11 Crest factor reduction techniques

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11.1 Introduction

Signal-to-noise ratio (SNR) is the principal figure of merit for any electronic system. The relation between the signal power and the noise power has been the main challenge for electronic engineers.

In order to study and optimize this type of system, the system design engineers have looked mainly to the noise optimization, that is, to the minimization of all sorts of noise in communication systems. This noise arises mainly from thermal noise [1], nonlinear distortion noise [2], and/or quantization noise in digital systems [3].

Thus the correct identification of the noise contributions is of fundamental importance to the calculation of the noise budget. This is one of the reasons why most of the communication engineers start by worrying about the time domain waveform characteristics of electronic systems. Mainly what they have called the peak of the signal versus its average value, or what is normally known as the peak-to-average ratio (PAR) [4] or crest factor (CF) [5].

Actually the first known reference to this problem, as far as we are aware, was made by Landon in 1936 [1], when he was studying the noise characteristics. A more intensive work [6] was also published by the same author in 1941, related to the distribution of amplitude with noise.

Several other authors have considered that problem since then and in fact in 1964 an instrument called the "PAR Meter" was even proposed by Andersson and Favin [4] for measuring the PAR in loudspeaker tests. What they were searching for was mainly the distortion generated at the loud speakers by the value of the peak-to-average ratio. At that time they proposed a factor of merit called the PAR Rating, that is a relation between the peak value, P, divided by the full-wave average of the received signal, A, and the corresponding values of the undistorted signal, P_0 and A_0, as expressed in the following.

$$PARRating(R) = 2\frac{P/A}{P_0/A_0} - 1 \tag{11.1}$$

This rating has a value of unity when the signal is undistorted and it tends to zero if the signal is somehow compressed.

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This work has been studied in depth by other authors [7, 8]. The PAR problem continues then to be studied, mainly due to the fact that some signals behave strangely and present some values of peak power to average power that are quite high.

This fact was recognized by the design engineers of radar, sonar and many other applications, which has also driven the mathematician Schroeder [9] to publish a first study on the arrangements of phases angles combinations in periodic signals. In that paper [9] he proposed a phase arrangement specially tailored for multi-sines. This phase arrangement algorithm is called the Schroeder Phase Algorithm [9]. In order to understand the Schroeder algorithm, consider a periodic signal of the form:

$$r(t) = \sum_{k=1}^{N} (\frac{p_k}{2})^{1/2} cos(\frac{2\pi kt}{T} + \theta_k). \tag{11.2}$$

The Schroeder phases should be:

$$\phi_n = \phi_1 - 2\pi \sum_{l=1}^{n-1} p_l n = 1, 2, \ldots, N \tag{11.3}$$

Schroeder proved that this phase arrangement algorithm is better than an in-phase arrangement of the periodic signal in terms of peak value.

This started the research work on algorithms to minimize the PAR, via a careful choice of the phase arrangements in a periodic signal. In fact in [10] Greenstein *et al*., proposed an algorithm based on an iterative method to obtain those phases, either at baseband, or at RF, showing that there are differences between the two approaches. This was then further improved by Boyd in [5] where he has defined the Crest Factor, as:

$$CF(u) \equiv \frac{\| u \|_\infty}{\| u \|_2} \tag{11.4}$$

where

$$\begin{aligned} \| u \|_\infty &\equiv sup|u(t)| \\ \| u \|_2 &\equiv (\textstyle\int_0^T ((u(t)^2))(1/2)). \end{aligned} \tag{11.5}$$

In this work he states that a signal has a CF of one only if the signal is squared. He then proves that different arrangements of the phase between signal harmonics present some good results, such as the Rudin–Shapiro Signs, and the Newman phases.

An important aspect of [5] is that the minimization of the phases is considered in the framework of modeling and for nonparametric measurements of the frequency response of linear systems. So it is a fundamental requirement that a low PAR signal be generated in order not to distort the measured signal, and only capture the linear behaviour of the device under test.

In this line of study, Van Den Bos [11] proposes a new iterative algorithm for CF minimization, comparing the obtained results with the ones of Schroeder, stating that the new method achieves a low value of CF.

In Ouderaa *et al*. [12] there are some comments on the paper from Boyd stating that the correct formula for measuring the CF should be:

$$K_r = \frac{M^+ - M^-}{2 \parallel u \parallel_2} \tag{11.6}$$

where M^+ and M^- are the largest positive and the largest negative value of a multi-sine. The same authors then propose in [13] a new type of algorithm for the CF minimization, based on an iterative method that commutes between time and frequency for minimizing the peak value of the time waveform.

The same group later published a large amount of improvements to those methods of CF minimization, mainly and fundamentally in order to extract the underlying linear model for a complex system. In those scenarios the minimization of the CF is fundamental in order to not excite the nonlinear behaviour of those systems. In that respect the selection and design of excitation signals with constant signal-to-noise ratios [14], with minimum crest factors for MIMO systems [15], etc. were studied.

Other scientific groups have also studied this phenomenon, like Evans [2], where specially tailored test signals with reduced CF are designed for the identification of the best linear model in systems presenting nonlinear distortion.

The CF minimization has been a very important line of study since the end of the 1980s and beginning of the 1990s.

In 1992, some authors started to give more importance to the fact that complex digital modulated signals present large values of peak-to-average ratios, and that could degrade the communication path [3, 16]. Since in the presence of high values of PAR, the efficiency of the Power Amplifiers (PAs) were extremely degraded, and the dynamic range of DAC/ADC increase enormously when the value of PAR is high.

This fact separates the study of the PAR in two directions. One is completely devoted to system identification, which continues the previous works of Schouckens and Evans, being mainly in the Instrumentation and Measurement Journals. The other direction is related to communications systems, both wired like x-DSL and wireless systems, mainly those based on multi-carrier modulations, such as OFDM (Orthogonal Frequency Division Multiplexing) or MC-CDMA (Multi-carrier CDMA).

These two areas of study, despite focusing on different scenarios, somehow are complementary, since they both seek the minimization of the time domain waveform peaks.

The rest of this chapter is organized into four sections. Section 11.2 is devoted to multi-carrier communication systems. In Section 11.3, we will focus on CF reduction using clipping plus filtering. In the final section, we will provide a summary and further discussions.

11.2 Multi-carrier communication systems

When the time domain waveforms in the communication signals start to approximate the multi-sine shape usually used in ULS measurement instruments, the problem of CF, or peak-to-average power ratio (PAPR), as it is known in the communication world, starts to attract a greater attention.

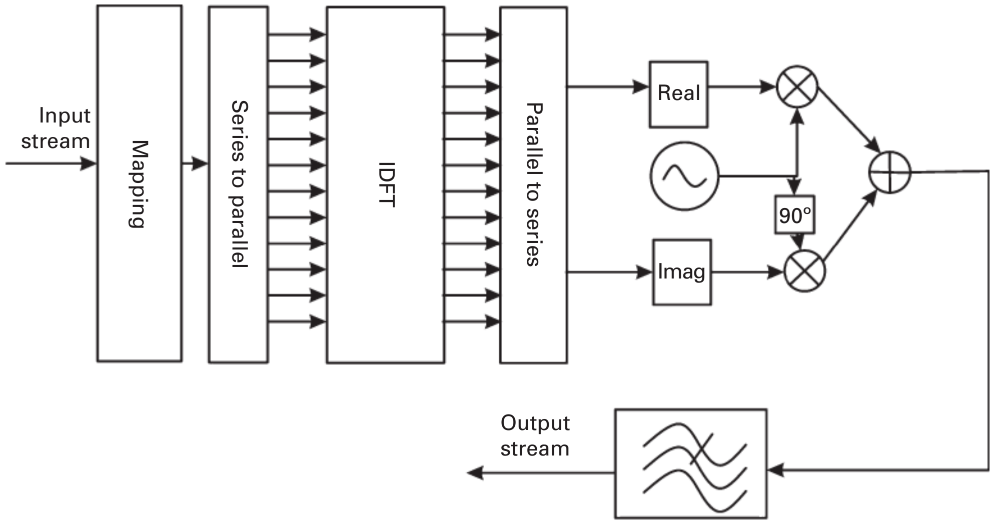

Figure 11.1 OFDM system blocks.

If we look back, the problem in this case is similar to the system identification area, since most of the high values of PAPR arise in multi-carrier communication signals, as in OFDM, or xDSL systems.

An OFDM system can be represented as in Figure 11.1. As can be seen from the figure, the time domain waveform is nothing more than a summation of sines at different frequencies for each time instant, so the problem to address for PAPR reduction is exactly the same as in the system identification case. In fact this problem starts to be first discussed, in relation to communication systems, in [16] where a method for measuring the PAPR of Digital TV was presented and the value of those PAPR values compared for different modulation formats. At that time this problem was considered very important, since it imposed more care in the design of linear PA's and an increase of the DAC/ADC dynamic range.

Related to this dynamic range problem, reference [3] presents some requirements of the A/D converter in QAM receivers, where some guidelines for the selection of the appropriate bit-precision of the A/D converter is given for QAM modulated signals when they preset high values of PAPR. At this time the problem also starts to have industrial significance, with some measuring companies presenting some application notes referring to the problem of PAPR [17], and how to measure it.

The increase in PAPR in multi-carrier systems becomes so complex that all the possible schemes for its minimization have become a goal for wireless system design engineers.

The approaches for that minimization span from special coding for PAPR minimization [18], tone reservation [19], tone injection [20], clipping [21], clipping followed by filtering [22], change of the constellation diagram [23] to others based on representation of the signals to transmit [24], like partial transmit sequences, PTS, interleaving and selected mapping SLM, etc.

Recently the study has centered on not on how to minimize the PAPR exclusively, but on how to have a good compromise among the minimum PAPR, bit-rate, and BER optimization [25].

11.2.1 Definition of the relation between peak power and average power

In order to better understand the different approaches followed, let us now first define the diverse definitions on the relation between the peaks and the average power of the time domain waveforms.

Some authors [26] normally used PAPR as the ratio of the power that would result if the envelope were sustained at its peak magnitude to the average power of the overall signal. The PAPR has thus the mathematical form of:

$$PAPR = \frac{\max\limits_{0 \le t \le NT} |x(t)|^2}{1/NT \int_0^{NT} |x(t)|^2 dt}. \tag{11.7}$$

A version of the PAPR for sampled signal can also be used and is defined as:

$$PAPR_s = \frac{\max\limits_{0 \le k \le NL-1} |x_k|^2}{E[|x(k)|^2]}. \tag{11.8}$$

Despite there being some controversy about the relation of the sampled PAPR to its continuous version, the overall discussion states that we are allowed to choose either one to use. Although one could be different from the other, or at least the minimization of one could not mean the minimization of the other [27, 28], they all converge to a solution where the PAPR will be in fact minimized when compared to the previous results.

Other authors have also studied and proposed new forms of PAPR specially tailored for multi-carrier communication systems. One of those examples is what was called by Wulich [25], the Efficient PAPR, where the measurement of the PAPR is not only regarded as a reduction of the peak power, but as an optimum relation of the PAPR that minimizes the BER.

This way a better value of compromise could be obtained among system complexity, peak reduction, bit-rate optimization, and BER values.

11.2.2 PAPR/CF reduction techniques

After the observation that a high value of PAPR can appear in multi-carrier systems, and thus degrade the SNDR, the scientific community started to search for ways to minimize the PAPR. A very large amount of work has been undertaken on this.

In [29] Seung made a very good overview of the PAPR reduction techniques that were available to that date for multi-carrier transmission. The techniques used span from hardware based schemes to software exclusively based ones. These include Active Constellation Extension [23, 30–33], Tone Injection [20, 34], and Tone Reservation [19, 35–37].

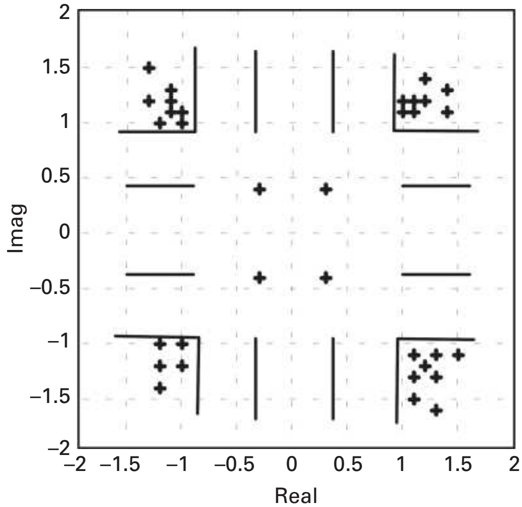

Figure 11.2 ACE scheme.

One obvious solution is Amplitude Clipping and all its combinations with other techniques although it has some major problems as pointed out in [21, 22, 38–43]. More recently also Companding techniques [44–48] was considered.

In addition to these, we also have reduction techniques based on the multiple signal representation such as Partial Transmit Sequences, PTS [21, 49–52], Selective Mapping [49, 53, 54] and Interleaving [55, 56]. This list will not be complete without referring to Coding [18, 57–63] and other more orthodox techniques such as Symbol Shaping [64, 65].

We will now briefly address those different techniques individually.

Active Constellation Extension

In the Active Constellation Extension (ACE) scheme [23, 30–33], the main idea is to change the constellation diagram in order to reduce the PAPR with a low cost impact on the BER. If we look at Figure 11.2, and consider the case of QPSK modulation, the transmitted signal is extended towards the out bounds of the constellation diagram, which can have a slight impact on the BER but it can be proved that has a strong impact on the reduction of PAPR.

What the technique does is nothing more than to transmit a slightly different value of the modulator symbol in order to minimize the PAPR. This changed value should be made at certain boundaries in order that the demodulator could demodulate the signal with low values of BER. Otherwise the BER could be completely degraded due to the reduction of the distance between symbols in the constellation diagram.

Tone Injection

A similar solution to ACE is so-called Tone Injection (TI) [20, 34, 66]. In this scheme the main idea is the creation of several constellation diagrams which can be dynamically chosen in order to reduce the PAPR.

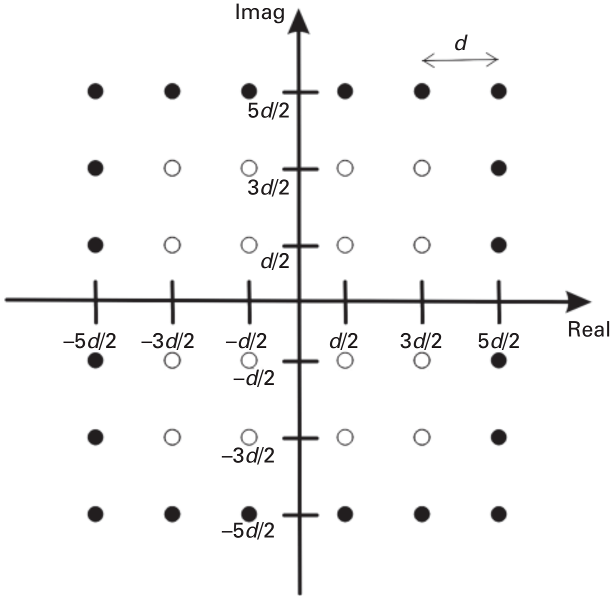

Figure 11.3 TI based on extra constellation diagrams.

In fact the addition of new constellation points in the constellation diagram is the equivalent of injecting new tones. This new constellation point in the augmented constellation could be selected as to maintain the time waveform at a stable PAPR. The idea from an OFDM point of view is nothing more than to add constants C's to the OFDM symbol, which are carefully selected to reduce the PAPR and not to increase the BER. The effect of the added constant is to increase the constellation magnitude so that each of the points in the original constellation is mapped into the expanded constellation.

The values of C's are $C_k = p_k D + jq_k D$, p_k and q_k are chosen to minimize the PAPR, and the constant D is known both at the transmitter and the receiver. Figure 11.3 presents this solution for the case of a 16-QAM. In the figure, what we can see is that the black points could also be transmitted, but no new information is added. This means that we can transmit the same digital symbol either using the white points or the black points for the same base information bit, so the modulator could have some redundancy, which is chosen in order to minimize the PAPR.

The main problem is the increase in BER. Nevertheless the augmented capability to reduce PAPR is quite satisfactory.

Tone Reservation

In the case of Tone Reservation [19, 35–37] the underlying idea is to reserve, or say, to select some sub carriers in order that the overall RF signal has a reduced PAPR. In DSL communication systems this is normally done in the low SNR tones, since they will not be very important for the overall signal demodulation. So in this case we will add some information, C, to the un-used tones to reduce the overall PAPR in the time domain scenario.

The unused tones are called the reserved tones and normally do not carry data or they cannot carry data reliably due to their low SNR. It is exactly these tones that are used to send the optimum vector C that was selected to reduce large peak power samples of OFDM symbols. The method is very simple to implement, and the receiver could ignore the symbols carried on the un-used tones, without any complex demodulation process, or extra tail bits.

Amplitude clipping

Amplitude clipping [21, 22, 38–43] is obviously the one that can achieve improved results and is less complex to apply. Nevertheless the clipping increases the occupied bandwidth and simultaneously degrades significantly the in-band and out-of-band distortion, giving rise to the increase of BER, due to its nonlinearity nature.

The technique is based mainly on the following procedure: if the signal is below a certain threshold, then the output is exactly the signal itself. Nevertheless if it passes that threshold then the signal should be clipped as is presented in the next expression.

$$y = \begin{cases} x, & |x| \leq A \\ Ae^{j\phi(x)}, & |x| > A \end{cases} \tag{11.9}$$

where $\phi(x)$ is the phase of x. The main problem of this technique is that somehow we are distorting the signal generating nonlinear distortion both in-band and out-of-band. The in-band distortion cannot be filtered out, and some form of linearizer should be used [67], or other form of reconstruction of the signal prior to the reception block as was done in [68], which somehow has the opposite effect, since one of the reasons to reduce the PAPR is exactly to minimize the impact of the nonlinear PA.

The out-of-band emission, usually called spectral regrowth, can be filtered out but the filtering process will increase again the PAPR. That is why some algorithms are used sequentially with clipping and filtering in order to converge to a minimum value.

This technique can be further associated with other schemes to improve the PAPR overall solution.

Companding techniques

Companding technique [44–48] is a similar technique to clipping, but the signal is not actually clipped, but rather companded or expanded according to its amplitude.

This is probably the oldest technique for reducing PAPR, and actually it was used in the old analog telephone lines, where the voice was companded in order to reduce the need for high dynamic range. Most of the authors have dedicated their time to select the optimum form of the companding function in order to simultaneously reduce the PAPR and improve the BER. Figure 11.4 presents one implementation of these schemes [48].

One possibility for the companding function is the well-known relationship, as expressed in the following:

$$F(x) = \operatorname{sgn}(x)\frac{\ln(1 + u|x|)}{1 + u} \quad -1 \leq x \leq 1. \tag{11.10}$$

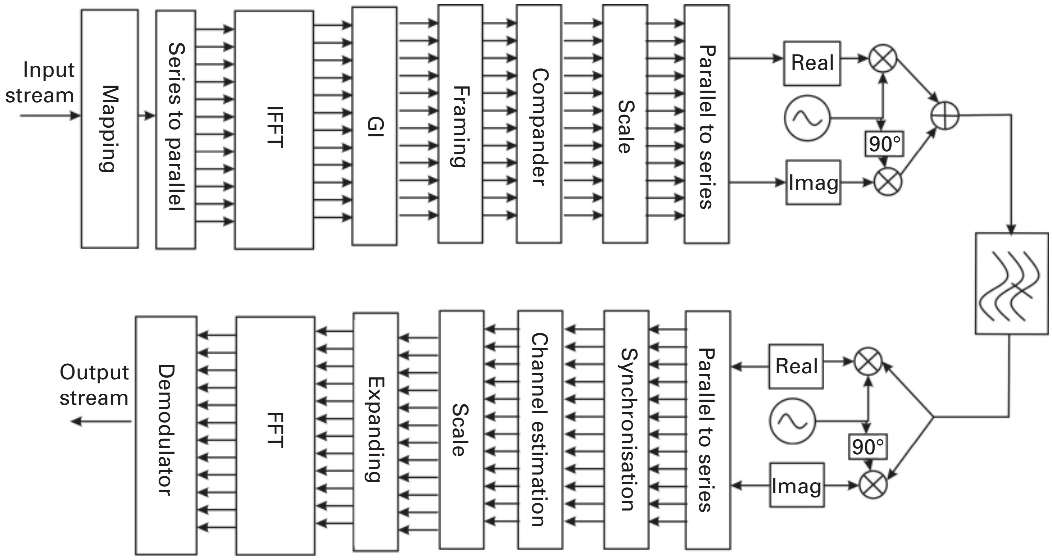

Figure 11.4 Companding implementation.

The drawbacks of this solution are similar to those of the clipping technique, but in this case the nonlinear distortion can be somehow post-distorted at the receiver more efficiently, since the nonlinearity is not as severe as that in the clipping form.

Partial transmit sequences

Other forms of PAPR minimization are related to transmitting only the selected small PAPR waveforms. That can be achieved, for instance, in the Partial Transmit Sequence, PTS [24, 49–52], technique.

In this case the information to be transmitted is first sub-divided into several blocks, Figure 11.5. The blocks are then multiplied with a phase arrangement weighting factor that will allow the reduction of the PAPR. The main objective of the algorithm is to select those B vectors that will minimize the PAPR. The receiving part of the overall system should receive those values in order to demodulate conveniently the information signal. This is one of the drawbacks of this technique since it will need to use an extra carrier to send this information, or an extra overhead, reducing the bit-rate.

Selected mapping

In the select mapping solution [49, 53, 54] the basic idea is to choose from a large amount of possible data blocks and to transmit the one with lowest PAPR. If we look at Figure 11.6, the data information is first replicated into several equal blocks. Those blocks are further multiplied by a B vector, corresponding to different phase sequences.

Then these different blocks are converted to the time domain, by using the IDFT, and the one with lowest PAPR is selected and transmitted. Some side information should also be transmitted to the receiver, in order to demodulate the data sequence.

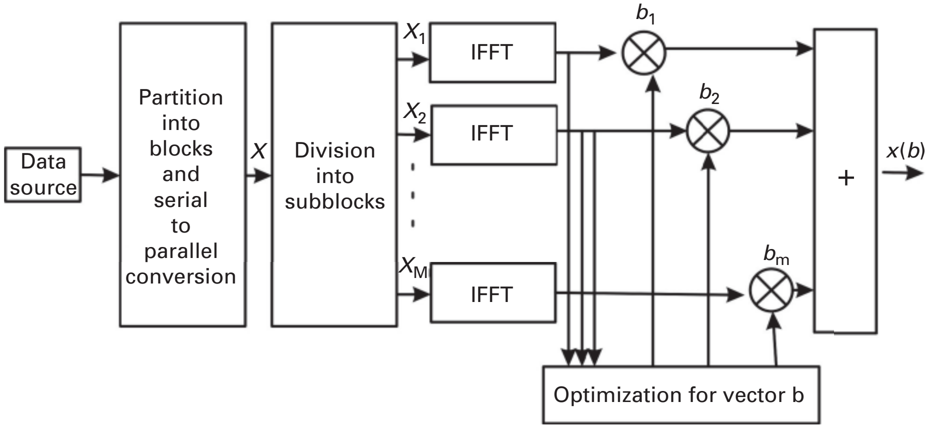

Figure 11.5 PTS technique.

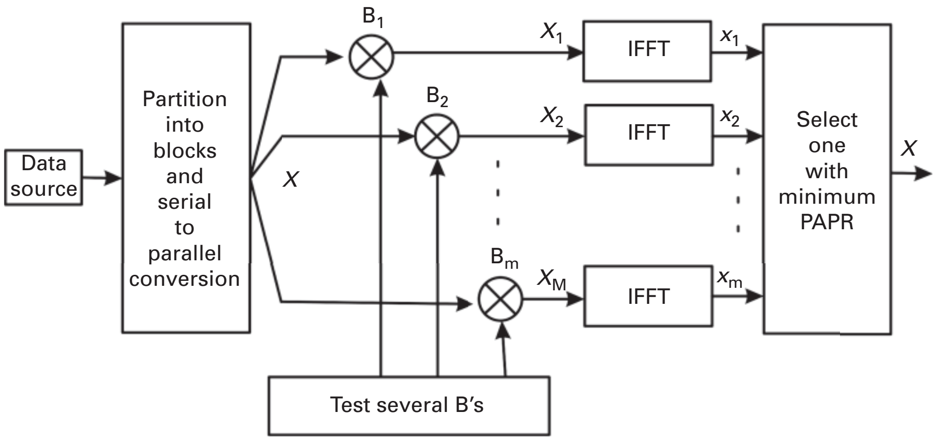

Figure 11.6 SLM technique.

This is one of the drawbacks of this technique since we will lose some bit-rate due to the side information.

Interleaving

In the case of interleaving [55, 56], the approach is similar to the SLM technique, since in this case we should have several interleavers at the transmitter and at the receiver.

The algorithm starts by first replicating the data to be transmitted into several equal blocks, then each one of the interleavers is applied to the data blocks and after the IDFT the one with low PAPR is selected for transmission, Figure 11.7. Again at the receiver we must know which interleaver was selected, so the amount of side information to be sent is on the order of $\log_2(K)$, where K is the number of interleavers used.

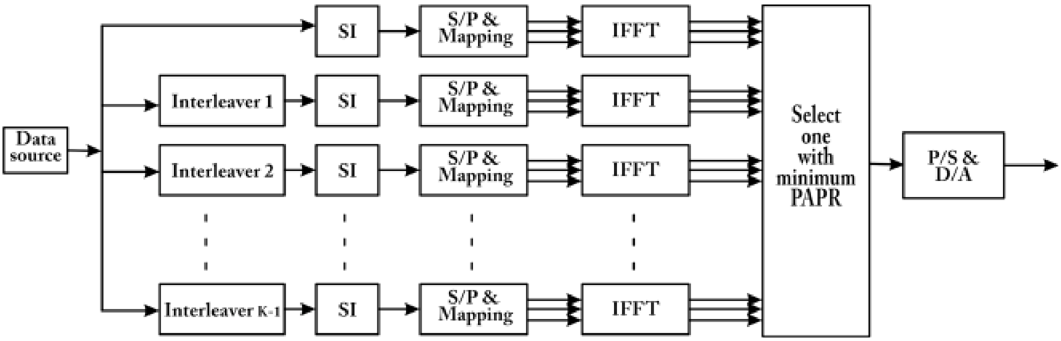

Figure 11.7 Interleaving technique.

Coding

Coding [18, 57–63] is one of the simplest techniques that was used for PAPR reduction. In this case the main idea is to select from a possible set of words to transmit, the one with low PAPR. This fact is achieved by adding to the data words a number of block codes in order to generate a minimum PAPR final word. Consider Figure 11.8, where the PAPR of the 4 word bit is presented. We can see that a maximum of 16W PEP (Peak Envelope Power) is achieved.

The main idea of the block coding is to select the sequences that have a higher PAPR and do not consider them for transmission. That can be done, for instance in the case of Figure 11.8, by block coding with encoding the data into 3 bit word and then add an extra fourth bit, so that only the sequences with low PAPR are transmitted.

The main problem with this technique is the large amount of look-up tables needed for coding and decoding of the block coding. Nevertheless this technique could be associated with other coding mechanisms available at the wireless system. For example we can use specific error code sequences to reduce the PAPR, or as some authors present recently we can use only specific Walsh codes in a CDMA system to reduce the PAPR, and do not allow other codes that will degrade the PAPR for that user.

Symbol shaping

More recently some authors have dedicated their work to the study of how the base band filter shaping [64, 65] of the bit pulses affects the PAPR, and how to use that information for minimizing the PAPR.

This proposal is similar to clipping and filtering, since what those authors are proposing is a pre-filter that is able to be designed not for ISI, but for PAPR reduction.

11.3 CF reduction using clipping plus filtering

In order to better understand some of the concepts previously presented, in this section one of the most frequently used techniques for the RF/Wireless system engineers

d_sec	d_1	d_2	d_3	d_4	PEP
0	0	0	0	0	w 16.00
1	1	0	0	0	7.07
2	0	1	0	0	7.07
3	1	1	0	0	9.45
4	0	0	1	0	7.07
5	1	0	1	0	16.00
6	0	1	1	0	9.45
7	1	1	1	0	7.07
8	0	0	0	1	7.07
9	1	0	0	1	9.45
10	0	1	0	1	16.00
11	1	1	0	1	7.07
12	0	0	1	1	9.45
13	1	0	1	1	7.07
14	0	1	1	1	7.07
15	1	1	1	1	16.00

Figure 11.8 Coding sequence.

will be presented. The technique is called the clipping plus filtering approach, and we will show how to apply it at the baseband waveforms, and at the RF waveforms when in presence of OFDM signals.

The first point that we should account for is that clipping on OFDM signals causes in-band and out-of-band distortions on the frequency domain. The out-of-band distortion can be filtered out but the in-band distortion remains after filtering and it contributes to error vector magnitude (EVM) on the receiver side. The in-band distortion on the constellation diagram can be described as amplitude scaling and scattering around desired constellation points [69]–[70]. Reduction in average amplitude of received symbols around every desired constellation point increases and more scattering occurs as clipping goes intense. Unlike scattering in single carrier systems, it is observed that circle-shaped scattering occurs at each desired constellation point, which means that received symbols are uniformly distributed in angle. Two-dimensional distributions of the received symbols at each desired constellation point are identical and are not Gaussian. As clipping goes deep the distributions come close to Gaussian.

Clipping can be conducted on an envelope or bandpass signal. Other terms for envelope clipping include baseband clipping and soft envelope clipping. The envelope clipping is described in (11.9) and bandpass clipping can be described as

$$y = \begin{cases} x, & |x| \le A \\ A, & x > A \\ -A, & x < -A \end{cases} . \tag{11.11}$$

In the following subsections both the clipping and filtering techniques are simulated using Matlab and Simulink and then the results are compared.

11.3.1 System models

Model for baseband clipping-and-filtering simulation

A model used for envelope clipping-and-filtering simulation as in Figure 11.9 consists of a transmitter part, a receiver part, and a clipping block in between. At the transmitter side, a random binary signal is mapped into a 16-QAM signal and the QAM signal is changed to a complex envelope signal by zero-padding and IFFT in the OFDM transmitter. There are 203 zeros padded to 53 numbers of constellation vectors in each frame, which results in a frequency bandwidth of around 70 MHz, in order to capture up to fifth-order nonlinear distortion around the baseband. The resulting size of IFFT is 256. The reverse processes are conducted at the receiver side. The clipping-and-filtering block between the transmitter and the receiver makes an output of a clipped-and-filtered complex envelope signal. All blocks in the model are built and simulated in Simulink.

Model for bandpass clipping-and-filtering simulation

A model for bandpass clipping-and-filtering simulation as shown in Figure 11.10 has two parts; a baseband and an RF. The baseband part is the same as in the previous model for envelope clipping-and-filtering simulation. The RF part includes a quadrature modulator, a clipping-and-filtering block, a quadrature demodulator, and a low-pass filter in the order that a signal flows. A complex envelope taken from the output of the OFDM transmitter is up-converted to the center frequency of 80 MHz by the quadrature modulator. The resulting time-domain signal is clipped-and-filtered, down-converted by the quadrature demodulator, and then filtered before the signal reaches the baseband blocks. The

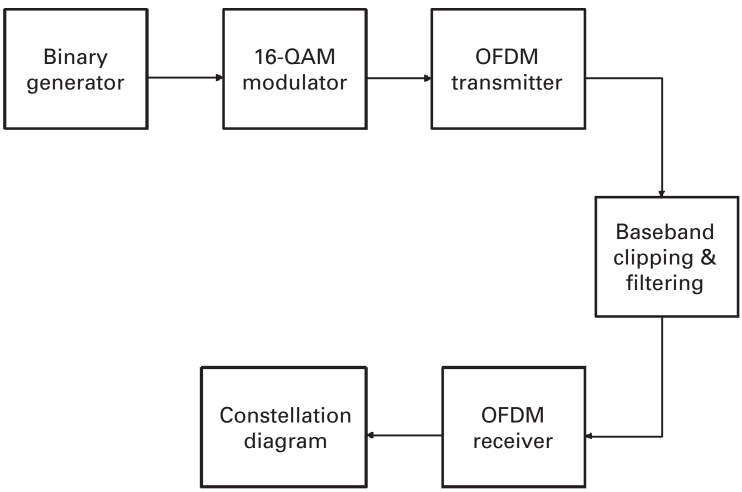

Figure 11.9 A model for simulating envelope-clipping effects.

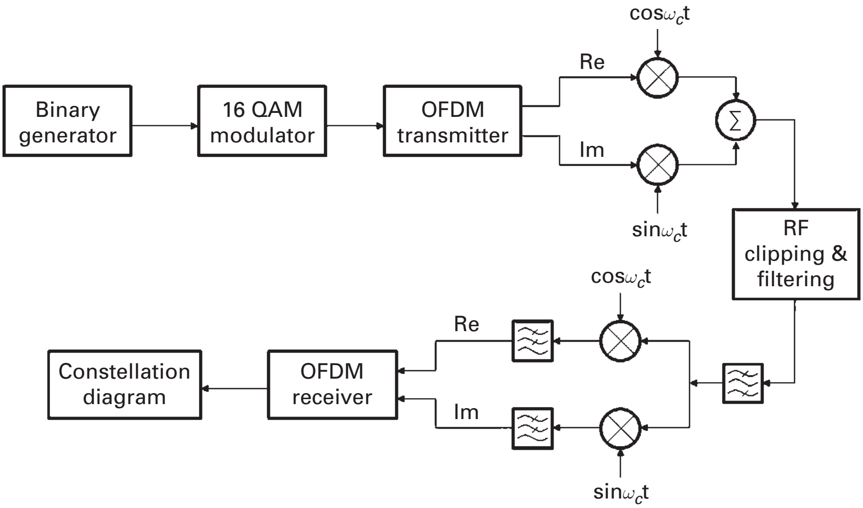

Figure 11.10 A model for simulating bandpass-clipping effects.

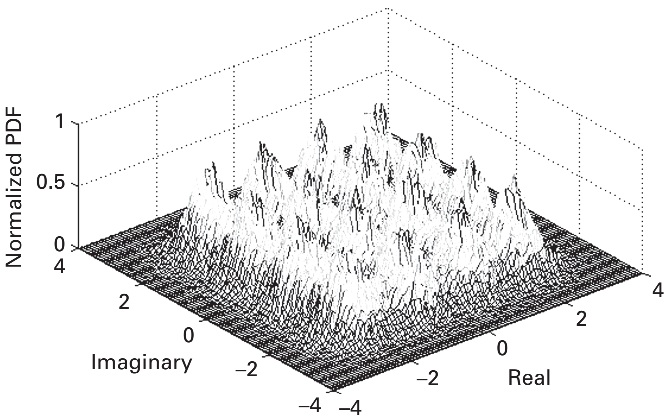

Figure 11.11 Constellation PDF of an envelope-clipped signal.

baseband and RF parts are built and simulated in the Simulink and Matlab environments respectively.

The total frequency bandwidth taken into account is about 760 MHz including the negative and positive frequency bands. This bandwidth is wide enough to capture up to fifth-order nonlinearities.

11.3.2 Simulation results and comparison

In each simulation 250 000 symbols are transmitted and received. Figures 11.11 and 11.12 show the resulting constellation diagrams at the receiver side from envelope and

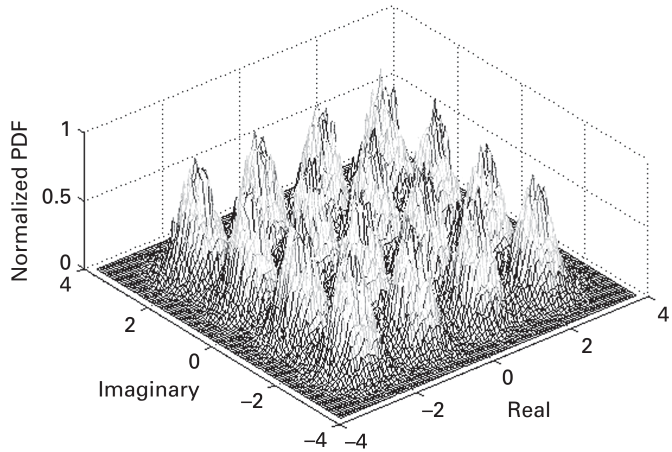

Figure 11.12 Constellation PDF of a bandpass-clipped signal.

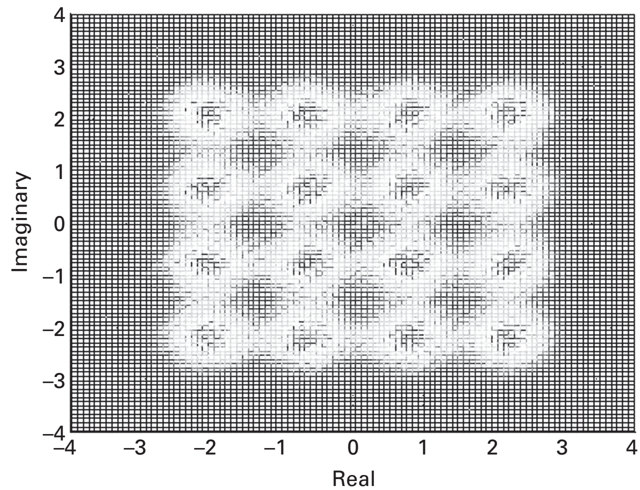

Figure 11.13 Top view of Figure 11.11.

bandpass clipping-and-filtering simulations respectively. Comparing these two figures, symbols in Figure 11.11 are scattered more around the centers of each constellation groups than those in Figure 11.12 This is observed more clearly in Figure 11.13 and Figure 11.14 which are respectively top views of Figure 11.11 and Figure 11.12. It is also observed that the centers of each constellation groups in Figure 11.13 moved more toward the origin than those in Figure 11.14. This means that at the same level of clipping, envelope clipping-and-filtering creates more distortion and BER than bandpass clipping-and-filtering.

In order to observe the difference between the two techniques in terms of in-band and out-of-band distortions, clipped only signals without being filtered are compared.

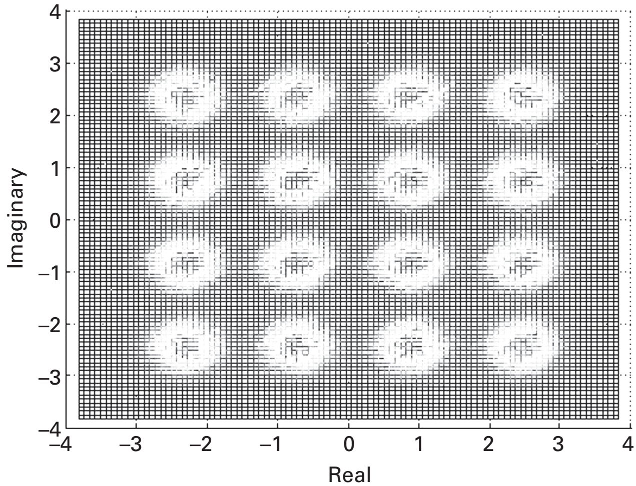

Figure 11.14 Top view of Figure 11.12.

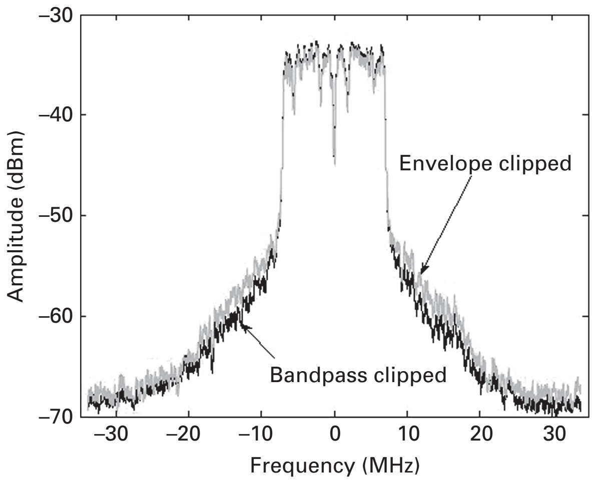

Figure 11.15 Frequency spectra of envelope- and bandpass-clipped signals with the same clipping level.

Frequency spectra of the clipped signals are shown in Figure 11.15. The envelope-clipped spectrum has more in-band suppression than the bandpass-clipped spectrum by about 0.5 dB, which is expected from the constellation diagrams in Figure 11.13 and Figure 11.14.

The spectrum of envelope clipping also has more spectral regrowth (or out-of-band distortion) than that of bandpass clipping. The difference of the spectral regrowths is about 2 dB.

So far, it seems that bandpass clipping-and-filtering has more advantages over the envelope-processed version due to lower EVM and spectral regrowth at the same clipping level. However, it can be observed where the "advantages" come from by

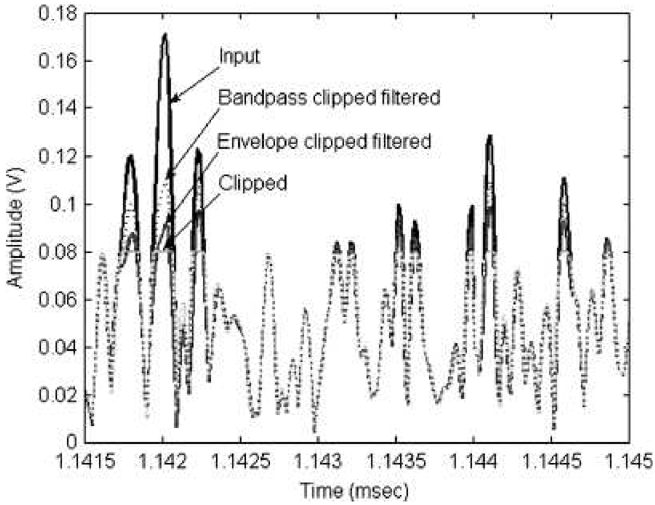

Figure 11.16 Envelope- and bandpass-clipped-and-filtered signals with the same clipping level.

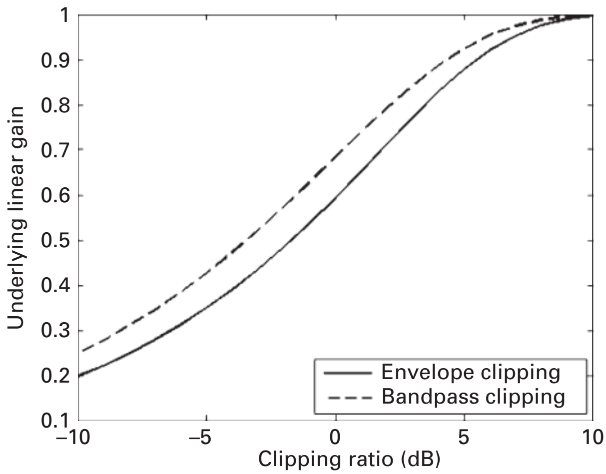

Figure 11.17 Underlying linear gains of envelope and bandpass clipping depending on clipping ratios.

comparing time-domain signals after clipping-and-filtering. As shown in Figure 11.16, the bandpass-clipped-and-filtered signal has higher peak values (or PAPR) than the envelope processed signal does even though both signals are clipped at the same level.

This can be explained in the same concept as underlying linear gain of a memoryless system. Given a clipping level, the underlying linear gain of the bandpass-clipped signal is larger than that of the envelope-clipped signal [71]–[73]. Figure 11.17 shows underlying linear gains of envelope and bandpass clipping depending on clipping ratios. Therefore, a lower EVM of the bandpass clipping results in the cost of a higher PAPR.

11.4 Conclusion

Recently multiband-multi-carrier systems have been attracting more attention as future candidates for communications. One of the foreseeable issues with the systems is the high PAPR problem. In the chapter we have revisited various techniques for CFR to address the problem, and briefly explained how they work and what are the disadvantages. Each technique has different positives and negatives in terms of throughput, EVM (or BER), and system complexity so it is difficult to conclude that a specific one is better than another due to trade-offs in performance and cost factors. Therefore, depending on system requirements it is expected that one technique or a combination of the techniques would be chosen in future systems.

Clipping-and-filtering is one of the most simple and intuitive techniques for CFR. An example of clipping plus filtering simulation is included to show how clipping-and-filtering affects frequency spectra and constellations of symbols. Differences between envelope and bandpass clipping-and-filtering are also shown.

Future schemes for CF reduction are mainly concentrated on reduction techniques focusing not on the value of the peak but combining with linearizers in order to optimize the values of BER in transmission chains. Moreover the combination of these techniques with software defined radio approaches is expected to improve the overall transmission quality in future wireless systems.

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