Monte Carlo Methods 1

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(1) Inverse Transformation Method for standard Laplace sample generation

Method:

$$f(x) = 0.5e^{-|x|} \text{ for } x \in \mathbb{R}$$

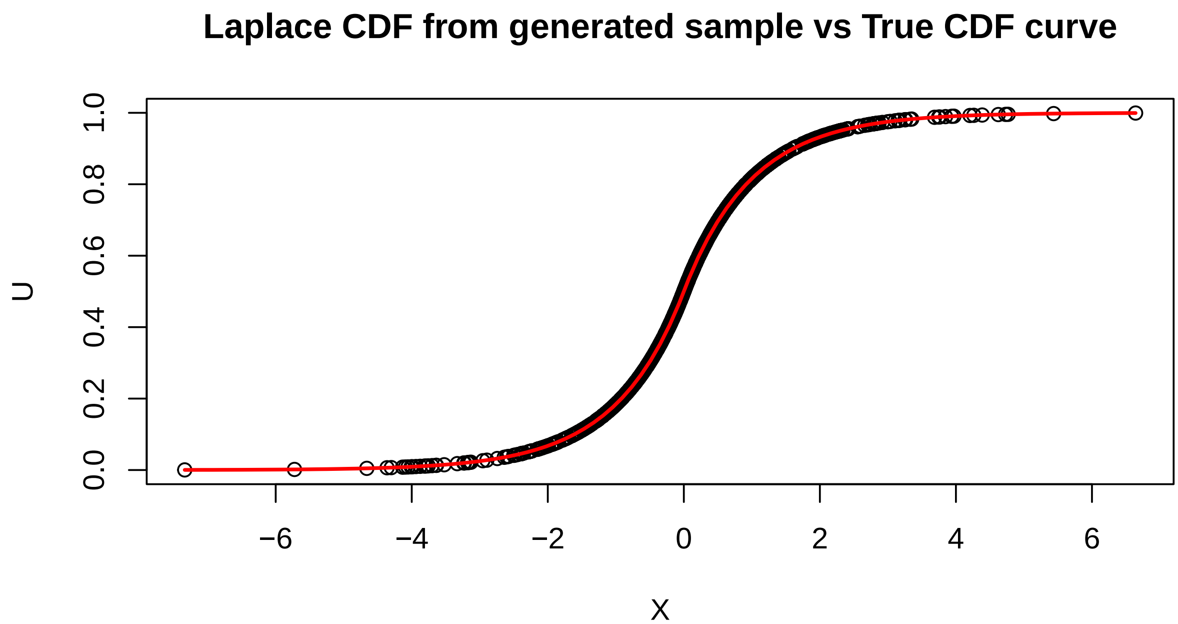
$$\text{If } x \geq 0$$
$$\int_{-\infty}^{0} 0.5e^{t}dt + \int_{0}^{x} 0.5e^{-t}dt = 1 - \tfrac{1}{2}e^{-x}$$

