

Chapter 1

Subject Name: Digital Fundamentals

Subject code:-3130704

Learning Outcomes

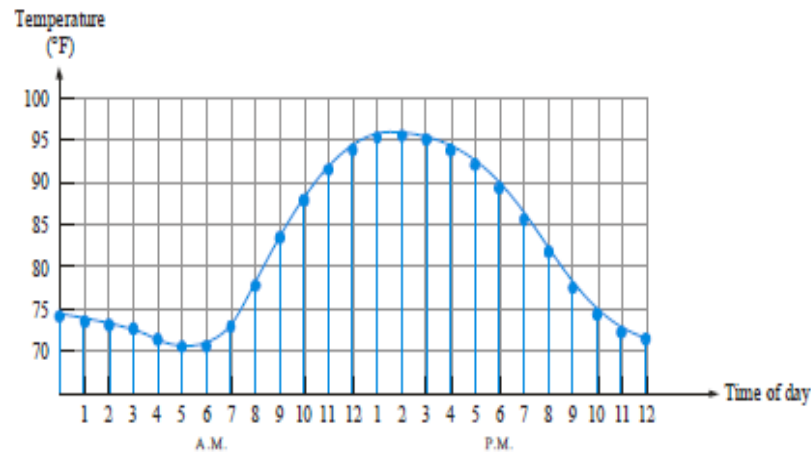
- **At the end of this chapter, you must be able to**
 - Distinguish between analog and digital systems.
 - Use of different types of basic gates
 - Boolean algebraic Laws
 - How to reduce Boolean function
 - Conversion of different number systems
 - Methods of error detecting and correcting
 - Comparison of logic families

Signal:

It is physical quantity ,which contains information & a function of one or more independent variables.

-Two types of signal: analog & digital

1. Analog signal
2. Digital signals



Analog signal versus digital signal

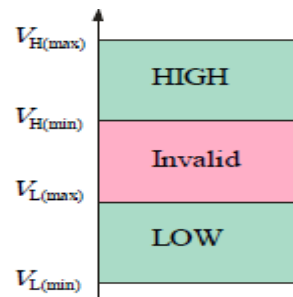
- Analog=continuous
- Digital=discrete
- Analog signals can have infinite no. of different signals. Example: Temperature, sound, pressure, distance, sound.
- A digital signal is not a continuous signal. example: octal, hexadecimal.

Advantage of digital signals

- Digital signals can be processed & transmitted more efficiently and reliably.
- Can be stored.
- Playback & further processing possible.
- Noise effect is less.

Binary Digits and Logic Levels

- Digital electronics uses circuits that have two states, which are represented by two different voltage levels called HIGH and LOW. The voltage represents numbers in the binary systems.
- In binary a single number is called a bit (for binary digit). A bit can have the value of either a 0 or 1, depending on if the voltage is HIGH or LOW.



Digital & Analog systems

Digital systems

Combination of device designed to manipulate logical information or physical quantity that are represented in digital form.

Example: digital calculator

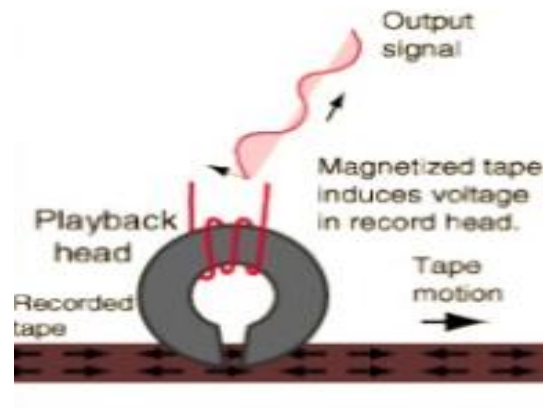
Analog system

Devices that manipulate physical quantities that are represented in analog form.

Example: magnetic tape recording, playback equipment

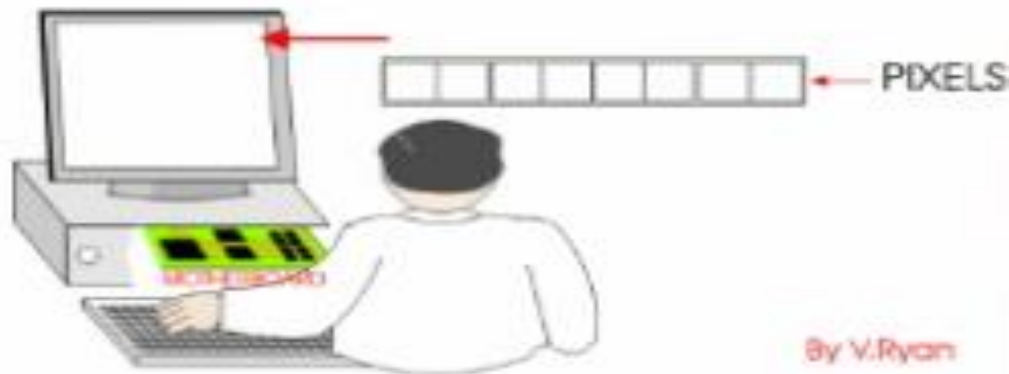
Example 1 Tape

- During playback ,a magnetic material in the tape head is magnetized as the magnetic tape passes.
- Then ,the magnetic field penetrates a coil of wire is wrapped around it.
- Change in magnetic field will induce a voltage in the coil. This induced voltage forms an electrical image of the signal which is recorded on the tape.



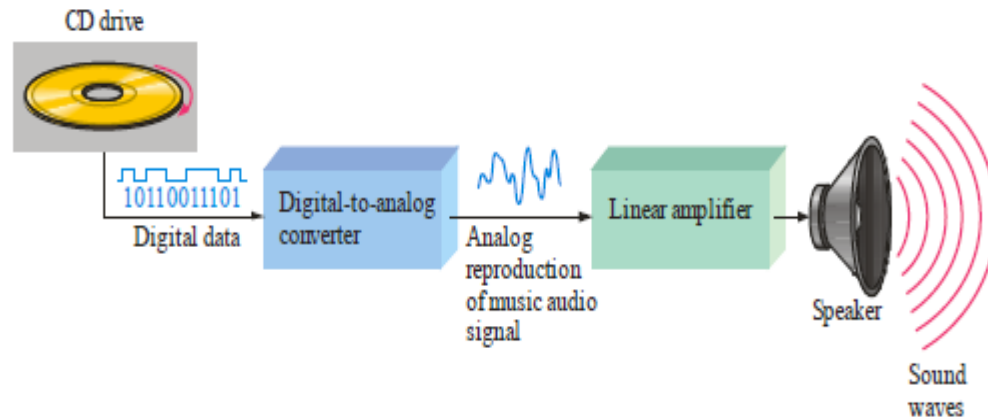
Example2 computer

- All the stored and processed data are in binary form. Why?
- Digital circuit/device only concerns about two operating states /logic levels.
- This system allows computers to perform complex calculations very quickly and efficiently.



Example 3 CD player

- CD player: Digital and analog parts co-exist together. Many systems use a mix of analog and digital electronics to take advantage of each technology . A typical CD player accepts digital data from the CD drive and convert it to an analog signal for amplification



Advantage of digital Techniques

- Digital systems are easier to design.
- Information storage is easy.
- Accuracy & precision are easier to maintain throughout the system.
- They are more versatile.
- Digital circuit are less affected by noise
- Digital circuitry can be fabricated on IC chips

Disadvantage of digital techniques

- The real world is analogue.
- Digital systems can be fragile.
- Processing digitized signals takes time.
- Digital circuits use more energy than analogue circuits & produce more heat.
- Digital circuits are made from analogue components –must make sure the digital behavior –must make sure the digital behavior is not affected by the analogue.
- Digital circuit are sometimes more expensive

Digital systems overcomes the drawback of analog systems

When dealing with analogue inputs and outputs four steps must be followed:

1. Convert the physical variable to an electrical signal(analogue)
- 2.Convert the electrical(analogue)signal into digital form- ADC (Analogue digital converter)
- 3.Process (operate on)the digital information
4. Convert the digital output back to real –world analogue form- DCA (Digital Analogue Converter)

Gate

- The Most basic digital device are called gates.
- Gates got their name from their function of allowing or blocking (gating)the flow of digital information.
- A gate has one or more inputs and produces an output depending on the inputs
- A gate is called a combinational circuit
- Three most important gate are : AND,OR,NOT

Logic gates

- Most basic logical unit of the digital system is gate circuit
- Types of gate circuit are as follows:

① AND Gate

- It has an o/p which is at logic level '0' and only goes 'HIGH' to logic level '1' when ALL of its i/p are at logic level '1'



