

Chapter 2 Properties of Gases

Perfect Gas:-

It obeys the Boyle's law and Charles law and the characteristic equation of a gas which is obtained by the above two laws.

No gas is perfect but many gases behaves nearly as perfect gas in the temperature and pressure ranges of applied thermodynamics.

Many gases like Air, Oxygen, Nitrogen, Hydrogen etc can be regarded as perfect gases.

They are known as real gases.

Vapour:-

It is a gaseous state of the fluid but at a temperature not too away from its boiling point. It may have liquid particles in suspension.

Such Vapours are known as wet vapours. Whereas gases remain in the gaseous state except under extreme pressure and temperature.

Behavior of wet vapour can not be determined by the Boyle's and Charles's laws.

Vapour do not obey these laws but one can use for the approximate solution under certain condition.

Gas laws:- Boyle's Law, Charles's Law

Boyle's Law:-

It states that “The volume of a given mass of a perfect gas varies inversely as the absolute pressure when the temperature is held constant”.

$$P \propto \frac{1}{V}$$

If P is the absolute pressure of the gas and V is the volume occupied by the gas from the Boyle's law

$$PV = C$$

Where ($T = \text{Constant}$)

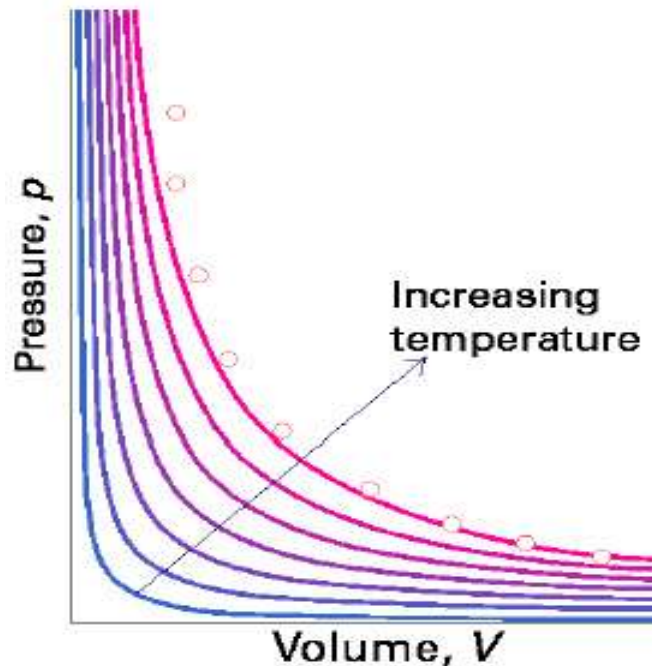
$$P = \frac{C}{V}$$

$$PV = C$$

The product of absolute pressure and volume of a given mass of the gas is constant when temperature is held constant.

Boyle's law, stated in mathematical terms for a gas whose pressure and volume is measured at two different pressure/volume states at a constant temperature is then,

$$P_1 V_1 = P_2 V_2$$



Charle's Law:-

“If any gas is heated at the constant pressure , the change in volume is directly proportional to the temperature range.”

This change in volume is same for all gases.

The change in this volume is $\frac{1}{273}$ times of volume at 0°C per degree centigrade temperature rise.

If V_0 is the volume of gas at 0°C , and T_0 is the temperature at 0°C , volume of the gas (V) at $T^{\circ}\text{C}$ can be written as

$$\frac{V}{V_o} = \left(1 + \frac{t}{273}\right)$$

$$V = V_o \cdot \left(1 + \frac{t}{273}\right)$$

$$V = V_o \cdot \left(\frac{273 + t}{273}\right)$$

$$V = V_o \cdot \left(\frac{T}{T_o}\right)$$

Where $(273 + t) = T$

$$\frac{V}{V_o} = \frac{T}{T_o}$$

Where T is the absolute temperature of gas in K and T_0 is the absolute temperature equal to 0°C that is 273 K.

The volume of a gas varies directly as the absolute temperature when pressure is constant.

$V \propto T$ when P is the constant.

Combined gas law:-

Pressure, Temperature, and Volume of the gas may change simultaneously.

In such condition any one of the above laws cannot be applied directly.

By combination of above laws we can derive an equation which can be used in such cases.

From Boyle's law $V \propto \frac{1}{p}$ if T is kept constant.

From Charles' law $V \propto T$ if p is kept constant

So if both p and T varies

$$V \propto \frac{T}{p}$$

$$V = \frac{CT}{p}$$

Where C is constant of proportionality

$$PV = CT$$

The constant C depends on the mass, properties of the gas used and the temperature scale used.

If the gas changes its state from 1 to 2 and similarly the change in properties from P_1, V_1, T_1 to P_2, V_2, T_2 we can have

$$P_1 V_1 = C T_1, P_2 V_2 = C T_2$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = C$$

The constant C is found to be mR Where m is the mass of the gas and R is the specific or characteristics gas constant, so the above equation can be written as

$$PV = mRT$$

This equation is known as “characteristics gas equation”.

If we replace total volume by specific volume the above equation can be written as $PV=RT$

Gas constant (R):-

In the above equation if the pressure is in $\frac{\text{N}}{\text{m}^2}$, temperature is in kelvin (K) , volume is in m^3 and mass is in kg, the unit of R will be in $\frac{\text{J}}{\text{kg.K}}$.

$$\begin{aligned} R &= \frac{VP}{mT} = \frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^3}{1} \cdot \frac{1}{\text{kg}} \cdot \frac{1}{\text{K}} \\ &= \frac{\text{N}}{\text{kg}} \cdot \frac{\text{m}}{\text{K}} = \frac{\text{J}}{\text{kg.K}} \end{aligned}$$

Here R represents the work done against external resistance when 1 kg of mass is heat by one degree temperature at constant pressure.

Universal Gas Constant:-

The product of the molecular mass of the gas and its specific gas constant (R) is same for all the gases.

This constant is known as Universal gas constant (R_o).

The numerical value of universal gas constant is

$$8.3143 \frac{\text{kJ}}{\text{kg mol K}}$$

$$R_o = MR$$

Where M is the molecular mass of the gas and R is the specific gas constant .

The equation of state for the kg mole of any gas can be written as $PV = 8.3143 T$

Avogadro's Law:-

It states that “ Equal volumes of all gases at the same pressure and temperature contain the same number of molecules”.

“ Molecular masses of all the gases occupy the same volume at the NTP. “

We have $pV = MRT$

$$\text{Now } MR = 8.3143 \frac{\text{Kj}}{\text{kg.mol.K}}$$

Normal pressure and temperature are taken as 273 K and 101.325 Kpa.

On the substitution of this values we can have

$$V = \frac{8.3143 \times 273}{101.325}$$
$$= 22.4012 \frac{\text{m}^3}{\text{kg mol}}$$

So we can write as “one kg mole of all gases occupy the volume of 22.4 m^3 at NTP.

Standard pressure and temperature are taken as 101.325 kPa and 15°C .

Joule's law of Internal Energy:-

The internal energy is a property of the system and hence it depends only on the state of the system.

Change in the internal energy does not depend on the path followed by the system.

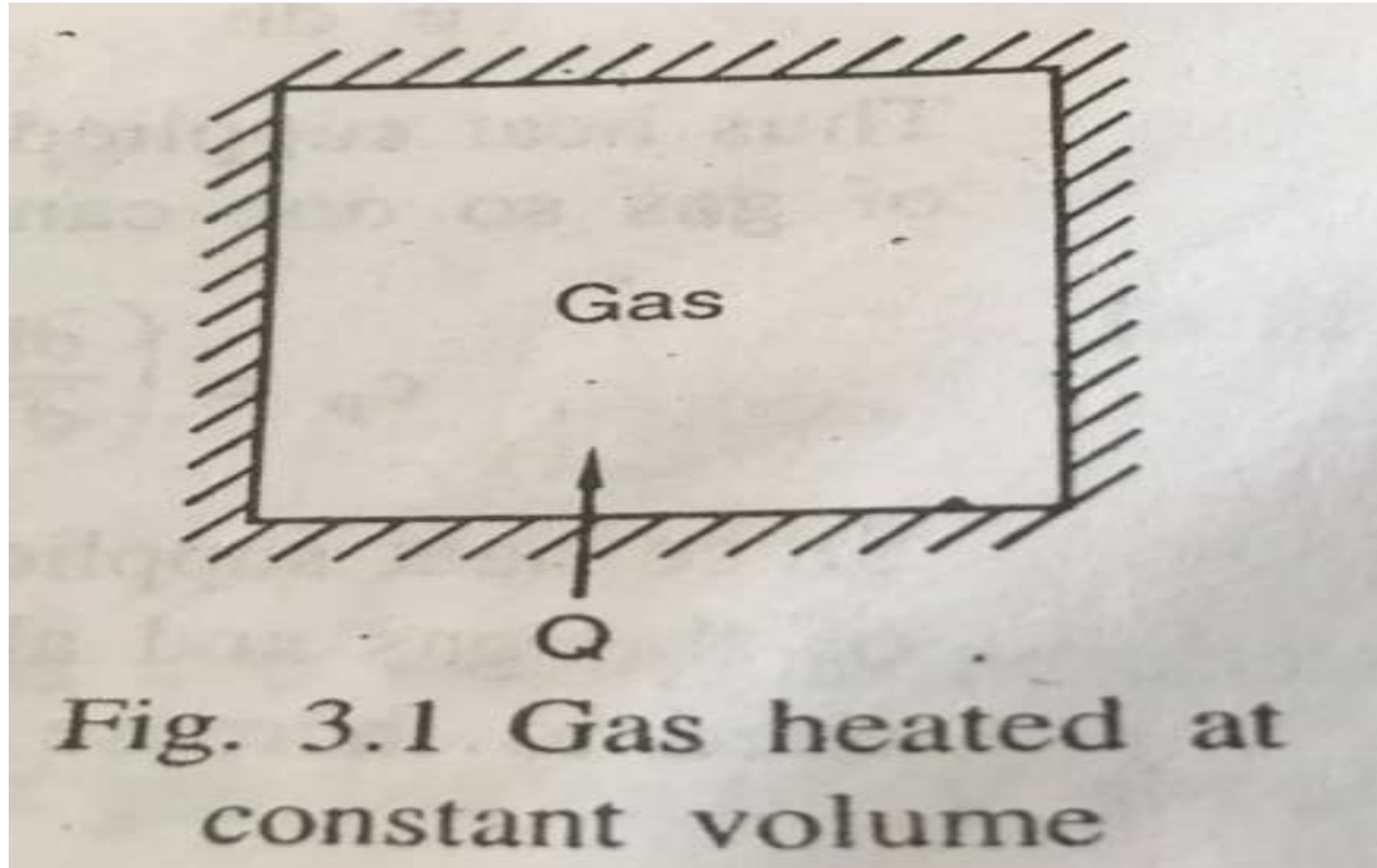
When any of the system parameters like pressure, volume, temperature etc changes there may be change in the internal energy because of the change in the state.

