0/1 Knapsack using Branch & Bound.

Question

	1	2	3	4
profit	10	10	12	18
weight	2	4	6	9

$W = 15$

$n = 4$

→ Two things to be calculated

① $u = \sum_{i=1}^{n} p_i x_i \leq m$

$c = \sum_{i=1}^{n} p_i x_i$ (with fraction)

Solution

Assume $u = \infty$ and start preparing the tree.

(1)

How to calculate u and c for node 1

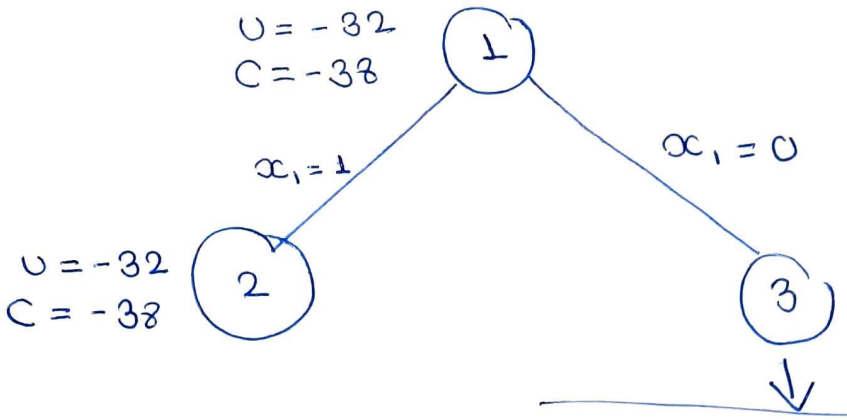
$u = \underbrace{\begin{matrix} 10 & 10 & 12 \\ 2 & 4 & 6 \end{matrix}}_{} = 32$

$c = \begin{matrix} 10 & 10 & 12 & 18/9 \times 3 \\ 2 & 4 & 6 & \end{matrix} = 10 + 10 + 12 + 6$

$= 38$

U = -32
C = -38 (1)

