Transmitter Techniques for Multi-Carrier On-Off Keying

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***Abstract*—The IEEE 802.11ba Task Group is developing a wake-up radio standard which introduces a physical layer based on a signalling technique named Multi-Carrier On-Off Keying. This form of modulation is a hybrid between Orthogonal Frequency Division Multiplexing and On-Off Keying. It brings in both new challenges and possibilities. In this paper we present novel transmitter techniques that improve the performance, either by addressing some shortcomings or by exploiting some of the unique characteristics of Multi-Carrier On-Off Keying.**

***Index Terms*—wake-up radio, WUR, 802.11ba, MC-OOK.**

I. Introduction

Internet of Things (IoT) is expected to vastly increase the number of connected devices. To enable IoT devices to operate on e.g. coin-cell batteries for years, energy consumption is of major concern. The energy consumption for the device is typically dominated by the energy required by the receiver to perform scanning of the channel [1]. This problem has motivated the introduction of wake-up receivers (WURs) as a means to significantly reduce the power consumption in wireless devices. WURs are companion radios to the primary communication radio and are implemented as a separate receiver. The exceptionally low power consumption of a WUR is to a large extent achieved by a reduction of the receiver requirements. The trade-off between power consumption and receiver sensitivity is well-known [2], and poor sensitivity can largely be compensated by using a low data rate for the wake-up packet (WUP). A commonly used modulation for the WUP is on-off keying (OOK). OOK is a binary modulation, where a logical one is represented by sending a signal (on) with a certain time duration, whereas a logical zero is represented by not sending a signal (off) with the same duration. This allows for power efficient receiver implementations, although the sensitivity performance is somewhat compromised.

There are currently activities ongoing in the IEEE 802.11ba task group (TGba) to standardize the PHY and MAC layers for a WUR [3], [4] to complement primary connectivity radios based on the 802.11a/g/n/ac amendments. In TGba, an unusual form of OOK is used, referred to as Multi Carrier OOK (MC-OOK). The idea with MC-OOK is to simplify implementation and accelerate time to market by re-using the IFFT readily available in Wi-Fi Access Points (APs). With the IFFT, an on-signal is generated by populating several subcarriers with non-zero coefficients, while an off-part is generated by not sending anything at all. Throughout the work in TGba it became evident that the use of MC-OOK introduces new challenges and opportunities, in terms for transmitter techniques.

In this paper, we look into some MC-OOK transmitter techniques that result in improved performance. All these techniques depend in a fundamental way on particular characteristics of MC-OOK which, to our knowledge, are not found in other wireless systems based on only either OFDM or OOK. In Section III we study the impact of local geographical regulations that impose limits on the Power Spectral Density (PSD) of any signal transmitted in a specific frequency band. It is shown that suppression of spectral lines and flattening of the PSD allows for increased output power. In Section IV, we consider the problem of designing waveforms that yield MC-OOK signals exhibiting good link performance. We design 256-QAM frequency domain symbols that lead to waveforms with good performance and desirable time domain properties. In Section V, we look at new transmit diversity techniques that are useful for multi-antenna MC-OOK transmitters. Finally, in Section VI, we briefly review Partial OOK, a modification of OOK described in [1], and give some new theoretical results that illustrate the potential advantages of Partial OOK over OOK. But first, we introduce the physical layer used in TGba in Section II.

II. The Physical Layer in IEEE 802.11ba

The TGba proposes to generate the WUP by means of an IFFT, as this block is already available in Wi-Fi transmitters supporting e.g. 802.11a/g/n/ac. A block diagram of a WUP transmitter is exemplified in Fig. 1. To construct the WUP, on-signals are generated by populating the 13 subcarriers in the center, excluding the DC subcarrier. This is illustrated in Fig. 1, where the frequency domain symbols are labelled $\{X_k : k = -6, \cdots -1, 1, \cdots, 6\}$. Off-signals are generated by not transmitting anything. The IFFT has 64 points and is operating at a sampling rate of 20 MHz. Just as in ordinary OFDM, a cyclic prefix (CP) is added after the IFFT to generate symbols with a duration $T = 4\ \mu s$. Since the subcarrier spacing is 312.5 kHz, this generates a 312.5 kHz $\cdot\, 13 \approx 4$ MHz signal.

