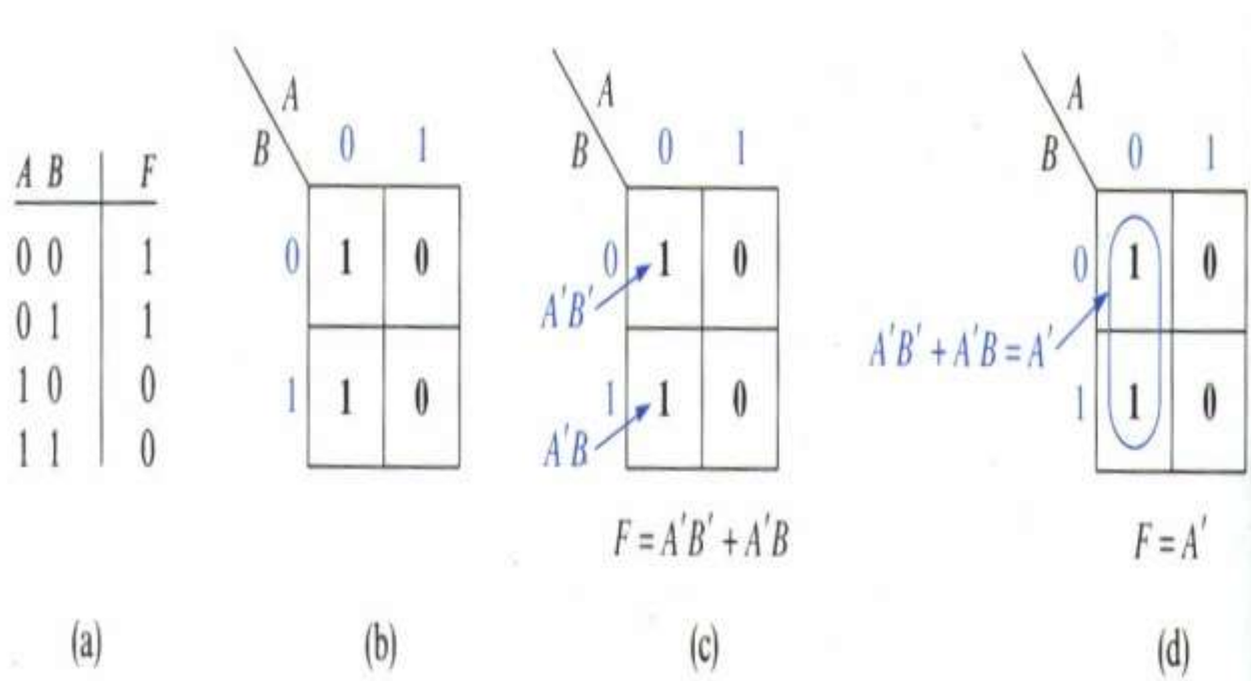
Karnaugh Maps

- Algebraic procedures:
    - Difficult to apply in a systematic way.
    - Difficult to tell when you have arrived at a minimum solution.
- Karnaugh map (K-map) can be used to minimize functions of up to 6 variables.
    - K-map is directly applied to two-level networks composed of AND and OR gates.
        - Sum-of-products, (SOP)
        - Product-of-sum, (POS).

2-Variable K-map

– Place 1s and 0s from the truth table in the K-map.
– Each square of 1s = minterms.
– Minterms in adjacent squares can be combined since they differ in only one variable. Use XY’ + XY = X.

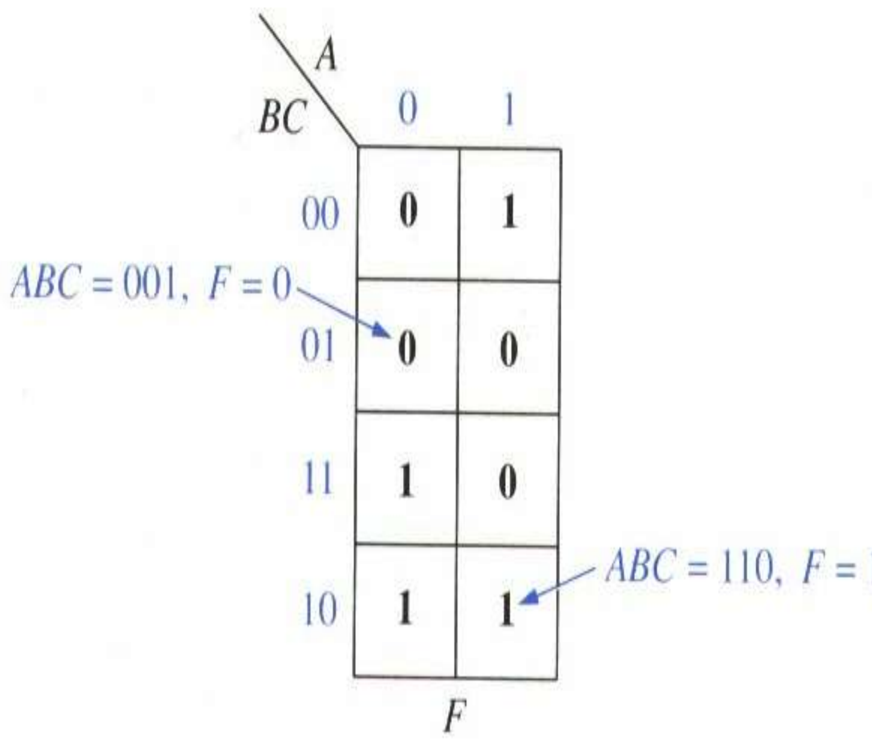


3-Variable K-map

– Note BC is listed in the order of 00, 01, 11, 10. (Gray code)
– Minterms in adjacent squares that differ in only one variable can be combined using XY’ + XY = X.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

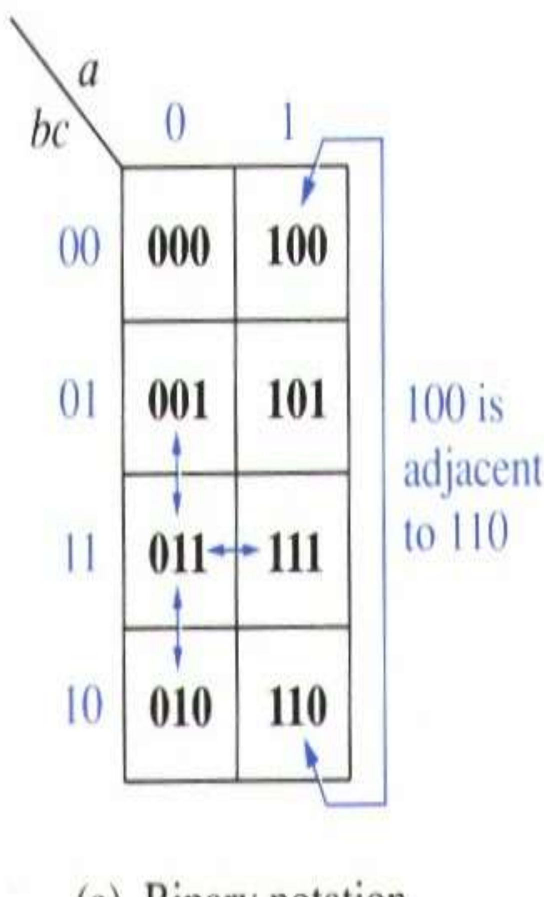
(a)



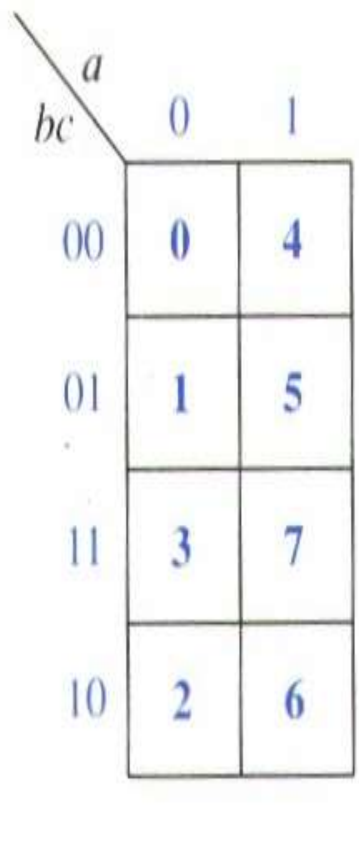
(b)

Location of Minterms

– Adjacent terms in 3-variable K map.



