ENGINEERING CURVES
Part-II
(Point undergoing two types of displacements)

INVOLUTE

1. Involute of a circle
   a)String Length = πD

   b)String Length > πD

   c)String Length < πD

2. Pole having Composite shape.

3. Rod Rolling over a Semicircular Pole.

CYCLOID

1. General Cycloid

2. Trochoid (superior)

3. Trochoid (Inferior)

4. Epi-Cycloid

5. Hypo-Cycloid

SPIRAL

1. Spiral of One Convolution.

2. Spiral of Two Convolutions.

HELIX

1. On Cylinder

2. On a Cone

AND

Methods of Drawing Tangents & Normals To These Curves.

DEFINITIONS

CYCLOID:

IT IS A LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A STRAIGHT LINE PATH.

SUPERIORTROCHOID:
IF THE POINT IN THE DEFINATION OF CYCLOID IS OUTSIDE THE CIRCLE

INFERIOR TROCHOID.:
IF IT IS INSIDE THE CIRCLE

EPI-CYCLOID
IF THE CIRCLE IS ROLLING ON ANOTHER CIRCLE FROM OUTSIDE

HYPO-CYCLOID.
IF THE CIRCLE IS ROLLING FROM INSIDE THE OTHER CIRCLE,

INVOLUTE:

IT IS A LOCUS OF A FREE END OF A STRING WHEN IT IS WOUND ROUND A CIRCULAR POLE

SPIRAL:

IT IS A CURVE GENERATED BY A POINT WHICH REVOLVES AROUND A FIXED POINT AND AT THE SAME MOVES TOWARDS IT.

HELIX:

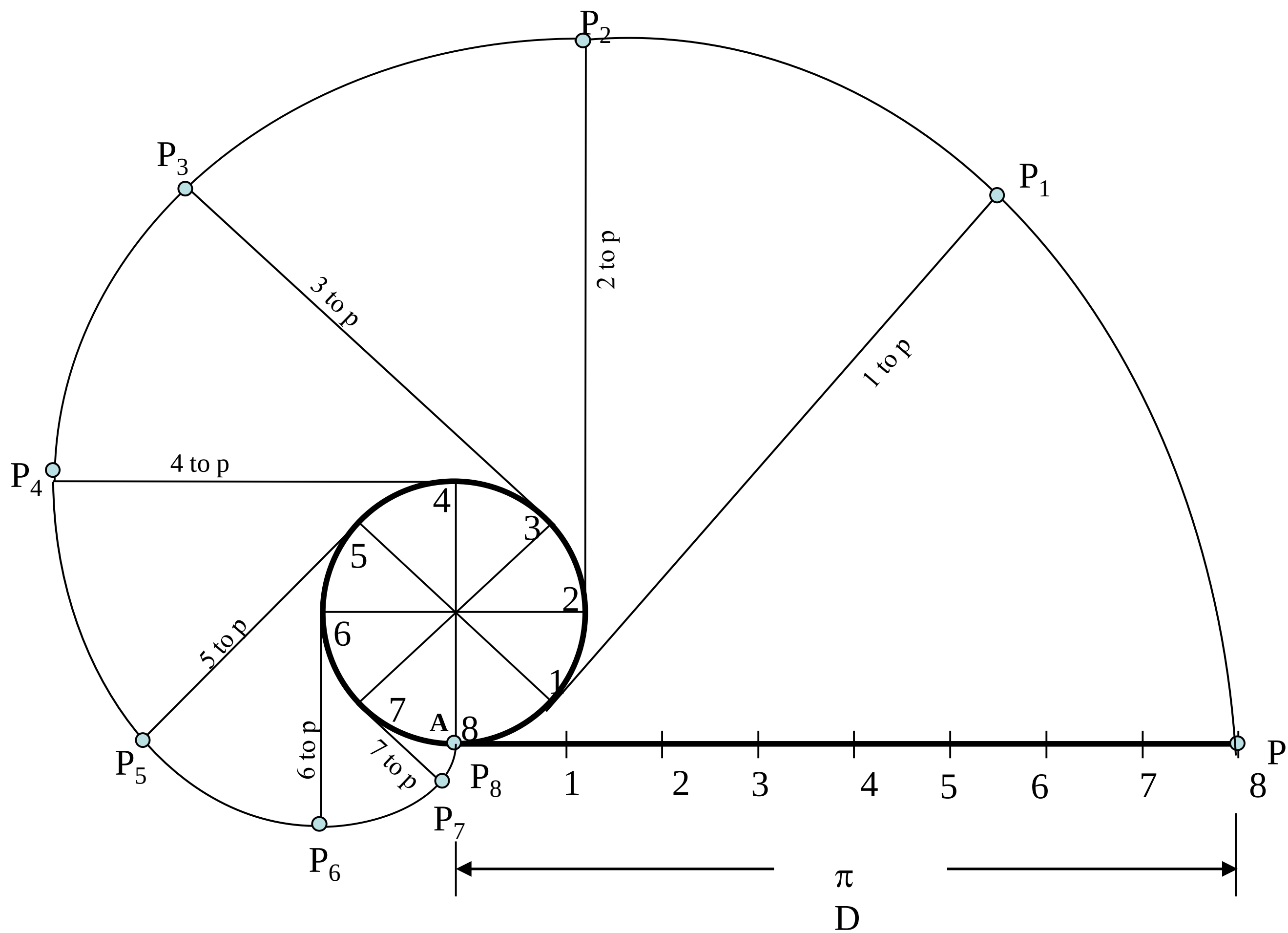
IT IS A CURVE GENERATED BY A POINT WHICH MOVES AROUND THE SURFACE OF A RIGHT CIRCULAR CYLINDER / CONE AND AT THE SAME TIME ADVANCES IN AXIAL DIRECTION AT A SPEED BEARING A CONSTANT RATIO TO THE SPPED OF ROTATION.

(for problems refer topic Development of surfaces)

Problem no 17: Draw Involute of a circle.
String length is equal to the circumference of circle.

Solution Steps:

1) Point or end P of string AP is exactly πD distance away from A. Means if this string is wound round the circle, it will completely cover given circle. B will meet A after winding.
2) Divide πD (AP) distance into 8 number of equal parts.
3) Divide circle also into 8 number of equal parts.
4) Name after A, 1, 2, 3, 4, etc. up to 8 on πD line AP as well as on circle (in anticlockwise direction).
5) To radius C-1, C-2, C-3 up to C-8 draw tangents (from 1,2,3,4,etc to circle).
6) Take distance 1 to P in compass and mark it on tangent from point 1 on circle (means one division less than distance AP).
7) Name this point P1
8) Take 2-B distance in compass and mark it on the tangent from point 2. Name it point P2.
9) Similarly take 3 to P, 4 to P, 5 to P up to 7 to P distance in compass and mark on respective tangents and locate P3, P4, P5 up to P8 (i.e. A) points and join them in smooth curve it is an INVOLUTE of a given circle.



