(1)

Laplace Transform

Definition:

Let $f(t)$ be a given function that is defined for all $t \geq 0$ we multiplying $f(t)$ by e^{-st} and integerate with respect to t from "0" to "infinity (∞)". Then if the resulting integral exists (that is has some finit value). it is the function of s, say $F(s)$.

$$\int_0^\infty f(t) \cdot e^{-st} dt = F(s)$$

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^\infty f(t) \cdot e^{-st} dt = F(s)}$$

$$\boxed{\mathcal{L}\{f(t)\} = F(s)}$$

$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\}}$$

Ex1 Let $f(t) = 1$ when $t \geq 0$

Find $F(s)$.

Sol:

$$F(s) = \mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{1\} = \int_0^\infty 1\, e^{-st}\, dt$$

$$= \left| \frac{e^{-st}}{-s} \right|_0^\infty$$

$$= -\frac{1}{s}[0 - 1]$$

$$= \frac{1}{s}$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

Ex2

(i) $\mathcal{L}\{t\} = \int_0^\infty t\, e^{-st}\, dt$

$$= t\, \frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} \cdot 1\, dt$$

$$= -\frac{1}{s}[0 - 0] + \frac{1}{s}\int_0^\infty e^{-st}\, dt$$

$$= \frac{1}{s}\mathcal{L}\{1\} = \frac{1}{s}\cdot\frac{1}{s} = \frac{1}{s^2}$$

$$\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}}$$

$$\boxed{\mathcal{L}\{t^2\} = \frac{2}{s^3}}$$

(ii) $\mathcal{L}\{t^2\} = \int_0^\infty e^{-st} \cdot t^2\, dt$

$$= t^2\, \frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} \cdot 2t\, dt$$

$$= -\frac{1}{s}(0 - 0) + \frac{2}{s}\int_0^\infty t\, e^{-st}\, dt$$

$$= \frac{2}{s}\mathcal{L}\{t\} = \frac{2}{s}\cdot\frac{1}{s^2}$$

iii, $\mathcal{L}\{t^3\} = \int_0^\infty t^3 e^{-st}\,dt$

$= t^3 \frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} \cdot 3t^2\,dt$

$\frac{1}{e^\infty}$

$\frac{1}{\infty}$

$= 0$

$= -\frac{1}{s}(0-0) + \frac{3}{s}\mathcal{L}\{t^2\}$

$= \frac{3}{s}\frac{2}{s^3} = \frac{3!}{s^4}$

$\int x^3\,dx$

$\Big| \frac{x^4}{4}\Big)$ $\int t\sin x\,dx = \text{- - - -}$

Similarly

$\boxed{\mathcal{L}\{t^4\} = \frac{4!}{s^5}}$

$\boxed{\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}}$