

Chapter- 5 Heat Engines

Elementary heat engines :-

Heat engine is a device in which net heat is added to the engine and net work transfer from the engine.

Thus for the heat engine net heat transfer and the net work transfer both are positive in a cycle executed by the engine.

Heat engines can be classified in two categories

1. External combustion engines
2. Internal combustion engines

External Combustion Engine:- The combustion of the fuel take place outside the engine.

Heat liberated by the combustion of the fuel is transferred to the working fluid of the engine.

Such as Steam engines, steam turbines, close cycle gas turbines.

The combustion of the fuel take place in the boiler which produces high pressure steam, this steam is expanded which produce the mechanical work.

Internal Combustion Engine:- The combustion of the fuel take place inside the engine.

Combustion of the fuel takes place with the help of air.

Combustion product it self are the working fluid of the engines.

Such as Petrol engine, Diesel engine, Open cycle gas turbines.

Difference between Internal combustion engine & External combustion engine

Internal combustion engine:-

1. They are compact in size and hence suitable for the small capacity.
2. More refined fuel is required which is costly. Generally used with liquid and gaseous fuel.
3. Due to less components, complexity is less.
4. Lower efficiency.
5. More pollutant.

External combustion engine:-

1. They are bulky and hence designed for the large capacity.
2. Cheaper fuels can be used. Generally solid, liquid, and gaseous fuels are used.
3. More complex.
4. Higher efficiency.
5. Less pollutant.

Essential components of Heat engine:-

A thermodynamic cycle is executed by the heat engine to produce net positive work from net positive heat addition to the engine.

This cycle is known as the heat engine cycle. Sometimes it is also referred to as a power cycle. A heat engine cycle consists of the series of the processes which operate with particular devices.

The essential components of heat engine are as follows:-

1.

Heat source:- It is a reservoir of the heat from which heat is supplied to the working fluid.

Example as a furnace of boiler, combustion chamber of I.C engine.

2.

Heat sink:- A heat sink is a low temperature reservoir where heat is rejected by the working fluid.

3.

Working fluid:- Substance which receive and reject heat and undergoes various processes of heat engine cycle is called working fluid.

4. Expander:- It is the device in which working fluid is expanded and work is available.
5. Compressor:- It is a device in which pressure of the working fluid increases. It may be coupled to the expander or it may be connected to the separate power source.

Working substances:-

All the thermodynamic systems requires some working substance in order to perform various operations to execute a thermodynamic cycle. These working substance are known as working fluids.

They can readily be compressed or expanded.

They also receive or reject the heat.

Common examples of the working fluids are air and the steam.

Analysis of all heat engine cycles involve calculation of various properties of these working fluids.

Classification of heat engines :-

Heat engines may be classified in the different types are as follows

1. Carnot Cycle Engine
2. Rankine Cycle Engine
3. Otto Cycle Engine
4. Diesel Cycle Engine

Carnot Cycle

Carnot conceive an ideal cycle in which all processes are reversible on P-V diagram.

Process of the Carnot cycle are shown as below.

conceived an ideal cycle in which all processes are reversible represented on p - V diagram. Processes of Carnot cycle are

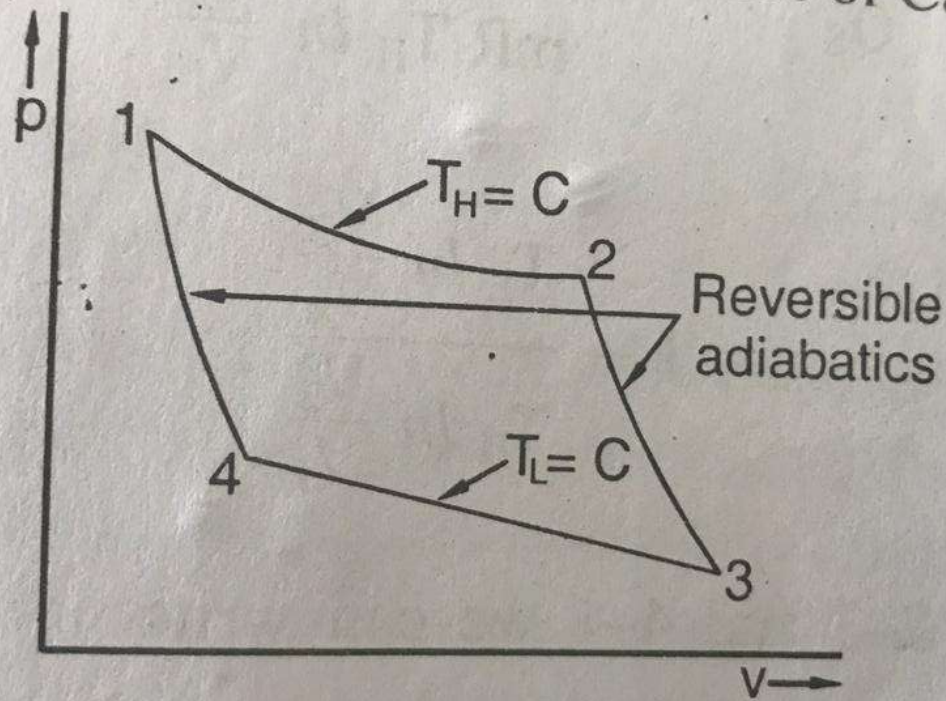


Fig. 5.1 Carnot cycle

1-2 : Heat is added to the system (of working fluid) at temperature T_H from the source. Working substance is expanded

Process 1-2 :

Heat is add to the system of working fluid at constant temperature T_H from the source.

Working substance is expanded reversibly at the constant temperature.

Process 2-3 :

Working fluid undergoes a reversible adiabatic expansion. Temperature will reduce from T_H to T_L .

Process 3-4 :

Working fluid rejects heat at the constant temperature T_L to sink reversible. Working fluid is said to be compressed at the constant temperature T_L .

Process 4-1 :

Working fluid undergoes a reversible adiabatic compression and its temperature will increase from T_L to T_H .

