BASIC ELECTRICAL ENGINEERING
THREE PHASE POWER MEASUREMENT

Next

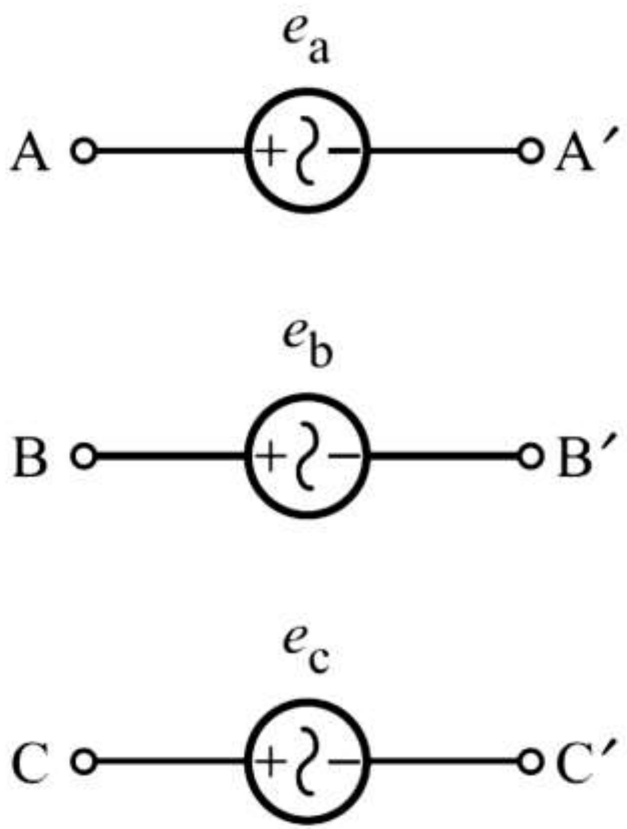
Topics to be Discussed

- Three-Phase System.
  - Advantages.
  - Concept.
  - Generation.
- Unbalanced Three-Phase System.
- Voltages And Currents Relations.
  - (1) Star-Connected System.
  - (2) Delta-Connected System
- Power In Three-phase System.
- Power Measurement.
  - Two-Wattmeter Method.
- Power Factor Measurement.
  - Important Points.

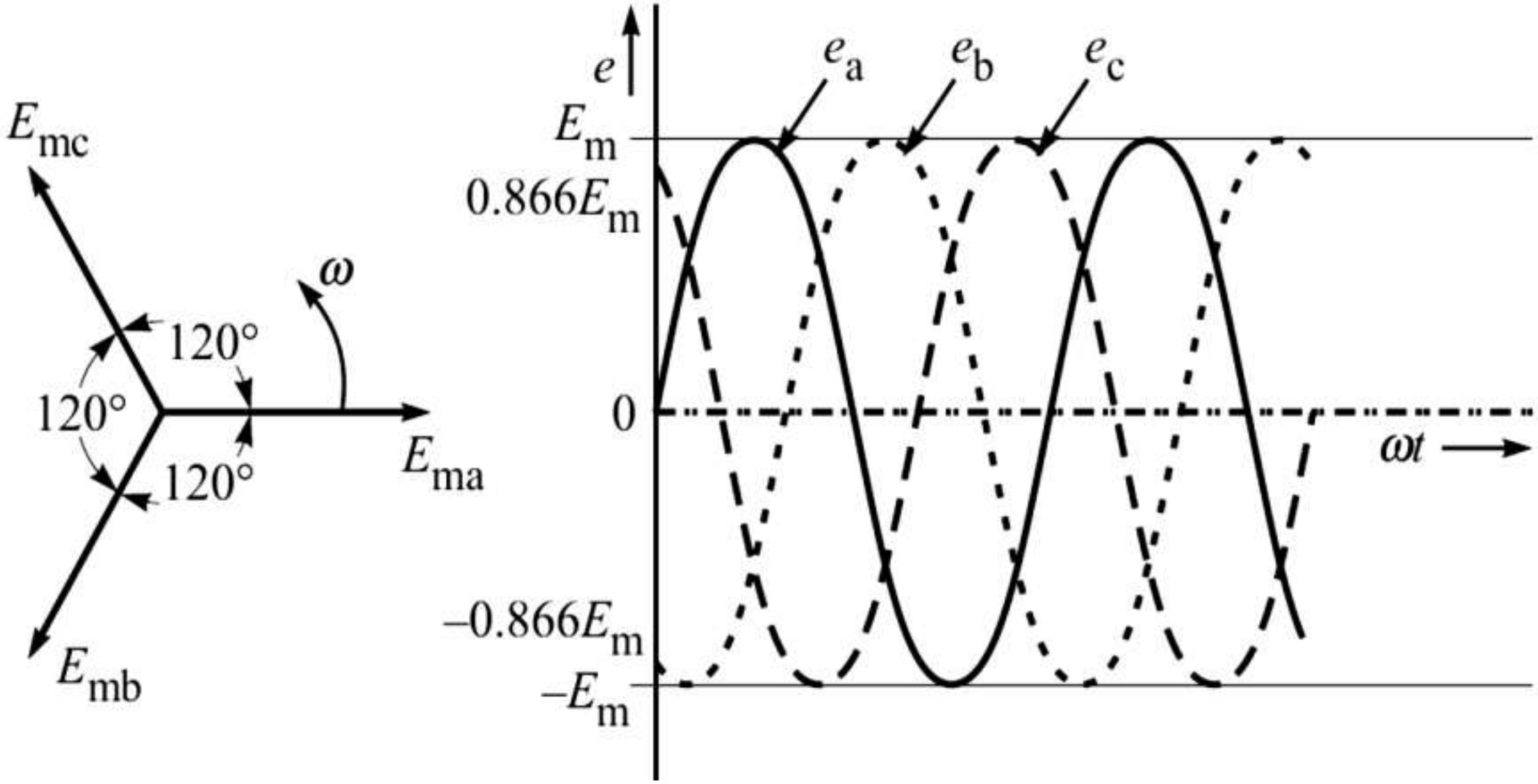
Advantages of Three-Phase System

- Transmission lines require much less conductor material.
- A three-phase machine gives a higher output.
- A three-phase motor develops a uniform (not a pulsating) torque.
- The three-phase induction motors are self-starting.
- Can be used to supply domestic as well as industrial (or commercial) power.
- The voltage regulation is better.

Concept of Three-phase Voltages



- The ***phase order*** or ***phase sequence*** or ***phase rotation*** is *abc*.



E_{mc}
ω
120°
120°
120°
E_{ma}
E_{mb}
e
e_a
e_b
e_c
E_m
$0.866E_m$
0
ωt
$-0.866E_m$
$-E_m$

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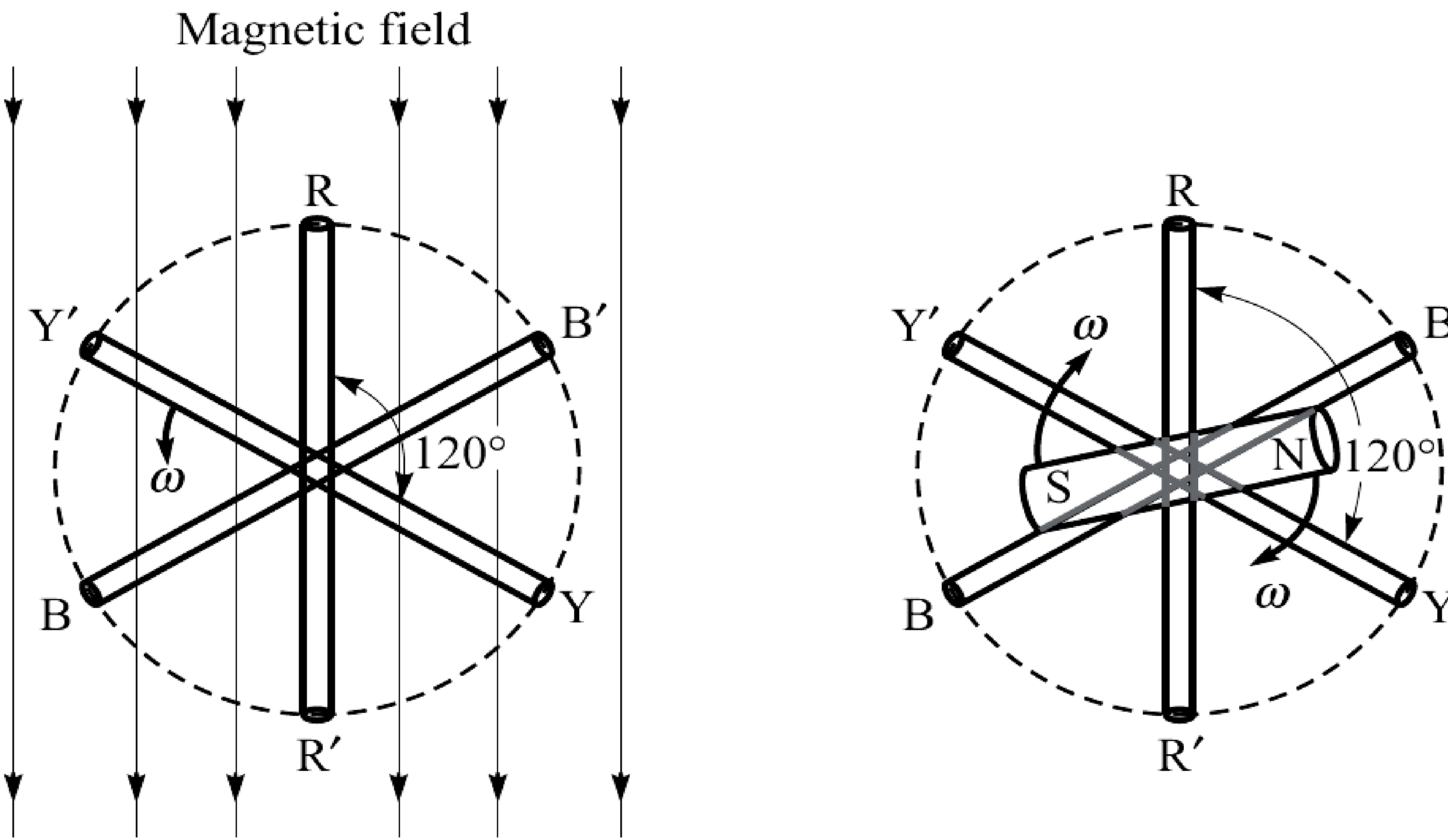
Generation of Three-phase Voltages



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- Whether the coils rotate anti-clockwise or the magnet on the rotor rotates clockwise, the effect is the same.
- But the latter is safer and easier to make external connections to stationary coils.

Three windings connected to three loads using six line conductors

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- Thus, the terminals on the periphery appear in the order : R, B’, Y, R’, B, Y’.
- The three emfs generated e_R, e_Y and e_B connected to three respective loads L_1, L_2 and L_3.
- This necessitates the use of six line conductors.
- Obviously, it is cumbersome and expensive.
- Let us now consider how it may be simplified.

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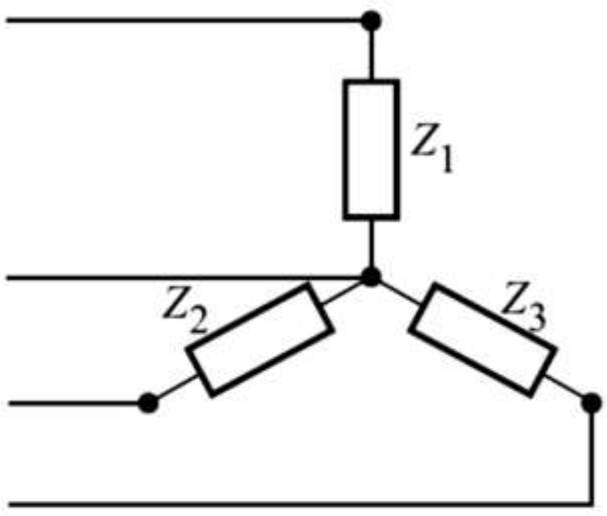
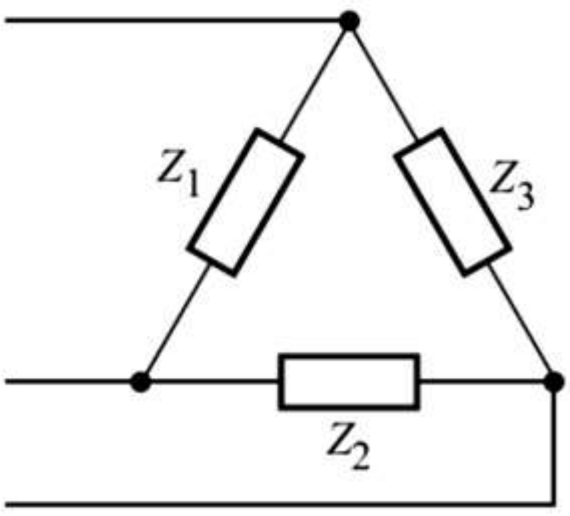
Three-phase Loads

There are *two* kinds of three-phase systems :

