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```
In [19]: import networkx as nx
import numpy as np
from numpy import *
import matplotlib.pyplot as plt
plt.ioff()
import sys
sys.path.append('../d3networkx/')
import d3networkx as d3nx
from d3graph import D3Graph, D3DiGraph
import asyncio
import random
import randomnet
```

The `randomnet` import statement provides functions to build local attachment and small world random networks. This small world network is slightly different from the version that is implemented in NetowrkX.

## Section 15.1.0: Small World

### Small World Networks

This function generates a small world network, where `n` nodes are connected to `q` neighboring nodes "around the circle" and with probability `p` to all other nodes. Even though the `randomnet.py` file contains a very similar function, this version has some extra code to lay out the network in an intuitive way (with the nodes in a circle).

```
In [2]: async def small_world(n,q,p,G=None,d3=None,x0=300,y0=300):
    ...
    q must be even
    ...
    if d3:
        d3.set_interactive(False)
    if G is None:
        G = D3Graph()
    for i in range(n):
        G.add_node(i)
        if d3:
            x = 200*cos(2*pi*i/n) + x0
            y = 200*sin(2*pi*i/n) + y0
            d3.position_node(i,x,y)
    # add the regular edges
    for u in range(n):
        for v in range(u+1,int(u+1+q/2)):
            v = v % n
            G.add_edge(u,v)
    print(nx.diameter(G))
    if d3:
        d3.update()
        await asyncio.sleep(3)
```

```

d3.set_interactive(True)

# add the random edges
for u in range(n):
    for v in range(u+1,n):
        if not G.has_edge(u,v):
            if random.random() <= p:
                G.add_edge(u,v)
print(nx.diameter(G))
return G

```

In [3]: `d3 = await d3nx.create_d3nx_visualizer()`

websocket server started...visualizer connected...

Now with the visualizer running, we will visualize a small world network

In [4]: `G = D3Graph()`  
`d3.set_graph(G)`  
`G = await small_world(20,4,0.1,G,d3)`

5  
networkx connected...4

In [5]: `print('HIGHLIGHTED QUESTION - write down your (visual) observations about the diameter of the network with only the local connections and then with the random connections added in. In the first case, without random connections, why is the diameter so high? With local connections we have a higher value for the diameter as compared to with random connection where we have a lower value for the diameter. Without random connections the diameter is higher since our longest shortest path between any two nodes is n/q nodes long since we add edges between nodes u and v with v being v % n, whereas the random graph will always have a path of length <= n/q.`

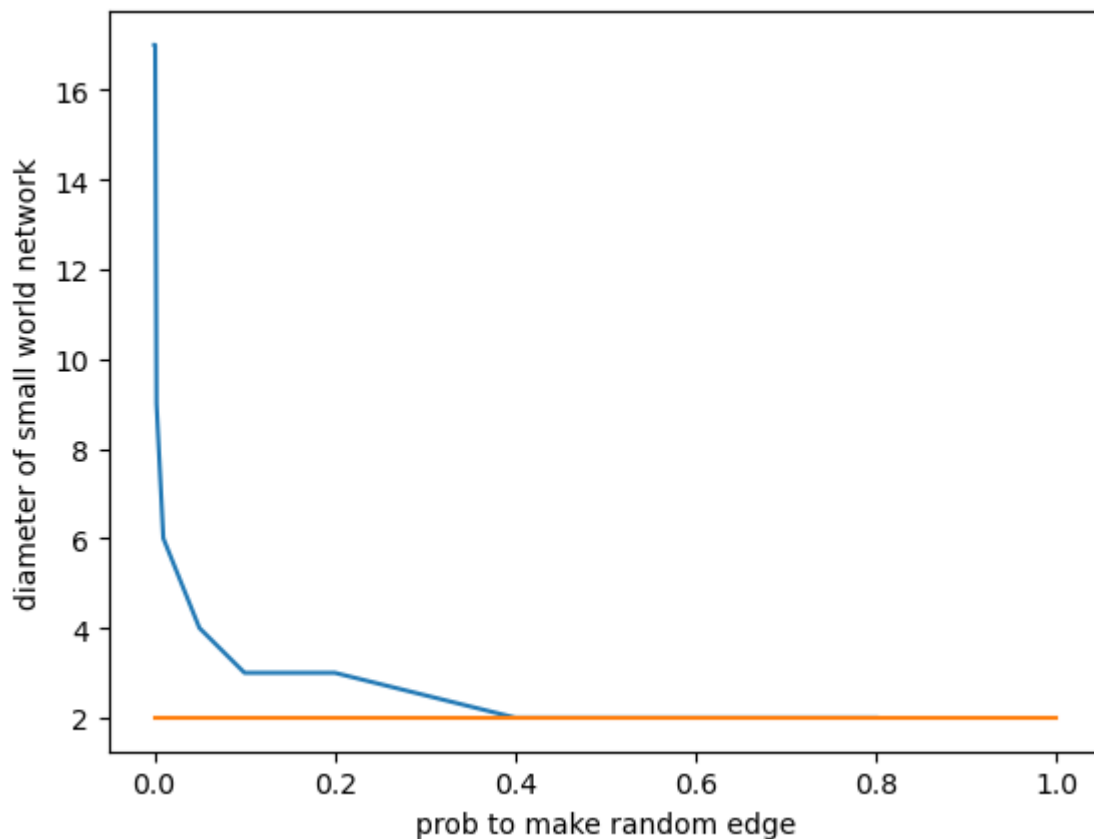
HIGHLIGHTED QUESTION - write down your (visual) observations about the diameter of the network with only the local connections and then with the random connections added in. In the first case, without random connections, why is the diameter so high? With local connections we have a higher value for the diameter as compared to with random connection where we have a lower value for the diameter. Without random connections the diameter is higher since our longest shortest path between any two nodes is  $n/q$  nodes long since we add edges between nodes  $u$  and  $v$  with  $v$  being  $v \% n$ , whereas the random graph will always have a path of length  $\leq n/q$ .

In [ ]:

Now let's plot the convergence of the small world effect.

In [6]: `n = 100`  
`P = [0,0.0001,0.001,0.0025,0.005,0.01,0.05,0.1,0.2,0.4,0.6,0.8]`  
`d = []`  
`for p in P:`  
 `G = randomnet.small_world_graph(n,6,p)`  
 `d.append(nx.diameter(G))`  
 `# calculate the diameter and store it for plotting below`  
  
`print('HIGHLIGHTED QUESTION - plot the diameter versus the p values.')`  
`## Plot the Convergence`  
`plt.figure()`  
`plt.plot(P, d) # change x and y to your "x" and "y" values`  
`plt.plot([0.0001,1],[log10(n),log10(n)])`  
`plt.xlabel('prob to make random edge')`  
`plt.ylabel('diameter of small world network')`  
`plt.show()`

HIGHLIGHTED QUESTION - plot the diameter versus the p values.



In [162...

```
print('HIGHLIGHTED QUESTION - Make a brief comment on the how and why these random connections (qualitatively) lead to the small world effect.')
print('Since the small world effect claims the diameter should scale proportionally to log(n), we see that the diameter of small word network converges to the orange line which represents log(n). So, it is telling us that the diameter is decreasing as we increase our probability to make an edge between any two nodes which causes a shorter average path between any two nodes which causes the small world effect.'
```

HIGHLIGHTED QUESTION - Make a brief comment on the how and why these random connections (qualitatively) lead to the small world effect.

Since the small world effect claims the diameter should scale proportionally to  $\log(n)$ , we see that the diameter of small word network converges to the orange line which represents  $\log(n)$ . So, it is telling us that the diameter is decreasing as we increase our probability to make an edge between any two nodes which causes a shorter average path between any two nodes which causes the small world effect.

## Sections 8.1-8.4.1: Power Law Networks

### Fitting Power Law

The following helper functions provide easy access to the degree sequence and the degree and cumulative degree distributions.

```
In [8]: def degree_sequence(G):
        return [d for n, d in G.degree()]

        def degree_distribution(G, normalize=True):
            deg_sequence = degree_sequence(G)
            max_degree = max(deg_sequence)
            ddist = zeros((max_degree+1,))
            for d in deg_sequence:
                ddist[d] += 1
            if normalize:
```

```

        ddist = ddist/float(G.number_of_nodes())
    return ddist

def cumulative_degree_distribution(G):
    ddist = degree_distribution(G)
    cdist = [ ddist[k:].sum() for k in range(len(ddist)) ]
    return cdist

```

The following function, which you must complete, plots the degree distribution and calculates the power law coefficient,  $\alpha$ .

In [201...

```

def calc_powerlaw(G,kmin=None):
    ddist = degree_distribution(G,normalize=False)
    cdist = cumulative_degree_distribution(G)
    k = arange(len(ddist))

    N = 0
    k_vec = []
    for d in degree_sequence(G):
        if d >= kmin:
            k_vec.append(d)
            N += 1

    # N = cdist[kmin] * G.number_of_nodes()
    # print(N)

    # sum (ki / kmin - 0.5) where ki >= kmin
    inside = 0
    for ki in k_vec:
        if ki >= kmin:
            inside += np.log(ki / (kmin - 0.5))

    alpha = 1 + (N * (1/inside)) # calculate using Newman (8.6)!
    sigma = float(alpha - 1) / float(np.sqrt(N)) # calculate using Newman (8.7)!
    print('HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for')
    print(' %1.2f +/- %1.2f' % (alpha,sigma) )

    print('HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the')
    plt.figure(figsize=(8,12))
    plt.subplot(211)
    plt.bar(k,ddist, width=0.8, bottom=0, color='b') # replace xvalues and barheights!

    plt.subplot(212)
    plt.loglog(k,cdist) # replace xvalues and yvalues!
    plt.grid(True)

```

In [78]:

```

G = nx.read_weighted_edgelist('japanese.edgelist',create_using=nx.DiGraph)
print('Japanese Network')
calc_powerlaw(G,10) # select kmin!
plt.show()
G = nx.read_weighted_edgelist('ca-HepTh.edgelist',create_using=nx.Graph)
print('ca-HepTh Network')
calc_powerlaw(G,45) # select kmin!
plt.show()
G = nx.read_weighted_edgelist('soc-Epinions1.edgelist',create_using=nx.DiGraph)
print('soc-Epinions1 Network')

```

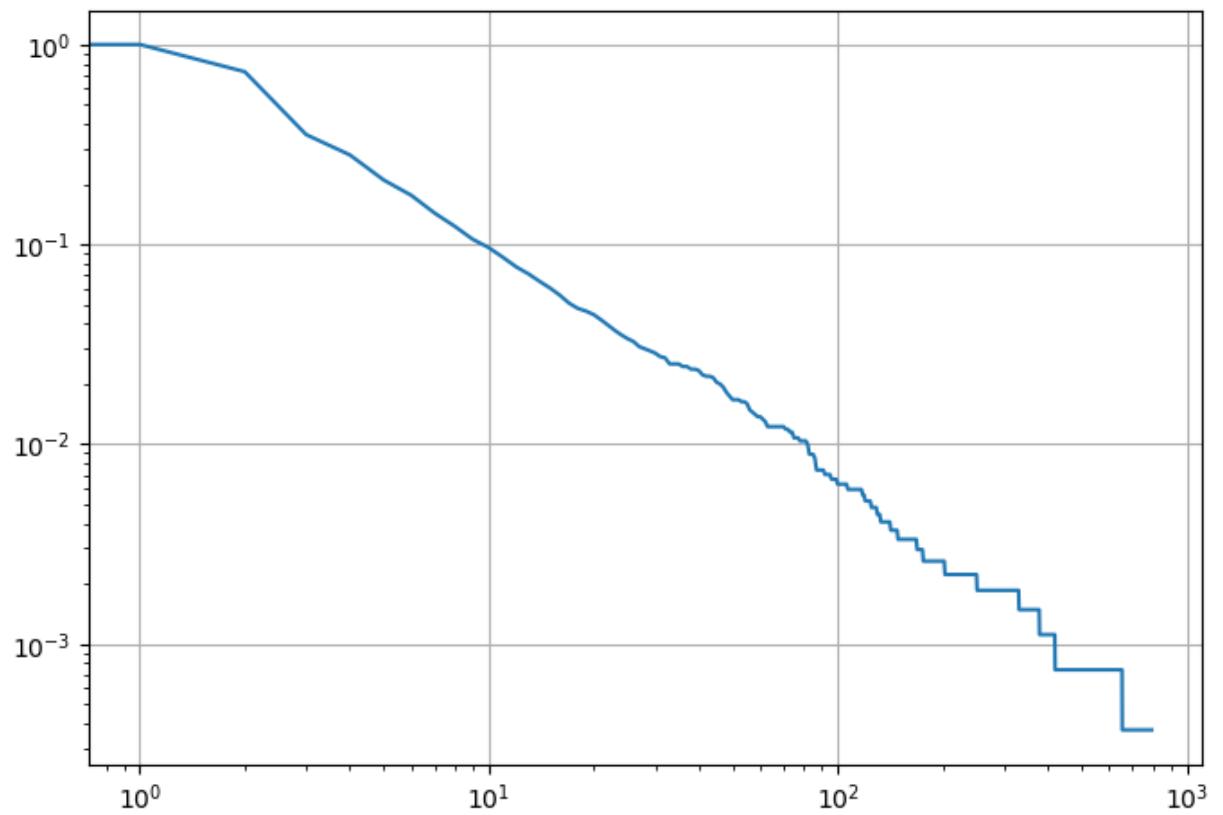
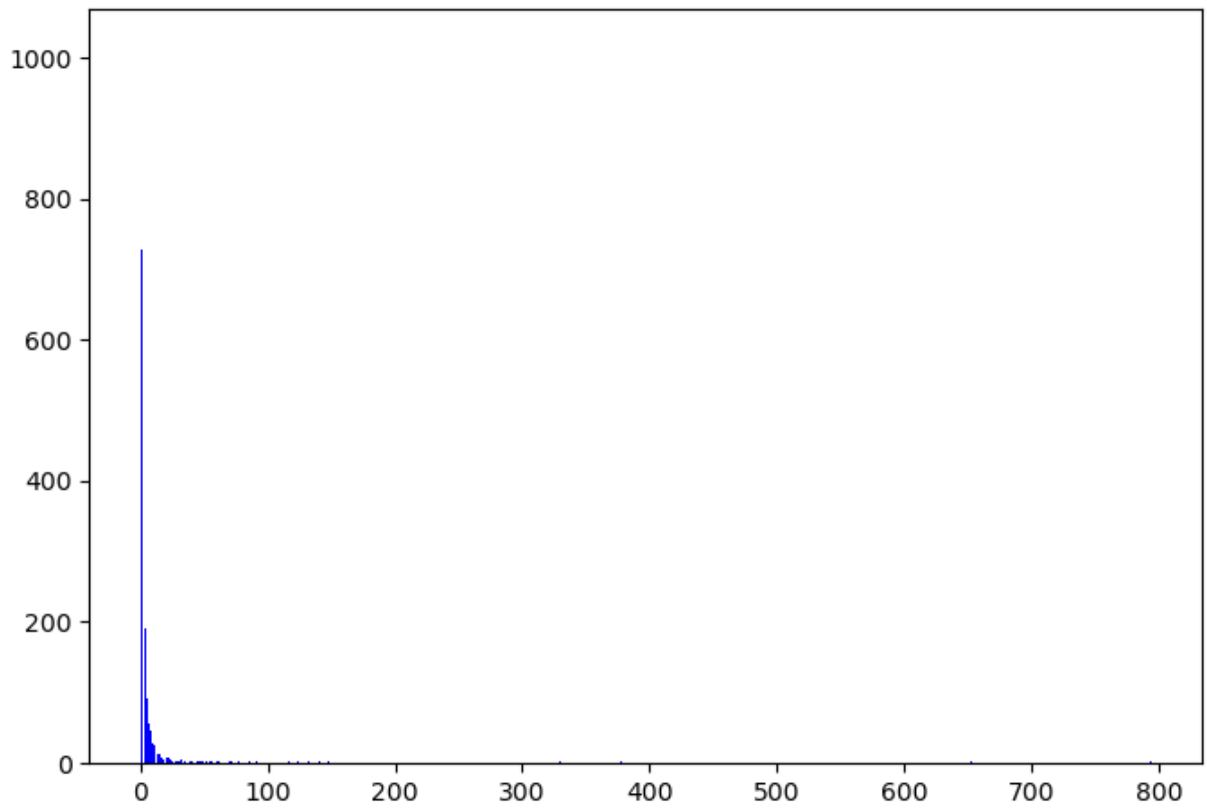
```
calc_powerlaw(G,10) # select kmin!  
plt.show()
```

Japanese Network

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.11 +/- 0.07

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

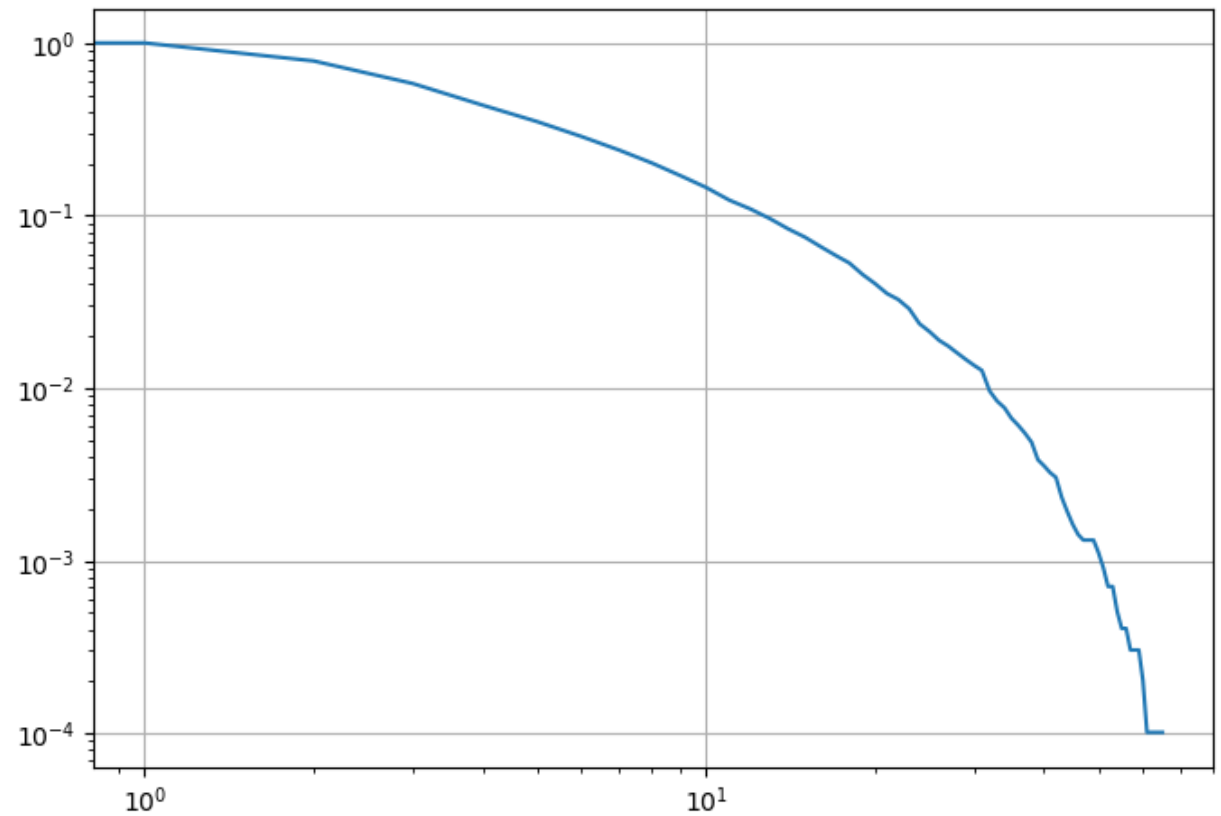
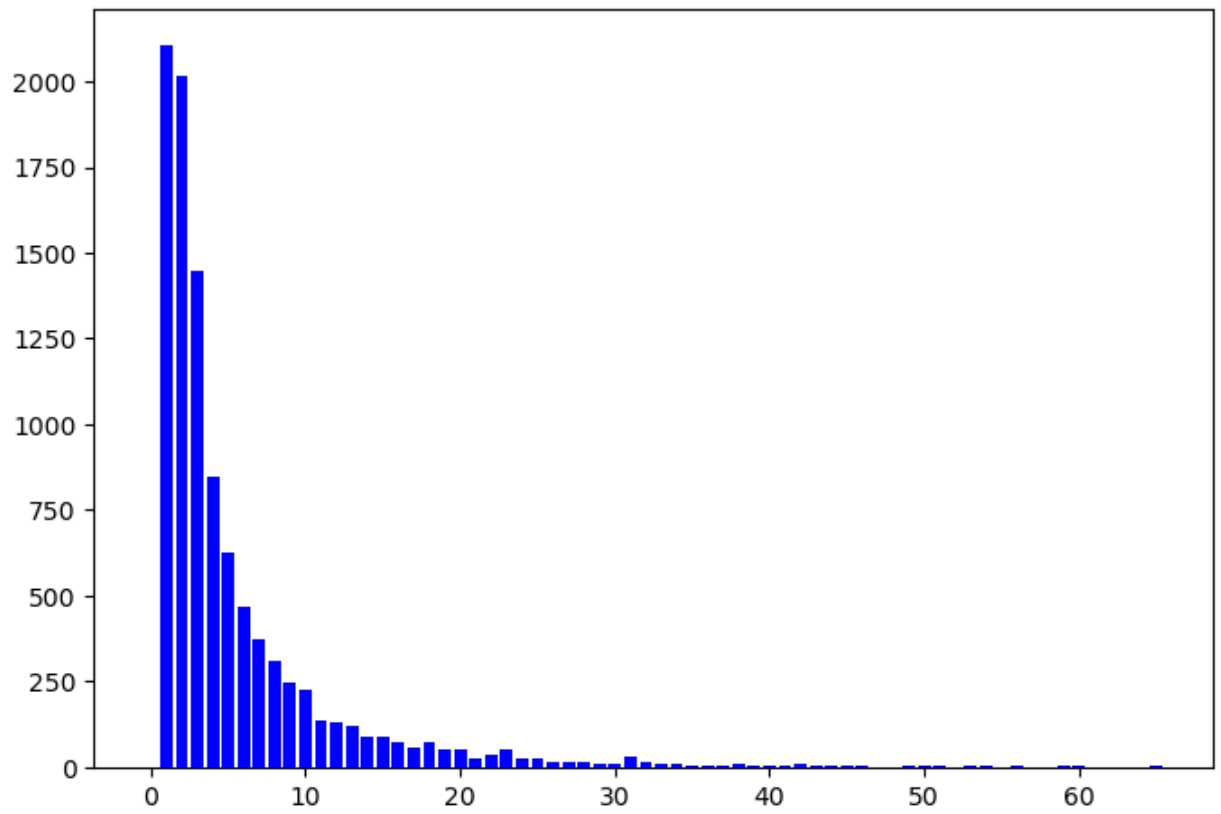


ca-HepTh Network

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

7.43 +/- 1.61

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



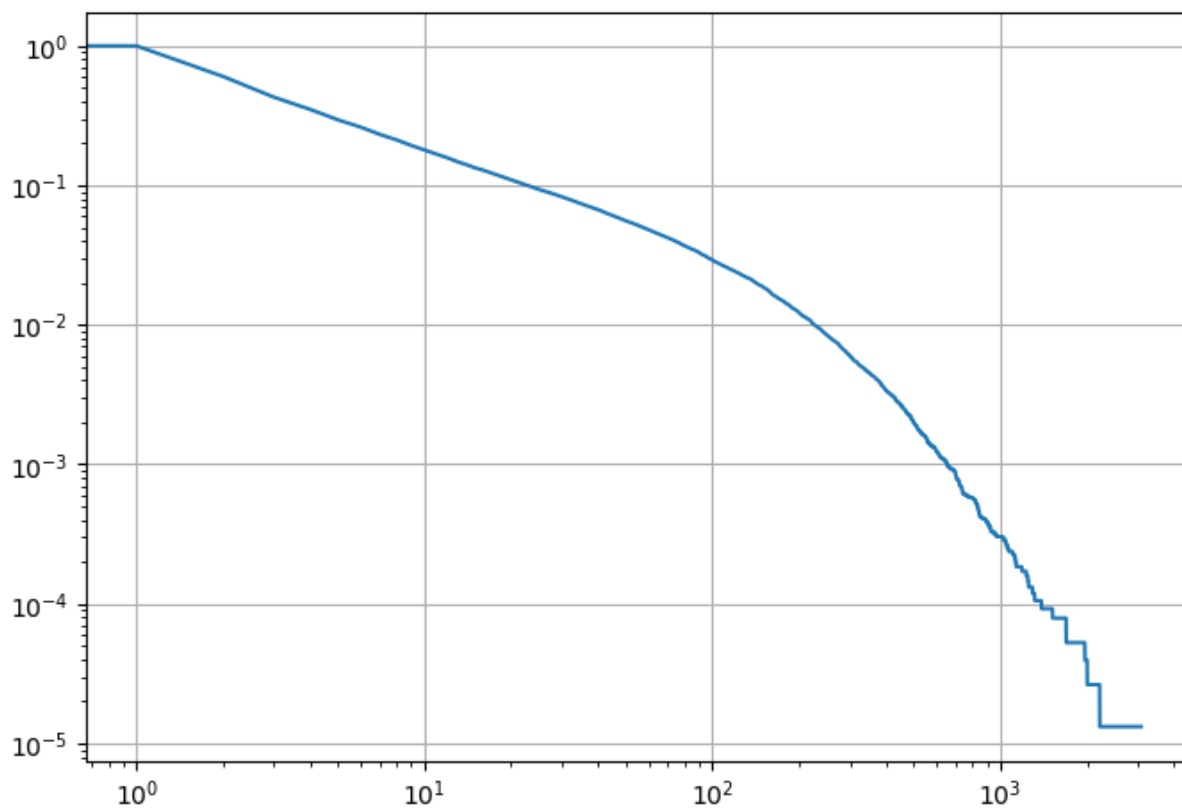
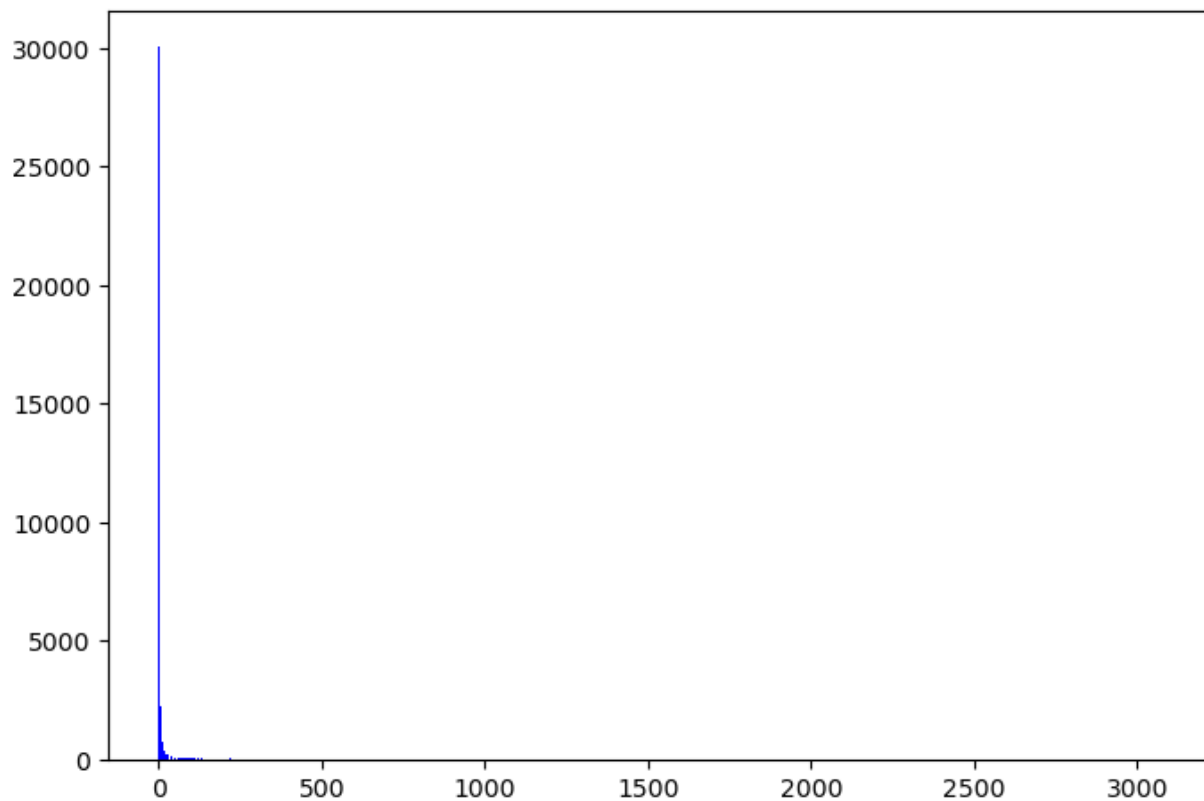


soc-Epinions1 Network

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

1.79 +/- 0.01

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Giant Component

In [150...

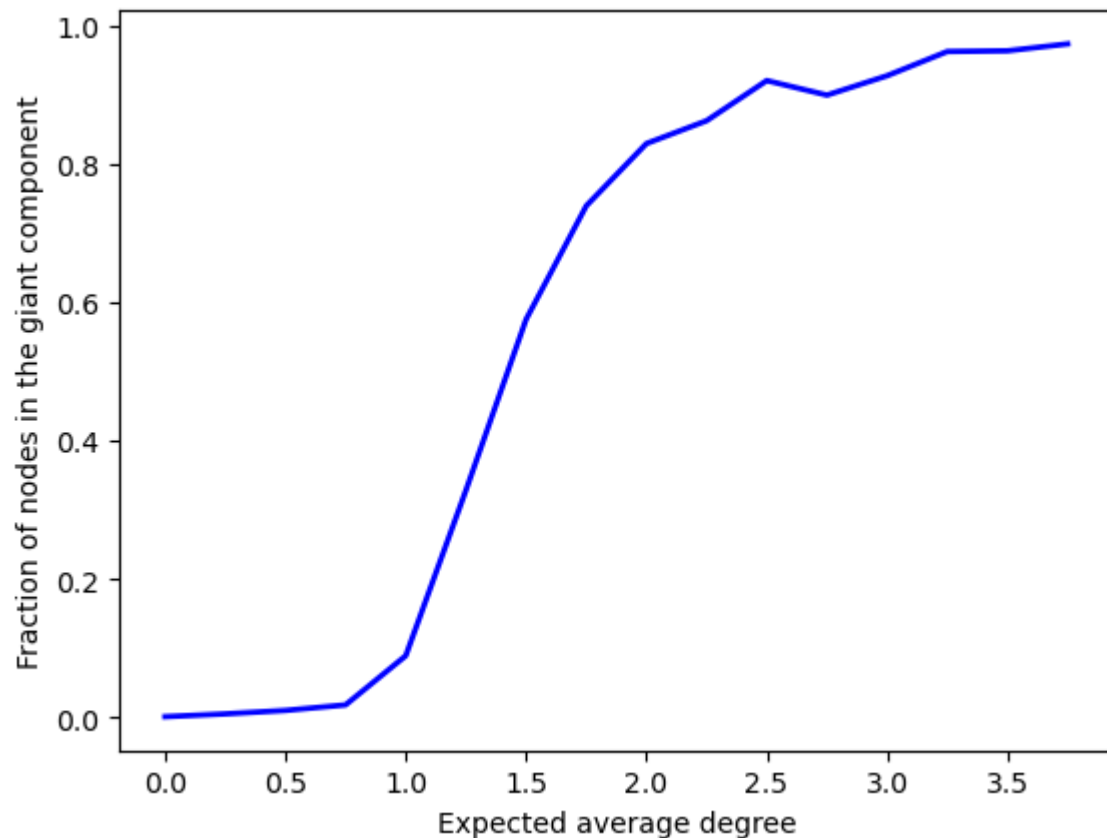
```
n = 1000  
c = np.arange(0, 4, 0.25)
```

