

# ENGINEERING CURVES

## Part- I {Conic Sections}

### ELLIPSE

1. Concentric Circle Method
2. Rectangle Method
3. Oblong Method
4. Arcs of Circle Method
5. Rhombus Method
6. Basic Locus Method  
(Directrix – focus)

### PARABOLA

1. Rectangle Method
2. Method of Tangents  
(Triangle Method)
3. Basic Locus Method  
(Directrix – focus)

### HYPERBOLA

1. Rectangular Hyperbola  
(coordinates given)
2. Rectangular Hyperbola  
(P-V diagram - Equation given)
3. Basic Locus Method  
(Directrix – focus)

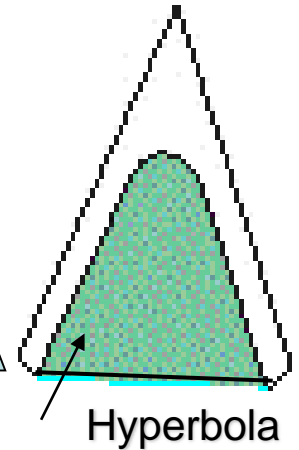
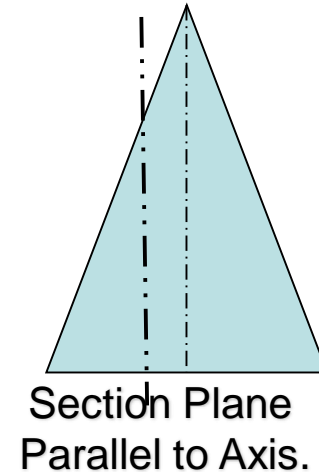
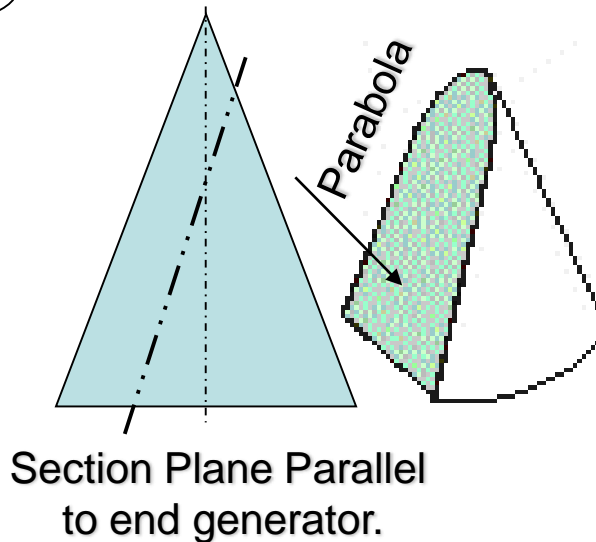
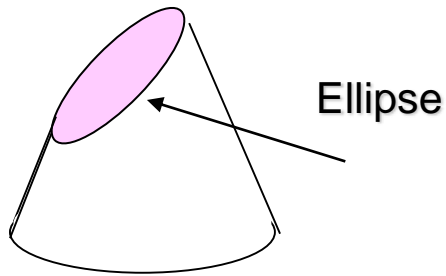
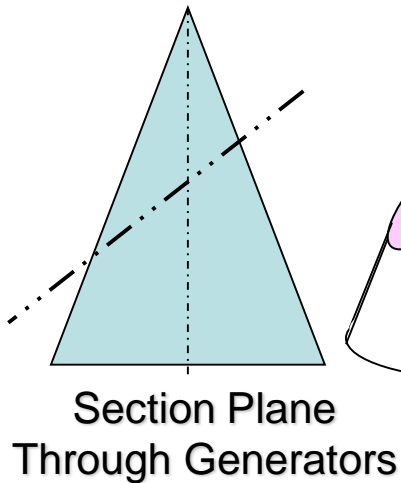
Methods of Drawing  
Tangents & Normals  
To These Curves.

## CONIC SECTIONS

**ELLIPSE, PARABOLA AND HYPERBOLA ARE CALLED CONIC SECTIONS  
BECAUSE**

**THESE CURVES APPEAR ON THE SURFACE OF A CONE  
WHEN IT IS CUT BY SOME TYPICAL CUTTING PLANES.**

**OBSERVE  
ILLUSTRATIONS  
GIVEN BELOW..**



## COMMON DEFINATION OF ELLIPSE, PARABOLA & HYPERBOLA:

These are the loci of points moving in a plane such that the ratio of it's distances from a *fixed point* And a *fixed line* always remains constant.

The Ratio is called **ECCENTRICITY. (E)**

A) For Ellipse  $E < 1$

B) For Parabola  $E = 1$

C) For Hyperbola  $E > 1$

**Refer Problem nos. 6. 9 & 12**

## SECOND DEFINATION OF AN ELLIPSE:-

It is a locus of a point moving in a plane such that the SUM of it's distances from TWO fixed points always remains constant.

{ And this *sum equals* to the length of *major axis*. }

These TWO fixed points are FOCUS 1 & FOCUS 2

**Refer Problem no.4**  
**Ellipse by Arcs of Circles Method.**

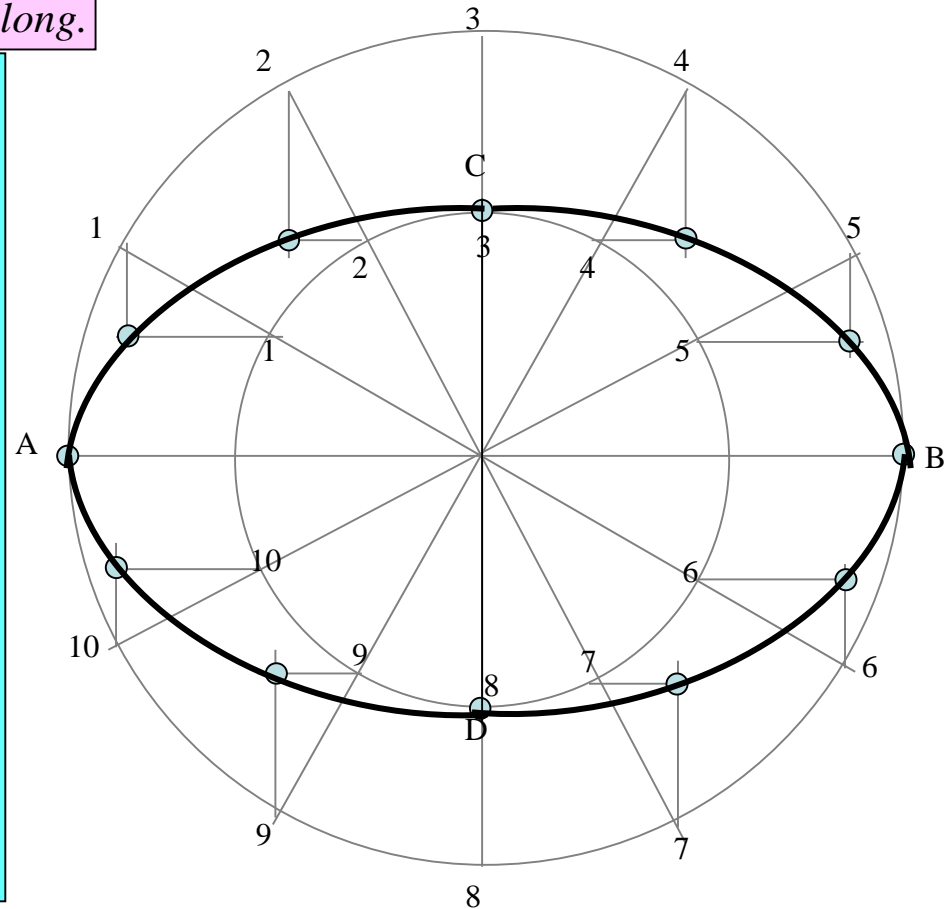
**Problem 1 :-**

*Draw ellipse by concentric circle method.*

*Take major axis 100 mm and minor axis 70 mm long.*

**Steps:**

1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
3. Divide both circles in 12 equal parts & name as shown.
4. From all points of outer circle draw vertical lines downwards and upwards respectively.
5. From all points of inner circle draw horizontal lines to intersect those vertical lines.
6. Mark all intersecting points properly as those are the points on ellipse.
7. Join all these points along with the ends of both axes in smooth possible curve. It is required ellipse.