(i) Star or wye (Y) connection, and

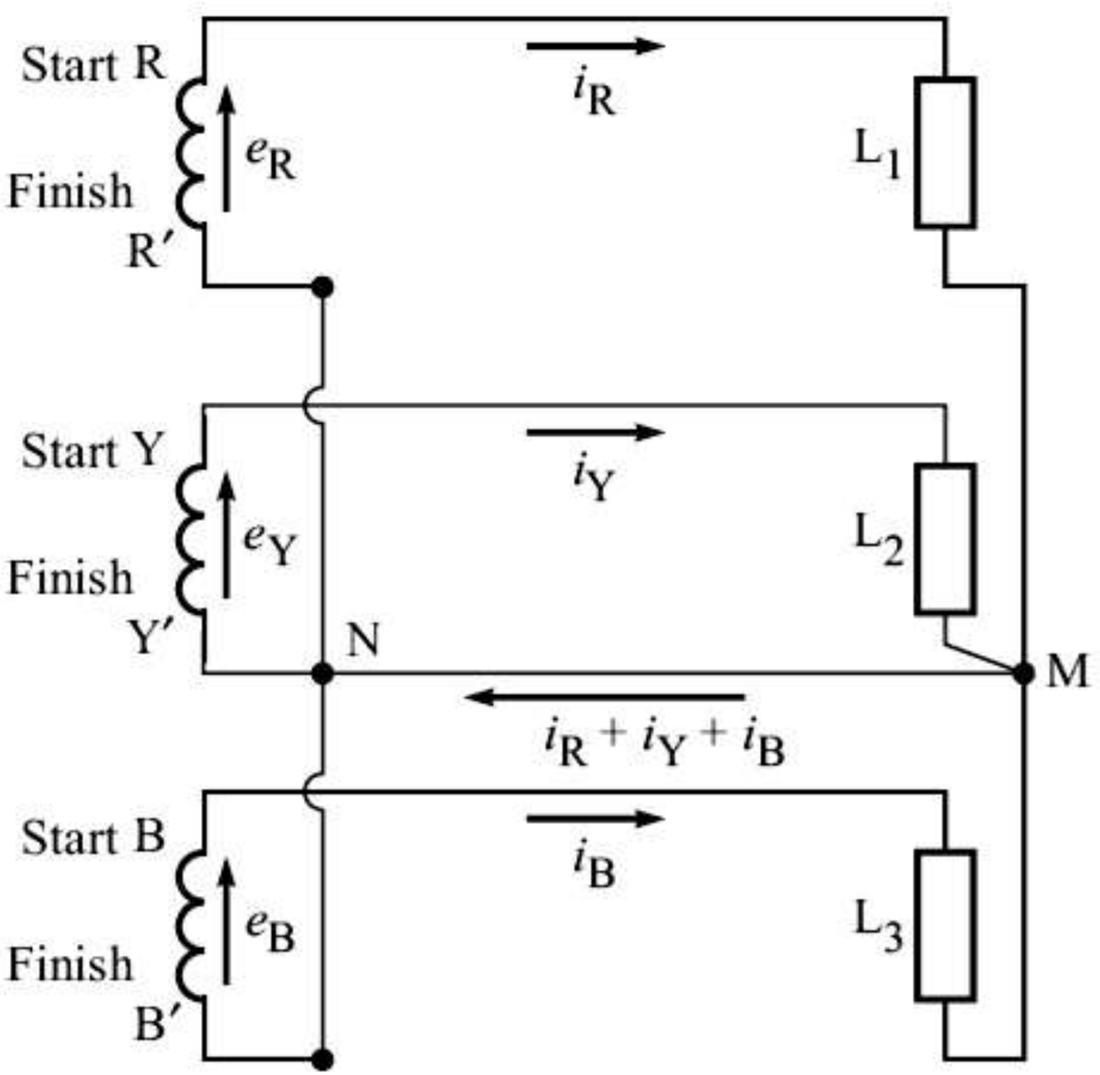
(ii)Delta (Δ) or mesh connection





If all the three impedances are equal, the load is said to be a *balanced load.*

Star (Y) Connected Three-phase System



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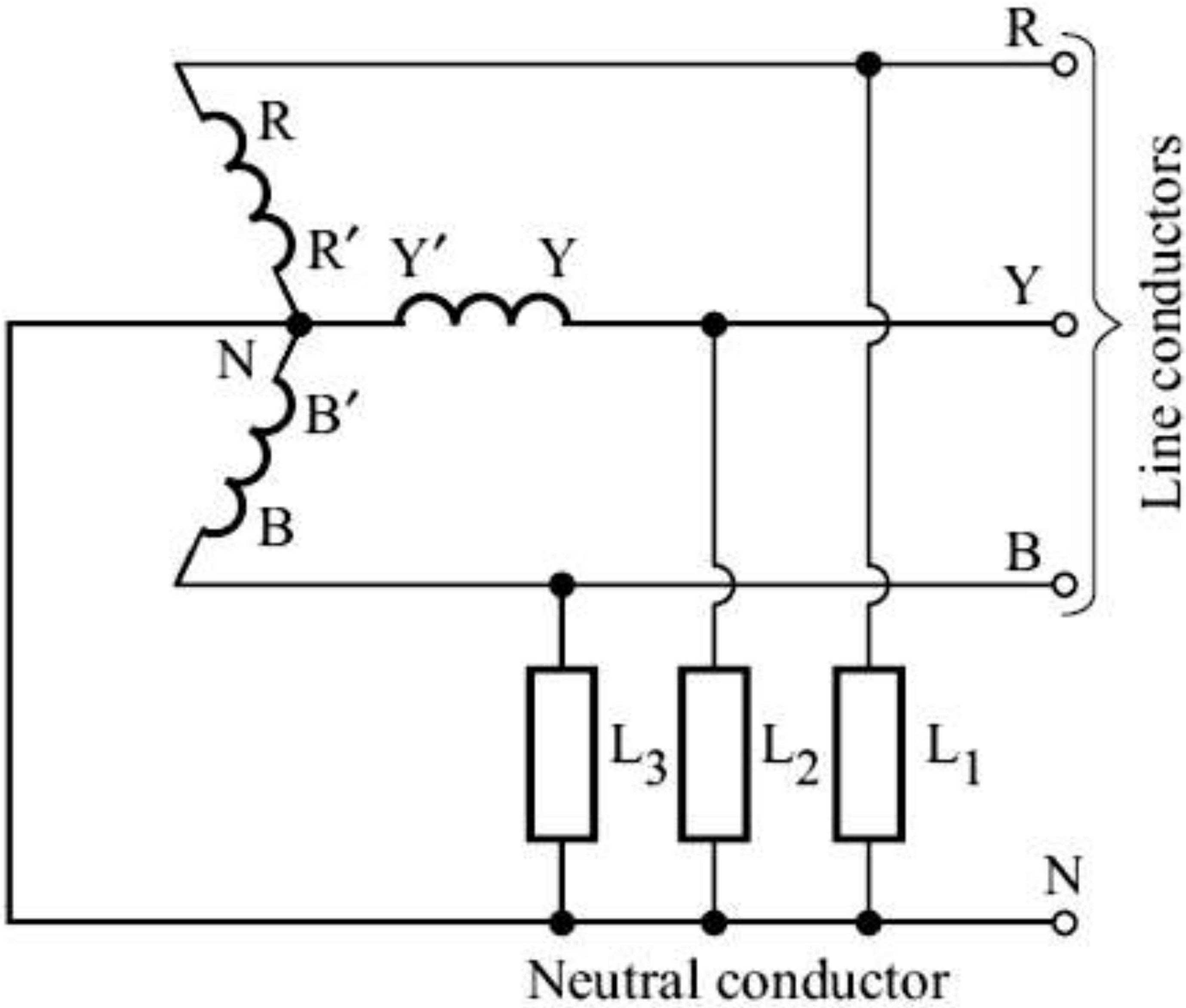

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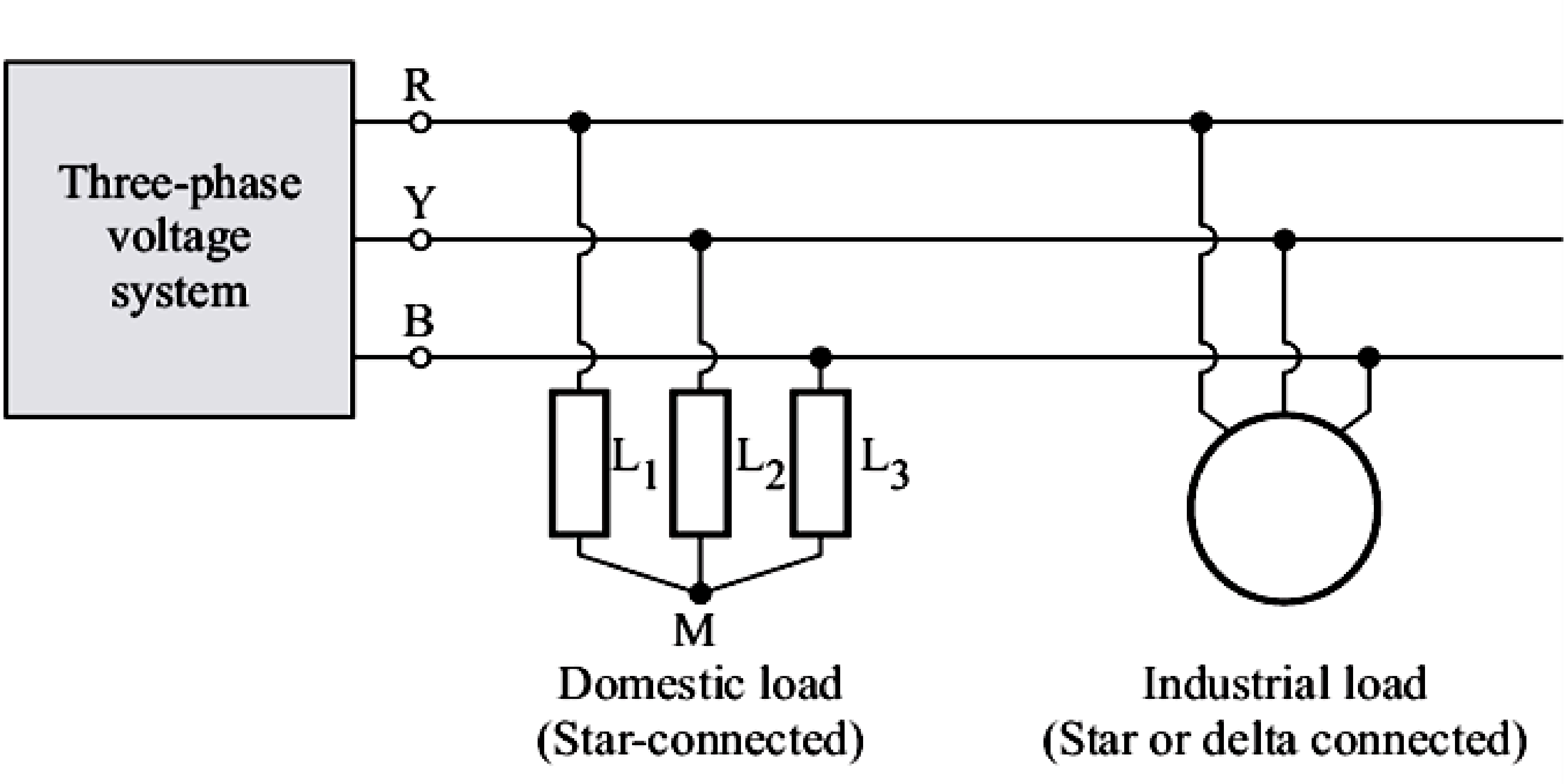
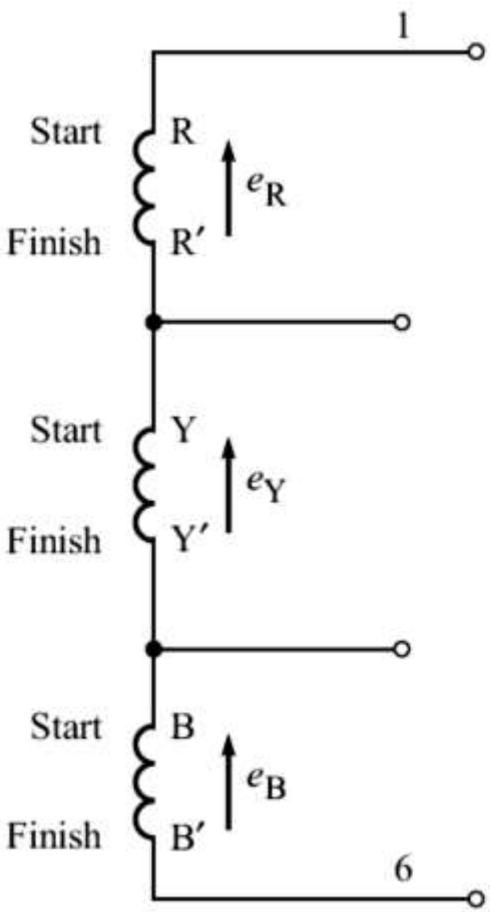
Unbalanced Three-Phase System

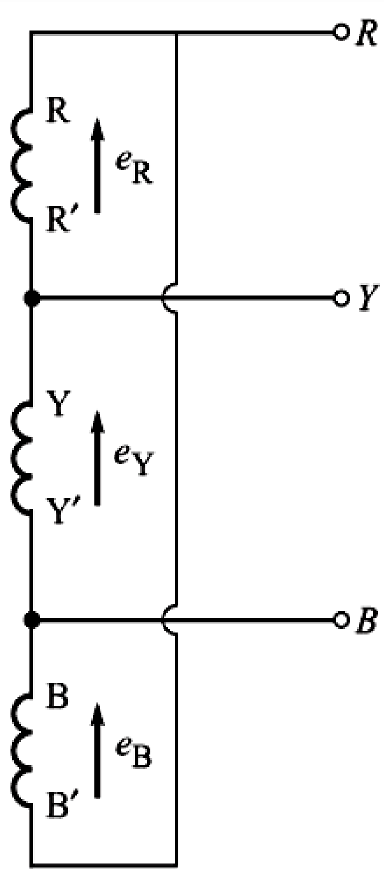


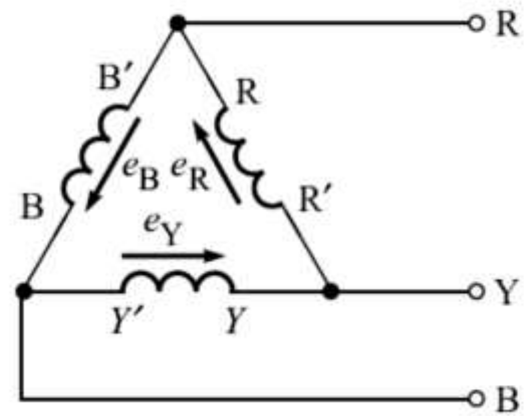
The common point M is called ***local neutral point***.

- Practically, it may not be possible always to make this star-connected domestic load balanced.
- However, as per KCL, the sum of the three line currents must still be zero.
- Hence, the voltages across the three loads get adjusted, resulting in ***neutral shift*** or ***floating neutral***.
- This situation is definitely undesirable.
- Therefore, we use ***Four-Wire Three-Phase Voltage System***.

Delta (Δ) Connected Three-phase System







Note that the 'finish' of one phase is connected to the 'start' of another phase.

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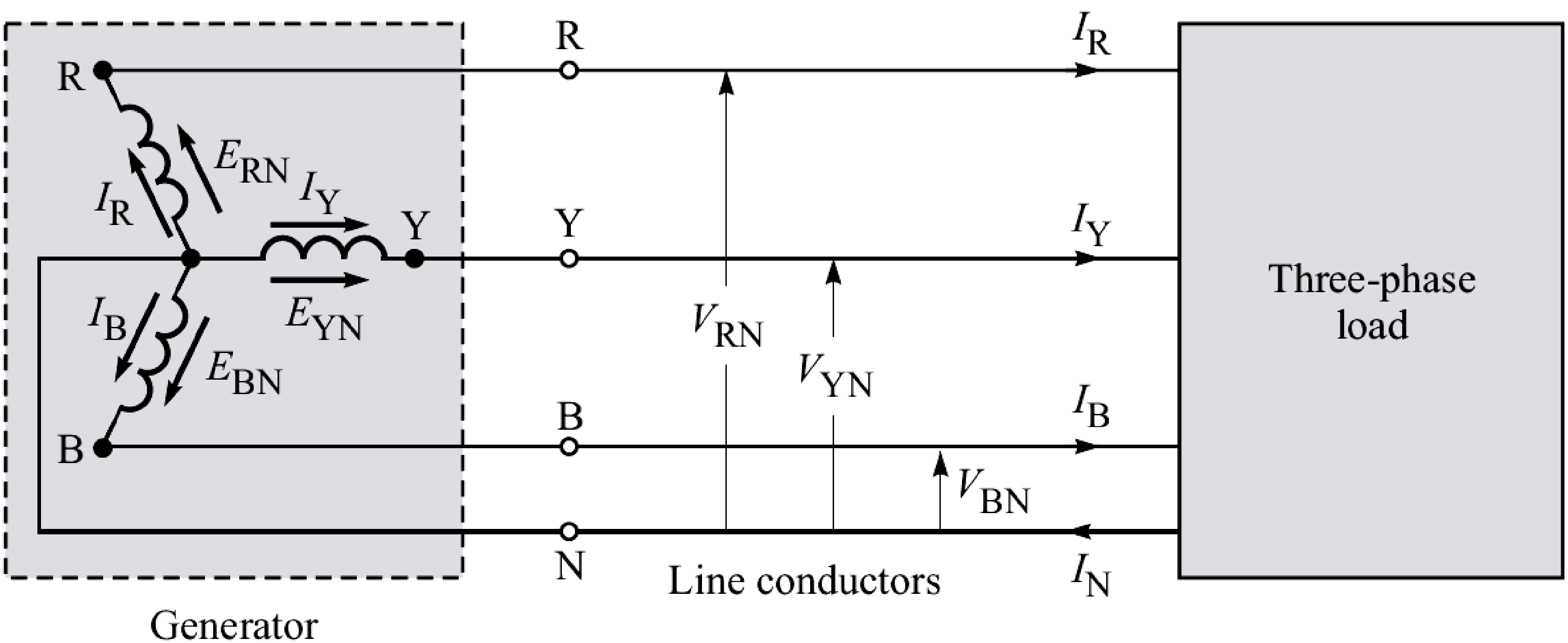
Voltages And Currents Relations in 3-φ Systems

