CHAPTER 1

# Decision Trees

The analysis of complex decisions with significant uncertainty can be confusing because 1) the consequence that will result from selecting any specified decision alternative cannot be predicted with certainty, 2) there are often a large number of different factors that must be taken into account when making the decision, 3) it may be useful to consider the possibility of reducing the uncertainty in the decision by collecting additional information, and 4) a decision maker s attitude toward risk taking can impact the relative desirability of different alternatives. This chapter reviews decision tree analysis procedures for addressing such complexities.

## 1.1 Decision Trees

To illustrate the analysis approach, a decision tree is used in the following example to help make a decision.

**Example 1.1**

**Product decision.** To absorb some short-term excess production capacity at its Arizona plant, Special Instrument Products is considering a short manufacturing run for either of two new products, a temperature sensor or a pressure sensor. The market for each product is known if the products can be successfully developed. However, there is some chance that it will not be possible to successfully develop them.

Revenue of \$1,000,000 would be realized from selling the temperature sensor and revenue of \$400,000 would be realized from selling the pressure sensor. Both of these amounts are net of production cost but do not include development cost. If development is unsuccessful for a product, then there will be no sales, and the development cost will be totally lost. Development cost would be \$100,000 for the temperature sensor and \$10,000 for the pressure sensor.

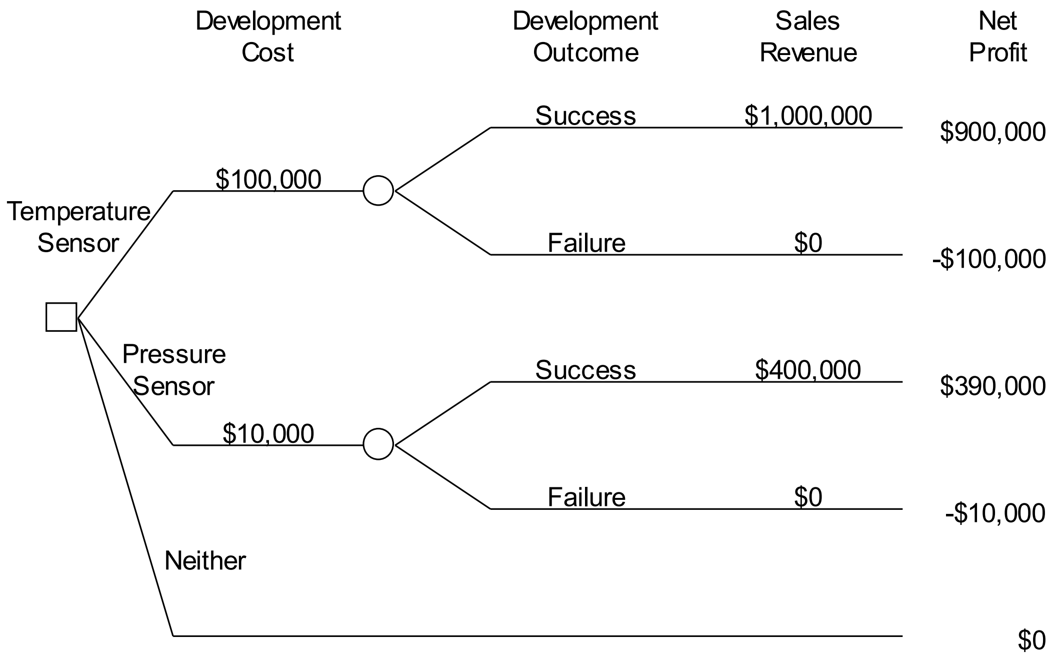


**Figure 1.1** *Special Instrument Products decision*

> **Question 1.1:** Which, if either, of these products should Special Instrument Products attempt to develop?

To answer Question 1.1 it is useful to represent the decision as shown in Figure 1.1. The tree-like diagram in this figure is read from left to right. At the left, indicated with a small square, is the decision to select among the three available alternatives, which are 1) the temperature sensor, 2) the pressure sensor, or 3) neither. The development costs for the develop temperature sensor and develop pressure sensor alternatives are shown on the branches for those alternatives. At the right of the development costs are small circles which represent the uncertainty about whether the development outcome will be a success or a failure. The branches to the right of each circle show the possible development outcomes. On the branch representing each possible development outcome, the sales revenue is shown for the alternative, assuming either success or failure for the development. Finally, the net profit is shown at the far right of the tree for each possible combination of development alternative and development outcome. For example, the topmost result of $900,000 is calculated as $\$1\ 000\ 000 - \$100\ 000 = \$900\ 000$. (Profits with negative signs indicate losses.)

The notation used in Figure 1.1 will be discussed in more detail shortly, but for now concentrate on determining the alternative Special Instrument Products should select. We can see from Figure 1.1 that developing the temperature sensor could yield the largest net profit ($900,000), but it could also yield the largest loss ($100,000). Developing the pressure sensor could only yield a net profit of \$390,000, but the possible loss is limited to \$10,000. On the other hand, not developing either of the sensors is risk free in the sense that there is no possibility of a loss. However, if Special Instrument Products decides not to attempt to develop one of the sensors, then the company is giving up the potential opportunity to make either \$900,000 or \$390,000. Question 1.1 will be answered in a following example after we discuss the criterion for making such a decision. ■

You may be thinking that the decision about which alternative is preferred depends on the probabilities that development will be successful for the temperature or pressure sensors. This is indeed the case, although knowing the probabilities will not by itself always make the best alternative in a decision immediately clear. However, if the outcomes are the same for the different alternatives, and only the probabilities differ, then probabilities alone are sufficient to determine the best alternative, as illustrated by Example 1.2.