```

S = []
for ci in c:
    p = float(ci) / (float(n - 1))
    ERG = nx.erdos_renyi_graph(n, p)
    giant_component = list(max(nx.connected_components(ERG), key=len))
    S.append(len(giant_component) / float(n))

plt.plot(c, S, 'b', linewidth=2)
plt.xlabel("Expected average degree")
plt.ylabel("Fraction of nodes in the giant component")
plt.show()

```



In [115...

```

print("HIGHLIGHTED QUESTION - Once you have your plot, you'll notice that the line is pretty rough and in some cases it may go up and down, even though the theoretical results predict a curve for the expected giant component size that is monotonic with increasing average degree. Why are you seeing this non-monotonic behavior? This is due to the random nature of ER graphs. Edges between nodes are formed with a probability p = c / (N - 1), and assuming we keep N = 1000, then as the expected average degree c rises we find that there are more edges being formed and therefore more nodes in the giant component. However as the network grows there is still a likelihood of forming components that are not connected to the giant component. If these components are large in size (but still smaller than the giant component), then the fluctuation will be greatly noticeable by a dip in the graph. This can cause the nonmonotonic behavior that we see in the graph. As we increase the size of N then the graph will be closer to the theoretical result.")

```

HIGHLIGHTED QUESTION - Once you have your plot, you'll notice that the line is pretty rough and in some cases it may go up and down, even though the theoretical results predict a curve for the expected giant component size that is monotonic with increasing average degree. Why are you seeing this non-monotonic behavior?

This is due to the random nature of ER graphs. Edges between nodes are formed with a probability  $p = c / (N - 1)$ , and assuming we keep  $N = 1000$ , then as the expected average degree  $c$  rises we find that there are more edges being formed and therefore more nodes in the giant component. However as the network grows there is still a likelihood of forming components that are not connected to the giant component. If these components are large in size (but still smaller than the giant component), then the fluctuation will be greatly noticeable by a dip in the graph. This can cause the nonmonotonic behavior that we see in the graph. As we increase the size of  $N$  then the graph will be closer to the theoretical result.

In [151...

```

def plot_giant_component(n, num_graphs):
    c = np.arange(0, 4, 0.25)
    S_mean = []

```

```

num_graphs = 100
p = 0.5

for ci in c:
    S = []
    for i in range(num_graphs):
        p = float(ci) / (float(n - 1))
        ERG = nx.erdos_renyi_graph(n, p)
        giant_component = list(max(nx.connected_components(ERG), key=len))
        S.append(len(giant_component) / float(n))

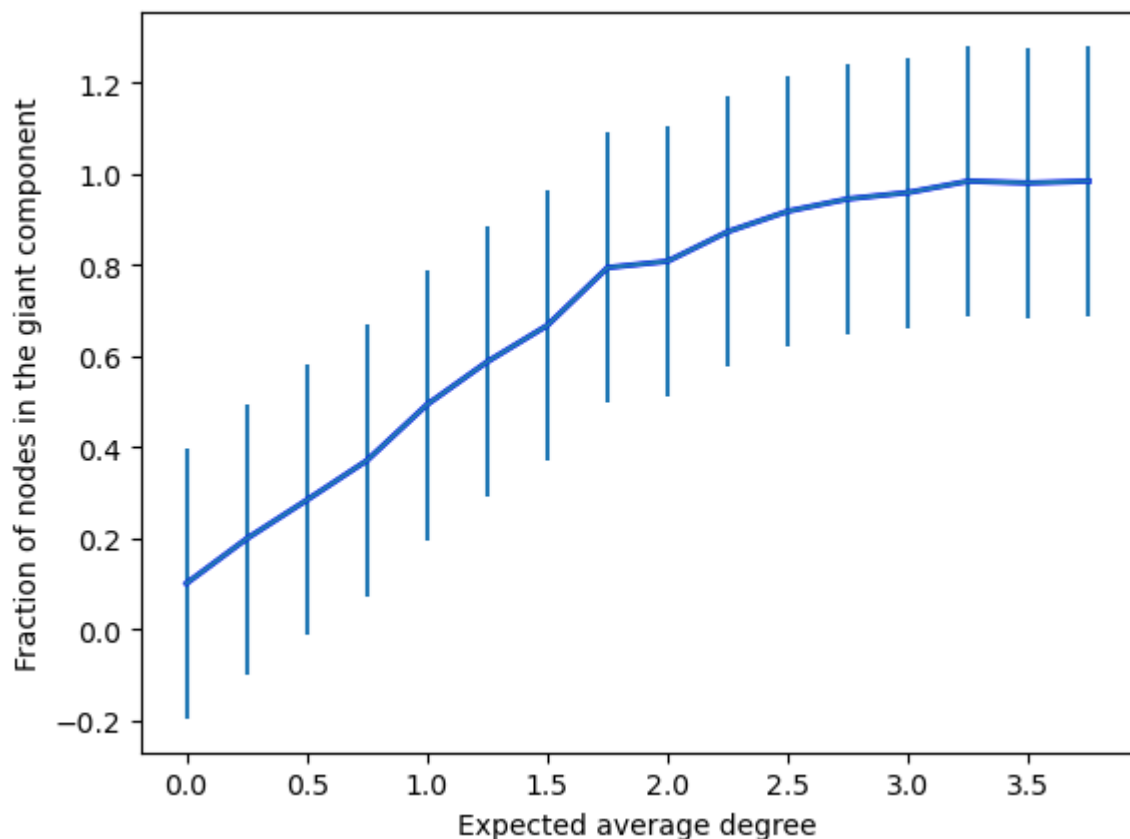
    S_mean.append(np.mean(S))
    S_std = np.std(S_mean)
    plt.errorbar(c, S_mean, yerr=S_std)
    plt.plot(c, S_mean, 'b', linewidth=2)
    plt.xlabel("Expected average degree")
    plt.ylabel("Fraction of nodes in the giant component")
    plt.show()

print('HIGHLIGHTED QUESTION - Make three plots, one for n=10, one for n=100, and one')
print('n = 10')
plot_giant_component(n=10, num_graphs=100)
print('n = 100')
plot_giant_component(n=100, num_graphs=100)
# print('n = 1000')
# plot_giant_component(n=1000, num_graphs=100)

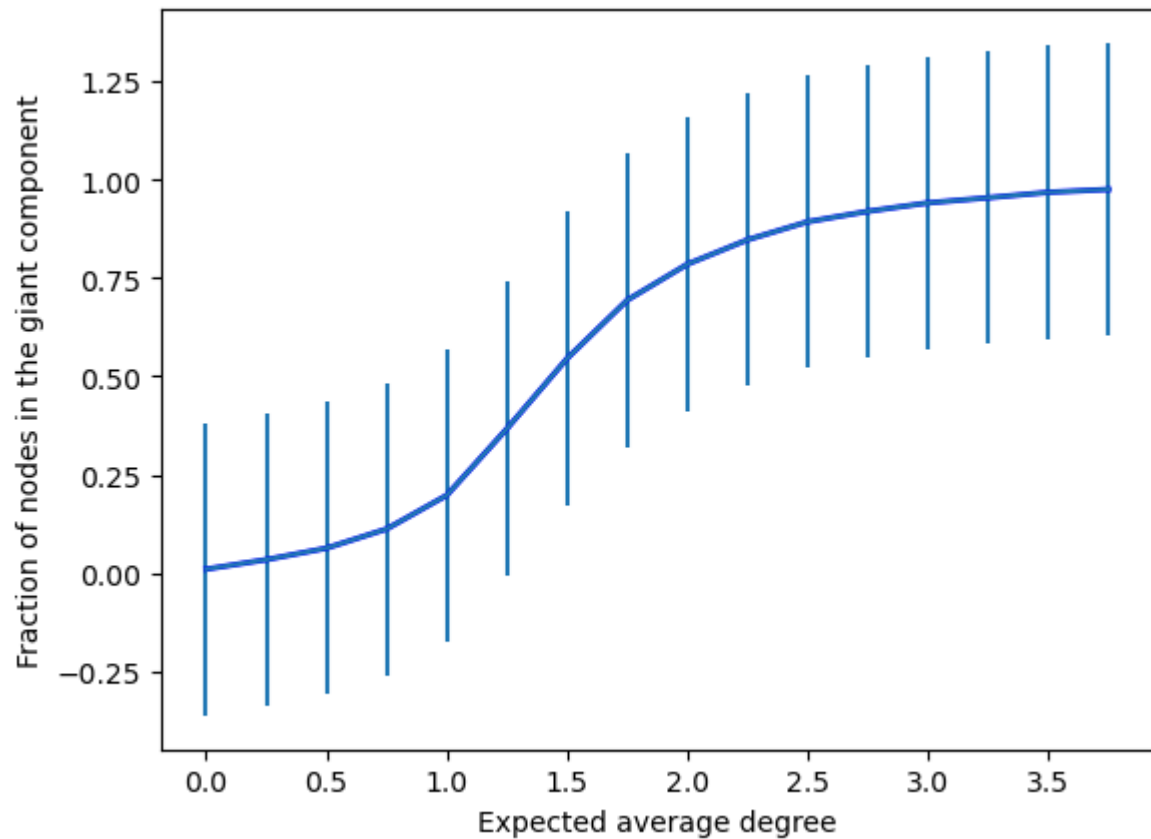
```

HIGHLIGHTED QUESTION - Make three plots, one for n=10, one for n=100, and one for n=1000.

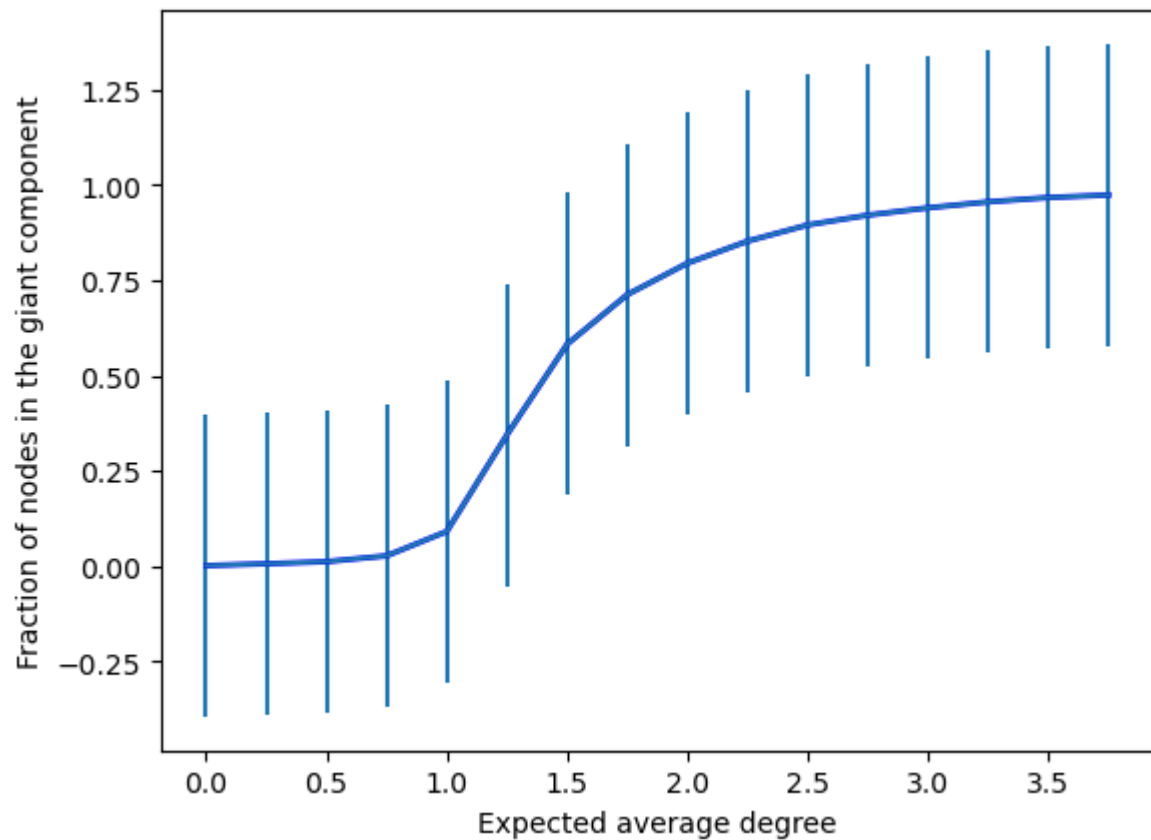
n = 10



n = 100

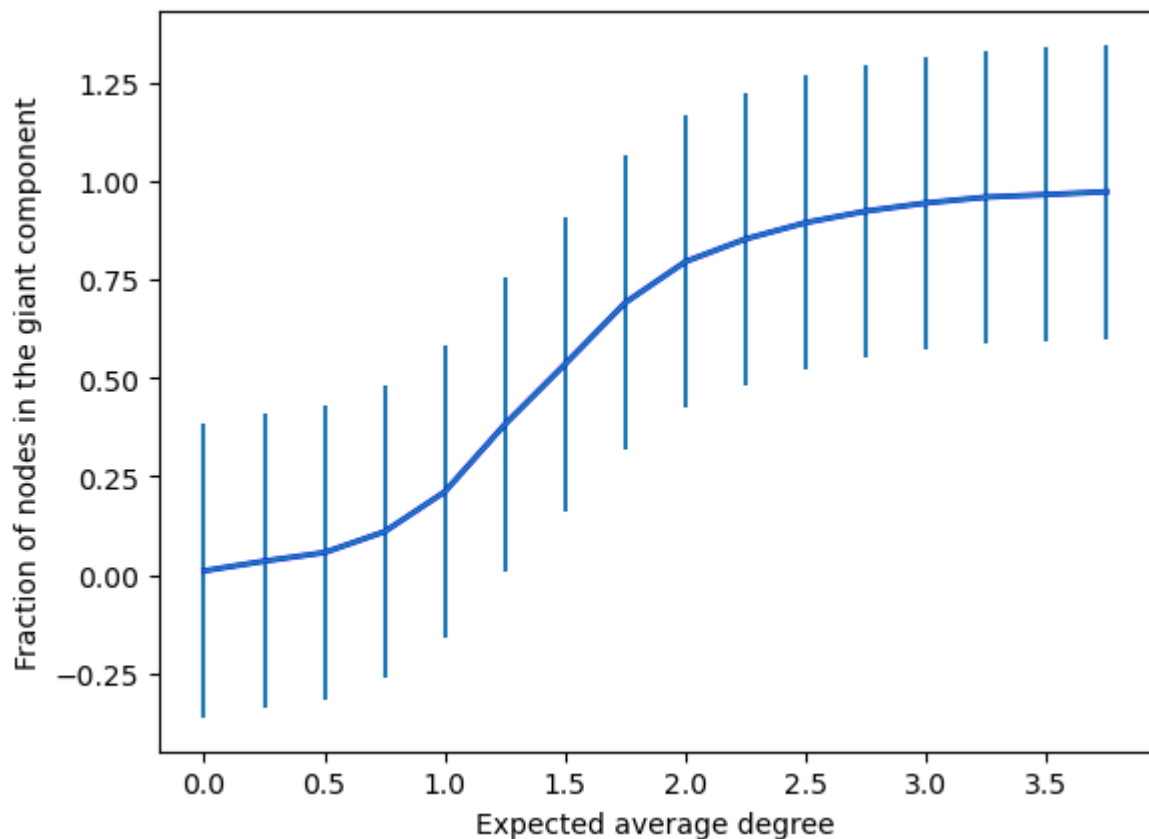


$n = 1000$



