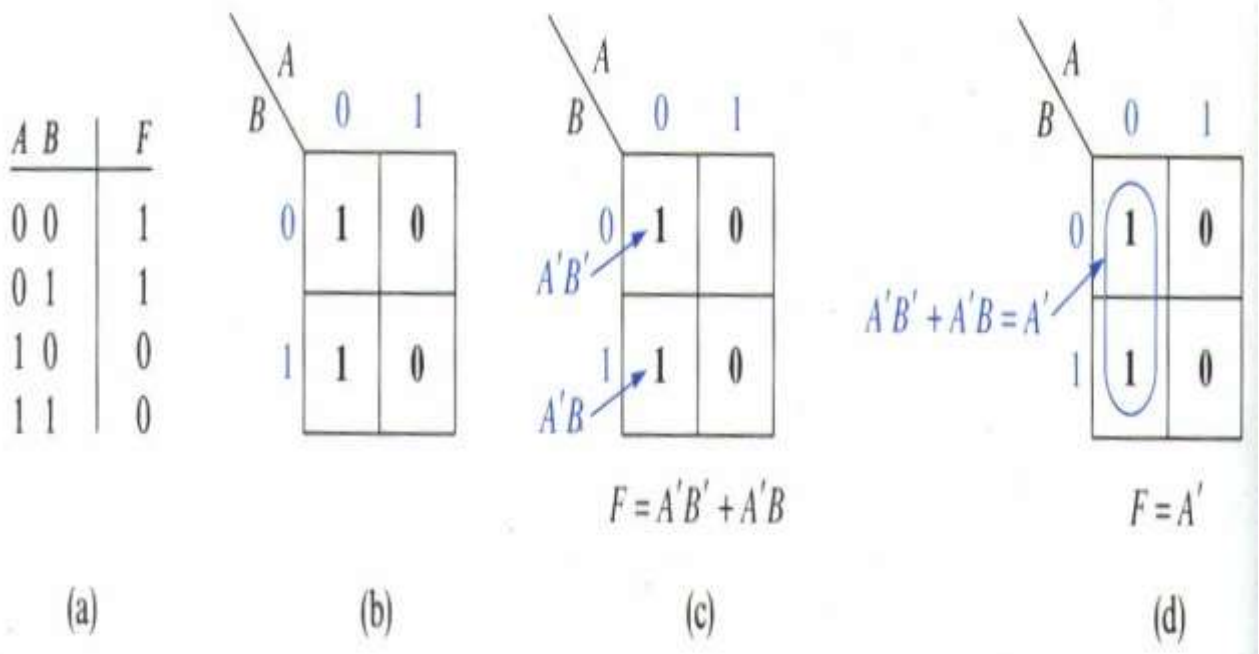


Karnaugh Maps

- Algebraic procedures:
 - Difficult to apply in a systematic way.
 - Difficult to tell when you have arrived at a minimum solution.
- Karnaugh map (K-map) can be used to minimize functions of up to 6 variables.
 - K-map is directly applied to two-level networks composed of AND and OR gates.
 - Sum-of-products, (SOP)
 - Product-of-sum, (POS).

2-Variable K-map

- Place 1s and 0s from the truth table in the K-map.
- Each square of 1s = minterms.
- Minterms in adjacent squares can be combined since they differ in only one variable. Use $XY' + XY = X$.



3-Variable K-map

- Note BC is listed in the order of 00, 01, 11, 10. (Gray code)
- Minterms in adjacent squares that differ in only one variable can be combined using $XY' + XY = X$.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

(a)

A \ BC		0	1
BC	00	0	1
	01	0	0
	11	1	0
	10	1	1
F			

$ABC = 001, F = 0$

$ABC = 110, F = 1$

(b)

Location of Minterms

- Adjacent terms in 3-variable K map.

A 3-variable Karnaugh map in binary notation. The vertical axis is labeled 'a' with values 00, 01, 11, 10. The horizontal axis is labeled 'bc' with values 0 and 1. The cells contain the following binary strings: (00,0) is 000, (00,1) is 100, (01,0) is 001, (01,1) is 101, (11,0) is 011, (11,1) is 111, (10,0) is 010, and (10,1) is 110. Blue arrows indicate adjacencies: a vertical double-headed arrow between 001 and 011, a horizontal double-headed arrow between 011 and 111, a vertical double-headed arrow between 011 and 010, a blue line connecting 100 and 110, and a blue line connecting 100 and 111. A text label '100 is adjacent to 110' is placed to the right of the map.

$a \backslash bc$	0	1
00	000	100
01	001	101
11	011	111
10	010	110

(a) Binary notation

A 3-variable Karnaugh map in decimal notation. The vertical axis is labeled 'a' with values 00, 01, 11, 10. The horizontal axis is labeled 'bc' with values 0 and 1. The cells contain the following decimal values: (00,0) is 0, (00,1) is 4, (01,0) is 1, (01,1) is 5, (11,0) is 3, (11,1) is 7, (10,0) is 2, and (10,1) is 6.