**Example 1.2**

**Tossing a die.** Suppose you are offered two alternatives, each of which consists of a single toss of a fair die. With the first alternative, you will win \$10 if any number greater than 4 is thrown, and lose \$5 otherwise. With the second alternative, you will win \$10 if any number greater than 3 is thrown, and lose \$5 otherwise. In this case, since there are 6 faces on a die, the probability of winning is $2\ 6 = 1\ 3$ for the first alternative and $3\ 6 = 1\ 2$ for the second alternative. Since the consequences are the same for both alternatives and the probability of winning is greater for the second alternative, you should select the second alternative. ■

However, the possible outcomes are often not the same in realistic business decisions and this causes additional complexities, as illustrated by further consideration of the Special Instruments Product decision in Example 1.3.

**Example 1.3**

**Product decision.** Suppose that in Example 1.1 the probability of development success is 0.5 for the temperature sensor and 0.8 for the pressure sensor. Figure 1.2 is a diagram with these probabilities shown in parentheses on the branches representing the possible outcomes for each sensor development effort. (While only the probability of success is specified for each development effort, the probability of failure is determined by the rules of probability since the probabilities of development success and development failure must add up to one.)

A study of Figure 1.2 shows that having the probabilities does not resolve this decision for us. The figure shows that although the probability of development success is considerably lower for the temperature sensor than it is for the pressure sensor (0.5 versus 0.8), the net profit from successful development of the temperature sensor is considerably higher than the net profit from successful development

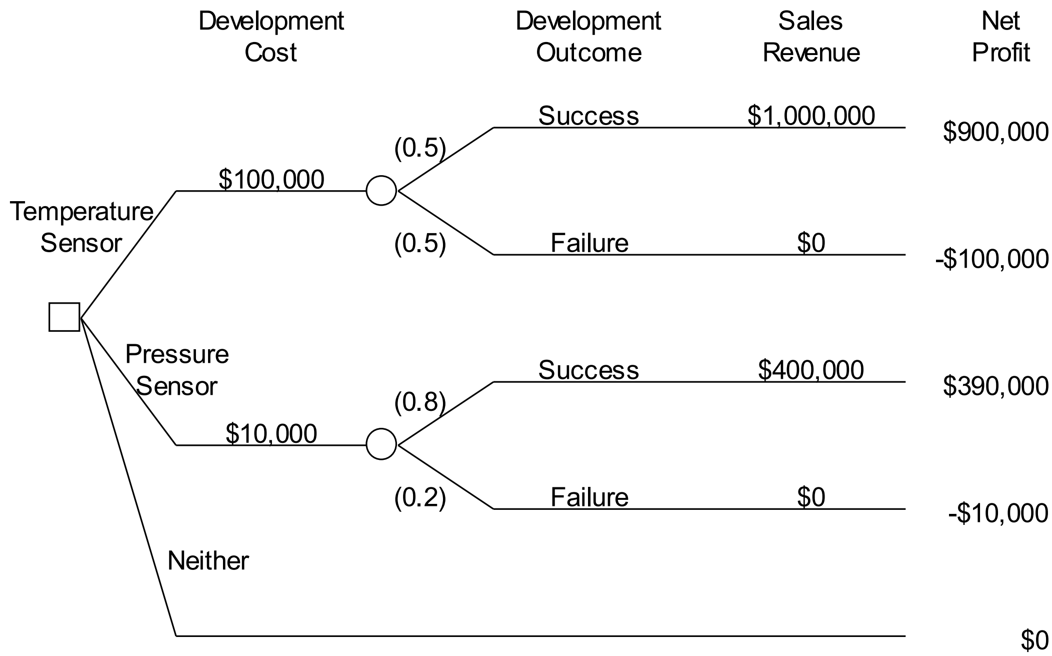


**Figure 1.2** *Special Instrument Products decision tree*

of the pressure sensor ($900,000 versus $390,000). That is, the alternative with the higher potential payoff has a lower probability that this payoff will actually be realized. ■

The resolution of this decision dilemma is addressed in the next section, but before doing this, Definition 1.1 clarifies the notation in Figures 1.1 and 1.2.

**Definition 1.1: Decision tree notation**

A diagram of a decision, as illustrated in Figure 1.2, is called a **decision tree**. This diagram is read from left to right. The leftmost node in a decision tree is called the **root node**. In Figure 1.2, this is a small square called a **decision node**. The branches emanating to the right from a decision node represent the set of decision alternatives that are available. One, and only one, of these alternatives can be selected. The small circles in the tree are called **chance nodes**. The number shown in parentheses on each branch of a chance node is the probability that the outcome shown on that branch will occur at the chance node. The right end of each path through the tree is called an **endpoint**, and each endpoint represents the final outcome of following a path from the root node of the decision tree to that endpoint.

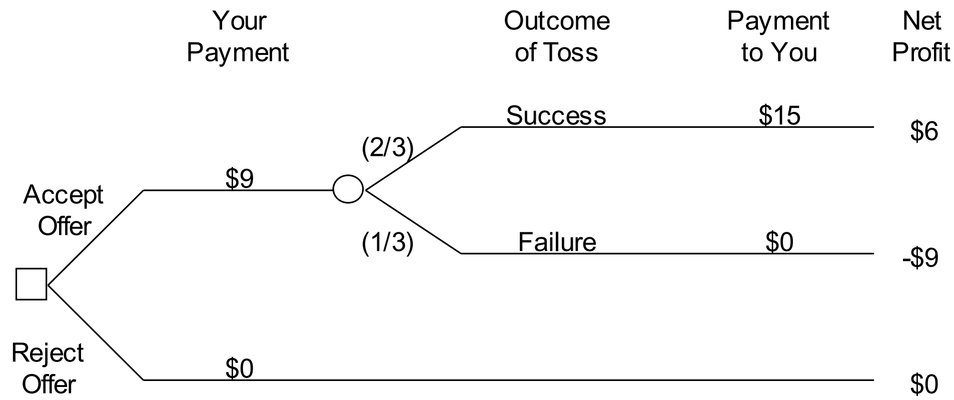


**Figure 1.3** *Die toss decision tree*

## 1.2 Expected Value

In order to decide which alternative to select in a decision problem, we need a decision criterion; that is, a rule for making a decision. Expected value is a criterion for making a decision that takes into account both the possible outcomes for each decision alternative and the probability that each outcome will occur. To illustrate the concept of expected value, we consider a simpler decision with lower stakes than the Special Instrument Products decision.