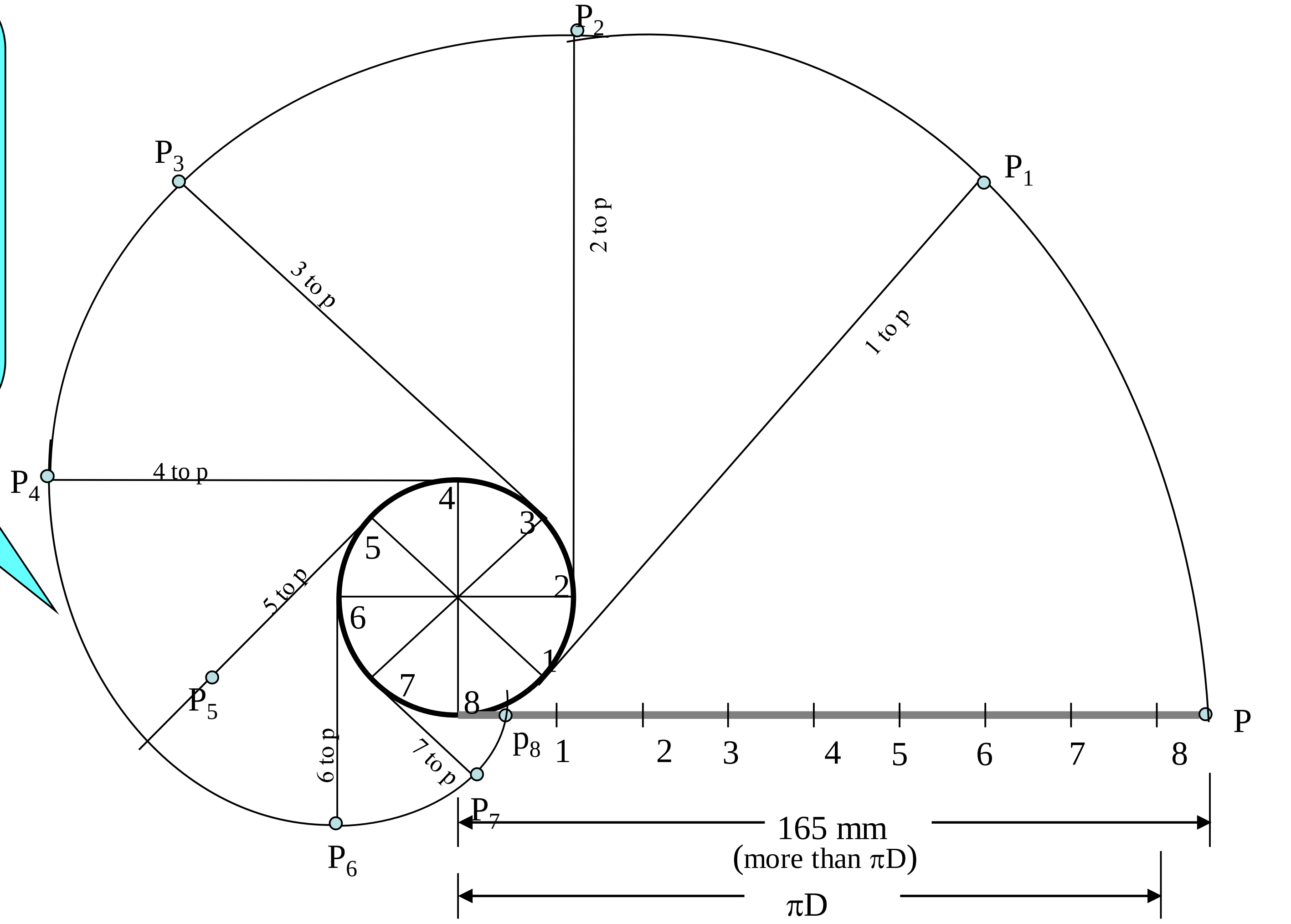
Problem 18: Draw Involute of a circle.
String length is MORE than the circumference of circle.

INVOLUTE OF A CIRCLE
String length MORE than πD

Solution Steps:
In this case string length is more than Π D.

But remember!

Whatever may be the length of string, mark Π D distance horizontal i.e.along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.



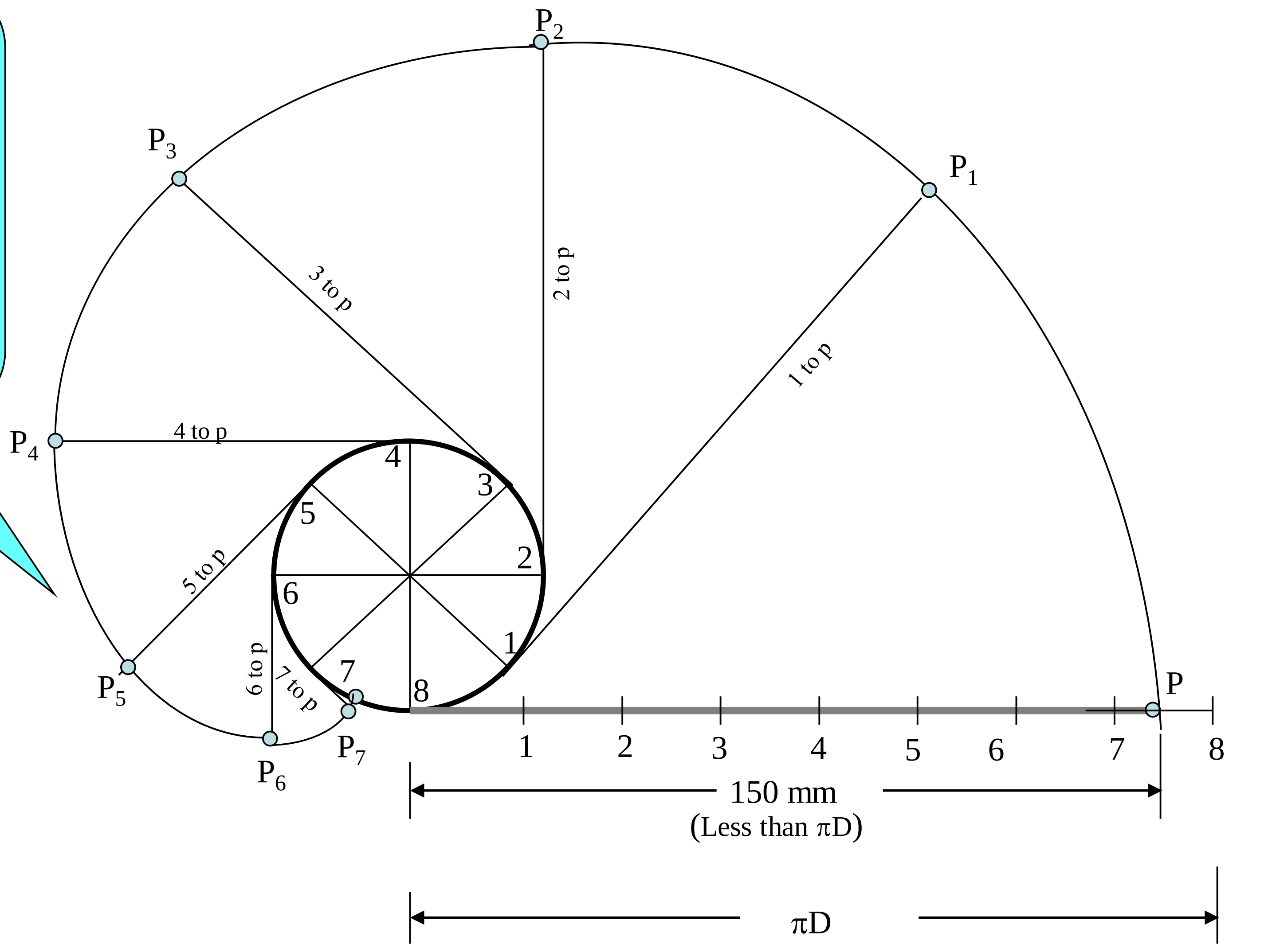
Problem 19: Draw Involute of a circle.
String length is LESS than the circumference of circle.

INVOLUTE OF A CIRCLE

String length LESS than πD

Solution Steps:
In this case string length is Less than Π D.