To use OOK directly for information exchange requires a receiver to estimate a threshold value in order to distinguish between the on- and off-signal. Determining the threshold may be a challenge because it depends on the signal-to-noise ratio (SNR). To avoid this problem, TGba uses Manchester coding, which is a modulation means where the information

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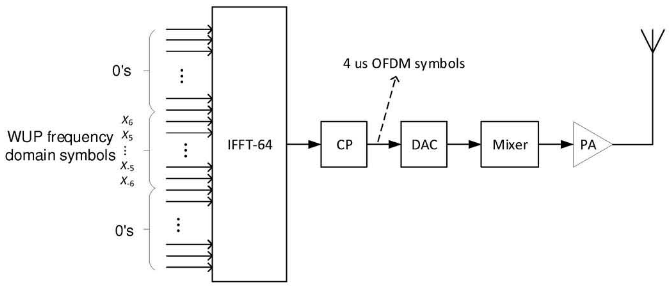


Fig. 1: Block diagram of the WUP transmitter.

is encoded in the transition from states. When Manchester coding is applied to OOK, the information is represented in a code-symbol by a transition from on-signal to off-signal, or vice versa. The decoding of a Manchester coded symbol is done by comparing the first and the second half of the symbols and decide in favour of a logical one if the first half of the symbol is smaller than the second half.

TGba defines two data rates, a Low Date Rate (LDR) and a High Data Rate (HDR), both employ Manchester coding (MC). The LDR adds a repetition code by duplicating each Manchester coded symbol. Since each on/off-signal is 4 μs, this results in a data rate of 62.5 kb/s. In the HDR, the on and off-signals have a duration of 2 μs and are uncoded, except for the MC. The on-signal can be generated by muting the odd-numbered frequency domain symbols $\{X_k : k = -5, -3, -1, 1, 3, 5\}$, using only the samples corresponding to the first 1.6 μs at the output of the IFFT, and inserting a 0.4 μs cyclic prefix. The data rate of HDR is 250 kb/s.

Simulations [5] show that the performance of MC-OOK depends strongly on the choice of the on-signal, with poor MC-OOK signals experiencing a 3 dB disadvantage compared to good MC-OOK signals. Since the MC-OOK signals are not protected by strong channel codes, the packet error rate is largely determined by the performance of the worst MC-OOK on-signal. As a consequence, TGba initially proposed that the same on-signal should be used in all MC-OOK symbols. The rationale is that this allows chipset manufacturers to optimize the on-signal for performance, and also in principle allows the use of coherent receivers in future implementations of WURs.

III. Spectral Lines and Spectrum Flatness

The PSD of the transmitted signal is subject to local geographical regulatory constraints.

First, consider the FCC regulations where, in addition to a maximum TX power of 30 dBm, the PSD P_{xx} is limited to 8 dBm in any 3 kHz bandwidth [6]. Repetition of the on-signal $h(t)$ in every MC-OOK symbol (see Section II) results in spectral lines at multiples of the frequency $F_l = 1/T = 250\ kHz$. This can be verified by direct computation of the PSD. Assuming that $h(t)$ is zero outside of the interval $[0, T]$, an MC-OOK signal $x_N(t)$ comprising N on-signals can be written in the form $x_N(t) = \sum_{n=-N}^{N} b_n h(t - nT)$, where $b_n \in \{0, 1\}$. Denoting the Fourier transform of $h(t)$ by $H(f)$ it can be shown that

$$P_{xx}(f) = \lim_{N\to\infty} \frac{1}{2NT} E\left[\left|\int_{-\infty}^{\infty} x_N(t)e^{-j\pi f t}dt\right|^2\right] = \frac{|H(f)|^2}{T}\left(\frac{1}{4} + \frac{\cos(2\pi fT)}{8} + \frac{3\cos(2\pi 2fT)}{4} + \frac{3\cos(2\pi 3fT)}{8} + \sum_{k=4}^{\infty}\frac{\cos(2\pi kfT)}{2}\right). \quad (1)$$

The divergence of the series in (1) is a problem because it turns out that the FCC regulations are not fulfilled if the full TX power (of 30 dBm) is used.