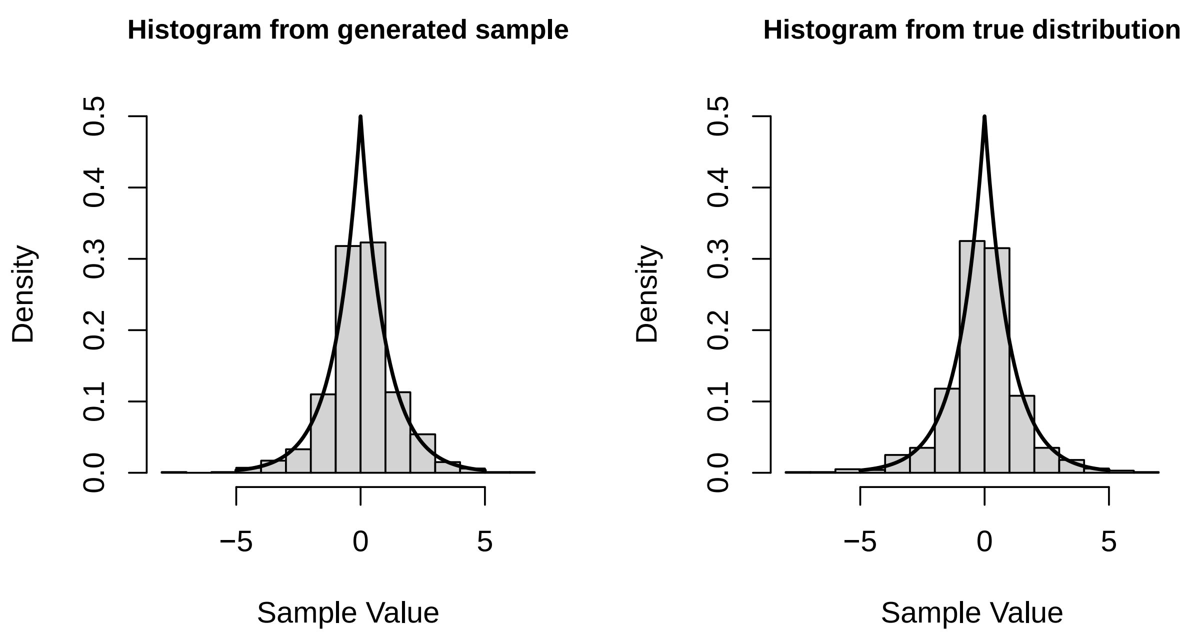
$$\text{If } x < 0$$
$$\int_{-\infty}^{x} 0.5e^{t}dt = \tfrac{1}{2}e^{x}$$

$$U = \begin{cases} 1 - \frac{1}{2}e^{-x} & x \geq 0 \\ \frac{1}{2}e^{x} & x < 0 \end{cases}$$

$$F^{-1}(U) = \begin{cases} log(2 - 2U)^{-1} & U \geq \frac{1}{2} \\ log2U & U < \frac{1}{2} \end{cases}$$

```r
library(ExtDist) # to use the true Laplace distribution
set.seed(77)
n = 1000
U <- runif(n)

X <- (U < 0.5) * log(2 * U) + (U >= 0.5) * -log(2 - 2*U)
T <- rLaplace(1000, mu = 0, b = 1) #sampling from the true standard Laplace distribution

plot(X,U, main = "Laplace CDF from generated sample vs True CDF curve")
curve(pLaplace(x, mu = 0, b = 1), lwd = 2, xlab = "", ylab = "", add = TRUE, col = "red")
```





The histogram generated from a random sample using the inverse transformation method looks very close to the histogram generated from the true standard Laplace distribution. Plus, the overlaid curve on the histogram is the true density curve of Pareto distribution and the shape of the curve aligns well with the shape of the histogram. Hence, it is reasonable to say the random numbers generated from my function truly follows the target distribution.

(2) Inverse Transformation Method for Pareto sample generation

Method:

$$f(x;\alpha,\gamma) = \frac{\gamma\alpha^\gamma}{x^{\gamma+1}} I\{x \geq \alpha\}$$