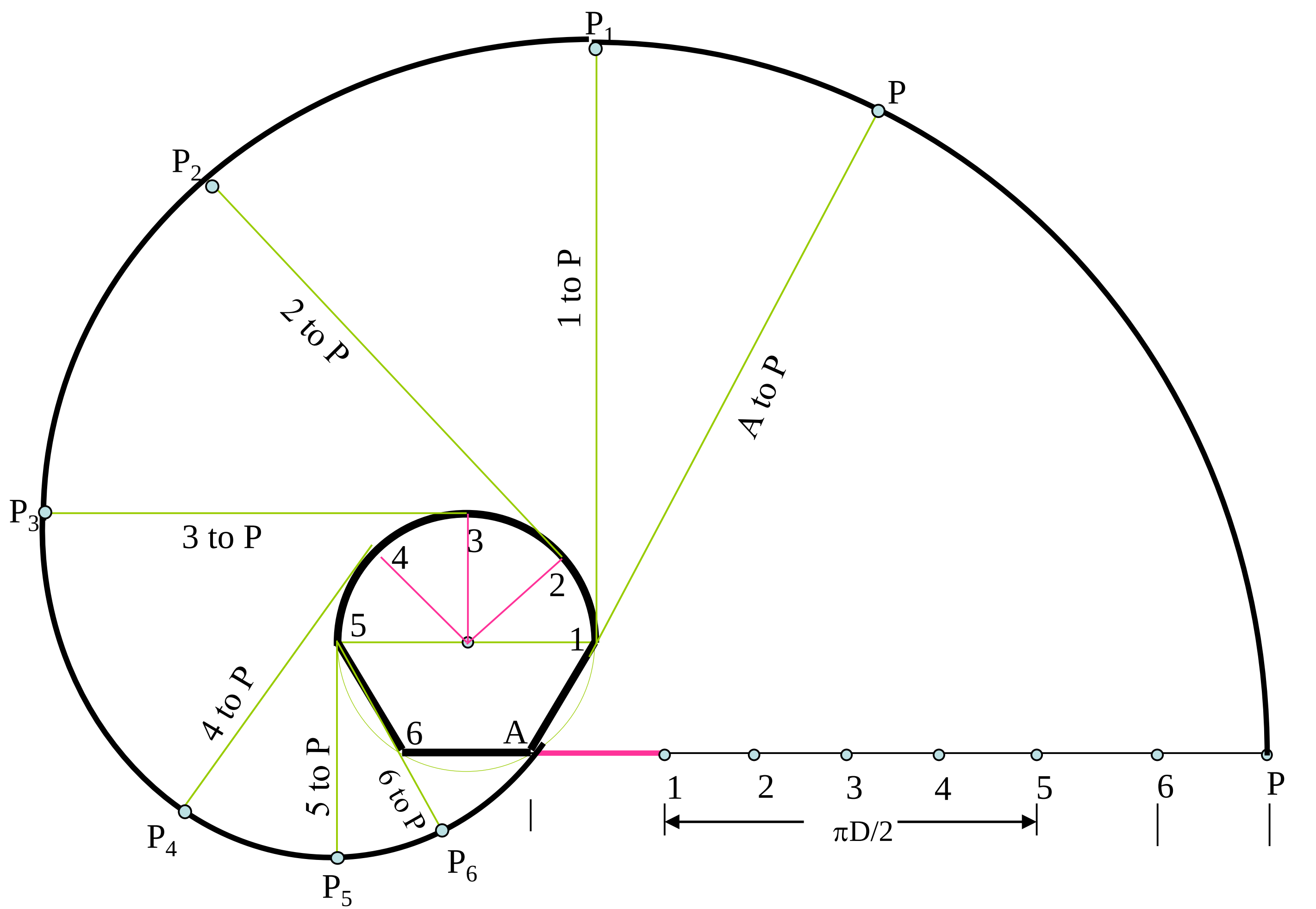
But remember!
Whatever may be the length of string, mark Π D distance horizontal i.e.along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.



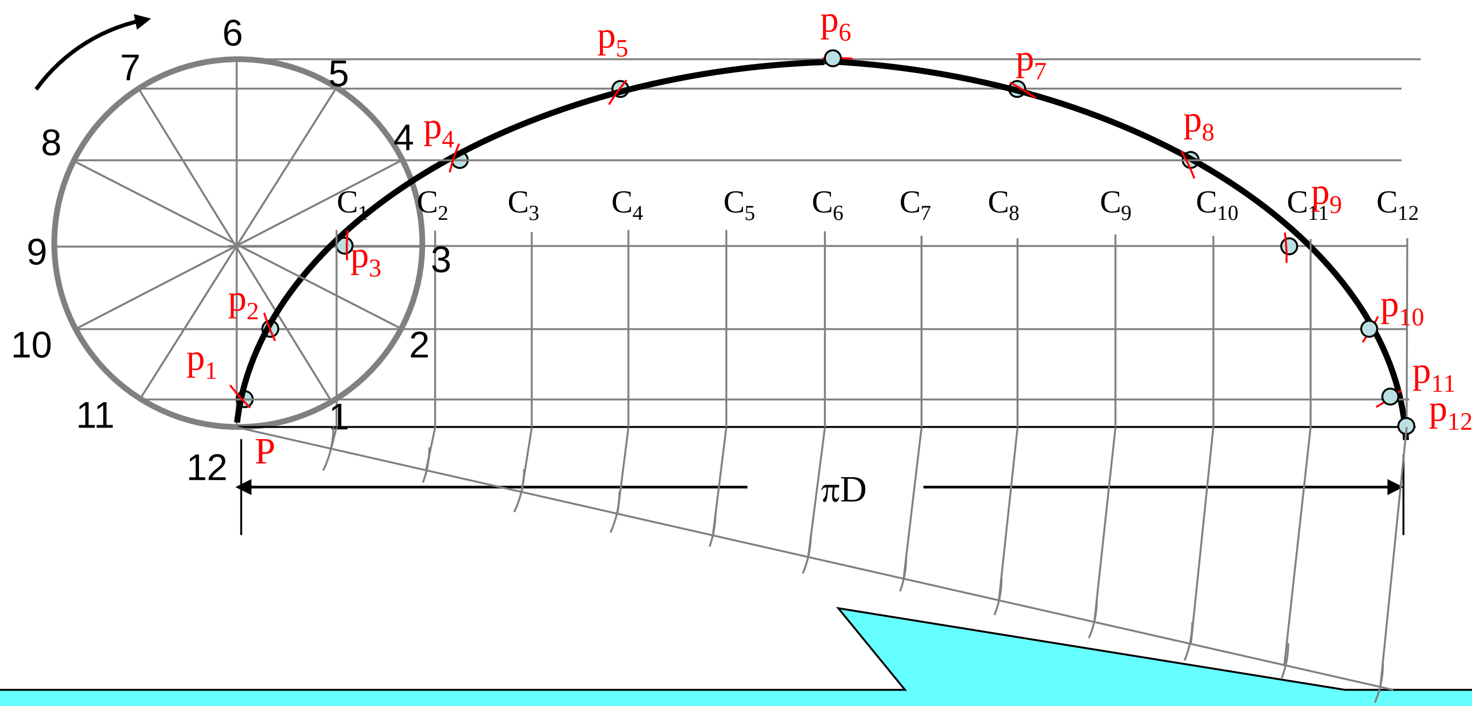
PROBLEM 20 : A POLE IS OF A SHAPE OF HALF HEXABON AND SEMICIRCLE. ASTRING IS TO BE WOUND HAVING LENGTH EQUAL TO THE POLE PERIMETER DRAW PATH OF FREE END *P* OF STRING WHEN WOUND COMPLETELY.
(Take hex 30 mm sides and semicircle of 60 mm diameter.)

INVOLUTE OF COMPOSIT SHAPED POLE

SOLUTION STEPS:
Draw pole shape as per dimensions.
Divide semicircle in 4 parts and name those along with corners of hexagon.
Calculate perimeter length.
Show it as string AP.
On this line mark 30mm from A
Mark and name it 1
Mark πD/2 distance on it from 1
And dividing it in 4 parts name 2,3,4,5.
Mark point 6 on line 30 mm from 5
Now draw tangents from all points of pole and proper lengths as done in all previous involute's problems and complete the curve.