**Example 1.4**

**Rolling a die.** A friend proposes a wager: You will pay her \$9.00, and then a fair die will be rolled. If the die comes up a 3, 4, 5, or 6, then your friend will pay you \$15.00. If the die comes up 1 or 2, she will pay you nothing. Furthermore, your friend agrees to repeat this game as many times as you wish to play.

**Question 1.2:** Should you agree to play this game?

If a six-sided die is fair, there is a 1/6 probability that any specified side will come up on a roll. Therefore there is a 4/6 (= 2 3) probability that a 3, 4, 5, or 6 will come up and you will win. Figure 1.3 shows the decision tree for one play of this game.

At first glance, this may not look like a good bet since you can lose \$9.00, while you can only win \$6.00. However, the probability of winning the \$6.00 is 2/3, while the probability of losing the \$9.00 is only 1/3. Perhaps this isn t such a bad bet after all since the probability of winning is greater than the probability of losing.

The key to logically analyzing this decision is to realize that your friend will let you play this game as many times as you want. For example, how often would you expect to win if you play the game 1,500 times? Based on what you have learned about probability, you know that the proportion of games in which you will win over the long run is approximately equal to the probability of winning a single game. Thus, out of the 1,500 games, you would expect to win approximately $(2\ \ 3) \times 1\ \ 500 = 1\ \ 000$ times. Therefore, over the 1,500 games, you would expect to win a total of approximately $1\ \ 000 \times \$6 + 500 \times (-\$9) = \$1\ \ 500$. So this game looks like a good deal!

Based on this logic, what is each play of the game worth? Well, if 1,500 plays of the game are worth \$1,500, then one play of the game should be worth $\$1\ \ 500\ \ 1\ \ 500 = \$1\ 00$. Putting this another way, you will make an average of \$1.00 each time you play the game.

A little thought about the logic of these calculations shows that you can directly determine the average payoff from one play of the game by multiplying each possible payoff from the game by the probability of that payoff, and then adding up the results. For the die tossing game, this calculation is $(2\ \ 3) \times \$6 + (1\ \ 3) \times (-\$9) = \$1$. ■

The quantity calculated in the manner illustrated in Example 1.4 is called the **expected value** for an alternative, as shown in Definition 1.2. Expected value is often a good measure of the value of an alternative since over the long run this is the average amount that you expect to make from selecting the alternative.

**Definition 1.2: Expected Value**

The **expected value** for an uncertain alternative is calculated by multiplying each possible outcome of the uncertain alternative by its probability, and summing the results. The **expected value decision criterion** selects the alternative that has the best expected value. In situations involving profits where  more is better,  the alternative with the highest expected value is best, and in situations involving costs, where  less is better,  the alternative with the lowest expected value is best.

**Example 1.5**

**Product decision.** The expected values for the Special Instrument Products decision are designated by  EV  in Figure 1.4. These are determined as follows: 1) For the temperature sensor alternative, $0\ 5 \times \$900\ \ 000 + 0\ 5 \times (-\$100\ \ 000) = \$400\ \ 000$, 2) for the pressure sensor alternative, $0\ 8 \times \$390\ \ 000 + 0\ 2 \times (-\$10\ \ 000) = \$310\ \ 000$, and 3) for doing neither of these \$0. Thus, the alternative with the highest expected value is developing the temperature sensor, and if the expected value criterion is applied, then the temperature sensor should be developed. ■

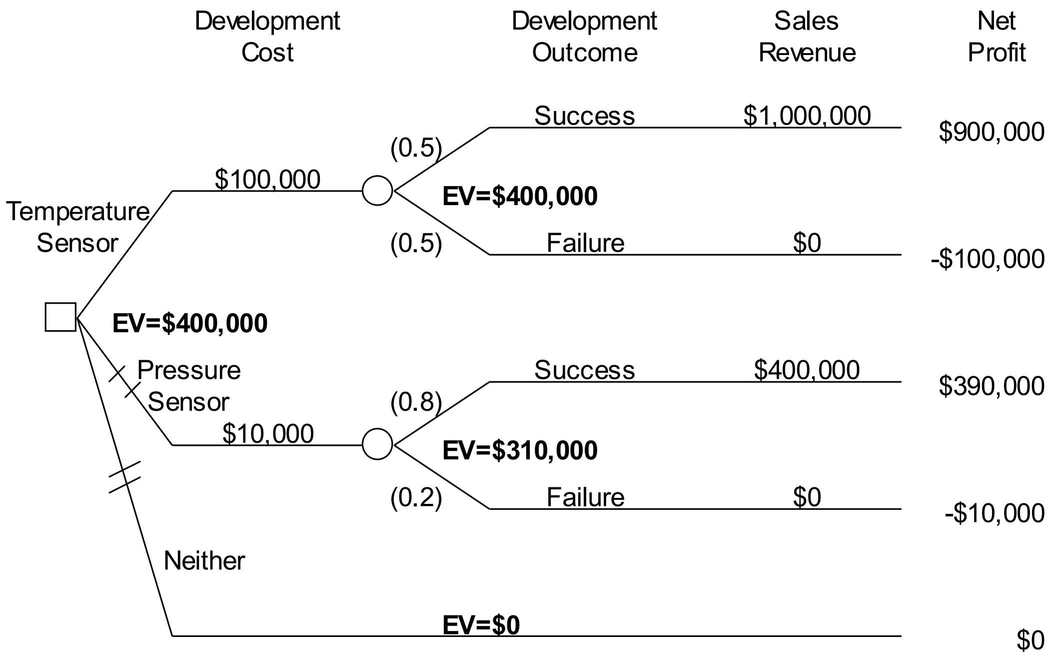


**Figure 1.4** *Special Instrument Products decision tree, with expected values*

Figure 1.4 illustrates some additional notation that is often used in decision trees. The branches representing the two alternatives that are less preferred are shown with crosshatching (//) on their branches. The expected value for each chance node is designated by EV . Finally, the expected value at the decision node on the left is shown as equal to the expected value of the selected alternative.