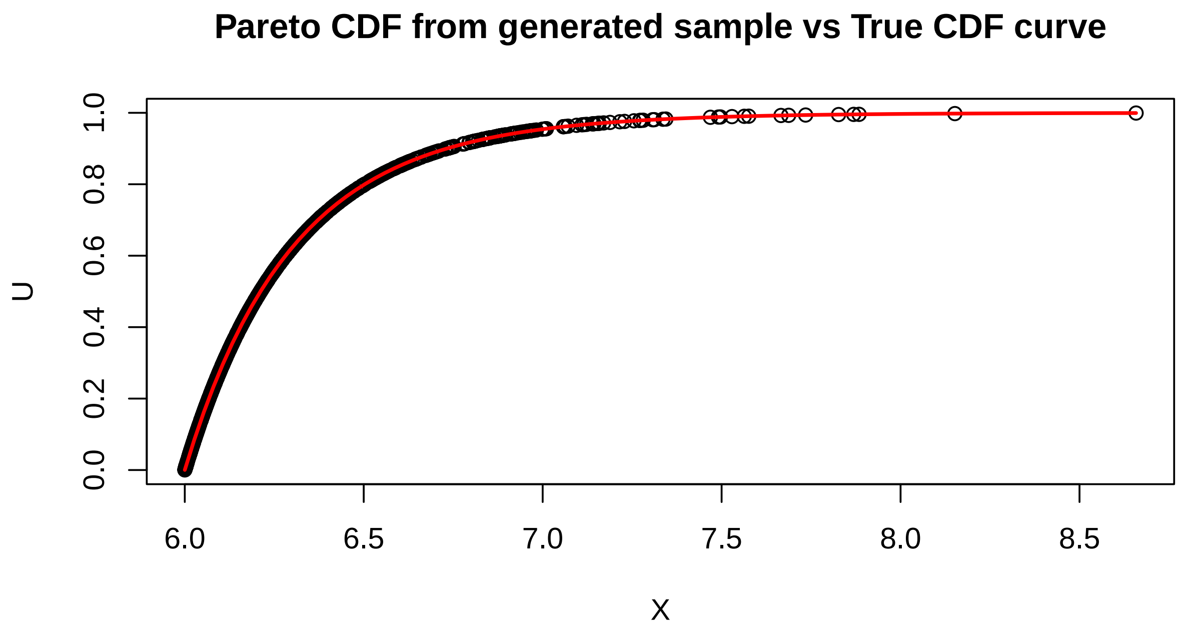
$$\int_\alpha^x \gamma\alpha^\gamma t^{-\gamma-1} dt = \gamma\alpha^\gamma \int_\alpha^x t^{-\gamma-1} dt = 1 - (\frac{\alpha}{x})^\gamma$$