Efficiency of the Carnot Cycle:

Process 1-2 is reversible isothermal heat addition process.

From 1st law of Thermodynamics

$$Q = \Delta U + W$$

For ideal gas internal energy depends only on temperature hence for this process

$$\Delta U = 0$$

So that $Q = W$

W for the constant temperature process and hence hyperbolic process, $PV = C$

$$Q_s = W = P_1 V_1 \ln \frac{v_2}{v_1}$$

$$= mRT_h \ln \frac{v_2}{v_1}$$

Where Q_s is the heat supplied.

Process 3-4 is the isothermal heat rejection process so similar to process 1-2

We can write as

$$Q_R = W = P_3 V_3 \ln \frac{v_3}{v_4} = mRT_l \ln \frac{v_3}{v_4}$$

Where Q_R is the heat rejected.

Equation is written in such a way that Q_R is the positive number.

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$= 1 - \frac{mrT_L \ln \frac{V_3}{V_4}}{mrT_H \ln \frac{V_2}{V_1}}$$

Now for the process 2-3 and 4-1

We can write like as

$$\frac{T_H}{T_L} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)}$$

Because $(V_3 = V_4)$ and $(V_2 = V_1)$

$$\text{So that } \left(\frac{V_3}{V_2}\right) = \left(\frac{V_4}{V_1}\right)$$

$$\text{So that } \left(\frac{V_3}{V_4}\right) = \left(\frac{V_2}{V_1}\right)$$

Put this value in the carnot cycle efficiency equation

We get

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$= 1 - \frac{mRT_L \ln \frac{V_3}{V_4}}{mRT_H \ln \frac{V_2}{V_1}} \quad \left(\text{But } \left(\frac{V_3}{V_4} \right) = \left(\frac{V_2}{V_1} \right) \right)$$

$$= 1 - \frac{mRT_L \ln \frac{V_2}{V_1}}{mRT_H \ln \frac{V_2}{V_1}}$$

$$= 1 - \frac{T_L}{T_H}$$

$$\eta_{\text{carnot}} = 1 - \frac{T_L}{T_H}$$

From the above equation it is evident that the efficiency of the carnot cycle does not depend on the working substance but depends only on the temperature of the heat addition and heat rejection.

Carnot Vapour Cycle

Components of carnot vapour cycle is shown in to the figure and same is represent on p-V diagram.

5.5 CARNOT VAPOUR CYCLE :

5.5

Components of Carnot vapour cycle is shown in Fig. 5.2 and same is represented on p-V diagram in Fig. 5.3.

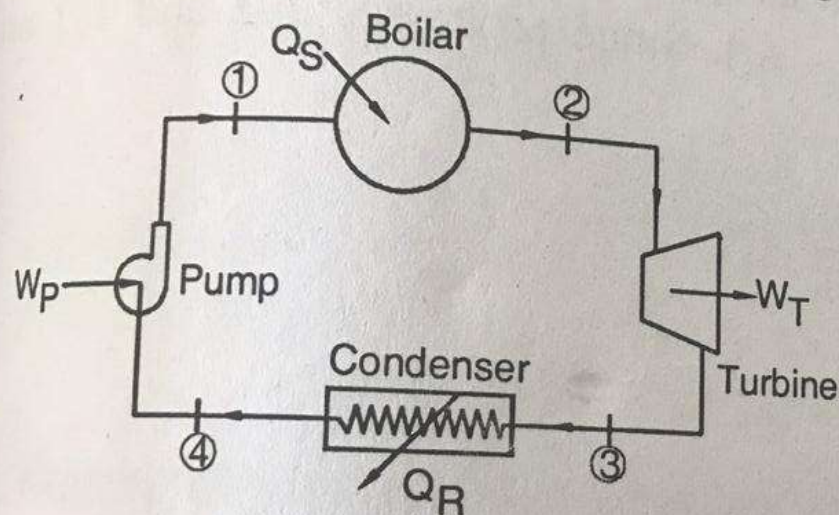


Fig. 5.2 Components of Carnot vapour cycle

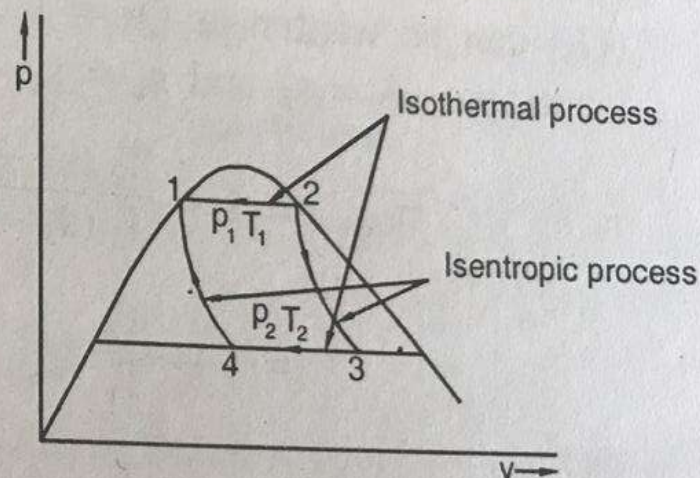


Fig. 5.3 Carnot vapour cycle on p-V diagram

As shown, during process 1-2 heat is added in boiler at constant pressure which is also an isothermal process in wet region. Process 2-3 is expansion of steam in turbine which develops work W_T . During process 3-4 heat is rejected in condenser to cooling medium (generally water) at constant pressure and temperature. Process 4-1 is pumping/compression of wet steam from condenser pressure to boiler pressure which requires work input of W_P .

Process 1-2:

Heat is add in the boiler at the constant pressure which is also an isothermal process in the wet region.

Process 2-3:

The expansion of the steam in the turbine which develops the work of $\mathbf{W_t}$.

Process 3-4:

Heat is reject in to the condenser to the cooling medium at constant pressure and temperature.

Process 4-1:

The pumping of wet steam from condenser pressure to the boiler pressure which requires the work input of $\mathbf{W_p}$.

