Digital Logic Design
Chapter 2

Boolean Algebra and Logic Gate

2.2 BASIC DEFINITIONS

- A set is collection of elements having the same property.
  - S: set, x and y: element or event
  - For example: $S = \{1, 2, 3, 4\}$
    - If $x = 2$, then $x \in S$.
    - If $y = 5$, then $y \notin S$.
- A binary operator defines on a set S of elements is a <u>rule</u> that assigns, to each pair of elements from S, a unique element from S.
  - For example: given a set S, consider $\boldsymbol{a*b = c}$ and * is a binary operator.
  - If (a, b) through * get $\boldsymbol{c}$ and $\boldsymbol{a, b, c \in S}$, then * is a binary operator of S.
  - On the other hand, if * is not a binary operator of S and $\boldsymbol{a, b \in S}$, then $\boldsymbol{c \notin S}$.

2.1 Algebras

- **What is an algebra?**
  - Mathematical system consisting of
    - Set of elements (example: $N = \{1,2,3,4,\ldots\}$)
    - Set of operators (+, -, $\times$, $\div$)
    - Axioms or postulates (associativity, distributivity, closure, identity elements, etc.)
- **Why is it important?**
  - Defines rules of "calculations"
- **Note: operators with two inputs are called *binary***
  - Does not mean they are restricted to binary numbers!
  - Operator(s) with one input are called *unary*

BASIC DEFINITIONS

- The common postulates used to formulate algebraic structures are:

1. **Closure**: a set S is closed with respect to a binary operator if, for every pair of elements of S, the binary operator specifies a rule for obtaining a unique element of S.
   - ***For example, natural numbers $N=\{1,2,3,...\}$ is closed w.r.t. the binary operator + by the rule of arithmetic addition, since, for any $a, b \in N$, there is a unique $c \in N$ such that***
     - $a+b = c$
     - **But operator – is not closed for N, because 2-3 = -1 and 2, 3 $\in N$, but $(-1) \notin N$.**
2. **Associative law**: a binary operator * on a set S is said to be associative whenever
   - $(x * y) * z = x * (y * z)$ **for all** $x, y, z \in S$
     - $(x+y)+z = x+(y+z)$
3. **Commutative law**: a binary operator * on a set S is said to be commutative whenever
   - $x * y = y * x$ **for all** $x, y \in S$
     - $x+y = y+x$

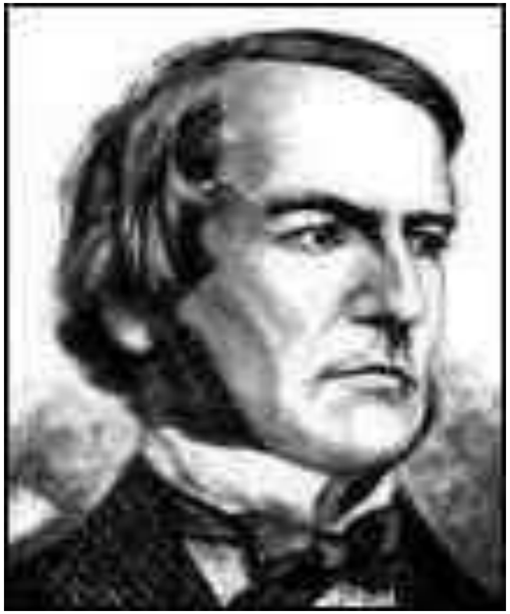
BASIC DEFINITIONS

4. ***Identity element***: a set S is said to have an identity element with respect to a binary operation * on S if there exists an element $e \in S$ with the property that
   - $e * x = x * e = x$ **for every** $x \in S$
     - » $0+x = x+0 = x$ **for every** $x \in I$ $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
     - » $1 \times x = x \times 1 = x$ **for every** $x \in I$ $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
5. ***Inverse***: a set having the identity element **e** with respect to the binary operator to have an inverse whenever, for every $x \in S$, there exists an element $y \in S$ such that
   - $x * y = e$
     - » **The operator + over *I*, with *e* = 0, the inverse of an element *a* is (-*a*), since *a*+(-*a*) = 0.**
6. ***Distributive law***: if (*) and (.) are two binary operators on a set S, (*) is said to be distributive over (.) whenever
   - $x * (y.z) = (x * y).(x * z)$

George Boole

Father of Boolean algebra

- He came up with a type of linguistic algebra, the three most basic operations of which were (and still are) **AND, OR and NOT**. It was these three functions that formed the basis of his premise, and were the only operations necessary to perform comparisons or basic mathematical functions.
- Boole's system was based on a binary approach, **processing only two objects - the yes-no, true-false, on-off, zero-one approach**.
- Surprisingly, given his standing in the academic community, Boole's idea was either criticized or completely ignored by the majority of his peers.
- Eventually, one bright student, claude shunnon(1916-2001), picked up the idea and ran with it



George Boole (1815 - 1864)

2.3 Axiomatic Definition of Boolean Algebra

- We need to define algebra for binary values
  - Developed by George Boole in 1854
- Huntington postulates (1904) for Boolean algebra :
- $B = \{0, 1\}$ and two binary operations, (+) and (.)
  - Closure with respect to operator (+) and operator (.)
  - Identity element 0 for operator (+) and 1 for operator (.)
  - Commutativity with respect to (+) and (.)

$$x+y = y+x, \quad x \cdot y = y \cdot x$$

  - Distributivity of (.) over (+), and (+) over (.)

$$x \cdot (y+z) = (x \cdot y)+(x \cdot z) \quad \text{and} \quad x+(y \cdot z) = (x+y) \cdot (x+z)$$

    - **Complement for every element x is x' with $x+x'=1$, $x \cdot x'=0$**
  - There are at least two elements $x,y \in B$ such that $x \neq y$

Boolean Algebra

- Terminology:
  - *Literal:* A variable or its complement
  - *Product term:* literals connected by (·)
  - *Sum term:* literals connected by (+)

Postulates of Two-Valued Boolean Algebra

- $B = \{0, 1\}$ and two binary operations, (+) and (.)
- The rules of operations: AND、OR and NOT.