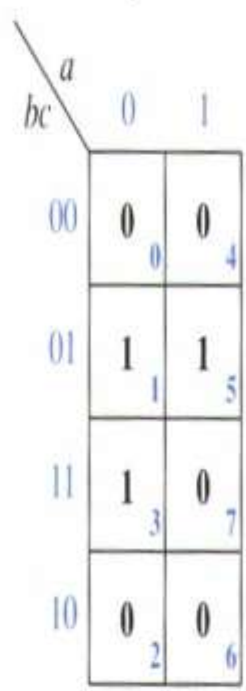
(a) Binary notation



(b) Decimal notation

K Map Example

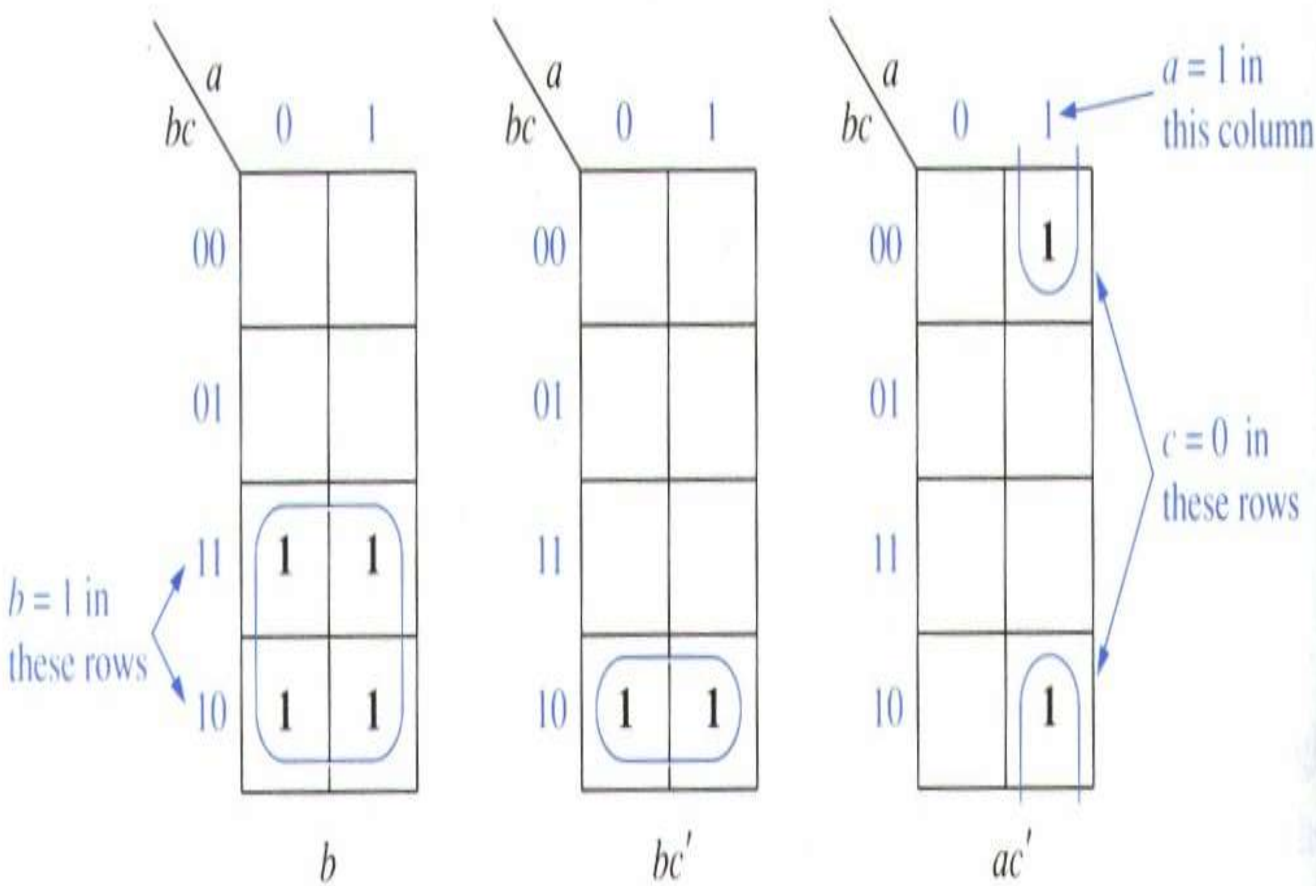
– K-map of $F(a,b,c) = \sum m(1,3,5)$
  $=\prod M(0,2,4,6,7)$



Karnaugh Map of
$F(a, b, c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$

Place Product Terms on K Map

- Example
  - Place b, bc’ and ac’ in the 3-variable K map.



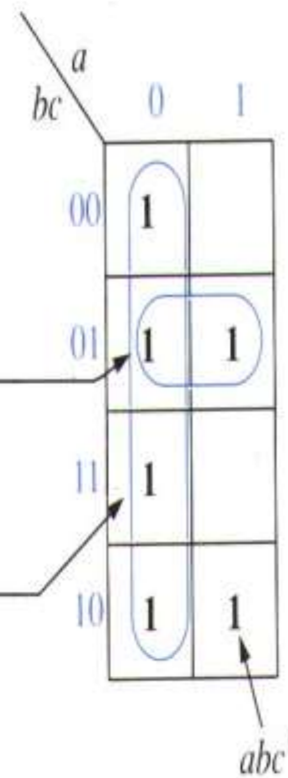
More Example

– Exercise. Plot f(a, b, c) = abc’ + b’c + a’ into the K-map.

$$f(a, b, c) = abc' + b'c + a'$$

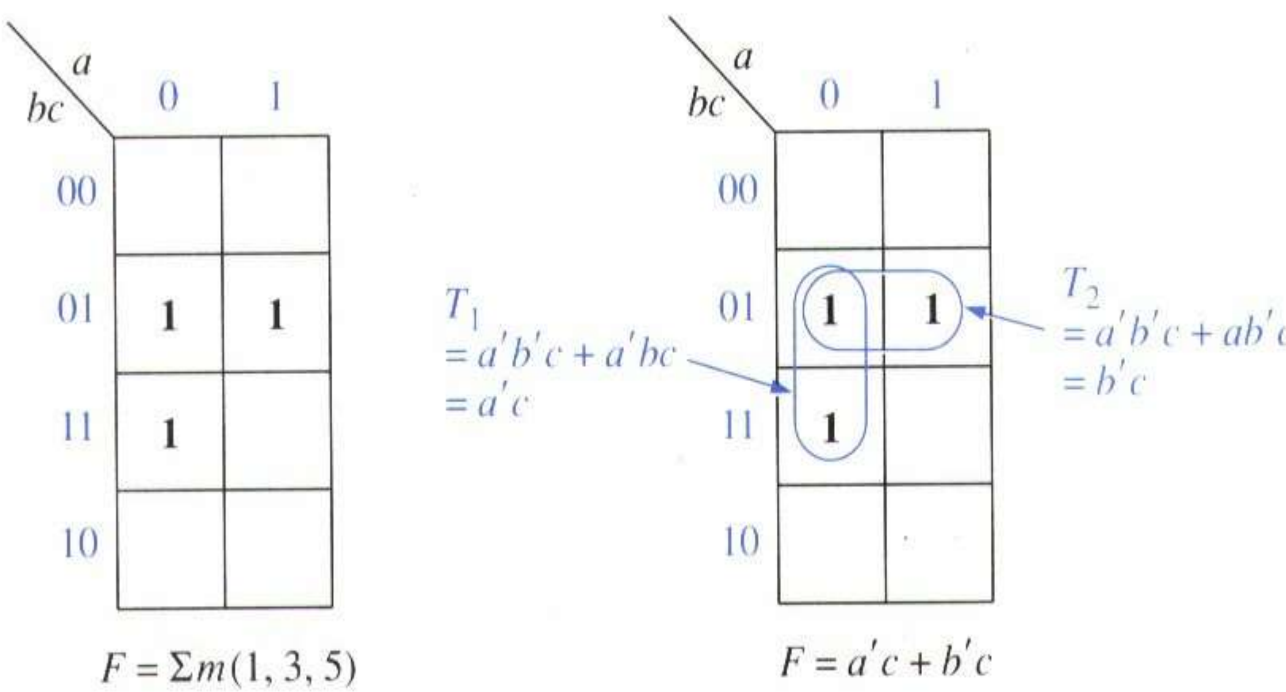
we would plot the map as follows:

