Humza Salman mhs180007

In [74]:
```
import networkx as nx
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as plt
import pandas as pd
```

This function prints the top five (or `num`) nodes according to the centrality vector `v`, where `v` takes the form: `v[nidx]` is the centrality of the node that is the `nidx`-th element of `G.nodes()`

In [90]:
```
def get_top_5(G,v, num=5):
    thenodes = list(G.nodes())
    idx_list = [(i,v[i]) for i in range(len(v))]
    idx_list = sorted(idx_list, key = lambda x: x[1], reverse=True)
    ret = []
    for i in range(min(num,len(idx_list))):
        nidx, score = idx_list[i]
        # print('  %i. %s (%1.4f)' % (i+1,thenodes[nidx],score))
        txt = ('%i.  %s   (%1.4f)' % (i+1,thenodes[nidx],score))
        ret.append(txt)
        #print '  %i. %s' % (i+1,G.node_object(idx))
    return ret
```

In [68]:
```
def print_top_5(G,v, num=5):
    thenodes = list(G.nodes())
    idx_list = [(i,v[i]) for i in range(len(v))]
    idx_list = sorted(idx_list, key = lambda x: x[1], reverse=True)
    for i in range(min(num,len(idx_list))):
        nidx, score = idx_list[i]
        print('  %i. %s (%1.4f)' % (i+1,thenodes[nidx],score))
        #print '  %i. %s' % (i+1,G.node_object(idx))
```

This function returns the index of the maximum of the array. If two or more indices have the same max value, the first index is returned.

In [69]:
```
def index_of_max(v):
    return np.where(v == max(v))[0]
```

This function accepts a dictionary of nodes with centrality values and returns a centrality vector

In [70]:
```
def centrality_vector(G,d):
    thenodes = list(G.nodes())
    v = np.zeros((G.number_of_nodes(),))
    for i,u in enumerate(thenodes):
        v[i] = d[u]
    return v
```

This function provides the index of a node based on its order in `G.nodes()`

In [71]:
```
def node_index(G,n):
    thenodes = list(G.nodes())
```

```
    return thenodes.index(n)
```

Now we read in the edgelist file that contains the coappearance network we will analyze. We will look at two different networks, corresponding to only the *Lord of the Rings* series and the *Lord of the Rings* series plus the prequel, The *Hobbit*. The `unweighted` boolean, if set to `True` will set all the edge weights to one. Recall that setting all weights to 1 is different (in NetworkX) from having no weights assigned, which could be accomplished instead by: `G = nx.read_edgelist('LoTR_characters.edgelist',data=False)`.

In [72]:
```
def create_graph(edgelist, unweighted):
    # G = nx.read_weighted_edgelist('LotR_characters.edgelist') # just Lord of the Rir
    #G = nx.read_weighted_edgelist('hobbit_LotR_characters.edgelist') # with the Hobbi
    G = nx.read_weighted_edgelist(edgelist) # just Lord of the Rings
    if unweighted:
        for u,v in G.edges():
            G[u][v]['weight'] = 1
    A = np.array(nx.adjacency_matrix(G).todense().T)
    N = G.number_of_nodes()

    return G, A, N
```

In [123…
```
G_lotr_unweighted, A_lotr_unweighted, N_lotr_unweighted = create_graph(edgelist='LotR_
G_lotr_weighted, A_lotr_weighted, N_lotr_weighted = create_graph(edgelist='LotR_charac
G_lotr_hobbit_weighted, A_lotr_hobbit_weighted, N_lotr_hobbit_weighted = create_graph(
```

```
C:\Users\Humza\AppData\Local\Temp\ipykernel_8236\251722499.py:8: FutureWarning: adjac
ency_matrix will return a scipy.sparse array instead of a matrix in Networkx 3.0.
  A = np.array(nx.adjacency_matrix(G).todense().T)
```

Section 7.1 Degree Centrality

In [124…
```
print('HIGHLIGHTED QUESTION - print out the top 5 characters with highest centrality i
degree_df = pd.DataFrame()
degree_df['LotR_unweighted'] = get_top_5(G, A_lotr_unweighted.sum(axis=1))
degree_df['LotR_weighted'] = get_top_5(G, A_lotr_weighted.sum(axis=1))
degree_df['LotR+Hobbit_weighted'] = get_top_5(G, A_lotr_hobbit_weighted.sum(axis=1))
degree_df.head()
```

```
HIGHLIGHTED QUESTION - print out the top 5 characters with highest centrality in term
s of degree
```

Out[124]:

	LotR_unweighted	LotR_weighted	LotR+Hobbit_weighted
0	1. shagrat (153.0000)	1. shagrat (762.0000)	1. gandalf (901.0000)
1	2. smeagol (144.0000)	2. smeagol (661.0000)	2. frodo (661.0000)
2	3. frodo (140.0000)	3. frodo (632.0000)	3. aragorn (632.0000)
3	4. eomer (137.0000)	4. eomer (606.0000)	4. pippin (606.0000)
4	5. bilbo (125.0000)	5. bilbo (484.0000)	5. bilbo (602.0000)

Section 7.2 Eigenvector Centrality

In [ ]:

```
```

In [125…

```
print('HIGHLIGHTED QUESTION - Eigenvector Centrality (by NetworkX):')
eig_cen_nx_df = pd.DataFrame()
eig_cen_nx_df['LotR_unweighted'] = get_top_5(G_lotr_unweighted, list(nx.eigenvector_ce
eig_cen_nx_df['LotR_weighted'] = get_top_5(G_lotr_weighted, list(nx.eigenvector_centra
eig_cen_nx_df['LotR+Hobbit_weighted'] = get_top_5(G_lotr_hobbit_weighted, list(nx.eige
eig_cen_nx_df.head()
```

```
HIGHLIGHTED QUESTION - Eigenvector Centrality (by NetworkX):
```

Out[125]:

	LotR_unweighted	LotR_weighted	LotR+Hobbit_weighted
0	1. gandalf (0.1682)	1. gandalf (0.3412)	1. gandalf (0.3570)
1	2. aragorn (0.1641)	2. frodo (0.2920)	2. frodo (0.2827)
2	3. frodo (0.1618)	3. aragorn (0.2908)	3. aragorn (0.2783)
3	4. elrond (0.1541)	4. pippin (0.2839)	4. pippin (0.2723)
4	5. pippin (0.1533)	5. sauron (0.2300)	5. bilbo (0.2287)

In [304…

```
def eig_cen_la(A):
    k, v = la.eig(A)
    k = np.abs(k)
    v = np.abs(v)

    k1_idx = index_of_max(k)

    return v[:,k1_idx]
```

In [305…

```
print('HIGHLIGHTED QUESTION - Eigenvector Centrality (by linear algebra):')
eig_cen_la_df = pd.DataFrame()
eig_cen_la_df['LotR_unweighted'] = get_top_5(G_lotr_unweighted, eig_cen_la(A_lotr_unwe
eig_cen_la_df['LotR_weighted'] = get_top_5(G_lotr_weighted, eig_cen_la(A_lotr_weighted
eig_cen_la_df['LotR+Hobbit_weighted'] = get_top_5(G_lotr_hobbit_weighted, eig_cen_la(A
eig_cen_la_df.head()
```

```
HIGHLIGHTED QUESTION - Eigenvector Centrality (by linear algebra):
```

Out[305]:

	LotR_unweighted	LotR_weighted	LotR+Hobbit_weighted
0	1. gandalf (0.1682)	1. gandalf (0.3412)	1. gandalf (0.3570)
1	2. aragorn (0.1641)	2. frodo (0.2920)	2. frodo (0.2827)
2	3. frodo (0.1618)	3. aragorn (0.2908)	3. aragorn (0.2783)
3	4. elrond (0.1541)	4. pippin (0.2839)	4. pippin (0.2723)
4	5. pippin (0.1533)	5. sauron (0.2300)	5. bilbo (0.2287)

In [320…

```
noi = 'arwen'
noi_idx = node_index(G_lotr_unweighted,noi)
print('HIGHLIGHTED QUESTION - Confirm that this is the case for a node index of your c
print('HIGHLIGHTED QUESTION - Confirming that eigenvector centrality is a steady-state
# compare the eigenvector centrality of arwen to the sum of the centralities of its ne
print(f'{noi}\'s eigenvector centrality value: {float(eig_cen_la(A_lotr_unweighted)[no

neighbors_of_noi = list(G_lotr_unweighted.neighbors(noi))
```

```
weighted_sum = 0
k, v = la.eig(A_lotr_unweighted)
k = np.real(k)
v = np.real(v)
k1_idx = index_of_max(k)
lambda1 = k[k1_idx]

for n in neighbors_of_noi:
    weighted_sum += eig_cen_la(A_lotr_unweighted)[node_index(G_lotr_unweighted, n)]

print(f'weighted sum of eigenvector centrality values of {noi}\'s neighbors: {float(we
```

```
HIGHLIGHTED QUESTION - Confirm that this is the case for a node index of your choosin
g by printing these two quantities: the centrality of that character and the (weighte
d) sum of the centralities of that character's neighbors (normalized by the largest e
igenvalue).
HIGHLIGHTED QUESTION - Confirming that eigenvector centrality is a steady-state of so
rts for node arwen:
arwen's eigenvector centrality value: 0.11265370336918339
weighted sum of eigenvector centrality values of arwen's neighbors: 0.112653703369183
22
```

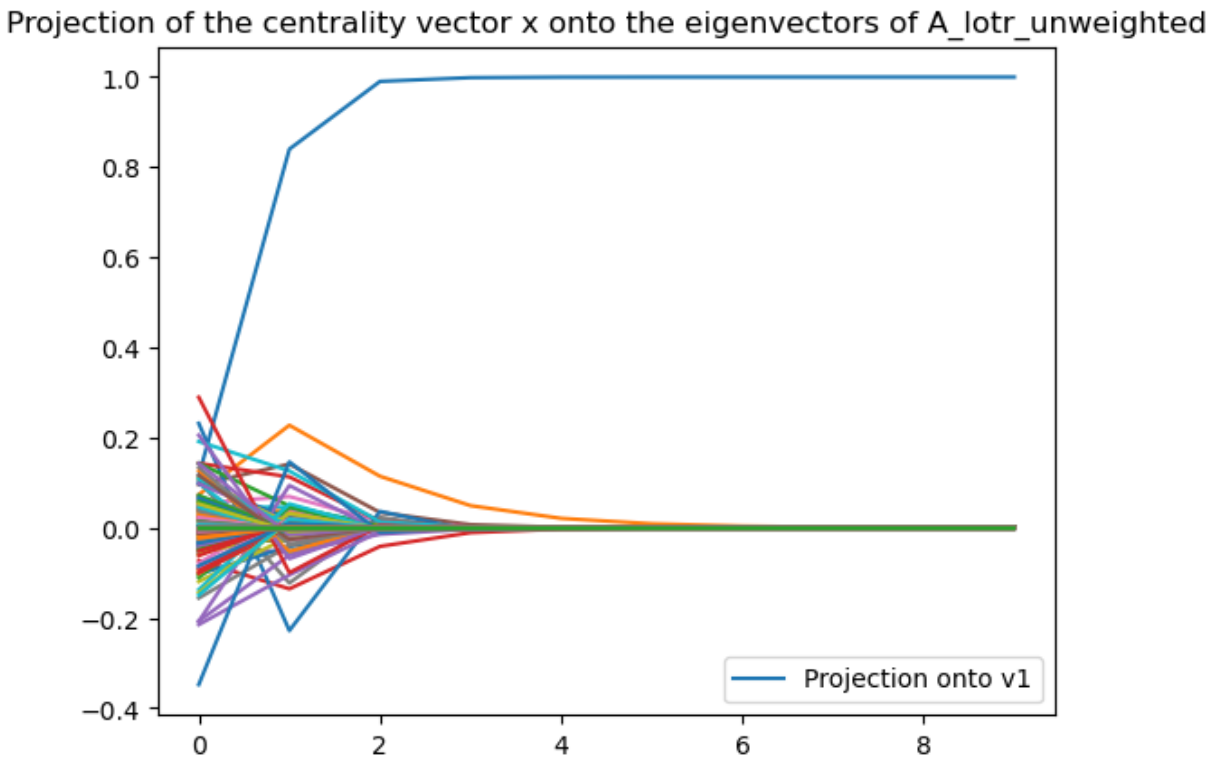
The following section should run and produce a plot that you need to interpret.