```
In [161... print('n = 100 but averaging over 1 graph')
plot_giant_component(n=100, num_graphs=1)
```

$n = 100$  but averaging over 1 graph



In [138... `print('HIGHLIGHTED QUESTION - Observe the differences in the plots and explain how this supports the notion that the statistics of the model converge to their expected values as n gets larger.`  
`print('As n increases we see that the fluctuations in the giant component of the ER graph decrease significantly. For n = 10 and averaging over 100 graph iterations the graph exhibits some non-monotonic behavior and the curve is not smooth. For n=1000 and averaging over 100 graph iterations the graph is closer to the monotonic behavior that we expect and also exhibits a smoother curve. This tells us that as the graph size increases, the statistics of the ER model give us a better approximation of a random graph for a given average degree.'`

HIGHLIGHTED QUESTION - Observe the differences in the plots and explain how this supports the notion that the statistics of the model converge to their expected values as n gets larger.

As n increases we see that the fluctuations in the giant component of the ER graph decrease significantly. For n = 10 and averaging over 100 graph iterations the graph exhibits some non-monotonic behavior and the curve is not smooth. For n=1000 and averaging over 100 graph iterations the graph is closer to the monotonic behavior that we expect and also exhibits a smoother curve. This tells us that as the graph size increases, the statistics of the ER model give us a better approximation of a random graph for a given average degree.

## Degree Distributions of Random Network Models

In [164... `N = 5000`

In [220... `print('HIGHLIGHTED QUESTION - Use this same code to look at the degree distributions and cumulative degree distributions of these networks. Do this for small, intermediate, and large values of the various parameters that determine these networks`

HIGHLIGHTED QUESTION - Use this same code to look at the degree distributions and cumulative degree distributions of these networks. Do this for small, intermediate, and large values of the various parameters that determine these networks

## Erdos-Renyi

In [170... `p = [0.1, 0.2, 0.4]`  
`kmin = [500, 1000, 2000]`  
`for pi, kmini in zip(p, kmin):`  
`print(f'ERG with N = {N} and p = {pi} and kmin = {kmini}')`

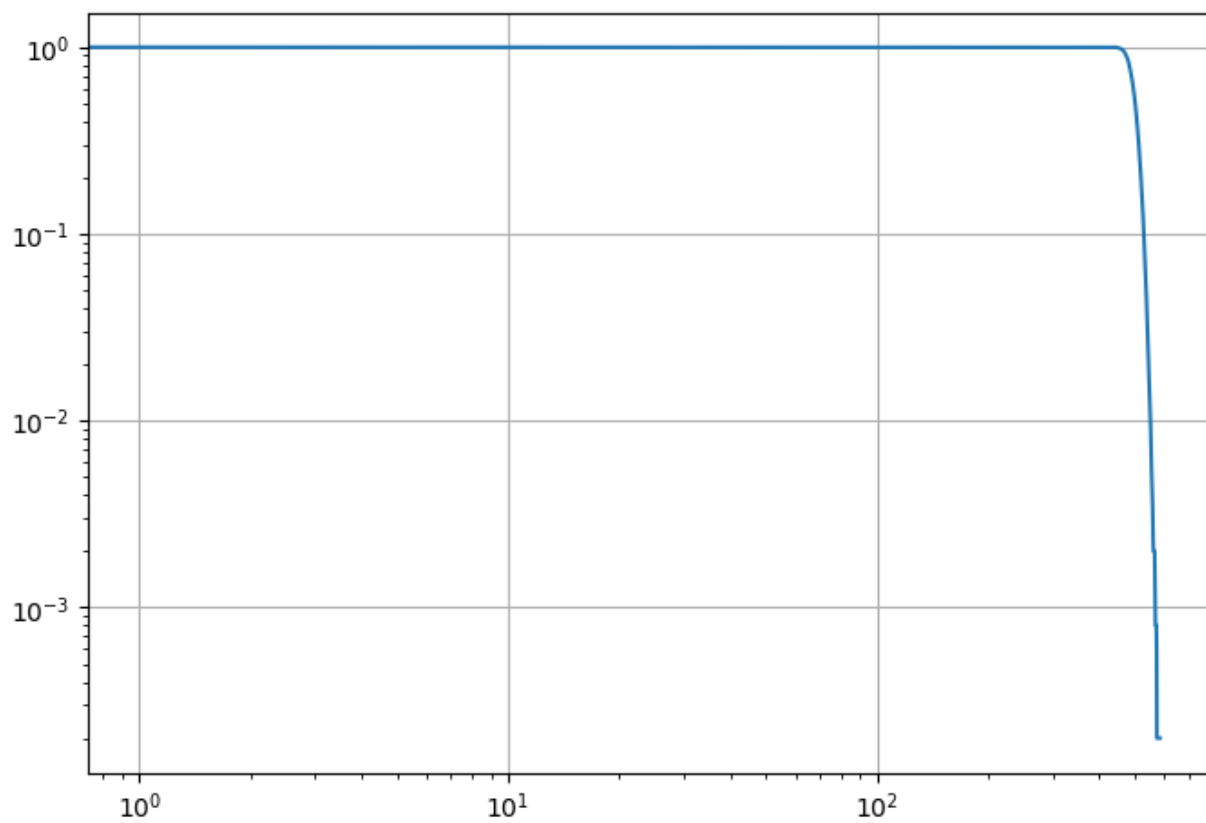
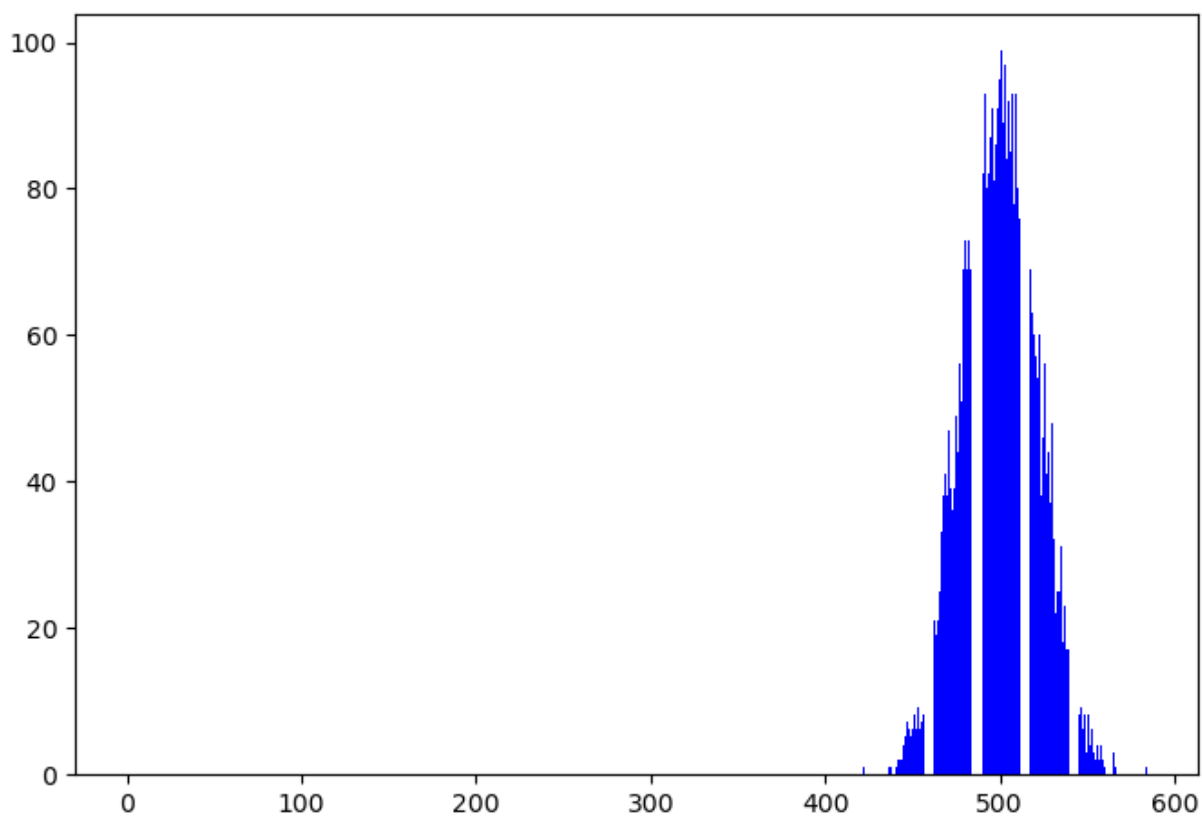
```
ERG = nx.erdos_renyi_graph(N, pi)
calc_powerlaw(ERG, kmini)
plt.show()
```

ERG with  $N = 5000$  and  $p = 0.1$  and  $k_{\min} = 500$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

30.57 +/- 0.59

