

Ex 5.7

$$\begin{aligned} \text{Q1 } y_1' &= -y_1 + y_2 \\ y_2' &= -y_1 - y_2 \end{aligned} \quad \begin{aligned} y_1(0) &= 1 \\ y_2(0) &= 0 \end{aligned}$$

① ②

Sol: $\Rightarrow 0 \Rightarrow$

$$\begin{aligned} \mathcal{L}\{y_1'\} &= -\mathcal{L}\{y_1\} + \mathcal{L}\{y_2\} \\ s\mathcal{L}\{y_1\} - y_1(0) &= -Y_1 + Y_2 \\ sY_1 - 1 &= -Y_1 + Y_2 \end{aligned}$$

①

$$\begin{aligned} (s+1)Y_1 - Y_2 &= 1 \\ (s+1)Y_1 - Y_2 - 1 &= 0 \end{aligned} \quad \text{③}$$

$\Rightarrow 0 \Rightarrow$

$$\begin{aligned} \mathcal{L}\{y_2'\} &= -\mathcal{L}\{y_1\} - \mathcal{L}\{y_2\} \\ s\mathcal{L}\{y_2\} - y_2(0) &= -Y_1 - Y_2 \\ sY_2 - 0 &= -Y_1 - Y_2 \\ Y_1 + (s+1)Y_2 &= 0 \end{aligned}$$

④

$$\begin{aligned} (s+1)Y_1 - Y_2 - 1 &= 0 \quad \text{③} \\ Y_1 + (s+1)Y_2 - 0 &= 0 \quad \text{④} \end{aligned}$$

$$\begin{aligned} \frac{Y_1}{[0+(s+1)]} &= -\frac{Y_2}{[0+1]} = \frac{1}{(s+1)^2+1} \\ Y_1 &= \frac{s+1}{(s+1)^2+1} \Rightarrow \mathcal{L}\{y_1\} = \frac{s+1}{(s+1)^2+1} \\ Y_2 &= -\frac{1}{(s+1)^2+1} \Rightarrow \mathcal{L}\{y_2\} = -\frac{1}{(s+1)^2+1} \\ y_1 &= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} = e^{-t} \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \\ y_2 &= \mathcal{L}^{-1}\left\{-\frac{1}{(s+1)^2+1}\right\} = e^{-t} \mathcal{L}^{-1}\left\{-\frac{1}{s^2+1}\right\} = -e^{-t} \sin t \end{aligned}$$