PROBLEM 22: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

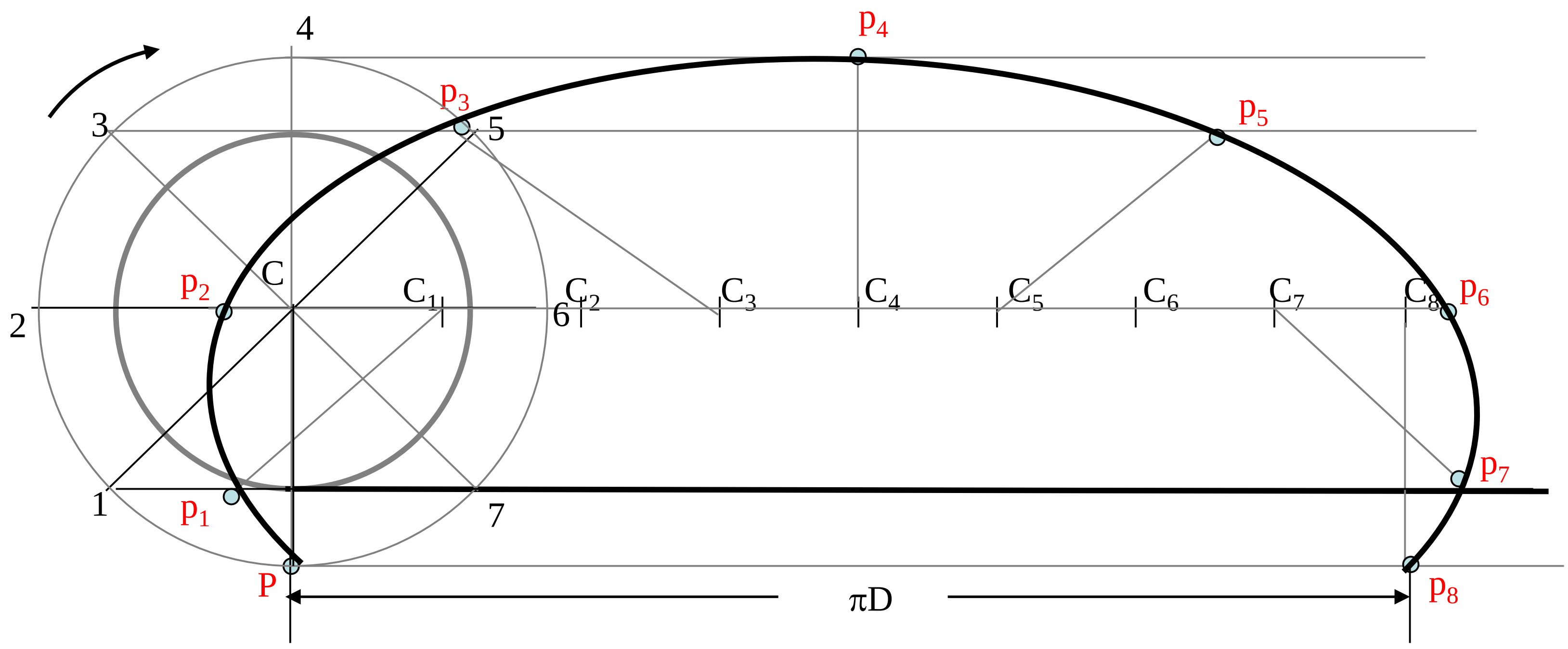


Solution Steps:

1) From center C draw a horizontal line equal to πD distance.
2) Divide πD distance into 12 number of equal parts and name them C1, C2, C3__ etc.
3) Divide the circle also into 12 number of equal parts and in clock wise direction, after P name 1, 2, 3 up to 12.
4) From all these points on circle draw horizontal lines. (parallel to locus of C)
5) With a fixed distance C-P in compass, C1 as center, mark a point on horizontal line from 1. Name it P.
6) Repeat this procedure from C2, C3, C4 upto C12 as centers. Mark points P2, P3, P4, P5 up to P8 on the horizontal lines drawn from 1,2, 3, 4, 5, 6, 7 respectively.
7) Join all these points by curve. **It is Cycloid**.

PROBLEM 23: DRAW LOCUS OF A POINT , **5 MM** AWAY FROM THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. **Take Circle diameter as 50 mm**

SUPERIOR TROCHOID

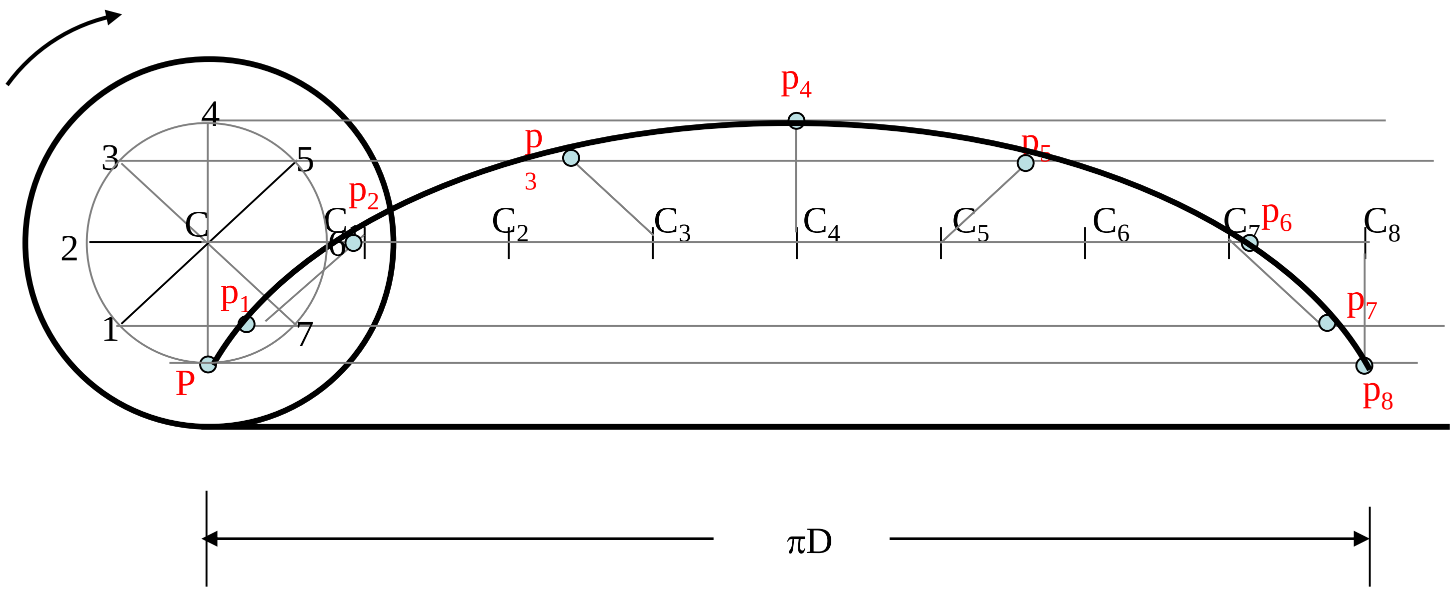


Solution Steps:

1) Draw circle of given diameter and draw a horizontal line from it's center C of length Π D and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
2) Draw circle by CP radius, as in this case CP is larger than radius of circle.
3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius wit different positions of C as centers, cut these lines and get different positions of P and join
4) This curve is called **Superior Trochoid.**

PROBLEM 24: DRAW LOCUS OF A POINT , **5 MM** INSIDE THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. **Take Circle diameter as 50 mm**

INFERIOR TROCHOID



Solution Steps:

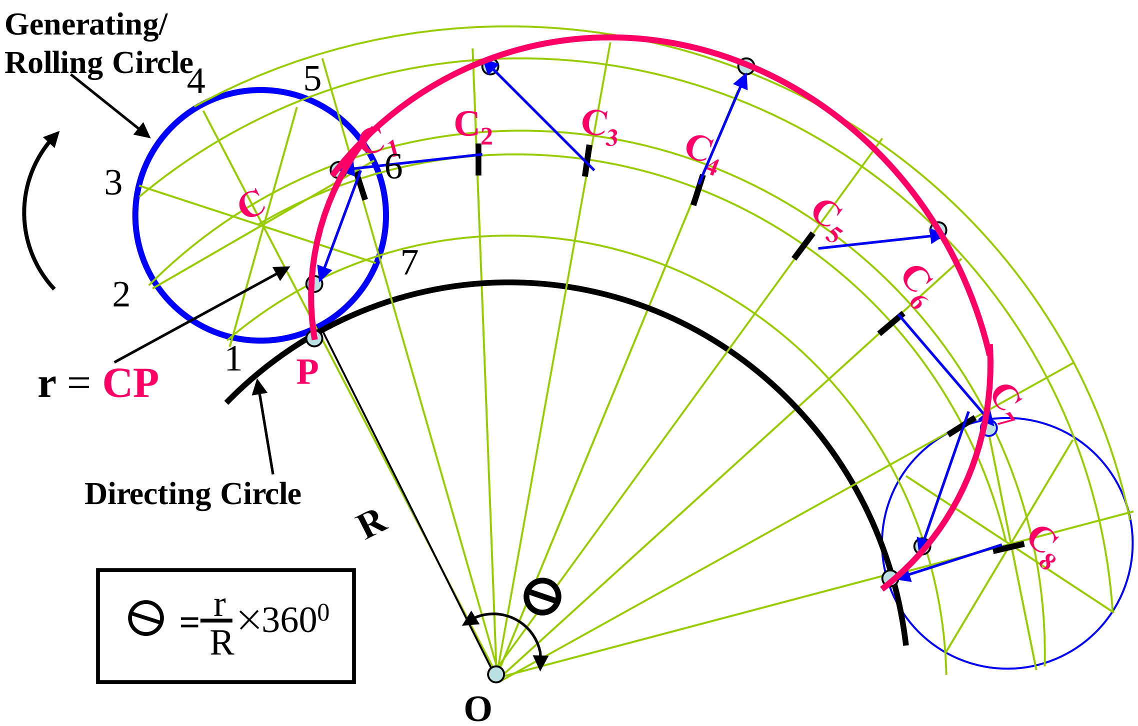
1) Draw circle of given diameter and draw a horizontal line from it's center C of length Π D and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
2) Draw circle by CP radius, as in this case CP is SHORTER than radius of circle.
3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join those in curvature.
4) This curve is called **Inferior Trochoid**.