(1) Star-Connected System



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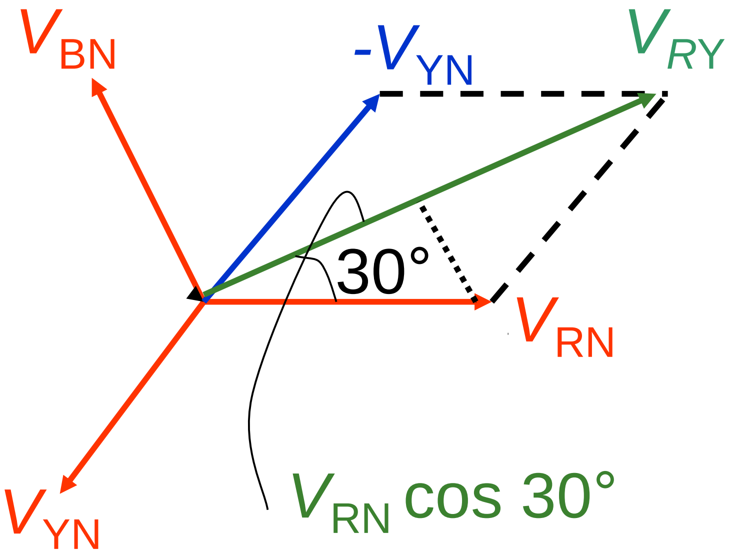
- In a three-phase system, there are two sets of voltages :
  - the set of ***phase voltages***, and
  - the other is the set of ***line voltages***.
- $\boldsymbol{V}_{RN}$, $\boldsymbol{V}_{YN}$ and $\boldsymbol{V}_{BN}$ denote the set of three phase voltages.
- The term 'line voltage' is used to denote the voltage between two lines.
- $\boldsymbol{V}_{RY}$ represents line voltage between the lines R and Y.

$$\mathbf{V}_{\mathrm{RY}} = \mathbf{V}_{\mathrm{RNY}} = \mathbf{V}_{\mathrm{RN}} + \mathbf{V}_{\mathrm{NY}}$$

$$= \mathbf{V}_{\mathrm{RN}} - \mathbf{V}_{\mathrm{YN}} = \mathbf{V}_{\mathrm{RN}} + (-\mathbf{V}_{\mathrm{YN}})$$



$$V_{\mathrm{RY}} = 2(V_{\mathrm{RN}} \cos 30^\circ)$$

or $$V_{\mathrm{L}} = 2V_{\mathrm{ph}}\left(\sqrt{3}/2\right) = \sqrt{3}V_{\mathrm{ph}}$$

$$\boxed{V_{\mathrm{L}} = \sqrt{3}V_{\mathrm{ph}}}$$

and $$\boxed{I_{\mathrm{L}} = I_{\mathrm{ph}}}$$

Click

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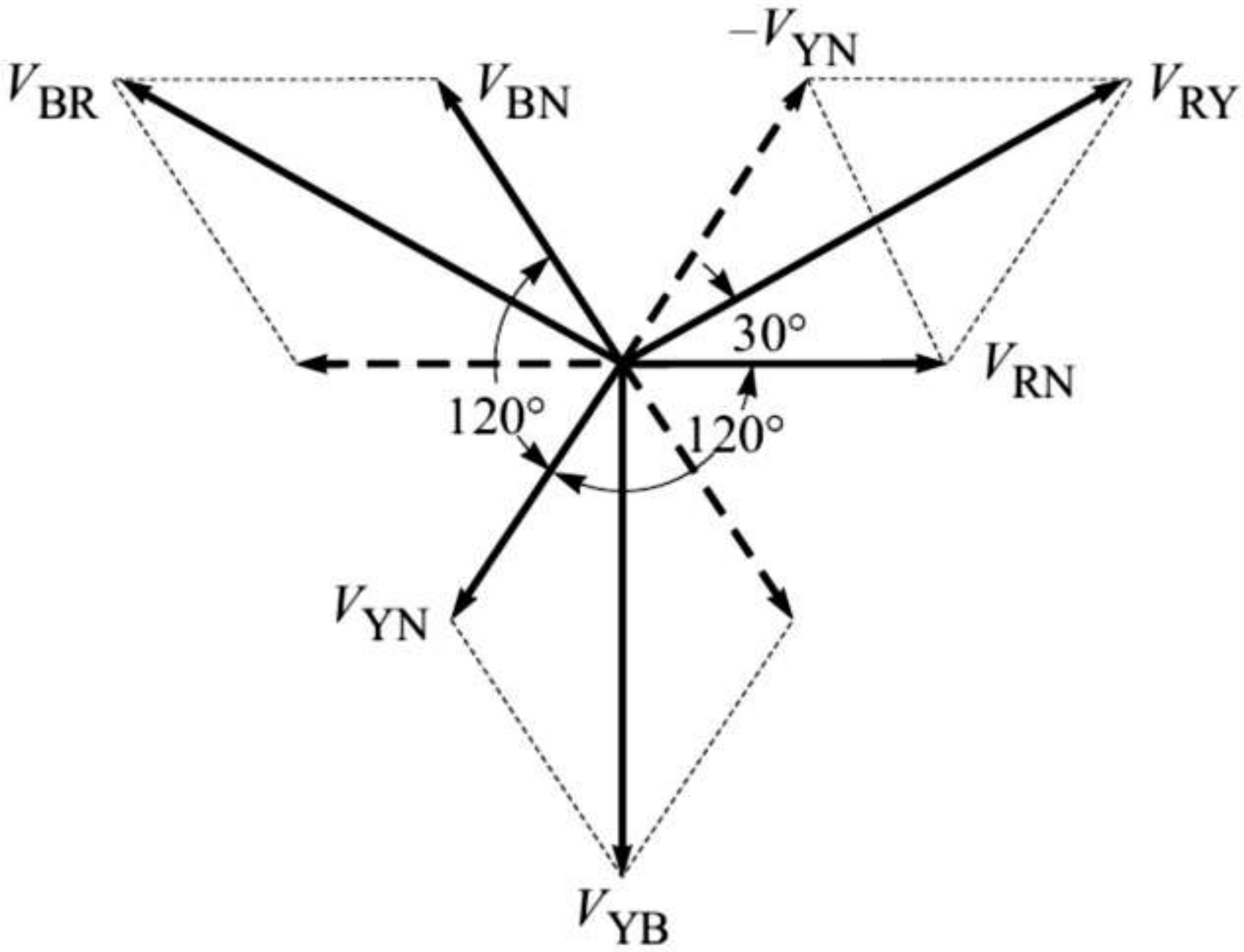
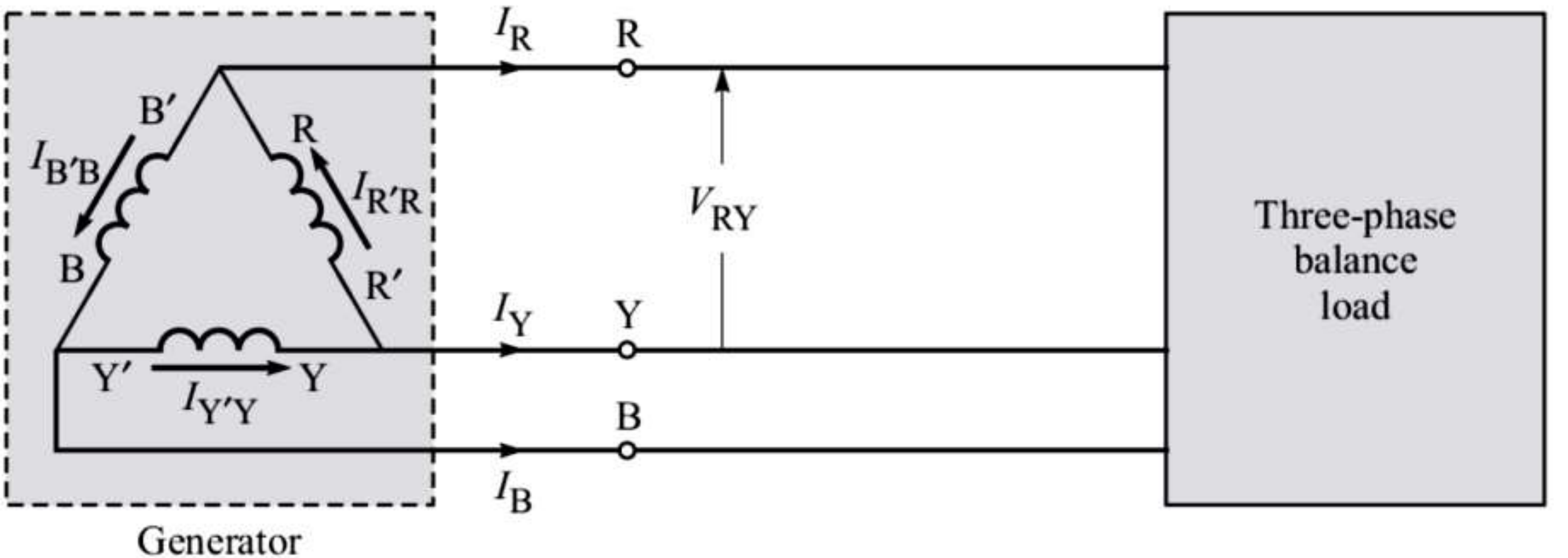


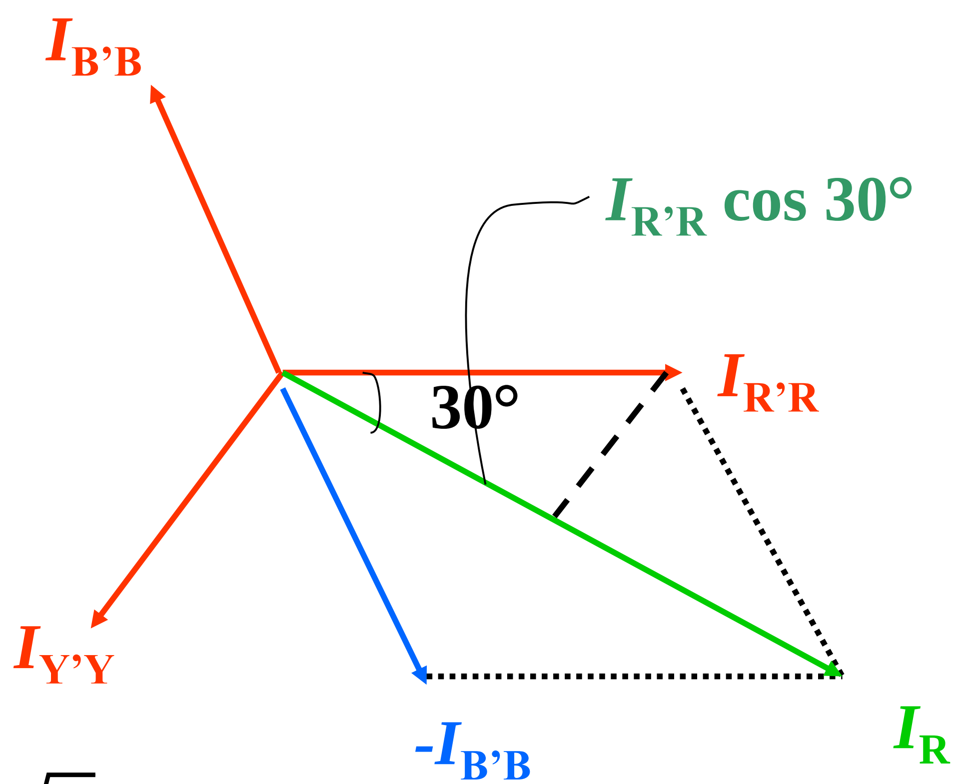
Star-connected System

(2) Delta-Connected System



$$\left|\mathbf{I}_{R'R}\right| = \left|\mathbf{I}_{Y'Y}\right| = \left|\mathbf{I}_{B'B}\right| = I_{ph}\,(\text{say})$$

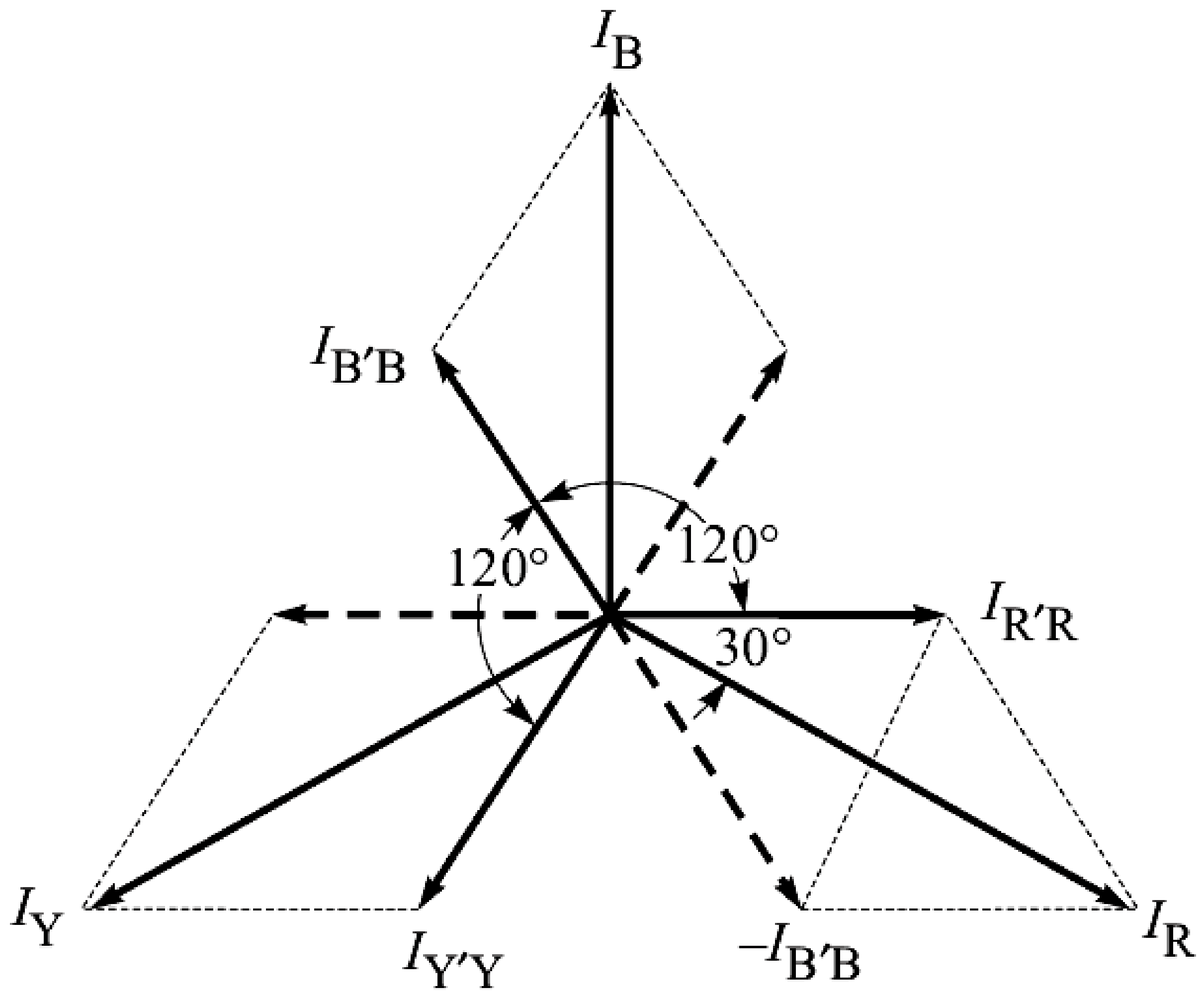
$$\mathbf{I}_{\mathrm{R}} = \mathbf{I}_{\mathrm{R'R}} - \mathbf{I}_{\mathrm{B'B}}$$



$$I_{\mathrm{R}} = 2(I_{\mathrm{ph}} \cos 30^{\circ}) = \sqrt{3} I_{\mathrm{ph}}$$

Click

$$\boxed{I_{\mathrm{L}} = \sqrt{3} I_{\mathrm{ph}}} \quad \text{and} \quad \boxed{V_{\mathrm{L}} = V_{\mathrm{ph}}}$$



Delta-connected System

Important Points about Three-Phase Systems

1. It is normal practice to specify the values of the line voltages and line currents.
2. The current in any phase can be determined by dividing the phase voltage by its impedance.

