George Mason University, Computer Science

Homework 7

Yongxin Wang
ywang51@mason.gmu.edu

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1. Sensor Model
   Based on the assumption of independent measurements.

$$
\begin{aligned}
p(\mathbf{z} \mid \mathbf{x}, \mathbf{l}) &= \prod_{k=1}^{K} p(z_k \mid \mathbf{x}, \mathbf{l}) \\
&= p(z_r \mid \mathbf{x}, \mathbf{l})\, p(z_\theta \mid \mathbf{x}, \mathbf{l}) \\
&= \frac{1}{\sigma_r \sqrt{2\pi}} e^{-(z_r - \|\mathbf{x}-\mathbf{l}\|)^2 \big/ 2\sigma_r^2} \cdot \frac{1}{\sigma_\theta \sqrt{2\pi}} e^{-\left(z_\theta - \theta_x^1\right)^2 \big/ 2\sigma_\theta^2}
\end{aligned}
$$

   In the equation above, $\theta_x^l = arctan(\mathbf{l}(2) - \mathbf{x}(2), \mathbf{l}(1) - \mathbf{x}(1))$.

2. Bayesian Inference

   a)

$$
\begin{aligned}
p(\mathbf{x} \mid d_0, d_1) &= \frac{p(d_0, d_1 \mid \mathbf{x})\, p(\mathbf{x})}{p(d_0, d_1)} \\
&= p(d_0 \mid \mathbf{x})\, p(d_1 \mid \mathbf{x}) \frac{p(\mathbf{x})}{p(d_0, d_1)} \\
&= \frac{1}{\sigma_0 \sqrt{2\pi}} e^{-(d_0 - \|\mathbf{x}-\mathbf{x_0}\|)^2 \big/ 2\sigma_0^2} \cdot \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-(d_1 - \|\mathbf{x}-\mathbf{x_1}\|)^2 \big/ 2\sigma_1^2} \cdot \frac{p(\mathbf{x})}{p(d_0, d_1)} \\
p(\mathbf{p_0} \mid d_0, d_1) &= 0.0979 \frac{p(\mathbf{p_0})}{p(d_0, d_1)} \\
p(\mathbf{p_1} \mid d_0, d_1) &= 0.0114 \frac{p(\mathbf{p_1})}{p(d_0, d_1)}
\end{aligned}
$$

My friend is more likely to be at university if no further information will be provided.

b)

$$p\left(\mathbf{p_0} \mid d_0, d_1\right) = 0.0294 \frac{1}{p\left(d_0, d_1\right)}$$
$$p\left(\mathbf{p_1} \mid d_0, d_1\right) = 0.0080 \frac{1}{p\left(d_0, d_1\right)}$$

Given that my friend can only be either at university or home, the total probabiliry is summed to 1.

$$p\left(\mathbf{p_0} \mid d_0, d_1\right) = 0.7861$$
$$p\left(\mathbf{p_1} \mid d_0, d_1\right) = 0.2193$$

3. Motion Model