$a \backslash bc$	0	1
00	0	4
01	1	5
11	3	7
10	2	6

(b) Decimal notation

K Map Example

- K-map of $F(a,b,c) = \sum m(1,3,5)$
 $= \prod M(0,2,4,6,7)$

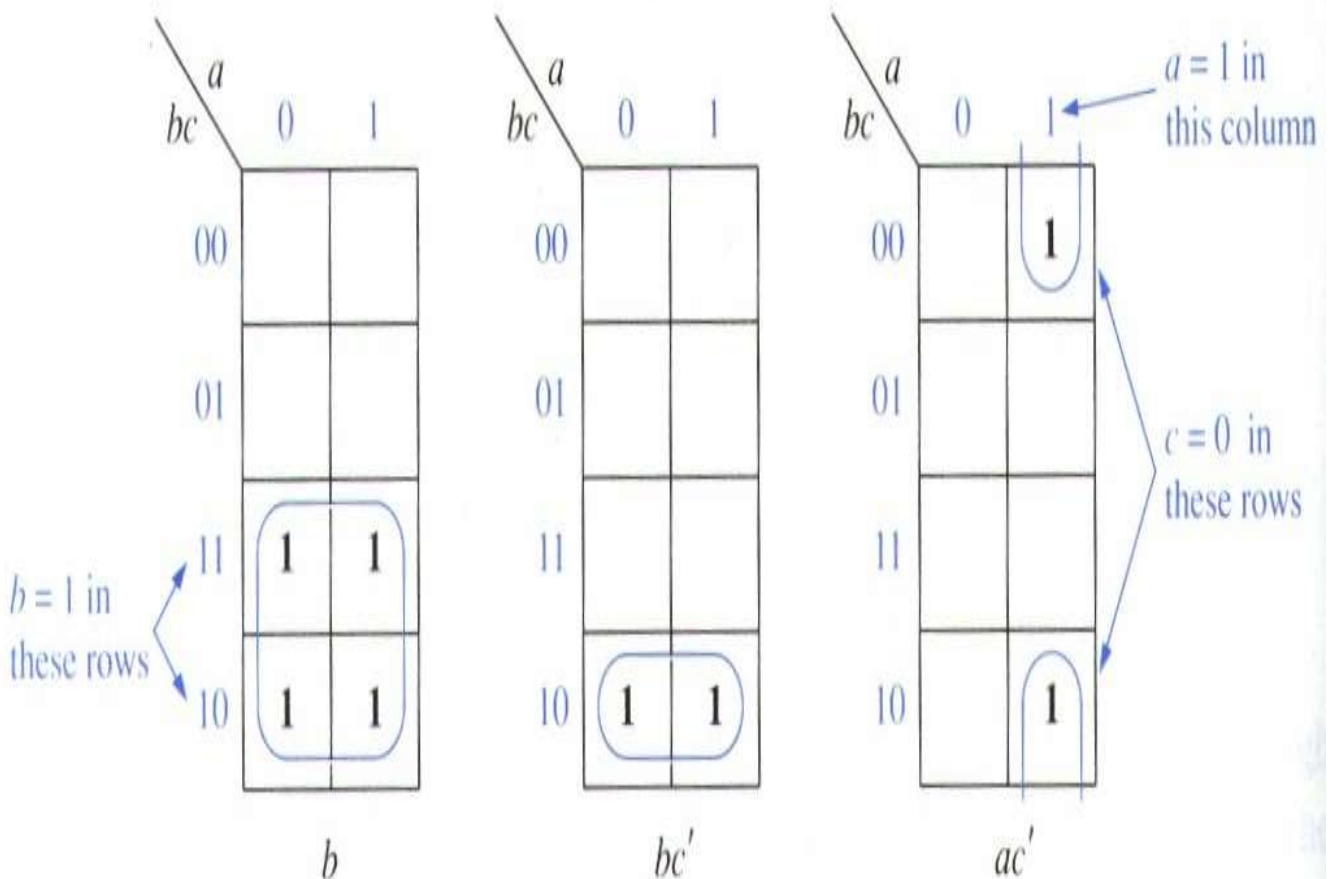
a bc		0	1
		00	01
00	0	0	0
01	1	1	1
11	1	0	0
10	0	0	0

Karnaugh Map of

$$F(a, b, c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$$

Place Product Terms on K Map

- Example
 - Place b , bc' and ac' in the 3-variable K map.



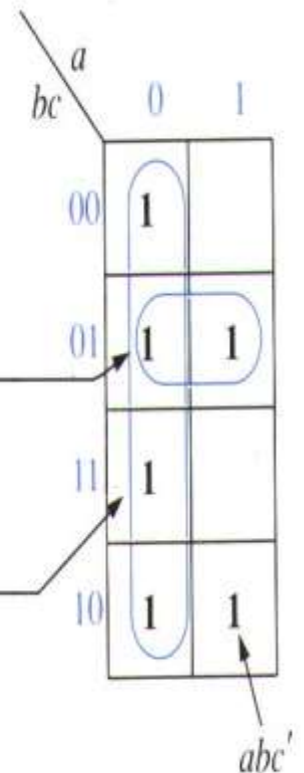
More Example

- Exercise. Plot $f(a, b, c) = abc' + b'c + a'$ into the K-map.

$$f(a, b, c) = abc' + b'c + a'$$

we would plot the map as follows:

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map.
(Note: since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)



Simplification Example

- Exercise. Simplify: $F(a,b,c) = \sum m(1,3,5)$
 - Procedure: place minterms into map.
 - Select adjacent 1's in group of two 1's or four 1's etc.
 - Kick off x and x' .

