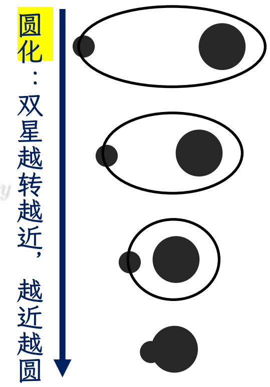
有了空间引力波探测器，就知道了离心率，就能知道某些双黑洞系统是怎么来的了

一篇工作的完成离不开导师和合作者们的悉心指导(｀・ω・´)



离心率越小，轨道越圆。圆形轨道e=0

太长不看：

- 想知道恒星级双黑洞怎么形成的，离心率e是个突破口
- 但光靠地面的探测器怕是不够用
- 还得是空间探测器，但是空间探测器有自己的难处

- 我们结合了一下二者的优点，搞了个叫档案搜索的多波段探测，这样探测离心率e就没那么费力了
- 话虽如此，但离心率e掺和进来后带来的额外计算负担可还是不小啊，得让大家知道知道
- 不管怎么样，我们首先把这个探测过程实现了，还把具体增加了多少负担估算出来了

I. 从盘古开天地到这篇文章要研究啥

2015年首例引力波事件被探测，随后更是惊喜不断，看到不少来自恒星级双黑洞的信号——但科学家对这些系统怎么形成的反而更迷惑了

离心率是个不错的突破口，但目前看到的系统可以说基本都是圆形轨道

为什么呢？双黑洞越绕越近，越损失能量，轨道越圆，因此就算之前有离心率，等快并合的时候再去看早没了！——这就是地面探测器的局限性

已经结束咧！.jpg

所以怎么才能看到并合前的信号呢？

空间探测器表示：我来助你！.gif

双黑洞还在绕转的时候是在空间探测器灵敏范围内的，那会儿还早，还没来得及圆化

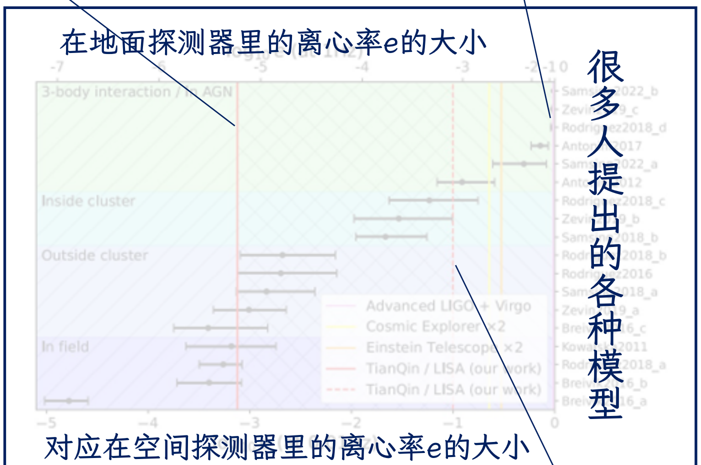
看图一↓！科学家们搞了各种模型，比如：

- 双黑洞自己玩自己的（孤立演化），没人打扰，轨道就会很圆；
- 如果在星团这种热闹的地方，互相拉扯，离心率就会大上不少；但如果中途被甩出星团，则和孤立演化差别不大；
- 要是考虑活动星系核啥的就更热闹了，那样离心率会非常接近于1

晋西北都乱成一锅粥了！

我们的工作证明了：空间探测器比那些地面探测器能探测到小得多的e

LIGO探测器这条线已经快贴上坐标轴了，在它左边还有这么多模型等着区分呢



我们的工作：因为资源限制就先算到这么大了，但这不是空间探测器的上限

妙啊，所以具体咋探测？匹配滤波。

这玩意跟听歌识曲似的，首先你得有个覆盖面广的曲库，即模板库。地面探测器就测几秒钟信号，10万个足矣，但是空间探测器信号动辄几年，真这么搞得要10^{30}个，这就是真·天文数字了！