Example 1

- A 400-V, 3-ϕ supply is connected across a balanced load of three impedances each consisting of a 32-Ω resistance and 24-Ω inductive reactance. Determine the current drawn from the power mains, if the three impedances are
  - (*a*) Y-connected, and
  - (*b*) Δ-connected.

Next

Solution : **Z** = $R + jX = (32 + j24)\ \Omega$.

Click

$$\therefore \qquad Z = \sqrt{R^2 + X^2} = \sqrt{32^2 + 24^2} = 40\ \Omega$$

Click

(*a*) Y-*connection* :

$$V_{\text{ph}} = \frac{V_{\text{L}}}{\sqrt{3}} = \frac{400}{\sqrt{3}}\,\text{V} \qquad \Rightarrow \qquad I_{\text{ph}} = \frac{V_{\text{ph}}}{Z} = \frac{400/\sqrt{3}}{40} = \frac{10}{\sqrt{3}}\ \text{A}$$

$$\therefore \qquad I_{\text{L}} = I_{\text{ph}} = \frac{10}{\sqrt{3}} = \mathbf{5.78\ A}$$

Click

(*b*) *For* Δ-*connection* :

$$V_{\text{ph}} = V_{\text{L}} = 400\,\text{V} \qquad \Rightarrow \qquad I_{\text{ph}} = \frac{V_{\text{ph}}}{Z} = \frac{400}{40} = 10\ \text{A}$$

$$\therefore \qquad I_{\text{L}} = \sqrt{3} I_{\text{ph}} = \sqrt{3} \times 10 = \mathbf{17.32\ A}$$

Next

Power In Three-phase System With A Balanced Load

Consider one phase only, $P_1 = V_{ph} I_{ph} \cos\phi$

Click

Hence, the total power consumed,

$$P = 3P_1 = 3V_{ph} I_{ph} \cos\phi$$

For a *star-connected system,* $V_L = \sqrt{3} V_{ph}$ and $I_L = I_{ph}$

Click

$$P = 3(V_L/\sqrt{3}) I_L \cos\phi = \boxed{\sqrt{3} V_L I_L \cos\phi}$$

For a *delta-connected system,* $V_L = V_{ph}$ and $I_L = \sqrt{3}$

Click

$$P = 3V_L (I_L/\sqrt{3}) \cos\phi = \boxed{\sqrt{3} V_L I_L \cos\phi}$$

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Thus, for *any balanced load*,

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

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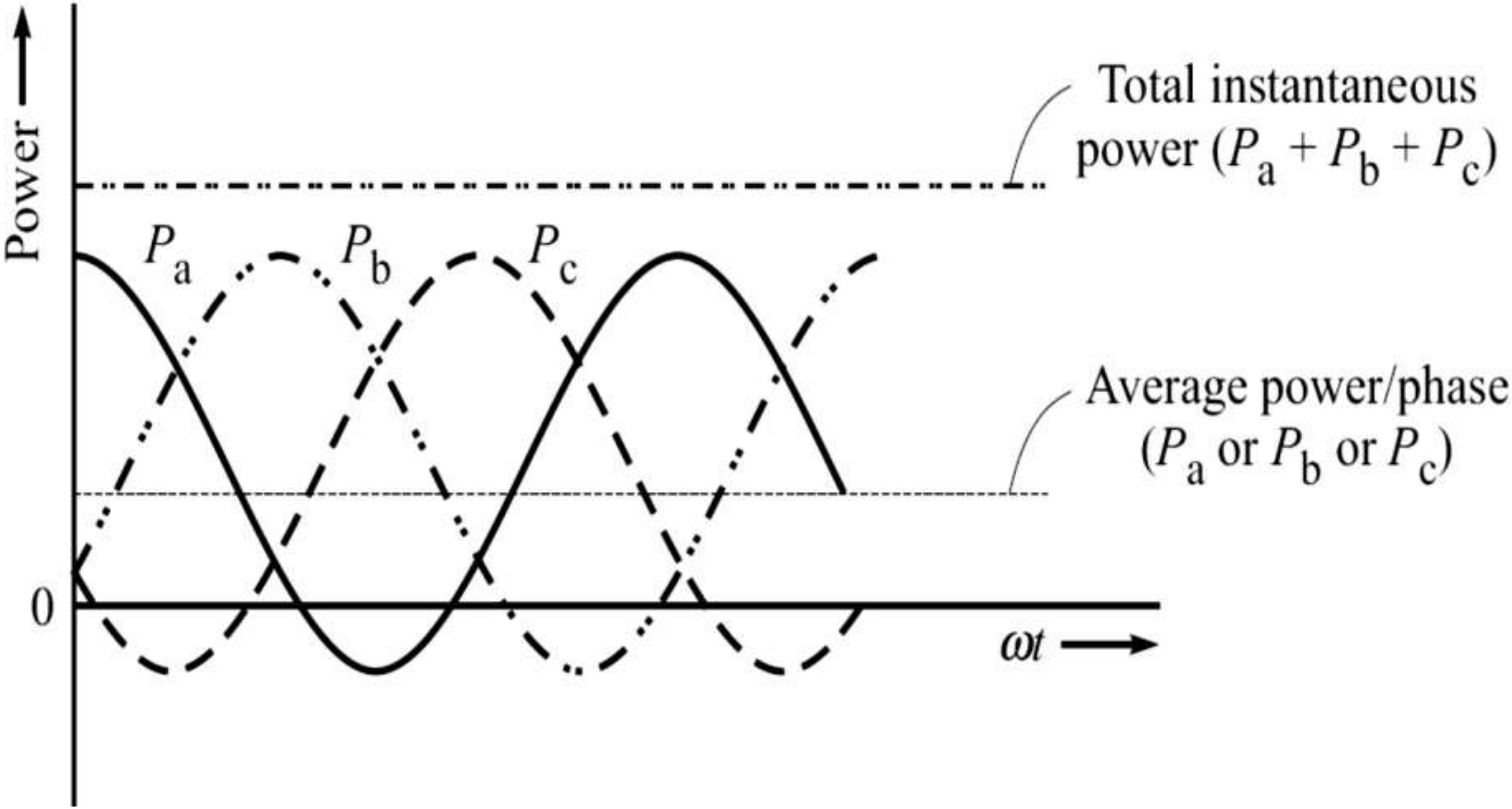
Power
Total instantaneous power ($P_a + P_b + P_c$)
P_a
P_b
P_c
Average power/phase (P_a or P_b or P_c)
0
ωt

Next

- In three-phase system with balanced load (such as a three-phase motor), there is no variation of power at all.
- This is the reason why for driving heavy mechanical loads we prefer a three-phase motor rather than a single-phase motor.

Example 2

- A 400-V, 3-ϕ supply is connected to a balanced network of three impedances each consisting of a 20-Ω resistance and a 15-Ω inductive reactance.
- If the three impedances are (*a*) star-connected, and (*b*) delta-connected, in each case determine
  - (*i*) the line current,
  - (*ii*) the power factor, and
  - (*iii*) the total power in kW.

(a) For *star-connected load :* $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$

Click

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Click

and $$Z_{ph} = \sqrt{20^2 + 15^2} = 25\ \Omega$$

Click

(i) $$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = \mathbf{9.24\ A}$$

Click

(ii) $$\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{20}{25} = \mathbf{0.8\ (lagging)}$$

(iii) $$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 = \mathbf{5.12\ kW}$$

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(b) For *delta-connected load* : $V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$

$$\therefore \qquad V_L = V_{ph} = 400 \text{ V}$$

Click

(i) $$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16 \text{ A}$$

Click

$$\therefore \qquad I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 16 = \mathbf{27.71 \ A}$$

(ii) The power factor is same as above,

$$pf = \mathbf{0.8 \ (lagging)}$$

Click

(iii) $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 27.71 \times 0.8 = \mathbf{15.36 \ kW}$

Click

Note that power consumed has become 3 times.

Next

No.	***Star-Connected System***	***Delta-Connected System***
1.	Similar ends are joined together.	Dissimilar ends are joined.
2.	$V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$	$V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$
3.	Neutral wire available.	Neutral wire not available.
4.	4-wire, 3-ϕ system possible.	4-wire, 3-ϕ system not possible.
5.	Both domestic and industrial loads can be handled.	Only industrial loads can be handled.
6.	By earthing the neutral wire, relays and protective devices can be provided in alternators for safety.	Due to absence of neutral wire, it is not possible.

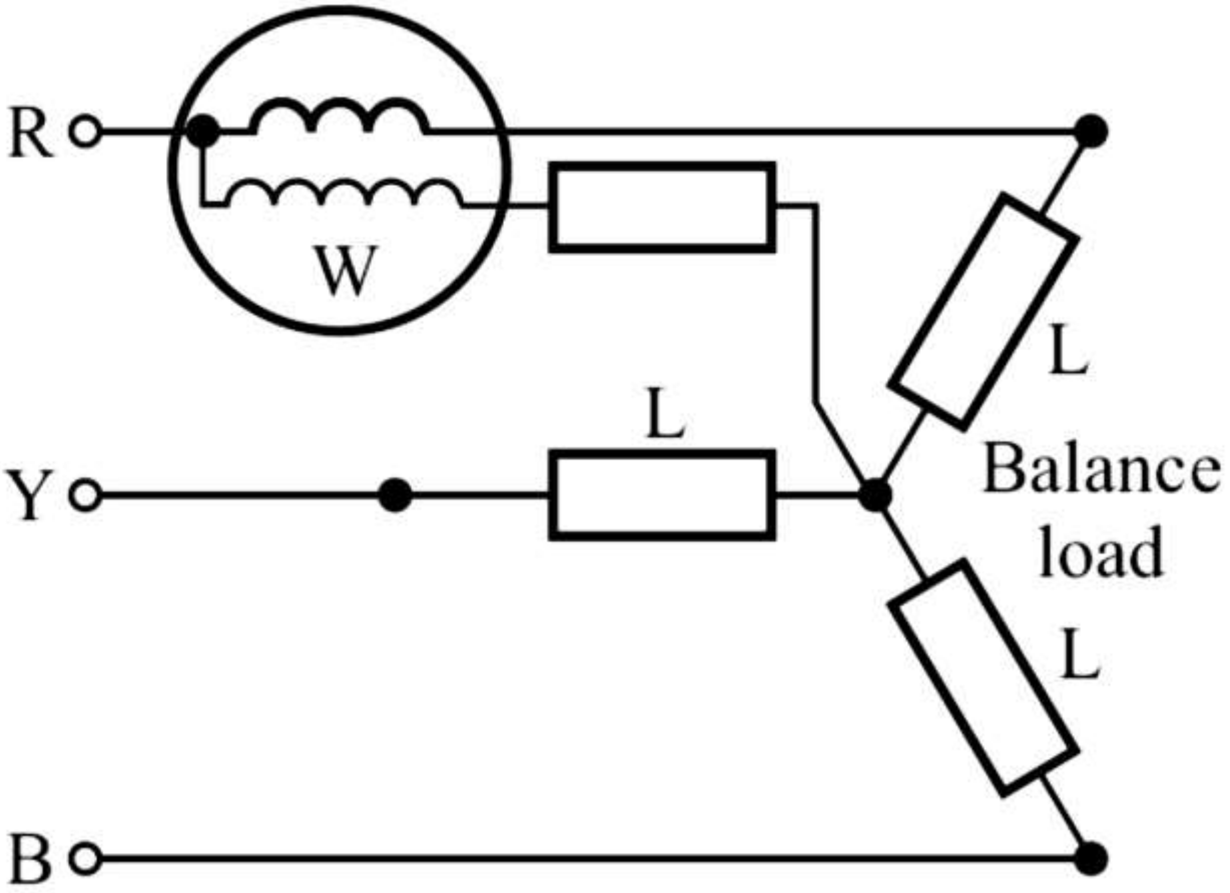
Next

Measurement of Power

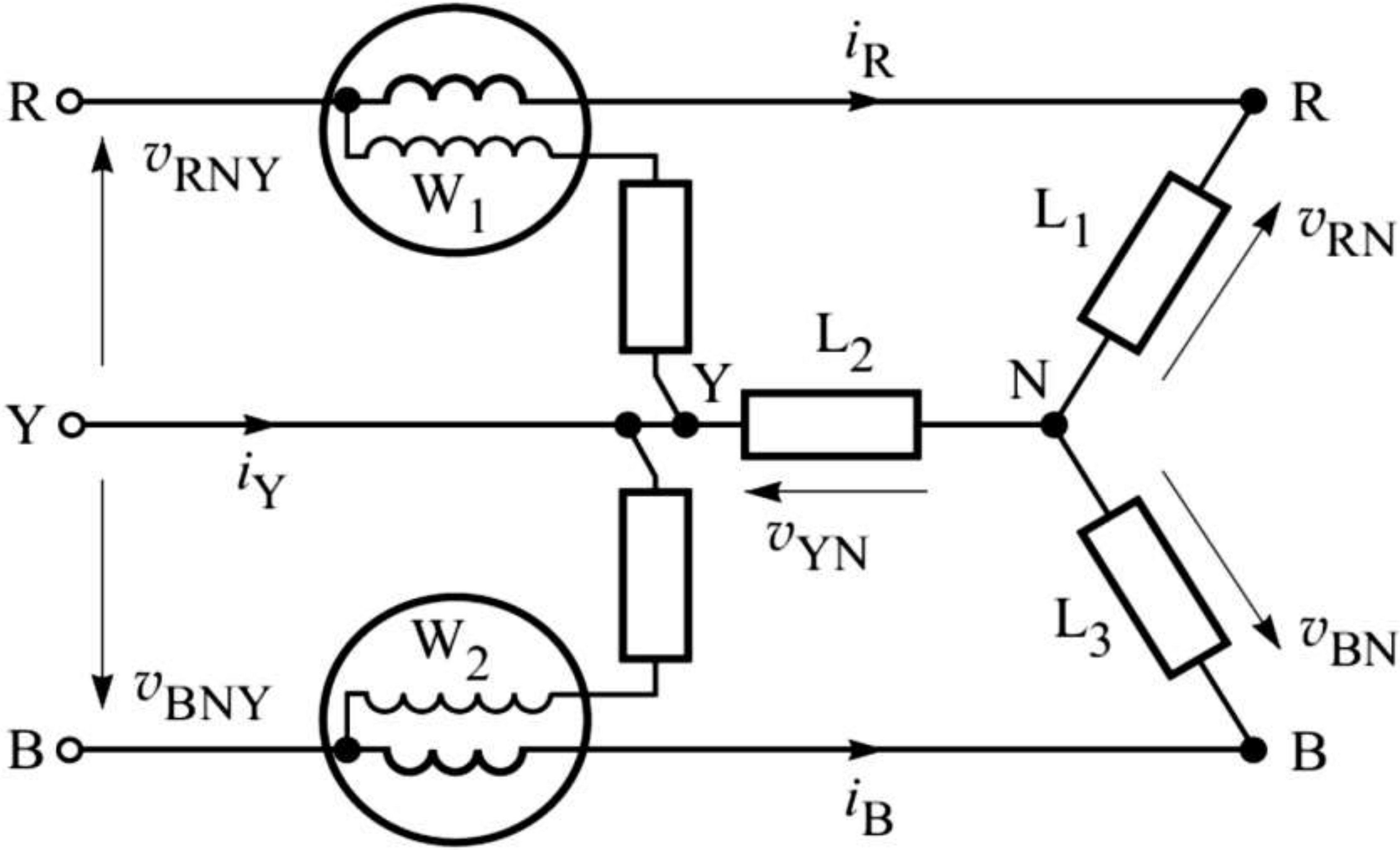
(i) Three-Wattmeter Method : This is simplest and straight forward method.

