Effects of Clipping and Filtering on the Performance of OFDM

Xiaodong Li[†]
Department of Electrical Engineering
Box 352500, University of Washington
Seattle, WA 98105
Email: xdli@ee.washington.edu

Leonard J. Cimini, Jr.
AT&T Labs - Research
791 Holmdel-Keyport Road
Holmdel, NJ 07733
Email: ljc@research.att.com

Abstract — **Orthogonal frequency division multiplexing (OFDM) is an attractive technique for wireless communication applications. However, an OFDM signal has a large peak-to-mean envelope power ratio (PMEPR), which can result in significant distortion when transmitted through a nonlinear device, such as a transmitter power amplifier. We investigate, through extensive computer simulations, the effects of clipping and filtering on the performance of OFDM, including the power spectral density, the bit error rate, and the PMEPR. Our results show that clipping and filtering is a promising technique for the transmission of OFDM signals using realistic linear amplifiers.**

I. Introduction

The transmission of high-bit-rate data in a radio environment is limited by multipath delay spread. Orthogonal frequency division multiplexing (OFDM) is a very attractive technique for overcoming this problem (for example, see [1]). The basic concept of OFDM is to divide the total bandwidth into many narrowband subchannels which are transmitted in parallel. The subchannels are chosen narrow enough so that the effects of multipath delay spread are minimized.

Any multicarrier signal, for example, OFDM, with a large number of subchannels has a large peak-to-mean envelope power ratio (PMEPR), or a large crest factor (CF), defined as the ratio of the peak voltage to the rms voltage. (For a baseband signal, PMEPR and CF are the same. However, for a bandpass signal with a carrier frequency larger than the signal bandwidth, PMEPR is 3 dB less than CF.) For example, a baseband OFDM signal with N subchannels has a PMEPR $= N^2/N = N$. For $N = 128$, PMEPR ≈ 21 dB. When passed through a nonlinear device, such as a transmitter power amplifier, the signal may suffer significant spectral spreading and in-band distortion. The conventional solutions to this problem are to use a linear amplifier or to back-off the operating point of a nonlinear amplifier; both approaches resulting in a significant power efficiency penalty.

Two alternative solutions have been proposed to reduce the PMEPR of the signal input to the amplifier: (i) nonlinear block coding which avoids the transmission of large amplitude sequences [2] and (ii) deliberately clipping the OFDM signal before amplification [3, 4]. In addition, transmit diversity, which is an attractive technique to improve the performance of OFDM over multipath fading channels [5], reduces the number of tones on each transmit branch and therefore also reduces the PMEPR.

Deliberate clipping is a simple approach and, since the large peaks occur with a very low probability, clipping could be an effective technique for the reduction of the PMEPR. However, clipping is a nonlinear process and may cause significant in-band distortion, which degrades the bit-error-rate performance, and out-of-band noise, which reduces the spectral efficiency. Filtering after clipping can reduce the spectral splatter but may also cause some peak regrowth.

Here, we study, through extensive computer simulations, the effects of clipping and filtering for the design of a high-bit-rate OFDM wireless communications system. In Section 2, we describe our basic OFDM system model and performance measures. Then, in Section 3, we provide some simulation results on the performance measurements and optimum design parameters derived from these results. We conclude with a summary in the final section.

II. System Description and Performance Measures

A. System Description

In our simulation, we assume an OFDM scheme with $N = 128$ subchannels. To eliminate the intersymbol interference caused by the multipath delay spread, we add a time guard interval of length 32. A block diagram of the OFDM transmitter is shown in Fig. 1. The QPSK block maps the input data bits into complex symbols. Then, the OFDM modulation is realized using an inverse fast Fourier transform (IFFT). With the complex QPSK symbol denoted as X_k $(0 \le k \le N-1)$, the complex baseband OFDM samples are

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, \quad 0 \le n \le N-1 \ . \qquad (1)$$

If these digital signals are clipped directly, the resulting clipping noise will all fall in-band and cannot be reduced by filtering. To address this aliasing problem, we oversample each OFDM block by padding the original input with zeros