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



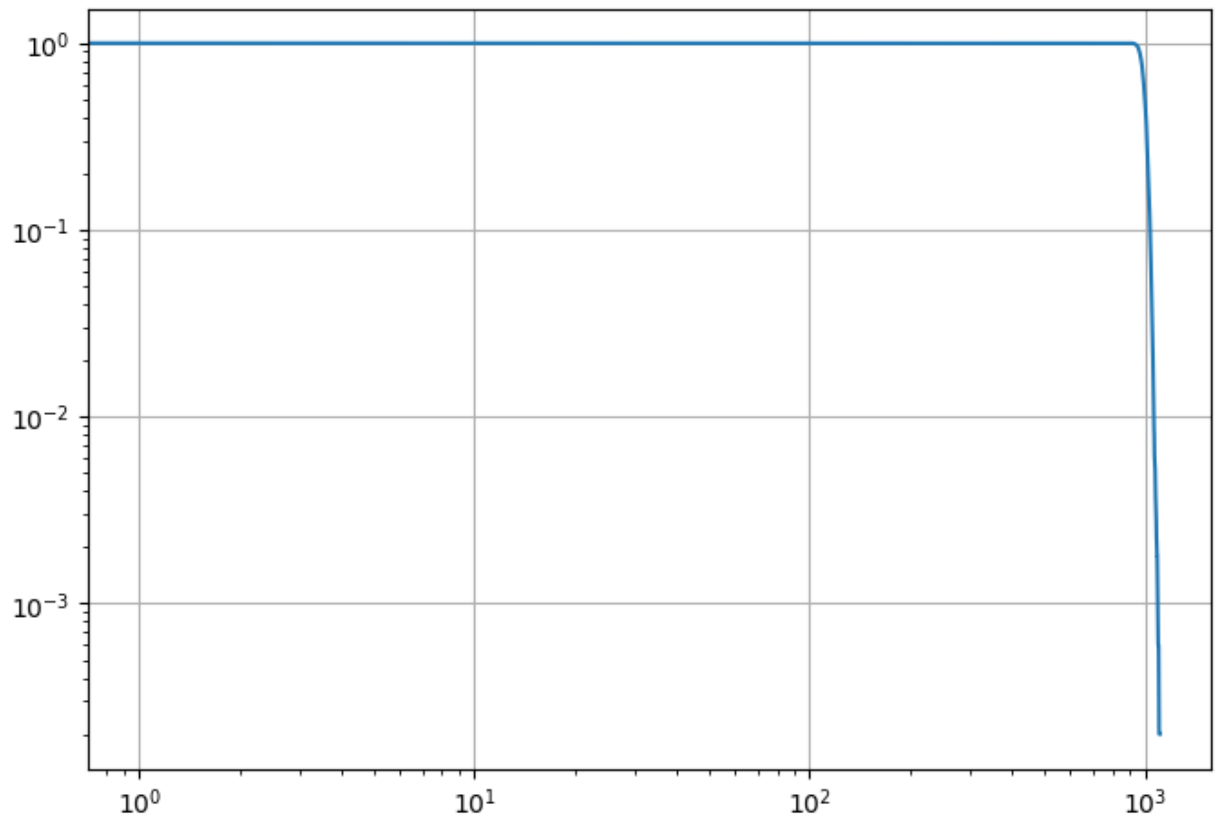
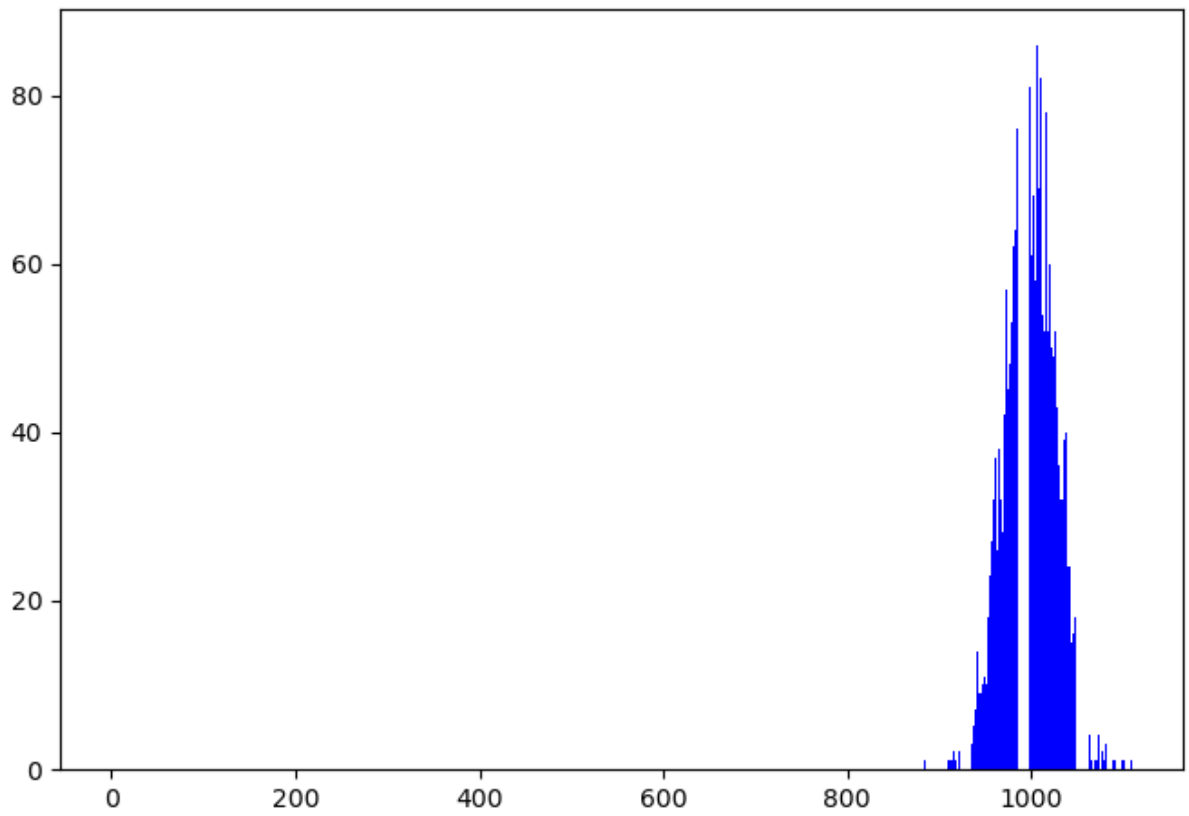


ERG with  $N = 5000$  and  $p = 0.2$  and  $k_{\min} = 1000$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

45.60 +/- 0.89

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

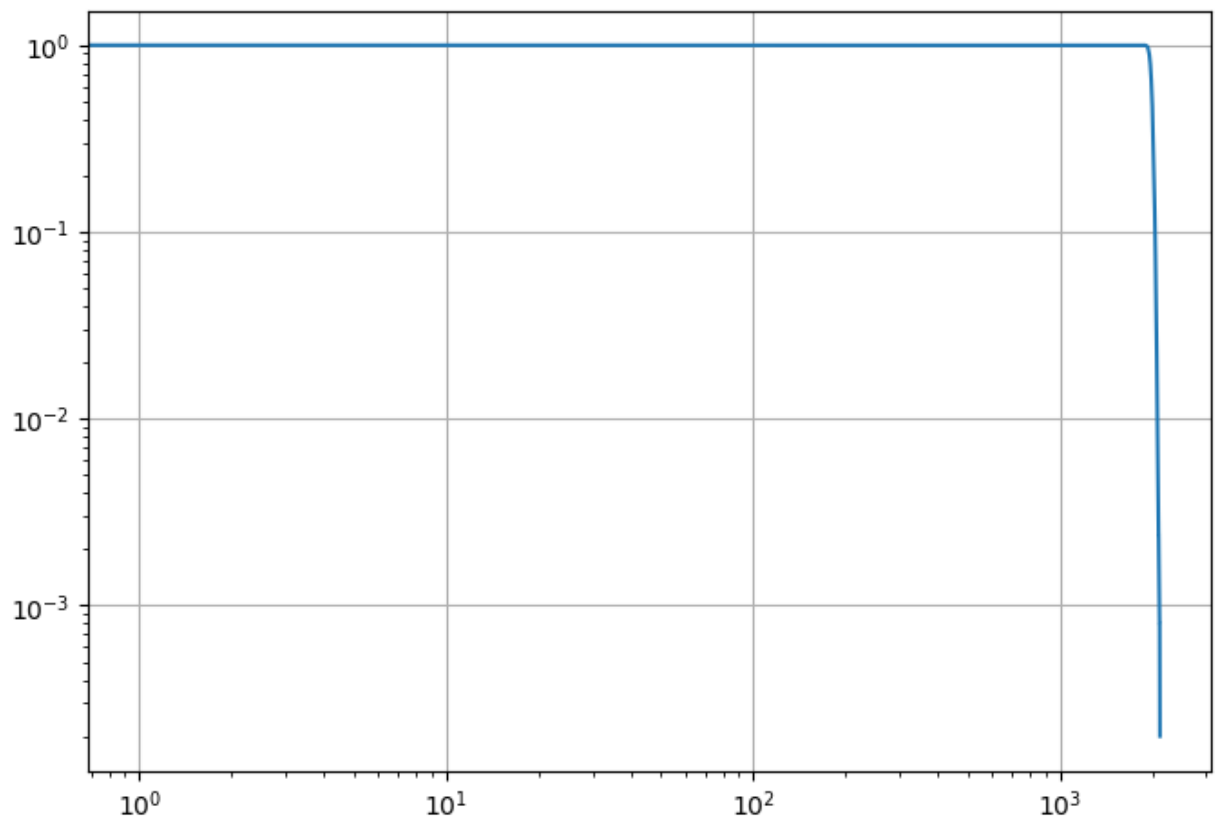
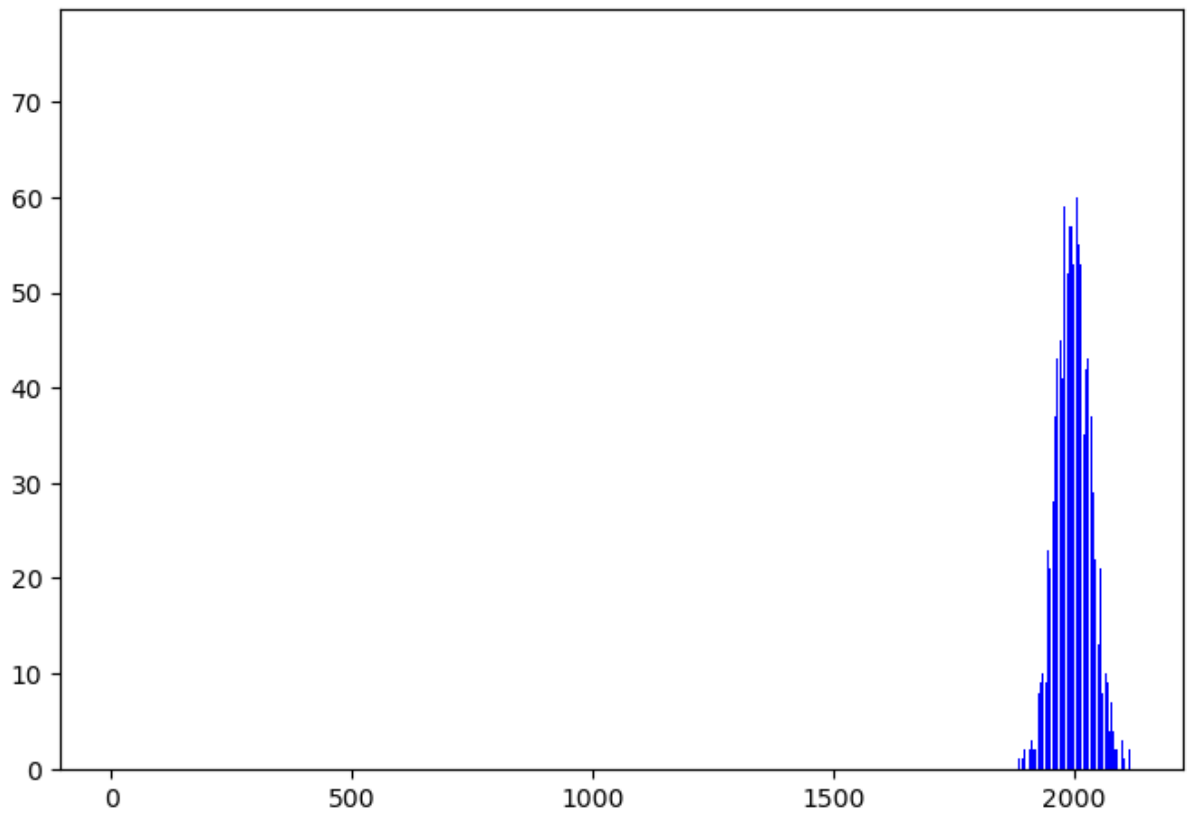


ERG with  $N = 5000$  and  $p = 0.4$  and  $k_{\min} = 2000$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

75.66 +/- 1.52

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Small-World

In [173...

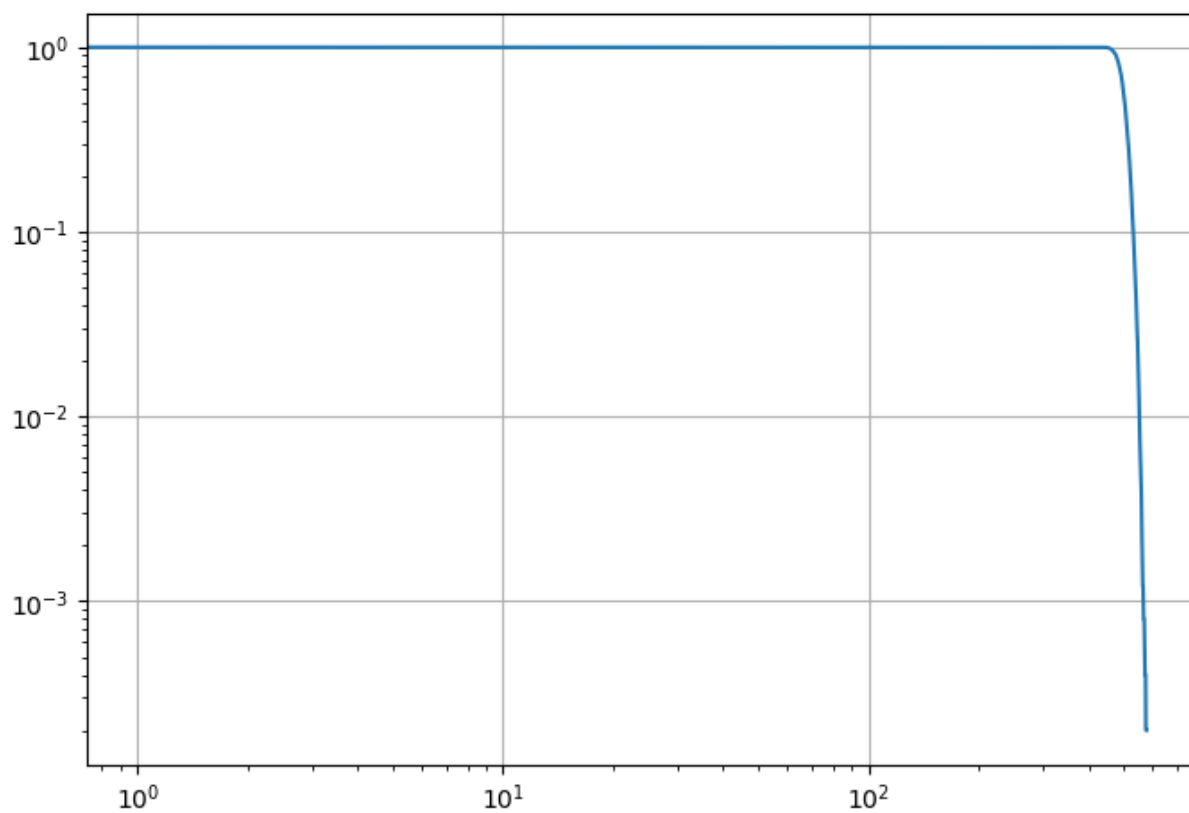
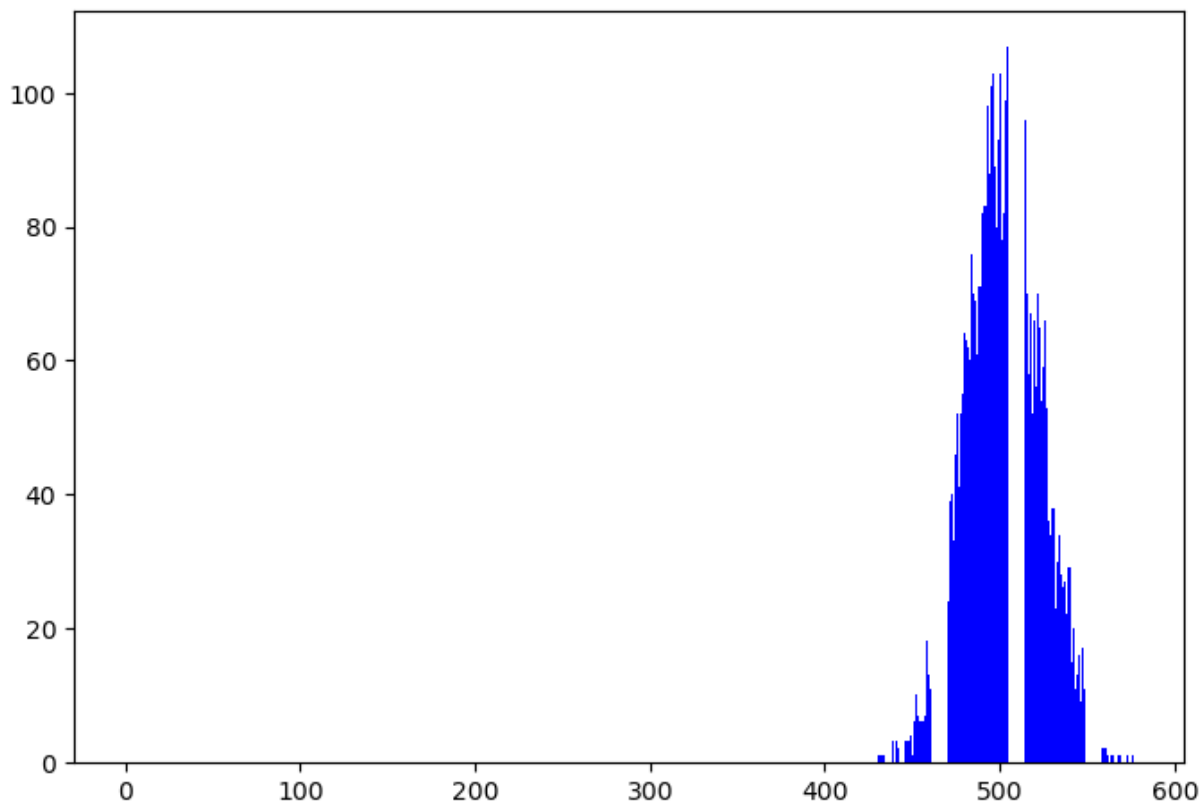
```
p = [0.1, 0.2, 0.4]
q = [2, 4, 10]
kmin = [500, 1000, 2000]
for pi, qi, kmini in zip(p, q, kmin):
    print(f'SW with N = {N} and p = {pi} | q = {qi} | kmin = {kmini}')
    SW = randomnet.small_world_graph(N, qi, pi)
    calc_powerlaw(SW, kmini)
    plt.show()
```

SW with N = 5000 and p = 0.1 | q = 2 | kmin = 500

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

29.16 +/- 0.54

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

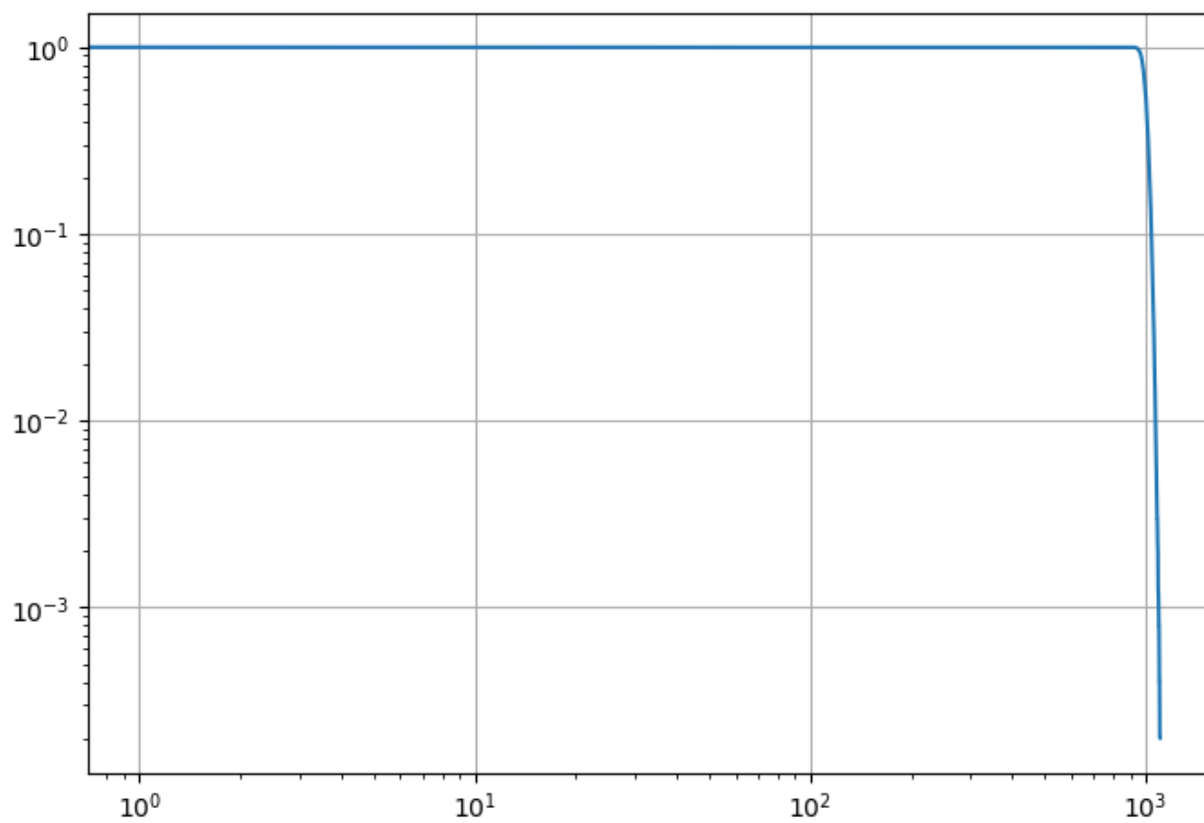
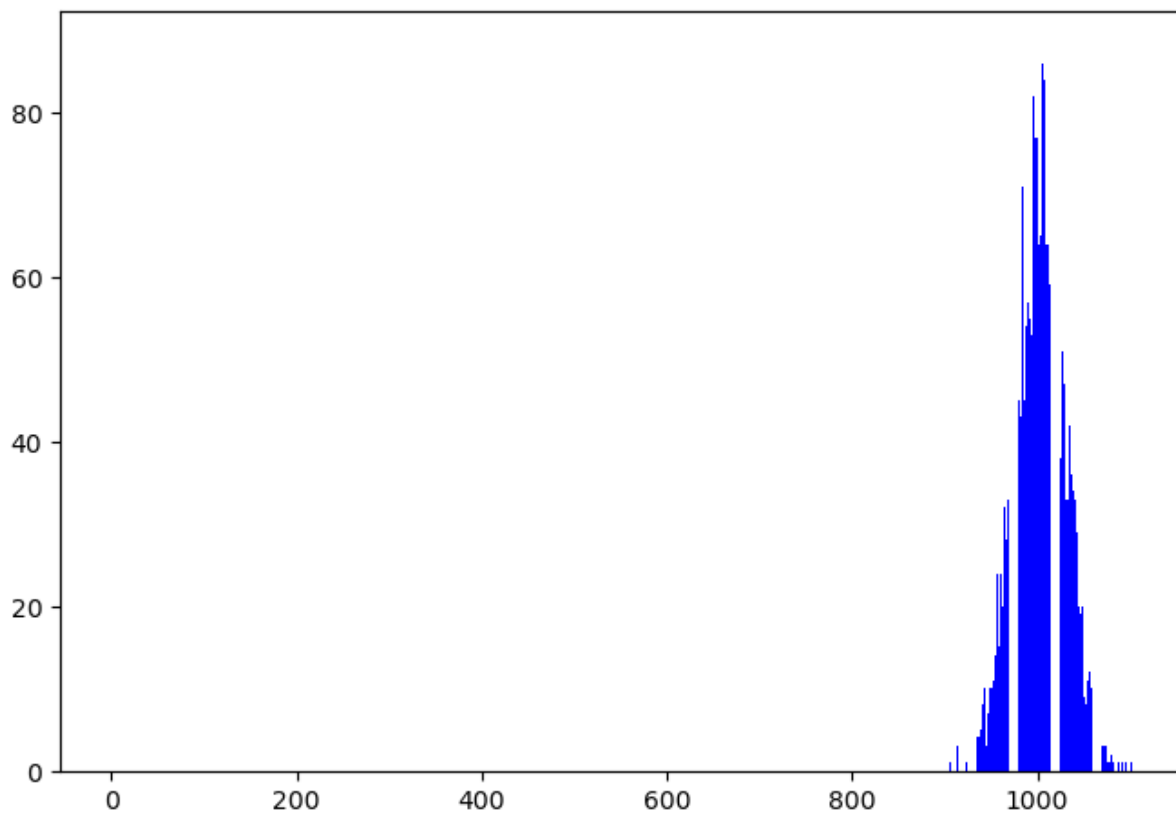


SW with  $N = 5000$  and  $p = 0.2$  |  $q = 4$  |  $k_{\min} = 1000$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

44.11 +/- 0.82

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



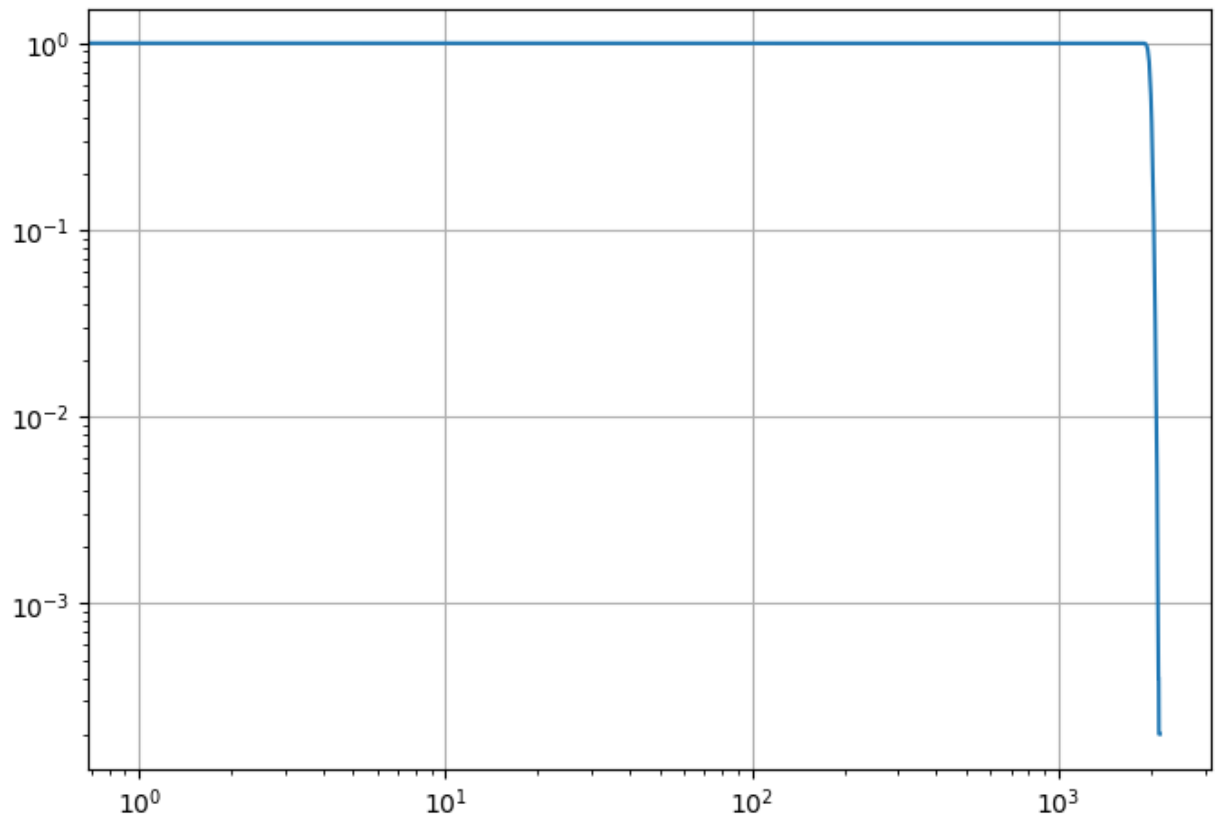
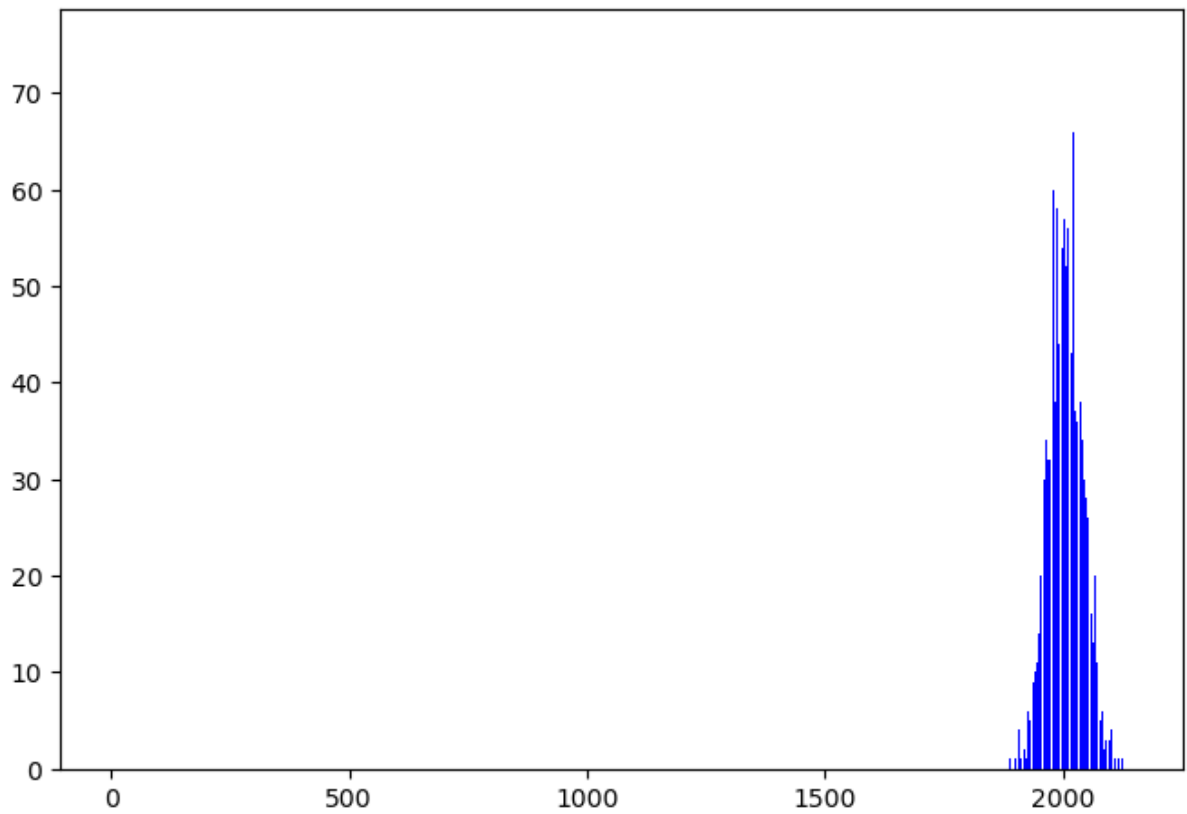


SW with  $N = 5000$  and  $p = 0.4$  |  $q = 10$  |  $k_{\min} = 2000$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

69.62 +/- 1.29

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Barabasi-Albert

In [208...

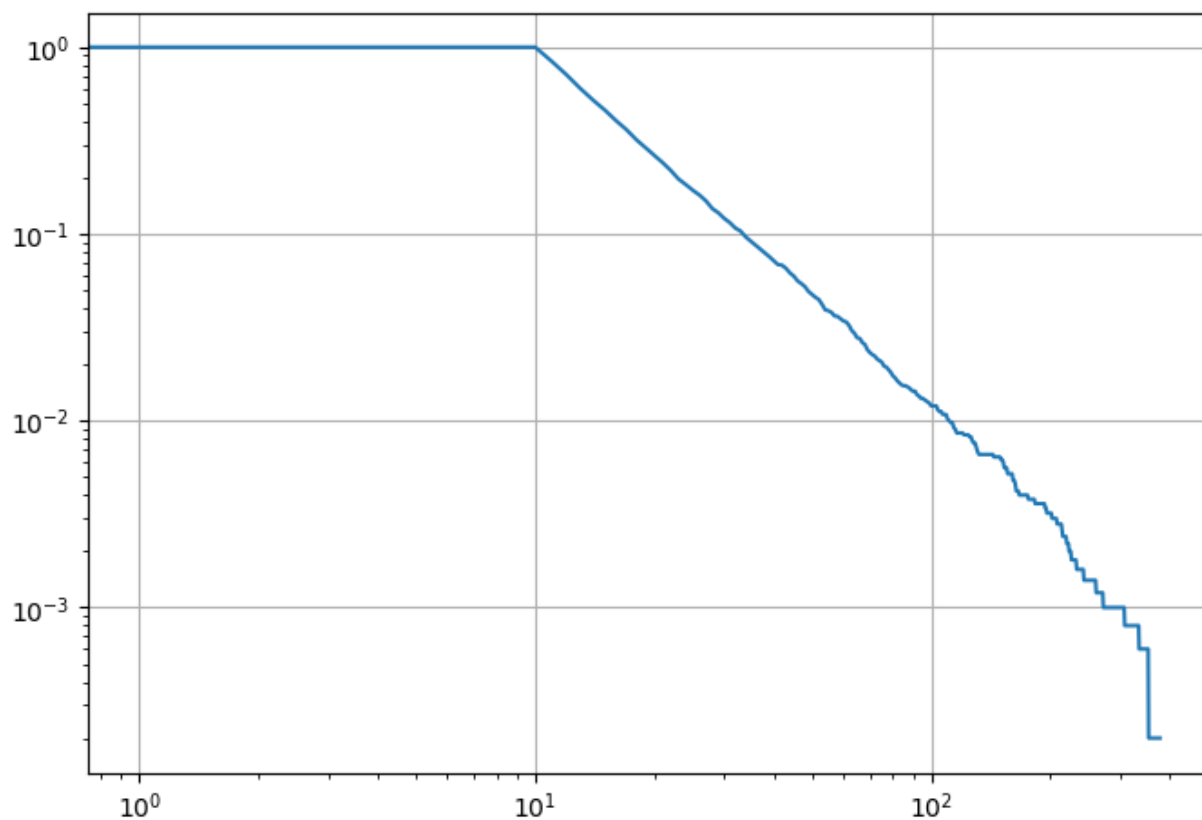
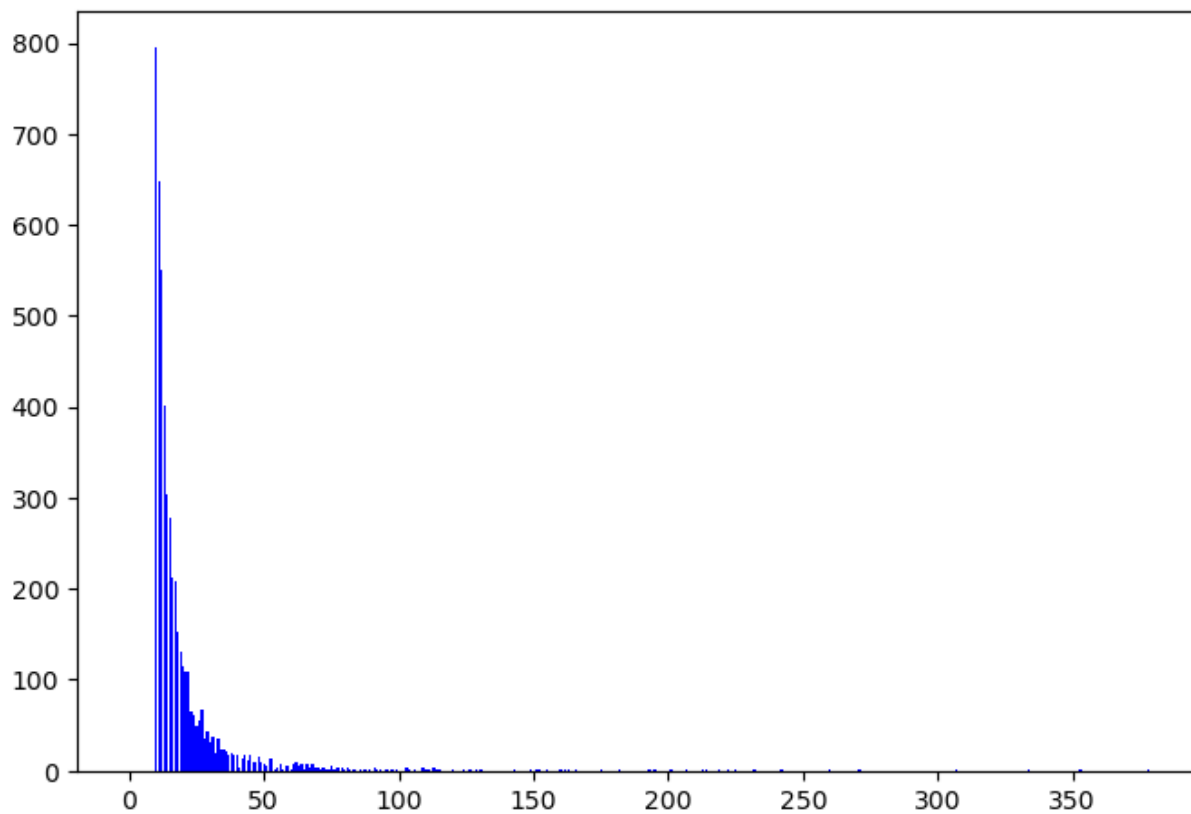
```
m = [10, 100, 1000]
kmin = [40, 200, 1000]
for mi, kmini in zip(m, kmin):
    print(f'BA with N = {N} | k = {kmini} | m = {mi}')
    BAG = nx.barabasi_albert_graph(N,mi)
    calc_powerlaw(BAG, kmini)
    plt.show()
```

BA with N = 5000 | k = 40 | m = 10

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.95 +/- 0.10

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

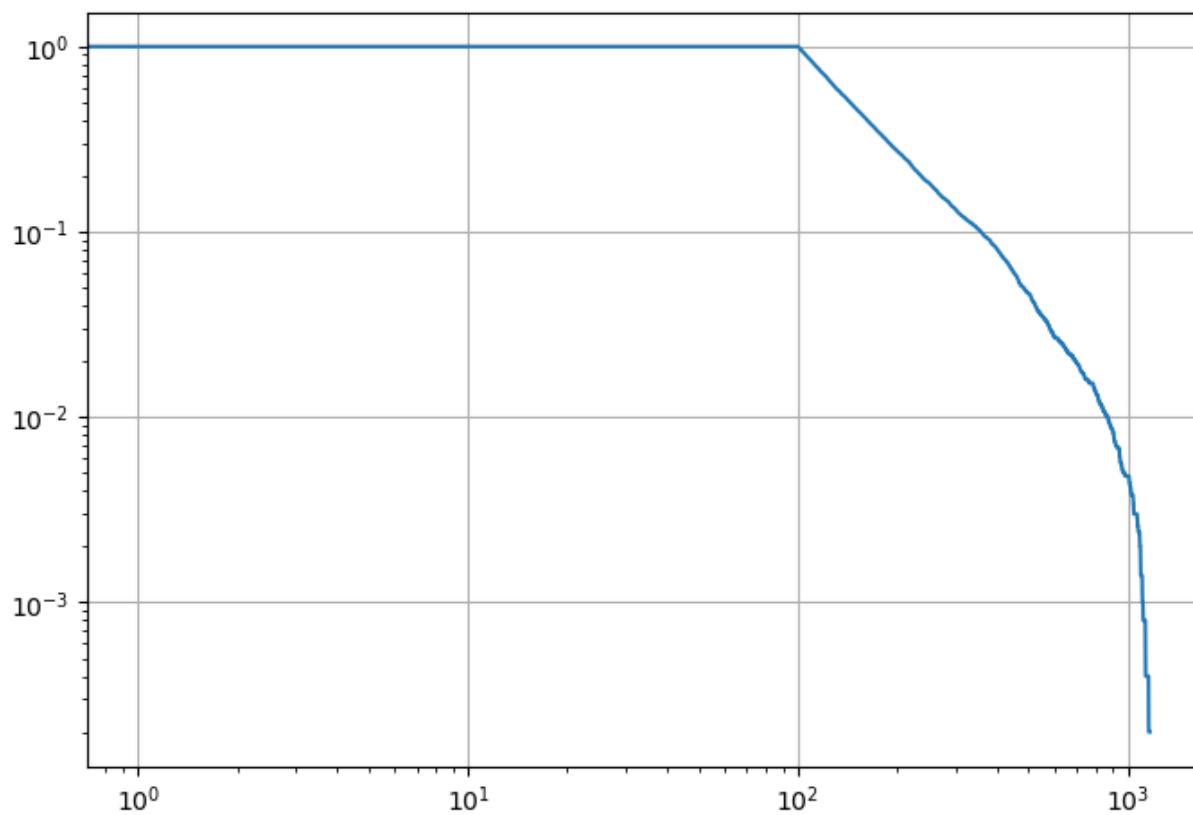
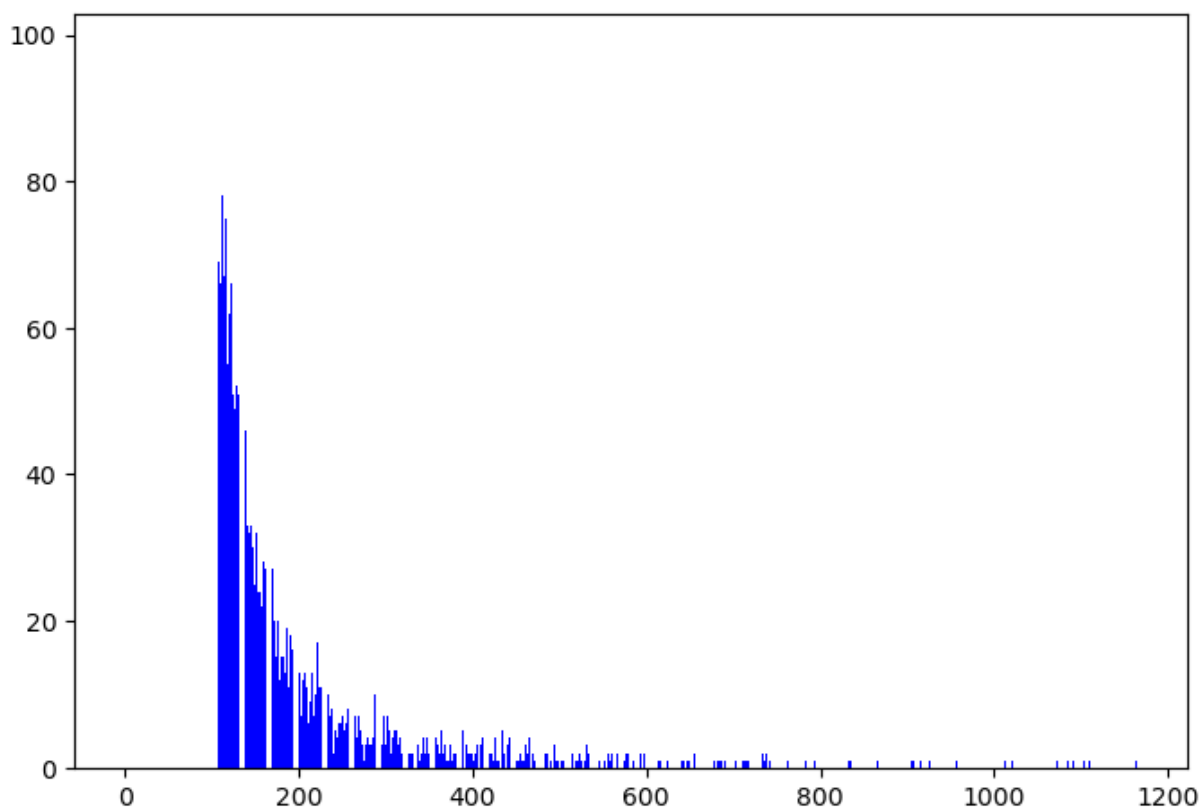


BA with  $N = 5000$  |  $k = 200$  |  $m = 100$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.98 +/- 0.05

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

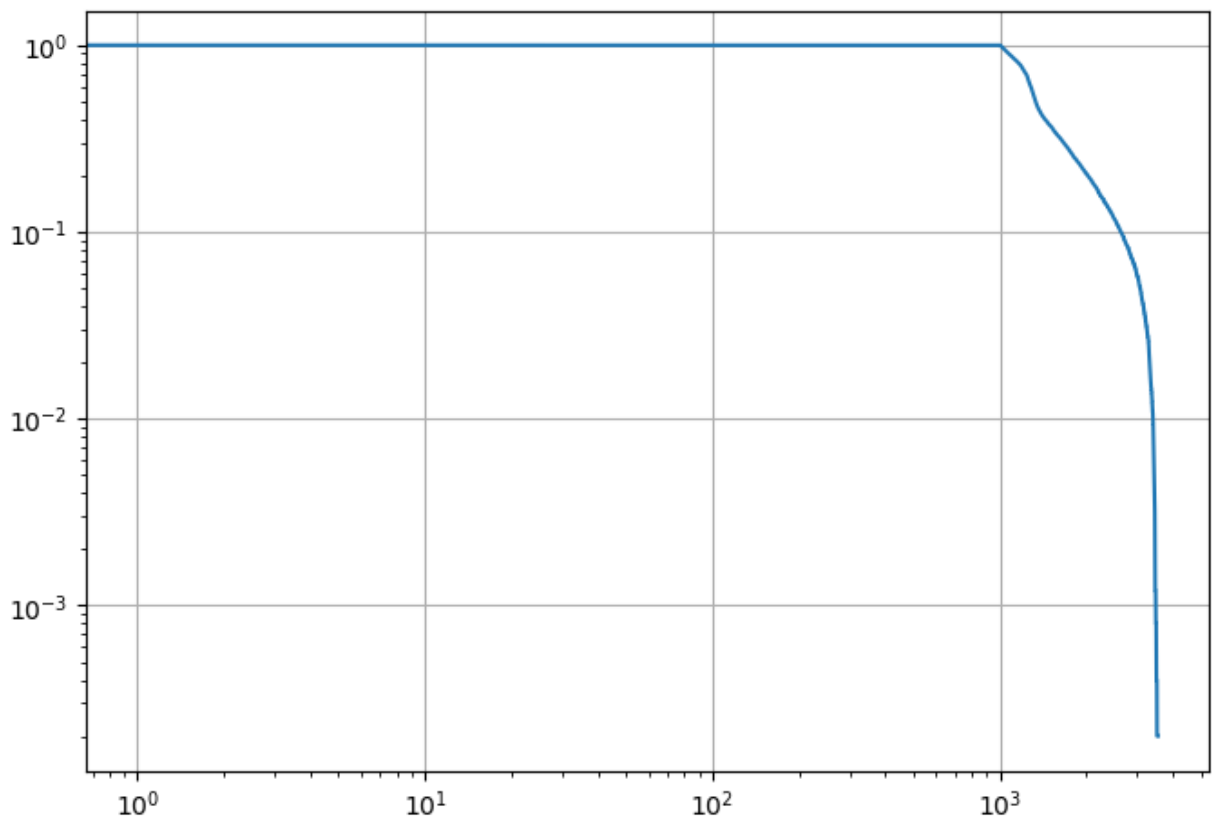
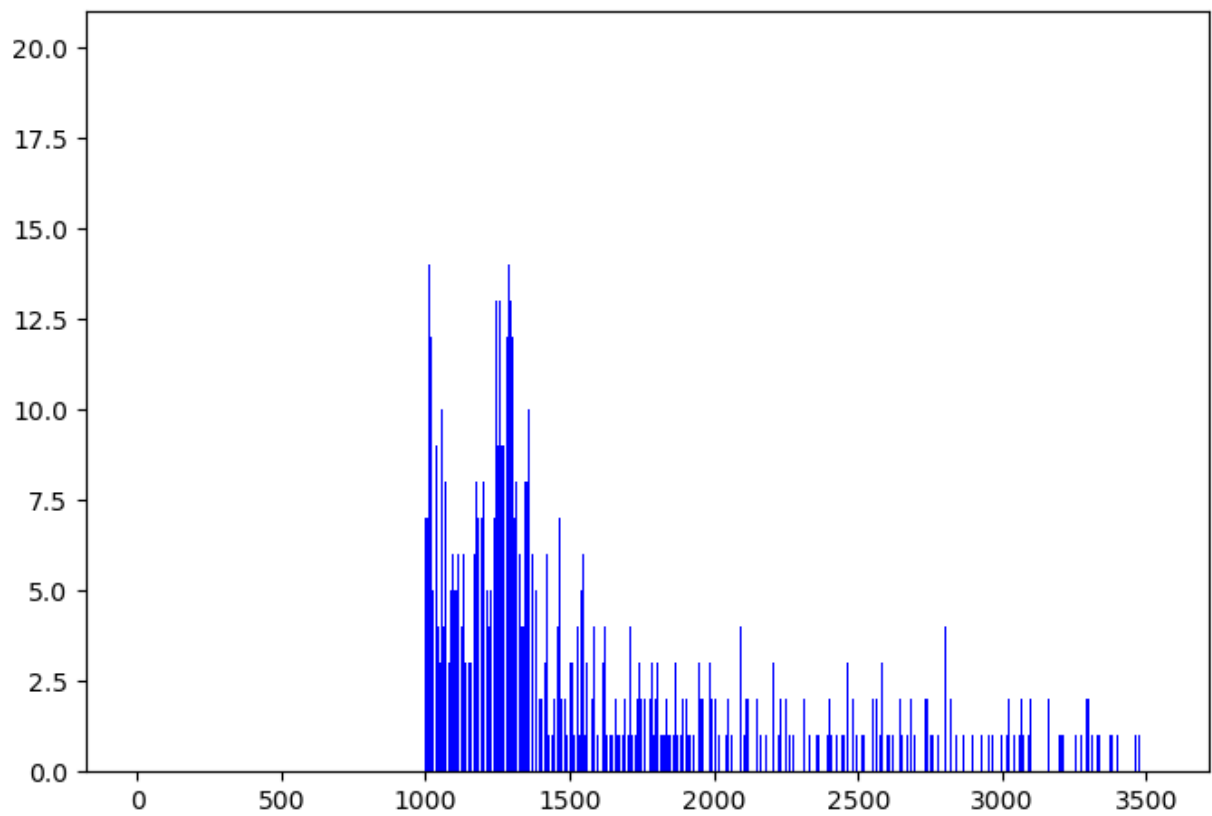


BA with  $N = 5000$  |  $k = 1000$  |  $m = 1000$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

3.43 +/- 0.03

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Local Attachment