In [131…

```
print('Showing the convergece of eigenvector centrality...')
num_steps = 10
x = np.zeros((N_lotr_unweighted,1)) # initial centrality vector
x[76] = 1
cs = np.zeros((N_lotr_unweighted,num_steps))
for i in range(num_steps):
    x = x/la.norm(x) # at each step we need to normalize the centrality vector
    for j in range(G_lotr_unweighted.number_of_nodes()):
        cs[j,i] = np.real(np.dot( x.T , v[:,j] ))[0] # project x onto each of the eige
    x = np.dot(A_lotr_unweighted,x) # "pass" the centrality one step forward

plt.figure() # this creates a figure to plot in
for i in range(G_lotr_unweighted.number_of_nodes()): # for each eigenvector plot the p
    if i == k1_idx:
        plt.plot(range(num_steps),cs[i,:],label='Projection onto v1') # only label the
    else:
        plt.plot(range(num_steps),cs[i,:])
#plt.ylim([-0.2,1.1]) # this sets the limits for the y axis
plt.legend(loc='best') # this attaches a legend
plt.title('Projection of the centrality vector x onto the eigenvectors of A_lotr_unwei
plt.show() # this makes the figure appear
```

```
Showing the convergece of eigenvector centrality...
```



In [321…
```
print('HIGHLIGHTED QUESTION - Turn in this plot. In your own words, make a short state
print('We find that all projections of centrality vector x onto eigenvectors of A exce
```

```
HIGHLIGHTED QUESTION - Turn in this plot. In your own words, make a short statement t
hat describes what is going.
We find that all projections of centrality vector x onto eigenvectors of A except for
v1 converge to 0 as t approaches infinity. This tells us that the centrality x ~ v1
(eigenvector corresponding to largest eigenvalue of A). In other words, because c1 an
d lambda1 are constants, the centrality vector x is proportional to v1 because lambda
1 is the largest eigenvalue and v1 is the eigenvector that corresponds to lambda1.
```

Section 7.3 Katz Centrality

In [199…
```
def calc_lambda1 (A):
    k, v = la.eig(A)
    k = np.real(k)
    v = np.real(v)
    k1_idx = index_of_max(k)
    lambda1 = k[k1_idx]
    return lambda1
```

In [202…
```
def katz_cen_la(G, A, N, alpha):
    beta = 1
    x = la.inv(np.eye(N) - alpha * A) @ (beta * np.ones((N,1)))
    return x
```

In [281…
```
print('Katz Centrality LA:')
print('HIGHLIGHTED QUESTION - Try a few values for α and print out the top 5 central c
katz_cen_la_df = pd.DataFrame()
# x = katz_cen_la(G_lotr_unweighted, A_lotr_unweighted, N_lotr_unweighted, lambda1)
# print(get_top_5(G_lotr_unweighted, x[0]))
# print(x)
```

```
katz_cen_la_df = pd.DataFrame(columns=['a1', 'LotR_unweighted', 'a2', 'LotR_weighted',

lambda1_lotr_unweighted = calc_lambda1(A_lotr_unweighted)
lambda1_lotr_weighted = calc_lambda1(A_lotr_weighted)
lambda1_lotr_hobbit_weighted = calc_lambda1(A_lotr_hobbit_weighted)

alpha_lotr_unweighted = np.linspace(0, 0.9*(1/lambda1_lotr_unweighted), num = 5)
alpha_lotr_weighted = np.linspace(0, 0.9*(1/lambda1_lotr_weighted), num = 5)
alpha_lotr_hobbit_weighted = np.linspace(0, 0.9*(1/lambda1_lotr_hobbit_weighted), num

for a1, a2, a3 in zip(alpha_lotr_unweighted, alpha_lotr_weighted, alpha_lotr_hobbit_we
    x1 = get_top_5(G_lotr_unweighted, katz_cen_la(G_lotr_unweighted, A_lotr_unweighted
    x2 = get_top_5(G_lotr_weighted, katz_cen_la(G_lotr_weighted, A_lotr_weighted, N_lo
    x3 = get_top_5(G_lotr_hobbit_weighted, katz_cen_la(G_lotr_hobbit_weighted, A_lotr_

    for x1i, x2i, x3i in zip(x1, x2, x3):
        temp_df = pd.DataFrame([[str(a1), x1i, str(a2), x2i, str(a3), x3i]], columns=[
        katz_cen_la_df = pd.concat([katz_cen_la_df, temp_df])

    katz_cen_la_df = pd.concat([katz_cen_la_df, pd.DataFrame([['---', '---', '---', '-
katz_cen_la_df.head(30)
```

Katz Centrality LA:
HIGHLIGHTED QUESTION - Try a few values for α and print out the top 5 central charact
ers in each case.

