Job Search Model with Nonlinear Expectations

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1. The job seeker uses power-transformed expectations
2. We assume that $w \in \mathsf{W} = [w_1, w_2]$ and $0 < c < w_1 < w_2$
3. We construct an ADP $(V, \mathbb{T})$ by setting
   3.1 $V = [c, \bar{v}] \subset bm\mathsf{W}$ where $\bar{v} := \frac{c+w_2}{1-\beta}$
   3.2 $T_\sigma \in \mathbb{T}$ defined as

$$T_\sigma v = \sigma e + (1 - \sigma)(c + \beta R v), \tag{1}$$

where

$$(Rv)(w) := \left(\int v^\gamma(w') P(w, \mathrm{d}w') \right)^{1/\gamma} \qquad (w \in \mathsf{W}, \gamma \neq 0),$$

and $e(w) := \frac{w}{1-\beta}$ is the stopping reward.

Two facts regarding

$$(Rv)(w) := \left(\int v^{\gamma}(w') P(w, \mathrm{d}w') \right)^{1/\gamma}$$

1. R is order-preserving on V and maps any constant function to itself
2. R is concave when $\gamma \leqslant 1$ and convex when $\gamma \geqslant 1$
    2.1 $R(\lambda v) = \lambda Rv$
    2.2 $\gamma \geqslant 1$: $R(v + w) \leqslant Rv + Rw$ (Minkowski inequality)
    2.3 $\gamma \leqslant 1$: $R(v + w) \geqslant Rv + Rw$ (reverse Minkowski inequality)

To see the optimality and algorithmic results:

1. bmW is countably Dedekind complete Banach lattice
2. T_σ is an order preserving self-map on V, and
    2.1 $\exists \varepsilon > 0$: $T_\sigma c \geqslant \frac{c}{1-\beta} \bigwedge (c + \beta c) \geqslant c + \varepsilon(\bar{v} - c)$
    2.2 $\exists \varepsilon > 0$: $T_\sigma \bar{v} \leqslant \frac{w_2}{1-\beta} \bigvee (c + \beta \bar{v}) = (\bar{v} - \frac{c}{1-\beta}) \bigvee (\bar{v} - w_2) \leqslant \bar{v} - \varepsilon(\bar{v} - c)$
3. Regularity:

$$\sigma(w) := \mathbb{1}\left\{\frac{w}{1-\beta} \geqslant c + (Rv)(w)\right\}$$

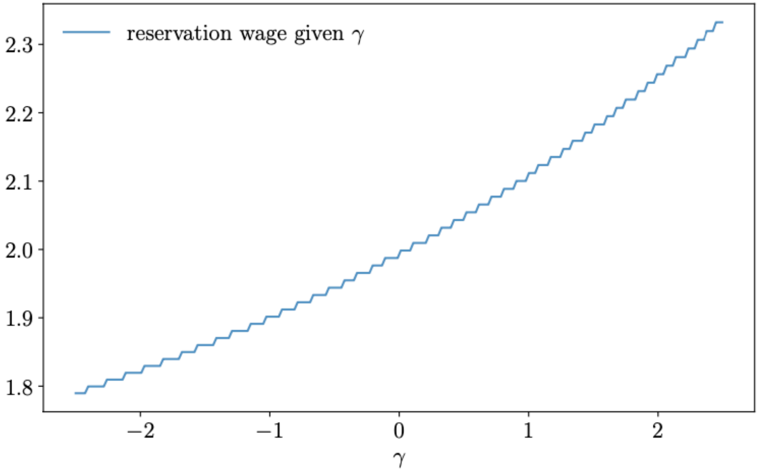

Figure 4.6: The reservation wage as a function of γ