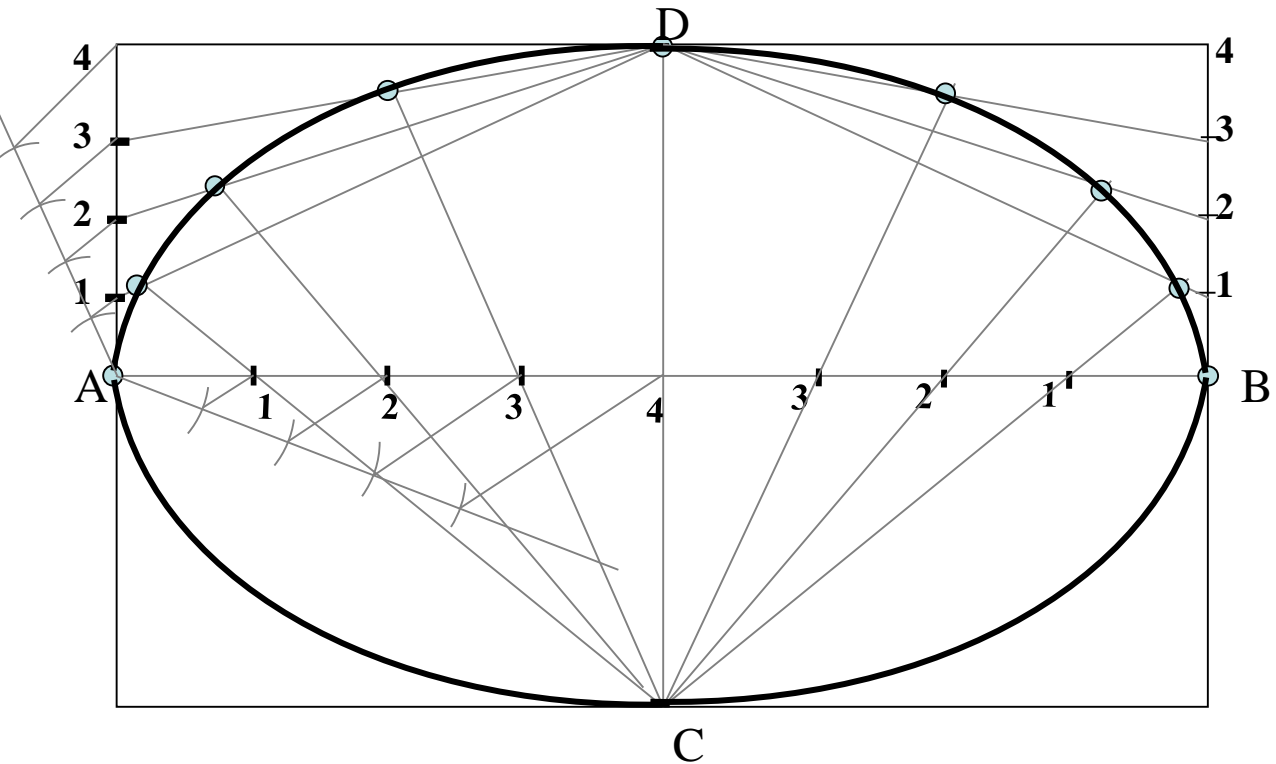
**Steps:**

- 1 Draw a rectangle taking major and minor axes as sides.
  2. In this rectangle draw both axes as perpendicular bisectors of each other.
  3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts.( here divided in four parts)
  4. Name those as shown..
  5. Now join all vertical points 1,2,3,4, to the upper end of minor axis. And all horizontal points i.e.1,2,3,4 to the lower end of minor axis.
  6. Then extend C-1 line upto D-1 and mark that point. Similarly extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
  7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side part.along with lower half of the rectangle.Join all points in smooth curve.
- It is required ellipse.

**Problem 2**

*Draw ellipse by **Rectangle method**.*

*Take major axis 100 mm and minor axis 70 mm long.*

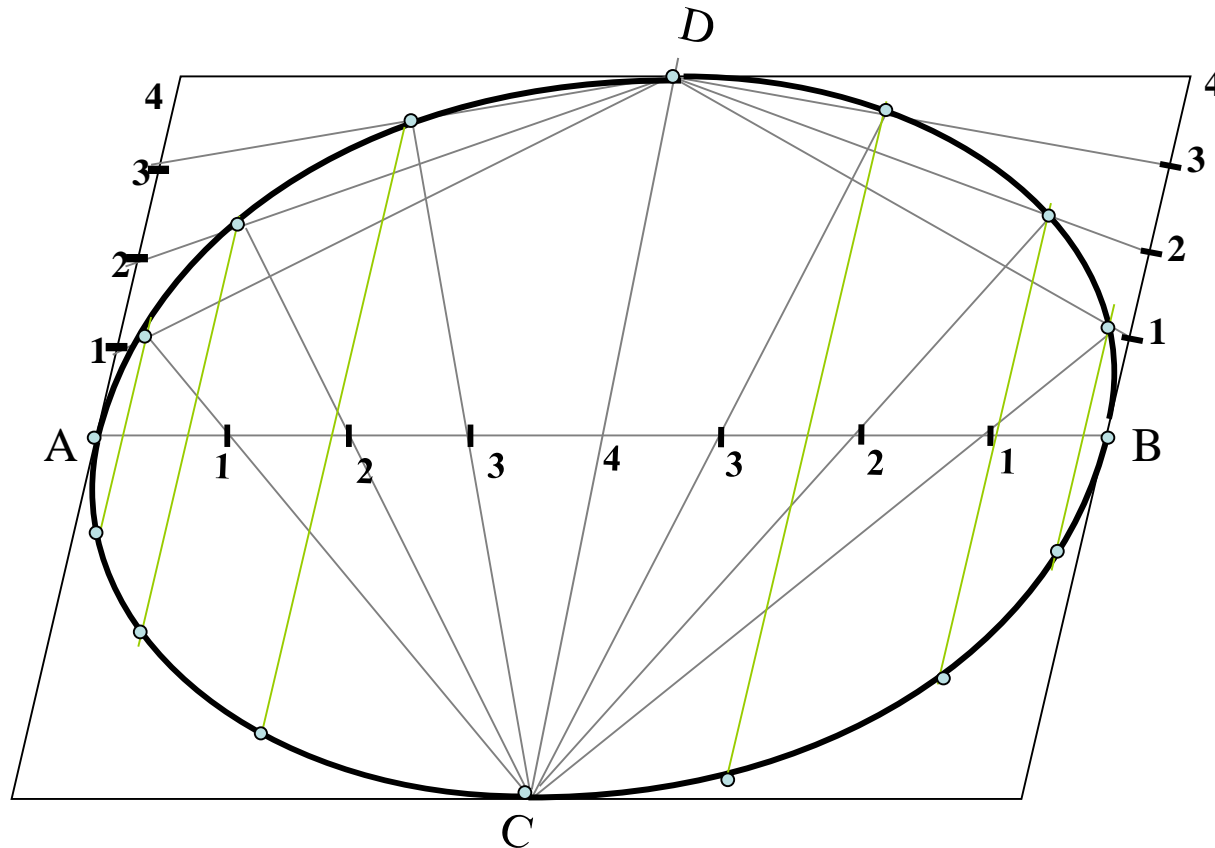


**Problem 3:-**

*Draw ellipse by Oblong method.*

*Draw a parallelogram of 100 mm and 70 mm long sides with included angle of  $75^\circ$ . Inscribe Ellipse in it.*