## Xanadu Traders

We conclude this section by analyzing a decision involving international commerce. This example will be extended in the remainder of this chapter

**Example 1.6**

**Xanadu Traders.** Xanadu Traders, a privately held U.S. metals broker, has acquired an option to purchase one million kilograms of partially refined molyzirconium ore from the Zeldavian government for $5.00 per kilogram. Molyzirconium can be processed into several different products which are used in semiconductor manufacturing, and George Xanadu, the owner of Xanadu Traders, estimates that he would be able to sell the ore for $8.00 per kilogram after importing it. However, the U.S. government is currently negotiating with Zeldavia over alleged dumping of certain manufactured goods which that country exports to the United States. As part of these negotiations, the U.S. government has threatened to ban the import from Zeldavia of a class of materials that includes molyzirconium. If the U.S. government refuses to issue an import license for the molyzirconium after Xanadu has purchased it, then Xanadu will have to pay a penalty of \$1.00 per kilogram to the Zeldavian government to annul the purchase of the molyzirconium.

Xanadu has used the services of Daniel A. Analyst, a decision analyst, to help in making decisions of this type in the past, and George Xanadu calls on him to assist with this analysis. From prior analyses, George Xanadu is well-versed in decision analysis terminology, and he is able to use decision analysis terms in his discussion with Analyst.

Analyst: As I understand it, you can buy the one million kilograms of molyzirconium ore for \$5.00 a kilogram and sell it for \$8.00, which gives a profit of $(\$8\ 00 - \$5\ 00) \times 1\ 000\ 000 = \$3\ 000\ 000$. However, there is some chance that you cannot obtain an import license, in which case you will have to pay \$1.00 per kilogram to annul the purchase contract. In that case, you will not have to actually take the molyzirconium and pay Zeldavia for it, but you will lose $\$1\ 00 \times 1\ 000\ 000 = \$1\ 000\ 000$ due to the cost of annulling the contract.

Xanadu: Actually, some chance may be an understatement. The internal politics of Zeldavia make it hard for their government to agree to stop selling their manufactured goods at very low prices here in the United States. The chances are only fifty-fifty that I will be able to obtain the import license. As you know, Xanadu Traders is not a very large company. The \$1,000,000 loss would be serious, although certainly not fatal. On the other hand, making \$3,000,000 would help the balance sheet

> **Question 1.3:** Which alternative should Xanadu select? Assume that Xanadu uses expected value as his decision criterion.

To answer this question, construct a decision tree. There are two possible alternatives, purchase the molyzirconium or don t purchase it. If the molyzirconium is purchased, then there is uncertainty about whether the import license will be issued or not. The decision tree is shown in Figure 1.5. Starting from the root node for this tree, it costs \$5 million to purchase the molyzirconium, and if the import license is issued, then the molyzirconium will be sold for \$8 million, yielding a net profit of \$3 million. On the other hand, if the import license is not issued then Xanadu will recover \$4 million of the \$5 million that it invested, but will lose the other \$1 million due to the cost of annulling the contract.

The endpoint net profits are shown in millions of dollars, and the expected value for the purchase alternative is $0\ 5 \times \$3 + 0\ 5 \times (-\$1) = \$1$, in millions of dollars. Therefore, if expected value is used as the decision criterion, then the preferred alternative is to purchase the molyzirconium. ■

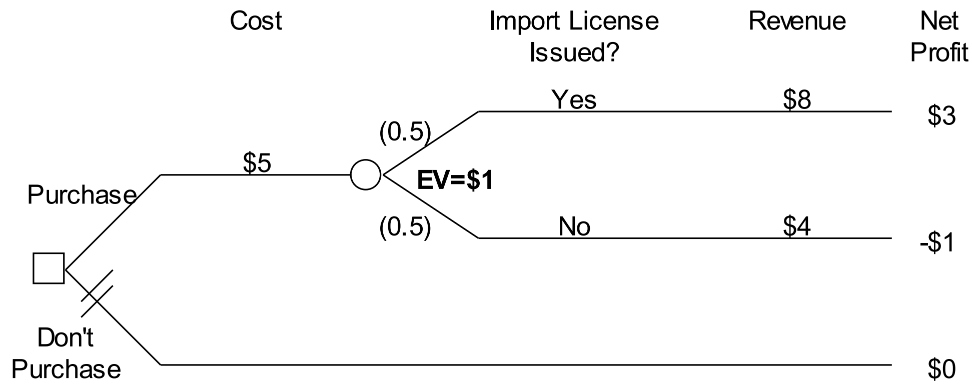


**Figure 1.5** *Xanadu Traders initial decision tree (dollar amounts in millions)*

## 1.3 Dependent Uncertainties

In this section, we consider an additional complexity that often occurs in business decisions: **dependent uncertainties**. Specifically, we will examine a case that illustrates this complexity of real world decisions, and review a procedure for analyzing decisions that include dependent uncertainties.

**Example 1.7**

**Xanadu Traders.** This is a continuation of Example 1.6. We now consider an expanded version of the decision that includes dependent uncertainties and extend the analysis procedure to handle this new issue. We continue to follow the discussion between Daniel Analyst and George Xanadu that started in Example 1.6.

Analyst: Maybe there is a way to reduce the risk. As I understand it, the reason you need to make a quick decision is that Zeldavia has also offered this deal to other brokers, and one of them may take it before you do. Is that really very likely? Perhaps you can apply for the import license and wait until you know whether it is approved before closing the deal with Zeldavia.

