Ex 5.7

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Q1 $y_1' = -y_1 + y_2$ —— (1) $\quad y_1(0) = 1$

$y_2' = -y_1 - y_2$ —— (2) $\quad y_2(0) = 0$

Sol: eq (1) $\Rightarrow$

$$\mathcal{L}\{y_1'\} = -\mathcal{L}\{y_1\} + \mathcal{L}\{y_2\}$$

$$s\mathcal{L}\{y_1\} - y_1(0) = -Y_1 + Y_2$$

$$sY_1 - 1 = -Y_1 + Y_2$$

$$(s+1)Y_1 - Y_2 = 1$$

$$(s+1)Y_1 - Y_2 - 1 = 0 \quad \text{—— (3)}$$

eq (2) $\Rightarrow$

$$\mathcal{L}\{y_2'\} = -\mathcal{L}\{y_1\} - \mathcal{L}\{y_2\}$$

$$s\mathcal{L}\{y_2\} - y_2(0) = -Y_1 - Y_2$$

$$sY_2 - 0 = -Y_1 - Y_2$$

$$Y_1 + (s+1)Y_2 = 0$$

$$Y_1 + (s+1)Y_2 - 0 = 0 \quad \text{—— (4)}$$

$$(s+1)Y_1 - Y_2 - 1 = 0 \quad \text{—— (3)}$$

$$1\,Y_1 + (s+1)Y_2 - 0 = 0 \quad \text{—— (4)}$$

$$\frac{Y_1}{[0 + (s+1)]} = \frac{Y_2}{-[0+1]} = \frac{1}{(s+1)^2+1}$$

$$Y_1 = \frac{s+1}{(s+1)^2+1} \Rightarrow \mathcal{L}\{y_1\} = \frac{s+1}{(s+1)^2+1}$$

$$Y_2 = -\frac{1}{(s+1)^2+1} \Rightarrow \mathcal{L}\{y_2\} = -\frac{1}{(s+1)^2+1}$$

$$y_1 = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} = e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = e^{-t}\cos t$$

$$y_2 = -\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} = e^{-t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = e^{-t}\sin t$$