini qanoatlantiradi.

$A\in\ell$, chunki $4(5/2)-3\cdot1-7=0$;

$B\notin\ell$, chunki $4\cdot3-3\cdot2-7\neq0$;

$C\in\ell$, chunki $4\cdot1-3(-1)-7=0$;

$D\notin\ell$, chunki $4\cdot0-3(-2)-7\neq0$;

$E\notin\ell$, chunki $4\cdot4-3\cdot3-7=0$;

$F\in\ell$, chunki $4\cdot5-3\cdot2-7\neq0$.

87. $M(-2;\ -5)$ nuqtadan o‘tib, $3x+4(5/2)-3\cdot1-7=0\,4y+2=0$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziq tenglamasi tuzilsin.

Yechish: To‘g‘ri chiziq tenglamasini y ga nisbatan yechamiz: $y=-(3/4)x-1/2$. Demak, izlangan to‘g‘ri chiziq berilgan to‘g‘ri chiziqqa parallel bo‘lgani uchun uning burchak koeffitsienti $-3/4$ ga teng. 5-chi banddagi (2) tenglamadan foydalanib topamiz:

$y-(-5)=(-3/4)[x-(-2)]$, ya’ni $3x+4y+26=0$.

88. $A(2;\ 2)$, $B(-2;\ -8)$ va $C(-6;\ -2)$ nuqtalar uchburchakning uchlari bo‘lsin. Uchburchak medianalari tenglamasini tuzing.

Yechish: BC, AC va AB tomonlar o‘rtalarining koordinatalarini topamiz:

$$x'=(-2-6)/2=-4,\ y'=(-8-2)/2=-5;\ A_1(-4;-5);$$

$$x''=(2-6)/2=-2,\ y''=(2-2)/2=0;\ \ B_1(-2;0);$$

$$x'''=(2-2)/2=0;\ y'''=(2-8)/2=-3;\ C_1(0;\ -3).$$

Medianalarni ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan foydalanib topamiz. AA_1 medirnaning tenglamasi

$(y-2)/(-5-2)=(x-2)/(-4-2)$, yoki

$(y-2)/7=(x-2)/6$ ya’ni $7x-6y-2=0$. $B(-2;\ -8)$ va $B_1(-2;\ 0)$ nuqtalarning abssissalari bir xil, BB_1 mediana ordinata o‘qiga parallel bo‘lgani uchun uning tenglamasi $x+2=0$ bo‘ladi. SS_1 mediana tenglamasi

$(y+2)/(-3+2)=(x+6)/(0+6)$ yoki $x+6y+18$.

89. Uchburchakning uchlari berilgan: $A(0;\ 1)$, $B(6;\ 5)$ va

$C(12; -1)$. C uchidan tushirilgan uchburchak balandligini tenglamasi topilsin.

Yechish: 5-chi banddagi (4) formuladan foydalanib, AB tomonning burchak koeffitsientini topamiz; $k = 5 - 1)/(6 - 0) = 4/6 = 2/3$. Perpendikularlik shartiga ko'ra, C uchidan tushirilgan burchak koeffitsienti $-3/2$. Bu balandlikning tenglamasi quyidagi ko'rinishda bo'ladi:

$$y + 1 = (- 3/2)(x - 12), \text{ yoki } 3x + 2y - 34 = 0.$$

90. Uchburchak tomonlari berilgan: $x + 3y - 7 = 0$ (AB), $4x - y - 2 = 0$ (BC), $6x + 8y - 35 = 0$ (AC). Uchburchakning B uchidan tushurilgan balandlik uzunligini toping.

Yechish:

B nuqta koordinatalarini aniqlaymiz: $x + 3y -7 = 0$ va $4x -y - 2 = 0$ tenglamalar sistemasini yechib $x = 1$, $y = 2$, ya'ni $B(1;2)$ nuqtani aniqlaymiz. BB_1 balandlik uzunligini B nuqtadan AC to'g'ri chiziqqacha bo'lgan masofani hisoblab topamiz:

$$\mathrm{BB_1} = \frac{|6 \cdot 1 + 8 \cdot 2 - 35|}{\sqrt{6^2 + 8^2}} = 1{,}3.$$

91. $3x + y - 3\sqrt{10} = 0$ va $6x + 2y + 5\sqrt{10} = 0$ o'zaro parallel to'g'ri chiziqlar orasidagi masofani aniqlang.

Yechish:

Masala bir to'g'ri chiziqning ixtiyoriy nuqtasidan ikkinchi to'g'ri chiziqqacha bo'lgan masofani topishga keltiriladi. Birinchi to'g'ri chiziqda $x = 0$ deb olsak, $y = 3\sqrt{10}$ bo'ladi. Shunday qilib

$M(0; 3\sqrt{10})$ — birinchi to'g'ri chiziqqa tegishli nuqta bo'ladi. M nuqtadan ikkinchi to'g'ri chiziqqacha bo'lgan masofani topamiz:

$$d = \frac{|6 \cdot 0 + 2 \cdot 3\sqrt{10} + 5\sqrt{10}|}{\sqrt{36 + 4}} = \frac{11\sqrt{10}}{2\sqrt{10}} = 5{,}5.$$

92. $x + y - 5 + 0$ va $7x - y - 19 = 0$ to'g'ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamalarini yozing (8-chizma).

Yechish:

Oldin bu masalani umumiy ko'rinishda yechamiz. Ikki to'g'ri chiziq orasidagi bissektrisa bu to'g'ri chiziqlardan teng uzoqlikda yotuvchi nuqtalar geometrik o'rnidir. Agar berilgan to'g'ri chiziqlar tenglamalari $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$

bo'lsa ($A_1/A_2 \neq B_1/B_2$, ya'ni to'g'ri chiziqlar parallel emaslar), u holda birorta bissektrisaga tegishli ixtiyoriy $M(x_0; y_0)$ nuqta uchun nuqtadan to'g'ri chiziqqacha bo'lgan masofa formulasidan foydalanib quyidagi tenglikni olamiz:

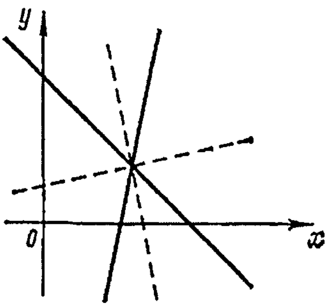
$$\frac{|A_1x_0 + B_1y_0 + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x_0 + B_2y_0 + C_2|}{\sqrt{A_2^2 + B_2^2}}.$$

$M(x_0; y_0)$ nuqta ixtiyoriy bo'lgani uchun uni $M(x; y)$ bilan belgilash mumkin. Absolut qiymat ostidagi ifodaning ishorasi har xil ishoraga ega bo'lishi mumkinligini hisobga olgan holda bissektrisalarning tenglamalaridan biri uchun:

$$\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = \frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}},$$

ikkinchisi uchun:

$$\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = -\frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}},$$



8-chizma

bo'ladi. Shunday qilib, ikkala bissektrisa uchun quyidagi tenglamani yoza olamiz:

$$\frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} \pm \frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}} = 0.$$

Endi aniq masalani yechamiz. A_1, B_1, C_1, A_2, B_2, C_2 larni berilgan to'g'ri chiziq tenglamasidagi ularning qiymatlari bilan almashtirib, topamiz:

$$\frac{x + y - 5}{\sqrt{1+1}} \pm \frac{7x - y - 19}{\sqrt{49+1}} = 0, \text{ ya'ni}$$

$$5(x + y - 5) \pm (7x - y - 19) = 0.$$

Bissektrisalardan birining tenglamasi $5(x + y - 5) + (7x - y - 19) = 0$, ya'ni $3x + y - 11 = 0$ ikkinchisiniki $5(x + y - 5) - (7x - y - 19) = 0$, ya'ni $x - 3y + 3 = 0$.

93. $A(1; 1)$, $B(10; 13)$, $C(13; 6)$ lar uchburchak uchlarining koordinatalari. A burchak bissektrisasining tenglamasini tuzing.

Yechish:

Bissektrisa tenglamasini tuzishda (yuqoridagi misolning yechilishida) boshqacha usulni ishlatamiz. D nuqta bissektrisaning BC tomon bilan kesishgan nuqtasi bo‘lsin. Uchburchak ichki burchak bissektrisasi xossasiga asosan $|BD|:|DC|=|AB|:|AC|$ bo‘ladi. Lekin

$$|AB|=\sqrt{(10-1)^2+(13-1)^2}=15,$$

$|AC|=\sqrt{(13-1)^2+(6-1)^2}=13$. Demak, $\lambda=|BD|:|DC|=15/13.$

BC kesmani D nuqta qanday nisbatda bo‘lishi ma’lum bo‘lgani uchun D nuqtaning koordinatalari quyidagi

$$x=\frac{10+(15/13)13}{1+15/13},\ y=\frac{13+(15/13)6}{1+15/13}$$

formulalar orqali topiladi yoki $x=325/28$, $y=259/28$ ya’ni $D(325/28,\ 259/28)$. Masala ikki A va D nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzishga keltiriladi:

$$\frac{y-1}{259/28-1}=\frac{x-1}{325/28-1},\ \text{ya'ni}\ 7x-9y+2=0.$$

94. ABC uchburchak balandliklarining tenglamalari berilgan: $x+y-2=0$, $9x-3y-4=0$ va $A(2;\ 2)$. Uchburchak tomonlarining tenglamasi tuzilsin.

Yechish: Nuqta balandliklarining birortasida yotmasligiga ishonish mumkin; uning koordinatalari balandlik tenglamalarini qanoatlantirmaydi. $9x-3y-4=0$ BB_1 balandlikning $x+y-2=0$ CC_1 balandlikning tenglamalari bo‘lsin. AC tomonning tenglamasini tuzishda uning A nuqtadan o‘tishini va BB_1 balandlikka perpendikulyarligini hisobga olamiz. BB_1 balandlikning burchak koeffitsienti 3 ga teng bo‘lgani uchun AC tomonning burchak koeffisienti $-1/3$ bo‘ladi, ya’ni $k_{AC}=-1/3$. Berilgan nuqtadan o‘tuvchi va burchak koeffitsienti $k=-1/3$ bo‘lgan to‘g‘ri chiziq tenglamasidan foydalanib, AC tomon tenglamasini hosil qilamiz: $y-2=(-1/3)(x-2)$ yoki $x+3y-8=0$

Yuqoridagi kabi $k_{CC_1}=-1$, $k_{AB}=1$ va AB tomon tenglamasi $y-2=x-2$, ya’ni $y=x$ bo‘ladi. AB va BB_1, AC va CC_1 to‘g‘ri chiziqlar tenglamasini birga echib, uchburchak uchlarining koordinatalarini topamiz: $B(2/3;\ 2/3)$ va $C(-1;\ 3)$. BC tomon tenglamasini tuzish qoladi. $\dfrac{y-2/3}{3-2/3}=\dfrac{x-2/3}{-1-2/3}$, ya’ni $7x+5y-8=0$.

95. $M(5;\ 1)$ nuqtadan o‘tib, $2x+y-4=0$ to‘g‘ri chiziq bilan $\pi/4$ burchak tashkil etuvchi to‘g‘ri chiziqlar tenglamasini tuzing (9-chizma).

Yechish:

Izlangan to‘g‘ri chiziqlardan birining burchak koeffitsienti k bo‘lsin. Berilgan to‘g‘ri chiziq burchak koeffisienti -2 ga teng. Bu to‘g‘ri chiziqlar orasidagi burchak $\pi/4$ ga teng bo‘lgani uchun:

$$tg(\pi/4)=\left|\frac{k+2}{1-2k}\right|, \text{ ya'ni } 1=\left|\frac{k+2}{1-2k}\right|$$

Bundan

$$\frac{k+2}{1-2k}=1 \quad \text{va} \quad \frac{k+2}{1-2k}=-1.$$

Bu tenglamalarni yechib, $k=-1/3$ va $k=3$ larni topamiz. Shunday qilib, izlangan to‘g‘ri chiziq tenglamalaridan biri $y-1=(-1/3)(x-5)$, ya’ni $x+3y-8=0$ ikkinchisiniki $y-1=3(x-5)$, ya’ni $3x-y-14=0$.

96. $2x+3y+5+1\ (x+8y+6)=0$ dastaga kirib, $M(1;\ 1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqni toping.

Yechish:

M nuqtaning koordinatalari izlangan to‘g‘ri chiziq tenglamasini qanoatlantiradi, shuning uchun λ ni quyidagicha topamiz:

$2\cdot 1+3\cdot 1+5+\lambda(1+8\cdot 1+6)=0$, yoki $10+15\lambda=0$, ya’ni $\lambda=-2/3$ λ ning qiymatini dasta tenglamasiga qo‘yib, izlangan to‘g‘ri chiziq tenglamasini hosil qilamiz: $2x+3y+5-(2/3)(x+8y+$ $+6)=0$, ya’ni

$$4x-7y+3=0.$$

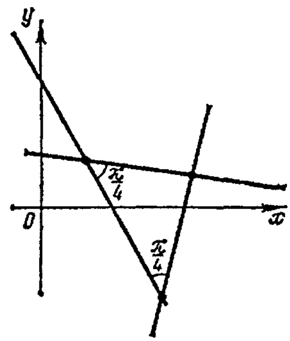
97. $3x-4y+7=0$ va $5x+2y+3=0$ to‘g‘ri chiziqlarning kesishgan nuqtasidan o‘tib, ordinata o‘qiga parallel to‘g‘ri chiziqni toping.

Yechish:

To‘g‘ri chiziq quyidagi dastaga tegishli:
$3x-4y+7+\lambda(5x+2y+3)=0$, ya’ni $(3+5\lambda)x+(-4+2\lambda)y+(7+3\lambda)=0$. Izlangan to‘g‘ri chiziq ordinata o‘qiga parallel bo‘lgani uchun y oldidagi koeffitsient nolga teng bo‘ladi: $-4+2\lambda=0$, ya’ni $\lambda=2$ ni dasta tenglamasiga qo‘yib topamiz: $x+\lambda=0$

98. Uchburchak tomonlarining tenglamalari berilgan:

$x+2y+5=0$ (AB), $3x+y+1=0$ (BC), $x+y+7=0$ (AC). AC tomonga tushurilgan balandlik tenglamasini tuzing.



9-chizma

Yechish:

Balandlik dastaga tegishli bo‘lgani uchun $x+2y+5+1(3x+y+1)=0$, ya’ni $(1+3\lambda)x+(2+\lambda)y+(5+\lambda)=0$ Dastaning burchak koeffitsienti $-(1+3\lambda)/(2+\lambda)$; AC ning burchak koeffitsienti -1 ga teng bo‘lgani uchun izlangan balandlikning burchak koeffisienti 1 ga teng va λ ni topish uchun $-(1+3\lambda)/(2+\lambda)=1$ ga ega bo‘lamiz. Bundan $1+3\lambda+2+\lambda=0$, ya’ni $\lambda = -3/4$ ning topilgan qiymatini dasta tenglamasiga qo‘yib topamiz:

$$\left(1-\frac{9}{4}\right)x+\left(2-\frac{3}{4}\right)y+\left(5-\frac{3}{4}\right)=0, \quad \text{ya'ni } 5x-5y-17=0.$$

99. ABC usburchakning uchlari berilgan: $A(0;\ 2)$, $B(7;\ 3)$, $C(1;\ 6)$. $B\hat{A}C=\alpha$ burchakni aniqlang.

100. Uchburchak tomonlarining tenglamasi berilgan: $x+y-6=0$, $3x-5y+14=0$ va $5x-3y-14=0$. Uning balandliklari tenglamasini toping.

101. $3x+4y-20=0$ va $8x+6y-5=0$ to‘g‘ri chiziqlar orasidagi burchaklar bissektrisalarining tenglamasini tuzing.

102. Uchburchak uchlarining koordinatalari berilgan: $A(0;\ 0)$, $B(-1;\ -3)$, $C(-5;\ -1)$. Uchburchak uchlaridan o‘tib va uning tomonlariga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

103. $M(2;\ 7)$ nuqtadan o‘tib va AB $(A(-1;\ 7)$, $B(8;-2))$ to‘g‘ri chizig‘i bilan 45^0 li burchak hosil qiluvchi to‘g‘ri chiziqlar tenglamasini tuzing.

104. $M(2;\ -1)$ nuqtadan koordinat o‘qlaridan $a=8$, $b=6$ kesmalar ajratuvchi to‘g‘ri chiziqqacha masofani aniqlang.

105. Uchlari $A(3/2;\ 1)$, $B(1;\ 5/3)$, $C(3;\ 3)$ nuqtalarda bo‘lgan uchburchakning C uchidan tushurilgan balandlikning uzunligini hisoblang.

106. M ning qanday qiymatida $7x-2y-5=0$, $x+7y-8=0$ va $mx+my-8=0$ to‘g‘ri chiziqlar bir nuqtada kesishadi.

107. Uchburchak tomonlari o'rtalarining koordinatalari berilgan: $A_1(-1;-1)$, $B_1(1;\ 9)$, $C_1(9;\ 1)$. Uchburchakning tomonlari o'rtasiga o'tkazilgan perpendikular tenglamasini tuzing.

108. $A(2;\sqrt{3})$ va $B(3;2\sqrt{3})$ nuqtalardan o'tuvchi to'g'ri chiziqning ordinata o'qi bilan tashkil qilgan o'tkir burchagini toping.

109. $A(1;\ 2)$ va $C(3;\ 6)$ nuqtalar kvadratning qarama-qarshi uchlarining koordinatalari. Kvadratning qolgan uchlari koordinatalari topilsin.

110. Absissa o'qida $8x+15y+10=0$ to'g'ri chiziqdan bir birlik masofada turgan nuqtani toping.

111. Uchburchak uchlarining koordinatalari berilgan: $A(1;\ 1)$, $B(4;\ 5)$ va $C(13;\ -4)$. B uchidan o'tkazilgan mediana va C uchidan tushirilgan balandlik tenglamasini tuzing. Uchburchak yuzasini hisoblang.

112. $2x+3y+6+\lambda(x-5y-6)=0$ dastaga tegishli va dastaning asosiy to'g'ri chiziqlariga perpendikular to'g'ri chiziqlarni toping (bu yerda $2x+3y+6=0$, $x-5y-6=0$ asosiy to'g'ri chiziqlar hisobladi).

113. $x+6y+5=0$, $3x-2y+1=0$ to'g'ri chiziqlarning kesishgan nuqtasidan va $M(-4/5;\ 1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

114. $x+2y+3=0$, $2x+3y+4=0$ to'g'ri chiziqlarning kesishgan nuqtasidan o'tadigan va $5x+8y=0$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

115. $3x-y-1=0$, $x+3y+1=0$ to'g'ri chiziqlarning kesishgan nuqtasidan o'tuvchi va abssissa o'qiga parallel to'g'ri chiziq tenglamasini toping.

116. $5x+3y+10=0$, $x+y-15=0$ to'g'ri chiziqlarning kesishgan nuqtasidan va koordinat boshidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

117. $x+2y+1=0$, $2x+y+2=0$ to'g'ri chiziqlarning kesishgan nuqtasidan o'tuvchi, abssissa o'qi bilan 135^0 burchak tashkil etuvchi to'g'ri chiziq tenglamasi tuzilsin.

118. $M(a;\ b)$ nuqtadan o'tib, $x+y+c=0$ to'g'ri chiziq bilan 45^0 burchak tashkil etuvchi to'g'ri chiziqlar tenglamasi tuzilsin.

119. Uchburchakning tomonlari berilgan: $x = 0(AB)$,

$x+y-2=0(BC)$, $y=0$ (AC). B uchidan tushirilgan mediana va A uchidan o'tuvchi balandlik tenglamasini tuzing.

120. Tomonlari $x+y\sqrt{3}+1=0$, $x\sqrt{3}+y+1=0$ va $x-y-10=0$ bo'lgan uchburchak teng yonli ekanligini isbotlang. Uning burchagini toping.

121. Parallelogrammning uchlari ketma-ket berilgan: $A(0; 0)$, $B(1; 3)$, $C(7; 1)$. Uning diagonallari orasidagi burchakni toping va parallelogrammning to'g'ri turtburchak ekanligini isbotlang.

122. Uchburchak tomonlarining tenglamasi berilgan: $x-y+2=0(AB)$, $x=2(BC)$, $x+y-2=0(AC)$. B nuqtadan va AC tomoni 1:3 nisbatda bo'luvchi nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

123. Uchlari $A(1;\ 1)$, $B(2;\ 17\sqrt{3})$, $C(3;\ 1)$ bo'lgan uchburchak teng tomonli ekanligini isbotlang va uning yuzasini hisoblang.

124. Tomonlari koeffitsientlari butun son bo'lgan tenglamalar bilan berilgan uchburchak teng tomonli bo'lishi mumkin emasligini isbotlang.

125. Uchburchakning uchi $A(3;\ 9)$ va mediana tenglamalari $y-6=0$ va $3x-4y+9=0$ bo'lsin. Qolgan uchlarining koordinatalarini toping.

126. Agar uchburchakning katetlari koordinata o'qlarida joylashgan va yuzasi 12 kv. birlikka teng bo'lsa, $M(2;\ 3)$ nuqtadan o'tuvchi to'g'ri burchakli uchburchak gipotenuzasining tenglamasini yozing.

127. Agar kvadratning tomoni oxirlari koordinata o'qlarida yotgan to'g'ri chiziq kesmasidan iborat bo'lsa, uning qolgan uchta tomonining tenglamasini tuzing.

3-§. IKKINCHI TARTIBLI EGRI CHIZIQLAR

1. Aylana.

Aylana deb tekislikdagi shunday nuqtalarning to'plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning markaz deb ataluvchi nuqtasigacha bo'lgan masofa o'zgarmas miqdordir, Agar bu o'zgarmas miqdor r –aylananing radiusi, $C(a;\ b)$–uning markazi bo'lsa, aylananing tenglamasi

$$(x-a)^2+(y-b)^2=r^2 \qquad (1)$$

ko'rinishda bo'ladi. Aylana markazi koordinata boshi bilan ustma-ust tushsa, aylana tenglamasi $x^2+y^2=r^2$ bo'ladi. Agar (1) tenglamada qavslarni ochsak:

$$x^2+y^2+\ell x+my+n=0, \qquad (2)$$

bu yerda $\ell=-2a,\ m=-2b,\ n=a^2+b^2-r^2$.

Agar $\ell^2+\mathrm{m}^2-4\mathrm{n}>0$ bo'lsa, (2) tenglama aylanani ifodalaydi. Agar $\ell^2+\mathrm{m}^2-4\mathrm{n}=0$ bo'lsa, tenglama $(-\ell/2;-\mathrm{m}/2)$ nuqtani aniqlaydi, agar $\ell^2+\mathrm{m}^2-4\mathrm{n}<0$ bo'lsa, u geometrik ma'noga ega emas. Bu holda tenglama mavhum aylanani aniqlaydi. Aylana tenglamasida x^2, y^2 oldidagi koeffitsientlar teng bo'lib, xy li had qatnashmaydi. Agar $x_1^2+y_1^2=r^2$ bo'lsa, $M(x_1;\ y_1)$ nuqta aylanada yotadi, $x_1^2+y_1^2>r^2$ bo'lsa, M nuqta aylanadan tashqarida, $x_1^2+y_1^2<r^2$ bo'lsa, M nuqta aylana ichida yotadi.

128. $2x^2+2y^2-8x+5y-4=0$ aylananing radiusi va markazining koordinatalarini toping.

Yechish:

Tenglamani 2 ga qisqartirib va hadlarini guruhlab $x^2-4x+y^2+(5/2)y=2$ tenglamani yozamiz. x^2-4x va $y^2+(5/2)y$ ni to'la kvadratga to'ldirib, birinchisiga 4 ni, ikkinchisiga $(5/4)^2$ ni, o'ng tomoniga y^2 va $(5/4)^2$ ni qo'shamiz:

$$(x^2-4x+4)+(y^2+\frac{5}{2}y+\frac{25}{16})=2+4+\frac{25}{16}$$

yoki

$$(x-2)^2+(y+5/2)^2=\frac{121}{16}.$$

Shunday qilib, aylana markazining koordinatalari $a=2$, $b=-5/4$, radiusi $r=11/4$.

129. Tomonlari $9x-2y-41=0$, $7x+4y+7=0$, $x-3y+1=0$ tenglamalar bilan berilgan uchburchakka tashqi chizilgan aylana tenglamasini tuzing.

Yechish:

Uchburchak uchlarining koordinatalarini topamiz:

$$\begin{cases}9x-2y-41=0,\\7x+4y+7=0;\end{cases} \begin{cases}9x-2y-41=0,\\x-3y+1=0;\end{cases} \begin{cases}7x+4y+7=0,\\x-3y+1=0.\end{cases}$$

Natijada $A(3;-7)$, $B(5;2)$, $C(-1;0)$ nuqtalarga ega bo‘lamiz. Izlangan aylana tenglamasi $(x-a)^2+(y-b)^2=r^2$ bo‘lsin. a, b, r noma’lumlarni topish uchun A, B, C nuqtalarning koordinatalarini izlangan aylana tenglamasiga qo‘yamiz, natijada
$(3-a)^2+(-7-b)^2=r^2$; $(5-a)^2+(2-b)^2=r^2$; $(-1-a)^2+b^2=r^2$ ga ega bo‘lamiz. r^2 ni yo‘qotib quyidagi tenglamalar sistemasiga kelamiz

$$\begin{cases}(3-a)^2+(-7-b)^2=(5-a)^2+(2-b)^2,\\(3-a)^2+(-7-b)^2=(-1-a)^2+b^2.\end{cases}$$

yoki

$$\begin{cases}4a+18b=-29,\\8a-14b=57.\end{cases}$$

Bundan $a=3{,}1$; $b=-2{,}3$. r^2 ni $(-1-a)^2+b^2=r^2$ tenglamadan topamiz, ya’ni $r^2=22{,}1$. Shunday qilib, izlangan tenglama $(x-3{,}1)^2+(y+2{,}3)^2=22{,}1$ bo‘ladi.

130. Agar aylananing markazi $x+y-3=0$ to‘g‘ri chiziqda yotsa, $A(5;0)$ va $B(1;4)$ nuqtalardan o‘tuvchi aylana tenglamasini tuzing.

Yechish:

AB kesuvchining o‘rtasi bo‘lgan M nuqtani topamiz: $x_M=(5+1)/2=3$, $y_M=(4+0)/2=2$, ya’ni $M(3;2)$.

Aylananing markazi AB ning o‘rtasiga o‘tkazilgan perpendikularda yotadi. AB to‘g‘ri chiziqning tenglamasi $(y-0)/(4-0)=(x-5)/(1-5)$ ya’ni $x+y-5=0$ bo‘ladi. Bu to‘g‘ri chiziqning burchak koeffitsienti -1 bo‘lgani uchun perpendikularning burchak koeffisienti 1 ga teng, perpendikularning tenglamasi

$$y-2=x-3, \text{ ya'ni } x-y-1=0.$$

C aylananing markazi AB to‘g‘ri chiziq bilan ko‘rsatilgan perpendikular kesishgan nuqtadir, uni $x+y-5=0$, $x-y-1=0$ tenglamalarni birga yechib topiladi. Demak, $x=2$, $y=1$, ya’ni $C(2;1)$. Aylananing radiusi CA kesmaning uzunligiga tengdir, ya’ni

$$r=\sqrt{(5-2)^2+(1-0)^2}=\sqrt{10}.$$

Demak, izlangan tenglama

$$(x-2)^2+(y-1)^2=10.$$

131. $x^2+y^2=49$ aylananing $A(1;2)$ nuqtada teng o‘rtasidan bo‘linadigan vatarning tenglamasi tuzilsin.

Yechish:

Aylananing $A(1;\ 2)$ nuqtadan o'tgan diametr tenglamasini tuzamiz. Uning tenglamasi $y=2x$. Izlangan vatar diametriga perpendikular va A nuqtadan o'tadi, ya'ni uning tenglamasi: $y-2=(-1/2)(x-1)$ yoki $x+2y-5=0$.

132. $x-y-3=0$ to'g'ri chiziqqa nisbatan $x^2+y^2=2x+4y-4$ aylanaga simmetrik bo'lgan aylana tenglamasini toping.

Yechish:

Berilgan aylana tenglamasini $(x-1)^2+(y-2)^2=1$ kanonik holga keltiramiz; aylananing markazi $C(1;\ 2)$ va radiusi 1 ga teng. Simmetrik aylananing markazi $C_1(x_1;\ y_1)$ ni topamiz, buning uchun C nuqtadan $x-y-3=0$ to'g'ri chiziqqa perpendikular to'g'ri chiziq o'tkazamiz. Uning tenglamasi $y-2=k(x-1)$, bu yerda $k=-1/1=-1$ bundan $y-2=-x+1$ yoki $x+y-3=0$. $x-y-3=0$ va $x+y-3=0$ tenglamalarni birga yechib $x=3$, $y=0$ topamiz, ya'ni $C(1;\ 2)$ nuqtaning berilgan to'g'ri chiziqqa proeksiyasi $P(3;\ 0)$. Simmetrik nuqtaning koordinatalarini kesmaning o'rtasini topish formulalaridan izlaymiz: $3=(1+x_1)/2$, $0=(2+y_1)2$; shunday qilib, $x_1=5$, $y_1=-2$. Demak, $C_1(5;\ -2)$ simmetrik aylananing markazi, u aylana $(x-5)^2+(y+2)^2=1$ tenglamaga ega bo'ladi.

133. $x^2+y^2=4(y+1)$ aylananing koordinata boshidan o'tuvchi vatarlar o'rtalari to'plamini toping.

Yechish:

Vatarlar to'plamining tenglamasi $y=kx$ ko'rinishga ega. Vatarlarning aylana bilan kesishgan nuqtalarining koordinatalarini k orqali ifodalaymiz, buning uchun $y=kx$ va $x^2+y^2-4y-4=0$ tenglamalarni birga yechamiz. $x^2(k^2+1)-4kx-4=0$ kvadrat tenglamani hosil qilamiz. Bu yerda $x_1+x_2=4k/(1+k^2)$. Abssissalar yig'indisining yarmi vatar o'rtasining abssissasini beradi, ya'ni $x=2k/(1+k^2)$, vatar o'rtasining ordinatasi $y=2k^2/(1+k^2)$ ga teng. Oxirgi ikkita tenglik izlangan nuqtalar to'plamining parametrik tenglamalaridir. Bu tengliklardan k ni yo'qotib (buning uchun $x=2k/(1+k^2)$ munosabatda $k=y/x$ deyish etarli), $x^2+y^2-2y=0$ ni hosil qilamiz. Shunday qilib, izlangan to'plam aylanadir.

134. 1) $x^2+y^2-8x+6y=0$; 2) $x^2+y^2+10x-4y+29=0$; 3) $x^2+y^2-4x+14y+54=0$ aylanalarning radiusi va markazi koordinatalarini toping.

135. $x^2+y^2+4x-6y=0$ aylananing Oy o‘qi bilan kesishgan nuqtasidan o‘tuvchi radiuslar orasidagi burchakni toping.

136. $A(1;\ 2)$, $B(0;\ -1)$ va $C(-3;\ 0)$ nuqtalardan o‘tuvchi aylana tenglamasini tuzing.

137. Markazi $2x-y-2=0$ to‘g‘ri chiziqda yotgan $A(7;\ 7)$ va $B(-2;\ 4)$ nuqtalardan o‘tuvchi aylana tenglamasini tuzing.

138. $x^2+y^2=16$ va $(x-5)^2+y^2=9$ aylanalar umumiy vatarining tenglamasini tuzing.

139. $(x-3)^2+(y+2)^2=25$ aylananing $x-y+2=0$ to‘g‘ri chiziq bilan kesishgan nuqtalaridan bu aylanaga o‘tkazilgan urinma tenglamasini tuzing.

140. $x^2+y^2=4$ aylana berilgan. $A(-2;\ 0)$ nuqtadan $|BM|=|AB|$ masofaga davom ettirilgan AB vatar o‘tkazilgan. M nuqtalar to‘plamini toping.

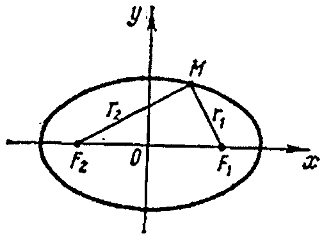
2. Ellips.

Ellips deb tekislikdagi shunday nuqtalarning to‘plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning *fokuslar* deb ataluvchi ikki nuqtasigacha bo‘lgan masofalar yig‘indisi o‘zgarmas miqdordir. U $2a$ bilan belgilanadi, bu o‘zgarmas miqdor fokuslar orasidagi masofadan katta bo‘ladi.

Agar koordinata o‘qlari ellipsga nisbatan 10-chizmada ko‘rsatilganidek joylashib, ellipsning fokuslari esa OX o‘qida koordinat boshidan bir xil masofada ($F_1(c;\ 0)$, $F_2(-c;\ 0)$) yotsa, ellipsning oddiy (kanonik) tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (1)$$

ko‘rinishda bo‘ladi. a – ellipsning katta, b – kichik yarim o‘qi, a, b, c (c – fokuslar orasidagi masofaning yarmi) lar o‘rtasida $a^2=b^2+c^2$ munosabat bor.



10-chizma

Ellipsning shakli (siqilish o‘lchovi) uning ekssentrisiteti $\ell=c/a$ ($c<a$ bo‘lgani uchun $\ell<1$) bilan xarakterlanadi. Ellipsning biror M nuqtasidan fokuslargacha bo‘lgan masofalar *nuqtaning fokal radi-*

us-vektorlari deb ataladi. Ular r_1, r_2 bilan belgilanadi (ellipsning ta'rifiga ko'ra, uning ixtiyoriy nuqtasi uchun $r_1+r_2=2a$ bo'ladi). Xususiy holda $a=b$ ($c=0$, $\ell=0$, fokuslar markaz bilan ustma-ust tushsa) bo'lsa, ellips aylanaga aylanib qoladi: $x^2+y^2=a^2$. $M(x_1;\ y_1)$ nuqta va $x^2/a^2+y^2/b^2=1$ ellipsning o'zaro joylanishi quyidagi shartlar bilan aniqlanadi: agar $x_1^2/a^2+y_1^2/b^2=1$ bo'lsa, M nuqta ellipsda yotadi; agar $x_1^2/a^2+y_1^2/b^2<1$ bo'lsa, M nuqta ellipsdan tashqarida; $x_1^2/a^2+y_1^2/b^2<1$ bo'lsa, M nuqta ellips ichida yotadi. Fokal radius-vektorlar ellips nuqtalarining absissasi orqali $r_1=a-ex$ (o'ng fokal radius-vektor), $r_2=a+ex$ (chap fokal radius vektor) ifodalanadi.

141. $M(5/2;\sqrt{6}/4)$ va $N(-2;\sqrt{15}/5)$ nuqtalardan o'tuvchi ellipsning kanonik tenglamasini tuzing.

Yechish: $x^2/a^2+y^2/b^2=1$ izlangan ellips tenglamasi bo'lsin. Bu tenglamani M, N nuqtalarning koordinatalari qanoatlantirish kerak. Demak,

$$\frac{25}{4a^2}+\frac{3}{8b^2}=1,\quad \frac{4}{a^2}+\frac{3}{5b^2}=1.$$

Bundan $a^2=10$, $b=1$ topamiz. Demak, ellips tenglamasi $x^2/10+y^2=1$ bo'ladi.

142. $x^2/25+y^2/9=1$ ellips fokal radius-vektorlar ayirmasiga teng bo'lgan nuqtani toping.

143. $x^2/a^2+y^2/b^2=1$ ellipsning fokusidan katta yarim o'qqa tushirilgan perpendikularning ellips bilan kesishgan nuqtasigacha uzunligini toping.

144. Chap fokusdan va $x^2/25+y^2/16=1$ ellipsning quyi uchidan o'tgan to'g'ri chiziq tenglamasini tuzing.

145. Koordinata o'qlariga joylashtirilgan ellips $M(1;\ 1)$ nuqtadan o'tadi va ekssentrisiteti $\ell=3/5$ ga teng. Ellips tenglamasini tuzing.

146. $M(7;\ 1)$, $N(-5;\ -4)$, $P(4;\ 5)$ nuqtalar $x^2/50+y^2/32=1$ ellipsga nisbatan qanday joylashgan.

147. Agar ellipsning fokal kesmasi uchidan α burchak ostida ko'rinsa, uning ekssentrisitetini toping.

148. $x+5=0$ to'g'ri chiziqda $x^2/20+y^2/4=1$ ellipsning chap fokusidan va yuqori uchidan barobar uzoqlikda turgan nuqtani toping.

149. Agar $F_1(0;\ 0)$ va $F_2(1;\ 1)$ ellipsning fokuslari katta o'qi 2 ga teng bo'lsa, ellips ta'rifidan foydalanib, uning tenglamasini tuzing.

150. $A(0;\ 1)$ nuqtadan masofasi $y-4=0$ to'g'ri chiziqqacha bo'lgan masofadan 2 marta kichik bo'lgan nuqtalar geometrik o'rnining tenglamasini tuzing.

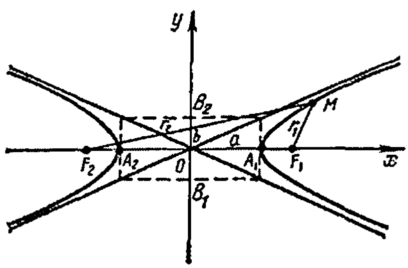
151. Uzunligi A ga teng bo'lgan AB kesmaning oxirlari to'g'ri burchakning tomonlari bo'yicha sirpanadi. Bu kesmani 1:2 nisbatda bo'luvchi M nuqta chizadigan egri chiziq tenglamasini tuzing.

3. Giperbola.

Giperbola deb tekislikdagi shunday nuqtalarning to'plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning *fokuslar* deb ataluvchi ikki nuqtasigacha bo'lgan masofalar ayirmalarining absolut qiymatlari o'zgarmas miqdordir. Bu o'zgarmas miqdorni 2a bilan belgilanadi, u fokuslar orasidagi masofadan kichik. Agar giperbolaning fokuslarini $F_1(c;\ 0)$, $F_2(-c;\ 0)$ nuqtalarga joylashtirsak, u holda giperbolaning $x^2/a^2 - y^2/b^2 = 1$ (1) kanonik tenglamaga ega bo'lamiz, bu yerda $b^2 = c^2 - a^2$. Giperbola ikki tarmoqdan iborat va koordinata o'qlariga simmetrik joylashgan.

$A_1(a;\ 0)$, $A_2(-a,\ 0)$ lar giperbolaning uchlari deb ataladi. $|A_1A_2|=a$ giperbolaning *haqiqiy*, $|A_1A_2|=b$ *mavhum o'qi* deyiladi (11-chizma).

Agar $M(x;\ y)$ nuqtadan biror to'g'ri chiziqqacha bo'lgan masofasi nolga intilsa ($x\to+\infty$ yoki $x\to-\infty$), u to'g'ri chiziq giperbolaning *asimptotasi* deyiladi. Giperbola ikkita asimptotaga ega, ular $y=\pm(b/a)x$. Giperbolaning asimptotalarini yasash uchun tomonlari $x=a$, $x=-a$, $y=b$, $y=-b$ bo'lgan to'g'ri turtburchak chizamiz. Bu to'g'ri turtburchakning qarama-qarshi uchlaridan o'tkazilgan to'g'ri chiziq giperbolaning asimptotalari bo'ladi.



11-chizma

11-chozmada giperbola va uning asimptotalari o'zaro joylanishi ko'rsatilgan. $\ell=c/a>1$ nisbat giperbolaning ekssentrisiteti deyiladi. $r_1\ell=x-a$ (o'ng fokal radius-vektor), $r_1\ell=x+a$ (chap fokal radius-vektori) giperbola o'ng tarmog'ining *fokal radius-vektorlari* deyiladi. Xuddi shunday chap tarmog'ining fokal radius-vektorlari $r_1=-\ell x+a$, $r_1(-\ell x-a)$ bo'ladi. Agar $a=b$ bo'lsa, giperbolaning tenglamasi $x^2-y^2=a^2$ bo'ladi. Bunday giperbola *teng tomonli* deb ataladi. Uning asimptotalari to'g'ri burchak hosil qiladi. Agar koordinat o'qlarini asimptotalar deb qarasak (teng tomonli giperbolada), uning tenglamasi $xy=m$ ($m=\pm a^2/2$; $m>0$ bo'lsa, giperbola I va III chorakda, $m<0$ bo'lsa, II va IV chorakda yotadi. $xy=m$ tenglamani $y=m/x$ ko'rinishda yozish mumkin bo'lgani uchun teng tomonli giperbola x, y miqdorlar orasidagi teskari proporsional bog'lanishni ifodalaydi.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \left(\frac{y^2}{b^2}-\frac{x^2}{a^2}=1\right)$$

tenglama ham giperbolani ifodalaydi, lekin haqiqiy o'q OY bo'ladi.

$x^2/a^2-y^2/b^2=1$ va $x^2/a^2-y^2/b^2=-1$ giperbolalar bir xil yarim o'qqa va bir xil asimptotaga ega, lekin haqiqiy va mavhum o'qlari almashinib keladi. Bunday giperbolalar *qo'shma* deb ataladi.

152. $x^2/16-y^2/9=1$ giperbolaning o'ng tarmog'ida shunday nuqtalarni topingki, undan o'ng fokusgacha masofa chap fokusgacha bo'lgan masofadan 2 marta kichik bo'lsin.

Yechish:

Giperbolaning o'ng tarmog'i uchun fokal radius-vektorlar $r_1\ell=x-a$ va $r_2\ell=x+a$ formulalar bilan aniqlanadi. Demak, $\ell x+a=2\ell(x-a)$ tenglamaga ega bo'lamiz, bundan $x=3a/\ell$; bu yerda $a=4$, $\ell=c/a=\sqrt{a^2+b^2}/a=5/4$, ya'ni $x=9{,}6$. Giperbola tenglamasidan ordinatani topamiz:

$$y=\pm\frac{3}{4}\sqrt{x^2-16}=\pm\frac{3}{4}\sqrt{\left(\frac{48}{5}\right)^2-16}=\pm\frac{3}{5}\sqrt{119}.$$

Shunday qilib, masalaning shartini ikki nuqta qanoatlantiradi:

$M_1(9{,}6;0{,}6\sqrt{119})$. va $M_2(9{,}6;-0{,}6\sqrt{119})$.

153. $A(1,\ 0)$ va $B(2,\ 0)$ nuqtalar berilgan. M nuqta shunday harakatlanadiki, AMB uchburchakda B burchak A burchakdan 2 marta kattaligicha qoladi. M nuqta chizadigan egri chiziq tenglamasini toping.

Yechish:

M nuqtaning koordinatalarini x, y bilan belgilab $tg\,\hat{B}$ va $tg\,\hat{A}$ larni A, B va M nuqtalarning koordinatalari bilan ifodalaymiz:

$$tg\,\hat{B} = -\frac{y}{x-2} = \frac{y}{2-x}, \quad tg\,\hat{A} = \frac{y}{x+1}.$$

Shart bo'yicha:

$tg\,\hat{B} = tg2\hat{A}$, ya'ni $tg\,\hat{B} = 2tg\,\hat{A}/(1 - tg^2\,\hat{A})$ tenglamaga ega bo'lamiz. Bu tenglamaga $tg\,\hat{B}$, $//tg\,\hat{A}$ larni qo'yib

$$\frac{y}{2-x} = \frac{2y(x+1)}{1 - y^2/(1+x)^2}$$

ga ega bo'lamiz, y ($y \neq 0$) ga qisqartirib va soddalashtirib $x^2 - y^2/3 = 1$ ega bo'lamiz. Izlangan egri chiziq giperboladir.

154. Giperbolaning ekssentrisiteti $\sqrt{2}$ ga teng. $M(\sqrt{3};\sqrt{2})$ nuqtadan o'tadigan giperbolaning tenglamasini tuzing.

Yechish:

Ekssentrisitetning aniqlanishiga ko'ra $c/a = \sqrt{2}$ yoki $c^2 = 2a^2$. Lekin $c^2 = a^2 + b^2$ bo'lgani uchun $a^2 + b^2 = 2a^2$, yoki $a^2 = b^2$, ya'ni giperbolateng tomonli. Ikkinchi tenglikni M nuqtaning giperbolada yotishidan keltirib chiqaramiz, ya'ni $(\sqrt{3})^2/a^2 + (\sqrt{2})^2/b^2 = 1$ yoki $3/a^2 + 2/b^2 = 1$. $a^2 = b^2$ bo'lgani uchun $3/a^2 - 2/a^2 = 1$, ya'ni $a^2 = 1$. Shunday qilib, izlangan giperbola tenglamasi $x^2 - y^2 = 1$ bo'ladi.

155. Agar giperbolaning asimptotalari $y = \pm(2\sqrt{2}/3)x$ bo'lsa, $M(9;\ 8)$ nuqtadan o'tgan giperbola tenglamasini tuzing.

156. Fokus va uchlari tenglamasi $x^2/8 + y^2/5 = 1$ bo'lgan ellipsning mos fokus va uchlaridayotgan giperbola tenglamasini tuzing.

157. $M(0;-1)$ nuqtadan va $3x^2 - 4y^2 = 12$ ning o'ng uchidan to'g'ri chiziq o'tkazilgan. To'g'ri chiziqning giperbola bilan kesishgan ikkinchi nuqtasi topilsin.

158. $x^2 - y^2 = 8$ giperbola berilgan. $M(4;\ 6)$ nuqtadan o'tuvchi va giperbola bilan bir xil fokusga ega bo'lgan ellips tenglamasini yozing.

159. $9x^2+25y^2=1$ ellips berilgan. Ellips bilan bir xil fokusga ega bo'lgan teng tomonli giperbola tenglamasini tuzing.

160. Giperbolaning asimptotalari orasidagi burchak 60^0. Giperbolaning ekssentrisiteti topilsin.

161. $x^2/64-y^2/36=1$ giperbolaning chap tarmag'ida shunday nuqtani topingki, uning o'ng fokal radius-vektori 18 ga teng bo'lsin.

162. Ekssentrisiteti 2 va fokuslari $x^2/25+y^2/9=1$ bo'lgan ellipsning fokuslari bilan ustma-ust tushadigan giperbola tenglamasini tuzing.

163. $x^2/16-y/9=1$ giperbolaning $x^2+y^2=91$ aylana bilan kesishgan nuqtalardagi fokal radius-vektorlari kesishgan nuqtalaridagi fokal radius-vektorlari topilsin.

164. Giperbolaning asimptotalaridan biriga o'tkazilgan perpendikulyar uzunligini mavhum o'qqa tengligini isbotlang.

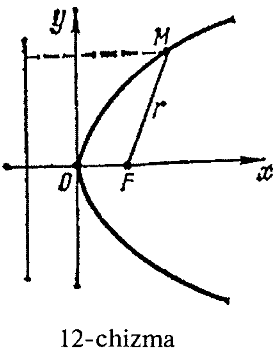
165. $x^2-y^2=1$ giperbolaning ixtiyoriy nuqtasidan uning asimptotalarigacha bo'lgan masofalar ko'paytmasi o'zgarmas songa tengligini isbotlang.

166. $x^2+4x+y^2=0$ aylanadan va $M(2;\ 0)$ nuqtadan barobar uzoqlikda turgan nuqtalar to'plamining tenglamasini tuzing.

4. Parabola.

Fokus deb ataluvchi nuqta va *direktrisa* deb ataluvchi to'g'ri chiziqdan barobar uzoqlikda turgan nuqtalar to'plami *parabola* deb ataladi. Agar parabolaning direktrisasi $x=-r/2$, fokusi $F(r/2;\ 0)$ nuqta bo'lsa, uning tenglamasi

$$y^2=2px \qquad (1)$$



12-chizma

Bu parabola abssissa o'qiga nisbatan simmetrik joylashgan (12-chizma)

$$(p>0)\ x^2=2py \qquad (2)$$

tenglama ordinata o'qiga nisbatan simmetrik bo'lgan parabola bo'ladi. $p>0$ da (1) va (2) parabola mos o'qlarining musbat tomoniga, $p<0$ bo'lsa, manfiy tomoniga qaragan bo'ladi. $y^2=2px$ parabolaning fokal radius-vektorining uzunligi $r=x+p/2$ $(r>0)$ formula bilan aniqlanadi.

167. Agar parabolaning Ox o'qiga perpendikular bo'lgan vatarning uzunligi 16 ga, bu vatarning parabola uchidan masofasi 6 ga teng bo'lsa, uchi koordinata boshida yotgan Ox o'qiga parallel bo'lgan parabola tenglamasini tuzing.

Yechish:

Vatarning uzunligi va uning parabola uchidan masofasi ma'lum bo'lgani uchun bu vatar oxirlarining koordinatalari ma'lum bo'ladi. Parabola tenglamasi $y^2=2px$; bunda $x=6$, $y=8$ deb $8^2=2p\cdot 6$ topamiz, bundan $2p=32/3$. Shunday qilib, izlangan parabola tenglamasi $y^2=32x/3$ bo'ladi.

168. Oy o'qiga nisbatan simmetrik va I, III koordinat burchak bissektrisasida uzunligi $8\sqrt{2}$ bo'lgan vatar ajratuvchi, uchi koordinata boshida yotgan parabola tenglamasini tuzing.

Yechish:

Izlangan parabola tenglamasi $x^2=2py$, bissektrisasining tenglamasi $y=x$. Shunday qilib, parabola bilan bissektrisaning kesishgan nuqtalarini hosil qilamiz: O(0; 0) va $M(2p;\ 2p)$. Vatarning uzunligi ikki nuqta orasidagi masofa kabi topiladi: $8\sqrt{2}=\sqrt{4p^2+4p^2}$, bundan $2p=8$. Demak, izlangan tenglama $x^2=8y$ bo'ladi.

169. Agar parabolaning fokusi $4x-3y-4=0$ to'g'ri chiziq bilan Ox o'qining kesishgan nuqtasida yotsa, parabolaning tenglamasini tuzing.

170. $y^2=8x$ parabolada direktrisadan 4 ga teng masofada turuvchi nuqtani toping.

171. Ox o'qiga nisbatan simmetrik, $y=x$ to'g'ri chiziqdan uzunligi $4\sqrt{2}$ ga teng vatar ajratuvchi, uchi koordinata boshida yotuvchi parabola tenglamasini tuzing.

172. $y^2=2x$ parabola koordinat boshidan o'tuvchi to'g'ri chiziqdan uzunligi 3/4 ga teng vatar ajratadi. Bu to'g'ri chiziq tenglamasini tuzing.

173. Agar simmetriya o'qiga perpendikular, fokus va parabola uchi orasidagi masofani teng o'rtasidan bo'luvchi vatarning uzunligi birga teng bo'lsa, parabola tenglamasini tuzing.

174. $y^2=32x$ parabolada $4x+3y+10=0$ to'g'ri chiziqdan 2 birlik masofada turuvchi nuqtani toping.

175. Uchi koordinat boshida yotuvchi, $M(4;\ 2)$ nuqtadan o'tuvchi va Ox o'qiga simmetrik o'tgan parabola tenglamasini tuzing; bu nuqtaning fokal radius-vektori bilan Ox orasidagi α burchakni aniqlang.

4-§. KOORDINATALARNI ALMASHTIRISH VA IKKINCHI TARTIBLI EGRI CHIZIQ TENGLAMALARINI SODDALASHTIRISH

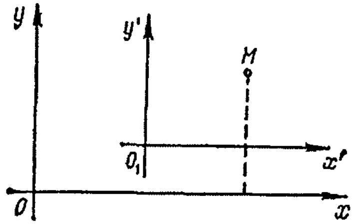
1. Koordinatalarni almashtirish.

xOy koordinat sistemasidan yangi $x'O_1y'$ (koordinat o'qlarining yo'nalishi o'zgarmaydi, ya'ni koordinat boshi deb O_1(a, b) nuqta olinadi; 13-chizma) sistemaga o'tishda qandaydir M nuqtaning eski va yangi koordinat sistemalari bilan bog'lanishi

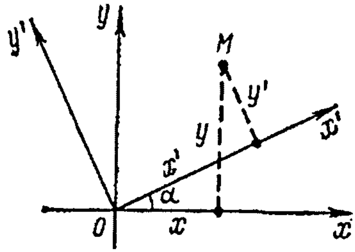
$$x = x' + a, \quad y = y' + b \tag{1}$$

$$x' = x - a, \quad y' = y - b \tag{2}$$

formulalar bilan aniqlanadi.



13-chizma



14-chizma

(1) formula orqali eski koordinatalar yangilari orqali, (2) formuladan yangi koordinatlar eskilari orqali aniqlanadi. Koordinat o'qlarini A burchakka burashda (koordinat boshi o'zgarmaydi) A soat miliga teskari yo'nalishda hisoblanadi (14-chizma). Eski x, y yangi x', y' koordinatalar bilan

$$x = x'\cos\alpha - y'\sin\alpha, \quad y = x'\sin\alpha + y'\cos\alpha \tag{3}$$

$$x' = x\cos\alpha - y\sin\alpha, \quad y' = -x\sin\alpha + y\cos\alpha \tag{4}$$

formulalar bilan aniqlanadi.

176. Koordinat o'qlari parallel ko'chirilgan, yangi koordinat boshi $O_1(3; -4)$ nuqtada joylashgan. Nuqtaning eski koordinatalari $M(7;\ 8)$ ma'lum. Bu nuqtaning yangi koordinatalarini toping.

Yechish:

Bu yerda $a = 3$, $b = -4$, $x = 7$, $y = 8$ teng. (2) formuladan $x' = 7 - 3 = 4$, $y' = 8 - (-4) = 12$ larni topamiz.

177. xOy tekisligida $M(4;\ 3)$ nuqta berilgan. Koordinat sistemasi shunday buralganki, yangi o‘q M nuqtadan o‘tsin. Agar A nuqtaning yangi koordinatalari $x`=5$, $y`=5$ bo‘lsa, A ning eski koordinatalarini toping.

Yechish:

$OM=\sqrt{4^2+3^2}=5$ bo‘lgani uchun $\sin\alpha=3/5, \cos\alpha=4/5$ teng, u holda (3) formulalar $x=(4/5)x`-(3/5)y`$, $y=(3/5)x`+(4/5)y`$ ko‘rinishni oladi. $x`=y`=5$ deb olib, $x=1$, $y=7$ ni topamiz.

178. Koordinat sistemasi $\alpha=\pi/6$ burchakka burilgan $M(\sqrt{3};3)$ nuqtaning yangi koordinatalarini toping.

Yechish:

(4) formuladan foydalanib, topamiz:

$x'=\sqrt{3}\cos(\pi/6)+3\sin(\pi/6)=3/2+3/2=3,$

$y'=-\sqrt{3}\sin(\pi/6)+3\cos(\pi/6)=-\sqrt{3}/2+3\sqrt{3}/2=\sqrt{3}$

179. $M(9/2;\ 11/2)$ nuqta berilgan. Yangi koordinata o‘qlari sifatida

$2x-1=0$ ($O_1y`$ o‘q), $2y-5=0$ ($O_1x`$ o‘q) chiziqlar olingan. M nuqtaning yangi sistemasidagi koordinatalari topilsin.

180. $M(4\sqrt{5};\ 2\sqrt{5})$ nuqta berilgan. Yangi abssissa o‘qi sifatida $y=2x$, ordinata o‘qi sifatida $y=-0{,}5x$ to‘g‘ri chiziqlar olingan. Yangi o‘qlar eskilari bilan o‘tkir burchak tashkil etadi. M nuqtaning yangi sistemadagi koordinatalarini toping.

2. Parabola: $y=Ax^2+Bx+C$ va giperbola: $y=(kx+l)/(px+q)$.

$y=Ax^2+Bx+C$ ko‘rinishdagi tenglama koordinat o‘qlarini parallel ko‘chirganda, ya’ni $x=x`+a$, $y=y`+b$ (a, B – yangi koordinat boshi, $x`$, $y`$ – yangi koordinatalar) formulalar yordamida parabolaning kanonik tenglamasiga keladi.

$y=Ax^2+Bx+C$ bilan aniqlanadigan parabola Oy o‘qiga parallel bo‘lgan simmetriya o‘qiga ega bo‘ladi. ($x=Ay^2+By+C$ esa Ox o‘qiga parallel bo‘lgan simmetriya o‘qiga ega). $y=(kx+l)/(px+q)$ kasr chiziqli funksiya, agar $kq-pl\neq0$, $p\neq0$ bo‘lsa, teng tomonli giperbolani aniqlaydi. Koordinat o‘qlarini parallel ko‘chirganda bu tenglama teng tomonli giperbolaning $xy=m$ kanonik tenglamasiga keladi, bu giperbolaning asimptotalari koordinat o‘qlarida bo‘ladi. $m>0$ bo‘lganda giperbola tarmoqlari I va III chorakda, $m<0$ bo‘lganda II va IV chorakda yotadi.

181. $y=9x^2-6x+2$ parabola tenglamasini kanonik holga keltiring.

Yechish:

x, y o‘rniga mos ravishda $x'+a$, $y'+b$ ni qo‘yamiz: $y'+b=$ $=9(x'+a)^2-6(x'+a)+2$ yoki $y'=9x'^2+6x'(3a-1)+(9a^2-6a+2-b)$. a, b larni shunday tanlaymizki, x' oldidagi koeffitsient va ozod xad nolga aylansin:

$$\begin{cases} 3a-1=0, \\ 9a^2-6a+2-b=0. \end{cases} \quad \text{ya'ni } a=1/3, \; b=1.$$

Demak, $x'^2=(1/9)y'$ parabolaning kanonik tenglamasi. Parabola uchi

$$Q_1(1/3;\ 1) \text{ da va } p=1/18.$$

Bunday masalani boshqacha usul bilan ham yechish mumkin. Berilgan $y=Ax^2+Bx+C$ (yoki $x=Ay^2+By+C$) tenglama $(x-a)^2=2p(y-b)[(y-b)^2=2p(x-a)]$ ko‘rinishga keltiriladi. U holda $O_1(a,\ b)$ nuqta parabolaning uchi, p parametrning ishorasi parabolaning qaysi tomonga yo‘nalganini ko‘rsatadi.

$y=9x^2-6x-2$ ni quyidagicha o‘zgartiramiz:

$$y=9(x^2-\frac{3}{2}x+\frac{1}{9})-1+2;$$

$$y-1=9(x-\frac{1}{3})^2; \quad (x-\frac{1}{3})^2=\frac{1}{9}(y-1).$$

Bundan yana parabola uchi $O_1(1/3;\ 1)$ nuqtada bo‘lib, parametr $p=1/18$ ga tengligini topamiz, parabola Oy o‘qining musbat tomoniga qarab yo‘nalgan.

182. $y=(4x+5)/(2x-1)$ giperbola tenglamasini $x'y'=k$ kurinishga keltiring. Giperbola asimptotalarining tenglamasini boshlang‘ich koordinat sistemasiga nisbatan yozing.

Yechish:

Koordinat o‘qlarini parallel ko‘chirish yordamida berilgan tenglama

$$(y'+b)(2x'+2a-1)=4x'+4a+5$$

yoki

$$2x'y'+(2b-4)x'+(2a-1)y'=4a+b-2AB+5$$

ko‘rinishga keladi. $2b-4=0$, $2a-1=0$ dan $a=0{,}5$, $b=2$ larni topamiz. U holda yangi koordinat sistemasida giperbola tenglamasi

$x'y' = 3{,}5$ ko'rinishga keladi. Giperbolaning asimptotalari sifatida yangi koordinatalar olingani uchun, $x' = 0{,}5$, $y' = 2$ lar asimptota bo'ladi. Bunday masalaning boshqacha yechilishi: $y = (kx + 1)/(px + q)$ tenglama $(x - a)(y - b) = m$ ko'rinishga keltiriladi, giperbolaning markazi $O_1(a, b)$ nuqtada yotadi; uning asimptotalari sifatida $x = a$ va $x = b$ lar olinadi, M ning ishorasi giperbolaning qaysi choraklarda yotishini ko'rsatadi. $y=(4x+5)/(2x-1)$ ni

$$2(x-\frac{1}{2})y-4(x-\frac{1}{2}+\frac{7}{4})=0.$$

yoki

$$(2x-1)y-(4x+5)=0;\; 2(x-0{,}5)(y-2)=7$$

ko'rinishga keltiramiz.

Shunday qilib, giperbola tenglamasi $(x - 0{,}5)(y - 2) = 3{,}5$ ko'rinishga keltirildi. Giperbola markazi $O_1(0{,}5;\ 2)$ nuqtada, giperbola tarmoqlari I va III chorakda $x - 0{,}5 = 0$, $y - 2 = 0$ asimptotalar orasida yotadi.

183. 1) $y = 4x - 2x^2$; 2) $y = -x^2 + 2x + 2$; 3) $x = -4y^2 + y$; 4) $x = y^2 + 4y + 5$ parabolalar tenglamasini kanonik ko'rinishga keltiring.

184. 1) $y = 2x/(4x - 1)$ 2) $y = (2x + 3)/(3x - 2)$ 3) $y = (10x + 2)/(5x + 4)$ giperbola tenglamalarini $x'y' = m$ ko'rinishga keltiring.

3. Ikkinchi tartibli egri chiziqning besh hadli tenglamasi.

$Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0$ tenglama *ikkinchi tartibli egri chiziqning besh xadli tenglamasi* deyiladi (xy had qatnashmaydi). Bu tenglama tekislikda ellips, giperbola yoki parabolani aniqlaydi (A, C koeffitsientlar ko'paytmasining ishorasiga qarab koordinat o'qlariga parallel bo'lgan simmetriya o'qlariga ega bo'ladi).

1. $AC > 0$ bo'lsa ellips, $A = C$ bo'lsa ellips aylanaga aylanadi.

2. $AC < 0$ bo'lsa giperbola, agar tenglamaning chap tomoni ikkita chiziqli ko'paytuvchiga ajralsa, giperbola ikkita kesishuvchi to'g'ri chiziqqa aylanadi:

$$Ax^2 + Cy^2 + 2Dx + 2Ey + F = (a_1x + b_1y + c_1)(a_2x + b_2y + c_2)$$

3. $AC = 0$ ($A = 0$, $C \neq 0$ yoki $A \neq 0$, $C = 0$) bo'lsa tenglama parabolani aniqlaydi, bu holda, agar chap tomon yoki x, yoki y ni o'z ichiga olmasa; (agar tenglama $Ax^2 + 2Dx + F = 0$ yoki

$Cy^2+2Ey+F=0$ ko'rinishga ega bo'lsa) parabola ikkita parallel to'g'ri chiziqqa ajralishi mumkin (*h*aqiqiy har xil, haqiqiy ust-ma-ust tushadigan yoki mavhum). Egri chiziqning turi va uning tekislikdagi joylanishi uni $A(x-x_0)^2+C(y-y_0)^2=f$ ($AC>0$ yoki $AC<0$) ko'rinishga keltirib osongina topiladi; hosil bo'lgan tenglama ko'rinishiga qarab ellips va giperbolalarning ajralishi yoki birlashishini ham aniqlash mumkin. Birlashmagan egri chiziq holida, koordinat boshini $O_1(x_0,\ y_0)$ ko'chirib, ellips yoki giperbola tenglamasini kanonik holga keltirish mumkin. $AC=0$ bo'lgan hol oldingi paragrafda to'la qaralgan, chunki bu holda parabola tenglamasini $y=a_1x^2+b_1x+c$ yoki $x=a_1y^2+b_1y+c_1$ ko'rinishda yozish mumkin.

185. $4x^2+9y^2-8x-36y+4=0$ tenglama qanday egri chiziqni ifodalaydi.

Yechish:

Berilgan tenglamani quyidagicha o'zgartiramiz:

$$4(x^2-2x)+9(y^2-4y)=-4;$$

$$4(x^2-2x+1-1)+9(y^2-4y+4-4)=-4;$$

$$4(x-1)^2+9(y-2)^2=-4+4+36;$$

$$4(x-1)^2+9(y-2)^2=36.$$

Yangi koordinata boshi deb $O`(1;\ 2)$ nuqtani olib, o'qlarni parallel ko'chiramiz. Koordinatalarni almashtirish formulalaridan foydalanamiz: $x=x`+1$, $y=y`+2$. Yangi o'qlarga nisbatan $4x`^2+9y`^2=36$, yoki $x`^2/9+y`^2/4=1$ tenglamaga ega bo'lamiz. Shunday qilib, berilgan tenglama ellips ekan.

186. $x^2-9y^2+2x+36-44=0$ tenglama qanday egri chiziqni ifodalaydi.

Yechish:

Berilgan tenglamani o'zgartiramiz:

$$(x^2+2x+1-1)-9(y^2-4y+4-4)=44,$$

$$(x+1)^2-9(y-2)^2=44+1-36,$$

$$(x+1)^2-9(y-2)^2=9.$$

Yangi koordinat boshi deb, $O_1(-1;\ 2)$ ni olib o'qlarni parallel ko'chiramiz. Koordinatalarni almashtirish formulalari $x=x`-1$, $y=y`+2$ bo'ladi. Koordinatalarni almashtirishlardan so'ng $x`^2-9y`^2=9$ yoki $x`^2/9-y`^2=1$ ga ega bo'lamiz. Bu egri chiziq giperboladir. Bu giperbolaning asimptotalari $y'=\pm(1/3)x'$ to'g'ri chiziqlardir.

Quyidagi misollarda keltirilgan tenglamalar qanday egri chiziqni aniqlaydi? Ularni chizing.

187. $36x^2 + 3y^2 - 36 - 24y - 23 = 0.$

188. $16x^2 + 25y^2 - 32x + 50y - 359 = 0.$

189. $\frac{1}{2}x^2 - \frac{1}{9}y^2 - x + \frac{2}{3}y - 1 = 0.$

190. $x^2 + 4y^2 - 4x - 8y + 8 = 0.$

191. $x^2 + 4y^2 + 8y + 5 = 0.$

192. $x^2 - y^2 - 6x + 10 = 0.$

193. $2x^2 - 4x + 2y - 3 = 0.$

194. $x^2 - 6x + 8 = 0.$

195. $x^2 + 2x + 5 = 0.$

4. Ikkinchi tartibli egri chiziqning umumiy tenglamasini kanonik holga keltirish.

Agar ikkinchi tartibli egri chiziq

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$$

tenglama bilan berilgan bo'lsa, koordinata o'qlarini burab, $x = x'\cos\alpha - y'\sin\alpha$, $y = x'\sin\alpha + y'\cos\alpha$ formuladan foydalanib, koordinatalar ko'paytmasi qatnashgan haddan ozod bo'lamiz. Qolgan almashtirishlar bundan oldingi paragrafda bajarilgan. Egri chiziqning ikkita chiziqqa ajralishi berilgan tenglamaga qarab quyidagicha bajarilishi mumkin. Tenglamani y ga nisbatan kvadrat tenglama deb qaraymiz (y^2 oldidagi koeffitsient noldan farqli); agar bu yerda kvadrat ildiz ostida qandaydir $ax + b$ ikki hadning to'la kvadrati tursa, ildizdan chiqib, u uchun ikkita qiymat: $y_1 = k_1x + b_1$, $y_2 = k_2x + b_2$ topiladi. Bu egri chiziq ikkita to'g'ri chiziqqa ajraladi. Berilgan tenglama x ga nisbatan ham yechilishi mumkin. Agar egri chiziqning tenglamasida $A = C = 0$ $(B \neq 0)$ bo'lsa, $B/D = 2E/F$ bo'lgan hadgina ikkita to'g'ri chiziqni aniqlaydi.

196. $9x^2 + 24xy + 16y^2 - 25 = 0$ tenglama ikkita to'g'ri chiziq to'plamini aniqlashini ko'rsating.

Yechish:

Tenglamani $(3x + 4y)^2 - 25 = 0$ ko'rinishda yozib olamiz; chap tomonni ko'paytuvchilarga ajratamiz: $(3x + 4y - 5)(3x + 4y + 5) = 0$. Shunday qilib, berilgan tenglama $3x + 4y - 5 = 0$, $3x + 4y + 5 = 0$ ko'rinishdagi to'g'ri chiziqlarni aniqlaydi.

197. $3x^2+8xy-3y^2-14x-2y+8=0$ tenglama ikkita to'g'ri chiziq tenglamasini aniqlashini ko'rsating.

Yechish:

Berilgan tenglamani $3y^2-2(4x-1)y-(3x^2-14x+8)=0$ ko'rinishda yozib olamiz, uni y ga nisbatan yechamiz:

$$y=\frac{4x-1\pm\sqrt{(4x-1)^2+(9x^2-4x+24)}}{3}$$

yoki

$$y=\frac{4x-1\pm(5x-5)}{3}.$$

Bundan esa $y=3x-2$ va $y=(-x+4)/3$ to'g'ri chiziq tenglamalariga ega bo'lamiz, ularni $3x-y-2=0$, $x+3y-4=0$ ko'rinishda yozish mumkin.

198. $xy+2x-4y-8=0$ tenglama bilan qanday chiziq aniqlanadi.

Yechish:

Tenglamani $x(y+2)-4(y+2)=0$ yoki $(x-4)(y+2)=0$ ko'rinishda yozib olamiz. Shunday qilib, tenglama $x-4=0$, $y+2=0$ to'g'ri chiziqlarni aniqlaydi, chiziqlarning biri Ox, ikkinchisi Oy o'qiga parallel.

199. $5x^2+4xy+8y^2+8x+14y+5=0$ tenglamani kanonik ko'rinishga keltiring.

