

# Inverse Probability Weighting

In backdoor criterion part, we learned that  $PA(X)$  always satisfies backdoor criterion. Hence, we can represent postintervention causal effect as such:

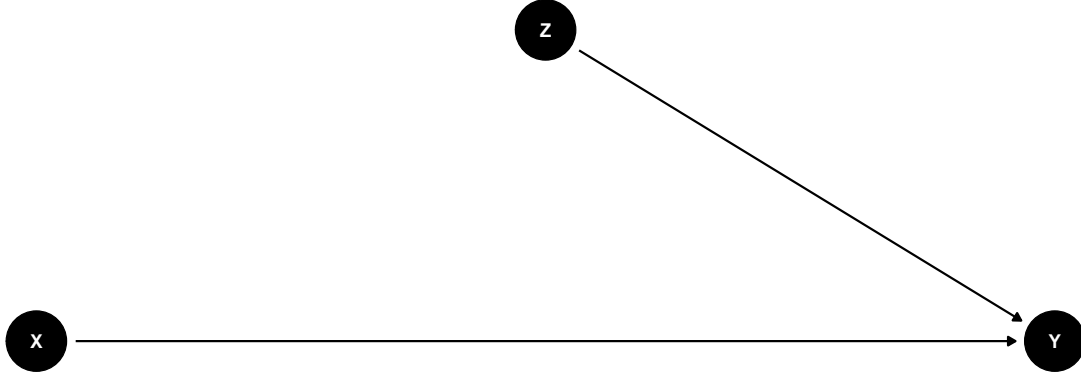
$$P(Y = y \mid do(X = x)) = \sum_z P(Y = y \mid X = x, PA = z)P(PA = z)$$

Modifying this equation, we can obtain the following:

$$P(Y = y \mid do(X = x)) = \sum_z \frac{P(Y=y, X=x, PA=z)}{P(X=x \mid PA=z)}$$

The factor  $P(X = x \mid PA = z)$  in the denominator is known as the “propensity score.”

Now, assume we have a graphical model with an intervention,  $do(X=x)$ , on the model.



$$\begin{aligned} &P(Y = y \mid do(X = x)) \\ &= P_m(Y = y \mid X = x) \quad (\text{by definition}) \\ &= \sum_z P_m(Y = y \mid X = x, Z = z)P(Z = z \mid X = x) \quad (\text{Bayes' rule}) \\ &= \sum_z P_m(Y = y \mid X = x, Z = z)P(Z = z) \quad (X \perp\!\!\!\perp Z) \\ &= \sum_z P(Y = y \mid X = x, Z = z)P(Z = z) \quad (\text{invariance relations}) \\ &= \sum_z \frac{P(Y=y \mid X=x, Z=z)P(X=x \mid Z=z)P(Z=z)}{P(X=x \mid Z=z)} \\ &= \sum_z \frac{P(Y=y, X=x, Z=z)}{P(X=x \mid Z=z)} \end{aligned}$$

From this result, we can see that postintervention causal effect can be computed by multiplying the pre-treatment distribution of  $(X, Y, Z)$  by a factor  $1/P(X = x \mid Z = z)$ , propensity score. Namely, each case  $(Y = y, X = x, Z = z)$  in the population should boost its probability by the inverse conditional probability of assignment to a treatment condition given a set of observed covariates. This is the reason why this method is called “inverse probability weighting.”

## Reference

Pearl, J., Glymour, M., & Jewell, N. P. (2019). Causal inference in statistics a primer. Wiley.

Hernán MA, Robins JM (2020). Causal Inference: What If. Boca Raton: Chapman & Hall/CRC.