AND

x	y	$X.y$
0	0	0
0	1	0
1	0	0
1	1	1

OR

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT

x	X'
0	1
1	0

1. Closure (+ and·)
2. The identity elements

   (1) + = 0

   (2) • = 1

Postulates of Two-Valued Boolean Algebra

3. The commutative laws $x+y = y+x, \quad x.y = y.x$
4. The distributive laws

x	y	z	$y+z$	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y)+(x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Postulates of Two-Valued Boolean Algebra

5. Complement
   - $x+x'=1 \rightarrow 0+0'=0+1=1;\ 1+1'=1+0=1$
   - $x \cdot x'=0 \rightarrow 0 \cdot 0'=0 \cdot 1=0;\ 1 \cdot 1'=1 \cdot 0=0$
6. Has two distinct elements 1 and 0, with $0 \neq 1$

- Note
  - A set of two elements
  - (+) : OR operation; (·) : AND operation
  - A complement operator: NOT operation
  - Binary logic is a two-valued Boolean algebra

Duality

- The principle of *duality* is an important concept. This says that if an expression is valid in Boolean algebra, the dual of that expression is also valid.
- To form the dual of an expression, replace all (+) operators with (·) operators, all (·) operators with (+) operators, all ones with zeros, and all zeros with ones.
- Following the replacement rules…

  a(b + c) = ab + ac
- Form the dual of the expression

  a + (bc) = (a + b)(a + c)
- Take care not to alter the location of the parentheses if they are present.

Basic Theorems

Table 2.1
Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Boolean Theorems

- Huntington's postulates define some rules

Post. 1:	closure
Post. 2:	(a) $x+0=x$, (b) $x \cdot 1=x$
Post. 3:	(a) $x+y=y+x$, (b) $x \cdot y=y \cdot x$
Post. 4:	(a) $x(y+z) = xy+xz$,
	(b) $x+yz = (x+y)(x+z)$
Post. 5:	(a) $x+x'=1$, (b) $x \cdot x'=0$

- Need more rules to modify algebraic expressions
  - Theorems that are derived from postulates
- What is a theorem?
  - A formula or statement that is derived from postulates (or other proven theorems)
- Basic theorems of Boolean algebra
  - Theorem 1 (a): $x + x = x$ (b): $x \cdot x = x$
  - Looks straightforward, but needs to be proven !

Proof of $x+x=x$

- We can only use Huntington postulates:

Huntington postulates:

Post. 2: (a) $x+0=x$, (b) $x \cdot 1=x$
Post. 3: (a) $x+y=y+x$, (b) $x \cdot y=y \cdot x$
Post. 4: (a) $x(y+z) = xy+xz$,
(b) $x+yz = (x+y)(x+z)$
Post. 5: (a) $x+x'=1$, (b) $x \cdot x'=0$

- Show that $x+x=x$.

$x+x$	$= (x+x)\cdot 1$	by 2(b)
	$= (x+x)(x+x')$	by 5(a)
	$= x+xx'$	by 4(b)
	$= x+0$	by 5(b)
	$= x$	by 2(a)
	Q.E.D.	

- We can now use Theorem 1(a) in future proofs

Proof of $x \cdot x = x$

- Similar to previous proof

Huntington postulates:

Post. 2: (a) $x+0=x$, (b) $x \cdot 1=x$
Post. 3: (a) $x+y=y+x$, (b) $x \cdot y=y \cdot x$
Post. 4: (a) $x(y+z) = xy+xz$,
(b) $x+yz = (x+y)(x+z)$
Post. 5: (a) $x+x'=1$, (b) $x \cdot x'=0$
Th. 1: (a) $x+x=x$

- Show that $x \cdot x = x$.

$x \cdot x = xx+0$ by 2(a)

$= xx+xx'$ by 5(b)

$= x(x+x')$ by 4(a)

$= x \cdot 1$ by 5(a)

$= x$ by 2(b)

Q.E.D.