Yechish:

Birinchi banddagi (3) formuladan foydalanib, tenglamani o'zgartiramiz:

$$5(x'\cos\alpha-y'\sin\alpha)^2+4(x'\cos\alpha-y'\sin\alpha)(x'\sin\alpha+y'\cos\alpha)+$$
$$+8(x'\sin\alpha+y'\cos\alpha)^2+8(x'\cos\alpha-y'\sin\alpha)+14(x'\sin\alpha+y'\cos\alpha)+5=0$$

yoki

$$(5\cos^2\alpha+4\sin\alpha\cos\alpha+8\sin^2\alpha)x'^2+(5\sin^2\alpha-4\sin\alpha\cos\alpha+$$
$$+8\cos^2\alpha)y'^2+6\sin\alpha\cos\alpha+4(\cos^2\alpha-\sin^2\alpha)x'y'+(8\cos\alpha+14\sin\alpha)x'+$$
$$+(14\cos\alpha-8\sin\alpha)y'+5=0,$$

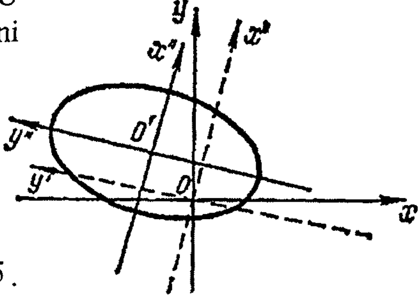
$4(\cos^2\alpha-\sin^2\alpha)+6\sin\alpha\cos\alpha=0$ shartidan (ya'ni $x'y'$ oldidagi koeffitsientni nolga tenglaymiz) α ni topamiz, $2\mathrm{tg}^2\alpha-3\mathrm{tg}\alpha-2=0$ ga ega bo'lamiz, bundan $tg\ \alpha_1=2,\ tg\ \alpha_2=-1/2$. $tg\ \alpha$ ning bu qiymati ikkita o'zaro perpendikular yo'nalishga to'g'ri keladi. $tg\ \alpha=-1/2$ o'rniga $tg\ \alpha=2$ olishimiz mumkin (x', y' lar o'rnini almashtiramiz, 15-chizma).

$tg\,\alpha=2$ dan $\sin\alpha=\pm2\sqrt{5}$, $\cos\alpha=\pm1/\sqrt{5}$; $\sin\alpha$, $\cos\alpha$ larning musbat qiymatini olamiz. U holda tenglama ushbu ko‘rinishni oladi:

$$9x'^2+4y'^2+\frac{36}{\sqrt{5}}x'-\frac{2}{\sqrt{5}}y'=0$$

yoki

$$9(x'^2+\frac{4}{\sqrt{5}}x')+4(y'^2+\frac{1}{2\sqrt{5}}y')=-5.$$



15-chizma

Qavs ichidagi ifodalarni to‘la kvadratga to‘ldiramiz:

$$9(x'+\frac{2}{\sqrt{5}})^2+4(y'-\frac{1}{4\sqrt{5}})^2=\frac{36}{5}+\frac{1}{20}-5$$

yoki

$$9(x'+\frac{2}{\sqrt{5}})^2+4(y'-\frac{1}{4\sqrt{5}})^2=\frac{9}{2}.$$

Koordinata boshi uchun $O'(-2/\sqrt{5},1/4\sqrt{5})$ nuqtani olib $x'=x''-2/\sqrt{5}$, $y'=y''+1/(4\sqrt{5})$ koordinat almashtirish formulalarini qo‘llab, $9x''^2+4y''^2=9/4$ yoki $\frac{x''^2}{1/4}+\frac{y''^2}{9/16}=1$ ga ega bo‘lamiz (bu ellips tenglamasi).

200. $6xy+8y^2-12x-26y+11=0$ tenglamani kanonik holga keltiring.

Yechish:

1. Birinchi banddagi (3) formuladan foydalanib tenglamani o‘zgartiramiz:

$$6(x'\cos\alpha-y'\sin\alpha)(x'\sin\alpha+y'\cos\alpha)+8(x'\sin\alpha+y'\cos\alpha)^2-$$
$$-12(x'\cos\alpha-y'\sin\alpha)-26(x'\sin\alpha+y'\cos\alpha)+11=0,$$
$$(6\sin\alpha\cos\alpha+8\sin^2\alpha)x'^2+(8\cos^2\alpha-6\sin\alpha\cos\alpha)y'^2+$$
$$+[16\sin\alpha\cos\alpha+6(\cos^2\alpha-\sin^2\alpha)]x'y'-(12\cos\alpha+26\sin\alpha)x'-$$
$$-(26\cos\alpha-12\sin\alpha)y'+11=0.$$

$x'y'$ had oldidagi koeffitsientni nolga tenglab topamiz.

$16\sin\alpha\cos\alpha + 6(\cos^2\alpha - \sin^2\alpha) = 0$ yoki $3tg^2\alpha - 8tg\alpha - 3 = 0$

Bundan $tg\alpha_1 = 3$, $tg\alpha_2 = -1/3$; $tg\alpha = 3$ desak, u holda $\sin\alpha = \pm 3/\sqrt{10}$, $\cos\alpha = \pm 1/\sqrt{10}$; $\sin\alpha$, $\cos\alpha$ ning musbat qiymatlarini topamiz. U holda tenglama quyidagi

$$9x'^2 - y'^2 - 9\sqrt{10}x' + \sqrt{10}y' + 11 = 0$$

yoki

$$9(x'^2 - \sqrt{10}x') - (y'^2 - \sqrt{10}y') = -11$$

ko'rinishga keladi.

2. Qavs ichidagi ifodalarni to'la kvadratgacha to'ldiramiz:

$$9\left(x' - \frac{\sqrt{10}}{2}\right)^2 - \left(y' - \frac{\sqrt{10}}{2}\right)^2 = \frac{45}{2} - \frac{5}{2} - 11,$$

yoki

$$9\left(x' - \frac{\sqrt{10}}{2}\right)^2 - \left(y' - \frac{\sqrt{10}}{2}\right)^2 = 9.$$

Yangi koordinat boshi uchun $O'(\sqrt{10}/2, \sqrt{10}/2)$ ni olib, $x' = x'' + \sqrt{10}/2$, $y' = y'' + \sqrt{10}/2$ ni qo'llab, $9x''^2 - y''^2 = 9$ yoki $x''^2 - y''^2/9 = 1$ ni hosil qilamiz (bu giperbola tenglamasidir).

201. $x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$ ni kanonik holga keltiring.

Yechish:

1). O'qlarni burash formulasidan foydalanib tenglamani o'zgartiramiz:

$$(x'\cos\alpha - y'\sin\alpha)^2 - 2(x'\cos\alpha - y'\sin\alpha)(x'\sin\alpha + y'\cos\alpha) +$$

$$+(x'\sin\alpha + y'\cos\alpha)^2 - 10(x'\cos\alpha - y'\sin\alpha) - 6(x'\sin\alpha + y'\cos\alpha) + 25 = 0,$$

yoki

$$(\cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha)x'^2 + (\sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha)y'^2 +$$

$$+2(\sin^2\alpha - \cos^2\alpha)x'y' - (10\cos\alpha + 6\sin\alpha)x' + (10\sin\alpha - 6\cos\alpha)y' + 25 = 0$$

x'y' oldidagi koeffitsientni nolga tenglab topamiz

$2\sin^2\alpha - \cos^2\alpha = 0$, bundan $tg^2\alpha = 1$, ya'ni

$tg\,\alpha_1 = 1$, $tg\,\alpha_2 = -1$. $tg\,\alpha = 1$ ni olsak, $\alpha_1 = \pi/4$ yoki $\sin\alpha = 1/\sqrt{2}$, $\cos\alpha = 1/\sqrt{2}$.

U holda tenglama $2y'^2 - 8\sqrt{2}x' + 2\sqrt{2}y' + 25 = 0$,

yoki $2(y'^2 + \sqrt{2}y') - 8\sqrt{2}x' + 25 = 0$.

2. Qavs ichidagi ifodani to'la kvadratga to'ldiramiz:

$$2\left(y'+\frac{\sqrt{2}}{2}\right)^2=8\sqrt{2}x'-24 \quad \text{yoki} \quad \left(y'+\frac{\sqrt{2}}{2}\right)^2=4\sqrt{2}\left(x'-\frac{3}{\sqrt{2}}\right).$$

Yangi koordinat boshini $O'(3/\sqrt{2},\ -\sqrt{2}/2)$ ga ko'chiramiz. $x'=x''-3/\sqrt{2}$, $y'=y''+\sqrt{2}/2$ ifodalardan foydalanib, $y''^2=4\sqrt{2}x''$ ni topamiz (bu parabola tenglamasi).

Quyidagi tenglamalar ikkita to'g'ri chiziqqa ajraluvchi egri chiziqni aniqlashini ko'rsating va bu to'g'ri chiziqlar tenglamasini toping.

202. $25x^2+10xy+y^2-1=0$

203. $x^2+2xy+y^2+2x+2y+1=0$

204. $8x^2-18xy+9y^2+2x-1=0$

Quyidagi egri chiziq tenglamalarini kanonik holga keltiring:

205. $14x^2+24xy+21y^2-4x+18y-139=0$

206. $4xy+3y^2+16x+12y-36=0$

207. $9x^2-24xy+16y^2-20x+110y-5=0$

5-§. IKKINCHI VA UCHINCHI TARTIBLI ANIQLOVCHILAR. IKKI VA UCH NOMA'LUMLI CHIZIQLI TENGLAMALAR SISTEMASI

1. Ikkinchi tartibli aniqlovchilar va chiziqli tenglamalar sistemasi.

$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ – jadvalga mos ikkinchi tartibli aniqlovchi

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=a_1b_2-a_2b_1$$

tenglik bilan aniqlanadi. Ikki noma'lumli ikkita chiziqli tenglamalar sistemasi

$$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases},$$

uning determinanti $D=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}\neq 0$ bo'lsa, u yagona yechimga ega bo‘lib, yechim Kramer formulalaridan topiladi:

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}; \quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}. \tag{1}$$

Agar $D=0$ bo‘lsa, sistema yoki birgalikda emas, $(D_x \neq 0,\ D_y \neq 0)$ yoki aniqlanmagan $(D_x = D_y = 0)$. Oxirgi holda sistema bitta tenglamaga keltiriladi. Sistemaning birgalikda bo‘lmaslik shartini $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, aniqmaslik shartini $a_1/a_2 = b_1/b_2 = c_1/c_2$ kabi yozish mumkin. Chiziqli sistemaning ozod hadi nolga teng bo‘lsa, u *bir jinsli* deb ataladi.

Uch noma’lumli ikkita bir jinsli

$$\begin{cases} a_1x+b_1y=c_1, \\ a_2x+b_2y=c_2. \end{cases}$$

tenglamalar sistemasini qaraymiz.

Agar $a_1/a_2 = b_1/b_2 = c_1/c_2$ bo‘lsa, sistema bitta tenglamaga keltiriladi, undan bitta noma’lum qolgan ikkita noma’lum orqali ifodalanadi (ularning qiymatlari ixtiyoriy aniqlanadi).

Agar $a_1/a_2 = b_1/b_2 = c_1/c_2$ bajarilmasa, sistemaning yechimlari

$$x=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}\cdot t,\quad y=\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}\cdot t,\quad z=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}\cdot t \tag{2}$$

formulalardan topiladi (t — ixtiyoriy qiymatni qabul qilishi mumkin). Bu yechimlarni

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}=\frac{y}{-\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}=\frac{z}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}=t,$$

proporsiyalar ko‘rinishida yozish mumkin.

Agar mahrajlardan biri nolga aylansa, mos suratni ham nolga tenglashtirish kerak.

208. $\begin{cases} (a+b)x-(a-b)y=4ab, \\ (a-b)x+(a+b)y=2(a^2-b^2) \end{cases}$ tenglamalar sistemasini yeching.

Yechish:

(1) formuladan D, D_x, D_y larni topamiz:

$$D=\begin{vmatrix} a+b & -(a-b) \\ a-b & a+b \end{vmatrix}=(a+b)^2+(a-b)^2=2(a^2+b^2),$$

$$D_x=\begin{vmatrix} 4ab & -(a-b) \\ 2(a^2-b^2) & a+b \end{vmatrix}=4a^2b+4ab^2+2a^3-2a^2b-2ab^2+2b^3=$$

$$=2(a^3+a^2b+ab^2+b^3)=2(a^2+b^2)(a+b),$$

$$D_y=\begin{vmatrix} a+b & 4ab \\ a-b & 2(a^2-b^2) \end{vmatrix}=2a^3+2a^2b-2ab^2-2b^3-4a^2b+4ab^2=$$

$$=2(a^3-a^2b+ab^2-b^3)=2(a^2+b^2)(a-b)$$

$$x=D_x/D=a+b,\quad y=D_y/D=a-b.$$

209. $\begin{cases} 3x+4y+5z=0, \\ x+2y-3z=0 \end{cases}$ bir jinsli chiziqli tenglamalar sistemasini yeching.

Yechish:

(2) formuladan foydalanib, topamiz

$$x=\begin{vmatrix} 4 & 5 \\ 2 & -3 \end{vmatrix}\cdot t=-22\cdot t,\quad y=-\begin{vmatrix} 3 & 5 \\ 1 & -3 \end{vmatrix}\cdot t=14\cdot t,\quad z=\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}\cdot t=2\cdot t,$$

t ga ixtiyoriy qiymat berish mumkin.

Tenglamalar sistemasini yeching:

210. $\begin{cases} 5x-3y=1, \\ x+11y=6. \end{cases}$ **211.** $\begin{cases} 2x+y=1/5, \\ 4x+2y=1/3. \end{cases}$

212. $\begin{cases} ax-by=a^2+b^2, \\ bx+ay=a^2+b^2. \end{cases}$ **213.** $\begin{cases} 3x+2y=1/6, \\ 9x+6y=1/2/. \end{cases}$

214. $\begin{cases} x-2y+z=0, \\ 3x-5y+2z=0. \end{cases}$ **215.** $\begin{cases} x\cos\alpha-y\sin\alpha=\cos 2\alpha, \\ x\sin\alpha+y\cos\alpha=\sin 2\alpha. \end{cases}$

216. $\begin{cases} a^2x-2(a^2+b^2)y+b^2z=0, \\ 2x+2y-3z=0. \end{cases}$

2. Uchinchi tartibli aniqlovchi va chiziqli tenglamalar sistemasi.

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

elementlar jadvaliga mos uchinchi tartibli determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \cdot \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \cdot \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

tenglik bilan aniqlanadi.

Agar berilgan aniqlovchida berilgan elementni o‘z ichiga olgan yo‘l va ustunni o‘chirishdan hosil bo‘lgan ikkinchi tartibli aniqlovchi *uchinchi tartibli aniqlovchining berilgan elementining minori* deb ataladi. Minorning $(-1)^k$ ga ko‘paytmasi *berilgan elementning algebraik to‘ldiruvchisi* deyiladi. (k – berilgan elementni o‘z ichiga olgan yo‘l va ustun elementlar yigindisi). Shunday qilib, aniqlovchining elementiga mos minor ishorasi quyidagi

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

jadval bilan aniqlanadi.

Yuqoridagi III tartibli aniqlovchini ifodalovchi tenglikning o‘ng tomonida I yo‘l elementlarini ularning o‘z algebraik to‘ldiruvchilariga ko‘paytmalarining yig‘indisiga teng.

Teorema 1. ***III tartibli aniqlovchi ixtiyoriy yo‘l (ustun) elementlarini o‘z algebraik to‘ldiruvchilariga ko‘paytmalarining yig‘indisiga teng.***

Bu teorema ixtiyoriy yo‘l elementlari bo‘yicha yoyib aniqlovchini hisoblashga yordam beradi.

Teorema 2. ***Ixtiyoriy yo‘l elementlarini boshqa yo‘l elementlarining algebraik to‘ldiruvchilariga ko‘paytmalarining yig‘indisi nolga teng.***

Aniqlovchining xossalari.

1. Agar aniqlovchining yo'llarini ustunlari bilan yoki ustunlarini yo'llari bilan almashtirsak, aniqlovchining qiymati o'zgarmaydi.

2. Aniqlovchining biror yo'lida umumiy ko'paytuvchiga ega bo'lsa, uni aniqlovchining tashqarisiga chiqarish mumkin.

3. Aniqlovchining biror yo'l elementlari boshqa yo'l elementlariga teng bo'lsa, unday aniqlovchi nolga teng.

4. Agar aniqlovchining ikkita yo'li o'rnini almashtirsak, uning ishorasi teskariga o'zgaradi.

5. Agar aniqlovchining biror yo'l elementlariga boshqa yo'l elementlarini noldan farqli songa ko'paytirib qo'shsak, uning qiymati o'zgarmaydi.

Uch noma'lumli uchta chiziqli

$$\begin{cases} a_1x+b_1y+c_1z=d_1, \\ a_2x+b_2y+c_2z=d_2, \\ a_3x+b_3y+c_3z=d_3. \end{cases}$$

tenglamalar sistemasini quyidagi Kramer formulalaridan foydalanib yechamiz.

$$x=D_x/D, \quad y=D_y/D, \quad z=D_z/D, \tag{1}$$

$$D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \quad D_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}.$$

Bu yerda $D\neq 0$ deb faraz qilamiz (agar $D=0$ bo'lsa, sistema aniqlanmagan yoki birgalikda bo'lmaydi).

Agar bir jinsli sistema

$$\begin{cases} a_1x+b_1y+c_1z=0, \\ a_2x+b_2y+c_2z=0, \\ a_3x+b_3y+c_3z=0. \end{cases}$$

ning aniqlovchisi noldan farqli bo‘lsa, u yagona $x=0$, $y=0$, $z=0$ yechimga ega bo‘ladi. Agar bir jinsli sistemaning aniqlovchisi nolga teng bo‘lsa, sistema ikkita tenglamaga yoki bitta tenglamaga keladi. Agar sistemaning minorlaridan kamida biri noldan farqli bo‘lsa, birinchi hol, hamma minorlar nol bo‘lsa, ikkinchi hol ro‘y beradi. Bu ikki holda ham (birinchi bandga qarang) sistema cheksiz ko‘p yechimlarga ega bo‘ladi.

217. $\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}$ uchinchi tartibli aniqlovchini hisoblang.

Yechish:

Aniqlovchini birinchi yo‘l elementlari bo‘yicha yoyamiz

$$\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix} = 5\cdot\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} - 3\cdot\begin{vmatrix} -1 & 4 \\ 7 & 6 \end{vmatrix} + 2\cdot\begin{vmatrix} -1 & 2 \\ 7 & 3 \end{vmatrix} =$$

$$= 5\cdot 0 - 3\cdot(-34) + 2\cdot(-17) = 68.$$

218. Yuqoridagi aniqlovchini yo‘l (ustun) elementlarining chiziqli kombinatsiyasi haqidagi teoremadan foydalanib hisoblang.

Yechish:

II yo‘l elementlarini 5 ga ko‘paytirib, I yo‘l elementlariga, 7 ga ko‘paytirib III yo‘l elementlariga qo‘shamiz:

$$\begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 13 & 22 \\ -1 & 2 & 4 \\ 0 & 17 & 34 \end{vmatrix}.$$

I ustun elementlari bo‘yicha yoyib hisoblaymiz:

$$\begin{vmatrix} 0 & 13 & 22 \\ -1 & 2 & 4 \\ 0 & 17 & 34 \end{vmatrix} = 0\begin{vmatrix} 2 & 4 \\ 17 & 34 \end{vmatrix} + 1\begin{vmatrix} 13 & 22 \\ 17 & 34 \end{vmatrix} + 0\begin{vmatrix} 13 & 22 \\ 2 & 4 \end{vmatrix} = 13\cdot 34 - 17\cdot 22 = 68.$$

219. $\begin{cases} x+2y+z=8, \\ 3x+2y+z=10, \\ 4x+3y-2z=4 \end{cases}$ tenglamalar sistemasini yeching.

Yechish:

(1) formuladan topamiz:

$$x=\frac{\begin{vmatrix}8 & 2 & 1\\ 10 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}}{\begin{vmatrix}1 & 2 & 1\\ 3 & 2 & 1\\ 4 & 3 & -2\end{vmatrix}}=\frac{8\cdot\begin{vmatrix}2 & 1\\ 3 & -2\end{vmatrix}-2\cdot\begin{vmatrix}10 & 1\\ 4 & -2\end{vmatrix}+1\cdot\begin{vmatrix}10 & 2\\ 4 & 3\end{vmatrix}}{1\cdot\begin{vmatrix}2 & 1\\ 3 & -2\end{vmatrix}-2\cdot\begin{vmatrix}3 & 1\\ 4 & -2\end{vmatrix}+1\cdot\begin{vmatrix}3 & 2\\ 4 & 3\end{vmatrix}}=\frac{14}{14}=1,$$

$$y=\frac{\begin{vmatrix}1 & 8 & 1\\ 3 & 10 & 1\\ 4 & 4 & 2\end{vmatrix}}{14}=\frac{1\cdot\begin{vmatrix}10 & 1\\ 4 & -2\end{vmatrix}-8\cdot\begin{vmatrix}3 & 1\\ 4 & -2\end{vmatrix}+1\cdot\begin{vmatrix}3 & 10\\ 4 & 4\end{vmatrix}}{14}=\frac{28}{14}=2,$$

$$z=\frac{\begin{vmatrix}1 & 2 & 8\\ 3 & 2 & 10\\ 4 & 3 & 4\end{vmatrix}}{14}=\frac{1\cdot\begin{vmatrix}2 & 10\\ 3 & 4\end{vmatrix}-2\cdot\begin{vmatrix}3 & 10\\ 4 & 4\end{vmatrix}+8\cdot\begin{vmatrix}3 & 2\\ 4 & 3\end{vmatrix}}{14}=\frac{42}{14}=3.$$

220. $\begin{cases}4x+y+z=0,\\ x+3y+z=0,\\ x+y+2z=0\end{cases}$ bir jinsli tenglamalar sistemasini yeching.

Yechish:

Bu yerda $D=\begin{vmatrix}4 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 2\end{vmatrix}$. Buni hisoblash uchun I yo‘l elementlariga III yo‘l elementlarini -4 ga, II yo‘l elementlariga III yo‘l elementlarini -1 ga ko‘paytirib qo‘shamiz:

$$D=\begin{vmatrix}0 & -3 & -7\\ 0 & 2 & -1\\ 1 & 1 & 2\end{vmatrix}=1\cdot\begin{vmatrix}-3 & -7\\ 2 & -1\end{vmatrix}=17.$$

$D \neq 0$ bo'lgani uchun sistema $x = y = z = 0$ yechimga ega.

221. $\begin{cases} 3x + 2y - z = 0, \\ x + 2y + 9z = 0, \\ x + y + 2z = 0 \end{cases}$ sistemani yeching.

Yechish:

$$D = \begin{vmatrix} 3 & 2 & -1 \\ 1 & 2 & 9 \\ 1 & 1 & 2 \end{vmatrix} = 3 \cdot \begin{vmatrix} 2 & 9 \\ 1 & 2 \end{vmatrix} - 2 \cdot \begin{vmatrix} 1 & 9 \\ 1 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -15 + 14 + 1 = 0 .$$

Demak, sistema noldan farqli yechimga ega. Birinchi ikkita sistemani yechamiz (chunki uchinchisi dastlabki ikkitasining natijasi):

$$\begin{cases} 3x + 2y - z = 0, \\ x + 2y + 9z = 0. \end{cases}$$

I banddagi (2) formuladan topamiz:

$$x = \begin{vmatrix} 2 & -1 \\ 2 & 9 \end{vmatrix} \cdot t = 20 \cdot t, \quad y = -\begin{vmatrix} 3 & -1 \\ 1 & 9 \end{vmatrix} \cdot t = -28 \cdot t, \quad z = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \cdot t = 4 \cdot t.$$

222. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}$ ni III yo'l elementlari bo'yicha yoyib hisoblang.

223. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}$ ni yo'l (ustun)lar chiziqli kombinasiya haqidagi teoremani qo'llab hisoblang.

224. $\begin{vmatrix} 2 & 3 & 4 \\ 2 & a+3 & b+4 \\ 2 & c+3 & d+4 \end{vmatrix}$ ni hisoblang.

Tenglamalar sistemasini yeching.

225. $\begin{cases} 5x - y - z = 0, \\ x + 2y + 3z = 14, \\ 4x + 3y + 2z = 16, \end{cases}$

226. $\begin{cases} x + 3y - 6z = 12, \\ 3x + 2y + 5z = -10, \\ 2x + 5y - 3z = 6. \end{cases}$

227. $\begin{cases} -5x + y + z = 0, \\ x - 6y + z = 0, \\ x + y - 7z = 0. \end{cases}$

228. $\begin{cases} x + y + z = 0, \\ 3x + 6y + 5z = 0, \\ x + 4y + 3z = 0. \end{cases}$

229. $\begin{cases} ax + by + cz = a - b, \\ bx + cy + az = b - c, \\ cx + ay + bz = c - a, \end{cases}$

agar $a + b + c \neq 0$ bo'lsa.

230. $\begin{cases} ax + by + (a + b)z = 0, \\ bx + ay + (a + b)z = 0, \\ x + y + 2z = 0. \end{cases}$

II BOB
VEKTORLAR ALGEBRASINING ELEMENTLARI

1-§. FAZODA TO'G'RI BURCHAKLI KOORDINATALAR

Agar fazoda $Oxyz$ to'g'ri burchakli dekart koordinat sistemasi berilgan bo'lsa, u xolda koordinatalari x (abssissa), y (ordinata) va z (aplikata) bo'lgan M nuqta $M(x;\ y;\ z)$ bilan belgilanadi. $A(x_1;\ y_1;\ z_1)$ va $B(x_2;\ y_2;\ z_2)$ nuqtalar orasidagi masofa

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

formula bilan aniqlanadi.

$M(x;\ y;\ z)$ nuqtadan koordinat boshigacha bo'lgan masofa

$$d = \sqrt{x^2 + y^2 + z^2} \tag{2}$$

formula bilan topiladi.

Agar oxirlari $A(x_1;\ y_1;\ z_1)$ va $B(x_2;\ y_2;\ z_2)$ bo'lgan kesma $C(x;y;z)$ nuqta orqali λ nisbatda (1-*b*ob, 1-§) bo'lingan bo'lsa, C nuqtaning koordinatalari

$$\bar{x} = \frac{x_1 + \lambda x_2}{1 + \lambda};\quad \bar{y} = \frac{y_1 + \lambda y_2}{1 + \lambda};\quad \bar{z} = \frac{z_1 + \lambda z_2}{1 + \lambda}. \tag{3}$$

Kesma o'rtasining koordinatalari

$$\bar{x} = \frac{x_1 + x_2}{2};\quad \bar{y} = \frac{y_1 + y_2}{2};\quad \bar{z} = \frac{z_1 + z_2}{2} \tag{4}$$

formulalar bilan topiladi.

231. $M_1(2;\ 4;\ -2)$ va $M_2(-2;\ 4;\ 2)$ nuqtalar berilgan. M_1M_2 to'g'ri chiziqni $\lambda = 3$ nisbatda bo'luvchi M nuqtani toping.

Yechish:

Kesmani berilgan nisbatda bo'luvchi formulalardan foydalanamiz:

$$x_M = \frac{x_1 + \lambda x_2}{1 + \lambda} = \frac{2 + 3(-2)}{1 + 3} = -1;\quad y_M = \frac{y_1 + \lambda y_2}{1 + \lambda} = \frac{4 + 3 \cdot 4}{1 + 3} = 4;$$

$$z_M = \frac{z_1 + \lambda z_2}{1 + \lambda} = \frac{-2 + 3 \cdot 2}{1 + 3} = 1.$$

Demak, izlangan nuqta $M(-1;\ 4;\ 1)$ ekan.

232. Uchlari $A(1;\ 1;\ 1)$, $B(5;\ 1;\ -2)$, $C(7;\ 9;\ 1)$ bo‘lgan uchburchak berilgan. A burchak bissektrisasining CB tomon bilan kesishgan D nuqtaning koordinatalarini toping.

Yechish: A burchakni tashkil etuvchi tomonlar uzunliklarini topamiz.

$$|AC|=\sqrt{(x_C-x_A)^2+(y_C-y_A)^2+(z_C-z_A)^2}=\sqrt{(7-1)^2+(9-1)^2+(1-1)^2}=10,$$

$$|AB|=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+(z_B-z_A)^2}=\sqrt{(5-1)^2+(1-1)^2+(-2-1)^2}=5.$$

Demak, $|CD|:|DB|=10:5=2$, chunki bissektrissa CB tomonni o‘ziga yopishgan tomonlarga proporsional bo‘laklarga ajratadi.

Shunday qilib,

$$x_D=\frac{x_C+\lambda x_B}{1+\lambda}=\frac{7+2\cdot5}{1+2}=\frac{17}{3};\quad y_D=\frac{y_c+\lambda y_B}{1+\lambda}=\frac{9+2\cdot1}{1+2}=\frac{11}{3};$$

$$z_D=\frac{z_c+\lambda z_B}{1+\lambda}=\frac{1+2(-2)}{1+2}=-1.$$

Izlangan nuqta $D(17/3;\ 11/3;\ -1)$ ekan.

233. *Ox* o‘qida $A(2;\ -4;\ 5)$ va $B(-3;\ 2;\ 7)$ nuqtalardan barobar uzunlikda turgan nuqtani toping.

Yechish:

M izlangan nuqta bo‘lsin. Uning uchun $|AM|=|MB|$ shart bajarilishi kerak. Bu nuqta Ox o‘qida yotgani uchun, uning koordinatalari $(x;\ 0;\ 0)$, shuning uchun

$$|AM|=\sqrt{(x-2)^2+(-4)^2+5^2},\qquad |MB|=\sqrt{(x+3)^2+2^2+7^2}$$

bo‘ladi, bu tengliklarning ikki tomonini kvadratga oshirib, quyidagini topamiz: $(x-2)^2+41=(x+3)^2+53$ yoki $10x=-17$, ya’ni $x=-1{,}7$. Shunday qilib, izlangan nuqta $M(-1{,}7;\ 0;\ 0)$.

234. $A(3;\ 3;\ 3)$ va $B(-1;\ 5;\ 7)$ nuqtalar berilgan. AB kesmani teng uch bo‘lakka bo‘luvchi C, D nuqtalarning koordinatalarini toping.

235. Uchlari $A(1;\ 2;\ 3)$, $B(7;\ 10;\ 3)$, $C(-1;\ 3;\ 1)$ bo‘lgan uchburchak berilgan. A burchakni o‘tmas ekanligini isbotlang.

236. Uchlari $A(2;\ 3;\ 4)$, $B(3;\ 1;\ 2)$, $C(4;\ -1;\ 3)$ bo‘lgan uchburchak og‘irlik markazining koordinatalarini toping.

237. $A(3;\ 1;\ 4)$ va $B(-4;\ 5;\ 3)$ nuqtalardan baravar uzoqlikda turgan M nuqta, koordinat boshidan $C(0;\ 6;\ 0)$ nuqtagacha bo‘lgan Oy o‘qidagi kesmani qanday nisbatda bo‘ladi.

238. Oz o'qida $M_1(2;\ 4;\ 1)$ va $M_2(-3;\ 2;\ 5)$ nuqtalardan baravar uzoqlikda turgan nuqtani toping.

239. xOy tekislikda $A(1;\ -1;\ 5)$, $B(3;\ 4;\ 4)$ va $C(4;\ 6;\ 1)$ nuqtalardan baravar uzoqlikda turgan nuqtani toping.

2-§. VEKTORLAR VA ULAR USTIDA AMALLAR

$Oxyz$ koordinatlar fazosida berilgan erkin vektor $\overline{a}$ ni $\overline{a} = a_x \cdot \overline{i} + a_y \cdot \overline{j} + a_z \cdot \overline{k}$ ko'rinishida tasvirlash mumkin. $\overline{a}$ vektorni bunday tasvirlash uni *koordinata o'qlari yoki ortlar bo'yicha yoyish* deb ataladi. Bu yerda a_x, a_y, a_z lar $\overline{a}$ *vektorning mos o'qlardagi proyeksiyalari* ($\overline{a}$ vektorning koordinatalari) deyiladi, $\overline{i}$, $\overline{j}$, $\overline{k}$ lar esa o'qlarning ortlari (mos o'qlarning musbat yunalish bilan ustma-ust tushgan birlik vektorlar).

$a_x\overline{i}$, $a_y\overline{j}$, $a_z\overline{k}$ lar $\overline{a}$ *vektorning koordinat o'qlari bo'yicha tashkil etuvchilari* (komponentalari) deb ataladi. $\overline{a}$ vektorning kattaligi a yoki $|\overline{a}|$ bilan belgilanib $|\overline{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ formuladan topiladi.

$\overline{a}$ vektorning yo'nalishi uning koordinat o'qlari bilan tashkil qilgan α, β, γ burchaklar orqali belgilanadi. Bu burchaklarning kosinusi (vektorning yo'naltiruvchi kosinusi)

$$\cos\alpha = \frac{a_x}{a} = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}; \quad \cos\beta = \frac{a_y}{a} = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}};$$

$$\cos\gamma = \frac{a_z}{a} = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

formuladan aniqlanadi.

Vektorning yo'naltiruvchi kosinuslari $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ munosabat bilan bog'langan. Agar $\overline{a}$ va $\overline{b}$ vektorlar ortlar bo'yicha yoyilmasi bilan berilgan bo'lsa, ularning yigindisi va ayirmasi

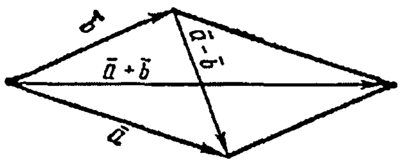
$$\overline{a} + \overline{b} = (a_x + b_x)\cdot\overline{i} + (a_y + b_y)\cdot\overline{j} + (a_z + b_z)\cdot\overline{k},$$

$$\overline{a} - \overline{b} = (a_x - b_x)\cdot\overline{i} + (a_y - b_y)\cdot\overline{j} + (a_z - b_z)\cdot\overline{k}$$

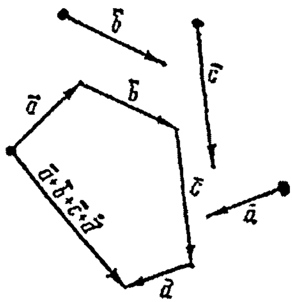
formulalardan aniqlanadi.

Boshlari ustma-ust tushadigan $\overline{a}$ va $\overline{b}$ vektorlar yig'indisi tomonlari $\overline{a}$ va $\overline{b}$ bo'lgan parallelogram diagonali bilan ustma-ust tushadigan vektor orqali tasvirlanadi. $\overline{a} - \overline{b}$ ayirma shu parallelogramning ikkinchi diagonali bilan ustma-ust tushib, vektorning boshi $\bar{b}$ ning oxirida, oxiri $\bar{a}$ ning oxirida yotadi (16-chizma).

Ixtiyoriy sondagi vektorlar yig'indisi ko'pburchaklar qoidasi bo'yicha topiladi (17-chizma). $\bar{a}$ vektorni m skalyarga ko'paytmasining $m\cdot\bar{a}=m\cdot a_x\cdot\bar{i}+m\cdot a_y\cdot\bar{j}+m\cdot a_z\cdot\bar{k}$ formuladan topiladi. Agar m>0 bo'lsa, $\bar{a}$ va $m\cdot\bar{a}$ vektorlar parallel (kollinear) va bir tomonga yo'nalgan, m<0 bo'lsa, qarama-qarshi tomonga yo'nalgan bo'ladi. Agar $m=1/a$ bo'lsa, $\bar{a}/a$ vektor uzunligi birga teng bo'lib yo'nalishi $\bar{a}$ ning yo'nalishi bilan ustma-ust tushadi. Bu vektor $\bar{a}$ vektorning birlik vektori (ort) deyilib, $\bar{a}_0$



16-chizma



17-chizma

bilan belgilanadi. $\bar{a}$ vektor yo'nalishidagi birlik vektorni topish $\bar{a}$ vektorni normallashtirish deyiladi. Shunday qilib, $\bar{a}_0=\bar{a}/a$, yoki $\bar{a}=a\bar{a}_0$.

Boshi koordinat boshida, oxiri M nuqtada yotgan $\overline{OM}$ vektor *M nuqtaning radius-vektori* deyilib, $\bar{r}(M)$ yoki $\bar{r}$ bilan belgilanadi. Uning koordinatalari M nuqtaning koordinatalari bilan ustma-ust tushgani uchun uning ort bo'yicha yoyilmasi $\bar{z}=x\bar{i}+y\bar{j}+z\bar{k}$ ko'rinishda bo'ladi. Boshi $A(x_1;\ y_1;\ z_1)$, oxiri $B(x_2;\ y_2;\ z_2)$ nuqtada bo'lgan $\overline{AB}$ vektor $\overline{AB}=\bar{r}_2-\bar{r}_1$ ko'rinishda yoziladi, bu yerda $\bar{r}_2$ B nuqtaning, $\bar{r}_1$ A nuqtaning radius vektori. Shuning uchun $\overline{AB}$ vektorning ortlar bo'yicha yoyilmasi $\overline{AB}=(x_2-x_1)\bar{i}+(y_2-y_1)\bar{j}+(z_2-z_1)\bar{k}$ ko'rinishda bo'ladi. Uning uzunligi A va B nuqtalar orasidagi masofaga teng. $\left|\overline{AB}\right|=d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ $\overline{AB}$ vektorning yo'nalishi $\cos\alpha = \frac{x_2 - x_1}{d}$; $\cos\beta = \frac{y_2 - y_1}{d}$; $\cos\gamma = \frac{z_2 - z_1}{d}$ yo'naltiruvchi kosinuslar bilan aniqlanadi.

240. *ABC* uchburchakka *AB* tomon *M* va *N* nuqtalar orqali teng uch bo'lakka bo'lingan: $|AM| = |MN| = |NB|$. Agar $\overline{CA} = \overline{a}, \overline{CB} = b$ bo'lsa, $\overline{CM}$ vektorni toping.

Yechish:

$\overline{AB} = \overline{b} - \overline{a}$ ga egamiz. Demak, $\overline{AM} = (\overline{b} - \overline{a})/3$. $\overline{CM} = \overline{CA} + \overline{AM}$ bo'lgani uchun $\overline{CM} = \overline{a} + (\overline{b} - \overline{a})/3 = (2\overline{a} + \overline{b})/3$.

241. *ABC* uchburchakda *AM* to'g'ri chiziq *BAC* burchakning bissektrisasi, *M* nuqta *BC* tomonda yotadi. Agar $\overline{AB} = \overline{b}$, $\overline{AC} = \overline{c}$ bo'lsa, $\overline{AM}$ ni toping.

Yechish:

$\overline{BC} = \overline{c} - \overline{b}$ ga egamiz. Uchburchak ichki burchaklarining bissektrisasi xossasiga asosan $|\overline{BM}|:|MC| = b:c$, ya'ni $|BM|:|BC| = b:(b+c)$. Bundan $\overline{BM} = \frac{b}{b+c}(\overline{c} - \overline{b})$. $\overline{AM} = \overline{AB} + \overline{BM}$ bo'lgani uchun

$$\overline{AM} = b + \frac{b}{b+c}(\overline{c} - \overline{b}) = \frac{b\overline{c} + c\overline{b}}{b+c}$$

242. *ABC* uchburchak uchlarining radius-vektorlari $\overline{r}_1$, $\overline{r}_2$, $\overline{r}_3$ bo'lsin. Uchburchak medianalari kesishgan nuqtasining radius-vektorini toping.

Yechish:

$\overline{BC} = \overline{r}_3 - \overline{r}_2$; $\overline{BD} = (\overline{r}_3 - \overline{r}_2)/2$ (*DBC* tomonning o'rtasi);
$\overline{AB} = \overline{r}_2 - \overline{r}_1$; $\overline{AD} = \overline{BD} + \overline{AB} = (\overline{r}_3 - \overline{r}_2)/2 + \overline{r}_2 - \overline{r}_1 = (\overline{r}_2 + \overline{r}_3 - 2\overline{r}_1)/2$;
$\overline{AM} = (2/3)\overline{AD}$ (*M* – medianalar kesishgan nuqtasi), shuning uchun $\overline{AM} = (\overline{r}_2 + \overline{r}_3 - 2\overline{r}_1)/3$.

Shunday qilib;

$\overline{r} = \overline{OM} = \overline{r}_1 + \overline{AM} = (\overline{r}_2 + \overline{r}_3 - 2\overline{r}_1)/3 + \overline{r}_1$, yoki $\overline{r} = (\overline{r}_1 + \overline{r}_2 - \overline{r}_3)/3$.

243. $\overline{a} = 20\overline{i} + 30\overline{j} - 60\overline{k}$ vektorning uzunligi va yo'naltiruvchi kosinuslarini toping.

Yechish:

$$a = \sqrt{20^2 + 30^2 + 60^2} = 70,$$
$$\cos\alpha = 20/70 = 2/7;\ \cos\beta = 30/70 = 3/7;\ \cos\gamma = -60/70 = -6/7.$$

244. Agar $A(1;\ 3;\ 2)$ va $B(5;\ 8;\ -1)$ bo'lsa, $\bar{a}=\overline{AB}$ vektorni topíng.

Yechish:

$\overline{AB}$ vektorning koordinat o'qlariga proyeksiyalari B va A nuqtalarning mos proyeksiyalari ayirmasiga teng:

$$a_x=5-1=4,\ a_y=8-3=5,\ a_z=-1-2=-3.$$

Demak, $\overline{AB}=4\cdot\bar{i}+5\cdot\bar{j}-3\cdot\bar{k}.$

245. $\bar{a}=3\bar{i}+4\bar{j}-12\bar{k}$ ni normallashtiring.

Yechish:

$\bar{a}$ vektorning uzunligini topamiz:

$$|\bar{a}|=\sqrt{a_x^2+a_y^2+a_z^2}=\sqrt{3^2+4^2+(-12)^2}=13$$

Izlangan birlik vektor

$$\bar{a}_0=\frac{\bar{a}}{|\bar{a}|}=\frac{3\cdot\bar{i}+4\cdot\bar{j}-12\cdot\bar{k}}{13}=\frac{3}{13}\cdot\bar{i}+\frac{4}{13}\cdot\bar{j}-\frac{12}{13}\cdot\bar{k}\quad \text{bo'ladi.}$$

246. ABC uchburchak berilgan. BC tomonida $|BM|:|MC|=\lambda$ nisbatda M nuqta joylashgan. Agar $\overline{AB}=\bar{b},\ \overline{AC}=\bar{c}$ bo'lsa, $\overline{AM}$ ni toping.

247. $\overline{AB}=\bar{a}+2\bar{b},\ \overline{BC}=-4\bar{a}-\bar{b},\ \overline{CD}=-5\bar{a}-3\bar{b}$ berilgan. $ABCD$ trapetsiya ekanligini isbotlang.

248. Agar $\bar{a}=\overline{AB}+\overline{CD}, A(0;0;1), B(3;2;1), C(4;6;5), D(1;6;3)$ bo'lsa, $\bar{a}$ vektorning koordinat o'qlariga proeksiyalarini toping.

249. $\bar{a}=m\bar{i}+(m+1)\bar{j}+m(m+1)\bar{k}$ vektorning uzunligini toping.

250. ABC uchburchak uchlarining radius-vektorlari berilgan:

$$\bar{r}_A=\bar{i}+2\bar{j}+3\bar{k},\ \bar{r}_B=3\bar{i}+2\bar{j}+\bar{k},\ \bar{r}_C=\bar{i}+4\bar{j}+\bar{k}.$$

ABC uchburchakning teng tomonli ekanligini isbotlang.

251. $\bar{a}=\bar{i}+2\bar{j}+\bar{k}-(1/5)(4\bar{i}+8\bar{j}+3\bar{k})$ vektorning moduli va yo'naltiruvchi kosinuslari topilsin.

252. $M_1(1;\ 2;\ 3)$ va $M_2(3;\ -4;\ 6)$ berilgan. $\overline{M_1M_2}$ vektorning kattaligi va yo'nalishi aniqlansin.

253. $\bar{a}=4\bar{i}-2\bar{j}+3\bar{k}$ vektor berilgan. Agar $b=a$, $b_y=a_y$ va $b_x=0$ bo'lsa, $\bar{b}$ ni toping.

254. M nuqtaning radius-vektori Oy o'qi bilan 60^0, Oz o'qi bilan 45^0 li burchak tashkil etadi, uning uzunligi $r=8$. M nuqtaning abssissasi manfiy bo'lsa, uni toping.

255. $\bar{a}=\bar{i}-2\bar{j}-2\bar{k}$ vektorni normallashtiring.

3-§. SKALYAR VA VEKTOR KO'PAYTMA. ARALASH KO'PAYTMA

1. Skalyar ko'paytma.

$\bar{a}$ va $\bar{b}$ *vektorlarning skalyar ko'paytmasi* deb shunday sonni aytamizki, u vektorlar uzunliklarini ular orasidagi burchak kosinusiga ko'paytmasiga teng: $\bar{a}\cdot\bar{b}=ab\cos\varphi$

Skalyar ko'paytmaning xossalari:

1. $\bar{a}\cdot\bar{b}=|a|^2$ yoki $\bar{a}^2=|a|^2$

2. Agar $\bar{a}=0$ yoki $\bar{b}=\bar{0}$, yoki $\bar{a}\perp\bar{b}$ (noldan farqli vektorlar ortonalligi) bo'lsa, $\bar{a}\cdot\bar{b}=0$.

3. $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{a}$ (o'rin almashtirish qonuni).

4. $\bar{a}(\bar{b}+\bar{c})=\overline{ab}+\overline{ac}$ (taqsimot qonuni)

5. $(m\bar{a})\bar{b}=\bar{a}(m\bar{b})=m(\overline{ab})$ (skalyar ko'paytuvchiga nisbatan guruhlash qonuni)

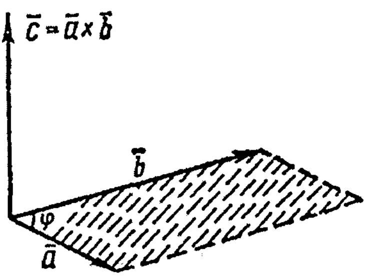
Koordinata o'qlari ortlarining skalyar ko'paytmasi

$\bar{i}^2=\bar{j}^2=\bar{k}^2=1$, $\bar{i}\cdot\bar{j}=\bar{j}\cdot\bar{k}=\bar{i}\cdot\bar{k}=0$.

$\bar{a}=a_x\bar{i}+a_y\bar{j}+a_z\bar{k}$, $\bar{b}=b_x\bar{i}+b_y\bar{j}+b_z\bar{k}$ lar berilgan bo'lsa, ularning skalyar ko'paytmasi $\overline{ab}=a_xb_x+a_yb_y+a_zb_z$ formuladan topiladi.

2. Vektor ko'paytma.

$\bar{a}$ *vektorning* $\bar{b}$ *vektorga ko'paytmasi* deb shunday $\bar{c}$ vektorni aytamizki, u quyidagi shartlarni qanoatlantirsin (18-chizma).



18-chizma

1. $\bar{c}$ ning kattaligi $\bar{a}$ va $\bar{b}$ vektorlardan yasalgan parallelogramning yuziga teng ($c=ab\sin\varphi$, $\varphi=\bar{a}\wedge\bar{b}$);

2. $\bar{c}$ vektor $\bar{a}$ va $\bar{b}$ vektorlarga perpendikular;

3. $\bar{a}, \bar{b}, \bar{c}$ vektorlar bitta nuqtaga keltirilgandan so'ng o'ng sistemani tashkil etsin. $\bar{a}$ vektorning $\bar{b}$ vektorga vektor ko'paytmasi $\bar{a}\times\bar{b}$ ko'rinishda yoziladi.

Vektor ko'paytmaning xossalari:

1. $\bar{b}\times\bar{a}=-\bar{a}\times\bar{b}$, o'rin almashtirish xossasiga ega emas.

2. Agar $\bar{a}=0$, yo $\bar{b}=0$, yo $\bar{a}\|\bar{b}$ bo'lsa, $\bar{a}\,\bar{b}$ $x=0$ bo'ladi.

3. $(m\bar{a})\times\bar{b}=\bar{a}\times(m\bar{b})=m(\bar{a}\times\bar{b})$ (skalyar ko'paytuvchining guruhlash qonuni)

4. $\bar{a}\times(\bar{b}+\bar{c})=\bar{a}\times\bar{b}+\bar{a}\times\bar{c}$ (taqsimot qonuni)

$\bar{i}, \bar{j}, \bar{k}$ ortlarning vektor ko'paytmasi uchun

$$\bar{i}\times\bar{i}=\bar{j}\times\bar{j}=\bar{k}\times\bar{k}=0,$$

$$\bar{i}\times\bar{j}=-\bar{j}\times\bar{i}=\bar{k};\ \bar{j}\times\bar{k}=-\bar{k}\times\bar{j}=\bar{i};\ \bar{k}\times\bar{i}=-\bar{i}\times\bar{k}=\bar{j}$$

tengliklar o'rinli.

$\bar{a}=x_1\bar{i}+y_1\bar{j}+z_1\bar{k}$, $\bar{b}=x_2\bar{i}+y_2\bar{j}+z_2\bar{k}$ vektorlarning vektor ko'paytmasi

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}$$

formula yordamida topiladi.

3. Aralash ko'paytma.

Uch $\bar{a}, \bar{b}, \bar{c}$ vektorning aralash ko'paytmasi $\bar{a}\times\bar{b}$ ni $\bar{c}$ ga skalyar ko'paytmasiga teng, ya'ni $\bar{a}\times\bar{b}\cdot\bar{c}$. Aralash ko'paytmaning moduli shu vektorlarga qurilgan parallelepipedning hajmiga teng. Aralash ko'paytmaning xossalari:

1. Agar: a) ko'paytiriluvchi vektorlardan biri nolga teng: b) ikkitasi kolleniar: b) uchta noldan farqli vektor bitta tekislikka parallel (komplanar) bo'lsa, aralash ko'paytma nolga teng.

2. Agar aralash ko'paytmada vektor ko'paytma (x) va skalyar ko'paytma ($\cdot$) larning o'rnini almashtirsak aralash ko'paytma o'zgarmaydi, ya'ni $\bar{a}\times\bar{b}\cdot\bar{c}=\bar{a}\cdot\bar{b}\times\bar{c}$. Shuni hisobga olib, aralash ko'paytma $\bar{a}\cdot\bar{b}\cdot\bar{c}$ kabi yoziladi.

3. Agar ko'paytiriladigan vektorlar o'rnini doiraviy shaklda almashtirsak, ko'paytma o'zgarmaydi: $\bar{a}\cdot\bar{b}\cdot\bar{c}=\bar{b}\cdot\bar{c}\cdot\bar{a}=\bar{c}\cdot\bar{a}\cdot\bar{b}$.

4. Ixtiyoriy ikkita vektor o'rnini almashtirsak, aralash ko'paytmaning ishorasi o'zgaradi.

$$\bar{b}\cdot\bar{a}\cdot\bar{c}=-\bar{a}\cdot\bar{b}\cdot\bar{c}\,;\quad \bar{c}\cdot\bar{b}\cdot\bar{a}=-\bar{a}\cdot\bar{b}\cdot\bar{c};\quad \bar{a}\cdot\bar{c}\cdot\bar{b}=-\bar{a}\cdot\bar{b}\cdot\bar{c}.$$

$$\bar{a}=x_1\bar{i}+y_1\bar{j}+z_1\bar{k};\quad \bar{b}=x_2\bar{i}+y_2\bar{j}+z_2\bar{k};\quad \bar{c}=x_3\bar{i}+y_3\bar{j}+z_3\bar{k}$$

larning aralash ko'paytmasi

$$\bar{a}\cdot\bar{b}\cdot\bar{c}=\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$ dan topiladi.

Aralash ko'paytmaning xossalaridan quyidagilar kelib chiqadi: uch vektor komplanarligining zarur va yetarli sharti $\bar{a}\cdot\bar{b}\cdot\bar{c}=0$. $\bar{a}, \bar{b}, \bar{c}$ larga qurilgan parallelepiped hajmi $V_1=|\bar{a}\cdot\bar{b}\cdot\bar{c}|$, uchburchakli piramidaning hajmi $V_2=\frac{1}{6}V_1=\frac{1}{6}|\bar{a}\cdot\bar{b}\cdot\bar{c}|$.

256. $\bar{a}=3\bar{i}+4\bar{j}+7\bar{k}$ va $\bar{b}=2\bar{i}-5\bar{j}+2\bar{k}$ larning skalyar ko'paytmasini toping.

Yechish:

$\bar{a}\cdot\bar{b}=3\cdot2+4\cdot(-5)+7\cdot2=0$ ni topamiz. $\bar{a}\cdot\bar{b}=0$ va $a\neq0, \bar{b}\neq0$ bo'lganligi uchun $\bar{a}\perp\bar{b}$.

257. $\bar{a}=m\bar{i}+3\bar{j}+4\bar{k}$ va $\bar{b}=4\bar{i}+m\bar{j}-7\bar{k}$ vektorlar berilgan. m ning qanday qiymatida vektorlar perpendikular bo'ladi.

Yechish:

Bu vektorlarning skalyar ko'paytmasini topamiz:

$\bar{a}\cdot\bar{b}=4m+3m-28;$ $\bar{a}\perp\bar{b}$ bo'lgani uchun $\overline{ab}=0$ bo'ladi. Bundan 7m−28=0, ya'ni m=4.

258. Agar a=2, b=3, $\bar{a}\perp\bar{b}$ bo'lsa, $(5\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b})$ ni toping.

Yechish:

$$(5\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b})=10\cdot\bar{a}^2-5\overline{ab}+6\overline{ab}-3\bar{b}^2=10\cdot\bar{a}^2-3\bar{b}^2=40-27=13.$$

259. $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$ va $\bar{b}=6\bar{i}+4\bar{j}-2\bar{k}$ vektorlar orasidagi burchakni hisoblang.

Yechish:

$\overline{ab}=ab\cos\varphi$ bo'lgani uchun $\cos\varphi=\dfrac{\overline{ab}}{ab}$

$$\overline{ab}=1\cdot6+2\cdot4+3(-2)=8,\qquad a=\sqrt{1+4+9}=\sqrt{14},$$

$$b=\sqrt{36+16+4}=2\sqrt{14}$$

Demak, $\cos\varphi=\dfrac{8}{\sqrt{14}\cdot2\sqrt{14}}=\dfrac{2}{7}$ va $\varphi=\arccos\dfrac{2}{7}$.

260. $\bar{a}=2\bar{i}+3\bar{j}+5\bar{k}$ va $\bar{b}=\bar{i}+2\bar{j}+\bar{k}$ larning vektor ko'paytmasini toping.

Yechish:

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 3 & 5\\ 1 & 2 & 1\end{vmatrix}=\bar{i}\cdot\begin{vmatrix}3 & 5\\ 2 & 1\end{vmatrix}-\bar{j}\cdot\begin{vmatrix}2 & 5\\ 1 & 1\end{vmatrix}+\bar{k}\cdot\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix},$$

ya'ni $\bar{a}\times\bar{b}=-7\bar{i}+3\bar{j}+\bar{k}$.

261. $\bar{a}=6\bar{i}+3\bar{j}-2\bar{k}$, $\bar{b}=3\bar{i}-2\bar{j}+6\bar{k}$ larga yasalgan parallelogramm yuzini hisoblang.

Yechish:

$\bar{a}$ ning $\bar{b}$ ga vektor ko'paytmasini topamiz:

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 6 & 3 & -2\\ 3 & -2 & 6\end{vmatrix}=\bar{i}\begin{vmatrix}3 & -2\\ -2 & 6\end{vmatrix}-\bar{j}\begin{vmatrix}6 & -2\\ 3 & 6\end{vmatrix}+\bar{k}\begin{vmatrix}6 & 3\\ 3 & -2\end{vmatrix}=14\bar{i}-42\bar{j}-21\bar{k}.$$

Ikki vektorning vektor ko'paytmasi moduli shulardan yasalgan parallogramm yuziga teng bo'lgani uchun

$$S=|a\times b|=\sqrt{14^2+42^2+21^2}=49.$$

262. Uchlari $A(1;\ 1;\ 1)$, $B(2;\ 3;\ 4)$, $C(4;\ 3;\ 2)$ bo'lgan uchburchak yuzini hisoblang.

Yechish:

$\overline{AB}, \overline{AC}$ vektorlarni topamiz:

$$\overline{AB}=(2-1)\bar{i}+(3-1)\bar{j}+(4-1)\bar{k}=\bar{i}+2\bar{j}+3\bar{k},$$
$$\overline{AC}=(4-1)\bar{i}+(3-1)\bar{j}+(2-1)\bar{k}=3\bar{i}+2\bar{j}+\bar{k}.$$

$\overline{AB}, \overline{AC}$ vektorlardan yasalgan parallelogram yuzining yarmi ABC uchburchak yuziga teng.

$$\overline{AB}\times\overline{AC}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=\bar{i}\begin{vmatrix}2 & 3\\ 2 & 1\end{vmatrix}-\bar{j}\begin{vmatrix}1 & 3\\ 3 & 1\end{vmatrix}+\bar{k}\begin{vmatrix}1 & 2\\ 3 & 2\end{vmatrix}=-4\bar{i}+8\bar{j}-4\bar{k}.$$

Demak, $S_{ABC}=\dfrac{1}{2}|\overline{AB}\times\overline{AC}|=\dfrac{1}{2}\sqrt{16+64+16}=\sqrt{24}$ (kv. birlik).

263. $\bar{a}+3\bar{b}$ va $3\bar{a}+\bar{b}$ vektorlardan yasalgan parallelogramm yuzini hisoblang, bu yerda $|\bar{a}|=|\bar{b}|=1, (\bar{a},\hat{\ }\bar{b})=30^0$.

Yechish:

$$(\bar{a}+3\bar{b})\times(3\bar{a}+\bar{b})=3\bar{a}\times a+\bar{a}\times\bar{b}+9\bar{b}\times\bar{a}+3\bar{b}\times\bar{b}=$$
$$=3\cdot 0+\bar{a}\times\bar{b}-9\bar{a}\times\bar{b}+3\cdot 0=-8\bar{a}\times\bar{b},$$
$$(\bar{a}\times\bar{a}=\bar{b}\times\bar{b}=0,\ \bar{b}\times\bar{a}=-\bar{a}\times\bar{b}).$$

Demak, $S = 8|\bar{a}\times\bar{b}| = 8\cdot 1\cdot 1\cdot \sin 30^0 = 4$ (kv. birlik).

264. $\bar{a} = 2\bar{i} - \bar{j} - \bar{k}$, $\bar{b} = \bar{i} + 3\bar{j} - \bar{k}$, $\bar{c} = \bar{i} + \bar{j} + 4\bar{k}$ larning aralash ko'paytmasini toping.

Yechish:

$$\bar{a}\cdot\bar{b}\cdot\bar{c} = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 3 & -1 \\ 1 & 1 & 4 \end{vmatrix} = 2\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 33.$$

265. $\bar{a} = 2\bar{i} - 5\bar{j} + 7\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$, $\bar{c} = \bar{i} + 2\bar{j} + 2\bar{k}$ vektorlarning komplanarligini ko'rsating.

Yechish:

Uch vektorning aralash ko'paytmasini topamiz:

$$\bar{a}\cdot\bar{b}\cdot\bar{c} = \begin{vmatrix} 2 & 5 & 7 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{vmatrix} = 2\cdot\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} - 5\cdot\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} + 7\cdot\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 8 - 15 + 7 = 0,$$

$\bar{a}\cdot\bar{b}\cdot\bar{c} = 0$ bo'lgani uchun $\bar{a}, \bar{b}, \bar{c}$ lar komplanar.

266. Uchlari $A(2;\ 2;\ 2)$, $B(4;\ 3;\ 3)$, $C(4;\ 5;\ 4)$, $D(5;\ 5;\ 6)$ bo'lgan uchburchakli piramida hajmini toping.

Yechish:

AB, AC, AD lar A nuqtada uchrashadigan piramidaning qirralari bilan ustma-ust tushadi, ularni topamiz:

$$\overline{AB} = 2\bar{i} + \bar{j} + \bar{k},\ \overline{AC} = 2\bar{i} + 3\bar{j} + 2\bar{k},\ \overline{AD} = 3\bar{i} + 3\bar{j} + 4\bar{k}.$$

Bu vektorlarning aralash ko'paytmasini topamiz

$$\overline{AB}\cdot\overline{AC}\cdot\overline{AD} = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{vmatrix} = 2\cdot\begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} - 1\cdot\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} + 1\cdot\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} = 7.$$

$\overline{AB}, \overline{AC}, \overline{AD}$ larga qurilgan parallelepipedning 1/6 qismi piramidaning hajmiga teng, shuning uchun $V = 7/6$ (kv.birlik).

267. $(\bar{a} - \bar{b})(\bar{b} - \bar{c})(\bar{c} - \bar{a})$ ni hisoblang.

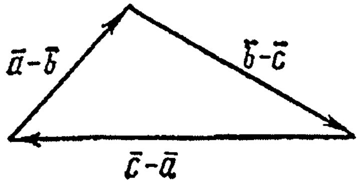
Yechish:

$(\bar{a}-\bar{b})+(\bar{b}-\bar{c})+(\bar{c}-\bar{a})=0$

bo'lgani uchun, bu vektorlar komplanar (19-rasm).

Demak, ularning aralash ko'paytmasi nolga teng:

$(\bar{a}-\bar{b})(\bar{b}-\bar{c})(\bar{c}-\bar{a})=0.$



19-chizma

268. Agar $a=4, b=6, (\widehat{\bar{a},\bar{b}})=\frac{\pi}{3}$ bo'lsa, $3\bar{a}-2\bar{b}$ va $5\bar{a}-6\bar{b}$ vektorlarning skalyar ko'paytmasini toping.

269. $\bar{a}=3\bar{i}+4\bar{j}+5\bar{k}$ va $\bar{b}=4\bar{i}+5\bar{j}-3\bar{k}$ vektorlar orasidagi burchakni toping.

270. m ning qanday qiymatida $\bar{a}=m\bar{i}+\bar{j}$ va $\bar{b}=3\bar{i}-3\bar{j}+4\bar{k}$ vektorlar perpendikular?

271. Agar $a=1,\ b=2,\ c=3, (\widehat{\bar{a},\bar{b}})=(\widehat{\bar{a},\bar{c}})=(\widehat{\bar{b},\bar{c}})=\frac{\pi}{3}$ bo'lsa, $2\bar{a}+3\bar{b}+4\bar{c}$ va $5\bar{a}+6\bar{b}+7\bar{c}$ vektorlarning skalyar ko'paytmasini toping.

272. Agar $F=2, S=5, \varphi=(\overline{F}\,\widehat{}\,\overline{S})=\pi/6$ bo'lsa, $\overline{F}$ kuchning $\overline{S}$ yo'lda bajargan ishini toping.

273. $\bar{a}=\bar{i}+\bar{j}+2\bar{k}$ va $\bar{b}=2\bar{i}+\bar{j}+\bar{k}$ larga perpendikular bo'lgan birlik vektorni toping.

274. $\bar{a}, \bar{b}, \bar{c}$ lar uzunliklari bir xil va jufti-jufti bilan teng burchaklar hosil qiladi. Agar $\bar{a}=\bar{i}+\bar{j},\ \bar{b}=\bar{j}+\bar{k}$ ga teng bo'lsa, $\bar{c}$ vektorni toping.

275. $\bar{a}=2\bar{i}+2\bar{j}+\bar{k}$ va $\bar{b}=6\bar{i}+3\bar{j}+2\bar{k}$ vektorlar berilgan. $pr_{\bar{a}}\bar{b}$ va $pr_{\bar{b}}\bar{a}$ larni toping.

276. *ABCD* parallelogramning ketma-ket uchta nuqtasini radius-vektorlari berilgan:

$\bar{r}_A = \bar{i} + \bar{j} + \bar{k}$, $\bar{r}_B = \bar{i} + 3\bar{j} + 5\bar{k}$, $\bar{r}_C = 7\bar{i} + 9\bar{j} + 11\bar{k}$. D nuqtaning radius-vektorini toping.

277. Agar $\bar{a} - \bar{i} > 0$, $\bar{a} - \bar{j} > 0$, $\bar{a} \cdot \bar{k} > 0$, $\bar{b} \cdot \bar{i} < 0$, $\bar{b} - \bar{j} < 0$, $\bar{b} \cdot \bar{k} < 0$ bo'lsa, vektorlar perpendikular emasligini isbotlang.

278. $\bar{a} = \bar{i} + m\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + (m+1)\bar{k}$, $\bar{c} = \bar{i} - \bar{j} + m\bar{k}$ vektorlar m ning qanday qiymatida komplanar bo'ladi?

279. Noldan farqli x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 sonlar

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0, \quad \begin{array}{l} x_1x_2 + y_1y_2 + z_1z_2 = 0, \\ x_1x_3 + y_1y_3 + z_1z_3 = 0, \\ x_2x_3 + y_2y_3 + z_2z_3 = 0 \end{array}$$

tenglamalarni qanoatlantirishi mumkinmi?

280. $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ va $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$ larning vektor ko'paytmasini toping.

281. Uchlari $A(2;\ 2;\ 2)$, $B(4;\ 0;\ 3)$, $C(0;\ 1;\ 0)$ bo'lgan uchburchak yuzasini hisoblang.

282. $\bar{a} = \bar{i} - \bar{j} + \bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = 2\bar{i} + 3\bar{j} + 4\bar{k}$ vektorlarning aralash ko'paytmasini toping.

283. $\bar{a} = 7\bar{i} - 3\bar{j} + 2\bar{k}$, $\bar{b} = 3\bar{i} - 7\bar{j} + 8\bar{k}$, $\bar{c} = \bar{i} - \bar{j} + \bar{k}$ vektorlarning komplanarligini isbotlang.

284. Uchlari $A(0;\ 0;\ 1)$, $B(2;\ 3;\ 5)$, $C(6;\ 2;\ 3)$, $D(3;\ 7;\ 2)$ bo'lgan uchburchakli piramidaning hajmini hisoblang. BCD yoqqa tushirilgan piramida balandligining uzunligini toping.

285. $A(5;\ 7;\ -2)$, $B(3;\ 1;\ -1)$, $C(9;\ 4;\ -4)$, $D(1;\ 5;\ 0)$ nuqtalarning bir tekislikda yotishini isbotlang.

III BOB
FAZODA ANALITIK GEOMETRIYA

1-§. TEKISLIK VA TO'G'RI CHIZIQ

1. Tekislik.

1) Tekislikning vektor tenglamasi

$$\bar{r}\cdot\bar{n}=p$$

ko'rinishda bo'ladi. Bu yerda $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$ vektor, tekislikdagi $M(x,y,z)$ nuktaning radius-vektori; $\bar{n}=\bar{i}\cos\alpha+\bar{j}\cos\beta+\bar{k}\cos\gamma$ koordinat boshidan tekislikka tushirilgan perpendikular yo'nalishiga ega bo'lgan birlik vektor; α, β, γ lar shu perpendikulyarning *Ox*, *Oy*, *Oz* o'qlari bilan tashkil qilgan burchaklari; r — perpendikular uzunligi. Yuqoridagi tenglamani koordinata ko'rinishida yozsak

$$x\cos\alpha+y\cos\beta+z\cos\gamma-p=0 \qquad (1)$$

ga ega bo'lamiz (tekislikning normal tenglamasi).

2) Agar $A^2+B^2+C^2\neq 0$ bo'lsa, ixtiyoriy tekislik tenglamasini

$$Ax+By+Cz+D=0 \qquad (2)$$

ko'rinishda yozish mumkin. *A, B, C* lar tekislikka perpendikulyar $\overline{N}(A,B,C)$ vektorning koordinatalari. Umumiy tenglamani normal holga keltirish uchun uni normallashtiruvchi ko'paytuvchi

$$\mu=\pm\frac{1}{|N|}=\pm\frac{1}{\sqrt{A^2+B^2+C^2}} \qquad (3)$$

ga ko'paytirish kerak, bu yerdagi ishora *D* ning ishorasiga teskari bo'ladi.

3) $Ax+By+Cz+D=0$ umumiy tenglamaning xususiy hollari:

$A=0$; bu holda tekislik *Ox* o'qiga parallel;

$B=0$; bu holda tekislik *Oy* o'qiga parallel;

$C=0$; bu holda tekislik *Oz* o'qiga parallel;

$D=0$; bu holda tekislik koordinat boshidan o'tadi;

$A=B=0$; bu holda tekislik *Oz* o'qiga perpendikular (*xOy* tekisligiga parallel);

$A = C = 0$; bu holda tekislik Oy o'qiga perpendikular (xOz tekisligiga parallel);

$B = C = 0$; bu holda tekislik Ox o'qiga perpendikulyar (yOz tekisligiga parallel);

$A = D = 0$; bu holda tekislik Ox o'qidan o'tadi;

$B = D = 0$; bu holda tekislik Oy o'qidan o'tadi;

$C = D = 0$; bu holda tekislik Oz o'qidan o'tadi;

$A = B = D = 0$; bu holda tekislik xOy ($z = 0$) tekisligi bilan ustma-ust tushadi;

$A = C = D = 0$; bu holda tekislik xOz ($y = 0$) tekisligi bilan ustma-ust tushadi;

$B = C = D = 0$; bu holda tekislik yOz ($x = 0$) tekisligi bilan ustma-ust tushadi.

Agar umumiy tenglamada $D \neq 0$ bo'lsa, tenglamani D ga bo'lib,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \text{ ga ega bo'lamiz.}$$

(bu yerda $a = -D/A$, $b = -D/B$, $C = -D/C$.) Bu *tekislikning kesmalarga nisbatan tenglamasi* deyiladi; a, b, c lar tekislikning Ox, Oy, Oz o'qlar bilan kesishgan nuqtalari.