Xanadu: That s not very likely. Some of those brokers are pretty big operators, and dropping $1,000,000 wouldn t make them lose any sleep. I d say there is a 0.70 probability that someone else will take Zeldavia s offer if I wait until the import license comes through. Of course, it doesn t cost anything to apply for an import license, so maybe it is worth waiting to see what happens

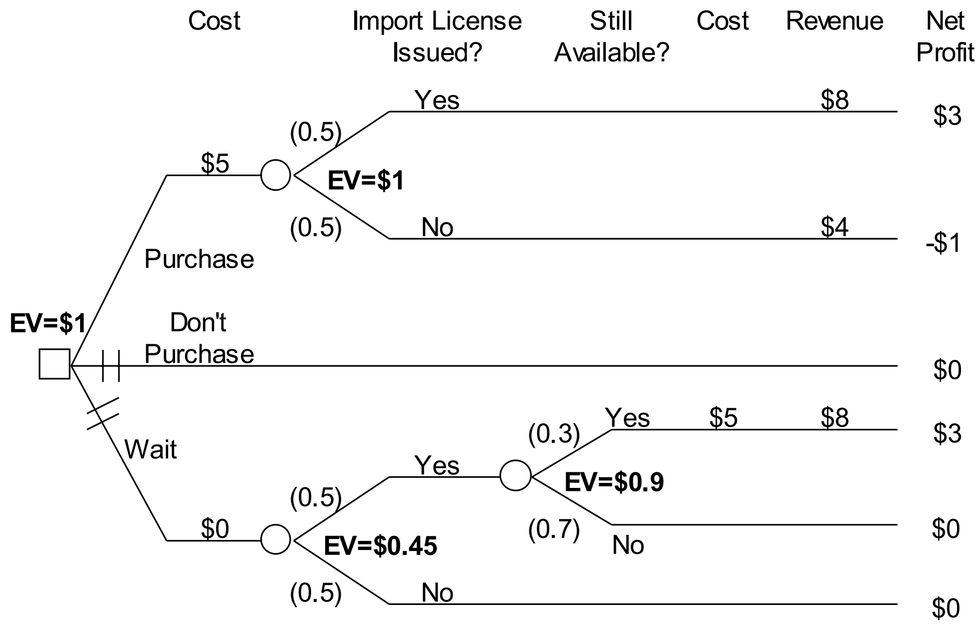


**Figure 1.6** *Xanadu Traders revised decision tree, with expected values*

> **Question 1.4:** Should Xanadu Traders wait to see if an import license is issued before purchasing the molyzirconium?

The decision tree for this revised problem is shown in Figure 1.6. The two alternatives at the top of this tree ( purchase and don t purchase ) are the same as the alternatives shown in Figure 1.5. The third alternative ( wait ) considers the situation where Xanadu waits to see whether it can obtain an import license before purchasing the molyzirconium. This alternative introduces a new analysis issue that must be addressed before the expected value for this alternative can be determined. This new issue concerns the fact that there are two stages of uncertainty for this alternative. First, the issue of an import license is resolved, and then there is a further uncertainty about whether the molyzirconium will still be available.

> **Question 1.5:** What is the expected value for the wait alternative?

The process of determining the expected value for this alternative involves two stages of calculation. In particular, it is necessary to start at the right side of the decision tree, and carry out successive calculations working toward the root node of the tree. Specifically, first determine the expected value for the alternative assuming that the import license is issued, and then use this result to calculate the expected value for the wait alternative prior to learning whether the import license is issued.

Examine Figure 1.6 to see how this calculation process works. As this figure shows, if the import license is issued, then there is a 0.3 probability that the molyzirconium will still be available. In this case, Xanadu will pay $5 million for the molyzirconium, and sell it for $8 million realizing $3 million in net profit. If the molyzirconium is not still available, then Xanadu will not have to pay anything and will realize no net profit. Thus, the expected value for the situation after the uncertainty about the import license has been resolved is $0\ 3 \times \$3 + 0\ 7 \times \$0 = \$0\ 9$. This expected value is shown next to the lower right chance node on the decision tree in Figure 1.6.

From the discussion regarding expected value in Section 1.2, it follows that this $0.9 million is the value of the alternative once the result of the import license application is known. Hence, this value should be used in the further expected value calculation needed to determine the overall value of the wait alternative. Thus, the expected value for the wait alternative is given by $0\ 5 \times \$0\ 9 + 0\ 5 \times \$0 = \$0\ 45$. This expected value is shown next to the lower left chance node on the decision tree in Figure 1.6. Since the expected value for the wait alternative is less than the $1 million expected value for purchasing the molyzirconium right now, this alternative is less preferred than purchasing the molyzirconium right now. Xanadu should not wait, assuming that expected value is used as the decision criterion. ■

The process of sequentially determining expected values when there are dependent uncertainties in a decision tree, as demonstrated in Example 1.6, is called **decision tree rollback**. This term is defined in Definition 1.3.

**Definition 1.3: Decision Tree Rollback**

> The process of successively calculating expected values from the endpoints of the decision tree to the root node, as demonstrated in this section, is called a **decision tree rollback**.

## 1.4 Sequential Decisions

In addition to dependent uncertainties, real business decisions often include **sequential decisions**. This section considers an example that demonstrates how to address sequential decisions.

**Example 1.8**

**ABC Computer Company.** ABC Computer Company is considering submission of a bid for a government contract to provide 10,000 specialized computers for use in computer-aided design. There is only one other potential bidder for this contract, Complex Computers, Inc., and the low bidder will receive the contract. ABC s bidding decision is complicated by the fact that ABC is currently working on a new process to manufacture the computers. If this process works as hoped, then it may substantially lower the cost of making the computers. However, there is some chance that the new process will actually be more expensive than the current manufacturing process. Unfortunately, ABC will not be able to determine the cost of the new process without actually using it to manufacture the computers.