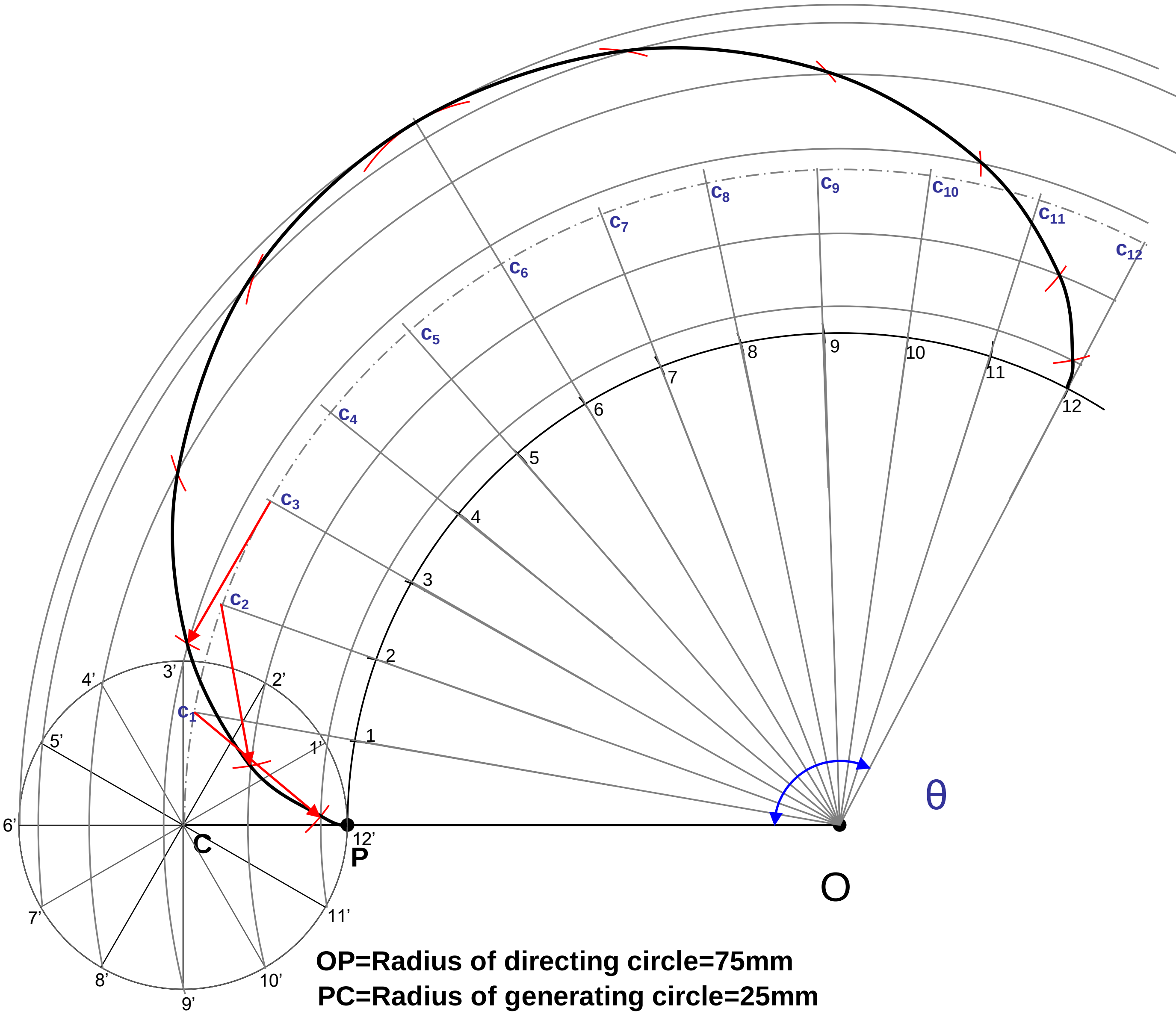
PROBLEM 25: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A CURVED PATH. **Take diameter of rolling Circle 50 mm And radius of directing circle i.e. curved path, 75 mm.**

EPI CYCLOID :

Solution Steps:

1) When smaller circle will roll on larger circle for one revolution it will cover Π D distance on arc and it will be decided by included arc angle θ.
2) Calculate θ by formula θ = (r/R) x 3600.
3) Construct angle θ with radius OC and draw an arc by taking O as center OC as radius and form sector of angle θ.
4) Divide this sector into 8 number of equal angular parts. And from C onward name them C1, C2, C3 up to C8.
5) Divide smaller circle (Generating circle) also in 8 number of equal parts. And next to P in clockwise direction name those 1, 2, 3, up to 8.
6) With O as center, O-1 as radius draw an arc in the sector. Take O-2, O-3, O-4, O-5 up to O-8 distances with center O, draw all concentric arcs in sector. Take fixed distance C-P in compass, C1 center, cut arc of 1 at P1. Repeat procedure and locate P2, P3, P4, P5 unto P8 (as in cycloid) and join them by smooth curve. This is EPI – CYCLOID.





OP=Radius of directing circle=75mm

PC=Radius of generating circle=25mm

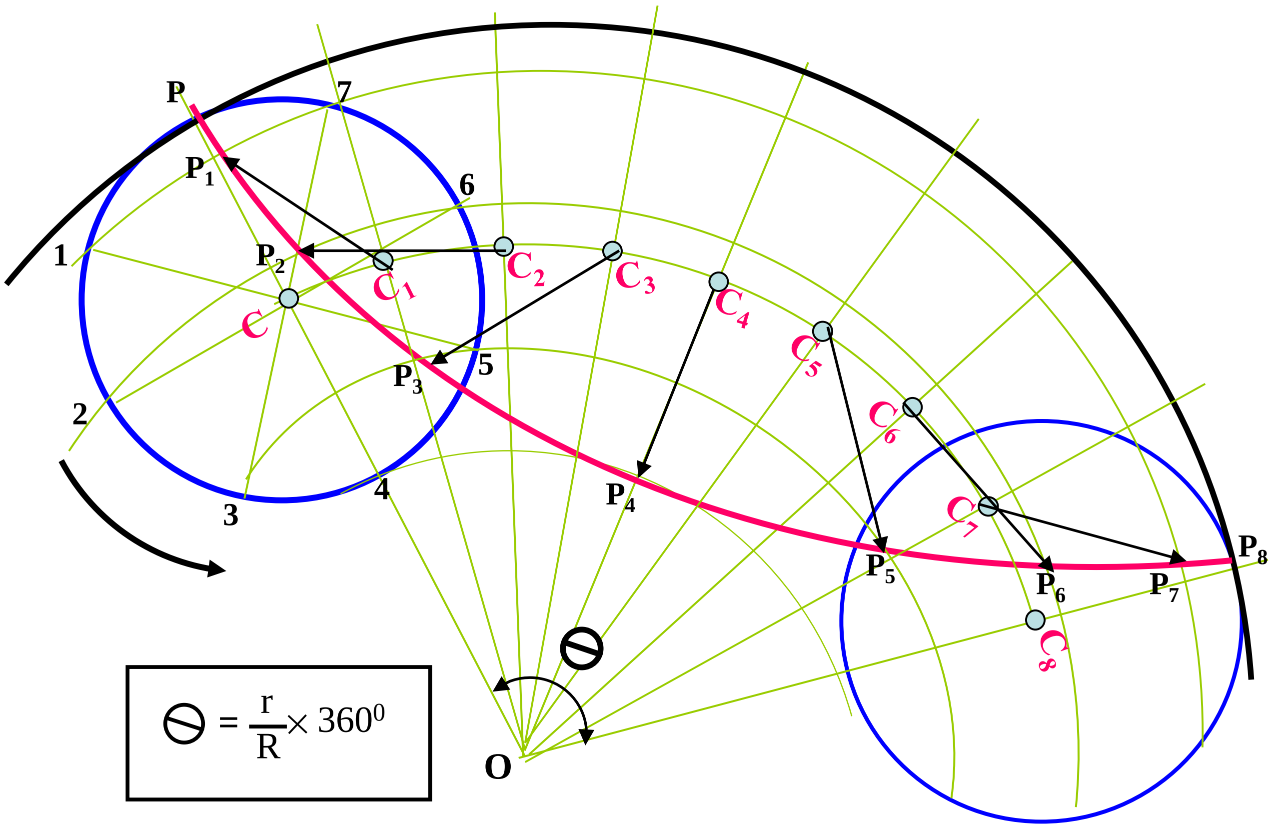
θ=r/R X360º= 25/75 X360º=120º

PROBLEM 26: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS FROM THE INSIDE OF A CURVED PATH. **Take diameter of rolling circle 50 mm and radius of directing circle (curved path) 75 mm.**