Second, consider the ETSI regulations, where the maximum transmit power is limited to 10 dBm in any 1 MHz bandwidth [8]. Another consequence of repeating the on-signal is that the PSD may exhibit an uneven distribution of power in the frequency domain. How bad the asymmetry is depends on the on-signal. This is a disadvantage because according to ETSI regulation, a flat PSD is required to achieve the maximum transmit power, which for a 4 MHz signal is roughly 16 dBm. Studies confirm that both spectral lines [9] and lack of PSD flatness [10] severely limit the allowed TX power of MC-OOK transmitters.

It follows from the preceding discussion that it is desirable to modify the on-signal in order to suppress the spectral lines and flatten the PSD. Specifically, modifications to the MC-OOK signal generation are sought such that the advantages of using only one on-signal are preserved, while suppressing the spectral lines and flattening the spectrum.

A. Phase and Cyclic-Shift Randomization

The spectral lines and spectrum asymmetry problems identified in the previous section are both due to the cyclo-stationarity inherent in the the use of a fixed OFDM symbol and MC-OOK. In order to eliminate the cyclo-stationarity it is necessary to randomize the MC-OOK symbols. We will discuss two randomization methods, referred to as phase randomization (PR) and cyclic shift randomization (CSR). PR was proposed in [9] and only addresses the spectral lines problem. In this method, the phase of each occurrence of an on-signal is randomly rotated by either π or 0. This transformation decorrelates the occurrences of an on-signal, thereby suppressing the spectral lines. The drawback is that it preserves any frequency asymmetry present in the PSD of the non-randomized MC-OOK signal. CSR which was first proposed in [11] also flattens the spectrum. It is also a symbol randomization technique for MC-OOK, and is implemented as follows for LDR (extension to HDR is straightforward).

1) Determine a set $\{T_{CS}^n : n = 0, \cdots, N-1\}$ of N delays.
2) For each on-signal, a delay is chosen randomly from T_{CS}^n.
3) Apply a cyclic delay of T_{CS}^n to the corresponding on-signal.

Let's examine CSR in more detail. Since the on-signal is an OFDM symbol, a cyclic delay by T_{CS}^n can be realized in the

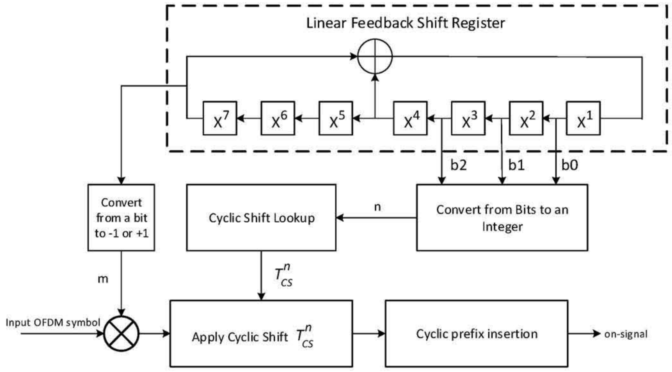


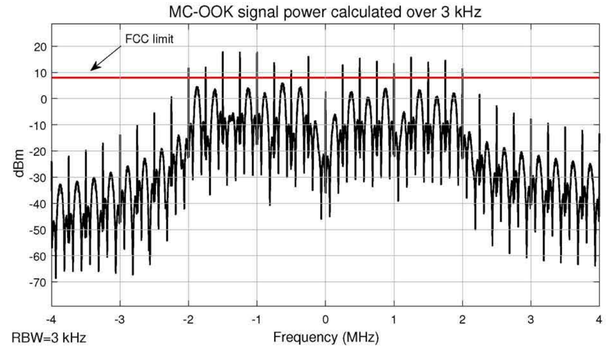
Fig. 2: Block diagram of the symbol randomizer.

frequency domain by multiplication of each frequency domain symbol X_k by the complex phase $e^{-j2\pi k\Delta f T^n_{CS}}$, where $\Delta f = 312.5$ kHz is the subcarrier spacing. Hence, when applying CSR, the k^{th} subcarrier in any occurrence of an on-signal is multiplied by $e^{j\theta_k}$, where θ_k is a phase chosen randomly (with equal probability) from the set of phases $\{-2\pi k\Delta f T^n_{CS} : n = 0, \cdots, N-1\}$.