Proof of x+1=1

- Theorem 2(a): $x + 1 = 1$

  $x + 1 = 1 \cdot (x + 1)$ by 2(b)

  $=(x + x')(x + 1)$ 5(a)

  $= x + x' 1$ 4(b)

  $= x + x'$ 2(b)

  $= 1$ 5(a)

<u>Huntington postulates</u>:

Post. 2: (a) $x+0=x$, (b) $x\cdot 1=x$
Post. 3: (a) $x+y=y+x$, (b) $x\cdot y=y\cdot x$
Post. 4: (a) $x(y+z) = xy+xz$,
(b) $x+yz = (x+y)(x+z)$
Post. 5: (a) $x+x'=1$, (b) $x\cdot x'=0$
Th. 1: (a) $x+x=x$

- Theorem 2(b): $x \cdot 0 = 0$ by duality
- Theorem 3: $(x')' = x$
  - Postulate 5 defines the complement of x, $x + x' = 1$ and $x x' = 0$
  - The complement of x' is x is also $(x')'$

Absorption Property (Covering)

- Theorem 6(a): $x + xy = x$
  - $x + xy = x \cdot 1 + xy$ by 2(b)
    - $= x(1 + y)$ 4(a)
    - $= x(y + 1)$ 3(a)
    - $= x \cdot 1$ Th 2(a)
    - $= x$ 2(b)

Huntington postulates:

Post. 2: (a) $x+0=x$, (b) $x \cdot 1=x$
Post. 3: (a) $x+y=y+x$, (b) $x \cdot y=y \cdot x$
Post. 4: (a) $x(y+z) = xy+xz$,
(b) $x+yz = (x+y)(x+z)$
Post. 5: (a) $x+x'=1$, (b) $x \cdot x'=0$
Th. 2: (a) $x+1=1$

- Theorem 6(b): $x(x + y) = x$ by duality
- By means of truth table (another way to proof)

x	y	xy	$x+xy$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

DeMorgan's Theorem

- Theorem 5(a): $(x + y)' = x'y'$
- Theorem 5(b): $(xy)' = x' + y'$
- By means of truth table

x	y	x'	y'	$x+y$	$(x+y)'$	$x'y'$	xy	$x'+y'$	$(xy)'$
0	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	1	0	0	1	0	0	1	0	0

THE END

THE END

Consensus Theorem

1. $xy + x'z + yz = xy + x'z$
2. $(x+y)\cdot(x'+z)\cdot(y+z) = (x+y)\cdot(x'+z)$ -- (dual)

- **Proof:**
  - $xy + x'z + yz$
    - $= xy + x'z + 1.yz$ 2(a)
    - $= xy + x'z + (x+x')yz$ 5(a)
    - $= xy + x'z + xyz + x'yz$ 3(b) &4(a)
    - $= (xy + xyz) + (x'z + x'zy)$ Th4(a)
    - $= x(y + yz) + x'(z + zy)$ 4(a)
    - $= xy + x'z$ Th6(a)
    - QED (2 true by duality).

Operator Precedence

- The operator precedence for evaluating Boolean Expression is
  - Parentheses
  - NOT
  - AND
  - OR
- Examples
  - $x\, y' + z$
  - $(x\, y + z)'$

2.5 Boolean Functions

- A Boolean function
  - Binary variables
  - Binary operators OR and AND
  - Unary operator NOT
  - Parentheses
- Examples
  - $F_1 = x\,y\,z'$
  - $F_2 = x + y'z$
  - $F_3 = x'\,y'\,z + x'\,y\,z + x\,y'$
  - $F_4 = x\,y' + x'\,z$

Boolean Functions

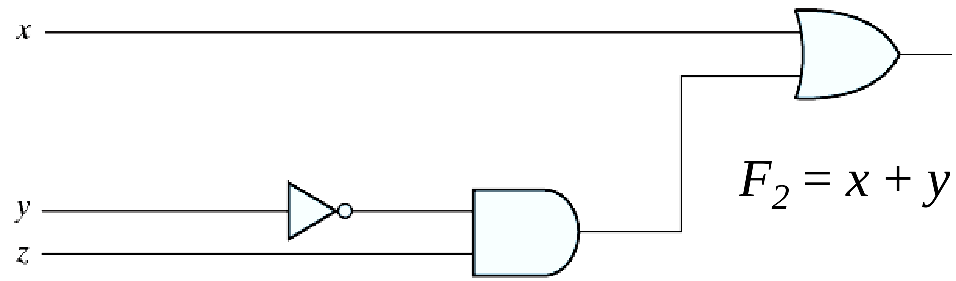
- The truth table of 2^n entries (n=number of variables)

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

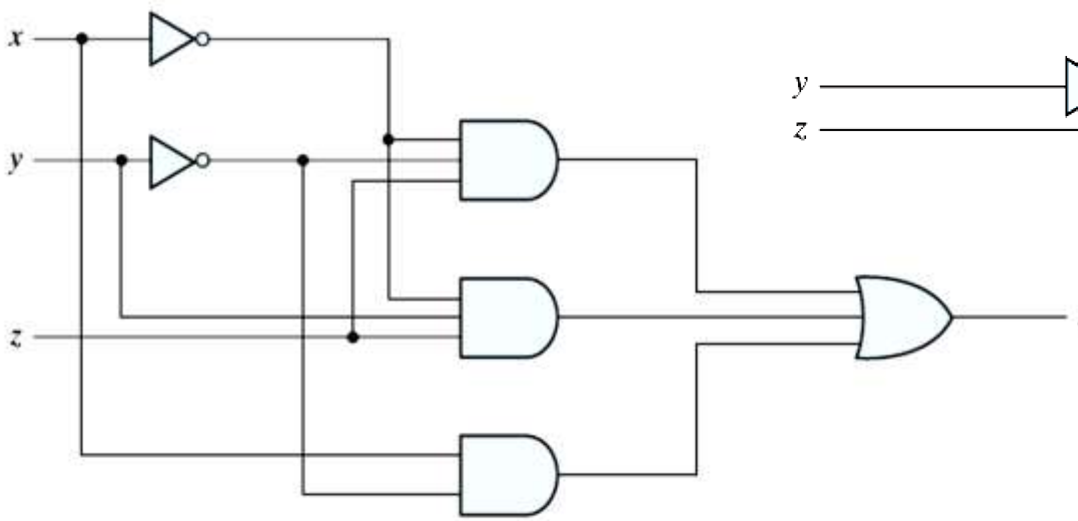
- Two Boolean expressions may specify the same function
  - $F_3 = F_4$