But Joule's law of internal energy states that if change in the internal energy ($U_2 - U_1$), then temperature changes from T_1 to T_2 will be same no matter how pressure P and volume V changes.

So from Joule's experiments internal energy is only function of temperature and not of pressure and volume.

Specific Heat of the gas at constant volume:-

Let certain quantity of gas is trapped in a rigid vessel as shown in the figure.



When heat is supplied to gas, it will be heated at the constant volume.

The whole heat energy is used to increase the internal energy of the gas which can be viewed as the increase in the temperature of the gas.

Thus heat required to increase the temperature of 1 kg of gas by 1 K when volume is kept constant is known as specific heat at the constant volume.

So mathematically we can write as below:

So that we can write from the theory as

$$\delta Q = dU \text{ -----(1)}$$

$$C_v = \frac{\delta Q}{dT}$$

But we know that $\delta Q = C_v \times dT$ -----(2)

So put the value of δQ from the equation no-2 in the equation no-1

$$C_v \times dT = dU$$

So we obtain the final equation as below for the constant volume process

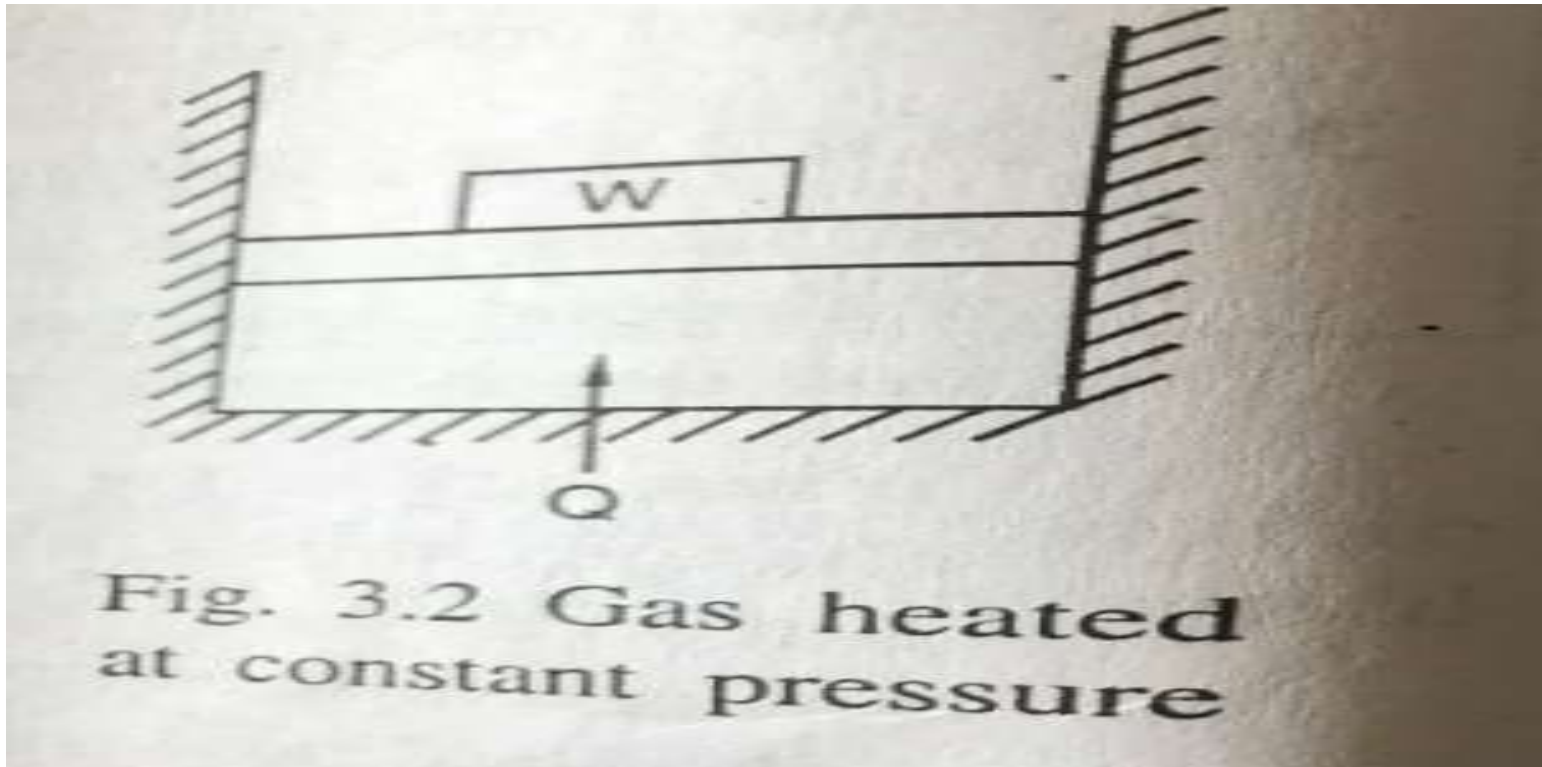
$$C_v = \left(\frac{dU}{dT} \right) \text{ at constant volume}$$

Specific heat of the gas at constant volume is denoted by the symbol C_v .

Specific Heat of the gas at constant Pressure:-

Let us certain quantity of heat is supplied to the gas ,it will be heated at the constant pressure.

The pressure will be net load on the piston divided by the piston area.



Here heat supplied will be utilized to increase the internal energy of the gas and also to lift the weight under the external pressure.

Thus heat supplied is used to increase the temperature and the volume of the gas.

Thus heat required to increase the temperature of 1 kg of gas by 1 K at the constant pressure is known as specific heat at the constant pressure. So mathematically we can write as below:

If we apply 1 st law of thermodynamics to this process

$$Q = \Delta u + W$$

Since only $W = pdv$ work is present

$$\begin{aligned}\delta Q &= du + pdv \\ &= du + d(pv) \\ &= d(u + pv)\end{aligned}$$

$$\delta Q = dh \text{ -----(1)}$$

Thus heat supply at the constant pressure is utilize to increases the enthalpy of the gas from the above equation.

$$\delta Q = dh \text{ -----(1)}$$

$$\text{But we know that } \delta Q = C_p \times dT \text{ -----(2)}$$

So put the value of δQ from the equation no-2 in the equation no-1

$$C_p \times dT = dh$$

So we obtain the final equation as below for the constant pressure process

$$C_p = \left(\frac{dh}{dT} \right) \text{ at constant pressure}$$

Specific heat of gas at constant pressure is denoted by the symbol C_p .

Conclusion of the theory:-

Heat supplied at the constant pressure is used to increase internal energy of the gas and to do the external work.

Heat supplied at the constant volume is used to increase internal energy of the gas.

The value of the C_p is always greater than the C_v .

Relationship between C_p and C_v :-

Let T_1 and T_2 are the initial and the final temperature of the gas respectively.

V_1 and V_2 are the initial and final specific volume of the gas,

p is the pressure of the gas,

R is the specific gas constant,

C_p and C_v are the specific heat at the constant pressure and volume.

Here 1 kg of the gas be heated at the constant volume and the temperature increase from T_1 and T_2

Heat supplied at the constant volume dQ

$$= C_v (T_2 - T_1)$$

Heat supplied at the constant volume increases the internal energy of the gas and no external work is done.

From the first law of thermodynamics

$$dQ = du + dw$$

But $dw = pdv$ and No external work is done

So change in volume is constant due to that $dv = 0$

$$dw = pdv = 0$$

So that $dQ = du$

Change in the internal energy $du = dQ$

$$= u_2 - u_1$$

$$= C_v (T_2 - T_1)$$

Here 1 kg of the gas is heated at the constant pressure and the temperature increase from the T_1 to T_2 .