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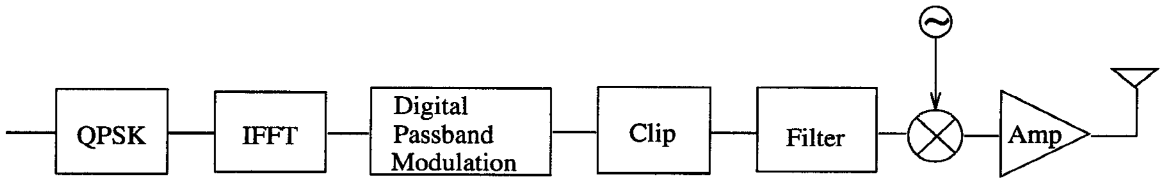

Fig. 1: OFDM transmitter block diagram.

and taking a longer IFFT. An oversampling rate of $m = 8$ is sufficient to reduce the effect of aliasing and is used throughout our simulation. We also rotate the input X_k so that the carrier frequency will be at the center of the passband in the next step. Thus, the input to the IFFT block is

$$X_k{}' = \begin{cases} X_{k+N/2}, & 0 \le k \le N/2-1 \\ 0, & N/2 \le k \le mN - N/2 - 1 \\ X_{k-(mN-N/2)}, & mN - N/2 \le k \le mN-1 \end{cases} \tag{2}$$

and the oversampled OFDM signal after IFFT is

$$x_n = \sum_{k=0}^{mN-1} X_k{}' e^{j2\pi nk/mN}, \quad 0 \le n \le mN-1 \ . \tag{3}$$

Of course, oversampling will increase the implementation complexity. When this is a concern, a smaller oversampling factor can be used together with interpolation.

We could clip the complex OFDM baseband signal by limiting its magnitude while maintaining its phase. However, such a clipper would require extra hardware, such as a divider, and hence might not be suitable for practical implementation. Therefore, in what follows, we clip the real bandpass signal instead of the complex baseband signal.

To generate the bandpass signal digitally, let the bandpass carrier frequency f_c be 1/4 of the sampling frequency f_s. Then, the real bandpass samples can be written as

$$\begin{aligned} y_n &= x_n \cos(2\pi \frac{f_c}{f_s} n) - x_n \sin(2\pi \frac{f_c}{f_s} n) \\ &= x_n \cos(\frac{n\pi}{2}) - x_n \sin(\frac{n\pi}{2}). \end{aligned} \tag{4}$$

This bandpass modulation can be easily realized by alternately taking the real and imaginary part of x_n:

$$y_n = \begin{cases} \mathrm{Re}\{x_n\}, & n = 4l \\ -\mathrm{Im}\{x_n\}, & n = 4l+1 \\ -\mathrm{Re}\{x_n\}, & n = 4l+2 \\ \mathrm{Im}\{x_n\}, & n = 4l+3 \end{cases} \quad l = 1, 2, 3, \cdots \tag{5}$$

The clipping operation on the real bandpass signal is simply

$$z_n = \begin{cases} -A, & \text{if } y_n < -A \\ y_n, & \text{if } -A \le y_n \le A \\ A, & \text{if } y_n > A \end{cases} . \tag{6}$$

where z_n is the clipped signal and A is the clipping level. In the following discussion, we will frequently use a normalized clipping level, which we call the clipping ratio (CR),

$$\mathrm{CR} = A/\sigma, \tag{7}$$

where σ is the rms power of the OFDM signal. It is easy to show that, for an OFDM signal with N subchannels, $\sigma = \sqrt{N}$ for a baseband signal and $\sigma = \sqrt{N/2}$ for a bandpass signal. For our example, $N = 128$ and $\sigma = \sqrt{128/2} = 8$. Therefore, CR $= 1$ is equivalent to $A = 8$, meaning the signal is clipped at the rms power level. A CR of 0.8 means the clipping level is about 2 dB lower than the rms level and a CR of 1.4 means the clipping level is about 3 dB higher than the rms level.

