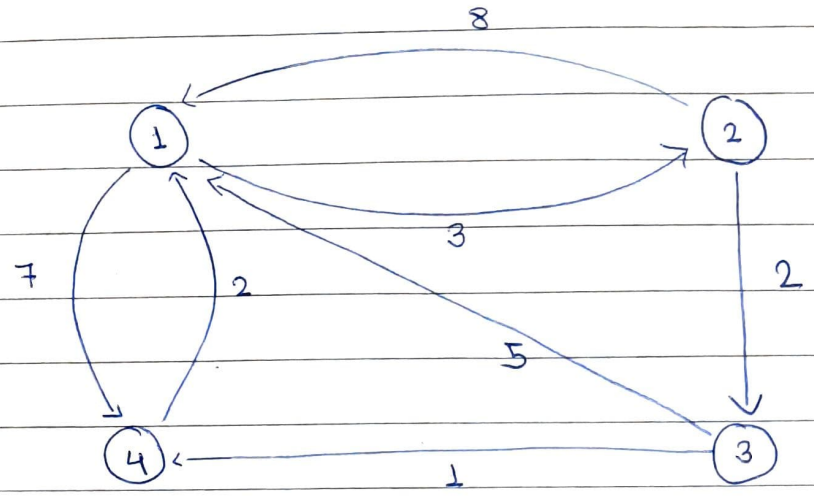
Date:

Floyd - Warshall Algorithm



Formula:-

$$A^{K}[i,j] = \min\left\{A^{K-1}[i,j],\ A^{K-1}[i,K] + A^{K-1}[K,j]\right\}$$

$$A^{0} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

Update A^0 to A^1 which contains distance betn each node with '1' as intermediate node.

→ 2 - 4
3 - 2
4 - 2

→ $A^1[2,4] = \min\left\{A^0[2,4],\ A^0[2,1] + A^0[1,4]\right\}$

$= \min\{\infty,\ 8+7\}$

$= \min\{\infty, 15\}$

$= 15$

→ $A^1[3,2] = \min\{A^0[3,2], A^0[3,1] + A^0[1,2]\}$
$= \min\{\infty, 5+3\}$
$= \min\{\infty, 8\}$
$= 11$

→ $A^1[4,2] = \min\{A^0[4,2], A^0[4,1] + A^0[1,2]\}$
$= \min\{\infty, 2+3\}$
$= \min\{\infty, 5\}$
$= 5$

④

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 4 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

Update A^1 to A^2 which contains distance bet^n each ~~each~~ node with '2' as intermediate node

→ 1 - 3
4 - 3

→ $A^2[1,3] = \min\{A^1[1,3], A^1[1,2] + A^1[2,3]\}$
$= \min\{\infty, 3+2\}$
$= 5$

→ $A^2[4,3] = \min\{A^1[4,3], A^1[4,2] + A^1[2,3]\}$
$= \min\{\infty, 5+2\}$
$= 7$

$A^2 =$

	1	2	3	4
1	0	3	5	7
2	8	0	2	15
3	5	8	0	1
4	2	5	7	0

Now update A^2 to A^3

→ 2 - 1

1 - 2 - 3 - 4

2 - 4

$A^3 =$

	1	2	3	4
1	0	3	5	6
2	7	0	2	3
3	5	8	0	1
4	2	5	7	0

→ Now update A^3 to A^4

- 2 - 1

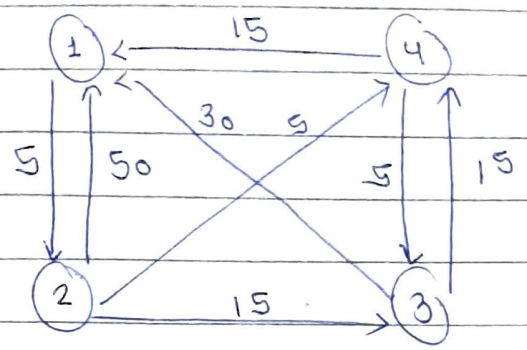
3 - 1

3 - 2

$A^4 =$

	1	2	3	4
1	0	3	5	6
2	5	0	2	3
3	3	6	0	1
4	2	5	7	0

(2)

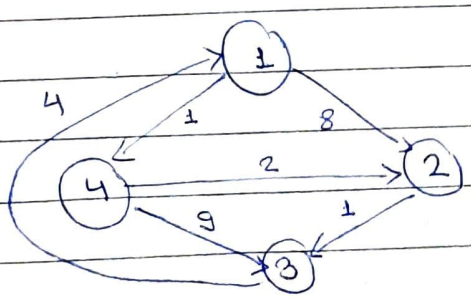


$$A^0 = \begin{bmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & \infty & 0 & 15 \\ 15 & \infty & 5 & 0 \end{bmatrix} \qquad A^1 = \begin{bmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 5 & 20 & 10 \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 0 & 5 & 20 & 10 \\ 45 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 5 & 15 & 10 \\ 20 & 0 & 10 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

(3)



$$A^0 = \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & \infty & 0 & \infty \\ \infty & 2 & 9 & 0 \end{bmatrix}$$

$$A^1 = \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 9 & 0 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 0 & 8 & 9 & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 3 & 0 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 0 & 8 & 9 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 12 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 7 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$$