Efficiency of the carnot vapour cycle:

Unit mass of the substance

Work develop by the turbine $\mathbf{W_T = (h_2 - h_3)}$

Work develop by the pump $\mathbf{W_P = (h_1 - h_4)}$

Net work developed $\mathbf{W_{net} = W_T - W_P}$
 $\mathbf{= (h_2 - h_3) - (h_1 - h_4)}$

Heat supplied = $\mathbf{(h_2 - h_1)}$

$$\eta_{\text{carnot vapour}} = \frac{\text{Net work Develop}}{\text{Heat Supply}}$$

$$= \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

$$= \frac{(h_2 - h_1) - (h_3 - h_4)}{(h_2 - h_1)}$$

$$= 1 - \frac{(h_3 - h_4)}{(h_2 - h_1)}$$

$$= 1 - \frac{Q_R}{Q_S}$$

Entropy is define as

$$ds = \int \frac{dQ_{\text{reversible}}}{T}$$

$$S = \frac{Q}{T}$$

So that $Q = T.S$

Here

Heat Supply $Q_s = T_1(s_2 - s_1)$

Heat Reject $Q_r = T_2(s_3 - s_4)$

$$\eta_{\text{carnot vapour}} = 1 - \frac{Q_R}{Q_S}$$

$$= 1 - \frac{T_2(s_3 - s_4)}{T_1(s_2 - s_1)}$$

But process of 2-3 and 4-1 are isentropic process

$$s_2 = s_3$$

$$s_1 = s_4$$

$$= 1 - \frac{T_2(s_3 - s_4)}{T_1(s_3 - s_4)}$$

$$= 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{T_L}{T_H}$$

Efficiency of the carnot vapour cycle does not depends on the working fluid but demands only on to the temperatures of heat addition and heat rejection.

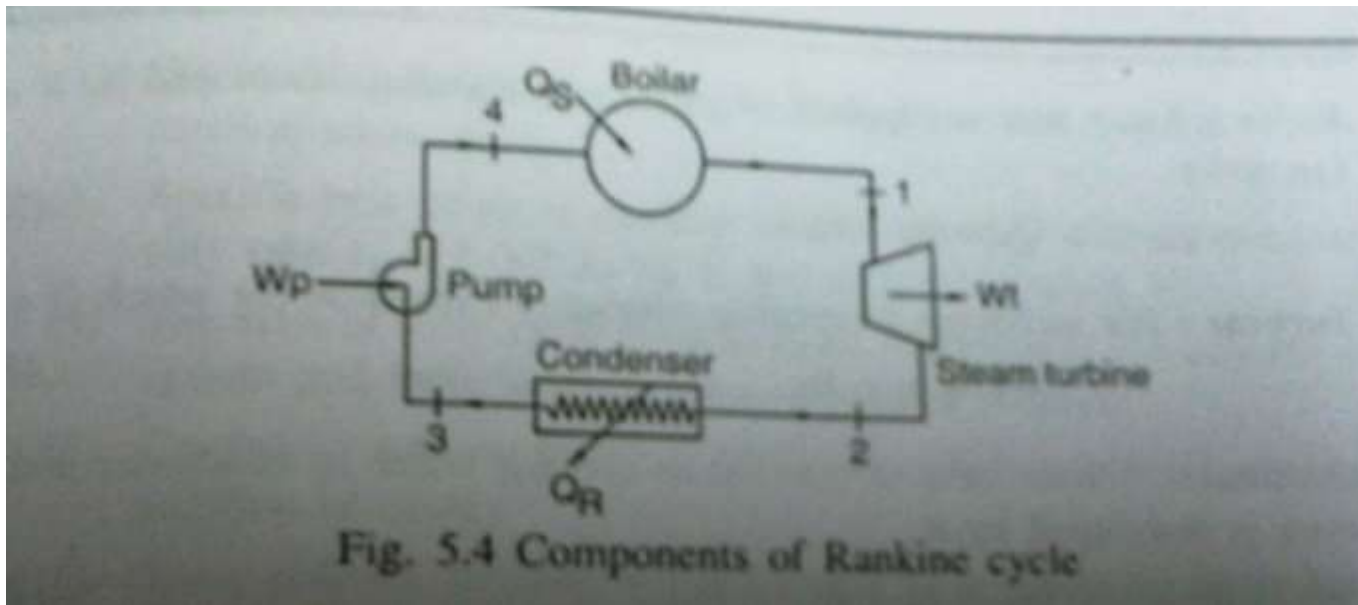
Rankine Cycle

It is difficult to pump the mixture of vapour and liquid and deliver it as a saturated liquid.

This is eliminated in the Rankine cycle by the complete condensation of vapour in the condenser and then pumping the water isentropically to the boiler pressure.

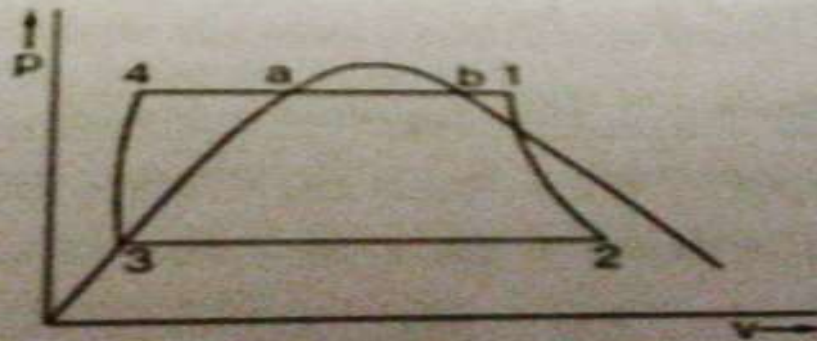
Similarly in the boiler heat is added at the constant pressure rather than at the constant temperature.

In the superheat region , heat addition at the constant temperature requires expansion of the steam which is eliminated in constant pressure heat addition.



Components of the Rankine Cycle

Working of Rankine cycle : Various processes of Rankine cycle are represented on p - V diagram in Fig. 5.5.



Rankine Cycle

Working of the Rankine Cycle:- Various processes of Rankine cycle is represented on the P-V diagram.