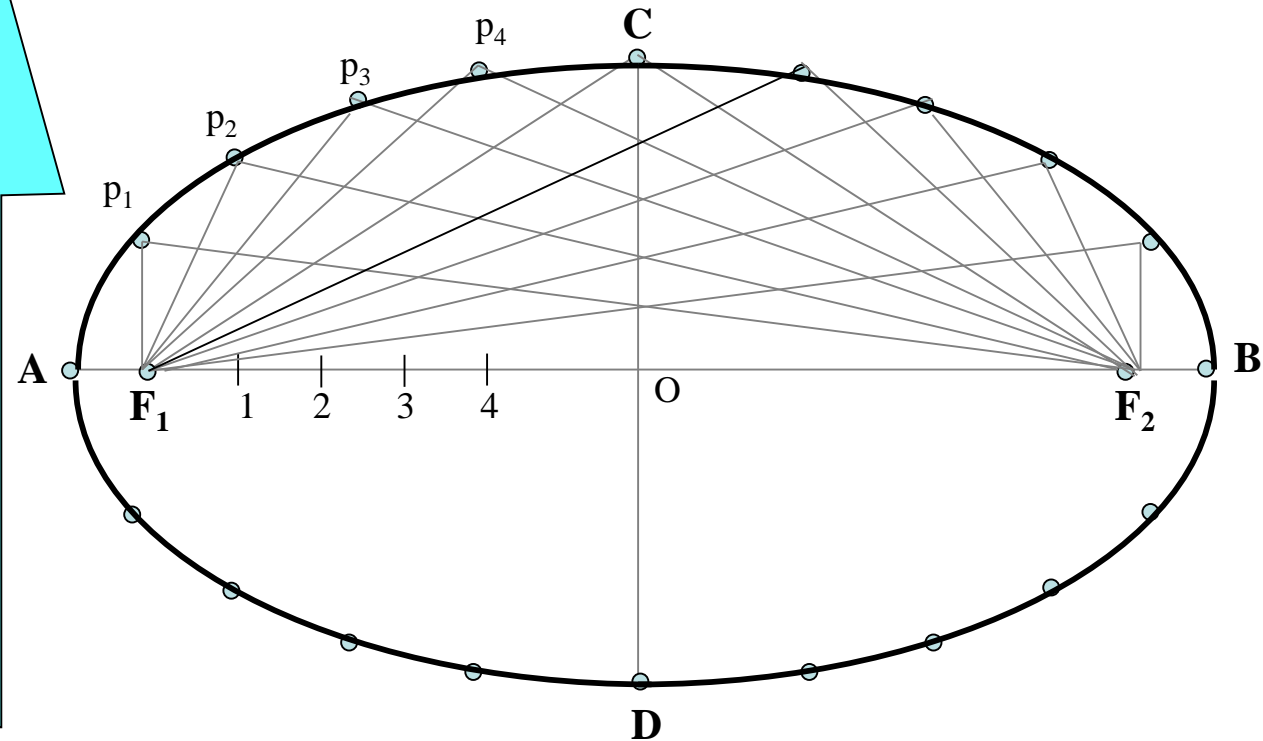
**STEPS ARE SIMILAR TO  
THE PREVIOUS CASE  
(RECTANGLE METHOD)  
ONLY IN PLACE OF RECTANGLE,  
HERE IS A PARALLELOGRAM.**



# ELLIPSE

## BY ARCS OF CIRCLE METHOD

As per the definition Ellipse is locus of point P moving in a plane such that the **SUM** of it's distances from two fixed points ( $F_1$  &  $F_2$ ) remains constant and equals to the length of major axis AB. (Note  $A_1 + B_1 = A_2 + B_2 = AB$ )



### PROBLEM 4.

MAJOR AXIS AB & MINOR AXIS CD ARE 100 AND 70MM LONG RESPECTIVELY .DRAW ELLIPSE BY ARCS OF CIRCLES METHOD.

#### STEPS:

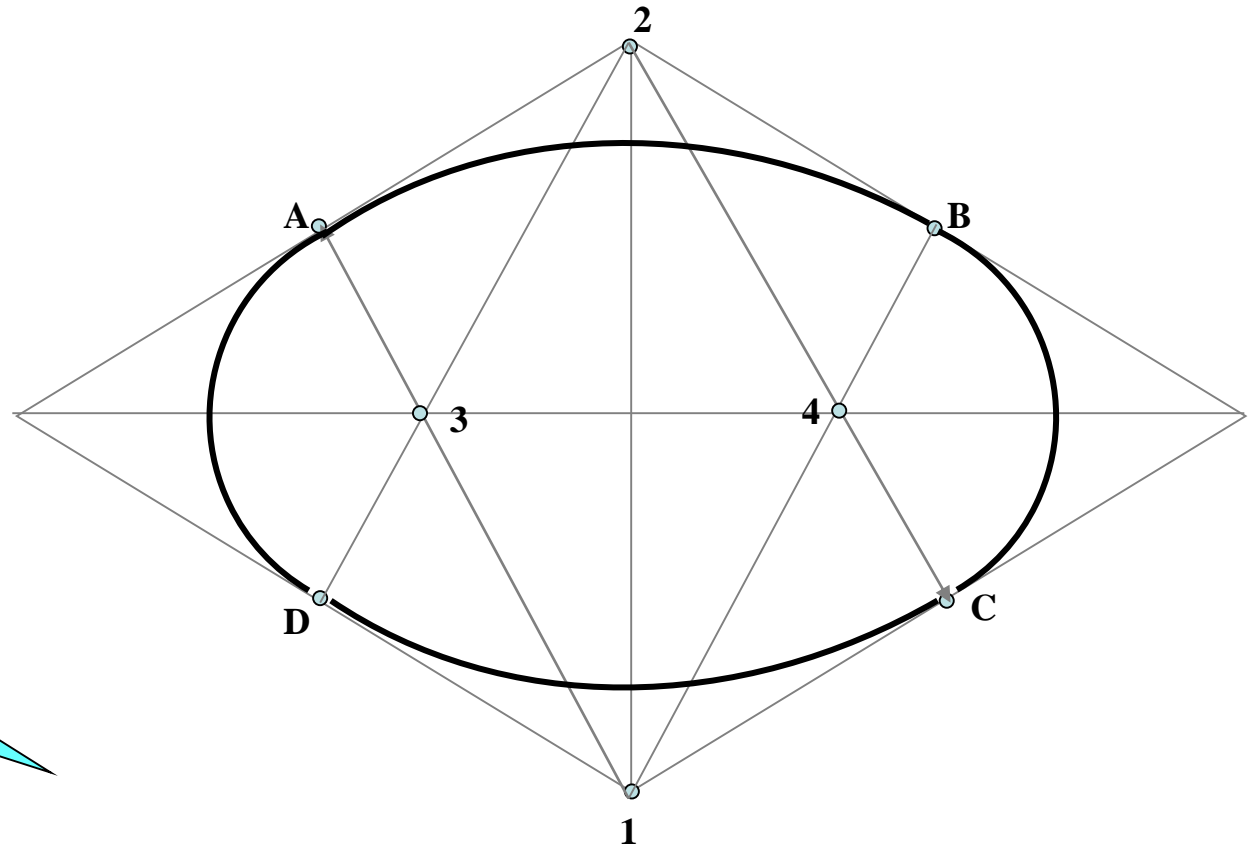
1. Draw both axes as usual. Name the ends & intersecting point
2. Taking AO distance i.e. half major axis, from C, mark  $F_1$  &  $F_2$  on AB . ( focus 1 and 2.)
3. On line  $F_1 - O$  taking any distance, mark points 1, 2, 3, & 4
4. Taking  $F_1$  center, with distance A-1 draw an arc above AB and taking  $F_2$  center, with B-1 distance cut this arc. Name the point  $p_1$
5. Repeat this step with same centers but taking now A-2 & B-2 distances for drawing arcs. Name the point  $p_2$
6. Similarly get all other P points.  
With same steps positions of P can be located below AB.
7. Join all points by smooth curve to get an ellipse/