```
In [13]: G = randomnet.local_attachment_graph(N,m,r)
```



In [215...

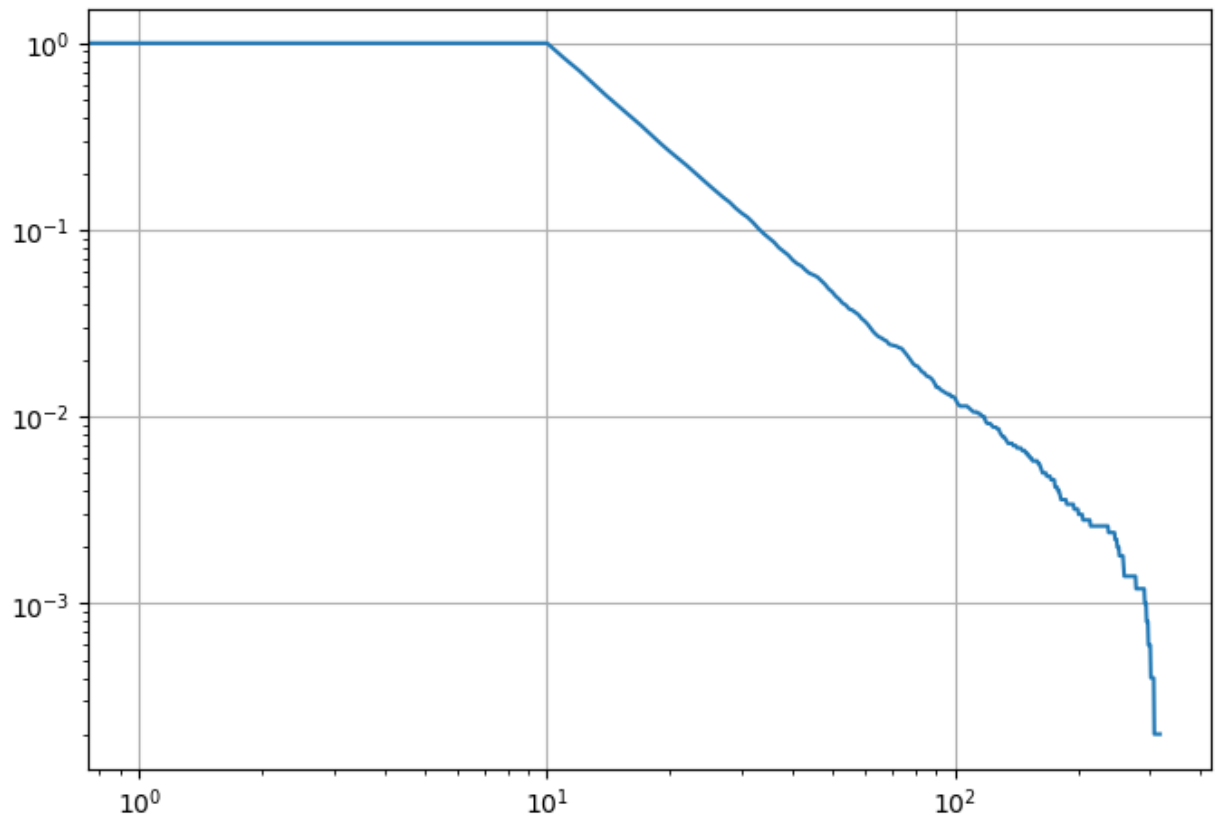
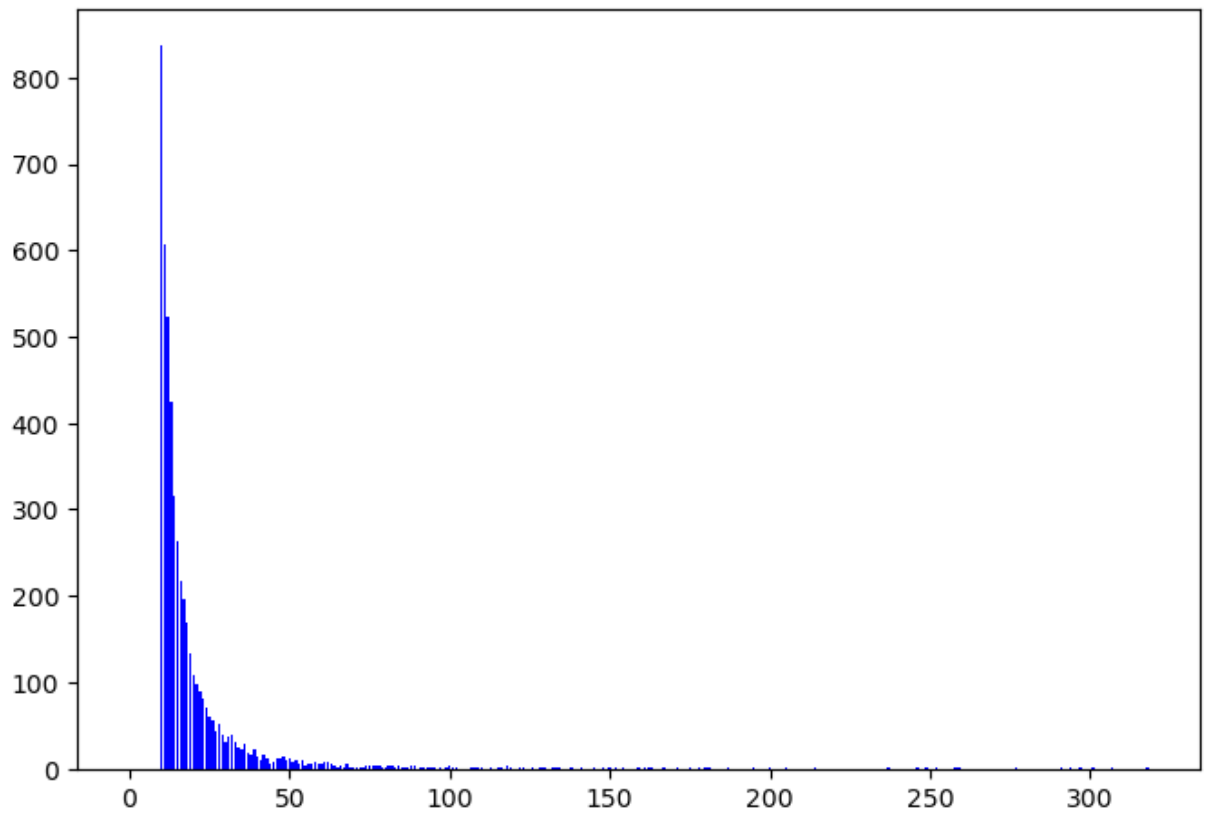
```
m = [10, 100, 1000]
r = [5, 50, 500]
kmin = [15, 120, 1000]
for mi, kmini, ri in zip(m, kmin, r):
    print(f'LAG with N = {N} | k = {kmini} | m = {mi} | r = {ri}')
    LAG = randomnet.local_attachment_graph(N,mi,ri)
    calc_powerlaw(LAG, kmini)
    plt.show()
```

LAG with N = 5000 | k = 15 | m = 10 | r = 5

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.87 +/- 0.04

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

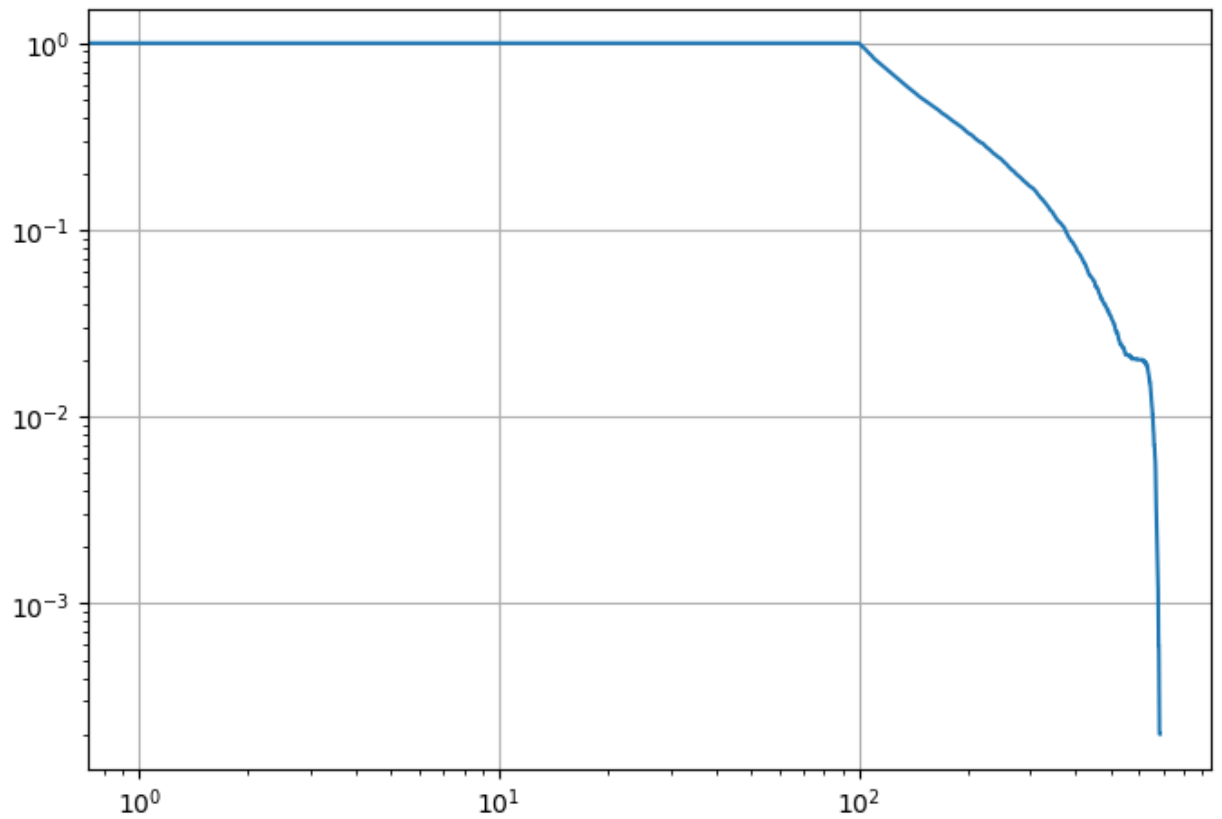
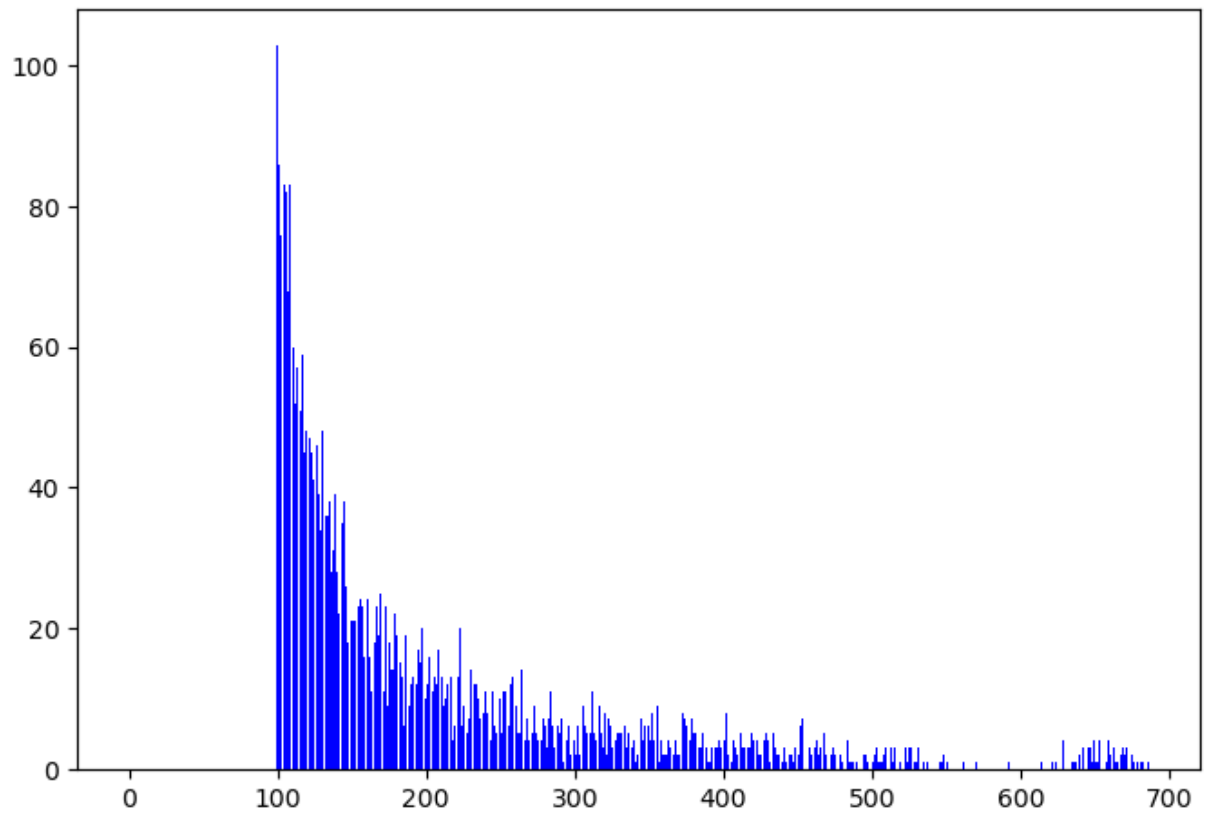


LAG with  $N = 5000$  |  $k = 120$  |  $m = 100$  |  $r = 50$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.76 +/- 0.03

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

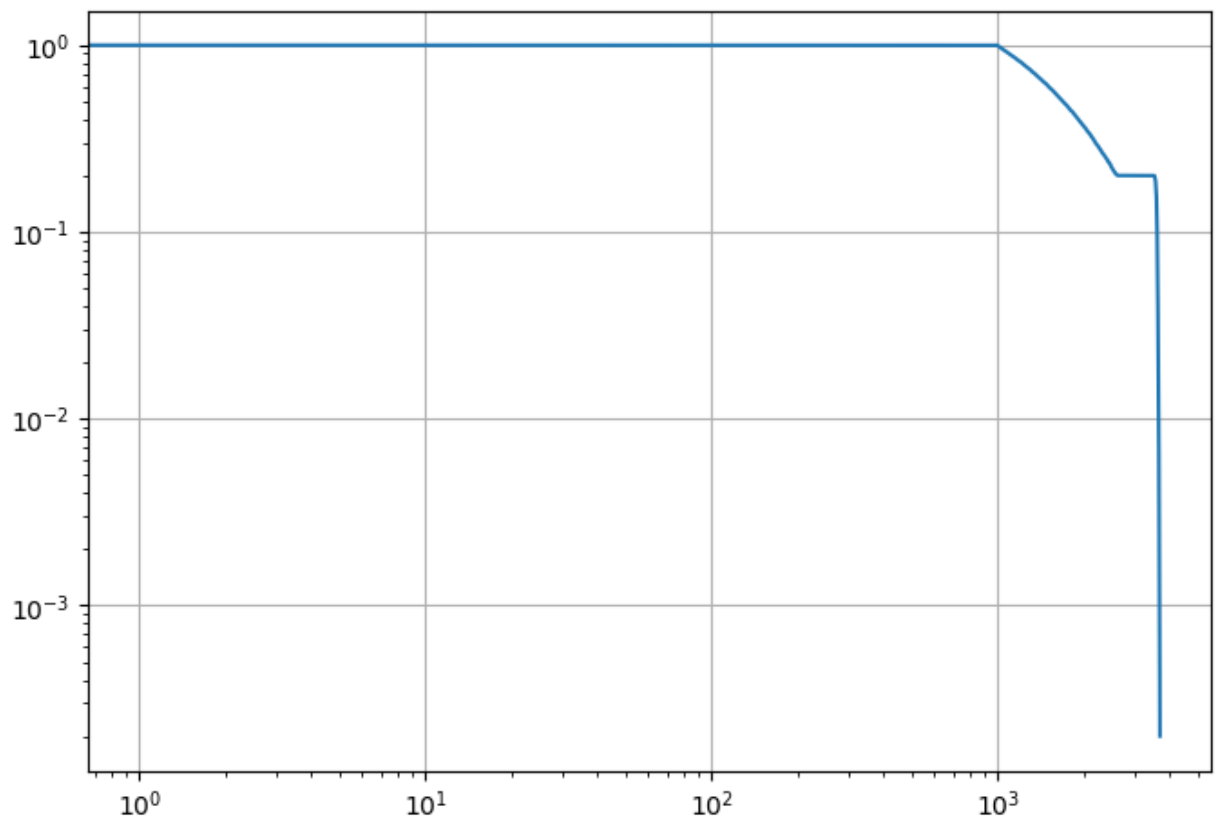
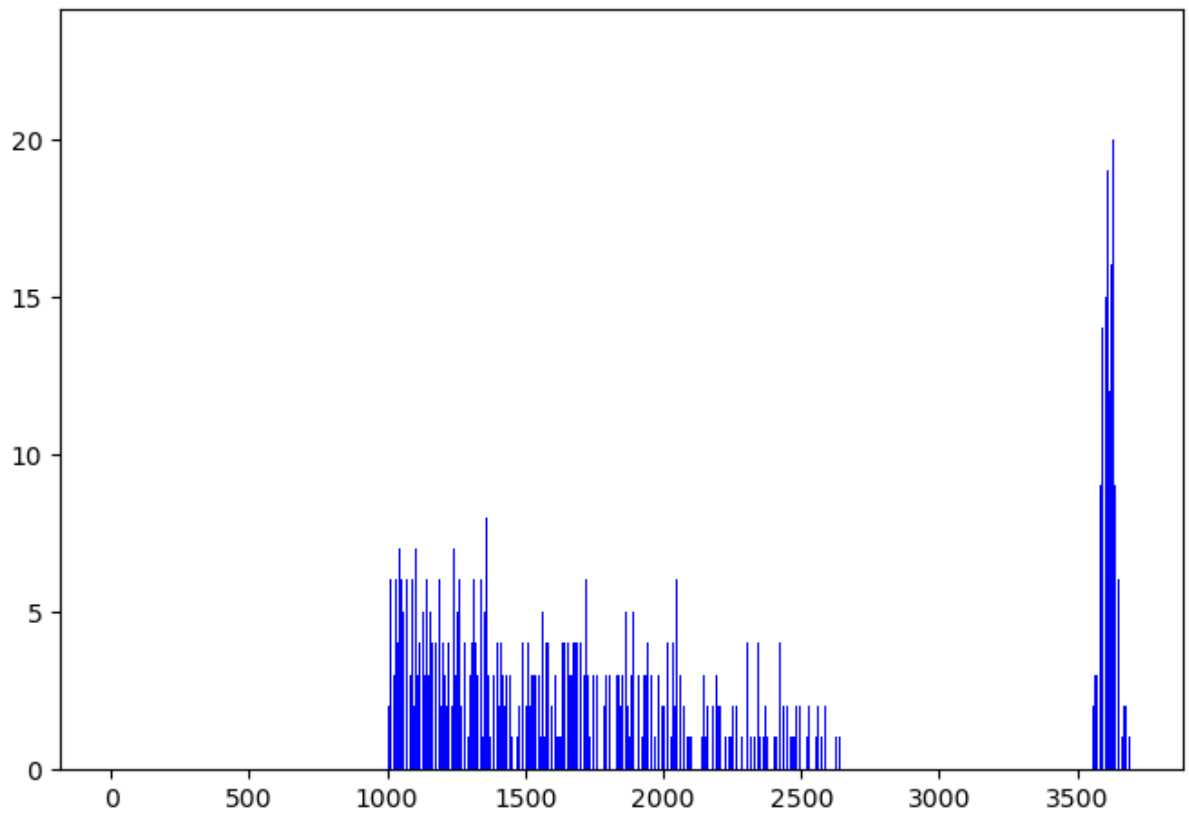


LAG with  $N = 5000$  |  $k = 1000$  |  $m = 1000$  |  $r = 500$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.66 +/- 0.02

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Duplication Divergence