Truth table

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

② OR gate



truth table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

③ NOT gate



A	C
0	1
1	0

④ NOR gate



truth table

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

⑤ NAND gate



truth table

A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

⑥ Ex-OR gate

- It has 1 state when one & only one of its two i/p assumes a logic 1 state & has 0 state when all its i/p are same.



truth table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

⑦ Ex-NOR gate



truth table

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

Boolean Algebra

AND laws

1. $A \cdot 0 = 0$ (Null law)
2. $A \cdot 1 = A$ (Identity law)
3. $A \cdot A = A$
4. $A \cdot \bar{A} = 0$

Distributive laws

1. $A(B+C) = AB + AC$
2. $A + BC = (A+B)(A+C)$

Commutative laws

1. $A + B = B + A$
2. $A \cdot B = B \cdot A$

Idempotent laws

1. $A \cdot A = A$
2. $A + A = A$

OR laws

1. $A + 0 = A$ (Null law)
2. $A + 1 = 1$ (Identity law)
3. $A + A = A$
4. $A + \bar{A} = 1$

Redundant Literal laws

1. $A + A\bar{B} = A + B$
2. $A(\bar{A} + B) = AB$

Associate laws

1. $(A+B)+C = A+(B+C)$
2. $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Absorption laws

1. $A + AB = A$
2. $A(A+B) = A$

De-Morgan's Theorem

① Law 1: - $\overline{A+B} = \overline{A} \cdot \overline{B}$

→ Complement of sum of variables is equal to the product of their individual complement

$$\begin{matrix} A \\ \text{---} \\ \text{---} \\ B \end{matrix} \rightarrow \text{OR Gate} \rightarrow Y = A + B = \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \rightarrow \text{NAND Gate} \rightarrow Y = \bar{A} \cdot \bar{B}$$



Truth table

A	B	$A+B$	\bar{A}	\bar{B}	$A \cdot \bar{B}$
0	0	0	1	1	0
0	1	1	1	0	0
1	0	1	0	1	0
1	1	1	0	0	0

\uparrow \uparrow
 L.H.S R.H.S

② Law:- $\overline{A \cdot B} = \overline{A} + \overline{B}$

→ Complement of a product of variable is equal to the sum of their individual complement.

A
 B  $Y = \overline{A \cdot B}$ $=$  $Y = \overline{A + B}$

Truth Table

A	B	$A \cdot B$	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
0	0	0	1	1	1
0	1	0	1	0	1
1	0	0	0	1	1
1	1	1	0	0	0

\uparrow \uparrow
 L.H.S R.H.S

Universal Gate

NAND as universal gate :

① NOT using NAND

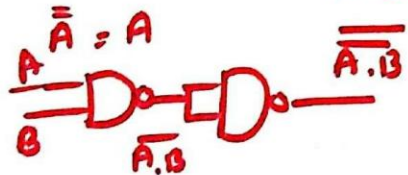


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

→ NAND

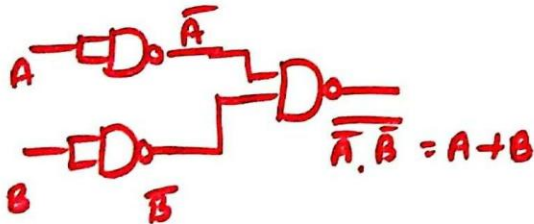
② AND using NAND

$$Y = A.B = \overline{\overline{A.B}}$$



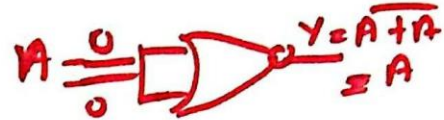
③ OR using NAND

$$Y = A+B = \overline{\overline{A+B}} = \overline{\overline{A}.\overline{B}}$$



NOR as Universal NOR :

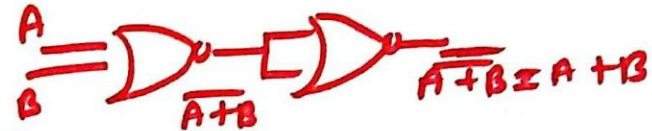
① NOT using NOR



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

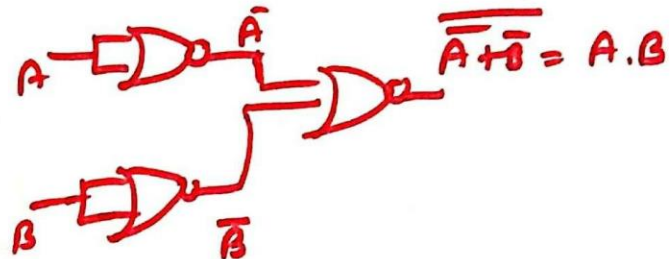
② OR gate using NOR

$$Y = A+B = \overline{\overline{A+B}}$$



③ AND gate using NOR

$$Y = A.B = \overline{\overline{A.B}} = \overline{\overline{A} + \overline{B}}$$



Reducing Boolean Expression

① $f = A + B[AC + (B + \bar{C})D]$

Ans $f = A + B[AC + BD + \bar{C}D]$
 $= A + BAC + BBD + B\bar{C}D$ (distributive)
 $= A + ABC + BD + B\bar{C}D$ (distributive)
 $= A(1 + BC) + BD(1 + \bar{C})$ ($A.A = A$)
 $= A(1 + BC) + BD(1 + \bar{C})$ ($A + 1 = 1$)

$f = A + BD$

③ Reduce the expression $(B + BC)(B + \bar{B}C)(B + D)$

Ans $F = A + B[AC + (B + \bar{C})D]$
 $= A + B(AC + BD + \bar{C}D)$ (Ans = B)
 $= A + BAC + BBD + B\bar{C}D$ (distributive)
 $= A + ABC + BD + B\bar{C}D$ (distributive)
 $= A(1 + BC) + BD(1 + \bar{C})$ (In order)
 $= A(1) + BD(1)$ (factor)
 $= A + BD$ ($1 + \bar{C} = 1$)

② $f = A[B + \bar{C}(AB + A\bar{C})]$ (GTV 3/12/19)

Ans $f = A[B + \bar{C}(\overline{AB} \cdot \overline{A\bar{C}})]$ (De-Morgan's)
 $= A[B + \bar{C}(\bar{A} + \bar{B})(\bar{A} + C)]$ (De-Morgan's)
 $= A[B + \bar{C}(\bar{A}\bar{A} + \bar{A}C + \bar{B}\bar{A} + \bar{B}C)]$
 $= A[B + \bar{C}\bar{A} + \bar{C}\bar{A}C + \bar{C}\bar{B}\bar{A} + \bar{C}\bar{B}C]$ (distributive)
 $= A[B + \bar{C}\bar{A} + 0 + \bar{C}\bar{B}\bar{A} + 0]$ (distributive)
 $= AB + A\bar{C}\bar{A} + A\bar{C}\bar{B}\bar{A}$ ($A.A' = 0$)
 $= AB + 0 + 0$ (distributive)
 $= AB$ ($A.A' = 0$)

④ Reduce the expression $(\overline{A + BC})(AB + ABC)$

Ans = 0

⑤ show that, $AB + A\bar{B}C + B\bar{C} = A\bar{C} + BC$

$$(1) (B + B^c)(B + \overline{B}C)(B + D)$$

$$(2) \overline{(A + \overline{B}C)} (A\overline{B} + ABC)$$

$$(3) \text{ show that } AB + A\overline{B}C + B\overline{C} \neq B\overline{C} + AC$$

$$(2) \text{ show that } A\overline{B}C + B + B\overline{D} + A\overline{B}\overline{D} + \overline{A}C = B + C$$

Classification of Number systems

Number System Classification

Positional/ Weighted Number System

- Decimal
- Octal
- Binary
- Hexadecimal
- BCD
- 8-4-2-1 Code

Non-Positional/ Non-Weighted Number System

- Excess-3 Code
- Cyclic Code
- Roma Code
- Gray Code

Non-positional Number Systems

Characteristics

- Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc.
- Each symbol represents the same value regardless of its position in the number.
- The symbols are simply added to find out the value of a particular number.

Difficulty

- It is difficult to perform arithmetic with such a number system.