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1’s in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1’s in all the squares of the $a = 0$ column of the map. (*Note:* since there already is a 1 in the abc = 001 square, we do not have to place a second 1 there because $x + x = x$.)



Simplication Example

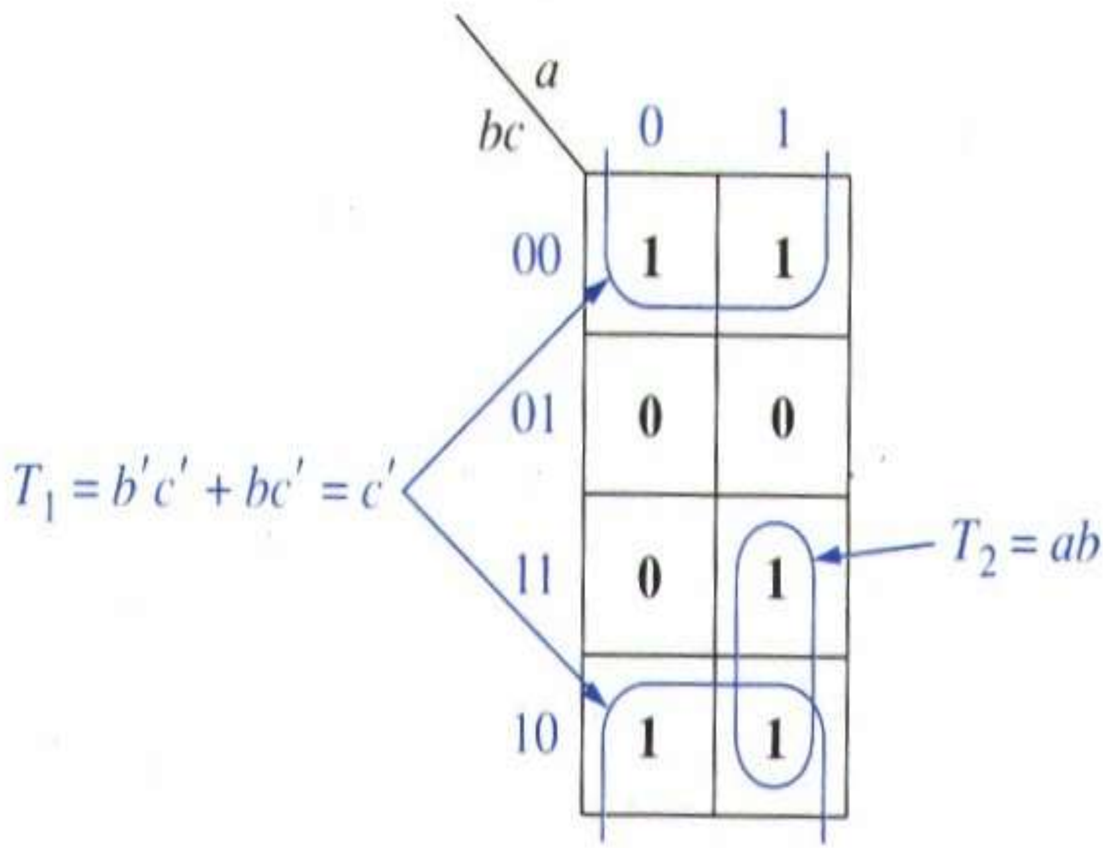
- Exercise. Simplify: F(a,b,c) = $\sum m(1,3,5)$
  - Procedure: place minterms into map.
  - Select adjacent 1's in group of two 1's or four 1's etc.
  - Kick off x and x'.



(a) Plot of minterms (b) Simplified form of F

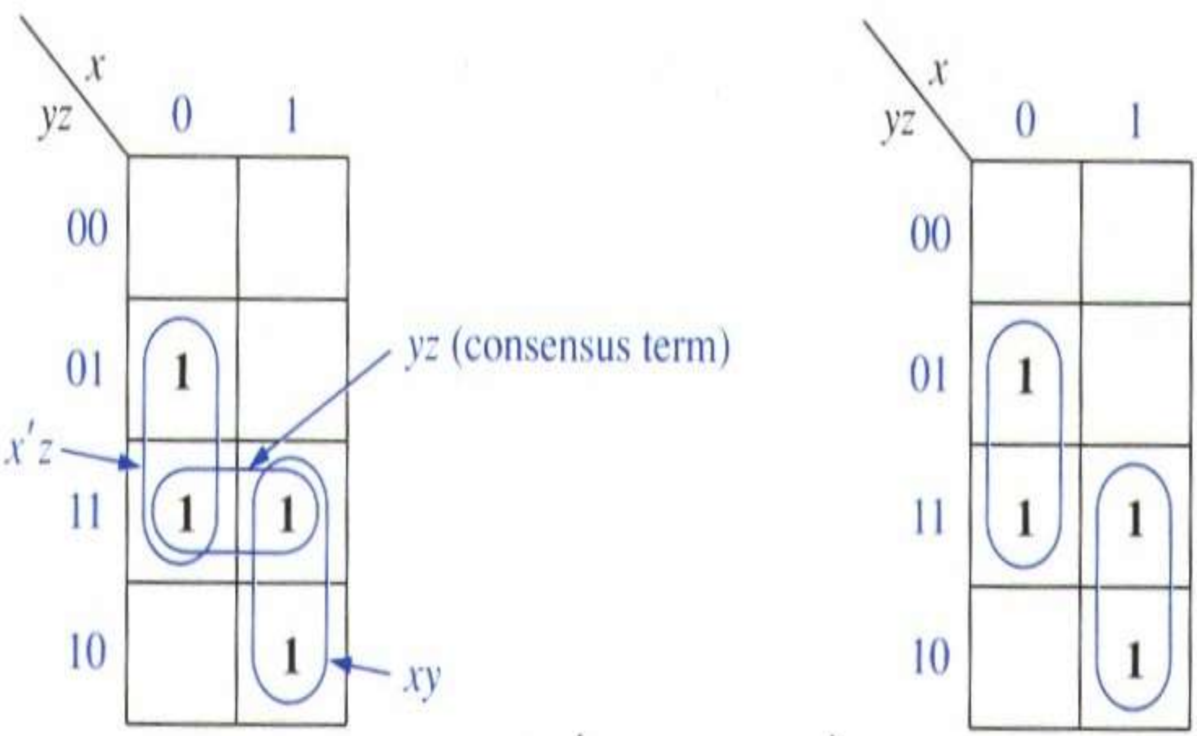
More Example

- The complement of F
  - Using four 1's to eliminate two variables.