(ii)Two-Wattmeter Method : This can be used for any balanced or unbalanced load, star- or delta-connected.

(iii)One-Wattmeter Method : This can be used only for a star-connected balanced load.


R
W
Y
B
L
L
L
Balance
load

Two-Wattmeter Method



The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases

Proof : Total instantaneous power

$$= i_{\mathrm{R}} v_{\mathrm{RN}} + i_{\mathrm{Y}} v_{\mathrm{YN}} + i_{\mathrm{B}} v_{\mathrm{BN}}.$$

The instantaneous power measured by W_1,

$$p_1 = i_{\mathrm{R}} \left(v_{\mathrm{RN}} - v_{\mathrm{YN}} \right)$$

The instantaneous power measured by W_2,

$$p_2 = i_{\mathrm{B}} \left(v_{\mathrm{BN}} - v_{\mathrm{YN}} \right)$$

$$\therefore \quad p_1 + p_2 = i_R(v_{RN} - v_{YN}) + i_B(v_{BN} - v_{YN})$$

$$= i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN}$$

By KCL, $i_R + i_Y + i_B = 0 \Rightarrow (i_R + i_B) = -i_Y$

$$\therefore \quad p_1 + p_2 = i_R v_{RN} + i_B v_{BN} + i_Y v_{YN}$$

$$= \text{total instantaneous power}$$

Since we did not assume a balanced load or a sinusoidal waveform, it follows that the sum of the two wattmeter readings gives the total power under all conditions.

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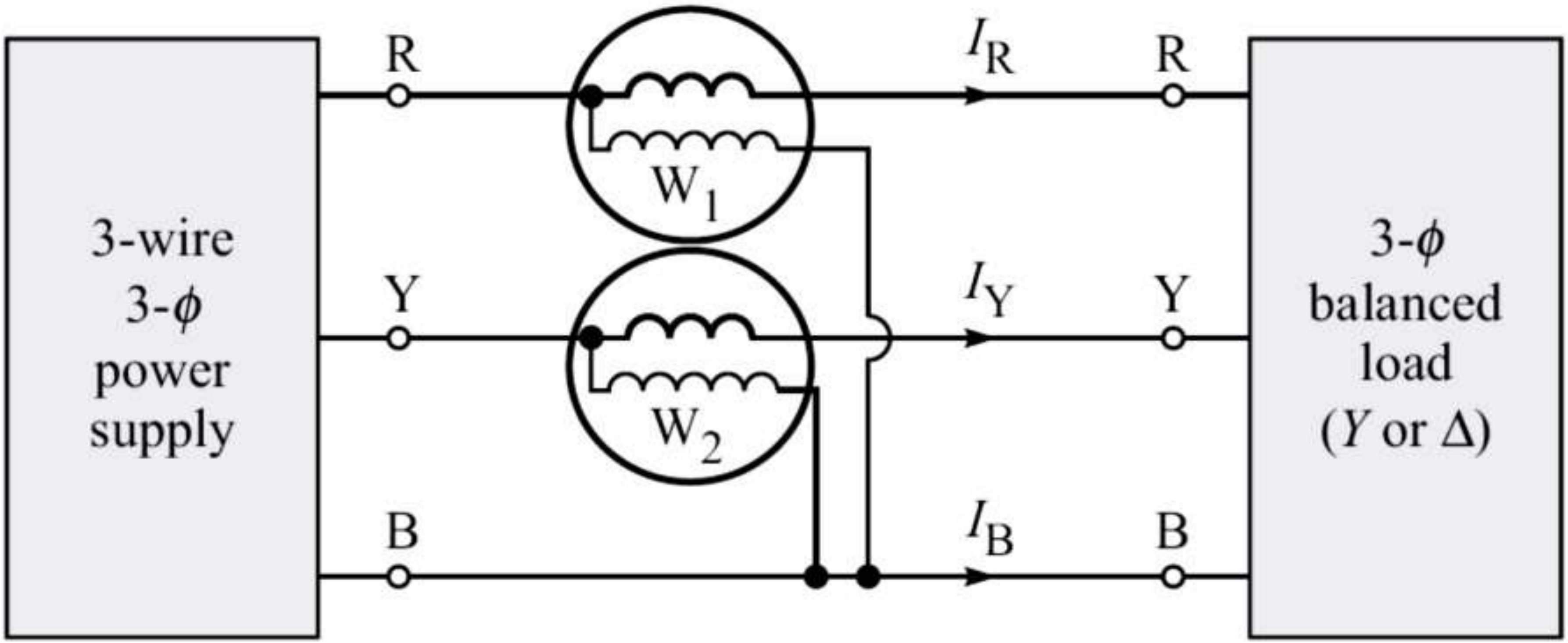
Power Factor Measurement by Two-Wattmeter Method

Concept of ‘power factor’ is meaningful only if the load is balanced.



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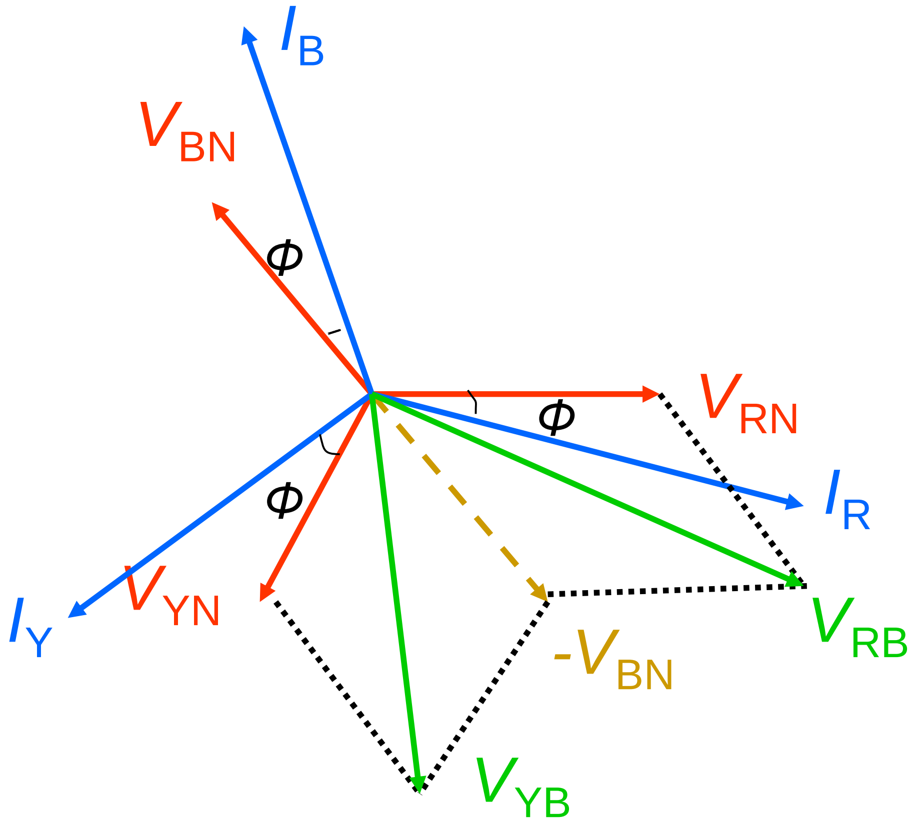
Since the load is balanced,

$$I_{\mathrm{R}} = I_{\mathrm{Y}} = I_{\mathrm{B}} = I_{\mathrm{L}} \text{ (say)}$$

$$\text{and} \quad V_{\mathrm{RN}} = V_{\mathrm{YN}} = V_{\mathrm{BN}} = V_{\mathrm{ph}} \text{ (say)}$$

Since $\mathbf{V}_{\mathrm{RB}} = \mathbf{V}_{\mathrm{RN}} - \mathbf{V}_{\mathrm{BN}}$, and $\mathbf{V}_{\mathrm{YB}} = \mathbf{V}_{\mathrm{YN}} - \mathbf{V}_{\mathrm{BN}}$, we can determine the line voltages V_{RB} and V_{YB} by phasor method,



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- It is seen that the line voltage V_{RB} lags the phase voltage V_{RN} by 30° and V_{YB} leads V_{YN} by 30°.
- Thus, the phase angle between V_{RB} and I_R is $(30° - \phi)$.
- Similarly, the phase angle between V_{YB} and I_Y is $(30° + \phi)$.
- Therefore, the readings of the two wattmeters are

$$P_1 = V_{\mathrm{RB}} I_{\mathrm{R}} \cos(30° - \phi) = V_{\mathrm{L}} I_{\mathrm{L}} \cos(30° - \phi)$$

Click

$$P_2 = V_{\mathrm{YB}} I_{\mathrm{Y}} \cos(30° + \phi) = V_{\mathrm{L}} I_{\mathrm{L}} \cos(30° + \phi)$$

Click

$$\therefore \quad \frac{P_1}{P_2} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

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By applying componendo and dividendo,

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$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\cos(30° - \phi) - \cos(30° + \phi)}{\cos(30° - \phi) + \cos(30° + \phi)}$$

$$= \frac{2\sin 30° \sin\phi}{2\cos 30° \cos\phi} = \tan 30° \tan\phi$$

Click

$$\therefore \quad \boxed{\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right]}$$

Calculate the phase angle ϕ, and then determine the power factor.

Next

Important Points :

- If $\Phi = 0°$, $P_1 = P_2$.
- If $\Phi < 60°$, both P_1 and P_2 are positive; and

$$P = P_1 + P_2$$

- If $\Phi = 60°$, P_2 is zero.
- If $\Phi > 60°$, (i.e., if $pf < 0.5$), P_2 is negative.

  In such case, the connection of either the current coil or the potential coil has to be reversed to make positive deflection.

  The value P_2 should then be taken as negative while calculating the power factor or the total power.

Example 3

- Two-wattmeter method was used to determine the input power to a three-phase motor. The readings were 5.2 kW and -1.7 kW, and the line voltage was 415 V. Calculate

  (*a*) the total power,

  (*b*) the power factor, and

  (*c*) the line current.

Solution :

Click

(*a*) The total power,

$$P = P_1 + P_2 = 5.2\text{ kW} - 1.7\text{ kW} = \mathbf{3.5\ kW}$$

Click

(*b*) $\tan\phi = \sqrt{3}\left[\dfrac{P_1 - P_2}{P_1 + P_2}\right] = \sqrt{3}\left[\dfrac{5.2-(-1.7)}{5.2+(-1.7)}\right] = 3.41$

