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Page

★ Assignment - 1 (Numerical Method)

1. The velocity v of falling parachutist is given by $v = \frac{gm}{c}(1 - e^{-(c/m)t})$

where $g = 9.8\ m/s^2$. $c = 15\ kg/s$, $v = 35\ m/s$ at $t = 9s$. Use False position Method to determine m to level of 0.1%.

$$35 = \frac{9.8m}{15}(1 - e^{-(15/m)\times 9})$$

⇒ let $m = x$

$$f(x) = 9.8x - 9.8x\, e^{-(15/x)\times 9} - 525 = 0$$

∴ Roots lie between 53 & 54

$f(53) = -5.600$ $\quad$ $f(54) = 4.200$

Using Regula-Falsi Method, we get

No. of iterations	a	b	f(a)	f(b)	x_n
1	53	54	-5.600	4.2	53.571
2	53.571	54	-0.004	4.2	53.571

Hence, the root for system is approximately

$$\boxed{x = 53.571}$$

2. The concentration of pollutant bacteria c in a lake decreases according to

$$c = 75e^{-1.5t} + 20e^{-0.075t}$$

Determine the time required for bacteria $c = 15$ using N-R method with $t = 6$ & stopping criterion of 0.5%.

⇒ & let $t = x$

$\therefore f(x) = 75e^{-1.5t} + 20e^{-0.075t} - 15$ ⇄

Now, $f'(x) = -112.5e^{-1.5t} - 1.5e^{-0.075t}$

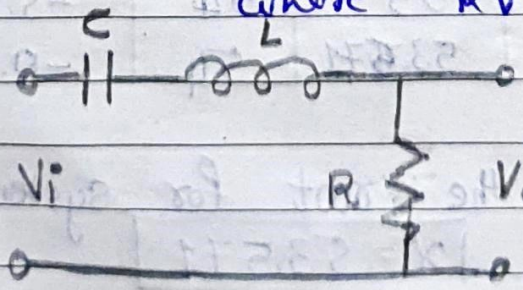
Now using N-R method we can solve it.

3. A band pass filter pases signals with frequences that are certain range.

$$RV = \left|\frac{V_o}{V_i}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$\therefore R = 1000$ $C = 8\,\mu F$

$L = 11\,mH$ where $RV \geq 0.87$



$\Rightarrow$ let $w = x$

After solving eq. we get,

$$f(x) = (7.74 \times 10^{-15})x^4 - (9.760 \times 10^{-6})x^2 + 1$$

$\therefore$ Then by using bisection method get approx. equal roots of it.

4 The lateral surface area, S of a cone is given by:

$$S = \pi r \sqrt{r^2 + h^2}$$

where r is the radius of the base & h is height. Surface Area is $1800 m^2$ & $h = 25m$. Solve by using N-R method. with

$$r = \frac{S}{(\pi \sqrt{r^2 + h^2})}$$

. Start with $r = 17m$.

$\Rightarrow$ let $r = x$, After solving eq. we get,

$\therefore f(x) = \pi(x)\sqrt{x^2 + 625} - 1800$

Then, the $f'(x)$ is

$f'(x) = 2\pi^2(2x^3 + x)$

$\therefore$ Hence, using N-R method we get approx equal roots of it.

5. The location of $\bar{x}$ of the centroid of an arc of a circle is $\bar{x} = \frac{r \sin\alpha}{\alpha}$

$$\bar{x} = \frac{3r}{4}$$

(a) Use bisection Method, a=0.5 b=1.5, first four iterations.

⟹ Let $\alpha = x$

$f(x) = 4\sin x - 3x$

∴ Roots lie between 0.5 & 1.5

Using Bisection Method we get,

No. of iterations	(-) a	(+) b	f(a)	f(b)	x_n
1	0.5	1.5	<0	>0	1
2	1	1.5	<0	>0	1.25
3	1.25	1.5	<0	>0	1.375
4	1.25	1.375	<0	>0	1.313

Hence, upto 4 iterations we get approx. root

$\boxed{x = 1.313}$

(b) Use secant method, a=0.5 & b=1.5, 1st four iterations.

⟹ $f(x) = 4\sin x - 3x$

Roots lie between 0.5 & 1.5

Using Secant Method, we get

No. of iterations	a	b	f(a)	f(b)	x_n
1	0.5	1.5	0.418	-0.510	0.950
2	0.950	1.5	0.404	-0.51	1.193
3	0.95	1.193	0.401	0.139	1.322
4	1.322	1.193	-0.089	0.139	1.272

Hence, upto 4 iterations we get approx. root $\boxed{x = 1.272}$

6. An icecream drum is made of a waffle cone filled with ice cream such that above cone forms a spherical cap. Volume is

$$V = \pi\left(\frac{r^2 h}{3} + \frac{r^2 H}{2} + \frac{H^3}{6}\right)$$

V

$V = \frac{1}{3}$ U.S pint (1 U.S pint = 28.875 in^3), $h = 4$ in, $r = 1.1$ in.

$\Rightarrow$ Let $H = x$ By solving eq. we get,

$f(x) = 0.525x^3 + 1.901x - 4.558$

$\therefore$ Roots lie between 1 & 2.

By using Second Method we get,

No. of iterations	a	b	f(a)	f(b)	x_n
1	1	2	-2.132	3.444	1.382
2	1.382	2	-0.545	3.444	1.467
3	1.382	1.467	-0.545	-0.112	1.490
4	1.490	1.487	0.011	-0.112	1.488
5	1.490	1.488	0.011	0	1.488

Hence, the given system has equal approx. root at $x = 1.488$

Now by using N-R Method we get,

$f(x) = 0.525H^3 + 1.901H - 4.558$

$f'(x) = 1.575H^2 + 1.901$

No. of iterations	a	f(x)	f'(x)	$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$
1	1	-2.132	2.994	1.712
2	1.712	1.331	6.517	1.508
3	1.508	0.109	5.483	1.488
4	1.488	0	5.388	1.488

Hence the given system has equal approx. root at $x = 1.488$.