Filtering after clipping is required to reduce the out-of-band clipping noise. In our simulation, we use an equiripple bandpass FIR filter with 103 coefficients. The stopband attenuation is designed to be 40 dB to guarantee a very low interference level to the neighboring OFDM channels. The passband ripple is 1 dB.

B. Performance Measures

In our simulation, we concentrate on the following performance measures: cumulative distribution function (CDF), power spectral density (PSD), crest factor (CF), and bit error rate (BER).

1) Cumulative Distribution Function

When the number of subchannels is large, the baseband OFDM signal can be closely approximated by a complex Gaussian distribution. Since we take the real and imaginary parts of the baseband signal alternately to form the real bandpass signal, the amplitude of the bandpass signal is well approximated by a one-sided Gaussian distribution. We study the CDF of the amplitude to see what percentage of the samples would be affected by the clipping operation. The *log*(1 − CDF) functions are used to show more details of the tails of the distribution.

In our simulation, the bandpass carrier frequency $f_c = f_s/4$, where f_s is the sampling frequency. So, there are only 4 samples per carrier period. Although this sampling rate is sufficient for BER and PSD measurements, it is not enough for the measurement of CDF and CF since some of the envelope peaks of the bandpass signals might be missed. We increase the sampling rate to 16 samples per carrier period by interpolating the clipped and filtered bandpass samples so that the measured CDF and CF are approximately the same as those of the analog envelope. Our study shows that a further increase of the sampling rate does not have much effect on the CDF and CF results.

2) Power Spectral Density

The actual power spectral density of the OFDM signal is quite complicated because of the guard interval and any spectral shaping. Our focus is on the spectral splatter caused by clipping and the effect of filtering. Therefore, the PSD is measured for each OFDM block and then averaged over many blocks to eliminate the effects of the rectangular window. The guard interval is excluded when measuring the PSD.

3) Crest Factor

Our goal is to reduce the CF of the OFDM signal. The peak power of the passband OFDM signal is N^2 and the average power is $N/2$. Therefore the CF without clipping is $2N$ or 24 dB for $N = 128$. Although this absolute number is extensively used in the literature, it does not characterize the OFDM signal very well since the probability of this value occurring is almost zero, as we will show. Here, we will look at the CF at different percentiles of the CDF.

4) Bit Error Rate

Without clipping distortion, the bit error rate is only a function of the received signal-to-noise ratio (SNR), where the noise is additive white Gaussian noise (AWGN) in the receiver. Clipping causes in-band noise, which is approximately white, and which causes a degradation in the BER performance. We investigate this degradation by measuring the BER against the SNR for various clipping ratios. The SNR we used for the BER measurements is the ratio of the average signal power per sample to the average channel noise power per sample. At the front end of the receiver, a filter matched to the filter in the transmitter is used. In our simulation, white Gaussian noise is first generated, then passed through the filter. The SNR is measured at the output of the filter.

III. Results

A. CDF and PSD of the Clipped Signal

In Fig. 2, we show the $log(1 - \text{CDF})$ functions of the clipped and unclipped signals. From Fig. 2, we see that most of the signals are concentrated at low amplitudes even without clipping. For example, for the unclipped signal, about 99% of the amplitudes are less than 20 and about 99.999% are less than 35. Conversely, only 1% of the amplitudes are greater than 20. Since no signal amplitude can go beyond the clipping level, the $log(1 - \text{CDF})$ curves of the clipped signal cut off at that point. Compared with an 24-dB absolute CF of the unclipped signal, the CF of the