$$\therefore \quad \phi = \tan^{-1} 3.41 = 73°39'$$

$$\therefore \quad pf = \cos\phi = \cos 73°39' = \mathbf{0.281}$$

Click

(*c*) $3500 = \sqrt{3} \times 415 \times I_L \times 0.281$

$$\Rightarrow \quad I_L = \mathbf{17.3\ A}$$

Next

Review

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Next

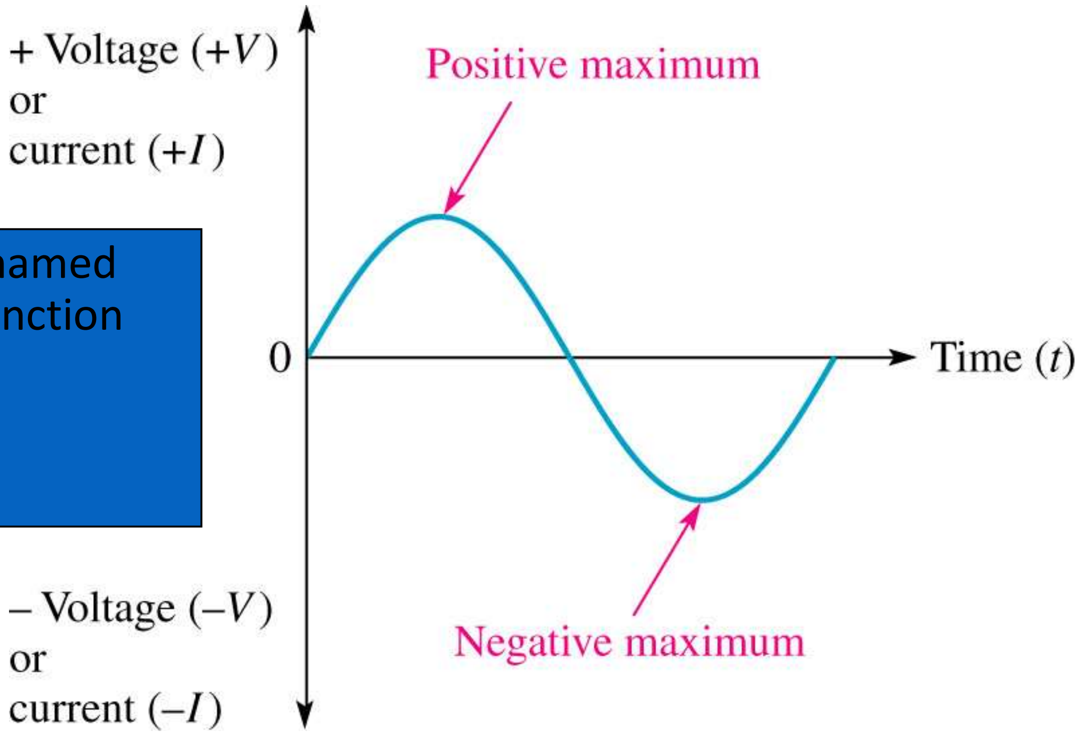
Principles of Electric Circuits

Summary

Sine waves

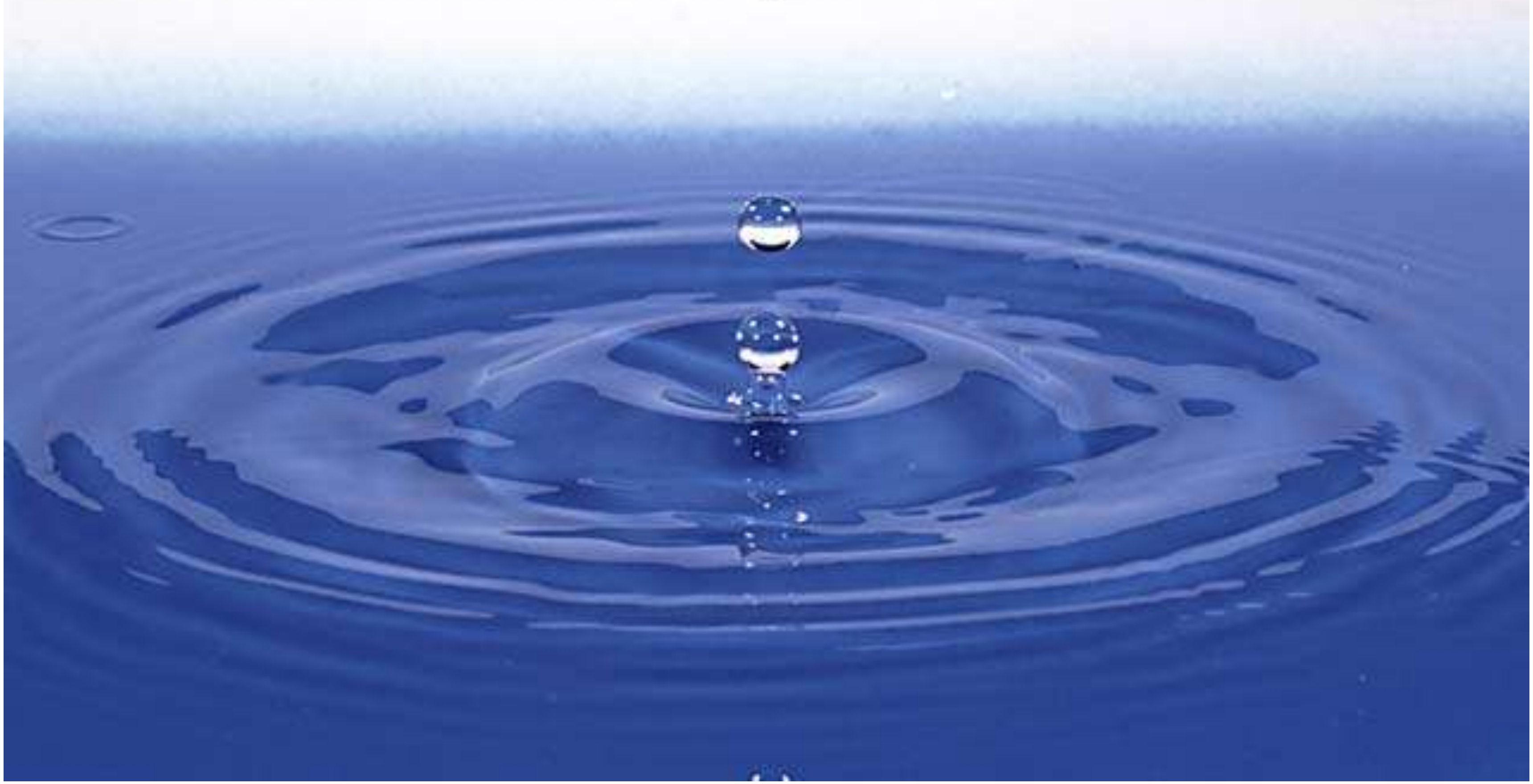
The sinusoidal waveform (sine wave) is the fundamental alternating current (ac) and alternating voltage waveform.

Electrical sine waves are named from the mathematical function with the same shape.



Summary

A wave is a disturbance. Unlike water waves, electrical waves cannot be seen directly but they have similar characteristics. All periodic waves can be constructed from **sine waves**, which is why sine waves are fundamental.



Summary

Sine waves

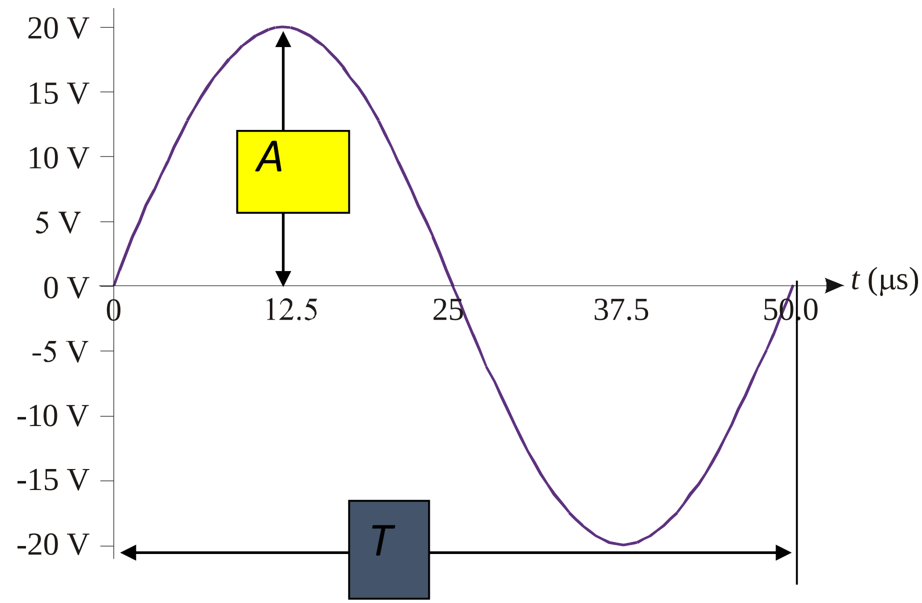
Sine waves are characterized by the amplitude and period. The **amplitude** is the maximum value of a voltage or current; the **period** is the time interval for one complete cycle.

Example

The amplitude (A) of this sine wave is

20 V

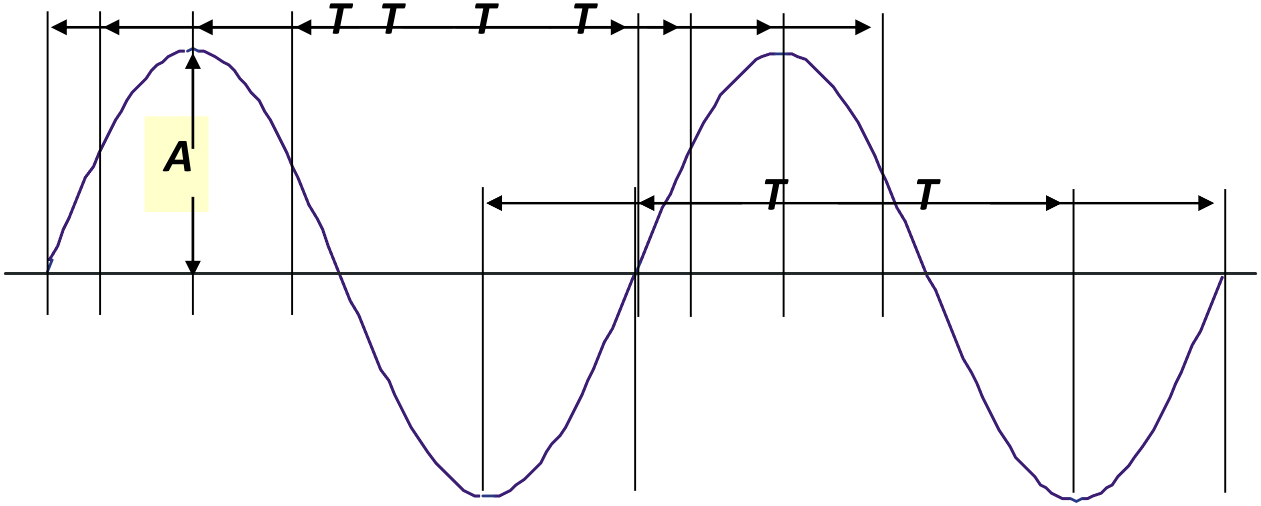
The period is 50.0 μs



Summary

Sine waves

The period of a sine wave can be measured between any two corresponding points on the waveform.



By contrast, the amplitude of a sine wave is only measured from the center to the maximum point.

Summary

Frequency

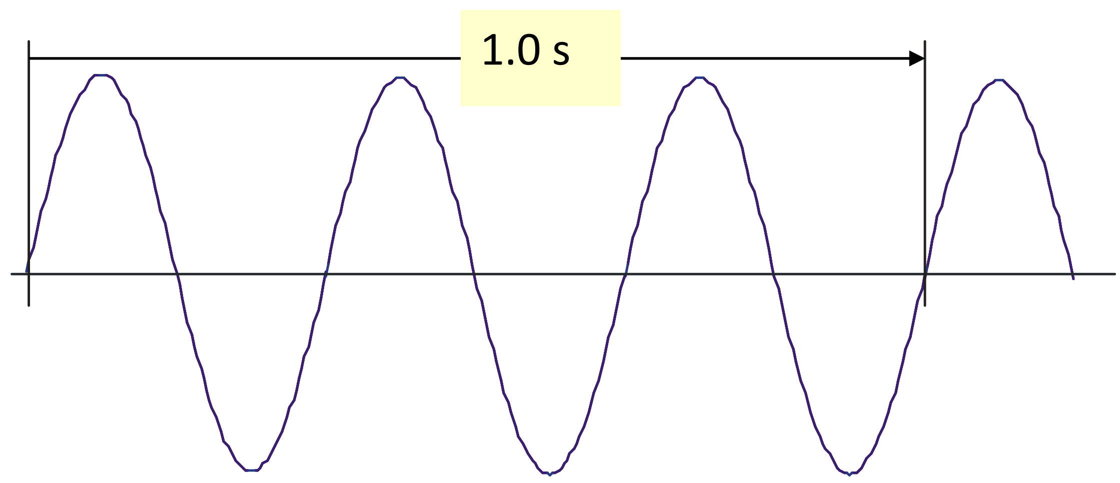
Frequency (f) is the number of cycles that a sine wave completes in one second.

Frequency is measured in **hertz** (Hz).

Example

If 3 cycles of a wave occur in one second, the frequency is

3.0 Hz



Summary

Period and frequency

The period and frequency are reciprocals of each other.

$$f = \frac{1}{T}$$

and

$$T = \frac{1}{f}$$

Thus, if you know one, you can easily find the other.

(The 1/*x* key on your calculator is handy for converting between *f* and *T*.)

Example

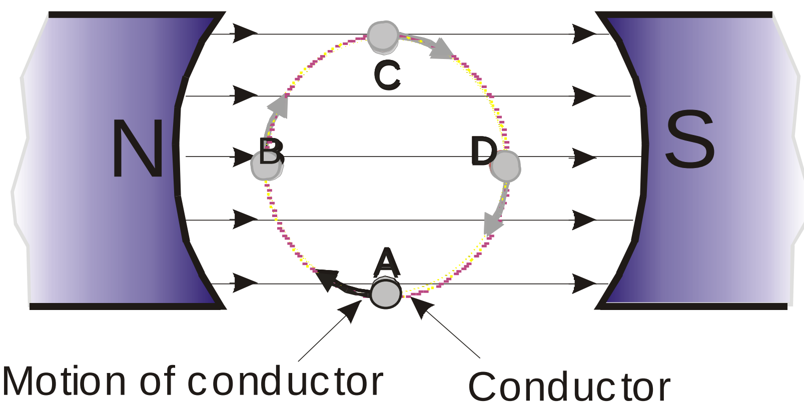
If the period is 50 μs, the frequency is 0.02 MHz = 20 kHz.

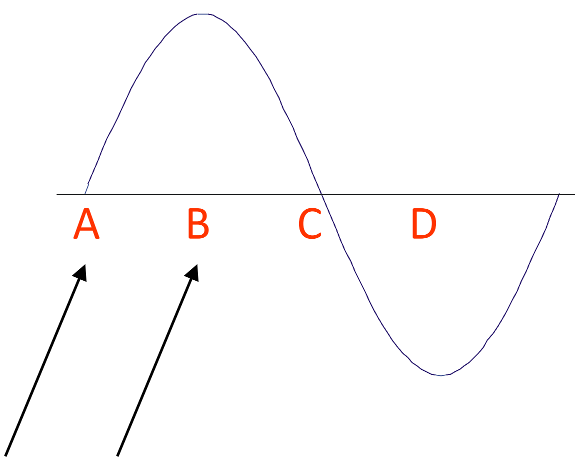
Summary

Generation of a sine wave

Sinusoidal voltages are produced by ac generators and electronic oscillators.

When a conductor rotates in a constant magnetic field, a sinusoidal wave is generated.





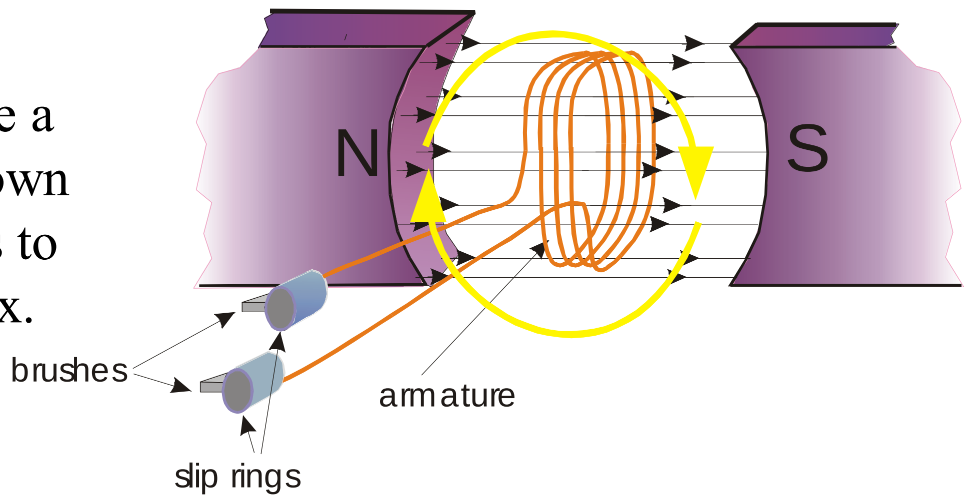
When the conductor is moving parallel with the lines of flux, no voltage is induced.

When the loop is moving perpendicular to the lines of flux, the maximum voltage is induced.

AC generator (alternator)

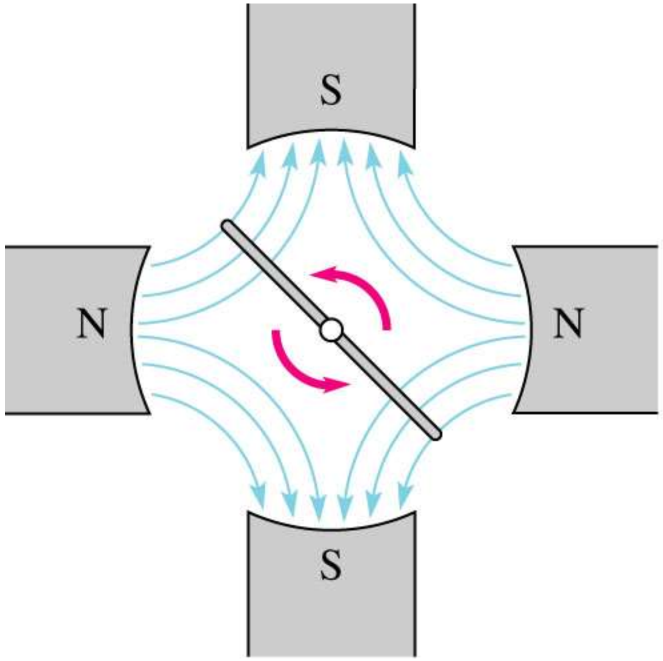
Generators convert rotational energy to electrical energy. A stationary field alternator with a rotating armature is shown. The armature has an induced voltage, which is connected through slip rings and brushes to a load. The armature loops are wound on a magnetic core (not shown for simplicity).