**PROBLEM 5.**

DRAW RHOMBUS OF 100 MM & 70 MM LONG  
DIAGONALS AND INSCRIBE AN ELLIPSE IN IT.

**STEPS:**

1. Draw rhombus of given dimensions.
2. Mark mid points of all sides & name Those A,B,C,& D
3. Join these points to the ends of smaller diagonals.
4. Mark points 1,2,3,4 as four centers.
5. Taking 1 as center and 1-A radius draw an arc AB.
6. Take 2 as center draw an arc CD.
7. Similarly taking 3 & 4 as centers and 3-D radius draw arcs DA & BC.



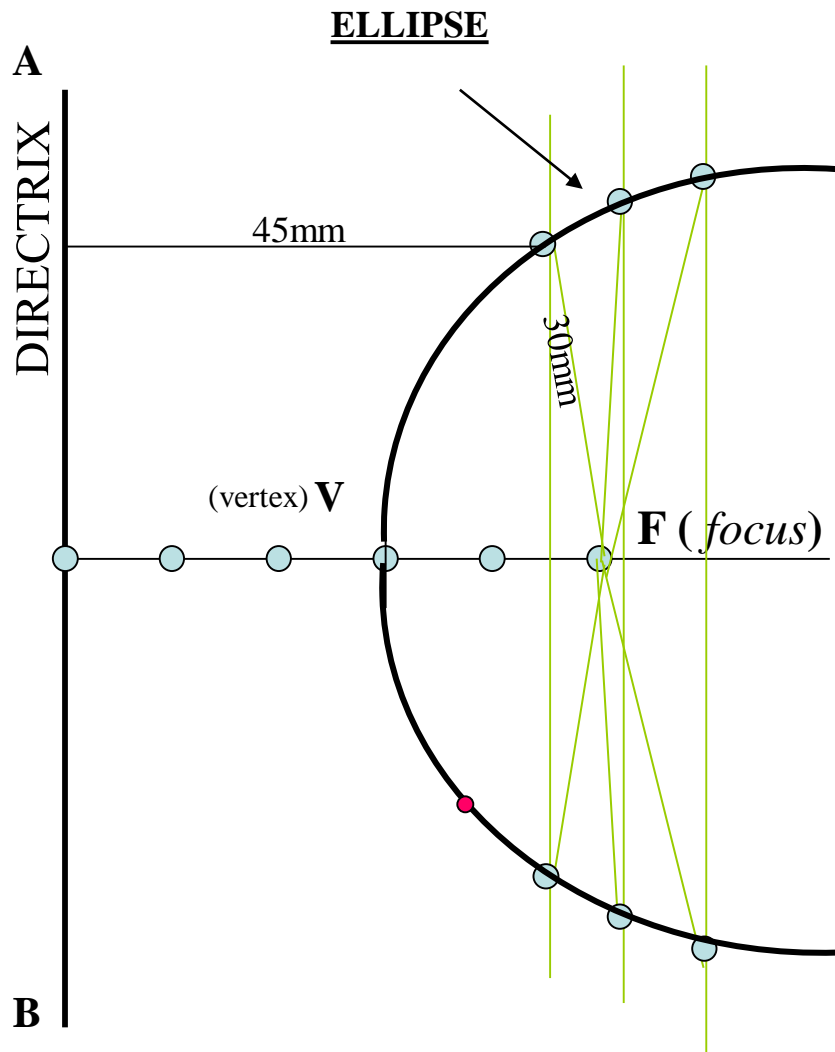


**PROBLEM 6:-** POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE **RATIO** OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO **2/3** DRAW LOCUS OF POINT P. { **ECCENTRICITY = 2/3** }

**STEPS:**

1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2<sup>nd</sup> part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB  $2/3$  i.e  $20/30$
4. Form more points giving same ratio such as  $30/45$ ,  $40/60$ ,  $50/75$  etc.
5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.

This is required locus of P. It is an ELLIPSE.