$a \backslash bc$	0	1
00		
01	1	1
11	1	
10		

$$F = \sum m(1, 3, 5)$$

(a) Plot of minterms

$a \backslash bc$	0	1
00		
01	1	1
11	1	
10		

$$T_1 = a'b'c + a'bc = a'c$$

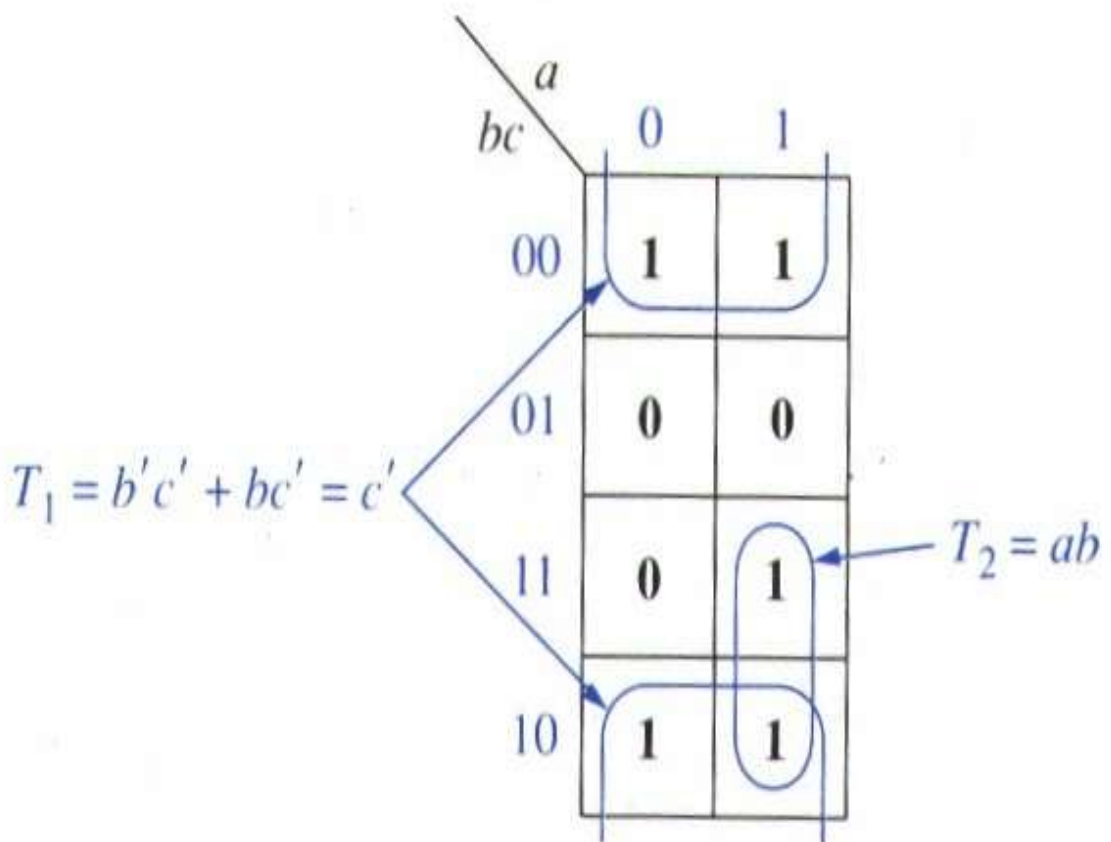
$$T_2 = a'b'c + ab'c = b'c$$

$$F = a'c + b'c$$

(b) Simplified form of F

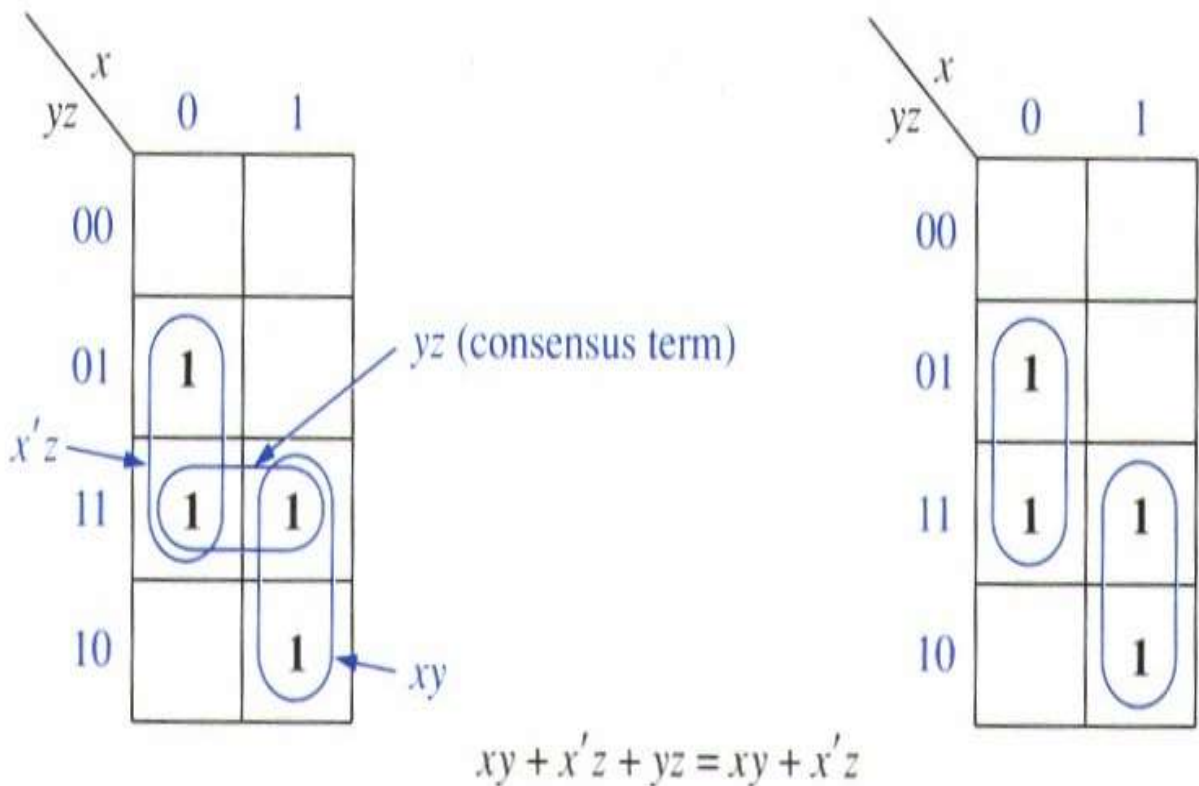
More Example

- The complement of F
 - Using four 1's to eliminate two variables.



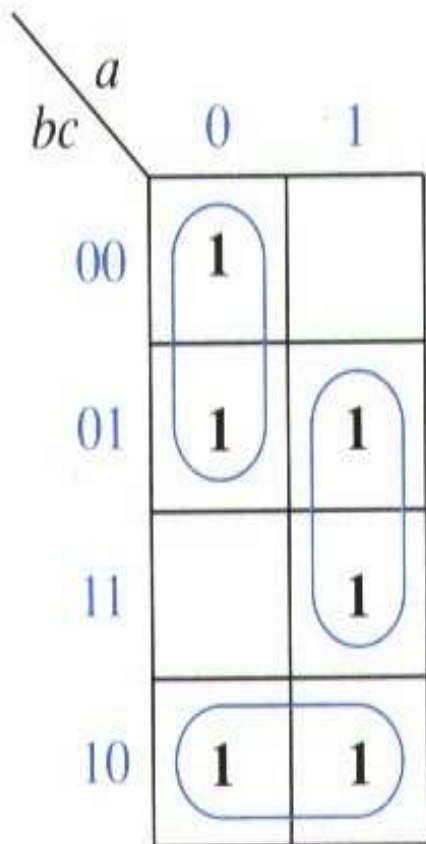
Redundant Terms

- If a term is covered by two other terms, that term is redundant. That is, it is a consensus term.
- Example: yz is the redundant.