Positional Number Systems

Characteristics

- Use only a few symbols called digits
- These symbols represent different values depending on the position they occupy in the number.
- The value of each digit is determined by:
 1. The digit itself
 2. The position of the digit in the number
 3. The base of the number system

- **Number system:** It is set of values used to represent a quantity.
- **Radix/Base:** No. of value that a digit can have is equal to the systems.
- **Weight :**each position represent a different multiple of base this multiplier called weight.
- **MSD:** Leftmost digit having the highest Weight known as most significant digit
- **LSD:** Rightmost digit having the highest Weight known as least significant digit

$$125_{10} \Rightarrow \begin{array}{rcl} 5 \times 10^0 & = & 5 \\ 2 \times 10^1 & = & 20 \\ 1 \times 10^2 & = & 100 \\ \hline & & 125 \end{array}$$

Weight

Base

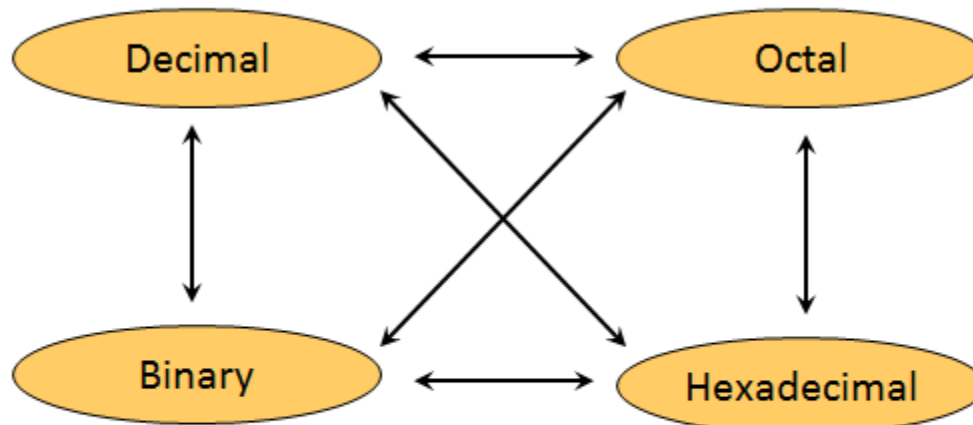
Common number systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, ... 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, ... 7	No	No
Hexa-decimal	16	0, 1, ... 9, A, B, ... F	No	No

- The maximum value of a single digit is always equal to one less than the value of the base.

Conversion among Bases

- Possibilities



- Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

Binary Number System

Characteristics

- A positional number system
- Has only 2 symbols or digits (0 and 1). Hence its base = 2.
- The maximum value of a single digit is 1 (one less than the value of the base).
- Each position of a digit represents a specific power of the base (2)
- This number system is used in computers

Decimal To Binary

- **Successive division for inter part:**

1. Divide the integer part of given decimal no. by the base & note down remainder.
2. Continue to divide the quotient by base until there is nothing left. Note remainder from each step.
3. List the remainder in reverse order from bottom to top .

$$125_{10} = ?_2$$

2	125	1
2	62	0
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

$$125_{10} = 1111101_2$$

- **Successive multiplication for fractional part**

1. Multiply given no. by base
2. Note down carry generated in this multiplication as MSD
3. Multiply only fractional no. of the product in step 2 by the base ,& note down carry as the next bit to MSD
4. Repeat Step 2 & 3 Upto The End.

$$0.6875_{10} = ?_2$$

	<u>integer</u>		<u>fraction</u>
$0.6875 \times 2 = 1.3750$	1	+	0.3750
$0.3750 \times 2 = 0.7500$	0	+	0.7500
$0.7500 \times 2 = 1.5000$	1	+	0.5000
$0.5000 \times 2 = 1.0000$	1	+	0.0000

$$0.6875_{10} = 0.1011_2$$

Exercise

$$1. (163.875)_{10} =$$

$$2. (52)_{10} =$$

$$3. (0.75)_{10} =$$

$$4. (105.15)_{10} =$$

Answer

1. 10100011

2. 110100

3. 0.11

4. 1101001.001001

Binary to Decimal

1. Write down weight corresponding to different position.
2. Multiply each digit in the given no. with corresponding weight to obtain product no.
3. Add all the product no. to get decimal equivalent.

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow \\ 1 \times 2^5 & + & 0 \times 2^4 & + & 1 \times 2^3 & + & 0 \times 2^2 & + & 1 \times 2^1 & + & 1 \times 2^0 \\ 32 & + & 0 & + & 8 & + & 0 & + & 2 & + & 1 \\ & & & & & & & & & & 43_{10} \end{array}$$

$$101011_2 = 43_{10}$$

$$\begin{array}{cccc} 1 & 1 & . & 1 & 1 \\ \swarrow & \swarrow & & \swarrow & \swarrow \\ 1 \times 2^1 & + & 1 \times 2^0 & + & 1 \times 2^{-1} & + & 1 \times 2^{-2} \\ 2 & + & 1 & + & 0.5 & + & 0.25 \\ & & & & & & 3.75_{10} \end{array}$$

Exercise

1. $(10101)_2$
2. $(11011.101)_2$
3. $(1001011)_2$
4. $(1011.01)_2$

Answer

1. 21_{10}

2. 27.625_{10}

3. 75_{10}

4. 11.25_{10}


Decimal to octal conversion

1. Divide by 8

2. Keep track of remainder

$$125_{10} = ?_8$$


8	125	5
8	15	7
8	1	1
	0	



$$125_{10} = 175_8$$

$$0.6875_{10} = ?_8$$

	<u>integer</u>		<u>fraction</u>
$0.6875 \times 8 = 5.5000$	5	+	0.5000
$0.5000 \times 8 = 4.0000$	4	+	0.0000



$$0.6875_{10} = 0.54_8$$

exercise

$$1. (3000.45)_{10} =$$

$$2. (378.93)_{10} =$$

$$3. (5497)_{10} =$$

$$4. (3025)_{10} =$$

Answer

1. 5870.3463

2. 572

3. 12571

4. 5721

Octal to Decimal

1. Multiply each bit by 8^n , where n is the weight of the bit
2. The weight is the position of the bit, starting from 0 on the right.
3. Add the result.

$$\begin{array}{ccccccc} 4 & 3 & . & 2 & 5 & & \\ & \swarrow & & \swarrow & \swarrow & & \\ 4 \times 8^1 & + & 3 \times 8^0 & + & 2 \times 8^{-1} & + & 5 \times 8^{-2} \\ 32 & + & 3 & + & 0.25 & + & 0.0781 \\ & & & & & & 35.3281_{10} \end{array}$$

$$43.25_8 = 35.3281_{10}$$

Exercise

1. $(314)_8$
2. $(4057.06)_8$
3. $(5721)_8$
4. $(630.4)_8$

Answer

1. 204_{10}

2. 2095.0937_{10}

3. 3025_{10}

4. 408.5_{10}

Decimal to Hexadecimal

1. Divide by 16

2. Keep track of remainder

$$1234_{10} = ?_{16}$$

16	1234	2
16	77	13=D
16	4	4
	0	



$$1234_{10} = 4D2_{16}$$

Exercise

1. $(2003.31)_{10} =$

2. $(2598.675)_{10} =$

3. $(49056)_{10} =$

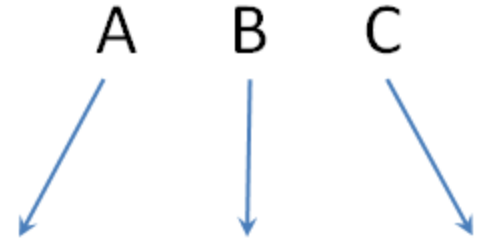
4. $(46687)_{10} =$

Answer

1. 7D3.4F5C2
2. A26.ACCC
3. BFA0
4. B65F

Hexadecimal To Decimal

1. Multiply each bit by 16^n , where n is the weight of the bit
2. The weight is the position of the bit, starting from 0 on the right.
3. Add the result.


$$\begin{array}{r} A \quad B \quad C \\ \swarrow \quad \downarrow \quad \searrow \\ A \times 16^2 + B \times 16^1 + C \times 16^0 \\ 10 \times 16^2 + 11 \times 16^1 + 12 \times 16^0 \\ 2560 + 176 + 12 \end{array}$$

$$2748_{10}$$

$$ABC_{16} = 2748_{10}$$

Exercise

1. $(4CB.2)_{16}$
2. $(A0F9.0EB)_{16}$
3. $(5C7)_{16}$
4. $(B65F)_{16}$

Answer

1. 1224.125_{10}

2. 41209.0572_{10}

3. 1479_{10}

4. 46687_{10}

Octal to Binary

- Convert each octal digit to a 3-bit equivalent binary representation.

$$705_8 = ?_2$$

7	0	5
↓	↓	↓
111	000	101

$$705_8 = 111000101_2$$

Exercise

1. $(364)_8$
2. $(5721)_8$
3. $(12571)_8$
4. $(26153.7406)_8$

Answer

1. 011110100_2

2. 101111010001_2

3. 001010101111001_2

4. $10110001101011.11110000110_2$

Binary to Octal

- Group bits in 3 starting from LSB

$$1011010111_2 = ?_8$$

001	011	010	111
↓	↓	↓	↓
1	3	2	7

$$1011010111_2 = 1327_8$$

Exercise

1. $(11010010)_2$
2. $(110101.101010)_2$
3. $(10101111001.0111)_2$
4. $(1100000110.1101)_2$

Answer

1. 322_8

2. 65.52_8

3. 2571.34_8

4. $306.D_8$

Hexadecimal to Binary

- Convert each hexadecimal digit to a 4 bits equivalent binary representation.