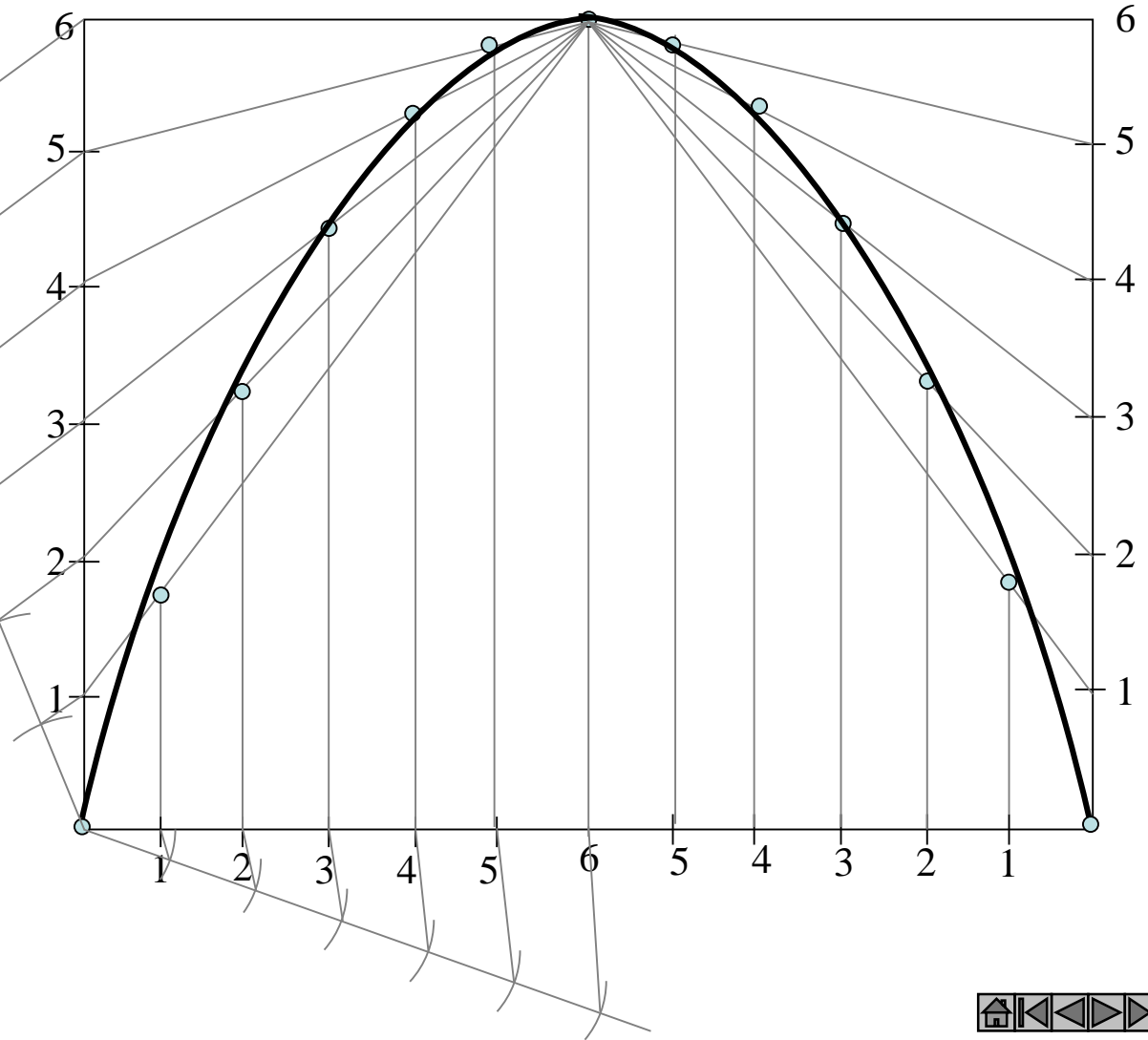


**PROBLEM 7: A BALL THROWN IN AIR ATTAINS 100 M HEIGHT AND COVERS HORIZONTAL DISTANCE 150 M ON GROUND.**  
 Draw the path of the ball (projectile)-

## PARABOLA RECTANGLE METHOD

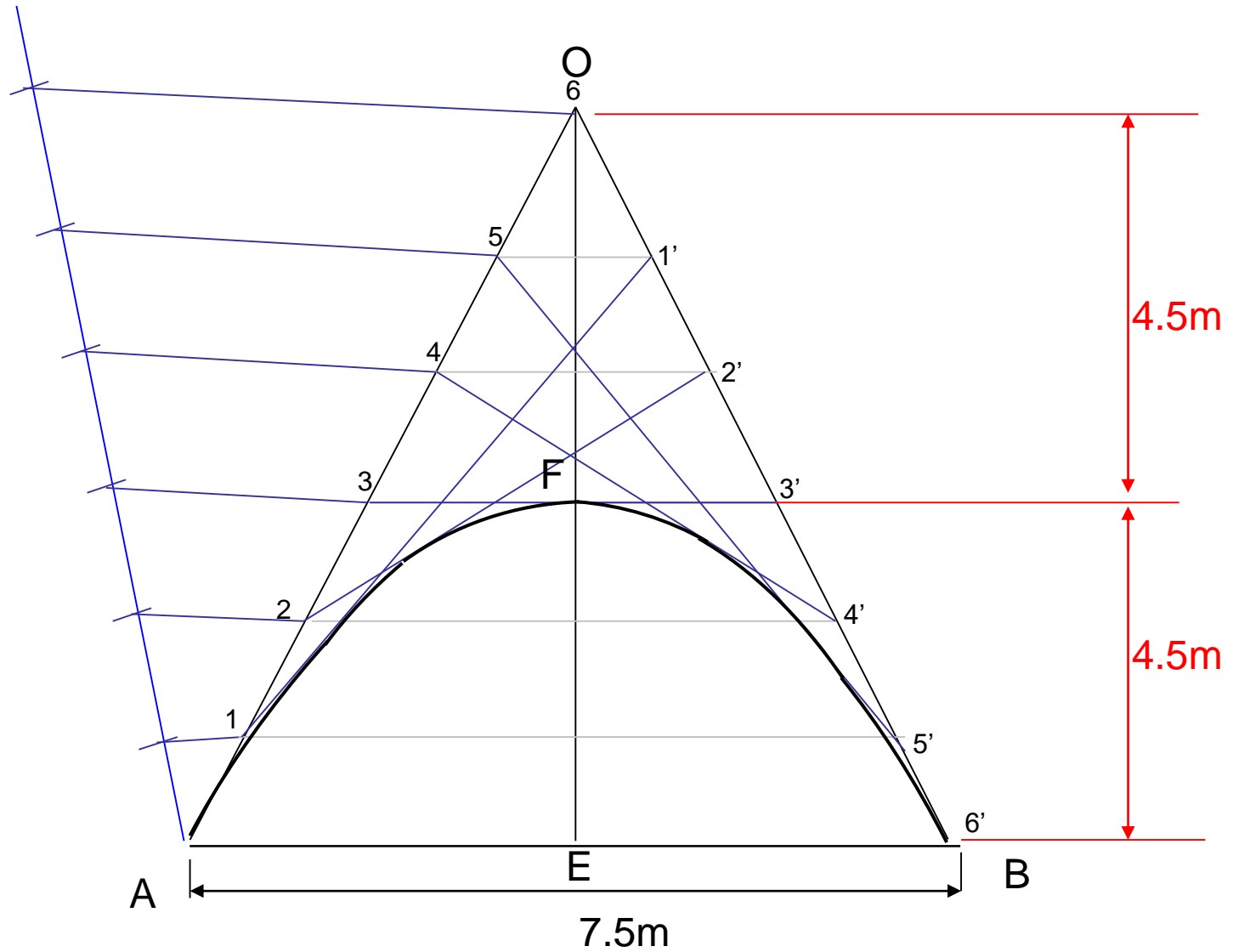
### STEPS:

1. Draw rectangle of above size and divide it in two equal vertical parts
  2. Consider left part for construction. Divide height and length in equal number of parts and name those 1,2,3,4,5 & 6
  3. Join vertical 1,2,3,4,5 & 6 to the top center of rectangle
  4. Similarly draw upward vertical lines from horizontal 1,2,3,4,5. And wherever these lines intersect previously drawn inclined lines in sequence Mark those points and further join in smooth possible curve.
  5. Repeat the construction on right side rectangle also. Join all in sequence.
- This locus is Parabola.**