Heat is supplied at constant pressure dQ
 $= C_p(T_2 - T_1)$ -----Equation No -1

Heat is supplied at constant pressure is utilize
for

1. To increase the internal energy of the gas:
Increase in the internal energy of the gas du
 $= C_v (T_2 - T_1)$ -----Equation No-2
2. For overcoming external resistance for doing work:
Energy require to overcome the external resistance dw
 $= p (v_2 - v_1)$ -----Equation No-3

From first law of thermodynamics we can write the statement as below:

$$Q = u + pv$$

Take the differentiation on both the side of equation

$$dq = du + pdv \text{ -----Equation No-4}$$

From the Equation No-1, Equation No-2, and from Equation No-3 put the value in the Equation No-4, we get the new Equation as below

$$C_p (T_2 - T_1) = C_v(T_2 - T_1) + p (v_2 - v_1) \text{ ----Equation No-5}$$

From the characteristics gas Equation we can write

$$pv_1 = RT_1 \text{ ----- Equation No -A}$$

$$pv_2 = RT_2 \text{ ----- Equation No -B}$$

Subtracting Equation No-A from Equation No-B

$$p(v_2 - v_1) = R (T_2 - T_1)$$

Put the value of $p(v_2 - v_1)$ in the Equation No -5

So finally we get the new form of Equation as below

$$C_p (T_2 - T_1) = C_v(T_2 - T_1) + R (T_2 - T_1)$$

Neglecting the term $(T_2 - T_1)$ from both the side of Equation

$$C_p = C_v + R$$

$$C_p - C_v = R$$

Ratio of the Specific Heats:-

The ratio of $\frac{C_p}{C_v}$ is denoted by the greek letter of γ .

It is also known as adiabatic index.

According to the classical kinetic theory of gases value of γ should be 1.67 monoatomic number, 1.4 diatomic number and 1.33 polyatomic number for the gases.

For polyatomic gases the variation from theoretical value 1.33 is large.

For the air value of γ is 1.4.

$C_p - C_v = R$ ----- Equation No-1

Divide the above Equation with the C_v

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

Now $\frac{C_p}{C_v} = \gamma$, so that

$$(\gamma - 1) = \frac{R}{C_v}$$

$$C_v = \frac{R}{(\gamma - 1)}$$

Put the value of C_v in Equation No-1

$$C_p - C_v = R$$

$$\text{But } C_v = \frac{R}{(\gamma - 1)},$$

So that We get

$$C_p - \frac{R}{(\gamma - 1)} = R$$

$$C_p = \frac{R}{(\gamma - 1)} + R$$

$$C_p = R\left(\frac{1}{(\gamma - 1)} + 1\right)$$

$$C_p = R\left(\frac{1 + \gamma - 1}{(\gamma - 1)}\right)$$

$$C_p = R\left(\frac{\gamma}{(\gamma - 1)}\right)$$

So that we get the values of C_p and C_v as below:

$$C_p = R\left(\frac{\gamma}{(\gamma - 1)}\right), C_v = \frac{R}{(\gamma - 1)}$$

For air value of C_p and C_v can be taken as below:

The specific heat at the constant pressure is taken

as $1.005 \frac{\text{kJ}}{\text{kg.k}}$

The specific heat at the constant volume is taken

as $0.718 \frac{\text{kJ}}{\text{kg.k}}$

Enthalpy:

Enthalpy of a substance at any point is quantification of energy content in it, which could be given by summation of internal energy and flow work.

$$H = u + pV$$

Flow Process:

Work interaction taking place in a system can be classified as flow work or non flow work base on the nature of process

Flow process :

Flow process is one in which fluid enters the system and leaves it after work interaction.

Such processes occur in the system having open boundary permitting across the mass interaction system boundary.

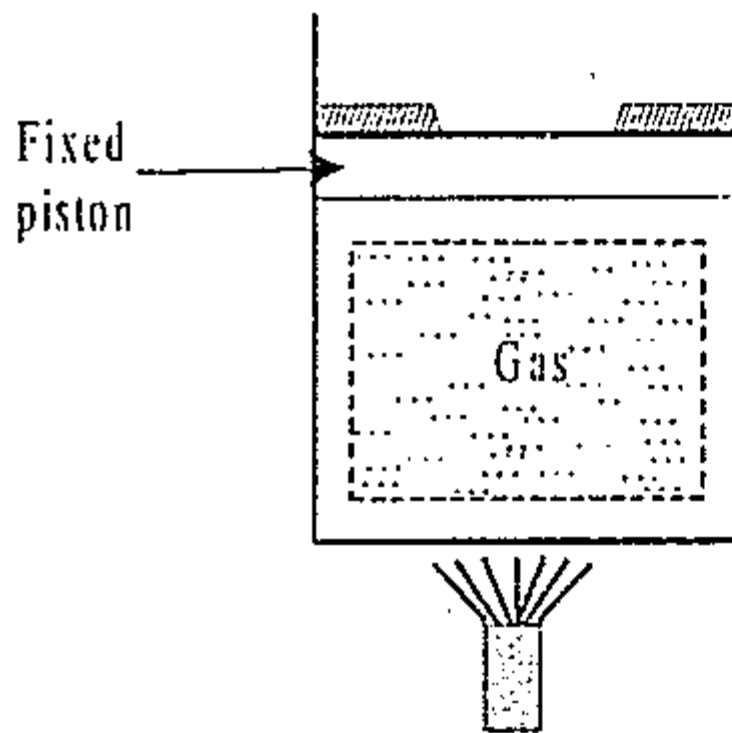
Non flow process:

Non flow process is the one in which there is no mass interaction across the system boundaries during the occurrence of the process.

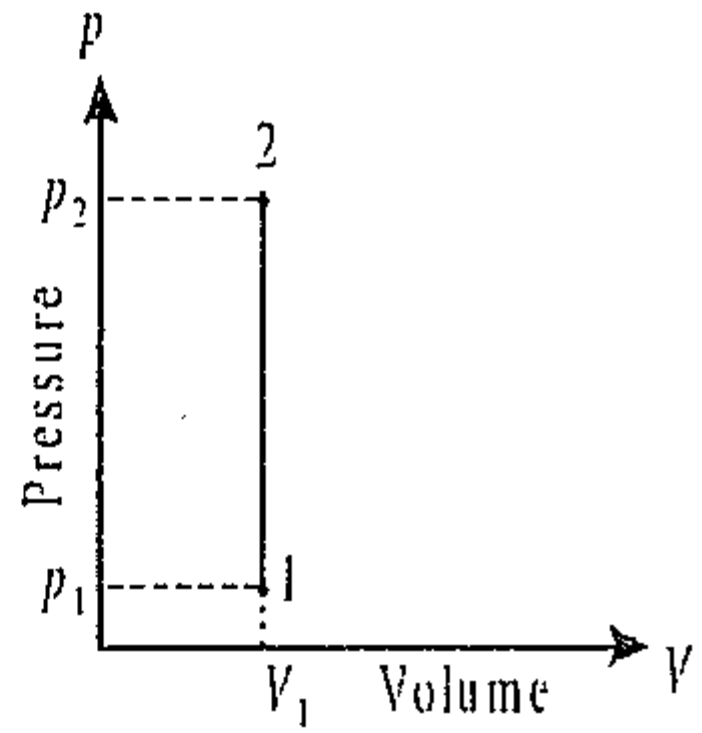
Different types of non flow process of perfect gases are :

- (a) constant volume process
- (b) constant pressure process
- (c) constant temperature process
- (d) adiabatic process
- (e) polytrophic process

(1) constant volume process:



(a)



(b)

Fig. 3.4 Constant volume process.

In constant volume process the working substance is contained in a rigid vessel, hence boundaries of a system are immovable and no work can be done on or by the system. This process is also known as **Isochoric process**.

Work done during process:

$$\text{Work done } dw = p dv$$

$$\text{Total work done } \int dw = \int_{v_1}^{v_2} p dv$$

Constant volume process, change in volume
 $dv=0$.

Work done = 0.

Relation Between p, V and T:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = C$$

But $V_1 = V_2 = \text{constant}$

Therefore $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Change in internal energy:

$$dU = m C_v (T_2 - T_1)$$

Heat transferred:

According to first law of thermodynamics

$$\delta Q = \delta W + dU$$

for constant volume process $dv = 0$,

therefore $\delta W = 0$

$$\delta Q = dU = m.C_v (T_2 - T_1)$$

Change in enthalpy:

$$H = U + pV$$

For state 1 & 2:

$$H_1 = U_1 + p_1 V_1$$

$$H_2 = U_2 + p_2 V_2$$

So that Change in enthalpy,

$$\begin{aligned}\Delta H &= H_2 - H_1 \\ &= (U_2 - U_1) + (p_2 V_2 - p_1 V_1)\end{aligned}$$

Put the value of U and ($pV = mRT$) in the above equation.

$$= mC_v(T_2 - T_1) + mR(T_2 - T_1)$$

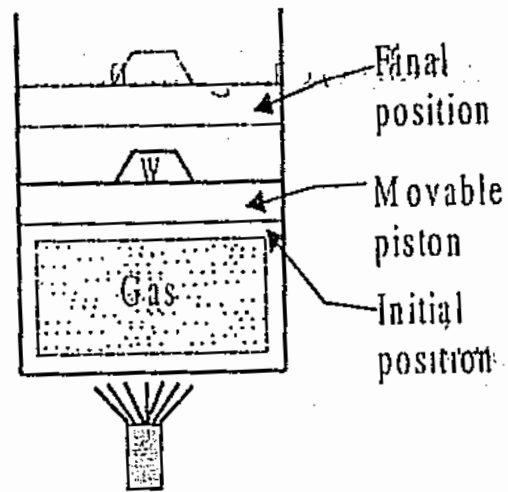
$$\text{But } R = C_p - C_v$$

$$\Delta H = m C_v(T_2 - T_1) + m(C_p - C_v) (T_2 - T_1)$$

$$\Delta H = m C_p(T_2 - T_1)$$

(2) Constant pressure process:

During this process the pressure of the gas remains constant.