于是乎有人想到了个点子：不如先看地面给出的并合信号，然后按图索骥去空间探测器那边的数据里看能不能把老底儿挖出来（即**档案搜索**）

这样很有针对性，大部分信息已经掌握了，给空间探测器造个小的模板库就可以了，岂不美哉？

点子不错，但目前还没人真刀真枪地把这事给做出来，更别提在这上加什么离心率了——诶，这不就来活儿了么↓

II. 各种技术细节

生成模板库需要一个引力波波形，我们挑了个带离心率的波形叫EccentricFD

我们考虑天琴和LISA两个空间探测器，它们都计划在2030s开始上天干活；

所以下一代地面探测器才是真的配合它们档案搜索的，例如Cosmic Explorer (CE)和Einstein Telescope (ET)。

但CE、ET它们也都还没真开始干活呢，得先想法子估计它们的探测水平，这个工具叫Fisher信息矩阵

sen. Since GW emission will cause a binary orbit to circularize over time [16], we assume that events are nonecc-

算完了发现和前人工作的结论相似，这就放心了。什么结论呢？

地面探测器能把大部分参数信息都测量得很准，即比空间探测器准，但这两样不行：
一个是之前提到的离心率e；另一个是核心参数啁啾质量M。

这样好办，那把别的参数都固定了，咱就盯着这俩参数生成模板库了

we assume that all the parameters except for chirp mass and eccentricity are known exactly when performing an archival search, and the chirp mass range is determined by the uncertainty from the network of the ET and two CEs, i.e., $\mathcal{M} \in [\mathcal{M}_0 - 10\sigma_{\mathcal{M}}, \mathcal{M}_0 + 10\sigma_{\mathcal{M}}]$. In the future,

这里面是啥原理？来看这个GitHub repo：
[HumphreyWang/sbank_simplified]

and conservative estimate.

所以用啥生成模板库？sbank，一个Python程序包。

现实中数据和模板一毛一样是不可能的，那怎么评估你这匹配滤波配没配上呢？

$$\mathrm{FF}\left(\lambda^{\mu}\right) \equiv \max_{\lambda^{\mu'}} \frac{\left(h(\lambda^{\mu})\middle|h(\lambda^{\mu'})\right)}{\sqrt{\left(h(\lambda^{\mu})|h(\lambda^{\mu})\right)\left(h(\lambda^{\mu'})|h(\lambda^{\mu'})\right)}}. \quad (1)$$

Here $\lambda^{\mu'}$ denotes the parameter set for a template in the bank, and λ^{μ} is the parameter set for the test waveform.

好问题，匹配因子FF就是干这个的，我们一般取0.97作为阈值，比这还低就说明没匹配上，你这个模板库对于该信号就是个失败的模板库

period of only seconds before coalescence, so that the Doppler frequency modulation from the movement at Earth's orbit can be ignored. However, the long observation time and the orbital motion of space-based observatories make the response time dependent, and one must consider these time-dependent response terms during bank generation.

难办？就别办了！.jpg
(╯°口°)╯┻━┻

the relative spacecraft motion results in unequal arm lengths. The method of time delay interferometry (TDI) has been proposed for canceling out the laser phase noise from different arms. It constructs particular combina-

tions to make virtual equal arm interferometers. This is further complicated when considering eccentric wave-

先别着急，还有好几个事得注意，空间探测器得考虑天线响应函数，还有离心率加进来得考虑谐频的问题，还有……总之比大家预想的要麻烦多了

Since different eccentric harmonics have different correspondences with the Fourier frequency, we should provide a frequency cutoff during the calculation to avoid the waveform generation exceeding the valid range for a specific GW detector: $\tilde{h}_{\rm det} = \sum_j \tilde{h}_j \times \Theta\left(j \cdot f_{\rm high} - 2f\right)\Theta\left(2f - j \cdot f_{\rm low}\right)$, where $\Theta(x)$ is the Heaviside step function and j denotes the jth eccentric harmonic [26]. For TianQin or LISA, we have $f_{\rm low} = \max\left[10^{-4}\mathrm{Hz}, f_0\right], f_{\rm high} = \min\left[f_{\rm ISCO}, 1\mathrm{Hz}\right]$, where $f_{\rm ISCO} = (6^{3/2}\pi(m_1+m_2))^{-1}$ is the quadrupolar frequency at innermost-stable circular orbit (ISCO).