Draw a parabola by tangent method given base 7.5m and axis 4.5m

Take scale 1cm = 0.5m



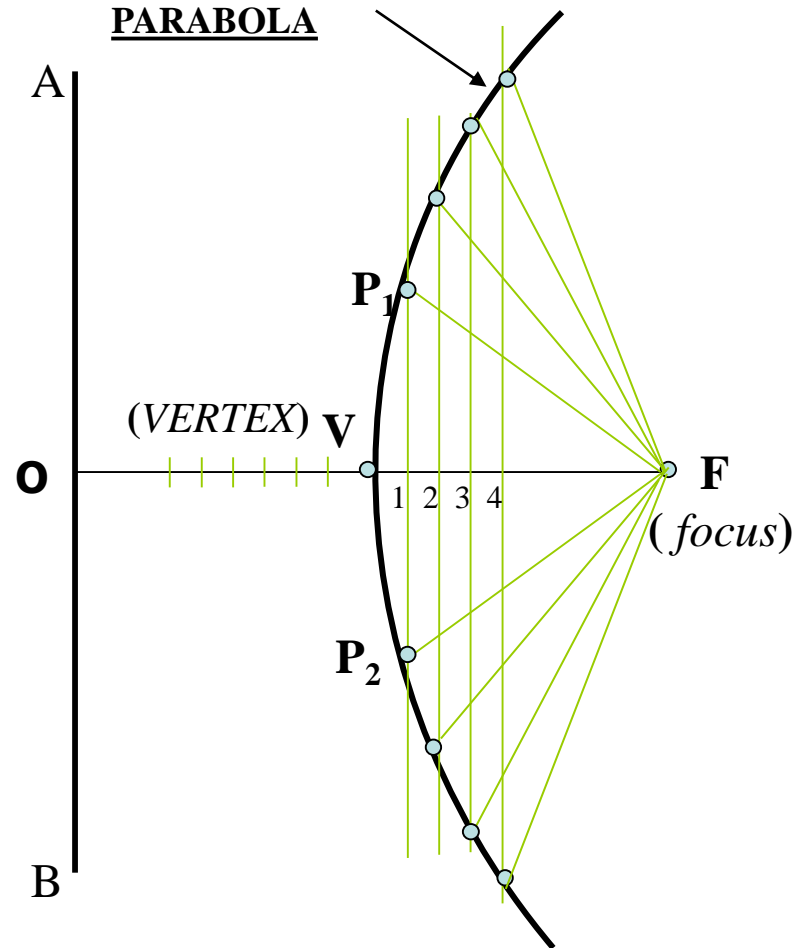
**PROBLEM 9:** Point F is 50 mm from a vertical straight line AB. Draw locus of point P, moving in a plane such that it always remains equidistant from point F and line AB.

## PARABOLA DIRECTRIX-FOCUS METHOD

### SOLUTION STEPS:

1. Locate center of line, perpendicular to AB from point F. This will be initial point P and also the vertex.
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.
3. Mark 5 mm distance to its left of P and name it 1.
4. Take O-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point  $P_1$  and lower point  $P_2$ . ( $FP_1 = O1$ )
5. Similarly repeat this process by taking again 5mm to right and left and locate  $P_3, P_4$ .
6. Join all these points in smooth curve.

It will be the locus of P equidistance from line AB and fixed point F.

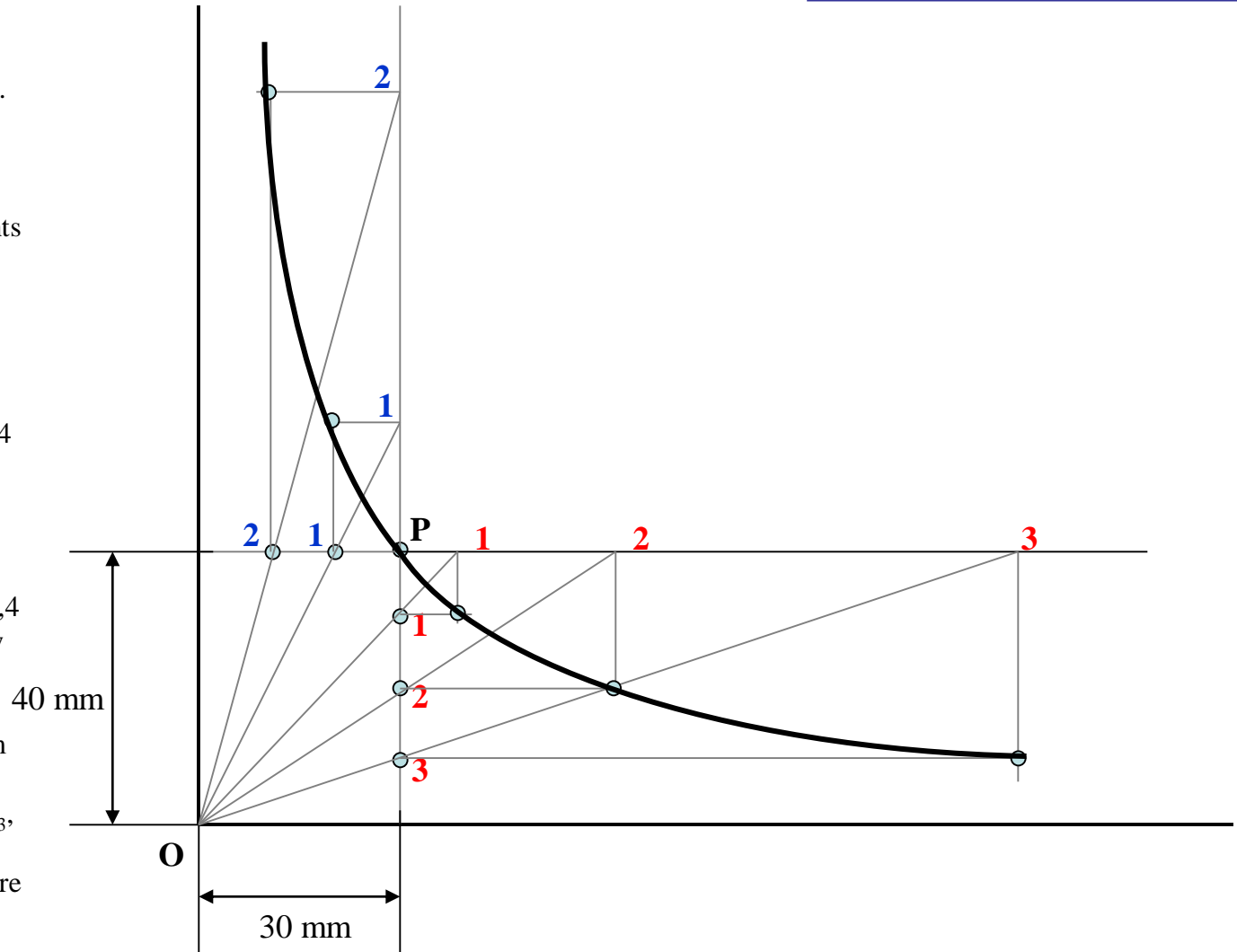


**Problem No.10:** Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.

## HYPERBOLA THROUGH A POINT OF KNOWN CO-ORDINATES

### *Solution Steps:*

- 1) Extend horizontal line from P to right side.
- 2) Extend vertical line from P upward.
- 3) On horizontal line from P, mark some points taking any distance and name them after P-1, 2,3,4 etc.
- 4) Join 1-2-3-4 points to pole O. Let them cut part [P-B] also at 1,2,3,4 points.
- 5) From horizontal 1,2,3,4 draw vertical lines downwards and
- 6) From vertical 1,2,3,4 points [from P-B] draw horizontal lines.
- 7) Line from 1 horizontal and line from 1 vertical will meet at  $P_1$ . Similarly mark  $P_2, P_3, P_4$  points.
- 8) Repeat the procedure by marking four points on upward vertical line from P and joining all those to pole O. Name this points  $P_6, P_7, P_8$  etc. and join them by smooth curve.