Processes 1-2 :- Superheated dry saturated or wet steam expanded isentropically by the steam turbine from the boiler pressure to the condenser pressure. In the practical cycle state 1 is always superheated steam.

Processes 2-3:- Steam is condensed in the condenser at the constant pressure. It leaves the condenser as the saturated liquid.

Steam is condensed by transferring heat to cooling water.

Processes 3-4:- Saturated water at the condenser pressure is pumped to the boiler pressure.

Processes 4-1:- Heat is supplied to the water at the constant pressure in the boiler.

In the process 4-a ,sensible heat is added to the water and its temperature is increased to the saturation temperature corresponding to the boiler pressure.

Feed water heater and the economizer is used for this process.

In the process a-b , latent heat is supplied to the water and saturated water converts in to the saturated vapour.

In the process b-1 steam is superheated in the super heater.

Thermal efficiency of the Rankine cycle can be obtained by applying the steady flow energy equation to each of the four components of the Rankine cycle.

Boiler :- Since heat is supplied in the boiler and no work developed by it we can write

$$Q_s = h_1 - h_4$$

Turbine:- For the isentropic reversible adiabatic expansion for which $Q = 0$

$$W_t = h_1 - h_2$$

Condenser:- Since heat is rejected to cooling water in the condenser and no work is developed by it.

$$Q_r = h_2 - h_3$$

Pump:- For the reversible adiabatic pumping $Q = 0$

$$W_p = h_4 - h_3$$

$$\eta_{\text{Rankine}} = \frac{\text{Net work output}}{\text{Heat supplied in the boiler}}$$

$$\text{Net work output} = W_t - W_p$$

$$= (h_1 - h_2) - (h_4 - h_3)$$

$$\text{Heat supplied in the boiler } Q_s = h_1 - h_4$$

$$\eta_{\text{Rankine}} = \frac{[(h_1 - h_2) - (h_4 - h_3)]}{(h_1 - h_4)}$$

Generally pump work is very small compare to the work developed by the turbine and hence it is neglected.

$$\eta_{\text{Rankine}} = \frac{(h_1 - h_2)}{(h_1 - h_4)}$$

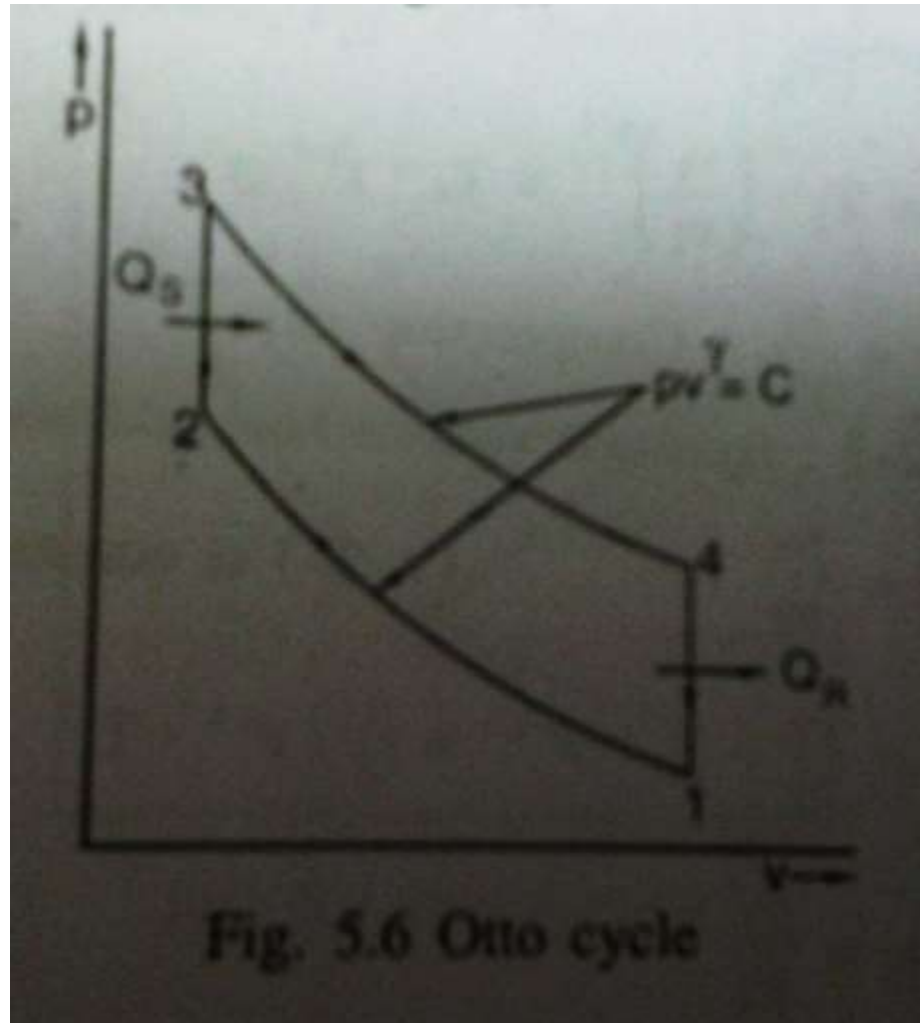
Otto cycle (Constant Volume Cycle)

An air standard Otto cycle which is also known as constant volume cycle consists of four processes.

Two of them are reversible adiabatic isentropic and the other two are constant volume processes.

An air standard Otto cycle on the p - V diagram is shown in the figure.

Otto Cycle



Various Processes are

1-2:- Reversible adiabatic isentropic compression of the air. Pressure and temperature of air will increase.

2-3:- Heat is added at the constant volume. Further rise is pressure and temperature of air.

3-4:- Reversible adiabatic isentropic expansion of air. Pressure and the temperature of air will decrease. Work is developed during the process.

4-1:- Constant volume heat rejection process. Pressure and temperature will restore to its initial value.