Out[281]:

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	[0.]	1. amroth (1.0000)	[0.]	1. amroth (1.0000)	[0.]	1. anborn (1.0000)
0	[0.]	2. aragorn (1.0000)	[0.]	2. aragorn (1.0000)	[0.]	2. frodo (1.0000)
0	[0.]	3. arathorn (1.0000)	[0.]	3. arathorn (1.0000)	[0.]	3. faramir (1.0000)
0	[0.]	4. arwen (1.0000)	[0.]	4. arwen (1.0000)	[0.]	4. gandalf (1.0000)
0	[0.]	5. beregond (1.0000)	[0.]	5. beregond (1.0000)	[0.]	5. samwise (1.0000)
0	---	---	---	---	---	---
0	[0.00356308]	1. gandalf (1.6614)	[0.00075133]	1. gandalf (1.7341)	[0.00072498]	1. gandalf (1.8306)
0	[0.00356308]	2. frodo (1.6254)	[0.00075133]	2. frodo (1.6351)	[0.00072498]	2. frodo (1.6188)
0	[0.00356308]	3. aragorn (1.6124)	[0.00075133]	3. aragorn (1.6129)	[0.00072498]	3. aragorn (1.5952)
0	[0.00356308]	4. pippin (1.5952)	[0.00075133]	4. pippin (1.5902)	[0.00072498]	4. pippin (1.5736)
0	[0.00356308]	5. elrond (1.5527)	[0.00075133]	5. sauron (1.4701)	[0.00072498]	5. bilbo (1.5536)
0	---	---	---	---	---	---
0	[0.00712616]	1. gandalf (2.7549)	[0.00150266]	1. gandalf (3.0568)	[0.00144996]	1. gandalf (3.3067)
0	[0.00712616]	2. frodo (2.6670)	[0.00150266]	2. frodo (2.7745)	[0.00144996]	2. frodo (2.7464)
0	[0.00712616]	3. aragorn (2.6461)	[0.00150266]	3. aragorn (2.7279)	[0.00144996]	3. aragorn (2.6901)
0	[0.00712616]	4. pippin (2.5863)	[0.00150266]	4. pippin (2.6710)	[0.00144996]	4. pippin (2.6369)
0	[0.00712616]	5. elrond (2.5022)	[0.00150266]	5. sauron (2.3379)	[0.00144996]	5. bilbo (2.5288)
0	---	---	---	---	---	---
0	[0.01068923]	1. gandalf (5.1961)	[0.00225399]	1. gandalf (6.1931)	[0.00217493]	1. gandalf (6.7694)
0	[0.01068923]	2. frodo (5.0052)	[0.00225399]	2. frodo (5.4667)	[0.00217493]	2. frodo (5.4433)
0	[0.01068923]	3. aragorn (4.9939)	[0.00225399]	3. aragorn (5.3888)	[0.00217493]	3. aragorn (5.3283)
0	[0.01068923]	4. pippin (4.8077)	[0.00225399]	4. pippin (5.2614)	[0.00217493]	4. pippin (5.2108)

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	[0.01068923]	5. elrond (4.6867)	[0.00225399]	5. sauron (4.4291)	[0.00217493]	5. bilbo (4.7870)
0	---	---	---	---	---	---
0	[0.01425231]	1. gandalf (18.1234)	[0.00300532]	1. gandalf (23.3913)	[0.00289991]	1. gandalf (25.6190)
0	[0.01425231]	2. aragorn (17.5677)	[0.00300532]	2. frodo (20.1953)	[0.00289991]	2. frodo (20.3180)
0	[0.01425231]	3. frodo (17.4287)	[0.00300532]	3. aragorn (20.0332)	[0.00289991]	3. aragorn (19.9538)
0	[0.01425231]	4. pippin (16.5844)	[0.00300532]	4. pippin (19.5519)	[0.00289991]	4. pippin (19.5149)
0	[0.01425231]	5. elrond (16.4761)	[0.00300532]	5. sauron (15.9987)	[0.00289991]	5. bilbo (16.9176)
0	---	---	---	---	---	---

In [283…

```
print('Katz Centrality networkx:')
katz_cen_nx_df = pd.DataFrame()
# x = katz_cen_la(G_lotr_unweighted, A_lotr_unweighted, N_lotr_unweighted, lambda1)
# print(get_top_5(G_lotr_unweighted, x[0]))
# print(x)
katz_cen_nx_df = pd.DataFrame(columns=['a1', 'LotR_unweighted', 'a2', 'LotR_weighted',

lambda1_lotr_unweighted = calc_lambda1(A_lotr_unweighted)
lambda1_lotr_weighted = calc_lambda1(A_lotr_weighted)
lambda1_lotr_hobbit_weighted = calc_lambda1(A_lotr_hobbit_weighted)

alpha_lotr_unweighted = np.linspace(0, 0.9*(1/lambda1_lotr_unweighted), num = 5)
alpha_lotr_weighted = np.linspace(0, 0.9*(1/lambda1_lotr_weighted), num = 5)
alpha_lotr_hobbit_weighted = np.linspace(0, 0.9*(1/lambda1_lotr_hobbit_weighted), num

for a1, a2, a3 in zip(alpha_lotr_unweighted, alpha_lotr_weighted, alpha_lotr_hobbit_we
    x1 = get_top_5(G_lotr_unweighted, list(nx.katz_centrality(G_lotr_unweighted, alpha
    x2 = get_top_5(G_lotr_weighted, list(nx.katz_centrality(G_lotr_weighted, alpha=a2,
    x3 = get_top_5(G_lotr_hobbit_weighted, list(nx.katz_centrality(G_lotr_hobbit_weigh

    for x1i, x2i, x3i in zip(x1, x2, x3):
        temp_df = pd.DataFrame([[str(a1), x1i, str(a2), x2i, str(a3), x3i]], columns=[
        katz_cen_nx_df = pd.concat([katz_cen_nx_df, temp_df])

    katz_cen_nx_df = pd.concat([katz_cen_la_df, pd.DataFrame([['---', '---', '---', '-
katz_cen_nx_df.head(30)
```

```
Katz Centrality networkx:
```

Out[283]:

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	[0.]	1. amroth (1.0000)	[0.]	1. amroth (1.0000)	[0.]	1. anborn (1.0000)
0	[0.]	2. aragorn (1.0000)	[0.]	2. aragorn (1.0000)	[0.]	2. frodo (1.0000)
0	[0.]	3. arathorn (1.0000)	[0.]	3. arathorn (1.0000)	[0.]	3. faramir (1.0000)
0	[0.]	4. arwen (1.0000)	[0.]	4. arwen (1.0000)	[0.]	4. gandalf (1.0000)
0	[0.]	5. beregond (1.0000)	[0.]	5. beregond (1.0000)	[0.]	5. samwise (1.0000)
0	---	---	---	---	---	---
0	[0.00356308]	1. gandalf (1.6614)	[0.00075133]	1. gandalf (1.7341)	[0.00072498]	1. gandalf (1.8306)
0	[0.00356308]	2. frodo (1.6254)	[0.00075133]	2. frodo (1.6351)	[0.00072498]	2. frodo (1.6188)
0	[0.00356308]	3. aragorn (1.6124)	[0.00075133]	3. aragorn (1.6129)	[0.00072498]	3. aragorn (1.5952)
0	[0.00356308]	4. pippin (1.5952)	[0.00075133]	4. pippin (1.5902)	[0.00072498]	4. pippin (1.5736)
0	[0.00356308]	5. elrond (1.5527)	[0.00075133]	5. sauron (1.4701)	[0.00072498]	5. bilbo (1.5536)
0	---	---	---	---	---	---
0	[0.00712616]	1. gandalf (2.7549)	[0.00150266]	1. gandalf (3.0568)	[0.00144996]	1. gandalf (3.3067)
0	[0.00712616]	2. frodo (2.6670)	[0.00150266]	2. frodo (2.7745)	[0.00144996]	2. frodo (2.7464)
0	[0.00712616]	3. aragorn (2.6461)	[0.00150266]	3. aragorn (2.7279)	[0.00144996]	3. aragorn (2.6901)
0	[0.00712616]	4. pippin (2.5863)	[0.00150266]	4. pippin (2.6710)	[0.00144996]	4. pippin (2.6369)
0	[0.00712616]	5. elrond (2.5022)	[0.00150266]	5. sauron (2.3379)	[0.00144996]	5. bilbo (2.5288)
0	---	---	---	---	---	---
0	[0.01068923]	1. gandalf (5.1961)	[0.00225399]	1. gandalf (6.1931)	[0.00217493]	1. gandalf (6.7694)
0	[0.01068923]	2. frodo (5.0052)	[0.00225399]	2. frodo (5.4667)	[0.00217493]	2. frodo (5.4433)
0	[0.01068923]	3. aragorn (4.9939)	[0.00225399]	3. aragorn (5.3888)	[0.00217493]	3. aragorn (5.3283)
0	[0.01068923]	4. pippin (4.8077)	[0.00225399]	4. pippin (5.2614)	[0.00217493]	4. pippin (5.2108)

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	[0.01068923]	5. elrond (4.6867)	[0.00225399]	5. sauron (4.4291)	[0.00217493]	5. bilbo (4.7870)
0	---	---	---	---	---	---
0	[0.01425231]	1. gandalf (18.1234)	[0.00300532]	1. gandalf (23.3913)	[0.00289991]	1. gandalf (25.6190)
0	[0.01425231]	2. aragorn (17.5677)	[0.00300532]	2. frodo (20.1953)	[0.00289991]	2. frodo (20.3180)
0	[0.01425231]	3. frodo (17.4287)	[0.00300532]	3. aragorn (20.0332)	[0.00289991]	3. aragorn (19.9538)
0	[0.01425231]	4. pippin (16.5844)	[0.00300532]	4. pippin (19.5519)	[0.00289991]	4. pippin (19.5149)
0	[0.01425231]	5. elrond (16.4761)	[0.00300532]	5. sauron (15.9987)	[0.00289991]	5. bilbo (16.9176)
0	---	---	---	---	---	---

Section 7.4 PageRank Centrality

In [318…
```
Aa = [[0, 1, 1], [1, 0, 0], [0, 0, 1]]
Dd = np.diag(np.sum(Aa, axis=1))
print(Aa)
print(la.inv(Dd))
print(Aa @ la.inv(Dd))
```

```
[[0, 1, 1], [1, 0, 0], [0, 0, 1]]
[[0.5 0.  0. ]
 [0.  1.  0. ]
 [0.  0.  1. ]]
[[0.  1.  1. ]
 [0.5 0.  0. ]
 [0.  0.  1. ]]
```

In [288…
```
def pagerank_cen_la(G, A, N, alpha):
    beta=1
    D = np.diag(np.sum(A, axis=1))
    x = la.inv(np.eye(N) - ((alpha * A) @ la.inv(D))) @ (beta * np.ones((N,1)))
    return x
```

In [289…
```
print('PageRank Centrality LA:')
print('HIGHLIGHTED QUESTION -  Calculate PageRank centrality (by linear algebra) for T

pagerank_cen_la_df = pd.DataFrame(columns=['a1', 'LotR_unweighted', 'a2', 'LotR_weight


alpha_lotr_unweighted = np.linspace(0, 0.9*(1), num = 5)
alpha_lotr_weighted = np.linspace(0, 0.9*(1), num = 5)
alpha_lotr_hobbit_weighted = np.linspace(0, 0.9*(1), num = 5)

for a1, a2, a3 in zip(alpha_lotr_unweighted, alpha_lotr_weighted, alpha_lotr_hobbit_we
    x1 = get_top_5(G_lotr_unweighted, pagerank_cen_la(G_lotr_unweighted, A_lotr_unweig
    x2 = get_top_5(G_lotr_weighted, pagerank_cen_la(G_lotr_weighted, A_lotr_weighted,
```

```
    x3 = get_top_5(G_lotr_hobbit_weighted, pagerank_cen_la(G_lotr_hobbit_weighted, A_l

    for x1i, x2i, x3i in zip(x1, x2, x3):
        temp_df = pd.DataFrame([[str(a1), x1i, str(a2), x2i, str(a3), x3i]], columns=[
        pagerank_cen_la_df = pd.concat([pagerank_cen_la_df, temp_df])

    pagerank_cen_la_df = pd.concat([pagerank_cen_la_df, pd.DataFrame([['---', '---', '
pagerank_cen_la_df.head(30)
```

```
PageRank Centrality LA:
HIGHLIGHTED QUESTION -  Calculate PageRank centrality (by linear algebra) for The Lor
d of the Rings network and print the top 5 characters with highest centrality.
```

Out[289]:

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	0.0	1. amroth (1.0000)	0.0	1. amroth (1.0000)	0.0	1. anborn (1.0000)
0	0.0	2. aragorn (1.0000)	0.0	2. aragorn (1.0000)	0.0	2. frodo (1.0000)
0	0.0	3. arathorn (1.0000)	0.0	3. arathorn (1.0000)	0.0	3. faramir (1.0000)
0	0.0	4. arwen (1.0000)	0.0	4. arwen (1.0000)	0.0	4. gandalf (1.0000)
0	0.0	5. beregond (1.0000)	0.0	5. beregond (1.0000)	0.0	5. samwise (1.0000)
0	---	---	---	---	---	---
0	0.225	1. gandalf (2.4502)	0.225	1. gandalf (3.2611)	0.225	1. gandalf (3.5224)
0	0.225	2. frodo (2.3676)	0.225	2. frodo (3.1394)	0.225	2. frodo (3.0161)
0	0.225	3. pippin (2.2534)	0.225	3. pippin (2.8401)	0.225	3. bilbo (2.7674)
0	0.225	4. aragorn (2.1374)	0.225	4. aragorn (2.6912)	0.225	4. pippin (2.7361)
0	0.225	5. bilbo (2.0645)	0.225	5. bilbo (2.3807)	0.225	5. aragorn (2.5755)
0	---	---	---	---	---	---
0	0.45	1. gandalf (4.7029)	0.45	1. gandalf (7.3358)	0.45	1. gandalf (8.0354)
0	0.45	2. frodo (4.4931)	0.45	2. frodo (6.8726)	0.45	2. frodo (6.5192)
0	0.45	3. pippin (4.2258)	0.45	3. pippin (6.1261)	0.45	3. bilbo (5.8681)
0	0.45	4. aragorn (3.9905)	0.45	4. aragorn (5.8597)	0.45	4. pippin (5.8264)
0	0.45	5. bilbo (3.7862)	0.45	5. elrond (4.8432)	0.45	5. aragorn (5.5337)
0	---	---	---	---	---	---
0	0.675	1. gandalf (9.4685)	0.675	1. gandalf (16.9806)	0.675	1. gandalf (18.6646)
0	0.675	2. frodo (8.9864)	0.675	2. frodo (15.4664)	0.675	2. frodo (14.5493)
0	0.675	3. pippin (8.4460)	0.675	3. pippin (13.8468)	0.675	3. pippin (13.0562)
0	0.675	4. aragorn (8.1115)	0.675	4. aragorn (13.6092)	0.675	4. bilbo (13.0527)
0	0.675	5. bilbo (7.5494)	0.675	5. elrond (10.8644)	0.675	5. aragorn (12.7677)
0	---	---	---	---	---	---
0	0.9	1. gandalf (33.6899)	0.9	1. gandalf (69.7152)	0.9	1. gandalf (76.6303)
0	0.9	2. frodo (31.7950)	0.9	2. frodo (61.4812)	0.9	2. frodo (57.3737)
0	0.9	3. pippin (30.0813)	0.9	3. aragorn (57.0390)	0.9	3. aragorn (53.1988)
0	0.9	4. aragorn (29.9833)	0.9	4. pippin (55.8421)	0.9	4. pippin (52.1734)
0	0.9	5. bilbo (27.0848)	0.9	5. elrond (44.2408)	0.9	5. bilbo (51.8432)
0	---	---	---	---	---	---

In [290…

```
print('PageRank Centrality networkx:')

pagerank_cen_la_df = pd.DataFrame(columns=['a1', 'LotR_unweighted', 'a2', 'LotR_weight
```

```
alpha_lotr_unweighted = np.linspace(0, 0.9*(1), num = 5)
alpha_lotr_weighted = np.linspace(0, 0.9*(1), num = 5)
alpha_lotr_hobbit_weighted = np.linspace(0, 0.9*(1), num = 5)

for a1, a2, a3 in zip(alpha_lotr_unweighted, alpha_lotr_weighted, alpha_lotr_hobbit_we
    x1 = get_top_5(G_lotr_unweighted, list(nx.pagerank(G_lotr_unweighted, alpha=a1).va
    x2 = get_top_5(G_lotr_weighted, list(nx.pagerank(G_lotr_weighted, alpha=a2).values
    x3 = get_top_5(G_lotr_hobbit_weighted, list(nx.pagerank(G_lotr_hobbit_weighted, al

    for x1i, x2i, x3i in zip(x1, x2, x3):
        temp_df = pd.DataFrame([[str(a1), x1i, str(a2), x2i, str(a3), x3i]], columns=[
        pagerank_cen_la_df = pd.concat([pagerank_cen_la_df, temp_df])

    pagerank_cen_la_df = pd.concat([pagerank_cen_la_df, pd.DataFrame([['---', '---', '
pagerank_cen_la_df.head(30)
```

PageRank Centrality networkx:

Out[290]:

	a1	LotR_unweighted	a2	LotR_weighted	a3	LotR+Hobbit_weighted
0	0.0	1. amroth (0.0061)	0.0	1. amroth (0.0061)	0.0	1. anborn (0.0058)
0	0.0	2. aragorn (0.0061)	0.0	2. aragorn (0.0061)	0.0	2. frodo (0.0058)
0	0.0	3. arathorn (0.0061)	0.0	3. arathorn (0.0061)	0.0	3. faramir (0.0058)
0	0.0	4. arwen (0.0061)	0.0	4. arwen (0.0061)	0.0	4. gandalf (0.0058)
0	0.0	5. beregond (0.0061)	0.0	5. beregond (0.0061)	0.0	5. samwise (0.0058)
0	---	---	---	---	---	---
0	0.225	1. gandalf (0.0116)	0.225	1. gandalf (0.0155)	0.225	1. gandalf (0.0158)
0	0.225	2. frodo (0.0113)	0.225	2. frodo (0.0149)	0.225	2. frodo (0.0135)
0	0.225	3. pippin (0.0107)	0.225	3. pippin (0.0135)	0.225	3. bilbo (0.0124)
0	0.225	4. aragorn (0.0102)	0.225	4. aragorn (0.0128)	0.225	4. pippin (0.0123)
0	0.225	5. bilbo (0.0098)	0.225	5. bilbo (0.0113)	0.225	5. aragorn (0.0115)
0	---	---	---	---	---	---
0	0.45	1. gandalf (0.0159)	0.45	1. gandalf (0.0248)	0.45	1. gandalf (0.0255)
0	0.45	2. frodo (0.0152)	0.45	2. frodo (0.0232)	0.45	2. frodo (0.0207)
0	0.45	3. pippin (0.0143)	0.45	3. pippin (0.0207)	0.45	3. bilbo (0.0187)
0	0.45	4. aragorn (0.0135)	0.45	4. aragorn (0.0198)	0.45	4. pippin (0.0185)
0	0.45	5. bilbo (0.0128)	0.45	5. elrond (0.0163)	0.45	5. aragorn (0.0176)
0	---	---	---	---	---	---
0	0.675	1. gandalf (0.0189)	0.675	1. gandalf (0.0339)	0.675	1. gandalf (0.0351)
0	0.675	2. frodo (0.0179)	0.675	2. frodo (0.0308)	0.675	2. frodo (0.0273)
0	0.675	3. pippin (0.0168)	0.675	3. pippin (0.0276)	0.675	3. pippin (0.0245)
0	0.675	4. aragorn (0.0162)	0.675	4. aragorn (0.0271)	0.675	4. bilbo (0.0245)
0	0.675	5. bilbo (0.0151)	0.675	5. elrond (0.0217)	0.675	5. aragorn (0.0240)
0	---	---	---	---	---	---
0	0.9	1. gandalf (0.0207)	0.9	1. gandalf (0.0428)	0.9	1. gandalf (0.0443)
0	0.9	2. frodo (0.0195)	0.9	2. frodo (0.0377)	0.9	2. frodo (0.0332)
0	0.9	3. pippin (0.0185)	0.9	3. aragorn (0.0350)	0.9	3. aragorn (0.0308)
0	0.9	4. aragorn (0.0184)	0.9	4. pippin (0.0343)	0.9	4. pippin (0.0302)
0	0.9	5. bilbo (0.0166)	0.9	5. elrond (0.0271)	0.9	5. bilbo (0.0300)
0	---	---	---	---	---	---

In [117…

```
print('HIGHLIGHTED QUESTION -  For a connected undirected graph, show that the vector
```

```
HIGHLIGHTED QUESTION -  For a connected undirected graph, show that the vector v1=(k
1,k2,…,kn), where ki is the degree of node i, is an eigenvector of AD^-1 and using th
is show why α is upper bounded by 1.
```

$$(AD^{-1})v_1 = \lambda v$$

$$= \begin{bmatrix} \Sigma a_{1i} \\ \Sigma a_{2i} \\ \vdots \\ \Sigma a_{ni} \end{bmatrix} \begin{bmatrix} 1/\Sigma a_{1i} \\ 1/\Sigma a_{2i} \\ \vdots \\ 1/\Sigma a_{ni} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} v_1 = \mathbb{1} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

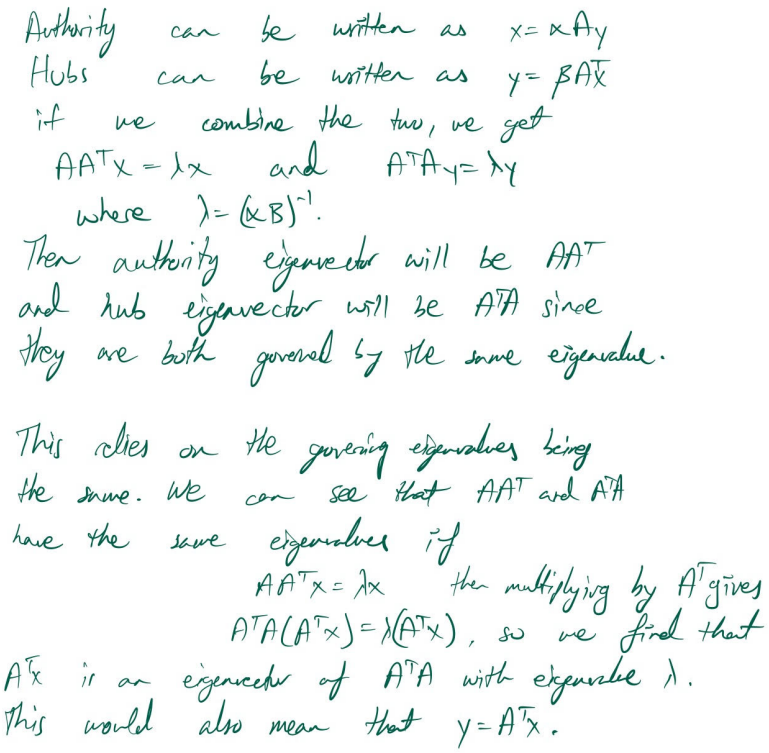
so we find that $\lambda = \mathbb{1}$ and $v_1 = [k_1\ k_2 \dots k_n]^T$

Since the largest eigenvalue = 1 and the corresponding eigenvector is $[k_1\ k_2 \dots k_n]^T$, we know that α should be less than the inverse of the lagest eigenvalue of AD^{-1}, so $1/1 = 1$ therefore $\alpha < 1$.

