

$$R = \frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n A_{ij} (x_i - \mu)(x_j - \mu) = \frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n A_{ij} (x_i - \mu_i)(x_j - \mu_j)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} (x_i x_j - x_i \mu_j - x_j \mu_i + \mu_i \mu_j)$$

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i x_j - \left(\sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i \right) \mu_j - \left(\sum_{j=1}^n \sum_{i=1}^n A_{ij} x_j \right) \mu_i + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} \mu_i \mu_j$$

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i x_j - \left(\frac{1}{2m} \sum_{i=1}^n k_i x_i \right) \mu_j - \left(\frac{1}{2m} \sum_{j=1}^n k_j x_j \right) \mu_i + \left(\frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n A_{ij} \right) \mu^2$$

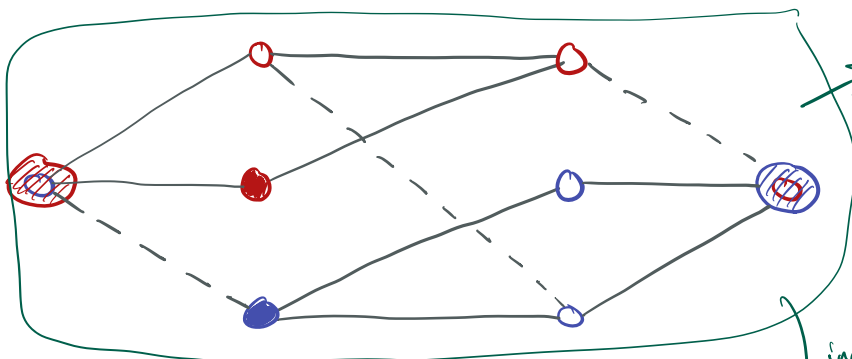
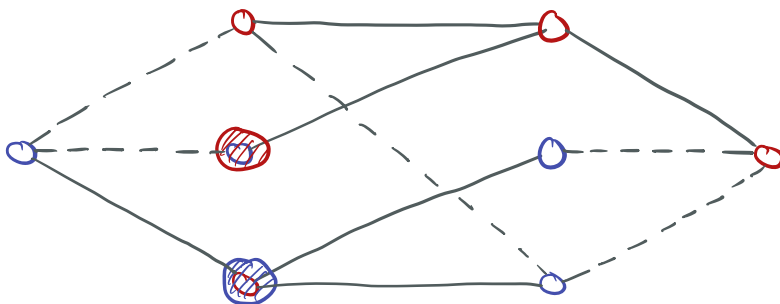
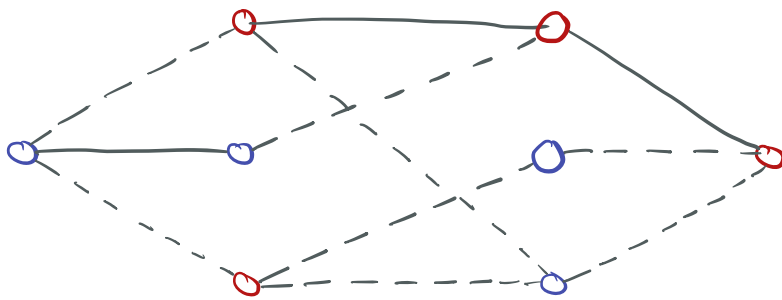
$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i x_j - \mu_i \mu_j - \mu_j \mu_i + \left(\frac{\sum_{i=1}^n \sum_{j=1}^n A_{ij}}{\sum_{i=1}^n \sum_{j=1}^n A_{ij}} \right) \mu^2$$

$$= \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i x_j - \mu^2 - \mu^2 + \mu^2$$

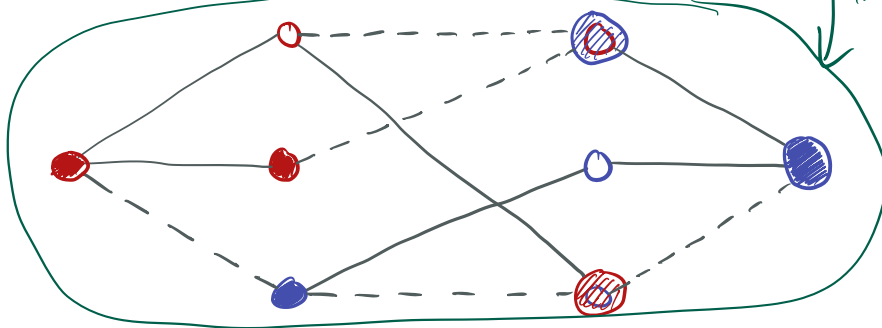
$$= \left(\sum_{i=1}^n \sum_{j=1}^n \frac{1}{2m} A_{ij} x_i x_j \right) - \mu^2$$

$$= \left(\frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i x_j \right) - \left(\frac{1}{4m} \sum_{i=1}^n \sum_{j=1}^n k_i k_j x_i x_j \right)$$

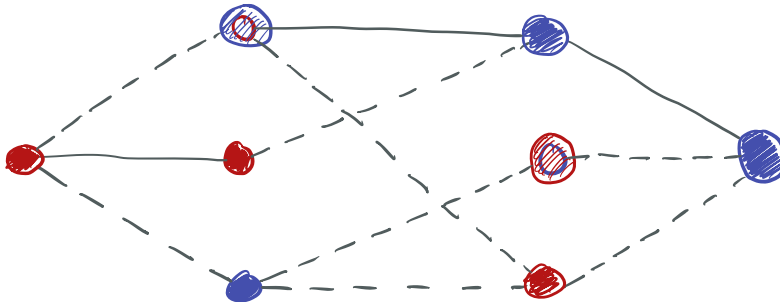
$$= \frac{1}{2m} \sum_{i=1}^n \sum_{j=1}^n \left(A_{ij} - \frac{k_i k_j}{2m} \right) x_i x_j$$



best cut size reduction



increase in cut size



at the end of creating all partitions, we saw our cut set size increase from a lower cut set but decreased to a smaller value than the original.

Show $\sum_{j=1}^n B_{ij} = 0$

where $B_{ij} = A_{ij} - \frac{k_i k_j}{2m}$

$$\sum_{j=1}^n B_{ij} = \sum_{j=1}^n A_{ij} - \frac{k_i k_j}{2m}$$

$$\sum_{j=1}^n B_{ij} = \sum_{j=1}^n A_{ij} - \sum_{j=1}^n \frac{k_i k_j}{2m}$$

$$\sum_{j=1}^n B_{ij} = \left(\sum_{j=1}^n A_{ij} \right) - \frac{k_i}{2m} \left(\sum_{j=1}^n k_j \right)$$

newman
eq. 6.19

newman
eq. 6.20

$$\sum_{j=1}^n B_{ij} = k_i - \frac{k_i}{2m} 2m$$

$$\sum_{j=1}^n B_{ij} = k_i - k_i$$

$$\sum_{j=1}^n B_{ij} = 0$$