Redundant Terms

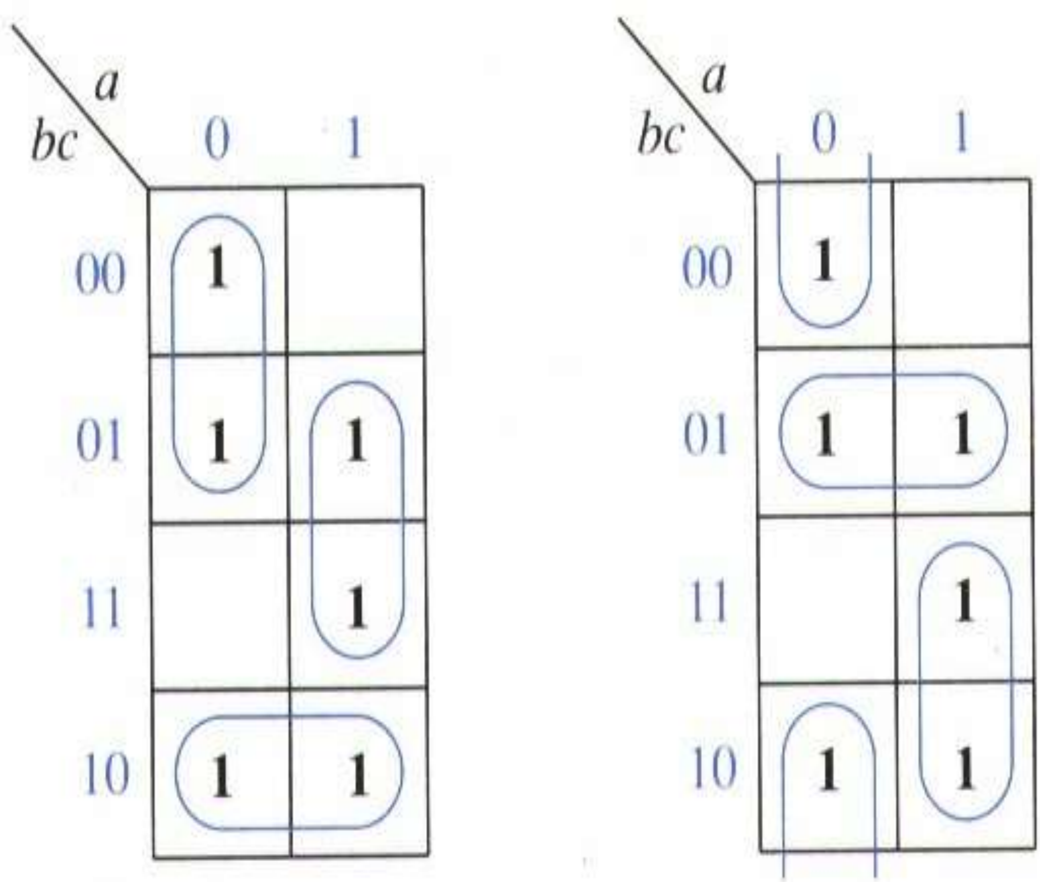
- If a term is covered by two other terms, that term is redundant. That is, it is a consensus term.
- Example: yz is the redundant.



$$xy + x'z + yz = xy + x'z$$

More Than Two Minimum Solutions

- $F = \sum m(0,1,2,5,6,7)$



$F = a'b' + bc' + ac$ $\qquad$ $F = a'c' + b'c + ab$

4-Variable K Map

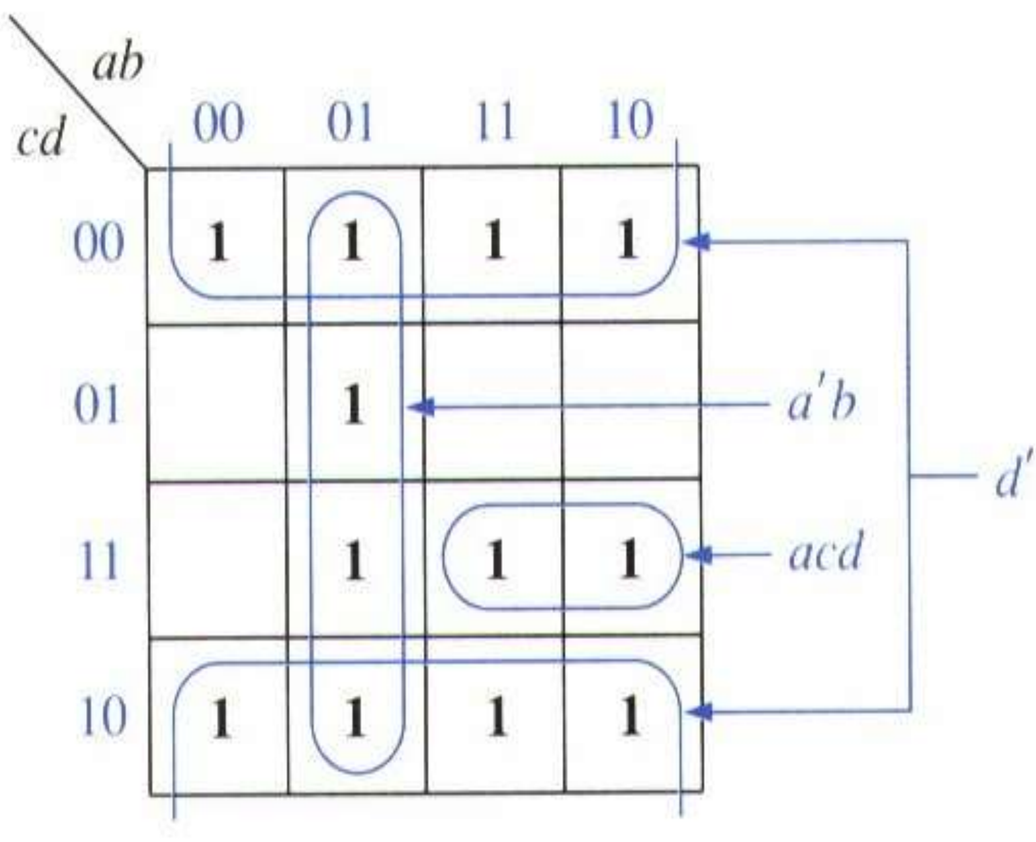
- Each minterm is adjacent to 4 terms with which it can combine.
  - 0, 8 are adjacent squares
  - 0, 2 are adjacent squares, etc.
  - 1, 4, 13, 7 are adjacent to 5.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Plot a 4-variable Expression