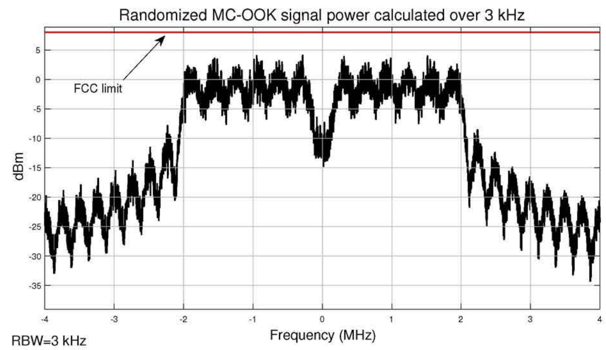
An appropriate choice of the delay set is $N = 8$ and $T^n_{CS} = -n \cdot 400\ ns$, because it yields the phases $-2\pi\Delta f T^n_{CS} = \frac{2\pi}{8}n$, for $n = 0, \cdots, 7$. Hence, the phase θ_k is chosen at random from the set $\Theta(k) = \{\frac{2\pi}{8}kn \mod 2\pi : n = 0, \cdots, 7\}$. Note that for some values of k, such as $k = 4$, the set $\Theta(k)$ may contain fewer than 8 different elements. However, since $\sum_{n=0}^{7} e^{j2\pi nk/8} = 0$ for any k not divisible by $N = 8$, it can be shown that these phase rotations are independent, zero mean random variables. Therefore, the result of CSR is a de-correlation of the occurrences of the on-signal and suppression of the spectral lines. This argument shows that CSR is a form of phase randomization, where possibly different random phase shifts are applied to different subcarriers. In addition, it is seen that each occurrence of the on-signal is an OFDM symbol generated from a set of 8 different sets of frequency domain symbols. Denoting the 8 different on-signals by $h_m(t), m = 1, \cdots, 8$, and their corresponding Fourier transforms by $H_m(f)$, it can be shown that the PSD after CSR is $P_{xx}(f) = \frac{1}{8T}\sum_{m=1}^{8}|H_m(f)|^2$. This smoothing results in a flatter PSD, as will be exemplified in Section III-B.

A few additional notes:

- In a practical implementation, the random numbers used in PR and CSR may be generated by means of a Linear Feedback Shift Register (LFSR). We use the LFSR with generator polynomial $p(x) = x^7 + x^4 + 1$ from 802.11.
- PR and CSR may be conveniently combined with Cyclic Shift Diversity, a technique commonly used in OFDM transmitters with multiple TX chains. In this case, a fixed per-antenna cyclic shift, say τ_a for antenna number a, is added to each T^n_{CS}, resulting in a cyclic shift by $T^n_{CS} + \tau_a$.
- The application of CSR does not change the I/Q-diagram of the transmitted signal. It follows that signal properties that depend on the I/Q trajectories, such as Peak to Average Power Ratio (PAPR) or cubic metric are preserved when CSR is applied.



(a) Using the same MC-OOK symbol for each on-signal.



(b) Using PR and CSR to randomize the on-signal.

Fig. 3: PSD of MC-OOK signal integrated over 3 kHz, assuming a total power of 30 dBm.

B. Numerical Evaluation

In this section, we present some numerical results showing the effects of PR and CSR as depicted in Fig. 2. To generate the OFDM on-signal, we use the coefficients $\{X_k : k = -6, \cdots, 6\} = \{1, 1, 1, -1, -1, -1, 0, -1, 1, -1, -1, 1, -1\}$, which provides good performance in a wide variety of propagation conditions [12], [13]. The corresponding PSD, integrated over 3 kHz, is illustrated in Fig. 3a. In the figure, it can be seen that when the total power is 30 dBm, the 8 dBm/3 kHz limit is exceeded by approximately 10 dB. As a consequence, a WUP transmission following the FCC regulations should be transmitted with a power not exceeding 20 dBm, which falls short of the possible maximum of 30 dBm. Moreover, the power accumulated over the negative frequencies exceeds by 1.5 dB the power accumulated over the positive frequencies. This asymmetry is a consequence of the uneven power distribution across frequency. By constraining this MC-OOK signal to obey the ETSI limit of maximum 10 dBm/MHz, the total output power turns out to be 14.5 dBm, which is 1.5 dB short of the 16 dBm maximum.