```
In [14]: G = nx.duplication_divergence_graph(N,s)
```

In [218...

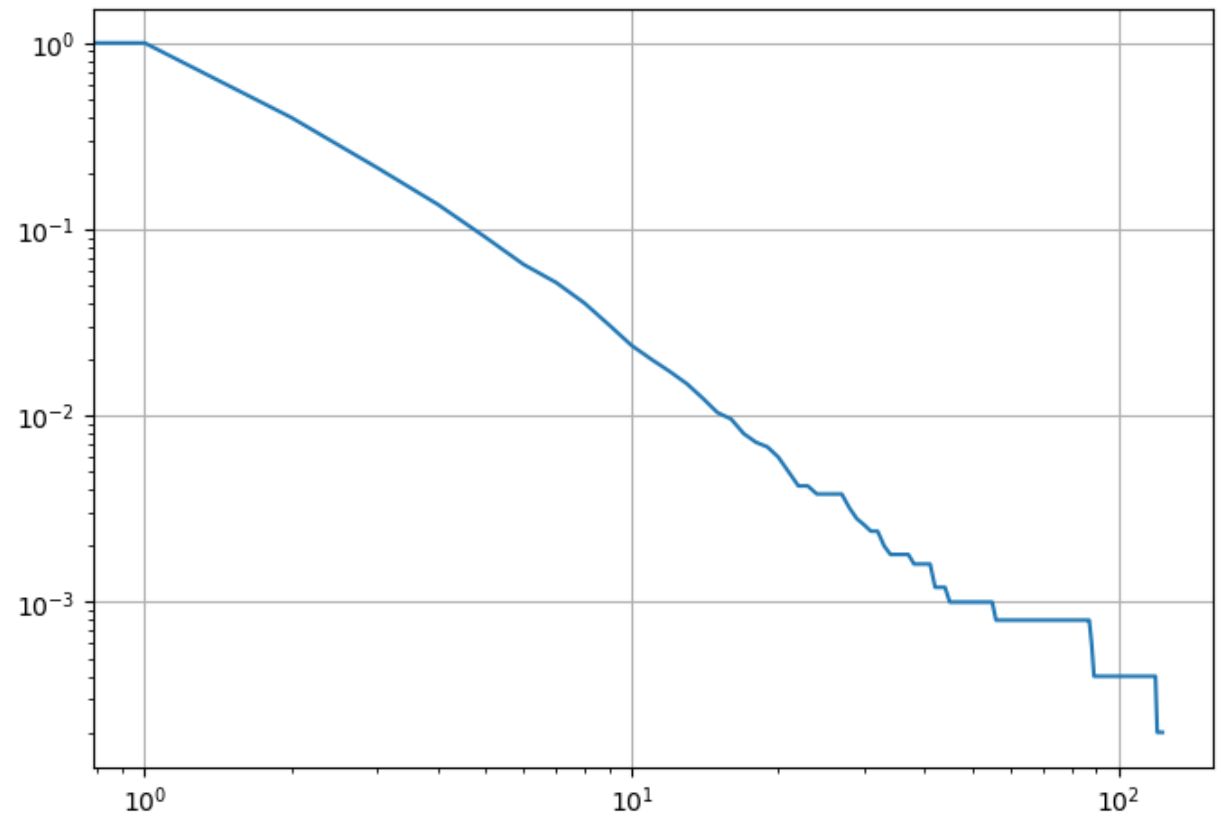
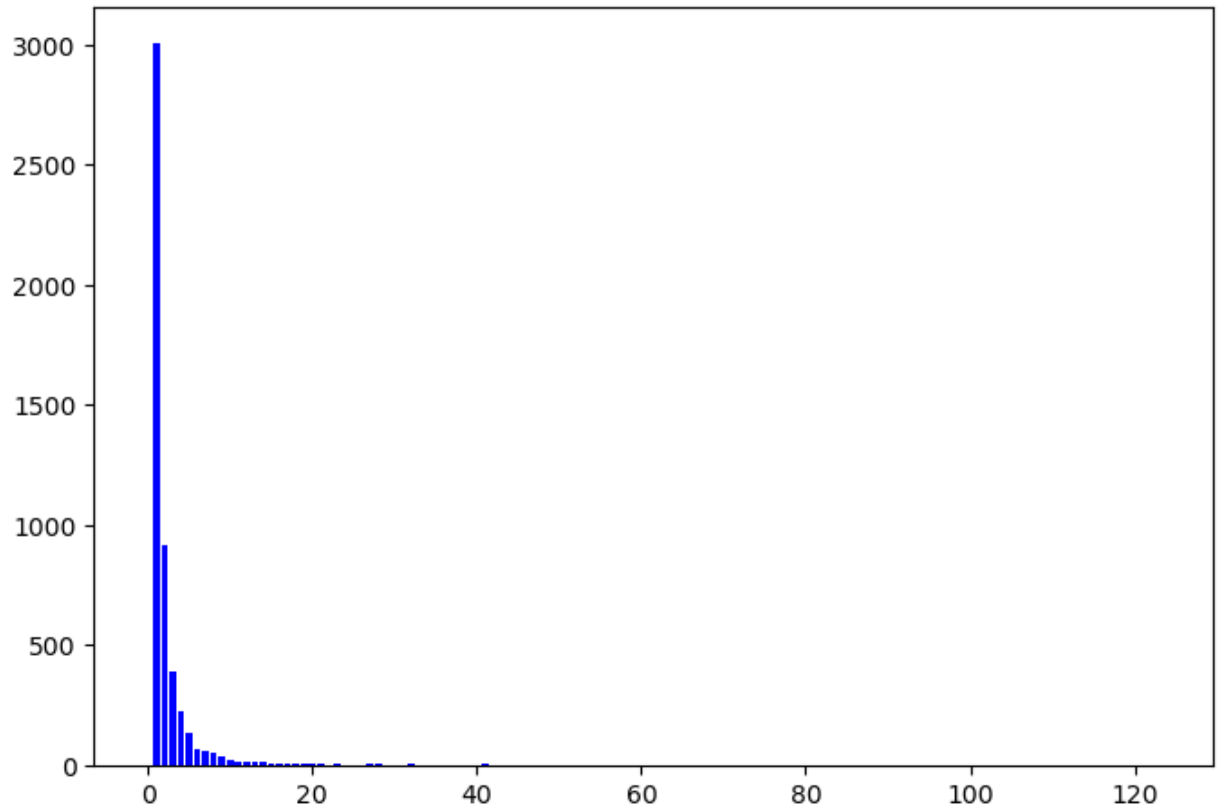
```
p = [0.05, 0.2, 0.4]
kmin = [5, 10, 50]
for pi, kmini in zip(p, kmin):
    print(f'BA with N = {N} | p = {pi} | k = {kmini}')
    DDG = nx.duplication_divergence_graph(N, pi)
    calc_powerlaw(DDG, kmini)
    plt.show()
```

BA with N = 5000 | p = 0.05 | k = 5

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.74 +/- 0.08

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



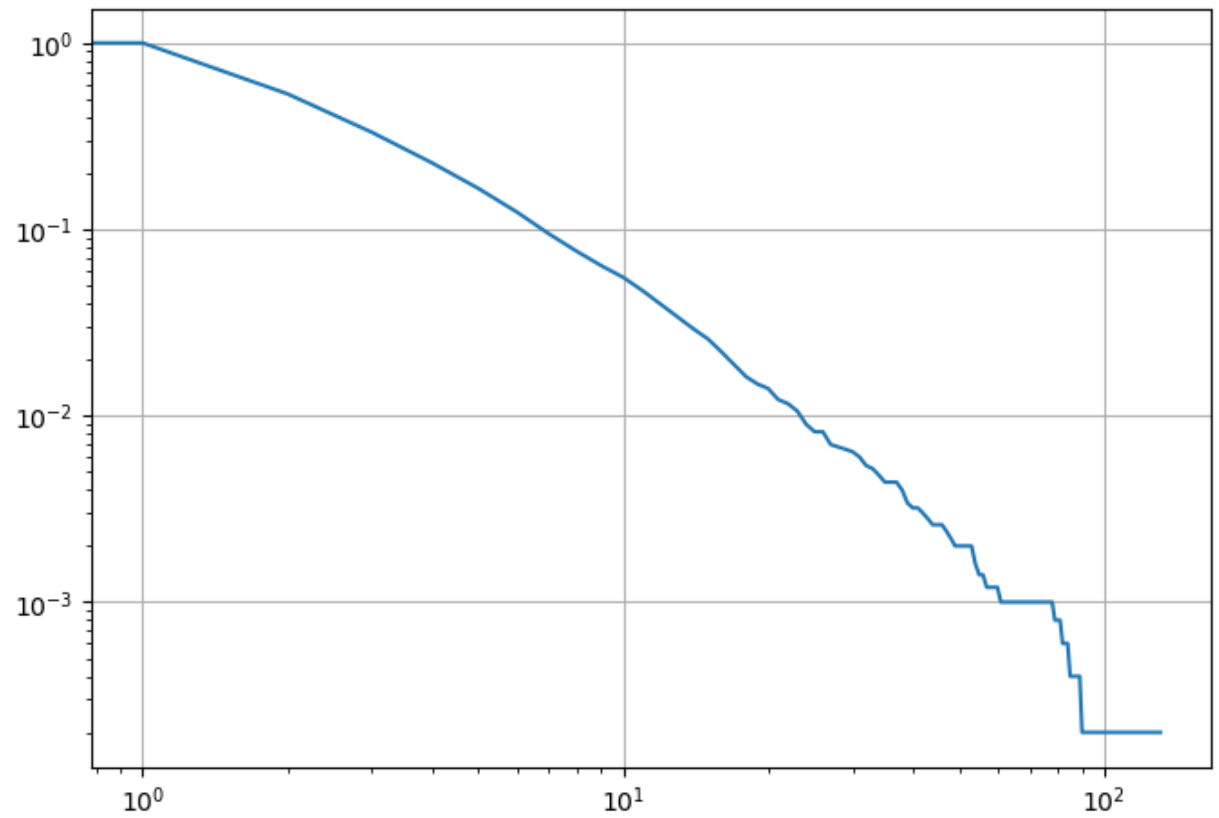
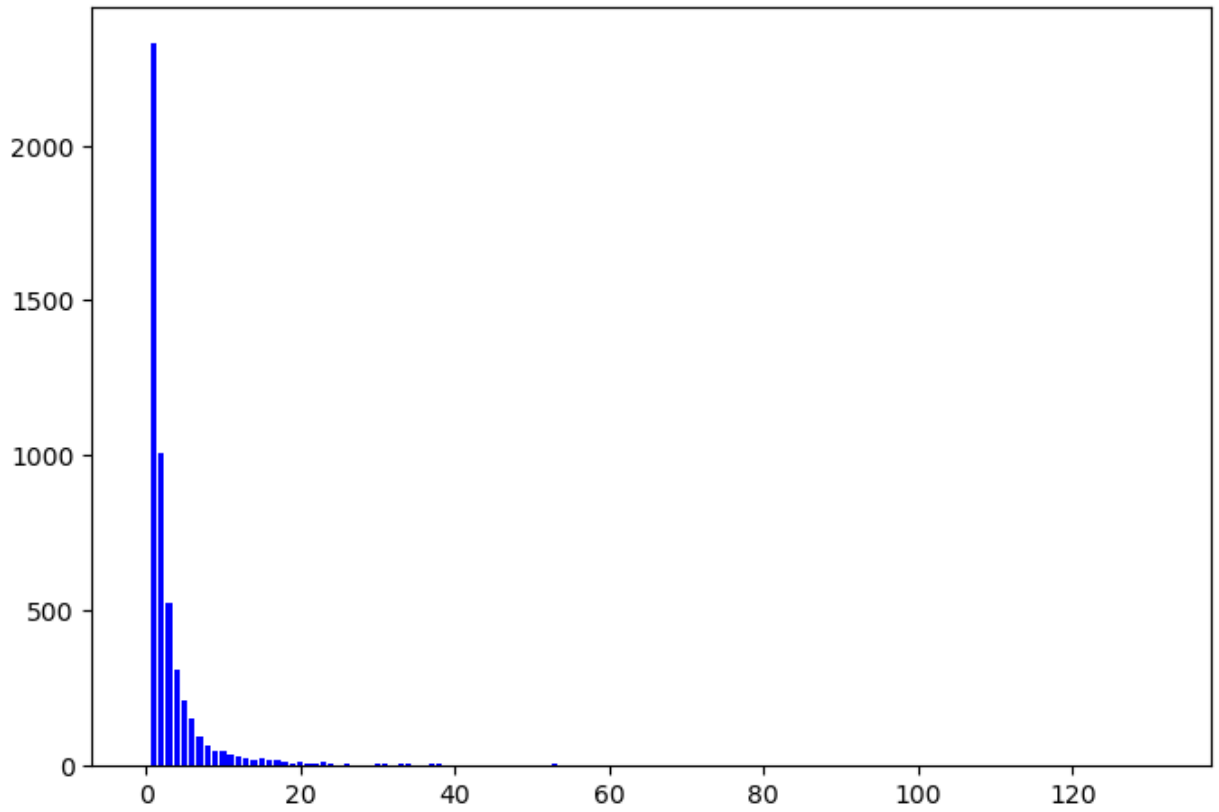


BA with  $N = 5000$  |  $p = 0.2$  |  $k = 10$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.93 +/- 0.12

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should

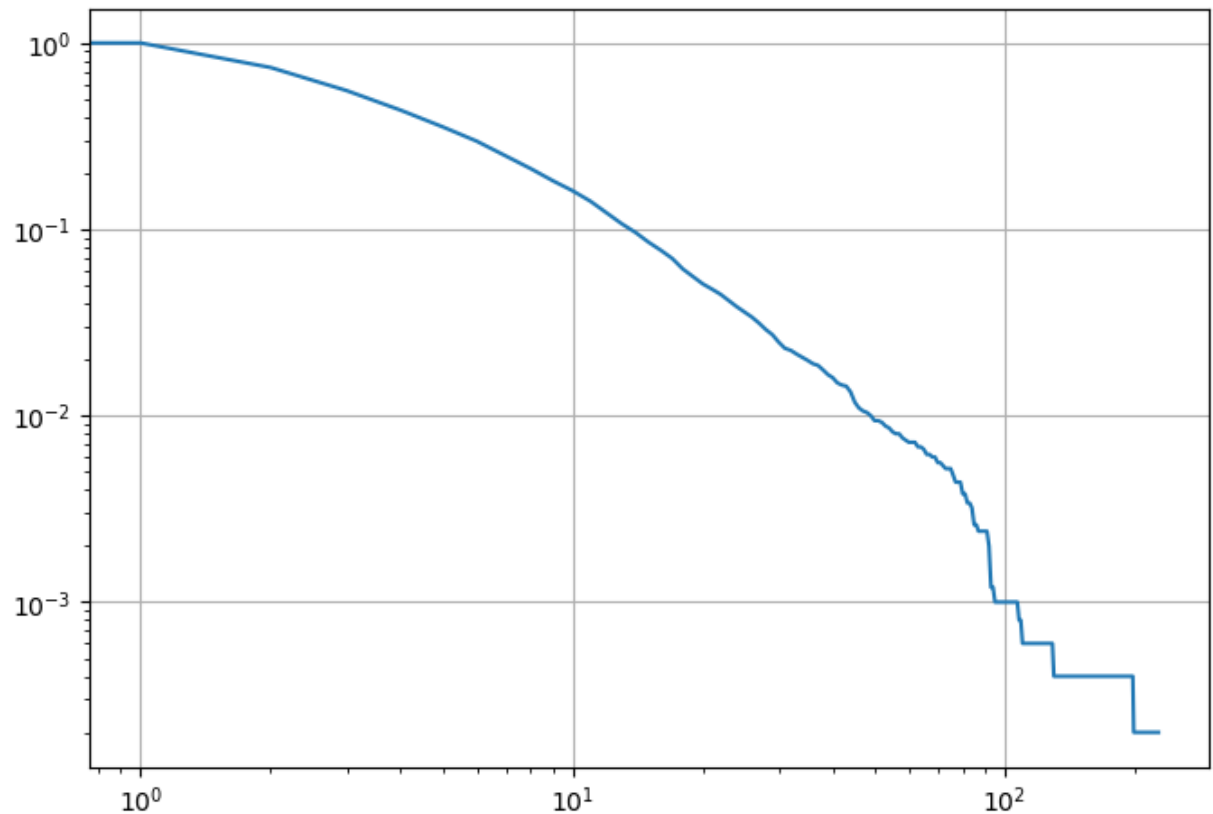
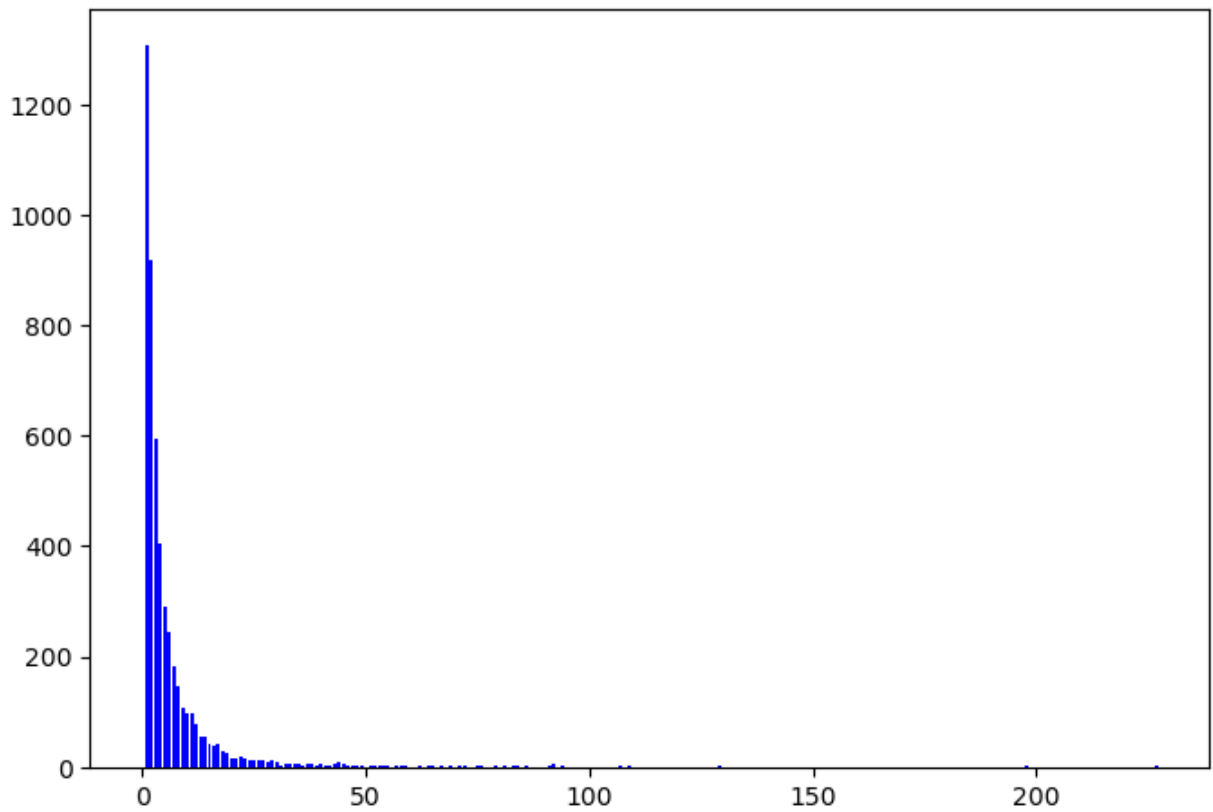


BA with  $N = 5000$  |  $p = 0.4$  |  $k = 50$

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

3.29 +/- 0.33

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