HYPO CYCLOID

Solution Steps:

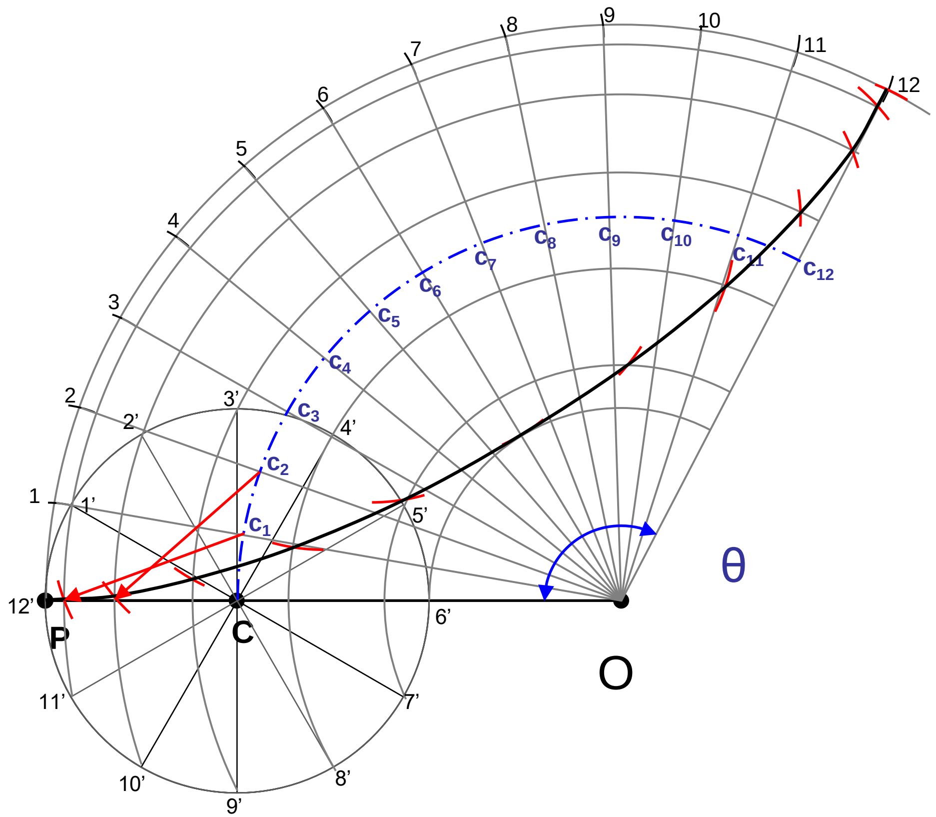
1) Smaller circle is rolling here, inside the larger circle. It has to rotate anticlockwise to move ahead.
2) Same steps should be taken as in case of EPI – CYCLOID. Only change is in numbering direction of 8 number of equal parts on the smaller circle.
3) From next to P in anticlockwise direction, name 1,2,3,4,5,6,7,8.
4) Further all steps are that of epi – cycloid. **This is called HYPO – CYCLOID**.



$$\theta = \frac{r}{R} \times 360^0$$

OC = R (Radius of Directing Circle)

CP = r (Radius of Generating Circle)



OP=Radius of directing circle=75mm

PC=Radius of generating circle=25mm

θ=r/R X360º= 25/75 X360º=120º

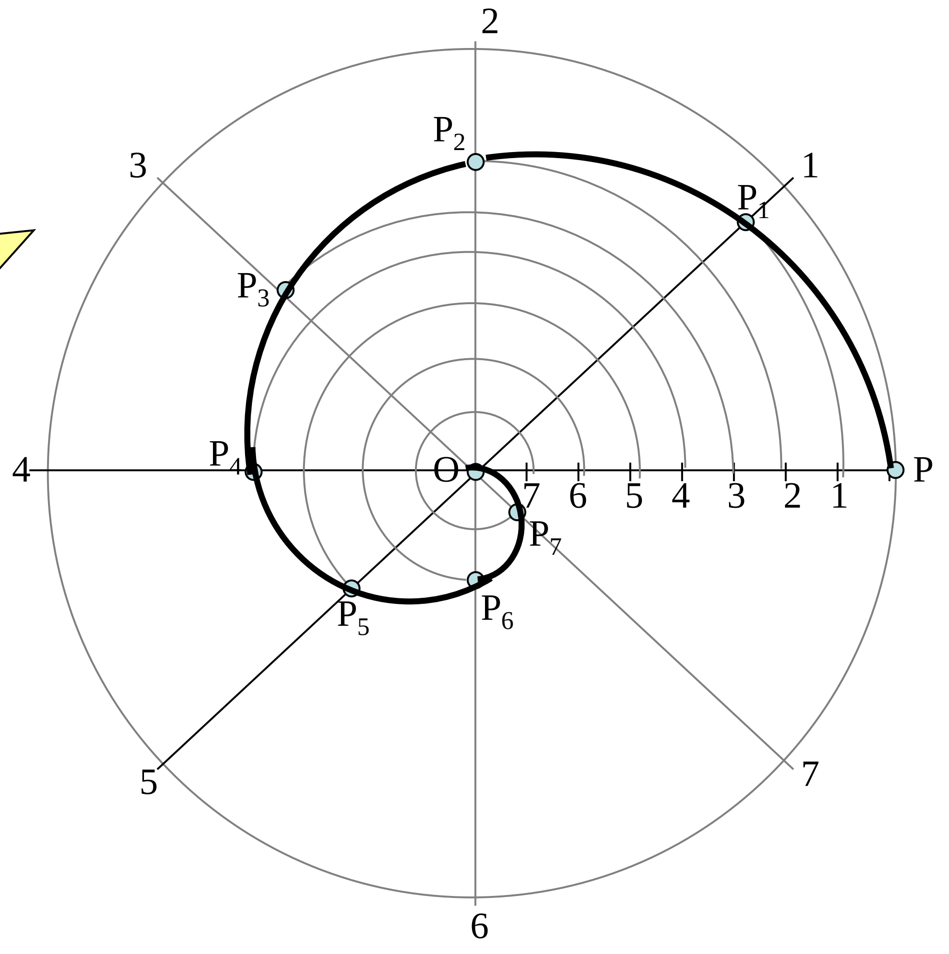
Problem 27: **Draw a spiral of one convolution. Take distance PO 40 mm.**

SPIRAL

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

Solution Steps

1. With PO radius draw a circle and divide it in EIGHT parts. Name those 1,2,3,4, etc. up to 8
2. Similarly divided line PO also in EIGHT parts and name those 1,2,3,-- as shown.
3. Take o-1 distance from op line and draw an arc up to O1 radius vector. Name the point P_1
4. Similarly mark points P_2, P_3, P_4 up to P_8
   And join those in a smooth curve.
   It is a SPIRAL of one convolution.