4) $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2$ tekisliklar orasidagi burchak

$$\cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \tag{5}$$

formula bilan aniqlanadi.

Ikki tekislikning parallellik sharti

$$A_1/A_2 = B_1/B_2 = C_1/C_2, \tag{6}$$

Perpendikularlik sharti

$$A_1A_2 + B_1B_2 + C_1C_2 = 0.$$

5) $M_0(x_0, y_0, z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tek... masofa

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

formula bilan aniqlanadi.

Bu tekislikning normal tenglamasiga koordinatalarini qo'yib, natijaning absolu...

tijaning musbat yoki manfiyligi nuqta va koordinata boshini berilgan tekislikka nisbatan joylanishini xarakterlaydi. Agar M_0 nuqta va koordinat boshi tekislikning turli tomonida yotsa, musbat, bir tomonida yotsa, manfiy ishora olinadi.

6) $M_0(x_0, y_0, z_0)$ nuqtadan o‘tib, $\bar{N} = A\bar{i} + B\bar{j} + C\bar{k}$ vektorga perpendikular tekislik tenglamasi

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \qquad (9)$$

ko‘rinishda bo‘ladi.

A, B, C larning ixtiyoriy qiymatlarida (9) tenglik M_0 nuqtadan o‘tuvchi dastaga tegishli tekislikni ifodalaydi. Shuning uchun uni *tekisliklar dastasining tenglamasi* deyiladi.

7) $$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0 \qquad (10)$$

tenglama λ ning ixtiyoriy qiymatida

$$A_1x+B_1y+C_1z+D_1=0 \ \ (\mathrm{I}) \qquad A_2x+B_2y+C_2z+D_2=0 \ \ (\mathrm{II})$$

tekisliklarning kesishgan chizig‘idan o‘tuvchi tekislikni aniqlaydi.

(I) va (II) tenglamalar bilan aniqlanadigan tekisliklar parallel bo‘lsa, u holda tekisliklar dastasi bu tekisliklarga parallel tekisliklar to‘plamiga aylanadi.

8) Berilgan $M_1(\bar{r}_1), M_2(\bar{r}_2), M_3(\bar{r}_3)$ uch nuqtadan o‘tuvchi tekislik tenglamasini (bu yerda

$$\bar{r}_1 = x_1\bar{i} + y_1\bar{j} + z_1\bar{k}, \quad \bar{r}_2 = x_2\bar{i} + y_2\bar{j} + z_2\bar{k}, \quad \bar{r}_3 = x_3\bar{i} + y_3\bar{j} + z_3\bar{k}),$$

$\bar{r} - \bar{r}_1, \ \bar{r}_2 - \bar{r}_1, \ \bar{r}_3 - \bar{r}_1$ vektorlarning komplanarlik shartidan ($\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ radius vektor) topamiz:

$$(\bar{r} - \bar{r}_1)(\bar{r}_2 - \bar{r}_1)(\bar{r}_3 - \bar{r}_1) = 0$$

yoki koordinat ko‘rinishda

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0. \qquad (11)$$

6. $2x + 3y - 6z + 21 = 0$ tekislik tenglamasini normal holga
g.

Yechish:

Normallashtiruvchi ko'paytuvchini topamiz ($D=21>0$ bo'lgani uchun manfiy ishorani olamiz):

$$\mu=-\frac{1}{\sqrt{2^2+3^2+6^2}}=-\frac{1}{7}.$$

Shunday qilib, tekislikning normal tenglamasi

$$(-2/7)x-(3/7)y+(6/7)z-3=0$$

ko'rinishda bo'ladi.

287. $M_0(3;\ 5;\ -8)$ nuqtadan $6x-3y+2z-28=0$ tekislikkacha bo'lgan masofani aniqlang.

Yechish:

Nuqtadan tekislikkacha masofa formulasidan foydalanib,

$$d=\frac{|6\cdot 3-3\cdot 5+2\cdot(-8)-28|}{\sqrt{6^2+3^2+2^2}}=\frac{41}{7}$$

ni topamiz. M_0 nuqtaning koordinatalarini normal tenglamaga qo'yganda natija manfiy bo'lgani uchun, M_0 nuqta va koordinat boshi tekislikning bir tomonida yotadi.

288. $M(2;\ 3;\ 5)$ nuqtadan o'tib, $\overline{N}=4\overline{i}+3\overline{j}+2\overline{k}$ vektorga perpendikular tekislik tenglamasini tuzing.

Yechish:

(9) formuladan foydalanamiz:

$4(x-2)+3(y-3)+2(z-5)=0$, ya'ni $4x+3y+2z-27=0$.

289. $M(2;\ 3;\ -1)$ nuqtadan o'tib, $5x-3y+2z-10=0$ tekislikka parallel tekislik tenglamasini tuzing.

Yechish:

(9) formuladan

$$A(x-2)+B(y-3)+C(z+1)=0$$

Berilgan tekislikning normali $\overline{n}=(5,-3,2)$ bilan izlangan tekislikning normal vektori ustma-ust tushadi, demak, $A=5$, $B=-3$, $C=2$ va izlangan tekislik tenglamasi $5(x-2)-3(y-3)+2(z+1)=0$ yoki $5x-3y+2z+1=0$ bo'ladi.

290. $P(2;\ 3;\ -5)$ nuqtadan koordinat o'qlariga perpendikular tushirilgan. Ularning asosidan o'tuvchi tekislik tenglamasini tuzing.

Yechish:

Koordinat tekisliklariga tushirilgan perpendikularlarning asosi $M_1(2;\ 3;\ 0)$, $M_2(2,\ 0;\ -5)$, $M_3(0,\ 3;\ -5)$ nuqtalar bo'ladi.

(II) formulani qo'llab, M_1, M_2, M_3 nuqtadan o'tadigan

$$\begin{vmatrix} x-2 & y-3 & z \\ 0 & -3 & -5 \\ -2 & 0 & -5 \end{vmatrix} = 0$$

yoki $15x + 10y - 6z - 60 = 0$ tekislik tenglamasini hosil qilamiz.

291. $A(5;\ 4;\ 3)$ nuqtadan o'tuvchi va koordinat o'qlaridan teng kesmalar ajratuvchi tekislik tenglamasini yozing.

Yechish:

(4) tekislikning kesmalarga nisbatan tenglamasidan foydalanib, $(a = b = c)$ $x/a + y/a + z/a = 1$ ga ega bo'lamiz. A nuqtaning koordinatalari izlangan tekislik tenglamasini qanoatlantiradi, shuning uchun $5/a + 4/a + 3/a = 1$, bundan $a = 12$. Shunday qilib, $x + y + z - 12 = 0$ tenglamaga ega bo'lamiz.

292. $x + y + 5z - 1 = 0$, $2x + 3y - z + 2 = 0$ tekisliklarning kesishgan chiziqlardan va $M(3;\ 2;\ 1)$ nuqtadan o'tuvchi tekislik tenglamasini yozing.

Yechish:

(10) formuladan foydalanib quyidagini yozamiz:

$$x + y + 5z - 1 + \lambda(2x + 3y - z + 2) = 0$$

M nuqtaning koordinatalari bu tenglamani qanoatlantirishidan λ ni topamiz: $3 + 2 + 5 - 1 + \lambda(6 + 6 - 1 + 2) = 0$, bundan $\lambda = -9/13$. Shunday qilib, izlangan tenglama

$x + y + 5z - 1 - \frac{9}{13}(2x + 3y - z + 2) = 0$, yoki $5x + 14y - 74z + 31 = 0$ bo'ladi.

293. $x + 3y + 5z - 4 = 0$ va $x - y - 2z + 7 = 0$ tekisliklarning kesishgan chizig'idan o'tuvchi va Oy o'qiga parallel bo'lgan tekislik tenglamasini tuzing.

Yechish:

Tekisliklar dastasining tenglamasidan foydalanamiz:

$$x + 3y + 5z - 4 + \lambda(x - y - 2z + 7) = 0$$

$$(1 + \lambda)x + (3 - \lambda)y + (5 - 2\lambda)z + 7\lambda - 4 = 0$$

izlangan tekislik Oy o'qiga parallel bo'lgani uchun uning oldidagi koeffisient nolga teng bo'ladi: $2-\lambda=0$, ya'ni, $\lambda=3$. Buning qiymatini dasta tenglamasiga qo'yib topamiz: $4x-z+17=0$.

294. $A(2;\ -1;\ 4)$ va $B(3;\ 2;\ -1)$ nuqtalardan o'tib, $x+y+2z-3=0$ tekislikka perpendikular tekislik tenglamasini toping.

Yechish:

Izlangan tekislikning normal vektori $\overline{N}$ sifatida $\overline{AB}=\{1;3;-5\}$ ga va berilgan tekislik normaliga perpendikulyar vektorni olamiz. Shuning uchun N sifatida $\overline{AB}$ va $\bar{n}$ larning vektor ko'paytmasini olamiz:

$$\overline{N}=\overline{AB}\times\bar{n}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & 3 & -5\\ 1 & 1 & 2\end{vmatrix}=\bar{i}\begin{vmatrix}-2 & 2\\ -4 & 3\end{vmatrix}+\bar{j}\begin{vmatrix}2 & 3\\ 3 & 5\end{vmatrix}+\bar{k}\begin{vmatrix}3 & -2\\ 5 & -4\end{vmatrix}=2\bar{i}+\bar{j}-2\bar{k}.$$

$M(3;-1;-5)$ nuqtadan o'tuvchi va $N=\{2;1;-2\}$ vektorga perpendikular tekislik tenglamasi formulasini qo'llab topamiz

$$2(x-3)+(y+1)-2(z+5)=0 \text{ yoki } 2x+y-2z-15=0$$

296. Quyidagi 1) $x+y-z-2=0$; 2) $3x+5y-4z+7=0$ tekislik tenglamalarini normal holga keltiring.

297. $M_0(1;\ 3;\ -2)$ nuqtadan $2x-3y-4z+12=0$ tekislikkacha masofani toping. M_0 nuqta tekislikka nisbatan qanday joylashgan bo'ladi.

298. $M_0(2;\ 3;\ -5)$ nuqtadan $4x-2y+5z+2=0$ tekislikka tushirilgan perpendikulyar uzunligini toping.

299. 1) Koordinat o'qlaridan teng kesmalar ajratib, $M(-2;\ 3;\ 4)$ nuqtadan o'tuvchi:

2) Oz o'qidan ajratgan kesmasi Ox, Oy lardan ajratgan kesmasidan 2 marta ortiq va $N(2;\ -1;\ 4)$ nuqtadan o'tadigan tekislik tenglamasini tuzing.

300. $3x+2y-z+5=0$ tekislikka perpendikular bo'lib va $P(2;\ 0;\ -1)$

$Q(1;\ -1;\ 3)$ nuqtalardan o'tuvchi tekislik tenglamasi tuzilsin.

301. $2x-5y+2z+5=0$ tekislikda shunday M nuqtani topingki, OM to'g'ri chiziq koordinat o'qlari bilan bir xil burchaklar tashkil etsin.

302. $P(4;\ -3;\ 12)$ nuqta koordinat boshidan tekislikka tushirilgan perpendikularning asosi ekanligini bilgan holda tekislik tenglamasini toping.

303. Koordinat o'qlaridan o'tib, $3x-4y+5z-12=0$ tekislikka perpendikulyar tekisliklar tenglamasi tuzilsin.

304. Nuqtalari $P(1;\ -4;\ 2)$, $Q(7;\ 1;\ -5)$ nuqtalardan baravar uzoqlikda turgan tekislik tenglamasini tuzing.

305. $P(0;\ 2;\ 0)$; $Q(2,\ 0,\ 0)$ nuqtalardan o'tib, $x=0$ tekisligi bilan 60^0 gradusli burchak tashkil etuvchi tekislik tenglamasi tuzilsin.

306. $M(1;\ -1;\ -1)$ nuqtadan o'tib, biri Ox o'qini, ikkinchisi Oz o'qini o'z ichiga olgan tekisliklar orasidagi burchakni aniqlang.

307. Koordinat boshidan va $P(4;\ -2;\ 1)$, $Q(2;\ 4;\ -3)$ nuqtalardan o'tuvchi tekislik tenglamasi tuzilsin.

308. $2x+2y+z-7=0$, $2x-y+3z-3=0$, $4x+5y-2z-12=0$ tekislikning kesishgan nuqtasidan va $M(0;\ 3;\ 0)$; $N(1;\ 1;\ 1)$ nuqtalardan o'tgan tekislik tenglamasini yozing.

309. $x+5y+9z-13=0$, $3x-y-5z+1=0$ tekisliklarning kesishgan chizig'idan va $M(0;\ 2;\ 1)$ nuqtadan o'tuvchi tekislik tenglamasi tuzilsin.

310. Ox, Oz o'qlardan bir xil kesmalar ajratuvchi va $x+2y+3z-5=0$, $3x-2y-z+1=0$ tekisliklarning kesishgan chizig'idan o'tuvchi tekislik tenglamasini yozing.

311. xOy tekisligi bilan 60^0 gradusli burchak hosil qilgan, $(1+\sqrt{2})x+2y+2z-4=0$, $x+y+z+1=0$ tekisliklarning kesishgan chizig'idan o'tuvchi tekislik tenglamasini tuzing.

312. $2x-y-12z-3=0$, $3x+y-7z-2=0$ tekisliklarning kesishgan chizig'idan o'tuvchi, $x+2y+5z-1=0$ tekislikka perpendikular tekislik tenglamasi topilsin.

313. $A_1x+B_1y+C_1z+D_1=0$, $A_2x+B_2y+C_2z+D_2=0$ tekisliklarning kesishgan chizig'idan, koordinat boshidan o'tuvchi tekislik tenglamasini yozing.

314. $M(0;\ 4;\ 1)$ nuqtadan o'tuvchi va $\bar{a}=\bar{i}+\bar{j}+\bar{k}$, $\bar{b}=\bar{i}+\bar{j}-\bar{k}$ vektorlarga parallel bo'lgan tekislikni toping.

315. $\bar{a}=\bar{i}+2\bar{j}+\bar{k}$ vektor $x+y+2z-4=0$ tekislik bilan qanday burchak tashkil etadi.

2. To‘g‘ri chiziq.

1) To‘g‘ri chiziqni *ikki tekislikning kesishgan chizig‘i* deb qarash mumkin:

$$\begin{cases} A_1x+B_1y+C_1z+D_1=0, \\ A_2x+B_2y+C_2z+D_2=0. \end{cases}$$

2) Bu tenglamada ketma-ket x va y ni yo‘qotib, $x=az+c$, $y=bz+d$ ga ega bo‘lamiz. Bu yerda to‘g‘ri chiziq uni xOz, yOz tekisligiga proyeksiyalovchi ikkita tekislik bilan aniqlangan.

3) Ikki $M_1(x_1;\ y_1;\ z_1)$, $M_2(x_2;\ y_2;\ z_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}. \qquad (1)$$

4) $M_1(x_1;\ y_1;\ z_1)$ nuqtadan o‘tib, $\overline{S}=\ell\bar{i}+m\bar{j}+n\bar{k}$ vektorga parallel to‘g‘ri chiziqning kanonik tenglamasi:

$$\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}. \qquad (2)$$

Xususiy holda, uni quyidagicha

$$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\cos\beta}=\frac{z-z_1}{\cos\gamma}$$

yozish mumkin, bu yerda α, β, γ to‘g‘ri chiziqning o‘qlar bilan tashkil qilgan burchaklari. To‘g‘ri chiziqning yo‘naltiruvchi kosinuslari

$$\cos\alpha=\frac{l}{\sqrt{l^2+m^2+n^2}}, \quad \cos\beta=\frac{m}{\sqrt{l^2+m^2+n^2}},$$

$$\cos\gamma=\frac{n}{\sqrt{l^2+m^2+n^2}} \qquad (3)$$

formulalar bilan aniqlanadi.

5) Kanonik tenglamalarda t parametr kiritib, parametrik tenglamalarga kelish mumkin:

$$\begin{cases} x=lt+x_1, \\ y=mt+y_1, \\ z=nt+z_1. \end{cases} \qquad (4)$$

6) Kanonik tenglamalar bilan berilgan ikki to‘g‘ri chiziq orasidagi burchak:

$$\cos\varphi = \frac{\ell_1\ell_2 + m_1m_2 + n_1n_2}{\sqrt{\ell_1^2 + m_1^2 + n_1^2}\cdot\sqrt{\ell_2^2 + m_2^2 + n_2^2}}, \tag{5}$$

$$\ell_1/\ell_2 = m_1/m_2 = n_1/n_2, \tag{6}$$

$$\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0. \tag{7}$$

(6) ikki to‘g‘ri chiziqning parallellik, (7) perpendikularlik shartidir.

7) Kanonik tenglamalar bilan berilgan ikki to‘g‘ri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0. \tag{8}$$

Agar ℓ_1, m_1, n_1 lar ℓ_2, m_2, n_2 larga proporsioanal bo‘lmasa, u holda ko‘rsatilgan munosabat ikki to‘g‘ri chiziqning fazoda kesishishining zaruriy va yetarli shartidir.

8) $(x-x_1)/\ell=(y-y_1)/m=(z-z_1)/n$ to‘g‘ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchak formulasi:

$$\sin\varphi = \frac{A\ell + Bm + Cn}{\sqrt{A^2 + B^2 + C^2}\cdot\sqrt{\ell^2 + m^2 + n^2}}, \tag{9}$$

$$A\ell + Bm + Cn = 0, \tag{10}$$

$$A/\ell = B/m = C/n. \tag{11}$$

(10) va (11) to‘g‘ri chiziq va tekislikning parallellik va perpendikularlik shartidir.

9) To‘g‘ri chiziq va tekislik kesishgan nuqtasini topish uchun ularning tenglamasini birga yechish kerak.

a) Agar $A\ell+Bm+Cn\neq0$ bo‘lsa, to‘g‘ri chiziq tekislikni kesadi.

b) Agar $A\ell+Bm+Cn=0$, $Ax_0+By_0+Cz_0+D\neq0$ bo‘lsa, to‘g‘ri chiziq tekislikka parallel bo‘ladi.

d) Agar $A\ell + Bm + Cn = 0$, $Ax_0 + By_0 + Cz_0 + D = 0$ bo'lsa, to'g'ri chiziq tekislikda yotadi.

316. $2x-y+3z-1=0$ va $5x+4y-z-7=0$ to'g'ri chiziq tenglamasini kanonik holga keltiring.

Yechish:

1-usul. Avval y, sungra z ni yo'qotib, quyidagi tenglamalarni topamiz:

$13x+11z-11=0$, $17x+11y-22=0$

Har bir tenglamani x ga nisbatan yechib:

$$x = \frac{11(y-2)}{-17} = \frac{11(z-1)}{-13}, \text{ ya'ni } \frac{x}{-11} = \frac{y-2}{17} = \frac{z-1}{13}$$

ni hosil qilamiz.

2-usul. Izlangan to'g'ri chiziqqa parallel $\overline{S} = \ell\bar{i} + m\bar{j} + n\bar{k}$ vektorni topamiz. U $\overline{N_1} = 2\bar{i} - \bar{j} + 3\bar{k}$, $\overline{N_2} = 5\bar{i} + 4\bar{j} - \bar{k}$ vektorlarga perpendikular bo'lgani uchun $\overline{S}$ ni $\overline{N_1}, \overline{N_2}$ larning *vektor ko'paytmasi* deb qarash mumkin:

$$\overline{S} = \overline{N_1} \times \overline{N_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} = -11\bar{i} + 17\bar{j} + 13\bar{k}.$$

Shunday qilib, $\ell=-11$; $m=17$; $n=13$. $M_1(x_1;\ y_1;\ z_1)$ sifatida (undan izlangan to'g'ri chiziq o'tadi) koordinat tekisliklaridan biri bilan (masalan, yOz tekisligi bilan) kesishgan nuqtani olish mumkin. Bunda $x_1=0$; y_1; z_1 larni berilgan tekislik tenglamalaridan topish mumkin:

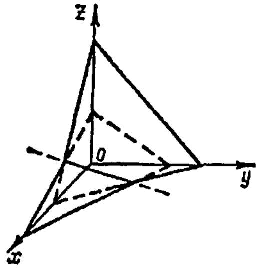
$$\begin{cases} -y + 3z - 1 = 0, \\ 4y - z - 7 = 0. \end{cases}$$

Bu sistemani yechib, $y_1=2$, $z_1=1$ larni topamiz. Demak, izlangan to'g'ri chiziq tenglamasi $x/(-1)=(y-2)/17=(z-1)/13$ bo'ladi.

317. $\begin{cases} 2x + 3y + 3z - 9 = 0, \\ 4x + 2y + z - 8 = 0 \end{cases}$ to'g'ri chiziqni chizing.

Yechish:

Izlangan to‘g‘ri chiziqni tekisliklarning kesishgan chizig‘i deb qarash mumkin. Buning uchun tekislik tenglamalarini kesmalarga nisbatan yozib olamiz: $x/4{,}5+y/3+z/3=1$, $x/2+y/4+z/8=1$. Berilgan tekisliklarni chizib, izlangan to‘g‘ri chiziqni hosil qilamiz (20-chizma).



20-chizma

318. Koordinat boshidan $(x-2)/2=(y-1)/3=(z-3)/1$ to‘g‘ri chiziqqa perpendikular tushiring.

Yechish:

To‘g‘ri chiziq va tekislikning perpendikularlik sharti (II) ni qo‘llab, koordinat boshidan o‘tuvchi va berilgan to‘g‘ri chiziqqa perpendikular tekislik tenglamasini tuzamiz. Bu tenglama $2x+3y+z=0$ bo‘ladi. Bu tekislik bilan berilgan to‘g‘ri chiziqning kesishgan nuqtasini topamiz. To‘g‘ri chiziqning parametrik tenglamasi $x=2t+2$, $y=3t+1$, $z=t+3$. t ni $2(2t+2)+3(3t+1)+t+3=0$ dan topamiz, bundan $t=-5/7$. Kesishish nuqtasining koordinatalari $x=4/7$, $y=-8/7$, $z=16/7$, ya’ni $M(4/7;\ -8/7;\ 16/7)$. Koordinat boshidan va M nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi $x/(4/7)=y/(-8/7)=z/(16/z)$ yoki $x/1=y/(-2)=z/4$ bo‘ladi.

319. To‘g‘ri chiziq $x/2=y/(-3)=z/n$ tenglamasida n ni shunday aniqlash kerakki, bu to‘g‘ri chiziq $(x+1)/3=(y+5)/2=z/1$ bilan kesishsin va kesishish nuqtasini toping.

Yechish:

n parametrini topish uchun ikki to‘g‘ri chiziqning kesishish sharti (8) dan:

$x_1=-1$, $y_1=-5$, $z_1=0$, $x_1=0$, $y_2=0$, $z_2=0$, $\ell_1=3$, $m_1=2$, $n_1=1$,

$\ell_2=2$, $m_2=-3$, $n_2=4$ deb faraz qilib, quyidagini topamiz:

$$\begin{vmatrix} 1 & 5 & 0 \\ 3 & 2 & 1 \\ 2 & -3 & n \end{vmatrix} = 0 \text{ yoki } 2n+10+3-15n=0, \text{ ya’ni } n=1.$$

Berilgan to‘g‘ri chiziqlarning kesishgan nuqtasini topish uchun birinchi tenglamadan x, y ni z orqali ifodalaymiz: $x=2z$, $y=-3z$. Bu qiymatlarni $(x+1)/3=(y+5)/2$ ga qo'yib topamiz:

$(2z+1)/3=(-3z+5)/2$, bundan $z=1$; $x=2z=2$, $y=-3z=-3$. Demak, $M(2;\ -3;\ 1)$.

320. $M(3;\ 2;\ -1)$ nuqtadan o'tib, Ox o'qi bilan to'g'ri burchak ostida kesishadigan to'g'ri chiziq tenglamasini tuzing.

Yechish:

Izlangan to'g'ri chiziq OX o'qiga perpendikular va uni kesgani uchun $N(3;\ 0;\ 0)$ nuqtadan o'tadi. M, N nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzamiz:

$$(x-3)/0=(y-2)/(-2)=(z+1)/1.$$

321. $x+y-2z-6=0$ tekislik va undan tashqarida $M(1;\ 1;\ 1)$ nuqta berilgan. Berilgan tekislikka nisbatan M nuqtaga simmetrik N nuqtani toping.

Yechish:

M nuqtadan o'tuvchi ixtiyoriy to'g'ri chiziq tenglamasini yozamiz: $(x-1)/e=(y-1)/m=(z-1)/n$.

Teksilikka perpendikular bo'lgan to'g'ri chiziqning yo'naltiruvchi vektori koordinatalari (l; m; n) ni tekislikning normali $n=\{1,\ 1,\ -2\}$ bilan almashtirish mumkin. U holda bu to'g'ri chiziqning tenglamasi $(x-1)/1=(y-1)/1=(z-1)/(-2)$ ko'rinishida yoziladi.

$x+y-2z-6=0$, $(x-1)/1=(y-1)/1=(z-1)/(-2)$ tenglamalarni birga yechib berilgan tekislikka proyeksiyasini topamiz. Bu to'g'ri chiziq tenglamasini $x=t+1$, $y=t+1$, $z=-2t+1$ yozib, uni tekislik tenglamasiga qo'yib, t=1 ni topamiz, bundan $x=2$, $y=2$, $z=-1$.

Simmetrik nuqtaning koordinatalari $\mathrm{x}=(x_M+\mathrm{x}_N)/2$, $y=(y_M+y_N)/2$, $z=(z_M+z_N)/2$ lardan topiladi, ya'ni:

$2=(1+x_N)/2$, $2=(1+y_N)/2$, $-1=(1+z_N)/2$, bundan $x_N=3$, $y_N=3$, $z_N=-3$.

Demak, $N(3;\ 3;\ -3)$.

322. $(x-1)/2=y/3=(z+1)/(-1)$ to'g'ri chiziq va undan tashqarida $M(1;\ 1;\ 1)$ nuqta berilgan. Berilgan to'g'ri chiziqqa nisbatan M nuqtaga simmetrik bo'lgan N nuqtani toping.

Yechish:

Nuqtani berilgan to'g'ri chiziqqa proyeksiyalovchi tekislik tenglamasi

$$A(x-1)+B(y-1)+C(z-1)=0$$

ko'rinishda bo'ladi. To'g'ri chiziqqa perpendikular normal vektor koordinatalari $\{A;\ B;\ C\}$ ni berilgan to'g'ri chiziqning yo'naltiruvchi vektori $\{2;\ 3;\ -1\}$ koordinatalari bilan almashtiramiz, u holda $2(x-1)+3(y-1)-(z-1)=0$ yoki $2x+3y-z-4=0$ ga ega bo'lamiz.

M nuqtani to'g'ri chiziqqa proeksiyasini topamiz. Buning uchun $2x+3y-z-4=0$, $(x-1)/2=y/3=(z+1)/(-1)$ tenglamalarni birga yechamiz. Berilgan to'g'ri chiziqning parametrik tenglamalari $x=2t+1$, $y=3t$, $z=-t-1$ bo'ladi. x, y, z larni tekislik tenglamasiga qo'yib, $t=1/14$ ni topamiz. Bundan $x=8/7$, $y=3/14$, $z=-15/14$. Kesma o'rtasini topish formulalaridan foydalanib, simmetrik nuqtaning koordinatalarini topish mumkin, ya'ni: $8/7=(1+x_N)/2$, $3/14=(1+y_N)/2$, $-15/14=(1+z_N)/2$, bundan $x_N=9/7$, $y_N=-4/7$, $z_N=-22/7$. Demak, $N(9/7, -4/7, -22/7)$.

323. $(x+1)/2=(y-1)/(-1)=(z-2)/3$ to'g'ri chiziqdan $x/(-1)=(y+2)/2=(z-3)/(-3)$ to'g'ri chiziqqa parallel tekislik o'tkazing.

Yechish:

Birinchi to'g'ri chiziq tenglamasini xOy va yOz tekisliklariga proyeksiyalanuvchi ikki tekislik tenglamalari yordamida yozamiz:

$$(x+1)/2=(y-1)/(-1) \text{ yoki } x+2y-1=0,$$

$$(y-1)/(-1)=(z-2)/3 \text{ yoki } 3y+z-5=0.$$

Bu to'g'ri chiziqdan o'tuvchi tekisliklar dastasining tenglamasi

$$x+2y-1+\lambda(3y+z-5)=0 \text{ yoki } x+(2+3\lambda)y+\lambda z-(1+5\lambda)=0$$

bo'ladi.

Bu to'g'ri chiziq tekislikning parallellik shartidan foydalanib λ ni shunday aniqlaymizki, dastaning unga mos tekisligi ikkinchi tekislikka parallel bo'lsin. $-1\cdot 1+2(2+3\lambda)=0$ ga ega bo'lamiz, yoki $3\lambda+3=0$, bundan $\lambda=-1$. Shunday qilib, $x-y-z+4=0$ izlangan tekislik tenglamasidir.

324. $(x-1)/1=(y+1)/2=z/3$ to'g'ri chiziqni $x+y-2z-5=0$ tekislikka proyeksiyasini toping.

Yechish:

Berilgan to‘g‘ri chiziq tenglamasini xOz va yOz tekisliklariga proyeksiyalovchi ikki tekislik tenglamasi ko‘rinishida yozamiz:

$(x-1)/1=(y+1)/2$ yoki $2x-y-3=0$,
$(x-1)/1=z/3$ yoki $3x-z-3=0$.

Berilgan to‘g‘ri chiziqdan o‘tuvchi tekisliklar dastasining tenglamasini ushbu

$2x-y-3+\lambda(3x-z-3)=0$ yoki $(2+3\lambda)x-y-\lambda z-3(1+\lambda)=0$ ko‘rinishida yozamiz. Tekisliklar perpendikularligining shartidan foydalanib, tekisliklar dastasidan berilgan chiziqni berilgan tekislikka proyeksiyalovchi tekislikni ajratib olamiz: $1(2+3\lambda)+1(-1)+2(-\lambda)=0$ yoki $\lambda+1=0$, bundan $\lambda=-1$. Demak, proyeksiyalovchi tekislik tenglamasi $2x-y-3+(-1)(3x-z-3)=0$ yoki $x+y-z=0$ ko‘rinishda bo‘ladi. Izlangan proyeksiyani ikki tekislikning kesishgan chizig‘i (berilgan va proyeksiyalanuvchi) kabi aniqlanishi mumkin.

325. $M(5;\ 3;\ 4)$ nuqtadan o‘tib, $\vec{s}=2\cdot\vec{i}+5\cdot\vec{j}-8\cdot\vec{k}$ vektorga parallel to‘g‘ri chiziqning tenglamasini tuzing.

Yechish:

To‘g‘ri chiziqning kononik tenglamalaridan foydalanamiz.

(2) tenglamalarda

$$\ell=2,\ m=5,\ n=-8,\ x_1=5,\ y_1=3,\ z_1=4$$

deb olib, $\frac{(x-5)}{2}=\frac{(y-3)}{5}=\frac{(z-4)}{-8}$ tenglamani hosil qilamiz.

326. $M(1;\ 1;\ 1)$ nuqtadan o‘tib, $\vec{s}_1=2\cdot\vec{i}+3\cdot\vec{j}+\vec{k}$ va $\vec{s}_2=3\cdot\vec{i}+\vec{j}+2\cdot\vec{k}$ vektorlarga parallel to‘g‘ri chiziqning tenglamasini tuzing.

Yechish:

To‘g‘ri chiziq $\vec{s}_1\times\vec{s}_2=5\cdot\vec{i}-\vec{j}-7\cdot\vec{k}$ vektorga parallel bo‘lganligi

uchun, u $\frac{(x-1)}{5}=\frac{(y-1)}{-1}=\frac{(z-1)}{-7}$ tenglama bilan aniqlanadi.

327. $\begin{cases} x+2y+3z-26=0, \\ 3x+y+4z-14=0 \end{cases}$ to'g'ri chiziqning koordinata tekisliklariga proyeksiyalari tenglamasi tuzilsin.

328. $\begin{cases} 2x+3y-16z-7=0, \\ 3x+y-17z=0 \end{cases}$ to'g'ri chizik tenglamasini kanonik ko'rinishga keltiring.

329. $\begin{cases} x-2y-5=0, \\ x-3z+8=0 \end{cases}$ to'g'ri chiziqning koordinata o'qlari bilan tashkil qilgan burchaklarini toping.

330. $M(1;\ -2;\ 3)$ nuqta o'tib, Ox va Oy o'qlari bilan 45°, 60° li burchak tashkil etuvchi to'g'ri chiziq tenglamasini toping.

331. $N(5;\ -1;\ -3)$ nuqtadan o'tib, $\begin{cases} 2x+3y+z-6=0, \\ 4x-5y-z+2=0 \end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini yozing.

332. $(x-1)/(-1)=(y-2)/5=(z+4)/2$ va $(x-2)/2=(y-5)/(-2)=$ $=(z-1)/3$ to'g'ri chiziqlarning kesishgan nuqtasini toping.

333. Parallelogrammning ketma-ket uchta uchi $A(3;\ 0;\ -1)$, $B(1;\ 2;\ -4)$, $C(0;\ 7;\ -2)$ berilgan. AD, CD tomonlari tenglamasini toping.

334. $M(2;\ -5;\ 1)$, $N(-1;\ 1;\ 2)$ nuqtalardan o'tuvchi to'g'ri chiziqning parametrik tenglamasini toping.

335. $x/1=(y-3)/2=(z-2)/1$ va $(x-3)/1=(y+1)/2=(z-2)/1$ parallel to'g'ri chiziqlar orasidagi masofani hisoblang.

336. $A(-1;\ 2;\ 3)$, $B(2;\ -3;\ 1)$ nuqtalar berilgan. $M(3;\ -1;\ 2)$ nuqtadan o'tib, AB ga parallel to'g'ri chiziq tenglamasini tuzing.

337. $\begin{cases} 4x-y-z-+12=0, \\ y-z-2=0 \end{cases}$ va $\begin{cases} 3x-2y+16=0, \\ 3x-z=0 \end{cases}$ to'g'ri chiziqlar orasidagi burchakni toping.

338. yOz tekisligida koordinata boshidan o'tib, $\begin{cases} 2x-y=2, \\ y+2z=-2 \end{cases}$ to'g'ri chiziqqa perpendikular to'g'ri chiziqni toping.

339. *ABCD* parallelogrammning ikki uchi: $C(-2;\ 3;\ 5)$, $D(0;\ 4;\ -7)$ diogonallarining kesishgan nuqtasi $M(1;\ 2;\ -3.5)$ berilgan. *AB* tomon tenglamasini toping.

340. *ABC* uchburchak $x+2y+4z-8=0$ tekislikning koordinata o‘qlari bilan kesishishidan hosil bo‘lgan uchburchakning xOy tekisligiga parallel o‘rta chizig‘ining tenglamasini toping.

341. $A(1;\ 1;\ 1)$, $B(2;\ 3;\ 3)$, $C(3;\ 3;\ 2)$ nuqtalar berilgan. *A* nuqtadan o‘tib, *AB* va *AC* vektorlarga parallel to‘g‘ri chiziq tenglamasini tuzing.

342. $M(0;\ 2;\ 1)$ nuqtadan o‘tib, $\bar{a}=\bar{i}+2\bar{j}+2\bar{k}$, $\bar{b}=3\bar{j}$, $c=3\bar{k}$ vektorlar bilan teng burchaklar tashkil etuvchi to‘g‘ri chiziq tenglamasini tuzing.

343. $(x+1)/3=(y-2)/(-1)=z/4$ to‘g‘ri chiziqdan o‘tib, $3x+y-z+2=0$ tekislikka perpendikular tekislik tenglamasini tuzing.

344. $x/2=(y+3)/1=(z-2)/(-2)$ to‘g‘ri chiziqning tekislikka proyeksiyasi tenglamasini yozing.

2-§. IKKINCHI TARTIBLI SIRTLAR

1. Sfera.

Dekart koordinata sistemasida markazi $C(a;\ b;\ c)$ va radiusi r bo‘lgan sfera

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2 \tag{1}$$

tenglama bilan aniqlanadi.

Agar markaz koordinata boshida yotsa, tenglama

$$x^2+y^2+z^2=r^2$$

ko‘rinishida bo‘ladi.

345. $x^2+y^2+z^2-x+2y+1=0$ sferaning markazi va radiusini toping.

Yechish:

Sfera tenglamasini (1) kanonik ko‘rinishga keltiramiz, x, y, z larni o‘z ichiga oluvchi xadlarini to‘la kvadratga to‘ldiramiz:

$$(x^2-x+\frac{1}{4})-\frac{1}{4}+(y^2+2y+1)-1+z^2+1=0$$

yoki

$$(x-\frac{1}{2})^2+(y+1)^2+z^2=\frac{1}{4}.$$

Demak, sferaning markazi $C(1/2;\ -1;\ 0)$, radiusi $r=1/2$.

346. Markazi xOy tekisligida yotuvchi, $A(1;\ 2;\ -4)$, $B(1;\ -3;\ 1)$, $C(2;\ 2;\ 3)$ nuqtalardan o‘tuvchi sfera tenglamasini tuzing.

Yechish:

A, B, C nuqtalar $(x-a)^2+(y-b)^2+z^2=r^2$ cferaga tegishli (markazi xOy tekisligida yotadi, ya’ni $c=0$) bo‘lgani uchun ularning koordinatalari izlangan tenglamani ayniyatga aylantiradi, shuning uchun

$$(1-a)^2+(2-b)^2+(-4)^2=r^2,\quad (1-a)^2+(-3-b)^2+1^2=r^2,$$
$$(2-a)^2+(2-b)^2+3^2=r^2$$

Bundan $(2-b)^2-(-3-b)^2=-15$, ya’ni $10b=10$, $(1-a)^2+(2-b)^2=-7$

Demak, $a=-2$, $b=1$. Sferaning markazi $C(-2;\ 1;\ 0)$ bo‘ladi. So‘ngra

$r^2=(-a)^2+(2-b)^2+16=(1+2)^2+(2-1)^2+16=26$. Shunday qilib, izlangan tenglama $(x+2)^2+(y-1)^2+z^2=26$ ko‘rinishda bo‘ladi.

347. $\begin{cases}(x-3)^2+(y+2)^2+(z-1)^2=100,\\ ABC2x-2y-z+9=0\end{cases}$ aylananing radiusi va markazini toping.

Yechish:

Sferaning markazi $C(3;\ -2;\ 1)$ dan $2x-2y-z+9=0$ tekislikka perpendikular tushiramiz, uning tenglamasi

$$(x-3)/2=(y+2)/(-2)=(z-1)/(-1) \qquad (*)$$

(perpendikularning yo‘naltiruvchi vektori sifatida berilgan tekislikning normal vektorini olish mumkin). (*) to‘g‘ri chiziq bilan berilgan tekislikning kesishgan nuqtasini topamiz. Bu nuqta aylananing markazidir. To‘g‘ri chiziq tenglamasini parametrik ko‘rinishda yozamiz: $x=2t+3$, $y=-2t-2$, $z=-t+1$. Bu *x, y, z* larni tekislik tenglamasiga qo‘yamiz: $2(2t+3)-2(-2t-2)-(-t+1)+9=0$, ya’ni $t=-2$. Demak, $x=2(-2)+3=-1$, $y=-2(-2)-2=2$, $z=-(-2)+1=3$ ya’ni aylananing markazi $C(-1;\ 2;\ 3)$ bo‘ladi. Sferaning markazi $C(3;\ -2;\ 1)$ nuqtadan $2x-2y-z+9=0$ tekislikkacha masofa d ni topamiz:

$$d=\frac{2\cdot3+2\cdot2-1+9}{\sqrt{2^2+2^2+1}}=6.$$

Aylana radiusini $r^2=R^2-d^2$ dan topamiz, bu yerda R sferaning radius. Shunday qilib, $r^2=100-36-64$, ya'ni $r=8$.

348. 1) $(x+1)^2+(y+2)^2+z^2=25$; 2) $x^2+y^2+z^2-4x+6y+2z=2$;
3) $2x^2+2y^2+2z^2+4y-3z+2=0$; 4) $x^2+y^2+z^2=2x$;
5) $x^2+y^2+z^2=4z-3$;
sferalarning markazi va radiusining koordinatalarini aniqlang.

349. M(1; −1; 3) nuqta 1) $(x-1)^2+(y+2)^2+z^2=19$;
2) $x^2+y^2+z^2-x+y=0$; 3) $x^2+y^2+z^2-4x+y-2z=0$ sferalarga nisbatan qanday joylashgan.

350. Agar M(4; −1; −3) va N(0; 3; −1) nuqtalar sfera diametrlaridan birining oxirlari bo'lsa, sferaning tenglamasini tuzing.

351. $z=0$ koordinata tekisligi bilan $(x-1)^2+(y-1)^2+(z-3)^2=25$ sferaning kesimida hosil bo'lgan aylana tenglamasini yozing.

352. $x^2+y^2+z^2=100$, $2x+2y-z=18$ aylananing radiusi va markazining koordinatalarini toping.

2. Silindrik sirtlar va ikkinchi tartibli konus.

$F(x; y)=0$ tenglama fazoda yasovchisi Oz o'qiga parallel bo'lgan silindrik sirtni ifodalaydi. Shunga o'xshash $F(x; z)=0$ yasovchisi Oy o'qiga, $F(y;z)=0$, Ox o'qiga parallel bo'lgan silindrik sirtni ifodalaydi.

Ikkinchi tartibli silindrik sirtlarning kanonik tenglamalari:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ – elliptik silindr;}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ – giperbolik silindr;}$$

$$y^2=2px \text{ – parabolik silindr.}$$

Uchala silindrning yasovchilari Oz o'qiga parallel, yo'naltiruvchisi esa, xOy tekisligida yotuvchi ikkinchi tartibli egri chiziq (ellips, giperbola, parabola). Fazoda egri chiziqni *parametrik shaklda* yoki *ikki sirtning kesishgan chizig'i* deb qarash mumkin.

Masalan, elliptik silindrning yo'naltiruvchisi, ya'ni xOy tekisligidagi ellips tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$
$$z=0$$

O'qi Oz, uchi koordinata boshida bo'lgan ikkinchi tartibli konus tenglamasi ushbu

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

ko'rinishda bo'ladi, xuddi shunga o'xshash,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=0, \quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$

larning o'qlari mos ravishda Oy, Ox lar hisoblanadi.

353. 1) $x^2=4y$; 2) $z^2=xz$ tenglamalar fazoda qanday sirtni ifodalaydi? Silindrik sirtning yo'naotiruvchisi $x^2=4y$, $z=0$ paraboladir.

Yechish:

1. $x^2=4y$ yasovchisi Oz o'qiga parallel bo'lgan parabolik silindrni ifodalaydi. Silindrik sirtni yo'naltiruvchisi $x^2=4y$, $z=0$ paraboladir.

2. $z^2=xz$ ni $z(z-x)=0$ ko'rinishda tasvirlash mumkin. U ikki $z=0$, $z=x$ tenglamaga ajraladi, ya'ni ikki tekislikni (xOy va bissektral tekislikni) aniqlaydi.

354. $x^2+y^2-2z^2=0$ konus $y=2$ tekislik bilan qanday chiziq bo'ylab kesishadi?

Yechish:

Tenglamalardan y ni yo'qotib, izlangan chiziq tenglamasini topamiz:

$x^2+4-2z^2=0$ yoki $z^2/-x^2/4=1$. Demak, izlangan kesishish chizig'i $y=2$ tekisligidayotuvchi giperboladir, uning haqiqiy o'qi Oz o'qiga, mavhum o'qi esa Ox o'qiga parallel.

355. Uchi $M(0;\ 0;\ 1)$ nuqtada, yasovchisi $x^2/25+y^2/9=1$, $z=3$ ellips bo'lgan konik sirt tenglamasini tuzing.

Yechish:

AM yasovchi tenglamasini tuzamiz, bu yerda $A(x_0,y_0,z_0)$ nuqta ellipsdayotadi. Bu yasovchining tenglamasi $x/x_0=y/y_0=(z-1)/(z_0-1)$.

A nuqta ellipsda yotgani uchun uning koordinatalari ellips tenglamasini qanoatlantiradi, ya'ni $x_0^2/25+y_0^2/9=1$, $z=3$.

$x/x_0=(z-1)/(z_0-1)$, $y/y_0=(z-1)/(z_0-1)$, $x_0^2/25+y_0^2/9=1$, $z_0=3$ sistemasidan x_0, y_0, z_0 larni yo'qotib, konik sirt tenglamasiga ega bo'lamiz: $x^2/25+y^2/9+(z-1)^2/4=0$.

356. 1) $x^2+y^2=4$; 2) $x^2/25+y^2/16=1$; 3) $x^2-y^2=1$; 4) $y^2=2x$; 5) $z^2=y$; 6) $z+x^2=0$; 7) $x^2+y^2=2y$; 8) $x^2+y^2=0$; 9) $x^2-z^2=0$; 10) $y^2=xy$ qanday sirtlarni tasvirlaydi. Ularni chizing.

357. $x^2-y^2+z^2=0$ konusning 1) $y=3$; 2) $z=1$; 3) $x=0$ tekisliklari bilan kesishgan chizig'ini toping.

358. Uchi koordinata boshida va yo'naltiruvchilari 1) $x=a$; $y^2+z^2=b^2$; 2) $y=b$; $x^2+z^2=a^2$; 3) $z=c$; $x^2/a^2+y^2/b^2=1$ bo'lgan konus tenglamasini tuzing.

3. Aylana sirt. Ikkinchi tartibli sirt.

Agar yOz tekisligida yotgan $F(y, z)=0$, $x=0$ egri chiziq Oz o'qi atrofida aylantirilsa, undan hosil bo'lgan aylanma sirt tenglamasi

$F(\sqrt{(x^2+y^2)}; z=0)$ ko'rinishida bo'ladi.

$F(x; \sqrt{(x^2+y^2)}=0)$ tenglama $F(x; y)=0$, $z=0$ egri chiziqning Ox o'qi atrofida, $F(\sqrt{(x^2+y^2)}; y=0)$ yuqoridagi egri chiziqning Oy o'qi atrofida aylanishidan hosil bo'lgan sirtni ifodalaydi.

Ellips, giperbola, parabolaning o'z simmetriya o'qi atrofida aylanishidan hosil bo'lgan ikkinchi tartibli aylanma sirt tenglamalarini keltiramiz.

$\frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1$ – *aylanma ellipsoid*, bu yerda aylanish o'qi Oz

$\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=1$ – *aylanma bir pallali giperboloid*, aylanish o'qi Oz (Oz giperbolaning mavhum o'qi).

$\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=-1$ – *aylanma ikki pallali giperboloid*, aylanish o'qi Oz (giperbolaning haqiqiy o'qi).

$x^2+y^2=2pz$ – *aylanma paraboloid.*

Ikkinchi tartibli aylanma sirtlar umumiy ko'rinishdagi ikkinchi tartibli sirtlarning xususiy holidir.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \text{ – } \textit{ellipsoid.}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \text{ – } \textit{bir pallali giperboloid.}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1 \text{ – } \textit{ikki pallali giperboloid.}$$

$$\frac{x^2}{p}+\frac{y^2}{q}=2z \quad (p>0,\ q>0) \text{ – } \textit{elliptik paraboloid.}$$

Bu to'rtta ikkinchi tartibli sirtlardan, ikkinchi tartibli uchta silindr (elliptik, giperbolik, parabolik) ikkinchi tartibli konusdan boshqa yana bitta n-tartibli sirt – *giperbolik paraboloid* mavjud:

$$\frac{x^2}{p}-\frac{y^2}{q}=2z \quad (p>0,\ q>0).$$

Shunday qilib, 9 ta ikkinchi tartibli sirt uchraydi.

359. $x+2y=4$, $z=0$ to'g'ri chiziqning Ox o'qi atrofida aylanishidan hosil bo'lgan sirt tenglamasini toping.

Yechish:

Aylanma sirt uchi $M(4;\ 0;\ 0)$ nuqtada bo'lgan konusdan iborat.

Izlangan sirt ixtiyoriy A nuqtasining koordinatalari X; Y; Z bo'lsin, unga berilgan to'g'ri chiziqda $B(x;\ y;\ 0)$ nuqta mos keladi.

A va B nuqtalar aylanish o'qi Ox ga perpendikular bo'lgan bitta tekislikda yotadi. U holda $X=x$, $Y^2+Z^2=y^2$ bo'ladi. x va y ning qiymatlarini berilgan to'g'ri chiziq tenglamasiga qo'yib izlangan aylanma sirt tenglamasini tuzamiz:

$$X+2\sqrt{Y^2+Z^2}=4 \text{ yoki } 4(Y^2+Z^2)-(X-4)^2=0, \text{ ya'ni}$$
$$(4Y^2+4Z^2)-(X-4)^2=0.$$

360. $x^2=yz$ tenglama qanday sirtni ifodalaydi?.

Yechish:

Koordinat o'qlarini Ox o'qi atrofida $\alpha=45°$ li burchakka buramiz (Oy o'qidan Oz o'qiga qarab soat miliga qarshi yo'nalishda).

Koordinat almashtirish formulalari:

$x=x'$, $y=y'\cos\alpha - z'\sin\alpha$, $z=y'\sin\alpha + z'\cos\alpha$, $\sin\alpha=\cos\alpha=\frac{\sqrt{2}}{2}$ bo‘lgani uchun

$$x=x',\ y=\frac{\sqrt{2}}{2}(y'-z'),\ z=\frac{\sqrt{2}}{2}(y'+z').$$

Bu ifodalarni sirt tenglamasiga qo‘yib, $x'^2=y'^2/2$ yoki $x'^2-y'^2+z'^2/2=0$ (uchi koordinat boshida yotgan, o‘qi ordinata bo‘lgan konus) ga ega bo‘lamiz.

361. $y+z-2=0$, $x=0$ to‘g‘ri chiziqning Oz o‘qi atrofida aylanishidan hosil bo‘lgan sirt tenglamasini yozing.

362. $z^2=x-y^2$ sirtning $z=1$, $y=1$, $x=1$, $z=-1$ tekisliklar bilan kesishish chizig‘i topilsin.

363. 1) $z=xy$, 2) $z^2=xy$ tenglamalar qanday sirtlarni aniqlaydi.

364. O‘qi Oz bo‘lib, uchi koordinat boshida va $M(-1;\ -2;\ 2)$, $N(1;\ 1;\ 1)$ nuqtalar sirtida yotgan elliptik paraboloid tenglamasini toping.

365. Agar sirtda uchta $A(3;\ 0;\ 0)$, $B(-2;\ 5/3;\ 0)$, $C(0;\ -1;\ 2/\sqrt{5})$ nuqta berilgan bo‘lsa, simmetriya o‘qlari koordinat o‘qlaridan iborat bo‘lgan ellipsoid tenglamasini tuzing.

366. $z=2-x^2-y^2$ va $z=x^2+y^2$ sirtlarning kesishgan chizig‘i tenglamasini toping.

367. $z^2+x^2=m(z^2+y^2)$ tenglama 1) $m=0$; 2) $0<m<1$; 3) $m>0$;

4) $m<0$; 5) $m=1$ bo‘lganda qanday sirt aniqlanishini tekshiring.

4. Ikkinchi tartibli sirtning umumiy tenglamasi.

x, y, z larga nisbatan ikkinchi darajali umumiy tenglama

$$A^2x+B^2y+C^2z+2Dyz+2Exz+2Fxy+2Gx+2Hy+2Kz+L=0$$

ko‘rinishda bo‘ladi. Bu tenglama sfera, ellipsoid, bir pallali yoki ikki pallali giperboloid, elliptik yoki giperbolik paraboloid, ikkinchi tartibli silindrik yoki konik sirtni aniqlaydi. U yana ikki tekislik, nuqta, to‘g‘ri chiziqlar to‘plamini aniqlashi yoki hech qanday geometrik ma’noga ega bo‘lmasligi mumkin (mavhum sirtni aniqlaydi).

$D=0$, $E=0$, $F=0$ da $A^2x+B^2y+C^2z+2Gx+2Hy+2Kz+L=0$ tenglamaga ega bo'lamiz. Bu holda tenglama o'qlarni parallel ko'chirish yordamida oson soddalashtiriladi, shunga qarab uning geometrik ma'nosini darrov aytish mumkin.

368. $x^2+4y^2+9z^2+12yz+6xz+4xy-4x-8y+12z+3=0$ tenglama qanday geometrik ma'noga ega?

Yechish:

Berilgan tenglamani $(x+2y+3z)^2-4(x+2y+3z)+3=0$ ko'rinishda yozish mumkin. Chap tomonni ko'paytuvchilarga ajratamiz:

$$(x+2y+3z-1)\ (x+2y+3z-3)=0.$$

Shunday qilib, tenglama ikki $(x+2y+3z)=0$ va $x+2y+3z-3=0$ tekislik to'plamini aniqlaydi.

369. $x^2-y^2+z^2-yz-xz-xy=0$ tenglama qanday geometrik ma'noga ega.

Yechish:

Tenglamani 2 ga ko'paytiramiz:

$2x^2+2y^2+2z^2-2yz-2xz-2xy=0$ yoki $(x-y)^2+(y-z)^2+(x-z)^2=0.$

Koordinatalari $x=y$, $y=z$, $x=z$ tengliklarni qanoatlantiruvchi nuqtalargina bu tenglamani qanoatlantiradi. Shunday qilib, tenglama $x=y=z$ to'g'ri chiziqni ifodalaydi.

370. $x^2+y^2+4z^2-2xy-8z+5=0$ tenglama qanday geometrik ma'noga ega.

Yechish:

Tenglamani $(x-y)^2+4(z-1)^2=-1$ ko'rinishida yozib olamiz. Bu tenglama hech qanday geometrik ma'noga ega emas, chunki chap tomon x; y; z ning hech qanday qiymatida manfiy bo'la olmaydi.

371. $4x^2+9y^2+36z^2-8x-18y-72z+13=0$ tenglamani kanonik ko'rinishga keltiring.

Yechish:

Bir xil koordinatali hadlarni guruhlaymiz

$$4(x^2-2x)+9(y^2-2y)+36(z^2-2z)=-13.$$

Qavs ichidagi ifodalarni to'la kvadratga to'ldirib topamiz:

$$4(x^2-2x+1)+9(y^2-2y+1)+36(z^2-2z+1)=-13+4+9+36$$

yoki

$$4(x^2-1)+9(y^2-1)+36(z^2-1)=36.$$

Yangi koordinata boshini $O'(1;\ 1;\ 1)$ nuqtada olib, koordinat o'qlarini parallel ko'chiramiz: koordinata almashtirish formulalari

$x=x`+1$, $y=y`+1$, $z=z`+1$ bo'ladi, u holda sirt tenglamasi quyidagi

$$4x`^2+9y`^2+36z`^2=36 \text{ yoki } x`^2/9+y`^2/4+z`^2=1$$

ko'rinishda yoziladi. Bu tenglama elli psoidni aniqlaydi, uning markazi koordinata boshida, yarim o'qlari mos ravishda 3; 2; 1 ga teng.

372. $x^2-y^2-4x+8y-2z=0$ tenglamani kanonik holga keltiring.

Yechish:

x, y larni o'z ichiga olgan hadlarni guruhlaymiz.

$(x^2-4)-(y^2-8y)=2z$ qavs ichidagi ifodalarni to'la kvadratgacha to'ldiramiz:

$(x^2-4x+4)-(y^2-8y+16)=2z+4-16$ yoki $(x-2)^2-(y-4)^2=2(z-6)$ yangi koordinat boshi sifatida O`(2; 4; 6) nuqtani olib, koordinat o'qlarini parallel ko'chiramiz, u holda $x=x`+2$, $y=y`+4$, $z=z`+6$. Giperbolik paraboloidning aniqlaydigan $x`^2-y`^2=2z`$ tenglamaga ega bo'lamiz.

373. $4x^2-y^2+4z^2-8y+4y+8z+4=0$ tenglama qanday sirtni ifodalaydi?

Yechish:

Mos almashtirishlarni bajarib:

$4(x^2-2x)-(y^2-4y)+4(z^2+2z)=-4;$

$4(x^2-2x+1)-(y^2-4y+4)+4(z^2+2z+1)=-4+4-4+4;$

$4(x-1)^2-(y^2-2)^2+4(z+1)^2=0$ tenglikni hosil qilamiz. Koordinata boshini O`(1; 2; -1) nuqtaga ko'chirib, koordinata o'qlarini parallel ko'chiramiz, $x=x'+1$, $y=y'+2$, $z=z'-1$ lar koordinata almashtirish formulalaridir. U holda berilgan tenglama $4x'^2-y'^2+4z'^2=0$ yoki $x'^2-y'^2/4+z'^2=0$ ko'rinishga keladi. Bu sirtning tenglamasidir.

Quyidagi tenglamalar bilan qanday sirt tasvirlanishini aniqlang:

374. $x^2-xy-xz+yz=0.$

375. $x^2+z^2-4x+4z+4=0.$

376. $x^2+2y^2+z^2-2xy-2y=0.$

377. $x^2+y^2+z^2-2y+2z=0.$

378. $x^2+2y^2+2z^2-4y+4z+4=0.$

379. $4x^2+y^2-z^2-24x-4y+2z+35=0.$

380. $x^2+y-z^2-2x-2y+2z+2=0.$

381. $x^2+y^2-6x+6y-4z+18=0.$

382. $9x^2-z^2-18x-18y-6z=0.$

IV BOB
DETERMINANT VA MATRITSALAR

1-§. n-TARTIBLI DETERMINANT HAQIDA

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

elementlar jadvaliga mos keluvchi 4-tartibli determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} +$$

$$+a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14} \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

tenglik bilan aniqlanadi.

4-tartibli determinant yordamida 5-tartibli va hokazo tartibli determinant tushunchasini kiritish mumkin.

Ixtiyoriy tartibli determinantlar uchun 3-tartibli determinantlar uchun kiritilgan algebraik yig'indilar uchun ikkita teorema va biror elementning minor va algebraik to'ldiruvchisi ta'rifi o'zgarmasdan qoladi. Shunday qilib, a_{ik} elementning minorini M_{ik}, algebraik to'ldiruvchisini A_{ik} bilan belgilab,

$$A_{ik} = (-1)^{i+k} M_{ik}$$

ifodaga ega bo'lamiz.

D n-tartibli determinant bo'lsin. Uni avval *i*-nchi yo'lning elementlari bo'yicha, so'ngra *k*-nchi ustunning elementlari bo'yicha yoyib, 1-teoremaga asosan:

$$D = a_{i1}A_{i1} + a_{i2}A_{i2} + \ldots + a_{in}A_{in},$$
$$D = a_{1k}A_{1k} + a_{2k}A_{2k} + \ldots + a_{nk}A_{nk}$$

ga ega bo'lamiz. Ikkinchi tomondan, 2-teoremaga asosan

$j \neq i$, $k \neq i$, bo'lganda

$$a_{j1}A_{i1} + a_{j2}A_{i2} + \ldots + a_{jn}A_{in} = 0,$$
$$a_{1k}A_{1i} + a_{2k}A_{2i} + \ldots + a_{nk}A_{ni} = 0$$

ga ega bo'lamiz.

Ikkinchi va uchinchi tartibli determinantlarning xossalari ixtiyoriy tartibli determinantlar uchun o'rinlidir.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$$

sistemaning determinanti

$$D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix} \neq 0$$

bo'lsa, uning yechimlari $x_1 = \frac{D_1}{D}$, $x_2 = \frac{D_2}{D}$, ..., $x_n = \frac{D_n}{D}$ formulalardan topiladi. Bu formulalarda D – sistemaning determinanti, D_k, ($k = 1,\ 2,\ \ldots,\ n$) sistemaning determinantida k-nchi tartibli ustunni (aniqlanadigan noma'lumlar oldidagi koeffitsientlar ustuni) ozod ustuni bilan almashtirishdan hosil bo'lgan determinantidir:

$$\begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1,k-1} & b_1 & a_{1,k+1} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2,k-1} & b_2 & a_{2,k+1} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{n,k-1} & b_n & a_{n,k+1} & \ldots & a_{nn} \end{vmatrix}.$$

383. Ushbu $\begin{vmatrix} 3 & 5 & 7 & 2 \\ 1 & 2 & 3 & 4 \\ -2 & -3 & 3 & 2 \\ 1 & 3 & 5 & 4 \end{vmatrix}$ determinantni hisoblang.

Yechish:

Quyidagi amallarni bajaramiz: 1) birinchi yo'l elementlaridan uchinchi yo'l elementlarini 3 ga ko'paytirib ayiramiz; 2) ikkinchi yo'l elementlarini 2 ga ko'paytirib, uchinchi yo'l elementlariga qo'shamiz; 3) to'rtinchi yo'l elementlaridan ikkinchi yo'l elementlarini ayiramiz. U holda berilgan determinant quyidagi ko'rinishga keladi:

$$D = \begin{vmatrix} 0 & -1 & -2 & -10 \\ 1 & 2 & 3 & 4 \\ 0 & 1 & 9 & 10 \\ 0 & 1 & 2 & 0 \end{vmatrix}.$$

Bu determinantni birinchi ustun elementlari bo'yicha yoyamiz:

$$D = -\begin{vmatrix} 0 & 0 & -10 \\ 0 & 7 & 10 \\ 1 & 2 & 0 \end{vmatrix}.$$

Hosil bo'lgan determinantni birinchi ustun elementlari bo'yicha yoyamiz:

$$D = -\begin{vmatrix} 0 & -10 \\ 7 & 10 \end{vmatrix} = -70.$$

384. Ushbu $\begin{vmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 2 & 3 & 0 & 0 \\ 0 & 4 & 3 & 4 & 0 \\ 0 & 0 & 5 & 4 & 5 \\ 0 & 0 & 0 & 6 & 5 \end{vmatrix}$ determinantni hisoblang.

Yechish:

2-nchi, 4-nchi, va 5-nchi ustunlardan umumiy ko'paytuvchilarni determinant belgisidan tashqariga chiqaramiz:

$$D = 2 \cdot 2 \cdot 5 \cdot \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 3 & 1 & 3 & 0 & 0 \\ 0 & 2 & 3 & 2 & 0 \\ 0 & 0 & 5 & 2 & 1 \\ 0 & 0 & 0 & 3 & 1 \end{vmatrix}.$$

Ikkinchi ustun elementlari birinchi ustun elementlaridan ayirib, hosil bo'lgan determinantni birinchi yo'l elementlari bo'yicha yoyamiz:

$$D = 20 \cdot \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & -2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 2 & 0 \\ 0 & 0 & 5 & 2 & 1 \\ 0 & 0 & 0 & 3 & 1 \end{vmatrix} = 20 \cdot \begin{vmatrix} -2 & 3 & 0 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & 5 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix}.$$

1-nchi qator elementlarini 2-nchi qator elementlariga qo'shamiz, -2 ni determinant tashqarisiga chiqaramiz, so'ngra hosil bo'lgan determinantni 1-nchi ustun elementlari bo'yicha yoyamiz:

$$D = -40 \cdot \begin{vmatrix} 1 & 3 & 0 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & 5 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{vmatrix} = -40 \begin{vmatrix} 6 & 2 & 0 \\ 5 & 2 & 1 \\ 0 & 3 & 1 \end{vmatrix}.$$

3-nchi qator elementlarini 2-nchi qator elementlaridan ayiramiz, 2 ni determinant tashqarisiga chiqaramiz va hosil bo'lgan determinantni 3-nchi ustun elementlari bo'yicha yoyamiz:

$$D = -80 \cdot \begin{vmatrix} 3 & 1 & 0 \\ 5 & -1 & 0 \\ 0 & 3 & 1 \end{vmatrix} = -80 \cdot \begin{vmatrix} 3 & 1 \\ 5 & -1 \end{vmatrix} = 640.$$

385.

$$\begin{cases} x+2y+3z=14, \\ y+2z+3t=20, \\ z+2t+3x=14, \\ t+2x+3y=12 \end{cases}$$

sistemadan y ni toping.

Yechish:

Berilgan sistemani

$$\begin{cases} x+2y+3z+0t=14, \\ 0x+y+2z+3t=20. \\ 3x+0y+z+2t=14, \\ 2x+3y+0z+t=12 \end{cases}$$

ko‘rinishda yozib olamiz.

$$D=\begin{vmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{vmatrix}$$

determinantni yozib olamiz. So‘ngra 2-nchi ustun elementlaridan 1-nchi ustun elementlarini 2 ga ko‘paytirib ayiramiz:

$$D=\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 3 & -6 & -8 & 2 \\ 2 & -1 & -6 & 1 \end{vmatrix}=\begin{vmatrix} 1 & 2 & 3 \\ -6 & -8 & 2 \\ -1 & -6 & 1 \end{vmatrix}=(-2)\cdot(-1)\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & -1 \\ 1 & 6 & -1 \end{vmatrix}\cdot$$

1-nchi ustun elementlarini 2 ga ko‘paytirib 2-nchi ustun elementlarini ayiramiz: 1-nchi ustun elementlarini 3 ga ko‘paytirib 3-nchi ustun elementlaridan ayiramiz:

$$D=2\begin{vmatrix} 1 & 0 & 0 \\ 3 & -2 & -10 \\ 1 & 4 & -4 \end{vmatrix}=2\begin{vmatrix} -2 & -10 \\ 4 & -4 \end{vmatrix}=2\ (8+40)=96,$$

$$D_y=\begin{vmatrix}1&14&3&0\\0&20&2&3\\3&14&1&2\\2&12&0&1\end{vmatrix}=2\cdot\begin{vmatrix}1&7&3&0\\0&10&2&3\\3&7&1&2\\2&6&0&1\end{vmatrix}$$

ni topamiz. 1-nchi qator elementlarini 3 ga ko‘paytirib, 3-nchi qator elementlaridan ayiramiz, 1-nchi qator elementlarini 2 ga ko‘paytirib 4-nchi qator elementlaridan ayiramiz:

$$D_y=2\cdot\begin{vmatrix}1&7&3&0\\0&10&2&3\\0&-14&-8&2\\0&-8&-6&1\end{vmatrix}=2\cdot\begin{vmatrix}10&2&3\\-14&-8&2\\-8&-6&1\end{vmatrix}=2\cdot2\cdot2\cdot\begin{vmatrix}5&1&3\\-7&-4&2\\-4&-3&1\end{vmatrix}.$$

3-nchi qator elementlarini 3 ga ko‘paytirib, 1-nchi qator elementlaridan ayiramiz; 3-nchi qator elementlarini 2 ga ko‘paytirib 2-nchi qator elementlaridan ayiramiz:

$$D_y=8\cdot\begin{vmatrix}17&10&0\\1&2&0\\-4&-3&1\end{vmatrix}=192.$$

Bundan $y=Dy/D=192/96=2$.

386. Hisoblang:

$$V=\begin{vmatrix}1&1&1&1\\a&b&c&d\\a^2&b^2&c^2&d^2\\a^3&b^3&c^3&d^3\end{vmatrix}.$$

Yechish:

1-nchi qatorni a ga ko‘paytirib 2-nchisidan, 2-nchi qatorni *a* ga ko‘paytirib 3-nchi qatordan; 3-nchi qatorni *a* ga ko‘paytirib 4-nchi qatordan ayiramiz:

$$V=\begin{vmatrix}1&1&1&1\\0&b-a&c-a&d-a\\0&b^2-ab&c^2-ac&d^2-ad\\0&b^3-ab^2&c^3-ac^2&d^3-ad^2\end{vmatrix}=(b-a)(c-a)(d-a)\begin{vmatrix}1&1&1\\b&c&d\\b^2&c^2&d^2\end{vmatrix}$$

1 nchi qatorni b ga; 2-nchi qatorni b ga ko‘paytirib 2-nchi qatordan, 3-nchi qatordan ayiramiz:

$$V=(b-a)(c-a)(d-a)\begin{vmatrix}1 & 1 & 1\\ 0 & c-b & d-b\\ 0 & c^2-bc & d^2-db\end{vmatrix}=$$

$$=(b-a)(c-a)(d-a)(c-b)(d-b)\begin{vmatrix}1 & 1\\ c & d\end{vmatrix}=$$

$$=(b-a)(c-a)(d-a)(c-b)(d-b)(d-a).$$

Agar a, b, c, d lar orasida tenglari bo‘lsa, berilgan determinant nolga teng va aksincha.

Ushbu determinantlarni hisoblang:

387. $\begin{vmatrix}1 & -2 & 3 & 4\\ 2 & 1 & -4 & 3\\ 3 & -4 & -1 & -2\\ 4 & 3 & 2 & -1\end{vmatrix}$. **388.** $\begin{vmatrix}-1 & -1 & -1 & -1\\ -1 & -2 & -4 & -8\\ -1 & -3 & -9 & -27\\ -1 & -4 & -16 & -64\end{vmatrix}$.

389. $\begin{vmatrix}10 & 2 & 0 & 0 & 0\\ 12 & 10 & 2 & 0 & 0\\ 0 & 12 & 10 & 2 & 0\\ 0 & 0 & 12 & 10 & 2\\ 0 & 0 & 0 & 12 & 10\end{vmatrix}$. **390.** $\begin{vmatrix}1+a & 1 & 1 & 1\\ 1 & 1-a & 1 & 1\\ 1 & 1 & 1+b & 1\\ 1 & 1 & 1 & 1-b\end{vmatrix}$.