Now the Efficiency of the Otto Cycle is obtain by the equation

For the Otto Cycle Heat supplied and heat rejection are constant volume processes

$$Q = \Delta u = C_v \cdot \Delta T$$

$$\text{Heat supplied} = Q_s = C_v(T_3 - T_2)$$

$$\text{Heat rejected} = Q_r = C_v(T_4 - T_1)$$

$$\begin{aligned}\text{Work done in the cycle} &= \text{Heat added} - \text{Heat rejected} \\ &= C_v (T_3 - T_2) - C_v (T_4 - T_1)\end{aligned}$$

$$\begin{aligned}\text{Air Standard Efficiency } \eta_a &= \frac{\text{Work done}}{\text{Heat added}} \\ &= \frac{[C_v(T_3 - T_2) - C_v(T_4 - T_1)]}{C_v(T_3 - T_2)} \\ &= \mathbf{1} - \frac{(T_4 - T_1)}{(T_3 - T_2)}\end{aligned}$$

Now we need to substitute for the T_2, T_3, T_4 in terms of T_1 .

For defining the efficiency equation put all the value in to the Air Standard cycle efficiency equation.

For the process 1-2 :- (reversible adiabatic compression)

$$(\gamma-1)$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$(\gamma-1)$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = T_1 \cdot r^{\gamma-1}$$

Where $r = \frac{V_1}{V_2}$ = Compression Ratio

For the process 2-3 :- (constant volume heat addition)

$$\frac{T_3}{T_2} = \frac{p_3}{p_2}$$

$$T_3 = \frac{p_3}{p_2} \cdot T_2$$

$$T_3 = r_p \cdot T_2 \quad \text{Where } r_p = \left(\frac{p_3}{p_2}\right)$$

$$T_3 = r_p.T_2$$

Put the value of $T_2 = T_1 . r^{\gamma-1}$ in the above equation of T_3 we get the new equation

$$T_3 = r_p . r^{\gamma-1} . T_1$$

For the Process 3-4 :- (reversible adiabatic expansion)

$$(\gamma - 1)$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)$$

Since $v_1 = v_4$

and $v_2 = v_3$

$$(\gamma - 1)$$

$$(\gamma - 1)$$

$$\frac{T_4}{T_3} = \left(\frac{V_2}{V_1} \right) = \left(\frac{1}{r} \right)$$

$$(\gamma-1)$$

$$T_4 = T_3 \cdot \left(\frac{1}{r}\right)$$

Now put the value of $T_3 = r_p \cdot T_2$ in the above equation

$$(\gamma-1)$$

$$T_4 = r_p \cdot T_2 \left(\frac{1}{r}\right)$$

Now put the value of $T_2 = r^{\gamma-1} \cdot T_1$ in the above equation

$$(\gamma-1)$$

$$T_4 = r_p \cdot r^{\gamma-1} \cdot T_1 \left(\frac{1}{r}\right)$$

$$(\gamma-1)$$

$$T_4 = r_p \cdot T_1 \cdot r^{\gamma-1} \cdot \left(\frac{1}{r}\right)$$

$$(\gamma - 1)$$

$$T_4 = r_p \cdot \left(\frac{r}{r}\right)^{\gamma-1} \cdot T_1$$

$$T_4 = r_p \cdot T_1$$

$$T_2 = T_1 . r^{\gamma-1}$$

$$T_3 = r_p . T_2$$

$$= r_p . r^{\gamma-1} . T_1$$

$$T_4 = r_p . T_1$$

Now Substitute the value of T_2, T_3, T_4 in the equation of air standard cycle efficiency, we get

$$\eta_a = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{(r_p \cdot T_1 - T_1)}{(r_p \cdot r^{\gamma-1} \cdot T_1 - r^{\gamma-1} \cdot T_1)}$$

$$= 1 - \frac{(r_p - 1)T_1}{(r_p \cdot r^{\gamma-1} - r^{\gamma-1})T_1}$$

$$= 1 - \frac{1}{r^{\gamma-1}} \frac{(r_p - 1)}{(r_p - 1)}$$

$$\eta_a (\text{Otto}) = 1 - \frac{1}{r^{\gamma-1}}$$

Efficiency of the Otto cycle is a function of the compression ratio only.

The compression ratio varies from 5 to 8.

Diesel cycle

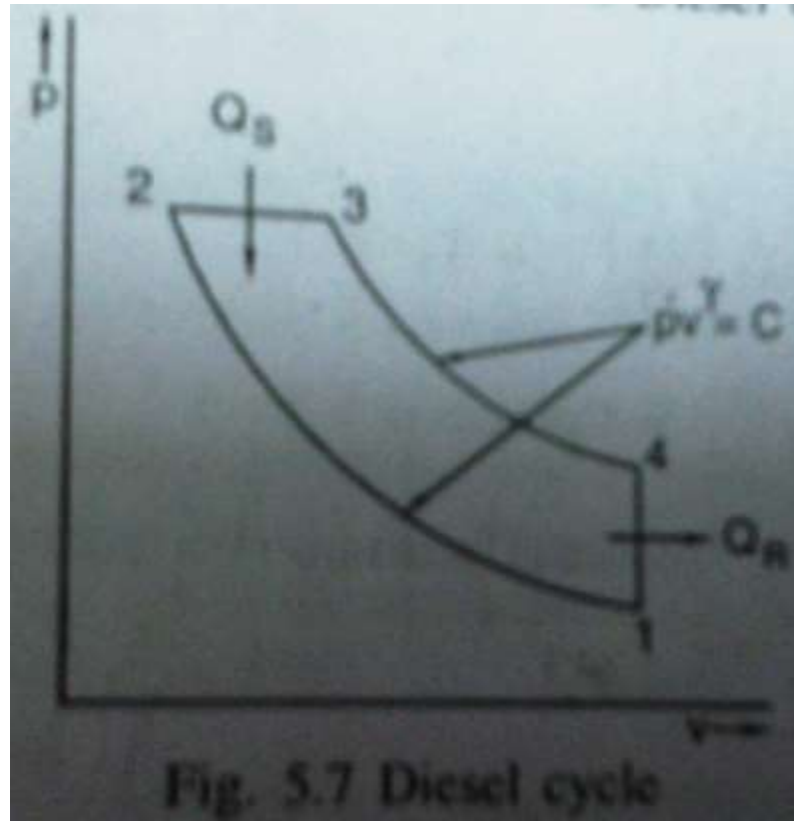
It is ideal cycle for the compression ignition internal combustion engines or the engines working on the diesel oil.

