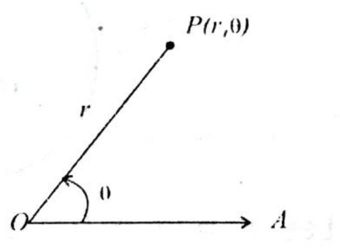
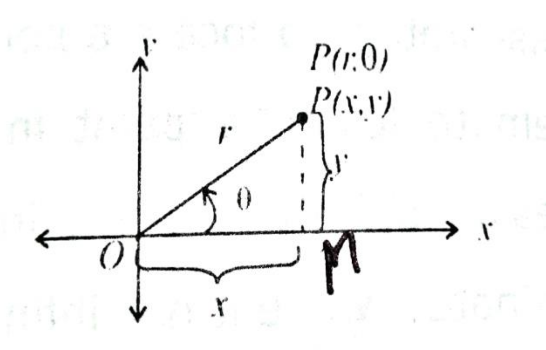
Tracing of Polar coordinates:

A point P in the plane, has polar coordinates (r, θ), where r is the distance of the point from the fixed origin O (Called the Pole) and θ is the angle between $\overrightarrow{OP}$ and initial ray $\overrightarrow{OA}$ (Called polar axis).



Relation between Polar and Cartesian coordinates:



Choose the polar axis along the positive x-axis and the pole at the origin , from right triangle PMO

$cos\theta = \frac{x}{r}$ and $sin\theta = \frac{x}{r}$

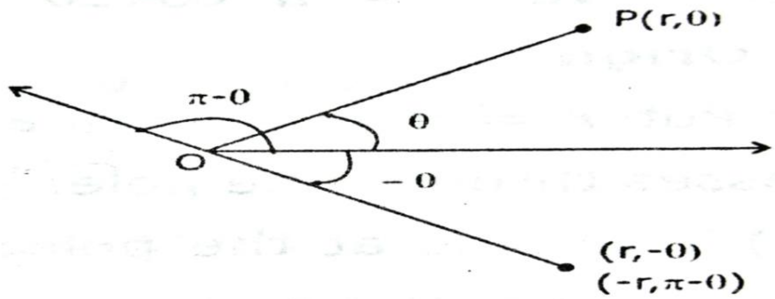
$x = rcos\theta$, $y = rsin\theta$, from these relation we have $r^2 = x^2 + y^2$ and $tan\theta = \frac{y}{x}$

Procedure for tracing polar curves:

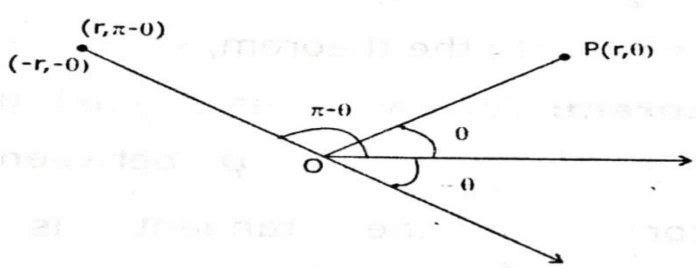
Let the equation of the curve be $f(r,\ \theta) = 0$

1.Symmetry:

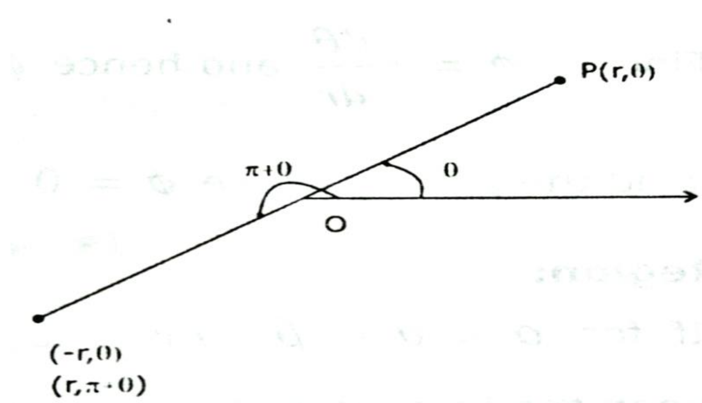
(i) Symmetry about the initial line (polar axis): If the equation of the curve remains unchanged when θ is replaced by $-\theta$ or when the equation remains unchanged on replacing r by –r and θ by $\pi - \theta$ then the curve is symmetric with respect to the initial line(polar axis).



(ii) Symmetry about the line $\theta = \frac{\pi}{2}$(Normal axis): If the equation of the curve remains unchanged when θ is replaced by $\pi - \theta$ or when the equation remains unchanged on replacing r by -r, and θ by $-\theta$ then the curve is symmetric with respect to the line $\theta = \frac{\pi}{2}$(normal axis).

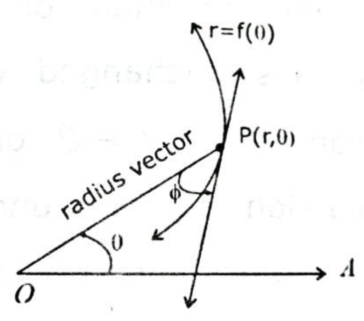


(iii) Symmetry about the pole : If the equation of the curve remains unchanged when r is replaced by -r (i.e. power of r is even) or when the equation remains unchanged on replacing θ by $\pi+\theta$ then the curve is symmetric with respect to the pole.