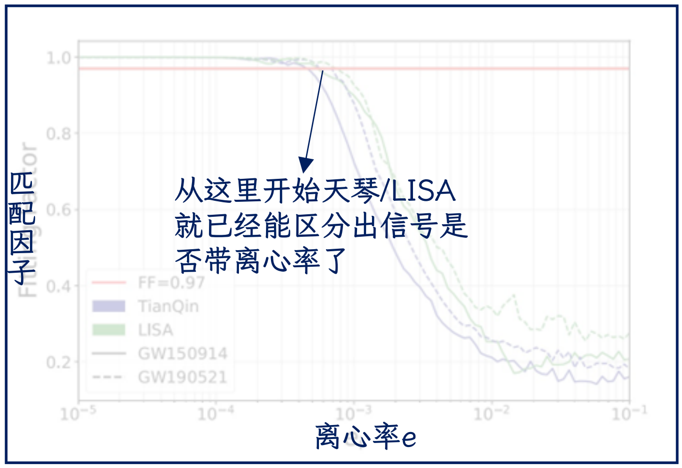


FIG. 2: The fitting factor between a noneccentric template bank and a signal with different eccentricities. The blue(green) lines denote the banks of TianQin(LISA), the solid(dashed) lines correspond to the banks of a GW150914-like(GW190521-like) scenario.

III. 模板库生成，启动！

等一下！上面说困难这么多，那要不咱就别考虑离心率了？干嘛给自己找罪受呢？

那就得看e小到啥程度我们就可以不管了，来看图二↑：一个没离心率的模板库去匹配带各种离心率e的信号，还不到0.001呢就已经配不上了。所以别想着偷懒了，再说e测不准的话对别的参数也不好

tial eccentricity at ~ 0.01Hz. We also investigate the bias between the injected and recovered chirp mass when neglecting eccentricity, which increases from $\lesssim 10^{-6} M_\odot$ at $e_i = 0$ to $\gtrsim 10^{-3} M_\odot$ at $e_i = 0.1$. Such systematic bias could be even larger in the full parameter space. It is therefore necessary for searches to take eccentricity into account.

TABLE I: Template bank sizes for GW150914- and GW190521-like events with different parameter spaces.

	Parameter space	GW150914-like	GW190521-like
TianQin	$e_i \in [0, 0.1]$	117202	49943
	$\mathcal{M} \in \mathcal{M}_0 \pm 10\sigma_{\mathcal{M}}$	3034	4250
LISA	$e_i \in [0, 0.1]$	100403	44867
	$\mathcal{M} \in \mathcal{M}_0 \pm 10\sigma_{\mathcal{M}}$	2070	3088

现在我们分别生成只有离心率和啁啾质量的模板库，看表一↑。单考虑离心率居然就要十万个模板？而且程序放服务器上跑了好久的。。。这还是离心率只算到0.1呢

of memory). Therefore, for TianQin/LISA, we consider $e_i = 5\times10^{-4}$ ($e_i = 0.1$) as the smallest distinguishable eccentricity (the upper limit by the current computational cost), which corresponds to the red solid (dashed) line in Fig. 1. Figure 3 shows the distribution of the ec-

先不说这个，这离心率分布还挺有趣的，和前人工作中的估计对上了，看图三↓：离心率越大，同样范围内需要的模板数越大

small, $\mathcal{M}$ appears to be uniformly distributed.

FIG. 3: The distribution of the eccentricity in the archival search template bank. The shaded regions represent the 1σ Poisson fluctuation.

既然如此，那我们可以估计一下同时考虑两个参数时的模板库大小，多大呢？要上亿个模板！
空间探测器的情况本就复杂，算得慢，这样要算到猴年马月去？！

只恨算力不足.jpg

eccentricity range increases, the full 2D archival search banks are expected to have $N_T \sim \mathcal{O}(10^8)$ templates, if we consider the maximal valid range for EccentricFD, i.e. $e_i \in [0, 0.4]$, N_T will be up to $\mathcal{O}(10^9)$.