The cycle consists of four reversible processes. Two processes are reversible adiabatic.

One is constant pressure and one is constant volume processes.

Figure shows the diesel cycle on the p - V diagram.

Diesel Cycle



The processes of the diesel cycle are as below:-

1-2:-Reversible adiabatic isentropic compression of air.

Pressure and temperature of air will increase.

2-3:- Heat addition at the constant pressure.

During this process pressure of the air remain constant and volume and the temperature will increase.

Volume ratio $\frac{v_3}{v_2}$ is called cut –off ratio.

3-4:- Reversible adiabatic isentropic expansion of the air.

Pressure and temperature of air will decrease.

4-1:-Heat rejection at the constant volume, Pressure and the temperature of the air will decrease.

As the $Q_s = \Delta h = C_p.\Delta T$ and

$$Q_r = \Delta u = C_v.\Delta T$$

Heat supplied $Q_s = C_p.(T_3-T_2)$

Heat rejected $Q_r = C_v.(T_4-T_1)$

Work done per cycle = $Q_s - Q_r$

$$= C_p.(T_3-T_2) - C_v. (T_4-T_1)$$

Air standard cycle efficiency :

$$\eta_a = \frac{\text{Workdone}}{\text{Heat supplied}}$$

$$= \frac{[C_p.(T_3-T_2) - C_v.(T_4-T_1)]}{C_p.(T_3-T_2)}$$

$$= 1 - \frac{C_v(T_4-T_1)}{C_p(T_3-T_2)}$$

$$= 1 - \frac{1(T_4-T_1)}{\gamma(T_3-T_2)}$$

Now we need to substitute for the T_2, T_3, T_4 in terms of T_1 .

For defining the efficiency equation put all the value in to the Air Standard cycle efficiency equation.

For the process 1-2 :- (reversible adiabatic compression)

$$\gamma-1$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\gamma-1$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= T_1 \cdot (r^{\gamma-1})$$

$$\text{Where } r = \left(\frac{V_1}{V_2}\right) = \text{Compression ratio}$$

For the process 2-3:- (constant pressure heat addition)

$$\left(\frac{T_3}{T_2}\right) = \left(\frac{V_3}{V_2}\right)$$

$$= Q$$

= cut off ratio

$$T_3 = Q \cdot T_2 \qquad \text{Now } T_2 = (r^{\gamma-1}) \cdot T_1$$

$$= Q \cdot (r^{\gamma-1}) \cdot T_1$$

For the process 3-4:- (reversible adiabatic expansion)

$$\gamma-1$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\gamma-1$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_4}\right)^{\gamma-1} \quad \text{multiply and divide with } v_2$$

$$\gamma-1$$

$$= T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_4}\right)^{\gamma-1}$$

$$\gamma-1$$

$$T_4 = T_3 \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_4} \right) \quad \text{Now } (v_1 = v_4)$$

Put instead of v_4 , v_1 in above equation, We get

$$\gamma-1$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1} \right) \quad \text{Now } \frac{V_3}{V_2} = 9 \text{ and } \frac{V_2}{V_1} = \frac{1}{r}$$

$$\gamma-1$$

$$T_4 = T_3 \cdot \left(9 \cdot \frac{1}{r} \right)$$

$$\gamma-1$$

$$T_4 = T_3 \cdot \left(\gamma \cdot \frac{1}{r} \right) \quad \text{But } T_3 = \gamma \cdot T_2$$

Put T_3 value in the above equation, we get

$$\gamma-1$$

$$T_4 = \gamma \cdot T_2 \cdot \left(\gamma \cdot \frac{1}{r} \right) \quad \text{But } T_2 = (r^{\gamma-1}) \cdot T_1$$

Put T_2 value in the above equation, we get

$$\gamma-1$$

$$T_4 = \gamma \cdot (r^{\gamma-1}) \cdot T_1 \cdot \left(\gamma \cdot \frac{1}{r} \right)$$

$$\gamma-1$$

$$T_4 = g \cdot (r^{\gamma-1}) \cdot T_1 \cdot (g \cdot \frac{1}{r})$$

Now simplify the above equation, we get the final equation for T_4

$$\gamma-1$$

$$T_4 = g \cdot (r^{\gamma-1}) \cdot T_1 \cdot (g^{\gamma-1}) \cdot (\frac{1}{r})$$

$$\gamma-1$$

$$T_4 = g^1 \cdot (g^{\gamma-1}) \left(\frac{r}{r}\right)^{\gamma-1} \times T_1$$

$$T_4 = g^\gamma \cdot T_1$$

$$T_2 = (r^{\gamma-1}).T_1$$

$$T_3 = \rho . T_2$$

$$= \rho . (r^{\gamma-1}).T_1$$

$$T_4 = \rho^{\gamma} . T_1$$

Now Substitute the value of T_2, T_3, T_4 in the equation of air standard cycle efficiency, we get

$$\begin{aligned}
 \eta_a &= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \\
 &= 1 - \frac{(g^\gamma \cdot T_1 - T_1)}{\gamma(g \cdot (r^{\gamma-1}) \cdot T_1 - (r^{\gamma-1}) \cdot T_1)} \\
 &= 1 - \frac{T_1(g^\gamma - 1)}{T_1 \cdot r^{\gamma-1} [\gamma(g - 1)]} \\
 &= 1 - \frac{(g^\gamma - 1)}{r^{\gamma-1} \cdot [\gamma(g - 1)]} \\
 \eta_{a(\text{Diesel})} &= 1 - \frac{1}{r^{\gamma-1}} \left[\frac{(g^\gamma - 1)}{\gamma(g - 1)} \right]
 \end{aligned}$$

Since $\gamma = 1.4$ for the air and ρ is always greater than 1, the quantity in the bracket is always greater than one.