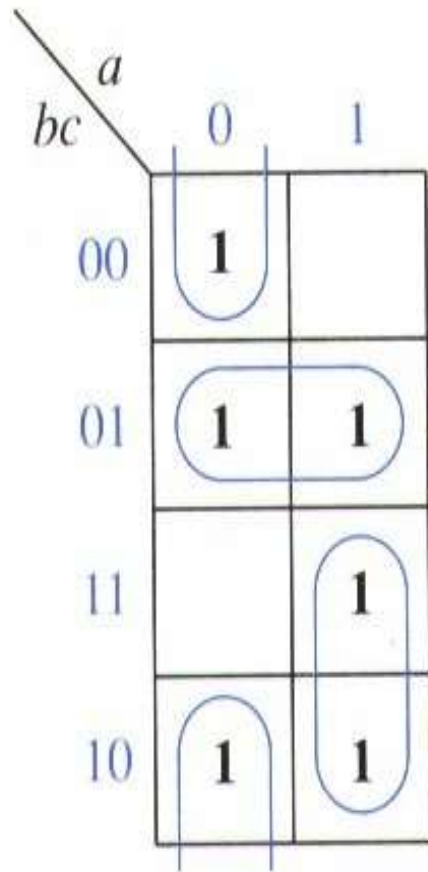


More Than Two Minimum Solutions

- $F = \sum m(0,1,2,5,6,7)$



$$F = a'b' + bc' + ac$$



$$F = a'c' + b'c + ab$$

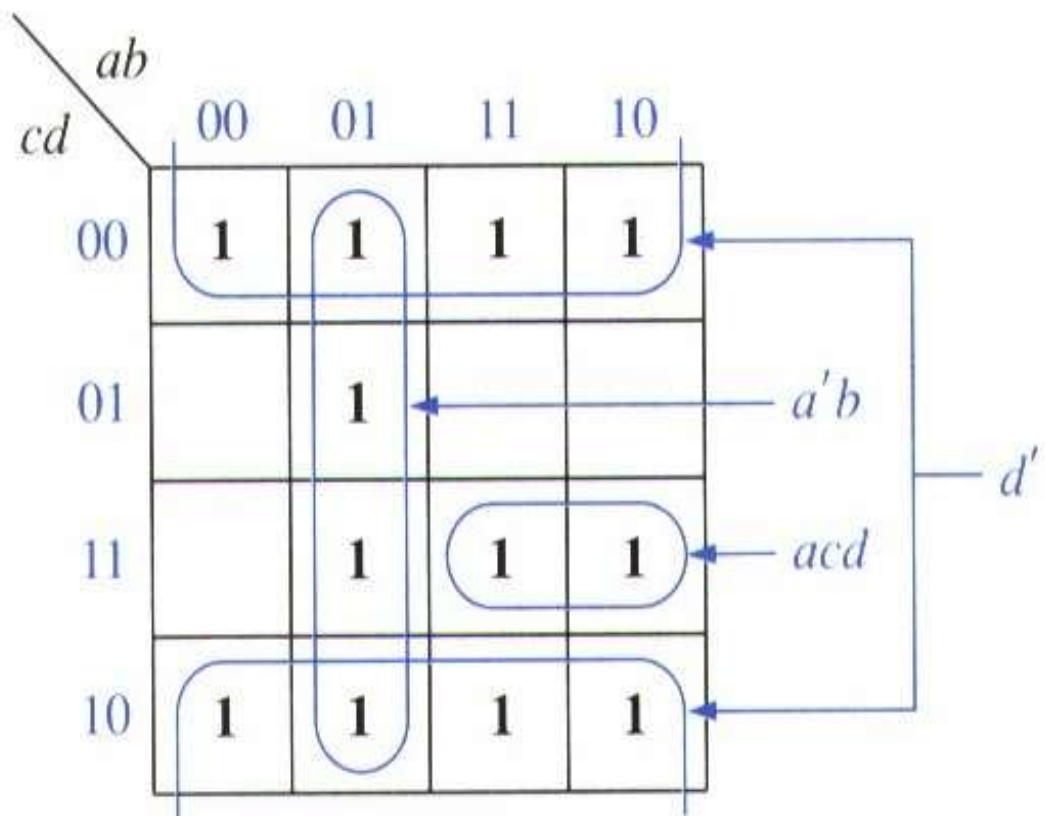
4-Variable K Map

- Each minterm is adjacent to 4 terms with which it can combine.
 - 0, 8 are adjacent squares
 - 0, 2 are adjacent squares, etc.
 - 1, 4, 13, 7 are adjacent to 5.

<i>AB</i>					
<i>CD</i>		00	01	11	10
	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

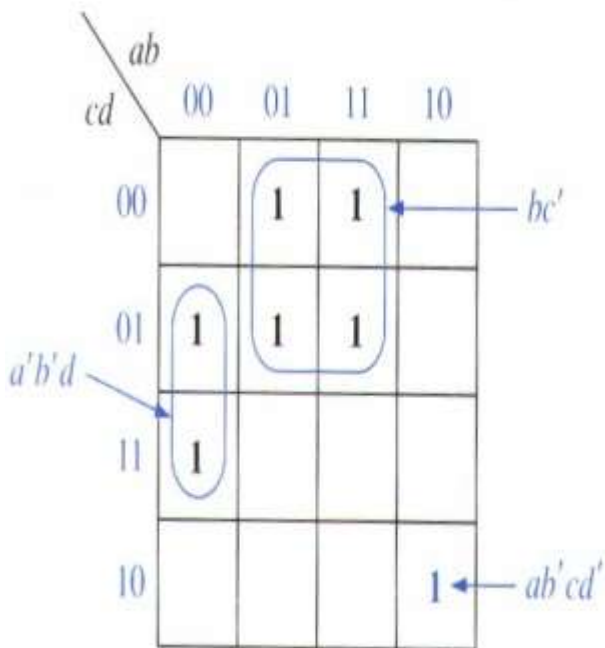
Plot a 4-variable Expression

- $F(a,b,c,d) = acd + a'b + d'$
 $acd = 1$ if $a=1, c=1, d=1$



Simplification Example

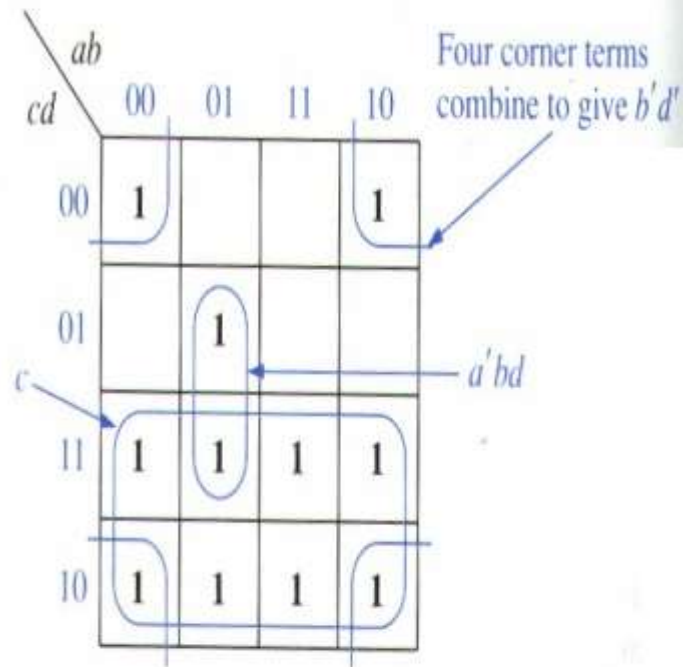
- Minterms are combined in groups of 2, 4, or 8 to eliminate 1, 2, 3 variables.
- Corner terms.



$$f_1 = \sum m(1, 3, 4, 5, 10, 12, 13)$$

$$= bc' + a'b'd + ab'cd'$$

(a)



$$f_2 = \sum m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$$

$$= c + b'd' + a'bd$$

(b)

Simplification with Don't Care

- Don't care “x” is covered if it helps. Otherwise leave it along.

