# *Fundamentals of Multimedia*

# Lecture 4
# Lossless Data Compression
# Fixed Length Coding

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# Outcomes of Lecture 3

- *Physical and perceptual aspects of color*
  - Human Vision
- *Color models in image*
  - RGB
  - CMYK
  - HSB
- *Gamma Correction*
- *Color models in video*
  - YUV
  - YCbCr

# Outline

- *Basics of Information Theory*
  - Data entropy
- *Fixed Length Coding*
  - Run Length Coding (RLC)
  - Dictionary-based Coding
    - *Lempel-Ziv-Welch (LZW) algorithm*

# Outline

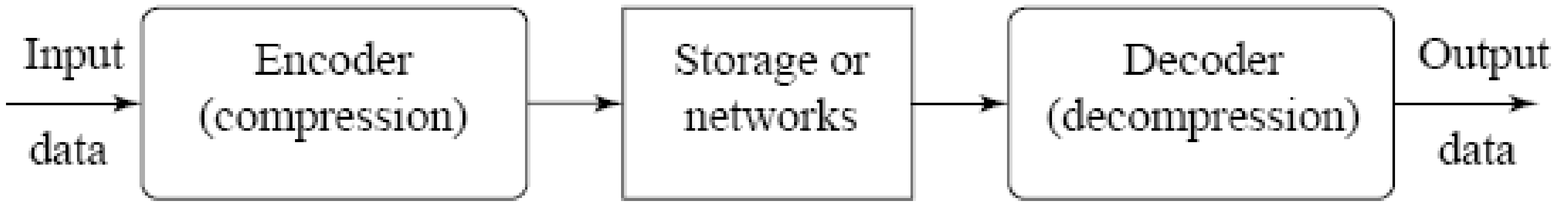
- ***Basics of Information Theory***
  - Data entropy
- *Fixed Length Coding*
  - Run Length Coding (RLC)
  - Dictionary-based Coding
    - *Lempel-Ziv-Welch (LZW) algorithm*

# Data Compression

- *What is Compression?*
  - The process of coding
  - Reduce the total number of bits needed to represent certain information.
- *Why?*
  - Huge volume of multimedia data
  - More efficient data storage, processing and transmission
- *Compression Ratio*
  - Compression ratio= $B_0$ / $B_1$
  - $B_0$ : number of bits before compression
  - $B_1$ : number of bits after compression

# Compression Schemes

*A General Data Compression Scheme.*




- *Lossy Compression*
  - The compression and decompression processes induce ***information loss***.
- *Lossless Compression*
  - The compression and decompression processes induce ***no information loss***.

# Example of Compression Schemes

- *Transmit the data {250, 251, 251, 252, 253, 253, 254, 255} by the network*
  - Rewrite the data sequence using binary: {11111010, 11111011, 11111011, 11111100, 11111101, 11111101, 11111110, 11111111}
  - Totaly require 8*8 = 64 bits for transmission
- *The available bandwidth is limited*
  - Only 16 bits available.
  - Compression is necessary.

# Example of Lossy Compression

- *Encode:* *Drop the least significant bits*
- *Encode data:* *8*2 bit = 16 bits*


| | | | | |
|---|---|---|---|---|
| 250 | 11/111010 | 11 | 11/000000 | 192 |
| 251 | 11/111011 | 11 | 11/000000 | 192 |
| 251 | 11/111011 | 11 | 11/000000 | 192 |
| 252 | 11/111100 | 11 | 11/000000 | 192 |
| 253 | 11/111101 | 11 | 11/000000 | 192 |
| 253 | 11/111101 | 11 | 11/000000 | 192 |
| 254 | 11/111110 | 11 | 11/000000 | 192 |
| 255 | 11/111111 | 11 | 11/000000 | 192 |

