Ch : 1 :- Probability.

→ **Outline :-**

1.1. Introduction.
1.2. Some Basic terms & Concepts.
1.3. Definitions of Probability.
1.4. Theorems on Probability.
1.5. Conditional Probability.
1.6. Multiplicative thm for independent events.
1.7. Baye's Theorem.

We will discussed one by one Sub-topics in this chapter.

1.1 Introduction :- Probability is the branch of mathematics concerning numerical ~~numerical~~ descriptions of how likely an event is to occur or how likely it is that a proposition is true.
The Concept of probability originated from the analysis of the games of Chance.

e.g. tossing of a coin
throwing of dice
playing of cards.

1.2 Basic Definitions :-

1. Random Experiment :- A process by which we observe something uncertain. (undecided).

e.g tossing a coin, roll a die.

2. Outcome :- An outcome is a result of a random experiment.

3. Sample Space :- The set of all possible outcomes is known as sample space.

e.g In a random experiment:

(i) toss a coin;
Sample space : $S = \{H, T\}$.

(ii) roll a die ; $S = \{1, 2, 3, 4, 5, 6\}$

(iii) The no. of goals in a soccer match;
$S = \{0, 1, 2, 3, \ldots\}$.

4. Trial :- When we repeat a random experiment several times, we call each one of them a trial.

i.e A trial is a particular performance of a random experiment.

In the example of tossing a coin, each trial will result in either heads or tails.

5. **Event :-** An event is a collection of possible outcomes. In other words, an event is a subset of the sample space to which are assign a probability.

e.g. Tossing of a coin is a trial & getting a head or tail is an event.

6. **Exhaustive Events :-** The total no. of possible outcomes in any trial of a random experiment is known as exhaustive events or exhaustive cases.

e.g.
(i) In a tossing of a coin, there are two exhaustive events namely head & tail.

(ii) In drawing 2 cards from a pack of cards the no. of exhaustive cases is : $^{52}C_2$.

7. **Mutually Exclusive Events :-** Events are said to be mutually exclusive if no two or more

them two of the events can happen simultaneously in the same trial.

i.e The joint occurrence is not possible.

e.g (i) In observation of seed germination the seed may either germinate or it will not. So germination & non-germination are mutually exclusive events.

(ii) In tossing a coin, the events head or tail are mutually exclusive, because both H & T cannot occur at the same time

8. Equally likely Events :— Two or more events are said to be equally likely if each one of them has an equal chance of occurring.

9. Independent Events :— Several events are said to be independent if the happening of an event is not affected by the happening of one or more events.

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e.g. In throwing a dice, the result of the first throw does not affect the result of the second throw.

10. **Favourable Events :-** The outcomes which make necessary the happening of an event in a trial are called favourable events.

e.g. If two dice are thrown, the no. of favourable events of getting a sum 5 is four (4).
i.e. (1, 4), (2, 3), (3, 2), (4, 1).

1.3 Definitions of Probability :-

1) Mathematical or Classical Probability:

If an experiment results in 'n' exhaustive cases which are mutually exclusive & equally likely cases out of which 'm' events are favourable to the happening of an event 'A', then the probability happening of 'A' is given by

$$P(A) = \frac{\text{Favourable number of cases}}{\text{Exhaustive number of cases}} = \frac{m}{n}$$

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Note:- 1. If $m = 0 \Rightarrow P(A) = 0$, then 'A' is called an impossible event.
i.e $P(\phi) = 0$.

2. If $m = n \Rightarrow P(A) = 1$, then 'A' is called certain (or) assure event.

3. The probability is a non-negative real number and it is lies between 0 to 1.

4. The probability of non-happening of the event 'A' is denoted by $P(\bar{A})$ or 'q'.

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{m}{n}$$

$$\Rightarrow q = 1 - p \quad (\because P(A) = p)$$

$$\Rightarrow p + q = 1.$$

or $P(A) + P(\bar{A}) = 1$.

2. Statistical or Empirical Probability:

If an experiment is repeated a (n) number of times, an event 'A' happens 'm' times then the statistical probability of

'A' is given by

$$p = P(A) = \lim_{n \to \infty} \frac{m}{n} \text{, provided}$$

the limit is finite & unique.

3). **Axioms for Probability :-** The probability $P(A)$ Satisfies the following three axioms.

I. For any event A, $P(A) \geq 0$.

II. Probability of the Sample Space S is $P(S) = 1$.

III. If $A_1, A_2, A_3, \ldots A_n$ are finite mutually exclusive events or disjoint events, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$

$$= \sum_{i=1}^{n} P(A_i) .$$

i.e the probability of a union of mutually exclusive events is the sum of probabilities of the events themselves.

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Examples :-

Ex: 1 Find the probability that there will be 5 Sundays in the month of October.

Sol^n :- In the month of october 4 weeks & 3 days.
These 3 days can occur in the following possible ways:

(i) Mon, Tue, Wed
(ii) Tue, Wed, Thurs
(iii) Wed, Thurs, Fri
(iv) Thurs, Fri, Sat
(v) Fri, Sat, Sun
(vi) Sat, Sun, Mon
(vii) Sun, Mon, Tue.

Number of exhaustive cases $n = 7$.
Number of favourable cases $m = 3$.

Let A be the event of getting 5 Sundays in the month of october.

So, $P(A) = \frac{m}{n} = \frac{3}{7}$.

Ex : 2 A Card is drawn at random from a pack of 52 Cards. Find the Probability that the Card drawn is (i) an ace Card
(ii) a club Card.

Sol :- Total number of Cards = 52
One Card is drawn at random out of 52 Cards.

$\therefore \; n(S) = {}^{52}C_1 = 52. \quad \left(\because \frac{52!}{(52-1)!\,1!} = 52\right)$

(i) Let A be the event of getting an ace card. There are 4 ace Cards & one of them can be drawn in ${}^{4}C_1$ ways.

$\therefore \; n(A) = {}^{4}C_1 = 4.$

$\& \; P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}.$

(ii) Let B be the event of getting a club Card. There are 13 club Cards & one of them can be drawn in ${}^{13}C_1$ ways.

$\therefore \; n(A) = {}^{13}C_1 = 13$

$\& \; P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$