Hexa-Decimal to Binary

Hexa-Decimal	Binary	Hexa-Decimal	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

$$10AF_{16} = ?_2$$

1
↓
0001

0
↓
0000

A
↓
1010

F
↓
1111

$$10AF_{16} = 100001010111_2$$

Exercise

- $(FA8)_{16} = (\quad)_2$
- $(9AC3)_{16} = (\quad)_2$
- $(1A74D)_{16} = (\quad)_2$
- $(1AC.9A)_{16} = (\quad)_2$
- $(ABC.5AC)_{16} = (\quad)_2$

Binary to Hexadecimal

- Group bits in 4 starting from LSB

$$1011010111_2 = ?_{16}$$

0010	1101	0111
↓	↓	↓
2	D	7

$$1011010111_2 = 2D7_{16}$$

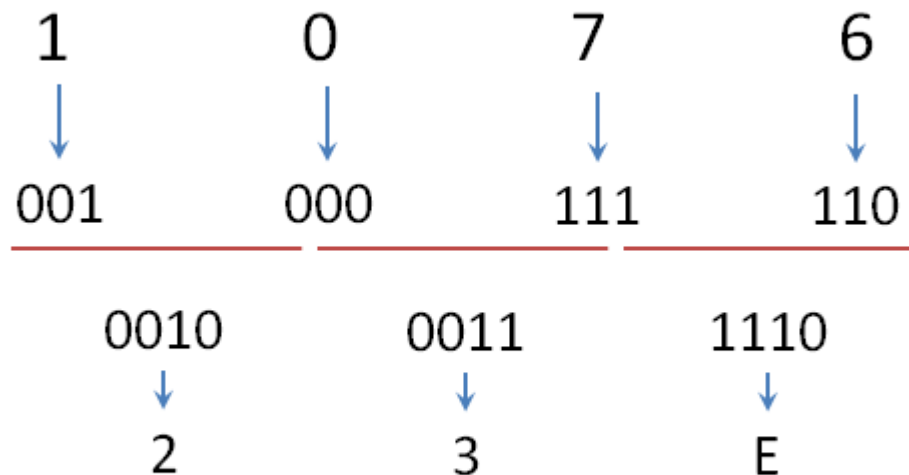
Exercise

- $(11011)_2 = (\quad)_{16}$
- $(101101)_2 = (\quad)_{16}$
- $(11101111)_2 = (\quad)_{16}$
- $(110.011)_2 = (\quad)_{16}$
- $(1001.0010)_2 = (\quad)_{16}$

Octal To Hexadecimal

- Convert octal to binary
- Regroup bits in 4 from LSB
- Convert Binary To hexadecimal

$$1076_8 = ?_{16}$$



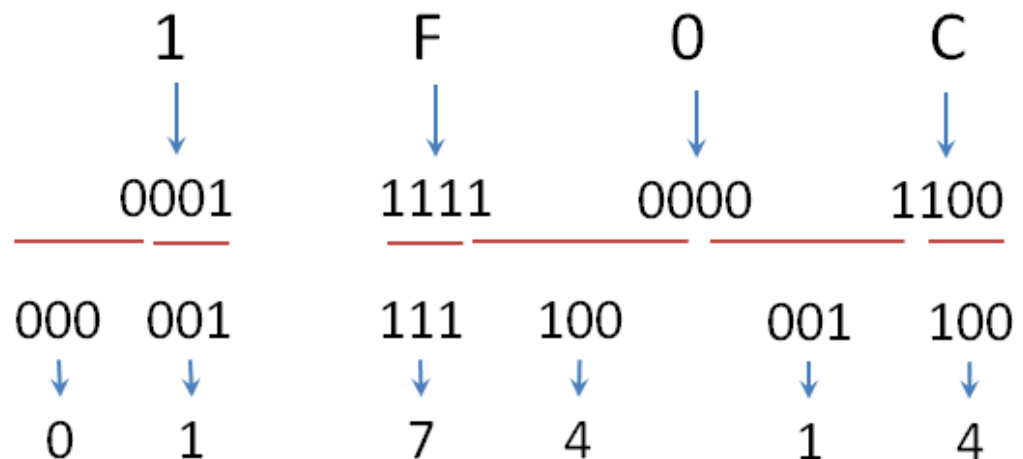
Exercise

- $(463)_8 = (\quad)_{16}$
- $(2056)_8 = (\quad)_{16}$
- $(2057.64)_8 = (\quad)_{16}$
- $(6543.04)_8 = (\quad)_{16}$
- $(7476.47)_8 = (\quad)_{16}$

Hexadecimal to Octal

- Convert hexadecimal to binary
- Regroup bits in 3 from LSB
- Convert Binary To Octal

$$1F0C_{16} = ?_8$$



$$1F0C_{16} = 17414_8$$

Exercise

$$(FA8)_{16} = (\quad)_8$$

$$(9AC3)_{16} = (\quad)_8$$

$$(1A74D)_{16} = (\quad)_8$$

$$(1AC.9A)_{16} = (\quad)_8$$

$$(ABC.5AC)_{16} = (\quad)_8$$

Sign binary Number

- Two ways of representing signed numbers:
 1. sign magnitude form
 2. complement form
- Most of computer use complement form for negative number notation.
- 1's and 2's complement are two different method this type.

1's complement

- It is obtained by subtracting each digit of that binary no. from 1.
- Or, change 1's to 0's and 0's to 1's. in binary
- - Example

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \\ - \ 1 \ 1 \ 0 \ 1 \\ \hline 0 \ 0 \ 1 \ 0 \\ \text{(1's complement of 1101)} \end{array}$$

$$\begin{array}{r} 1 \ 1 \ 1 \ . \ 1 \ 1 \\ - \ 1 \ 0 \ 1 \ . \ 0 \ 1 \\ \hline 0 \ 1 \ 0 \ . \ 1 \ 0 \\ \text{(1's complement of 101.01)} \end{array}$$

2's complement

- It is obtained by adding 1 to 1's complement.
- **Or**, copy all zeros ,working from LSB toward the MSB , until the 1st 1 is reached, copy that 1 & flip all the remaining bits.

▪ Example

$$\begin{array}{r} 1\ 1\ 1\ 1 \\ -\ 1\ 1\ 0\ 0 \\ \hline 0\ 0\ 1\ 1 \\ +1 \\ \hline 0\ 1\ 0\ 0 \\ \text{(2's complement of 1100)} \end{array}$$

$$\begin{array}{r} 1\ 1\ 1\ .\ 1\ 1 \\ -\ 1\ 0\ 1\ .\ 0\ 1 \\ \hline 0\ 1\ 0\ .\ 1\ 0 \\ +1 \\ \hline 0\ 1\ 0\ .\ 1\ 1 \\ \text{(2's complement of 101.01)} \end{array}$$

9's complement

- It's obtained by subtracting each digit of that decimal no. from 9

▪ Example

$$\begin{array}{r} 9 \ 9 \ 9 \ 9 \\ - \ 3 \ 4 \ 6 \ 5 \\ \hline 6 \ 5 \ 3 \ 4 \end{array}$$

(9's complement of 3465)

$$\begin{array}{r} 9 \ 9 \ 9 \ . \ 9 \ 9 \\ - \ 7 \ 8 \ 2 \ . \ 5 \ 4 \\ \hline 2 \ 1 \ 7 \ . \ 4 \ 5 \end{array}$$

(9's complement of 782.54)

10's complement

- It's obtained by adding 1's to 9's complement.


$$\begin{array}{r} 9 \ 9 \ 9 \ . \ 9 \ 9 \\ - \ 7 \ 8 \ 2 \ . \ 5 \ 4 \\ \hline 2 \ 1 \ 7 \ . \ 4 \ 5 \\ + 1 \\ \hline 2 \ 1 \ 7 \ . \ 4 \ 6 \end{array}$$

(10's complement of 782.54)

Sign number representation

- If no. is positive ,the magnitude is represented in its true binary form and a sign bit 0 is placed in front of the MSB
- If no. is negative ,magnitude is represented in its 2's complement form and sign bit 1 is placed in front of MSB
 - Express -65.5 in 12 bit 2's complement form.