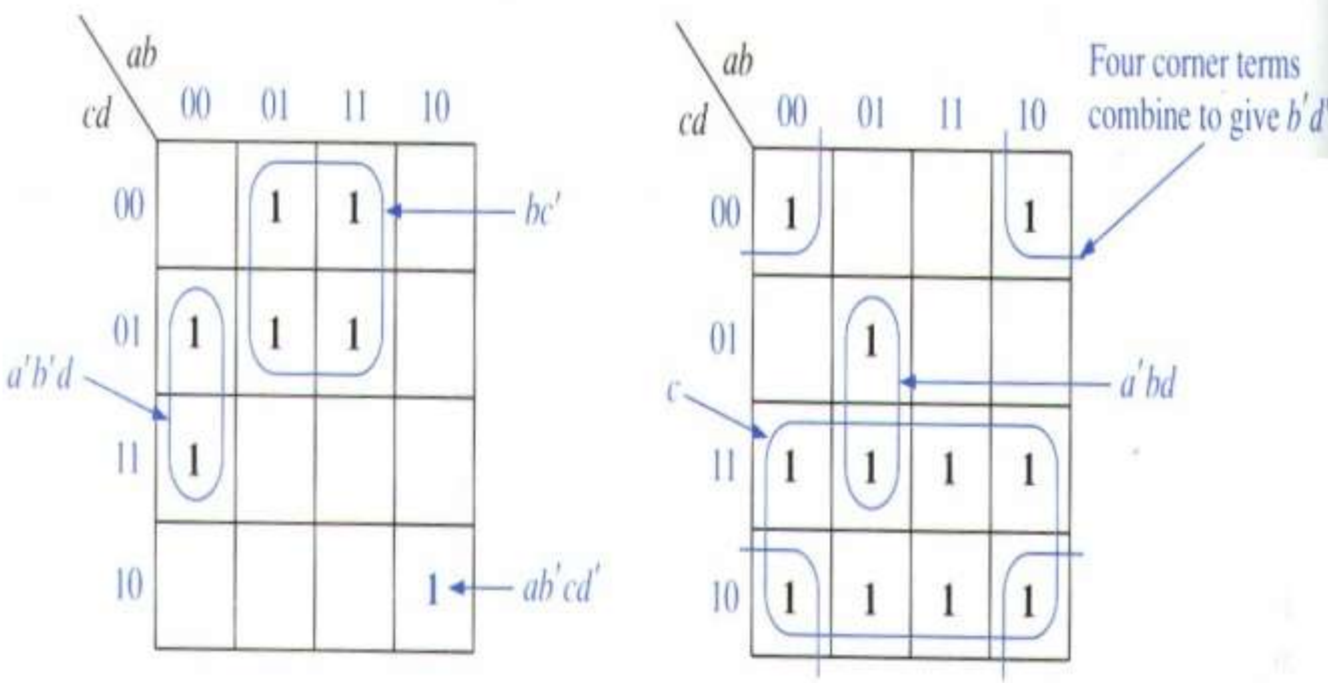
- F(a,b,c,d) = acd + a'b + d'

  acd = 1 if a=1, c=1, d=1



Simplification Example

- Minterms are combined in groups of 2, 4, or 8 to eliminate 1, 2, 3 variables.
- Corner terms.



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

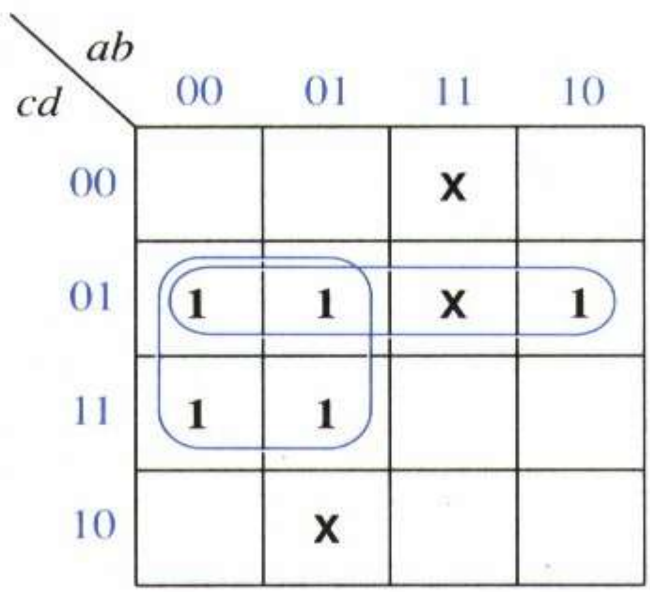
(a)

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

Simplification with Don’t Care

- Don’t care “x” is covered if it helps. Otherwise leave it along.



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Eg Simplify Boolean fn

$$F = A'B'C' + B'CD' + A'BCD' + AB'C'$$

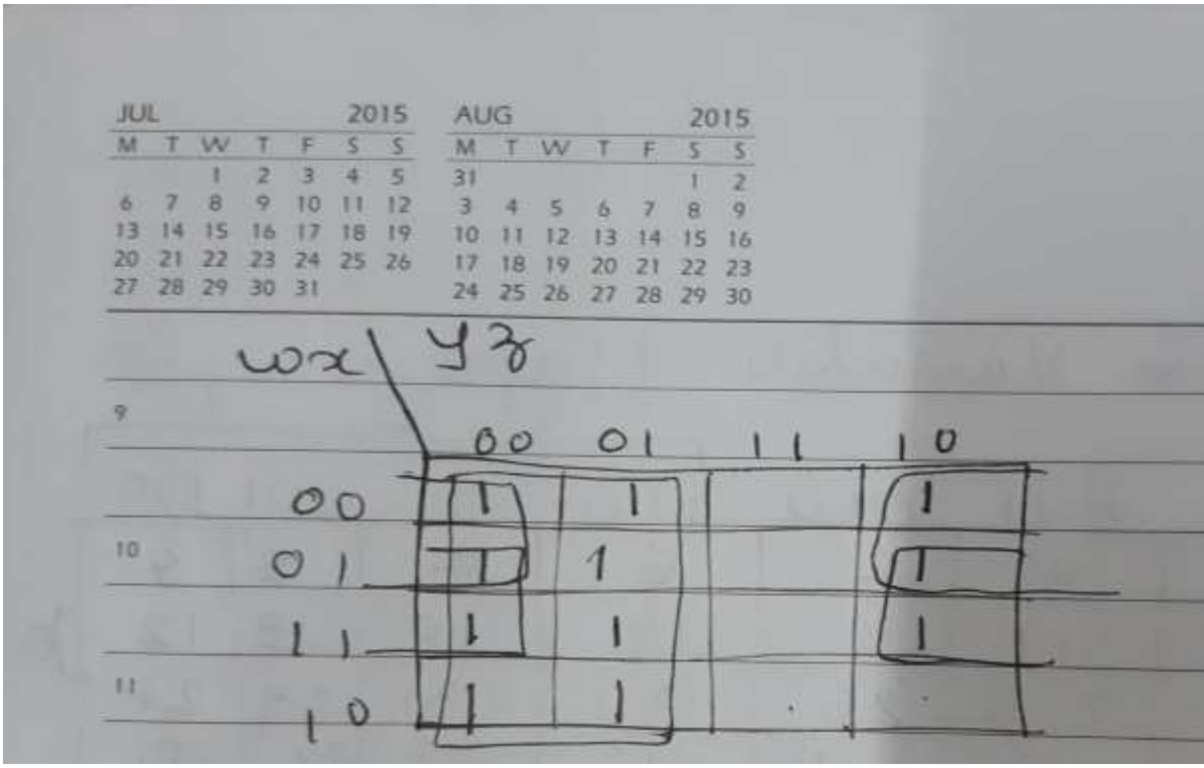
AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$$F = B'C' + B'D' + A'CD'$$

Eg Simplify Boolean fn

$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$$F = y' + w'z' + xz'$$


JUL 2015
M T W T F S S
1 2 3 4 5
6 7 8 9 10 11 12
13 14 15 16 17 18 19
20 21 22 23 24 25 26
27 28 29 30 31
AUG 2015
M T W T F S S
31 1 2
3 4 5 6 7 8 9
10 11 12 13 14 15 16
17 18 19 20 21 22 23
24 25 26 27 28 29 30
wx \ yz
00 01 11 10
00 1 1 1
01 1 1 1
11 1 1 1
10 1 1

Eg 3.3 Simplify the Boolean fn.

$$F = A'C + A'B + AB'C + BC$$

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

$$F = C + A'B$$

Eg Simplify the Boolean fn

$$F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

x \ yz	00	01	11	10
0	1			1
1	1	1	0	1

$$= z' + xy'$$

Eg Simplify the Boolean fn

$F = x'yz + x'yz' + xy'z' + xy'z$

$x \backslash yz$	00	01	11	10
0			1	1
1	1	1		

$F = x'y + xy'$

Eg Simplify Boolean fn

$F = x'yz + xy'z' + xyz + xyz'$

$x \backslash yz$	00	01	11	10
0			1	
1	1		1	1

$F = yz + xz'$

$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$.

Eg 4 Express Boolean fn $F = xy + x'z$ in a product of max term form. First convert fn into OR terms & using the distr law.