```
In [219... print('HIGHLIGHTED QUESTION - What are their power-law exponents?')
print('The following models exhibit scale-free distributions:')
print('These models\' power law exponents vary given their parameters, but here is an
print('Barabasi_Albert_graph with an alpha value of about 3')
```

```
print('Local_Attachment_graph with an alpha value of about 2.6 to 2.8')
print('Divergence_graph with an alpha value of about 3')
```

HIGHLIGHTED QUESTION - What are their power-law exponents?

The following models exhibit scale-free distributions:

These models' power law exponents vary given their parameters, but here is an approximate value

Barabasi\_Albert\_graph with an alpha value of about 3

Local\_Attachment\_graph with an alpha value of about 2.6 to 2.8

Divergence\_graph with an alpha value of about 3

## Fitting Random Models

In [238...

```
CA_G = nx.read_weighted_edgelist('ca-HepTh.edgelist')
n = CA_G.number_of_nodes()
m = CA_G.number_of_edges()
ds = degree_sequence(CA_G)
cc = nx.average_clustering(CA_G)
print(f'Number of Nodes: {n}')
print(f'Number of Edges: {m}')
print(f'Average Degree: {np.sum(ds)/n}')
print(f'Clustering Coefficient: {cc}')
calc_powerlaw(CA_G, 45)
plt.show()
print('We will be comparing our ER, LA, and DD networks to the statistics just printed')
```

Number of Nodes: 9877

Number of Edges: 25998

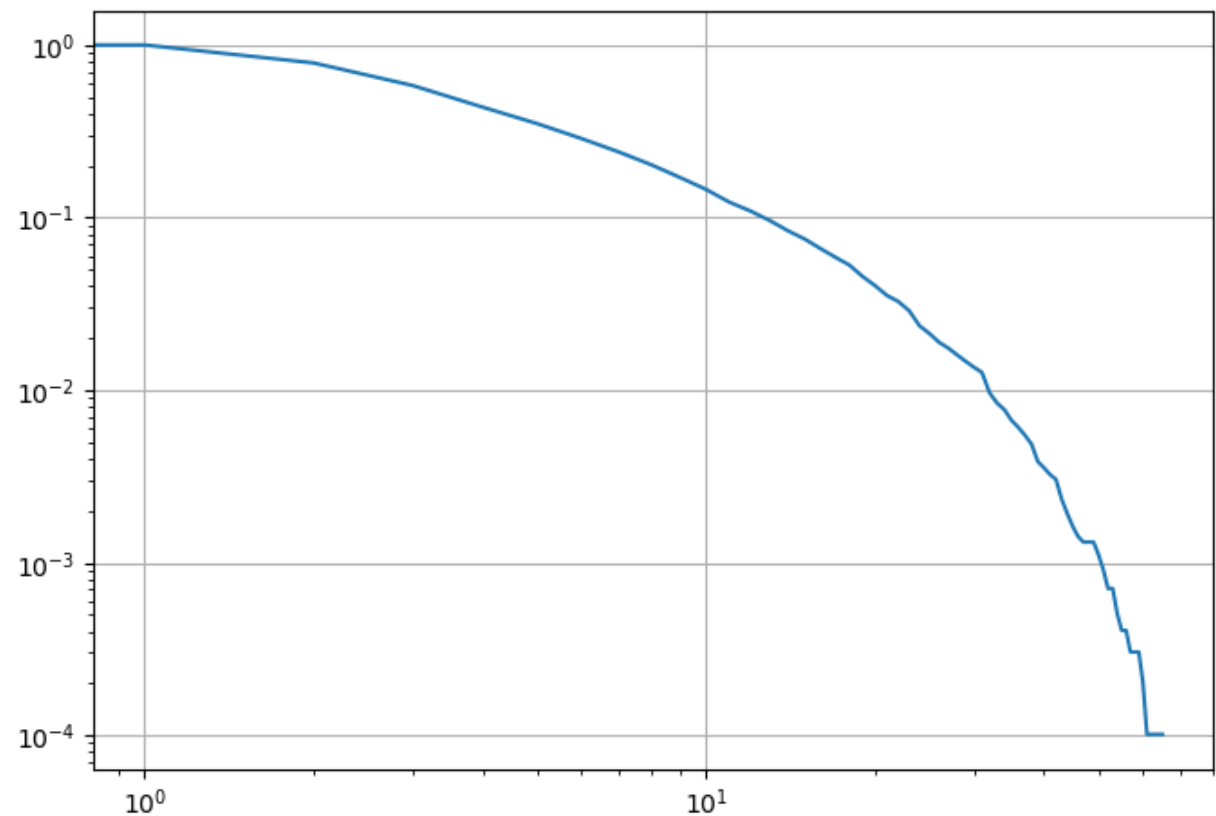
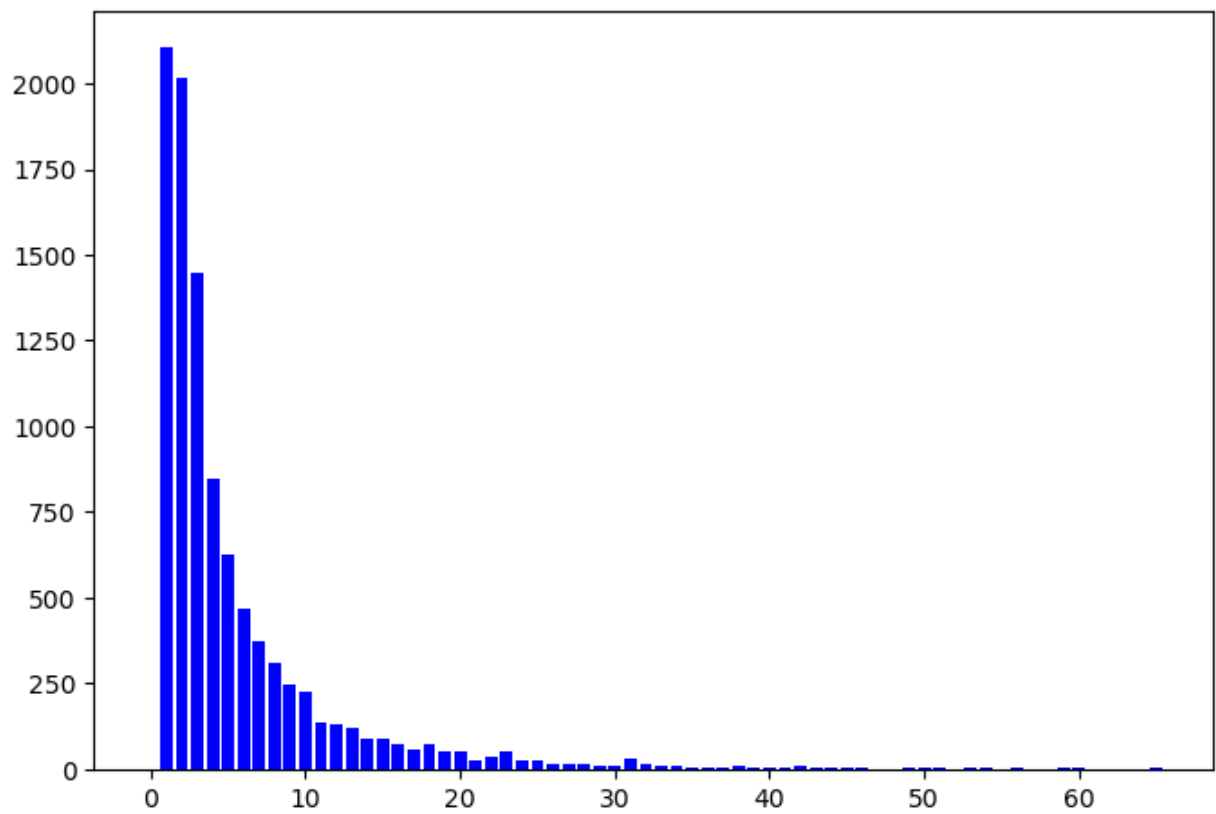
Average Degree: 5.264351523742027

Clustering Coefficient: 0.4714390529669332

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

7.43 +/- 1.61

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



We will be comparing our ER, LA, and DD networks to the statistics just printed

In [235...

```
ds = degree_sequence(CA_G)
print(np.sum(ds)/n)
```

5.264351523742027

In [239...

```
CA_ERG = nx.erdos_renyi_graph(n, m/(n*(n-1)/2))
print('Erdos Renyi ca-Hep tuning')
print(f'Number of Nodes: {CA_ERG.number_of_nodes()}')
print(f'Number of Edges: {CA_ERG.number_of_edges()}')
print(f'Average Degree: {np.sum(degree_sequence(CA_ERG))/n}')
print(f'Clustering Coefficient: {nx.average_clustering(CA_ERG)}')
```

Erdos Renyi ca-Hep tuning  
Number of Nodes: 9877  
Number of Edges: 26185  
Average Degree: 5.302217272451149  
Clustering Coefficient: 0.0003998993368235037

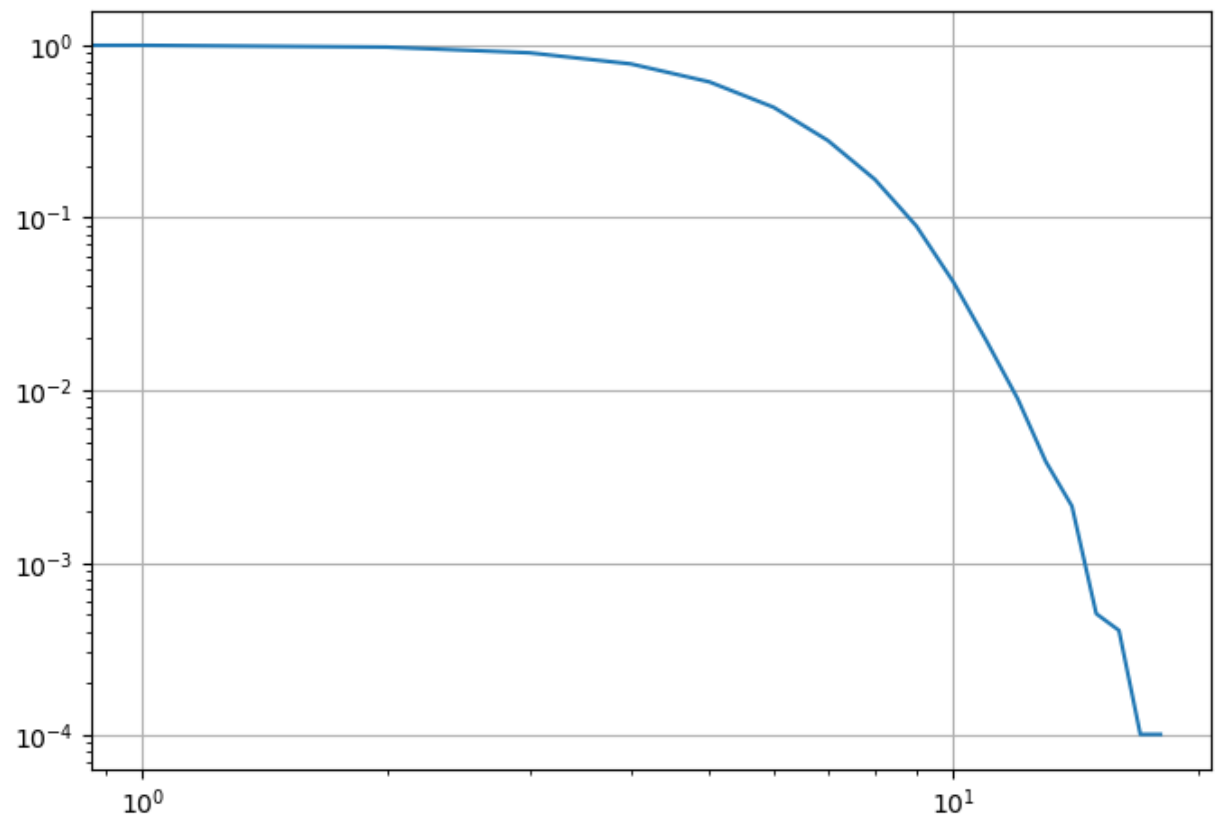
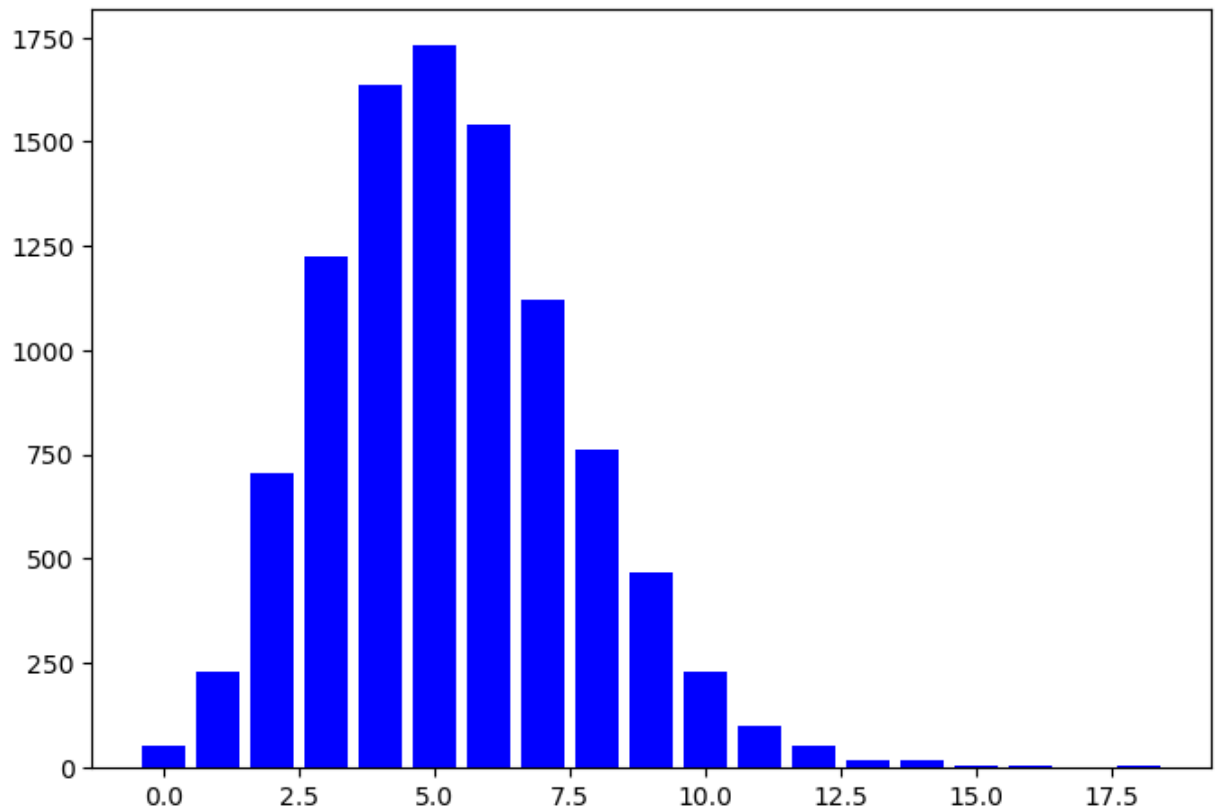
In [240...