2	65	1
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	



$$0.5 \times 2 = 1.0$$

So, result in 12-bit binary is as follows:

$$65.5_{10} = 01000001.1000_2$$

For negative number, we have to convert this into 2's complement form

$$-65.5_{10} = 10111110.1000_2$$

Exercise

1. Express -45 in 8 bit 2's complement form.
2. Express -73.25 in 12 bits 2's complement form.

Answer

1. 11010011

2. 10110110.0100

Subtraction using complement form

1. Using 9'S Complement

- obtain 9's complement of subtrahend

-Add the result to minuend and call it intermediate result.

If carry is generated then answer is positive and add the carry to LSD

-If There is no Carry Then Answer Is Negative And Take 9's Complement Of Intermediate Result And Place Negative Sign To The Result

1) $745.81 - 436.62$

7	4	5	.	8	1		7	4	5	.	8	1		
-	4	3	6	.	6	2	$\xrightarrow{\text{9's complement}}$	+	5	6	3	.	3	7
<hr/>								<hr/>						
3	0	9	.	1	9			1	3	0	9	.	1	8
								$\xrightarrow{\quad}$						
								+	1					
<hr/>								<hr/>						
								3	0	9	.	1	9	

Example

2) $436.62 - 745.81$

4	3	6	.	6	2			4	3	6	.	6							
-	7	4	5	.	8	1	$\xrightarrow{\text{9's complement}}$	+	2	5	4	.	1						
<hr/>																			
-	3	0	9	.	1	9													
													$\xrightarrow{\text{9's complement}}$						
														6	9	0	.	8	
														-	3	0	9	.	1

As carry is not generated, so take 9's complement of the intermediate result and add ' - ' sign to the result

2. Using 10'S Complement

- obtain 10's complement of subtrahend
- Add the result to minuend.
- If carry is generated then ignore it and result itself is answer.
- If There is no Carry Then Answer Is Negative And Take 10's Complement Of Result And Place Negative Sign To The Result .

Example

1) $745.81 - 436.62$

7	4	5	.	8	1		7	4	5	.	8	1			
-	4	3	6	.	6	2	$\xrightarrow{\text{10's complement}}$	+	5	6	3	.	3	8	
<hr/>								<hr/>							
3	0	9	.	1	9			1	3	0	9	.	1	9	
									\uparrow						
									Ignore the carry						

Example

2) $436.62 - 745.81$

$$\begin{array}{r}
 436.62 \\
 - 745.81 \\
 \hline
 -309.19
 \end{array}
 \xrightarrow{\text{10's complement}}
 \begin{array}{r}
 436.62 \\
 + 254.19 \\
 \hline
 690.81 \\
 - 309.19 \\
 \hline
 381.62
 \end{array}$$

As carry is not generated, so take 10's complement of the intermediate result and add ' - ' sign to the result

Binary addition

- Rules for binary addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10 \text{ i.e.}$$

0 with a carry
of 1

A diagram illustrating binary addition. It shows three rows of digits separated by horizontal lines. The top row is the first addend: 1 1 1 1 . 1 1. The middle row is the second addend, preceded by a plus sign: + 0 1 1 1 . 0 1 1. The bottom row is the result: 1 0 1 0 1 . 0 0 0. Blue curved arrows indicate the carry propagation from right to left. There are six upward-pointing arrows above the top row and six downward-pointing arrows below the bottom row, showing the path of the carry from the least significant bit to the most significant bit.

	1	1	1	1	.	1	1	
	1	1	0	1	.	1	0	1
+	0	1	1	1	.	0	1	1
	1	0	1	0	.	0	0	0

Binary Subtraction

$$\begin{array}{r}
 \leftarrow \text{borrow} \\
 1 \\
 (-) 1 \\
 \hline
 0 \\
 \hline
 \end{array}$$

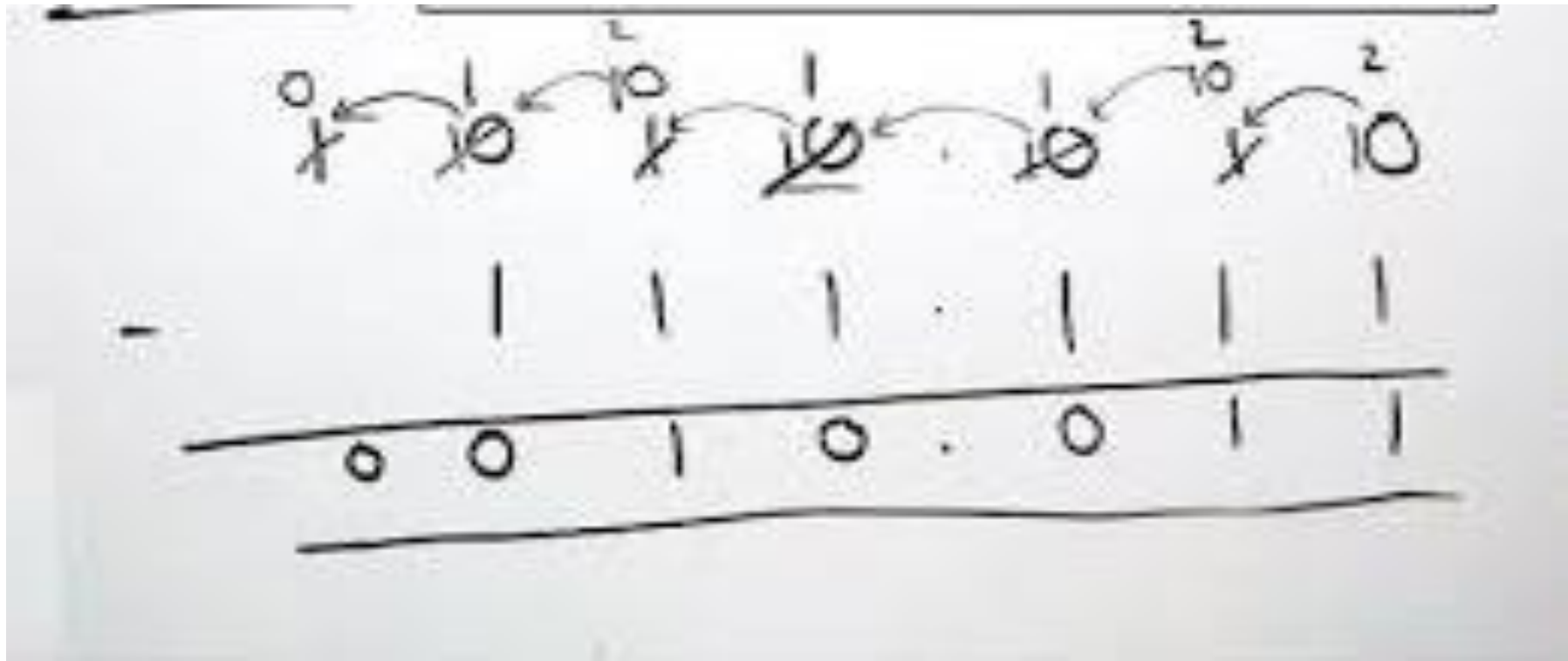
$$0 - 0 = 0$$

$$1 - 1 = 0$$

$$1 - 0 = 1$$

$$0 - 1 = 1, \text{ with a borrow 1}$$

Circuit Globe



3.Subtraction using 1's complement

- obtain 1's complement of subtrahend.
- Add the result to minuend and call it intermediate result.
- If carry is generated then answer is positive and add the carry to LSD
- If There Is No Carry Then Answer Is Negative And Take 1's Complement Of Intermediate Result And Place Negative Sign To The Result.

Example

1) 68.75 – 27.50

68.75		01000100.1100
- 27.50	1's complement →	+ 11100100.0111
<hr/>		<hr/>
+ 41.25		10010100.1001
		→ +1
		<hr/>
		00101001.0100

Example

2) 43.25 - 89.75

43.25		00101011.0100
- 89.75	$\xrightarrow{\text{1's complement}}$	+ 10100110.0011
<hr/>		<hr/>
- 46.50		11010001.0111
	$\xrightarrow{\text{1's complement}}$	00101110.1000

As carry is not generated, so take 1's complement of the intermediate result and add ' - ' sign to the result

3.Subtraction using 1's complement

- obtain 2's complement of subtrahend.
- Add the result to minuend.
- If carry is generated then ignore it and result is answer.
- If There Is No Carry Then Answer Is Negative And Take 2's Complement Of Result And Place Negative Sign To The Result.