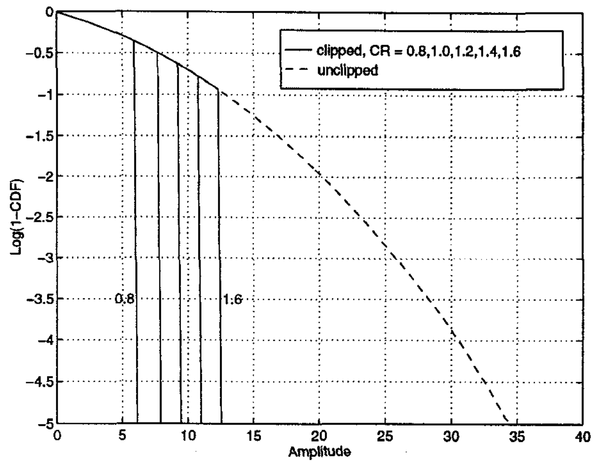

Fig. 2: Log(1 − CDF) function of the amplitude of the clipped signals.

clipped signal is drastically reduced (e.g., for CR = 1.6, CF = 5 dB).

In Fig. 3, we show the PSD of the clipped signal with various CRs from 0.8 to 1.6. The in-band signal attenuation as well as the out-of-band emission caused by clipping is evident. It is not obvious from the figure that clipping also causes in-band distortion, which cannot be reduced by filtering and is the main cause of the BER performance degradation. For CR = 1.4, the out-of-band noise emission power is only 16 dB lower than the signal power. This shows that filtering is necessary to suppress the spectral splatter caused by clipping.

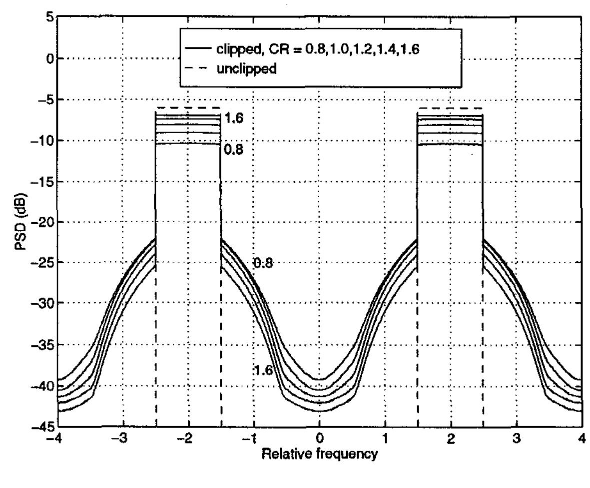

Fig. 3: PSD of the clipped signals.

B. PSD and CDF of the Clipped and Filtered Signal

From Fig. 4, we see that filtering greatly attenuates the out-of-band emission. With CR = 1.4, the spectral sidelobes after filtering are now at least 50 dB lower than the signal mainlobe. The 1-dB in-band ripple caused by the FIR filtering may boost the power of some subchannels while suppressing others. However, with forward error-correction coding across the subchannels, the effect of these ripples should be negligible.

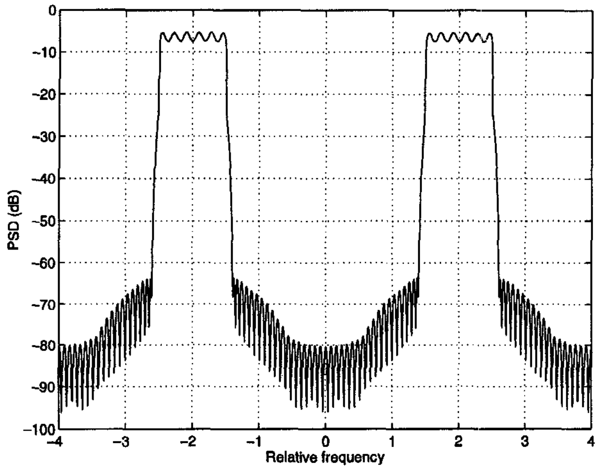

Fig. 4: PSD of the clipped and filtered signal, CR = 1.4.

Filtering will cause peak regrowth, but the question is how much. In Fig. 5, we show the $log(1 - \mathrm{CDF})$ functions with clipping and filtering for various values of CR. Compared with Fig. 2, we see obvious peak regrowth due to filtering. At the 99.9% point, the peak regrowth is 4 to 5 dB. However, the amplitude distribution after clipping and filtering is much more concentrated at low levels than the original unclipped signals.