If ABC decides to bid, it will make one of three bids: \$9,500 per computer, \$8,500 per computer, or \$7,500 per computer. Complex Computers is certain to bid, and it is equally likely that Complex will bid \$10,000, \$9,000, or \$8,000 per computer. If ABC decides to bid, then it will cost \$1,000,000 to prepare the bid due to the requirement that a prototype computer be included with the bid. This \$1,000,000 will be totally lost regardless of whether ABC wins or loses the bidding competition.

With ABC s current manufacturing process, it is certain to cost \$8,000 per computer to make each computer. With the proposed new manufacturing process, there is a 0.25 probability that the manufacturing cost will be \$5,000 per computer and a 0.50 probability that the cost will be \$7,500 per computer. Unfortunately, there is also a 0.25 probability that the cost will be \$8,500 per computer.

> **Question 1.6:** Should ABC Computer Company submit a bid, and if so, what should they bid per computer?

A decision tree for this situation is shown in Figure 1.7. First, ABC must decide whether to bid and how much to bid. If ABC s bid is lower than Complex Computer s, then ABC must decide which manufacturing process to use. If ABC uses the proposed new manufacturing process, then the cost of manufacturing the computers is uncertain. The net profit figures (in millions of dollars) shown at the endpoints of the Figure 1.7 tree take into account the cost of preparing the bid, the cost of manufacturing the computers, and the revenue that ABC will receive for the computers. For example, examine the topmost endpoint value. It costs \$1 million to prepare the bid, and ABC bids \$9,500, which is lower than Complex Computers  bid of \$10,000, and hence ABC wins the contract. Then the proposed new manufacturing process is used, and it costs \$8,500 per computer to manufacture the 10,000 computers. Therefore, at this endpoint, ABC makes a net profit of $-1\ 000\ 000 - 10\ 000 \times \$8\ 500 + 10\ 000 \times \$9\ 500 = \$9\ 000\ 000 = \$9$

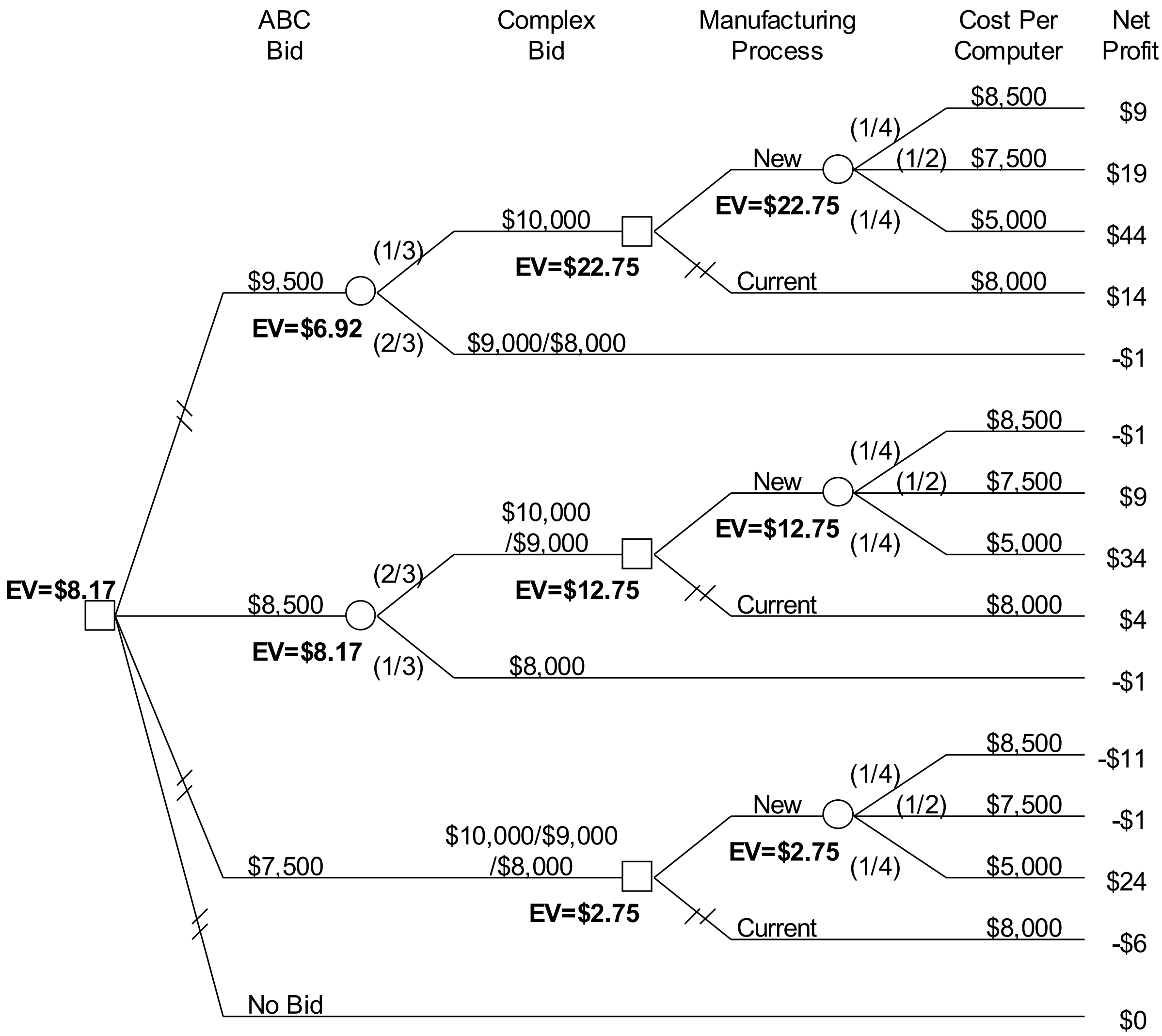


**Figure 1.7** *ABC Computer Company decision tree, with net profit in millions of dollars*

million. Verify the net profits shown at the other endpoints so that you better understand this process.