<i>ab</i> <i>cd</i> \	00	01	11	10
00			x	
01	1	1	x	1
11	1	1		
10		x		

$$\begin{aligned} f &= \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13) \\ &= a'd + c'd \end{aligned}$$

Eg Simplify Boolean fn

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

x

	AB \ CD	00	01	11	10
00	00	1	1		1
01	01				1
11	11				
10	10	1	1		1

$$F = B'C' + B'D' + A'CD'$$

Eg Simplify Boolean fn

$$F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$$F = y' + w'z' + xz'$$

JUL 2015							AUG 2015						
M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	31					1	2
6	7	8	9	10	11	12	3	4	5	6	7	8	9
13	14	15	16	17	18	19	10	11	12	13	14	15	16
20	21	22	23	24	25	26	17	18	19	20	21	22	23
27	28	29	30	31			24	25	26	27	28	29	30

wx / yz

		00	01	11	10
9	00	1	1		1
10	01	1	1		1
	11	1	1		1
11	10	1	1		

Eg 3.3 Simplify the Boolean fn.

$$F = A'B'C + A'B + AB'C + BC$$

A \ BC				
	00	01	11	10
0		1	1	1
1		1	1	

$$F = C + A'B$$

Eg Simplify the Boolean fn

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

x \ yz				
	00	01	11	10
0	1			1
1	1	1	0	1

$$= z' + xy'$$

Ex Simplify the Boolean fn

$$F = x'y'z + x'y'z' + xy'z' + xy'z$$

x	yz			
	00	01	11	10
0			1	1
1	1	1		

$$F = x'y + xy'$$

Ex Simplify Boolean fn

$$F = x'y'z + xy'z' + x'yz + x'yz'$$

x	yz			
	00	01	11	10
0			1	
1	1		1	1

$$F = yz + xz'$$

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

Ex & press Boolean fn $F = xy + x^2z$
 is a product of max term
 form First convert to OR
 terms & using the distrib law.

$$F = xy + x^2z$$

$$= (xy + x^2)(x + z)$$

$$= (x + x^2)(y + x^2)(x + z)(y + z)$$

$$= (x^2 + y)(x + z)(y + z)$$

$$x^2 + y = x^2 + y + xz z^2 = (x^2 + y + xz)(x^2 + y + z)$$

$$x + z = x + z + xy y^2$$

$$= (x + y + z)(x + y^2 + z)$$

$$y + z = y + z + x x^2$$

$$= (x + y + z)(x^2 + y + z)$$

$$f = m_0 m_2 m_4 m_5$$

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

Eg: Express the Boolean fn
 $F = A + B'C$ in a Sum of
min term.

15

$$\rightarrow F = A + B'C$$

16

$$A = A(B + B')$$

17

$$= AB + AB'$$

$$= AB(C + C') + AB'(C + C')$$

18

$$= ABC + ABC' + AB'C + AB'C'$$

$$B'C = B'C(A + A')$$

$$= B'CA + B'CA'$$

$$F = ABC + ABC' + \underline{AB'C} + \underline{AB'C'} + \underline{B'CA} + \underline{B'CA'}$$

Notes

$$= m_7 + m_6 + m_5 + m_4 + m_1$$

It is as hard to take success as it is failure

10 Wednesday

A Fine 6 Six Variable Maps

	000	001	011	010	110	111	101	100
AB	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

$\underbrace{\hspace{10em}}_E$
 $\underbrace{\hspace{10em}}_E$

Eg simplify the Boolean for

$$F(A, B, C, D, E) = \Sigma(0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)$$

AB	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10		1	1	

$$F = A'B'D' + BE + EA + D'E'$$

$$F = BE + A'D'E + A'B' \downarrow (D'E' + D'E)$$

$$A'B'E'(C)$$

Perhaps the reward of the spirit who tries is not the goal but the exercise

Eg Simplify the Boolean fn in
 a) Sum of product
 b) Product of sum

$$F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10)$$

AB \ CD		00	01	11	10
00	1	1	0	1	
01	0	1	0	0	
11	0	0	0	0	
10	1	1	0	1	

$$F = C'D' + B'D' + A'C'D'$$

$$= (C' + B')$$

$$F' = AB + CD + BD'$$

$$F = (A' + B') \cdot (C' + D') \cdot (B' + D)$$

Eg Simplify the Boolean fn

$$F(w, x, y, z) = \sum(1, 3, 7, 11, 15)$$

the don't care condition.

$$d(w, x, y, z) = \sum(0, 2, 5)$$

Notes

The best way to succeed in this world is to act on the advice you give to others.

wx	$y_0 z_0$	01	11	10
00	X	1	1	X
01		X	1	
11			1	
10			1	

$$F = \overline{z} y z + w' z$$

$$= z (y + w')$$

wx	$y_0 z_0$	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

$$F' = z' \bar{w} y'$$

$$F = z \cdot (w' + y)$$

Variable Mapping

$$f = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}D$$

A	BC			
	00	01	11	10
0	1	1	1	1
1				D

A	BC			
	00	01	11	10
0	0+1	0+1	0+1	0+1
1				1

$$f_{\text{min}} = \bar{A} + B\bar{C}D$$

Eg Reduce by Mapping

$$f = \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D}$$

$$F = m_1(0+1) + m_6(0+1) + m_7(0+1) + m_5\bar{D} + m_9\bar{D} + m_2\bar{D} + m_4\bar{D}$$

A	BC			
	00	01	11	10
0		1	1	1
1	1	1	1	1

A	BC			
	00	01	11	10
0		1	1	1
1	1	1	1	1

$$f = \bar{B}C\bar{D} + \bar{A}C\bar{D} + A\bar{B} + A\bar{C}\bar{D}$$

15

wk 25 - day 166

Monday

	1	2	3	4	5	6	7
4	5	6	7	8	9	10	11
11	12	13	14	15	16	17	18
18	19	20	21	22	23	24	25
25	26	27	28	29	30	31	29

eg Reduce by Mapping

$$f = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} C D + \overline{A} B C \overline{E} + \overline{A} B \overline{C} E + A \overline{B} C + A B C + A B \overline{C} \overline{D}$$

		BC			
A	D	00	01	11	10
	0	1	0	E	E
1	1		1	1	0

$$f = m_0 + m_1 D + m_3 E + m_2 E + m_5 + m_2 + m_6 \overline{D}$$

$$f = A C + A B \overline{D} + B C \overline{E} + \overline{A} \overline{B} D + \overline{A} C E + \overline{A} B \overline{C}$$

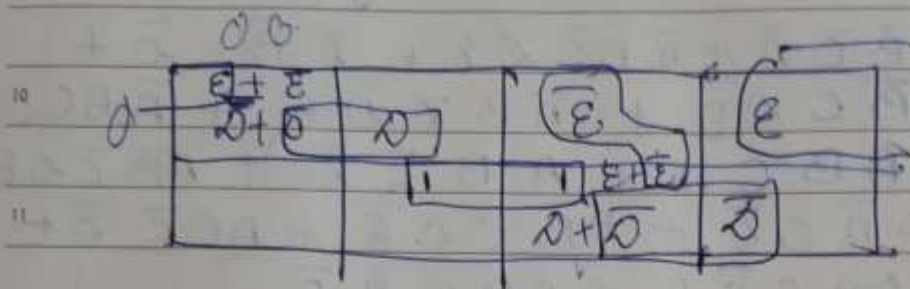
eg Reduce by Mapping

$$f = \overline{A} \overline{B} \overline{C} E + \overline{A} \overline{B} C + \overline{A} B C F + \overline{A} B \overline{C} + A \overline{B} C D + A B \overline{C} \overline{F}$$

$$f = m_0 E + m_1 + m_2 + m_3 F + m_5 D + m_6 \overline{F}$$

		BC			
A	D	00	01	11	10
	0	E	1	F	1
1	1		D		F

$$f = \overline{A} \overline{B} E + \overline{B} C D + B \overline{C} \overline{F} + \overline{A} B F + \overline{A} \overline{B} C$$