$F = xy + x'z$

$= (xy + x')(xy + z)$

$= (x + x')(y + x')(x + z)(y + z)$

$= (x' + y)(x + z)(y + z)$

$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$

$x + z = x + z + yy'$

$= (x + y + z)(x + y' + z)$

$y + z = y + z + xx'$

$= (x + y + z)(x' + y + z)$

$f = M_0 M_2 M_4 M_5$

$F(x, y, z) = \Pi(0, 2, 4, 5)$

The secret of success is constancy to purpose

Eg: Express the Boolean fn $F = A + B'C$ in a Sum of min term.

$\longrightarrow$ $F = A + B'C$

$A = A(B + B')$
$= AB + AB'$
$= AB(C + C') + AB'(C + C')$
$= ABC + ABC' + AB'C + AB'C'$

$B'C = B'C(A + A')$
$= B'CA + B'CA'$.

$F = ABC + ABC' + \underbrace{AB'C} + AB'C' + \underbrace{B'CA} + B'CA'$

$= m_7 + m_6 + m_5 + m_4 + m_1$

It is as hard to take success as it is failure

Five & Six Variable Maps

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

C (columns 110, 111, 101, 100); B (rows 01, 11); A (rows 11, 10); D (columns 011, 010, 110, 111); E (columns 001, 011); E (columns 111, 101)

Eg Simplify the Boolean fn

$F(A, B, C, D, E)$
$= \Sigma(0, 2, 4, 6, 9, 11, 13, 15, 12, 21, 25, 27, 29, 31)$

AB \ CDE								
	1			1	1			1
		1	1			1	1	
		1	1			1	1	
		1					1	

$F = A'B'D' + BE + EA + D'E'$

$F = BE + AD'E + A'B'(DE' + D'E')$
$\downarrow$
$A'B'E'(1)$

Eg Simplify the Boolean fn in
a) Sum of product
b) Product of Sum

$$F(A,B,C,D) = \Sigma(0,1,2,5,8,9,10)$$

AB\CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

$$F = C'B' + B'D' + A'C'D$$
$$= (C' + B')$$

$$F' = AB + CD + BD'$$

$$F = (A' + B') \cdot (C' + D') \cdot (B' + D)$$

Eg Simplify the Boolean fn

$F(w,x,y,z) = \Sigma(1,3,7,11,15)$ &
the don't care conditions
$d(w,x,y,z) = \Sigma(0,2,5)$

Notes

wx \ yz	00	01	11	10
00	X	1	1	X
01		X	1	
11			1	
10			1	

$$F = yz + w'z$$
$$= z(y + w')$$

wx \ yz	00	01	11	10
00	X	1	1	X
01	0	X	1	0
11	0	0	1	0
10	0	0	1	0

$$F' = z' + wy'$$

$$F = z \cdot (w' + y)$$

4 Variable Mapping

$$f = \bar A \bar B \bar C + \bar A \bar B C + \bar A B \bar C + \bar A BC + AB\bar C D$$

A \ BC	00	01	11	10
0	1	1	1	1
1				D

A \ BC				
	$D+\bar D$	$D+\bar D$	$D+\bar D$	$D+\bar D$
				D

$$f_{min} = \bar A + B\bar C D$$

Eg Reduce by Mapping

$$f = \bar A \bar B C D + \bar A \bar B C \bar D + AB\bar C D + AB\bar C\bar D + ABC\bar D + ABCD + \bar A\bar B C\bar D + \bar A B C\bar D + \bar A B\bar C\bar D + A\bar B\bar C D$$

$$F = m_1(D+\bar D) + m_6(B+\bar D) + m_7\,[illegible] + m_5\bar D + m_3 D + m_2\bar D + m_4 D.$$

A \ BC				
		1	D	$\bar D$
	D	$\bar D$	1	1

A \ BC	00	01	11	10
0		$D\bar D$	$D+$	$\bar D$
1	D	$\bar D$	$D+\bar D$	$\bar D+D$

$$f = \bar B C\bar D + \bar A C D + AB + A\bar C D$$

Eg Reduce by Mapping

$$f = \bar{A}B\bar{C} + \bar{A}\bar{B}CD + \bar{A}BC\bar{E} + \bar{A}B\bar{C}E + A\bar{B}C + ABC + AB\bar{C}\bar{D}$$

A \ BC	00	01	11	10
0	1	D	$\bar{E}$	E
1		1	1	$\bar{D}$

$$f = m_0 + m_1 D + m_3 \bar{E} + m_2 E + m_5 + m_7 + m_6 \bar{D}$$

$$f = AC + AB\bar{D} + BC\bar{E} + \bar{A}\bar{B}D + \bar{A}CE + \bar{A}B\bar{C}$$

Eg Reduce by Mapping

$$f = \bar{A}\bar{B}\bar{C}E + \bar{A}\bar{B}C + \bar{A}BCF + \bar{A}B\bar{C} + A\bar{B}CD + AB\bar{C}\bar{F}$$

$$f = m_0 E + m_1 + m_2 + m_3 F + m_5 D + m_6 \bar{F}$$

A \ BC				
	E	1	F	1
		D		$\bar{F}$

$$f = \bar{A}\bar{B}E + \bar{B}CD + B\bar{C}\bar{F} + \bar{A}BF + \bar{A}\bar{B}C$$

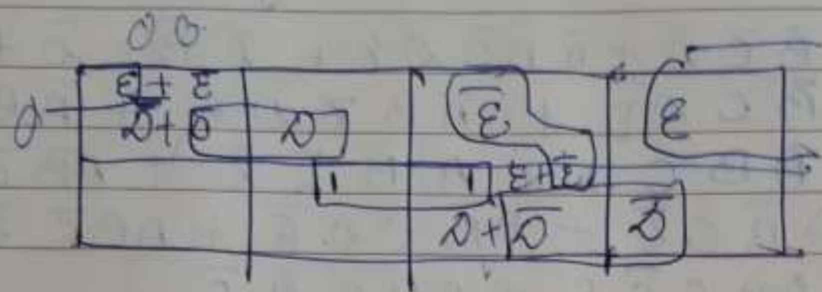
Eg Reduce by mapping

$$f = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}DE + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD\bar{E} + \bar{A}B\bar{C}DF + \bar{A}BC\bar{D}E + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}DE + A\bar{B}C\bar{D} + A\bar{B}CD\bar{E} + AB\bar{C}\bar{D} + ABC\bar{D}E + ABCD\bar{F}$$

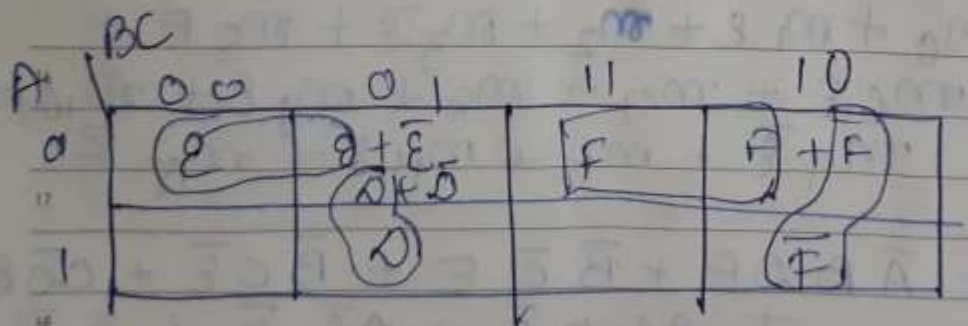
AB \ CD				
	1	E	$\bar{E}$	1
		F	1	E
	1		$\bar{F}$	E
	1	E	$\bar{E}$	1

$$f = m_0 + m_1 E + m_2 + m_3 \bar{E} + m_5 F + m_6 E + m_7 + m_8 + m_9 E + m_{10} + m_{11}\bar{E} + m_{12} + m_{14} E + m_{15}\bar{F}$$

$$f = \bar{A}BDF + \bar{B}\bar{C}E + \bar{B}C\bar{E} + C\bar{D}E + BCD\bar{F} + A\bar{C}\bar{D} + \bar{B}\bar{D}$$



$$\bar{A}\bar{B}D + AC + AB\bar{D} + E\bar{A}\bar{C} + BC\bar{E} + \bar{A}\bar{B}\bar{C}$$



$$\bar{A}\bar{B}E + \bar{A}BF + \bar{B}CD + B\bar{C}\bar{F} + \bar{A}\bar{B}C$$

AB\CD	00	01	11	10
00	$\bar{E}+E$	E	$\bar{E}$	$\bar{E}+E$
01		F	$F+\bar{F}$	E
11	1		$\bar{F}$	E
10	1 $\bar{E}+E$	E	$\bar{E}$	$\bar{E}+E$