Boolean Functions

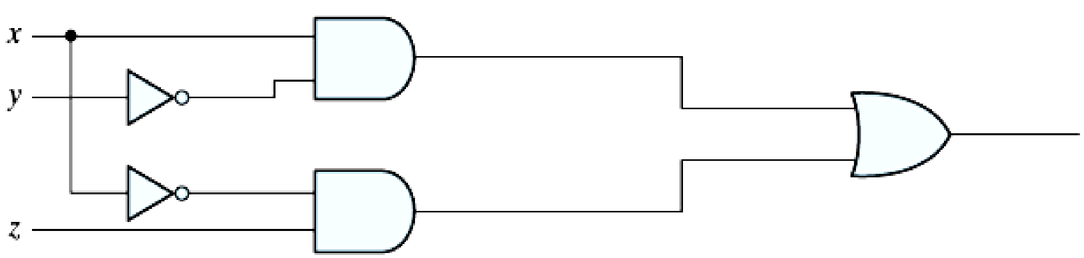
- Implementation with logic gates
  - F_4 is more economical



$F_2 = x + y'z$



$F_3 = x'y'z + x'yz + xy'$



$F_4 = xy' + x'z$

Algebraic Manipulation

- When a Boolean expression is implemented with logic gates, each **term** requires a gate and each variable (**Literal**) within the term designates an input to the gate. (F3 has 3 terms and 8 literal)
- To minimize Boolean expressions, minimize the number of literals and the number of terms → a circuit with less equipment
  - It is a hard problem (no specific rules to follow)
- Example 2.1
  1. $x(x'+y) = xx' + xy = 0+xy = xy$
  2. $x+x'y = (x+x')(x+y) = 1\,(x+y) = x+y$
  3. $(x+y)(x+y') = x+xy+xy'+yy' = x(1+y+y') = x$
  4. $xy + x'z + yz = xy + x'z + yz(x+x') = xy + x'z + yzx + yzx' = xy(1+z) + x'z(1+y) = xy + x'z$
  5. $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$, **by duality from function 4. (*consensus theorem* with duality)**

Complement of a Function

- An interchange of 0's for 1's and 1's for 0's in the value of F
  - By DeMorgan's theorem
  - $(A+B+C)' = (A+X)'$ let $B+C = X$

    $= A'X'$ by theorem 5(a) (DeMorgan's)

    $= A'(B+C)'$ substitute $B+C = X$

    $= A'(B'C')$ by theorem 5(a) (DeMorgan's)

    $= A'B'C'$ by theorem 4(b) (associative)
- Generalization: a function is obtained by interchanging AND and OR operators and complementing each literal.
  - $(A+B+C+D+ \ldots +F)' = A'B'C'D' \ldots F'$
  - $(ABCD \ldots F)' = A'+ B'+C'+D' \ldots +F'$

Examples

- Example 2.2
  - $F_1' = (x'yz' + x'y'z)' = (x'yz')' (x'y'z)' = (x+y'+z) (x+y+z')$
  - $F_2' = [x(y'z'+yz)]' = x' + (y'z'+yz)' = x' + (y'z')' (yz)'$

    $= x' + (y+z) (y'+z')$

    $= x' + yz'+y'z$
- Example 2.3: a simpler procedure
  - **Take the dual of the function and complement each literal**

1. $F_1 = x'yz' + x'y'z$.

   The dual of F_1 is $(x'+y+z') (x'+y'+z)$.

   Complement each literal: $(x+y'+z)(x+y+z') = F_1'$
2. $F_2 = x(y' z' + yz)$.

   The dual of F_2 is $x+(y'+z') (y+z)$.

   Complement each literal: $x'+(y+z)(y' +z') = F_2'$

2.6 Canonical and Standard Forms

Minterms and Maxterms

- A minterm (standard product): an AND term consists of all literals in their normal form or in their complement form.
  - For example, two binary variables *x* and *y*,
    - ***xy, xy', x'y, x'y'***
  - It is also called a standard product.
  - *n* variables can be combined to form 2^n minterms.
- A maxterm (standard sums): an OR term
  - It is also call a standard sum.
  - 2^n maxterms.

Minterms and Maxterms

- Each *maxterm* is the complement of its corresponding *minterm*, and vice versa.

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	**y**	**z**	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Minterms and Maxterms

- An Boolean function can be expressed by
  - A truth table
  - Sum of minterms for each combination of variables that produces a (1) in the function.
  - $f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$ (Minterms)
  - $f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$ (Minterms)

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Minterms and Maxterms

- The complement of a Boolean function
  - The minterms that produce a 0
  - $f_1' = m_0 + m_2 + m_3 + m_5 + m_6 \quad = x'y'z' + x'yz' + x'yz + xy'z + xyz'$
  - $f_1 = (f_1')'$
  - $= (x+y+z)(x+y'+z)\ (x+y'+z')\ (x'+y+z')(x'+y'+z) = M_0\, M_2\, M_3\, M_5\, M_6$
  - $f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z) = M_0M_1M_2M_4$
  - Any Boolean function can be expressed asterms).
  - A product of maxterms ("product" meaning the ANDing of terms).
  - A sum of minterms ("sum" meaning the ORing of Both boolean functions are said to be in Canonical form.