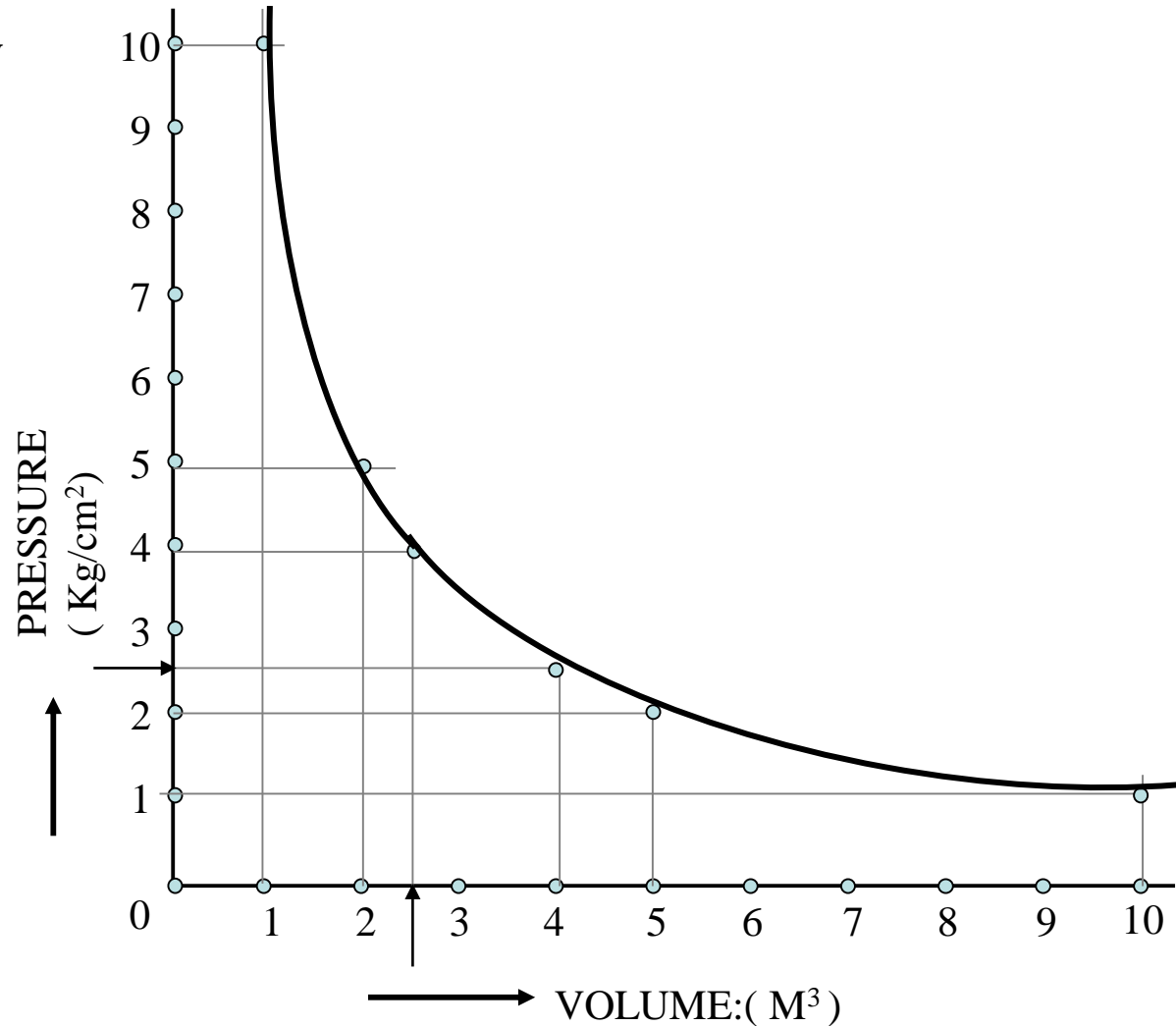
**Problem no.11:** A sample of gas is expanded in a cylinder from 10 unit pressure to 1 unit pressure. Expansion follows law  $PV=\text{Constant}$ . If initial volume being 1 unit, draw the curve of expansion. Also Name the curve.

## HYPERBOLA P-V DIAGRAM

Form a table giving few more values of P & V

$P \times V = C$		
10	$\times$	1 = 10
5	$\times$	2 = 10
4	$\times$	2.5 = 10
2.5	$\times$	4 = 10
2	$\times$	5 = 10
1	$\times$	10 = 10

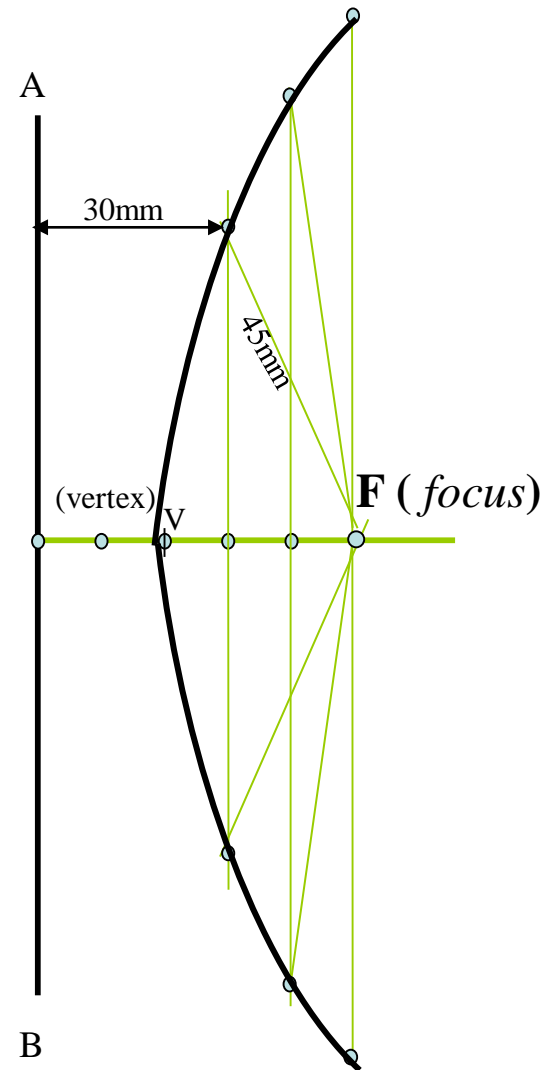
Now draw a Graph of Pressure against Volume.  
It is a PV Diagram and it is Hyperbola.  
Take pressure on vertical axis and Volume on horizontal axis.



**PROBLEM 12:-** POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE **RATIO** OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO **2/3** DRAW LOCUS OF POINT P. { **ECCENTRICITY = 2/3** }

**STEPS:**

1. Draw a vertical line AB and point F 50 mm from it.
  2. Divide 50 mm distance in 5 parts.
  3. Name 2<sup>nd</sup> part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB  $2/3$  i.e  $20/30$
  4. Form more points giving same ratio such as  $30/45$ ,  $40/60$ ,  $50/75$  etc.
  5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
  6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
  7. Join these points through V in smooth curve.
- This is required locus of P. It is an ELLIPSE.



1. JOIN POINT Q TO  $F_1$  &  $F_2$
2. BISECT ANGLE  $F_1QF_2$  THE ANGLE BISECTOR IS NORMAL
3. A PERPENDICULAR LINE DRAWN TO IT IS TANGENT TO THE CURVE.