So for the same compression ratio efficiency of Otto cycle is greater than that of the diesel cycle.

Numericals of Heat Engine

(1) In an engine working on the Otto cycle, air has a pressure of 1 bar and temperature of 27°C at the entry. Air is compressed with the compression ratio of 7. The heat is added at the constant volume until the temperature rises to 2000 K.

Find out

- (1) Air standard efficiency
- (2) Pressure and Temperature at the end of the compression
- (3) Heat supplied
- (4) Mean effective pressure in the Kpa.

Given:

$$r = \frac{V_1}{V_2} = 7$$

$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$

$$T_3 = 2000 \text{ K}$$

$$\begin{aligned} \text{Air Standard Cycle Efficiency } \eta_a &= 1 - \frac{1}{r^{\gamma-1}} \\ &= 1 - \frac{1}{7^{1.4-1}} \\ &= 1 - \frac{1}{7^{0.4}} \\ &= 54.1\% \end{aligned}$$

γ

Now we have $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$

γ

$$P_2 = P_1 \cdot \left(\frac{V_1}{V_2}\right)^\gamma$$

$$r = \frac{V_1}{V_2} = 7$$

$$= 1 \times 7^{1.4}$$

$$= 15.24 \text{ bar}$$

$$\gamma-1$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)$$

$$\gamma-1$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)$$

$$1.4-1$$

$$T_2 = 300 \cdot (7)$$

$$0.4$$

$$T_2 = 300 \cdot (7)$$

$$T_2 = 653.4 \text{ K}$$

Heat Supplied

$$\begin{aligned} Q_s &= C_v (T_3 - T_2) \\ &= 0.718 (2000 - 653.40) \\ &= 9669.9 \frac{kJ}{kg} \end{aligned}$$

Mean Effective pressure $p_m = \frac{\text{Work Done}}{\text{Swept Volume}}$

$$\begin{aligned} \text{Work done} &= \eta_a \times Q_s \\ &= 0.541 \times 9669.9 \\ &= 523.1 \frac{kJ}{kg} \end{aligned}$$

$$p_1 V_1 = mRT_1$$

$$V_1 = \frac{mRT_1}{p_1}$$

$$= \frac{1 \times 0.287 \times 300}{100}$$

$$= 0.861 \frac{m^3}{kg}$$

$$\text{Now } r = \frac{V_1}{V_2} = 7$$

$$V_2 = \frac{V_1}{r}$$

$$= \frac{0.861}{7}$$

$$= 0.123 \frac{m^3}{kg}$$

$$\begin{aligned}\text{Swept Volume } V_s &= V_1 - V_2 \\ &= 0.861 - 0.123 \\ &= 0.738 \frac{m^3}{kg}\end{aligned}$$

$$\begin{aligned}\text{Mean Effective pressure } p_m &= \frac{\text{Work Done}}{\text{Swept Volume}} \\ &= \frac{523.1}{0.738} \\ &= 708.67 \text{ kPa}\end{aligned}$$

(2)

The engine working on ideal otto cycle. The temperature at the beginning and at the end of the compression is 50°C and 400°C . Calculate the air standard cycle efficiency and also the compression ratio.

Given:

$$T_1 = 323 \text{ K}$$

$$T_2 = 673 \text{ K}$$

$$\gamma - 1$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$

$$= r^{\gamma - 1}$$

$$\begin{aligned}
 r^{\gamma-1} &= \frac{T_2}{T_1} \\
 &= \frac{673}{323} \\
 &= 2.08
 \end{aligned}$$

$$r = 6.27$$

$$\begin{aligned}
 \eta_a &= 1 - \frac{1}{r^{\gamma-1}} \\
 &= 1 - \frac{1}{6.27^{1.4-1}} \\
 &= 1 - \frac{1}{6.27^{0.4}} = 1 - \frac{1}{2.084} = 52\%
 \end{aligned}$$

(3) An oil engine works on the diesel cycle with the temperature of 27°C at the beginning of the compression. If the ratio of adiabatic compression is 14 and that of the adiabatic expansion is 8 then find out the thermal efficiency of the cycle.

Solution:

Now with the usual notations

$$r = \frac{V_1}{V_2} = 14$$

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} = 8$$

$$\rho = \text{cutoff ratio} = \frac{V_3}{V_2} = \frac{\frac{V_1}{V_2}}{\frac{V_1}{V_3}} = \frac{14}{8} = 1.75$$

$$\eta_{a(\text{Diesel})} = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{(\rho^{\gamma}-1)}{\gamma(\rho-1)} \right]$$

$$= 1 - \frac{1}{14^{1.4-1}} \left[\frac{(1.75^{1.4}-1)}{1.4(1.75-1)} \right]$$

$$= 0.606$$

$$= 60.6 \%$$

(4) An otto cycle has compression ratio of 6. Temperature at the beginning of the compression is 20°C and heat supplied by the combustion is $1900 \frac{\text{kJ}}{\text{kg}}$. Calculate the temperature at the remaining three salient points, η_a and net work output.

Solution:-

$$\begin{aligned}\eta_a &= 1 - \frac{1}{r^{\gamma-1}} \\ &= 1 - \frac{1}{6^{1.4-1}} \\ &= 51.16 \%\end{aligned}$$

Now we know the basic equation, that is

$$\frac{T_2}{T_1} = r^{\gamma - 1}$$

$$T_2 = T_1 \cdot r^{\gamma - 1}$$

$$= 293 \times 6^{(1.4 - 1)}$$

$$= 600 \text{ K}$$

Heat Supplied $Q_s = C_v (T_3 - T_2)$

$$1900 = 0.718 \times (T_3 - 600)$$

$$(T_3 - 600) = \frac{1900}{0.718}$$

$$T_3 = 600 + \frac{1900}{0.718}$$

$$= 3246.2 \text{ K}$$

$$\frac{T_3}{T_4} = r^{\gamma - 1}$$

$$\begin{aligned} T_4 &= \frac{T_3}{r^{\gamma - 1}} \\ &= \frac{3246.2}{6^{1.4 - 1}} \\ &= 1585.30 \text{ K} \end{aligned}$$

$$\begin{aligned}\text{Work output} &= \eta_a \times Q_s \\ &= 0.5116 \times 1900 \\ &= 972.0 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

(5)

An engine operating on an air standard Diesel cycle has maximum pressure and temperature of 39 bar and 1100°C. Pressure and the temperature at the beginning of the compression are 1 bar and 20°C.