Calculating the expected values shown on the Figure 1.7 decision tree requires addressing a new issue, namely what to do when there are multiple decision nodes in the tree. In this decision, the amount of the bid is the first decision, and if this is lower than the Complex Computers bid, then there is a second decision involving the type of manufacturing process to use. The calculation procedure for this situation is a straightforward extension of the calculation procedure that was demonstrated in the preceding section for dependent uncertainties.

This procedure will be illustrated by considering the topmost set of nodes in the Figure 1.7 tree. Start at the rightmost side of the tree, and calculate the expected value for the top rightmost chance node. This is determined as

$(1\ \ 4) \times \$9 + (1\ \ 2) \times \$19 + (1\ \ 4) \times \$44 = \$22\ 75$. At the top rightmost decision node, compare the expected values for the two branches. The expected value for the top branch of this decision node is \$22.75, and (since there is no uncertainty regarding the cost of the current manufacturing process) the expected value for the bottom branch is \$14. Since the top branch has the higher expected value, it is the preferred branch. That is, the proposed new manufacturing process should be used. Hence, the expected value for the  manufacturing process  decision node is equal to the expected value for the proposed new manufacturing process, which is \$22.75.

Now continue back toward the root of the decision tree by calculating the expected value for the top leftmost chance node in the tree. Since the expected value of the manufacturing process decision is \$22.75, and there is no uncertainty about the net profit if ABC loses the bid, the expected value for the top leftmost chance node is $(1\ \ 3) \times \$22\ 75 + (2\ \ 3) \times (-\$1) = \$6\ 92$.

A similar process is used to calculate the expected values for the other three branches of the root node, and the results are shown in Figure 1.7. These calculations show that an \$8,500 bid has the highest expected value, which is \$8.17 million. Hence, if ABC uses expected value as its decision criterion, then it should bid \$8,500. In addition, the calculations also show that ABC should use the proposed new manufacturing process if it wins the contract. The less preferred branches for each decision node have been indicated on the decision tree with cross hatching. ■

The complete specification of the alternatives that should be selected at all decision nodes in a decision tree is called a **decision strategy**.

**Definition 1.5: Decision Strategy**

> The complete specification of all the preferred decisions in a sequential decision problem is called the **decision strategy**. The decision strategy shown in Figure 1.7 can be summarized as follows: Bid \$8,500, and if you win the contract use the proposed new manufacturing process.

## 1.5 Exercises

**1.1** Arthrodax Company has been approached by Ranger Sound with a rush order offer to purchase 100 units of a customized version of Arthrodax s SoundScreamer audio mixer at \$5,000 per unit, and Arthrodax needs to decide how to respond. The electronic modifications of the standard SoundScreamer needed for this customized version are straightforward, but there will be a fixed cost of \$100,000 to design the modifications and set up for assembly of the customized SoundScreamers, regardless of the number of units produced. It will cost \$2,000 per unit to manufacture the circuit boards for the units. Since Arthrodax has some short term spare manufacturing capacity, the Ranger offer is potentially attractive. However, the circuit boards for the customized units will not fit into the standard SoundScreamer case, and Arthrodax must decide what to do about acquiring cases for the customized units as it decides whether to accept Ranger s purchase offer. An appropriate case can be purchased at \$500 per case, but Arthrodax could instead purchase an injection molder to make the cases. It will cost \$20,000 to purchase the molder, and there is a 0.6 probability that it will be possible to successfully make the cases using the molder. If the molder does not work, then the purchase price for the molder will be totally lost and Arthrodax must still purchase the cases at \$500 per case. If the molder works, then it will cost \$60 per case to make the cases using the molder. Regardless of which case is used, the cost of assembling the SoundScreamer circuit boards into the case is \$20 per unit. Unfortunately, there is no way to test the molder without purchasing it. Assume that there is no other use for the molder except to make the cases for the Ranger order.

(i) Draw a decision tree for Arthrodax s decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.2** This is a continuation of Exercise 1.1. Assume that all information given in that exercise is still valid, except as discussed in this exercise. Ranger now tells Arthrodax that there is uncertainty about the number of customized SoundScreamers that will be needed. Specifically, there is a 0.35 probability that it will need 100 units, and a 0.65 probability that it will need 50 units. If Arthrodax will agree now to produce either number of units, then Ranger will pay \$6,000 per unit if it ultimately orders 50 units, and will pay \$5,000 per unit if it ultimately orders 100 units. The timing is such on this rush order that Arthrodax will have to make a decision about purchasing the injection molder before it knows how many units Ranger will take. However, Arthrodax will only need to purchase or manufacture the number of circuit boards and cases needed for the final order of either 50 or 100 units.

(i) Draw a decision tree for Arthrodax s decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers. Note that this is a situation with dependent uncertainties, as discussed in Section 1.3.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.3** This is a continuation of Exercise 1.2. Assume that all information given in that exercise is still valid, except as discussed in this exercise. Assume now that Arthrodax could delay the decision about purchasing the injection molder until after it knows how many units Ranger will take.

(i) Draw a decision tree for Arthrodax s decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers. Note that this is a situation with sequential decisions, as discussed in Section 1.4.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.4** Aba Manufacturing has contracted to provide Zyz Electronics with printed circuit ( PC ) boards under the following terms: (1) 100,000 PC boards will be delivered to Zyz in one month, and (2) Zyz has an option to take delivery of an additional 100,000 boards in three months by giving Aba 30 days notice. Zyz will pay \$5.00 for each board that it purchases. Aba manufactures the PC boards using a batch process, and manufacturing costs are as follows: (1) there is a fixed setup cost of \$250,000 for any manufacturing batch run, regardless of the size of the run, and (2) there is a marginal manufacturing cost of \$2.00 per board regardless of the size of the batch run. Aba must decide whether to manufacture all 200,000 PC boards now or whether to only manufacture 100,000 now and manufacture the other 100,000 boards only if Zyz exercises its option to buy those boards. If Aba manufactures 200,000 now and Zyz does not exercise its option, then the manufacturing cost of the extra 100,000 boards will be totally lost. Aba believes there is a 50% chance Zyz will exercise its option to buy the additional 100,000 PC boards.