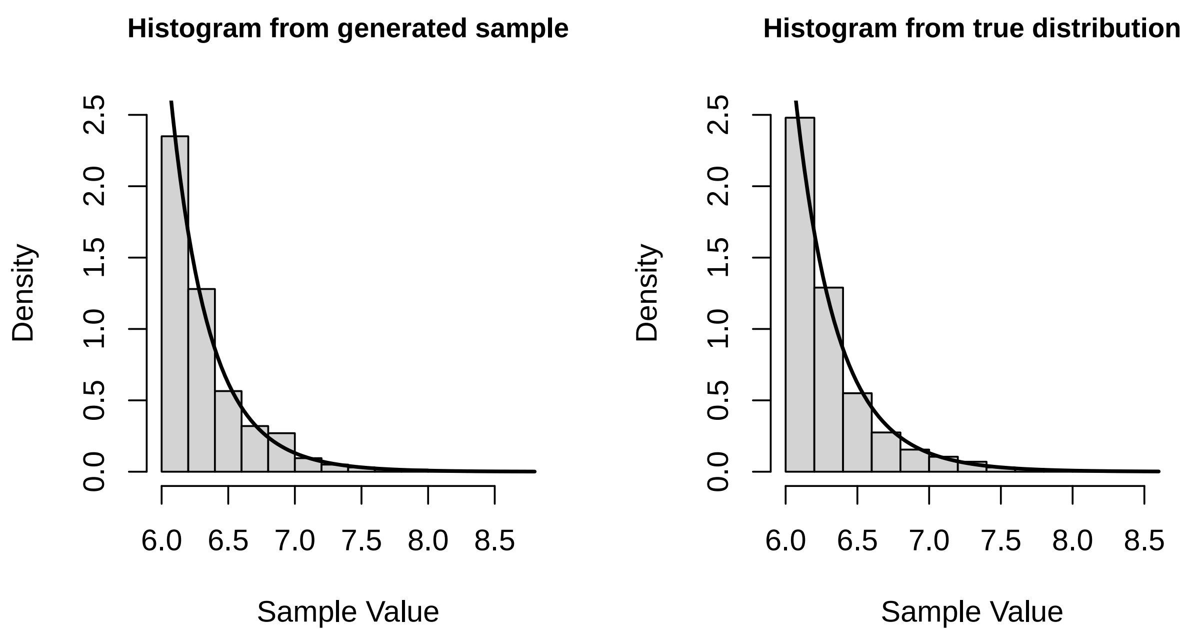
$$U = 1 - (\frac{\alpha}{x})^\gamma$$

$$F^{-1}(U) = \alpha(1-U)^{\frac{-1}{\gamma}}$$

```
library(extremefit) # to use the true Pareto distribution
set.seed(77)
n=1000
U <- runif(n)

Gpar <- function(n, r, a){
  set.seed(77)
  U <- runif(n)

  if (a < 0 | r < 0){
    stop("Wrong parameter value")}
  a*(1-U)^(-1/r)
}

X <- Gpar(1000, 20, 6)
T <- rpareto(1000, 20, 0, 6) #sampling from the true Pareto distribution
summary(X) #checking if x >= alpha
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   6.000   6.098   6.221   6.324   6.430   8.658
```





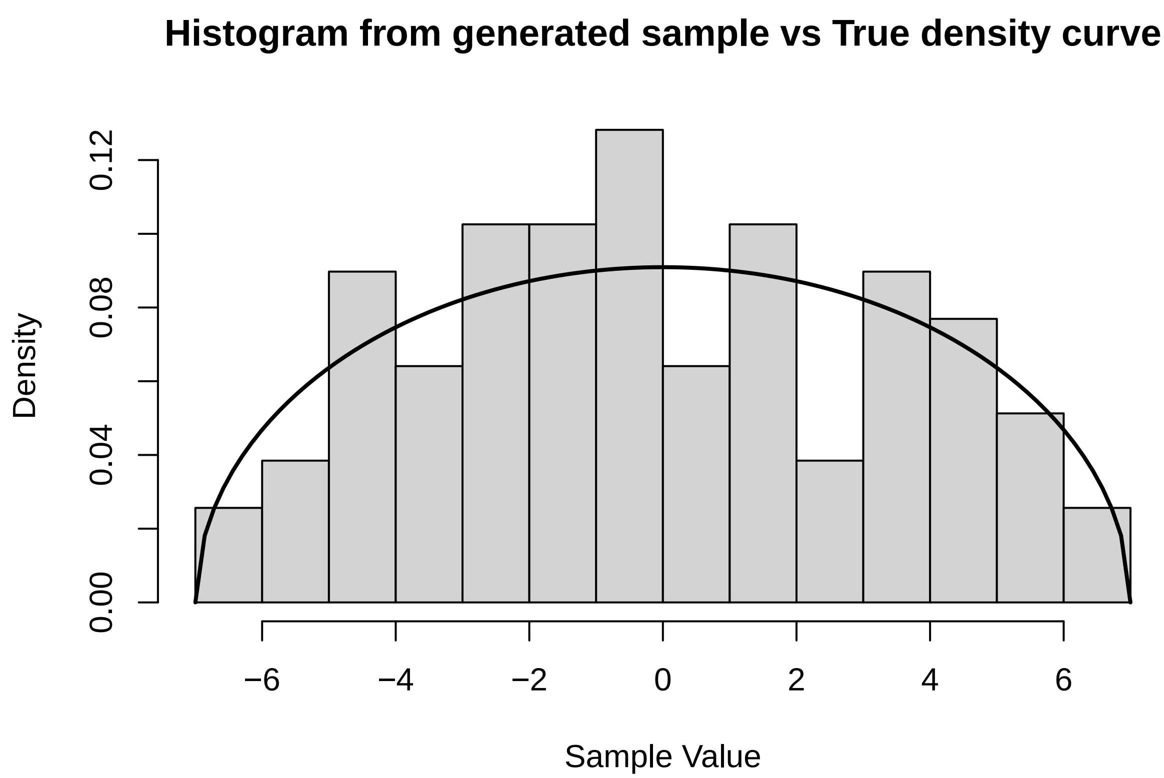
The histogram generated from a random sample using the inverse transformation method looks very close to the histogram generated from the true Pareto distribution. Plus, the overlaid curve on the histogram is the true density curve of Pareto distribution and the shape of the curve aligns well with the shape of the histogram. Hence, it is reasonable to say the random numbers generated from my function truly follows the target distribution.