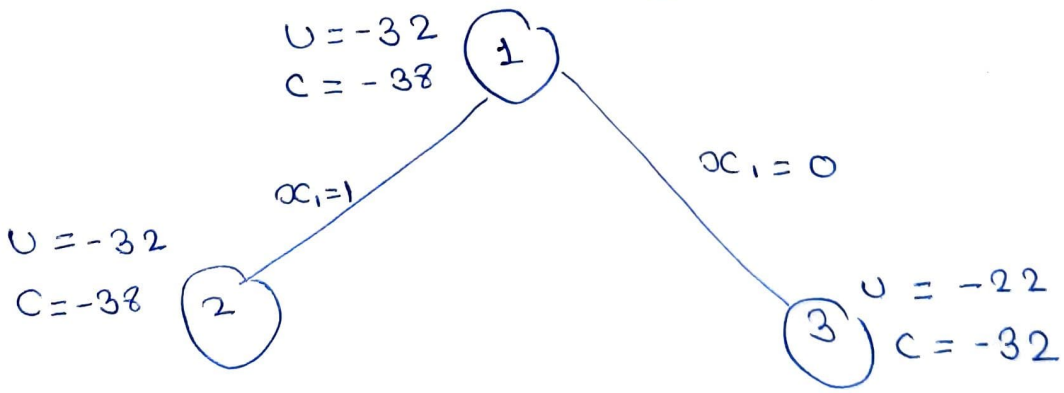
change U from ∞ to -32



$U = 10\ 12\ 4\ 6 = 22$

$C = 10\ 12\ 4\ 6\ 18/9 \times 5 = 32$

$U = -32$

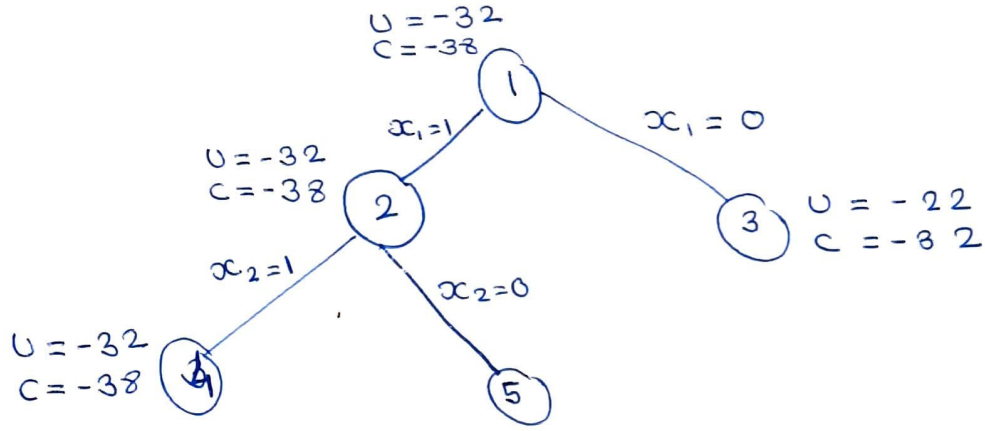


For every node you add check wheather c is greater than U. If c is greater than U then kill that node.

For node 2: C = -38 & U = -32

For node 3: C = -32 & U = -32

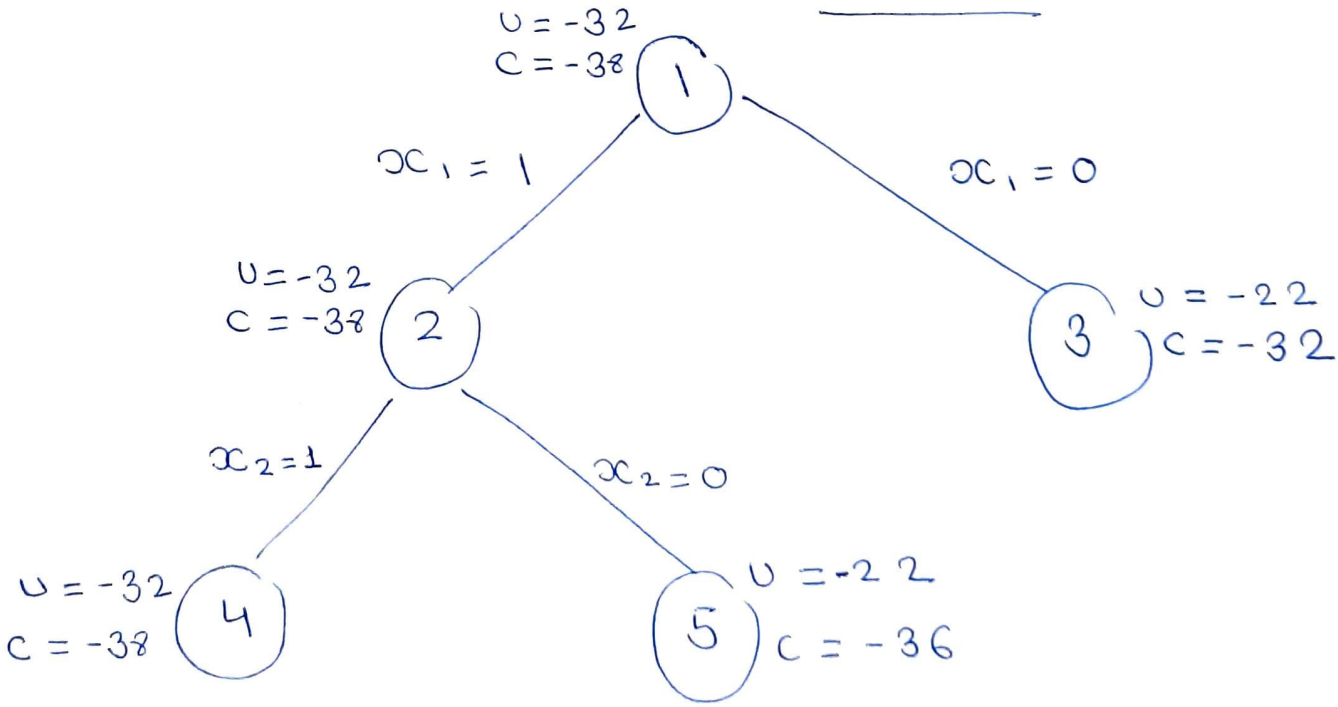
If we get less upper bound (u) for any node than update the universal upper bound U.