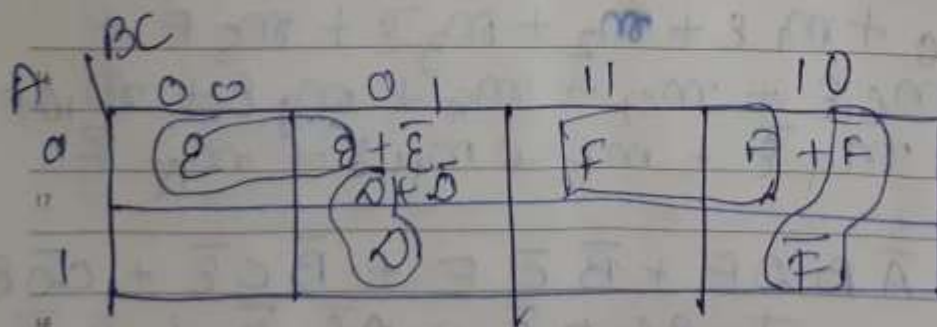


13

$$\bar{A} \bar{B} D + AC + AB\bar{D} + E \bar{A} \bar{C}$$

14

$$+ BC \bar{E} + \bar{A} \bar{B} \bar{C}$$



18

$$\bar{A} \bar{B} E + \bar{A} B F + \bar{B} C D + B \bar{C} \bar{F} + \bar{A} \bar{B} \bar{C}$$

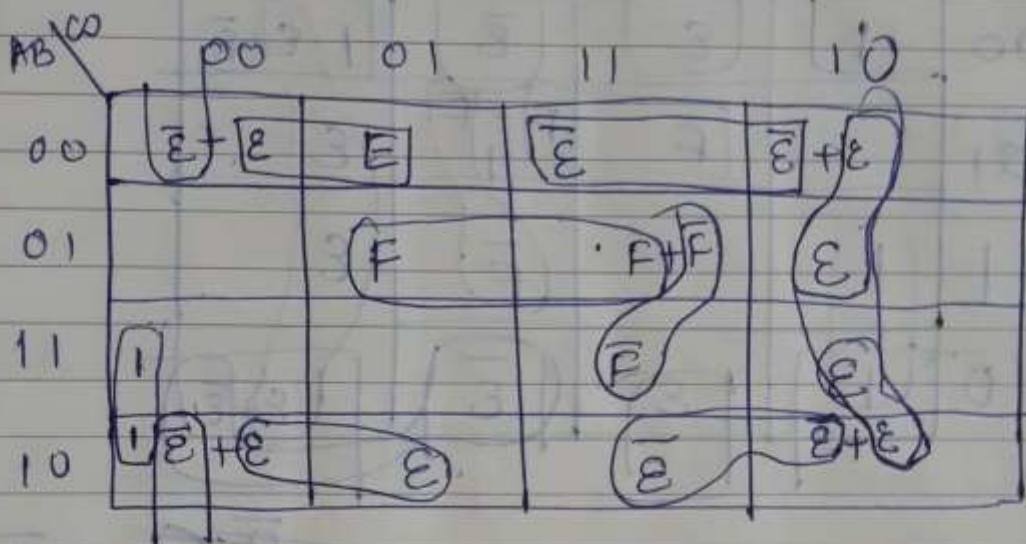
JUL 2015							AUG 2015						
M	T	W	T	F	S	S	M	T	W	T	F	S	S
			1	2	3	4	31					1	2
6	7	8	9	10	11	12	3	4	5	6	7	8	9
13	14	15	16	17	18	19	10	11	12	13	14	15	16
20	21	22	23	24	25	26	17	18	19	20	21	22	23
27	28	29	30	31			24	25	26	27	28	29	30

June 2015

wk 25 - day 169

Thursday

18



$$\begin{aligned}
 & E \bar{A} \bar{B} \bar{C} + \bar{A} \bar{B} \bar{E} C + \bar{A} E \bar{B} \bar{C} \bar{D} + \\
 & + \bar{B} \bar{E} \bar{E} \bar{D} + \boxed{\bar{A} B D F} + B C D \bar{E} + \\
 & A \bar{C} \bar{D} + A \bar{B} \bar{C} + A \bar{B} C \bar{E} + \cancel{E A E}
 \end{aligned}$$

$$E C \bar{D}$$

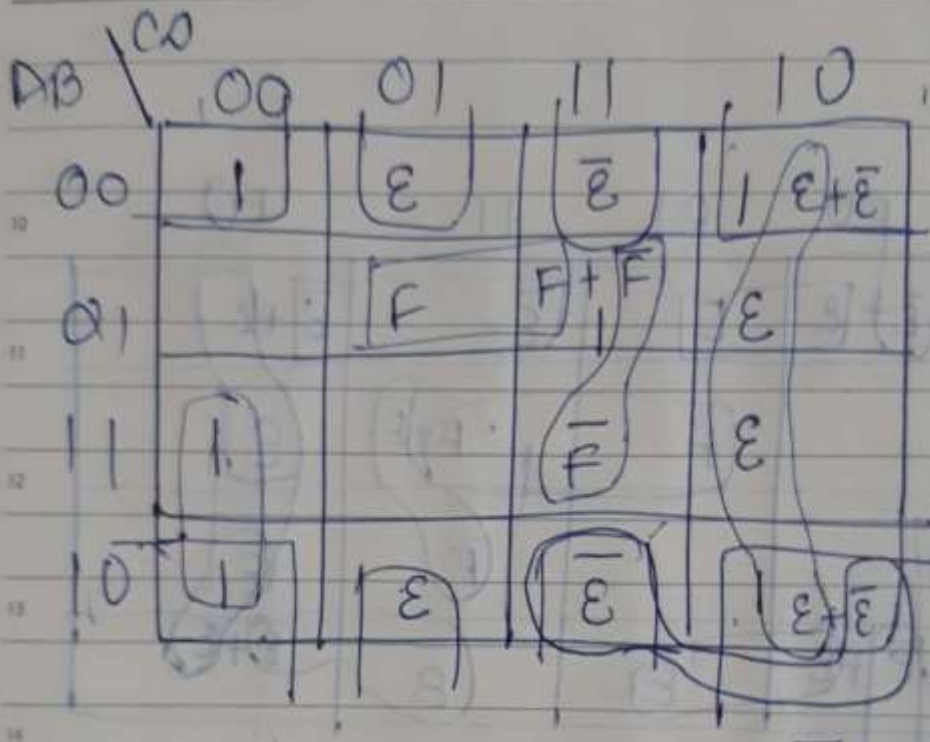
June 2015

19

WK 25 - day 170

Friday

MAY 2015							JUN						
M	T	W	T	F	S	S	M	T	W	T	F	S	S
					1	2	1	2	3	4	5	6	7
4	5	6	7	8	9	10	8	9	10	11	12	13	14
11	12	13	14	15	16	17	15	16	17	18	19	20	21
18	19	20	21	22	23	24	22	23	24	25	26	27	28
25	26	27	28	29	30	31	29	30					



$$\begin{aligned}
 & \overline{B} \overline{D} + \overline{B} E \overline{C} D + \overline{C} \overline{D} E + \overline{A} B F D + \overline{B} \overline{F} C D + \overline{A} \overline{C} \overline{D} \\
 & \overline{B} \overline{D} + \overline{B} E \overline{C} D + \overline{C} \overline{D} E + \overline{A} B F D + \overline{B} \overline{F} C D + \overline{A} \overline{C} \overline{D}
 \end{aligned}$$

2015	2015	2015
M T W T F S S	M T W T F S S	M T W T F S S
1 2 3 4 5 6 7	31 1 2 3 4 5 6	1 2 3 4 5 6 7
8 9 10 11 12 13 14	7 8 9 10 11 12 13	8 9 10 11 12 13 14
15 16 17 18 19 20 21	14 15 16 17 18 19 20	15 16 17 18 19 20 21
22 23 24 25 26 27 28	21 22 23 24 25 26 27	22 23 24 25 26 27 28
29 30 31	28 29 30 31	29 30 31

June 2015

Wk 25 - day 171

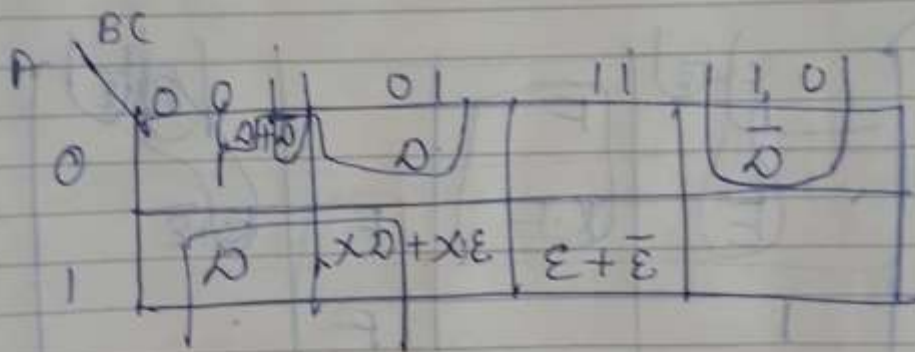
Saturday

20

eg Reduce key Mapping

$$f = \bar{A} \bar{B} \bar{C} + \bar{A} \bar{B} C D + \bar{A} B \bar{C} \bar{D} + A \bar{B} \bar{C} D + A B C E + A B C \bar{E} + d(A \bar{B} C D + A \bar{B} C E)$$

$$f = m_0 + m_1 D + m_2 \bar{D} + m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{18} + m_{19} + m_{20} + m_{21} + m_{22} + m_{23} + m_{24} + m_{25} + m_{26} + m_{27} + m_{28} + m_{29} + m_{30} + m_{31}$$



$$\bar{D} \bar{B} + \bar{A} \bar{C} \bar{D} + A B C$$

Sunday 21

June 2015

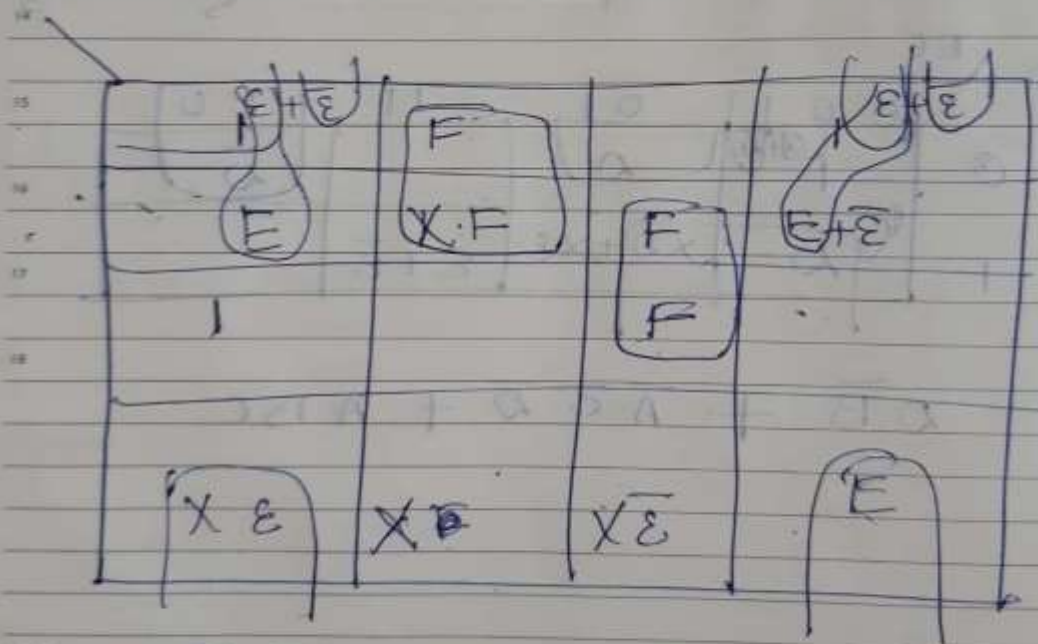
22

Mon 22 - Day 173
Monday

MAY 2015							JUN 2015						
M	T	W	T	F	S	S	M	T	W	T	F	S	S
				1	2	3	1	2	3	4	5	6	7
4	5	6	7	8	9	10	8	9	10	11	12	13	14
11	12	13	14	15	16	17	15	16	17	18	19	20	21
18	19	20	21	22	23	24	22	23	24	25	26	27	28
25	26	27	28	29	30	31	29	30					

eg Reduce by Mapping

$$f = m_0 + m_1 F + m_2 + m_4 E + m_6 (E + \bar{E}) + m_7 F + m_{10} E + m_{12} + m_{15} F + d(m_5 F + m_8 + m_{11} E + m_{13} E)$$



Notes

$$\begin{aligned} & E \bar{B} \bar{D} + \bar{C} D F \bar{A} + C D F B \\ & \bar{A} E \bar{D} + \bar{A} \bar{B} \bar{E} \bar{E} + \bar{A} \bar{C} \bar{D} E \\ & A B \bar{C} \bar{D} + \bar{A} E \bar{D} \end{aligned}$$

If at first you don't succeed, failure may be your style.