Tenglamalar sistemasini yeching:

391. $\begin{cases} y-3z+4t=-5,\\ x-2z+3t=-4,\\ 3x+2y-5t=12,\\ 4x+3y-5z=5.\end{cases}$ **392.** $\begin{cases} x-3y+5z-7t=12,\\ 3x-5y+7z-t=0,\\ 5x-7y+z-3t=4,\\ 7x-y+3z-5t=16.\end{cases}$

393. $\begin{cases} x+2y=5, \\ 3y+4z=18, \\ 5z+6u=39, \\ 7u+8u=68, \\ 9v+10x=55. \end{cases}$ **394.** $\begin{cases} 2x+3y-3z+4t=7, \\ 2x+y-z+2t=5, \\ 6x+2y+z=4, \\ 2x+3y-5t=-11. \end{cases}$

2-§. CHIZIQLI ALMASHTIRISH VA MATRITSALAR

$$x = a_{11}x' + a_{12}y',$$
$$y = a_{21}x' + a_{22}y'$$

tenglik orqali x, y o'zgaruvchilarni x', y' orqali ifodalash mumkin. Bu tenglikni x', y' o'zgaruvchilarni *chiziqli almashtirish* deyiladi. Ularning nuqta koordinatilarini chiziqli almashtirish kabi qarash mumkin.

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

jadval qaralayotgan *chiziqli almashtirish* matritsasi deyiladi.

$$D_A=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

chiziqli almashtirishlarning determinanti deyiladi.

Bundan so'ng $D_A \neq 0$ deb qaraladi.

Chiziqli almashtirishni uch o'zgaruvchili deb qarash mumkin:

$$\begin{cases} x = a_{11}x' + a_{12}y' + a_{13}z', \\ y = a_{21}x' + a_{22}y' + a_{23}z', \\ z = a_{31}x' + a_{23}y' + a_{33}z', \end{cases}$$

bu yerda $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, $\quad D_A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

lar bu *chiziqli almashtirishning mos ravishda matritsasi va determinanti* deyiladi.

Agar $D_A \neq 0$ $\left(D_A = 0\right)$ bo'lsa, A matritsa *xosmas* (*xos*) deb ataladi.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad va \quad \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

lar mos ravishda *2-nchi va 3-nchi tartibli kvadrat matritsa* deyiladi. Ko'p ta'riflarni umumlashtirish uchun ularni 3-nchi tartibli matritsa uchun beriladi. Ularni 2-nchi tartibli matritsa uchun qo'llash kiyinchilik tug'dirmaydi. Agar kvadrat matritsaning elementlari $a_{mn} = a_{nm}$ shartni qanoatlantirsa, matritsa simmetrik deyiladi.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \qquad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

matritsalar teng bo'lishi uchun $a_{mn} = b_{mn}$ shartning bajarilishi zarur va yetarlidir. *A, B* matritsalar yig'indisi quyidagicha aniqlanadi:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{pmatrix}.$$

A matritsani *m* soniga ko'paytirish uchun uning har bir elementini *m* ga ko'paytiramiz:

$$m \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} ma_{11} & ma_{12} & ma_{13} \\ ma_{21} & ma_{22} & ma_{23} \\ ma_{31} & ma_{32} & ma_{33} \end{pmatrix}.$$

A, B matritsalar ko'paytmasi quyidagicha aniqlanadi:

$$AB=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\cdot\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}=$$

$$=\begin{pmatrix} \sum_{j=1}^{3} a_{1j}b_{j1} & \sum_{j=1}^{3} a_{1j}b_{j2} & \sum_{j=1}^{3} a_{1j}b_{j3} \\ \sum_{j=1}^{3} a_{2j}b_{j1} & \sum_{j=1}^{3} a_{2j}b_{j2} & \sum_{j=1}^{3} a_{2j}b_{j3} \\ \sum_{j=1}^{3} a_{3j}b_{j1} & \sum_{j=1}^{3} a_{3j}b_{j2} & \sum_{j=1}^{3} a_{3j}b_{j3} \end{pmatrix}.$$

Ko'paytma matritsaning i-nchi qator va k-nchi ustunda turuvchi elementi, A matritsa i-nchi qatoridagi elementlarini B matritsa k-nchi ustunining mos elementlariga ko'paytmalari yig'indisiga teng.

Ikki matritsaning ko'paytmasi umuman o'rin almashtirish xossasiga bo'ysinmaydi. Ikki matritsa ko'paytmasiining determinanti bu matritsalar determinantlari ko'paytmalariga teng.

Hamma elementlari nollardan iborat bo'lgan matritsa *nol-matritsa* deyiladi.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}=0.$$

Bu matritsa uchun: $A+0=A$.

$$E=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ ni birlik matritsa deyiladi.

Bu matritsaning A ga chapdan va o'ngdan ko'paytmasi A ga teng: $AE=EA=A$. Birlik matritsaga ayniy chiziqli almashtirish to'g'ri keladi: $x=x'$, $y=y'$, $z=z'$. Agar $AB=BA=E$ ga teng bo'lsa, B matritsa A ga *teskari matritsa* deyiladi. A ga teskari matritsani A^{-1} bilan belgilanadi: $B=A^{-1}$. Har qanday xos emas matritsa teskari matritsaga ega. Teskari matritsa quyidagicha topiladi:

$$A^{-1}=\begin{pmatrix} \frac{A_{11}}{D_A} & \frac{A_{21}}{D_A} & \frac{A_{31}}{D_A} \\ \frac{A_{12}}{D_A} & \frac{A_{22}}{D_A} & \frac{A_{32}}{D_A} \\ \frac{A_{13}}{D_A} & \frac{A_{23}}{D_A} & \frac{A_{33}}{D_A} \end{pmatrix}.$$

A_{mn} ga A matritsa determinantidagi a_{mn} *elementning algebraik to'ldiruvchisi* deyiladi, ya'ni A_{mn} – A matritsa determinantidagi m-nchi qator va n-nchi ustunini o'chirishdan hosil bo'lgan ikkinchi tartibli determinant (minor) bilan $(-1)^{m+n}$ ifoda ko'paytmasidir.

$$x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$ matritsa *ustun matritsa* deyiladi.

AX ko'paytma quyidagicha aniqlanadi:

$$AX=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\cdot\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} a_{11}x_1 & a_{12}x_2 & a_{13}x_3 \\ a_{21}x_1 & a_{22}x_2 & a_{23}x_3 \\ a_{31}x_1 & a_{32}x_2 & a_{33}x_3 \end{pmatrix}.$$

$$\begin{cases} a_{11}x_1+a_{12}x_2+a_{13}x_3=b_1, \\ a_{21}x_1+a_{22}x_2+a_{23}x_3=b_2, \\ a_{31}x_1+a_{32}x_2+a_{33}x_3=b_3 \end{cases}$$

sistemani $AX=B$ ko'rinishda yozish mumkin, bu yerda

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Bu sistemaning yechimi $X = A^{-1} \cdot B \quad (D_A \neq 0)$ bo'ladi:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

matritsaning xarakteristik tenglamasi

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0.$$

Bu tenglamaning ildizlari λ_1, λ_2, λ_3 lar *matritsaning xarakteristik sonlari* deyiladi. Agar boshlang'ich matritsa simmetrik bo'lsa, λ_1 λ_2 λ_3 lar haqiqiy bo'ladi.

$$\begin{cases} (a_{11}-\lambda)\xi_1 + a_{12}\xi_2 + a_{13}\xi_3 = 0, \\ a_{21}\xi_1 + (a_{22}-\lambda)\xi_2 + a_{23}\xi_3 = 0, \\ a_{31}\xi_1 + a_{32}\xi_2 + (a_{33}-\lambda)\xi_3 = 0 \end{cases}$$

tenlamalar sistemasi, undagi λ xarakteristik son λ_1, λ_2, λ_3 lardan birini qabul qiladi va shuning uchun determinanti nolga teng bo'ladi. Shu xarakteristik songa mos uchta (ξ_1, ξ_2, ξ_3) sonni aniklaydi. Bu uchta sonlar to'plami (ξ_1, ξ_2, ξ_3) o'zgarmas ko'paytuvchi aniqligida noldan farqli $\bar{r} = \xi_1\bar{i} + \xi_2\bar{j} + \xi_3\bar{k}$ vektorni aniqlaydi, uni matritsaning xos vektori deb ataladi.

395. $x = x' + y' + z'$, $y = x' + y'$, $z' = x'$ chiziqli almashtirish va x', y', z' koordinat sistemasida (1; −1; 1), (3; −2; −1), (−1; −2; −3) nuqtalar berilgan. *x, y, z* sistemada bu nuqtalarning koordinatalarini aniqlang.

Yechish:

Nuqta koordinatalarini berilgan chiziqli almashtirish aniqlanadigan tenglikka qo'yamiz. Agar $x' = 1$, $y' = -1$, $z' = 1$ bo'lsa, u holda $x=1$, $y=0$, $z=1$ ya'ni (1; 0; 1); agar $x' = 3$, $y' = -2$, $z' = -1$ u holda $x=0$, $y=0$, $z=3$, ya'ni (0; 1; 3); agar

$x'=-1,\ y'=-2,\ z'=-3$, u holda $x=-6$, $y=-3$, $z=-1$, ya'ni $(-6;\ -3;\ -1)$.

396. Oldingi masaladagi x, y, z koordinatalardan x', y', z' koordinatalarga o'tishning chiziqli almashtirishini yozing.

Yechish:

Uchinchi tenglikdan $x'=z$ ni olamiz; ikkinchi tenglikdan uchinchi tenglikni ayirib $y'=y-z$ ni hosil qilamiz; birinchi tenglikdan ikkinchi tenglikni ayirib, $z'=x-y$ ni hosil qilamiz.

397. $x=x'+2y'$, $y=3x'+4y'$ chiziqli almashtirish berilgan. Qaysi nuqtalarning koordinatalarini bu almashtirish o'zgartirmaydi?

Yechish:

Agar $x=x'$, $y=y'$ bo'lsa, x va y larni topish kerak, ya'ni $x=x+2y$, $y=3x+4y$. Demak, $x=x'=0$, $y=y'=0$.

398. Quyidagi $x=3x'-2y'$, $y=5x'-4y'$ chiziqli almashtirish qaysi nuqtalarning koordinatalarini o'zgartirmaydi.

Yechish:

Oldingi misoldagi singari $x=3x-2y$, $y=5x-4y$ larga ega bo'lamiz. Bundan $x=y=x'=y'$, ya'ni chiziqli almashtirish bir xil koordinatali (t; t) nuqtalarning koordinatalarini o'zgartirmaydi.

399. Berilgan $A=\begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$.

$A+B$ ni toping.

Yechish:

$$A+B=\begin{pmatrix} 3+1 & 5+2 & 7+4 \\ 2+2 & -1+3 & 0-2 \\ 4-1 & 3+0 & 2+1 \end{pmatrix}=\begin{pmatrix} 4 & 7 & 11 \\ 4 & 2 & -2 \\ 3 & 3 & 3 \end{pmatrix}.$$

400. Agar $A=\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ bo'lsa $2A+5B$ matritsani toping.

Yechish:

$$2A=\begin{pmatrix}6 & 10\\ 8 & 2\end{pmatrix},\quad 5B=\begin{pmatrix}10 & 15\\ 5 & -10\end{pmatrix},\quad 2A+5B=\begin{pmatrix}16 & 25\\ 13 & -8\end{pmatrix}.$$

401. $A=\begin{pmatrix}1 & 3 & 1\\ 2 & 0 & 4\\ 1 & 2 & 3\end{pmatrix},\quad B=\begin{pmatrix}2 & 1 & 0\\ 1 & -1 & 2\\ 3 & 2 & 1\end{pmatrix}.$

bo'lsa, AB va BA matritsalar ko'paytmasini toping.

Yechish:

$$A\cdot B=\begin{pmatrix}1\cdot2+3\cdot1+1\cdot3 & 1\cdot1+3\cdot(-1)+1\cdot2 & 1\cdot0+3\cdot2+1\cdot1\\ 2\cdot2+0\cdot1+4\cdot3 & 2\cdot1+0\cdot(-1)+4\cdot2 & 2\cdot0+0\cdot2+4\cdot1\\ 1\cdot2+2\cdot1+3\cdot3 & 1\cdot1+2\cdot(-1)+3\cdot2 & 1\cdot0+2\cdot2+3\cdot1\end{pmatrix}=$$

$$=\begin{pmatrix}8 & 0 & 7\\ 16' & 10 & 4\\ 13 & 5 & 7\end{pmatrix},$$

$$B\cdot A=\begin{pmatrix}2\cdot1+1\cdot2+0\cdot1 & 2\cdot3+1\cdot0+0\cdot2 & 2\cdot1+1\cdot4+0\cdot3\\ 1\cdot1-1\cdot2+2\cdot1 & 1\cdot3-1\cdot0+2\cdot2 & 1\cdot1-1\cdot4+2\cdot3\\ 3\cdot1+2\cdot2+1\cdot1 & 3\cdot3+2\cdot0+1\cdot2 & 3\cdot1+2\cdot4+1\cdot3\end{pmatrix}=$$

$$=\begin{pmatrix}4 & 6 & 6\\ 1 & 7 & 3\\ 8 & 11 & 14\end{pmatrix}.$$

402. Agar $A=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$ bo'lsa, A^3 ni toping.

Yechish:

$$A^2=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}\cdot\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}9+2 & 6+8\\ 3+4 & 2+16\end{pmatrix}=\begin{pmatrix}11 & 14\\ 7 & 18\end{pmatrix},$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 11 & 14 \\ 7 & 18 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 33+14 & 22+56 \\ 21+18 & 14+72 \end{pmatrix} = \begin{pmatrix} 47 & 48 \\ 39 & 86 \end{pmatrix}.$$

403. Agar E – uchinchi tartibli birlik matritsa bo‘lib,

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$$

bo‘lsa, $2A+3A+5E$ matritsani toping.

Yechish:

$$A^2 = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 6 & 5 \\ 8 & 11 & 6 \\ 9 & 8 & 10 \end{pmatrix},$$

$$2A^2 = \begin{pmatrix} 20 & 12 & 10 \\ 15 & 22 & 12 \\ 18 & 16 & 20 \end{pmatrix}, \qquad 3A = \begin{pmatrix} 3 & 3 & 6 \\ 3 & 9 & 3 \\ 12 & 3 & 3 \end{pmatrix},$$

$$5 \cdot \mathrm{E} = 5 \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix},$$

$$2 \cdot A^2 + 3 \cdot A + 5 \cdot E = \begin{pmatrix} 28 & 15 & 16 \\ 19 & 36 & 15 \\ 30 & 19 & 28 \end{pmatrix}.$$

404. $$x = a_{11}x' + a_{12}y', \qquad y = a_{21}x' + a_{22}y'$$

va

$$x' = в_{11}x'' + в_{12}y'', \qquad y' = в_{21}x'' + в_{22}y''$$

ikkita chiziqli almashtirish berilgan. Ikkinchi chiziqli almashtirishlardan birinchisiga x' va y' larni qo‘yib, x va y larni x'' va y'' lar orqali ifodalovchi chiziqli almashtirishni hosil qilamiz. Hosil bo‘lgan almashtirish matritsasi birinchi va ikkinchi chiziqli almashtirish matritsalari ko‘paytmasiga tengligini ko‘rsating.

Yechish:

$$x = a_{11}(b_{11}x'' + b_{12}y'') + a_{12}(x''b_{21} + b_{22}y'') =$$
$$= (a_{11}b_{11} + a_{12}b_{21})x'' + (a_{11}b_{12} + a_{12}b_{22})y''.$$
$$y = a_{21}(b_{11}x'' + b_{12}y'') + a_{22}(x''b_{21} + b_{22}y'') =$$
$$= (a_{21}b_{11} + a_{22}b_{21})x'' + (a_{21}b_{12} + a_{22}b_{22})y''.$$

Hosil bo'lgan chiziqli almashtirish matritsasi

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{211}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

ko'rinishda bo'ladi, ya'ni u $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ va $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ matritsalar ko'paytmasiga teng.

405. $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{pmatrix}$ matritsa berilgan, uning teskarisini toping.

Yechish:

A matritsaning determinantini hisoblaymiz:

$$D_A = \begin{vmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 27 + 2 - 24 = 5\,.$$

Bu determinant elementlarining algebraik to'ldiruvchilarini topamiz:

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 3 & 4 \end{vmatrix} = 9; \quad A_{21} = \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -2; \quad A_{31} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4;$$

$$A_{12} = \begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix} = 1; \quad A_{22} = \begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 2; \quad A_{32} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -1;$$

$$A_{13} = \begin{vmatrix} 1 & 3 \\ 5 & 3 \end{vmatrix} = -12; \quad A_{23} = -\begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = 1; \quad A_{33} = \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} = 7.$$

Demak $A^{-1}=\begin{pmatrix} \frac{9}{5} & -\frac{2}{5}, & -\frac{4}{5} \\ \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} \\ -\frac{12}{5} & \frac{1}{5} & \frac{7}{5} \end{pmatrix}.$

406. Ushbu

$$\begin{cases} 2x+3y+2z=9, \\ x+2y-3z=14, \\ 3x+4y+z=16 \end{cases}$$

tenglamalar sistemasini matritsa ko'rinishidagi tenglamaga keltirib yeching.

Yechish:

Sistemani $AX=B$ ko'rinishda yozib olamiz, bu yerda:

$$A=\begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{pmatrix}, \qquad X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad B=\begin{pmatrix} 9 \\ 14 \\ 16 \end{pmatrix}.$$

Matritsa ko'rinishidagi tenglamaning yechimi $X=A^{-1}B$ bo'ladi.

A ni topamiz, buning uchun A ning aniqlovchisini hisoblaymiz:

$$D_A=\begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & -3 \\ 3 & 4 & 1 \end{vmatrix}=28+30-4=-6.$$

Bu aniqlovchining algebraik to'ldiruvchilarini topamiz:

$$A_{11}=\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix}=14; \qquad A_{21}=-\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}=5; \quad A_{31}=\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}=-13;$$

$$A_{12}=-\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}=-10; \quad A_{22}=\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix}=-4; \quad A_{32}=-\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix}=8;$$

$$A_{13}=\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}=-2; \qquad A_{23}=-\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}=1; \quad A_{33}=\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}=1.$$

Shunday qilib:
$$A^{-1}=-\frac{1}{6}\cdot\begin{pmatrix}14 & 5, & -13\\ -10 & -4 & 8\\ -2, & 1 & 1\end{pmatrix}.$$

Bundan:

$$X=-\frac{1}{6}\begin{pmatrix}14 & 5, & -13\\ -10 & -4 & 8\\ -2, & 1 & 1\end{pmatrix}\begin{pmatrix}9\\ 14\\ 16\end{pmatrix}=-\frac{1}{6}\begin{pmatrix}126+ & 70 & -208\\ -90 & -56 & +128\\ 18 & +14 & +16\end{pmatrix}=$$
$$=-\frac{1}{6}\begin{pmatrix}-12\\ -18\\ 12\end{pmatrix}=\begin{pmatrix}2\\ 3\\ 2\end{pmatrix}.$$

407. $\begin{pmatrix}5 & 2\\ 4 & 3\end{pmatrix}$ matritsa berilgan. Uning xarakteristik sonlari va xos vektorlari topilsin.

Yechish:

Xarakteristik tenglamani tuzamiz:

$$\begin{vmatrix}5-\lambda & 2\\ 4 & 3-\lambda\end{vmatrix}=0 \quad \text{yoki} \quad (5-\lambda)(3-\lambda)-8=0 \quad \text{ya'ni} \quad \lambda^2-8\lambda+7=0\cdot$$

Demak, xarakteristik sonlar $\lambda_1=1$ va $\lambda_2=7$ larga teng.

Birinchi xarakteristik songa mos xos vektorni
$$(5-\lambda_1)\cdot\xi_1'+2\cdot\xi_2'=0,$$
$$4\cdot\xi_1'+(3-\lambda_1)\cdot\xi_2'=0$$
tenglamalar sistemasidan topamiz. $\lambda_1=1$ bo'lgani uchun ξ_1' va ξ_2' larni $2\xi_1'+\xi_2'=0$ bog'lanishdan topamiz. $\xi_1'=\alpha$ ($\alpha\neq 0$ – ixtiyoriy son) deb olib, $\xi_2'=-2\alpha$ ni hosil qilamiz. $l_1=1$ ga mos vektor $\vec{r}_1=\alpha\cdot\vec{i}-2\alpha\cdot\vec{j}$ bo'ladi.

Ikkinchi xos vektorni topamiz:

$$\begin{cases}(5-\lambda_2)\xi_1'' + 2\xi_2'' = 0,\\ 4\xi_1'' + (3-\lambda_2)\xi_2'' = 0\end{cases}$$

sistemada $\lambda_2 = 7$ ni qo‘yib $\xi_1'' - \xi_2'' = 0$ tenglikka kelamiz, ya’ni $\xi'' = \xi_2'' = \beta \neq 0$. Ikkinchi xos xarakteristik songa mos xos vektor $\vec{r}_2 = \alpha \cdot \vec{i} - 2\alpha \cdot \vec{j}$ bo‘ladi.

408.

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

matritsaning xarakteristik sonlari va xos vektorlarni toping.

Yechish:

Xarakteristik tenglamani tuzamiz:

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

yoki

$$(3-\lambda)[(5-\lambda)(3-\lambda)-1] + (-3+\lambda+1) + (1-5+\lambda) = 0.$$

Elementar almashtirishlardan so‘ng

$$(3-\lambda)\cdot(\lambda^2 - 8\lambda + 12) = 0$$

ga ega bo‘lamiz, bundan l_1=2, l_2=3, l_3=6.

λ_1=2 xarakteristik songa mos xos vektorni topamiz:

$$\begin{cases} \xi_1' - \xi_2' + \xi_3' = 0, \\ -\xi_1' + 3\xi_2' - \xi_3' = 0, \\ \xi_1' - \xi_2' + \xi_3' = 0 \end{cases}$$

tenglamalar sistemasidan $\xi_2' = 0$, $\xi_3' = -\xi_1'$ larni topamiz. (tenglamalarning biri qolgan ikkisining natijasi bo‘lgani sababli, uni tashlab yuborish mumkin). $\xi_1' = \alpha$ desak, $\xi_2' = 0$, $\xi_3' = -\alpha$ va $\bar{r}_1 = \alpha \cdot i - \alpha \cdot j$. Endi λ_2=6 qiymatga mos xos vektorni topamiz.

$$\begin{cases} -\xi''_2+\xi''_3=0, \\ -\xi''_1+2\xi''_2-\xi''_3=0, \\ \xi''_1-\xi''_2=0 \end{cases}$$

tenglamalar sistemasini hosil qilamiz (bitta tenglama qolgan ikkisining natijasi). Bundan $\xi''_1=\xi''_2=\xi''_3=\beta$ va $\vec{r}_2=\beta\cdot\vec{i}+\beta\cdot\vec{j}+\beta\cdot\vec{k}$.

$\lambda_3=6$ qiymatga mos xos vektorni topamiz:

$$\begin{cases} \xi_1'''-\xi_2'''+\xi_3'''=0, \\ -\xi_1'''-\xi_2'''-\xi_3'''=0, \\ -\xi_1'''-\xi\ '''-3\xi_3'''=0 \end{cases}$$

(bitta tenglama qolgan ikkitasining natijasi). Bu sistemani yechib, quyidagilarni topamiz: $\xi_1'''=\gamma$, $\xi_2'''=-\gamma$, $\xi_3'''=\gamma$ va

$$\vec{r}_3=\gamma\cdot\vec{i}-2\gamma\cdot\vec{j}+\gamma\cdot\vec{k}.$$

Shunday qilib, berilgan matritsaning xos vektorlari

$$\vec{r}_1=\alpha\cdot(\vec{i}-\vec{k}),\ \vec{r}_2=\beta\cdot(i+j+k),\ \vec{r}_3=\gamma\cdot(i-2j-k).$$

Bu yerda *a, b, g* – lar ixtiyoriy noldan farqli sonlar.

409. Ikkita

$$x=a_{11}x'+a_{12}y'+a_{13}z',\quad y=a_{21}x'+a_{22}y'+a_{23}z',\quad z=a_{31}x'+a_{32}y'+a_{33}z',$$
$$x'=b_{11}x''+b_{12}y''+b_{13}z'',\quad y'=b_{21}x''+a_{22}y''+a_{23}z'',\quad z=a_{31}x''+a_{32}y''+a_{33}z''.$$

chiziqli almashtirish berilgan. Ikkinchi almashtirishdan x', y', z' larni birinchi chiziqli almashtirishga qo'yib x, y, z larni x'', y'', z'' lar orqali ifodalaymiz. Hosil bo'lgan almashtirish matritsasi I va II almashtirish matritsalarining ko'paytmasiga tengligini ko'rsating.

410. $x=6x'+y'-2z'$, $y=-18x'+2y'+6z'$, $z=2x'+2y'$ chiziqli almashtirish berilgan. Bu chiziqli almashtirish natijasida qaysi nuqtalarning koordinatalari ikkilanadi?

411. Ikkita chiziqli almashtirish berilgan;
$x=x'+y'+2z'$, $y=x'+2y'+6z'$, $z=2x'+3y'$, $x=2x'+2z'$, $y=x'+3y'+4z'$, $z=x'+3y'+2z'$.

Bu almashtirishlardan har biri bir xil bo'lgan bitta natija beradigan nuqtalarni toping.

412. $x = x'\cos\alpha - y'\sin\alpha,\quad y' = x'\sin\alpha + y'\cos\alpha$ chiziqli almashtirishni qo'llashda koordinatalari o'zgarmaydigan nuqtalarni toping.

413. $x = x'\cos\alpha - y'\sin\alpha,\quad y' = x'\sin\alpha + y'\cos\alpha$ chiziqli almashtirishni qo'llashda koordinatalarining joylari o'zgaradigan nuqtalar to'plamini toping.

414. $A = \begin{pmatrix} 5 & 8 & 4 \\ 3 & 2 & 5 \\ 7 & 6 & 0 \end{pmatrix}$ matritsa berilgan. Birlik matritsani hosil qilish uchun A ga qanday B matritsani qo'shish kerak.

415. $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ matritsa berilgan. A^2+A+E matritsalar yig'indisini toping.

416. $A = \begin{pmatrix} 10 & 20 & -30 \\ 0 & 10 & 20 \\ 0 & 0 & 10 \end{pmatrix}$ matritsa berilgan. Teskari matritsani toping.

417. $\begin{cases} 3x + 4y = 11, \\ 5y + 6z = 28, \\ x + 2z = 7 \end{cases}$ tenglamalar sistemasi berilgan, uni matritsa ko'rinishdagi tenglamasini yozib yeching.

418. $\begin{pmatrix} 7 & 4 \\ 5 & 6 \end{pmatrix}$ matritsaning xarakteristik sonlari va normallashtirilgan xos vektorini toping.

419. $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ matritsaning xarakteristik sonlari va xos vektorlarini toping.

3-§. IKKINCHI TARTIBLI EGRI CHIZIQ VA SIRTNING UMUMIY TENGLAMASINI KANONIK KO'RINISHGA KELTIRISH

$$a_{11}x^2+2a_{12}xy+a_{22}y^2,$$

$$a_{11}x^2+a_{22}y^2+a_{33}z^2+2a_{12}xy+2a_{13}xz+2a_{23}yz$$

ko'rinishdagi ifodalar mos ravishda *ikki va uch o'zgaruvchili kvadratik forma* deyiladi.

Ushbu $a_{21}=a_{12}$ shartni bajaruvchi

$$A_2^{(2)}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

va $a_{21}=a_{12}$, $a_{31}=a_{31}$, $a_{32}=a_{23}$ shartlarni bajaruvchi

$$A_3^{(3)}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

simmetrik matritsalar *bu formalarning matritsalari* deyiladi. Kvadratik formalarni, o'zgaruvchilarni chiziqli almashtirish yordamida, yangi o'zgaruvchilarning ko'paytmalarini o'z ichiga olmagan ko'rinishga keltirish mumkin, boshkacha aytganda ikki o'zgaruvchili kvadratik forma $\lambda_1 x'^2+\lambda_2 y'^2$, uch o'zgaruvchilisi esa $\lambda_1 x'^2+\lambda_2 y'^2+\lambda_3 z'^2$ ko'rinishga keltiriladi. x_i^2- lar oldidagi koeffitsientlar xarakteristik sonlardan iborat bo'lishi uchun chiziqli almashtirishni quyidagicha bajarish kerak: λ_1, λ_2, λ_3 xarakteristik sonlarga mos o'zaro ortogonal, normallashtirilgan xos vektorlar uchligi (ikki o'zgaruvchili kvadratik formalar uchun juftlik) aniqlanadi:

$$\vec{e}_1=\alpha_1\cdot\vec{i}+\beta_1\cdot\vec{j}+\gamma_1\cdot\vec{k},$$

$$\vec{e}_2=\alpha_2\cdot\vec{i}+\beta_2\cdot\vec{j}+\gamma_2\cdot\vec{k},$$

$$\vec{e}_3=\alpha_3\cdot\vec{i}+\beta_3\cdot\vec{j}+\gamma_3\cdot\vec{k}.$$

e_1, e_2, e_3 vektorlar ortogonal va normallashtirilgan bo'lgani sababli quyidagi ayniyat bajarilishi lozim:

$$\alpha_i^2+\beta_i^2+\gamma_i^2=1 \quad (i=1,\ 2,\ 3);$$
$$\alpha_i\alpha_j+\beta_i\beta_j+\gamma_i\gamma_j=0 \quad (i,j=1,2,3,\quad i\neq j).$$

U holda o'zgaruvchilarni almashtirish matritsasi quyidagi ko'rinishda bo'ladi:

$$S=\begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{pmatrix}.$$

Boshqacha qilib aytganda

$$x=\alpha_1\cdot x'+\alpha_2\cdot y'+\alpha_3\cdot z',$$
$$y=\beta_1\cdot x'+\beta_2\cdot y'+\beta_3\cdot z',$$
$$z=\gamma_1\cdot x'+\gamma_2\cdot y'+\gamma_3\cdot z'$$

deb olish kerak. O'zgaruvchilarni bunday almashtirish *chiziqli ortogonal almashtirish* deb yuritiladi. Bu holda S matritsaning determinantsi ± 1 ga teng: $D_S=\pm 1$.

Ikkinchi tartibli egri chiziq yoki sirtning umumiy tenglamalarini kanonik ko'rinishga keltirishga *ortogonal chiziqli almashtirish* deyiladi. Agar yangi koordinat o'qlarining o'zaro joylashishi saqlansa, u holda matritsaga qo'shimcha shart kiritiladi: $D_S=1$. Ikkinchi tartibli egri chiziq yoki sirt tenglamalarini kanonik ko'rinishga keltirish quyidagicha bajariladi:

a) Koordinatalarni shunday chiziqli ortogonal almashtirish kerakki, egri chiziq yoki sirt tenglamalarining yuqori darajali hadlari kvadratik formasi kvadratlar yig'indisiga keltiriladi, tenglamada mos almashtirish bajariladi. Bu bilan koordinat ko'paytmasi qatnashgan had yo'qoladi.

b) Yangi koordinat o'qlarini parallel ko'chirib, tenglama kanonik ko'rinishga keltiriladi.

420. $5x^2+4xy+8y^2-32x-56y+80=0$ egri chiziq tenglamasini kanonik ko'rinishga keltiring.

Yechish:

Yuqori darajali xadlar matritsasi:

$$A=\begin{pmatrix}5 & 2\\ 2 & 8\end{pmatrix}.$$

Matitsaning xarakteristik tenglamasini tuzamiz:

$\begin{vmatrix}5-\lambda & 2\\ 2 & 8-\lambda\end{vmatrix}=0$, ya'ni $\lambda^2-13\lambda+36=0$. Xarakteristik sonlarni topamiz: $\lambda_1=4$, $\lambda_2=9$.

$\lambda_1=4$ uchun xos vektorni topamiz:

$$\begin{cases}\xi_1+2\xi_2=0\\ 2\xi_1+4\xi_2=0\end{cases}$$

Bundan $\xi_1=-2\xi_2$, $\xi_2=-\alpha$ deb $\xi_1=2\alpha$ va $\vec{r}_1=\alpha\cdot(2\vec{i}-\vec{j})$ ni topamiz. $\vec{r}_1$ vektorni normallashtirib, $\vec{e}=\frac{2}{\sqrt{5}}\cdot\vec{i}-\frac{1}{\sqrt{5}}\cdot\vec{j}$ ni hosil qilamiz. $\lambda_2=9$ uchun

$$\begin{cases}4\eta_1+2\eta_2=0,\\ 2\eta_1-\eta_2=0\end{cases}$$

sistemadan ikkinchi xos vektorni topamiz. Bundan $\eta_2=2\eta_1$ va $\vec{r}_2=\beta(i+2\vec{j})$ topamiz. Normallashtirib $e_2=\frac{1}{\sqrt{5}}\vec{i}+\frac{2}{\sqrt{5}}\vec{j}$ ni hosil qilamiz. Ko'rsatish osonki, $e_1\cdot e_2=0$, ya'ni e_1 va e_2 vektorlar o'zaro ortogonaldir. Koordinatalarni almashtirish matritsasini tuzish uchun normallashtirilgan ortogonal xos vektorlardan foydalanamiz:

$$S=\begin{pmatrix}\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\end{pmatrix}, \qquad D_S=1.$$

Bundan

$$x=\frac{2}{\sqrt{5}}x'+\frac{1}{\sqrt{5}}y',$$

$$y=-\frac{1}{\sqrt{5}}y''+\frac{2}{\sqrt{5}}y'.$$

x, y lar uchun topilgan ifodalarni egri chiziq tenglamasiga qo‘yamiz:

$$5\cdot\left(\frac{2}{\sqrt{5}}x'+\frac{1}{\sqrt{5}}y'\right)^2+4\cdot\left(\frac{2}{\sqrt{5}}x'+\frac{1}{\sqrt{5}}y'\right)\cdot\left(-\frac{1}{\sqrt{5}}x'+\frac{2}{\sqrt{5}}y'\right)+$$

$$+8\cdot\left(-\frac{1}{\sqrt{5}}x'+\frac{2}{\sqrt{5}}y'\right)^2-32\cdot\left(\frac{2}{\sqrt{5}}x'+\frac{1}{\sqrt{5}}y'\right)-56\cdot\left(-\frac{1}{\sqrt{5}}x'+\frac{2}{\sqrt{5}}y'\right)+80=0.$$

Qavslarni ochib, soddalashtirib

$$4x'^2+9y'^2-\frac{8}{\sqrt{5}}x'-\frac{144}{\sqrt{5}}y'+80=0$$

ga ega bo‘lamiz. Bu tenglamada x'^2, y'^2 lar oldidagi koeffitsientlar λ_1, λ_2 xarakteristik sonlar ekanligini ko‘rib turibmiz. Tenglamani quyidagi $4\cdot(x'^2-\frac{2}{\sqrt{5}}x')+9\cdot(y'^2-\frac{16}{\sqrt{5}}y')+80=0$ ko‘rinishga keltiramiz. Qavs ichidagi ifodalarni to‘la kvadratga to‘ldiramiz:

$$4\cdot\left(x'^2-\frac{2}{\sqrt{5}}x'+\frac{1}{5}-\frac{1}{5}\right)+9\cdot\left(y'^2-\frac{16}{\sqrt{5}}y'+\frac{64}{5}-\frac{64}{5}\right)+80=0$$

yoki

$$4\cdot\left(x'-\frac{1}{\sqrt{5}}\right)^2-\frac{4}{9}+9\cdot\left(y'-\frac{8}{\sqrt{5}}\right)^2-\frac{576}{5}+80=0,$$

$$4\cdot\left(x'-\frac{1}{\sqrt{5}}\right)^2+9\cdot\left(y'-\frac{8}{5}\right)=36 \quad x''=x'-\frac{1}{\sqrt{5}}, \quad y''=y'-\frac{8}{\sqrt{5}}.$$

deb, koordinat o‘qlarini parallel ko‘chiramiz va $4x''^2+9y''^2=36$

yoki $\frac{x''^2}{9}+\frac{y''^2}{4}=1$ ni hosil qilamiz (ellipsning kanonik tenglamasi).

421. Ushbu

$$9x^2+24xy+16y^2+230x-110y-225=0$$

egri chiziq tenglamasini kanonik holga keltiring.

Yechish:

Xarakteristik tenglamani yezamiz:

$$\begin{vmatrix} 9-\lambda & 12 \\ 12 & 16-\lambda \end{vmatrix}=0$$

yoki $\lambda^2-25\lambda=0$, ya'ni $\lambda_1=0,\quad \lambda_2=25$. $\lambda=0$ da

$$\begin{cases} 9\xi_1+12\xi_2=0, \\ 12\xi_1+16\xi_2=0 \end{cases}$$

sistemani hosil qilamiz.

Bu tenglamalarning har biri $\frac{\xi_1}{4}=\frac{\xi_2}{-3}$ ga keladi. Demak, $\vec{r}_1=\alpha\cdot(4\vec{i}-3\vec{j})$, matritsaning xos vektori bo'lib xizmat qiladi.

$\alpha=\frac{1}{\sqrt{4^2+(-3)^2}}=\frac{1}{5}$ da normallashtirgan xos vektor $\vec{e}_1=\frac{4}{5}\cdot\vec{i}-\frac{3}{5}\cdot\vec{j}$ ni topamiz:

$\lambda=25$ da $\begin{cases} -16\eta_1+12\eta_2=0, \\ 12\eta_1-9\eta_2=0 \end{cases}$ sistemani hosil qilamiz. Bundan ikkinchi xos vektor $\vec{e}_2=\frac{1}{5}\cdot\vec{i}+\frac{4}{5}\cdot\vec{j}$, $(\vec{e}_1\cdot\vec{e}_2=0)$ ni topamiz.

Koordinatalarni almashtirish matritsasi

$$S=\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix},\quad (D_S=1).$$

Almashtirish formulalari: $x=x'(4/5)+y'(3/5)$, $y=(-3/5)x'+(4/5)y'$.

Egri chiziq tenglamasini ushbu $(3x+4y)^2-230x+110y-225=0$ ko'rinishda yangi koordinatalarga o'tkazib yozamiz:

$$25y'^2-230\left(\frac{4}{5}x'+\frac{3}{5}y'\right)+110\left(-\frac{3}{5}x'+\frac{4}{2}y'\right)-225=0.$$

Bu ifodani soddalashtirib, 25 ga qisqartirib topamiz:

$$y'^2-10x'-2y'-9=0 \text{ yoki } (y'-1)^2=10(x'+1).$$

Yangi koordinat boshi uchun O′(−1;1) nuqtani olib, o‘qlarni parallel ko‘chiramiz, natijada $y''^2=10x''$ ga kelamiz (parabola).

422. $3x^2+5y^2+3z^2-2xy-2xz-2yz-12x-10=0$ sirt tenglamasini kanonik ko‘rinishga keltiring.

Yechish:

Yuqori darajali hadlar koeffitsientlaridan tuzilgan matritsa:

$$\begin{vmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{vmatrix}=0.$$

Matritsaning xarakteristik sonlari

$$\begin{vmatrix} 3-\lambda & 1- & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix}=0$$

tenglamadan topiladi, uni $(3-\lambda)(\lambda^2-8\lambda+12)=0$ ko‘rinishga keltirish mumkin, bundan $\lambda_1=2$, $\lambda_2=3$, $\lambda_3=6$.

$\lambda_1=2$ da

$$\begin{cases} u_1-u_2+u_3=0, \\ -u_1+3u_2-u_3=0, \\ u_1-u_2+u_3=0 \end{cases}$$

sistemaga ega bo‘lamiz. λ ning bu qiymatiga $(\alpha;\ 0;\ -\alpha)$ xos vektor mos keladi.

Normallashtirib $\vec{e}_1=\frac{1}{\sqrt{2}}\vec{i}-\frac{1}{\sqrt{2}}\vec{k}$ vektorga kelamiz.

$\lambda_2=3$ da

$$\begin{cases} -v_2+v_3=0, \\ -v_1+2v_2-v_3=0, \\ v_1-v_2=0 \end{cases}$$

sistemaga kelamiz. Bundan ikkinchi normallashtirilgan vektor

$\vec{e}_2 = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$ ni topamiz. $\vec{e}_1$, $\vec{e}_2$ vektorlar ortogonal, ya'ni $\vec{e}_1 \cdot \vec{e}_2 = 0$.

$\lambda_3 = 6$ da

$$\begin{cases} -3w_1 - w_2 + w_3 = 0, \\ -w_1 - w_2 - w_3 = 0, \\ w_1 - w_2 - 3w_3 = 0 \end{cases}$$

ga ega bo'lamiz. Uchinchi mos xos vektor $\vec{e}_3 = \frac{1}{\sqrt{6}}\vec{i} - \frac{2}{\sqrt{6}}\vec{j} + \frac{1}{\sqrt{6}}\vec{k}$ oldingi $\vec{e}_1$ va $\vec{e}_2$ vektorlarga ortogonal, ya'ni $\vec{e}_1 \cdot \vec{e}_3 = 0$, $\vec{e}_3 \cdot \vec{e}_2 = 0$.

Koordinatalar almashtirish matritsasini topamiz:

$$S = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix},$$

Bundan koordinat almashtirish formulalarini topamiz:

$$x = \frac{1}{\sqrt{2}} \cdot x' + \frac{1}{\sqrt{3}} \cdot y' + \frac{1}{\sqrt{6}} \cdot z',$$

$$y = \frac{1}{\sqrt{3}} \cdot y' - \frac{2}{\sqrt{6}} \cdot z',$$

$$z = -\frac{1}{\sqrt{2}} \cdot x' + \frac{1}{\sqrt{3}} \cdot y' + \frac{1}{\sqrt{6}} \cdot z'.$$

x, y, z lar uchun topilgan ifodalarni sirt tenglamasiga qo'yib, soddalashtirib quyidagilarni topamiz:

$$2x'^2 + 3y'^2 + 6z'^2 - 6\sqrt{2}x' - 4\sqrt{3}y' - 2\sqrt{6}z' - 10 = 0.$$

x'^2, y'^2, z'^2 lar oldidagi koeffiesientlar λ_1, λ_2, λ_3 sonlar bo'lishi kerak.

Tenglamani ushbu

$$2\cdot\left(x'^2-\frac{6}{\sqrt{2}}x'\right)+3\cdot\left(y'^2-\frac{4}{\sqrt{3}}y'\right)+6\cdot\left(z'^2-\frac{2}{\sqrt{6}}z'\right)=10$$

ko'rinishda yozamiz va qavs ichidagi ifodani to'la kvadratga to'ldirib,

$2\cdot\left(x'-\frac{3}{\sqrt{2}}\right)^2+3\cdot\left(y'-\frac{2}{\sqrt{3}}y'\right)^2+6\cdot\left(z-\frac{1}{\sqrt{6}}\right)^2=24$ ga ega bo'lamiz.

$$x'=x''+3/\sqrt{2},\qquad y'=y''+2/\sqrt{3},\qquad z'=z''+1/\sqrt{6}$$

formulalar buyicha koordinat o'qlarini parallel ko'chirib va 24 ga bo'lib, ellipsoidning $\frac{x''^2}{12}+\frac{y''^2}{8}+\frac{z''^2}{4}=1$ kanonik tenglamasiga kelamiz.

Egri chiziq tenglamasini kanonik ko'rinishga keltiring:

423. $5x^2+6xy+5y^2-16x-16y-16=0$.

424. $7x^2+16xy-23y^2-14x-16y-218=0$.

425. $x^2+2xy+y^2-8x+4=0$.

Sirt tenlamalarini kanonik ko'rinishga keltiring:

426. $x^2+5y^2+z^2+2xy+6xz+2yz-6=0$.

Koordinat almashtirish formulalari:

$$x=\frac{1}{\sqrt{3}}x'+\frac{1}{\sqrt{6}}y'+\frac{1}{\sqrt{2}}z',$$

$$y=-\frac{1}{\sqrt{3}}x'+\frac{2}{\sqrt{6}}y',$$

$$z=\frac{1}{\sqrt{3}}x'+\frac{1}{\sqrt{6}}y'-\frac{1}{\sqrt{2}}z'.$$

427. $2x^2+y^2+2z^2-2xy-2yz+x-4y-3z+2=0$.

Koordinat almashtirish formulalari:

$$x=-(1/\sqrt{6})x'-(1/\sqrt{2})y'+(1/\sqrt{3})z',\qquad x'=x'',$$

$$y=(-2/\sqrt{6})x'-(1/\sqrt{3})z',\qquad y'=y''+1/\sqrt{2},$$

$$z=-(1/\sqrt{6})x'+(1/\sqrt{2})y'+(1/\sqrt{3})z',\qquad z'=z''+1/\sqrt{3}.$$

4-§. MATRITSANING RANGI. EKVIVALENT MATRITSALAR

To'g'ri burchakli matritsa berilgan bo'lsin:

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$

Bu matritsadan ixtiyoriy k-ta qator k-ta ustun ajratamiz $(k \le m,\ k \le n)$.

A matritsaning ajratilgan qator va ustunlarining kesishgan joyida turgan elementlaridan tuzilgan k-nchi tartibli determinant A ning k-nchi *tartibli minori* deyiladi. A matritsa $C_m^k \cdot C_n^k$ ta k-nchi tartibli minorlarga ega. A matritsaning noldan farqli hamma minorlarini qaraymiz. *A matritsaning rangi* deb uning noldan farqli minorlarining eng yuqori tartibiga aytamiz. Agar matritsaning hamma elementlari nollardan iborat bo'lsa, uning rangi nolga teng. Tartibi matritsaning rangiga teng bo'lgan noldan farqli har qanday minor matritsaning *bazis minori* deyiladi. Matritsaning rangini r(A) bilan belgilaymiz. Agar $r(A)=r(B)$ ga teng bo'lsa, A va B lar *ekvivalent matritsalar* deyiladi va $A \sim B$ kabi yoziladi. Elementar almashtirishlardan matritsaning rangi o'zgarmaydi.

Elementar almashtirishlarga quyidagilar kiradi:

1) matritsaning qatorlarini ustunlar bilan almashtirish;
2) matritsaning qatorlarini o'zaro almashtirish;
3) hamma elementlari nollardan iborat yo'llarni o'chirish;
4) birorta qatorini noldan farqli songa ko'paytirish;
5) biror qator elementlariga boshqa qatorning mos elementlarini qo'shish.

428. Ushbu

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{pmatrix}$$ matritsaning rangini aniqlang.

Yechish: Matitsaning ikkinchi va uchinchi tartibli hamma minorlari nolga teng, chunki bu minorlarning qator elementlari proporsional. Birinchi tartibli minor noldan farqli. Demak, matritsaning rangi birga teng.

429. Ushbu

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 11 \end{pmatrix} \text{ matritsaning rangini toping.}$$

Yechish:

Bu matritsada ikkinchi qatorni, so'ngra ikkinchi, uchinchi, turtinchi ustunlarni o'chirib, $\begin{pmatrix} 1 & 5 \\ 2 & 11 \end{pmatrix}$ matritsani hosil qilamiz, u berilgan matritsaga ekvivalent. $\begin{vmatrix} 1 & 5 \\ 2 & 11 \end{vmatrix} = 1 \neq 0$ bo'lgani uchun berilgan matritsaning rangi 2 ga teng.

430.

$$A = \begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix} \text{ matritsaning rangini hisoblang.}$$

Yechish: Birinchi va uchinchi qator elementlarini qo'shib, so'ngra birinchi yo'l elementlarini 4 ga bo'lamiz:

$$A = \begin{pmatrix} 4 & 8 & 12 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$$

birinchi yo'l elementlaridan ikkinchi yo'l elementlarini ayirib, so'ngra birinchi yo'l elementlarini o'chiramiz:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

Oxirgi matritsaning rangi 2 ga teng, chunki

$$\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \neq 0.$$

Demak, berilgan matritsaning rangi 2 ga teng.

431.

$$A=\begin{pmatrix}4&3&2&2\\0&2&1&1\\0&0&3&3\end{pmatrix}$$ matritsaning rangini aniqlang.

Yechish: 4-nchi ustun elementlaridan 3-nchi ustun elementlarini ayirib, 4-nchi ustunni o'chiramiz:

$$A=\begin{pmatrix}4&3&2&2\\0&2&1&1\\0&0&3&3\end{pmatrix}\sim\begin{pmatrix}4&3&2&0\\0&2&1&0\\0&0&3&0\end{pmatrix}\sim\begin{pmatrix}4&3&2\\0&2&1\\0&0&3\end{pmatrix}.$$

$$\begin{vmatrix}4&3&2\\0&2&1\\0&0&3\end{vmatrix}=24\neq 0$$

bo'lgani uchun matritsaning rangi 3 ga teng.

432.

$$A=\begin{pmatrix}1&0&2&0&0\\0&1&0&2&0\\2&0&4&0&0\end{pmatrix}$$

matritsaning rangi va bazis minorlarin toping.

Yechish:

$$\begin{pmatrix}1&0&2&0&0\\0&1&0&2&0\\2&0&4&0&0\end{pmatrix}\sim\begin{pmatrix}1&0&2&0\\0&1&0&2\\2&0&4&0\end{pmatrix}\sim\begin{pmatrix}1&0&2&0\\0&1&0&2\\1&0&2&0\end{pmatrix}\sim$$

$$\sim\begin{pmatrix}1&0&2&0\\0&1&0&2\\0&0&0&0\end{pmatrix}\sim\begin{pmatrix}1&0&2&0\\0&1&0&2\end{pmatrix};$$ $r(A)=2$. Bu matritsani

noldan farqli 2-nchi tartibli minorlari bazis minorlari bo'ladi:

$$\begin{vmatrix}1&0\\0&1\end{vmatrix},\quad \begin{vmatrix}1&0\\0&2\end{vmatrix},\quad \begin{vmatrix}0&2\\1&0\end{vmatrix},\quad \begin{vmatrix}2&0\\0&2\end{vmatrix},$$

$$\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}, \quad \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}, \quad \begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix}, \quad \begin{vmatrix} 0 & 2 \\ 4 & 0 \end{vmatrix}.$$

Shunday qilib, A matritsa 8 ta bazis minorlarga ega.

433.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

matritsa nechta ikkinchi tartibli minorlarga ega. Bu minorlarning hammasini yozing.

Yechish:

Matitsa $C_3^2 \cdot C_3^2 = 3 \cdot 3 = 9$ ta ikkinchi tartibli minorlarga ega:

$$\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \quad \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}, \quad \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix},$$

$$\begin{vmatrix} a_{12} & a_{13} \\ a_{23} & a_{33} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix},$$

$$\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

434. $A = \begin{pmatrix} \lambda & 5\lambda & -\lambda \\ 2\lambda & \lambda & 10\lambda \\ -\lambda & -2\lambda & -3\lambda \end{pmatrix}$ matritsaning rangini aniqlang.

435. $A = \begin{pmatrix} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 6 \\ 3 & 1 & 2 & 6 \end{pmatrix}$ matritsaning rangini aniqlang.

436. $A = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{pmatrix}$ matritsaning rangini va bazis minorlarini toping.

437. $A=\begin{pmatrix} 1 & 2 & 1 & 3 & 4 \\ 3 & 4 & 2 & 6 & 8 \\ 1 & 2 & 1 & 3 & 4 \end{pmatrix}$ matritsaning rangi va bazis minorlarini aniqlang.

5-§. *n* NOMA'LUMLI *m* TA CHIZIQLI TENGLAMALAR SISTEMASINI TEKSHIRISH

n noma'lumli m ta chiziqli tenglamalar sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m. \end{cases} \tag{1}$$

Tenglamalar sistemasining yechimi deb shunday n ta $(x_1, x_2, \dots, x_n)$ sonlar to'plamini aytamizki, ularni sistemadagi noma'lumlar o'rniga qo'yganimizda tenglamalar ayniyatga aylanadi. Agar sistema kamida bitta $(x_1, x_2, \dots, x_n)$ yechimga ega bo'lsa, uni *birgalikda*, aks holda *birgalikda emas* deyiladi. Agar sistema faqat bitta yechimga ega bo'lmasa *aniqlangan*, aks holda *aniqlanmagan* deyiladi.

$$A=\begin{pmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{m1} & a_{m2} \dots & a_{mn} \end{pmatrix}, \quad A_1=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n}b_1 \\ a_{21} & a_{22} & \dots & a_{2n}b_2 \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn}b_m \end{pmatrix},$$

matritsalar (1) *sistemaning matritsasi* va *kengaytirilgan matritsasi* deyiladi. (1) sistemaning birgalikda bo'lishi uchun A va A_1 matritsalar rangi teng bo'lishi zarur va yetarlidir (Kroneker-Kapelli teoremasi).

Shunday qilib, (1) sistema birgalikda bo'lishi uchun $r(A)=r(B)=r$ bo'lishi zarur va yetarlidir, Bu r soni (1) *sistemaning rangi* deb ataladi.

Agar $b_1 = b_2 = ... = b_m = 0$ bo‘lsa, *sistema bir jinsli* deyiladi. Bir jinsli tenglamalar sistemasi har doim birgalikda. Agar birgalikda bo‘lgan sistemaning rangi noma’lumlar soniga teng bo‘lsa, sistema aniqlangan bo‘ladi.

Agar sistemaning rangi noma’lumlar sonidan kam bo‘lsa sistema aniqlanmagan bo‘ladi. Sistema birgalikda, lekin $r << n$ bo‘lsin. A matritsaning qandaydir bazis minorini qaraymiz. Bu minorda ixtiyoriy yo‘lni ajratamiz. Bu yo‘lning elementlari (1) sistema tenglamalarining biridagi r ta noma’lumlari oldidagi koeffitsientlardan iborat. Bu r ta noma’lumlarni qaralayotgan tenglamalar sistemasining bazis noma’lumlari deb atashadi. Qolgan $n-r$ tasini (1) sistemaning erkli noma’lumlari deyishadi.

(1) sistemadan koeffitsientlari bazis minorning elementlarini o‘z ichiga olgan r ta tenglamalar sistemasini ajratamiz. Bunda bazis noma’lumlarini chap tomonda qoldirib, erkli noma’lumlarni o‘ng tomonga o‘tkazamiz. Hosil bo‘lgan sistemadagi bazis noma’lumlarni erkli noma’lumlar orqali ifodalaymiz (masalan, Kramer formulasi bo‘yicha).

Shunday qilib, erkli noma’lumlarga ixtiyoriy qiymatlar berib, bazis noma’lumlarining ularga mos qiymatlarini topish mumkin. Demak, (1) sistema cheksiz ko‘p yechimlarga ega bo‘ladi.

438. $$\begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 1, \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 5x_5 = 2, \\ 2x_1 + 11x_2 + 12x_3 + 25x_4 + 22x_5 = 4 \end{cases}$$

tenglamalar sistemasini tekshiring.

Yechish:

Sistema matritsasi va kengaytirilgan matritsalar rangini aniqlaymiz. Kengaytirilgan matritsani yozib olamiz:

$$A_1 = \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 1 & -2 & 3 & -4 & 5 & 2 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right).$$

Vertikal chiziq bilan sistemani ozod hadlardan ajratamiz. 2-qator elementlariga 3-qator yo‘lning mos elementlarini qo‘shib, 2-qator elementlarini 3 ga bo‘lamiz:

$$A_1 = \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 3 & 9 & 15 & 21 & 27 & 6 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 1 & 3 & 5 & 7 & 9 & 2 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right).$$

2-qator elementlaridan 1-qatorning mos elementlarini ayiramiz:

$$A_1 = \left(\begin{array}{ccccc|c} 1 & 3 & 5 & 7 & 9 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 11 & 12 & 25 & 22 & 4 \end{array}\right);$$

$$A \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 11 & 12 & 25 & 22 \end{pmatrix}.$$

Osongina ko‘rish mumkinki $r(A)=2$, $r(A_1)=3$, ya’ni $r(A) \neq r(A_1)$. Demak, sistema birgalikda emas.

439.

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 14, \\ 3x_1 + 2x_2 + x_3 = 10, \\ x_1 + x_2 + x_3 = 6, \\ 2x_1 + 3x_2 - x_3 = 5, \\ x_1 + x_2 = 3 \end{cases}$$

tenglamalr sistemasini tekshiring.

Yechish:

Kengaytirilgan matritsani yozamiz:

$$A_1 = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 3 & 2 & 1 & 10 \\ 1 & 1 & 1 & 6 \\ 2 & 3 & -1 & 5 \\ 1 & 1 & 0 & 3 \end{array}\right).$$

2-qator elementlarini 1- va 4-qator elementlariga qo‘shib, 1-qator elementlarini 4 ga, 4-qator elementlarini 5 ga bo‘lamiz:

$$A_1 \sim \left(\begin{array}{ccc|c} 4 & 4 & 4 & 24 \\ 3 & 2 & 1 & 10 \\ 1 & 1 & 1 & 6 \\ 5 & 5 & 0 & 15 \\ 1 & 1 & 0 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & 1 & 10 \\ 1 & 1 & 1 & 6 \\ 1 & 1 & 0 & 3 \\ 1 & 1 & 0 & 3 \end{array}\right)$$

3-qator elementlaridan 1-qator elementlarini, 5-qator elementlaridan 4-qator elementlarini ayiramiz, so'ngra 3- va 5-qatorni o'chiramiz:

$$A_1 \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & 1 & 10 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & 1 & 10 \\ 1 & 1 & 0 & 3 \end{array}\right); \quad A \sim \begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Oxirgi matritsaning aniqlovchisini topamiz:

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1 \neq 0 .$$

Demak, $r(A)=3$. Kengaytirilgan matritsaning rangi ham 3 ga teng, chunki topilgan determinant A_1 matritsaning minoridir. Shunday qilib sistema birgalikda bo'ladi. Uni yechish uchun 1, 3, 5-tenlamalarni olamiz:

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 14, \\ x_1 + x_2 + x_3 = 6, \\ x_1 + x_2 = 3. \end{cases}$$

Bundan osongina $x_1=1$, $x_2=2$, $x_3=3$ larni topish mumkin.

440.

$$\begin{cases} x_1 + 5x_2 + 4x_3 + 3x_4 = 1, \\ 2x_1 - x_2 + 2x_3 - x_4 = 0, \\ 5x_1 + 3x_2 + 8x_3 + x_4 = 1 \end{cases}$$

tenglamalr sistemasini tekshiring.

Yechish:

$$A_1=\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 5 & 3 & 8 & 1 & 1\end{array}\right).$$

3-qatordan 1-qatorni ayiramiz:

$$A_1\sim\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 4 & -2 & 4 & -2 & 0\end{array}\right).$$

3-qatorni 2 ga bo‘lib, hosil bo‘lgan 3-qatordan 2-qatorni ayiramiz, so‘ngra 3-qatorni o‘chiramiz:

$$A_1\sim\left(\begin{array}{cccc|c}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right). \quad A_1\sim\begin{pmatrix}1 & 5 & 4 & 3 & 1\\ 2 & -1 & 2 & -1 & 0\end{pmatrix}.$$

$r(A)=r(A_1)=2$ ligini oson ko‘rsatish mumkin. Demak, sistema birgalikda. Berilgan sistemadagi 1- va 2-tenglamalrni qaraymiz:

$$\begin{cases}x_1+5x_2+4x_3+3x_4=1,\\ 2x_1-x_2+2x_3-x_4=0.\end{cases}$$

Bazis noma’lumlar sifatida x_1 va x_2 larni olamiz, chunki bu noma’lumlar oldidagi koeffitsientlardan tuzilgan $\begin{vmatrix}1 & 5\\ 2 & -1\end{vmatrix}$ determinant noldan farqli. Ozod noma’lumlar x_3 va x_4 bo‘ladi. Sistemani ushbu

$$\begin{cases}x_1+5x_2=1-4x_3-3x_4,\\ 2x_1-x_2=-2x_3+x_4\end{cases}$$

ko‘rinishda yozib, x_1 va x_2 larni x_3 va x_4 lar orqali ifodalaymiz:

$$x_1=\frac{\begin{vmatrix}1-4x_3-3x_4 & 5\\ -2x_3+x_4 & -1\end{vmatrix}}{\begin{vmatrix}1 & 5\\ 2 & -1\end{vmatrix}}=-\frac{6}{11}x_3-\frac{8}{11}x_4-\frac{1}{11},$$

$$x_2 = \frac{\begin{vmatrix} 1 & 1-4x_3-3x_4 \\ 2 & -2x_3+x_4 \end{vmatrix}}{-11} = -\frac{6}{11}x_3 + \frac{7}{11}x_4 + \frac{2}{11}.$$

$x_3 = u$, $\quad x_4 = v$ deb olib, sistemaning yechimlarini quyidagicha hosil qilamiz:

$$x_1 = -\frac{6}{11}u + \frac{8}{11}v - \frac{1}{11}, \quad x_2 = -\frac{6}{11}u + \frac{7}{11}v + \frac{2}{11}, \quad x_3 = u, \quad x_4 = v.$$

u va v larga turli sonli qiymatlarni berib, sistemaning turli yechimlarini hosil qilamiz.

Berilgan tenglamalar sistemalarini tekshiring:

441. $\begin{cases} 3x_1 + 2x_2 = 4, \\ x_1 - 4x_2 = -1, \\ 7x_1 + 10x_2 = 12, \\ 5x_1 + 6x_2 = 8, \\ 3x_1 - 16x_2 = -5. \end{cases}$ **442.** $\begin{cases} x_1 + 5x_2 + 4x_3 = 1, \\ 2x_1 + 10x_2 + 8x_3 = 3, \\ 3x_1 + 15x_2 + 12x_3 = 5. \end{cases}$

443. $\begin{cases} x_1 - 3x_2 + 2x_3 = -1, \\ x_1 + 9x_2 + 6x_3 = 3, \\ x_1 + 3x_2 + 4x_3 = 1. \end{cases}$

6-§. GAUSS METODI BILAN CHIZIQLI TENGLAMALAR SISTEMASINI YECHISH

Chiziqli algebraik tenglamalarni determinant yordamida yechish ikki va uch noma'lumli tenglamalar sistemasi uchun qulay. Tenglamalar soni sistemada ko'p bo'lganda Gauss metodi qulay. Bu metodni 4 noma'lumli 4 ta tenglamalar sistemasida tahlil qilamiz:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z + a_{14}u = a_{15} & (a) \\ a_{21}x + a_{22}y + a_{23}z + a_{24}u = a_{25} & (b) \\ a_{31}x + a_{32}y + a_{33}z + a_{34}u = a_{35} & (d) \\ a_{41}x + a_{42}y + a_{43}z + a_{44}u = a_{45} & (e) \end{cases}$$

$a_{11} \neq 0$ deb faraz qilamiz (agar a_{11}=0 bo'lsa, tenglamalarning o'rnini almashtiramiz).

1-qadam. (a) tenglamani a_{11} ga bo'lib, hosil bo'lgan tenglamani a_{21} ga ko'paytirib (b) dan ayiramiz, so'ngra a_{31} ga ko'paytirib (d) dan ayiramiz, a_{41} ko'paytirib (e) dan ayiramiz. 1-qadamdan so'ng quyidagi sistemaga kelamiz:

$$\begin{cases} x + b_{12}y + b_{13}z + b_{14}u = b_{15}, & (f) \\ \quad b_{12}y + b_{13}z + b_{14}u = b_{15}, & (g) \\ \quad b_{12}y + b_{13}z + b_{14}u = b_{15}, & (h) \\ \quad b_{12}y + b_{13}z + b_{14}u = b_{15} & (j) \end{cases}$$

a_{ij} dan quyidagi formulalar bo'yicha b_{ij} topiladi:

$$b_{ij} = \frac{a_{ij}}{a_{11}} \qquad (j=2,\ 3,\ 4,\ 5).$$

$$b_{ij} = a_{ij} - a_{i1}b_{ij} \quad (\text{i}=2,3,4;\ \text{j}=2,3,4,5).$$

2-qadam. (a), (b), (d), (e) tenglamalarda nima qilgan bo'lsak, (f), (g), (h), (j) larda ham shularni qaytaramiz va hokazo. Natijada berilgan tenglama quyidagi ko'rinishga keladi:

$$\begin{cases} x + b_{12}y + b_{13}z + b_{14}u = b_{15}, \\ \qquad y + c_{23}z + c_{24}u = c_{25}, \\ \qquad\qquad z + d_{34}u = d_{35}, \\ \qquad\qquad\qquad u = e_{45}. \end{cases}$$

Hosil bo'lgan sistemadan barcha noma'lumlar ketma-ket topiladi.

444.

$$\begin{cases} 36,47x + 5,28y + 6,34z = 12,26, & (a) \\ 7,33x + 28,74y + 5,86z = 15,15, & (b) \\ 4,63x + 6,31y + 26,17z = 25,22 & (d) \end{cases}$$

tenglamalar sistemasini yeching.

Yechish:

(a) tenglamani 36,47 ga bo'lib, x+0,1447y+0,1738z=0,3361 (*) ga ega bo'lamiz. (*) ni 7,33 ga ko'paytirib, natijani (b) dan ayiramiz va $27{,}67y+4{,}586z=12{,}6864$ ga ega bo‘lamiz. Endi (*) ni 4,63 ga ko‘paytirib, natijani (s) dan ayiramiz va $5{,}64y+25{,}36z=23{,}6639$ ga ega bo‘lamiz. Shunday qilib,

$$\begin{cases} 27{,}67y+4{,}586z=12{,}6864, & (e) \\ 5{,}64y+25{,}3653z=23{,}6639 & (f) \end{cases}$$

tenglamalar sistemasini hosil qilamiz. (d) ni 27,68 ga bo‘lib

$$y+0{,}1657z=0{,}4583 \quad (**)$$

ni hosil qilamiz. (**) ni 5,64 ga ko‘paytirib, (e) dan ayiramiz va $24{,}4308z=21{,}0791$ ni topamiz. Demak, $z=0{,}8628$. U holda

$y=0{,}4583-0{,}1657\cdot 0{,}8628=0{,}3153$,

$x=0{,}3361-0{,}1447\cdot 0{,}3153-0{,}1738(x)0{,}8628=0{,}1405$.

Shunday qilib, $x=0{,}1405$, $y=0{,}3153$, $z=0{,}8628$.

Amalda sistemaning o‘zini emas, uning noma’lumlar oldidagi koeffitsientlari va ozod hadlaridan tuzilgan matritsasini pog‘onasimon ko‘rinishga keltirish maqsadga muvofiqdir:

$$\left(\begin{array}{ccc|c} 36{,}47 & 5{,}28 & 6{,}34 & 12{,}26 \\ 7{,}33 & 28{,}74 & 5{,}86 & 15{,}15 \\ 4{,}63 & 6{,}31 & 26{,}17 & 25{,}22 \end{array}\right).$$

5-tekshiruv ustunini hosil qilamiz. Uning har bir elementi mos qator elementlarining yig‘indisidan iborat:

$$\left(\begin{array}{ccc|c|c} 36{,}47 & 5{,}28 & 6{,}34 & 12{,}26 & 60{,}35 \\ 7{,}33 & 28{,}74 & 5{,}86 & 15{,}15 & 57{,}08 \\ 4{,}63 & 6{,}31 & 26{,}17 & 25{,}22 & 62{,}33 \end{array}\right).$$

Matritsa elementlari ustida chiziqli almashtirish bajarsak, bu chiziqli almashtirish tekshiruv ustunida ham bajarilishi kerak. Almashtirilgan matritsa tekshiruv ustunining har bir elementi mos qator elementlarining yig‘indisiga teng bo‘ladi. Bir matritsadan 2-nchi matritsaga o‘tishni ekvivalentlik belgisi orqali yozamiz:

$$\left(\begin{array}{ccc|c|c} 36{,}47 & 5{,}28 & 6{,}34 & 12{,}26 & 60{,}35 \\ 7{,}33 & 28{,}74 & 5{,}86 & 15{,}15 & 57{,}08 \\ 4{,}63 & 6{,}31 & 26{,}17 & 25{,}22 & 62{,}33 \end{array}\right) \sim \left(\begin{array}{ccc|c|c} 1 & 0{,}1447 & 0{,}1738 & 0{,}3361 & 0{,}6547 \\ 7{,}33 & 28{,}74 & 5{,}86 & 15{,}15 & 57{,}08 \\ 4{,}63 & 6{,}31 & 26{,}17 & 26{,}17 & 62{,}33 \end{array}\right) \sim$$

$$\sim\left(\begin{array}{ccc|c|c}1 & 0,1447 & 0,1738 & 0,3361 & 1,6547\\ 0 & 27,6793 & 4,586 & 12,6864 & 44,9516\\ 0 & 5,64 & 25,3653 & 23,6639 & 54,6688\end{array}\right)\sim\left(\begin{array}{ccc|c|c}1 & 0,1447 & 0,1738 & 0,3361 & 1,6547\\ 0 & 1 & 0,1657 & 0,4583 & 1,6240\\ 0 & 5,64 & 25,3653 & 23,6639 & 54,6688\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccc|c|c}1 & 0,1447 & 0,1738 & 0,3361 & 1,6547\\ 0 & 1 & 0,1657 & 0,4583 & 1,6240\\ 0 & 0 & 24.4308 & 21.0791 & 45,5094\end{array}\right)\sim\left(\begin{array}{ccc|c|c}1 & 0,1447 & 0,1738 & 0,3361 & 1,6547\\ 0 & 1 & 0,1657 & 0,4583 & 1,6240\\ 0 & 0 & 1 & 0,8828 & 1,8629\end{array}\right).$$

Hosil bo‘lgan matritsadan foydalanib, mos almashtirilgan sistemani yozamiz va yechimlarini topamiz:

$z=0,8628,$

$y=0,4583-0,1657\cdot 0,8628=0,3153,$

$x=0,3361-0,1738\cdot 0,8628-0,1447\cdot 0,3153=0,1405.$

Agar sistema yagona yechimga ega bo‘lsa, pog‘onasimon tenglamalar sistemasi uchburchak ko‘rinishga keladi, bunda oxirgi tenglama bitta noma’lumga ega bo‘ladi. Aniqmas sistema holida, ya’ni bunda noma’lumlar soni chiziqli erkli tenglamalar sonidan ko‘p bo‘lsa (cheksiz ko‘p yechimlar to‘plamiga ega), uchburchakli sistema hosil bo‘lmaydi, chunki oxirgi tenglama birdan ortiq noma’lumga ega. Agar tenglamalar sistemasi birgalikda bo‘lmasa, uni pog‘anasimon shaklga keltirganda u kamida bitta $0=1$ ko‘rinishdagi tenglamaga ega bo‘ladi, ya’ni tenglamadagi hamma noma’lumlar nolli koeffitsientlarga ega, o‘ng tomon noldan farqli. Bunday sistema yechimga ega emas.