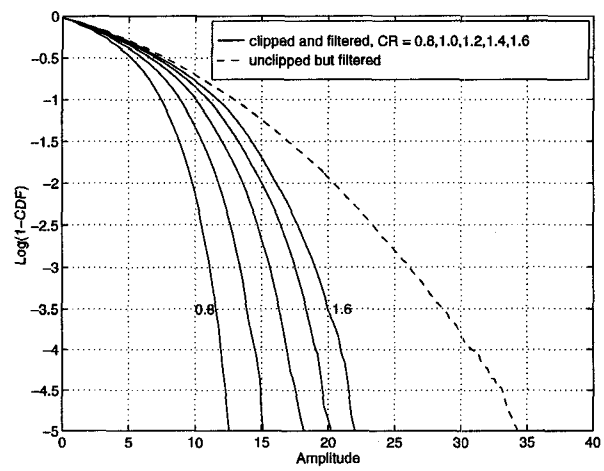

Fig. 5: Log$(1 - \mathrm{CDF})$ function of the amplitude of the clipped and filtered signals.

C. CF

In Fig. 6, we show the CFs of the signals in two categories: unclipped and clipped and filtered. The five dashed lines correspond to the CFs of the unclipped signals at the 90%, 99%, 99.9%, 99.99%, 99.999% points of the CDF respectively. The five solid lines show the corresponding CFs of the clipped and filtered signals.

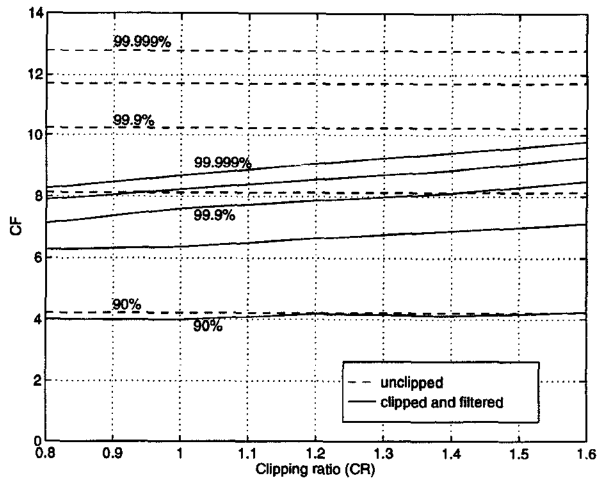

Fig. 6: Crest Factor of the clipped and filtered signals.

At the 90th-percentile, the CF of the clipped and filtered signal is the same as that of the unclipped one due to the peak regrowth caused by filtering. However, the improvement by clipping and filtering becomes greater as the target percentile increases. Comparing the CFs for the unclipped case with those of the clipped and filtered case, we see a compression in the dynamic range of the signal amplitudes. For the unclipped case, the CF increases from 4 to 13 dB when the target is raised from 90% to 99.999%. However, for the clipped and filtered case, the CF increases from 4 to only 9 dB at a CR = 1.4. We conclude that a 9-dB CF, or equivalently, a 6-dB PMEPR, at the 99.999% point is possible with clipping and filtering at a CR around 1.4. This compares favorably with a single-carrier QPSK signal using raised-cosine pulse shaping. In this case, with a roll-off factor of 0.5, we get a PMEPR of about 6 dB [6].

The use of multiple transmit antennas reduces the absolute CF on each antenna since the number of tones transmitted is reduced. However, the CF at the 99.999% point remains the same for both a single-antenna and a multiple-antenna system as long as the number of tones is large enough and the Gaussian distribution assumption of the OFDM signal is valid. This is also confirmed by our simulation with four transmit antennas.