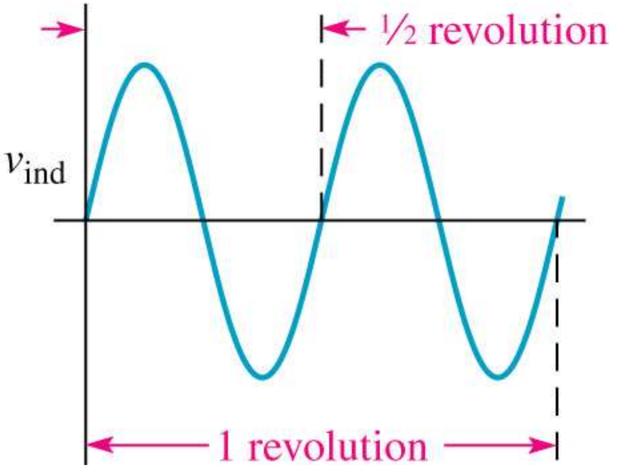
Small alternators may use a permanent magnet as shown here; other use field coils to produce the magnetic flux.



AC generator (alternator)

By increasing the number of poles, the number of cycles per revolution is increased. A four-pole generator will produce two complete cycles in each revolution.



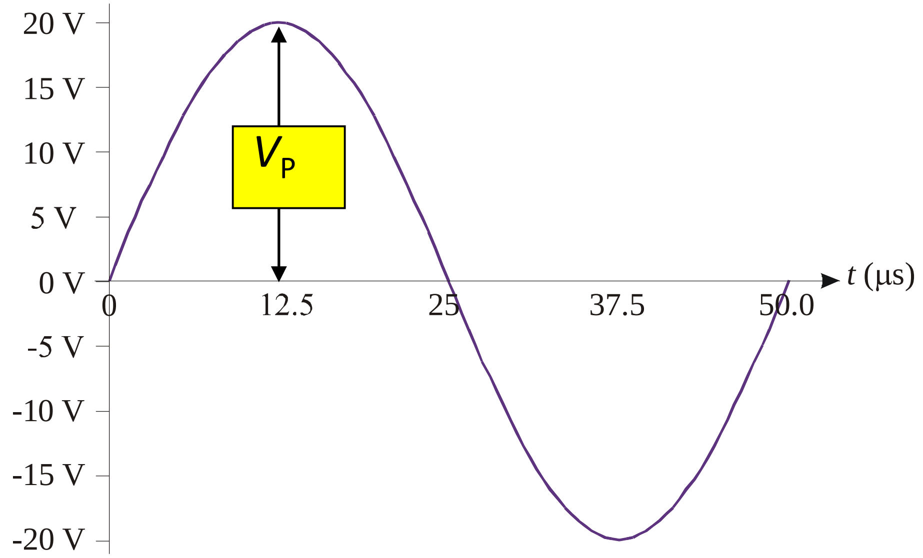


Sine wave voltage and current values

There are several ways to specify the voltage of a sinusoidal voltage waveform. The amplitude of a sine wave is also called the peak value, abbreviated as V_P for a voltage waveform.

Example

The peak voltage of this waveform is 20 V.



Sine wave voltage and current values

The voltage of a sine wave can also be specified as either the peak-to-peak or the rms value. The peak-to-peak is twice the peak value. The rms value is 0.707 times the peak value.

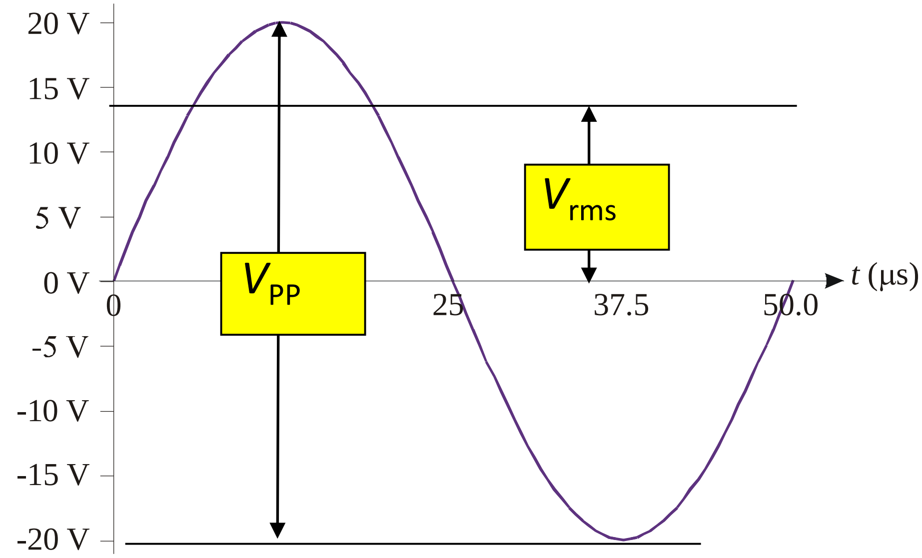
Example

The peak-to-peak voltage is

40 V.

The rms voltage is

14.1 V.



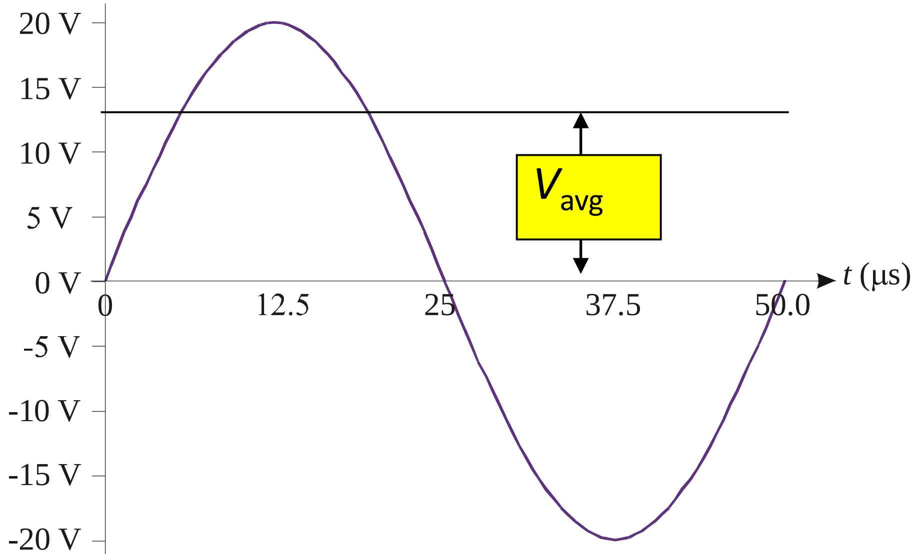
Sine wave voltage and current values

For some purposes, the average value (actually the half-wave average) is used to specify the voltage or current. By definition, the average value is as 0.637 times the peak value.

Example

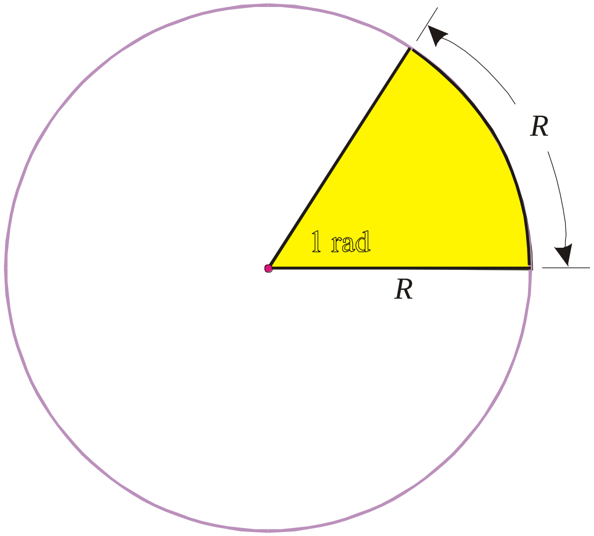
The average value for the sinusoidal voltage is

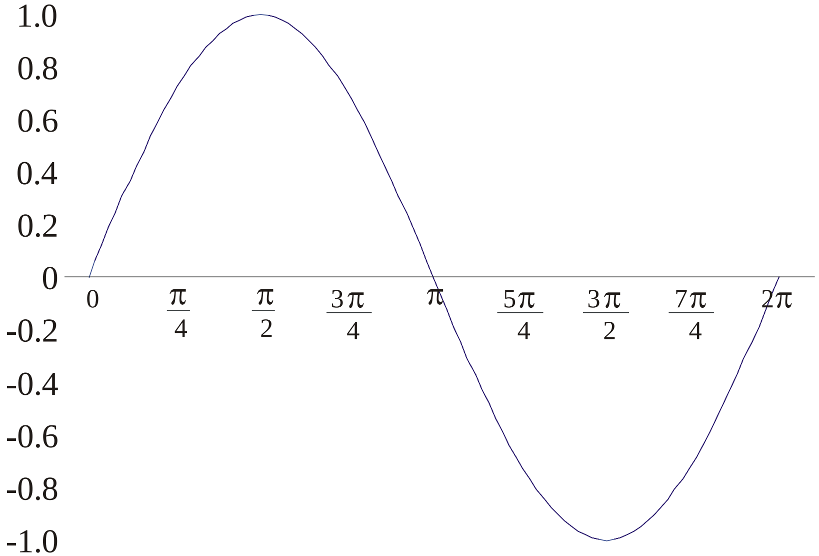
12.7 V.



Angular measurement

Angular measurements can be made in degrees (^{o}) or radians. The radian (rad) is the angle that is formed when the arc is equal to the radius of a circle. There are 360^o or 2π radians in one complete revolution.





Angular measurement

Because there are 2π radians in one complete revolution and 360^o in a revolution, the conversion between radians and degrees is easy to write. To find the number of radians, given the number of degrees:

$$\boxed{\text{rad} = \frac{2\pi \text{ rad}}{360^\circ} \times \text{degrees}}$$

To find the number of degrees, given the radians:

$$\boxed{\text{deg} = \frac{360^\circ}{2\pi \text{ rad}} \times \text{rad}}$$

Sine wave equation

Instantaneous values of a wave are shown as v or i. The equation for the instantaneous voltage (v) of a sine wave is

$$v = V_p \sin\theta$$

where

V_p = Peak voltage

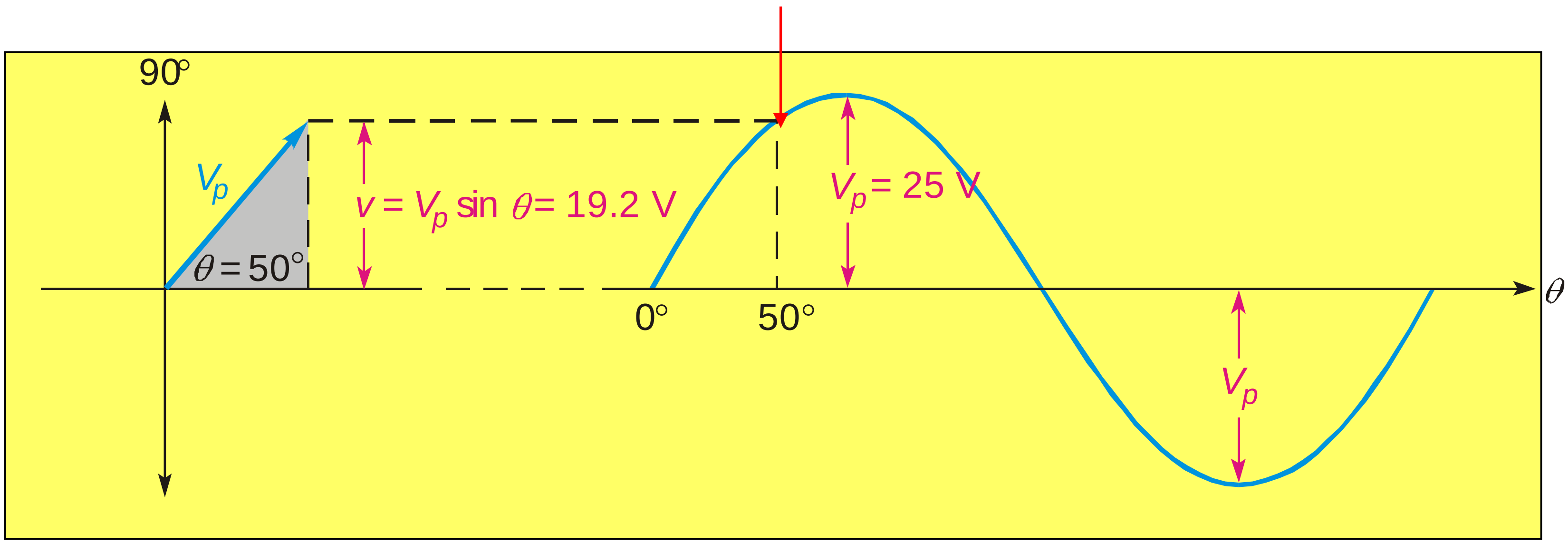
θ = Angle in rad or degrees

Example

If the peak voltage is 25 V, the instantaneous voltage at 50 degrees is 19.2 V

Sine wave equation

A plot of the example in the previous slide (peak at 25 V) is shown.
The instantaneous voltage at 50° is 19.2 V as previously calculated.



Phase shift

The phase of a sine wave is an angular measurement that specifies the position of a sine wave relative to a reference. To show that a sine wave is shifted to the left or right of this reference, a term is added to the equation given previously.