445. Ushbu

$$\begin{cases}3x+2y+z=5,\\ x+y-z=0,\\ 4x-y+5z=3.\end{cases}$$

tenglamalar sistemasini yeching.

Yechish: Matitsani ekvivalenti bilan almashtiramiz:

$$\left(\begin{array}{ccc|c|c}3 & 2 & 1 & 5 & 11\\ 1 & 1 & -1 & 0 & 1\\ 4 & -1 & 5 & 3 & 11\end{array}\right)\sim\left(\begin{array}{ccc|cc}1 & 1 & -1 & 0 & 1\\ 3 & 2 & 1 & 5 & 11\\ 4 & -1 & 5 & 3 & 11\end{array}\right)$$

(hisoblashni soddalashtirish uchun birinchi va ikkinchi tenglama o'rinlarini almashtirdik). Qolgan ikki qatordan birinchi yo'lni 3 ga, 4 ga ko'paytirib ayiramiz:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & -4 & -8 \\ 0 & 0 & -11 & -33 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 1 & -1 & 0 & 1 \\ 0 & 1 & -4 & -5 & -8 \\ 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

(oxirgi qatorni 2 ga qisqartirdik). Sistema uchburchak ko'rinishga keladi:

$$\begin{cases} x+y-z=0, \\ y-4z=-5, \\ z=2. \end{cases}$$

U yagona yechimga ega, ya'ni $z=2$, $y=3$, $x=-1$.

Tenglamalar sistemasini yeching:

446. $\begin{cases} 2x_1+x_2-x_3=5, \\ x_1-2x_2+2x_3=-3, \\ 7x_1+x_2-x_3=10. \end{cases}$

447. $\begin{cases} x_1-x_2-x_3+x_4=4, \\ 2x_1-x_2+3x_3-2x_4=1, \\ x_1-x_2+2x_4=6, \\ 3x_1-x_2+x_3-x_4=0. \end{cases}$

448. $\begin{cases} 3x_1-x_2+x_3+2x_5=18, \\ 2x_1-5x_2+x_4+x_5=-7, \\ x_1-x_4+2x_5=8, \\ 2x_2+x_3+x_4-x_5=10, \\ x_1+x_2-3x_3+x_4=1. \end{cases}$

449. $\begin{cases} 4x_1+2x_2+3x_3=-2, \\ 2x_1+8x_2-x_3=8, \\ 9x_1+x_2+8x_3=0. \end{cases}$

450. $\begin{cases} 0,04x-0,08y+4z=20, \\ 4x+0,24y-0,08z=8, \\ 0,09x+3y-0,15z=9. \end{cases}$

451. $\begin{cases} 3,21x+0,71y+0,34z=6,12, \\ 0,43x+4,11y+0,22z=5,71, \\ 0,17x+0,16y+4,73z=7,06. \end{cases}$

7-§. CHIZIQLI TENGLAMALAR SISTEMASINI JORDAN-GAUSS USULIDA YECHISH

Chiziqli tenglamalar sistemasini Gauss usulida yechishda tekshiruv ustuniga ega bo'lgan matritsa usuli ko'rildiki, natijada berilgan tenglamalar sistemasi uchburchak ko'rinishiga keltirildi. Keyingi bayon uchun Jordan—Gaussning takomillashgan usuli bilan tanishish muhim ahamiyatga ega, bunda noma'lumlarning qiymatlari to'g'ridan to'g'ri topiladi.

Bizga quyidagi chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3, \\ a_{41}x_1 + a_{42}x_2 + \ldots + a_{4n}x_n = b_4. \end{cases} \qquad (1)$$

Bu sistemaning A matritsasidan 0 dan farqli a_{qp} elementini tanlaymiz.

Bu element hal qiluvchi element deb ataladi. A matritsaning p-ustuni *hal qiluvchi ustun* deb, q-qatori *hal qiluvchi qator* deb ataladi.

Yangi tenglamalar sistemasini qaraymiz:

$$\begin{cases} a'_{11}x_1 + a'_{12}x_2 + \ldots + a'_{1n}x_n = b'_1, \\ a'_{21}x_1 + a'_{22}x_2 + \ldots + a'_{2n}x_n = b'_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a'_{m1}x_1 + a'_{m2}x_2 + \ldots + a'_{mn}x_n = b'_m. \end{cases} \qquad (2)$$

Bu sistemaning matritsasi A'. Bu sistemaning koeffitsientlari va ozod hadlari quyidagi formulalardan aniqlanadi:

$$\left.\begin{aligned} a'_{ij} &= a_{ij} - \frac{a_{ip}a_{qj}}{a_{qp}} \\ b'_i &= b_i - \frac{a_{ip}b_q}{a_{qp}} \end{aligned}\right\}, \qquad \text{агар} \qquad i \neq j.$$

Xususan, agar $i \neq q$ bo'lsa, $a_{ip} = 0$ bo'ladi. Agarda $i = q$ bo'lsa, u holda $a'_{qj} = a_{qj}, b'_q = b_q$ deb qabul qilamiz. Shunday qilib (1) va (2) sistemalardagi q-nchi tenglamalar bir xil bo'lib, (2) sistemaning q-nchi tenglamasidan boshqa barcha tenglamalaridagi x_p oldidagi koeffitsientlari 0 ga teng. Shuni ko‘zda tutish lozimki, (1) va (2) sistemalar bir vaqtda yoki birgalikda, yoki birgalikda emas. Ular birgalikda bo‘lgan holda teng kuchli sistemalardir (ularning yechimlari ustma-ust tushadi).

A' matritsaning a_{ij} elementini aniqlashda “to‘rtburchak usuli” ni ko‘zda tutish foydalidir.

A matritsaning 4 elementini qaraymiz: a_{ij} (almashtirishga tanlangan element), a_{qp} (hal qiluvchi element) va a_{ip}, a_{qj} elementlar. a'_{ij} elementni topish uchun to‘rtburchakning qarama-qarshi uchlaridagi a_{ip} va a_{qj} elementlar ko‘paytmasini a_{qp} elementga bo‘lib a_{ij} elementdan ayiramiz:

$$\begin{matrix} a_{ij} & \cdots\cdots\cdots\cdots & a_{ip} \\ \vdots & & \vdots \\ a_{qj} & \cdots\cdots\cdots\cdots & a_{qp} \end{matrix}$$

Xuddi shu tariqa (2) sistemani ham almashtirish mumkin, bunda A' matritsaning hal qiluvchi elementi sifatida $a'_{sr} \neq 0$ elementini qabul qilamiz ($s \neq q$, $r \neq p$). Bu almashtirishdan so‘ng x_p lar oldidagi barcha koeffitsientlar 0 ga teng bo‘ladi. Hosil bo‘lgan sistema yana almashtirilishi mumkin va hokazo. Agar $r=n$ (sistemaning rangi noma’lumlar soniga teng) bo‘lsa, u holda bir qator almashtirishlardan so‘ng quyidagi tenglamalar sistemasiga kelamiz:

$$\begin{gathered} k_1x_1=l_1, \\ k_2x_2=l_2, \\ \cdots\cdots \\ k_nx_n=l_n, \end{gathered}$$

va bu tengliklardan noma’lumlarning qiymatlarini topamiz. Noma’lumlarni ketma-ket yo‘qotishga asoslangan bu yechish usuli *Jordan-Gauss usuli* deb ataladi.

452.
Quyidagi chiziqli tenglamalar sistemasining matritsasi berilgan:

$$A=\begin{pmatrix} 5 & 4 & 6 & -1 & 7 \\ 8 & 1 & 3 & 2 & 0 \\ 0 & 1 & 5 & 3 & -1 \\ 7 & -6 & 5 & -4 & 3 \end{pmatrix}.$$

Bu sistemani Jordan-Gauss usulida yechishda hal qiluvchi element sifatida $a_{23}=3$ ni qabul qilamiz. Almashtirilgan matritsaning a'_{24}, a'_{13}, a'_{44} elementlarini toping.

Yechish:

a_{24} element hal qiluvchi qatordan bo'lgani uchun $a'_{24}=a_{24}=2$. a_{13} element hal qiluvchi ustundan bo'lgani uchun $a'_{13}=0$. a'_{44} elementni to'rtburchak qoidasi bilan topamiz:

$$A=\begin{pmatrix} 5 & 4 & 6 & -1 & 7 \\ 8 & 1 & 3 & 2 & 0 \\ 0 & 1 & 5 & 3 & -1 \\ 7 & -6 & 5 & -4 & 3 \end{pmatrix},$$

$$a'_{44}=a_{44}-\frac{a_{24}a_{13}}{a_{23}}=-4-\frac{2\cdot 5}{3}=-7\frac{1}{3}.$$

453. Quyidagi tenglamalar sistemasini yeching:

$$\begin{cases} x_1+x_2-3x_3+2x_4=6, \\ x_1-2x_2-x_4=-6, \\ x_2+x_3+3x_4=16, \\ 2x_1-3x_2+2x_3=6. \end{cases}$$

Yechish:

Bu sistemaning koeffitsientlarini, ozod hadlarini va koeffitsientlar bilan ozod hadlari yig'indilarini quyidagi jadvalga yozamiz (Σ–tekshiruv ustuni):

x_1	x_2	x_3	x_4	b	Σ
1	1	−3	2	6	7
1	−2	0	−1	−6	−8
0	1	1	3	16	21
2	−3	2	0	6	7

Hal qiluvchi element sifatida biz birinchi tenglamadagi x_1 oldidagi koeffitsientni olamiz. Jadvalning bu element turgan qatorini (hal qiluvchi qator) o'zgarishsiz keyingi jadvalga yozamiz, 1-ustunning hal qiluvchi elementidan boshqa barcha elementlarini 0 bilan almashtiramiz. To'rtburchak qoidasini qo'llab, jadvalning qolgan kataklarini to'ldiramiz (bu qoidani Σ ustunga ham qo'llaymiz):

x_1	x_2	x_3	x_4	b	Σ
1	1	−3	2	6	7
0	−3	3	−3	−12	−15
0	1	1	3	16	21
0	−5	8	−4	−6	−7

Σ−tekshiruv ustunida mos qator elementlarining yig'indisi hosil bo'ladi.

2-qator elemenlarini −3 ga bo'lib, quyidagi jadvalni hosil qilamiz:

x_1	x_2	x_3	x_4	b	Σ
1	1	−3	2	6	7
0	1	−1	1	4	5
0	1	1	3	16	21
0	−5	8	−4	−6	−7

Hal qiluvchi element sifatida 2-qatorning 2-elementini olamiz. 1-nchi ustunni o'zgarishsiz yozamiz, 2-ustun elementlarining hal qiluvchisidan tashqari barchasini 0 bilan almashtiramiz, 2-(hal qiluvchi) qatorni o'zgarishsiz yozamiz, qolgan katakda turgan elementlarni to'rtburchak qoidasiga ko'ra almashtiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	−2	1	2	2
0	1	−1	1	4	5
0	0	2	2	12	16
0	0	3	1	14	18

3-qator elementlarini 2 ga bo'lamiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	-2	1	2	2
0	1	-1	1	4	5
0	0	1	1	6	8
0	0	3	1	14	18

3-ustunning 3-elementini hal qiluvchi sifatida olib jadvalni almashtiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	0	3	14	18
0	1	0	2	10	13
0	0	1	1	6	8
0	0	0	−2	−4	-6

4-qator elementlarini −2 ga bo‘lamiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	0	3	14	18
0	1	0	2	10	13
0	0	1	1	6	8
0	0	0	1	2	3

4-qatorning 4-elementini hal qiluvchi sifatida olib, jadvalni o‘zgartiramiz:

x_1	x_2	x_3	x_4	b	Σ
1	0	0	0	8	9
0	1	0	0	6	7
0	0	1	0	4	5
0	0	0	1	2	3

Natijada quyidagi tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} 1\cdot x_1+0\cdot x_2+0\cdot x_3+0\cdot x_4=8, \\ 0\cdot x_1+1\cdot x_2+0\cdot x+0\cdot x_4=6, \\ 0\cdot x_1+0\cdot x_2+1\cdot x_3+0\cdot x_4=4, \\ 0\cdot x_1+0\cdot x_2+0\cdot x_3+1\cdot x_4=2. \end{cases}$$

ya’ni $x_1=8$, $x_2=6$, $x_3=4$, $x_4=2$.

454. Tenglamalar sistemasini yeching:

$$\begin{cases} x_1+x_2-2x_3+x_4=1, \\ x_1-3x_2+x_3+x_4=0, \\ 4x_1-x_2-x_3-x_4=1, \\ 4x_1+3x_2-4x_3-x_4=2. \end{cases}$$

Yechish:

Jadval tuzamiz:

1	1	−2	1	1	2
1	−3	1	1	0	0
4	−1	−1	−1	1	2
4	3	−1	−1	2	4

1-ustunning 1-elementi – hal qiluvchi:

1	1	−2	1	1	2
0	−4	3	0	−1	−2
0	−5	7	−5	−3	−6
0	−1	4	−5	−2	−4

4-qatordagi ishoralarni o‘zgartiramiz:

1	1	−2	1	1	2
0	−4	3	0	−1	−2
0	−5	7	−5	−3	−6
0	1	−4	5	2	4

2-ustunning 4-elementi – hal qiluvchi:

1	0	2	−4	−1	−2
0	0	−13	20	7	14
0	0	−13	20	7	14
0	1	−4	5	2	4

3-qatordan 2-qatorni ayiramiz va 3-qatorni o‘chiramiz:

1	0	2	−4	−1	−2
0	0	−13	20	7	14
0	1	−4	5	2	4

2-qatorning 4-elementi – hal qiluvchi:

1	0	−0,6	0	0,4	0,8
0	0	−13	20	7	14
0	1	−0,75	0	0,25	0,5

Matritsaning rangi 3 ga teng. Demak, sistema uchta bazis noma’lumlar – x_1, x_2 va x_4 lar va bitta ozod noma’lum – x_3 ga ega.

Natijada quyidagi tenglamalar sistemasiga ega bo‘lamiz:

$$\begin{cases} 1\cdot x_1+0\cdot x_2-0,6\cdot x_3+0\cdot x_4=0,4, \\ 0\cdot x_1+0\cdot x_2-13\cdot x+20\cdot x_4=7, \\ 0\cdot x_1+1\cdot x_2-0,75\cdot x_3+0\cdot x_4=0,25. \end{cases}$$

Bundan: x_1=0,4+0,6x_3, x_2=0,25+0,75x_3, x_4=0,35+0,65x_3.

Shunday qilib, sistema quyidagi yechimga ega: x_1=0,4+0,6u, x_2=0,25+0,75u, x_4=0,35+0,65u, bu yerda u – ixtiyoriy son.

455. Tenglamalar sistemasini yeching:

$$\begin{cases} 6x-5y+7z+8t=3, \\ 3x+11y+2z+4t=6, \\ 3x+2y+3z+4t=1, \\ x+y+z=0. \end{cases}$$

Yechish:

Jadval tuzamiz:

6	−5	7	8	3	19
3	11	2	4	6	26
3	2	3	4	1	13
1	1	1	0	0	3

1-ustunning 4-elementi – hal qiluvchi:

0	−11	1	8	3	1
0	8	−1	4	6	17
0	−1	0	4	1	4
1	1	1	0	0	3

3-ustunning 1-elementi – hal qiluvchi:

0	−11	1	8	3	1
0	−3	0	12	9	18
0	−1	0	4	1	4
1	12	0	−8	−3	2

3-qator elementlarining ishoralarini qarama-qarshisiga o‘zgartiramiz:

0	−11	1	8	3	1
0	−3	0	12	9	18
0	1	0	−4	−1	−4
1	12	0	−8	−3	2

2-ustunning 3-elementi – hal qiluvchi:

0	0	1	−36	−8	−43
0	0	0	0	6	6
0	$\boxed{1}$	0	−4	−1	−4
1	0	0	40	9	50

Natijada quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} 0\cdot x+0\cdot y+1\cdot z-36\cdot t=-8, \\ 0\cdot x+0\cdot y+0\cdot z+0\cdot t=6, \\ 0\cdot x+1\cdot y+0\cdot z-4\cdot t=-1, \\ 1\cdot x+0\cdot y+0\cdot z+40\cdot t=9. \end{cases}$$

Osongina ko'rish mumkinki, ikkinchi tenglamani *x, y, z* va t larning xech bir qiymatlari qanoatlantirmaydilar. Shunday qilib, hosil bo'lgan sistema va berilgan sitema birgalikda emas.

456. Berilgan matritsaning rangini aniqlashga Jordan-Gauss usulini qo'llang:

$$A=\begin{pmatrix} 7 & -1 & 3 & 5 \\ 1 & 3 & 5 & 7 \\ 4 & 1 & 4 & 6 \\ 3 & -2 & -1 & -1 \end{pmatrix}.$$

Yechish:

Jadval tuzamiz:

7	−1	3	5	14
$\boxed{1}$	3	5	7	16
4	1	4	6	15
3	−2	−1	−1	−1

Oxirgi (nazorat) ustunda mos qator elementlarining yig'indisi yozilgan, 1-ustunning 2-elementi — hal qiluvchi:

0	−22	−32	−44	−98
1	3	5	7	16
0	−11	−16	−22	−49
0	−11	−16	−22	−49

3- va 4-qatorning mos elementlaidan 1-qator mos elementlarini −2 ga bo'lib ayiramiz va 3- va 4-qatorni o'chiramiz:

0	11	16	22	49
1	3	5	7	16

Hosil bo'lgan matritsaning ixtiyoriy ikkinchi tartibli determinanti 0 dan farqli. Demak, A matritsaning rangi 2 ga teng.

Tenglamalar sistemalarini Jordan-Gauss usulida yeching:

457. $\begin{cases} x_1+2x_2+x_3=8, \\ x_2+3x_3+x_4=15, \\ 4x_1+x_3+x_4=11, \\ x_1+x_2+5x_4=23. \end{cases}$ **458.** $\begin{cases} x_2-x_1+x_3-x_4=-2, \\ x_1+2x_2-2x_3-x_4=-5, \\ 2x_1-x_2-3x_3+2x_4=-1, \\ x_1+2x_2+3x_3-6x_4=-10, \end{cases}$

459. $\begin{cases} x_1+5x_2-2x_3-3x_4=1, \\ 7x_1+2x_2-3x_3-4x_4=2, \\ x_1+x_2+x_3+x_4=5, \\ 2x_1+3x_2+2x_3-3x_4=4, \\ x_1-x_2-x_3-x_4=-2. \end{cases}$

460. Matritsaning rangini Jordan-Gauss usulida toping:

$$A=\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 1 & 3 \\ 3 & 1 & 2 & 5 \\ 2 & 2 & 2 & 3 \end{pmatrix}.$$

V BOB
CHIZIQLI ALGEBRA ASOSLARI

1-§. CHIZIQLI FAZO

1. Asosiy tushunchalar.

Elementlari $\overline{x}, \overline{y}, \overline{z}...$ bo'lgan shunday R to'plamni qaraymizki, uning ixtiyoriy ikki $\overline{x} \in R$ va $\overline{y} \in R$ elementlari uchun $\overline{x} + \overline{y} \in R$ yig'indi, ixtiyoriy $\overline{x} \in R$ elementi va ixtiyoriy haqiqiy λ son uchun $\lambda\overline{x} \in R$ ko'paytma aniqlangan bo'lsin.

Agar R to'plamning elementlarini qo'shish va bu to'plamning elementlarini haqiqiy songa ko'paytirishda quyidagi shartlarni qanoatlantirsa:

1) $\overline{x} + \overline{y} = \overline{y} + \overline{x}$;

2) $(\overline{x} + \overline{y}) + \overline{z} = \overline{x} + (\overline{y} + \overline{z})$;

3) ixtiyoriy $x \in R$ element uchun shunday $\overline{0} \in R$ (nol element) mavjudki, $\overline{x} + \overline{0} = \overline{x}$;

4) har bir $\overline{x} \in R$ element uchun shunday $\overline{y} \in R$ element mavjud bo'lsaki, bunda $\overline{x} + \overline{y} = \overline{0}$ (kelgusida $\overline{y} = -\overline{x}$ ko'rinishda yozamiz, ya'ni $\overline{x} + (-\overline{x}) = \overline{0}$);

5) $1 \cdot \overline{x} = \overline{x}$;

6) $\lambda(m\overline{x}) = (\lambda m)\overline{x}$;

7) $(\lambda + m)\overline{x} = \lambda\overline{x} + m\overline{x}$;

8) $\lambda(\overline{x} + \overline{y}) = \lambda\overline{x} + \lambda\overline{y}$,

u holda R to'plam chiziqli (yoki vektor) fazo, bu fazoning elementlari $\overline{x}, \overline{y}, \overline{z}...$ esa *vektorlar* deyiladi.

Masalan, barcha geometrik vektorlar to'plami chiziqli fazo bo'ladi, chunki bu to'plamning elementlari uchun keltirilgan shartlarni qanoatlantiruvchi amallar aniqlangan.

Chiziqli fazodagi ikki $\overline{x}$ va $\overline{y}$ *vektorlarning ayirmasi* deb, bu fazoning shunday $\overline{v}$ vektoriga aytiladiki, $\overline{y} + \overline{v} = \overline{x}$ bo'ladi. $\overline{x}$ va $\overline{y}$ larning ayirmasi $\overline{x} - \overline{y}$ bilan belgilanadi, ya'ni $\overline{x} - \overline{y} = \overline{v}$. Isbotlash osonki, $\overline{x} - \overline{y} = \overline{x} + (-\overline{y})$.

Quyidagi teoremalar ham o'rinli:

1. Har bir chiziqli fazoda faqat bitta nol-element mavjud.

2. Chiziqli fazoning har bir elementi uchun faqat bitta teskari element mavjud.

3. Har bir $\overline{y} \in R$ uchun $0\overline{x} = \overline{0}$ tenglik bajariladi.

4. Har bir haqiqiy son λ va $\overline{0} \in R$ uchun $\lambda \bullet \overline{0} = \overline{0}$ tenglik bajariladi.

5. $\lambda\overline{x} = \overline{0}$ tenglikdan quyidagi ikki tenglikning biri kelib chiqadi; $\lambda = 0$ yoki $\overline{x} = \overline{0}$.

6. $(-1)\overline{x}$ element $\overline{x}$ element uchun qarama-qarshi element bo'ladi.

461. $(\xi_1; \xi_2; ...; \xi_n)$, $(\eta_1, \eta_2, ..., \eta_n)$, ... haqiqiy sonlarning barcha sistemalar to'plami berilgan bo'lsin. Har qanday ikki elementining yig'indisi $(\xi_1; \xi_2; ...; \xi_n), + (\eta_1, \eta_2, ..., \eta_n) = = (\xi_1 + \eta_1;\ \xi_2 + \eta_2;\ ...;\ \xi_n + \eta_n)$ tenglik bilan; har qanday elementning ixtiyoriy songa ko'paytmasi $(\xi_1;\ \xi_2; ...;\ \xi_n) = = (\lambda\xi_1;\ \lambda\xi_2;\ ...;\ \lambda\xi_n)$ tenglik bilan aniqlanadi. Bu to'plamning chiziqli fazo ekanligi isbotlansin.

Yechish:

$\overline{x} = (\xi_1;\ \xi_2;\ ...;\ \xi_n)$, $\overline{y} = (\eta_1;\ \eta_2;\ ...;\ \eta_n)$, $\overline{z} = (\zeta_1;\ \zeta_2;\ ...;\ \zeta_n); ...$ belgilashlarni kiritamiz. Yuqorida keltirilgan 1–8 shartlarning bajarilishini tekshiramiz.

1. $\overline{x} + \overline{y} = (\xi_1 + \eta_1;\ \xi_2 + \eta_2;\ ...;\ \xi_n + \eta_n)$;

$\overline{y} + \overline{x} = (\eta_1 + \xi_1;\ \eta_2 + \xi_2;\ ....;\ \eta_n + \xi_n)$, ya'ni $\overline{x} + \overline{y} = \overline{y} + \overline{x}$.

2. $\overline{x} + \overline{y} = (\xi_1 + \eta_1;\ \xi_2 + \eta_2;\ ...;\ \xi_n + \eta_n)$, $\overline{y} + \overline{z} = (\eta_1 + \xi_1;\ \eta_2 + \xi_2;\ ...;\ \eta_n + \xi_n)$,

$(\overline{x} + \overline{y}) + \overline{z} = (\xi_1 + \eta_1 + \zeta_1;\ \xi_2 + \eta_2 + \zeta_2;\ ...;\ \xi_n + \eta_n + \zeta_n)$,

$\overline{x} + (\overline{y} + \overline{z}) = (\xi_1 + \eta_1 + \zeta_1;\ \xi_2 + \eta_2 + \zeta_2;\ ...;\ \xi_n + \eta_n + \zeta_n)$.

Shunday qilib, $(\overline{x} + \overline{y}) + \overline{z} = \overline{x} + (\overline{y} + \overline{z})$.

3. $\overline{0} = (0;\ 0;\ ...;\ 0)$ – nol element bo'ladi.

Haqiqatan ham $\overline{x} + \overline{0} = (\xi_1 + 0;\ \xi_2 + 0; ...,\ \xi_n + 0) = \overline{x}$.

4. $(-\xi_1; -\xi_2; ...; -\xi_n)$ element $(\xi_1;\ \xi_2;\ ...;\ \xi_n)$ elementga qarama-qarshi element bo'ladi, chunki

$(\xi_1;\ \xi_2;\ ...;\ \xi_n)+(-\xi_1;-\xi_2;\ ...;\ -\xi_n)=(0;\ 0;...;\ 0)=\overline{0}$

5. $1\cdot\overline{x}=(1\xi_1;\ 1\xi_2;\ ...;\ 1\xi_n)=\overline{x}$

6. $\lambda(\mu\overline{x})=\lambda(\mu\xi_1;\ \mu\xi_2;\ ...;\ \mu\xi_n)=(\lambda\mu\xi_1;\ \lambda\mu\xi_2;\ ...;\ \lambda\mu\xi_n)=(\lambda\mu\overline{x})$

7. $(\lambda+\mu)\overline{x}=((\lambda+\mu)\xi_1;\ (\lambda+\mu)\xi_2;\ ...;(\lambda+\mu)\xi_n)=(\lambda\xi_1+\mu\xi_1;\ \lambda\xi_2+\mu\xi_2;$
$...;\ \lambda\xi_n+\mu\xi_n)=(\lambda\xi_1;\ \lambda\beta\xi_2;\ ...;\ \lambda\xi_n)+(\mu\xi_1;\ \mu\xi_2;...;\ \mu\xi_n)=$
$=\lambda(\xi_1;\ \xi_2;\ ...;\ \xi_n)+\mu(\xi_1;\ \xi_2;\ ...;\ \xi_n)=\lambda\overline{x}+\mu\overline{x}$

8. $\lambda(\overline{x}+\overline{y})=\lambda(\xi_1+\eta_1;\ \xi_2+\eta_2;\ ...;\ \xi_n+\eta_n)=(\lambda\xi_1+\lambda\eta_1;\ \lambda\xi_2+\lambda\eta_2;\ ...;\ \lambda\xi_n+\lambda\eta_n)=$
$(\lambda\xi_1;\lambda\xi_2;\ ...;\ \lambda\xi_n)+(\lambda\eta_1;\ \lambda\eta_2;\ ...;\ \lambda\eta_n)=\lambda(\xi_1;\ \xi_2;\ ...;\xi_n)+\lambda(\eta_1;\eta_2;...;\eta_n)=\lambda\overline{x}+\lambda\overline{y}$

462. Barcha kompleks sonlar to'plami chiziqli fazo bo'lishligini isbotlang.

463. $\xi_1,\ \xi_2,\ \eta_1,\ \eta_2,\ \zeta_1,\ \zeta_2$ – lar har hil haqiqiy sonlar bo'lganda, to'rtta haqiqiy sonlar $(\xi_1;\ \xi_2;\ 0;\ 0)$; $(\eta_1;\ \eta_2;\ 0;\ 0)$; $(\zeta_1;\ \zeta_2;\ 0;\ 0)$ sistemasining to'plami chiziqli fazo bo'ladimi? Elementlarni qo'shish va haqiqiy songa ko'paytirish 461 masaladagi kabi aniqlanadi.

464. $(\xi_1;\ \xi_2;\ 1;\ 1)$, $(\eta_1;\ \eta_2;\ 1;\ 1)$, $(\zeta_1,\ \zeta_2;\ 1;\ 1)$ elementlar to'plami chiziqli fazo bo'ladimi?

465. Barcha ikkinchi darajali ko'phadlar $\alpha_0t^2+\alpha_1t+\alpha_2$, $\beta_0t^2+\beta_1t+\beta_2$, $\gamma_0t^2+\gamma_1t+\gamma_2$,.... to'plami chiziqli fazo bo'ladimi?

466. Darajasi uchdan oshmagan barcha ko'phadlar to'plami chiziqli fazo tashkil etadimi?

467. $f_1(t),\ f_2(t),\ f_3(t),...$ funksiyalar berilgan bo'lsin. Agar bu funksiyalar:

1) $[a,b]$ kesmada aniqlangan barcha uzluksiz funksiyalar to'plamini;

2) $[a,b]$ kesmada differensiallanuvchi barcha funksiyalar to'plamini;

3) barcha elementar funksiyalar to'plamini;

4) barcha elementar bo'lmagan funksiyalar to'plamini tashkil qilsa, bu funksiyalar to'plamlari chiziqli fazo bo'ladimi?

468. Musbat sonlarning barcha juftligidan iborat to'plam berilgan: $\overline{x}=(\varphi_1,\ \varphi_2)$, $\overline{y}=(\eta_1,\ \eta_2)$, $\overline{z}=(\xi_1,\xi_2)$, ... Agar ikki elementni qo'shish $\overline{x}+\overline{y}=(\varphi_1\eta_1;\ \varphi_2\eta_2)$ tenglik bilan haqiqiy songa ko'paytirish esa $\lambda\overline{x}=(\varphi_1^{\lambda},\ \varphi_2^{\lambda})$ tenglik bilan aniqlansa, bu to'plam chiziqli bo'ladimi?

469. Chiziqli fazo: 1) bitta vektordan; 2) ikkita har hil vektordan tuzilgan bo'lishi mumkinmi?

470. Chiziqli fazodan $\overline{x}$ vektor yo'qotilgan bo'lsin. Bu yo'qotishdan keyin hosil bo'lgan vektorlar to'plami chiziqli fazoligicha qolishi mumkinmi?

471. Chiziqli fazodan sanoqsiz vektorlar to'plami yo'qotilgan. Bu yo'qotishdan keyin hosil bo'lgan vektorlar to'plami chiziqli fazo bo'lishi mumkinmi?

472. Vagon provodniklarining rezerviga ular tarqatishi uchun har kuni skladdan: 1) qand; 2) choy; 3) pechene; 4) quritilgan non; 5) pista ko'mir keltiriladi. Faraz qilaylik, φ_1, φ_2, φ_3, φ_4, φ_5 mos ravishda bir kunda keltiriladigan bu mahsulotlar miqdorining kilogrammlardagi orttirmasi bo'lsin.

Agar $\varphi_i > 0$ bo'lsa, u holda mos ravishda oziq-ovqat yoki ko'mir shu kuni tarqatilganidan ko'p keltirildi, agar $\varphi_i < 0$ bo'lsa, u holda oziq-ovqat yoki ko'mir skladdan keltirilganiga qaraganda ko'p tarqatilgan. φ_1, φ_2, φ_3, φ_4, φ_5 sonlar sistemasining to'plami chiziqli fazo bo'ladimi? (−100; 5; 0; −200; 3) vektor nimani bildiradi?

473. φ_1, φ_2, $\varphi_3$butun sonlar uchliklarining to'plami chiziqli fazo tashkil etadimi?

474. Vagon deposining parkiga har kuni turli tipdagi vagonlar keladi: yuk, aloqa, qattiq o'rinli, kupeli va yumshoq, ulardan har kuni passajir va tez yurar poyezdlar tuziladi va yo'lga chiqadi. Faraz qilayliq φ_1, φ_2, φ_3, φ_4, φ_5 mos ravishda vagonlar sonining bir sutkadagi orttirmasi. φ_1, φ_2, φ_3, φ_4, φ_5 sonlar to'plami chiziqli fazo bo'ladimi?

475. Umumiy boshi koordinata boshida bo'lgan va 1-nchi oktantada joylashgan, barcha geometrik vektorlar chiziqli fazo tashkil qiladimi?

476. Chiziqli bir jinsli tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y + c_1z = 0, \\ a_2x + b_2y + c_2z = 0 \end{cases}$$

ning barcha yechimlari to'plami chiziqli fazo tashkil qilishini isbotlang.

Ko'rsatma:

Agar $(x_1;\ y_1;\ z_1)$ va $(x_2;\ y_2;\ z_2)$ berilgan sistemaning yechimi bo'lsa, u holda ixtiyoriy λ uchun $(x_1+x_2;\ y_1+y_2;\ z_1+z_2)$ va $(\lambda x_1;\ \lambda y_1;\ \lambda z_1)$ ham sistemaning yechimi bo'ladi.

477. $A_0 y^{(n)} + A_1 y^{(n-1)} + \ldots + A_n y = 0$ (A_0, A_1, ... , A_n – x ning funksiyalari) differensial tenglamani qanoatlantiruvchi barcha $y_1(x)$, $y_2(x)$, $y_3(x)$, ... funksiyalar chiziqli fazoni tashkil qilishini isbotlang.

2. Chiziqli bog'liqsiz vektorlar.

$\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ lar R chiziqli fazoning vektorlari bo'lsin. Quyidagi tenglik bilan aniqlangan vektor

$$\overline{v} = \alpha \cdot \overline{x} + \beta \cdot \overline{y} + \gamma \cdot \overline{z} + \ldots + \lambda \cdot \overline{u}$$

ham R chiziqli fazoga tegishli bo'ladi, bunda α, β, γ, ..., λ – haqiqiy sonlar. Bu vektor $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlarning *chiziqli kombinatsiyasi* deyiladi.

Faraz qilaylik, $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlarning chiziqli kombinatsiyasi nol-vektor bo'lsin, ya'ni

$$\alpha \cdot \overline{x} + \beta \cdot \overline{y} + \gamma \cdot \overline{z} + \ldots + \lambda \cdot \overline{u} = \overline{0}. \qquad (1)$$

$\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlar *chiziqli bog'liqsiz* deyiladi, agarda (1) tenglik $\alpha = \beta = \gamma = \ldots = \lambda = 0$ bo'lgan holdagina bajarilsa.

Agarda (1) tenglik $\alpha, \beta, \gamma, \ldots, \lambda$ larning kamida bittasi noldan farqli bo'lgan holda ham bajarilsa, $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlar *chiziqli bog'liq* deyiladi. Osongina isbotlash mumkinki, $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlar chiziqli bog'liq bo'ladilar, shunda va faqat shundaki, agarda bu vektorlarning birini qolganlarining chiziqli kombinasiyasi ko'rinishida ifodalab bo'lsa.

478. Agarda $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlar orasida $\overline{0}$ vektor mavjud bo'lsa, bu vektorlarning chiziqli bog'liq ekanligini ko'rsating.

Yechish:

Faraz qilaylik, $\overline{x} = \overline{0}$ bo'lsin. $\alpha \cdot \overline{x} + \beta \cdot \overline{y} + \gamma \cdot \overline{z} + \ldots + \lambda \cdot \overline{u} = \overline{0}$ tenglik $\alpha \neq 0$, $\beta = \gamma = \ldots = \lambda = 0$ bo'lganda bajarilgani uchun berilgan vektorlar chiziqli bog'liqdir.

479. Chiziqli fazoning elementlari tartiblangan haqiqiy sonlar $\overline{x}_i = (\xi_{1i}, \xi_{2i}, \ldots, \xi_{ni})$ $(i = 1, 2, 3, \ldots)$ sistemasidan iborat bo'lsin.

Agar vektorlarning yig'indisi va vektorning songa ko'paytmasi $\overline{x}_i + \overline{x}_k = (\xi_{1i} + \xi_{1k};\ \xi_{2i} + \xi_{2k};\ \ldots;\ \xi_{ni} + \xi_{nk})$, $\lambda \overline{x}_i = (\lambda\xi_{1i};\ \lambda\xi_{2i};\ \ldots;\ \lambda\xi_{ni})$ tengliklar bilan aniqlanganda, $\overline{x}_1$, $\overline{x}_2$, ..., $\overline{x}_n$ vektorlarning chiziqli bog'liqsiz bo'lishligi uchun ξ_{ik} $(i = 1, 2, \ldots, n;\ k = 1, 2, \ldots, n)$ sonlar qanday shartni qanoatlantirishi kerak?

Yechish:

$\alpha_1\overline{x}_1+\alpha_2\overline{x}_2+...+\alpha_3\overline{x}_3=0$ tenglikni ko'ramiz. U quyidagi tenglamalar sistemasiga teng kuchli.

$$\alpha_1\xi_{11}+\alpha_2\xi_{12}+...+\alpha_n\xi_{1n}=0,$$
$$\alpha_1\xi_{21}+\alpha_2\xi_{22}+...+\alpha_n\xi_{2n}=0,$$
$$\alpha_1\xi_{n1}+\alpha_2\xi_{n2}+...+\alpha_n\xi_{nn}=0.$$

x_1, x_2, ..., x_n vektorlar chiziqli bog'liq bo'lmagan holda bu sistema yagona $\alpha_1=\alpha_2=...=\alpha_n=0$ yechimga ega bo'lishi lozim, ya'ni:

$$\begin{vmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1n} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2n} \\ \xi_{n1} & \xi_{n2} & \cdots & \xi_{nn} \end{vmatrix} \neq 0$$

Xususiy holda (ξ_{11}, ξ_{21}) va (ξ_{12}, ξ_{22}) vektorlar chiziqli bog'liqsiz bo'ladi, shunda va faqat shundaki, $\xi_{11}\xi_{22}-\xi_{12}\xi_{21}\neq 0$ bo'lsa.

480. Darajasi ikkidan oshmagan ko'phadlarning chiziqli fazosini qaraymiz. $\overline{P}_1=1+2\cdot t+3\cdot t^2$, $\overline{P}_2=2+3\cdot t+4\cdot t^2$ va $\overline{P}_3=3+5\cdot t+7\cdot t^2$ vektorlarning chiziqli bog'liqligi isbotlansin.

Yechish:

Bu holda $\overline{P}_3=1\cdot\overline{p}_1+1\cdot\overline{P}_2$ ekanligini ko'rish qiyinlik tug'dirmaydi. Demak, $\overline{P}_1$, $\overline{P}_2$ va $\overline{P}_3$ vektorlar chiziqli bog'liq bo'ladi.

481. 468-masalaning shartida aniqlangan $\overline{x}=(\xi_1, \xi_2)$ va $\overline{y}=(\eta_1, \eta_2)$ vektorlar qanday hollarda chiziqli bog'liq bo'ladi?

Yechish:

$\overline{x}=\lambda\overline{y}$ tenglikdan $(\xi_1, \xi_2)=\lambda(\eta_1, \eta_2)$ yoki $(\xi_1, \xi_2)=(\eta_1^\lambda, \eta_2^\lambda)$ ekanligi kelib chiqadi, ya'ni $\xi_1=\eta_1^\lambda$, $\xi_2=\eta_2^\lambda$. Bundan quyidagi tenglikka kelamiz: $\ln\xi_1\cdot\ln\eta_2=\ln\eta_1\cdot\ln\xi_2$.

482. Uchta komplanar $\overline{a}$, $\overline{b}$ va $\overline{c}$ – vektorlarning chiziqli bog'liq ekanligini isbotlang.

Ko'rsatma.

Vektorlarni umumiy boshlang'ich nuqtaga keltiring va vektorlardan birini boshqa ikkita vektorlarga mos ravishda kollinear tashkil etuvchilarga yoying.

483. Uchta komplanar bo‘lmagan $\overline{a}$, $\overline{b}$ va $\overline{c}$ vektorlarning chiziqli erkli ekanligini isbotlang.

484. Ixtiyoriy to‘rtta $\overline{a}$, $\overline{b}$, $\overline{c}$ va $\overline{d}$ vektorlarning chiziqli bog‘liq ekanligini isbotlang.

Yechish:

Agar vektorning uchtasi komplanar bo‘lsa, masala oson yechiladi. Faraz qilaylik, bu vektorlar komplanar bo‘lmasin. Hamma to‘rtta vektorni umumiy boshlang‘ich O nuqtaga keltiramiz. Diagonali $\overline{d}$ vektor bo‘lgan, qirralari $\overline{a}$, $\overline{b}$ va $\overline{c}$ ni o‘z ichiga olgan to‘g‘ri chiziqdajoylashgan parallelepipedni yasaymiz. Bundan esa $\overline{d}=\alpha\overline{a}+\beta\overline{b}+\gamma\overline{c}$ ekanligini ko‘rish qiyin emas.

485. Agar chiziqli fazoning n ta $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$ vektorlari chiziqli bog‘liq bo‘lsa, u holda shu fazoning $n+1$ ta $\overline{x}$, $\overline{y}$, $\overline{z}$, ..., $\overline{u}$, $\overline{v}$ vektorlari ham chiziqli bog‘liq bo‘lishligini isbot qiling.

3. Chiziqli fazoning o‘lchovi va bazisi.

Agar R chiziqli fazoda n ta chiziqli erkli vektor mavjud bo‘lib, lekin shu fazoning ixtiyoriy $n+1$ ta vektori chiziqli bog‘liq bo‘lsa, u holda R fazo *n o‘lchovli* deyiladi. R fazoning o‘lchovi n ga teng deb aytish va $d(R)=n$ ko‘rinishda yozish qabul qilingan, istalgancha ko‘p chiziqli erkli vektorlarni topish mumkin bo‘lgan fazo *cheksiz o‘lchovli* deyiladi. Agar R cheksiz o‘lchovli fazo bo‘lsa, u holda $d(R)=\infty$.

n o‘lchovli chiziqli fazoning n ta chiziqli erkli vektorlari to‘plami *bazis* deyiladi. Quyidagi teorema o‘rinli: n o‘lchovli chiziqli fazodagi har bir vektorni bazis vektorlarning chiziqli kombinatsiyasi ko‘rinishda yagona usulda ifodalash mumkin.

Agar $\overline{e}_1,\overline{e}_2,...,\overline{e}_n$ lar n o‘lchovli chiziqli fazo R ning bazisi bo‘lsa, u holda ixtiyoriy $\overline{x}\in R$ vektorni yagona usulda $\overline{x}=\xi_1e_1+\xi_2e_2+.....+\xi_ne_n$ ko‘rinishda ifodalash mumkin.

Shunday qilib, $\overline{x}$ vektor $\overline{e}_1$, $\overline{e}_2$, ..., $\overline{e}_n$ bazisda ξ_1, ξ_2, ..., ξ_n sonlar yordamida yagona usulda aniqlanadi. Bu sonlar $\overline{x}$ *vektorning berilgan bazisdagi koordinatalari* deyiladi.

Agar $\overline{x}=\xi_1e_1+\xi_2e_2+.....+\xi_ne_n$, $\overline{y}=\eta_1e_1+\eta_2e_2+.....+\eta_ne_n$ bo‘lsa, u holda: $\overline{x}+\overline{y}=(\xi_1+\eta_1)\cdot\overline{e}_1+(\xi_2+\eta_2)\cdot\overline{e}_2+.....+(\xi_n+\eta_n)\cdot\overline{e}_n$, $\lambda\overline{x}=\lambda\xi_1\overline{e}_1+\lambda\xi_2\overline{e}_2+.....+\lambda\xi_n\overline{e}_n$ bo‘ladi.

Chiziqli fazoning o'lchovini aniqlashda quyidagi teoremadan foydalanish foydalidir: *Agar R chiziqli fazodagi ixtiyoriy vektor* $\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_n$ *chiziqli erkli vektorlarning chiziqli kombinatsiyasi ko'rinishda ifodalangan bo'lsa, u holda d(R)=n bo'ladi (demak,* $\overline{e}_1, \overline{e}_2, \ldots, \overline{e}_n$ *vektorlar R fazoda bazisni tashkil etadi).*

486. Tartiblangan haqiqiy sonlarning turli xil juftliklaridan tuzilgan chiziqli fazo berilgan:

$$\overline{x}_1 = (\xi_{11};\ \xi_{21}),\ \overline{x}_2 = (\xi_{12};\ \xi_{22}),\ \overline{x}_3 = (\xi_{13};\ \xi_{23}),\ \ldots,$$

bunda vektorlarni qo'shish va haqiqiy songa ko'paytirish $\overline{x}_i + \overline{x}_k = (\xi_{1i} + \xi_{1k};\ \xi_{2i} + \xi_{2k})$; $\lambda \overline{x}_i = (\lambda \xi_{1i};\ \lambda \xi_{2i})$ tengliklar bilan aniqlangan.

$\overline{e}_1 = (1;\ 2)$ va $\overline{e}_2 = (3;4)$ vektorlar berilgan chiziqli fazoning bazisini tashkil qilishini isbotlang. $\overline{x} = (7;\ 10)$ vektorning bu bazisdagi koordinatalarini toping.

Yechish:

$\overline{e}_1 = (1;\ 2)$ va $\overline{e}_2 = (3;4)$ vektorlar chiziqli erkli (479-masalaga qarang). Biror $\overline{y} = (\eta_1;\ \eta_2)$ vektorni qaraymiz. Ixtiyoriy η_1 va η_2 lar uchun shunday λ va μ sonlar mavjudligini ko'rsatamizki, $\overline{y} = \lambda \overline{e}_1 + \mu \overline{e}_2$ yoki $(\eta_1;\ \eta_2) = (\lambda + 3\mu;\ 2\lambda + 4\mu)$ tengliklar bajariladi. Osongina ko'rinadiki, bu tenglik bajariladigan $(\lambda;\ \mu)$ qiymatlarning yagona jufti mavjud. Bu

$$\begin{cases} \lambda + 3\mu = \eta_1, \\ 2\lambda + 4\mu = \eta_2 \end{cases}$$

tenglamalar sistemasining aniqlangan ekanligidan kelib chiqadi. Shunday qilib, $\overline{e}_1$ va $\overline{e}_2$ vektorlar bazisni tashkil qiladi. Bu bazisda $\overline{x} = (7;\ 10)$ vektorning koordinatalarini aniqlaymiz.

Masala quyidagi tenglamalar sistemasidan λ va μ ni aniqlashga keltiriladi:

$$\begin{cases} \lambda + 3\mu = 7, \\ 2\lambda + 4\mu = 10. \end{cases}$$

Bundan $\lambda = 1$, $\mu = 2$ larni topamiz, ya'ni $\overline{x} = \overline{e}_1 + 2\overline{e}_2$.

487. Elementlari $\overline{x} = (\xi_1;\ \xi_2;\ \ldots;\ \xi_n)$ vektorlardan iborat chiziqli fazo (479-masalaga qarang) o'zining bazisi sifatida quyidagi vektorlarga $\overline{e}_1 = (1;\ 0;\ 0;\ \ldots;\ 0)$, $\overline{e}_2 = (0;\ 1;\ 0;\ \ldots;\ 0)$, $\overline{e}_3(0;\ 0;\ 1;\ \ldots;\ 0)$, ..., $\overline{e}_n(0;\ 0;\ 0;\ \ldots;\ 1)$ ega ekanligini ko'rsating.

Yechish:

$\overline{x}=\xi_1(1;\ 0;\ 0;\ ...;\ 0),\ \xi_2(0;\ 1;\ 0;\ ...;\ 0)+...+\xi_n(0;\ 0;\ 0;...;1)$ ekanligini ko'rish qiyin emas, ya'ni $\overline{x}=\xi_1\overline{e}_1+\xi_2\overline{e}_2+...+\xi_n\overline{e}_n$. Shunday qilib, har qanday vektorni $\overline{e}_1, \overline{e}_2, ..., \overline{e}_n$ vektorlarning chiziqli kombinatsiyasi ko'rinishida ifodalash mumkin. $\overline{e}_1, \overline{e}_2, ..., \overline{e}_n$ vektorlar chiziqli erkli, chunki bu vektorlarning koordinatalaridan tuzilgan determinant 1 ga teng, ya'ni noldan farqli. Shunday qilib, bu vektorlar bazisni tashkil qiladi, R fazo esa n o'lchovli bo'ladi.

488. Agar elementlarini qo'shish va haqiqiy songa ko'paytirishni odatdagidek ma'noda tushunilganda, bazisi $1, t, t^2, t^3, ..., t^{n-1}, t^n$ bo'lgan chiziqli fazo qanday elementlardan tuzilgan bo'ladi?

489. Ikkinchi tartibli barcha matritsalar to'plami to'rt o'lchovli chiziqli fazo ekanligini ko'rsating.

490. $\overline{e}_1=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \overline{e}_2=\begin{pmatrix}0 & 2\\ 0 & 0\end{pmatrix}, \overline{e}_3=\begin{pmatrix}0 & 0\\ 3 & 0\end{pmatrix}, \overline{e}_4=\begin{pmatrix}0 & 0\\ 0 & 4\end{pmatrix}$ matritsalar 489-masalada qaralgan chiziqli fazoning bazisi bo'lishini ko'rsating.

491. 468-masalada qaralgan chiziqli fazoning $\overline{e}_1(1; 10)$ va $\overline{e}_2(10; 1)$ elementlari bazis bo'lishini ko'rsating. $\overline{x}=(2;\ 3)$ vektorning shu bazisdagi koordinatalarini toping.

Yechish:

$\ln 1\cdot\ln 1-\ln 10\cdot\ln 10\neq 0$ bo'lganligi uchun $\overline{e}_1$ va $\overline{e}_2$ vektorlar chiziqli erkli bo'ladi (481-masalaga qarang). Faraz qilaylik, ixtiyoriy $\overline{y}=(\eta_1;\ \eta_2)$ vektor $\overline{e}_1$ va $\overline{e}_2$ vektorlarning chiziqli kombinatsiyasi ko'rinishida ifodalangan bo'lsin. $\overline{y}=\lambda\overline{e}_1+\mu\overline{e}_2$, yoki $(\eta_1;\eta_2)=(1^\lambda\cdot 10^\mu;10^\lambda\cdot 1^\mu)$ bajariladigan shunday sonlar juftligi mavjudligini ko'rsatamiz. Demak, $\mu=\lg\eta_1, \lambda=\lg\eta_2$. Xususan, $\overline{x}=\overline{e}_1\lg 3+\overline{e}_2\lg 2$. Shunday qilib, $(\lg 3;\lg 2)$ – $\overline{x}$ vektorning $\overline{e}_1;\ \overline{e}_2$ bazisdagi koordinatalaridir.

492. 479-masalada qaralgan n o'lchovli fazoning bazislari sifatida

$\overline{e}_1=(1;\ 1;\ 1;\ ...;\ 1;\ 1),\ \overline{e}_2=(0;\ 1;\ 1;\ ...;\ 1;\ 1),\ \overline{e}_3=(0;\ 0;\ 1;\ ...;\ 1;\ 1), ...$

$...,\ \overline{e}_{n-1}=(0;\ 0;\ 0;\ ...;\ 0;\ 1),\ \overline{e}_n=(1;\ 1;\ 1;\ ...;\ 1;\ 1)$

vektorlarni qabul qilish mumkinligini ko'rsating.

Ko'rsatma:

$\overline{e}_1' = \overline{e}_1 - \overline{e}_2$, $\overline{e}_2' = \overline{e}_2 - \overline{e}_3$, ..., $\overline{e}_{n-1}' = \overline{e}_2 - \overline{e}_3$, $\overline{e}_n' = \overline{e}_2 - \overline{e}_3$ vektorlar qaralsin.

4. Chiziqli fazolarning izomorfizmi.

Ikkita R va R' chiziqli fazolarni qarymiz. R fazoning elementlarini $\overline{x}, \overline{y}, \overline{z}, \ldots$ bilan belgilaymiz. R' fazoning elementlarini esa $\overline{x}', \overline{y}', \overline{z}', \ldots$bilan belgilaymiz. Agar $\overline{x}, \overline{y}, \overline{x}', \overline{y}'$ elementlar orasida shunday o'zaro bir qiymatli moslik $\overline{x} \leftrightarrow \overline{x}'$; $\overline{y} \leftrightarrow \overline{y}'$ o'rnatish mumkin bo'lsaki, bunda $\overline{x} + \overline{y} \leftrightarrow \overline{x}'' + \overline{y}'$, $\lambda\overline{x} \leftrightarrow \lambda\overline{y}$ bo'lsa (λ–ixtiyoriy haqiqiy son), u holda R va R' fazolar *o'zaro izomorf* deyiladi. Quyidagi muhim teoremani eslatib o'tamiz. Uning yordamida chekli o'lchovli chiziqli fazolarning izomorfligi oson o'rnatiladi: *Ikkita chekli o'lchovli R va R' fazolarning izomorf bo'lishi uchun, ularning o'lchovlari bir xil bo'lishi zarur va yetarlidir.*

493. Ikkita chiziqli fazo R va R' berilgan. R fazoning elementlari t argumentning barcha differensiallanuvchi funksiyalari bo'lib, ular t=0 da 0 ga aylanadi. R' fazoning elementlari esa R fazodagi funksiyalarning hosilalaridan iborat bo'lsin. R va R' fazolarning izomorf ekanligini isbotlang.

Yechish:

Faraz qilaylik, $f_1(t)$, $f_2(t)$, $f_3(t)$, ... lar R fazoning funksiyalari, $\varphi_1(t)$, $\varphi_2(t)$, $\varphi_3(t)$, ... esa R' fazoning funksiyalari bo'lsin. Bu funksiyalarning indekslar bilan ta'minlanganligidan R va R' sanoqli to'plam degan xulosa kelib chiqmaydi. Faraz qilaylik, $\varphi_i(t) = f_i'(t)$ bo'lsin, u holda

$$f_i(t) = \int_0^t \varphi_i(t)dt$$

bo'ladi. Shunday qilib, R va R' chiziqli fazolar orasida (ularning chiziqli ekanligini mustaqil ravishda isbotlang) o'zaro bir qiymatli moslik o'rnatildi. Quyidagi:

$$\varphi_i(t) + \varphi_k(t) = \left[f_i(t) + f_k(t)\right]', \; f_i(t) + f_k(t) = \int_0^t [\varphi_i(t) + \varphi_k(t)]dt$$

$$\lambda \cdot \varphi_i(t) = \left[\lambda \cdot f_i(t)\right]', \; \lambda \cdot f_i(t) = \int_0^t \lambda \cdot \varphi_i(t)dt$$

tengliklar yordamida o'zaro bir qiymatli moslik o'rnatildi:

$$f_i(t)+f_k(t) \leftrightarrow \varphi_i(t)+\varphi_k(t), \quad \lambda f_i(t) \leftrightarrow \lambda\varphi_i(t).$$

Shunday qilib, R va R' – izomorf fazolar.

494. Barcha geometrik vektorlar va darajasi ikkidan oshmagan ko'phadlar to'plamlari izomorf chiziqli fazolar ekanligini isbotlang.

495. R va R' izomorf chiziqli fazolar berilgan. Bu fazolar elementlari orasida o'zaro bir qiymatli moslik o'rnatilgan: $\overline{x} \leftrightarrow \overline{x}$, $\overline{y} \leftrightarrow \overline{y}, \ldots$ Har qanday haqiqiy α, β, γ sonlar uchun $\alpha\overline{x}+\beta\overline{y}+\gamma\overline{z} \leftrightarrow \alpha\overline{x}'+\beta\overline{y}'+\gamma\overline{z}'$ ekanligini isbotlang.

496. Faraz qilaylik, R va R' izomorf chiziqli fazolar bo'lib, $\overline{x} \leftrightarrow \overline{x}'$ moslik o'rinli bo'lsin. Bu holda $(-\overline{x}) \leftrightarrow (-\overline{x}')$ ekanligini isbotlang.

497. R va R' izomorf fazolar berilgan, shu bilan birga $\overline{0}$ va $\overline{0}'$ bu fazolarning nol elementlaridir. $\overline{0} \leftrightarrow \overline{0}'$ bo'lishi bu fazolarning boshqa elementlari orasida bir qiymati moslik qanday o'rnatilganligiga bog'liq emasligini isbotlang.

498. Haqiqiy sonlarning turlicha juftliklari berilgan: $(\xi_1;\, \eta_1)$, $(\xi_2;\, \eta_2)$, $(\xi_3;\, \eta_3), \ldots$ Ikkita chiziqli fazo quyidagicha tuzilgan: elementlari $\overline{x}_1=(\xi_1;\, \eta_1)$, $\overline{x}_2=(\xi_2;\, \eta_2)$, $\overline{x}_3=(\xi_3;\, \eta_3), \ldots$ bo'lgan R fazo, unda vektorlarni qo'shish va songa ko'paytirish

$$\overline{x}_1+\overline{x}_2=(\xi_1+\xi_2;\; \eta_1+\eta_2), \quad \lambda\overline{x}_1=(\lambda\xi_1;\; \lambda\eta_1)$$

tenglikdan aniqlanadi va

$$\overline{x}_1'=(\overline{e^{-\xi_1}};\; \overline{e^{-\eta_1}}), \quad \overline{x}_2'=(\overline{e^{-\xi_2}};\; \overline{e^{-\eta_2}}), \quad \overline{x}_3'=(\overline{e^{-\xi_{33}}};\; \overline{e^{-\eta_3}}), \ldots$$

vektorlardan tuzilgan R' fazo, unda mos amallar

$$\overline{x}_1'+\overline{x}_2'=(e^{-\xi_1-\xi_2};\; e^{-\eta_1-\eta_2}), \quad \lambda\overline{x}_1=(e^{-\lambda\xi_1};\; e^{-\lambda\eta_1})$$

tengliklardan aniqlanadi. R va R' fazolar izomorf ekanligini isbotlang.

499. Agar R fazoning elementlari $\overline{x}, \overline{y}, \overline{z}, \ldots$ vektorlar, R' fazoning elementlari esa $2\overline{x}, 2\overline{y}, 2\overline{z}, \ldots$ bo'lsa, R va R' chiziqli fazolar izomorf bo'ladimi? R va R' fazolar bir xil elementlardan tashkil topganligini ko'rsating.

2-§. YANGI BAZISGA O'TISHDA KOORDINAT ALMASHTIRISH

n o'lchovli chiziqli fazo R^n ning ikkita bazisi: $\overline{e}_1, \overline{e}_2, \overline{e}_3, \ldots$ (eski) va $\overline{e}'_1, \overline{e}'_2, \overline{e}'_3$ (yangi) mavjud bo'lsin. Har bir yangi bazisdagi vektorni eski bazisdagi vektorlar orqali ifodalaydigan bog'lanishlar berilgan:

$$e'_1 = a_{11}e_1 + a_{21}e_2 + \ldots + a_{n1}e_n.$$
$$e'_2 = a_{12}e_1 + a_{22}e_2 + \ldots + a_{n2}e_n,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$e'_n = a_{1n}e_1 + a_{2n}e_2 + \ldots + a_{nn}e_n.$$

Quyidagi matritsani

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}$$

eski bazisdan yangi bazisga o'tish matritsasi deyiladi.

Qandaydir $\overline{x}$ vektorini olaylik. $(\xi_1; \xi_2; \ldots; \xi_n)$ bu vektorning eski bazisdagi koordinatalari, $(\xi'_1; \xi'_2; \ldots; \xi'_n)$ esa bu vektorning yangi bazisdagi koordinatalari bo'lsin. Bunda $\overline{x}$ vektorning eski koordinatalari yangi koordainatalari orqali quyidagi formulalar orqali ifodalanadi:

$$e_1 = a_{11}e'_1 + a_{12}e'_2 + \ldots + a_{1n}e'_n,$$
$$e_2 = a_{21}e'_1 + a_{22}e'_2 + \ldots + a_{2n}e'_n,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$e_n = a_{n1}e'_1 + a_{n2}e'_2 + \ldots + a_{nn}e'_n$$

va ular *koordinatalarni almashtirish formulalari* deyiladi.

A matritsaning ustunlari eski bazisdan yangi bazisga o'tish formulalaridagi koordinatalar, bu matritsaning yo'llari esa eski koordinatalarni yangilari orqali almashtirish formulalaridagi koordinatalar ekanligini ko'rish qiyin emas.

500. $\overline{x} = \overline{e}_1 + \overline{e}_2 + \overline{e}_3 + \overline{e}_4$ vektor berilgan. Bu vektorni yangi bazis $\overline{e}'_1, \overline{e}'_2, \overline{e}'_3, \overline{e}'_4$ orqali yoying, agar $\overline{e}'_1 = \overline{e}_2 + \overline{e}_3 + \overline{e}_4$, $\overline{e}'_2 = \overline{e}_1 + \overline{e}_3 + \overline{e}_4$, $\overline{e}'_3 = \overline{e}_1 + \overline{e}_2 + \overline{e}_4$, $\overline{e}'_4 = \overline{e}_1 + \overline{e}_2 + \overline{e}_3$ bo'lsa.

Yechish:

1-usul:

Eski bazisdan yangisiga o'tish matritsasini yozamiz:

$$A=\begin{pmatrix}0 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\\ 1 & 1 & 0 & 1\\ 1 & 1 & 1 & 0\end{pmatrix}.$$

Bu matritsaning qatorlari koordinatalarni almashtirish formulalarining koeffitsientlari bo'ladi:

$$\xi_1=\xi_2'+\xi_3'+\xi_4',\ \xi_2=\xi_1'+\xi_3'+\xi_4',\ \xi_3=\xi_1'+\xi_2'+\xi_4',\ \xi_4=\xi_1'+\xi_2'+\xi_3'.$$

$\xi_1=\xi_2=\xi_3=\xi_4=1$ bo'lganligi uchun tenglamalar sistemasini yechib, $\xi_1'=\xi_2'=\xi_3'=\xi_4'=\frac{1}{3}$ ni topamiz va $\overline{x}=\frac{1}{3}(\overline{e}_1'+\overline{e}_2'+\overline{e}_3'+\overline{e}_4')$.

2-usul:

$$\begin{aligned}
&\overline{x}=\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4\\
&\overline{e}_1'=0\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4\\
&\overline{e}_2'=\overline{e}_1+0\overline{e}_2+\overline{e}_3+\overline{e}_4\\
&\overline{e}_3'=\overline{e}_1+\overline{e}_2+0\overline{e}_3+\overline{e}_4\\
&\overline{e}_4'=\overline{e}_1+\overline{e}_2+\overline{e}_3+0\overline{e}_4
\end{aligned}$$

tenglamalar sistemasidan $\overline{e}_1,\ \overline{e}_2,\ \overline{e}_3,\ \overline{e}_4$ larni yo'qotib,

$$\begin{vmatrix}\overline{x} & 1 & 1 & 1 & 1\\ \overline{e}_1' & 0 & 1 & 1 & 1\\ \overline{e}_2' & 1 & 0 & 1 & 1\\ \overline{e}_3' & 1 & 1 & 0 & 1\\ \overline{e}_4' & 1 & 1 & 1 & 0\end{vmatrix}=0$$

ni hosil qilamiz. Bu determinantni 1-ustun elementlari bo'yicha yoyib, $\overline{x}$ ni $\overline{e}_1',\ \overline{e}_2',\ \overline{e}_3',\ \overline{e}_4'$ lar orqali ifodalanadi.

3-usul.

$\overline{e}_1'+\overline{e}_2'+\overline{e}_3'+\overline{e}_4'=3\overline{e}_1+3\overline{e}_2+3\overline{e}_3+3\overline{e}_4$ ekanligidan

$$\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}=\frac{1}{3}(\overline{e}_1'+\overline{e}_2'+\overline{e}_3'+\overline{e}_4')$$

bo‘ladi. Bundan:

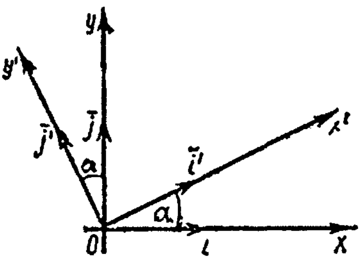
$$\overline{x}=\frac{1}{3}(\overline{e}_1'+\overline{e}_2'+\overline{e}_3'+\overline{e}_4').$$

501. $\overline{x}=8\overline{e}_1+6\overline{e}_2+4\overline{e}_3-18\overline{e}_4$ vektor berilgan. Bu vektorni yangi bazis bo‘yicha yoying. Bu yangi bazis eski bazis bilan quyidagi tenglamalar orqali bog‘langan:

$$\overline{e}_1'=-3\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4,$$
$$\overline{e}_2'=2\overline{e}_1-4\overline{e}_2+\overline{e}_3+\overline{e}_4,$$
$$\overline{e}_3'=\overline{e}_1+3\overline{e}_2-5\overline{e}_3+\overline{e}_4,$$
$$\overline{e}'=\overline{e}_1+\overline{e}_2+4\overline{e}_4-6\overline{e}_4.$$

502. $\overline{x}=2(\overline{e}_1+\overline{e}_2+...+\overline{e}_n)$ vektor berilgan. Agar $\overline{e}_1'=\overline{e}_1+\overline{e}_2$, $\overline{e}_2'=\overline{e}_1+\overline{e}_2$, $\overline{e}_3'=\overline{e}_1+\overline{e}_2$, ..., $\overline{e}_{n-1}'=\overline{e}_{n-1}+\overline{e}_n$, $\overline{e}_n'=\overline{e}_n+\overline{e}_1$ bo‘lsa, $\overline{x}$ vektorni $\overline{e}_1'$, $\overline{e}_2'$, ..., $\overline{e}_n'$ bazis bo‘yicha yoying.

503. xOy koordinatalar sistemasi koordinata boshi atrofida α burchakka burilgan (21-rasm). Yangi sistemadagi $\overline{a}=x\cdot\overline{i}+y\cdot\overline{j}$ vektorning koordinatalarini uning eski sistemadagi koordinatalari orqali ifodalang.



21-rasm.

Yechish:

$\overline{i}'$ va $\overline{j}'$ vektorlarni i va j ortlar bo‘yicha yozamiz:

$$\overline{i}'=\overline{i}\cdot\cos\alpha+\overline{j}\cdot\sin\alpha,$$
$$\overline{j}'=\overline{i}\cdot\cos\left(\frac{\pi}{2}+\alpha\right)+\overline{j}\cdot\sin(\frac{\pi}{2}+\alpha).$$

Eski $\overline{i}$, $\overline{j}$ bazisdan yangi $\overline{i}'$, $\overline{j}'$ bazisga o‘tish matritsasini yozamiz:

$$A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}.$$

Bundan

$$x = x'\cos\alpha - y'\sin\alpha,$$

$$y = x'\sin\alpha + y'\cos\alpha$$

ekanligi kelib chiqadi, ya'ni

$$x' = x\cos\alpha + y\sin\alpha,$$

$$y' = -x\sin\alpha + y\cos\alpha.$$

504. $\bar{e}_1' = \alpha\bar{e}_2$, $\bar{e}_2' = \beta\bar{e}_3$, $\bar{e}_3' = \gamma\bar{e}_4$, $\bar{e}_4' = \delta\bar{e}_5$, $\bar{e}_5' = \varepsilon\bar{e}_1$ bog'lanishlar berilgan. $\bar{x}$ vektorning ξ_1, ξ_2, ξ_3, ξ_4, ξ_5 – eski koordinatalarini shu vektorning ξ_1', ξ_2', ξ_3', ξ_4', ξ_5' – yangi koordinatalari bilan bog'lovchi formulalarini yozing.

505. Eski bazis $\bar{e}_1$, $\bar{e}_2$, $\bar{e}_3$ – bilan yangi bazis $\bar{e}_1'$, $\bar{e}_2'$, $\bar{e}_3'$ orasida quyidagi bog'liqlik bo'lishi mumkinmi:

$$\bar{e}_1' = \bar{e}_2 - \bar{e}_3,\ \bar{e}_2' = \bar{e}_3 - \bar{e}_1,\ \bar{e}_3' = \bar{e}_1 - \bar{e}_2?$$

3-§. QISM TO'PLAM

1. Chiziqli fazoning qism to'plami.

R' chiziqli fazo R *chiziqli fazoning qism fazosi* deyiladi, agarda R' ning elementlari faqatgina R ning elementlaridan tashkil topgan bo'lsa. Masalan, biror tekislikka parallel bo'lgan barcha vektorlar to'plami barcha geometrik vektorlar fazosining qism fazosi bo'ladi.

Agar $\bar{x}$, $\bar{y}$, $\bar{z}$, ..., $\bar{u}$ lar R chiziqli fazoning biror vektorlari bo'lsa, u holda barcha $\alpha\bar{x} + \beta\bar{y} + \gamma\bar{z} + ... + \lambda\bar{u}$ ko'rinishdagi vektorlar (bu yerda α, β, γ, ..., λ – barcha mumkin bo'lgan haqiqiy sonlardir) R chiziqli fazoning qism fazosini tashkil etadi.