D. BER

In Fig. 7, we show the BER performance, averaged across all the subchannels, with clipping and filtering over an AWGN channel. Clipping causes in-band noise, which results in a BER performance degradation. For hard clipping (CR = 0.8), the degradation is more than 4 dB at the 10^{-2} BER level. But when CR $\geq$ 1.4, less than 1 dB penalty is encountered. Other results, not shown here, indicate that the BER performance on different subchannels is unequal. This is due to the passband ripple of the filter and the fact that the central subchannels are subject to more clipping distortion than the outer ones. However, forward error-correction coding across subchannels will reduce the error floor caused by clipping and compensate for the variation between subchannels.

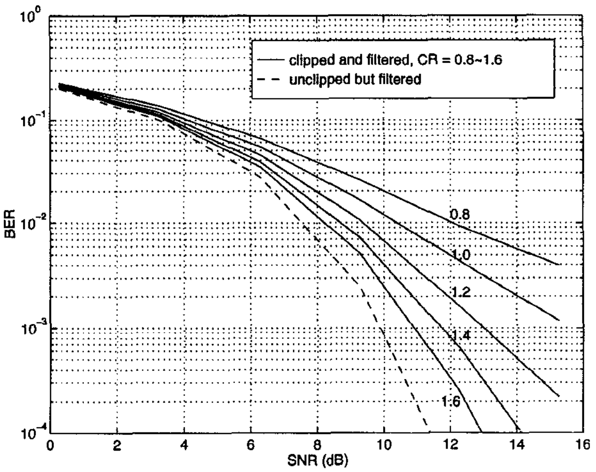

Fig. 7: BER of the the clipped and filtered signals.

E. Optimum Clipping Ratio

A lower CR results in a lower CF, but also causes a larger BER performance degradation due to the clipping noise. So, there is a trade-off in the selection of CR. In Fig. 8, the solid curve shows the reduction of the CF at the 99.999% point offset by the BER degradation at the 10^{-2} level. From this figure, we conclude that an optimum CR might be drawn from the range 1.4-1.6. If the CR is lower than this, the gain in the reduction of CF cannot be justified by the loss in the BER degradation. On the other hand, the BER degradation for CR = 1.6 is already very small, further increasing the CR may not deliver sufficient improvement in BER to compensate the growth in CF.

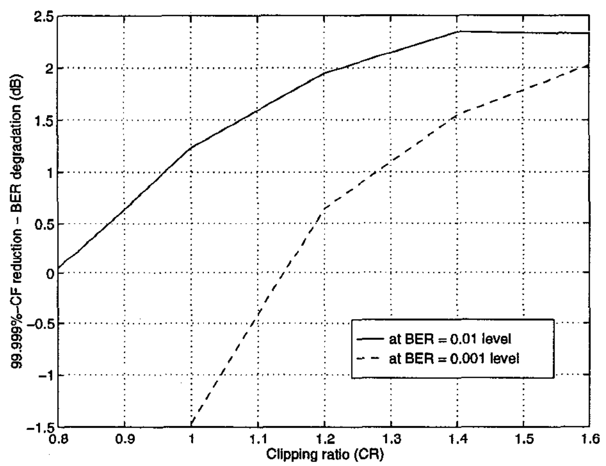

Fig. 8: CF reduction at the 99.999% point offset by the BER degradation.

IV. Summary

In this paper, we investigated the effects of clipping and filtering on the performance of OFDM. Instead of using an absolute CF, the CFs at various percentiles of the CDF are used to better characterize the "peakiness" of an OFDM signal. With a clipping ratio around 1.4 and filtering, the CF of a bandpass OFDM signal with 128 tones at the 99.999% point is reduced from 13 dB to about 9 dB, which is comparable to the absolute CF of a raised-cosine pulse shaped bandpass QPSK signal. A bandpass FIR filter is used to suppress the out-of-band clipping noise to at least 50 dB below the in-band signal level. The SNR degradation at BER = 10^{-2} caused by clipping distortion is less than 1 dB for a clipping ratio above 1.4. We conclude that clipping and filtering is a promising technique to reduce the CF of OFDM signals using realistic linear amplifiers.

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