(a)

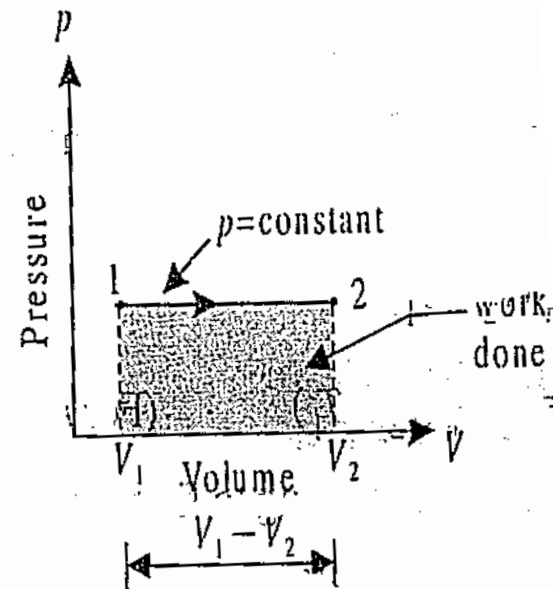


Fig. 3.5 Constant pressure process.

Work done during the process:

Work done $dW = p dv$

Total work done $\int dw = \int_{v_1}^{v_2} p dv$

but p is constant

Work done

$$W = p \int_{V_1}^{V_2} dv = p(V_2 - V_1)$$

Relation between P, V & T:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{But } (P_1 = P_2)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{charle's law})$$

Change in internal energy:

$$dU = mC_v(T_2 - T_1)$$

[Internal energy is a function of temperature alone]

Heat transfer:

According to first law of thermodynamics

$$\delta Q = \delta W + dU$$

$$= p(V_2 - V_1) + mC_v(T_2 - T_1)$$

But [$PV = mRT$]

$$= mR(T_2 - T_1) + mC_v(T_2 - T_1)$$

But $R = C_p - C_v$

$$= m(C_p - C_v)(T_2 - T_1) + mC_v(T_2 - T_1)$$

$$\delta Q = mC_p(T_2 - T_1)$$

Change in enthalpy

$$H = U + pV$$

For state 1 & 2:

$$H_1 = U_1 + p_1 V_1$$

$$H_2 = U_2 + p_2 V_2$$

So that Change in enthalpy,

$$\Delta H = H_2 - H_1$$

$$= (U_2 - U_1) + (p_2 V_2 - p_1 V_1)$$

$$= mC_v(T_2 - T_1) + mR(T_2 - T_1) \quad [PV = mRT]$$

$$\text{But } R = C_p - C_v$$

$$\Delta H = mC_v(T_2 - T_1) + m(C_p - C_v)(T_2 - T_1)$$

$$\Delta H = m C_p(T_2 - T_1)$$

Constant Temperature Process:

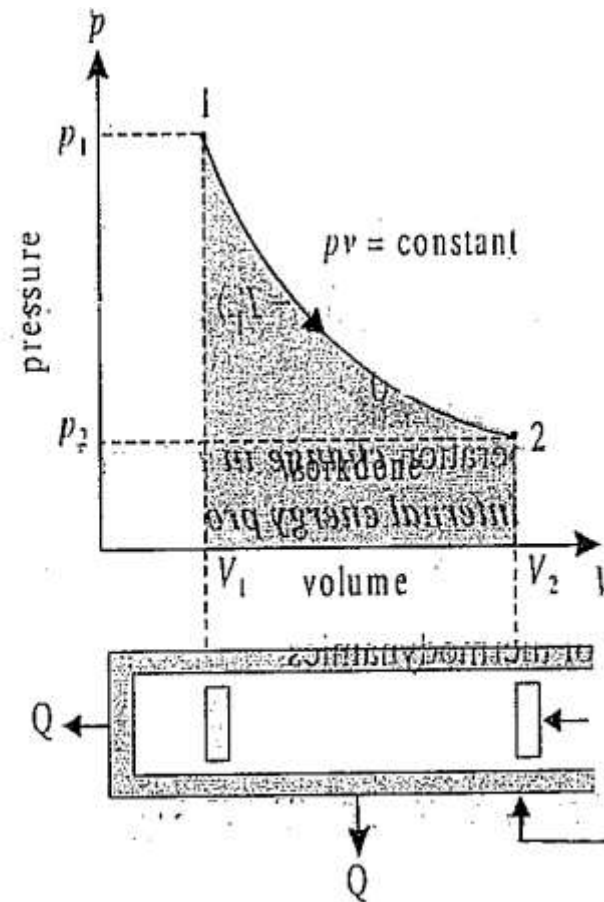


Fig. 3.6 Isothermal process.

In isothermal process temperature remains constant during the process .

This process follows Boyle's law.

Other name of this process is constant internal energy process.

Work done during process:

$$\text{Work done } dw = p dv$$

$$\text{Total work done } \int dw = \int_{V_1}^{V_2} p dv$$

$$\text{Now } pv = p_1 v_1 = p_2 v_2 = C$$

$$\text{So that } p = \frac{C}{v}$$

Put the value of $p = \frac{C}{v}$ in the work done Equation

$$\int dw = \int_{v_1}^{v_2} p dv$$

$$\begin{aligned} W &= \int_{V_1}^{V_2} dv \frac{c}{v} \\ &= c \int_{V_1}^{V_2} 1 \left(\frac{dV}{V} \right) \\ &= c \log_e \frac{V_2}{V_1} \end{aligned}$$

Substituting the value of $c = pv$ in the above equation we get

$$W = pv \log_e \frac{V_2}{V_1} = p_1 v_1 \log_e \frac{V_2}{V_1} = p_2 v_2 \log_e \frac{V_2}{V_1}$$

$$W = p v \log_e \frac{V_2}{V_1} = p_1 v_1 \log_e \frac{V_2}{V_1} = p_2 v_2 \log_e \frac{V_2}{V_1}$$

$$\text{Now } p v = m R T$$

$$p_1 v_1 = m R T_1$$

$$p_2 v_2 = m R T_2$$

So put the value in the above Equation

$$W = m R T \log_e \frac{V_2}{V_1}$$

$$= m R T_1 \log_e \frac{V_2}{V_1}$$

$$= m R T_2 \log_e \frac{V_2}{V_1}$$

Relation Between p, V and T:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, \text{ But } T_1 = T_2$$

So that

$$P_1 V_1 = p_2 V_2$$

Change in the Internal Energy:

$$\text{Change in the Internal Energy } du = mC_v(T_2 - T_1)$$

$$\text{But } T_1 = T_2$$

So that

$$du = 0$$

During an Isothermal Process change in the internal energy is zero.

So that this process is known as Constant Internal energy process.