The addition of the CSR and PR to the MC-OOK transmitter has a significant impact on the PSD. Consider Fig. 3b showing the PSD of the randomized MC-OOK signal integrated over 3 kHz. It can be seen that there is ample margin to the 8 dBm/3 kHz FCC limit. The PSD is also symmetric. By means of this symbol randomization technique, the WUP can have a power of 30 dBm if subject to FCC regulations, and of 15.6 dBm if subject to ETSI regulations. In other words, the technique

TABLE I: Waveform design for the HDR and LDR on-signals.

SC	LDR	HDR
-6	$-0.6903 - 0.3835i$	$0.2301 + 0.5369i$
-5	$-0.5369 + 0.6903i$	$0 + 0i$
-4	$-0.0767 + 0.0767i$	$0.0767 + 1.1504i$
-3	$0.6903 + 1.1504i$	$0 + 0i$
-2	$1.1504 - 0.6903i$	$-0.3835 + 0.9971i$
-1	$-0.6903 + 0.0767i$	$0 + 0i$
0	$0 + 0i$	$0 + 0i$
1	$0.0767 - 0.6903i$	$0 + 0i$
2	$0.6903 - 1.1504i$	$0.9971 - 0.3835i$
3	$1.1504 + 0.6903i$	$0 + 0i$
4	$-0.0767 + 0.0767i$	$-1.1504 - 0.0767i$
5	$0.6903 - 0.5369i$	$0 + 0i$
6	$0.3835 + 0.6903i$	$0.5369 + 0.2301i$

allows TX power increases of 10 dB or 1.1 dB depending on the regulatory domain. Moreover, the link performance is identical to the performance obtained using an optimized (non-randomized) MC-OOK signal design.

IV. Waveform Design

Beside the flat frequency response condition mentioned in the previous section, another desirable characteristic for MC-OOK waveforms is low PAPR. Waveforms with a flat frequency response give the best performance in frequency selective channels, due to improved frequency diversity. On the other hand, waveforms with low PAPR allow for relaxed linearity requirements in the transmitter, and when an envelope detector is used at the receiver, give the best performance in additive white Gaussian noise (AWGN), or frequency flat channels. Since OFDM signals are well known for their high PAPR, it is challenging to design an on-signal possessing both a flat frequency response and low PAPR at the same time. Using the following methodology results in on-signals with the desirable characteristics.

1) Generate a time domain single carrier waveform with constant envelope and 4 MHz bandwidth by means of continuous phase modulation, such as Gaussian minimum shift keying.
2) Transform the time domain waveform to the frequency domain via a discrete Fourier transform.
3) Quantize the Fourier coefficients to the nearest 256-QAM symbol (or any other desirable modulation order).
4) The QAM symbols corresponding to inactive subcarriers are muted.

By performing the above, OFDM frequency domain coefficients are obtained for either LDR or HDR, depending on which subcarriers are set to zero.

A. Numerical Evaluation

Examples of on-signal for the LDR and HDR on-signals are shown in Table I, where the column labelled SC contains the subcarrier number. The PAPR is 1 dB for the LDR and 0.6 dB for the HDR. The I/Q-diagram of the HDR waveform is shown in Fig. 4. The link performance for the HDR is illustrated in Fig. 5, which for comparison also includes the performance of an on-signal found by computer search (and with a PAPR of 3.5 dB) [5]. We can see that in the AWGN case, a link performance increase of about 0.6 dB is obtained. Further simulations [10] show that the proposed on-signal design also exhibits excellent performance in fading channels.

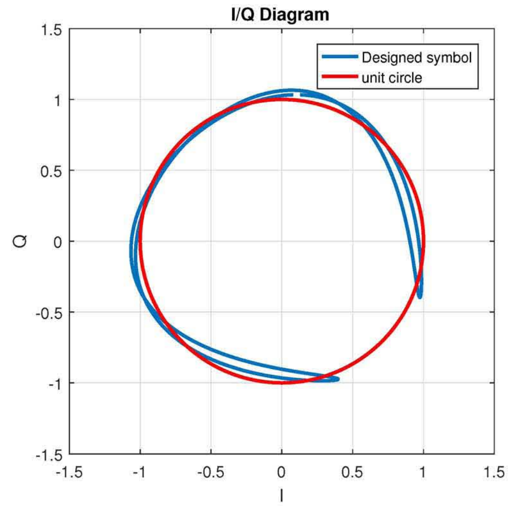


Fig. 4: I/Q diagram of designed on-signal, HDR.