2. Pole(Origin): put r=0 in the equation of curve , If we get real values of θ then the curve passes through the pole, real values of θ are tangents at pole

3. **Angle between the radius vector and tangents:**
The angle $\emptyset$ between the radius vector and the tangent is given by $tan\emptyset = \frac{r}{\frac{dr}{d\theta}}$, if $tan\emptyset = 0$ tangent is coincides with the radius vector and $tan\emptyset = \infty$, tangent is perpendicular to radius vector.



(4)Tabular Values: corresponding to different values of θ find the values of r and $tan\emptyset$.

Example:1 Trace the curve $r = a(1 + cos\theta)\ (a > 0)$

1. symmetry : Science the equation remains unchanged when θ is replaced by $-\theta$,curve is symmetrical about the initial line .

2 pole: if r=0 $\therefore$ $a(1 + cos\theta) = 0$ $\therefore$ $cos\theta = -1$ we get $\theta = (2k + 1)\pi\ , k \in Z$,

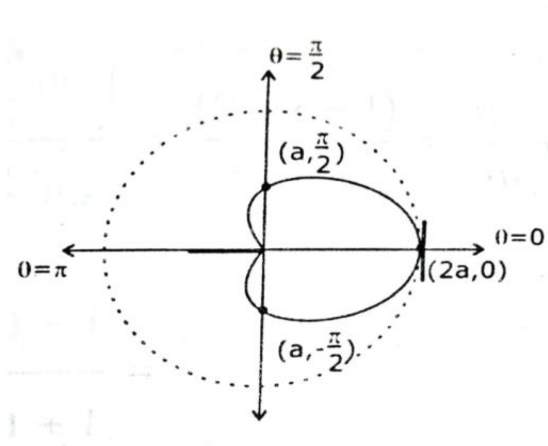
$\therefore$ curve is passes through the pole and $\theta = (2k + 1)\pi\ , k \in Z$ are tangents at pole

3 Angle between the tangent and radius vector:

$$tan\emptyset = \frac{r}{\frac{dr}{d\theta}} \quad \therefore\ tan\emptyset = \frac{a(1+cos\theta)}{-asin\theta} \quad \therefore\ tan\emptyset = -cot\frac{\theta}{2}$$

4 Tabular Values :

θ :	0	$\frac{\pi}{2}$	π
r :	2a	a	0
$tan\emptyset$:	∞	-1	0



Example:2 Trace the Curve $r^2 = a^2cos2\theta$.

1. Symmetry: Science the equation remains unchanged when θ is replaced by $-\theta$,curve is symmetrical about the initial line.

Since the equation remains unchanged when θ is replaced by $\pi - \theta$,curve is symmetrical about normal line.

Since power of r is even , curve is symmetrical about pole.

2.Pole: if r=0 $\therefore cos2\theta = 0$ $\therefore$ $\theta = (2k+1)\frac{\pi}{4}, k \in Z$ $\therefore$ Curve is passes through pole and $\theta = (2k+1)\frac{\pi}{4}, k \in Z$ are tangents at pole.

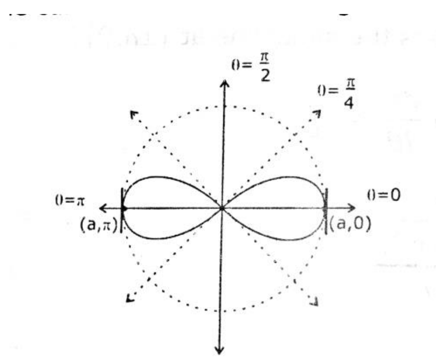
3. Angle between the tangent and radius vector:

From given equation $2r\frac{dr}{d\theta} = -2a^2 sin2\theta$

Now $tan\emptyset = \frac{r}{\frac{dr}{d\theta}}$ $\therefore tan\emptyset = \frac{r}{\frac{-a^2 sin2\theta}{r}} = -\frac{r^2}{a^2 sin2\theta} = -\frac{a^2 cos2\theta}{a^2 sin2\theta} = -cot2\theta$

4 Tabular values:

θ :	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
r :	a	0	-	0	a
$tan\emptyset$:	∞	0	-	0	∞



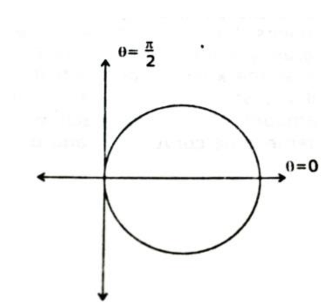
Example:3 Trace the lemniscates $r = 2acos\theta$

1. Symmetry: Since the equation remain unchanged when $\theta$is replaced by $-\theta$, curve is symmetrical about initial line.

2.Pole: if r=0, $\therefore cos\theta = 0$ $\therefore \theta = (2k+1)\frac{\pi}{2}, k \in Z$ $\therefore$ Curve is passing through pole and

$\therefore \theta = (2k+1)\frac{\pi}{2}, k \in Z$ are tangents at pole.

3. Angle between the tangent and radius vector: $tan\emptyset = \frac{r}{\frac{dr}{d\theta}}$ $\therefore tan\emptyset = \frac{2acos\theta}{-2asin\theta} = -cot\theta$



4. Tabular Values:

θ:	0	$\frac{\pi}{2}$	π
r:	2a	0	-2a
$tan\emptyset$:	∞	0	∞

Exercise: Trace the following Curves:

(1) $r = a(1 - cos\theta)$ (2) $r = a(1 + sin\theta)$ (3) $r = a(1 - sin\theta)$

Ans: (1) (2) (3)