$\alpha\bar{x} + \beta\bar{y} + \gamma\bar{z} + ... + \lambda\bar{u}$ vektorlarning barcha chiziqli kombinatsiyalari to'plami, $\bar{x}, \bar{y}, \bar{z}, ..., \bar{u}$ – *vektorlarning chiziqli qobig'i* deb ataladi va $L(\bar{x}, \bar{y}, \bar{z}, ..., \bar{u})$ – bilan belgilanadi.

Agar R_1 – chiziqli fazo R ning qism fazosi bo'lsa, u holda

$d(R_1) \le d(R)$ bo'ladi. R chiziqli fazoda R_1 va R_2 ikkita qism fazo berilgan bo'lsin. Barcha elementlari bir vaqtda R_1 va R_2 ga tegishli bo'lgan R_3 to'plam, R_1 va R_2 *qism fazolarning kesishmasi* deyiladi. $R_3 = R_1 \cap R_2$ yozuv, R_1 va R_2 qism fazolarning kesishmasi R_3 ekanligini bildiradi. Barcha elementlari $\overline{x} + \overline{y}$ ko'rinishda bo'lgan R_4 to'plam R_1 va R_2 *qism fazoning yig'indisi* deyiladi, bunda $x \in R_1$, $y \in R_2$. $R_4 = R_1 + R_2$ yozuv R_1 va R_2 qism fazolarining yig'indisi R_4 ekanligini bildiradi. R_3 kesishma va R_4 yig'indi R fazoning qism fazolari ekanligini isbotlash mumkin, $d(R_1) + d(R_2) = d(R_3) + d(R_4)$ ekanligini hisobga olish o'rinli.

506. R chiziqli fazoning qism fazosi bitta elementdan iborat bo'lishi mumkinmi?

507. Elementlari haqiqiy sonlarning turli sistemalari bo'lgan R chiziqli fazo berilgan:

$$\overline{x} = (\xi_1;\ \xi_2;\ \xi_3;\ \xi_4),\quad \overline{y} = (\eta_1;\ \eta_2;\ \eta_3;\ \eta_4),\quad \overline{z} = (\zeta_1;\ \zeta_2;\ \zeta_3;\ \zeta_4),\ \dots .$$

Ikki elementni qo'shish va elementni songa ko'paytirish quyidagi tengliklar bilan aniqlangan:

$$\overline{x} + \overline{y} = (\xi_1 + \eta_1;\ \xi_2 + \eta_2;\ \xi_3 + \eta_3,\ \xi_4 + \eta_4),$$

$$\lambda\overline{x} = (\lambda\xi_1;\ \lambda\xi_2;\ \lambda\xi_3;\ \lambda\xi_4).$$

Elementlari $\overline{x}_1 = (0;\ \xi_2;\ \xi_3;\ \xi_4)$, $\overline{y}_1 = (0;\ \eta_2;\ \eta_3;\ \eta_4)$, $\overline{z}_1 = (0;\ \zeta_2;\ \zeta_3;\ \zeta_4), \dots$ bo'lgan R_1 to'plam va elementlari $\overline{x}_2 = (\xi_1;\ 0;\ \xi_3;\ \xi_4)$, $\overline{y}_2 = (\eta_1;\ 0;\ \eta_3;\ \eta_4)$, $\overline{z}_2 = (\zeta_1;\ 0;\ \zeta_3;\ \zeta_4), \dots$ bo'lgan R_2 tuplam R chiziqli fazoning qism fazosi ekanligini isbotlang.

508. 507-masalada qaralgan R chiziqli fazo uchun R_1 va R_2 qism fazolarning kesishmasi R_3 va yig'indisi R_4 ni toping.

509. 504 va 505-masalalardagi qism fazo uchun $d(R_1) + d(R_2) = d(R_3) + d(R_4)$ tenglikning bajarilishini ko'rsating.

510. Barcha geometrik vektorlardan tuzilgan chiziqli fazo berilgan bo'lsin. Boshlanishi koordinata boshida va 1 oktantda joylashgan vektorlar to'plami bu fazoning qism fazosi bo'la oladimi?

511. Elementlari 1 oktantning koordinata tekisliklarida yotmagan $R=(x,y,z)$ nuqtalarning koordinatalaridan tashkil topgan R chiziqli fazo berilgan bo'lsin. Ixtiyoriy ikki $P_1=(x_1;\ y_1;\ z_1)$ va $P_2=(x_2;\ y_2;\ z_2)$ elementlarning yig'indisi $P_1+P_2=(x_1x_2;\ y_1y_2;\ z_1z_2)$ tenglik bilan, $P=(x,y,z)$ elementni haqiqiy son λ ga ko'paytirish esa $\lambda P=(x^{\lambda};y^{\lambda};z^{\lambda})$ tenglik bilan aniqlanadi. $z=1$ tekislikda joylashgan bu fazoning R_1 nuqtalar to'plami R fazoning qism fazosi ekanligini isbotlang.

512. Darajasi beshdan oshmagan ko'phadlarning R chiziqli fazosi berilgan. Agar elementlarni qo'shish va elementlarni songa ko'paytirish oddiy ma'noda tushinilganda a_0t+a_1 ko'rinishdagi ko'phadlarning R_1 to'plami va $b_0t^4+b_1t^2+b_2$ ko'phadlarning R_2 to'plami R fazoning qism fazosi ekanligini isbotlang.

513. Avvalgi masalaning shartiga ko'ra, $R_3=R_1\cap R_2$ va $R_4=R_1+R_2$ fazo ostilarini toping.

514. Barcha geometrik vektorlar R fazosining ikkita qism fazosini qaraymiz: xOy koordinata tekisligiga parallel vektorlar to'plami R_1 va xOz tekisligiga parallel vektorlar to'plami R_2. $R_3=R_1\cap R_2$ va $R_4=R_1+R_2$ to'plamlarni toping.

515. R_1 va R_2 lar R chiziqli fazoning fazo ostlari. R_1' va R_2' -lar esa $-R'$ chiziqli fazoning fazo ostlari bo'lsin. Ma'lumki R_1 va R_1' fazo ostlari, shuningdek R_2 va R_2' fazo ostlari ham izomorfdirlar. $R_3=R_1\cap R_2$ va $R_3'=R_1'\cap R_2'$ qism fazolarning, shuningdek $R_4=R_1+R_2$ va $R_4'=R_1'+R_2'$ – qism fazolarning izomorfligini ko'rsating.

516. $[-a,\ a]$ kesmada uzluksiz va musbat $f(x)$ funksiyalar to'plami berilgan. Agar vektorlarning yig'indisi sifatida mos funksiyalarning ko'paytmasini, vektorning haqiqiy son λ ga ko'paytmasi sifatida esa mos funksiyalarni λ darajaga oshirgandagi natija qabul qilinsa, bu to'plam chiziqli fazo bo'lishini isbotlang. Bu fazoning barcha juft funksiyalar to'plami qism fazo bo'ladimi? Bu fazoning barcha toq funksiyalar to'plamichi?

517. Geometrik vektorlarning chiziqli fazosi qaraladi. X,Y,Z rasional sonlar bo'lganda $\bar{a}=X\bar{i}+Y\bar{j}+Z\bar{k}$ ko'rinishdagi barcha vektorlar to'plami bu fazoning qism fazosini tashkil qiladimi?

2. Bir jinsli chiziqli tenglamalar sistemasining yechimlaridan tashkil topgan qism fazo.

Bir jinsli chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = 0. \end{cases} \qquad (1)$$

$x_1 = \lambda_1,\ x_2 = \lambda_2,\ \ldots,\ x_n = \lambda_n$ – (1) sistemaning birorta yechimi bo‘lsin. Bu yechimni $\overline{f} = (\lambda_1;\ \lambda_2;\ \ldots;\ \lambda_n)$ vektor ko‘rinishda yozamiz. Agar (1) tenglamalar sistemasining har qanday yechimini $\overline{f_1},\ \overline{f_2},\ \ldots,\ \overline{f_n}$ vektorlarning chiziqli kombinatsiyasi ko‘rinishida ifodalash mumkin bo‘lsa, u holda (1) tenglamalar sistemasining $\overline{f_1},\ \overline{f_2},\ \ldots,\ \overline{f_n}$ chiziqli erkli yechimlar to‘plami *fundamental yechimlar sistemasi* deyiladi.

Fundamental yechimlar sistemasining mavjudligi haqidagi teorema: *Agar*

$$\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

matritsaning rangi n dan kichik bo‘lsa, u holda (1) sistema nol bo‘lmagan yechimga ega. Fundamental yechimlar sistemasini aniqlaydigan vektorlar soni k=n−r formula bo‘yicha topiladi, bunda r—matritsaning rangi.

Shunday qilib, agar qaralayotgan R^n chiziqli fazo, n ta haqiqiy sonlarning barcha sistemalaridan tashkil topgan bo‘lsa, u holda (1) sistemaning barcha yechimlari to‘plami R^n fazoning qism fazosi bo‘ladi. Bu qism fazoning o‘lchovi k ga teng bo‘ladi.

518. Chiziqli bir jinsli tenglamalar sistemasining yechimlari bo‘lgan qism fazoning bazisi va o‘lchovini toping:

$$x_1+2\cdot x_2+3\cdot x_3+4\cdot x_4=0,$$
$$\frac{1}{2}\cdot x_1+x_2+\frac{3}{2}\cdot x_3+2\cdot x_4=0,$$
$$\frac{1}{3}\cdot x_1+\frac{2}{3}\cdot x_2+x_3+\frac{4}{3}\cdot x_4=0,$$
$$\frac{1}{4}\cdot x_1+\frac{1}{2}\cdot x_2+\frac{3}{4}\cdot x_3+x_4=0.$$

Yechish:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1/2 & 1 & 3/2 & 2 \\ 1/3 & 2/3 & 1 & 4/3 \\ 1/4 & 1/2 & 3/4 & 1 \end{pmatrix}$$

matritsaning rangi 1 ga teng, madomiki matritsaning birinchi tartibli minorlaridan boshqa barcha minorlari nolga teng. Noma'lumlar soni 4 ga teng, shuning uchun yechimlar qism fazosining o'lchovi $k=n-2=4-1=3$, ya'ni bu qism fazo uch o'lchovli bo'ladi. $r=1$ bo'lganligi uchun bu sistemadan qandaydir bitta tenglamani olish yetarli. Sistemaning birinchi tenglamasini olamiz va uni $x_1=-2\cdot x_2-3\cdot x_3-4\cdot x_4$ ko'rinishda yozamiz. Agar $x_2=1,\ x_3=0,\ x_4=0$ bo'lsa, u holda $x_1=-2$, agar $x_2=0,\ x_3=1,$ $x_4=0$ bo'lsa, u holda $x_1=-3$, agar $x_2=0,\ x_3=0,\ x_4=1$ bo'lsa, u holda, $x_1=-4$. Shunday qilib, biz berilgan sistema yechimlarining uch o'lchovli qism fazosining bazisini tashkil qiladigan $\overline{f_1}=(-2;\ 1;\ 0;\ 0)$, $\overline{f_2}=(-3;\ 0;\ 1;\ 0)$, $\overline{f_3}=(-4;\ 0;\ 0;\ 1)$ chiziqli erkli vektorlarni hosil qildik.

519. $\overline{f}=\overline{f_1}-2\overline{f_2}+\overline{f_3}$ vektor 518-masaladagi tenglamalar sistemasini qanoatlantirishini ko'rsating.

520. Quyidagi tenglamalar sistemasi yechimlari qism fazosining bazisi va o'lchovini toping.

$$\begin{cases} x_1-2x_2+x_3=0, \\ 2x_1-x_2-x_3=0, \\ -2x_1+4x_2-2x_3=0. \end{cases}$$

Yechish:

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & -1 & -1 \\ -2 & 4 & 2 \end{pmatrix}$$

matritsaning rangi 2 ga teng, chunki matritsa elementlaridan tuzilgan determinant nolga teng, ikkinchi tartibli minorlari ichidan noldan farqlisi mavjud. Yechimlar qism fazosining o'lchovi $k=n-r=3-2=1$ bo'lganligi uchun berilgan uchta tenglamadan ikkita tenglamani olish yetarli. Birinchi tenglamaning koeffitsientlari uchinchi tenglamaning mos koeffitsientlariga proporsional bo'lganligi sababli uchinchi tenglamani tashlab yuboramiz.

Faraz qilaylik

$$\begin{cases} x_1 - 2\cdot x_2 = -x_3, \\ 2\cdot x_1 - x_2 = x_3 \end{cases}$$

sistemada $x_3=1$ bo'lsin, u holda

$$\begin{cases} x_1 - 2\cdot x_2 = -1, \\ 2\cdot x_1 - x_2 = 1 \end{cases}$$

sistemaning yechimi $x_1=1$, $x_2=1$ bo'ladi. Shunday qilib, yechimlar qism fazosi bitta bazis vektor $\bar{f}=(1;\ 1;\ 1)$ bilan aniqlanadi.

521. Berilgan tenglamalar sistemasi yechimlari qism fazosining o'lchovi va bazisini toping:

$$\begin{cases} x_1 + x_2 - x_3 + x_4 = 0, \\ x_1 - x_2 + x_3 - x_4 = 0, \\ 3x_1 + x_2 - x_3 + x_4 = 0, \\ 3x_1 - x_2 + x_3 - x_4 = 0. \end{cases}$$

Yechish:

Matritsaning rangini aniqlaymiz:

$$A=\begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & 1 & -1 & 1 \\ 3 & -1 & 1 & -1 \end{pmatrix}.$$

3-qatordan 2-nchini, 4-satrdan esa 1-nchini ayiramiz:

$$A\sim\begin{pmatrix}1 & 1 & -1 & 1\\ 1 & -1 & 1 & -1\\ 2 & 2 & -2 & 2\\ 2 & -2 & 2 & -2\end{pmatrix}.$$

3-qatorning mos elementlari 1-qatorning mos elementlariga proporsional, 4-qatorning mos elementlari esa 2-qatorning mos elementlariga proporsional bo'lganligi uchun, 3 va 4-qatorlarni o'chirish mumkin:

$$A\sim\begin{pmatrix}1 & 1 & -1 & 1\\ 1 & -1 & 1 & -1\end{pmatrix}.$$

Shunday qilib, A matritsaning rangi 2 ga teng va

$$k=n-2=4-2=2.$$

Demak, yechimlar qism fazosining o'lchovi 2 ga teng. $r=2$ bo'lganligi uchun, to'rtta tenglamadan ikkitasini olamiz:

$$\begin{cases}x_1+x_2-x_3+x_4=0,\\ x_1-x_2+x_3-x_4=0\end{cases} \quad \text{yoki} \quad \begin{cases}x_1+x_2=x_3-x_4,\\ x_1-x_2=-x_3+x_4.\end{cases}$$

$x_3=1$, $x_4=0$ deb faraz qilib, $\begin{cases}x_1+x_2=1,\\ x_1-x_2=-1\end{cases}$ sistemani hosil qilamiz. Shuning uchun, $x_1=0$, $x_2=1$ va $\overline{f_1}=(0;\ 1;\ 1;\ 0)$ bo'ladi.

Endi $x_3=1$, $x_4=0$ deb faraz qilsak bo'ladi. Shunday qilib, $x_4=0$, $x_2=-1$ va $\overline{f_2}=(0;\ -1;\ 0;\ 1)$.

Qism fazosining bazis vektorlari sifatida $\overline{f_1}=(0;\ 1;\ 1;\ 0)$, $\overline{f_2}=(0;\ -1;\ 0;\ 1)$ vektorlarni qabul qilish mumkin. Tenglamalar sistemasining umumiy yechimi $\overline{f}=c_1\overline{f_1}+c_2\overline{f_2}$ vektor bilan aniqlanadi, ya'ni $\overline{f}=(0;\ c_1-c_2;\ c_1;\ c_2)$.

522. Tenglamalar sistemasining yechimlari qism fazosining o'lchovi, bazisi va umumiy yechimini aniqlang:

$$\begin{cases}x_1+2x_2+x_3+x_4+x_5=0,\\ x_1-2x_2+x_3+x_4-x_5=0.\end{cases}$$

1. Umumiy tushunchalar

Agar har bir $\overline{x} \in R$ vektorga biror qoidaga ko'ra $A\overline{x} \in R$ vektor mos kelsa, chiziqli fazo R da *A almashtirish berilgan* deyiladi.

Agar har qanday $\overline{x}$ va $\overline{y}$ vektorlar uchun va har qanday λ haqiqiy son uchun $A(\overline{x}+\overline{y})=A\overline{x}+A\overline{y}$, $A(\lambda\overline{x})=\lambda A\overline{x}$ tengliklar bajarilsa, *A almashtirish chiziqli* deyiladi. Agar u har qanday $\overline{x}$ vektorni o'zini o'ziga almashtirsa, *ayniy almashtirish* deyiladi. Ayniy chiziqli almashtirishlar E bilan belgilanadi. Shunday qilib, $E\overline{x}=\overline{x}$.

523. $A\overline{x}=\alpha\overline{x}$ almashtirishning chiziqli ekanligini ko'rsating, bunda α haqiqiy son.

Yechish:

Almashtirishning berilishiga ko'ra:

$$A(\overline{x}+\overline{y})=\alpha(\overline{x}+\overline{y})=\alpha\overline{x}+\alpha\overline{y}=A\overline{x}+A\overline{y},$$

$$A(\lambda\overline{x})=\alpha(\lambda\overline{x})=\lambda(\alpha\overline{x})=\lambda A\overline{x}.$$

Shunday qilib, chiziqli almashtirishning ikkala sharti ham bajarilayapti.

Bundan qaralayotgan A almashtirish chiziqlidir.

524. A almashtirish R chiziqli fazoda $A\overline{x}=\overline{x}+\overline{x}_0$ tenglik bilan aniqlangan, bu yerda $\overline{x}_0 \in R$ fiksirlangan noldan farqli vektor. A almashtirish chiziqli bo'ladimi?

Yechish:

$$A(\overline{x})=\overline{x}+\overline{x}_0, A(\overline{y})=\overline{y}+\overline{x}_0, A(\overline{x}+\overline{y})=\overline{x}+\overline{y}+\overline{x}_0,$$

$$A(\overline{x}+\overline{y})=A(\overline{x})+A(\overline{x})$$

tengliklardan $\overline{x}+\overline{y}+\overline{x}_0=(\overline{x}+\overline{x}_0)+(\overline{y}+\overline{x}_0)$ degan xulosaga kelamiz. Bundan esa $\overline{x}_0=0$ ni olamiz, lekin bu shartga ziddir. Demak, A almashtirish chiziqli emas.

525. Geometrik vektorlarning chiziqli fazosi berilgan.

Almashtirish har bir vektorni Ox o'qi bo'yicha tuzuvchilarga almashtirishdan iborat. Bu almashtirish chiziqli bo'ladimi?

Yechish:

Faraz qilaylik, $\overline{a}=x_1\overline{i}+y_1\overline{j}+z_1\overline{k}$ va $\overline{b}=x_2\overline{i}+y_2\overline{j}+z_2\overline{k}$ ixtiyoriy vektorlar, λ esa ixtiyoriy haqiqiy son bo'lsin.

$\bar{a}+\bar{b}=(x_1+x_2)\bar{i}+(y_1+y_2)\bar{j}+(z_1+z_2)\bar{k}$, $\lambda\bar{a}=\lambda x_1\bar{i}+\lambda y_1\bar{j}+\lambda z_1\bar{k}$ bo'lganligi uchun, $A(\bar{a}+\bar{b})=(x_1+x_2)\bar{i}=x_1\bar{i}+x_2\bar{i}=A\bar{a}+A\bar{b}$, $A(\lambda\bar{a})=Ax_1\bar{i}=\lambda A\bar{a}$ bo'ladi. Demak, A – chiziqli almashtirish.

526. xOy koordinata tekisligiga nisbatan har bir geometrik vektorni uni simmetrik akslantirishga almashtirish chiziqli almashtirish bo'ladi.

527. Har bir geometrik vektorni o'zining uzunligiga ko'paytirish chiziqli almashtirish bo'ladimi?

528. Agar $A\bar{x}=\bar{x}_0$ bo'lib, x element R chiziqli fazoning ixtiyoriy vektori, $\bar{x}_0$ esa fiksirlangan vektor bo'lsin. A almashtirish qanday hollarda chiziqli bo'ladi?

529. $\bar{x}=\xi_1\bar{e}_1+\xi_2\bar{e}_2+\xi_3\bar{e}_3+\xi_4\bar{e}_4$ vektorlar chiziqli fazosi berilgan, bunda $\xi_1, \xi_2, \xi_3, \xi_4$ turli haqiqiy sonlar. A – fiksirlangan haqiqiy son. $A\bar{x}=\xi_1\bar{e}_1+\xi_2\bar{e}_2+\xi_3\bar{e}_3+\xi_4\bar{e}_4$ tenglik bilan aniqlanadigan A almashtirish chiziqli bo'ladimi?

530. $\bar{x}=\xi_1\bar{e}_1+\xi_2\bar{e}_2+\xi_3\bar{e}_3+\xi_4\bar{e}_4$ vektorlarning chiziqli fazosi berilgan A almashtirish har bir vektorning ikkinchi va uchinchi koordinatalari o'rinlarini almashtirishdan iborat, ya'ni $A\bar{x}=\xi_1\bar{e}_1+\xi_2\bar{e}_2+\xi_3\bar{e}_3+\xi_4\bar{e}_4$ A alimashtirish chiziqli bo'ladimi?

531. A chiziqli almashtirishning matritsasi. $B\vec{x}=A\vec{x}-2\vec{x}$ tenglik bilan aniqlanadigan B almashtirish chiziqli bo'lishini isbotlang.

2. Chiziqli almashtirishning matritsasi.

Bazisi $\bar{e}_1$, $\bar{e}_2$, ..., $\bar{e}_n$ bo'lgan, n o'lchovli chiziqli fazo R da, A chiziqli almashtirish berilgan bo'lsin. $A\bar{e}_1$, $A\bar{e}_2$, ... , $A\bar{e}_n$ R fazosining vektorlari bo'lgani uchun, ularning har birini yagona usul bilan vektorlar bo'yicha yoyish mumkin:

$$A\overline{e_1}=a_{11}\overline{e_1}+a_{21}\overline{e_2}+\ldots+a_{n1}\overline{e_n},$$
$$A\overline{e_2}=a_{12}\overline{e_1}+a_{22}\overline{e_2}+\ldots+a_{n2}\overline{e_n},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$A\overline{e_n}=a_{1n}\overline{e_1}+a_{2n}\overline{e_2}+\ldots+a_{nn}\overline{e_n},$$

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

matritsa $\overline{e}_1$, $\overline{e}_2$, ..., $\overline{e}_n$ bazisdagi chiziqli almashtirishning matritsasi deiladi. Bu matritsaning ustunlari bazis vektorlarini almashtirish formulalarining koeffitsientlaridan tuzilgan. R fazoda qandaydir $\overline{x}=x_1\overline{e}_1+x_2\overline{e}_2+\dots+x_n\overline{e}_n$ vektorni olamiz. $A\overline{x}\in R$ bo'lganligi uchun, $A\overline{x}$ vektorni ham bazis vektori bo'yicha yoyish mumkin: $A\overline{x}=x_1'\overline{e}_1+x_2\overline{e}_2+\dots+x_n'e_n \bullet A\overline{x}$ vektorning $(x'_1, x'_2, \dots, x'_n)$ koordinatalari $\overline{x}$ vektorning $(x_1, x_2, \dots, x_n)$ koordinatalari orqali quyidagi formulalar bo'yicha ifodalanadi:

$$x'_1=a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n,$$
$$x'_2=a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n,$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$x'_n=a_{n1}x_1+a_{n2}x_2+\dots+a_{nn}x_n.$$

Bu n ta tenglikni $\overline{e}_1$, $\overline{e}_2$, ..., $\overline{e}_n$ bazisdagi A chiziqli almashtirish deyish mumkin. Bu chiziqli almashtirish formulalarining koeffitsientlari A matritsa yo'llarining elementlari bo'ladi.

532. n o'lchovli fazoda E ayniy almashtirishning matritsasini toping.

Yechish:

Ayniy almashtirish bazis vektorlarini o'zgartirmaydi:

$$\overline{e}_1'=\overline{e}_1,\ \overline{e}_2'=\overline{e}_2,\ \dots,\overline{e}_n'=\overline{e}_n,$$

ya'ni

$$\overline{e}_1'=1\cdot\overline{e}_1+0\cdot\overline{e}_2+\dots+0\cdot\overline{e}_n,$$
$$\overline{e}_1'=0\cdot\overline{e}_1+1\cdot\overline{e}_2+\dots+0\cdot\overline{e}_n,$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\overline{e}_n'=0\cdot\overline{e}_1+0\cdot\overline{e}_n+\dots+1\cdot\overline{e}_n.$$

Shunday qilib, birlik matritsa chiziqli almashtirishning matritsasi bo'lib xizmat qiladi:

$$E=\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}.$$

533. n o‘lchovli fazoda $A\overline{x}=\alpha\cdot\overline{x}$ o‘xshash almashtirishning matritsasini toping.

To‘rt o‘lchovli chiziqli fazoda A chiziqli almashtirish qaraladi. Agar $A\ell_1=\ell_3+\ell_4$, $A\ell_2=e_1+e_4$, $Ae_3=e_4+e_2$, $Ae_4=e_2+e_3$ bo‘lsa, bu almashtirishni koordinatalar formasida yozing.

Yechish:

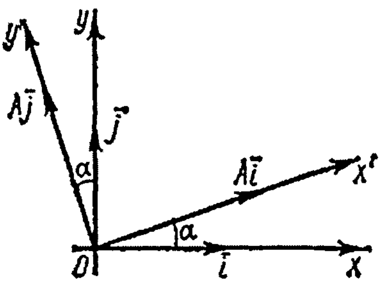
A almashtirishning matritsasi quyidagi ko‘rinishda bo‘ladi:

$$A=\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}.$$

Shuningdek, A almashtirish koordinata formasida yoziladi:

$$x'_1=x_2+x_3\,,\quad x'_2=x_3+x_4\,,\quad x'_3=x_1+x_4\,,\quad x'_4=x_1+x_2\,.$$

534. xOy tekisligidagi barcha vektorlar to‘plamining chiziqli almashtirishi har bir vektorni soat strelkasiga teskari yo‘nalishda α burchakka burilishdan iborat (22-rasm). Bu chiziqli almashtirish matritsasini koordinata formasida toping.



22-chizma

Yechish:

$A\overline{i}=\overline{i}\cos\alpha+\overline{j}\sin\alpha,$

$A\overline{j}=-\overline{i}\sin\alpha+\overline{j}\cos\alpha$ bo‘lganligi uchun

$$A=\begin{pmatrix} \cos\alpha-\sin\alpha \\ \sin\alpha+\cos\alpha \end{pmatrix}$$

bo'ladi. Shunday qilib, qaralayotgan chiziqli almashtirish quyidagi ko'rinishda bo'ladi:

$$x'=x\cos\alpha+y\sin\alpha;\ y'=x\sin\alpha+\cos\alpha.$$

536. $\overline{x}=x_1\overline{e}_1+x_2\overline{e}_2+x_3\overline{e}_3+x_4\overline{e}_4$ vektorlarning chiziqli fazosi qaraladi, bunda x_1, x_2, x_3, x_4 — turli haqiqiy sonlar. $A\overline{x}=x_1\overline{e}_1+x_2\overline{e}_2+x_3\overline{e}_3+x_4\overline{e}_4$ tenglik bilan aniqlangan A almashtirishning chiziqli ekanligini isbotlang va uning matritsasini toping.

3. Chiziqli almashtirishlar ustida amallar.

Quyida keltirilgan ta'riflarda quyidagi belgilashlarni qabul qilamiz: A va B — R chiziqli fazodagi ixtiyoriy chiziqli almashtirishlar, λ — ixtiyoriy haqiqiy son, $\overline{x}\in R$ — ixtiyoriy element.

$C_1\overline{x}=A\overline{x}+B\overline{x}$ tenglik bilan aniqlanadigan C_1 almashtirishni A va B *chiziqli almashtirishning yig'indisi* deyiladi va quyidagicha belgilanadi: $C_1=A+B$.

$C_2\overline{x}=\lambda A\overline{x}$ tenglik bilan aniqlanadigan C_2 almashtirish *A chiziqli almashtirishni λ songa ko'paitirish* deyiladi. Belgilash: $C_2=\lambda A$.

$C_3\overline{x}=AB\overline{x}$ tenglik bilan aniqlanadigan C_3 almashtirish *A chiziqli almashtirishni B chiziqli almashtirishga ko'paytmasi* deyiladi.

C_1, C_2 va C_3 almashtirishlar chiziqli bo'ladi. C_1, C_2 va C_3 chiziqli almashtirishning matritsasi $C_1=A+B$, $C_2=\lambda A$, $C_3=AB$ tengliklardan aniqlanadi.

Chiziqli almashtirishni qo'shishda o'rin almashtirish qonuni bajariladi; umumiy aytganda, AB ko'paytma BA ko'paytmadan farq qiladi.

R fazodagi chiziqli almashtirish ustidagi amallarning ba'zi xossalarini sanab o'tamiz:

$$A(BC)=(AB)C;\ AE=EA=A;\ (A+B)C=AC+BC;$$
$$C(A+B)=CA+CB.$$

Agar A chiziqli almashtirish uchun shunday B va C chiziqli almashtirishlar topilsaki, $BA=E$, $AC=E$ tengliklar bajarilsa, u holda $B=C$ bo'ladi. Bu holda $B=C=A^{-1}$ bilan belgilanadi, A^{-1} chiziqli almashtirish esa *A chiziqli almashtirishga nisbatan teskari* *chiziqli almashtirish* deyiladi. Shunday qilib, $A^{-1}A=AA^{-1}=E$.

Agar chekli o'lchovli fazoda A chiziqli almashtirish matritsasining determinanti noldan farqli bo'lsa, A chiziqli almashtirish *maxsusmas* deyladi.

Ixtiyoriy A maxsusmas chiziqli almashtirish A^{-1} teskari almashtirishga ega va faqat bitta ekanligini hisobga olish kerak.

Agar A maxsusmas chiziqli almashtirish koordinata formasida quyidagi tengliklar bilan aniqlansa:

$$
\begin{aligned}
x' &= a_{11}x + a_{12}y + \ldots + a_{1n}u, \\
y' &= a_{21}x + a_{22}y + \ldots + a_{2n}u, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
u' &= a_{n1}x + a_{n2}y + \ldots + a_{nn}u
\end{aligned}
$$

u holda A^{-1} teskari chiziqli almashtirish quyidagi ko'rinishda bo'ladi:

$$
\begin{aligned}
x &= \frac{A_{11}}{|A|}x' + \frac{A_{21}}{|A|}y' + \ldots + \frac{A_{n1}}{|A|}u', \\
y &= \frac{A_{12}}{|A|}x' + \frac{A_{22}}{|A|}y' + \ldots + \frac{A_{n2}}{|A|}u', \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
u &= \frac{A_{1n}}{|A|}x' + \frac{A_{2n}}{|A|}y' + \ldots + \frac{A_{nn}}{|A|}u'
\end{aligned}
$$

Bunda A_{ij} – A matritsaning a_{ij} elementining algebraik to'ldiruvchisi, $|A|$– A matritsaning determinanti.

A^{-1} chiziqli almashtirishning matritsasi A matritsaga nisbatan teskari bo'ladi va quyidagi tenglik bilan aniqlanadi:

$$
A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & \ldots & A_{n1} \\ A_{12} & A_{22} & \ldots & A_{n2} \\ \ldots & \ldots & \ldots & \ldots \\ A_{1n} & A_{2n} & \ldots & A_{nn} \end{pmatrix}.
$$

537. A almashtirish xOy tekislikdagi har bir vektorni $\alpha = \frac{\pi}{4}$

burchakka burishdan iborat. $A+E$ almashtirishni koordinata formasini toping.

Yechish: $$A\bar{i}=\bar{i}\cos(\frac{\pi}{4})+\bar{j}\sin(\frac{\pi}{4})=(\frac{\sqrt{2}}{2})\bar{i}+(\frac{\sqrt{2}}{2})\bar{j};$$

$$A\bar{j}=\bar{i}\cos(\frac{3\pi}{4})+\bar{j}\sin(\frac{3\pi}{4})=-(\frac{\sqrt{2}}{2})\bar{i}+(\frac{\sqrt{2}}{2})\bar{j}$$

bo‘ladi.

Shuningdek,
$$A=\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}.$$

$E=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ bo‘lganligi uchun:

$$A+E=\begin{pmatrix}\frac{\sqrt{2}}{2}+1 & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}+1\end{pmatrix}$$

bo‘ladi.

Shunday qilib, $A+E$ chiziqli almashtirishni

$$x'=(\frac{\sqrt{2}}{2}+1)x-(\frac{\sqrt{2}}{2})y, \quad y'=(\frac{\sqrt{2}}{2})x+(\frac{\sqrt{2}}{2}+1)y$$

tengliklar yordamida yozish mumkin.

538. Ikkita chiziqli almashtirish berilgan:

$$\begin{aligned}x'&=x+2y+3z,\\ y'&=4x+5y+6z,\\ z'&=7x+8y+9z,\end{aligned}\quad (A) \text{ va } \quad \begin{aligned}x'&=x+3y+4,5z,\\ y'&=6x+7y+9z,\\ z'&=10,5x+12y+13z.\end{aligned}\quad (B)$$

$3A-2B$ ni toping.

539. Chiziqli almashtirishlar berilgan:

$$\begin{aligned}x'&=x+y,\\ y'&=y+z,\\ z'&=z+x,\end{aligned}\quad (A) \text{ va } \quad \begin{aligned}x'&=x+y,\\ y'&=y+z,\\ z'&=z+x.\end{aligned}\quad (B)$$

AB va BA almashtirishlarni toping.

Yechish: Berilgan almashtirishlarning matritsasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 0 & 1\end{pmatrix},\qquad \mathrm{B}=\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}.$$

Bu matritsalarning ko'paytmasini topamiz:

$$AB=\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 1\\ 1 & 2 & 1\end{pmatrix},\qquad BA=\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 1\\ 1 & 2 & 1\end{pmatrix}.$$

Bu holda AB=BA, shuning uchun AB va BA chiziqli almashtirishlar ustma-ust tushadi. AB almashtirishning koordinat formasi quyidagicha yoziladi:

$$x'=x+y+2z,$$
$$y'=2x+y+z,$$
$$z'=x+2y+z.$$

540. xOy tekisligidagi $\bar{u}=x\bar{i}+y\bar{j}$ vektorlar to'plami ustida ikkita chiziqli almashtirish bajariladi: A – vektorni uning Ox o'qi bo'yicha tuzuvchisiga almashtirish; B – vektorni I va III koordinata burchaklarining bissektrisasiga nisbatan simmetrik akslantirish. AB va BA almashtirishlarni toping.

Yechish:

Shartga ko'ra $A\bar{u}=x\bar{i}$, $B\bar{u}=x\bar{i}+y\bar{j}$. Shunday qilib, $A\bar{i}=\bar{i}$, $A\bar{j}=0$, $B\bar{i}=\bar{j}$, $B\bar{j}=\bar{i}$.

ya'ni

$$A=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix},\ B=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ AB=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix},\ BA=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}.$$

Demak, AB almashtirish $x'=y,\ y'=0$ tengliklar bilan, BA almashtirish esa $x'=y,\ y'=0$ tengliklar bilan aniqlanadi. Bu tengliklarni geometrik mulohazalardan hosil qilishni tavsiya etamiz.

541. A almashtirish xOy tekislikdagi har bir vektorni α burchakka burishdan iborat. A^2 (ya'ni $A\cdot A$) almashtirishning matritsasini toping.

Yechish:

$$A\bar{i} = \bar{i}\cos\alpha + \bar{j}\sin\alpha,$$
$$A\bar{j} = -\bar{i}\sin\alpha + \bar{j}\cos\alpha$$

bo'lgani uchun

$$A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

bo'ladi. Shunga ko'ra:

$$A^2 = \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & -2\cdot\sin\alpha\cdot\cos\alpha \\ 2\cdot\sin\alpha\cdot\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}.$$

Demak, koordinata formasida A^2 almashtirish

$$y' = x\sin 2\alpha + y\cos 2\alpha,$$
$$x' = x\cos 2\alpha - y\sin 2\alpha$$

tengliklar bilan aniqlanadi. Bu natijalarni sof geometrik mulohazalardan ham hosil qilish mumkin.

542. A almashtirish xOy tekislikdagi har bir vektorni $\frac{\pi}{4}$ burchakka burishdan iborat. Chiziqli almashtirishning $B = A^2 + \sqrt{2}A + E$ matritsasini toping.

Yechish:

$$A = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix};$$

$$B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} = 2\cdot\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

543. Geometrik vektorlar fazosi berilgan. A almashtirish fazoni Oz o'qi atrofida $\frac{\pi}{4}$ burchakka burishdan iborat, B chiziqli almashtirish esa fazoni Ox o'qi atrofida xuddi shu burchakka burish bo'lsin. AB chiziqli almashtirishning matritsasini toping.

Yechish: $A\cdot\bar{i} = \bar{i}\cdot\cos\left(\frac{\pi}{4}\right) + \bar{j}\cdot\sin\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)\cdot\bar{i} + \left(\frac{\sqrt{2}}{2}\right)\cdot\bar{j}$

$$A\cdot\bar{j}=-\bar{i}\cdot\sin\left(\frac{\pi}{4}\right)+\bar{j}\cdot\cos\left(\frac{\pi}{4}\right)=-\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{i}+\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{j},$$

$$A\bar{k}=\bar{k},\qquad B\cdot\bar{i}=\bar{i},\qquad B\bar{j}=\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{j}+\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{k},$$

$$B\cdot\bar{k}=-\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{j}+\left(\frac{\sqrt{2}}{2}\right)\cdot\bar{k}.$$

Shunga ko'ra:

$$A=\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0\\ 0 & 0 & 1\end{pmatrix},\quad B=\begin{pmatrix}1 & 0 & 0\\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix},\quad AB=\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{1}{2}\\ \frac{\sqrt{2}}{2} & \frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}.$$

544. Chiziqli almashtirish berilgan:

$$x'=-0,5(y+z),\quad y'=-0,5(x+z),\quad z'=-0,5(x+y).$$

Teskari chiziqli almashtirishning matritsasini toping.

545. Barcha geometrik vektorlar to'plami qaraladi. A chiziqli almashtirish bu vektorlarni P tekislikka nisbatan simmetrik akslantirish. A^{-1} ni toping.

546. Bazisi $\bar{e}_1, \bar{e}_2$ bo'lgan chiziqli fazoda A chiziqli almashtirish berilgan. Agar $A\bar{e}_1=\bar{e}_2$, $A\bar{e}_2=\bar{e}_1$ bo'lsa, teskari almashtirishning matrisasini toping.

547. A chiziqli almashtirish xOy tekislikdagi har bir vektorni α burchakka burishdan iborat. $B=A+A^{-1}$ matrisani toping.

548. A: $x'=x+y$, $y'=2(x+y)$ chiziqli almashtirish berilgan. Teskari chiziqli almashtirishni toping.

549. A chiziqli almashtirish xOy tekislikdagi har bir vektorni $\frac{\pi}{4}$ burchakka burishdan iborat. A^{-2} matritsani toping.

550. λ ning qanday qiymatlarida $x'=-2x+y+z$, $y'=x-2y+z$, $z'=x+y\lambda+z$ chiziqli almashtirishning teskarisi bo'lmaydi.

4. Chiziqli almashtirishning xarakteristik sonlari va xos vektorlari.

Faraz qilaylik, R — berilgan n o'lchovli chiziqli fazo bo'lsin. Nol bo'lmagan $\overline{x} \in R$ vektor *A chiziqli almashtirishning xos vektori* deyiladi. Agar $\overline{x} = \lambda \overline{x}$ tenglikni qanoatlantiradigan λ son topilsa, λ soni esa chiziqli almashtirishning *$\overline{x} \in R$ vektoriga mos xarakteristik soni* deyiladi.

Agar $\overline{e}_1, \overline{e}_2, ..., \overline{e}_n$ bazisdagi A chiziqli almashtirish matritsaga ega bo'lsa:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix},$$

u holda quyidagi

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn}-\lambda \end{vmatrix} = 0$$

ko'rinishda yozish mumkin bo'lgan n-tartibli tenglamaning $\lambda_1, \lambda_2, ..., \lambda_n$ haqiqiy ildizlari A chiziqli almashtirishning xarakteristik sonlari bo'ladi. Bu tenglama *xarakteristik tenglama* deb atalib, uning chap tomoni esa *A chiziqli almashtirishning xarakteristik ko'phadi* deyiladi. Koordinatalari quyidagi

$$\begin{cases} (a_{11}-\lambda_k)\cdot\xi_1 + a_{22}\cdot\xi_2 + \dots + a_{1n}\cdot\xi_n = 0, \\ a_{21}\cdot\xi_1 + (a_{22}-\lambda_k)\cdot\xi_2 + \dots + a_{2n}\cdot\xi_n = 0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\cdot\xi_1 + a_{n2}\cdot\xi_2 + \dots + (a_{nn}-\lambda_k)\cdot\xi_n = 0 \end{cases}$$

bir jinsli chiziqli tenglamalar sistemasini qanoatlantiradigan har qanday $\xi_1\overline{e}_1+\xi_2\overline{e}_2+....+\xi_n\overline{e}_n$ vektor, λ_k xarakteristik songa mos keladigan $\overline{x}_k$ xos vektor bo'ladi.

Quyidagi muhim teoremalarni keltiramiz:

Chiziqli almashtirishning xarakteristik ko'phadi bazisni tanlab olishga bog'lik emas.

Agar A chiziqli almashtirishning matritsasi A simmetrik bo'lsa, u holda $|A-\lambda\cdot \mathrm{E}|=0$ *xarakteristik tenglamaning barcha ildizlari haqiqiy sonlar bo'ladi.*

551. $x'=5x+4y$, $y'=6x+9y$ tenglamalar bilan aniqlanadigan A chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini toping.

Yechish: Almashtirishning matritsasi quyidagicha yoziladi:

$$A=\begin{pmatrix}5 & 4\\ 8 & 9\end{pmatrix}.$$

Xarakteristik tenglama esa quyidagi ko'rinishda bo'ladi:

$$\begin{vmatrix}5-\lambda & 4\\ 8 & 9-\lambda\end{vmatrix}=0$$

yoki $\lambda^2-14\lambda+13=0$.

Uning ildizlari $\lambda_1=1$, $\lambda_2=13-$ xarakteristik sonlar.

Xos vektorlarning koordinatalarini aniqlash uchun ikkita chiziqli tenglamalar sistemasini hosil qilamiz:

$$\begin{cases}(5-\lambda_1)\xi_1+4\xi_2=0,\\ 8\xi_1+(9-\lambda_1)\xi_2=0,\end{cases}\qquad \begin{cases}(5-\lambda_2)\xi_1+4\xi_2=0,\\ 8\xi_1+(9-\lambda_2)\xi_2=0.\end{cases}$$

$\lambda_1=1$ bo'lganligi uchun birinchi sistemani quyidagicha yozish mumkin:

$$\begin{cases}4\xi_1+4\xi_2=0.\\ 8\xi_1+8\xi_2=0.\end{cases}$$

Shunday qilib, ξ_1 va ξ_2 qiymatlar $\xi_1+\xi_2=0$ tenglamani qanoatlantirishi kerak yoki $\xi_2=-\xi_1$. Shunga ko'ra, bu sistemaning yechimi $\xi_1=c_1\xi_2=-c_1$ ko'rinishda bo'ladi. Bunda c_1- ixtiyoriy miqdor. Shuning uchun $\lambda=1$ xarakteristik songa $\overline{u}=c_1(\overline{e}_1-\overline{e}_2)$ xos vektorlar oilasi mos keladi, ya'ni $\overline{u}=c_1(\overline{e}_1-\overline{e}_2)$.

$\lambda_2 = 13$ qiymat quyidagi tenglamalar sistemasiga keltiradi.

$$\begin{cases} -8\cdot\xi_1 + 4\cdot\xi_2 = 0, \\ -8\cdot\xi_1 - 4\cdot\xi_2 = 0, \end{cases}$$

ya'ni $\xi_2 = 2\xi_1$. $\xi_1 = c_2$ deb faraz qilib, $\xi_2 = 2c_2$ ni hosil qilamiz. Shuningdek, $\lambda = 13$ xarakteristik songa $\overline{v} = c_2(\overline{e}_1 + 2\overline{e}_2)$ xos vektorlar oilasi mos keladi.

Demak, $\overline{u} = c_1(\overline{e}_1 - \overline{e}_2), \overline{v} = c_2(\overline{e}_1 + 2\overline{e}_2)$ tengliklarda c_1 va c_2 miqdorlarga turli son qiymatlarini berib, A chiziqli almashtirishning turli xos vektorlarini topamiz.

552. Matritsasi $A = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$ bo'lgan chiziqli almashtirish berilgan. Bu almashtirishning xarakteristik sonlarini va xos vektorlarini toping.

553. Matritsasi $A = \begin{pmatrix} 6 & -4 \\ 4 & -2 \end{pmatrix}$ bo'lgan chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini toping.

554. Matritsasi $A = \begin{pmatrix} a & -b \\ b & -a \end{pmatrix}$ bo'lgan chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini toping.

555. Matritsasi $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ bo'lgan chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini aniqlang.

Yechish: Xarakteristik tenglamani tuzamiz:

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0,$$

ya'ni

$$(1-\lambda)\cdot\left[(2-\lambda)^2 - 1\right] = 0,\ (1-\lambda)^2\cdot(3-\lambda) = 0,\ \lambda_{1,2} = 1,\ \lambda_3 = 3.$$

Agar $\lambda = 1$ bo'lsa, xos vektorning kordinatalarini aniqlash uchun tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \xi_1 - \xi_2 + \xi_3 = 0, \\ -\xi_1 + \xi_2 - \xi_3 = 0, \\ \xi_3 = 0. \end{cases}$$

Shunday qilib, $\lambda = 1$ xarakteristik songa $\overline{u} = c_1(e_1 - e_2)$ xos vektorlar oilasi mos keladi.

Agar $\lambda = 3$ bo'lsa, xos vektorning koordinatalarini aniqlash uchun tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} -\xi_1 - \xi_2 + \xi_3 = 0, \\ -\xi_1 - \xi_2 + \xi_3 = 0, \\ \xi_3 = 0. \end{cases}$$

Bu xarakteristik songa mos keladigan xos vektorlar oilasi $\overline{v} = c_2\left(\overline{e}_1 - \overline{e}_2\right)$ tenglikdan aniqlanadi.

556. Matritsasi $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ bo'lgan chiziqli almashtirishning xarakteristik sonlarini va xos vektorlarini aniqlang.

557. Agar $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ simmetrik matritsa va α, β, γ esa noldan farqli haqiqiy sonlar bo'lsa, u holda

$$A = \begin{pmatrix} a_{11} & a_{12}\dfrac{\alpha}{\beta} & a_{13}\dfrac{\alpha}{\gamma} \\ a_{21}\dfrac{\beta}{\alpha} & a_{22} & a_{22}\dfrac{\beta}{\gamma} \\ a_{31}\dfrac{\gamma}{\alpha} & a_{32}\dfrac{\gamma}{\beta} & a_{33} \end{pmatrix}$$

matritsa xarakteristik tenglamasining hamma ildizlari haqiqiy sonlar bo'lishini isbotlang.

Yechish: Matritsasi A bo'lgan $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$ chiziqli almashtirishni qaraymiz. U holda:

$$A\overline{e}_1 = a_{11}\cdot\overline{e}_1 + \left(a_{21}\cdot\frac{\beta}{\alpha}\right)\cdot\overline{e}_2 + \left(a_{31}\cdot\frac{\gamma}{\alpha}\right)\cdot\overline{e}_3,$$

$$A\overline{e}_2 = \left(a_{12}\cdot\frac{\alpha}{\beta}\right)\cdot\overline{e}_1 + a_{22}\cdot\overline{e}_2 + \left(a_{33}\cdot\frac{\gamma}{\beta}\right)\cdot\overline{e}_3$$

$$A\overline{e}_3 = \left(a_{13}\cdot\frac{\alpha}{\gamma}\right)\cdot\overline{e}_1 + \left(a_{23}\cdot\frac{\beta}{\gamma}\right)\cdot\overline{e}_2 + a_{33}\cdot\overline{e}_3$$

yoki

$$A(\alpha\cdot\overline{e}_1) = a_{11}\cdot\alpha\cdot\overline{e}_1 + a_{21}\cdot\beta\cdot\overline{e}_2 + a_{31}\cdot\gamma\cdot\overline{e}_3$$
$$A(\beta\cdot\overline{e}_1) = a_{12}\cdot\alpha\cdot\overline{e}_1 + a_{22}\cdot\beta\cdot\overline{e}_2 + a_{32}\cdot\gamma\cdot\overline{e}_3$$
$$A(\gamma\cdot\overline{e}_1) = a_{13}\cdot\alpha\cdot\overline{e}_1 + a_{23}\cdot\beta\cdot\overline{e}_2 + a_{33}\cdot\gamma\cdot\overline{e}_3$$

ga ega bo‘lamiz. $\alpha\cdot e_1 = \overline{e}_1'$, $\beta\cdot e_2 = \overline{e}_2'$, $\gamma\cdot e_3 = \overline{e}_3'$ deb faraz qilib, quyidagini topamiz:

$$A\overline{e}_1' = a_{11}\overline{e}_1 + a_{21}\overline{e}_2 + a_{31}\overline{e}_3,$$

$$A\overline{e}_2' = a_{12}\overline{e}_1 + a_{22}\overline{e}_2 + a_{32}\overline{e}_3,$$

$$A\overline{e}_3' = a_{13}\overline{e}_1 + a_{23}\overline{e}_2 + a_{33}\overline{e}_3$$

Shunday qilib, $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$ bazisdagi A chiziqli almashtirishning matritsasi quyidagi simmetrik matritsa bo‘ladi:

$$A' = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

Shuningdek, $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$, bazisdagi A chiziqli almashtirishning xarakteristik tenglamasining yechimlari faqat haqiqiy sonlardan iborat bo‘ladi. $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$ bazisga o‘tilganda xarakteristik sonlar o‘zgarmaganligi uchun bu ildizlar A matritsaning xarakteristik tenglamasiga ham ildiz bo‘ladi.

558. A chiziqli almashtirish fazoni Oz o‘qi atrofida $\frac{\pi}{3}$ burchakka burishdan iborat. Bu almashtirishning xarakteristik sonlari va xos vektorlarini toping.

Yechish: Bu chiziqli almashtirishning matritsasi:

$$A=\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ekanligini ko‘rsating.

559. A chiziqli almashtirishning xarakteristik sonlarini bilgan holda A^{-1} teskari chiziqli almashtirishning xarakteristik sonlarini toping.

560. Matritsasi $A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ bo‘lgan chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini toping.

561. Matritsasi $A=\begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix}$ bo‘lgan chiziqli almashtirishning xarakteristik sonlari va xos vektorlarini toping.

5-§. EVKLID FAZOSI

Agar chiziqli fazo R dagi ixtiyoriy ikki vektor $\overline{x}$, $\overline{y}$ lar uchun $(\overline{x},\overline{y})$ bilan belgilanadigan skalyar ko‘paytma — haqiqiy sonni aniqlaydigan shunday qoida mavjud bo‘lsa, R chiziqli fazo *Evklid fazosi* deyiladi, shu bilan birga bu qoida quyidagi shartlarni kanoatlantiradi:

1. $(\overline{x},\overline{y})=(\overline{y},\overline{x})$;

2. $(\overline{x},\overline{y}+\overline{z})=(\overline{x},\overline{y})+(\overline{x},\overline{z})$;

3. $(\lambda\overline{x}, \overline{y}) = \lambda(\overline{x}, \overline{y})$ ixtiyoriy haqiqiy son uchun;

4. $(\overline{x}, \overline{x}) > 0$ agar $\overline{x} \neq 0$ bo'lsa.

1–4 shartlardan quyidagilar kelib chiqadi:

a) $(\overline{y}+\overline{z},\ \overline{x}) = (\overline{y},\ \overline{x}) + (\overline{z},\ \overline{x})$;

b) $(\overline{x},\ \lambda\overline{y}) = \lambda(\overline{x},\ \overline{y})$;

d) har qanday $\overline{x}$ vektor uchun $(\overline{0},\ \overline{x}) = 0$.

Har qanday $\overline{x} \in R$ vektorni o'ziga skalyar ko'paytmasi $\overline{x}$ *vektorning skalyar kvadrati* deyiladi.

Evklid fazosida $\overline{x}$ *vektorning uzunligi* deb, shu vektor skalyar kvadratining kvadrat ildiziga aytiladi, ya'ni $|\overline{x}| = \sqrt{(\overline{x},\ \overline{x})}$.

Agar λ har qanday haqiqiy son bo'lsa, $\overline{x}$ esa – Evklid fazosining ixtiyoriy vektori bo'lsa, u holda $|\lambda\overline{x}| = |\lambda| \cdot |\overline{x}|$ bo'ladi.

Uzunligi birga teng bo'lgan vektor *normallashtirilgan* deyiladi.

Agar $\overline{x} \in R$ – nol bulmagan vektor bo'lsa, $\frac{1}{|\overline{x}|} \cdot \overline{x}$ (*yoki* $\frac{\overline{x}}{|\overline{x}|}$ bilan belgilash mumkin) normallashtirilgan vektor bo'ladi.

Evklid fazosidagi har qanday ikkita $\overline{x}$ va $\overline{y}$ vektor uchun Koshi-Bunyakovskiy tengsizligi deb ataladigan, $(\overline{x},\ \overline{y})^2 \leq (\overline{x},\ \overline{x})(\overline{y},\ \overline{y})$ tengsizlik urinli bo'ladi.

$(\overline{x},\ \overline{y})^2 \leq (\overline{x},\ \overline{x})(\overline{y},\ \overline{y})$ tengsizlik o'rinli bo'ladi, shunda va faqat shundaki, $\overline{x}$ va $\overline{y}$ vektorlar chiziqli bog'liqsiz bo'lsa.

Koshi-Bunyakovskiy tengsizligidan $-1 \leq \frac{(\overline{x}, \overline{y})}{|\overline{x}| \cdot |\overline{y}|} \leq 1$ kelib chiqadi.

$\cos\varphi = \frac{(\overline{x}, \overline{y})}{|\overline{x}| \cdot |\overline{y}|}$ tenglikdan aniqlanadigan va $[0,\ \pi]$ kesmaga tegishli bo'lgan φ burchak, $\overline{x}$ va $\overline{y}$ *vektorlar orasidagi burchak* deyiladi.
Agar nol bulmagan $\overline{x}$ va $\overline{y}$ vektorlar uchun $\varphi = \frac{\pi}{2}$ bo'lsa u holda $(\overline{x}, \overline{y}) = 0$ bo'ladi. Bu holda $\overline{x}$ va $\overline{y}$ *vektorlar ortogonal* deyiladi

va $\bar{x} \perp \bar{y}$ ko'rinishda yoziladi. Evklid fazosining ixtiyoriy $\bar{x}$ va $\bar{y}$ vektorlari uchun quyidagi muhim munosabatlar o'rinli:

1) $|\bar{x}+\bar{y}| \leq |\bar{x}|+|\bar{y}|$ – uchburchak tengsizligi

2) φ– $\bar{x}$ va $\bar{y}$ vektorlar orasidagi burchak bo'lsin. U holda $|\bar{x}-\bar{y}|^2 = |\bar{x}|^2 + |\bar{y}|^2 - 2|\bar{x}|\cdot|\bar{y}|\cdot cos\varphi$ tenglik o'rinli bo'ladi (kosinuslar teoremasi). Agar $\bar{x} \perp \bar{y}$ bo'lsa, u holda $|\bar{x}-\bar{y}|^2 = |\bar{x}|^2 + |\bar{y}|^2$ tenglik hosil bo'ladi. Oxirgi tenglikda $\bar{y}$ ni $-\bar{y}$ ga almashtirib $|\bar{x}+\bar{y}|^2 = |\bar{x}|^2 + |\bar{y}|^2$ tenglikni hosil qilamiz (Pifagor teoremasi).

561. 461-masalada ko'rilgan chiziqli fazo berilgan bo'lsin. $\bar{x} = (\xi_1;\ \xi_2;\ ...;\ \xi_n)$ va $\bar{y} = (\eta_1;\ \eta_2;\ ...;\ \eta_n)$ ikkita ixtiyoriy vektorlarning skalyar ko'paytmasini $(\bar{x},\ \bar{y}) = \xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n$ tenglik bilan aniqlash mumkinmi (bu fazo Evklid fazosi bo'lishi uchun)?

Yechish: 1–4 shartlar bajarilishini tekshiramiz:

1) $(\bar{y},\ \bar{x}) = \eta_1\xi_1 + \eta_2\xi_2 + ... + \eta_n\xi_n$ bo'lganligi uchun $(\bar{x}, \bar{y}) = (\bar{y}, \bar{x})$.

2) $\bar{z} = (\zeta_1; \zeta_2;\ ...;\ \zeta_n)$ bo'lsin, u holda

$$\bar{y}+\bar{z} = (\eta_1+\xi_1;\ \eta_2+\xi_2;\ ...;\ \eta_n+\xi_n) \text{ va}$$

$$(\bar{x},\ \bar{y}+\bar{z}) = \xi_1\eta_1 + \xi_1\zeta_1 + \xi_2\eta_2 + \xi_2\zeta_2 + ... + \xi_n\eta_n + \xi_n\zeta_n 0 =$$

$$= (\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n) + (\xi_1\zeta_1 + \xi_2\zeta_2 + ... + \xi_n\zeta_n) = (\bar{x},\ \bar{y}) + (\bar{x},\ \bar{z}).$$

3) $(\lambda\bar{x},\ \bar{y}) = \lambda\xi_1\eta_1 + \lambda\xi_2\eta_2 + ... + \lambda\xi_n\eta_n =$

$$= \lambda(\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n) = \lambda(\bar{x},\ \bar{y})$$

4) $(\bar{x},\ \bar{x}) = \xi_1^2 + \xi^2{}_2 + ... + \xi_n^2 \neq 0$, agar $\xi_1,\ \xi_2,\ ...,\ \xi_n$ sonlardan hech bo'lmaganda bittasi noldan farqli bo'lsa.

Demak, berilgan fazoda ko'rsatilgan tengliklar yordamida skalyar ko'paytmani aniqlash mumkin.

563. 562-masalada ko'rsatilgan Evklid fazosi berilgan. $\xi_1, \xi_2, ..., \xi_n$ lar har kuni zavodda ishlab chiqariladigan n ta ko'rinishdagi mahsulotlarning miqdori, η_1, η_2, η_n esa mos ravishda bu mahsulotlarning narxi bo'lsin. $\overline{x}=(\xi_1, \xi_2, ..., \xi_n)$ va $\overline{y}=(\eta_1, \eta_2, ..., \eta_n)$ vektorlarning skalyar ko'paytmasini qanday ma'noda tushuntirish mumkin?

564. Vektorlari n ta musbat sonlardan tashkil topgan turli sistemalar bo'lgan chiziqli fazo berilgan:

$$\overline{x}=(\xi_1; \xi_2; ...; \xi_n),\ \overline{y}=(\eta_1, \eta_2, ..., \eta_n),\ \overline{z}=(\zeta_1; \zeta_2; ...; \zeta_n), ...$$

vektorlarni qo'shish va vektorlarni songa ko'paytirish $\overline{x}+\overline{y}=(\xi_1\eta_1, \xi_2\eta_2, ..., \xi_n\eta_n)$, $\lambda\overline{x}=(\xi_1^\lambda, \xi_2^\lambda, ..., \xi_n^\lambda)$ tengliklar bilan aniqlanadi. Skalyar ko'paytmani boshqa $(\overline{x}, \overline{y})=\ln\xi_1\ln\eta_1+\ln\xi_2\ln\eta_2+...+\ln\xi_n\ln\eta_n$ tenglik bilan aniqlab, bu fazoni Evklid fazosi qilish mumkinmi?

Yechish:

1–4 shartlarning bajarilishini tekshiramiz:

1. $(\overline{x}, \overline{y})=\ln\xi_1\ln\eta_1+\ln\xi_2\ln\eta_2+...+\ln\xi_n\ln\eta_n,$

$$(\overline{x}, \overline{y})=\ln\eta_1\ln\xi_1+\ln\eta_2\ln\xi_2+...+\ln\eta_n\ln\xi_n,$$

ya'ni $(\overline{x},\overline{y})=(\overline{y},\overline{x})$

2. $\overline{y}+\overline{z}=\eta_1\zeta_1; \eta_2\zeta_2; ...; \eta_n\zeta_n$ bo'lganligi uchun,

$$(\overline{x}, \overline{y}+\overline{z})=\ln\xi_1\ln(\eta_1\zeta_1)+\ln\xi_2\ln(\eta_2\zeta_2)+...+\ln\xi_n\ln(\eta_n\zeta_n)=$$
$$=\ln\xi_1\ln\eta_1+\ln\xi_2\ln\eta_2+...+\ln\xi_n\ln\eta_n+\ln\xi_1\ln\zeta_1+\ln\xi_2\ln\zeta_2+$$
$$+...+\ln\xi_n\ln\zeta_n=(\overline{x}, \overline{y})+(\overline{x},\overline{z}).$$

3. $\lambda\overline{x}=(\xi^\lambda{}_1; \xi^\lambda{}_2; ...; \xi^\lambda{}_n)$ bo'lganligi uchun

$$(\lambda\overline{x}, \overline{y})=\ln\xi^\lambda{}_1\ln\eta_1+\ln\xi_2{}^\lambda\ln\eta_2+...+\ln\xi^\lambda{}_n\ln\eta_n=$$
$$=\lambda(\ln\xi_1\ln\eta_1+\ln\xi_2\ln\eta_2+...+\ln\xi_n\ln\eta_n)=\lambda(\overline{x}, \overline{y}).$$

4. $(\overline{x}, \overline{x})=\ln^2\xi_1+\ln^2\xi_2+...+\ln^2\xi_n\geq 0.$

Demak, qaralayotgan fazo Evklid fazosi bo'lar ekan.

565. $[a,b]$ oraliqdagi $\overline{x}=\overline{x}(t)$, $\overline{y}=\overline{y}(t)$, $\overline{z}=\overline{z}(t), ...$ uzluksiz funksiyalarning chiziqli fazosi qaraladi. Har qanday ikki $\overline{x}$

va $\bar{y}$ vektorlarning skalyar ko'paytmasini $(\bar{x},\ \bar{y}) = \int_a^b \bar{x}(t)\cdot\bar{y}(t)dt$ tenglikdan aniqlab, bu fazoni Evklid fazosi kilish mumkinmi?

566. Agar ikki vektorning skalyar ko'paytmasini ularning uzunliklarining ko'paytmasi sifatida aniqlansa, barcha geometrik vektorlar tuplami Evklid fazosi bo'ladimi?

567. Agar ikkita ixtiyoriy $\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko'paytmasi $\bar{a}$ vektorning uzunligi va $\bar{b}$ vektorning $\bar{a}$ vektor yo'nalishidagi proyeksiyasi uchlanganligining ko'paytmasi sifatida aniqlansa, barcha geometrik vektorlar to'plami Evklid fazo bo'ladimi?

568. 562-masalada qaralgan chiziqli fazoda $n=4$ bo'lsin. $\bar{x} = (4;\ 1;\ 2;\ 2)$ va $\bar{y} = (1;\ 3;\ 3;\ -9)$ vektorlar orasidagi burchakni aniqlang.

Yechish:

$$|\bar{x}| = \sqrt{(\bar{x},\ \bar{x})} = \sqrt{16+1+4+4} = 5;$$

$$|\bar{y}| = \sqrt{(\bar{y},\ \bar{y})} = \sqrt{1+9+9+81} = 10;$$

$$(\bar{x},\ \bar{y}) = 4+3+6-18 = -5;$$

$$\cos\varphi = \frac{(\bar{x},\ \bar{y})}{|\bar{x}|\cdot|\bar{y}|} = \frac{-5}{5\cdot 10} = -0{,}1; \qquad \varphi = \arccos(-0{,}1) = 174^0 15'.$$

569. 562-masalada qaralgan Evklid fazosi berilgan. $\bar{x} = (1,\ \sqrt{3},\ \sqrt{5},\ ...,\ \sqrt{2n-1})$ va $\bar{y} = (1,\ 0,\ 0,\ ...,\ 0)$ vektorlar orasidagi burchakni aniqlang.

$[-1,\ 1]$ kesmada $\bar{x}(\mathrm{t})$, $\bar{y}(\mathrm{t})$, $\bar{z}(\mathrm{t})$, ... uzluksiz funksiyalarning Evklid fazosi qaraladi. Skalyar ko'paytma $(\bar{x},\ \bar{y}) = \int_{-1}^{1} \bar{x}(t)\cdot\bar{y}(t)dt$ tenglik bilan aniqlanadi. $\bar{x} = 3t^2 - 1$, $\bar{y} = 3t - 5t^3$ vektorlar orasidagi burchakni toping.

Yechish:

$(\bar{x},\ \bar{y}) = \int_{-1}^{1} (3t^2 - 1)\cdot(3t - 5t^3)dt$ bo'ladi. Integral ostidagi funksiya toq bo'lganligi uchun $(\bar{x},\ \bar{y})=0$ ekanligini ko'rish qiyin emas. Demak $\bar{x}$ va $\bar{y}$ vektorlar ortogonal.

571. 562-masalada ko'rilgan Evklid fazosi n=6 da berilgan. $\bar{x}=(1,0,2,0,2,0)$ va $\bar{y}=(0;\ 6;\ 0;\ 3;\ 0;\ 2)$ ortogonal vektorlar uchun Pifagor teoremasining o'rinli ekanligini tekshiring.

Yechish:

$$|\bar{x}|=\sqrt{1+0+4+0+4+0}=3,\quad |\bar{y}|=\sqrt{0+36+0+9+0+4}=7;$$

$$\bar{x}+\bar{y}=(1,6,2,3,2,2);\quad |\bar{x}+\bar{y}|=\sqrt{1+36+4+9+4+4}=\sqrt{58}.$$

Demak, $|\bar{x}|^2+|\bar{y}|^2=|\bar{x}+\bar{y}|^2$.

572. 565-masalaning shartlariga mos keladigan uzluksiz funksiyalarning Evklid fazosida ikkita vektor ko'riladi $\bar{x}=t^2+1$, $\bar{y}=\lambda t^2+1$. $[0,1]$ kesmada $\bar{x}$ va $\bar{y}$ vektorlar ortogonal bo'ladigan λ ning kiymatini toping va bu vektorlar uchun Pifagor teoremasining urinli ekanligini tekshiring.

Yechish: Skalyar ko'paytmani tuzamiz

$$(\bar{x},\ \bar{y})=\int_0^1(t^2+1)(\lambda t^2+1)dt=\lambda/5+(\lambda+1)/3+1.$$

$(\bar{x},\ \bar{y})=0$ shartdan λ ni aniqlaymiz: $\lambda/5+(\lambda+1)/3+1=0$, bo'ladi bundan $\lambda=-5/2$. Endi $x=t^2+1$, $y=-5/2\,t^2+1$ va $(\bar{x}+\bar{y})=-(3/2)t^2+2$ vektorlarning uzunligini topamiz;

$$|\bar{x}|=\sqrt{\int_0^1(t^4+2t^2+1)dt}=\sqrt{\frac{1}{5}+\frac{2}{5}+1}=\sqrt{\frac{28}{15}};$$

$$|\bar{y}|=\sqrt{\int_0^1\left(25/4\,t^4-5t^2+1\right)dt}=\sqrt{\frac{5}{4}-\frac{5}{3}+1}=\sqrt{\frac{7}{12}};$$

$$|\bar{x}+\bar{y}|=\sqrt{\int_0^1\left(\frac{9}{4}t^4-6t^2+4\right)dt}=\sqrt{\frac{9}{20}-2+4}=\sqrt{\frac{49}{20}};$$

Shunday qilib, $|\bar{x}|^2=\dfrac{28}{15}$, $|\bar{y}|^2=\dfrac{7}{12}$, $|\bar{x}+\bar{y}|^2=\dfrac{49}{20}$, ya'ni $|\bar{x}|^2+|\bar{y}|^2=|\bar{x}+\bar{y}|^2$.

573. $\bar{a}^*=(\bar{a}_1;\bar{a}_2;...;\bar{a}_n)$, $\bar{b}^*=(\bar{b}_1;\bar{b}_2;...;\bar{b}_n)$, ... turli tartiblangan geometrik vektorlar sistemasining to'plami qaraladi. Agar elementlarni qo'shish, elementlarni songa ko'paytirish va skalyar ko'paytma $\bar{a}^*+\bar{b}^*=(\bar{a}_1+\bar{b}_1;\bar{a}_2+\bar{b}_2;...;\bar{a}_n+\bar{b}_n)$, $\lambda\bar{a}^*=(\lambda\bar{a}_1;\lambda\bar{a}_2;...;\lambda\bar{a}_n)$, $(\bar{a}^*+\bar{b}^*)=\bar{a}_1\bar{b}_1+\bar{a}_2\bar{b}_2+...+\bar{a}_n\bar{b}_n$ tengliklar bilan aniqlanganda, bu to'plam Evkilid fazosi bo'ladimi? Ya'ni oxirgi tenglikning o'ng tomoni geometrik vektorlar skalyar ko'paytmasining yigindisini beradimi?