Get a Minimum POS Using K Map

- Cover 0's to get simplified POS.
 - We want 0 in each term.

$$f = x'z' + wyz + w'y'z' + x'y$$

First, the 1's of f are plotted in Fig. 5-14. Then, from the 0's,

$$f' = y'z + wxz' + w'xy$$

and the minimum product of sums for f is

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

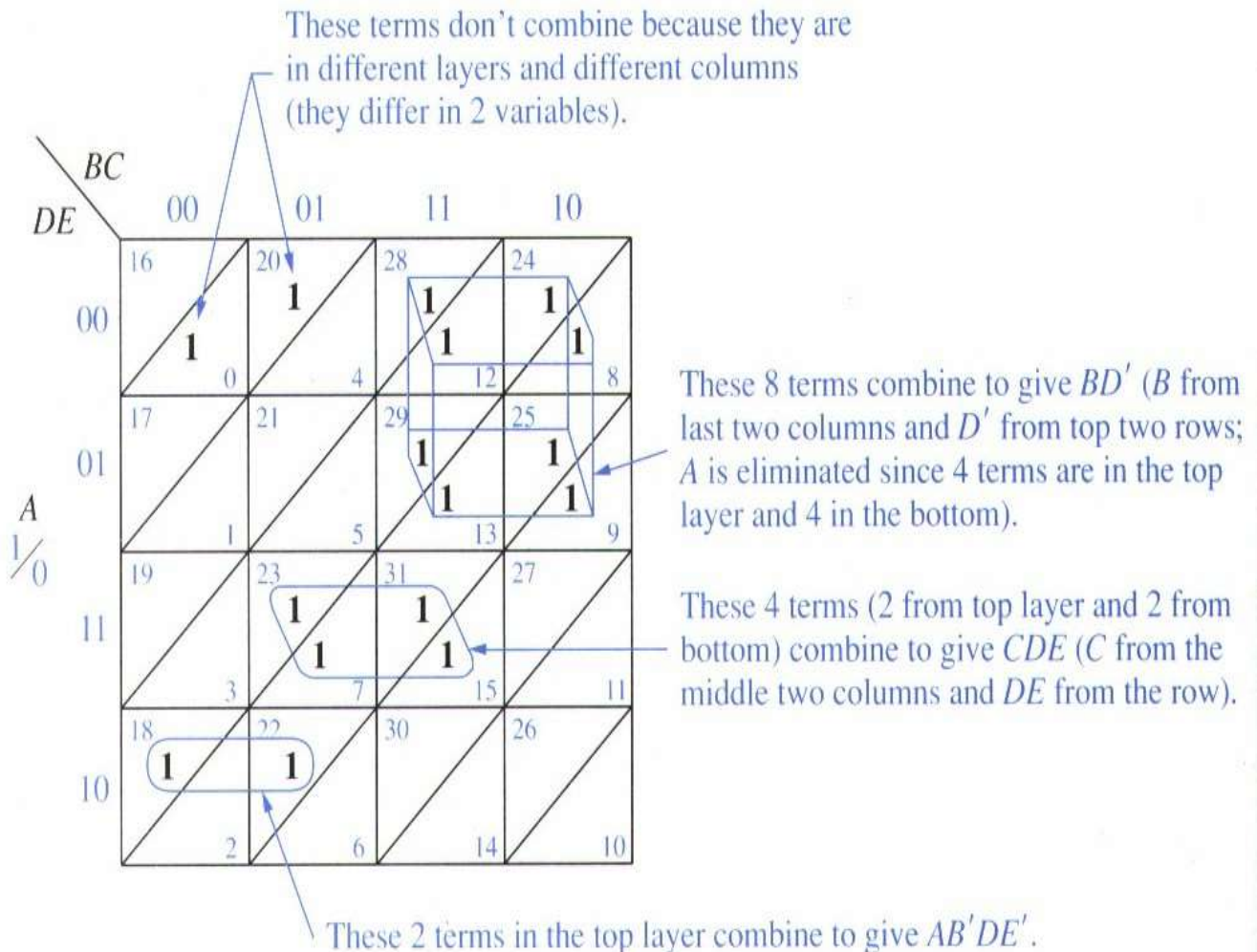
		wx			
		00	01	11	10
yz	00	1	1	0	1
	01	0	0	0	0
	11	1	0	1	1
	10	1	0	0	1

5-Variable K Map

- Use two 4-variable map to form a 5-variable K map ($16 + 16 = 32$)

(A,B,C,D,E)

- A' in the bottom layer
- A in the top layer.



Simplification Using Map-Entered Variables

- Extend K-map for more variables.
 - When E appears in a square, if E = 1, then the corresponding minterm is present in the function G.
 - $G(A,B,C,D,E,F) = m_0 + m_2 + m_3 + \text{Em}_5 + \text{Em}_7 + \text{Fm}_9 + m_{11} + m_{15} + (\text{don't care terms})$

CD \ AB	AB			
	00	01	11	10
00	1			
01	X	E	X	F
11	1	E	1	1
10	1			X

G

(a)

CD \ AB	AB			
	00	01	11	10
00	1			
01	X		X	
11	1		1	1
10	1			X

$E = F = 0$

$$MS_0 = A'B' + ACD$$

(b)

CD \ AB	AB			
	00	01	11	10
00	X			
01	X	1	X	
11	X	1	X	X
10	X			X

$E = 1, F = 0$

$$MS_1 = A'D$$

(c)

CD \ AB	AB			
	00	01	11	10
00	X			
01	X		X	1
11	X		X	X
10	X			X

$E = 0, F = 1$

$$MS_2 = AD$$

(d)

Map-Entered Variable

- $F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$, (don't care)
 - Choose D as a map-entered variable.
 - When $D = 0$, $F = A'C$ (Fig. a)
 - When $D = 1$, $F = C + A'B$ (Fig. b)
 - two 1's are changed to x's since they are covered in Fig. a.
- $F = A'C + D(C + A'B) = A'C + CD + A'BD$

		A	
		0	1
BC	00		
	01	1	x
	11	1	D
	10	D	

(a)

		A	
		0	1
BC	00		
	01	x	x
	11	x	1
	10	1	

(b)

		DA			
		00	01	11	10
BC	00				
	01	1	x	x	1
	11	1		1	1
	10				1

(c)

General View for Map-Entered Variable Method

- Given a map with variables P_1, P_2 etc, entered into some of the squares, the minimum SOP form of F is as follows:
- $F = MS_0 + P_1 MS_1 + P_2 MS_2 + \dots$
where
 - MS_0 is minimum sum obtained by setting $P_1 = P_2 \dots = 0$
 - MS_1 is minimum sum obtained by setting $P_1 = 1, P_j = 0 (j \neq 1)$, and replacing all 1's on the map with don't cares.
 - Previously, $G = A'B' + ACD + EA'D + FAD$.