Heat transfer

According to first law of thermodynamics

$$\delta Q = \delta W + dU$$

But Change in the Internal Energy $du = 0$

$$\text{So that } \delta Q = \delta W$$

$\delta Q = \delta W$ So that

$$Q = W = p v \log_e \frac{V_2}{V_1} = mRT \log_e \frac{V_2}{V_1}$$

$$= p_1 v_1 \log_e \frac{V_2}{V_1} = mRT_1 \log_e \frac{V_2}{V_1}$$

$$= p_2 v_2 \log_e \frac{V_2}{V_1} = mRT_2 \log_e \frac{V_2}{V_1}$$

Change in enthalpy,

$$\Delta H = H_2 - H_1$$

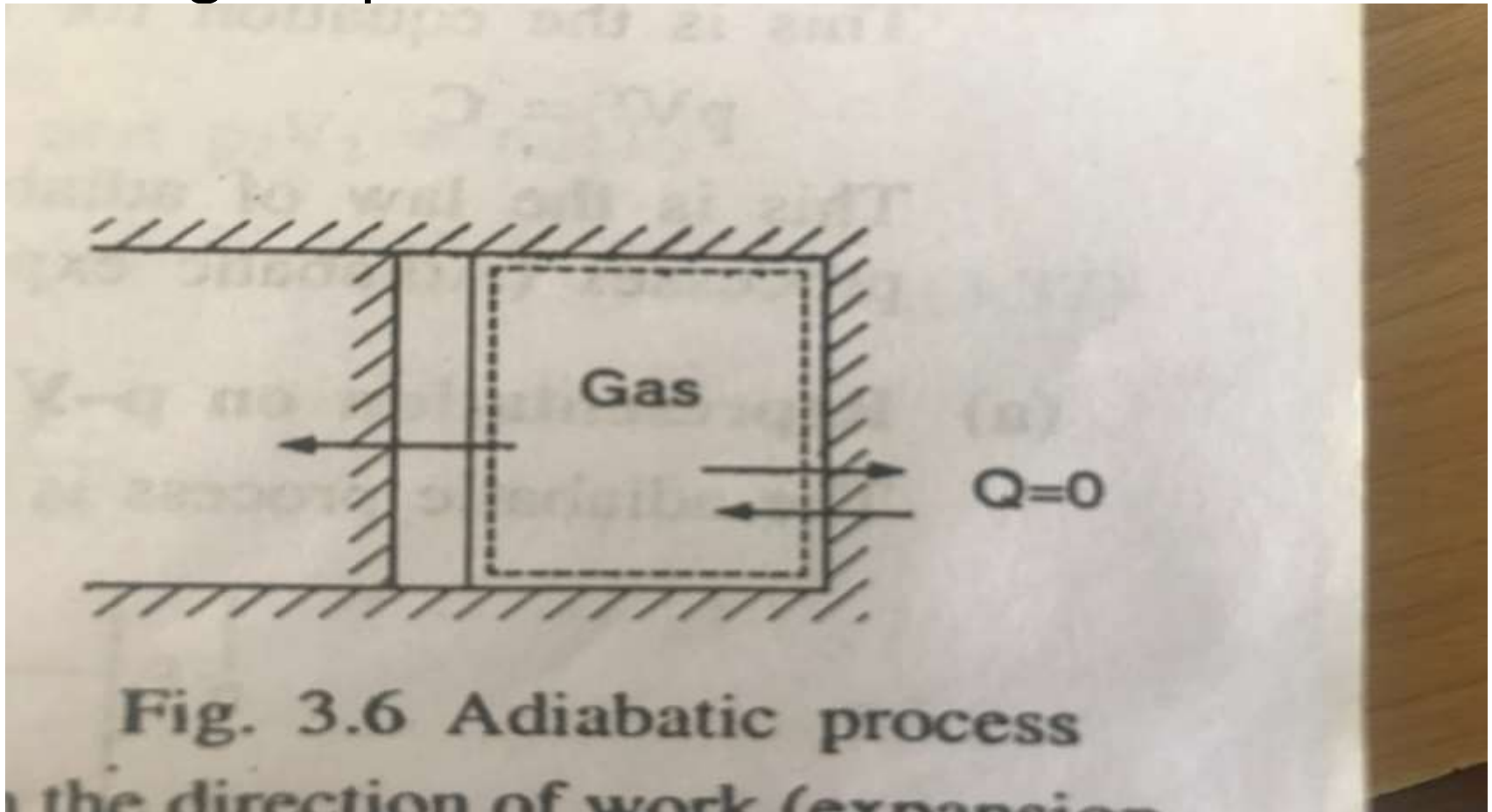
$$= m C_p (T_2 - T_1)$$

But $\Delta T = 0$

So that $\Delta H = 0$

(4) Adiabatic Process:

In an adiabatic process is an thermodynamic process in which there is no heat interaction during the process so that $Q = 0$



The work interaction is there at the expense of the internal energy.

There is no supply of heat takes place during the compression process.

Frictionless adiabatic process is also known as an isentropic process.

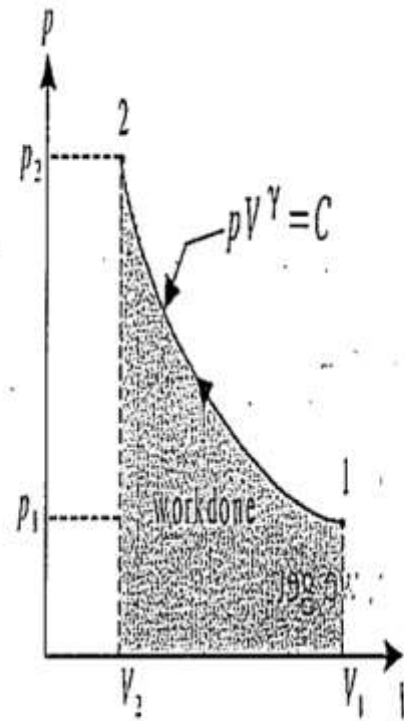


Fig. 3.7 Adiabatic Process

The adiabatic process on p-V diagram is represented in Fig. 3.7.

Law of Adiabatic Process

First law of thermodynamics in the differential form can be written as for the non flow process

$$\delta Q = dU + \delta W$$

Now we know that $du = C_v dT$

$$\delta W = p dV$$

Also for the Adiabatic Process $\delta Q = 0$

Put all the value in the first law of thermodynamics Equation.

We get

$$0 = C_v dT + p dV \text{ -----Equation No-1}$$

Now we have characteristic gas equation

$$pv = RT \text{ for the } (m=1)$$

Now differencing the above equation we get

$$pdv + vdp = RdT$$

$$dT = \frac{(pdv + vdp)}{R}$$

Now put the value of dT in the Equation No 1

We get

$$0 = C_v dT + pdv$$

$$0 = C_v \frac{(pdv + vdp)}{R} + pdv$$

$$C_v \frac{(pdv + vdp)}{R} + pdv = 0$$

$$C_v(pd v + v dp) + R p d v = 0$$

$$\text{Now } R = C_p - C_v,$$

So put the value in above Equation we get

$$C_v(pd v + v dp) + (C_p - C_v)p d v = 0$$

$$C_v p d v + C_v v dp + C_p p d v - C_v p d v = 0$$

$$C_v p d v + C_v v dp + C_p p d v - C_v p d v = 0$$

$$C_v v dp + C_p p dv = 0$$

$$C_v v dp + C_p p dv = 0$$

Divide the above equation with the term C_v

$$\frac{C_v v dp}{C_v} + C_p p dv = 0$$

$$vdp + \frac{C_p p dv}{C_v} = 0$$

$$\frac{dp}{p} + \frac{C_p}{C_v} \cdot \frac{dv}{v} = 0 \text{ -----Equation No -1}$$

Now We know that ratio of $\frac{C_p}{C_v}$ is = γ

$$\frac{dp}{p} + \gamma \cdot \frac{dv}{v} = 0 \text{ -----Equation No-1}$$

Now Integrate the Equation No-1 on both the sides we get the term

$$\int \frac{dp}{p} + \gamma \int \frac{dv}{v} = 0$$

$$\ln p + \gamma \ln v = C$$

Where C is kept constant

$$\text{So that } pv^\gamma = C$$

This is the equation for the unit mass.

For the total mass the equation can be written as below:

$$PV^\gamma = \text{Constant}$$

This is the law of the adiabatic process or the governing equation for adiabatic process.

Work done during process:

$$\text{Work done } W = \int_{v_1}^{v_2} p dv$$

Now we have the Adiabatic process law is as

$$p v^\gamma = P_1 V_1^\gamma = P_2 V_2^\gamma = C$$

$$\text{So that } p = \frac{C}{v^\gamma}$$

Put the value of p in the Work done Equation we get

$$\text{Work done } W = \int_{v_1}^{v_2} p dv$$

$$= \int_{v_1}^{v_2} \frac{C}{V^\gamma} dv$$

$$= C \int_{v_1}^{v_2} \frac{dV}{V^\gamma}$$

$$= \frac{C}{1} \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]$$

V_2

V_1

$$= \frac{C}{-\gamma+1} (V_2^{-\gamma+1} - V_1^{-\gamma+1})$$

$$\text{Now } p v^\gamma = P_1 V_1^\gamma = P_2 V_2^\gamma = C$$

$$= \frac{1}{-\gamma+1} (P_2 V_2^\gamma V_2^{-\gamma+1} - P_1 V_1^\gamma V_1^{-\gamma+1})$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$$

$$\text{Work Done } W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

Now we know that $P_1V_1 = mRT_1$

$$P_2V_2 = mRT_2$$

$$\begin{aligned}\text{Work Done } W &= \frac{P_1V_1 - P_2V_2}{\gamma - 1} \\ &= \frac{mRT_1 - mRT_2}{\gamma - 1} \\ &= \frac{mR(T_1 - T_2)}{\gamma - 1}\end{aligned}$$

$$\text{Work Done} = \frac{R(T_1 - T_2)}{\gamma - 1} \cdot m$$

Now $C_p - C_v = R$, and For Unit mass $m = 1$

$$\text{Work Done} = (C_p - C_v) \frac{(T_1 - T_2)}{\gamma - 1} \cdot 1$$

$$= \frac{(C_p - C_v)(T_1 - T_2)}{\gamma - 1}$$

$$\text{Now } \gamma = \frac{C_p}{C_v}$$

$$\begin{aligned}
 \text{Work Done} &= \frac{(C_p - C_v)(T_1 - T_2)}{\gamma - 1} \\
 &= \frac{(C_p - C_v)(T_1 - T_2)}{\frac{C_p}{C_v} - 1} \\
 &= \frac{(C_p - C_v)(T_1 - T_2)}{\frac{(C_p - C_v)}{C_v}} \\
 &= \frac{C_v(C_p - C_v)(T_1 - T_2)}{(C_p - C_v)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Work Done} &= C_v(T_1 - T_2) \\
 &= \text{change in internal energy}
 \end{aligned}$$