574. Tengsizliklarni o'rinli ekanligini ibotlang:

$$\sqrt{(\xi_1+\eta_1)^2+(\xi_2+\eta_2)^3+...+(\xi_n+\eta_n)^2}\le\sqrt{\xi^2{}_1+\xi^2{}_2+...+\xi^2{}_n}+\sqrt{\eta_1{}^2+\eta^2{}_2+...+\eta^2{}_n};$$

$(\xi_1{}^2+\xi^2{}_1+...+\xi^2{}_n)(\eta^2{}_1+\eta^2{}_2+...+\eta^2{}_n)\le(\xi_1\eta_1+\xi_2\eta_2+...+\xi_n\eta_n)^2$ bunda $\xi_1, \xi_2, \xi_3, ..., \xi_n; \eta_1, \eta_2, \eta_3, ..., \eta_n$ – haqiqiy sonlar.

Ko'rsatma: 562 masalada kurilgan Evklid fazosi uchun uchburchak va Koshi-Bunyakovskiy tengsizliklaridan foydalaning.

575. $[0;1]$ kesmada *x(t), y(t), ...* – turli uzluksiz funksiyalar qaraladi. $\sqrt{\int_0^1(x+y)^2dt}\le\sqrt{\int_0^1x^2dt}\sqrt{\int_0^1y^2dt}$, va agar $x(0)\ne0$ bo'lsa,

$$\int_0^1\left(\frac{y^2}{x^2}\right)dt\ge\frac{\left(\int_0^1 ydt\right)^2}{\left(\int_0^1 x^2dt\right)}$$ tengsizliklarning o'rinli ekanligini isbotlang.

6-§. ORTOGONAL BAZIS VA ORTOGONAL ALMASHTIRISHLAR

1. Ortogonal bazis.

Agar $i \neq k$ da $(\overline{e}_i; \overline{e}_k) = 0$ bo'lsa, Evklid fazosining $\overline{e}_1; \overline{e}_2; ...; \overline{e}_n$ bazisi *ortogonal* deyiladi. Quyidagi teorema o'rinli: *har qanday Evklid fazosi ortogonal bazisga ega.*

Agar ortogonal bazis normallashtirilgan vektorlardan tashkil topgan bo'lsa, u holda bu bazis *ortonormal* deyiladi. $e_1, e_2, e_3..., e_n$ – ortogonal bazis uchun quyidagi tenglik o'rinli:

$$(\overline{e}_i; \overline{e}_k) = \begin{cases} 0, i \neq k \text{ da,} \\ 1, i = k \text{ da.} \end{cases}$$

Agar n o'lchovli Evklid fazosida biror $\overline{f}_1 \overline{f}_2; ...; \overline{f}_n$ bazis ma'lum bo'lsa u holda bu fazoda har doim $\overline{e}_1; \overline{e}_2; ...; \overline{e}_n$ ortogonal bazisni ham topish mumkin. Ortogonal bazisda berilgan evklid fazosining har qanday $\overline{x}$ vektori $\overline{x} = \xi_1\overline{e}_1 + \xi_2\overline{e}_2 + ... + \xi_n\overline{e}_n$ tenglikdan aniqlanadi.

$\overline{x}$ vektorning uzunligi quyidagi formula bo'yicha topiladi:

$$|\overline{x}| = \sqrt{\xi^2{}_1 + \xi^2{}_2 + ... + \xi_n{}^2}.$$

Ikkita

$$\overline{x} = \xi_1\overline{e}_1 + \xi_2\overline{e}_2 + ... + \xi_n\overline{e}_n \text{ va } \overline{y} = \eta_1\overline{e}_1 + \eta_2\overline{e}_2 + ... + \eta_n\overline{e}_n$$

vektorlar chiziqli erkli (kollinear, proporsional) bo'ladi, faqat va faqat shundaki, agar $\frac{\xi_1}{\eta_1} = \frac{\xi_2}{\eta_2} = ... = \frac{\xi_n}{\eta_n}$, bo'lsa.

$\overline{x}$ va $\overline{y}$ vektorlarning ortogonallik sharti quyidagi ko'rinishda bo'ladi:

$$\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_m = 0.$$

Ikki $\overline{x}$ va $\overline{y}$ vektorlar orasidagi burchak quyidagi formula bo'yicha topiladi:

$$\cos\varphi \frac{\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n}{\sqrt{\xi^2{}_1 + \xi_2{}^2 + ... + \xi^2} * \sqrt{\eta^2{}_1 + \eta^2{}_2 + ... + \eta^2{}_n}}.$$

Quyidagi masalalarda n o'lchovli Evklid fazosining ortonormal bazisi $\overline{e}_1, \overline{e}_2, ..., \overline{e}_n$ – bilan belgilanadi.

576. $\overline{x} = 4\overline{e}_1 - 2\overline{e}_2 + 2\overline{e}_3 - \overline{e}_4$ vektorning uzunligini toping

577. $\overline{x} = \overline{e}_1 + 2\sqrt{2}\overline{e}_2 + 3\sqrt{3}\overline{e}_3 + 8\overline{e}_4 + 5\sqrt{5}\overline{e}_5$ vektorni normallashtiring.

578. Ortogonal bazis $\overline{e}_1; \overline{e}_2; \overline{e}_3$ dan $\overline{e}_1, \overline{e}_2, \overline{e}_3$ bazisga o'tish

matritsasi $A = \begin{pmatrix} \frac{-2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{-2}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{6}{7} & \frac{-2}{7} \end{pmatrix}$ berilgan. $\overline{e}_1; \overline{e}_2; \overline{e}_3$ bazis ortogonal

ekanligini isbotlang

579. $\overline{x} = \overline{e}_1 \sin^3\alpha + \overline{e}_2 \sin^2\alpha \cos\alpha + \overline{e}_3 \sin\alpha \cos\alpha + \overline{e}_4 \cos\alpha$ vektorni normallashtiring

580. $\overline{x} = \overline{e}_1\sqrt{7} + \overline{e}_2\sqrt{5} + \overline{e}_3\sqrt{3}$ va $\overline{y} = \overline{e}_1\sqrt{7} + \overline{e}_2\sqrt{5}$ vektorlar orasidagi burchakni toping

581. $\overline{x} = 3\cdot\overline{e}_1 - \overline{e}_2 - \overline{e}_3 - \overline{e}_4$, $\overline{y} = \overline{e}_1 - 3\cdot\overline{e}_2 + \overline{e}_3 + \overline{e}_4$, $z = \overline{e}_1 + \overline{e}_2 - 3\overline{e}_3 + \overline{e}_4$ vektorlarga ortogonal bo'lgan normallashtirilgan vektorni toping

582. λ ning qanday qiymatlarida $\overline{x} = \lambda\overline{e}_1 + \lambda\overline{e}_2 - \overline{e}_3 - \lambda\overline{e}_4$ va $\overline{y} = \overline{e}_1 - \overline{e}_2 - \lambda\overline{e}_3 - \overline{e}_4$ vektorlar bir xil uzunlikda bo'ladi?

583. To'rt o'lchovli fazoda $\overline{f}_1, \overline{f}_2, \overline{f}_3, \overline{f}_4$ bazis berilgan. Bu bazisning vektorlari yordamida shu fazoning ortonormal bazisini quring.

Yechish: Avval berilgan fazoda biror $\overline{g}_1, \overline{g}_2, \overline{g}_3, \overline{g}_4$ ortogonal bazisni quramiz. Faraz qilaylik, $\overline{g}_1 = \overline{f}_1$, $\overline{g}_2 = \overline{f}_2 + \alpha\overline{g}_1$ bo'lsin.

Shunday α haqiqiy sonni tanlab olamizki $\overline{g}_2 \perp \overline{g}_1$ shart bajarilsin. Oxirgi tenglikni ikkala tomonini $\overline{g}_1$ ga skalyar ko'paytiramiz va

$$(\overline{g}_1, \overline{g}_2) = (\overline{g}_1, \overline{f}_2) + \alpha(\overline{g}_1, \overline{g}_1)$$

tenglikni hosil qilamiz.

$(\bar{g}_1,\bar{g}_2)=0$ bo‘lganligi uchun, $\alpha=-(\bar{g}_1,\bar{f}_2)/(\bar{g}_1,\bar{g}_1)$ bo‘ladi.

So‘ngra $\bar{g}_3=\bar{f}_3+\beta_1\bar{g}_1+\beta_2\bar{g}_2$ tenglikda β_1 va β_2 ni shunday qilib tanlab olamizki, $\bar{g}_3\perp\bar{g}_1$,; $\bar{g}_3\perp\bar{g}_1$ shart bajarilsin.

$$(\bar{g}_1;\bar{g}_3)=(\bar{g}_1,\bar{f}_3)+\beta_1(\bar{g}_1;\bar{g}_1)+\beta(\bar{g}_1;\bar{g}_2),$$

$$(\bar{g}_2;\bar{g}_3)=(\bar{g}_2;\bar{f}_3)+\beta_1(\bar{g}_1;\bar{g}_2)+\beta_2(\bar{g}_2;\bar{g}_2)$$

tengliklardan, $\beta_1=-(\bar{g}_1;\bar{f}_3)/(\bar{g}_1;\bar{g}_1)$, $\beta_2=-(\bar{g}_2;\bar{f}_3)/(\bar{g}_2;\bar{g}_2)$

Va nihoyat $g_4=\bar{f}_4+\gamma_1\bar{g}_1+\gamma_2\bar{g}_2+\gamma_3\bar{g}_3$ tenglikdan

$$\gamma_1=-(\bar{g}_1;\bar{f}_4)/(\bar{g}_1;\bar{g}_1),\quad \gamma_2=-(\bar{g}_2;\bar{f}_4)/(\bar{g}_2;\bar{g}_2),$$

$$\gamma_3=-(\bar{g}_3;\bar{f}_4)/(\bar{g}_3;\bar{g}_3)$$

larni topamiz.

Shunday qilib, α, β_1, β_2, γ_1, γ_2, γ_3 larni yuqoridadek tanlab olganimizda, $\bar{g}_1$, $\bar{g}_2$ $\bar{g}_3$, $\bar{g}_4$ vektorlar juft-jufti bilan ortogonal vektorlar bo‘ladi. Demak,

$$\bar{e}_1=\frac{\bar{g}_1}{|\bar{g}_1|},\ \bar{e}_2=\frac{\bar{g}_2}{|\bar{g}_2|},\ \bar{e}_3=\frac{\bar{g}_3}{|\bar{g}_3|},\ \bar{e}_4=\frac{\bar{g}_4}{|\bar{g}_4|}$$

vektorlar ortogonallashgan bazis tashkil kiladi.

584. Darajasi ikkidan oshmagan ko‘phadlar to‘plami qaraladi. Ikki ko‘phadning skalyar ko‘paytmasi quyidagi tenglikdan aniqlanadi:

$$(\bar{x},\bar{y})=\int_0^1\bar{x}(t)\bar{y}(t)dt.$$

$\bar{f}_1=t^2$, $\bar{f}_2=t$, $\bar{f}_3=1$ bazisdan foydalanib va 583-masalada ko‘rilgan yechish usulidan fosdalanib, bu fazo uchun ortogonal bazisni quring.

Yechish: Avval $\bar{g}_1$, $\bar{g}_2$, $\bar{g}_3$ ortogonal bazisni ko‘ramiz. Faraz qilaylik $\bar{g}_1=\bar{f}_1$, ya’ni $\bar{g}_1=t^2$, $\bar{g}_2=\bar{f}_2+\alpha\bar{g}_1=t+\alpha t^2$. U holda

$\int_0^1 g_2 t^2 dt = \int_0^1 t^3 dt + \alpha \int_0^1 t^4 dt$. $\bar{g}_1$ va $\bar{g}_2$ vektorlarning ortogonalligidan oxirgi tenglikning chap tomoni nolga aylanadi. Shunday qilib $\alpha = \frac{-5}{4}$ va $\bar{g}_2 = t - \frac{5t^2}{4}$ Endi $\bar{g}_3$ ni topamiz. $\bar{g}_3 = 1 + \beta_1 t^2 + |\beta_2 \left(t - \frac{5t}{4}\right)$ tenglikda β_1 va β_2 ning qiymatlarini ortogonallik shartidan aniqlaymiz:

$\int_0^1 g_3 t^2 dt = 0;\ \int_0^1 g_3 \left(t - \frac{5}{4} t^2\right) dt = 0$. Shunday qilib, 0= $\int_0^1 t^2 dt + \beta_1 \int_0^1 t^4 dt$

va $0 = \int_0^1 \left(t - \frac{5}{4} t^2\right) dt + \beta_2 \int_0^1 \left(t - \frac{5}{4} t^2\right) dt$.

Bundan $\beta_1 = -\frac{5}{3};\ \beta_2 = -4;\ \bar{g}_3 = 1 - \frac{5t^2}{3} - 4\left(t - \frac{5t^2}{4}\right)$, ya'ni

$$\bar{g}_3 = 1 - 4t + \frac{10t^2}{3}.\ \text{Endi}\ \bar{g}_1 = t^2,\quad \bar{g}_2 = t - \frac{5t^2}{4}\ \text{va}$$

$\bar{g}_3 = 1 - 4t + \frac{10t^2}{3}$ vektorlarning uzunligini topamiz.

$$|\bar{g}_1| = \sqrt{\int_0^1 t^4 dt} = \frac{1}{\sqrt{5}},\quad |\bar{g}_2| = \sqrt{\int_0^1 \left(t - \frac{5}{4} t^2\right)^2 dt} = \frac{1}{4\sqrt{3}},$$

$$|\bar{g}_3| = \sqrt{\int_0^1 \left(1 - 4t + \frac{10}{3} t^2\right) dt} = \sqrt{\int_0^1 \left(1 - 8t + \frac{68}{3} t^2 - \frac{80}{3} t^3 + \frac{100}{9} t^4\right) dt} = \frac{1}{3}.$$

Shunday qilib,

$$\bar{e}_1 = \frac{\bar{g}_1}{|\bar{g}_1|} = \sqrt{5} t^2,\ \bar{e}_2 = \frac{\bar{g}_2}{|\bar{g}_2|} = \sqrt{3}\left(4 - 5t^2\right),$$

$$\bar{e}_3 = \frac{\bar{g}_3}{|\bar{g}_3|} = 3 - 12t + 10t^2$$

vektorlar ortonormal bazisni tashkil qiladi.

585. λ ning qanday qiymatlarida

$$\overline{g}_1=\lambda\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4,\qquad \overline{g}_2=\overline{e}_1+\lambda\overline{e}_2+\overline{e}_3+\overline{e}_4,$$
$$\overline{g}_3=\overline{e}_1+\overline{e}_2+\lambda\overline{e}_3+\overline{e}_4,\qquad \overline{g}_4=\overline{e}_1+\overline{e}_2+\overline{e}_3+\lambda\overline{e}_4.$$

vektorlardan tuzilgan bazis ortogonal bo'ladi? Bu bazisni normallashtiring.

Yechish: $(\overline{e}_i;\overline{e}_k)=0,\ (i\neq k)$ shartdan $\lambda+\lambda+1+1=0$ tenglamani hosil qilamiz. Shuningdek, $\lambda=-1$ va

$$\overline{g}_1=-\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4,\ \overline{g}_2=\overline{e}_1-\overline{e}_2+\overline{e}_3+\overline{e}_4,\ \overline{g}_3=\overline{e}_1+\overline{e}_2--\overline{e}_3+\overline{e}_4,$$
$$\overline{g}_4=\overline{e}_1+\overline{e}_2+\overline{e}_3-\overline{e}_4,\ |\overline{g}_i|=\sqrt{1+1+1+1}=2.$$

Shunday qilib,

$$\overline{e}_1=0{,}5\cdot(-\overline{e}_1+\overline{e}_2+\overline{e}_3+\overline{e}_4),\ \overline{e}_2=0{,}5\cdot(\overline{e}_1-\overline{e}_2+\overline{e}_3+\overline{e}_4),$$
$$\overline{e}_3=0{,}5\cdot(\overline{e}_1+\overline{e}_2-\overline{e}_3+\overline{e}_4),;\ \overline{e}_4=0{,}5\cdot(\overline{e}_1+\overline{e}_2+\overline{e}_3-\overline{e}_4)$$

ortonormal bazis tashkil qiladi.

586. α va β ning qanday qiymatlarida

$$\overline{e}_1'=\frac{\alpha}{3}\overline{e}_1+\frac{1-\alpha}{3}\overline{e}_2+\beta\cdot\overline{e}_3\qquad \overline{e}_2'=\frac{1-\alpha}{3}\overline{e}_1+\beta\overline{e}_2+\frac{\alpha}{3}\overline{e}_3\qquad \text{va}$$

$\overline{e}_3'=\beta\overline{e}_1+\frac{\alpha}{3}\overline{e}_2+\frac{1-\alpha}{3}\overline{e}_3$ vektorlardan tashkil topgan bazis ortonormal bo'ladi?

Yechish: $|\overline{e}_i|=1,\ (\overline{e}_1\quad \overline{e}_k)=0\ (i\neq k)$ da shartlardan tenglamalar sistemasini hosil qilamiz:

$$\begin{cases}\alpha^2+(1-\alpha)^2+9\beta^2=0,\\ \alpha(1-\alpha)+3(1-\alpha)\beta+3\alpha\beta=0.\end{cases}$$

Oxirgi tenglamadan $\beta=-\alpha\cdot(\alpha-1)/3$ ni topamiz. β ning bu qiymatini birinchi tenglamaga qo'yib, quyidagilarni hosil qilamiz: $\alpha^2+(1-\alpha)^2+\alpha^2(1-\alpha^2)^2=9;\ 1-2(1-\alpha)\alpha+\alpha^2(1-\alpha^2)=9;$ $(1-\alpha+\alpha^2)=9$. α ning haqiqiy qiymatlarida $1-\alpha+\alpha^2>0$

bo‘lganligi uchun, $1-\alpha+\alpha^2=3$, ya’ni $\alpha^2-\alpha-2=0$. Shuningdek $\alpha_1=-1;\ \alpha_2=2;\ \beta_1=\frac{-2}{3};\ \beta_2=\frac{2}{3}$.

Demak, ikkita ortonormal bazis hosil qilamiz:

$$\bar{e}_1^{(1)}=-\frac{1}{3}\bar{e}_1+\frac{2}{3}\bar{e}_2-\frac{2}{3}\bar{e}_3;\quad \bar{e}_2^{(1)}=\frac{2}{3}\bar{e}_1-\frac{2}{3}\bar{e}_2-\frac{1}{3}\bar{e}_3;$$

$$\bar{e}_3^{(1)}=-\frac{2}{3}\bar{e}_1-\frac{1}{3}\bar{e}_2+\frac{2}{3}\bar{e}_3.$$

$$\bar{e}_1^{(2)}=\frac{2}{3}\bar{e}_1-\frac{1}{3}\bar{e}_2+\frac{2}{3}\bar{e}_3;\quad \bar{e}_2^{(2)}=-\frac{1}{3}\bar{e}_1+\frac{2}{3}\bar{e}_2+\frac{2}{3}\bar{e}_3;$$

$$\bar{e}_3^{(2)}=\frac{2}{3}\bar{e}_1+\frac{2}{3}\bar{e}_2-\frac{1}{3}\bar{e}_3.$$

1. Ortogonal almashtirishlar.

Evklid fazodagi A chiziqli almashtirish ortogonal deyiladi, agar u bu fazodagi har qanday ikkita $\bar{x}$ va $\bar{y}$ vektorlarning skalyar ko‘paytmasini saqlasa, ya’ni $(A\bar{x},A\bar{y})=(\bar{x},\bar{y})$. Bunda $\bar{x}$ vektorning uzunligi o‘zgarmaydi, ya’ni $|A\bar{x}|=|\bar{x}|$. Shunday qilib,

$$\frac{\bar{x},\bar{y}}{|\bar{x}|\cdot|\bar{y}|}=\frac{(A\bar{x},A\bar{y})}{|A\bar{x}|\cdot|A\bar{y}|}$$

Oxirgi tenglikdan A chiziqli almashtirish har qanday ikkita $\bar{x}$ va $\bar{y}$ vektorlar orasidagi burchakni o‘zgartirmasligi kelib chiqadi. Ortogonal almashtirish ixtiyoriy ortonormal bazisni ortonormalga o‘tkazadi. Aksincha, agar chiziqli almashtirish biror ortonormal bazisni ortonormalga o‘tkazsa, u holda u ortogonal bo‘ladi.

587. Har bir geometrik vektorni biror fiksirlangan tekislikka nisbatan simmetrik vektorga o‘tkazadigan almashtirish ortogonal bo‘ladimi?

588. xOy tekisligida yotgan har qanday vektorni fiksirlangan α burchakka burishdan iborat bo‘lgan almashtirish ortogonal bo‘ladimi?

589. λ ning qanday qiymatlarida $A\overline{x}=\lambda\overline{x}$ tenglikdan aniqlanadigan A almashtirish ortogonal bo‘ladi?

590. Biror ortonormal $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$ bazisda

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

matrisa orqali aniqlangan A almashtirish, agar

$$a_{11}a_{12}+a_{21}a_{22}+a_{31}a_{32}=0\,, \qquad a_{11}a_{13}+a_{21}a_{23}+a_{31}a_{33}=0\,,$$

$$a_{12}a_{13}+a_{22}a_{23}+a_{32}a_{33}=0\,, \qquad a^2_{11}+a^2_{21}+a^2_{31}=1\,,$$

$$a^2_{12}+a^2_{22}+a^2_{32}=1\,, \qquad a^2_{13}+a^2_{23}+a^2_{33}=1$$

bo‘lsa, ortogonal bo‘ladimi?

591. $A\overline{x}=-\xi_1\overline{e}_1+\xi_2\overline{e}_2+\xi_3\overline{e}_3+\xi_4\overline{e}_4$ almashtirish ortogonal bo‘ladimi, bunda $\overline{x}=\xi_1\overline{e}_1+\xi_2\overline{e}_2+\xi_3\overline{e}_3+\xi_4\overline{e}_4$ ihtiyoriy vektorlar, $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$, $\overline{e}_4$ – esa ortonormal bazis?

592. $\overline{e}_1$, $\overline{e}_2$, $\overline{e}_3$, $\overline{e}_4$, $\overline{e}_5$, $\overline{e}_6$ – ortonormal bazis bo‘lsin. Agar $A\overline{e}_1=\overline{e}_2$, $A\overline{e}_2=-\overline{e}_2$, $A\overline{e}_3=\overline{e}_3\cos\alpha+\overline{e}_4\sin\alpha$,, $A\overline{e}_4=-\overline{e}_3\sin\alpha+\overline{e}_4\cos\alpha$, $A\overline{e}_5=\overline{e}_5\cos\beta+\overline{e}_6\sin\beta$, $A\overline{e}_6=-\overline{e}_5\sin\beta+\overline{e}_6\cos\beta$ bo‘lsa, A ortonormal almashtirish ekanligini isbotlang.

7-§. KVADRATIK FORMALAR

x_1, x_2, ... , x_p haqiqiy o‘zgaruvchilarning *kvadratik formasi* deb, birinchi darajali had va ozod had qatnashmagan, bu o‘zgaruvchilarga nisbatan ikkinchi darajali ko‘phadga aytiladi.

Agar $f(x_1, x_2, ..., x_n)$ – x_1, x_2, ..., x_n o‘zgaruvchilarning kvadratik formasi, λ esa qandaydir haqiqiy son bo‘lsa, u holda $f(\lambda x_1, \lambda x_2, ..., \lambda x_4)=\lambda^2 f(x_1, x_2, ..., x_n)$ bo‘ladi. Agar n=2 bo‘lsa, u holda $f(x_1, x_2) = a_{11}x^2{}_1 + a_{22}x^2{}_2 + 2a_{12}x_1x_2$. Agar $n=3$ bo'lsa, u holda

$$f(x_1, x_2, x_3) = a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$$

bo'ladi. Kelgusida barcha zaruriy ifodalashlar va ta'riflarni uch o'zgaruvchili kvadratik forma uchun keltiramiz.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

matritsada $a_{ik} = a_{ki}$ bo'lsa, $f(x_1, x_2, x_3)$ *kvadratik formaning matritsasi* deb ataladi, unga mos kelgan determinant esa *kvadratik formaning determinanti* deb ataladi. A – simmetrik matrisa bo'lganligi uchun

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0$$

xarakteristik tenglamaning λ_1, λ_2 va λ_3 ildizlari haqiqiy sonlar bo'ladi.

Faraz qilaylik,

$$\overline{e}_1' = b_{11}\overline{e}_1 + b_{21}\overline{e}_2 + b_{31}\overline{e}_3,$$
$$\overline{e}_2' = b_{12}\overline{e}_1 + b_{22}\overline{e}_2 + b_{32}\overline{e}_3,$$
$$\overline{e}_3' = b_{13}\overline{e}_1 + b_{23}\overline{e}_2 + b_{33}\overline{e}_3$$

vektorlar, $\overline{e}_1, \overline{e}_2, \overline{e}_3$ ortonormal bazisdagi $\lambda_1, \lambda_2, \lambda_3$ xarakteristik sonlarga mos keluvchi normallashtirilgan xos vektorlari bo'lsin.

O'z navbatida $\overline{e}_1', \overline{e}_2', \overline{e}_3'$ vektorlar ortonormal bazis tashkil etadilar.

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

matritsa esa $\overline{e}_1, \overline{e}_2, \overline{e}_3$ bazisdan $\overline{e}_1', \overline{e}_2', \overline{e}_3'$ bazisga o'tish matritsasi bo'ladi. Yangi ortonormal bazisga o'tishda koordinatalarni almashtirish formulalari quyidagi ko‘rinishda bo‘ladi:

$$x_1 = b_{11}x_1' + b_{12}x_2' + b_{13}x_3',$$
$$x_2 = b_{21}x_1' + b_{22}x_2' + b_{23}x_3',$$
$$x_3 = b_{31}x_1' + b_{32}x_2' + b_{33}x_3'.$$

Bu formulalar yordamida $f(x_1,\ x_2,\ x_3)$ kvadratik formani almashtirib, $x_1'x_2',\ x_1'x_3',\ x_2'x_3'$ ko‘paytmali hadlar kirmagan $f(x_1',\ x_2',\ x_3') = \lambda_1 x_1'^2 + \lambda_2 x_2'^2 + \lambda_3 x_3'^2$ kvadratik formani hosil qilamiz.

B ortogonal almashtirishlar yordamida $f(x_1,\ x_2,\ x_3)$ kvadratik forma *kanonik ko‘rinishga* keltirildi deb aytish qabul qilingan.

$\lambda_1,\ \lambda_2$ va λ_3 xarakteristik sonlar turli degan farazda mulohazalar yuritildi. Agar xarakteristik sonlar ichida bir xillari bo‘lsa nima qilish kerakligini masala yechish davomida ko‘rsatiladi.

593. $f = 27x_1^2 - 10x_1x_2 + 3x_2^2$ kvadratik formani kanonik *ko‘rinishga keltiring*

Yechish: Bunda $a_{11} = 27,\ a_{12} = -5,\ a_{22} = -3$. Xarakteristik tenglamani tuzamiz

$$\begin{vmatrix} 27-\lambda & -5 \\ -5 & 3-\lambda \end{vmatrix} = 0 \text{ yoki } \lambda^2 - 30\lambda + 56 = 0, \text{ ya'ni } \lambda_1 = 2,\ \lambda_2 = 28$$

xarakteristik sonlar. Xos vektorlarni aniqlaymiz.

Agar $\lambda = 2$ bo‘lsa, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} 25\xi_1 - 5\xi_2 = 0, \\ -5\xi_1 + \xi_2 = 0. \end{cases}$$

Bundan esa $\xi_2 = 5\xi_1$ ni hosil qilamiz. $\xi_1 = c$ deb olib, $\xi_2 = 5c$ ga bo‘lamiz, ya’ni xos vektor $\overline{u} = c(\overline{e}_1 + 5 \cdot \overline{e}_2)$ ga teng bo‘ladi.

Agar $\lambda = 28$ bo‘lsa, quyidagi sistemaga kelamiz:

$$\begin{cases} -\xi_1 - 5\xi_2 = 0, \\ -5\xi_1 - 25\xi_2 = 0. \end{cases}$$

Bu holda $\overline{v}=c\left(-5\cdot\overline{e}_1+\overline{e}_2\right)$ xos vektorni hosil qilamiz. $\overline{u}$ va $\overline{v}$ vektorlarni normallashtirish uchun s=$1/\sqrt{1^2+5^2}=1/\sqrt{26}$ deb qabul qilish kerak.

Demak, biz $\overline{e}_1'=\dfrac{\left(\overline{e}_1+5\overline{e}_2\right)}{\sqrt{26}}$, $\overline{e}_2'=\dfrac{\left(-5\overline{e}_1+\overline{e}_2\right)}{\sqrt{26}}$ normallashtirilgan xos vektorni topdik.

$\overline{e}_1$, $\overline{e}_2$ ortonormal bazisdan $\overline{e}_1'$, $\overline{e}_2'$ ortonormal bazisga o'tish matritsasi quyidagi ko'rinishga ega:

$$B=\begin{pmatrix} \frac{1}{\sqrt{26}} & \frac{-5}{\sqrt{26}} \\ \frac{5}{\sqrt{26}} & \frac{1}{\sqrt{26}} \end{pmatrix}.$$

Bundan koordinatalarni almashtirish formulalarini hosil qilamiz:

$$x_1=\frac{1}{\sqrt{26}}x_1'-\frac{5}{\sqrt{26}}x_2',\quad x_2=\frac{5}{\sqrt{26}}x_1'+\frac{1}{\sqrt{26}}x_2'.$$

Shunday qilib,

$$f=27\cdot\left(\frac{1}{\sqrt{26}}x_1'-\frac{5}{\sqrt{26}}x_2'\right)^2-10\cdot\left(\frac{1}{\sqrt{26}}x_1'-\frac{5}{\sqrt{26}}x_2'\right)\cdot$$

$$\cdot\left(\frac{5}{\sqrt{26}}x_1'+\frac{1}{\sqrt{26}}x_2'\right)+3\cdot\left(\frac{5}{\sqrt{26}}x_1'+\frac{1}{\sqrt{26}}x_2'\right)^2=2x_1'^2+28x_2'^2.$$

$f=\lambda_1 x_1'^2+\lambda_2 x_2'^2$ bo'lganligi uchun bu natijani birdaniga hosil qilish mumkin edi.

594. $f=2x_1^2+8x_1x_2+8x_2^2$ kvadratik formani kanonik ko'rinishga keltiring.

Yechish: Bunda $a_{11}=2,\ a_{12}=4,\ a_{22}=8$.

Xarakteristik tenglamani yechamiz:

$$\begin{vmatrix} 2-\lambda & 4 \\ 4 & 8-\lambda \end{vmatrix}=0,\quad \lambda_1=0,\ \lambda_2=10.$$

Xos vektorlarni aniqlaymiz.

$\lambda = 0$ da quyidagi sistemani hosil qilamiz:

$$\begin{cases} 2\xi_1 + 4\xi_2 = 0, \\ 4\xi_1 + 8\xi_2 = 0, \end{cases}$$

uning yechimlari $\xi_1 = 2c$, $\xi_2 = -c$ bo‘ladi, ya’ni $\bar{u} = c\cdot(2\bar{e}_1 - \bar{e}_2)$.

$\lambda = 10$ da quyidagi sistemani hosil qilamiz:

$$\begin{cases} -8\xi_1 + 4\xi_2 = 0 \\ 4\xi_1 - 2\xi_2 = 0 \end{cases}$$

uning yechimlari $\xi_1 = c, \xi_2 = 2c$, ya’ni $\bar{v} = c\cdot\left(\bar{e}_1 + 2\bar{e}_2\right)$.

$c = \dfrac{1}{\sqrt{2^2+1^2}} = \dfrac{1}{\sqrt{5}}$ deb qabul qilib $\vec{e}_1' = \dfrac{(2\bar{e}_1 - \bar{e}_2)}{\sqrt{5}}$,

$\vec{e}_2' = \dfrac{(\bar{e}_1 + 2\bar{e}_2)}{\sqrt{5}}$ normallashtirilgan xos vektorlarni topamiz.

Yangi bazisga o‘tish matritsasi (ortogonal almashtirish matritsasi) quyidagi ko‘rinishda bo‘ladi:

$$B = \begin{pmatrix} \dfrac{2}{\sqrt{5}} & \dfrac{1}{\sqrt{5}} \\ \dfrac{-1}{\sqrt{5}} & \dfrac{2}{\sqrt{5}} \end{pmatrix}.$$

Koordinatalarni almashtirish formulalari quyidagicha yoziladi:

$$x_1 = \frac{2}{\sqrt{5}}x_1' + \frac{2}{\sqrt{5}}x_2', x_2 = -\frac{1}{\sqrt{5}}x_1' + \frac{2}{\sqrt{5}}x_2'.$$

Shunga ko‘ra

$$f = 2\cdot\left(\frac{1}{\sqrt{5}}x_1' + \frac{1}{\sqrt{5}}x_2'\right)^2 + 8\cdot\left(\frac{2}{\sqrt{5}}x_1' + \frac{1}{\sqrt{5}}x\cdot_2\right)\cdot\left(-\frac{1}{\sqrt{5}}x_1' + \frac{2}{\sqrt{5}}x_2'\right) +$$

$$+8\cdot\left(-\frac{1}{\sqrt{5}}x_1' + \frac{2}{\sqrt{5}}x_2'\right)^2 = 10x_2'^2.$$

Bu masalani soddaroq yechish mumkin. $f=2(x_1+2x_2)^2$ ekanligini ko'rish qiyin emas. Shuning uchun

$x_2'=\frac{(x_1+2x_2)}{\sqrt{1+4}}=\frac{(x_1+2x_2)}{\sqrt{5}}$, $x_1'=\frac{(2x_1-x_2)}{\sqrt{5}}$ deb qabul qilish mumkin (ikkinchi tenglik almashtirishning ortogonalligini hisobga olib yozilgan). $x_1+2x_2=\sqrt{5}\cdot x_2'$ bo'lganligi uchun, $f=10x_2'^2$ bo'ladi.

595. $f=3x_1^2+2x_2^2+x_3^2+4x_1x_2+4x_2x_3$ kvadratik formani kanonik ko'rinishga keltiring.

Yechish: Bu yerda $a_{11}=3,\ a_{22}=2,\ a_{33}=1,\ a_{12}=2,\ a_{13}=0,\ a_{23}=2.$ Xarakteristik tenglamasini tuzamiz:

$$\begin{vmatrix} 3-\lambda & 2 & 0 \\ 2 & 2-\lambda & 2 \\ 0 & 2 & 1-\lambda \end{vmatrix}=0,$$

ya'ni $(3-\lambda)(2-\lambda)(1-\lambda)-4(1-\lambda)-4(3-\lambda)=0$. Uning yechimlari:

$$\lambda_1=2;\ \lambda_2=-1;\ \lambda_3=5.$$

Topilgan xarakteristik sonlarga mos xos vektorlarni aniqlaymiz. Xos vektorlarning koordinatalarini aniqlash uchun uchta chiziqli tenglamalar sistemasini hosil qilamiz:

1) $\lambda=2$, 2) $\lambda=-1$, 3) $\lambda=5$

$$\begin{cases}\xi_1+2\xi_2=0,\\ 2\xi_1+2\xi_3=0,\\ 2\xi_2-\xi_3=0;\end{cases}\qquad \begin{cases}4\xi_1+2\xi_2=0,\\ 2\xi_1+3\xi_2+2\xi_3=0,\\ 2\xi_2+2\xi_3=0;\end{cases}\qquad \begin{cases}-2\xi_1+2\xi_2=0,\\ 2\xi_1-3\xi_2+2\xi_3=0,\\ 2\xi_2-4\xi_3=0;\end{cases}$$

$$\begin{matrix}\xi_1=2c, & \xi_1=c, & \xi_1=2c,\\ \xi_2=-c, & \xi_2=-2c, & \xi_2=2c,\\ \xi_3=-2c, & \xi_3=2c, & \xi_3=c\end{matrix}$$

$$\bar{u}=c(2\bar{e}_1-\bar{e}_2-2\bar{e}_3),\quad \bar{v}=c(\bar{e}_1-2\bar{e}_2+2\bar{e}_3),\quad \bar{w}=c(2\bar{e}_1+2\bar{e}_2+\bar{e}_3)$$

$$\overline{e}_1'=\frac{1}{3}(2\overline{e}_1-\overline{e}_2-2\overline{e}_3),\quad \overline{e}_2'=\frac{1}{3}(\overline{e}_1-2\overline{e}_2+2\overline{e}_3),\quad \overline{e}_3'=\frac{1}{3}(2\overline{e}_1+2\overline{e}_2+\overline{e}_3)$$

Ortogonal almashtirish matritsasi quyidagi ko'rinishda bo'ladi:

$$B=\begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Koordinatalarni almashtirish formulalari quyidagicha:

$$x_1=\frac{2}{3}x_1'+\frac{1}{3}x_2'+\frac{2}{3}x_3',x_2=-\frac{1}{3}x_1'-\frac{2}{3}x_2'+\frac{2}{3}x_3',x_3=-\frac{2}{3}x_1'+\frac{2}{3}x_2'+\frac{1}{3}x_3'.$$

Shunga ko'ra: $f=2x_1'^2-x_2'^2+5x_3'^2$.

596. $f=6x_1^2+3x_2^2+3x_3^2+4x_1x_2+4x_1x_3-8x_2x_3$ kvadratik formani kanonik ko'rinishga keltiring.

Yechish: Bu yerda $a_{11}=6,\ a_{22}=3,\ a_{33}=3,\ a_{12}=2,\ a_{13}=2,\ a_{23}=-4$. Xarakteristik tenglamani yechib:

$$\begin{vmatrix} 6-\lambda & 2 & 2 \\ 2 & 3-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix}=0,$$

$\lambda_1=\lambda_2=7,\lambda_3=-2$ xarakteristik sonlarini topamiz. $\lambda=7$ da quyidagi sistemaga kelamiz:

$$\begin{cases} -\xi_1+2\xi_2+2\xi_3=0, \\ 2\xi_1-4\xi_2-4\xi_3=0, \\ 2\xi_1-4\xi_2-4\xi_3=0. \end{cases}$$

U bitta $\xi_1=2\xi_2+2\xi_3$ tenglamaga keltiriladi. Bu tenglamaning yechimini $\xi_1=2a+2b,\xi_2=a,\xi_3=b$ ko'rinishda yozish mumkin. Natijada ikkita a va b parametrlarga bog'liq bo'lgan $\overline{u}=2\cdot(a+b)\cdot\overline{e}_1+a\cdot\overline{e}_2+b\cdot\overline{e}_3$ xos vektorlar oilasini hosil qilamiz.

$\lambda=-2$ da quyidagi sistemani hosil qilamiz:

$$\begin{cases}8\xi_1+2\xi_2+2\xi_3=0,\\ 2\xi_1+5\xi_2-4\xi_3=0,\\ 2\xi_1-4\xi_2+5\xi_3=0.\end{cases}$$

Masalan, ikkita oxirgi tenglamani yechib $\frac{\xi_1}{9}=\frac{\xi_2}{-18}=\frac{\xi_3}{-18}$ yoki $\xi_1=-\frac{\xi_2}{2}=-\frac{\xi_3}{2}$, $\xi_1=c$, $\xi_2=-2c$, $\xi_3=-2c$ larni hosil qilamiz. Shunday qilib, $\overline{v}=c\cdot(\overline{e}_1-2\overline{e}_2-2\overline{e}_3)$ bir parametrli xos vektorlar oilasini topamiz. $\overline{u}=2\cdot(a+b)\cdot\overline{e}_1+a\cdot\overline{e}_2+b\cdot\overline{e}_3$ xos vektorlar oilasidan ikkita qandaydir ortogonal vektorni ajratamiz. Masalan, a=0, b=1 deb faraz qilib, $\overline{u}=2\cdot\overline{e}_1+\overline{e}_2$ xos vektorlarni hosil qilamiz. a va b parametrlarni shunday tanlaymizki, $(\overline{u},\overline{u}_1)=0$ tenglik bajarilsin. U holda $2\cdot2(a+b)+b=0$ tenglamani hosil qilamiz, ya'ni $4a+5b=0$ Endi $a=5$ b=−4 deb qabul qilish mumkin, bundan qaralayotgan oilaning boshqa xos vektorlarini topamiz: $\overline{u}_2=2\overline{e}_1+5\overline{e}_2-4\overline{e}_3$.

Demak, biz uchta o'zaro ortogonal vektorlarni hosil qildik: $\overline{u}_1=2\overline{e}_1+\overline{e}_3$, $\overline{u}_2=2\overline{e}_1+5\overline{e}_2-4\overline{e}_3$, $\overline{v}=\overline{e}_1-2\overline{e}_2-2\overline{e}_3$. $\overline{u}_1$ va $\overline{u}_2$ xos vektorlar $\lambda=7$ xarakteristik songa, $\overline{v}$ xos vektor s=1 da $\lambda=-2$ xarakteristik songa mos keladi. Bu vektorlarni normallashtirib, yangi ortonormal bazisni hosil qilamiz, bunda yangi bazisga o'tish matritsasi quyidagi ko'rinishda bo'ladi:

$$B=\begin{pmatrix}\frac{2}{\sqrt5} & \frac{2}{3\sqrt5} & \frac{1}{3}\\ 0 & \frac{\sqrt5}{3} & \frac{-2}{3}\\ \frac{1}{\sqrt5} & \frac{-4}{3\sqrt5} & \frac{-2}{3}\end{pmatrix}.$$

Berilgan kvadratik formaga

$$x_1 = \frac{2}{\sqrt{5}}x_1' + \frac{2}{3\sqrt{5}}x_2' + \frac{1}{3}x_3',\ x_2 = \frac{\sqrt{5}}{3}x_2' - \frac{2}{3}x_3',\ x_3 = \frac{1}{\sqrt{5}}x_1' - \frac{4}{3\sqrt{5}}x_2' - \frac{2}{3}x_3'$$

koordinatalarni almashtirish formulalarini qo'llab

$f = 7x_1'^2 + 7x_2'^2 - 2x_3'^2$ ni hosil qilamiz.

597. $17x^2 + 12xu + 8u^2 - 80 = 0$ egri chiziq tenglamasini kanonik ko'rinishga keltiring.

Yechish: Tenglamaning yuqori darajali hadlari guruhi, matrisasi

$$A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$$

bo'lgan $17x^2 + 12xu + 8u^2$ kvadratik formani hosil qiladi. Xarakteristik tenglamani tuzamiz:

$\begin{vmatrix} 17-\lambda & 6 \\ 6 & 8-\lambda \end{vmatrix} = 0$ yoki $\lambda^2 - 25\lambda + 100 = 0$. Uning yechimlari

$\lambda_1 = 5,\ \lambda_2 = 20$ xarakteristik sonlar. Shunga ko'ra $17x^2 + 12xu + 8u^2$

kvadratik forma $5x'^2 + 20y'^2$ kanonik ko'rinishga keladi, berilgan

tenglama esa quyidagi ko'rinishga keladi: $5x'^2 + 20y'^2 - 80 = 0$ yoki

$\frac{x'^2}{16} + \frac{y'^2}{4} = 1$, ya'ni berilgan egri chiziq ellips ekan.

Elipsning tenglamasini kanonik ko'rinishga keltiradigan bazisni topaylik, buning uchun xos vektorlarni aniqlaymiz.

$\lambda = 5$ da

$$\begin{cases} 12\xi_1 + 6\xi_2 = 0, \\ 6\xi_1 + 3\xi_2 = 0 \end{cases}$$

tenglamalar sistemasiga ega bo'lamiz va uni yechib $\xi_2 = -2\xi_1$ ni hosil qilamiz. $\xi_1 = c$ deb faraz qilib, $\xi_2 = -2c$ ni olamiz, ya'ni $\overline{u} = c \cdot (\overline{e}_1 - 2\overline{e}_2)$ xos vektor.

$$\lambda = 0 \text{ da} \quad \begin{cases} -3\xi_1 + 6\xi_2 = 0, \\ 6\xi_1 - 12\xi_2 = 0 \end{cases}$$

tenglamalar sistemasiga ega bo‘lamiz, bundan $\xi_1 = 2\xi_2$ ni hosil qilamiz, ya’ni $\overline{v} = c\cdot(2\overline{e}_1 + \overline{e}_2)$ xos vektor.

$c = \dfrac{1}{\sqrt{1^2+2^2}} = \dfrac{1}{\sqrt{5}}$ deb qabul qilib, $\overline{e}_1' = \dfrac{\overline{e}_1 - 2\overline{e}_2}{\sqrt{5}}$ va $\overline{e}_2' \dfrac{2\overline{e}_1 + \overline{e}_2}{\sqrt{5}}$ normallashtirilgan xos vektorlarni topamiz.

Ortogonal almashtirish matritsasi quyidagi ko‘rinishga ega:

$$B = \begin{pmatrix} \dfrac{1}{\sqrt{5}} & \dfrac{2}{\sqrt{5}} \\ \dfrac{-2}{\sqrt{5}} & \dfrac{1}{\sqrt{5}} \end{pmatrix}.$$

Koordinatalarni almashtirish formulalari quyidagicha yoziladi:

$$x = \frac{1}{\sqrt{5}}x' + \frac{2}{\sqrt{5}}y', \quad y = -\frac{2}{\sqrt{5}}x' + \frac{1}{\sqrt{5}}y'.$$

$$17x^2 + 12xy + 8y^2 - 80 = \frac{17}{5}(x' + 2y')^2 + \frac{12}{5}(x' + 2y')(-2x' + y') +$$

$$+\frac{8}{5}(-2x' + y')^2 - 80 = 5x'^2 + 20y'^2 - 80.$$

Bu natijani λ_1 va λ_2 lar topilganda darhol hosil qilish mumkin edi: $\lambda_1 x'^2 + \lambda_2 y'^2 - 8 = 0$.

Quyidagi egri chiziqlar tenglamalarini kononik ko‘rinishga keltiring:

598. $6x^2 + 2\sqrt{5}xy + 2y^2 - 21 = 0$.

599. $4xu + 3u^2 + 16 = 0$.

600. $5x^2 + 4\sqrt{6}xy + 7y^2 - 44 = 0$.

VI BOB
ANALIZGA KIRISH

1-§. ABSOLUT VA NISBIY XATOLIKLAR

Faraz qilaylik α – hisoblashlarda A aniq sonni almashtiradigan taqribiy son. α – taqribiy sonning absolut xatoligi deb, α taqribiy son va unga mos aniq A soni orasidagi ayirmaning absolut qiymatiga aytiladi: $|A-\alpha|$. $|A-\alpha|<\Delta$ tenglsizlikni qanoatlantiruvchi, mumkin bo‘lgan Δ kichik songa, limit absolut xatolik deyiladi.

A aniq son $a-\Delta \leq A \leq a+\Delta$ chegaralarda joylashadi, yoki $A=a\pm\Delta$. a taqribiy sonning nisbiy xatoligi deb, bu sonning absolut xatoligini mos aniq songa nisbatiga aytiladi:, $\frac{A-\alpha}{A}$. $\frac{A-\alpha}{A}\leq\delta$ tengsizlikni qanoatlantiruvchi, mumkin bo‘lgan δ dan kichik songa limit nisbiy xatolik deyiladi. Deyarli $A\approx\alpha$ bo‘lganligi uchun, limit nisbiy xatolik sifatida $\delta=\frac{\Delta}{a}$ son qabul qilinadi (odatda prosentlarda ifodalanadi). $\alpha(1-\delta)\leq A\leq\alpha(1+\delta)$ tengsizlik o‘rinli. O‘nli kasr ko‘rinishida yozilgan, musbat α taqribiy son absolut xatoligi n-xonasi birliklarining yarimdan oshmasa, bu son n ta ishonchli belgi (raqam) ga ega deyiladi.

$n>1$ da birinchi ishonchli raqami k bo‘lgan, α taqribiy sonning limit nisbiy xatoligi sifatida $\delta=\frac{1}{2\kappa}\left(\frac{1}{10}\right)^{\nu-1}$ sonni qabul qilish mumkin. Agar

$$\delta\leq\frac{1}{2(k+1)}\left(\frac{1}{10}\right)^{\nu-1} \tag{1}$$

ma’lum bo‘lsa, u holda a soni n ta ishonchli belgiga ega bo‘ladi.

Bir necha sonlar algebraik yig'indisining limit absolut xatoligi qo'shiluvchilar limit absolut xatoligining yig'indisiga teng. Musbat qo'shiluvchilar yig'indisining nisbiy xatoligi bu qo'shiluvchilar nisbiy xatoligining eng kattasidan oshmaydi. Taqribiy sonlar ko'paytmasining va yig'indisining limit nisbiy xatoligi bu sonlar limit nisbiy xatoligining yig'indisiga teng. Taqribiy son darajasining limit nisbiy xatoligi bu sonning limit nisbiy xatoligining daraja ko'rsatkichiga ko'paytmasiga teng.

601. Teodolitda o'lchangan burchak $22°20'30'' \pm 30''$ ga teng bo'lib chiqdi. O'lchamning nisbiy xatoligi qanday?

Yechish:

$\Delta = 30''$ absolut xatolik. U holda nisbiy xatolik

$$\delta = \frac{\Delta}{\alpha} = \frac{30''}{22°20'30''} \cdot 100\% = 0.04\% .$$

602. Nisbiy xatoligi 0,5% bo'lganda og'irlik kuchining tezlanishi g=0,806... ning ishonchli belgilar sonini aniqlang va taqribiy miqdorning mos yozuvini bering.

Yechish:

Birinchi qiymatli raqam 9 bo'lganligi uchun (I) tengsizlikdan foydalanib, $0,005 < \frac{1}{2\cdot 10}\left(\frac{1}{10}\right)^{v-1}$ ni hosil qilamiz. Ya'ni $n=2$. Demak, $g=9,8$

603. $\sqrt{19}$ sonining limit nisbiy xatoligi 0,1% ekanligi ma'lum. Bu songa nechta ishonchli belgilar kiradi?

Yechish:

Bunda birinchi ishonchli raqam 4 bo'ladi. $\delta = 0,001 = 10^{-3}$ limit nisbiy xatolik. (I) tengsizlikka asosan $0,001 \le \frac{1}{2\cdot 5}\cdot\left(\frac{1}{10}\right)^{n-1}$ bo'ladi. Bundan $n=3$ ni olamiz. Shunga ko'ra $\sqrt{19} = 4,36$ (to'rt xonali jadvaldan $\sqrt{19} = 4,3589$).

604. Agar nisbiy xatolik 1% ga teng bo'lsa, A=3,7563 son nechta ishonchli belgiga ega bo'ladi?

Yechish:

Birinchi ishonchli raqam 3 bo'ladi, shuning uchun

$$0,01 \le \frac{1}{2\cdot 4}\cdot\left(\frac{1}{10}\right)^{n-1},$$

bundan $n=2$. A sonni shunday yozish lozim: $A=3,8$.

605. Kvadratning yuzi 25,16 sm² ga teng (0,01 sm² gacha aniqlikda).

Kvadratning tomonini qanday nisbiy xatolik bilan va nechta ishonchli belgilari bilan aniqlash mumkin?

Yechish:

$x=\sqrt{25.16}$ izlanayotgan tomon. $\delta=(1(2)\,(0.01/25.16))$ kvadrat tomonining nisbiy xatoligi, bu yerda 0,01 yuzaning absolut xatoligi, ya'ni $\delta=0,0002$. Kvadratning o'lchanadigan tomoni uzunligining birinchi ishonchli raqami 5 ga teng. $k=5$ da (1) tengsizlikni echib, $(5+1)\cdot 0,0002<\frac{1}{10^{n-1}}$ ni hosil qilamiz, yoki $1,2\cdot 10^{-3}\le\frac{1}{10^{n-1}}$. Bundan $n=3$ ni olamiz.

606. Agar doiraning yuzi 124.35 sm² ga teng ekanligi ma'lum bo'lsa, (0,01 sm² gacha aniqlik bilan), uning radiusini nechta ishonchli belgilar bilan aniqlash mumkin?

607. Agar kesik konus asoslarining radiuslari $R=23,64\pm0,01$(sm), $r=17,31\pm0,01$ (sm), yasovchisi $\ell=10,21\pm0,01$ (sm), $\pi=3,14$ bo'lsa, uning to'la sirtini hisoblashlardagi limit nisbiy xatoligini toping.

608. $g=9,8066$ con beshta ishonchli belgili og'irlik kuchi tezlanishining (45° kenglik uchun) taqribiy qiymati bo'lsin. Uni nisbiy xatoligini toping.

609. Tomonlari $92,73\pm0,01$ va $94,5\pm0,01$ (sm) bo'lgan to'g'ri to'rtburchakning yuzini hisoblang. Natijaning nisbiy xatoligini va ishonchli belgilar sonini aniqlang.

2-§. BIR ERKLI O‘ZGARUVCHINING FUNKSIYASI

Ratsional va irrasional sonlar haqiqiy sonlar deyiladi. Barcha haqiqiy sonlar to‘plami R bilan belgilanadi. Har bir haqiqiy sonni sonlar o‘qidagi nuqtada tasvirlash mumkin.

X va Y ikkita bo‘sh bo‘lmagan to‘plamlar bo‘lsin. Agar X to‘plamning har bir x elementiga biror aniq qoidaga asosan Y ning yagona elementi u mos kelsa, bu holda X to‘plamda qiymatlar to‘plami Y bo‘lgan funksiya yoki asklantirish berilgan deyiladi. Bu funksiyaning ko‘rinishini shunday yozish mumkin: $x \in X, X \xrightarrow{f} Y$ yoki $f: x \to Y$ bunda X funksiyaning aniqlanish sohasi, $y = f(x)$ ko‘rinishdagi sonlardan tuzilgan Y to‘plam esa – funksiyaning qiymatlar to‘plami deyiladi. Agar u o‘zgaruvchi, x erkli o‘zgaruvchining funksiyasi bo‘lsa, u holda $y = f(x)$ yoki $y = \varphi(x)$ ko‘rinishda ham yoziladi. f va φ harflar shunday qoidani xarakterlaydiki, bunda berilgan x argumentga y ning qiymatlari mos keladi. f funksiyaning aniqlanish sohasi $D(f)$ bilan, qiymatlar to‘plami esa, $E(f)$ bilan belgilanadi. $f(x)$ funksiyaning $x=a$ dagi qiymati, bunda $a \in D(f)$, funksiyaning xususiy qiymati deyiladi va $f(a)$ bilan belgilanadi. Oddiy hollarda funksiyaning aniqlanish sohasi: $]a,b[$ interval (ochiq oraliq), ya’ni $a<x<b$ shartini qanoatlantiruvchi x ning qiymatlar to‘plami; $[a,b]$ segment (kesma yoki yopiq oraliq) ya’ni $a \leq x \leq b$ shartni qanoatlantiruvchi x ning qiymatlar to‘plami; $]a,b]$ (ya’ni $a \leq x \leq b$) yoki $[a,b[$ (ya’ni $a \leq x \leq b$) yarim interval, cheksiz interval $[a,+\infty[$ (ya’ni $a \leq x \leq b$) yoki $]-\infty,b[$ (ya’ni $-\infty < x \leq b$) yoki $]-\infty,+\infty[$ (ya’ni $-\infty \leq x \leq \infty$) bir necha intervallar va segmentlar to‘plami va h.k.

Quyidagi funksiyalar asosiy elementar funksiyalar deyiladi:

1. $y = x^{\alpha}$ darajali funksiya, bunda $x \in R$

2. $y = \alpha^{x}$ ko‘rsatkichli funksiya, bunda, a – birdan farqli ixtiyoriy musbat son: $a>0$, $a \neq 1$

3. $y=\log_a x$ logarifmik funksiya, bunda a – birdan farqli ixtiyoriy musbat son: $a>0$, $a \neq 1$.

4. $y=\sin x$, $y=\cos x$, $y=\mathrm{tg} x$, $y=\mathrm{ctg} x$, $y=\sec x$, $y=\mathrm{cosec} x$ trigonometrik funksiyalar:

5. $y=\arcsin x$, $y=\arccos x$, $y=\mathrm{arctg} x$, $y=\mathrm{arcctg} x$ teskari trigonometrik funksiyalar

Elementar funksiyalar deb asosiy elementar funksiyalardan to'rtta arifmetik amal va superpozisiyalash (ya'ni murakkab funksiyalarni hosil) qilishni, chekli son marta qo'llash yordamida hosil bo'ladigan funksiyalarga aytiladi. Haqiqiy son x ning absolut qiymati (moduli) elementar bo'lmagan funksiyaga misol bo'lib xizmat qiladi:

$$y=|x|=\begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$

$|x|$ geometrik nuqtai nazardan sonlar o'qida koordinatasi x bo'lgan nuqtadan sanoq boshigacha bo'lgan masofaga teng. Koordinatalari $(x, f(x))$ bo'lgan xOy tekislikdagi nuqtalar to'plami $y=f(x)$ funksiyaning grafigi deyiladi, bu yerda $x \in D(f)$

Agar har qanday $x \in D(f)$ uchun $f(-x)=f(x)$ [mos ravishda $f(-x)=-f(x)$] bo'lsa, aniqlanish sohasi nolga nisbatan simmetrik bo'lgan $f(x)$ funksiya, juft (toq) funksiya deyiladi. Juft funksiyaning grafigi ordinata o'qiga nisbatan, toq funksiyaning grafigi – koordinata boshiga nisbatan simmetrik bo'ladi. Agar shunday musbat T son mavjud bo'lsaki, $x \in D(f)$ va $(x+T) \in D$ da $f(x+T)=f(x)$ tenglik bajarilsa, $f(x)$ funksiya davriy funksiya deyiladi. Ko'rsatilgan hossalarga ega bo'lgan eng kichik musbat τ soniga funksiyaning asosiy davri deyiladi.

610. Agar $f(x)=x^2$ bulsa, $\dfrac{f(b)-f(a)}{b-a}$ ni toping.

Yechish:

Berilgan funksiyani $x=a$ va $x=b$ da qiymatini topamiz: $f(a)=a^2$, $f(b)=b^2$. U holda quyidagini hosil qilamiz:

$$\frac{f(b)-f(a)}{b-a}=\frac{b^2-a^2}{b-a}=a+b.$$

611. $f(x)=\dfrac{x-2}{2x-1}$ funksiyaning aniqlanish sohasini toping.

Yechish:

Agar $2x-1\neq 0$, ya'ni $x\neq 1/2$ bo'lsa, berilgan funksiya aniqlangan. Shunday qilib, funksiyaning aniqlanish sohasi quyidagi ikki intervalning birlashmasi bo'ladi: $D(f)=]-\infty,1/2[\cup]1/2,+\infty[$

612. $f(x)=\dfrac{\ln(1+x)}{x-1}$ funksiyaning aniqlanish sohasini toping.

Yechish:

Agar $x-1\neq 0$ va $1+x>0$, ya'ni agar $x\neq 1$ va $x>-1$ bo'lsa funksiya aniqlangan. Funksiyaning aniqlanish sohasi quyidagi ikki intervalning birlashmasi bo'ladi: $D(f)=]-1,1[\cup]1,+\infty[$

613. $f(x)=\sqrt{1-2x}+3\arcsin\dfrac{3x-1}{2}$ funksiyaning aniqlanish sohasini toping.

Yechish:

$1-2x\geq 0$ da birinchi qo'shiluvchi, $-1\leq(3x-1)/2\leq 0$ da esa ikkinchisi haqiqiy qiymatlarni qabul qiladi. Shunday qilib, berilgan funksiyaning aniqlanish sohasini topish uchun quyidagi tengsizlik sistemasini echish zarur:

$1-2x\geq 0$, $(3x-1)/2\leq 1$, $(3x-1)/2\geq -1$. Natijada $x\leq 1/2$, $x\leq 1$, $x\geq -1/3$ ni hosil qilamiz. Shunday qilib, funksiyaning aniqlanish sohasi $[-1/3,\ 1/2]$ kesma bo'ladi.

614. Funksiyalarning qiymatlar to'plamini toping:

1) $f(x)=x^2-6x+5$ 2) $f(x)=2+3\sin x$

Yechish:

1) Kvadratik uchxaddan to'la kvadrat ajratib, $f(x)=x^2-6x+$ $+9-4=(x-3)^2-4$ ni hosil qilamiz. O'ng tomonda turgan ifodaning birinchi xadi x ning barcha qiymatlarida musbat bo'lgani uchun funksiya -4 dan kichik bo'lmagan qiymat hosil qiladi. Shunday qilib funksiyaning qiymatlar sohasi $[-4,+\infty)$ ga teng.

2) Sinusning qiymatlari modul bo'yicha birdan oshmagani uchun $|\sin x|\leq 1$ yoki $-1\leq\sin x\leq 1$ tengsizlikni hosil qilamiz. Bu tengsizlikning ikkala tomonini 3 ga ko'paytirib va ularga 2 ni qo'shib, $-1\leq 2+3\sin x\leq 5$ ni hosil qilamiz. Shunday qilib $E(f)=[-1,5]$.

615. Funksiyalarning asosiy davrlarini toping.

1) $f(x)=\cos 8x$ 2) $f(x)=\sin 6x+tg4x$

Yechish:

1) $\cos x$ funksiyaning asosiy davri 2π bo‘lganligi uchun, $f(x)=\cos 8x$ funksiyaning asosiy davri $\frac{2\pi}{8}$ ga, yoki $\frac{\pi}{4}$ ga teng.

2) Bu yerda birinchi qo‘shiluvchining asosiy davri $\frac{2\pi}{6}=\frac{\pi}{3}$ ga teng, ikkinchisi uchun esa, u $\frac{\pi}{4}$ga teng. Berilgan funksiyaning asosiy davri $\frac{\pi}{3}$ va $\frac{\pi}{4}$ sonlarning eng kichik umumiy karralisiga, ya’ni π ga teng ekanligi ko‘rinib turibdi.

616. Funksiyalarning juft yoki toqligini aniqlang:

1) $f(x)=x^2\cdot\sqrt[3]{x}+2\sin x$; 2) $f(x)=2^x+2^{-x}$; 3) $f(x)=|x|-5e^{x^2}$;

4) $f(x)=x^2+5x$; 5) $f(x)=\lg\frac{x+3}{x-2}$.

Yechish:

Ko‘rilayotgan masalalarda har bir funksiyaning aniqlanish sohasi nolga nisbatan simmetrik: birinchi to‘rtta masalada $D(f)=(-\infty,+\infty)$, oxirgi masalada esa $D(f)=(-\infty,-3)\cup(3,\infty)$.

1) x ni $-x$ ga almashtirib

$f(-x)=(-x)^2\cdot\sqrt[3]{x}+2\sin(-x)=-x^2\sqrt[3]{x}-2\sin x$ ni hosil qilamiz, ya’ni $f(-x)=-f(x)$. Demak, berilgan funksiya toq funksiya ekan.

2) $f(-x)=2^{-x}+2^{-(-x)}=2^{-x}+2^{x}$ bo‘ladi, ya’ni $f(-x)=f(x)$ Demak, funksiya juft ekan.

3) Bu yerda $f(-x)=|-x|-5e^{(-x)^2}=|x|-5e^{x^2}$, ya’ni $f(-x)=f(x)$. Demak funksiya juft funksiya ekan.

4) $f(x)=(-x)^2+5(-x)=x^2-5x$ Shunday qilib $f(-x)\neq f(x)$ va $f(-x)\neq -f(x)$, ya’ni berilgan funksiya na juft, na toq bo‘lmaydi. Juft ham, toq ham emas.

5) $f(-x)=\lg\frac{-x+3}{-x-3}=\lg\frac{x-3}{x+3}=\lg\frac{(x+3)^{-1}}{x-3}=-\lg\frac{x+3}{x-3}$

ni topamiz, ya'ni $f(-x)=-f(x)$ va shuning uchun berilgan funksiya toqdir.

617. Funksiyalarning aniqlanish sohasini toping.

1) $f(x)=\sqrt{4-x^2}+\frac{1}{x}$; 2) $f(x)=\arccos\left(\frac{x}{2}-1\right)$;

3) $f(x)=\frac{1}{xe^x}$; 4) $f(x)=\frac{x-2}{\cos 2x}$; 5) $f(x)=\frac{2x^2+3}{x-\sqrt{x^2-4}}$;

6) $f(x)=\lg(3x-1)+2\lg(x+1)$; 7) $f(x)=\sqrt{\frac{x}{2-x}}-\sqrt{\sin x}$.

618. Funksiyalarning qiymatlar to'plamini toping.

1) $f(x)=|x|+1$; 2) $f(x)=5/x$; 3) $f(x)=\sqrt{16-x^2}$;

4) $f(x)=-x^2+8x-13$; 5) $f(x)=1-3\cos x$; 6) $f(x)=4^{-x^2}$

619.

1) $f(x)=x^4\sin 7x$; 2) $f(x)=5|x|-3\sqrt[3]{x^2}$; 3) $f(x)=x^4-3x^2+x$;

4) $f(x)=|x|+2$; 5) $f(x)=|x+2|$; 6) $f(x)=\lg\cos x$;

7) $f(x)=\frac{16^x-1}{4^x}$;

620. Funksiyalarning asosiy davrlarini toping.

1) $f(x)=\sin 5x$; 2) $f(x)=-2\cos(x/3)+1$;

3) $f(x)=\lg\cos 2x$; 4) $f(x)=tg3x+\cos 4x$.

3-§. FUNKSIYALARNING GRAFIKLARINI YASASH

Funksiyalarning grafiklarini yasashda quyidagi usullar qo'llaniladi: "nuqtalar bo'yicha" yasash; grafiklar ustida amallar (qo'shish, ayirish va grafiklarni ko'paytirish); grafiklarni almashtirish (siljitish, cho'zish).

$y=f(x)$ funksiyaning grafigidan foydalanib, quyidagi funksiyalarning grafiklarini yasash mumkin:

1) $y=f(x-a)$ – Ox o'qi bo'yicha a birlikka surilgan boshlang'ich grafik:

2) $y=f(x)+b$ – Oy o'qi bo'yicha b birlikka surilgan huddi o'sha grafik.

3) $y=Af(x)$ – Oy o'qi bo'yicha A marta cho'zilgan boshlangich grafik.

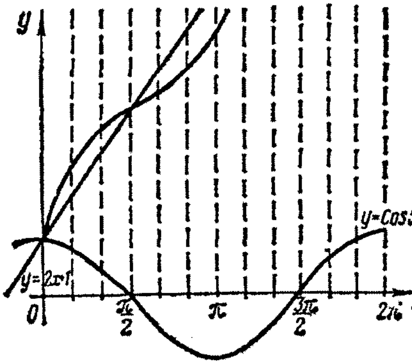
$y=f(kx)$ – Ox o'qi bo'yicha $1/k$ marta cho'zilgan, xuddi o'sha grafik.

Shunday qilib, $y=f(x)$ funksiyaning grafigiga ko'ra $y=Af[k(x-a)]+b$ ko'rinishdagi funksiyalarning grafigini yasash mumkin.

621. $y=2x+1+\cos x$ funksiyaning grafigini yasang.

Yechish:

Berilgan funksiya grafigini ikki funksiya grafiklarini qo'shish bilan yasaladi. $y=2x+1$ va $y=\cos x$. Birinchi funksiyaning grafigi to'g'ri chiziq bo'ladi va uni ikki nuqta bo'yicha yasash mumkin; ikkinchi funksiyaning grafigi – kosinusoida (23-rasm)



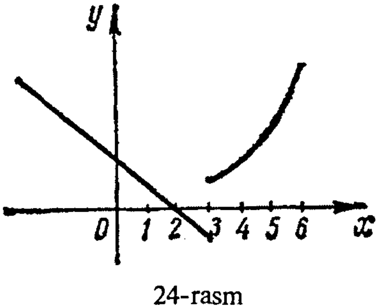
23-rasm

622. $y=\begin{cases} 2-x & x\le 3 \\ 0,1x^2 & x>3 \end{cases}$

funksiyaning grafigini yasang.

Yechish:

Grafik x<3 da nur, x≥3 da parabolaning bir tarmog'i bo'ladi. 24-rasmda izlanayotgan grafik tasvirlangan.



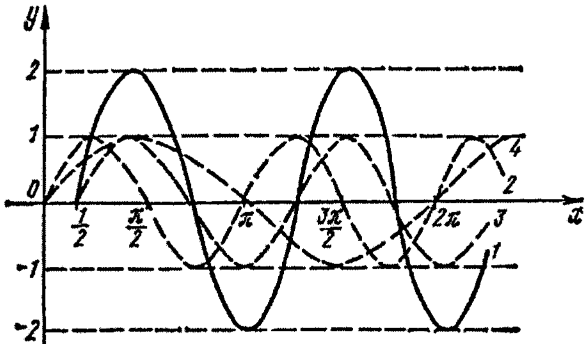
24-rasm

623. y=2sin(2x-1) funksiyaning grafigini yasang.

Yechish:

Berilgan funksiyani

$y=2\sin[2(x-\frac{1}{2})]$ ko'rinishga almashtiramiz. Bunda $A=2$, $k=2$, $a=1/2$ $y=\sin x$ ning grafigini berilgan deb olamiz. Keyin uni absissa o'qi bo'ylab ikki marta siqib, $y=\sin 2x$ funksiyaning grafigini yasaymiz. Bundan keyin hosil bo'lgan grafikni 1/2 birlikka o'ngga surib, $y=\sin 2(x-0{,}5)$ funksiyaning grafigini yasaymiz va nihoyat, oxirgi grafikni ordinata o'qi bo'yicha ikki marta cho'zib, $y=2\sin(2x-1)$ funksiyaning izlanayotgan grafigini hosil qilamiz (25-rasm).