Section 7.5 Hubs & Authorities

In [46]:
```
print('HIGHLIGHTED QUESTION -  Now derive the relationship between the corresponding e
```

```
HIGHLIGHTED QUESTION -  Now derive the relationship between the corresponding eigenve
ctors for hubs and authorities – i.e., express the hub eigenvector in terms of the au
thorities eigenvector.
```



Section 7.6 Closeness Centrality

Section 7.7 Betweenness Centrality

In [291…

```
print('Betweenness Centrality')
print('HIGHLIGHTED QUESTION - Using this function print the top 5 characters with high
bet_cen_nx_df = pd.DataFrame()
```

```
bet_cen_nx_df['LotR_unweighted'] = get_top_5(G_lotr_unweighted, list(nx.betweenness_ce
bet_cen_nx_df['LotR_weighted'] = get_top_5(G_lotr_weighted, list(nx.betweenness_centra
bet_cen_nx_df['LotR+Hobbit_weighted'] = get_top_5(G_lotr_hobbit_weighted, list(nx.betw
bet_cen_nx_df.head()
```

```
Betweenness Centrality
HIGHLIGHTED QUESTION - Using this function print the top 5 characters with highest be
tweenness centrality.
```

Out[291]:

	LotR_unweighted	LotR_weighted	LotR+Hobbit_weighted
0	1. gandalf (0.0797)	1. gandalf (0.0437)	1. gandalf (0.0498)
1	2. frodo (0.0675)	2. pippin (0.0366)	2. pippin (0.0370)
2	3. pippin (0.0613)	3. aragorn (0.0345)	3. aragorn (0.0339)
3	4. aragorn (0.0486)	4. frodo (0.0289)	4. frodo (0.0296)
4	5. bilbo (0.0393)	5. bilbo (0.0270)	5. bilbo (0.0292)

Weighted Analysis

In [313…

```
print('HIGHLIGHTED QUESTION - comment on the changes that you see due to including the
print()
print('Degree Centrality Changes:')
print('We find that the degree rankings stay the same, although the weight could have
print()
print('Eigenvector Centrality Changes:')
print('Since we use our adjacency matrix to calculate eigenvector centralities, we fir
print()
print('Katz Centrality Changes:')
print('For Katz centrality we find that gandalf, frodo, aragorn, and pippin are in the
print()
print('PageRank Centrality Changes:')
print('PageRank is similar to Katz, but normalizes by the out degree. We find that for
print()
print('Betweenness Centrality Changes:')
print('We find that the top 5 characters remain the same, but their ordering changes s
```

HIGHLIGHTED QUESTION - comment on the changes that you see due to including the weigh
ts in the analysis. In particular, offer explanations for why some of the rankings ch
ange and some do not (Comparing lotr_unweighted to lotr_weighted)

Degree Centrality Changes:
We find that the degree rankings stay the same, although the weight could have influe
nced the rankings if certain connections were assigned more significance. Since we ca
lculate degree centrality using the adjacency matrix (which is influenced by weight)
the top 5 ranking could have changed, but did not since the weighted graph had edges
that did not blow up the signifiance of a connection.

Eigenvector Centrality Changes:
Since we use our adjacency matrix to calculate eigenvector centralities, we find that
there are changes made in the key players for the lotr_unweighted and lotr_weighted v
ersion. Noticably, sauron is shown to be more of a key player than elrond (kicking el
rond out of the top 5). We find that gandalf is the most significant character as his
ranking does not change at all. Rankings for aragorn, frodo, and pippin shift slightl
y due to the weight of their connections (frodo having more weight than aragorn makes
him more important). Pippin also takes spot 4.

Katz Centrality Changes:
For Katz centrality we find that gandalf, frodo, aragorn, and pippin are in the top 4
spots consecutively. A noticeable change occurs for higher values of alpha in lotr_un
weighted where aragorn is more significant than frodo. However, we see that the fifth
spot has elrond in lotr_unweighted and sauron in lotr_weighted. This can be explained
due to the added weights making sauron more significant compared to elrond.

PageRank Centrality Changes:
PageRank is similar to Katz, but normalizes by the out degree. We find that for highe
r values of alpha, the number 3, 4, and 5 spots change. This is because alpha is "blo
wing up" how certain nodes with the same number of connections may have less weight i
n their connections being made to them, so they have a higher centrality.

Betweenness Centrality Changes:
We find that the top 5 characters remain the same, but their ordering changes slightl
y. Pippin and Aragorn take the 2 and 3 spots in lotr_weighted with frodo taking the 4
spot. This likely happens since the weights associated with the connections alter the
shortest path for some nodes, which would mean that pippin is closet to being a part
of the shortest path for a certain character knowing another character because the we
ight connections associated with pippin are overall lower. This same line of reasonin
g applies to aragorn, but for frodo we can see that the weight connections have decre
ased his betweenness since his rank fell.

A Sequel

In [314…

```
print('HIGHLIGHTED QUESTION - Do some of the changes make sense based on what you know
print()
print('Degree Centrality Changes:')
print('It makes sense that gandalf, frodo, aragorn, pippin, and bilbo are ranked highe
print()
print('Eigenvector Centrality Changes:')
print('We find that the only change occurs with bilbo taking the #5 spot in lotr_hobbi
print()
print('Katz Centrality Changes:')
print('Again, we notice a change in the #5 spot from sauron to bilo when looking at lo
print()
print('PageRank Centrality Changes:')
print('In lotr_hobbit_weighted we find that elrond is no longer in the top 5 character
```

```
print()
print('Betweenness Centrality Changes:')
print('We see no change from lotr_weighted to lotr_hobbit_weighted in the betweenness
```

HIGHLIGHTED QUESTION - Do some of the changes make sense based on what you know about The Hobbit (Comparing lotr_weighted to lotr_hobbit_weighted)

Degree Centrality Changes:
It makes sense that gandalf, frodo, aragorn, pippin, and bilbo are ranked higher up i n the degree network since they are referenced in both books as either main or suppor ting characters

Eigenvector Centrality Changes:
We find that the only change occurs with bilbo taking the #5 spot in lotr_hobbit_weig hted and seeing how bilbo is a main character in the hobbit and a suporting character in the lotr it makes sense he is ranked highly.

Katz Centrality Changes:
Again, we notice a change in the #5 spot from sauron to bilo when looking at lotr_hob bit_weighted katz centrality rankings. This happens because of the introduction of ne w connections that were previously not made.

PageRank Centrality Changes:
In lotr_hobbit_weighted we find that elrond is no longer in the top 5 characters but instead has been replaced by bilbo. This is due to the connections bilbo creates to o ther characters being the main character in the hobbit, so this makes sense.

Betweenness Centrality Changes:
We see no change from lotr_weighted to lotr_hobbit_weighted in the betweenness centra lity. This implies that all characters shown are central to the story and can be used to connect a multitude of other characters in the shortest path.

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: