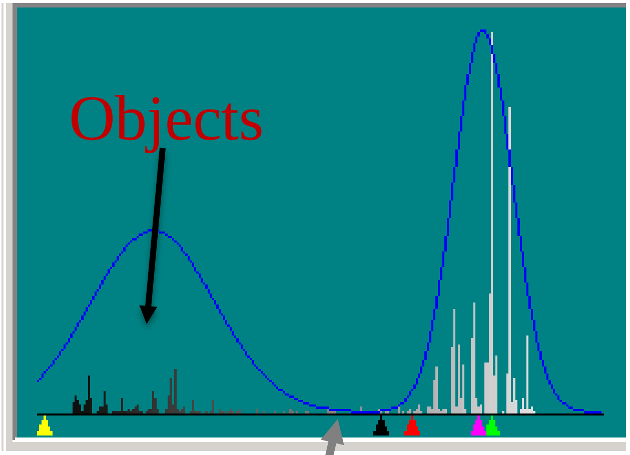
Lecture 9:

MORPHOLOGICAL IMAGE PROCESSING

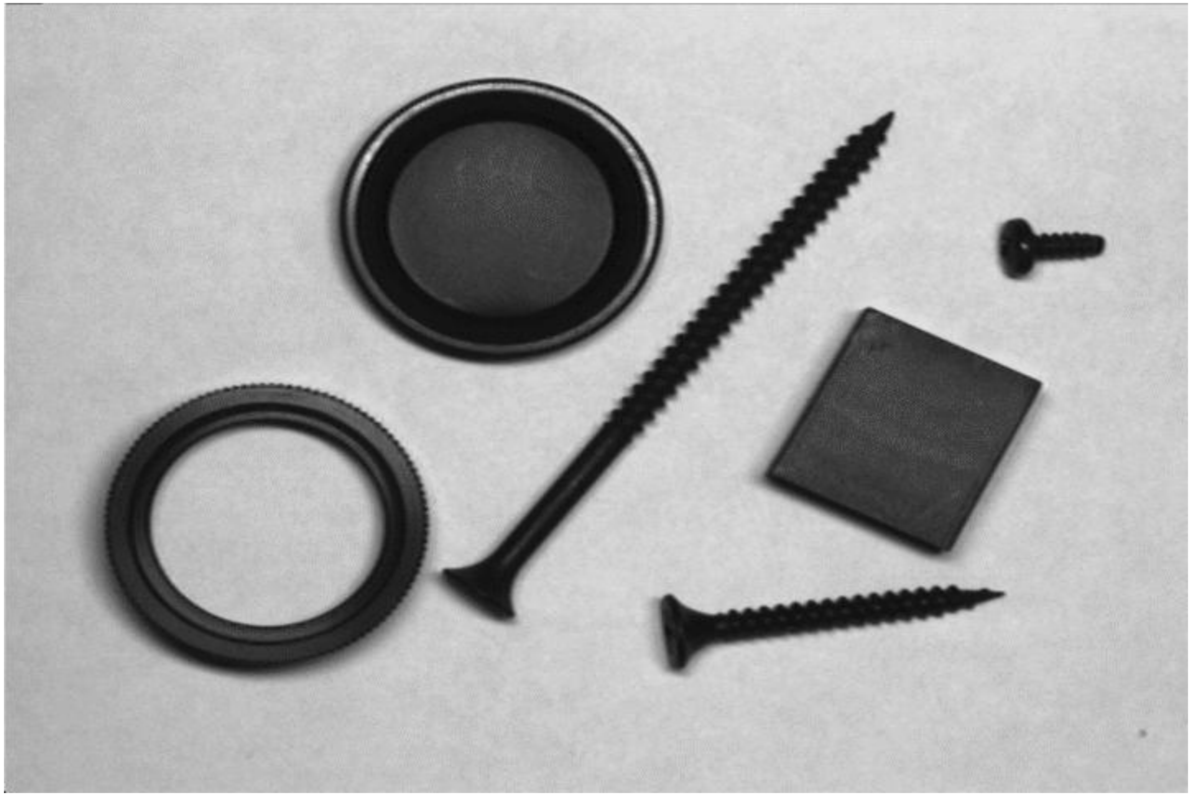
Morphological Image Processing

- Binary dilation and erosion
- Set-theoretic interpretation
- Opening, closing, morphological edge detectors

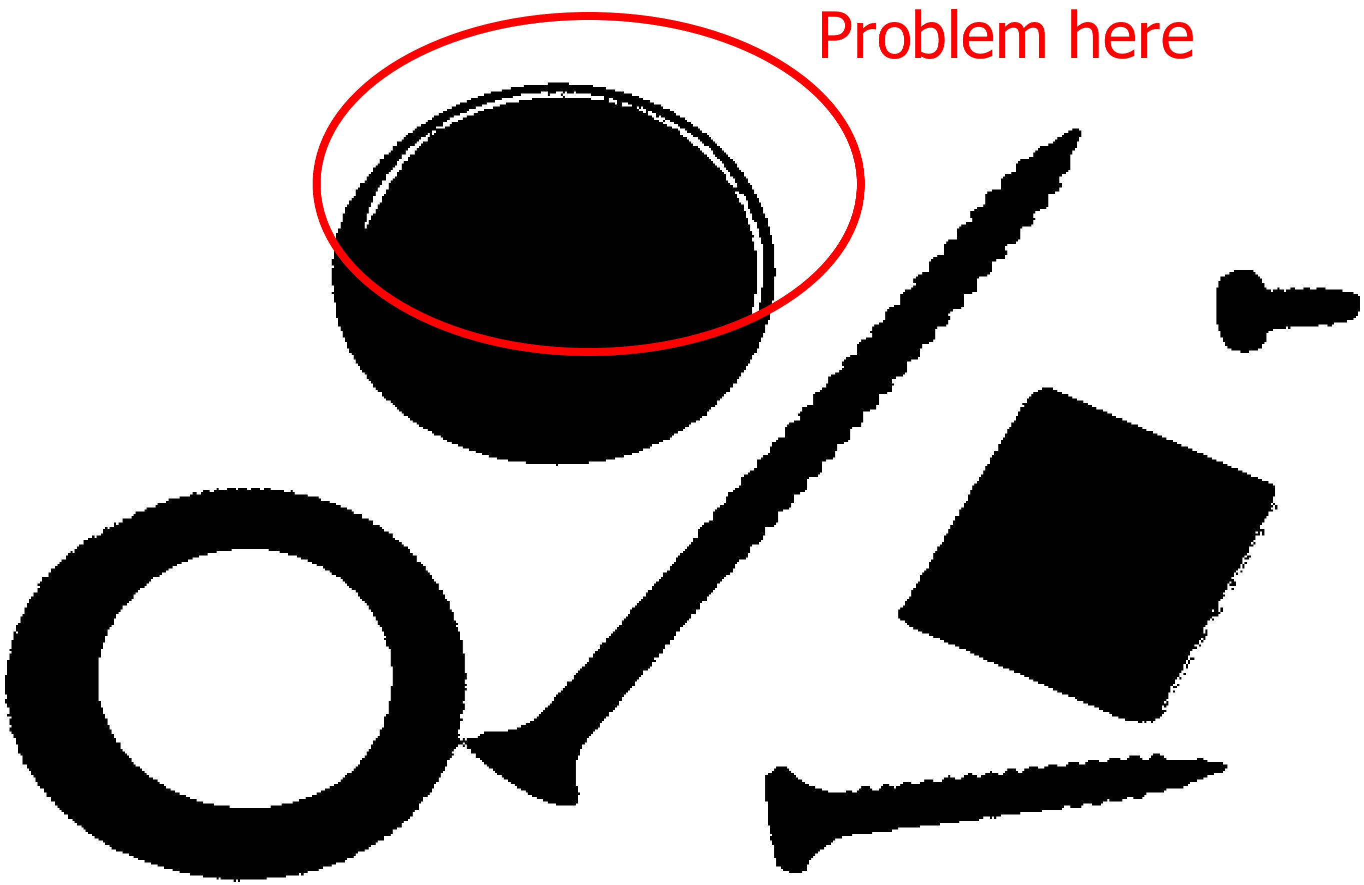
Gray Level Thresholding



Set threshold here

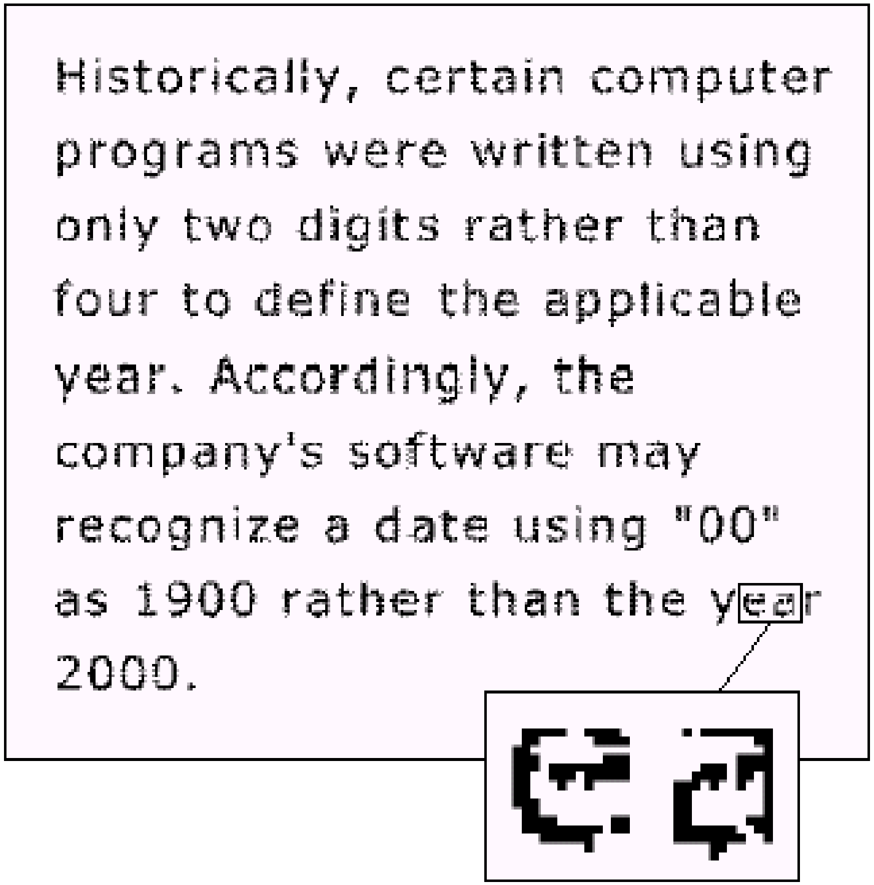


Binary Image

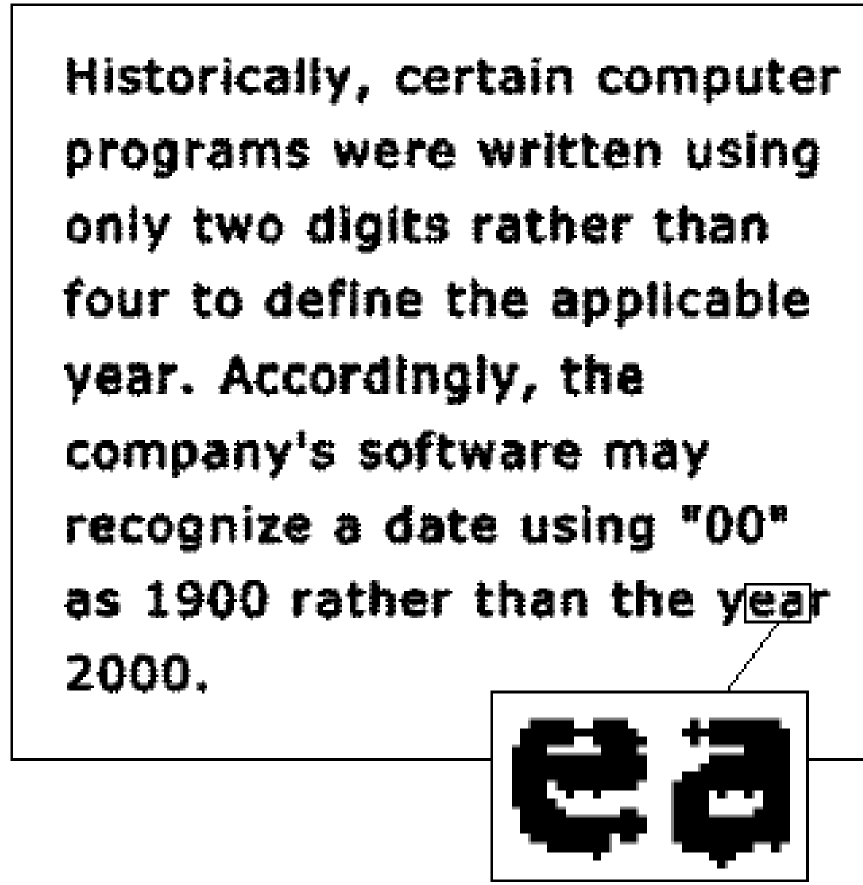


How do we fill "missing pixels"?

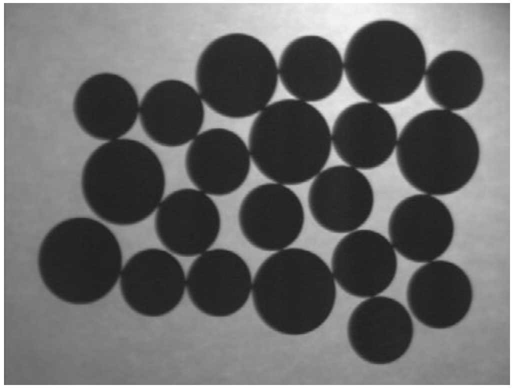
Original image

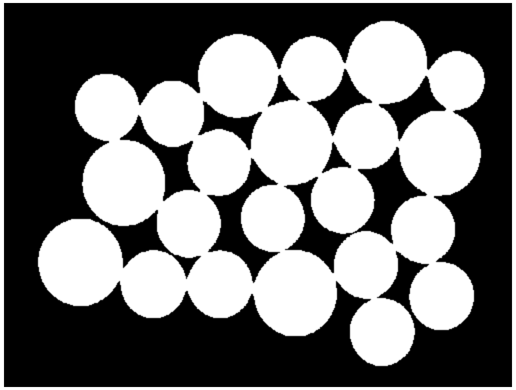


After dilation



Counting Coins





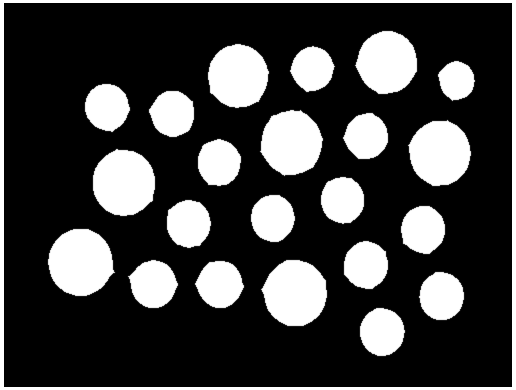


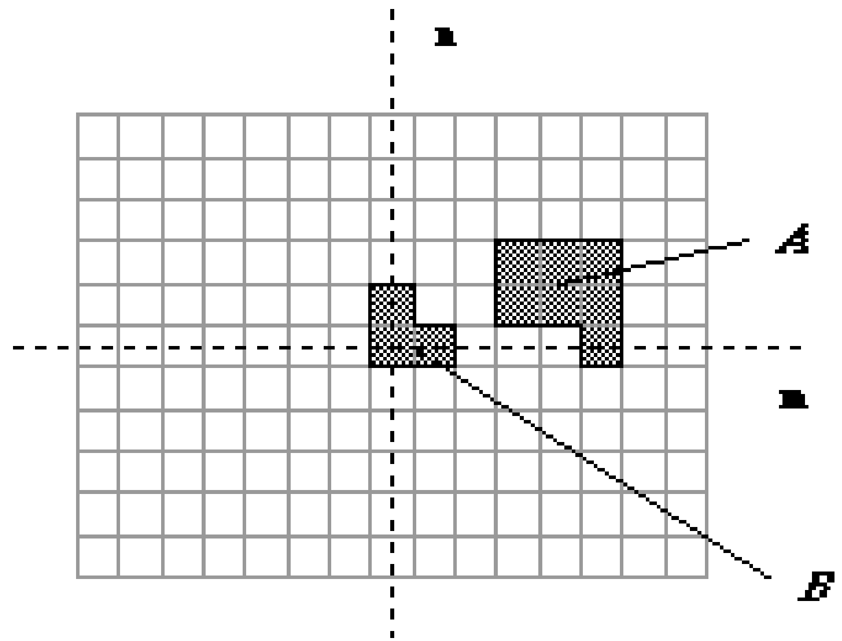
Image after segmentation

Image after segmentation and morphological processing

Mathematical Morphology

- Ảnh là một hàm hai chiều, f(x, y), của các biến tọa độ rời rạc (x, y).
- Một định nghĩa thay thế có thể dựa trên khái niệm rằng ảnh bao gồm một tập hợp các tọa độ rời rạc.

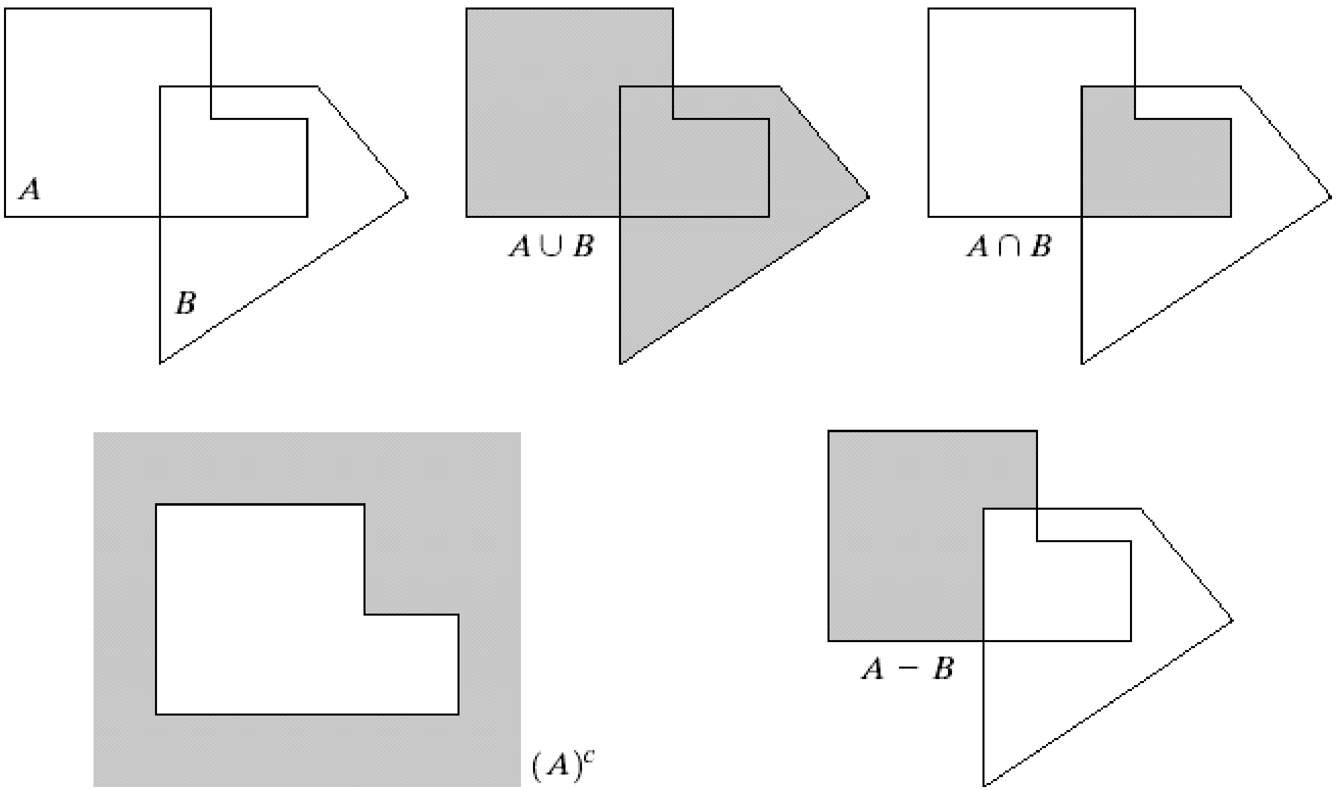
Morphology



A binary image containing two object sets A and B

- B = {(0,0), (0,1), (1,0)}
- A = {(5,0), (3,1), (4,1), (5,1), (3,2), (4,2), (5,2)}

Basic Set Theory



a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Logic Operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

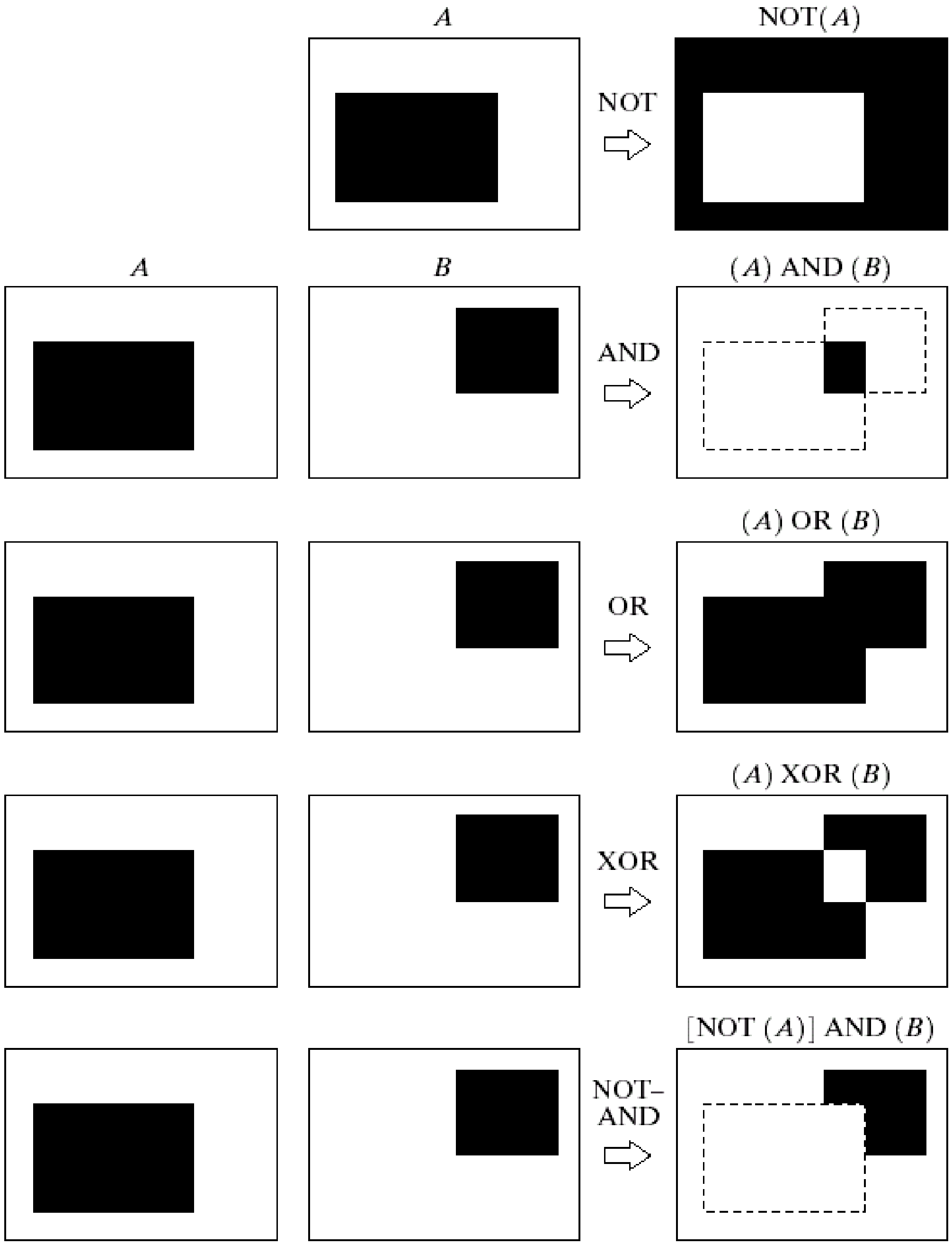
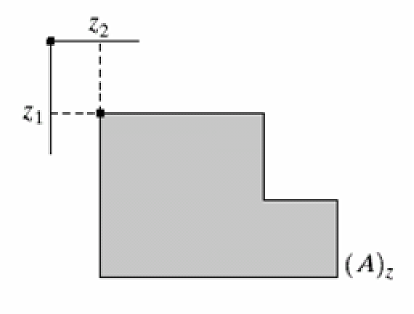


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Some Basic Definitions

- Let A and B be sets with components a=(a1,a2) and b=(b1,b2), respectively.
- The *translation* of A by x=(x1,x2) is

  $A + x = \{c \mid c = a + x, \text{ for } a \in A\}$



- The *reflection* of A is

  $A^r = \{x \mid x = -a \text{ for } a \in A\}$

- The *complement* of A is

  $A^c = \{x \mid x \notin A\}$

- The *union* of A and B is

  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

- The *intersection* of A and B is

  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Some Basic Definitions

- The difference of A and B is.

$$A - B = A \cap B^c = \{x \mid x \in A \text{ and } x \notin B\}$$

Binary image processing

- Representation of individual pixels as 0 or 1
  - Object = 1 (white)
  - background = 0 (black)

– Processing by logical functions is fast and simple

Structuring Element (SE)

- SE có thể thay đổi kích thước
- Giá trị của các phần tử là 0, 1

Examples of SE

Square/Box

1	1	1
1	1	1
1	1	1

Disc

	1	
1	1	1
	1	

Some Basic Definitions

- **Dilation – giãn/mở rộng**

  $$A \oplus B = \{x \mid (B + x) \cap A \neq \emptyset\}$$

- Dilation expands a region.

$$g[x,y] = OR\left[W\{f[x,y]\}\right] := dilate(f,W)$$

Original (701x781)

dilation with
3x3 structuring element

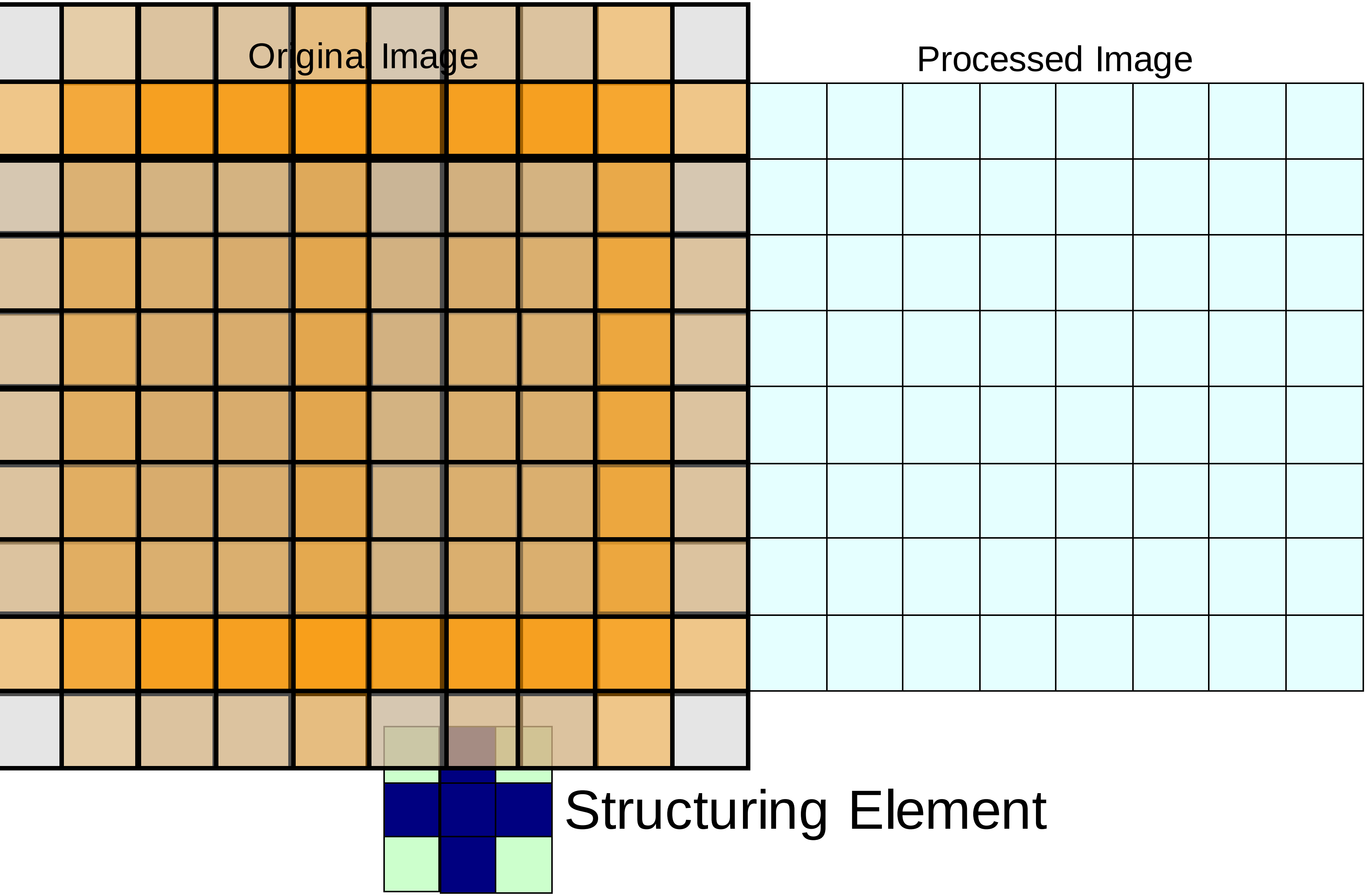
dilation with
7x7 structuring element

- Expands the size of 1-valued objects
- Smoothes object boundaries
- Closes holes and gaps

Dilation expands a region

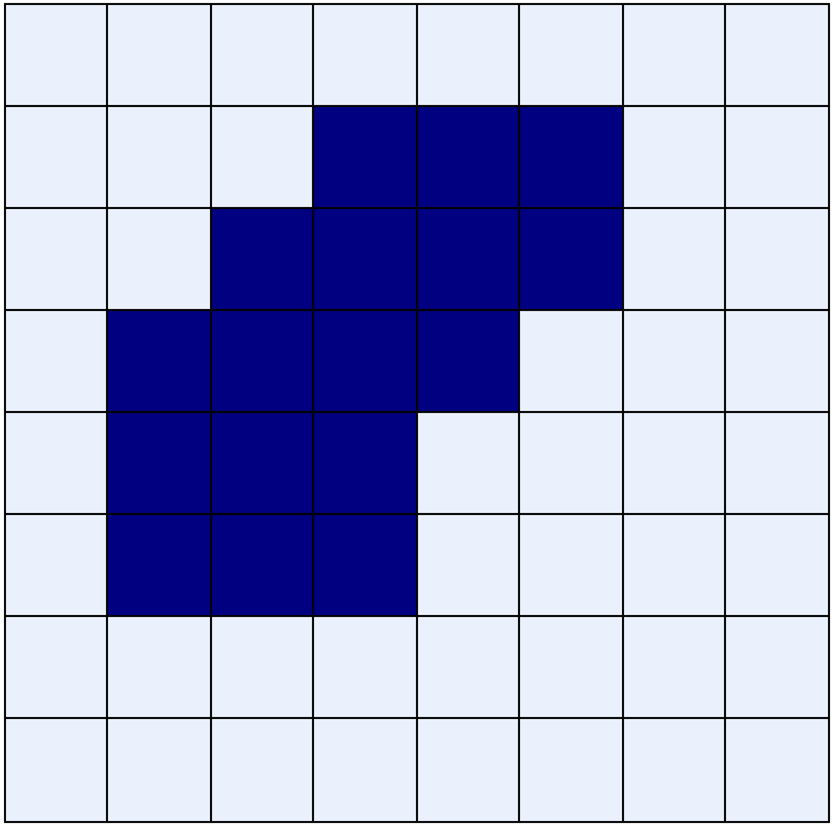
$$g[x,y] = OR[W\{f[x,y]\}] := dilate(f,W)$$

Dilation Example

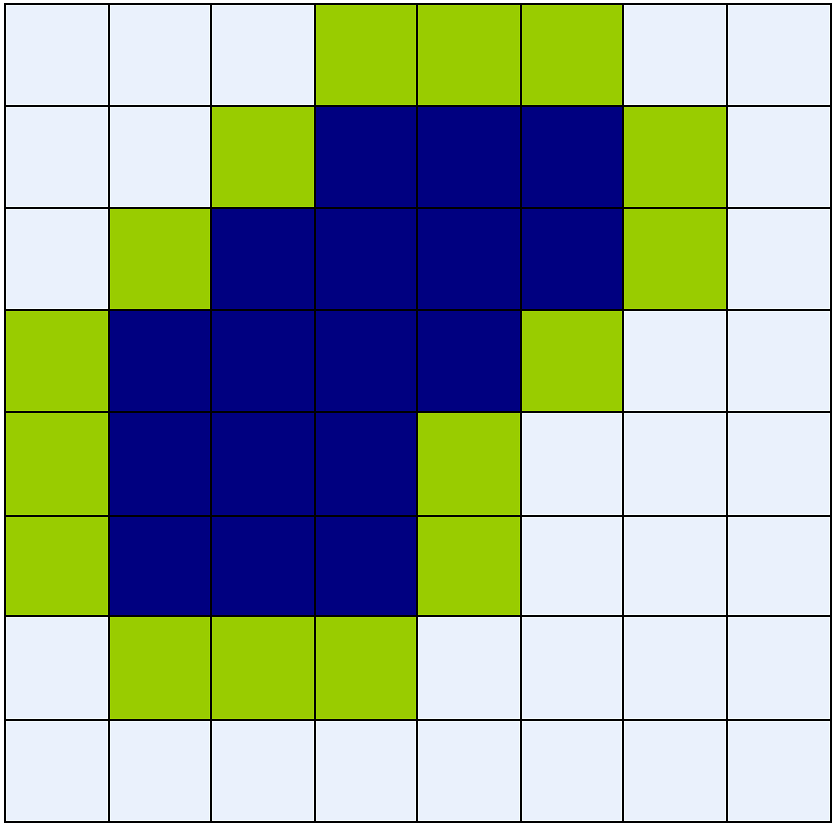


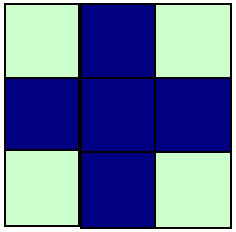
Dilation Example

Original Image



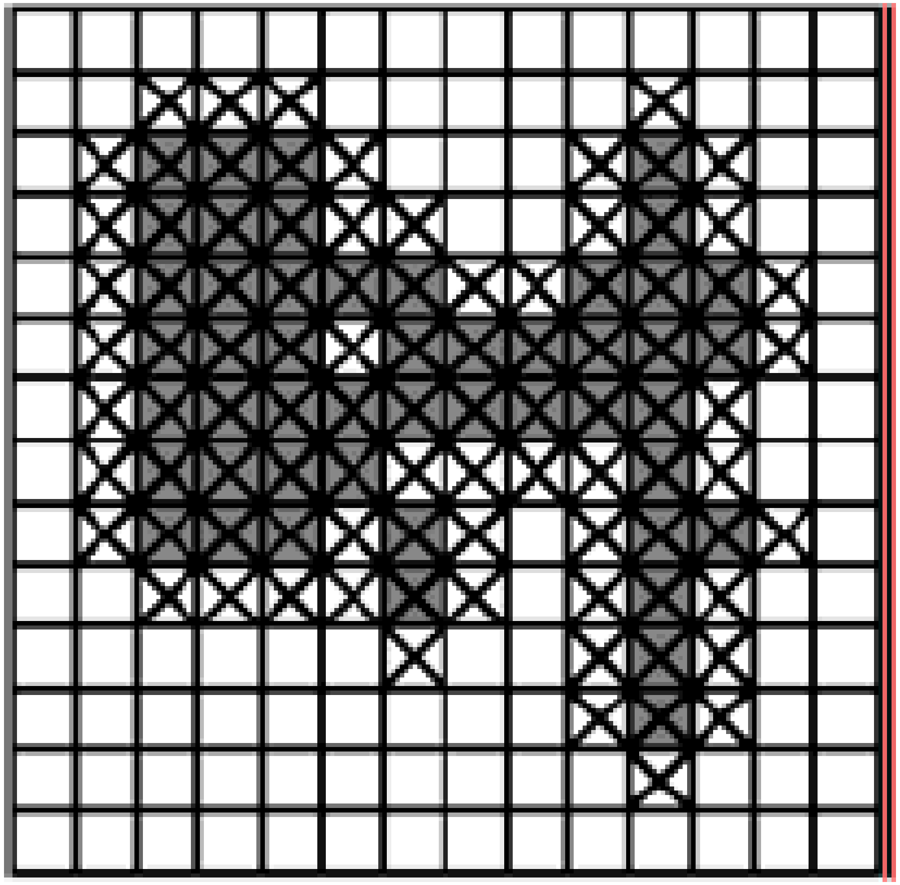
Processed Image With Dilated Pixels

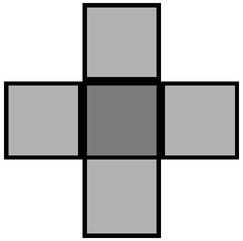




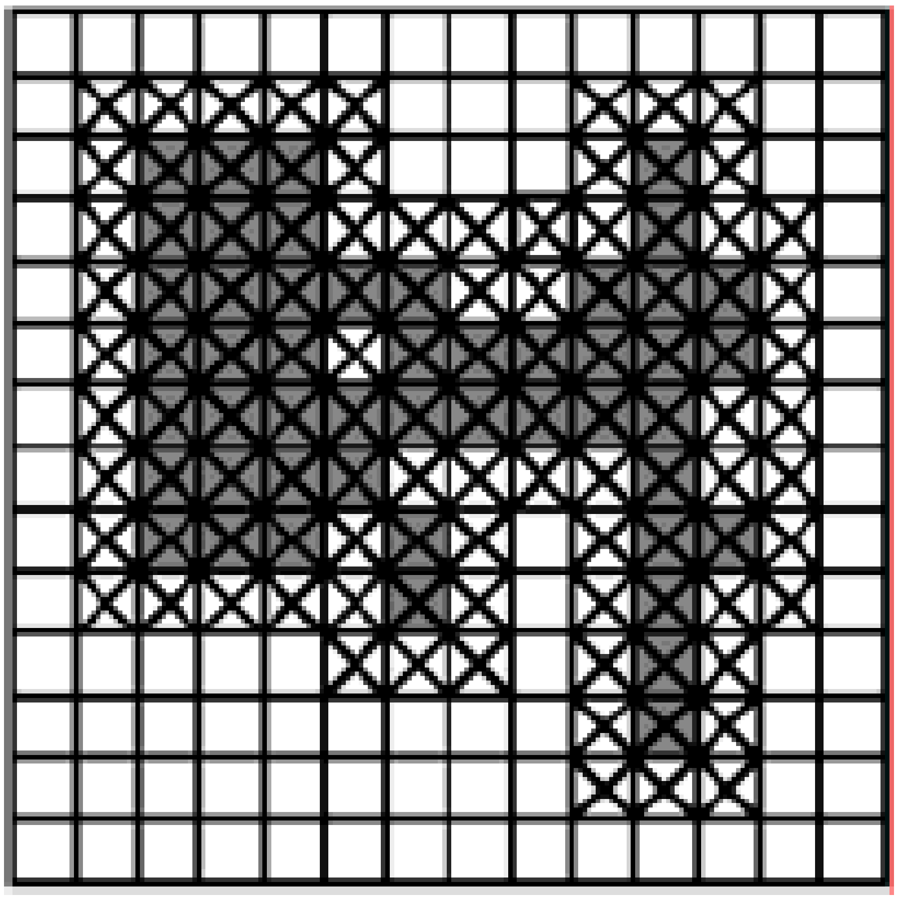
Structuring Element

Dilation

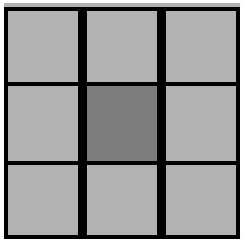


SE= 

Dilation

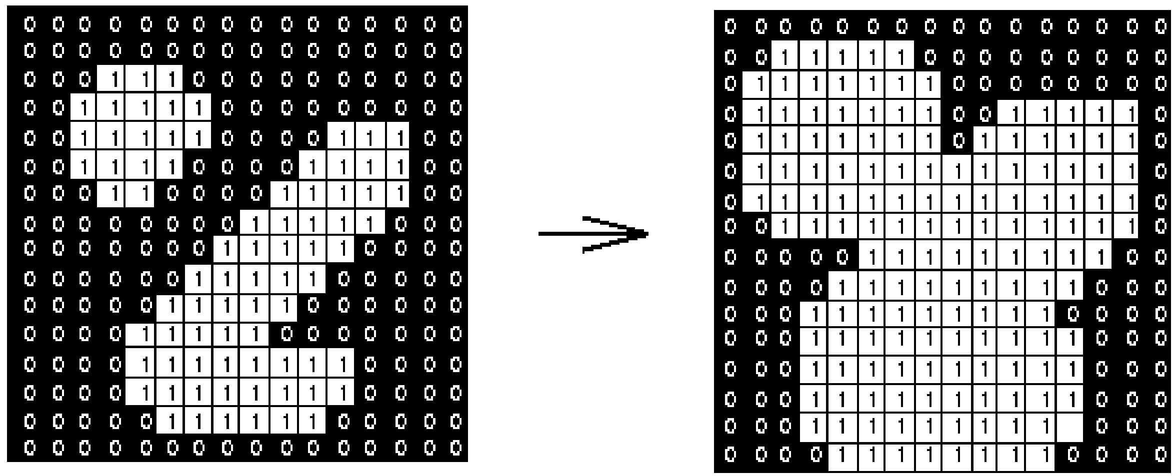


SE=



Example: Dilation

- **Dilation** is an important morphological operation



- Applied **Structuring Element**:

1	1	1
1	1	1
1	1	1

Dilation Example 1

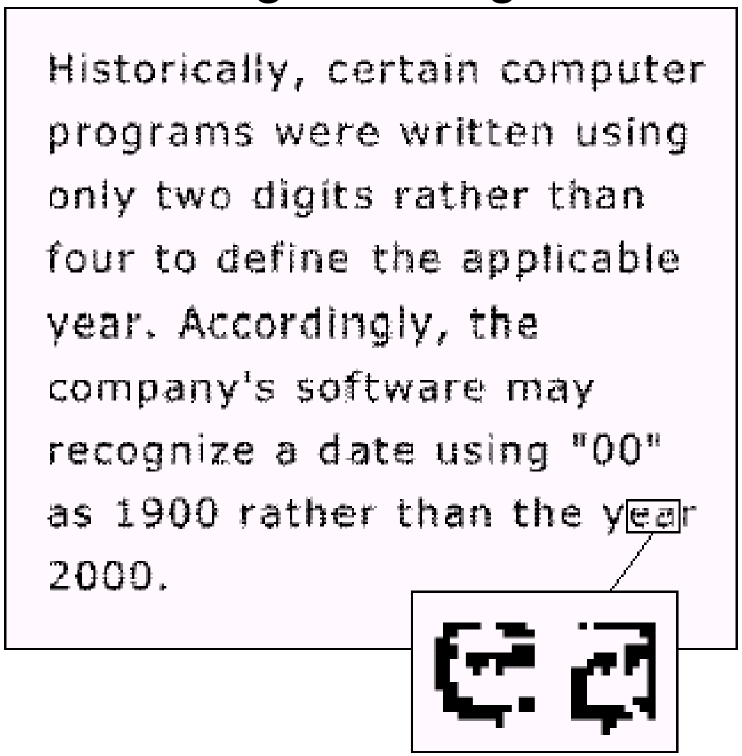
Original image

Dilation by 3*3 square structuring element

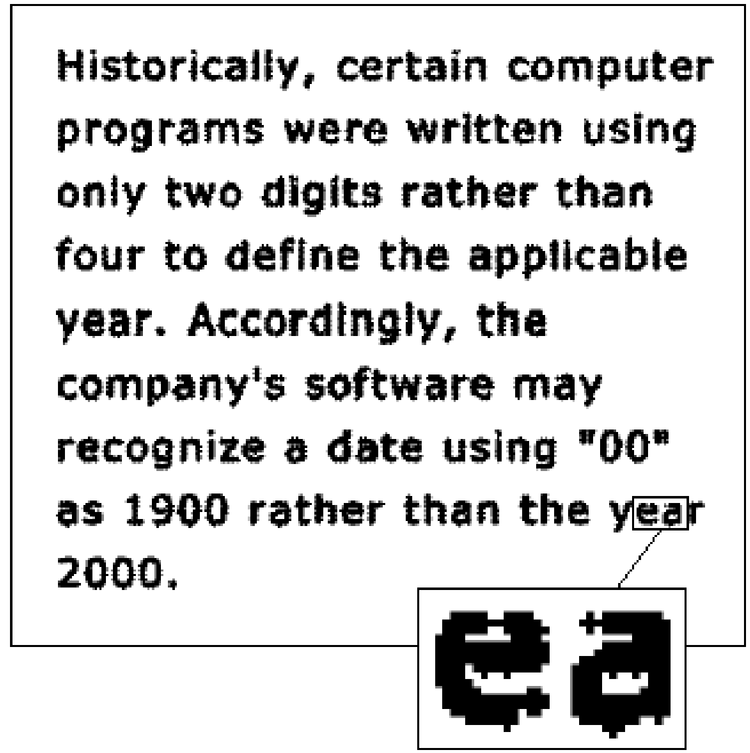
Dilation by 5*5 square structuring element

Application of dilation: bridging gaps in images

Original image


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

After dilation


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

0	1	0
1	1	1
0	1	0

Structuring element

What Is Dilation For?

Dilation can repair breaks

Dilation can repair intrusions

Some Basic Definitions

- **Erosion – co/thu hẹp**

$$A \ominus B = \{x \mid (B + x) \subseteq A\}$$

- Erosion shrinks a region.

$$g[x,y] = AND\left[\hat{W}\left\{f[x,y]\right\}\right] := erode(f,W)$$

Original (701x781)

erosion with
3x3 structuring element

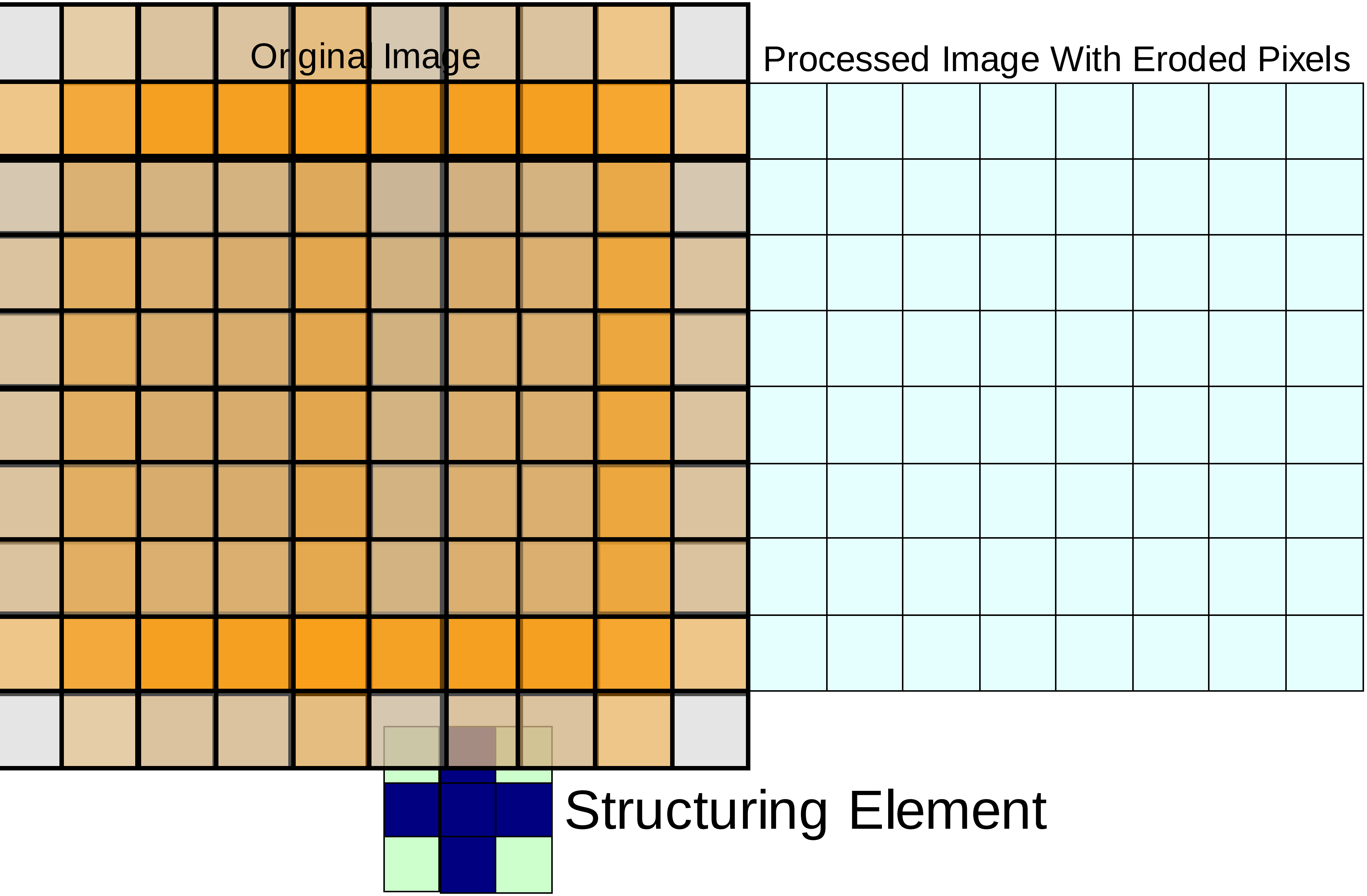
erosion with
7x7 structuring element

- Shrinks the size of 1-valued objects
- Smoothes object boundaries
- Removes peninsulas, fingers, and small objects

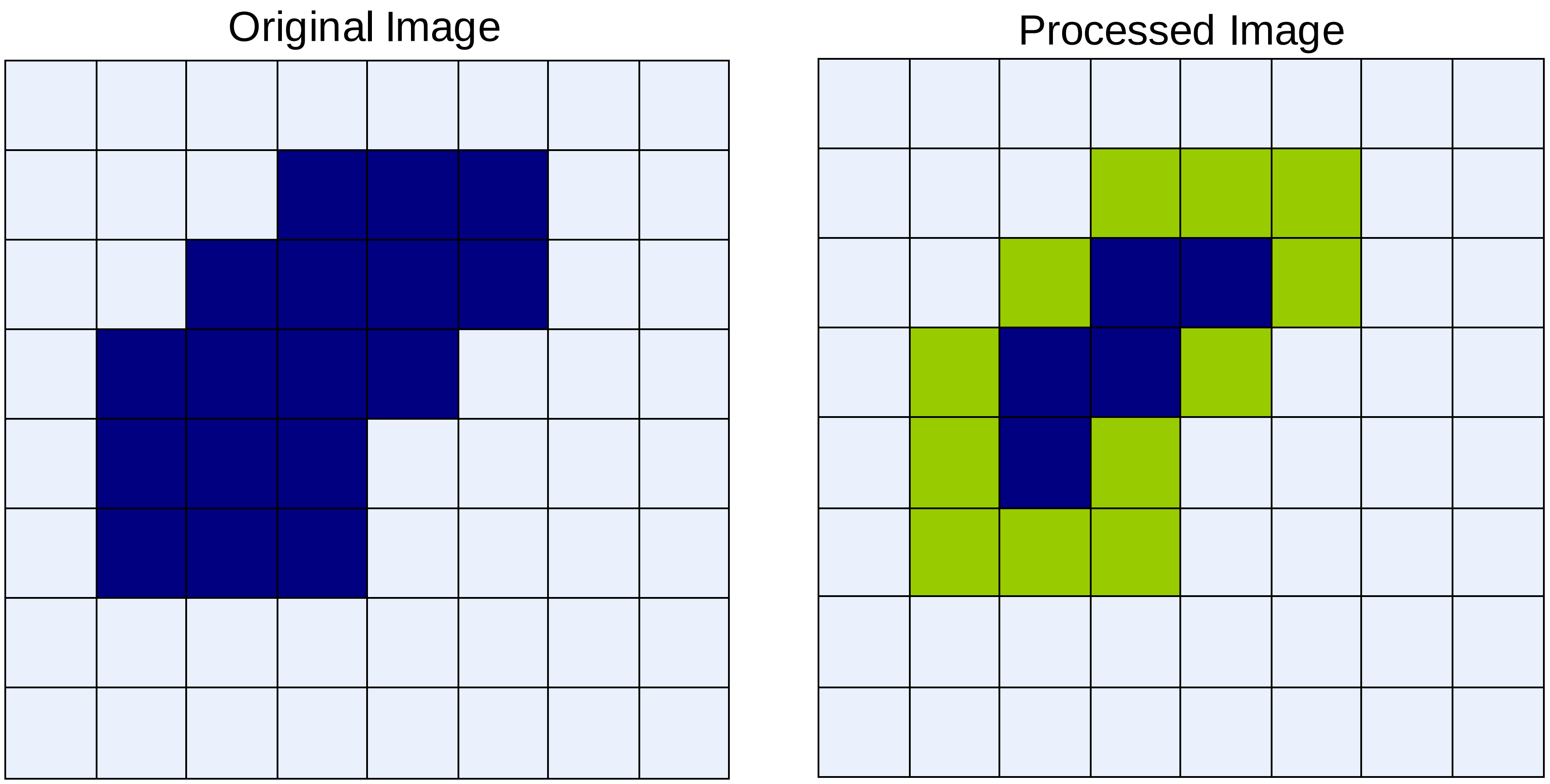
Erosion (shrinking foreground)

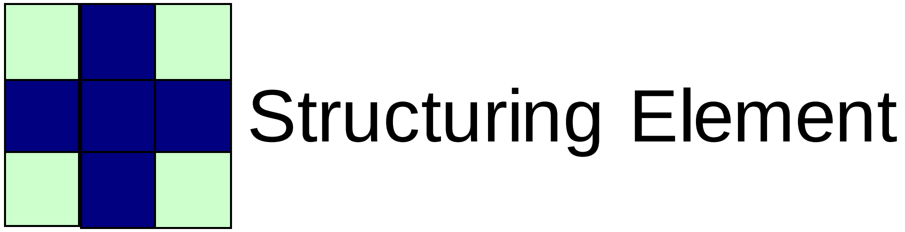
$$g[x,y] = AND\left[\hat{W}\left\{f[x,y]\right\}\right] := erode(f, W)$$

Erosion Example

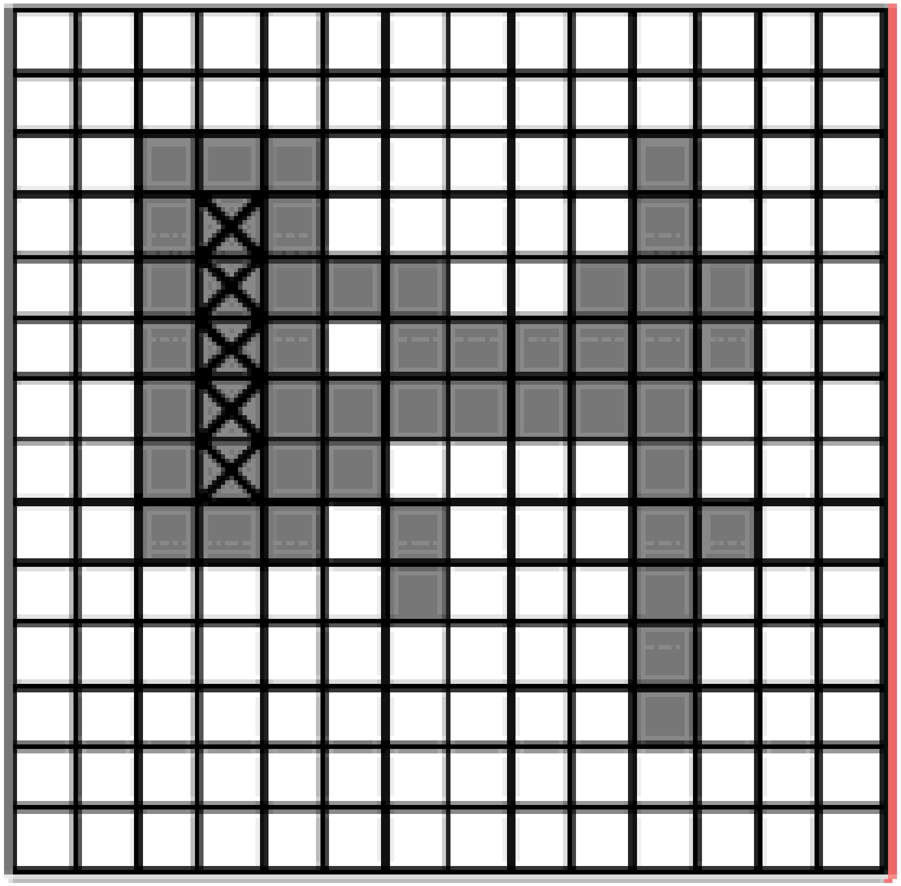


Erosion Example

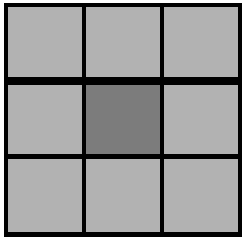




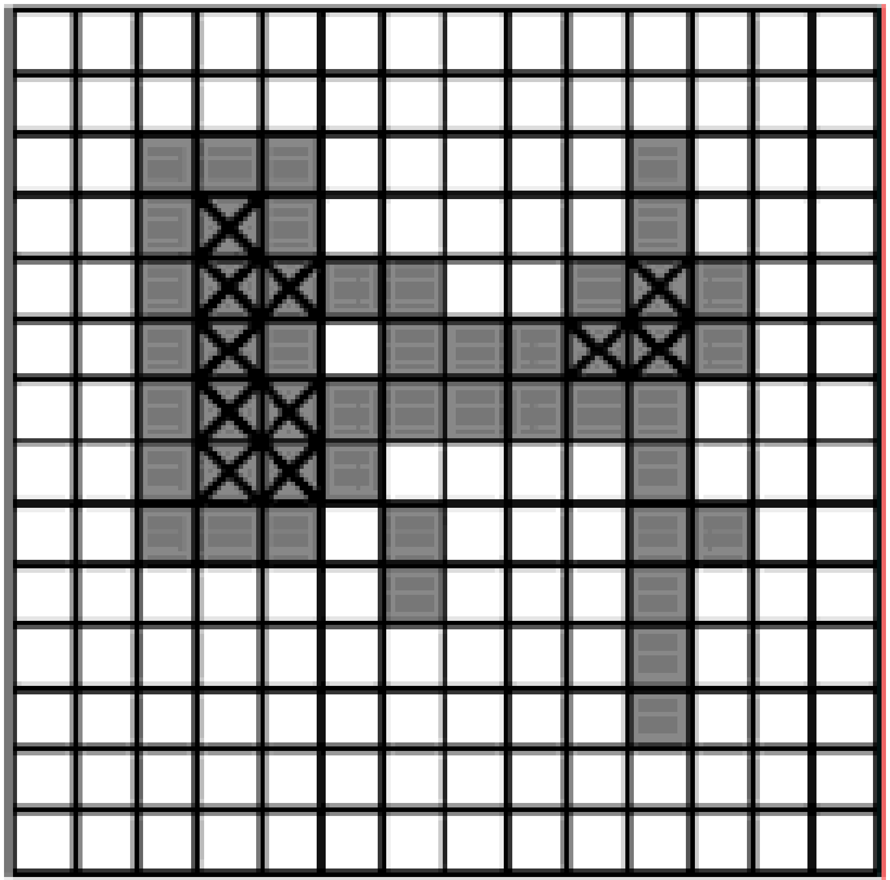
Erosion



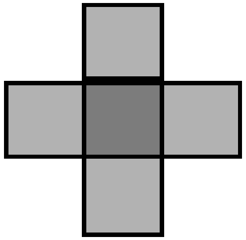
SE=



Erosion

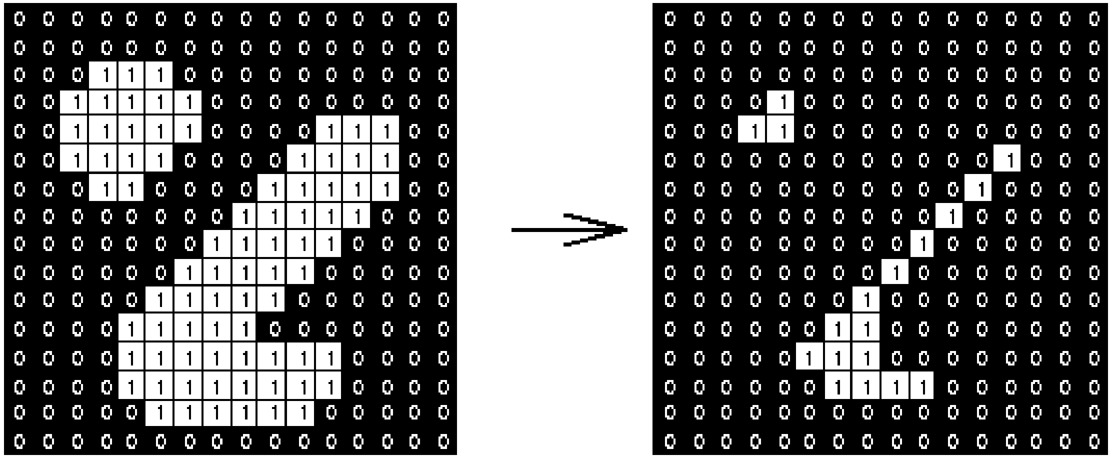


SE=



A first Example: Erosion

- **Erosion** is an important morphological operation

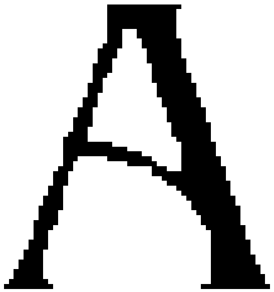


- Applied **Structuring Element**:

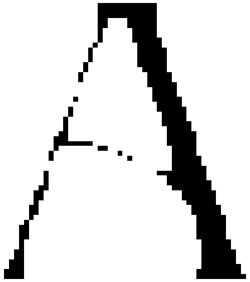
1	1	1
1	1	1
1	1	1

Erosion Example 1

Original image



Erosion by 3*3 square structuring element

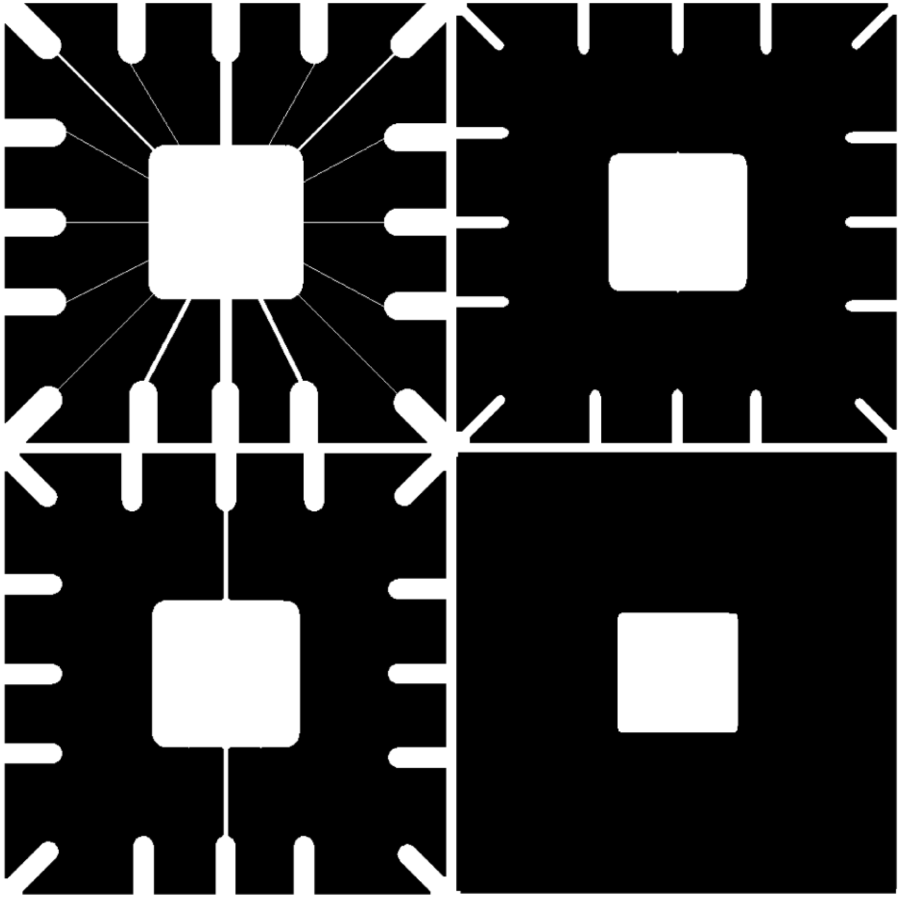


Erosion by 5*5 square structuring element

Erosion Example 2

Original image

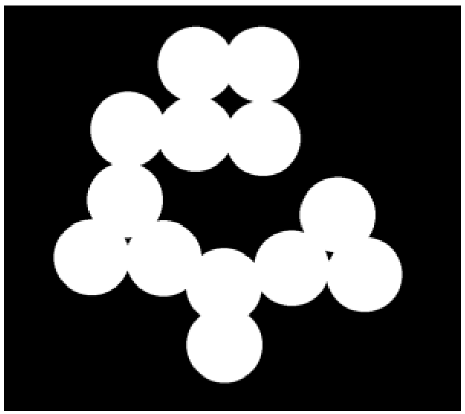
After erosion with a disc of radius 10



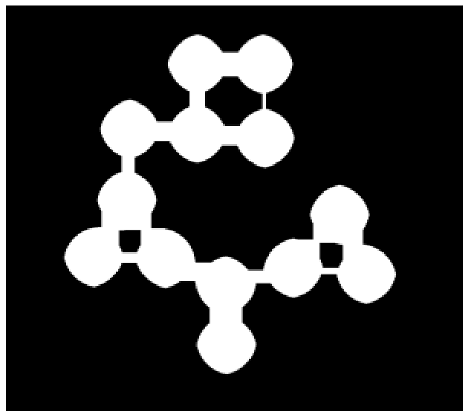
After erosion with a disc of radius 5

After erosion with a disc of radius 20

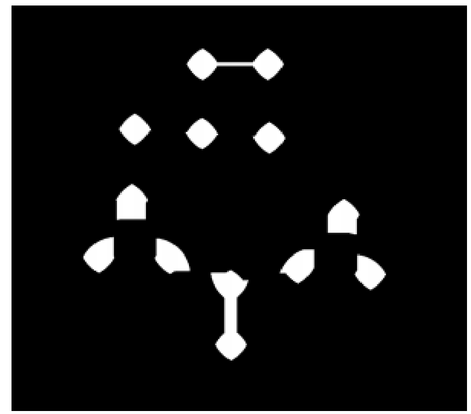
Erosion Example 3



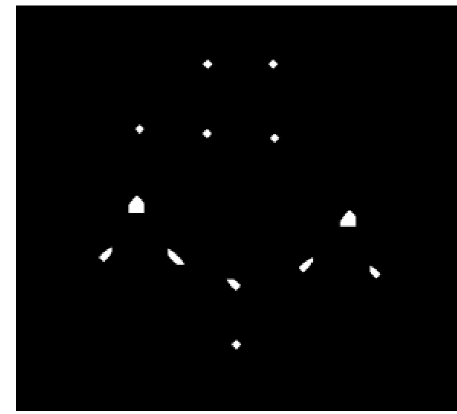
Original binary image
Circles (792x892)