Calculate the air standard cycle efficiency of the cycle.

Given:

$$P_1 = 1 \text{ bar}$$

$$T_1 = 20^\circ\text{C}$$

$$T_3 = 1100^\circ\text{C}$$

$$P_1 = P_2 = 39 \text{ bar}$$

For the Reversible Adiabatic Process 1-2:

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 293 \cdot \left(\frac{39}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\frac{0.4}{1.4}$$

$$T_2 = 293 \cdot \left(\frac{39}{1}\right)$$

$$T_2 = 834.6 \text{ K}$$

$$\frac{1}{\gamma}$$

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)$$

$$\frac{1}{\gamma}$$

$$r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)$$

$$\frac{1}{\gamma}$$

$$r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)$$

$$\frac{1}{1.4}$$

$$r = \frac{V_1}{V_2} = \left(\frac{39}{1} \right)$$

$$r = \frac{V_1}{V_2} = 13.7$$

For the constant heat addition

$$\varrho = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1373}{834.6} = 1.65$$

$$\eta_{a(\text{Diesel})} = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{(\varrho^{\gamma}-1)}{\gamma(\varrho-1)} \right]$$

$$= 1 - \frac{1}{13.7^{1.4-1}} \left[\frac{(1.65^{1.4}-1)}{1.4(1.65-1)} \right]$$

$$= 60.46 \%$$

(6)

A Diesel engine has the compression ratio of 19 and expansion ratio 9.1.

Calculate its air standard efficiency.

Calculation:

Air standard cycle efficiency for the Diesel engine is

$$\eta_{a(\text{Diesel})} = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{(g^{\gamma}-1)}{\gamma(g-1)} \right]$$

We have the $r = 19 = \frac{V_1}{V_2}$

Expansion ratio $= \frac{V_4}{V_3} = \frac{V_1}{V_3} = 9.1$

$$\begin{aligned} Q &= \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \frac{V_1}{V_2} \\ &= \frac{1}{9.1} \times \frac{19}{1} \\ &= 2.008 \end{aligned}$$

$$\eta_{a(\text{Diesel})} = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{(g^{\gamma}-1)}{\gamma(g-1)} \right]$$

$$= 1 - \frac{1}{19^{1.4-1}} \left[\frac{(2.008^{1.4}-1)}{1.4(2.008-1)} \right]$$

$$= 63.55 \%$$

(7)

A hot air engine works on the carnot cycle with the thermal efficiency of 60 %. If the final temperature of the air is 10°C. Find out the initial Temperature.

Given Data

$$\eta_c = 0.6, T_2 = 283 \text{ K}$$

$$\eta_c = 1 - \frac{T_2}{T_1}$$

$$0.6 = 1 - \frac{283}{T_1}$$

$$0.6 \cdot T_1 = \frac{T_1 - 283}{1}$$

$$\mathbf{T_1 = 707.5 \text{ K}}$$

(8)

A hot air engine works on the Carnot cycle with the thermal efficiency of 70%. If the final temperature of air is 20°C. Determine initial temperature.

For the Carnot Cycle $\eta_c = 1 - \frac{T_2}{T_1}$

$$0.7 = 1 - \frac{293}{T_1}$$

$$T_1 = 976.67 \text{ K}$$

(9)

Following data is available for the air standard Carnot Cycle.

Minimum Temperature in cycle = 15°C

Minimum Pressure in the cycle = 1.1 bar

Pressure after the isothermal compression = 4 bar

Pressure after the isentropic compression = 12 bar

Engine speed = 150 rpm

Calculate the efficiency of the engine and power output of the engine.

Given

$$T_3 = T_4 = 15^\circ\text{C} = 288 \text{ K}$$

$$P_3 = 1.1 \text{ bar}$$

$$P_4 = 4 \text{ bar}$$

$$P_1 = 12 \text{ bar}$$

$$N = 150 \text{ rpm}$$

For the reversible adiabatic compression 4-1

$$\frac{\gamma-1}{\gamma}$$

$$\frac{T_1}{T_4} = \left(\frac{P_1}{P_4}\right)$$

$$\frac{\gamma-1}{\gamma}$$

$$\frac{T_1}{1} = T_4 \cdot \left(\frac{P_1}{P_4}\right)$$

$$\frac{1.4-1}{1.4}$$

$$\frac{T_1}{1} = 288 \cdot \left(\frac{12}{4}\right)$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_1}{288} = \left(\frac{12}{4} \right)^{\frac{1.4-1}{1.4}}$$

$$T_1 = 394.2 \text{ K}$$

$$\begin{aligned} \text{Efficiency of the Carnot Cycle} &= 1 - \frac{T_2}{T_1} \\ &= 1 - \frac{288}{394.2} \\ &= 26.94 \% \end{aligned}$$

$$\begin{aligned}\text{Work done} &= \text{net heat transfer} \\ &= mR(T_1 - T_3) \ln r\end{aligned}$$

$$\text{Where } r = \frac{v_3}{v_4} = \frac{p_4}{p_3} = \frac{4}{1.1} = 3.64$$

$$\begin{aligned}\text{Work done} &= mR(T_1 - T_3) \ln r \\ &= 1.0 \cdot 287(394.2 - 288) \ln 3.64 \\ &= 39.4 \frac{\text{kJ}}{\text{cycle}}\end{aligned}$$

$$\begin{aligned}\text{Power} &= \frac{\text{work}}{\text{cycle}} \times \text{rps} \\ &= 39.4 \times \frac{150}{60} \\ &= 98.5 \text{ kW}\end{aligned}$$