To evaluate if we have overestimated the magnitude of 2D bank size due to any degeneracy between the eccentricity and the chirp mass [60–62], we generate a 2D

我们还真试了试算个小点的二维模板库，结果更加印证了上文提出的量级估计

which is smaller but of the same order as the direct multiplication of bank sizes that are calculated separately in their parameter spaces. Such results do not change our magnitude estimation of the full 2D archival search bank size. This indicates the challenge of computational cost: an example 2D bank with $e_i \in [0, 0.001]$ includes 13372 templates, and would need ~ 80hr for one core (and 18 GB of memory to cache waveforms) to generate. By slicing the full parameter space along eccentricity and generating the 2D bank in parallel, a bank with $N_T \sim \mathcal{O}(10^8)$ needs $\sim 8 \times 10^5$ core hours (and $\sim 10^5$GB of memory).

等一下！这些数字可靠吗，万一事情没那么严重，是我们高估了呢？

看图四↓：我们来做个有效性和冗余度检验！

- 有效性，通过√
- 冗余度，低，通过√

所以，我们的结论仍未被动摇

of templates should be smaller than the minimal match threshold. In Fig 4, following the validity test, for each template we present the histogram of the fitting factor, which is calculated on a bank that excludes the template itself. We find that only 6.22% of all templates are redundant. This brings marginal extra computational cost.

IV. 从这篇文章研究了啥到给未来画个饼

Numerous studies pointed out that the eccentricity of sBBHs will play a significant role in unveiling their origin.

我们真的把档案搜索这个饼给实现了，还在此基础上加了离心率，而且具体增加了多少负担也估算出来了

generation process that includes eccentricity.

We generate one-dimensional template banks for either initial eccentricity or for chirp mass. The upper limit of

FIG. 4: Validity and redundancy test of the example 2D template bank. The histogram in purple (cyan) shows the result of the validity test (redundancy test). The vertical red line corresponds to the match criteria $M = 0.97$.

大家好像觉得双黑洞那么多参数，再多一个离心率难不了太多，不如下次一定

initial eccentricity at a system five years before merger is 0.1. The range of chirp mass is determined by the estimation with the ground-based network. By extrapolating the one-dimensional bank results, we conclude that a two-dimensional eccentric bank will comprise $N_T \sim \mathcal{O}(10^{8-9})$ templates, which is $\sim \mathcal{O}(10^5)$ larger compared to the zero eccentricity case, and will require $\sim \mathcal{O}(10^6)$ core hours [illegible] larger compared to noneccentric case]. [illegible] here [illegible] 0.001. Constructing and filtering a template bank of 10^{8-9} waveforms will therefore be a challenging task, but it is not outside the scope of the expected computational facilities in the late 2030s, and could be further improved with additional optimization of the relevant software techniques.

确实，我们解决了一个问题，却带来了新的挑战。但计算资源和效率问题在2030s很可能就不是问题了哈哈哈

我们的工作还是证明了，这种方法确实能帮助区分恒星级双黑洞系统的形成机制

Ou[illegible] solution [illegible] realistic multiband CW observation scenario with which one can determine the formation mechanism of sBBHs with successful [illegible] searches.

It should be noted that we use a nonspinning eccentric waveform model in the paper. It is already known that spin effects are largely negligible during the inspiral[63] pha[illegible] not a concern at all [illegible] constrained by ground-based facilities. Our space-based archival search wou[illegible] eccentricities using the measured black hole spins. It is important to note though that for both ground- and space-based detectors, more precise waveform models will be needed in the future to avoid potential systematic errors [58, 64–67].

但有一点，我们用了个波形，有离心率但是没有自旋，这篇文章里倒也没什么，但在未来更精确的波形肯定不能少啊

同行们，别下次一定了！

One caveat in the study is the duty cycle. We consider ET + dual CE for the ground-based detectors, whereas in reality the duty cycle cannot reach 100%; so the sky localization from realistic future networks might be worse than our calculation. Space-based observatories will also be limited by duty cycles[18, 68]. We leave the detailed calculation to future studies.

V. ACKNOWLEDGMENTS

V. 谢谢您嘞！

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感谢一起聊天的老师和师兄们，
感谢~~金主爸爸~~各种基金项目

这里是巨人们的肩膀↓
（后面还有三页但是就截到这里吧）

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