Sum of Minterms

- Sum of minterms: there are 2^n minterms and 2^{2^n} combinations of functions with n Boolean variables.
- Example 2.4: express $F = A+B'C$ as a sum of minterms.
  - $F = A+B'C = A(B+B') + B'C = AB + AB' + B'C = AB(C+C') + AB'(C+C') + (A+A')B'C = ABC+ABC'+AB'C+AB'C'+A'B'C$
  - $F = A'B'C + AB'C' + AB'C + ABC' + ABC = m_1 + m_4 + m_5 + m_6 + m_7$
  - $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$
  - or, built the truth table first

Table 2.5
Truth Table for $F = A + B'C$

A	*B*	*C*	*F*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Product of Maxterms

- Product of maxterms: using distributive law to expand.
  - $x + yz = (x + y)(x + z) = (x+y+zz')(x+z+yy') = (x+y+z)(x+y+z')(x+y'+z)$
- Example 2.5: express $F = xy + x'z$ as a product of maxterms.
  - $F = xy + x'z = (xy + x')(xy + z) = (x+x')(y+x')(x+z)(y+z) = (x'+y)(x+z)(y+z)$
  - $x'+y = x' + y + zz' = (x'+y+z)(x'+y+z')$
  - $F = (x+y+z)(x+y'+z)(x'+y+z)(x'+y+z') = M_0M_2M_4M_5$
  - $F(x, y, z) = \Pi(0, 2, 4, 5)$

Conversion between Canonical Forms

- The complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function.
  - $F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$
  - Thus, $F'(A, B, C) = \Sigma(0, 2, 3)$
  - By DeMorgan's theorem

    $F(A, B, C) = \Pi(0, 2, 3)$

    $F'(A, B, C) = \Pi(1, 4, 5, 6, 7)$
  - $m_j' = M_j$
- To convert from one canonical form to another: **interchange** the symbols Σ and Π and list those numbers **missing** from the original form
  - » Σ of 1's
  - » Π of 0's

- Example
  - $F = xy + x'z$
  - $F(x, y, z) = \Sigma(1, 3, 6, 7)$
  - $F(x, y, z) = \Pi\ (0, 2, 4, 6)$

Table 2.6
Truth Table for $F = xy + x'z$

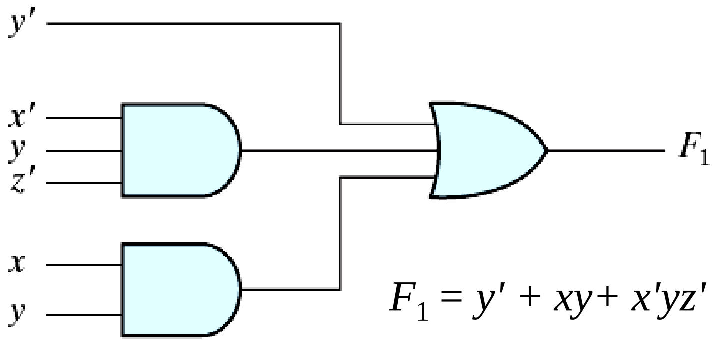
x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Standard Forms

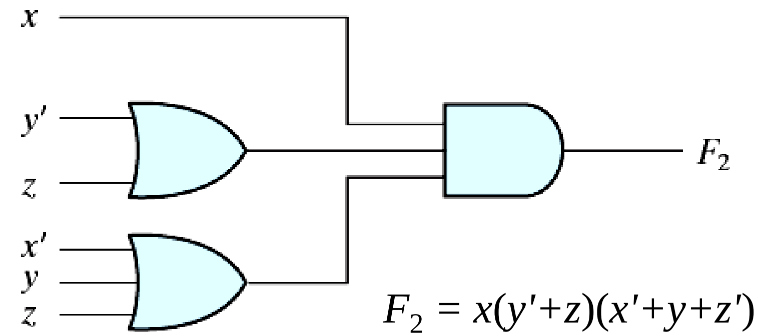
- In canonical forms each minterm or maxterm must contain **all the variables** either complemented or uncomplemented, thus these forms are very seldom the ones with the least number of literals.
- Standard forms: the terms that form the function may obtain **one, two, or any number** of literals, .There are two types of standard forms:
  - Sum of products: $F_1 = y' + xy + x'yz'$
  - Product of sums: $F_2 = x(y'+z)(x'+y+z')$
- A Boolean function may be expressed in a nonstandard form
  - $F_3 = AB + C(D + E)$
- But it can be changed to a standard form by using The distributive law
  - $F3 = AB + C(D + E) = AB + CD + CE$