(i) Explain why it might potentially be more profitable to manufacture all 200,000 boards now.

(ii) Draw a decision tree for the decision that Aba faces.

(iii) Determine the preferred course of action for Aba assuming it uses expected profit as its decision criterion.

**1.5** Kezo Systems has agreed to supply 500,000 PC FAX systems to Tarja Stores in 90 days at a fixed price. A key component in the FAX systems is a programmable array logic integrated circuit chip ( PAL chip ), one of which is required in each FAX system. Kezo has bought these chips in the past from an American chip manufacturer AM Chips. However, Kezo has been approached by a Korean manufacturer, KEC Electronics, which is offering a lower price on the chips. This offer is open for only 10 days, and Kezo must decide whether to buy some or all of the PAL chips from KEC. Any chips that Kezo does not buy from KEC will be bought from AM. AM Chips will sell PAL chips to Kezo for \$3.00 per chip in any quantity. KEC will accept orders only in multiples of 250,000 PAL chips, and is offering to sell the chips for \$2.00 per chip for 250,000 chips, and for \$1.50 per chip in quantities of 500,000 or more chips. However, the situation is complicated by a dumping charge that has been filed by AM Chips against KEC. If this charge is upheld by the U. S. government, then the KEC chips will be subject to an antidumping tax. This case will not be resolved until after the point in time when Kezo must make the purchase decision. If Kezo buys the KEC chips, these will not be shipped until after the antidumping tax would go into effect and the chips would be subject to the tax. Under the terms offered by KEC, Kezo would have to pay any antidumping tax that is imposed. Kezo believes there is a 60%

chance the antidumping tax will be imposed. If it is imposed, then it is equally likely that the tax will be 50%, 100%, or 200% of the sale price for each PAL chip.

(i) Draw a decision tree for this decision.

(ii) Using expected value as the decision criterion, determine Kezo s preferred ordering alternative for the PAL chips.

**1.6** **Intermodular Semiconductor Systems case.** The Special Products Division of Intermodular Semiconductor Systems has received a Request for Quotation from Allied Intercontinental Corporation for 100 deep sea semiconductor electrotransponders, a specialized instrument used in testing undersea engineered structures. While Intermodular Semiconductor Systems has never produced deep sea electrotransponders, they have manufactured subsurface towed transponders, and it is clear that they could make an electrotransponder that meets Allied s specifications. However, the production cost is uncertain due to their lack of experience with this particular type of transponder. Furthermore, Allied has also requested a quotation from the Undersea Systems Division of General Electrodevices. Intermodular Semiconductor Systems and General Electrodevices are the only companies capable of producing the electrotransponders within the time frame required to meet the construction schedule for Allied s new undersea habitat project.

Mack Reynolds, the Manager of the Special Products Division, must decide whether to bid or not, and if Intermodular Semiconductor Systems does submit a bid, what the quoted price should be. He has assembled a project team consisting of Elizabeth Iron from manufacturing and John Traveler from marketing to assist with the analysis. Daniel A. Analyst, a consulting decision analyst, has also been called in to assist with the analysis.

Analyst: For this preliminary analysis, we have agreed to consider only a small number of different possible bids, production costs, and General Electrodevices bids.

Reynolds: That s correct. We will look at possible per-unit bids of $3,000, $5,000, and $7,000. We will look at possible production costs of $2,000, $4,000, and $6,000 per unit, and possible per-unit bids by General Electrodevices of $4,000, $6,000, and $8,000.

Iron: There is quite a bit of uncertainty about the cost of producing the electrotransponders. I d say there is a 50% chance we can produce them in a volume of 100 units at $4,000 per unit. However, that still leaves a 50% chance that they will either be $2,000 or $6,000 per unit.

Analyst: Is one of these more likely than the other?

Iron: No. It s equally likely to be either $2,000 or $6,000. We don t have much experience with deep sea transponders. Our experience with subsurface towed transponders is relevant, but it may take some effort to make units that hold up to the pressure down deep. I m sure we can do it, but it may be expensive.

Analyst: Could you do some type of cost-plus contract?

Reynolds: No way! This isn t the defense business. Once we commit, we have to produce at a fixed price. Allied would take us to court otherwise. They re tough cookies, but they pay their bills on time.

Iron: I want to emphasize that there is no problem making the electrotransponders and meeting Allied s schedule. The real issue is what type of material we have to use to take the pressure. We may be able to use molyaluminum like we do in the subsurface towed units in which case the cost will be lower. If we have to go to molyzirconium, then it will be more expensive. Most likely, we will end up using some of each, which will put the price in the middle.

Analyst: What is General Electrodevices likely to bid?

Traveler: They have more experience than we do with this sort of product. They have never made deep sea electrotransponders, but they have done a variety of other deep sea products. I spent some time with Elizabeth discussing their experience, and also reviewed what they did on a couple of recent bids. I d say there is a 50% chance they will bid $6,000 per unit. If not, they are more likely to bid low than high there is about a 35% percent chance they will bid $4,000 per unit.

Analyst: So that means there is 15% chance they will bid $8,000.

Traveler: Yes.

Reynolds: Suppose we had a better handle on our production costs. Would that give us more of an idea what General Electrodevices would bid?

Iron: No. They use graphite-based materials to reinforce their transponders. The cost structure for that type of production doesn t have any relationship to our system using moly alloys.

(i) Draw a decision tree for the decision that Reynolds must make.

(ii) Determine the expected values for each of the alternatives, and specify which alternative Reynolds should select if he uses expected value as a decision criterion.