Relation between P, V and T

We have two basic Equations: $PV^\gamma = C$ -----EQ-1

$$\frac{PV}{T} = C \quad \text{-----EQ-2}$$

Divide the EQ-2 from EQ-1 we get

$$\begin{aligned} &= \frac{PV^\gamma}{\frac{PV}{T}} \\ &= T \times V^{\gamma-1} = \text{Constant} \end{aligned}$$

From the above Equation we get for state 1 and state 2

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\gamma-1$$

So that $\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ -----Equation No-1

We know that $P_1 V_1^\gamma = P_2 V_2^\gamma = C$

So that $\frac{P_1}{P_2} = \frac{V_2^\gamma}{V_1^\gamma}$ -----Equation No- 2

$$\frac{P_1}{P_2} = \frac{V_2^\gamma}{V_1^\gamma}$$

$$\frac{1}{\gamma}$$

So that $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma}}$ -----Equation No-3

Put the value of Equation No-3 in the Equation No-1 we get

$$\gamma-1$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)$$

$$\frac{1}{\gamma}$$

So that $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$ in above Equation

$$\frac{\gamma-1}{\gamma}$$

$$\gamma-1$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

Change in the Internal Energy:

According to first law of thermodynamics

$$\delta Q = \delta W + dU$$

But for the Adiabatic Process $\delta Q = 0$

So that $dU = -\delta W$

Change in the internal energy = - Work Done

Change in the internal energy is numerically equal to work done.

When the work is done by the gas it loses internal energy

When the work is done on the gas it gains internal energy.

Heat Transfer:-

During Adiabatic Process Heat Transfer is zero.

So that $\delta Q = 0$

Change in the Enthalpy:-

$$\Delta H = mC_p (T_1 - T_2)$$

(5) Polytropic Process:

This process is the most common use process in the practice.

Thermodynamic process is said to be govern by the law $PV^n = \text{constant}$.

Where $n = \text{index}$

Which can be vary from $-\infty$ to $+\infty$

n is lie within the range of 1 to 1.7.

Various thermodynamics processes discuss above are the special cases of the polytropic process.

The main difference in the equation between isentropic process and the polytropic process is that if we replacing γ by n in the relation of adiabatic operation we get the relation for the polytropic process.

That is why we have the equation like as follows:

$$\text{So that } \left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \text{-----Equation No-1}$$

$$\text{So that } \left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{n-1} \text{-----Equation No-2}$$

$$\text{So that } \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \text{-----Equation No-3}$$

Where n is the polytropic index for any process.

When $n = 0$

Put the value in Equation No-3 we get

n

$$\text{So that } \left(\frac{P_2}{P_1} \right) = \left(\frac{V_1}{V_2} \right)^0$$

$$\text{So that } \left(\frac{P_2}{P_1} \right) = \left(\frac{V_1}{V_2} \right) = 1$$

So that $P_1 = P_2$

Process is called as the Constant Pressure Process

When $n = 1$

Put the value in Equation No-2 we get

$$n-1$$

$$\text{So that } \left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)$$

$$1-1 \quad 0$$

$$\text{So that } \left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{1-1} = \left(\frac{V_1}{V_2}\right)^0 = 1$$

$$\text{So that } T_1 = T_2$$

Process is called as the Constant Temperature Process

When $n = \gamma$

Put the value in Equation No-3 we get

n

So that $\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)$

$$\left(\frac{V_1}{V_2}\right)^{\frac{1}{n}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = 1$$

So that

Process is called as the Constant Volume Process

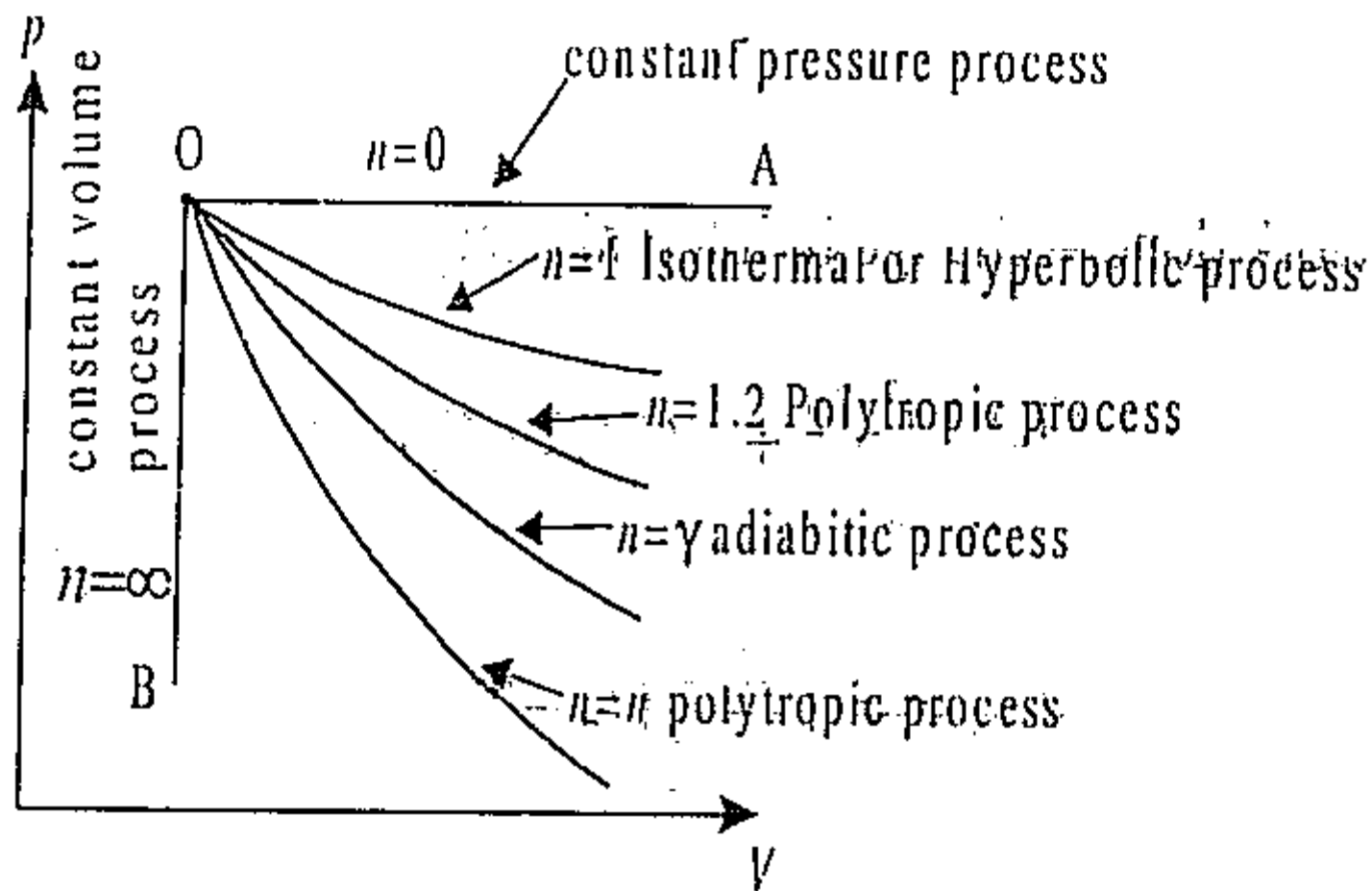


Fig. 3.9. Polytropic processes on p - v diagram.

Work done during process:

$$\text{Work done } W = \int_{v_1}^{v_2} p dv$$

Now we have the Adiabatic process law is as

$$p v^\gamma = P_1 V_1^\gamma = P_2 V_2^\gamma = C$$

$$\text{So that } p = \frac{C}{v^\gamma} = \frac{P_1 V_1^\gamma}{v^\gamma}$$

So that put the value of p in the Work done Equation we get

$$\text{Work done } W = \int_{V_1}^{V_2} p dV$$

$$= \int_{V_1}^{V_2} \frac{P_1 V_1^n}{V^n} dV$$

$$= P_1 V_1^n \int_{V_1}^{V_2} \frac{dV}{V^n}$$

$$= \frac{P_1 V_1^n}{-n+1} (V_2^{-n+1} - V_1^{-n+1})$$

$$P_1 V_1^n = P_2 V_2^n$$

$$= \frac{1}{-n+1} (P_2 V_2^n V_2^{-n+1} - P_1 V_1^n V_1^{-n+1})$$

By substituting $pV^n = P_1 V_1^n = P_2 V_2^n = C$

$$\text{Work Done } W = \frac{1}{-n+1} (P_2 V_2^n V_2^{-n+1} - P_1 V_1^n V_1^{-n+1})$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-n}$$

$$\text{Work Done } W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

Now we know that $P_1V_1 = mRT_1$

$$P_2V_2 = mRT_2$$

$$\begin{aligned}\text{Work Done } W &= \frac{P_1V_1 - P_2V_2}{\gamma - 1} \\ &= \frac{mRT_1 - mRT_2}{n - 1} \\ &= \frac{mR(T_1 - T_2)}{n - 1}\end{aligned}$$

$$\text{Work Done} = \frac{R(T_1 - T_2)}{n-1} \cdot m$$

Now $C_p - C_v = R$, and For Unit mass $m = 1$

$$\text{Work Done} = (C_p - C_v) \frac{(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{(C_p - C_v)(T_1 - T_2)}{n - 1}$$

$$\text{Now } \gamma = \frac{C_p}{C_v} = n$$

$$\begin{aligned}
 \text{Work Done} &= \frac{(C_p - C_v)(T_1 - T_2)}{\gamma - 1} \\
 &= \frac{(C_p - C_v)(T_1 - T_2)}{\frac{C_p}{C_v} - 1} \\
 &= \frac{(C_p - C_v)(T_1 - T_2)}{\frac{(C_p - C_v)}{C_v}} \\
 &= \frac{C_v(C_p - C_v)(T_1 - T_2)}{(C_p - C_v)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Work Done} &= C_v(T_1 - T_2) \\
 &= \text{change in internal energy}
 \end{aligned}$$