Implementation

- Two-level implementation

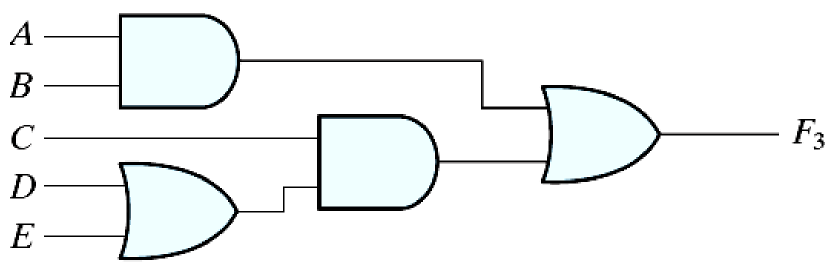


(a) Sum of Products

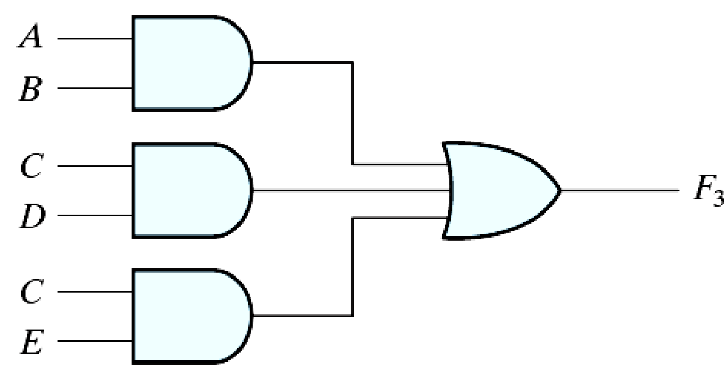


(b) Product of Sums

- Multi-level implementation



(a) $AB + C(D + E)$



(b) $AB + CD + CE$

2.7 Other Logic Operations

- 2^n rows in the truth table of n binary variables.
- 2^{2^n} functions for n binary variables.
- 16 functions of two binary variables.

Table 2.7
Truth Tables for the 16 Functions of Two Binary Variables

x	*y*	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

- All the new symbols except for the exclusive-OR symbol are not in common use by digital designers.

Boolean Expressions

Table 2.8
Boolean Expressions for the 16 Functions of Two Variables

Boolean Functions	Operator Symbol	Name	Comments
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_3 = x$		Transfer	x
$F_4 = x'y$	y/x	Inhibition	y, but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$(x \oplus y)'$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y, then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x, then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1

2.8 Digital Logic Gates

- Boolean expression: AND, OR and NOT operations
- Constructing gates of other logic operations
  - The feasibility and economy;
  - The possibility of extending gate's inputs;
  - The basic properties of the binary operations (commutative and associative);
  - The ability of the gate to implement Boolean functions.

Standard Gates

- Consider the 16 functions in Table 2.8
  - **Two** functions produce a constant : (F_0 and F_{15}).
  - **Four** functions with unary operations: complement and transfer: (F_3, F_5, F_{10} and F_{12}).
  - The other **ten** functions with binary operators
- **Eight** function are used as standard gates : complement (F_{12}), transfer (F_3), AND (F_1), OR (F_7), NAND (F_{14}), NOR (F_8), XOR (F_6), and equivalence (XNOR) (F_9).
  - Complement: inverter.
  - Transfer: buffer (increasing drive strength).
  - Equivalence: XNOR.

Summary of Logic Gates

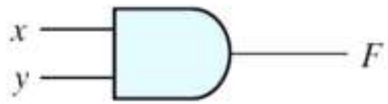
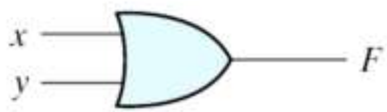
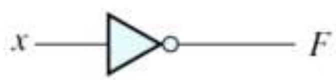
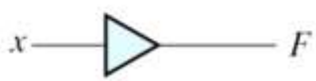
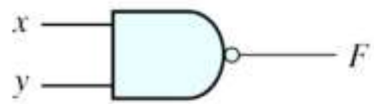
Name	Graphic symbol	Algebraic function	Truth table
AND		$F = xy$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1

Figure 2.5 Digital logic gates

Summary of Logic Gates

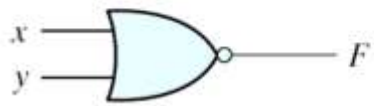
NAND



$F = (xy)'$

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

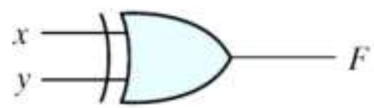
NOR



$F = (x + y)'$

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

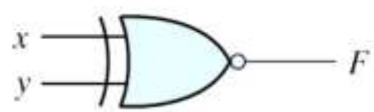
Exclusive-OR (XOR)



$F = xy' + x'y$
$= x \oplus y$

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive-NOR or equivalence



$F = xy + x'y'$
$= (x \oplus y)'$

x	y	F
0	0	1
0	1	0
1	0	0
1	1	1

Figure 2.5 Digital logic gates

Multiple Inputs

- Extension to multiple inputs
  - A gate can be extended to multiple inputs.
    - If its binary operation is commutative and associative.
  - AND and OR are commutative and associative.
    - OR
      - $x+y = y+x$
      - $(x+y)+z = x+(y+z) = x+y+z$
    - AND
      - $xy = yx$
      - $(x\,y)z = x(y\,z) = x\,y\,z$

Multiple Inputs

- Multiple NOR = a complement of OR gate, Multiple NAND = a complement of AND.
- The cascaded NAND operations = sum of products.
- The cascaded NOR operations = product of sums.