$$\boxed{v = V_{\mathrm{P}} \sin(\theta \pm \phi)}$$

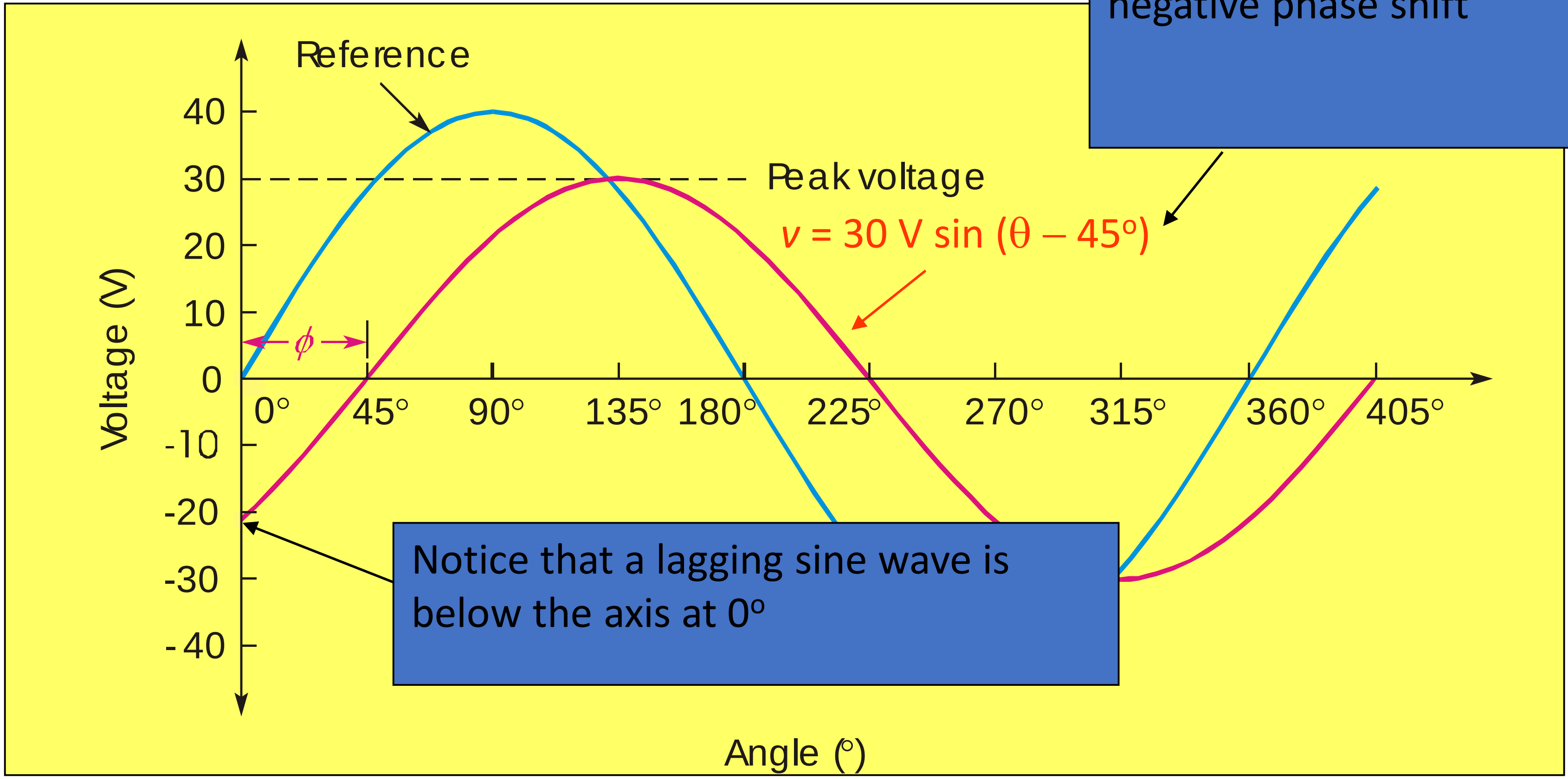
where

ϕ = Phase shift

Phase shift

Example of a wave that lags the reference

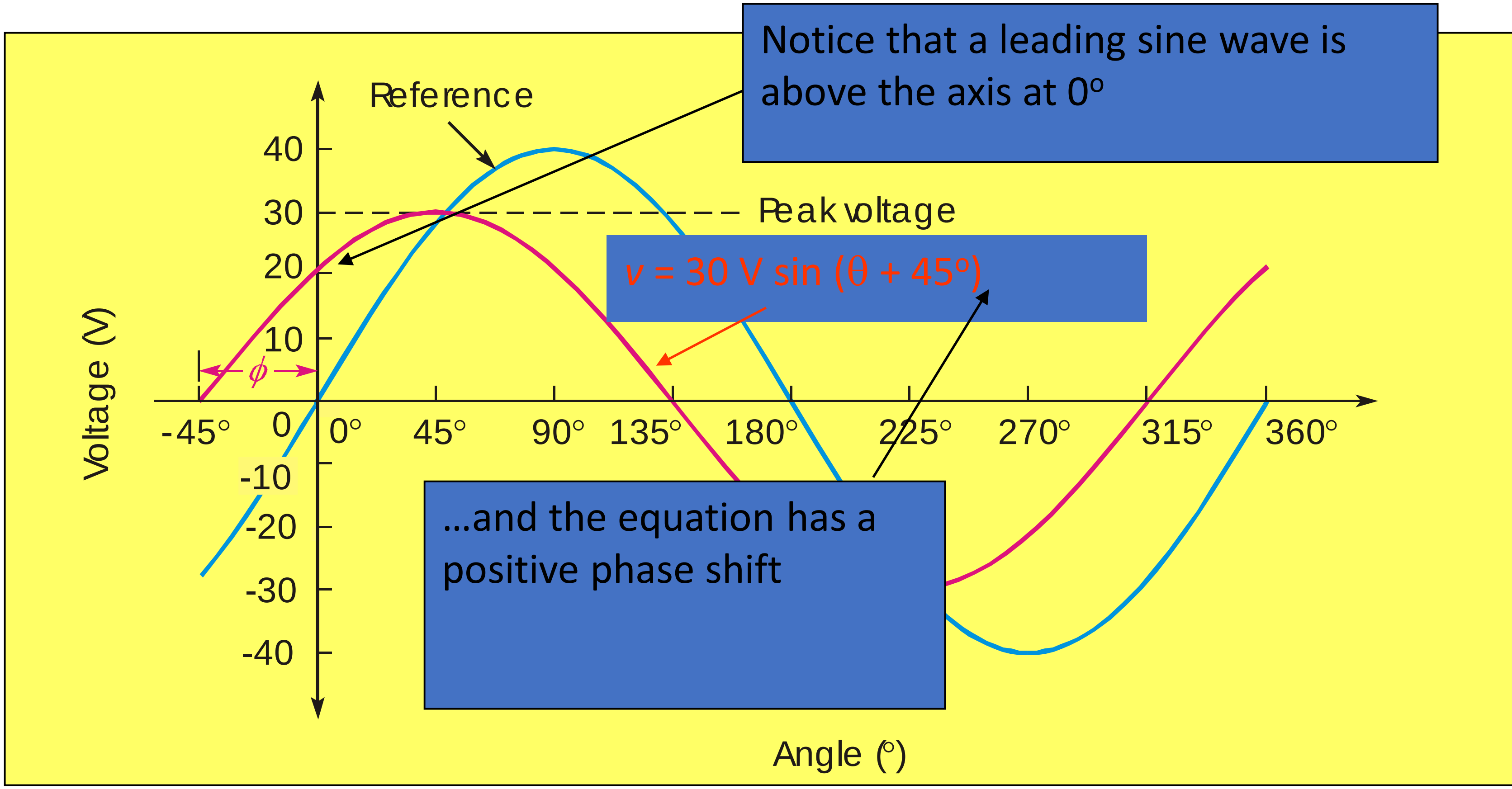
…and the equation has a negative phase shift



Notice that a lagging sine wave is below the axis at 0°

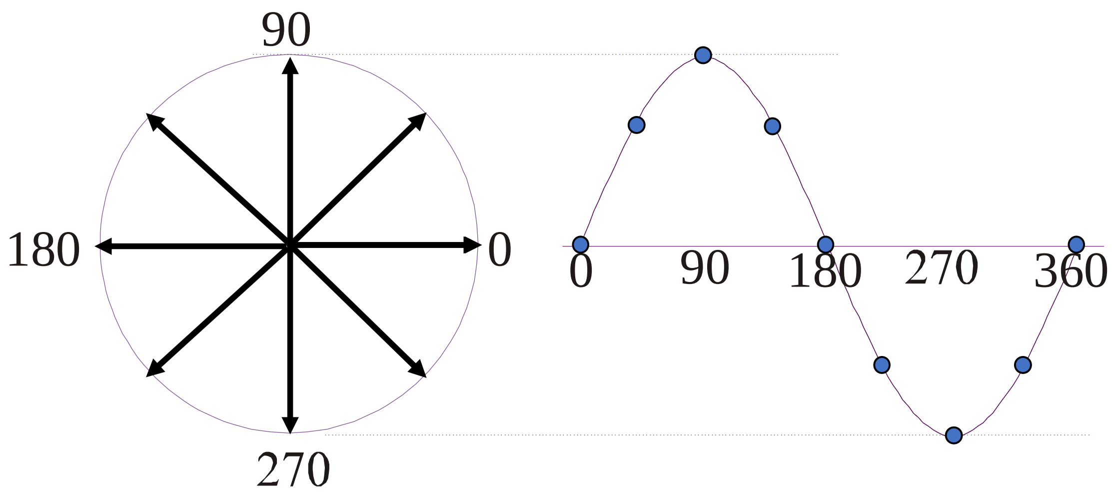
Phase shift

Example of a wave that leads the reference



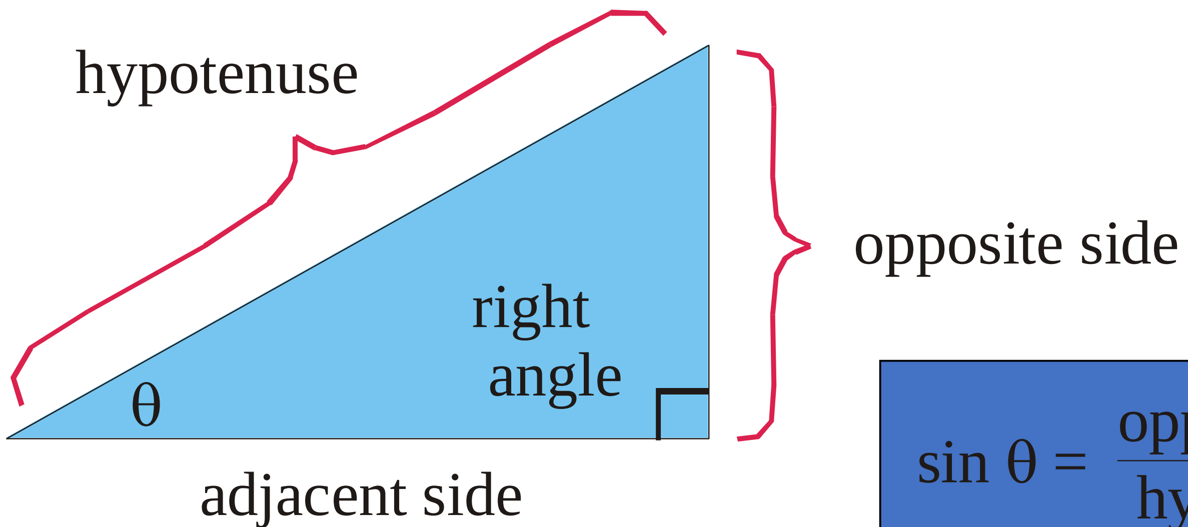
Phasors

The sine wave can be represented as the projection of a vector rotating at a constant rate. This rotating vector is called a **phasor**.



Phasors

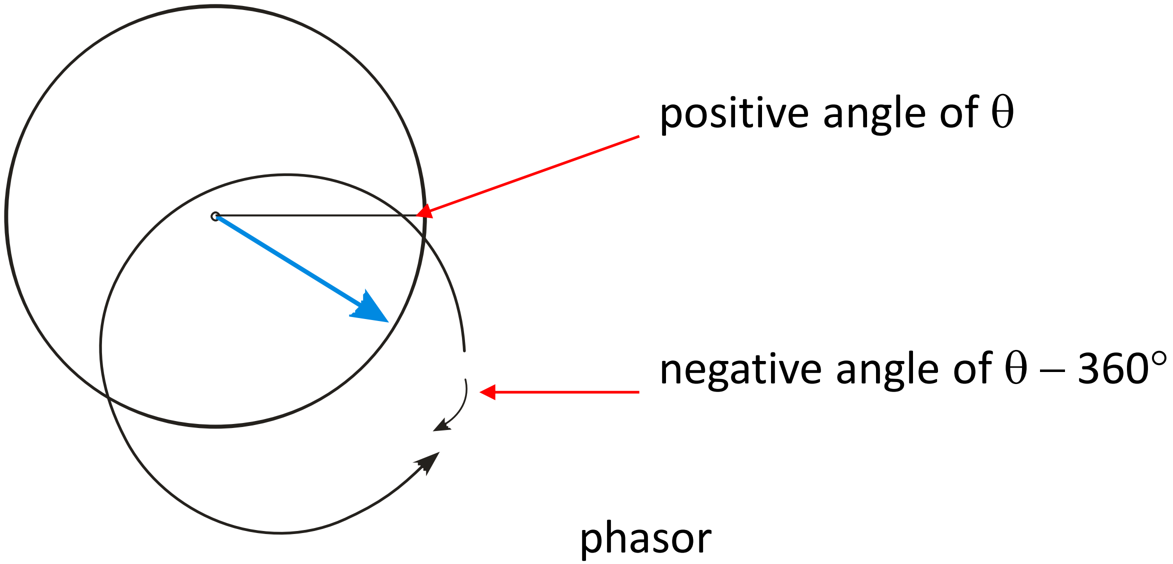
Phasors allow ac calculations to use basic trigonometry. The sine function in trigonometry is the ratio of the opposite side of a right triangle to the adjacent side.



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Phasors

The position of a phasor at any instant can be expressed as a positive angle, measured counterclockwise from 0° or as a negative angle equal to $\theta - 360°$.



Angular velocity of a phasor

When a phasor rotates through 360° or 2π radians, one complete cycle is traced out.
The velocity of rotation is called the **angular velocity** (ω).

$$\boxed{\omega = 2\pi f}$$

(Note that this angular velocity is expressed in radians per second.)

The instantaneous voltage at any point in time is given by

$$\boxed{v = V_p \sin 2\pi f}$$

Quiz

1. In North America, the frequency of ac utility voltage is 60 Hz. The period is

   a. 8.3 ms

   b. 16.7 ms

   c. 60 ms

   d. 60 s

Quiz

2. The amplitude of a sine wave is measured

a. at the maximum point

b. between the minimum and maximum points

c. at the midpoint

d. anywhere on the wave

Quiz

3. An example of an equation for a waveform that lags the reference is

a. $v = -40\text{ V} \sin(\theta)$

b. $v = 100\text{ V} \sin(\theta + 35^{\circ})$

c. $v = 5.0\text{ V} \sin(\theta - 27^{\circ})$

d. $v = 27\text{ V}$

Quiz

4. In the equation $v = V_p \sin \theta$, the letter v stands for the

a. peak value

b. average value

c. rms value

d. instantaneous value

Quiz

5. A sawtooth waveform has

a. equal positive and negative going ramps

b. two ramps - one much longer than the other

c. two equal pulses

d. two unequal pulses

Quiz

6. The number of radians in 90^o are

a. $\pi/2$

b. π

c. $2\pi/3$

d. 2π

Quiz

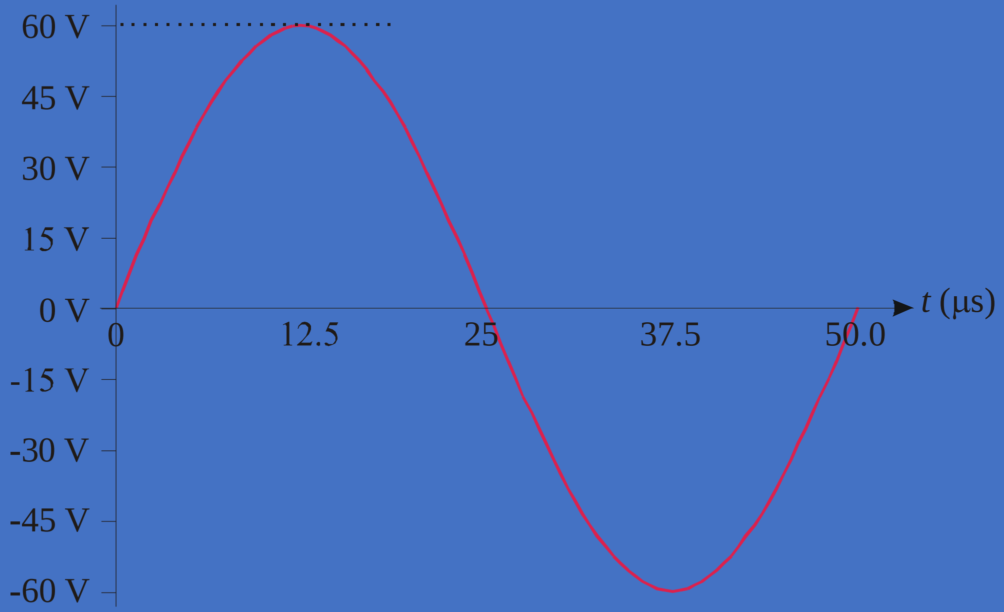
7. For the waveform shown, the same power would be delivered to a load with a dc voltage of

a. 21.2 V

b. 37.8 V

c. 42.4 V

d. 60.0 V



Quiz

Answers:

1. b
2. a
3. c
4. d
5. b
6. a
7. c