$E\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{E}C + \cancel{\bar{A}EB\bar{C}\bar{D}+}$

$\bar{B}\bar{E}\bar{E}\bar{D} + \boxed{\bar{A}BDF} + BCD\bar{F} +$

$A\bar{C}\bar{D} + A\bar{B}\bar{C} + A\bar{B}C\bar{E} + \cancel{EA\bar{D}}$

$EC\bar{D}$

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

AB \ CD	00	01	11	10
00	1	ε	$\bar{\varepsilon}$	1 $\varepsilon+\bar{\varepsilon}$
01		F	F + F 1	ε
11	1		$\bar{F}$	ε
10	1	ε	$\bar{\varepsilon}$	1 $\varepsilon+\bar{\varepsilon}$

$\cancel{\bar{B}\,\varepsilon}$ $\quad C\bar{\varepsilon}\bar{B}$

$\bar{B}\bar{D} + \bar{B}\,\varepsilon\,\bar{C}D + \cancel{CD\bar{B}\bar{\varepsilon}} +$

$B\,C\bar{D}\,\varepsilon + \bar{A}BFD +$

$B\bar{F}CD + A\bar{C}\bar{D}$

Eg Reduce key Mapping

$$f = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,\bar{B}\,C\,D + \bar{A}B\bar{C}\,\bar{D} + A\bar{B}\bar{C}\,D + ABC\,E + ABC\bar{E} + d\,(A\bar{B}\,C\,D + A\bar{B}\,C\,E)$$

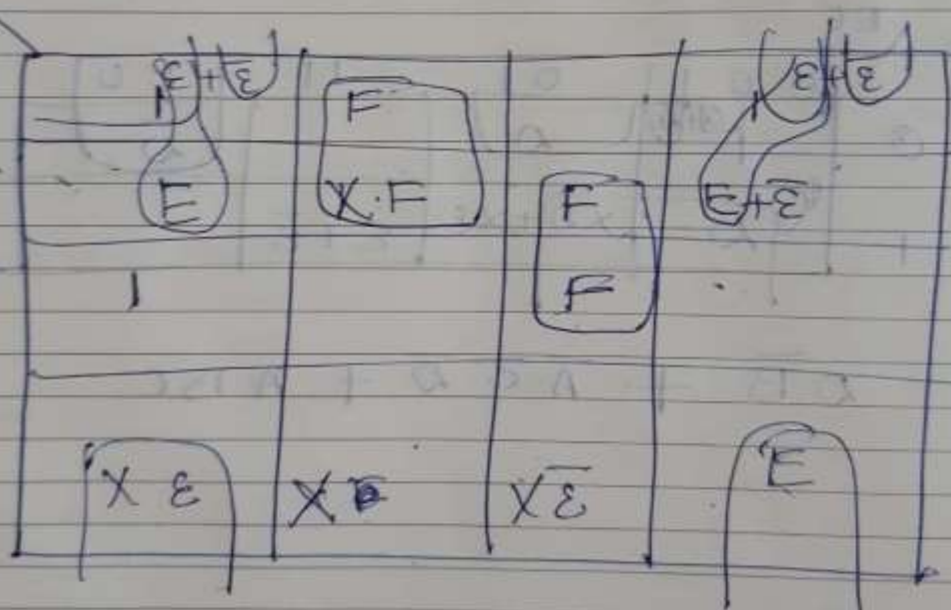
$$f = m_0 + m_1 D + m_2 D + m_4 D + m_7 E + m_7 \bar{E} + d\,(m_5 D + m_5 E)$$

A \ BC	00	01	11	10
0	$D+\bar{D}$	D		$\bar{D}$
1	D	$xD + xE$	$E + \bar{E}$	

$$D\bar{B} + \bar{A}\,\bar{C}\,\bar{D} + ABC$$

eg Reduce by Mapping

$$f = m_0 + m_1 F + m_2 + m_4 E + m_6 (E + \bar{E}) + m_7 F + m_{10} E + m_{12} + m_{15} F + d(m_5 F + m_9 + m_{11} E + m_8 E)$$



Notes

$E \bar{B} \bar{D} + \bar{C} D F \bar{A} + C D F B$

$\bar{A} E \bar{D} + \bar{A} \bar{B} \bar{E} (E) + \bar{A} \bar{C} \bar{D} E$

$A B \bar{C} \bar{D} + \bar{A} E \bar{D}$

If at first you don't succeed, failure may be your style

Get a Minimum POS Using K Map

- Cover 0’s to get simplified POS.
  - We want 0 in each term.

$$f = x'z' + wyz + w'y'z' + x'y$$

First, the 1’s of f are plotted in Fig. 5–14. Then, from the 0’s,

$$f' = y'z + wxz' + w'xy$$

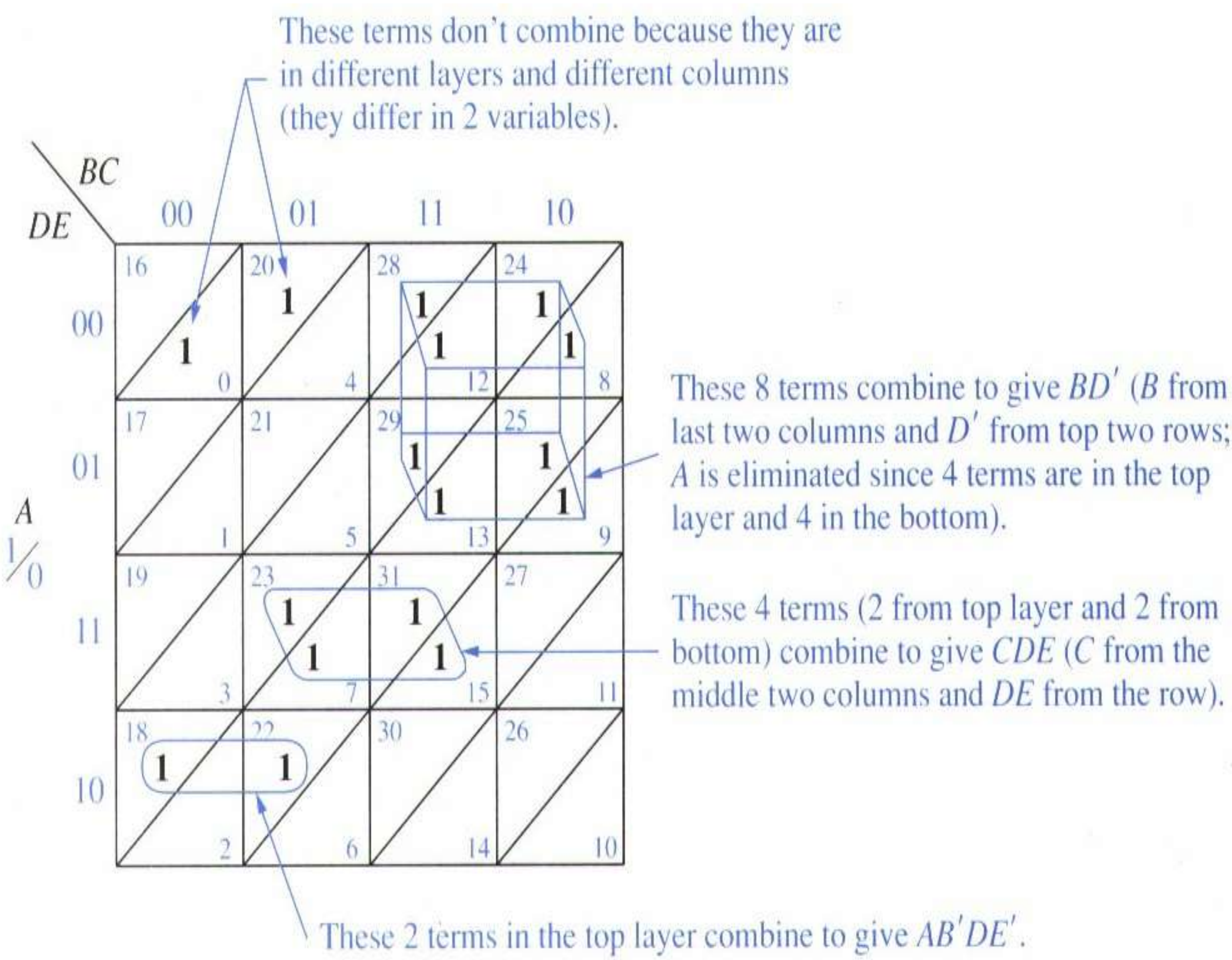
and the minimum product of sums for f is