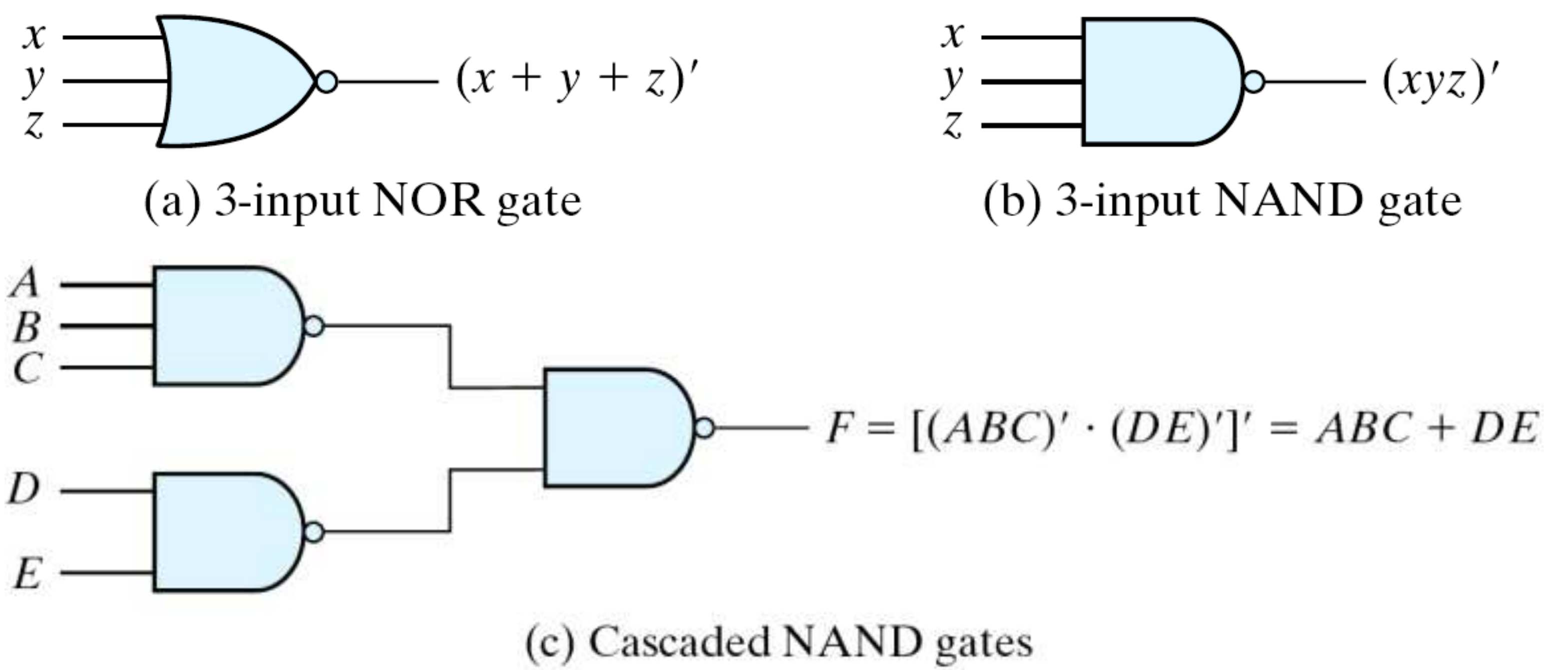
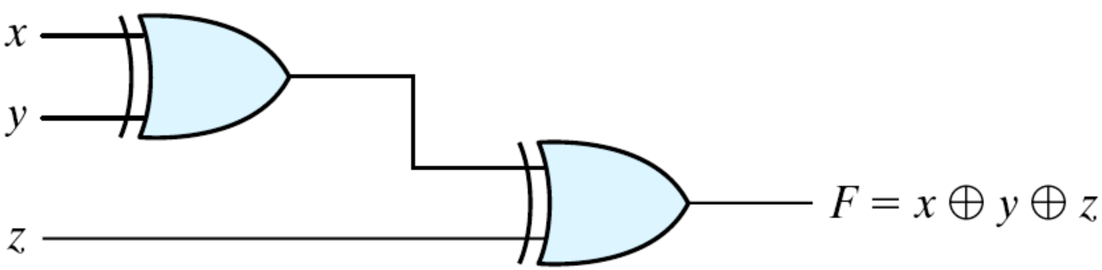


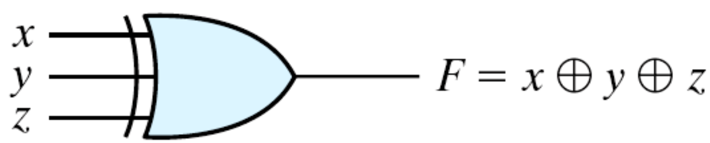
Figure 2.7 Multiple-input and cascated NOR and NAND gates

Multiple Inputs

- The XOR and XNOR gates are commutative and associative.
- Multiple-input XOR gates are uncommon?
- XOR is an odd function: it is equal to 1 if the inputs variables have an odd number of 1's.