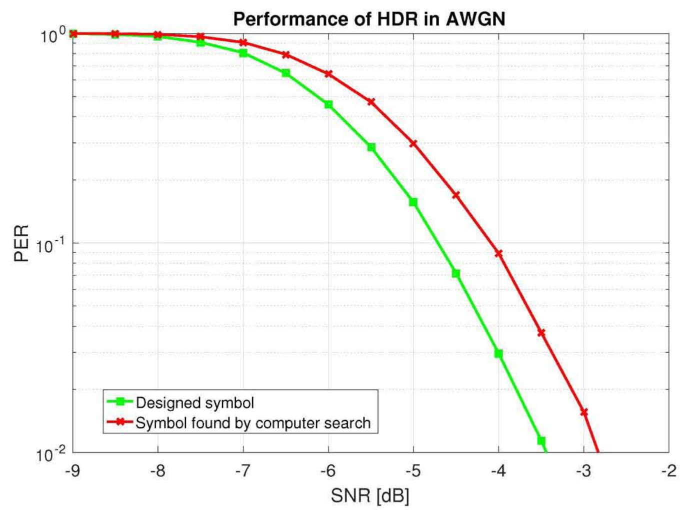


Fig. 5: Performance of designed on-signal.

V. Transmit Diversity

Since many APs have multiple transmit antennas, it is important to consider transmit diversity techniques for TGba WUPs. Cyclic shift diversity (CSD) is a commonly used transmit diversity technique. When CSD is applied, the signal on each antenna is cyclically shifted by a fixed antenna specific delay. The cyclic shifts are chosen as to remove any unintended beamforming. For MC-OOK, there are mainly two differences compared to the flavour of OFDM otherwise used in 802.11. The first difference is that the WUP has a bandwidth of 4 MHz instead of 20 MHz, meaning that the best cyclic shifts for a WUP might be different from the best shifts for an ordinary OFDM packet. This has been studied in TGba and new sets of cyclic shifts for 2-8 antennas have been proposed [14]. The second difference is that, unlike OFDM, information bits do not modulate the phases or amplitudes of the subcarriers, and the receiver only detects the envelope or power of the OFDM symbol. This fact allows a new TX diversity technique, where antenna specific symbols are transmitted from each antenna

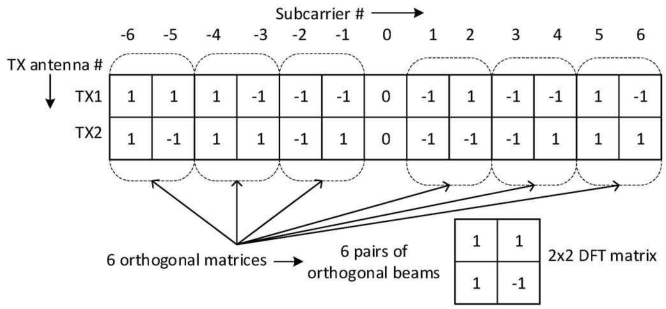


Fig. 6: Example of OSD, showing mapping of frequency domain symbols to TX antennas, LDR.

[15]. These OFDM symbols can be generated by choosing the frequency domain symbols corresponding to any given subcarrier from different entries in the same column of a DFT matrix. By assigning different columns of the DFT matrix to different subcarriers, orthogonal beams can be generated. Note that CSD also generates one beam for each subcarrier, but these beams are generally not orthogonal. We call this technique Orthogonal Symbol Diversity (OSD) and illustrate it by means of an example in the following subsection.

A. Numerical Evaluation

As a simple proof of concept, we provide a numerical evaluation of the 2-antenna case. Fig. 6 shows two sets of frequency domain symbols, and how they are mapped to the antennas. The symbol assigned to the first antenna is the same optimized symbol used to exemplify symbol randomization in Section 2. The frequency domain symbols assigned to the second antenna have been chosen so that orthogonal beams are generated. A total of 12 beams are created, 6 of which point in the same direction, and the other 6 point in the orthogonal direction. This methodology can be extended to more TX antennas and non-binary frequency domain symbols.

In Fig. 7, we see the performance using the fading channel models TGnB and TGnD [16]. We see that OSD performs better than traditional CSD.