**Induce Information Loss**

# Example of Lossless Compression

- *Encode:* *Encode the difference*
- *Encode data:* *8-bit + 7* 1-bit = 15 bits*


| | | | | |
|---|---|---|---|---|
| 250 | 250 | 11111010 | 250 | 250 |
| 251 | 1 | 1 | +1 | 251 |
| 251 | 0 | 0 | +0 | 251 |
| 252 | 1 | 1 | +1 | 252 |
| 253 | 1 | 1 | +1 | 253 |
| 253 | 0 | 0 | +0 | 253 |
| 254 | 1 | 1 | +1 | 254 |
| 255 | 1 | 1 | +1 | 255 |

**No Information Loss**

# Bound of Lossless Compression

- *The user expects*
  - Compression ratio as much as it can be
  - Without influence the recovery of the original file.
- *But! Compression ration can't be infinite.*
- *Entropy defines the bound of lossless compression*
  - The number of bits should be used to represent the information source on average
- *It can be interpreted as the* ***average shortest message length****, in bits, that can be sent to communicate the true value to a recipient.*

# Definition of Entropy

$$\eta = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i}$$

- *Alphabet:* $S = \{s_1, s_2, \ldots s_n\}$
  - Possible values of the information source
- *Probability:* $P = \{p_1, p_2, \ldots p_n\}$
  - Relevant probability that the $s_i$ occurs.
- *Self-information:* $log_2 \frac{1}{p_i}$
  - The amount of information contained in $s_i$
  - A value that occurs with very high probability carries little “surprise” or very little information.

# Example of Entropy Calculation

- *Message: {abcdabaa}*
- *Alphabet={a, b, c, d} with probability {4/8, 2/8, 1/8, 1/8}*

*a => 00*

*b => 01*

*c => 10*

*d => 11*

- *Message: {abcdabaa} => {00 01 10 11 00 01 00 00}*
- *Average lenght=16 bits / 8 chars = 2*

# Example of Entropy Calculation

- *Alphabet={a, b, c, d} with probability {4/8, 2/8, 1/8, 1/8}*
- $\eta = 4/8 * \log_2 2 + 2/8 * \log_2 4 + 1/8 * \log_2 8 + 1/8 * \log_2 8$
- $\eta = 1/2 + 1/2 + 3/8 + 3/8 = 1.75$ *average length*
- *a => 0     b => 10     c => 110     d => 111*
- *Message: {abcdabaa} => {0 10 110 111 0 10 0 0}*
- *average length = 14 bits / 8 chars = 1.75*

# Outline

- ***Basics of Information Theory***
  - Data entropy
- ***Fixed Length Coding***
  - Run Length Coding (RLC)
  - Dictionary-based Coding
    - *Lempel-Ziv-Welch (LZW) algorithm*

# Run-Length Coding

- ***Rationale for RLC:*** *if the information source has the property that symbols tend to form continuous groups, then such symbol and the length of the group can be coded.*
- ***Memoryless Source:*** *Namely, the value of the current symbol does not depend on the values of the previously appeared symbols.*
- *Instead of assuming memoryless source, Run-Length Coding (RLC) exploits **memory present** in the information source.*

# Run-Length Coding

- *RLE is a very simple form of data compression in which* ***runs of data*** *(that is, sequences in which the same data value occurs in many consecutive data elements) are stored as a single data value and count, rather than as the original run.*

**WWWWWWBWWWWWWWWWWWWBBBWWWWWWWWWWWWWW**

↓

6W1B12W3B14W

- *Compression Ratio 36/10= 3.6*

# Run-Length Coding

- *Extreme Cases:*
  - Best Case: AAAAAAAA → 8A
    - *Compression Ratio: 8/2=4*
  - Worst case: ABABABAB → 1A1B1A1B1A1B1A1B
    - *Compression Ratio: 8/16=0.5*
    - ***Negative compression:*** *the resulting compressed file is larger than the original one.*

# Dictionary-based Coding

- *Use **fixed-length** codeword*
    - Represent variable-length strings of possible values (symbols or characters) that commonly occur together, such as words in English text.
- *Limpel-Ziv-Welch (LZW) is an adaptive, dictionary-based technique*
    - Unix compress, GIF files.
    - The LZW encoder and decoder build up the same dictionary dynamically while receiving the data