(a) Using 2-input gates



(b) 3-input gate

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(c) Truth table

Figure 2.8 3-input XOR gate

Positive and Negative Logic

- Positive and Negative Logic
  - Two signal values <=> two logic values
  - Positive logic: H=1; L=0
  - Negative logic: H=0; L=1
- Consider a TTL gates
  - A positive logic AND gate
  - A negative logic OR gate

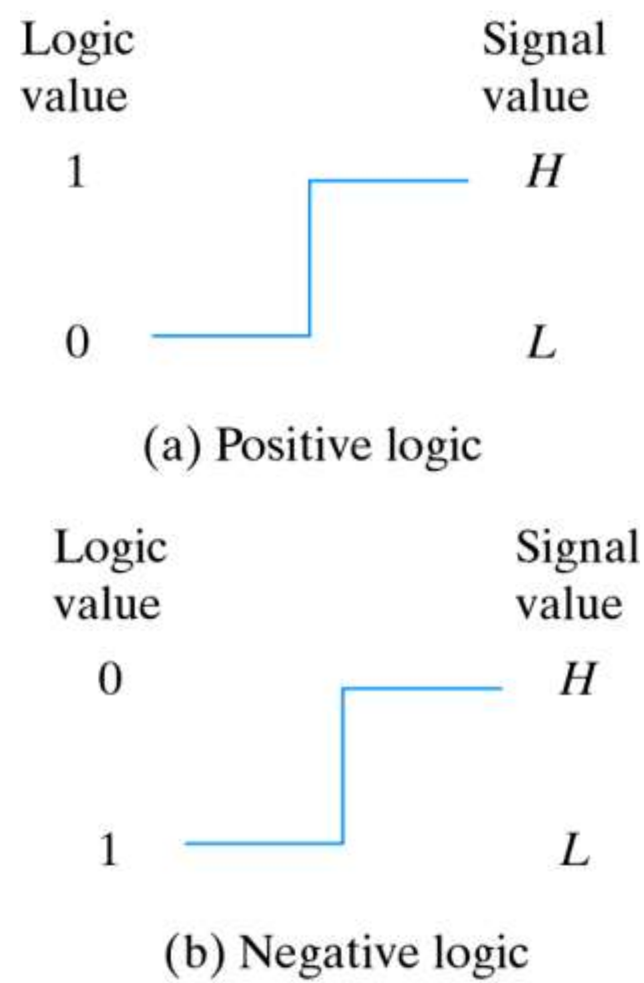
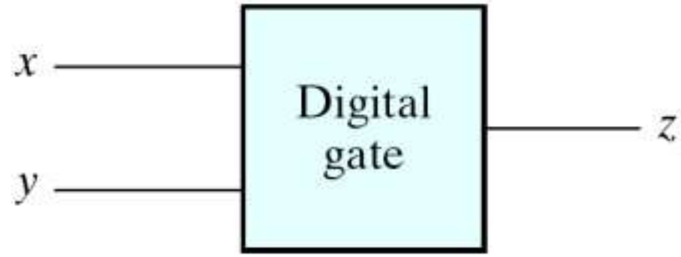


Figure 2.9 Signal assignment and logic polarity

Positive and Negative Logic

x	y	z
L	L	L
L	H	L
H	L	L
H	H	H

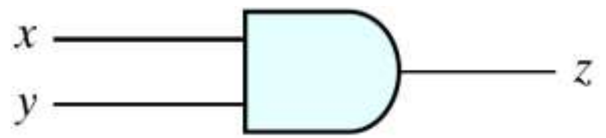
(a) Truth table with H and L



(b) Gate block diagram

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

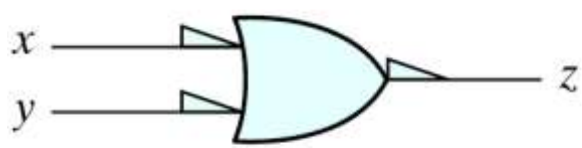
(c) Truth table for positive logic



(d) Positive logic AND gate

x	y	z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic



(f) Negative logic OR gate

Figure 2.10 Demonstration of positive and negative logic

2.9 Integrated Circuits

Level of Integration

- An IC (a chip)
- Examples:
  - Small-scale Integration (SSI): < 10 gates
  - Medium-scale Integration (MSI): 10 ~ 100 gates
  - Large-scale Integration (LSI): 100 ~ xk gates
  - Very Large-scale Integration (VLSI): > xk gates
- VLSI
  - Small size (compact size)
  - Low cost
  - Low power consumption
  - High reliability
  - High speed

Digital Logic Families

- Digital logic families: circuit technology
  - TTL: transistor-transistor logic (dying?)
  - ECL: emitter-coupled logic (high speed, high power consumption)
  - MOS: metal-oxide semiconductor (NMOS, high density)
  - CMOS: complementary MOS (low power)
  - BiCMOS: high speed, high density

Digital Logic Families

- The characteristics of digital logic families
  - Fan-out: the number of standard loads that the output of a typical gate can drive.
  - Power dissipation.
  - Propagation delay: the average transition delay time for the signal to propagate from input to output.
  - Noise margin: the minimum of external noise voltage that caused an undesirable change in the circuit output.

CAD

- CAD – Computer-Aided Design
  - Software programs that support computer-based representation of circuits of millions of gates.
  - Automate the design process
  - Two design entry:
    - » Schematic capture
    - » HDL – Hardware Description Language
      - – Verilog, VHDL
  - Simulation
  - Physical realization

Home Work (4)

Digital Design (**4th**)- Morris Mano-Page 66-Problems:

2.3 d,f
2.4 d,e
2.6 Only for (2.3 d,f)
2.7 Only for (2.4 d,f)
2.9
2.20
2.22

Home Work (5)

Digital Design (**4th**)- Morris Mano-Page 66-Problems:

2.13
2.14
2.15
2.27
2.28