VI. Partial MC-OOK

We studied Partial MC-OOK (P-OOK) in [1]. By shortening the on-signal in time, the instantaneous power can be increased while keeping the average power constant. The idea is depicted for the HDR in Figure 8. It is particularly interesting for the MC-OOK used in TGba, since the signal is typically bandlimited by the multicarrier bandwidth, and not by the information rate. In this section we present a theoretical analysis for P-OOK.

To understand the performance of P-OOK, we derive theoretical expressions for the bit error rate (BER) and the HDR packet error rate (PER) in AWGN. To do this we assume: the on-symbol is a constant envelope signal (see Section IV), the receiver employs a square-law detector, a synchronization error results in an offset of k samples from the ideal synchronization position, and the digital baseband signal comprises N_Z samples taken during the period T_Z (defined in

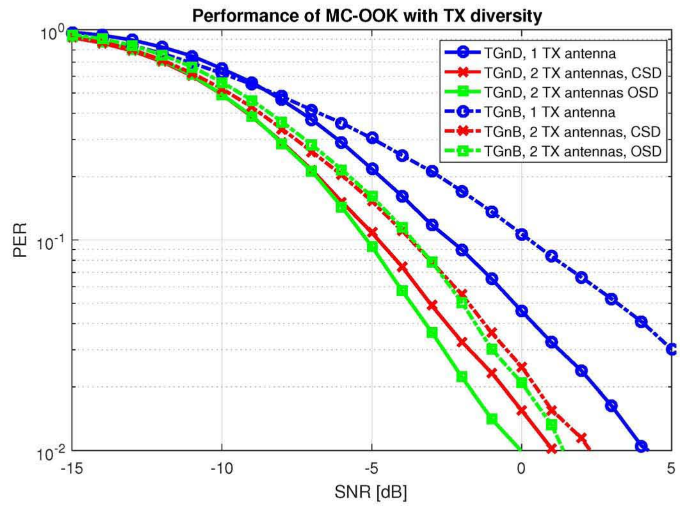


Fig. 7: Link simulation of a WUR. In the CSD and OSD case, two antennas are used.

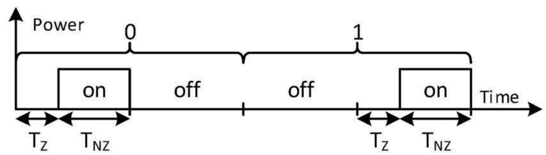


Fig. 8: Partial OOK in time domain.

Fig. 8) and N_{NZ} samples taken during the period T_{NZ} (also defined in Fig. 8). For simplicity we assume $k \leq N_Z$. With these assumptions, we derive bit-error rate expression (2) in the Appendix. Since the HDR transmissions are uncoded, then $\text{PER} = 1 - (1 - BER)^{N_{syms}}$, where N_{syms} is the number of symbols in the packet.

A. Numerical Evaluation

Suppose that the receiver bandwidth is 4 MHz, the payload is 6 bytes and that there is a timing error of $0.25\mu s$ with respect to the ideal synchronization position. Fig. 9 illustrates the performance of OOK and compares it with P-OOK. In this figure, we use HDR for the OOK curve, $T_{NZ} = 2\mu s$ and $T_Z = 0$. For P-OOK we use $T_{NZ} = 1.25\mu s$ and $T_Z = 0.75$.

The PER is calculated theoretically with the help of (2) and also by monte-carlo simulations. It can be seen that P-OOK offers a 2 dB performance advantage, and that the theoretical and simulated packet error rates are in excellent agreement.

VII. Conclusions

OOK is among the simplest forms of signaling and is well suited for WURs. The introduction of WUR in TGba created a desire to re-use OFDM transmitters to generate OOK, which led to the introduction of MC-OOK. With it, a host of practical problems arose, including power limitations imposed by local regulations, the need to optimize the MC-OOK waveforms for performance, the need for multi-antenna transmission schemes, as well as the ever present need to improve coverage. In this paper we have presented and evaluated simple, low

$$\text{BER}=\int_{-\infty}^{0}\int_{-\infty}^{\infty}\left(\frac{1}{1-j\omega\sigma^2}\right)^{N_{NZ}-k}\left(\frac{1}{1+j\omega^2\sigma^4}\right)^{k}\left[\int_0^{\infty}\frac{I_0\left(\frac{2A}{\sigma^2}\sqrt{x\frac{N_Z+N_{NZ}}{N_{NZ}}}\right)}{\sigma^2}e^{-\frac{x+A^2\frac{N_Z+N_{NZ}}{N_{NZ}}}{\sigma^2}-j\omega x}dx\right]^{N_{NZ}-k}\frac{e^{j\omega z}}{2\pi}d\omega dz \quad (2)$$