Example

1) $68.75 - 27.50$

$$\begin{array}{r} 68.75 \\ - 27.50 \\ \hline + 41.25 \end{array} \quad \begin{array}{l} \xrightarrow{\text{2's complement}} \\ + \end{array} \begin{array}{r} 01000100.1100 \\ 11100100.1000 \\ \hline 10010100.10100 \\ 00101001.0100 \end{array}$$

Ignore Carry bit

Example

2) 43.25 - 89.75

43.25		00101011.0100
- 89.75	$\xrightarrow{\text{2's complement}}$	+ 10100110.0100
<hr/>		<hr/>
- 46.50		11010001.1000
	$\xrightarrow{\text{2's complement}}$	00101110.1000

As carry is not generated, so take 2's complement of the intermediate result and add ' - ' sign to the result

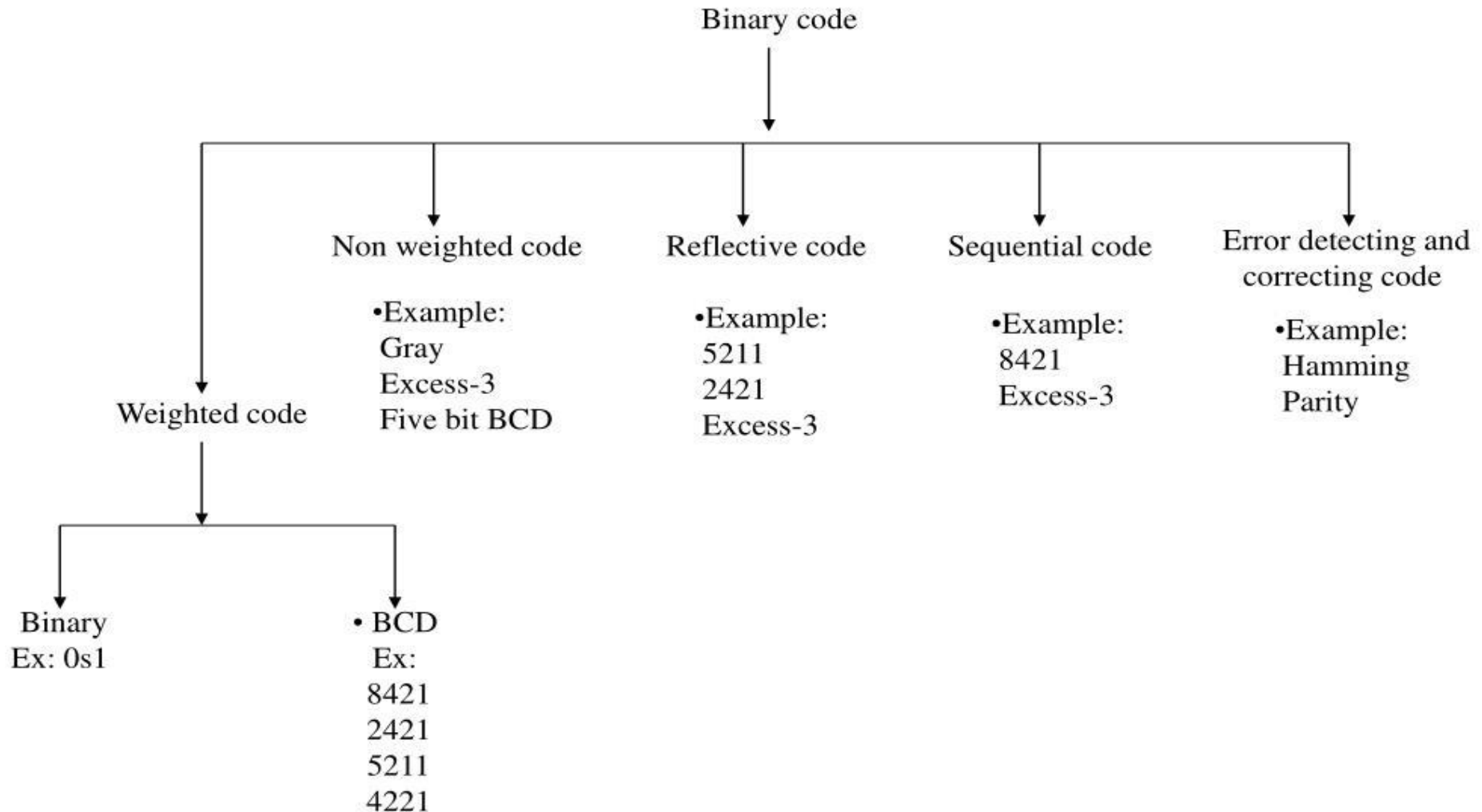
Binary code

- Group of symbol called code.

Advantages:

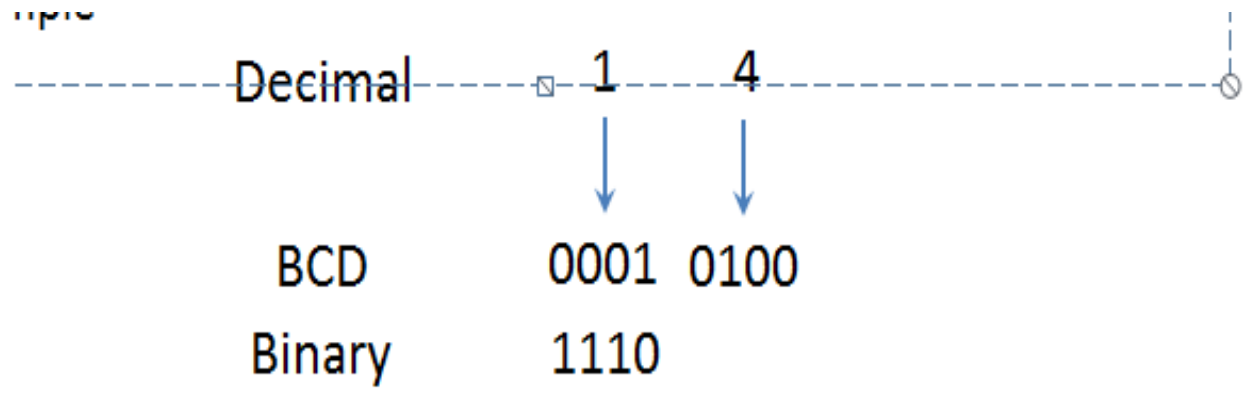
- 1.They are suitable For computer application.
- 2.Analysis and designing of digital circuit become easy .if we use binary code.
3. As only 1 & 0 are being used ,implementation of binary code become easy.

CLASSIFICATION OF BINARY CODE



BCD code

- **8421 BCD Code:**
- Each decimal digit , 0 through 9, is coded by 4 –bit binary number.
- They are weighted code.
- 1010 to 1111 are invalid in BCD.
- Less efficient than pure binary.
- Arithmetic operation are more complex than in pure binary,



Question- Comparison of BCD & Binary.

Packed BCD:

BCD number corresponding to decimal no. beyond are called packed BCD.

BCD Addition:

if there is no carry & sum term is not an illegal code, no correction is needed.

$$\begin{array}{r} 25 \\ + 13 \\ \hline 38 \end{array}$$

$$\begin{array}{r} 00100101 \\ + 00010011 \\ \hline 00111000 \end{array}$$

No carry, no illegal code. So, this is the correct sum.

- If there is a carry out of one group to the next group, or if the sum term is illegal code, then 6 (0110) is added to the sum term of that group & the resulting carry is added to the next group

679.6	0110	0111	1001	.0110		
+ 536.8	+ 0101	0011	0110	.1000		
<hr/>						
1216.4	1011	1010	1111	.1110	All are illegal codes	
	+0110	+0110	+0110	+.0110	Add 0110 to each	
	<hr/>					
	10001	10000	10101	1.0100	Propagate carry	
	+1 ↙	+1 ↙	+1 ↙	+1 ↙		
	<hr/>					
	0001	0010	0001	0110	.0100	Corrected Sum

Binary Subtraction

- If there is no borrow from the next higher group then no correction is required.
- If there is a borrow from the next group, then 6(0110) is subtracted from the difference term of this group.

$$\begin{array}{r}
 38 \\
 - 15 \\
 \hline
 23
 \end{array}
 \qquad
 \begin{array}{r}
 00111000 \\
 - 00010101 \\
 \hline
 00100011
 \end{array}$$

No borrow. So, this is the correct difference.

$$\begin{array}{r}
 206.7 \\
 - 147.8 \\
 \hline
 58.9
 \end{array}
 \qquad
 \begin{array}{r}
 0010\ 0000\ 0110\ .0111 \\
 - 0001\ 0100\ 0111\ .1000 \\
 \hline
 0000\ 1011\ 1110\ .1111 \\
 -0110\ -0110\ -.0110 \\
 \hline
 0101\ 1000\ .1001
 \end{array}$$

Borrows are present

Subtract 0110

Corrected difference

Excess-3 Code

- Excess -3 code=8421 BCD+0011(3)
- It is a self- complementing code.
- 0000,0001,0010,1101,1110,1111 are illegal codes.