```
calc_powerlaw(CA_ERG, 10)
plt.show()
```

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

8.98 +/- 0.39

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



```
In [279... CA_LA = randomnet.local_attachment_graph(n,3,1)
print('Local Attachment ca-Hep tuning')
print(f'Number of Nodes: {CA_LA.number_of_nodes()}')
print(f'Number of Edges: {CA_LA.number_of_edges()}')
```



```
print(f'Average Degree: {np.sum(degree_sequence(CA_LA))/n}')  
print(f'Clustering Coefficient: {nx.average_clustering(CA_LA)}')
```

Local Attachment ca-Hep tuning  
Number of Nodes: 9877  
Number of Edges: 29631  
Average Degree: 6.0  
Clustering Coefficient: 0.4110777725194516

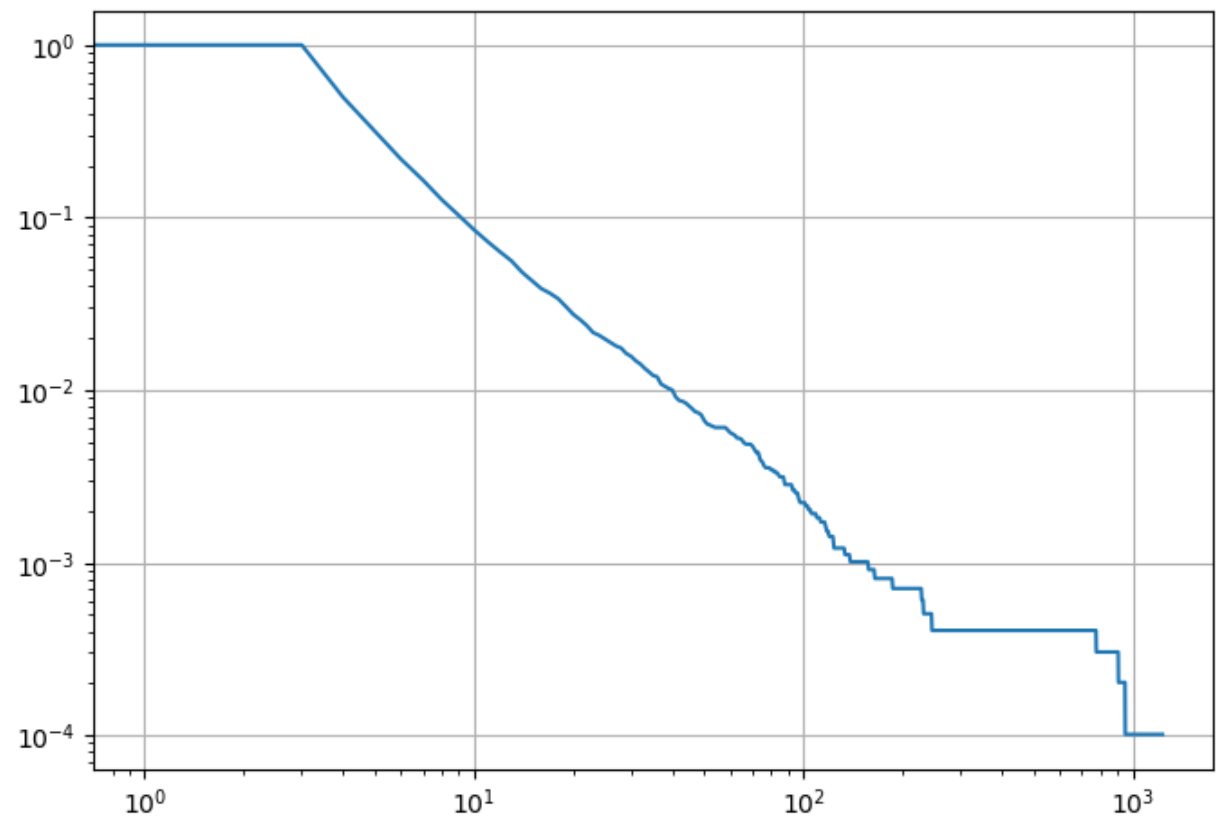
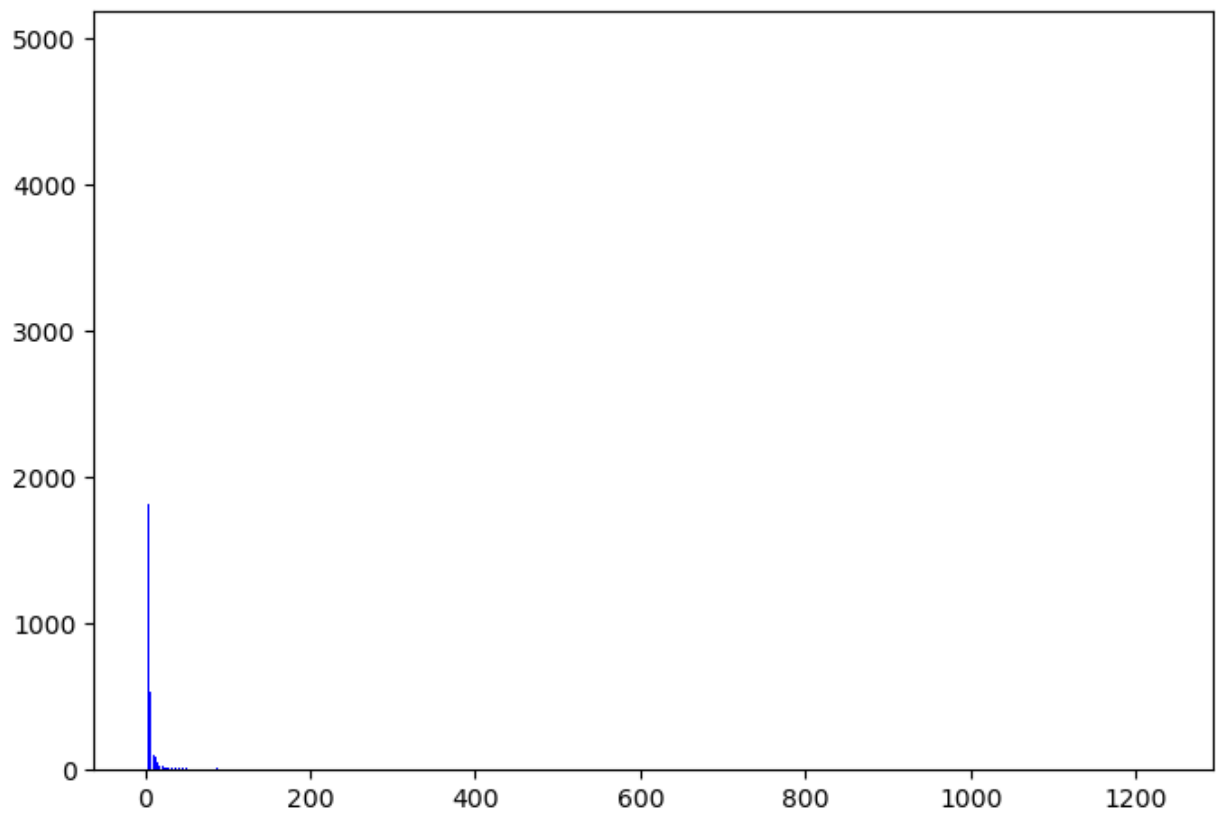
In [262...

```
calc_powerlaw(CA_LA, 10)  
plt.show()
```

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.53 +/- 0.05

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



In [265...

```
CA_DD = nx.duplication_divergence_graph(n, 0.35)
print('Duplication Divergence ca-Hep tuning')
print(f'Number of Nodes: {CA_DD.number_of_nodes()}')
print(f'Number of Edges: {CA_DD.number_of_edges()}')
```

```
print(f'Average Degree: {np.sum(degree_sequence(CA_DD))/n}')  
print(f'Clustering Coefficient: {nx.average_clustering(CA_DD)}')
```

Duplication Divergence ca-Hep tuning  
Number of Nodes: 9877  
Number of Edges: 26696  
Average Degree: 5.405689986838109  
Clustering Coefficient: 0.0

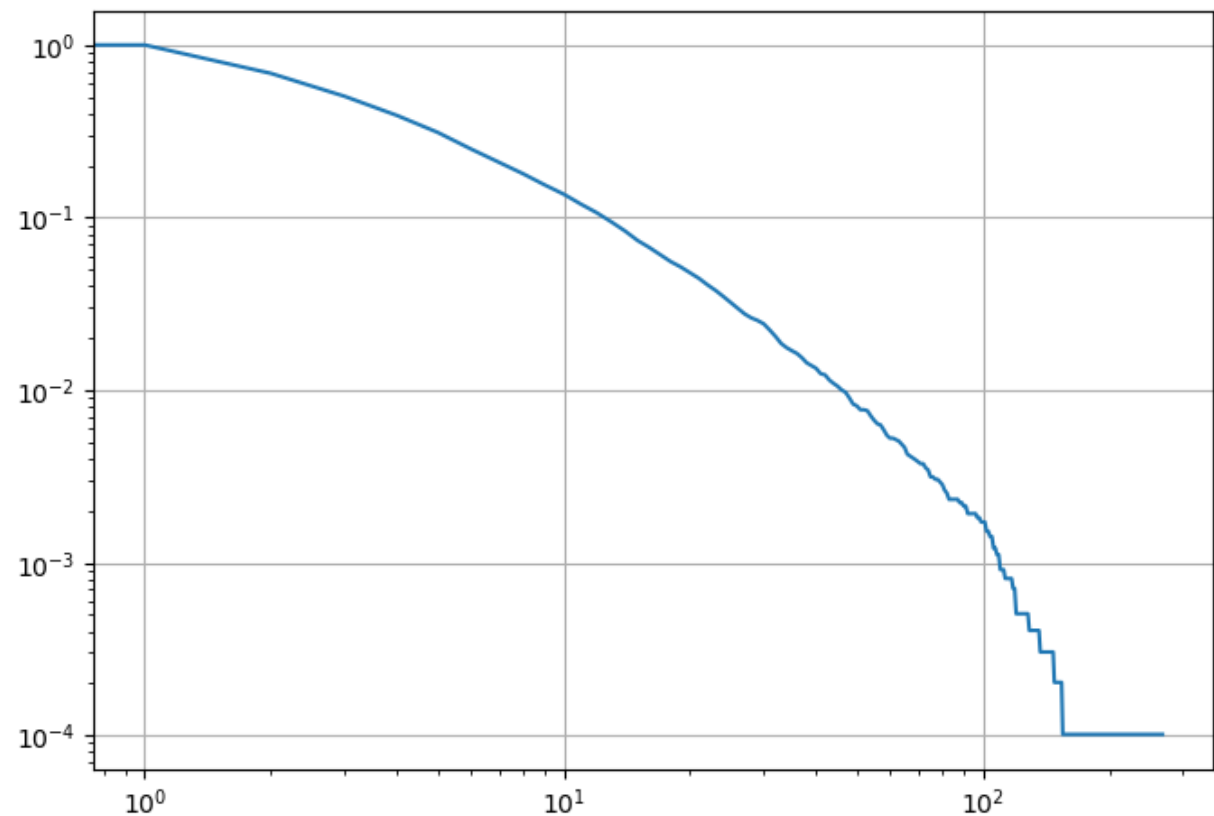
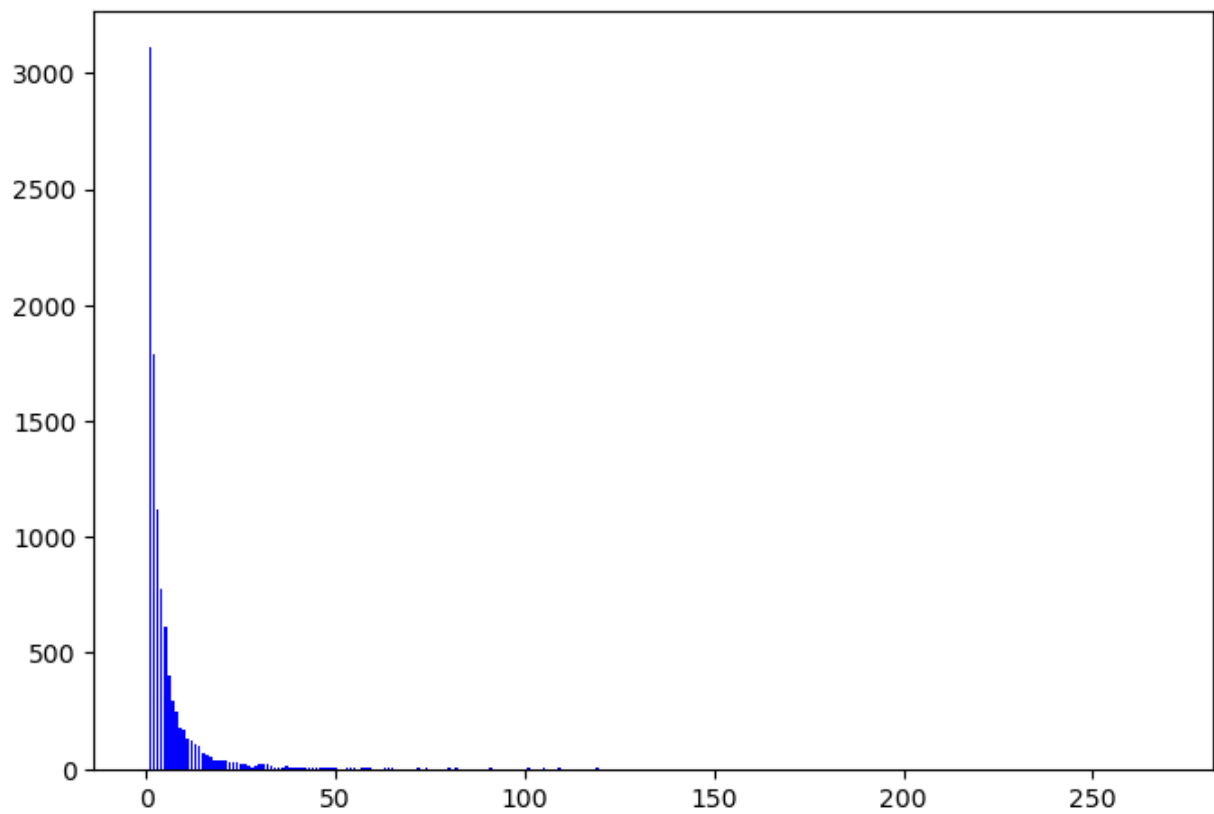
In [267...

```
calc_powerlaw(CA_DD, 10)  
plt.show()
```

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.57 +/- 0.04

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



```
In [288... CA_CM = nx.configuration_model(ds, create_using=nx.Graph())
print('Configuration Model ca-Hep tuning')
print(f'Number of Nodes: {CA_CM.number_of_nodes()}')
print(f'Number of Edges: {CA_CM.number_of_edges()}')
```

```
print(f'Average Degree: {np.sum(degree_sequence(CA_CM))/n}')  
print(f'Clustering Coefficient: {nx.average_clustering(CA_CM)}')
```

Configuration Model ca-Hep tuning  
Number of Nodes: 9877  
Number of Edges: 25973  
Average Degree: 5.259289257871823  
Clustering Coefficient: 0.0014703018933473328

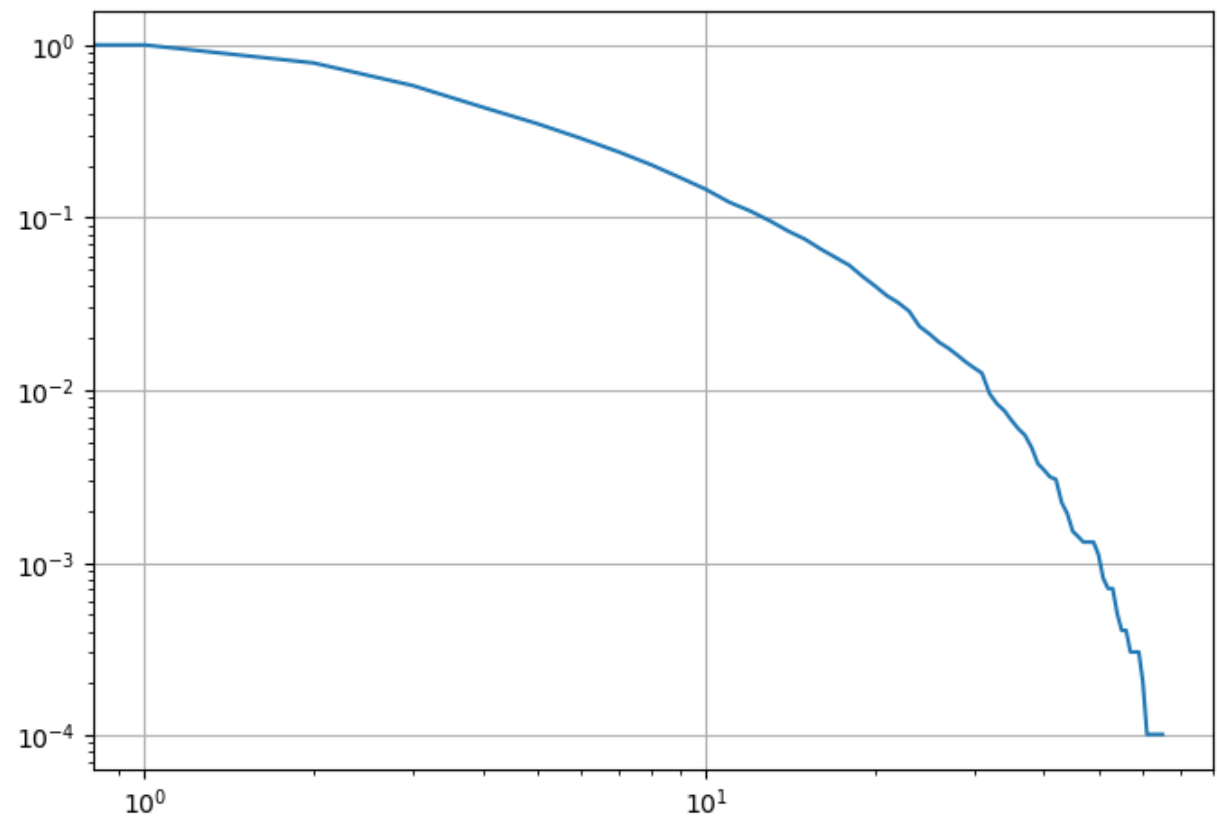
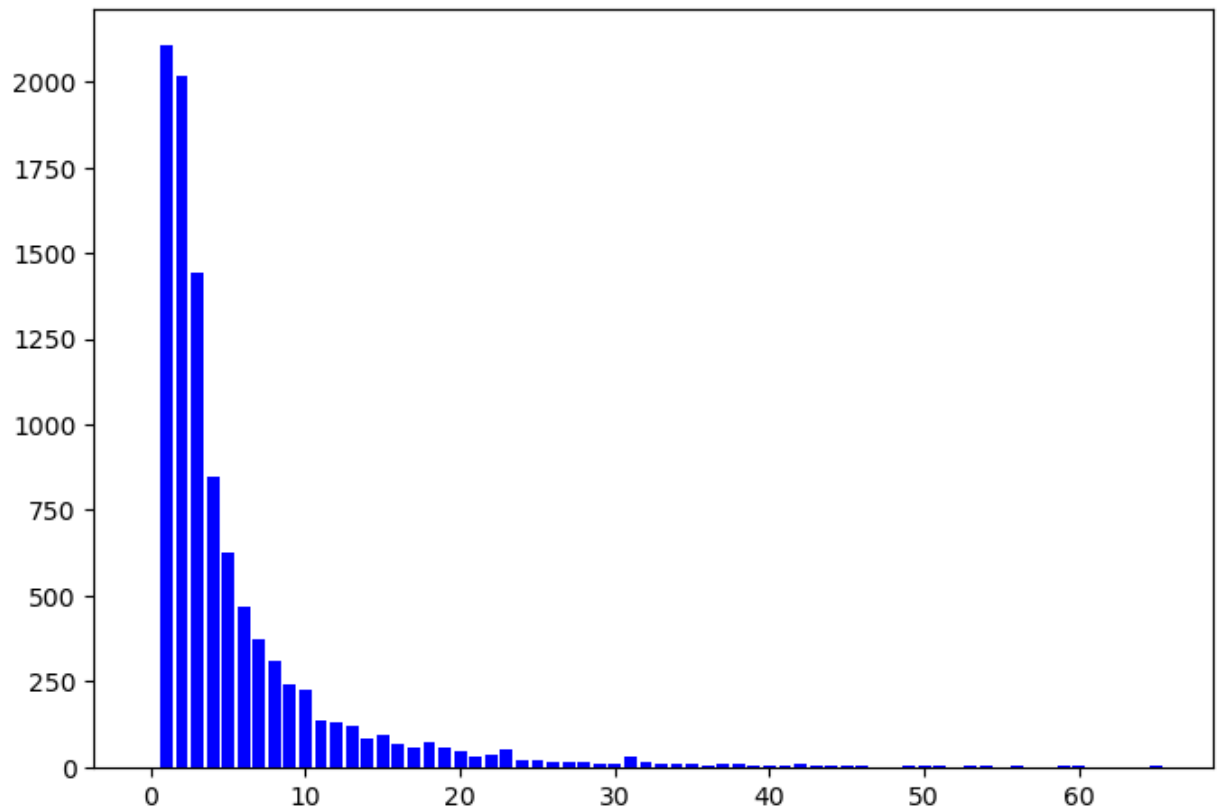
In [289...

```
calc_powerlaw(CA_CM, 10)  
plt.show()
```

HIGHLIGHTED QUESTION - calculate  $\alpha$  and the corresponding uncertainty,  $\sigma$  for each of the networks.

2.94 +/- 0.05

HIGHLIGHTED QUESTION - Use these functions to create two plots - one of the degree distribution, which will be a bar plot, and one of the cumulative degree distribution, which should



## Sections 13.2-13.4 Configuration Model

```
In [310... G = nx.read_weighted_edgelist('texas_road_sample.edgelist')
```

```
In [312... print('HIGHLIGHTED QUESTION - compute the average degree of the network and compare it
print('Texas Road Sample')
avg_degree = np.sum(degree_sequence(G)) / G.number_of_nodes()
print(f'Average Degree of Network: {avg_degree}')
average_degree_neighbor = list(dict(nx.average_neighbor_degree(G)).values())
# print(average_degree_neighbor)
mean = sum(average_degree_neighbor) / len(average_degree_neighbor)
print(f'Mean of Average Degrees of Neighbors: {mean}')
```

HIGHLIGHTED QUESTION - compute the average degree of the network and compare it to the average degree of neighbor nodes.

Texas Road Sample

Average Degree of Network: 2.498556304138595

Mean of Average Degrees of Neighbors: 2.973788899582932

```
In [313... G = nx.read_weighted_edgelist('international_airports.edgelist')
print('HIGHLIGHTED QUESTION - compute the average degree of the network and compare it
print('International Airports Sample Sample')
avg_degree = np.sum(degree_sequence(G)) / G.number_of_nodes()
print(f'Average Degree of Network: {avg_degree}')
average_degree_neighbor = list(dict(nx.average_neighbor_degree(G)).values())
# print(average_degree_neighbor)
mean = sum(average_degree_neighbor) / len(average_degree_neighbor)
print(f'Mean of Average Degrees of Neighbors: {mean}')
```

HIGHLIGHTED QUESTION - compute the average degree of the network and compare it to the average degree of neighbor nodes.

International Airports Sample Sample

Average Degree of Network: 10.668254508336169

Mean of Average Degrees of Neighbors: 45.069840918826

```
In [314... print('HIGHLIGHTED QUESTION - Explain why we see this effect much more strongly in one
print('If we think about the texas road sample, we realize that this is similar a square
```

HIGHLIGHTED QUESTION - Explain why we see this effect much more strongly in one network than the other.

If we think about the texas road sample, we realize that this is similar a square grid network, so at most any node is connected to 4 other nodes (going forward, taking a left, a right, or a u-turn). So, it makes sense that the average degree of the network would be similar to the average mean degree of its neighbors. It is because of the design of roads can only have them connect to so many intersections at a given time. However, International Airports are not constrained by petty 2 dimensional grids. Rather, there can be multiple hubs an airplane can fly to. So, we might see that there are some small international airports that do not connect to many other places international airports. However, large hubs such as DFW, LaGuardia, or JFK are centrally located and have access and power to fly to many other airports, hence they are also connected to these smaller airports. So, when these smaller airports are connected to the big-degree airports then the mean average degree of neighbors will be greater than the average degree of the network. It is because these big-degree networks are connected to a significant portion of other small-degree networks.

```
In [ ]:
```