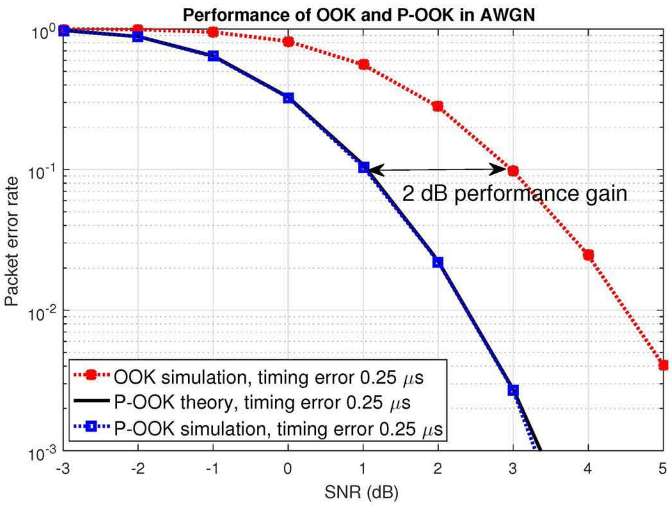


Fig. 9: Performance of OOK and P-OOK for HDR.

complexity, yet ingenious techniques that solve these problems and enhance the link performance of the 802.11 physical layer.

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APPENDIX

To derive the BER, we let the term sample refer to a digital sample at the output of the square-law detector. Suppose that σ^2 is the noise power, obtained by integrating the power spectral density of the thermal noise over the receiver bandwidth. Recall from the definition of k that a synchronization error results in an offset of k samples from the ideal synchronization position. $k=0$ corresponds to ideal synchronization, while a non-zero value of k means that the receiver estimates that the start of the ON part of a symbol lies $|k|$ samples from the optimum sampling position. Under the reasonable assumption that the noise samples are independent identically distributed with a probability density function (PDF) $p_W(w)$ given by

$$p_W(w)=\frac{1}{\sigma^2}e^{-\frac{w}{\sigma^2}}, w\geq 0. \quad (3)$$

The samples corresponding to a signal with constant amplitude A plus noise are i.i.d. with PDF [17]

$$p_X(x)=\frac{1}{\sigma^2}I_0\left(\frac{2A}{\sigma^2}\sqrt{x}\right)e^{-\frac{x+A^2}{\sigma^2}}, x\geq 0, \quad (4)$$

where I_0 is the 0-th modified Bessel function of the first kind.

Assuming that the Manchester coded symbol is on-off and that the synchronization error is $k\geq 0$ samples, a decision statistic Z can be computed as the sum of N_{NZ} samples corresponding to a hypothesized on-part of the signal minus N_{NZ} samples corresponding to a hypothesized off-part of the symbol. The decision is correct whenever $Z>0$, and incorrect if $Z<0$. Due to the synchronization error, Z consists of a sum of $N_{NZ}-k$ i.i.d. samples with a PDF given by (4) plus k i.i.d. samples whose PDF given by (3) minus the sum of N_{NZ} i.i.d. samples whose PDF is given by (3). If $P_W(\omega)$ and $P_X(\omega)$ denote the Fourier transforms of $p_W(w)$ and $p_X(s)$ respectively, then the Fourier inversion formula yields the PDF $p_Z(z)$ of the decision statistic Z, as follows.

$$p_Z(z)=\int_{-\infty}^{\infty}P_W(\omega)^kP_X(\omega)^{N_{NZ}-k}\left(P_W(\omega)^*\right)^{N_{NZ}}e^{j\omega z}\frac{d\omega}{2\pi}. \quad (5)$$

Moreover, by symmetry considerations we have that

$$\text{BER}=\int_{-\infty}^{0}p_Z(z)dz. \quad (6)$$

Combining (3), (4), (5) and (6), and using the fact that the P-OOK transmitter boosts the amplitude A of the ON part of the signal by a factor $\sqrt{\frac{N_Z+N_{NZ}}{N_{NZ}}}$, we arrive at the expression in (2).