Relation between P, V and T

We have two basic Equations: $PV^n = C$ -----EQ-1

$$\frac{PV}{T} = C \quad \text{-----EQ-2}$$

Divide the EQ-2 from EQ-1 we get

$$\begin{aligned} &= \frac{PV^n}{\frac{PV}{T}} \\ &= T \times V^{n-1} = \text{Constant} \end{aligned}$$

From the above Equation we get for state 1 and state 2

$$T_1 V_1^{n-1} = T_2 V_2^{n-1}$$

$$\frac{n-1}{n} \quad n-1$$

We have the Equation $\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_2}{V_1}\right)^{n-1}$

$$P_1 V_1^n = P_2 V_2^n = C$$

So that $\frac{P_1}{P_2} = \frac{V_2^n}{V_1^n}$ -----Equation No- 2

Change in the Internal Energy:

$$\Delta U = mC_v (T_2 - T_1)$$

Change in the Enthalpy:-

$$\Delta H = mC_p (T_2 - T_1)$$

Heat Transfer:-

$$Q = \Delta U + W$$

$$= mC_v (T_2 - T_1) + \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$\text{Now } C_v = \frac{R}{\gamma-1}$$

$$= \frac{mR (T_2 - T_1)}{\gamma-1} + \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$= \frac{P_2 V_2 - P_1 V_1}{\gamma-1} + \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$= \frac{P_1 V_1 - P_2 V_2}{n-1} - \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

$$= (P_1 V_1 - P_2 V_2) \left(\frac{1}{n-1} - \frac{1}{\gamma-1} \right)$$

$$=(P_1 V_1 - P_2 V_2) \left(\frac{1}{n-1} - \frac{1}{\gamma-1} \right)$$

$$=(P_1 V_1 - P_2 V_2) \left(\frac{\gamma-1-n+1}{(n-1)(\gamma-1)} \right)$$

$$= \left(\frac{P_1 V_1 - P_2 V_2}{n-1} \right) \left(\frac{\gamma-n}{\gamma-1} \right)$$

$$Q = \left(\frac{\gamma-n}{\gamma-1} \right) \times \text{Work Done during process}$$

Q. 3 Prove $n = \frac{C_p}{C_v} = \gamma$ for adiabatic process.

Ans. According to first law of thermodynamics

Heat supplied = Work done + change in internal energy

$$\therefore Q = \frac{p_1 V_1 - p_2 V_2}{n-1} + mC_v(T_2 - T_1)$$

But for adiabatic process, $Q = 0$

$$\therefore 0 = \frac{(p_1 V_1 - p_2 V_2)}{n-1} + mC_v(T_2 - T_1)$$

$$p_1 V_1 = mRT_1, \quad p_2 V_2 = mRT_2$$

$$\therefore 0 = \frac{mR(T_1 - T_2)}{n-1} + mC_v(T_2 - T_1)$$

$$\therefore mR(T_2 - T_1) = (n-1)mC_v(T_2 - T_1)$$

Putting $R = C_p - C_v$

$$\therefore (C_p - C_v) = (n-1)C_v$$

$$\therefore C_p - C_v = nC_v - C_v$$

$$\therefore C_p = nC_v$$

Index of the Compression or Expansion:-

System changes its states from 1 to 2 the index of the expansion or the compression can be calculate as follows:

Polytropic process law $PV^n = C$,

So we can write for the change of the state from P_1, V_1 to P_2, V_2

$$P_1 V_1^n = P_2 V_2^n$$

So that $\frac{P_1}{P_2} = \frac{V_2^n}{V_1^n}$

Now we take the logarithm on both the side

$$\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{V_2^n}{V_1^n}\right)$$

$$\ln\left(\frac{P_1}{P_2}\right) = n \ln\left(\frac{V_2}{V_1}\right)$$

$$n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)}$$

Properties of Gases Numericals

(1). Tank contains 3 m^3 of air at 25 bar with an absolute pressure. This air is cool until its pressure and temperature decrease to 15 bar and 21°C respectively. Determine change in the internal energy, change in enthalpy, and the heat transfer.

Take a $C_p = 1.005 \frac{\text{kJ}}{\text{kg.k}}$ and $C_v = 0.718 \frac{\text{kJ}}{\text{kg.k}}$

for the air.

Here the air is contained in a tank so it is a constant volume process.

$$V = \text{constant} = 3\text{m}^3$$

$$P_1 = 25 \text{ bar}$$

$$P_2 = 15 \text{ bar}$$

$$T_2 = 21^\circ\text{C} = 294 \text{ K}$$

Now for the Constant volume process

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}$$

$$T_1 = T_2 \cdot \frac{P_1}{P_2}$$

$$T_1 = 294 \times \frac{25}{15}$$

$$= 490 \text{ K}$$

Mass of the air in the tank can be calculate using the characteristic gas equation

$$pV = mRT$$

$$R = C_p - C_v$$

$$= 1.005 - 0.718 = 0.287 \frac{\text{kJ}}{\text{kg.k}}$$

Change in the Enthalpy

$$\begin{aligned} H_2 - H_1 &= m C_p (T_2 - T_1) \\ &= 53.33 \times 1.005 (294 - 490) \\ &= -10504 \text{ Kj} \end{aligned}$$

Enthalpy of the air is decrease by 10504.9 Kj during the process.

Change in the internal energy

$$\begin{aligned} U_2 - U_1 &= m C_v (T_2 - T_1) \\ &= 53.33 \times 0.718 \times (294 - 490) \\ &= -7505 \text{ kJ} \end{aligned}$$

Internal energy of air is decrease by 7505 Kj during the process.

Here it is an constant volume process so that $W=0$

So from the first law of thermodynamics

$$Q = \Delta U + W$$

But $W = 0$

So that $Q = \Delta U$

$$Q = -7505 \text{ Kj}$$

So that it is clear from the negative sign that system rejecting 7505 Kj of heat.

(2). 1 kg of air at 7 bar pressure and 90°C temperature undergoes a non flow polytropic process. The law of the expansion is $pV^{1.1} = \text{Constant}$.

The pressure falls to 1.4 bar during the process.

Calculate (1) Final temperature

(2) Work done

(3) Change in the internal energy

(4) Heat exchange (transfer)

Take $R = 287 \frac{\text{J}}{\text{Kg.K}}$ and $\gamma = 1.4$ for the air.

Given:

$$m = 1$$

$$P_1 = 7 \text{ bar}$$

$$T_1 = 90^\circ\text{C} = 363 \text{ K}$$

$$P_2 = 1.4 \text{ bar}$$

$$pV^{1.1} = \text{Constant}$$

For the polytropic process we have

$$\frac{n-1}{n}$$

So that $\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}$

$$\left(\frac{n-1}{n}\right)$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)}$$

$$\left(\frac{0.1}{1.1}\right)$$

$$T_2 = 363 \times \left(\frac{1.4}{7}\right)$$

$$T_2 = 313.6 \text{ K}$$

$$\begin{aligned}
 \text{Work Done } W &= \frac{P_1 V_1 - P_2 V_2}{n-1} \\
 &= \frac{mRT_1 - mRT_2}{n-1} \\
 &= \frac{mR(T_1 - T_2)}{n-1} \\
 &= \frac{1 \times 0.287 (363 - 313.6)}{1.1-1} \\
 &= 141.8 \text{ Kj}
 \end{aligned}$$

Now we have $C_p - C_v = R$ and $\frac{C_p}{C_v} = \gamma$

$$C_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.718 \frac{\text{kJ}}{\text{kg.K}}$$

$$C_p = 1.005 \frac{\text{kJ}}{\text{kg.K}}$$

$$\begin{aligned} \text{Change in the internal energy} &= mC_v (T_2 - T_1) \\ &= 1 \times 0.718 \times (313.6 - 363) \\ &= -35.47 \text{ KJ} \end{aligned}$$

$$\begin{aligned} \text{Heat Transfer } Q &= \frac{\gamma - n}{\gamma - 1} \times W \\ &= \frac{1.4 - 1.1}{1.4 - 1} \times 141.8 \\ &= 106.35 \text{ kJ} \end{aligned}$$

Which can also be calculate from first law of Thermodynamics

$$Q = \Delta U + W$$

$$= -35.47 + 141.80$$

$$= 106.33 \text{ Kj}$$

(3) A gas whose pressure, volume, temperature are 2.75 bar, 0.09 m^3 , 185°C respectively has the state change at constant pressure until its temperature becomes 15°C .