For node 5 : $U = 10\ \ 12 = 22$
$2\ \ 6$

$C = 10\ \ 12\ \ {}^{18}/_{9} \times 7 = 36$
$2\ \ 6$

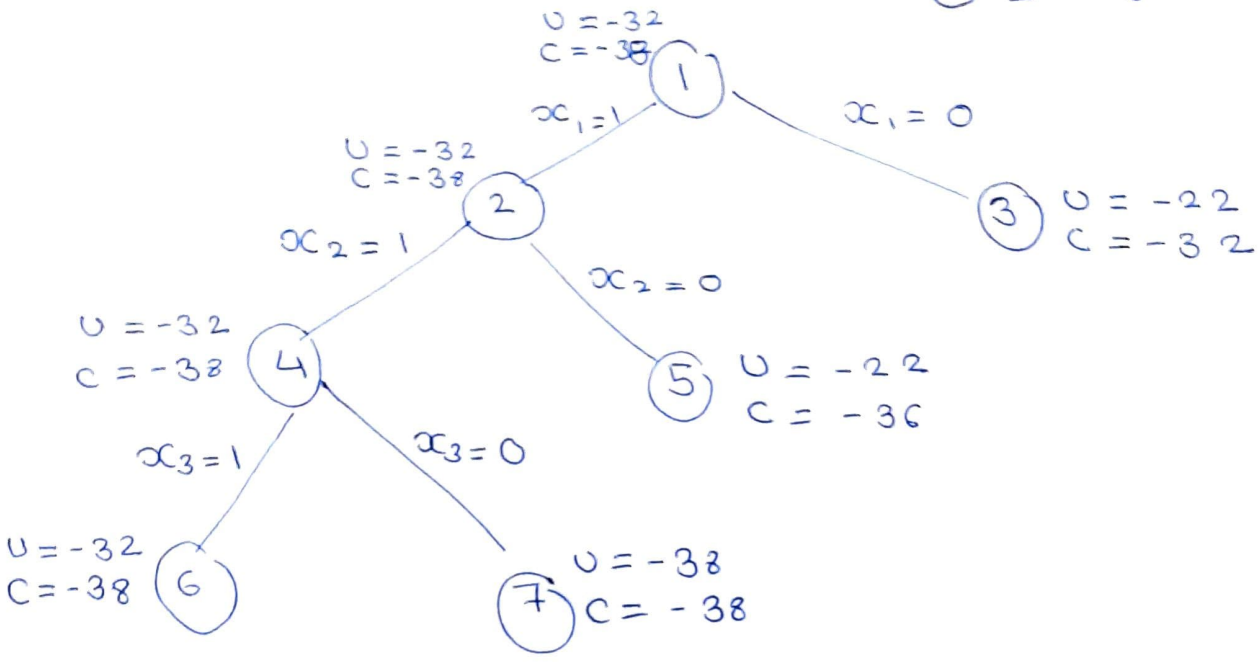
$U = -32$



For node 4 & 5 check if c is greater than U.

For node 4 & 5 check if U is smaller than U.

U = -38



Calculate U & C for node 7.