▪ Example

Decimal	1	4
	↓	↓
BCD	0001	0100
XS-3	0100	0111

Excess-3 addition

- If there is no carry out from the addition of any of the 4 bit group, subtract 0011 from the sum term of these group. If carry out, add 0011 to the sum term of those group.

$\begin{array}{r} 3 \quad 7 \\ + 2 \quad 8 \\ \hline 6 \quad 5 \end{array}$	$\begin{array}{r} 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \\ + 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\ \hline 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \end{array}$
	$\begin{array}{r} +1 \end{array}$
Propagate carry to next group	$\begin{array}{r} 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \\ - 0 \quad 0 \quad 1 \quad 1 \quad + 0 \quad 0 \quad 1 \quad 1 \\ \hline 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \end{array}$

- Add 0011 to group which generated carry
- Subtract 0011 to group which do not generated carry

247.6
+ 359.4

607.0

0101 0111 1010 .1001
+ 0110 1000 1100 .0111

1011 1111 10110 1.0000

+1 +1

1011 10000 0111 .0000

+1

1100 0000 0111 .0000

- 0011 +0011 +0011 +.0011

1001 0011 1010 .0011

Carry generated

Propagate carry

Corrected Sum
in XS-3

- Add 0011 to group which generated carry
- Subtract 0011 to group which do not generated carry

Excess-3 Subtraction

- If there is no borrow from the next 4 – bit group & add 0011 to the difference term of such group.
- If there is a borrow ,subtract 0011 from the difference term.

$$\begin{array}{r}
 267 \quad 0101 \ 1001 \ 1010 \\
 - 175 \quad - 0100 \ 1010 \ 1000 \\
 \hline
 092 \quad 0000 \ 1111 \ 0010 \\
 \quad +0011 -0011 +0011 \\
 \hline
 \quad 0011 \ 1100 \ 0101
 \end{array}$$

- Subtract 0011 to group which generated borrow
- Add 0011 to group which do not generated borrow

$$\begin{array}{r}
 57.6 \quad 1000 \ 1010 \ .1001 \\
 - 27.8 \quad - 0101 \ 1010 \ .1011 \\
 \hline
 29.8 \quad 0010 \ 1111 \ .1110 \\
 \quad +0011 -0011 -.0011 \\
 \hline
 \quad 0101 \ 1100 \ .1011
 \end{array}$$

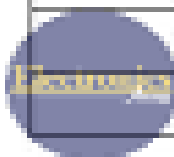
- Subtract 0011 to group which generated borrow
- Add 0011 to group which do not generated borrow

Gray code

- The gray code is a non-weighted code.
- It is a cyclic code because successive code words in this code differ in one bit position only, i.e. it is a unit distance code.
- It is also a reflective code.
- The n least significant bits for 2^n through $2^{n+1}-1$ are the mirror images of those for 0 through 2^n-1 .
- An N -bit gray code can be obtained by reflecting an $N-1$ bit code about an axis at the end of the code, and putting the MSB of 0 above the axis and the MSB of 1 below the axis.
- One reason for the popularity of the gray code is its ease of conversion to and from binary.
- Reflection of gray code is shown in table.

Gray Code				Decimal	4-bit binary
1-bit	2-bit	3-bit	4-bit		
0	00	000	0000	0	0000
1	01	001	0001	1	0001
	11	011	0011	2	0010
	10	010	0010	3	0011
		110	0110	4	0100
		111	0111	5	0101
		101	0101	6	0110
		100	0100	7	0111
			1100	8	1000
			1101	9	1001
			1111	10	1010
			1110	11	1011
			1010	12	1100
			1011	13	1101
			1001	14	1110
			1000	15	1111

Decimal	Binary Code	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000



Binary to grey code conversion

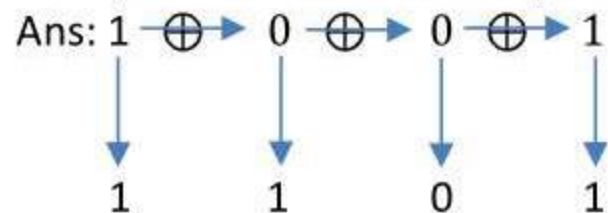
- If an n-bit binary number is represented by $B_n B_{n-1} \dots B_1$ and its gray code equivalent $G_n G_{n-1} \dots G_1$, where B_n and G_n are the MSBs, then the gray code bits are obtained from the binary code as follows:

$G_n = B_n$	$G_{n-1} = B_n \oplus B_{n-1}$	$G_{n-2} = B_{n-1} \oplus B_{n-2}$	$G_1 = B_2 \oplus B_1$
-------------	--------------------------------	------------------------------------	-------	------------------------

The conversion procedure is as follows:

1. Record the MSB of the binary as the MSB of the gray code.
2. Perform X-ORing between the MSB of the binary and the next bit in binary. This answer is the next bit of the gray code.
3. Perform X-ORing between 2nd bit of the binary and 3rd bit of the binary, the 3rd bit with the 4th bit, and so on.
4. Record the successive answer bits as the successive bits of the gray code until all the bits of the binary number are exhausted.

Example:- Convert the binary 1001 to Gray code.



Grey To Binary code conversion

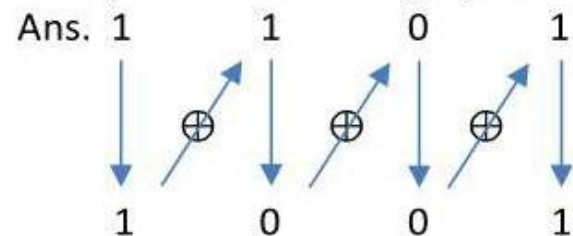
If an n-bit gray number is represented by $G_n G_{n-1} \dots G_1$ and its binary code equivalent $B_n B_{n-1} \dots B_1$, where G_n and B_n are the MSBs, then the binary bits are obtained from the gray bits as follows:

$B_n = G_n$	$B_{n-1} = B_n \oplus G_{n-1}$	$B_{n-2} = B_{n-1} \oplus G_{n-2}$	$B_1 = B_2 \oplus G_1$
-------------	--------------------------------	------------------------------------	-------	------------------------

The conversion procedure is as follows:

1. The MSB of the binary number is the same as the MSB of the gray code number.
2. Perform X-ORing between the MSB of the binary and next significant bit of gray code. This answer is the next bit of binary.
3. Perform X-ORing between the 2nd bit of the binary and 3rd bit of the gray code, the 3rd bit of the binary with the 4th bit of gray code, and so on.
4. Record the successive answers as the successive bits of the binary until all the bits of the gray code are exhausted.

Example:- Convert the gray code 1101 to binary.



Error detecting code

- Noise can alter or distort the data in transmission.
- The 1s may get changed to 0s and 0s to 1s.
- Because digital systems must be accurate to the digit, errors can pose a serious problem.
- Single bit error should be detect & correct by different schemes.
- Parity, Check Sums and Block Parity are the examples of error detecting code.

Parity

- Parity bit is the simplest technique.
- There are two types of parity – Odd parity and Even parity.
- For odd parity, the parity is set to a 0 or a 1 at the transmitter such that the total number of 1 bits in the word including the parity bit is an odd number.
- For even parity, the parity is set to a 0 or a 1 at the transmitter such that the total number of 1 bits in the word including the parity bit is an even number.

✓

- Detect a single-bit error but can not detect two or more errors within the same word.
- In any practical system, there is always a finite probability of the occurrence of single error.
- E.g. In an even-parity scheme, code 10111001 is erroneous because number of 1s is odd(5), while code 11110110 is error free because number of 1s is even(6).

Check sum

- Simple parity can not detect two errors within the same word.
- Added to the sum of the previously transmitted words
- At the transmission, the *check sum* up to that time is sent to the receiver.
- The receiver can check its sum with the transmitted sum.
- If the two sums are the same, then no errors were detected at the receiver end.
- If there is an error, the receiving location can ask for retransmission of the entire data.
- This type of transmission is used in teleprocessing system.