# LZW Compression for String

- *Input data*
  - ABABBABCABABBA
- *Initial simple dictionary only includes the possible values of the alphabet*

| code | string |
|---|---|
| 1 | A |
| 2 | B |
| 3 | C |

- *Then, apply the following algorithm*

# LZW Compression Algorithm

```
BEGIN
s = first input character;
while not EOF{
      c = next input character;
      if s + c exists in the dictionary
            s = s + c;
     else{
            output the code for s;
            add string s + c to the dictionary with a new code;
            s = c;
       }
    }
output the code for s;
END
```

# LZW Compression Algorithm

“ABABBABCABABBA”

s=next char

c=next char

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | | | |

The output codes are:

# LZW Compression Algorithm

"ABABBABCABABBA"

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | | | | |

Check s+c

s+c (AB) is not in the Dic.

--- output the code for s
A => 1

--- insert AB in Dic.

--- s=c

The output codes are: 1

# LZW Compression Algorithm

“ABABBABCABABBA”

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | A | | | |

The output codes are: 1

# LZW Compression Algorithm

“ABABBABCABABBA”

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | A | 2 | 5 | BA |
| A | | | | |

**Check s+c**

**s+c (BA) is not in the Dic.**

**--- output the code for s**
**B => 2**

**--- insert BA in Dic.**

**--- s=c**

The output codes are: 1

# LZW Compression Algorithm

"ABABBABCABABBA"

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | A | 2 | 5 | BA |
| A | B | | | |

The output codes are: 1 2

# LZW Compression Algorithm

"ABABBABCABABBA"

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | A | 2 | 5 | BA |
| A | B | | | |
| AB | | | | |

**Check s+c**

**s+c (AB) is in the Dic.**

**--- s=s+c**

The output codes are: 1 2

# LZW Compression Algorithm

"ABABBABCABABBA"

| s | c | output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| A | B | 1 | 4 | AB |
| B | A | 2 | 5 | BA |
| A | B | | | |
| AB | B | 4 | 6 | ABB |
| B | A | | | |
| BA | B | 5 | 7 | BAB |
| B | C | 2 | 8 | BC |
| C | A | 3 | 9 | CA |
| A | B | | | |
| AB | A | 4 | 10 | ABA |
| A | B | | | |
| AB | B | | | |
| ABB | A | 6 | 11 | ABBA |
| A | EOF | 1 | | |

- output codes are: 1 2 4 5 2 3 4 6 1
- From 14 characters, only 9 codes are sent
- compression ratio = 14/9 = 1.56

# LZW Decompression

```
BEGIN
s = NIL;
while not EOF{
      k = next input code;
      entry = dictionary entry for k;
      output entry;
      if (s != NIL)
            add s + entry[0] to dictionary with a new code;
      s = entry;
   }
END
```

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
| --- | --- | --- | --- | --- |
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | | | |

**S=nil**
**K=1**

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |

**Entry = A**
**Output = A**

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |

if (s != NIL)
    add string s + entry[0]
    to dictionary with a new code

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | | | | |

S= entry

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | | | |

K = next input

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | **B** | | |

**Entry = B**
**Output = B**

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | B | 4 | AB |

if (s != NIL)
add string s + entry[0]
to dictionary with a new code

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | B | 4 | AB |
| B | | | | |

S= entry

# LZW Decompression

1 2 4 5 2 3 4 6 1

| S | k | entry/output | code | string |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | B | 4 | AB |
| B | 4 | AB | 5 | BA |
| AB | 5 | BA | 6 | ABB |
| BA | 2 | B | 7 | BAB |
| B | 3 | C | 8 | BC |
| C | 4 | AB | 9 | CA |
| AB | 6 | ABB | 10 | ABA |
| ABB | 1 | A | 11 | ABBA |
| A | EOF | | | |

S + entry[0]

- Output: "ABABBABCABABBA",
- Truly lossless result!

# Summary

- *Basics of Information Theory*
  - Data entropy
- *Fixed Length Coding*
  - Run Length Coding (RLC)
  - Dictionary-based Coding
    - *Lempel-Ziv-Welch (LZW) algorithm*