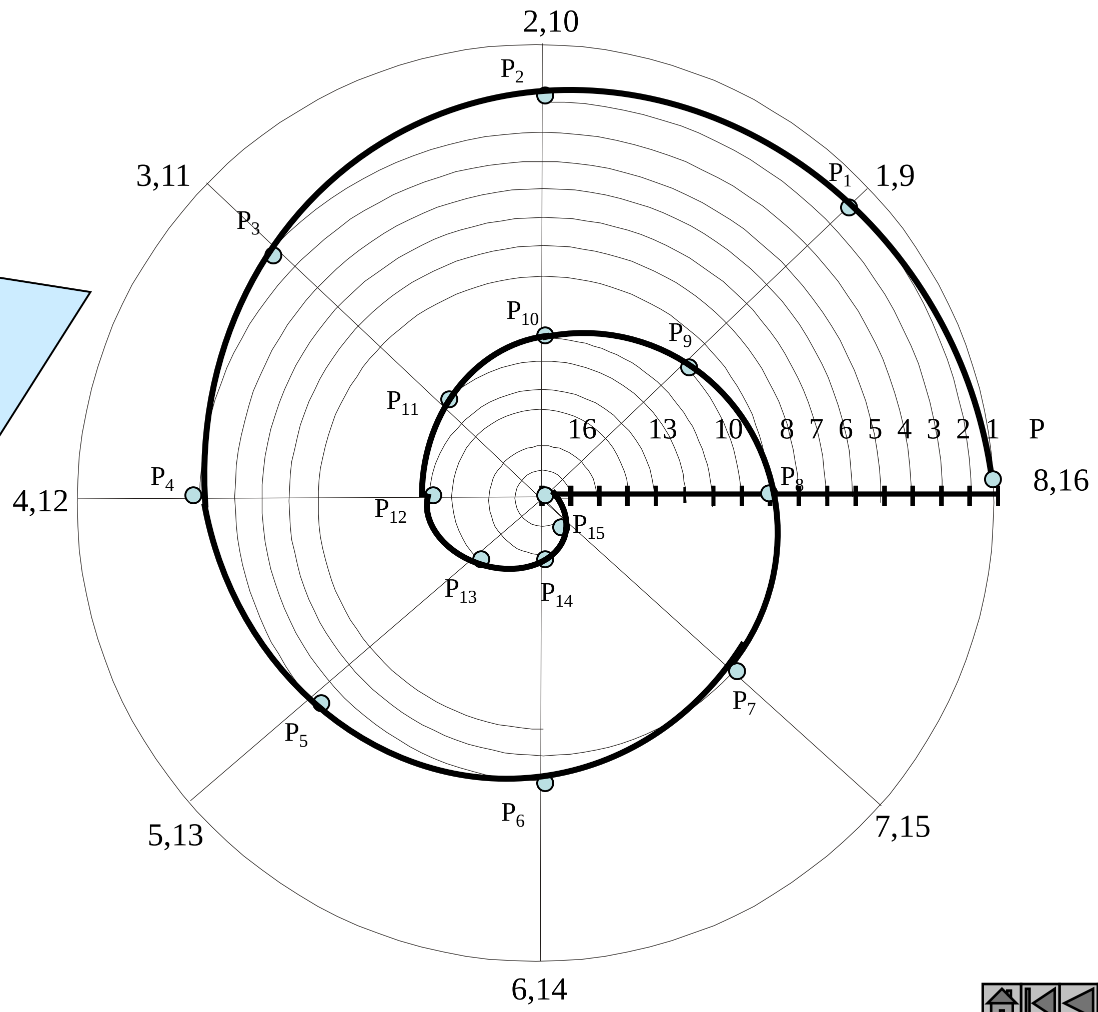


Problem 28
Point P is 80 mm from point O. It starts moving towards O and reaches it in two revolutions around.it Draw locus of point P (To draw a Spiral of TWO convolutions).

SPIRAL of two convolutions

IMPORTANT APPROACH FOR CONSTRUCTION!
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

SOLUTION STEPS:
Total angular displacement here is two revolutions And
Total Linear displacement here is distance PO.
Just divide both in same parts i.e.
Circle in EIGHT parts.
(means total angular displacement in SIXTEEN parts)
Divide PO also in SIXTEEN parts.
Rest steps are similar to the previous problem.



Involute
Method of Drawing
Tangent & Normal

STEPS:
DRAW INVOLUTE AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

JOIN **Q** TO THE CENTER OF CIRCLE **C**.
CONSIDERING **CQ** DIAMETER, DRAW
A SEMICIRCLE AS SHOWN.

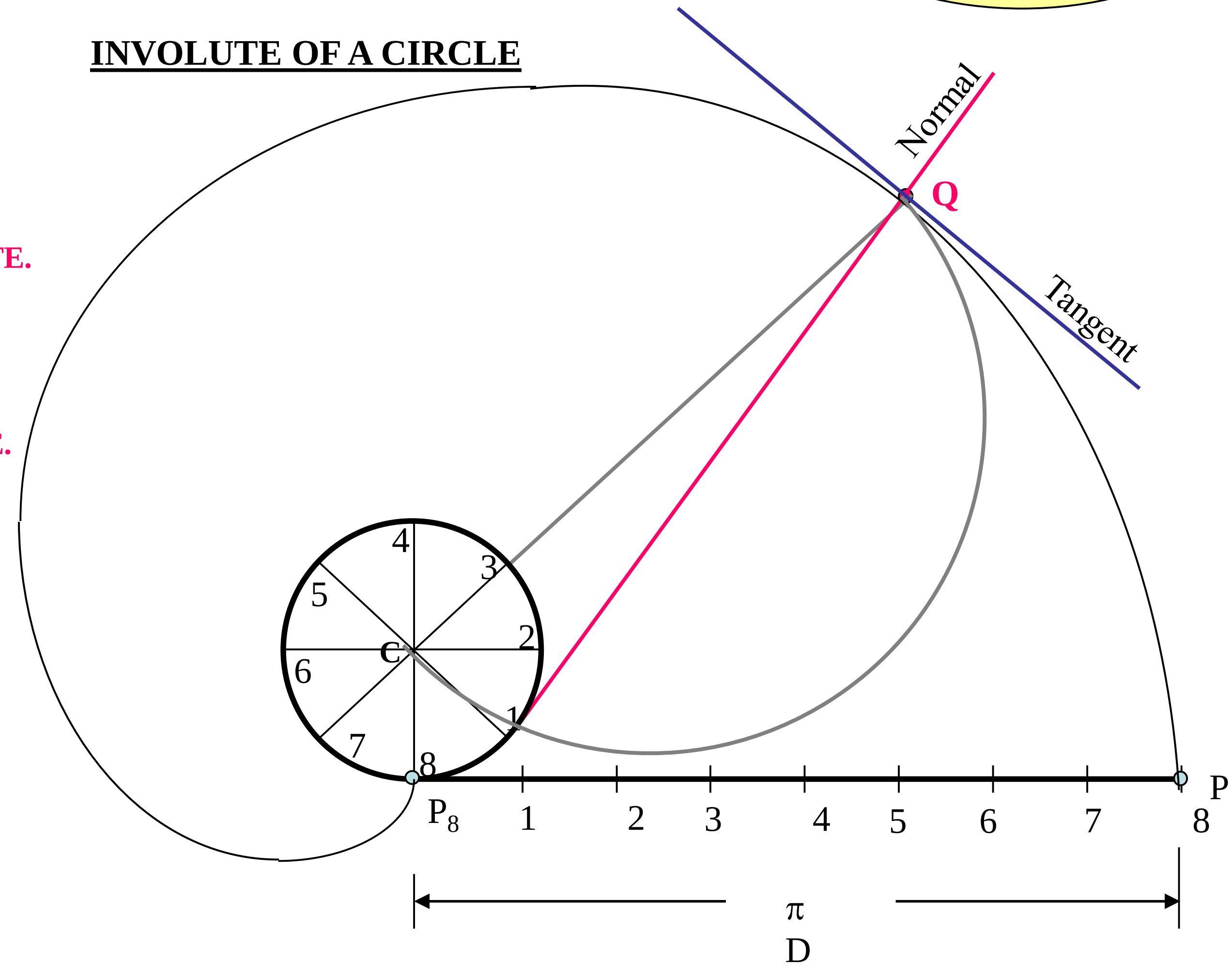
MARK POINT OF INTERSECTION OF
THIS SEMICIRCLE AND POLE CIRCLE
AND JOIN IT TO **Q.**

THIS WILL BE **NORMAL TO INVOLUTE.**

DRAW A LINE AT RIGHT ANGLE TO
THIS LINE FROM **Q.**

IT WILL BE TANGENT TO INVOLUTE.