Block parity

- When several binary words are transmitted or stored in succession, the resulting collection of bits can be regarded as a block of data, having rows and columns.
- Parity bits can then be assigned to both rows and columns.
- This scheme makes it possible to *correct* any single error occurring in a data word and to *detect* any two errors in a word.
- This technique also called word parity, is widely used for data stored on magnetic tapes.
- For example, six 8-bit words in succession can be formed into a 6x8 block for transmission.
- Parity bits are added so that odd parity is maintained both row-wise and column-wise and the block is transmitted as a 7x9 block as shown in Figure 1.
- At the receiving end, parity is checked both row-wise and column-wise and suppose errors are detected as shown in Figure 2.
- These single-bit errors detected can be corrected by complementing the error bit.
- In Figure 2, parity errors in the 3rd row and 5th column mean that the 5th bit in 3rd row is in error.
- It can be corrected by complementing it.
- Two errors as shown in Figure 3 can only be detected but not corrected.
- In Figure 3, parity errors are observed in both columns 2 and 4.
- It indicated that in one row there two errors.

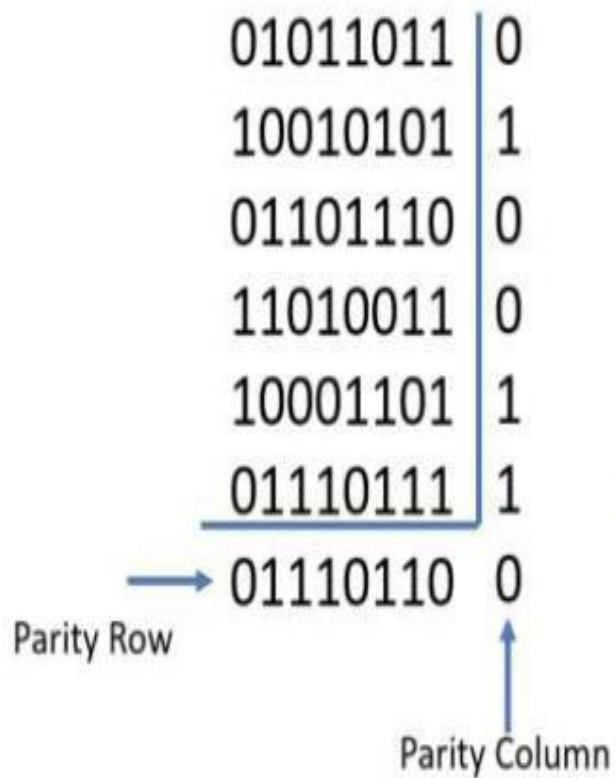


Figure 1

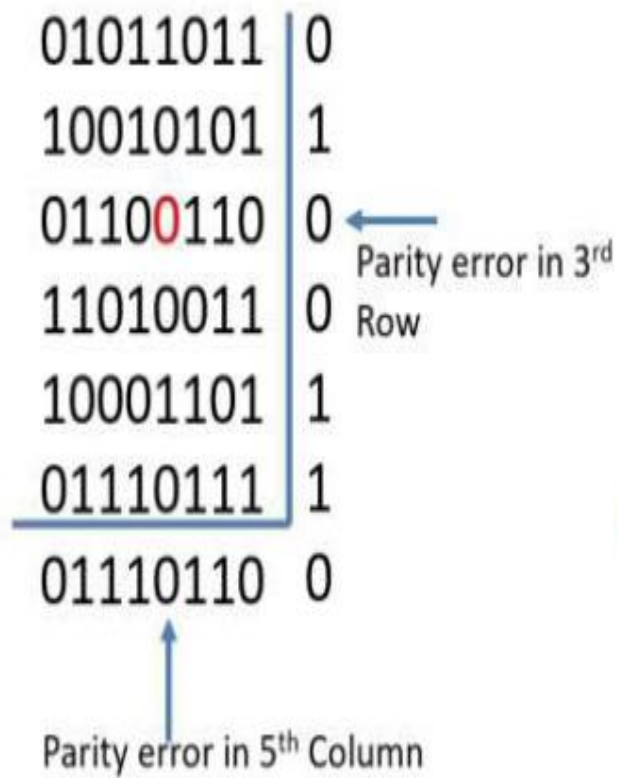


Figure 2

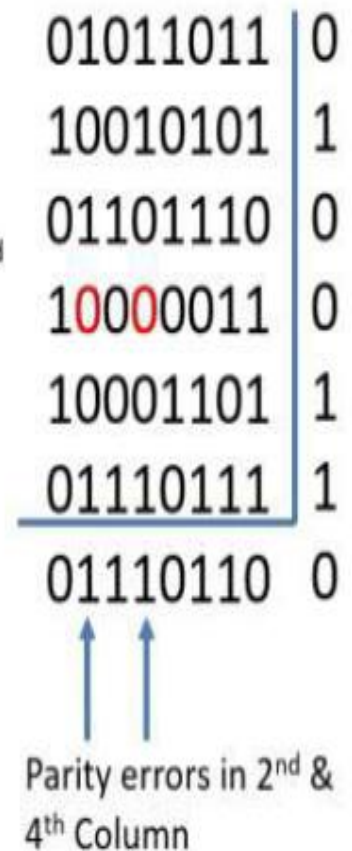


Figure 3

Error correcting code

- 7-bit Hamming Code is widely used error correcting code, containing 4 bits of data and 3 bits of even parity.
- Pattern: $P_1 P_2 D_3 P_4 D_5 D_6 D_7$
- Group-1: $P_1 D_3 D_5 D_7$, Group-2: $P_2 D_3 D_6 D_7$, Group-3: $P_4 D_5 D_6 D_7$

- Example: Data = 1101

$$P_1 P_2 D_3 P_4 D_5 D_6 D_7 = P_1 P_2 1 P_4 1 0 1$$

$$P_1 D_3 D_5 D_7 = 1 1 1 1$$

$$P_2 D_3 D_6 D_7 = 0 1 0 1$$

$$P_4 D_5 D_6 D_7 = 0 1 0 1$$

- 7-bit Hamming Code is 1 0 1 0 1 0 1

Error correcting code

- How to detect error?
- Example: Received data = 1001001

$$P_1 P_2 D_3 P_4 D_5 D_6 D_7 = 1 0 0 1 0 0 1$$

$$P_1 D_3 D_5 D_7 = 1 0 0 1 \text{ (No Error)}$$

$$P_2 D_3 D_6 D_7 = 0 0 0 1 \text{ (Error)}$$

$$P_4 D_5 D_6 D_7 = 1 0 0 1 \text{ (No Error)}$$

- The error word is $0 1 0 = 2_{10}$.
- Complement the 2^{nd} bit (from left).
- Correct code is 1 **1** 0 1 0 0 1

Digital IC specification

- Threshold voltage
- Propagation Delay
- Power dissipation
- Fan-in
- Fan-out
- Voltage & Current parameters
- Noise Margin
- Operating Temperatures
- Speed power products

Comparison of logic families

Characteristic	TTL	CMOS	ECL
Power Input	Moderate	Low	Moderate-High
Frequency limit	High	Moderate	Very high
Circuit density	Moderate-high	High-very high	Moderate
Circuit types per family	High	High	Moderate

Logic Family	Propagation delay time (ns)	Power dissipation per gate (mW)	Noise Margin (V)	Fan-in	Fan-out	Cost
TTL	9	10	0.4	8	10	Low
CMOS	<50	0.01	5	10	50	Low
ECL	1	50	0.25	5	10	High

Transistor transistor logic

- Dependence on transistors alone to perform basic logic operations.
- Most popular logic family.
- Most widely useful bipolar digital IC family.
- The TTL uses transistors operating in saturated mode.
- It is the fastest of the saturated logic families.
- Good speed, low manufacturing cost, wide range of circuits, and the availability in SSI and MSI are its merits.

Schottky TTL

-
- When a transistor is saturated, excess charge carriers will be stored in the base region and they must be removed before the transistor can be turned off.
 - So, owing to storage time delay, the speed is reduced.
 - The Schottky TTL series reduces this storage time delay by not allowing the transistor to go into full saturation.
 - This is accomplished by using a Schottky barrier diode(SBD) between the base and the collector of each transistor.
 - More than three times the switching speed of standard TTL, at the expense of approximately doubling the power consumption.

Tri state TTL

- It utilizes the advantage of the high speed of operation of the totem-pole configuration and wire ANDing of the open-collector configuration.
- It is called the tri-state TTL, because it allows three possible output states: HIGH, LOW, and HIGH Impedance (Hi-Z).
- In the Hi-Z state, both the transistors in the totem-pole arrangement are turned off, so that the output terminal is a HIGH impedance to ground or V_{CC} .
- In fact, the output is an open or floating terminal, that is, neither a LOW nor a HIGH.