Calculate:

(1) Heat transfer

(2) Work Done during the process

(3) Change in the internal energy

Take the value of $R = 0.29 \frac{\text{KJ}}{\text{Kg.k}}$

$C_p = 1.005 \frac{\text{KJ}}{\text{Kg.k}}$, $C_v = 0.715 \frac{\text{KJ}}{\text{Kg.k}}$

Given:

$$P_1 = 2.75 \text{ bar}$$

$$V_1 = 0.09 \text{ m}^3$$

$$T_1 = 185^\circ \text{ C} = 458 \text{ K}$$

$$T_2 = 15^\circ \text{ C} = 288 \text{ K}$$

$$Q = ?$$

$$W = ?$$

Constant Pressure Process

First of all we can calculate the mass undergoes a process from the characteristic gas equation.

$$\begin{aligned} M &= \frac{P.V}{R.T} \\ &= \frac{275 \times 0.09}{0.29 \times 458} \\ &= 0.186 \text{ KG} \end{aligned}$$

Due to the Constant Pressure Process

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\begin{aligned} V_2 &= \frac{T_2}{T_1} \cdot V_1 \\ &= \frac{288}{458} \cdot 0.09 \\ &= 0.057 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{Work Done } W &= P(V_2 - V_1) \\ &= 275(0.057 - 0.09) \\ &= -9.075 \text{ Kj} \end{aligned}$$

Work is done on the system.

Heat transfer during constant pressure

$$\begin{aligned} Q &= mC_p (T_2 - T_1) \\ &= 0.186 \times 1.005 (288 - 458) \\ &= -31.75 \text{ Kj} \end{aligned}$$

Heat is rejected by the system.

$$\begin{aligned} \Delta U &= mC_v (T_2 - T_1) \\ &= 0.186 \times 0.715 \times (288 - 458) \\ &= -22.61 \text{ Kj} \end{aligned}$$

From first law of Thermodynamics

$$\begin{aligned} Q &= \Delta U + W \\ &= -31.78 + 9.08 \\ &= -22.7 \text{ kJ} \end{aligned}$$

(4) 0.67 kg of gas at 14 bar and 290°C is expanded to four times the original volume according to the law $PV^{1.3} = \text{constant}$.

Calculate

(1) The original and final volume of the gas

(2) The final temperature of the gas

(3) The final pressure of the gas

Take the value of $R = 0.287 \frac{\text{KJ}}{\text{KG.K}}$

Given:

$$m = 0.67 \text{ kg.}$$

$$p_1 = 14 \text{ bar}$$

$$T_1 = 290^\circ \text{ C} = 563 \text{ K}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg.K}}$$

$$pV^{1.3} = \text{Constant}$$

Initial volume can be calculate by ideal gas equation.

$$pV_1 = mRT_1$$

$$\begin{aligned} V_1 &= \frac{mRT_1}{p} \\ &= \frac{0.67 \times 287 \times 563}{14 \times 10^5} \\ &= 0.077 \text{ m}^3 \end{aligned}$$

Final volume is four times that the initial volume.

$$V_2 = 4 V_1 = 0.308 \text{ m}^3$$

For the polytrophic process we have the equation

n

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n$$

n

1.3

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n$$

$$P_2 = 14 \left(\frac{1}{4}\right)^{1.3}$$

$$P_2 = 2.31 \text{ bar}$$

$$n-1$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$n-1$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$1.3 - 1$$

$$T_2 = 563 \cdot \left(\frac{1}{4}\right)^{1.3-1}$$

$$T_2 = 371.44 \text{ K}$$

(5) Determine the work done in compressing one kg of air from a volume of 0.15 m^3 at a pressure of 1.0 bar to a volume of 0.05 m^3 when the compression is there (i) isothermal (ii) adiabatic.

Take $\gamma = 1.4$

Also comment on your answer.

Given:

$$m = 1 \text{ kg}$$

$$V_1 = 0.15 \text{ m}^3$$

$$p_1 = 1 \text{ bar}$$

$$V_2 = 0.05 \text{ m}^3$$

$$W = ?$$

Isothermal compression

$$W = P_1 V_1 \ln \frac{p_1}{p_2}$$

$$\frac{p_2}{p_1} = \frac{V_1}{V_2}$$

$$\begin{aligned} P_2 &= P_1 \cdot \ln \frac{V_1}{V_2} \\ &= 1 \cdot \ln \frac{0.15}{0.05} \\ &= 3 \text{ bar} \end{aligned}$$

$$W = -16.48 \text{ kJ}$$

Adiabatic compression

γ

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

γ

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma$$

1.4

$$P_2 = 1. \left(\frac{0.15}{0.05}\right)^{1.4}$$

$$P_2 = 4.66 \text{ bar}$$

$$\begin{aligned}
 \text{Work done} &= \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \\
 &= \frac{100 \times 0.15 - 466 \times 0.05}{1.4 - 1} \\
 &= -20.75 \text{ Kj}
 \end{aligned}$$

Work required for the isothermal compression is lesser than that required for the adiabatic compression. This can also be seen from p-V diagram as area under isothermal process is lesser than area under adiabatic process.

(6) One kg of gas at $100 \frac{kN}{m^2}$ and $17^\circ C$ is compressed isothermally to a pressure of $2500 \frac{kN}{m^2}$ in a cylinder.

The characteristic equation of the gas is given by the equation $pV = 260 \frac{T}{kg}$ where T is in Kelvin.

Find out: (1) The final temperature

(2) Final Volume

(3) Compression ratio

(4) Change in the enthalpy

(5) Work done on the gas.

Given:

$$P_1 = 10 \frac{kN}{m^2}$$

$$T_1 = 17^{\circ}C$$

$$pV = 260T$$

(i) Process is an isothermal so that final temperature equal to initial temperature
 $= 17^{\circ} \text{ C}$

(ii) Isothermal process

$$P_1 V_1 = P_2 V_2$$

To calculate V_2 , first one need to calculate V_1

$$P_1 V_1 = 260 T_1$$

$$V_1 = \frac{260 T_1}{P_1}$$

$$= \frac{260 \times 0.754}{2500}$$

$$= 0.03016 \text{ m}^3$$

$$\begin{aligned}\text{Compression Ratio} &= \frac{V_1}{V_2} = \frac{p_2}{p_1} \text{ (for the isothermal process)} \\ &= \frac{2500}{100} = 25\end{aligned}$$

$$\text{Change in the enthalpy} = \Delta H = 0$$

$$\begin{aligned}W &= P_1 V_1 \ln \frac{p_1}{p_2} \\ &= 100 \times 0.754 \ln 25 \\ &= 242.70 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

(7) One kg of gas is compressed polytropically from 150 kPa pressure and 290 K temperature to 750 kPa. The compression is according to the law $pV^{1.3} = \text{constant}$.

Find out:

- (a) Final temperature
- (b) Work done
- (c) Change in the internal energy
- (d) Amount of heat transfer
- (e) Change in the enthalpy

$$R = 0.287 \frac{\text{kJ}}{\text{kg.K}}$$

$$C_p = 1.001 \frac{\text{kJ}}{\text{kg.K}}$$

Given:

$$P_1 = 150 \text{ kPa}$$

$$T_1 = 290 \text{ K}$$

$$P_2 = 750 \text{ Kpa}$$

$$n = 1.3$$

$$m = 1 \text{ kg}$$

$$R = 0.287 \frac{\text{kJ}}{\text{kg.K}}$$

$$C_p = 1.001 \frac{\text{kJ}}{\text{kg.K}}$$

(a) Final Temperature

$$\left(\frac{n-1}{n}\right)$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)$$

$$\left(\frac{n-1}{n}\right)$$

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)$$

$$\left(\frac{1.3-1}{1.3}\right)$$

$$T_2 = 290 \times \left(\frac{750}{150}\right)$$

(b) Work done

$$\begin{aligned} W &= \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{R(T_1 - T_2)}{n-1} \\ &= \frac{0.287(290 - 420.4)}{1.3 - 1} \\ &= \frac{0.287(290 - 420.4)}{0.3} \\ &= -124.75 \frac{kJ}{kg} \end{aligned}$$

Negative sign indicates that work is done on the gas.

(c) Change in the internal energy

$$\Delta U = C_v (T_2 - T_1)$$

$$C_v = C_p - R$$

$$= 1.001 - 0.287$$

$$= 0.714 \frac{kJ}{kg.K}$$

So that

$$\Delta U = 0.714 (420.4 - 290)$$

$$= 93.11 \frac{kJ}{kg}$$

(d) Amount of Heat Transfer

$$Q = \left(\frac{P_1 V_1 - P_2 V_2}{n-1} \right) \left(\frac{\gamma - n}{\gamma - 1} \right)$$

$$\gamma = \frac{C_p}{C_v} = \frac{1.001}{0.714} = 1.402$$

So that

$$\begin{aligned} Q &= \left(\frac{P_1 V_1 - P_2 V_2}{n-1} \right) \left(\frac{\gamma - n}{\gamma - 1} \right) \\ &= (-124.75) \times \left(\frac{1.402 - 1.3}{1.402 - 1} \right) \\ &= -31.65 \frac{kJ}{kg} \end{aligned}$$

(e) Change in the enthalpy

$$\begin{aligned}\Delta h &= C_p \times \Delta T \\ &= 1.001 (420 - 290) \\ &= 130.5 \frac{kJ}{kg}\end{aligned}$$