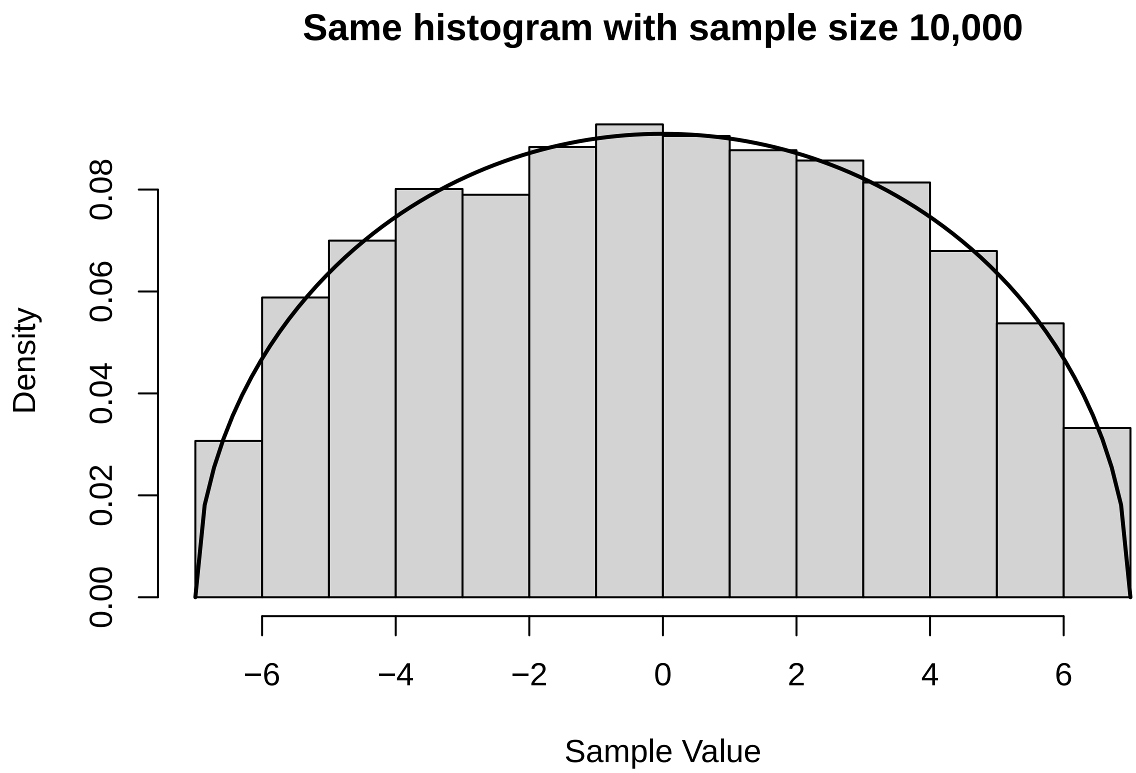
(3) Acceptance/rejection method to generate 100 pseudorandom variable from the pdf

Method:

Given probability density function: $f(x) = \frac{2}{\pi\beta^2}\sqrt{\beta^2 - x^2}, -\beta \leq x \leq \beta \quad \& \quad g(x) = U(-\beta, \beta)$

```
unifdens <- function(x, B){
  return((1/(2*B))*(x >= -B & x <= B))}

given_den <- function(x, B){
  2/(pi*B^2)*sqrt(B^2-x^2)*(-B <= x & x <= B)}

acc_rej <- function(fdens, gdens, B, n){
  set.seed(7)
  x <- runif(n, min = -B, max = B)
  return(x[runif(length(x)) <= fdens(x,B)/(max(fdens(x,B)/gdens(x,B))* gdens(x,B))])}

y <- acc_rej(given_den, unifdens, 7, 100) #beta is 7 and sample size is 100
Large_sample_size <- acc_rej(given_den, unifdens, 7, 10000)
```





The overlaid curve on the histogram is the true density curve of the given function and its shape is semi-circle. The shape of the curve aligns well with the shape of the histogram generated from a random sample using the acceptance/rejection method. Hence, it is reasonable to say the random numbers generated from my function truly follows the target distribution.