25-rasm

Funksiyalarning grafiklarini yasang:

624. $y=(x^3-x)/3$ $[-4,4]$ kesmada

625. $y=x^2(2-x)^2$ $[-3,3]$ kesmada

626. $y=\sqrt{x}+\sqrt{4-x}$ aniqlanish sohasida

627. $y=0{,}5x+2^{-x}$ $[0,5]$ kesmada

628. $y=2(x-1)^3$, $y=x^3$ funksiyadan foydalanib

629. $y=1/(x^2+4)$ **630.** $y=(x^2+1)/x$

631. $y=\sin(3x-2)+1$ **632.** $y=-2\cos(2x+1)$

633. $y=\arcsin(x-2)$ **634.** $y=x+1+\sin(x-1)$

635. $y=\sin x+\cos x$ **636.** $y=\begin{cases}-x^2 & x<0\\ 3x & x\geq 0\end{cases}$

637. $y=\begin{cases}4-x & x<-1\\ 5 & -1\leq x\leq 0\\ 3x^2+5 & x<0\end{cases}$

4-§. LIMITLAR

Agar har qanday istalgancha kichik ε musbat son uchun, shunday musbat N son topilsaki, barcha $n>N$ larda $|x_n-a|<\varepsilon$ bo'lsa, a son x_1, x_2, .., x_n, ... ketma-ketlikning limiti deyiladi va

$$\lim_{n\to\infty} x_n = a$$

ko'rinishda yoziladi.

Agar har qanday istalgancha kichik $\varepsilon>0$ uchun shunday $\delta>0$ topilsaki, $0<|x-a|<\delta$ da $|f(x)-A|<\varepsilon$ bo'lsa, u holda A soniga $f(x)$ funksiyaning limiti deyiladi va shunday yoziladi:

$$\lim_{n\to a} f(x) = A$$

Huddi shunga o'xshash, agar $|x|>N$ da $|f(x)-A|<\varepsilon$ bo'lsa,

$$\lim_{n\to\infty} f(x) = A$$

Agar $0<|x-a|<\delta$ da $|f(x)|>M$ bo'lsa, shartli ravishda quyidagicha yoziladi

$$\lim_{n\to a} f(x) = \infty$$

bunda M ixtiyoriy musbat son.

Bu holda $x\to a$ da $f(x)$ funksiya cheksiz katta deyiladi.

Agar $\lim\limits_{n\to a} \alpha(x) = 0$ bo'lsa, u holda $\alpha(x)$ funksiya $x\to a$ da cheksiz kichik deyiladi. Agar $x<a$ va $x\to a$ bo'lsa, u holda $x\to a-0$ yozuv; agar $x>a$ va $x\to a$ bo'lsa, u holda $x\to a+0$ yozuv ishlatiladi. $f(a-0)=\lim\limits_{n\to a-0} f(x)$ va $f(a+0)=\lim\limits_{n\to a+0} f(x)$ sonlar mos ravishda $f(x)$ funksiyaning a nuqtadagi chap va o'ng limitlari deyiladi.

Limitlarni amaliy hisoblash quyidagi teoremalarga asoslanadi.

Agar $\lim\limits_{n\to a} f(x)$ va $\lim\limits_{n\to a} g(x)$ mavjud bo'lsa, u holda

1. $\lim\limits_{n\to a}[f(x)+g(x) = \lim\limits_{n\to a} f(x) + \lim\limits_{n\to a} g(x)$,

2. $\lim\limits_{n\to a}[f(x)\cdot g(x)] = \lim\limits_{n\to a} f(x)\cdot\lim\limits_{n\to a} g(x)$,

3. $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}\quad\left(\lim\limits_{x\to a}g(x)\neq 0\right)$,

shuningdek quyidagi limitlardan foydalaniladi:

$$\lim_{x\to 0}\frac{\sin x}{x}=1,$$

birinchi ajoyib limit.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e\approx 2,71828$$

(ikkinchi ajoyib limit).

x sonning e asosga ko'ra logarifmi natural logarifm deyiladi va lnx bilan belgilanadi. Misollarni echishda quyidagi tengliklarni hisobga olish foydali:

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1,\ \lim_{x\to 0}\frac{a^x-1}{x}=\ln a,\ \lim_{x\to 0}\frac{(1+x)^m-1}{x}=m$$

638. $n\to\infty$ da $3, 2\frac{1}{2}, 2\frac{1}{3}, ..., 2+\frac{1}{n},...$ ketma-ketlikning limiti 2 soni ekanligini ko'rsating.

Yechish:

Bunda ketma-ketlikning n-xadi $x_n=2+\frac{1}{n}$ bo'ladi. Shuningdek, $x_{n-2}=\frac{1}{n}$. Avvaldan ε musbat sonni beramiz. n ni shunday yetarlicha katta tanlab olamizki, $\frac{1}{n}<\varepsilon$ tengsizlik bajarilsin. Buning uchun $n>\frac{1}{\varepsilon}$ deb qabul qilish etarli. Bunday tanlashda $|x_n-2|<\varepsilon$ hosil bo'ladi. Demak, $\lim\limits_{x\to\infty}x_n=2$.

639. $n\to\infty$ da $\frac{7}{3},\frac{10}{5},\frac{13}{7},...,\frac{3n+4}{2n+1},...$ ketma-ketlikning limiti 3/2 soni ekanligini ko'rsating.

Yechish:

Bunda $x_n-\frac{3}{2}=\frac{3n+4}{2n+1}-\frac{3}{2}=\frac{5}{2(2n+1)}\cdot\frac{5}{2(2n+1)}<\varepsilon$ tengsizlik n ning qanday qiymatlarida bajarilishini aniqlaymiz. $2(2n+1)>\frac{5}{\varepsilon}$ bo'lganligi uchun, $n>\frac{5}{4\varepsilon}-\frac{1}{2}$ bo'ladi. Demak $\left|x_n-\frac{3}{2}\right|<\varepsilon$ bo'ladi, ya'ni $\lim\limits_{x\to\infty} x_n=\frac{3}{2}$. $\varepsilon=0,1$ faraz qilib, $\left|x_n-\frac{3}{2}\right|<0,1$ tengsizlik $n>12$ da bajariladi (masalan, $n=13$ da) degan xulosaga kelamiz. Xuddi shunga o'xshash, $\left|x_n-\frac{3}{2}\right|<0,01$ tengsizlik n>124,5 (masalan, $n=125$ da) da, $\left|x_n-\frac{3}{2}\right|<0,001$ esa $n>1249,5$ (masalan, $n=1250$ da) da bajariladi.

Quyidagi limitlarni toping:

640. $\lim\limits_{x\to 4}\frac{5x+2}{2x+3}$.

Yechish:

$x\to 4$ bo'lgani uchun, kasrning surati $5\cdot 4+2=22$ songa, maxraji esa $2\cdot 4+3=11$ songa intiladi. Shunga ko'ra $\lim\limits_{x\to 4}\frac{5x+2}{2x+3}=\frac{22}{11}=2$.

641. $\lim\limits_{x\to\infty}\frac{3x+5}{2x+7}$.

Yechish:

Kasrning surat va mahraji $x\to\infty$ da chegaralanmagan holda o'sadi. Bu holda $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslik o'rinli deyiladi. Kasrni surat va mahrajini x ga bo'lib quyidagini hosil qilamiz

$$\lim_{x\to\infty}\frac{3x+5}{2x+7}=\lim_{x\to\infty}\frac{3+\frac{5}{x}}{2+\frac{7}{x}}=\frac{3}{2},$$

bunda $x\to\infty$ da 5/x va 7/x kasrlardan har biri nolga intiladi.

642. $\lim\limits_{x\to 3}\frac{x^2-9}{x^2-3x}$.

Yechish:

$x \to 3$ da kasrning surat va mahraji nolga intiladi (0/0 ko'rinishidagi aniqmaslik). Agar $x \neq 3$ bo'lsa u holda

$$\frac{x^2-9}{x^2-3x} = \frac{(x-3)(x+3)}{x(x-3)} = \frac{x+3}{x}$$

ni hosil qilamiz, shunga ko'ra

$$\lim_{x\to 3}\frac{x^2-9}{x^2-3x} = \lim_{x\to 3}\frac{x+3}{x}.$$

Lekin $x \to 3$ da $\frac{x+3}{x}$ kasr $\frac{3+3}{3}=2$ songa intiladi. Demak,

$$\lim_{x\to 3}\frac{x^2-9}{x^2-3x} = 2.$$

643. $\lim\limits_{x\to 1}\frac{x^3-x^2-x+1}{x^3+x^2-x-1}$.

Yechish:

Bunda 0/0 ko'rinishdagi aniqmaslik o'rinli. Kasrning surat va mahrajini ko'paytuvchilarga ajratamiz:

$$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3+x^2-x-1} = \lim_{x\to 1}\frac{x^2(x-1)-(x-1)}{x^2(x+1)-(x+1)} = \lim_{x\to 1}\frac{(x-1)^2(x+1)}{(x-1)(x+1)^2} = \lim_{x\to 1}\frac{x-1}{x+1} = \frac{0}{2} = 0$$

644. $\lim\limits_{x\to 10}\frac{x^3-100}{x^3-20x^2+100x}$.

Yechish:

Bu ham 0/0 ko'rinishdagi aniqmaslik. Kasrning surati 300 ga intiladi, maxraji esa nolga, ya'ni cheksiz kichik miqdor bo'ladi, shunga ko'ra qaralayotgan kasr – cheksiz katta miqdor bo'ladi va

$$\lim_{x\to 10}\frac{x^3-1000}{x^3-20x^2+100x} = \infty.$$

645. $\lim\limits_{x\to 0}\frac{\sqrt{x+4}-2}{x}$.

Yechish:

Kasrning surat va maxrajini $\sqrt{x+4}+2$ yig'indiga ko'paytiramiz:

$$\lim_{x\to 0}\frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{x(\sqrt{x+4}+2)}=\lim_{x\to 0}\frac{x+4-4}{\sqrt{x+4}+2}=$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}=\frac{1}{4}.$$

646. $\lim\limits_{x\to 0}\dfrac{\sqrt[5]{(1+x)^3}-1}{x}.$

Yechish:

Faraz qilaylik, $1+x=y^5$, u holda $x\to 0$ da $y\to 1$. Demak,

$$\lim_{x\to 0}\frac{\sqrt[5]{(1+x)^3}-1}{x}=\lim_{x\to 1}\frac{y^3-1}{y^5-1}=\lim_{x\to 1}\frac{y^2+y+1}{y^4+y^3+y^2+y+1}=\frac{3}{5}.$$

647. $\lim\limits_{x\to 0}\dfrac{\sin mx}{x}.$

Yechish:

Birinchi ajoyib limitni qo'llab, hosil qilamiz

$$\lim_{x\to 0}\frac{\sin mx}{x}=\lim_{x\to 0}\frac{m\sin mx}{mx}=m\lim_{x\to 0}\frac{\sin mx}{mx}=m.$$

648. $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{x^2}.$

Yechish:

$$\lim_{x\to 0}\frac{1-\cos 5x}{x^2}=\lim_{x\to 0}\frac{2\sin^2(5x/2)}{x^2}=2\lim_{x\to 0}\left(\frac{\sin(5x/2)}{x}\right)^2=2\cdot\left(\frac{5}{2}\right)^2=\frac{25}{2}.$$

Biz bunda, $m=5/2$ qabul qilib, avvalgi misolning natijasidan foydalandik.

649. $\lim\limits_{x\to\infty}\dfrac{x^3+2x^2+3x+4}{4x^3+3x^2+2x+1}.$

Yechish:

Bu $\dfrac{\infty}{\infty}$ ko'rinishdagi aniqmaslik, kasrning surat va mahrajini katta darajaga bo'lamiz, ya'ni x^3 ga:

$$\lim_{x\to\infty}\frac{x^3+2x^2+3x+4}{4x^3+3x^2+2x+1}=\lim_{x\to\infty}\frac{1+2/x+3/x^2+4/x^3}{4+3/x+2/x^2+1/x^3}=\frac{1}{4}.$$

650. $\lim\limits_{x\to\infty}\dfrac{3x^4-2}{\sqrt{x^8+3x+4}}$.

Yechish:

Surat va mahrajini x^4 ga bo'lamiz:

$$\lim_{x\to\infty}\frac{3x^4-2}{\sqrt{x^8+3x-4}}=\lim_{x\to\infty}\frac{3-2/x^4}{\sqrt{1+3/x^7+4/x^8}}=\frac{3}{1}=3.$$

651. $\lim\limits_{x\to\infty}(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3})$.

Yechish:

Bunda $\infty-\infty$ ko'rinishdagi aniqmaslik o'rinli. Berilgan ifodani $\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}$ ga ko'paytiramiz va bo'lamiz:

$$\lim_{x\to\infty}(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3})=$$

$$=\lim_{x\to\infty}\frac{(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3})(\sqrt{x^2+8x-3}-\sqrt{x^2+4x+3})}{\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}=$$

$$=\lim_{x\to\infty}\frac{x^2+8x+3-x^2-4x-3}{\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}=\lim_{x\to\infty}\frac{4x}{\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}=$$

$$=\lim_{x\to\infty}\frac{4}{\sqrt{1+8/x+3/x^2}+\sqrt{1+4/x+3/x^2}}=\frac{4}{2}=2.$$

652. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+5x+4}{x^2-3x+7}\right)^x$.

Yechish:

Kasrning suratini mahrajiga bo'lib, butun qismini ajratamiz:

$$\frac{x^2+5x+4}{x^2-3x+7}=1+\frac{8x-3}{x^2-3x+7}.$$

Shunday qilib berilgan funksiya $x\to\infty$ da, asosi birga, daraja ko'rsatkichi – cheksizlikka (1^∞ ko'rinishdagi aniqmaslik) intiladigan darajani ifodalaydi. Ikkinchi ajoyib limitni qo'llash uchun, funksiyani shunday almashtiramizki, natijada quyidagini hosil qilamiz:

$$\lim_{x\to\infty}\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x=\lim_{x\to\infty}\left(1+\frac{8x-3}{x^2-3x+7}\right)^x=$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{8x-3}{x^2-3x+7}\right)^{\frac{x^2-3x+7}{8x-3}}\right]^{\frac{x(8x-3)}{x^2-3x+7}}=$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{8x-3}{x^2-3x+7}\right)^{\frac{x^2-3x+7}{8x-3}}\right]^{\frac{8-3/x}{1-3/x+7/x^2}}.$$

$x\to\infty$ da $\frac{8x-3}{x^2-3x+7}\to 0$ bo'lganligi uchun $\lim\limits_{x\to\infty}\left(1+\frac{8x-3}{x^2-3x+7}\right)^{\frac{x^2-3x+7}{8x-3}}=e$ bo'ladi. $\lim\limits_{x\to\infty}\frac{8-3/x}{1-3/x+7/x^2}=8$ ekanligini hisobga olib, $\lim\limits_{x\to\infty}\left(\frac{x^2+5x+4}{x^2-3x+7}\right)^x=e^\infty$ ni topamiz.

653. $x\to 3$ da $f(x)=\frac{1}{x+2^{1/(x-3)}}$ funksiyaning chap va o'ng limitlarini toping.

Yechish:

Agar $x\to 3-0$ bo'lsa, u holda $1/(x-3)\to-\infty$ va $2^{1/(x-3)}\to 0$. Shuningdek, $\lim\limits_{x\to 3-0}f(x)=1/3$. Agar $x+3+0$, u holda $1/(x-3)\to\infty$, $2^{1/(x-3)}\to\infty$ va $\lim\limits_{x\to 3-0}f(x)=0$.

654. $x\to a$ da $f(x)=e^{1/(x-a)}$ funksiyaning chap va o'ng limitlarini toping.

Yechish:

Agar $x\to a-0$ bo'lsa, u holda $1/(x-a)\to-\infty$ u holda $1/(x-a)\to+\infty$ va $\lim\limits_{x\to a+0}f(x)=+\infty$.

655. $n\to\infty$ da $1/2,\ 5/3,9/4,...,(4n-3)/(n+1),...$ ketma-ketlikning 4 ga teng limitga ega ekanligini ko'rsating.

656. $n\to\infty$ da $1, 1/3, 1/5, ..., 1/(2n-1), ...$ ketma-ketlik cheksiz kichik miqdor ekanligini ko'rsating.

Quyidagi limitlarni toping:

657. $\lim\limits_{x\to 2}\dfrac{x^2-6x+8}{x^2-8x+12}$.

658. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x+x^2}-\sqrt{1+x+x^2}}{x^2-x}$.

659. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{x^2-9}$. **660.** $\lim\limits_{x\to 1}\dfrac{x^3-6x^2+11x-6}{x^2-3x+2}$.

661. $\lim\limits_{h\to 0}\dfrac{\sin(a+2h)-2\sin(a+h)+\sin a}{h^2}$.

662. $\lim\limits_{x\to 0}\dfrac{tgmx}{\sin nx}$. **663.** $\lim\limits_{x\to x_0}\dfrac{tgx-tgx_0}{x-x_0}$. **664.** $\lim\limits_{x\to\pi/4}\dfrac{\sin x-\cos x}{\pi-4x}$.

665. $\lim\limits_{x\to\pi/2}\dfrac{\cos x}{\pi-2x}$, $\pi/2-x=\alpha$ belgilang.

666. $\lim\limits_{x\to\infty}\dfrac{2x^4+3x^2+5x-6}{x^3+3x^2+7x-1}$. **667.** $\lim\limits_{x\to\infty}\dfrac{(2x^2+4x+5)(x^2+x+1)}{(x+2)(x^4+2x^3+7x^2+x-1)}$.

668. $\lim\limits_{x\to 2}\dfrac{x^2-7x+10}{x^2-8x+12}$. **669.** $\lim\limits_{x\to -1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}$.

670. $\lim\limits_{x\to 2}\dfrac{\sqrt{1+x\sin x}-1}{x^2}$. **671.** $\lim\limits_{x\to 0}\dfrac{x}{\sqrt[3]{1+x}-1}$.

672. $\lim\limits_{x\to 2}\dfrac{\sqrt{1+x+x^2}-\sqrt{7x+2x-x^2}}{x^2-2x}$.

673. $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{1-\cos 3x}$. **674.** $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{x^3}$.

675. $\lim\limits_{x\to 0}\dfrac{\ln(1+mx)}{x}$. **676.** $\lim\limits_{x\to\pm\infty}\dfrac{2^x+3}{2^x-3}$.

677. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+ax+b}-\sqrt{x^2+cx+d}\right)$.

678. $\lim\limits_{x\to\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$. **679.** $\lim\limits_{x\to\infty}\left(\sqrt[3]{x+1}-\sqrt[3]{x}\right)$.

680. $\lim\limits_{x\to\infty}\dfrac{1-5^x}{1-e^x}$. **681.** $\lim\limits_{x\to\infty}\dfrac{8^x-7^x}{6^x-5^x}$.

682. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\ln(+x)}$. **683.** $\lim\limits_{x\to 0}\dfrac{5^x-1}{x}$.

684. $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{x}-1}{x-1}$, $x=t^4$ belgilang. **685.** $\lim\limits_{x\to\pm 0}\dfrac{\sin x}{|x|}$

686. $\lim\limits_{t\to 0}\dfrac{t+\sin t}{t-\sin t}$. **687.** $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin x}{\ln(x+1)}$. **688.** $\lim\limits_{x\to 5-0}10^{1/(x-5)}$.

689. $\lim\limits_{x\to\infty}\sin x$. **690.** $\lim\limits_{x\to 1}\dfrac{x^5-1}{x^4-1}$.

691. $\lim t(\sqrt[4]{a}-1)$ (bunda t>0) $x\to 0$ da, $x=1/t$ bilan belgilang

692. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x^2}\right)^{x+1}$. **693.** $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^x$.

694. $\lim\limits_{x\to 3}\left(\dfrac{1}{x-3}-\dfrac{6}{x^2-9}\right)$. **695.** $\lim\limits_{x\to 0}\dfrac{5^x-4^x}{x^2+x}$.

696. $\lim\limits_{x\to 1}\dfrac{x^x-1}{x\ln x}$, $x^x=e^{x\ln x}$ ekanligini hisobga oling.

697. $\lim\limits_{x\to 0}\dfrac{\ln(1-3x)}{x}$. **698.** $\lim\limits_{x\to\infty}\dfrac{x^4+5x^3+7}{2x^5+3x^4+1}$. **699.** $\lim\limits_{x\to 0}\dfrac{\ln(x+2)-\ln 2}{x}$.

700. $\lim\limits_{x\to\infty}\left(\dfrac{x+8}{x-2}\right)^x$. **701.** $\lim\limits_{\alpha\to 0}(2-\cos\alpha)^{\operatorname{cosec}^2\alpha}$.

702. $\lim\limits_{x\to\infty}\left(\dfrac{x+a}{x+b}\right)^{x+c}$. **703.** $\lim\limits_{x\to 2}\left(\dfrac{x}{2}\right)^{1/(x-2)}$.

5-§. CHEKSIZ KICHIK MIQDORLARNI ANIQLASH

Faraz qilaylik, $x \to a$ da $\alpha(x)$, $\beta(x)$ lar cheksiz kichik miqdorlar bo'lsin.

1. Agar $\lim\limits_{x\to a}\frac{\alpha}{\beta}=0$ bo'lsa, u holda α, β ga nisbatan yuqori tartibli cheksiz kichik miqdor deyiladi. Bu holda $\alpha=o(\beta)$ ko'rinishda yoziladi.

2. Agar $\lim\limits_{x\to a}\frac{\alpha}{\beta}=m$ bo'lib, bunda m noldan farqli son bo'lsa, u holda α va β bir xil tartibdagi cheksiz kichik miqdorlar deyiladi va $\alpha\sim\beta$ yozuv α va β lar ekvivalent cheksiz kichik miqdorlar ekanligini bildiradi. Agar $\lim\limits_{x\to a}\frac{\alpha}{\beta}=\infty$ bo'lsa, u holda $\lim\limits_{x\to a}\frac{\beta}{\alpha}=0$ bo'lib β, α ga nisbatan yuqori tartibli cheksiz kichik miqdor bo'ladi, ya'ni $\beta=o(\alpha)$.

3. Agar α^{κ} va β bir xil tartibdagi cheksiz kichik miqdorlar bo'lsa, bunda $\kappa>0$, u holda β cheksiz kichik miqdor α ga nisbatan κ – tartibda bo'ladi deyiladi.

Cheksiz kichik miqdorlarning ba'zi hossalarini ta'kidlab o'tamiz.

1. Ikkita cheksiz kichik miqdorlarning ko'paytmasi ko'paytuvchilarga nisbatan yuqori tartibli cheksiz kichik miqdor bo'ladi, ya'ni agar $\gamma=\alpha\beta$ bo'lsa, u holda $\gamma=o(\alpha)$ va $\gamma=o(\beta)$

2. α va β cheksiz kichik miqdorlar ekvivalent bo'ladi faqat va faqat shundaki, agar ularning ayirmasi $\alpha-\beta=\gamma$ α va α ga nisbatan yuqori tartibli cheksiz kichik miqdor bo'lsa, ya'ni agar $\gamma=o(\alpha)$ va $\gamma=o(\beta)$ bo'lsa, u holda $\alpha\sim\beta$.

3. Agar ikkita cheksiz kichik miqdorlarning nisbati limitga ega bo'lsa, u holda har bir cheksiz kichik miqdorni unga ekvivalent cheksiz kichik miqdor bilan almashtirilsa bu limitning qiymati o'zgarmaydi, ya'ni, agar $\lim\limits_{x\to a}\frac{\alpha}{\beta}=m$, $\alpha\sim\alpha_1$, $\beta\sim\beta_1$ bo'lsa, u holda

$$\lim_{x\to a}\frac{\alpha_1}{\beta_1}=m \text{ bo'ladi.}$$

Quyidagi cheksiz kichik miqdorlarni ekvivalentligini nazarda tutish foydali, agar $x\to a$ bo'lsa, u holda $\sin x\sim x$, $\mathrm{tg}x\sim x$, $\arcsin x\sim x$, $\mathrm{arctg}x\sim x$, $\ln(1+x)\sim x$

704. Faraz qilaylik, t – cheksiz kichik miqdor bo'lsin. $\alpha=5t^2+2t^5$ ва $\beta=3t^2+2t^3$ cheksiz kichik miqdorlarni taqqoslang.

Yechish:

$$\lim_{t\to 0}\frac{\alpha}{\beta}=\lim_{t\to 0}\frac{5t^2+2t^5}{3t^2+2t^3}=\lim_{t\to 0}\frac{5+2t^3}{3+2t}=\frac{5}{3}$$

bo'ladi. α va β larning nisbati noldan farqli son bo'lgani uchun, α va β bir xil tartibdagi cheksiz kichik miqdorlar bo'ladi.

705. $t\to 0$ da $\alpha=t\sin^2 t$ va $\beta=2t\sin t$ cheksiz kichik miqdorlarni taqqoslang.

Yechish:

Bunda $\lim\limits_{t\to 0}\frac{\alpha}{\beta}=\lim\limits_{t\to 0}\frac{t\sin t^2}{2t\sin t}=\frac{1}{2}\lim\limits_{t\to 0}\sin t=0$ ya'ni $\alpha=o(\beta)$

706. $t\to 0$ da $\alpha=t\ln(1+t)$, β=tsint cheksiz kichik miqdorlarni taqqoslang.

Yechish:

$$\lim_{t\to 0}\frac{\alpha}{\beta}=\lim_{t\to 0}\frac{t\ln(1+t)}{t\sin t}=\lim_{t\to 0}\frac{\ln(1+t)}{\sin t}=\lim_{t\to 0}\frac{\frac{\ln(1+t)}{t}}{\frac{\sin t}{t}}=1$$

ni topamiz, ya'ni $\alpha\sim\beta$

707. Quyidagi limitni toping:

$$\lim_{x\to 0}\frac{\ln(1+3x\sin x)}{tgx^2}.$$

Yechish:

Kasrning surat va mahrajini ekvivalent cheksiz kichik miqdorlarga almashtiramiz:

$$\lim_{x\to 0}\frac{\ln(1+3x\sin x)}{tgx^2}=\lim_{x\to 0}\frac{3x\sin x}{x^2}=3\lim_{x\to 0}\frac{\sin x}{x}=3$$

708. $y=xe^x$ cheksiz kichik miqdorni cheksiz kichik x ga nisbatan tartibini aniqlang.

709. y=$\sqrt{1+x\sin x}-1$ cheksiz kichik miqdorni cheksiz kichik x ga nisbatan tartibini aniqlang.

710. $x\to 0$ da y=$\sqrt{\sin 2x}$ cheksiz kichik miqdorni cheksiz kichik x ga nisbatan tartibini aniqlang.

711. Агар $t \to 0$ bo'lsa, $\alpha=t^2\sin^2 t$ va $\beta=t\cdot \mathrm{tg}\, t$ cheksiz kichik miqdorlarni taqqoslang.

712. Agar $x \to 0$ va m – ratsional musbat son bo'lsa, $\alpha=(1+x)^m$ va $\beta=mx$ cheksiz kichik miqdorlarni taqqoslang.

713. $x \to 0$ da, $\alpha=\alpha^x-1$ va $\beta=x\ln a$ cheksiz kichik miqdorlarni taqqoslang.

Quyidagi limitlarni toping:

714. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-1}{tg3x}$. **715.** $\lim\limits_{x\to 0}\dfrac{\sin^2 3x}{\ln^2(1+2x)}$.

Surat va mahrajini ekvivalent cheksiz kichik miqdorlarga almashtiring.

716. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\ln(1-4x)}$. **717.** $\lim\limits_{x\to 1}\dfrac{\ln(1+x-3x^2+2x^3)}{\ln(1+3x-4x^2+x^3)}$.

718. $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{\ln(1+x^2)}$. **719.** $\lim\limits_{x\to 1}\dfrac{\sin(e^{x-1}-1)}{\ln x}$.

cosx ni 1-(1-cosx) кo'rinishida ifodalang.

720. $\lim\limits_{x\to 0}\dfrac{\sqrt[5]{(1+x)^3}-1}{(1+x)\sqrt[3]{(1+x)^2}-1}$. **721.** $\lim\limits_{\alpha\to 0}\dfrac{(5^\alpha-1)(4^\alpha-1)}{(3^\alpha-1)(6^\alpha-1)}$.

722. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8+3x}-2}{\sqrt[4]{16+5x}-2}$.

Surat va mahrajini 2 ga bo'ling.

6-§. FUNKSIYANING UZLUKSIZLIGI

$f(x)$ funksiya a nuqtada uzluksiz deyiladi, agar:

1) bu funksiya a nuqtaning biron bir atrofida aniqlangan;

2) $\lim\limits_{x\to a} f(x)$ limit mavjud;

3) bu limit funksiyaning a nuqtadagi qiymatiga teng bo'lsa, ya'ni $\lim\limits_{x\to a} f(x)=f(a)$ bo'lsa.

$x-a=\Delta x$ (argument orttirmasi) va $f(x)-f(a)=\Delta y$ (funksiya orttirmasi) deb belgilab, uzluksizlik shartini shunday yozish mumkin: $\lim\limits_{\Delta x\to 0}\Delta y=0$, ya'ni $f(x)$ funksiya a nuqtada uzluksiz bo'ladi shunda ва faqat shundaki, agar bu nuqtada argumentning cheksiz kichik orttirmasiga funksianing cheksiz kichik orttirmasi mos kelsa.

Agar funksiya biror sohaning har bir nuqtasida uzluksiz (intervalda, segmentda va h.к.) bo'lsa, u holda funksiya bu sohada uzluksiz deyiladi.

Funksiyaning aniqlanish sohasiga tegishli yoki bu soha uchun chegaraviy bo'lgan a nuqta uzulish nuqtasi deyiladi, agar bu nuqtada funksiyaning uzluksizlik sharti buzilsa.

Agar $\lim\limits_{x\to a-0}f(x)=f(a-0)$ va $\lim\limits_{x\to a+0}f(x)=f(a+0)$ chekli limitlar mavjud bo'lib, $f(a)$, $f(a-0)$, $f(a+0)$ sonlarning kamida bittasi qolganlariga teng bo'lmasa, u holda a nuqtaga 1-tur uzilish nuqtasi deyiladi.

1-tur uzilish nuqtalari o'z navbatida, yo'qotish mumkin bo'lgan nuqtalarga: ($f(a-0)=f(a+0)\neq f(a)$ bo'lganda, ya'ni funksiyaning chap va o'ng limitlari bir-biriga teng, lekin funksiyaning bu nuqtadagi qiymatiga teng emas) va sakrash nuqtalariga ($f(a-0)\neq f(a+0)$ bo'lganda, ya'ni funksiyaning a nuqtadagi chap va o'ng limitlari turli) bo'linadi, oxirgi holda $f(a+0)-f(a-0)$ ayirma funksiyaning a nuqtadagi sakrashi deyiladi.

I tur uzilish nuqtalari bo'lmagan uzilish nuqtalar, II tur uzilish nuqtalari deyiladi. II tur uzilish nuqtalarida hech bo'lmaganda bitta bir tomonlama limit mavjud bo'lmaydi.

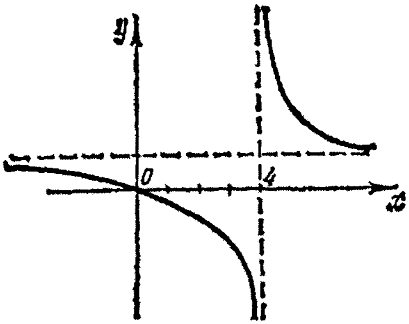
Chekli sondagi uzluksiz funksiyalarning yig'indisi va ko'paytmasi uzluksiz funksiya bo'ladi. Ikkita uzluksiz funksiya bo'linmasi, bo'luvchi nolga teng bo'lmagan barcha nuqtalarda uzluksiz bo'ladi.

723. $x=4$ da $y=x/(x-4)$ funksiya uzilishga ega ekanligini ko'rsating.

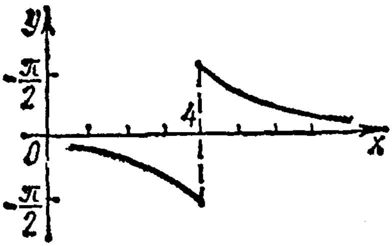
Yechish:

$$\lim_{x\to 4-0}\frac{x}{x-4}=-\infty,\quad \lim_{x\to 4+0}\frac{x}{x-4}=+\infty \text{ larni topamiz.}$$

Shunday qilib, $x \to 4$ da funksiya na chap, na o'ng limitga ega emas.Shuning uchun $x=4$ nuqta bu funksiya uchun II тур uzilish nuqtasi bo'ladi. (26-rasm)



26-rasm



27-rasm

724. $x=4$ da $y=\operatorname{arctg}\dfrac{1}{x-4}$ funksiya uzilishga ega ekanligini ko'rsating.

Yechish:

Agar $x \to 4-0$ bo'lsa, u holda $\dfrac{1}{x-4} \to -\infty$ va $\lim\limits_{x\to 4-0} y = -\dfrac{\pi}{2}$ bo'ladi. Agar $x \to 4+0$ bo'lsa, u holda $\dfrac{1}{x-4} \to +\infty$ va $\lim\limits_{x\to 4+0} y = \pi/2$ bo'ladi. Demak, $x \to 4$ da funksiya ham chap, ham o'ng chekli limitlarga ega va bu limitlar turli. Shunga ko'ra $x=4$ nuqta I tur uzilish nuqtasi – sakrash nuqtasi bo'ladi.

Funksiyaning bu sakrashi $\pi/2-(\pi/2)= \pi$ ga teng (27-rasm)

725. $x=5$ da $y=\dfrac{x^2-25}{x-5}$ funksiya uzilishga ega ekanligini ko'rsating.

Yechish:

$x=5$ nuqtada funksiya aniqlanmagan, chunki bu qiymatni funksiyaga qo'ysak 0/0 aniqmaslikni hosil qilamiz.

Boshqa nuqtalarda $x-5\neq0$ bo'lganligi uchun, kasrni $x-5$ ga qisqartirish mumkin. Shunga ko'ra, $x\neq5$ da $y=x+5$. Bundan esa

$\lim\limits_{x\to5-0} y=\lim\limits_{x\to5+0} y=10$ ekanligini ko'rish oson.

Shunday qilib, $x=5$ da funksiya yo'qotish mumkin bo'lgan uzilishga ega. Agar $x=5$ da $y=10$ deb shartlashilsa, uni yo'qotish mumkin bo'ladi. Demak, agar $x=5$ da ham, $(x^2-25)/(x-5)=x+5$ tenglik o'rinli deb olsak, x ning barcham qiymatlarida $y=(x^2-25)/(x-5)$ funksiya uzluksiz deb hisoblash mumkin. Bu holda funksiyaning grafigi $y=x+5$ to'g'ri chiziq bo'ladi.

726. $y=\dfrac{2^{1/(x-2)}-1}{2^{1/(x-2)}+1}$ funksiyaning uzilish nuqtalarini toping.

727. $y=\dfrac{1}{(x-1)(x-5)}$ funksiyaning uzilish nuqtalarini toping.

728. $x=1$ nuqtada $y=\dfrac{1}{1-e^{1-x}}$ funksiya uzilishining xarakteri qanday?

729. $x=0$ nuqtada $y=\dfrac{\sin x}{x}$ funksiya uzilishining xarakteri qanday?

730. $y=\dfrac{tgx\cdot arctg\dfrac{1}{x-3}}{x(x-5)}$ funksiyaning uzilish nuqtalarini toping.

731. $y=\dfrac{x^3-6x^2+11x-6}{3x^2-3x+2}$ funksiyaning uzilish nuqtalarini toping.

732. $y=\dfrac{x+1}{x^3+6x^2+11x+6}$ funksiyaning uzilish nuqtalarini toping.

733. $y=\dfrac{1}{(x-1)(x-6)}$ funksiyaning uzilish nuqtalarini toping.

734. $y=\dfrac{1}{(x-1)(x-6)}$ funksiyani quyidagi kesmalarda uzliksizlikka tekshiring.

1) $[2,5]$; 2) $[4,10]$ 3) $[0,7]$.

735. $y=\dfrac{1}{x^4+26x^2+25}$ funksiyani quyidagi kesmalarda uzliksizlikka tekshiring.

1) $[6,10]$; 2) $[-2,2]$; 3) $[-6,6]$.

I-BOB

4. 1) 8; 2) 3. **5.** 1) $\frac{1}{2}$; 2) $-\frac{9}{4}$. **6.** $M(7)$. **7.** $C(1)$, $D(2)$. **8.** $C(-9)$, $D(-1)$. **16.** 1) 13. 2) 3. **19.** 5. **20.** $(-1; 8)$, $(1; 9)$, $(3; 10)$. **21.** S=0, ya'ni A, B, C nuqtalar bir to'g'ri chiziqda yotadi. **22.** $D(17; 22)$. **23.** $C(-10; -7)$. **24.** $\sqrt{53}$, $\sqrt{82}$, $\sqrt{185}$. **25.** 24 kv.b. **29.** $A(4; \frac{\pi}{6})$; $B(3; -\frac{\pi}{2})$; $C(4\sqrt{2}; \frac{3\pi}{4})$; $D(2; -\frac{\pi}{4})$; $E(2\sqrt{2}; \frac{4\pi}{4})$; $F(7; \pi)$. **30.** $A(0; 10)$; $B(-\sqrt{2}; -\sqrt{2})$; $C(0; 0)$; $D(\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$; $E(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$; $F(-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$.
31. $\sqrt{\rho_1^2+\rho_2^2-2\rho_1\rho_2 cos(\theta_1-\theta_2)}$. **32.** 5. **33.** $M_1(\rho; -\theta)$. **34.** $M_1(\rho; \pi+\theta)$.
35. 1) $(3; \frac{7\pi}{4})$; $(5; -\frac{\pi}{3})$ va $(2; \frac{5\pi}{6})$; 2) $(3; -\frac{\pi}{6})$; $(5; \frac{2\pi}{3})$ va $(2; \frac{\pi}{6})$.
36. $M_1(\rho; \pi-\theta)$. **44.** $y=2x-1{,}5$. **45.** I va III koordinatalar burchaklarining bissektrisasi. **46.** II va IV koordinatalar burchaklarining bissektrisasi. **47.** $x^2+y^2-2x-2y=0$. **48.** $3x^2+2xy+3y^2-4x-4y=0$. **49.** $\rho=a$. **50.** $\theta=\alpha$. **51.** $\rho=a cos\theta$. **57.** $y=2x$ to'g'ri chiziq. **58.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (bu egri chiziq ellips deyiladi). **59.** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (bu egri chiziq giperbola deyiladi). **60.** AB to'g'ri chiziqning kesmasi, bu yerda $A(1; 0)$, $B(0; 1)$. **61.** $x^{2/3}+y^{2/3}=a^{2/3}$. **62.** $x=a(t sint+cos t)$, $y=a(sint-t cos t)$ (bu egri chiziq aylana evolventasi deyiladi). **67.** 1) $x+2y-2\sqrt{5}=0$; 2) $y=(-\frac{1}{2})x+\sqrt{5}$;

3) $\frac{x}{2\sqrt{5}}+\frac{y}{\sqrt{5}}=1$; 4) $(\frac{1}{\sqrt{5}})x+(\frac{2}{\sqrt{5}})y-2=0$. **68.** 135^0. **69.** 54. kv. b. **70.** Yo'q. **72.** $\sqrt{3}x+y-1=0$. **73.** $x+y-4=0$. **74.** $3x-2y=0$. **75.** $x+y-7=0$. **76.** $x+3=0$, $y+4=0$. **77.** $x+y-5=0$, $x+y+5=0$. **99.** $tg\alpha=\frac{27}{11}$. **100.** $x-y=0$, $5x+3y-26=0$, $3x+5y-26=0$. **101.** $14x+14y-45=0$, $2x-2y+35=0$. **102.** $3x-y+14=0$, $x-5y-14=0$, $x+2y=0$. **103.** $x-2=0$, $y-7=0$. **104.** 4,4. **105.** 2,4. **106.** $m=4$. **107.** $x-y=0$, $x+5y-14=0$, $5x+y-14=0$. **108.** $\frac{\pi}{6}$. **109.** (0; 5) va (4;3). **110.** (7/8; 0) va (−27/8; 0). **111.** $13x+6y-82=0$, $3x+4y-23=0$, $S=31{,}5$ kv.b. **112.** $3x-2y=0$, $5x+y+6=0$. **113.** $5x+4=0$. **114.** $5x+8y+11=0$. **115.** $5y+2=0$. **116.** $17x+11y=0$. **117.** $x+y+1=0$. **118.** $x=a$, $y=b$. **119.** $x=1$, $y=x$. **120.** 30^0. **121.** $\varphi=53^0 8'$. **122.** $5x-3y+2=0$. **123.** $\sqrt{3}$ kv.b. **125.** $B(1;\ 3)$, $C(11;\ 6)$. **126.** 1) $\frac{x}{4}+\frac{y}{6}=1$; 2) $\frac{x}{4(\sqrt{2}-1)}+\frac{y}{(-6)(\sqrt{2}+1)}=1$; $\frac{x}{(-4)(\sqrt{2}+1)}+\frac{y}{6(\sqrt{2}-1)}=1$ **127.** $3x-4y-9=0$; $3x-4y+16=0$, $4x+3y-37=0$ yoki $4x+3y+13=0$. **134.** 1) $a=4$, $b=-3, r=5$; 2) $a=-5$, $b=2$, $r=0$; tenglama nuqtani aniqlaydi. 3) $a=2$, $b=-7$, $r^2=-1$; tenglama geometrik ma'noga ega emas (mavhum aylana). **135.** $tg\varphi=-2{,}4$. **136.** $(x+1)^2+(y-1)^2=5$. **137.** $(x-3)^2+(y-4)^2=25$. **138.** $x=3{,}2$. **139.** $3x-4y+8=0$, $4x-3y+7=0$. **140.** $(x-2)^2+y^2=16$. **142.** (4; 1,8); (4; −1,8); (−4; 1,8); (−4; −1,8). **143.** b^2/a. **144.** $4x+3y+12=0$. **145.** $16x^2+25y^2=41$. **146.** M nuqta – ellipsdan tashqari; N nuqta – ellipsda; P nuqta – ellips ichida. **147.** $e=\sin(\alpha/2)$. **148.** $M(-5;\ 7)$.

149. $3x^2+3y^2-2xy-2x-2y-1=0$. **150**. $\frac{x^2}{3}+\frac{y^2}{4}=1$. **151**. Izlanayotgan egri chiziq – ellips. Agar koordinata o‘qlarini to‘g‘ri burchak tomonlari bo‘yicha yo‘naltirsa (A nuqta Ox o‘qida yotadi), bu ellipsning tenglamasi $9x^2+36y^2=4a^2$.**155**. $\frac{x^2}{9}-\frac{y^2}{8}=1$. **156**. $\frac{x^2}{3}-\frac{y^2}{5}=1$. **157**. (−4; −3). **158**. $\frac{x^2}{64}+\frac{y^2}{48}=1$.**159**. $x^2-y^2=\frac{8}{225}$. **160**. $\ell=\frac{2}{\sqrt{3}}$.**161**. (−8; 0). **162**. $\frac{x^2}{4}-\frac{y^2}{12}=1$. **163**. 6 va 14. **166**. $x^2-\frac{y^2}{3}=1$ giperbolaning o‘ng shoxi.

169. $y^2=4x$. **170**. $M_1(2;4)$ va $M_2(2;-4)$. **171**. $y^2=4x,\ y^2=-4x$. **172**. $y=\pm2\sqrt{2}x$. **173**. $y^2=\sqrt{2}x$. **174**. $M(0;0)$ va $M_1(18;-24)$. **175**. $y^2=x,\ tg\alpha=\frac{8}{15}$. **179**. (3;2). **180**. (8;−6). **183**. 1) $O_1(1;2),\ p=-\frac{1}{4};$ $x'^2=-(\frac{1}{2})y'$; 2) $O_1(1;3),\ p=-\frac{1}{2};\ x'^2=-y'$; 3) $O_1(\frac{1}{16};\frac{1}{8}),\ p=-\frac{1}{8};$ $y'^2=-(\frac{1}{4})x'$; 4) $O_1(1;-2),\ p=\frac{1}{2};\ y'^2=x'$. **184**. 1) $x'y'=\frac{1}{8}$; 2) $x'y'=\frac{13}{9}$; 3) $x'y'=-\frac{6}{5}$; 4) $x'y'=\frac{1}{2}$. **187**. $(x-\frac{1}{2})^2+(y-\frac{1}{3})^2=1$ aylana. **188**. $\frac{x'^2}{25}+\frac{y'^2}{16}=1$ ellips, yangi markaz $O'(1;\ -1)$. **189**. $\frac{x'^2}{4}-\frac{y'^2}{9}=1$ giperbola, yangi markaz $O'(2;\ 3)$. **190**. $O'(2;\ 1)$ nuqta. **191**. $\frac{x'^2}{(-1)}+\frac{y'^2}{(-\frac{1}{4})}=1$ mavhum ellips, $x'=x,\ \ y'=y+1$. **192**. $y'^2-x'^2=1$ giperbola, yangi markaz $O'(3;\ 0)$. **193**. $x'^2=-y'$ parabola, yangi markaz $O'(1;\ \frac{5}{2})$. **194**. x=2 va x=4 to‘g‘ri chiziqlar. **195**. Mavhum to‘g‘ri chiziqlar.

202. $5x+y+1=0$ va $5x+y-1=0$ ikki parallel to‘g‘ri chiziqlar. **203.** $x+y+1=0$ ikkita qo‘shilgan to‘g‘ri chiziqlar. **204.** $2x-3y+1=0$, $4x-3y-1=0$ ikkita kesishuvchi to‘g‘ri chiziqlar. **205.** $\frac{x''^2}{30}+\frac{y''^2}{5}=1$. **206.** $\frac{x''^2}{9}-\frac{y''^2}{36}=1$. **207.** $y''^2=-2x''$. **210.** $x=1/2$, $y=1/2$. **211.** Sistema qarama-qarshi (yechimi yo‘q). **212.** $x=a+в$, $y=a-в$. **213.** Sistema aniqlanmagan (cheksiz ko‘p yechimga ega; x – ixtiyoriy, $y=\left(-\frac{3}{2}\right)x+\frac{1}{12}$.

214. $x=y=z=t$. **215.** $x=\cos\alpha$ $y=\sin\alpha$. **216.** $x=2t, y=t,$ $z=2t$. **222.** 0. **223.** 2. **224.** $2(ad-bc)$. **225.** $x=1,$ $y=2,$ $z=3$. **226.** $x=0,$ $y=0,$ $z=-2$. **227.** $x=0, y=0,$ $z=0$. **228.** $x=t,$ $y=2t,$ $z=-3t$. **229.** $x=1,$ $y=-1,$ $z=0$ **230.** $x=t,$ $y=t,$ $z=-t$.

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234. $C(5/3;11/3;13/3)$, $D(1/3;\ 13/3;\ 17/3)$. **236.** $M(3;\ 1;\ 3)$. **237.** Teng ikkiga. **238.** $M(0;0;17/8)$. **239.** $M(16;\ -5;\ 0)$. **246.** $\overline{AM}=\frac{(b+\lambda c)}{(1+\lambda)}$. **248.** $a_x=0,$ $a_y=2,$ $a_z=-2$. **249.** m^2+m+1. **251.** $a=\frac{3}{5}$; $\cos\alpha=\frac{1}{3}$, $\cos\beta=\cos\gamma=\frac{2}{3}$. **252.** $|\overline{M_1M}|=7$, $\cos\alpha=\frac{2}{7}$, $\cos\beta=-\frac{6}{7}$, $\cos\gamma=\frac{3}{7}$. **253.** $\bar{b}=-2\bar{j}+5\bar{k}$ yoki $\bar{b}=-2\bar{j}-5\bar{k}$. **254.** $M(-4;4;\ 4\sqrt{2})$.

255. $\bar{a}_0=(1/3)\bar{i}-(2/3)\bar{j}-(2/3)\bar{k}$. **268.** -96. **269.** $\arccos(17/50)$. **270.** $m=1$. **271.** 547. **272.** $A=\vec{F}\cdot\vec{s}=F\cdot s\cdot\cos\varphi=5\sqrt{3}$. **273.** $(\pm1/\sqrt{11})(\bar{i}-3\bar{j}+\bar{k})$. **274.** $\bar{c}=\bar{i}+\bar{k}$ yoki $c=(1/3)(-\bar{i}+4\bar{j}-\bar{k})$. **275.** 20/3 va 20/7. **276.** $\bar{r}_D=7\bar{i}+7\bar{j}+7\bar{k}$. **279.** Yo‘q, chunki komplanar vektorlar o'zaro perpendikulyar bo'la olmaydi. **280**. $a\times b - 17\vec{i} + 7\vec{j} - \vec{k}$. **281.** $\sqrt{65}/2$ kv.b. **282.** 4. **284**. 20 kub. b.; $4\sqrt{510}/17$.

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296. 1) $\frac{(x+y-z-2)}{\sqrt{3}}=0$; 2) $\frac{-3}{(5\sqrt{2})^x}-(1-\sqrt{2})y+\frac{4}{(5\sqrt{2})^z}-\frac{7}{(5\sqrt{2})}=0$.

297. $\alpha=\frac{13}{\sqrt{29}}$; koordinatalar boshi va M_0 nuqta tesiklikning turli tomonida yotadi. **298.** $\alpha=7\sqrt{5}/3$. **299.** 1) $x+y+z-5=0$; 2) $2x+2y+z-6=0$ $2x+2y+z-6=0$. **301.** $M(5;\ 5;\ 5)$. **302.** $4x-3y+12z-169=0$. **303.** $5y+4z=0$, $5x-3z=0$, $4x+3y=0$. **304.** $6x+5y-7z-27=0$. **305.** $\frac{x}{2}+\frac{y}{2}+\frac{z}{(\pm\sqrt{2})}=1$. **306.** 60°. **307.** $x+7y+10z=0$. **308.** $x-y=0$. **309.** $x+y+z-3=0$. **310.** $5x+2y-9=0$. **311.** $\sqrt{2}x+y+z-5=0$. **312.** $4x+3y-2z-1=0$. **313.** $(A_1D_2-A_2D_1)x+$ $+(B_1D_2-B_2D_1)y+(C_1D_2-C_2D_1)z=0$. **314.** $x-y+2=0$. **315.** $\arcsin(5/6)$. **327.** $5y+5z-64=0$, $x=0$ (YOZ); $5x+5z-2=0$, $y=0$ (XOZ). **328.** $(x+1)/5=(y-3)/2=z/1$. **329.** $\cos\alpha=6/7$, $\cos\beta=3/7$, $\cos\gamma=2/7$. **330.** $(x-1)/\sqrt{2}=(y+3)/1=(z-3)/(\pm1)$. **331.** $(x-5)/1=(y+1)/3=$ $=(z+3)/(-11)$. **332.** $M(0;7;-2)$. **334.** $x=-3t-1$; $y=6t+1$; $z=t+2$. **335.** $5\sqrt{30}/6$. **336.** $(x-3)/3=(y+1)/(-5)=(z-2)(-2)$. **337.** $\cos\varphi=20/21$. **338.** $x/0=y/1=z/2$. **339.** $(x-4)/2=(y-1)/1=(z+2)/(-2)$. **340.** $x/2=(y-2)/(-1)=(z-1)/0$. **341.** $(x-1)/2=(y-1)/(-3)=(z-1)/2$. **342.** $x/1=(y-2)(-1)=(z-1)/(-1)$. **343.** $x-5y-2z+11=0$. **344.** $x/(-10)=(y-3,4)/13=(z-5,2)/19$. **348.** 1) $C(-1;\ -2;\ 0)$, r=5;

2) $C(2; -3; -1)$, $r=4$; 3) $C(0; -1; 3/4)$, $r=3/4$; 4) $C(1; 0; 0)$, $r=1$; 5) $C(0; 0; 2)$, $r=1$. **349.** 1) Sfera ichida; 2) Sfera tashqarisida; 3) Sfera ustida. **350.** $(x-2)^2+(y-1)^2+(z+2)^2=9$. **356.** 1) Aylanma silindr; 2) elliptik silindr; 3) giperbolik silindr; 4) parabolik silindr; 5) parabolik silindr; 6) prarabolik silindr; 7) aylanma silindr; 8) applikatalar o'qi $x=0$, $y=0$; 9) bissektral tekisliklar $x=z$ va $x=-z$; 10) $y=0$ va $y=x$ tekisliklar. **357.** 1) $x^2+z^2=9$, $y=3$ (aylanma); 2) $y^2-x^2=1$, $z=1$ (giperbola); 3) $z^2-y^2=0$, $x=0$ (ikki to'g'ri chiziq). **358.** 1) $y^2/b^2+z^2/b^2-x^2/a^2=0$. **363.** 1) Giperbolik paraboloid; 2) uchi koordinatalar boshida bo'lgan konus. **364.** $3z=2x^2+y^2$. **365.** $\frac{x^2}{9}+\frac{y^2}{5}+\frac{z^2}{1}=1$. **366.** (aylana) $x^2+y^2=1$, $z=1$. **367.** 1) Ordinatalar o'qi; 2) o'qi Oy bo'lgan va uchi koordinatalar. boshida bo'lgan konus; 3) o'qi Ox bo'lgan va uchi koordinatalar boshida bo'lgan konus; 4) koordinatalar boshi; 5) Oz o'qi bo'yicha kesishuvchi ikki tekislik. **374.** $x=y$ va $x=z$ ikki tekislik. **375.** $(x-2)^2+(z-2)^2=4$ aylanma silindr. **376.** $x=y=z$ to'g'ri chiziq. **377.** $x^2+(y-1)^2-(z-1)^2=0$ uchi $S(0; 1; 1)$ nuqtada bo'lgan ikkinchi tartibli konus. **378.** $(0; 1; -1)$ nuqta. **379.** $x'^2+y'^2/4-z'^2=1$ kanonik tenglamali bir pallali giperboloid. **380.** $x'^2+y'^2+z'^2=-1$ kanonik tenglamali ikki pallali giperboloid. **381.** $x'^2+y'^2=4z'$ kanonik tenglamali aylanma paraboloid. **382.** $x'^2-z'^2/9=2y$ kanonik tenglamali giperbolik paraboloid.

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387. 900. **388.** 12. **389.** 21280. **390.** a^2b^2. **391.** $x=1, y=-1,$. $z=0, t=2$. **410.** $(t; 2t; 3t)$ bu yerda t–ixtiyoriy haqiqiy son. **411.** $(2t; 2t; t)$ bu yerda t-ixtiyoriy haqiqiy son. **412.** (0; 0). **413.** $x'\cos\alpha-y'(1+\sin\alpha)=0$ to'g'ri chiziq. **414.** $B=\begin{pmatrix}-4 & -8 & -4\\ -3 & -1 & -5\\ -7 & -6 & 1\end{pmatrix}$.

415. $\begin{pmatrix} 9 & 6 & 6 \\ 6 & 9 & 6 \\ 6 & 6 & 9 \end{pmatrix}$. **416.** $\begin{pmatrix} 0,1 & -0,2 & 0,7 \\ 0 & 0,1 & -0,2 \\ 0 & 0 & 0,1 \end{pmatrix}$. **417.** $x=1,\quad y=2,\quad z=3$.

418. $\lambda_1=2,\quad \lambda_2=1;\ \ell_1=\left(4/\sqrt{41}\right)i-\left(5/\sqrt{41}\right)j;\quad \ell_2=\left(1/\sqrt{2}\right)i+\left(1/\sqrt{2}\right)j$.

419. $\lambda_1=-2,\quad \lambda_2=3,\quad \lambda_3=6\quad r_1=\alpha(i-k),\quad r_2=\beta(i-j+k),\quad r_3=\gamma(i+2j+k)$.

423. $x''^2/16+y''^2/4=1$. **424.** $x''^2/25-y''^2/9=1$. **425.** $y''^2=2\sqrt{2}x''^2$.

426. $x'^2+y'^2/1-z'^2/3=1$ (bir pallali giperboloid).

427. $2y''^2+3z''^2=\sqrt{6}x''$ (elliptik paraboloid). **434.** agar $\lambda\neq 0$ bo'lsa, $r(A)=2$. **435.** $r(A)=3$. **436.** $r(A)=3$ bazis minorlar $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ va $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 3 & 0 \end{pmatrix}$. **437.** $r(A)=2$; bazis minorlar $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$, $\begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix}$, $\begin{pmatrix} 1 & 4 \\ 3 & 8 \end{pmatrix}$, $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$, $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 3 & 6 \\ 1 & 3 \end{pmatrix}$ va $\begin{pmatrix} 3 & 8 \\ 1 & 4 \end{pmatrix}$. **441.** Sistema o'rinli, $r(A)=r(A_1)=2$; $x_1=1,\ x_2=1/2$. **442.** $r(A)=1,\quad r(A_1)=2$. Sistema o'rinli emas. **443.** Sistema o'rinli, $r(A)=r(A_1)=2$. **446.** $x_1=1,\quad x_2=5,\quad x_3=2$. **447.** $x_1=1,\quad x_2=2,\quad x_3=3,\quad x_4=4$. **448.** $x_1=5, x_2=4,\ x_3=3, x_4=1,\ x_5=2$. **449.** Sistema o'rinli emas. **450.** $x=1{,}96,\quad y=2{,}96,\quad z=5{,}04$. **451.** $x=1{,}50,\ y=1{,}16,\ z=1{,}40$. **458.** $x_1=u, x_2<u+1,\ x_2=u+2,\ x_4=u+3$. **459.** Sistema o'rinli emas. **460.** $r(A)=3$.

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463. Ha. **464.** Yo'q, chunki to'plamning ikki elementi yig'indisi shu to'plamning elementi emas. **465.** Yo'q, chunki ikkinchi darajali ikki ko'phadlarning yig'indisi birinchi darajali ko'phad yoki o'zgarmas son bo'lishi mumkin. **466.** Ha. **467.** 1) Ha; 2) Ha; 3) Ha; 4) Yo'q.

468. Ha. **469.** 1) nol vektor bo‘lgandagina; 2) yo‘q, chunki bu fazoda x va y vektorlardan tashqari boshqa $\lambda x+\mu y$ ko‘rinishdagi vektorlar ham bo‘lishi kerak. **470.** Yo‘q, chunki hosil bo‘lgan vektorlar to‘plamida yig‘indisi x bo‘lgan vektorlar topiladi, masalan, $(x-y/2)$ va $(x+y)/2$ vektorlar. **471.** Mumkin. Masalan, geometrik vektorlar to‘plamidan Oz o‘qiga perpendikular bo‘lmagan vektorlarni chiqarib tashlasak, chiziqli fazoni tashkil etuvchi $\lambda\bar{i}+\mu\bar{j}$ vektorlar to‘plami hosil bo‘ladi. **473.** Yo‘q, chunki $\lambda(\xi_1,\ \xi_2,\ \xi_3)$, agar λ – butun bo‘lmagan to‘plamga kirmasa. **474.** Yo‘q. **475.** Yo‘q, chunki qarama-qarshi vektorlar I oktantada joylashmagan. **488.** Darajasi n dan oshmaydigan barcha ko‘phadlar to‘plami. **501.** $x=e_1+2e_2+3e_3+4e_4$. **502.** $x=e_1+e_2+e_3+\ldots+e_n$. **504.** $\xi_1=\sum\xi_1$, $\xi_2=\alpha\xi_1$, $\xi_3=\beta\xi_2$, $\xi_4=\gamma\xi_3$, $\xi_5=\delta\xi_4$. **505.** Yo‘q, chunki $e_1'+e_2'+e_3'=0$ bajarilishi kerak, bu esa e_1', e_2', e_3' bazis vektorlarning chiziqli erkli bo‘lgani sababli mumkin emas. **506.** Bu element nol vektor bo‘lgandagina mumkin. **508.** Kesishgan $x_{12}=(0;0;\xi_3;\xi_4)$, $y_{12}=(0;0;\eta_3;\eta_4)\,z_{12}=(0;0;\xi_3;\xi_4)$ elementlar to‘plami yig‘indisi R fazo bilan ustma-ust tushadi. **509.** $d(R_1)=3,\ d(R_2)=3,\ d(R_3)=2,\ d(R_4)=4$. **510.** Yo‘q. **513.** R_3 o‘zgarmas kattaliklar to‘plami, $R_4=C_0t^4+C_1t^2+C_2t+C_3$ ko‘rinishdagi ko‘phadlar to‘plami. **514.** R_3-Ox o‘qiga parallel bo‘lgan barcha vektorlar to‘plami, $R_4=R$. **516.** Barcha juft funksiyalar to‘plami fazo osti tashkil qiladi, toqlarining to‘plami yo‘q, chunki ikki toq funksiyalarning ko‘paytmasi juft funksiya. **517.** Yo‘q, chunki ixtiyoriy λa vektor bu to‘plamga tegishli emas, agar λ irrastional son bo‘lsa. **522.** $k=3$; $f_1=(-1;0;1;0;0;)$, $f_2=(-1;0;0;1;0)$, $f=(0;-1/2;0;1)$, $f=\left(-\bar{\bar{C}};\ -\bar{\bar{C}};\ -0{,}5\bar{\bar{C}};\ \bar{\bar{C}}_1;\ \bar{\bar{C}}_2;\ \bar{\bar{C}}_3\right)$. **526.** Ha. **527.** Yo‘q, chunki $ab\neq 0$ bo‘lganda $|a+b|\cdot|a+b|=a\cdot a+b\cdot b$ tenglik bajarilmaydi. **528.** $x_0=0$

bo'lgandagina. **529.** $\alpha=0$ bo'lgandagina. **530.** Ha. **533.** $\begin{pmatrix} \alpha\, 0 \ \ 0 \ldots 0 \\ 0\, \alpha \ \ 0 \ldots 0 \\ \ldots\ldots\ldots \\ 0 \ \ 0 \ \ 0 \ \ \alpha \end{pmatrix}$.

536. $A=\begin{pmatrix} 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \\ 1\,0\,0\,0 \end{pmatrix}$. **538.** $3A-2B=E$. **544.** $A^{-1}=\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$.

545. $A^{-1}=A$. **546.** $A^{-1}=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. **547.** $B=2E\cos\alpha$. **548.** A chiziqli akslantirish teskarisiga emas, chunki $|A|=0$. **549.** $A^2=\left(A^{-1}\right)^2=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. **550.** $d<-2$ da. **552.** 1) Agar $\alpha\neq\beta$ bo'lsa, $\lambda_1=\alpha$, $\bar{u}=C_1\bar{e}_1$, $\lambda_2=\beta$, $\bar{\upsilon}=C_2\bar{e}_2$; 2) Agar $\alpha=\beta$ bo'lsa, $\lambda_1=\lambda_2=\alpha$, $\bar{u}=C_1\bar{e}_1+C_2\bar{e}_2$. **553.** $\lambda_1=\lambda_2=\alpha$, $\bar{u}=C_1\bar{e}_1+C_2e_2$. **556.** $\lambda=2$, $\bar{u}=C_1\left(\bar{e}_1-\bar{e}_3\right)$; $\lambda=3$, $\bar{v}=C_2\left(\bar{e}_1-\bar{e}_2+\bar{e}_3\right)$; $\lambda=6$, $\bar{w}=C_3\left(\bar{e}_1+2\bar{e}_2+\bar{e}_3\right)$ **558.** $\lambda=-1$, $\bar{u}=C_1\bar{i}+C_2\bar{j}$. **560.** $\lambda=1, \bar{u}=C_1\left(\bar{e}_1+\bar{e}_2+\bar{e}_3\ +\bar{e}_4\right)$; $\lambda=-1$, $\bar{\upsilon}=C_2\left(\bar{e}_1-\bar{e}_2+\bar{e}_3-\backslash\bar{e}_4\right)$. **561.** $\lambda=\alpha+\beta+\gamma$, $\bar{u}=C\left(\bar{e}_1+\bar{e}_2+\bar{e}_3\right)$. **563.** $(\bar{x},\bar{y})$–korxona ishlab chiqarayotgan barcha mahsulotning umumiy bahosi. **565.** Ha. **566.** Yo'q, chunki, $\lambda=0$ da 2° va 3° shartlar bajarilmaydi. **567.** Ha. **569.** $\arccos(1/n)$. **573.** Ha. **577.** $|\bar{x}|=5$. **578.** $\bar{x}/|\bar{x}|=(1/15)\bar{e}_1+(2\sqrt{2}/15)\bar{e}_2+(\sqrt{3}/5)\bar{e}_3+(8/15)\bar{e}_4+(\sqrt{5}/3)\bar{e}_5$. **579.** x – normirovanniy vektor. **580.** $\varphi=\frac{\pi}{3}$. **581.** $\pm0{,}5(\bar{e}_1+\bar{e}_2+\bar{e}_3\ +\bar{e}_4)$. **582.** $\lambda=\pm1$. **587.** Ha. **588.** Ha. **589.** $\lambda=\pm1$ da. **590.** Ha, chunki, $A\bar{e}_1, A\bar{e}_2$ *va* $A\bar{e}_3$ ortogonallashgan bazisni tashkil qiladi. **591.** Ha. **598.** $x'^2/21+y'^2/3=1$. **599.** $x'^2/16-y'^2/4=1$. **600.** $x'^2/44+y'^2/4=1$.

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606. $n=4$. **607.** $\delta=0.16\%$. **608.** $\delta=0.0005\%$. **609.** $\delta=0.022\%$; $n=4$; $S=8765\pm0,1m^2$. **617.** 1) $[-2, 0[\cup]0, 2]$; 2) $[0, 4]$; 3) $]-\infty, 0[\cup]0, +\infty[$; 4) $x\neq\pi(2n+1)/4, n\in Z$; 5) $]-\infty; -2] \cup [2; +\infty[$; 6) $]1/3; +\infty[$; 7) $[0; 2[$. **618.** 1) $[1, +\infty[$; 2) $]-\infty, 0[\cup]0, +\infty[$; 3) $[-4, 4]$; 4) $]-\infty, 3[$; 5) $[-2, 4]$; 6) $]0, 1]$. **619.** 1) Toq 2) juft; 3) toq ham emas, juft ham emas; 4) juft; 5) toq ham emas; 6) juft; 7) toq. **620.** 1) $2\pi/5$; 2) 6π; 3) π; 4) π. **657.** $1/2$. **658.** -1. **659.** $1/6$. **660.** -2. **661.** $-\sin\alpha$. **662.** m/n. **663.** $\sec^2 x_0$. **664.** $-\sqrt{2}/4$. **665.** $1/2$. **666.** ∞. **667.** 2. **668.** $3/4$. **669.** $-1/4$. **670.** $1/2$. **671.** 3. **672.** $\sqrt{7}/4$. **673.** $25/9$. **674.** $1/2$. **674.** $1/2$. **675.** m. **676.** 1 agar $x\to+\infty$; -1, agar $x\to-\infty$. **677.** $(a-c)/2$. **678.** 0. **679.** 0. **680.** $\ell n5$. **681.** $\ell n(8/7):\ell n(6/5)$. **682.** 2. **683.** $\ell n5$. **684.** 1/4. **685.** 1 agar $x\to+0$; -1, agar $x\to-0$. **686.** $+\infty$. **687.** 2. **688.** 0. **689.** Mavjud emas. **690.** 5/4. **691.** ℓna. **692.** e. **693.** e^3. **694.** 1/6. **695.** $\ell n(5/4)$. **696.** 1. **697.** -3. **698.** 0. **699.** 1/2. **700.** e^{10}. **701.** $\sqrt{e}$. **702.** e^{a-b}. **703.** $\sqrt{e}$. **708.** $y-x$. **709.** 2. **710.** 1/2. **711.** $\alpha=O(\beta)$. **712.** $\alpha-\beta$. **713.** $\alpha-\beta$. **714.** 1/3. **715.** 9/4. **716.** $-1/2$. **717.** $-1/2$. **718.** $-1/2$. **719.** 1. **720.** 9/25. **721.** $(\ell n5\cdot\ell n4)/(\ell n3\cdot\ell n6)$. **722.** 1,6. **726.** $x=2$ sakrash nuqtasi. **723.** $x=1$, $x=5$ II tur uzilish nuqtalari. **728.** II tur uzilishi. **729.** $x=0$ to'g'rilanadagan uzilish nuqtasi. **730.** $x=3$ sakrash nuqtasi $x=5$ II tur uzilish nuqtasi, $x=0$ to'g'rilanadagan uzilish nuqtasi $x=\pi/2+\pi n(n\in Z)$ II tur uzilish nuqtasi. **731.** $x=1$, $x=2$ to'g'rilanadagan uzilish nuqtalari. **732.** $x_1=-2$, $x=-3$ II tur uzilish nuqtalari; $x=-1$ .to'g'rilanadagan uzilish nuqtasi. **733.** Funksiya $]-\infty; +\infty[$ cheksiz oraliqda uzluksiz. **734.** 1) Funksiya uzluksiz; 2) bitta II tur uzilish nuqtasiga ega; 3) ikkita II tur uzilish nuqtasiga ega. **735.** 1. Funksiya uzluksiz, ikkita II tur uzilish nuqtasiga ega. 3. to'rtta II tur uzilish nuqtasiga ega.

P.E.DANKO, A.G.POPOV, T.Y.KOJEVNIKOVA

OLIY MATEMATIKA MISOL VA MASALALARDA

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ANALITIK GEOMETRIYA, CHIZIQLI ALGEBRA ASOSLARI VA ANALIZGA KIRISH

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Muharrir *A. Bahromov*
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O'zbekiston faylasuflari milliy jamiyati nashriyoti.
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