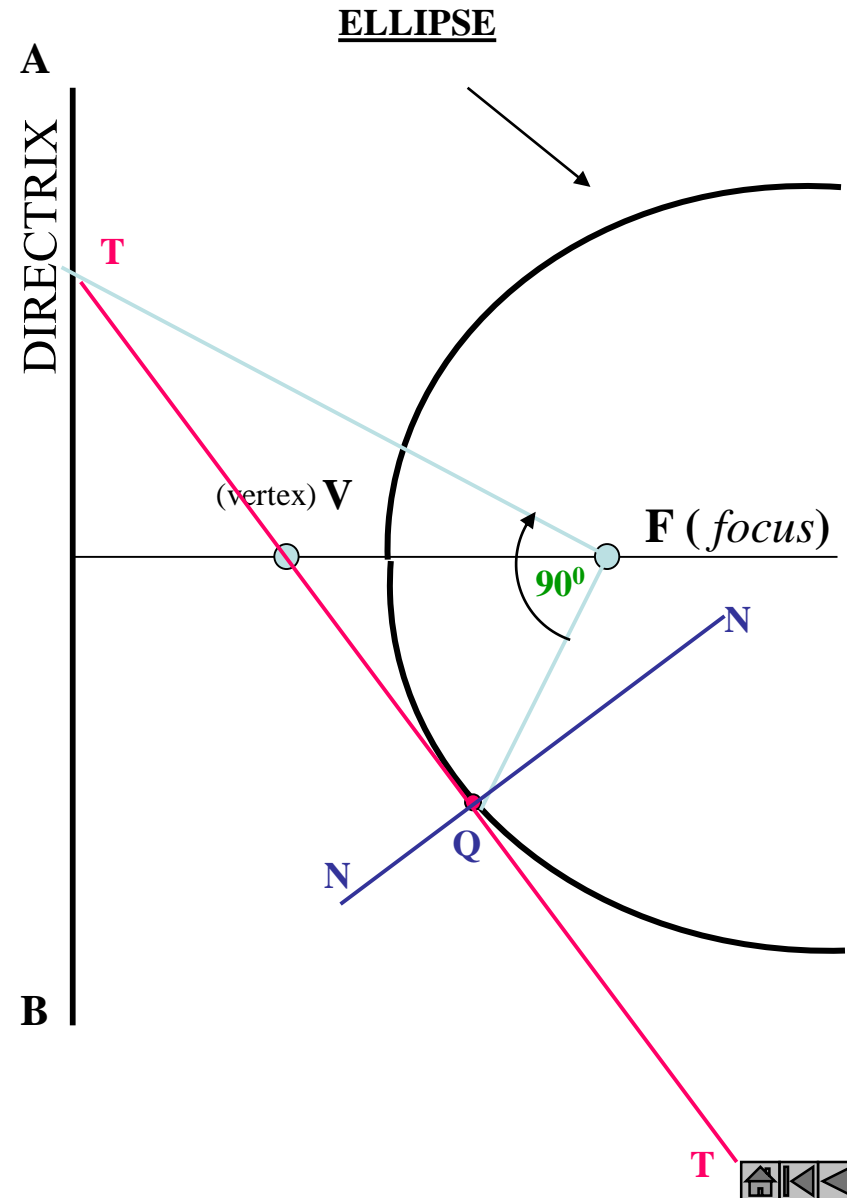


### Problem 14:

## TO DRAW TANGENT & NORMAL TO THE CURVE FROM A GIVEN POINT ( Q )

1. JOIN POINT Q TO F.
2. CONSTRUCT 90° ANGLE WITH THIS LINE AT POINT F
3. EXTEND THE LINE TO MEET DIRECTRIX AT T
4. JOIN THIS POINT TO Q AND EXTEND. THIS IS TANGENT TO ELLIPSE FROM Q
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM Q. IT IS NORMAL TO CURVE.

## ELLIPSE TANGENT & NORMAL

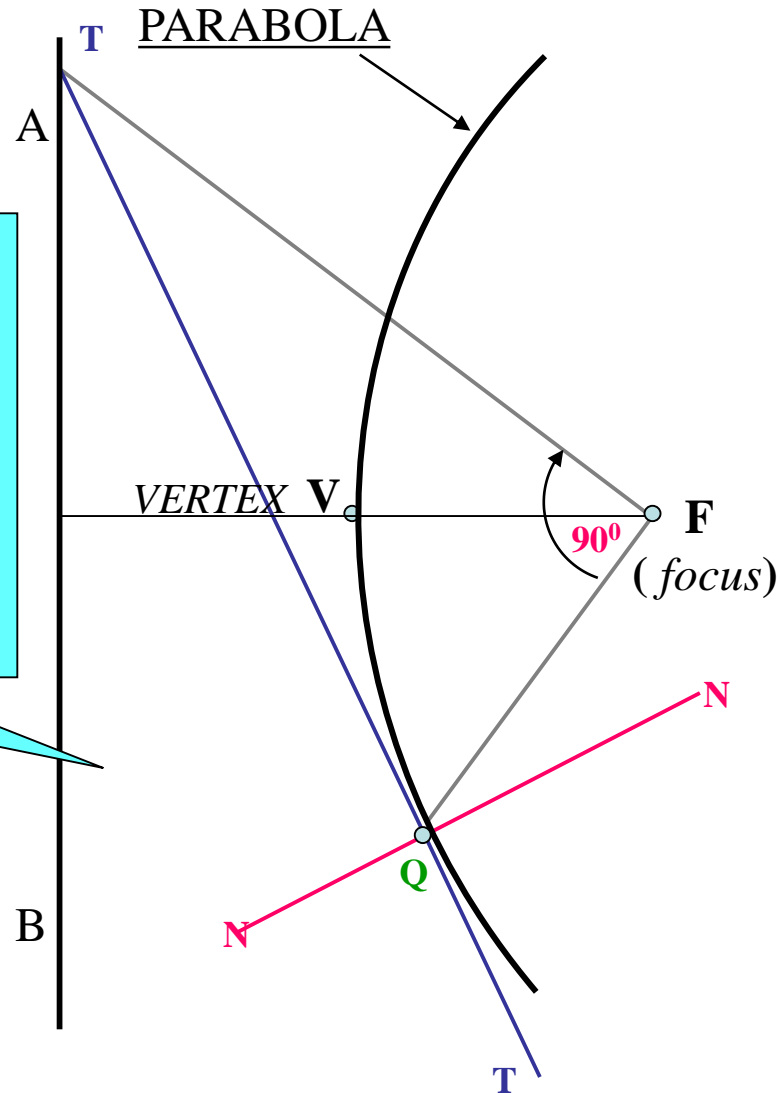


## Problem 15:

### TO DRAW TANGENT & NORMAL TO THE CURVE FROM A GIVEN POINT ( Q )

1. JOIN POINT **Q** TO **F**.
2. CONSTRUCT  $90^\circ$  ANGLE WITH THIS LINE AT POINT **F**
3. EXTEND THE LINE TO MEET DIRECTRIX AT **T**
4. JOIN THIS POINT TO **Q** AND EXTEND. THIS IS TANGENT TO THE CURVE FROM **Q**
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM **Q**. IT IS NORMAL TO CURVE.

## PARABOLA TANGENT & NORMAL



## Problem 16

### TO DRAW TANGENT & NORMAL TO THE CURVE FROM A GIVEN POINT ( Q )

1. JOIN POINT **Q** TO **F**.
2. CONSTRUCT  $90^\circ$  ANGLE WITH THIS LINE AT POINT **F**
3. EXTEND THE LINE TO MEET DIRECTRIX AT **T**
4. JOIN THIS POINT TO **Q** AND EXTEND. THIS IS TANGENT TO CURVE FROM **Q**
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM **Q**. IT IS NORMAL TO CURVE.

## HYPERBOLA TANGENT & NORMAL