$$U = 10 \quad 10 \quad 18 = 38$$
$$\quad\;\; 2 \quad\; 4 \quad\; 9$$

$$C = 38$$

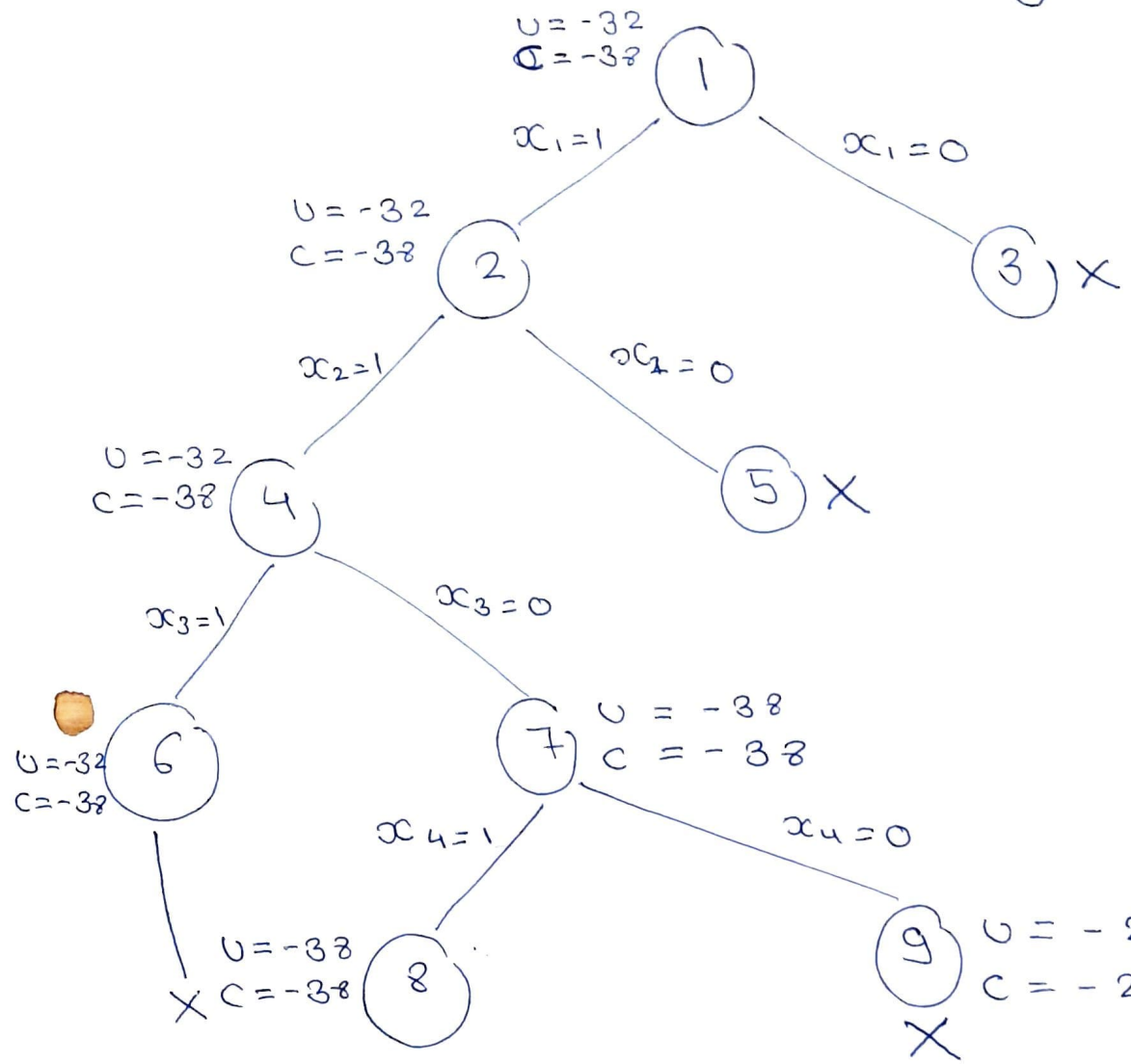
As U for node 7 is -38 which is less than universal bound (U), update U = -38

→ Now U = -38. Check the cost (c) of all the nodes. If c of any node is greater than U = -38, kill that node.

→ Here node 5 is having c = -36 which is greater than U. thus kill node 5 and same for node 3.

→ For node 6 now we cannot expand it further else it will exceed the capacity of knapsack. Hence we will not explore node no. 6.

$U = -38$



For node 9: $U = \frac{10}{2} + \frac{10}{4} = 20$

Here for node 9 $c = -20 > U$ Hence kill the node

Now only node 8 is alive which is the ans.

Hence the solution is $x\{1, 1, 0, 1\}$

Profit = 10 + 10 + 18 = 38

weight = 2 + 4 + 9 = 15