INVOLUTE OF A CIRCLE



CYCLOID Method of Drawing Tangent & Normal

STEPS:
DRAW CYCLOID AS USUAL.
MARK POINT **Q** ON IT AS DIRECTED.

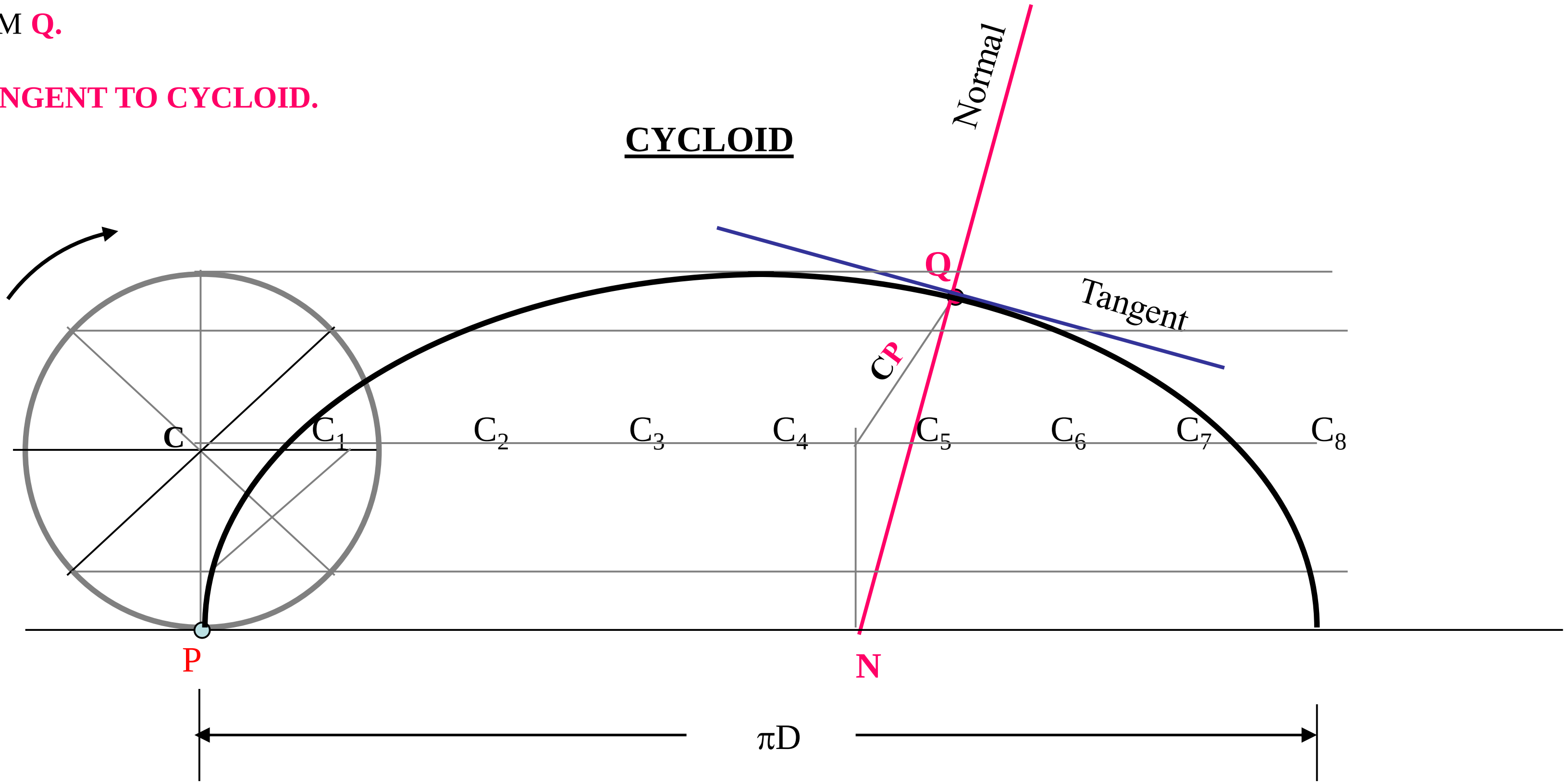
WITH CP DISTANCE, FROM **Q.** CUT THE POINT ON LOCUS OF **C** AND JOIN IT TO **Q.**

FROM THIS POINT DROP A PERPENDICULAR ON GROUND LINE AND NAME IT N

JOIN N WITH Q.THIS WILL BE **NORMAL TO CYCLOID.**

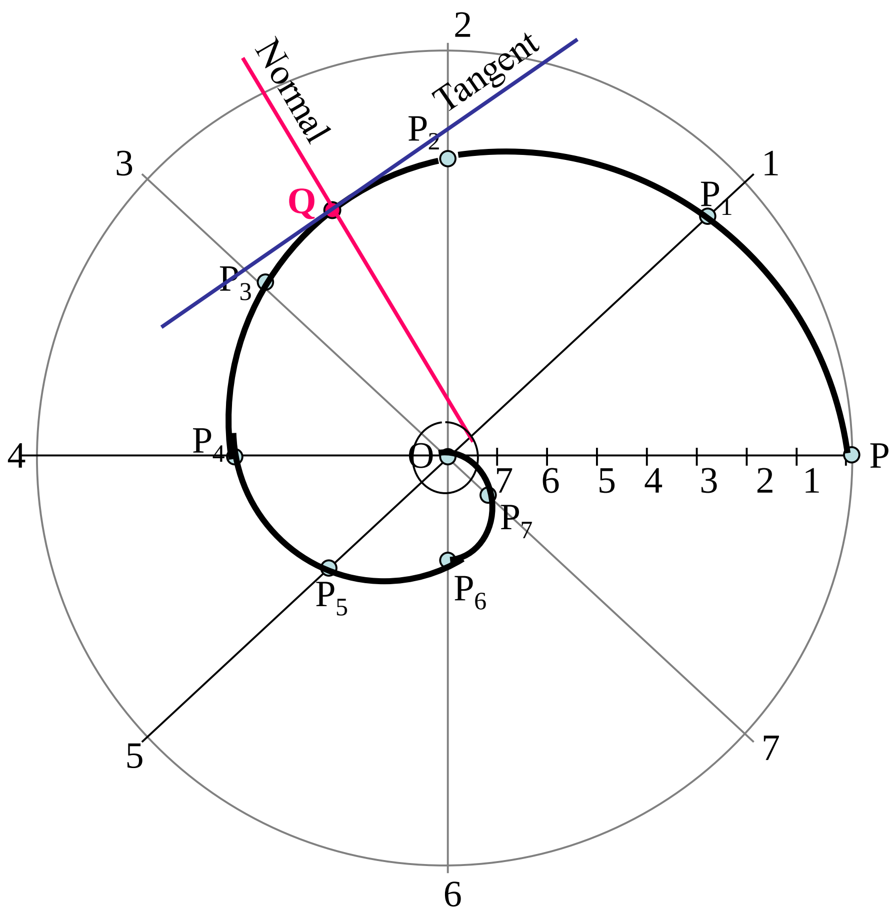
DRAW A LINE AT RIGHT ANGLE TO THIS LINE FROM **Q.**

IT WILL BE TANGENT TO CYCLOID.



Spiral.
Method of Drawing Tangent & Normal

SPIRAL (ONE CONVOLUSION.)



Constant of the Curve = $\dfrac{\text{Difference in length of any radius vectors}}{\text{Angle between the corresponding radius vector in radian.}}$

$$= \frac{OP - OP_2}{\pi/2} = \frac{OP - OP_2}{1.57}$$

= 3.185 m.m.

STEPS:

*DRAW SPIRAL AS USUAL.
DRAW A SMALL CIRCLE OF RADIUS EQUAL TO THE CONSTANT OF CURVE CALCULATED ABOVE.

* LOCATE POINT **Q** AS DISCRIBED IN PROBLEM AND THROUGH IT DRAW A TANGENTTO THIS SMALLER CIRCLE.THIS IS A **NORMAL** TO THE SPIRAL.

*DRAW A LINE AT RIGHT ANGLE

*TO THIS LINE FROM **Q.**
IT WILL BE TANGENT TO CYCLOID.