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

5-Variable K Map

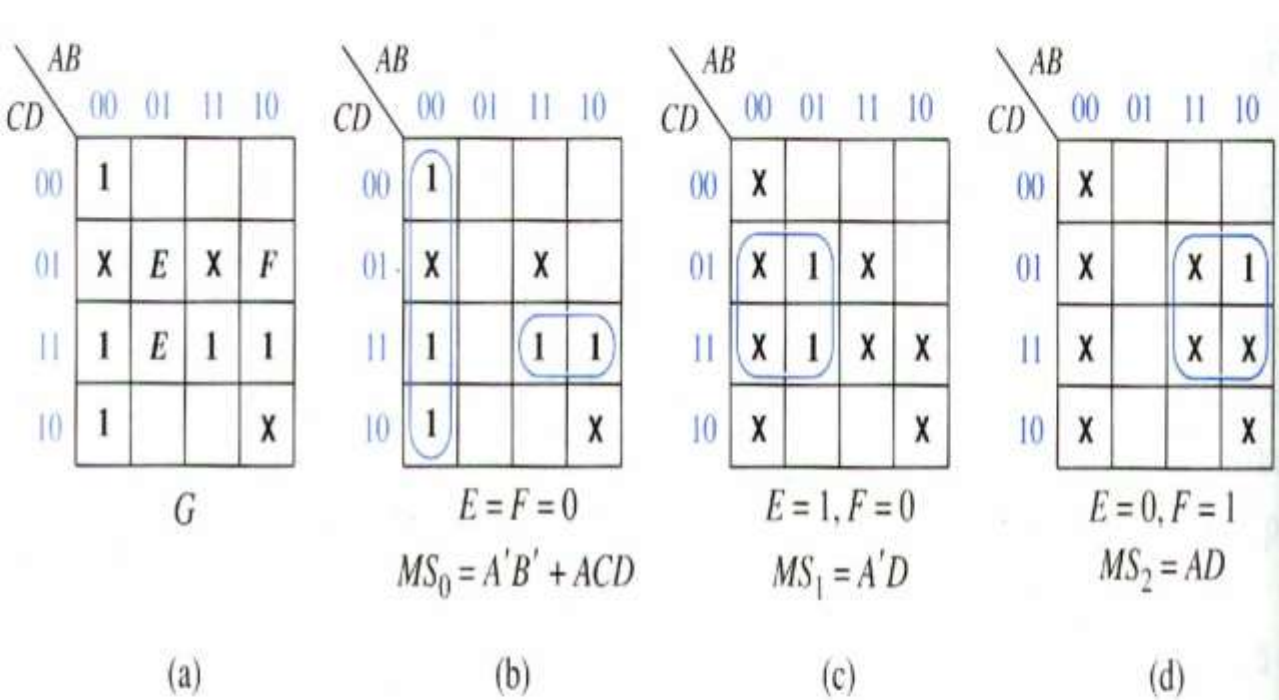
– Use two 4-variable map to form a 5-variable K map (16 + 16 = 32) (A,B,C,D,E)
  - A' in the bottom layer
  - A in the top layer.



Simplification Using Map-Entered Variables

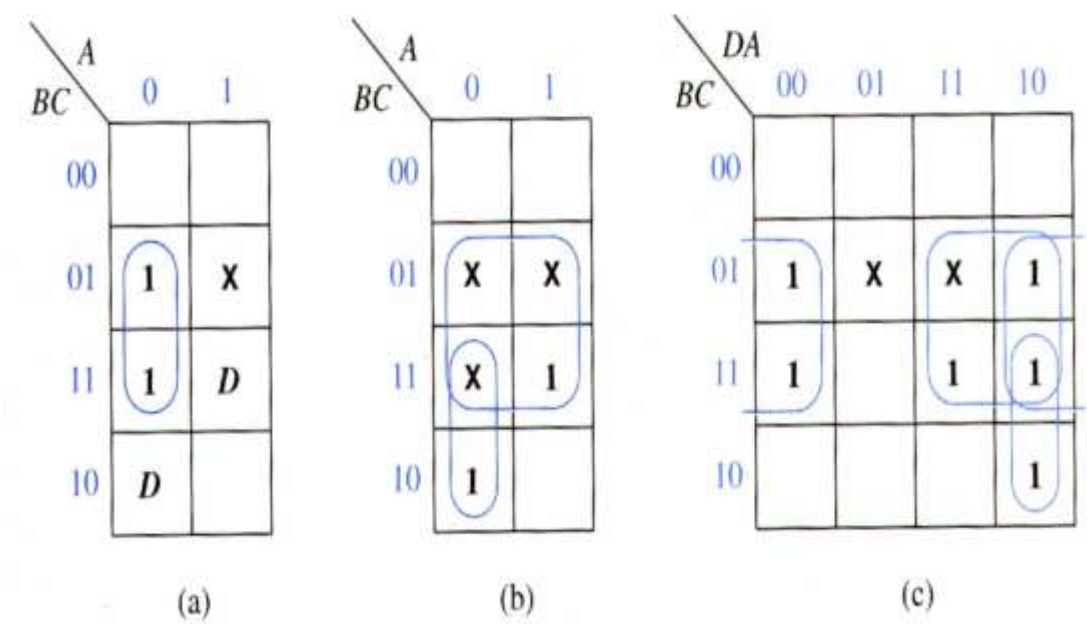
– Extend K-map for more variables.

  – When E appears in a square, if E = 1, then the corresponding minterm is present in the function G.
  – G (A,B,C,D,E,F) = $m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$ + (don't care terms)



Map-Entered Variable

- F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C), (don't care)
  - Choose D as a map-entered variable.
  - When D = 0, F = A'C (Fig. a)
  - When D = 1, F = C + A'B (Fig. b)
    - two 1's are changed to x's since they are covered in Fig. a.
- F = A'C + D(C+A'B) = A'C + CD + A'BD



General View for Map-Entered Variable Method

- Given a map with variables P1, P2 etc, entered into some of the squares, the minimum SOP form of F is as follows:
- F = MS0 + P1 MS1 + P2MS2 + … where
  - MS0 is minimum sum obtained by setting P1 = P2 .. =0
  - MS1 is minimum sum obtained by setting P1 = 1, Pj = 0 ($j \neq 1$), and replacing all 1's on the map with don't cares.

  - Previously, G = A'B' + ACD + EA'D + FAD.