Erosion by 30x30
structuring element



Erosion by 70x70
structuring element

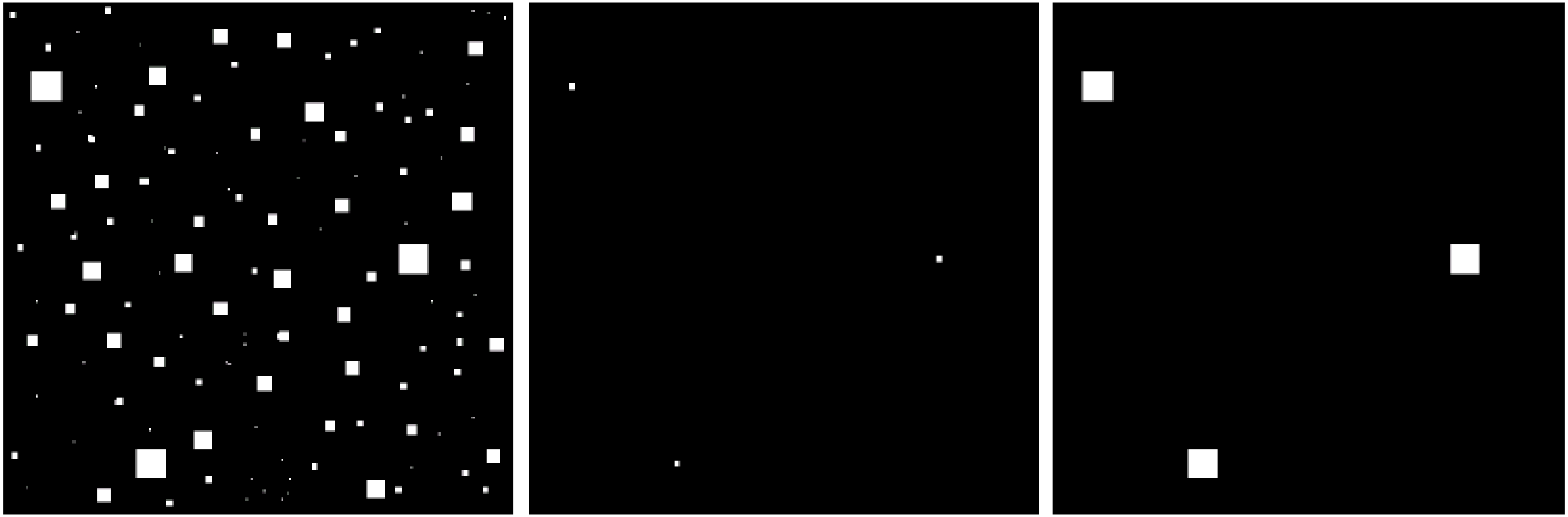


Erosion by 96x96
structuring element

Application of erosion: loại bỏ các chi tiết không liên quan

Squares of size 1,3,5,7,9,15 pels

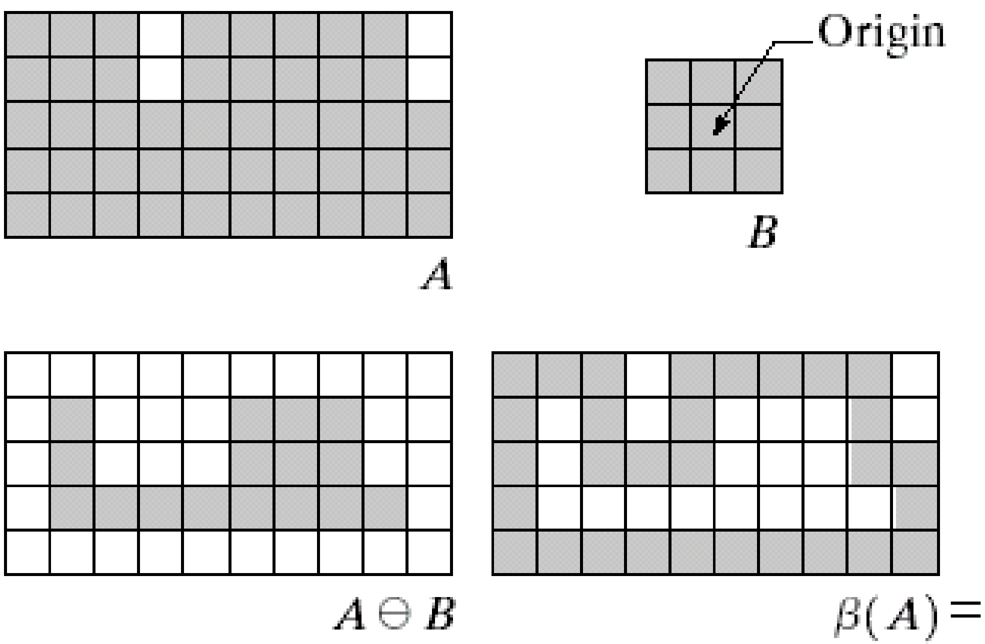
Erode with 13x13 square



original image → erosion → dilation

Dilation and erosion are duals

- Extract boundary of a set A:
  - First erode A (make A smaller)



$$A - (A \ominus B)$$

Application: boundary extraction

original image

Using 5x5 structuring element

Some Basic Definitions

- **Opening** is erosion followed by dilation:

  $$A \circ B = (A \ominus B) \oplus B$$

- Opening smoothes regions, removes spurs, breaks narrow lines.
- **Closing** is dilation followed by erosion:

  $$A \bullet B = (A \oplus B) \ominus B$$

- Closing fills narrow gaps and holes in a region.

Some Basic Definitions

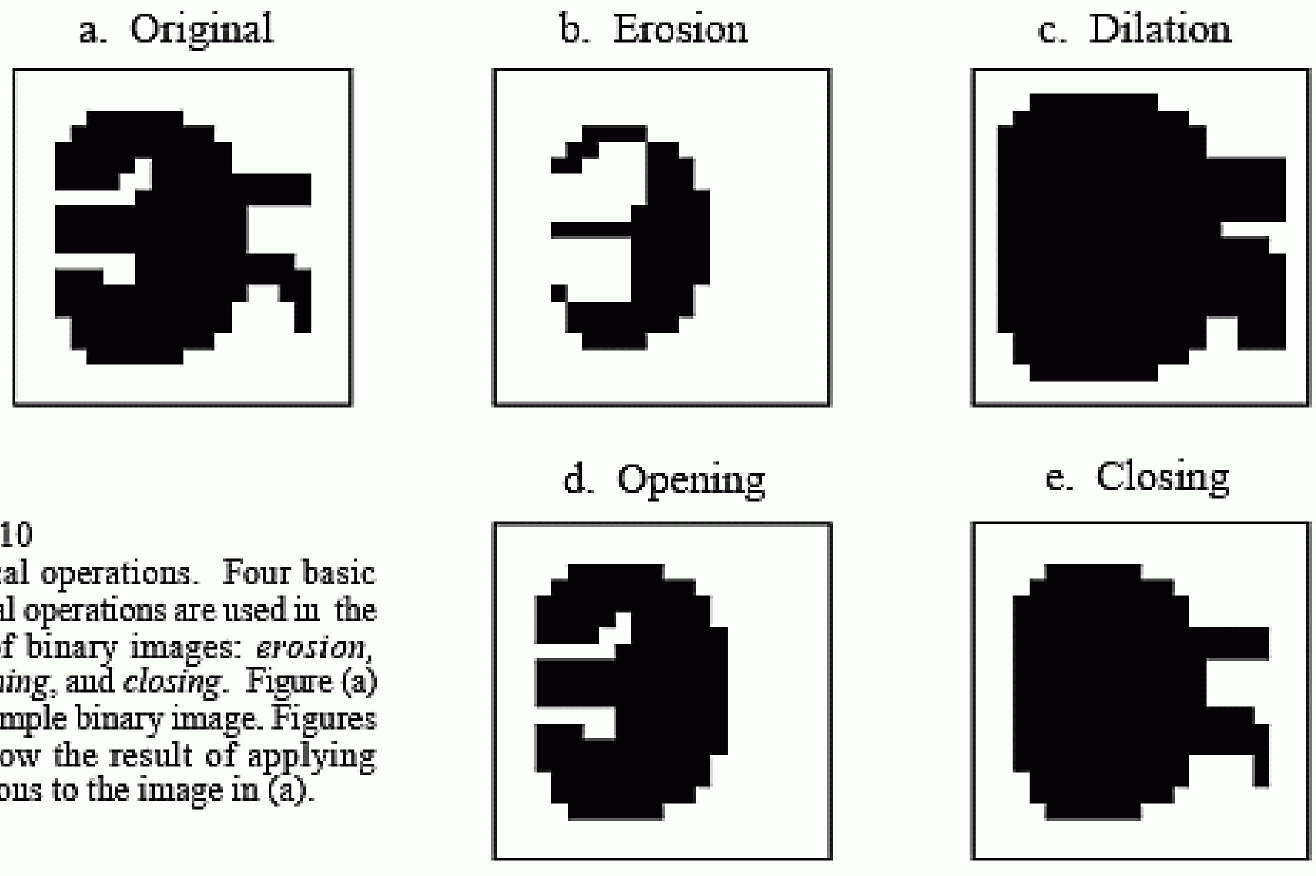
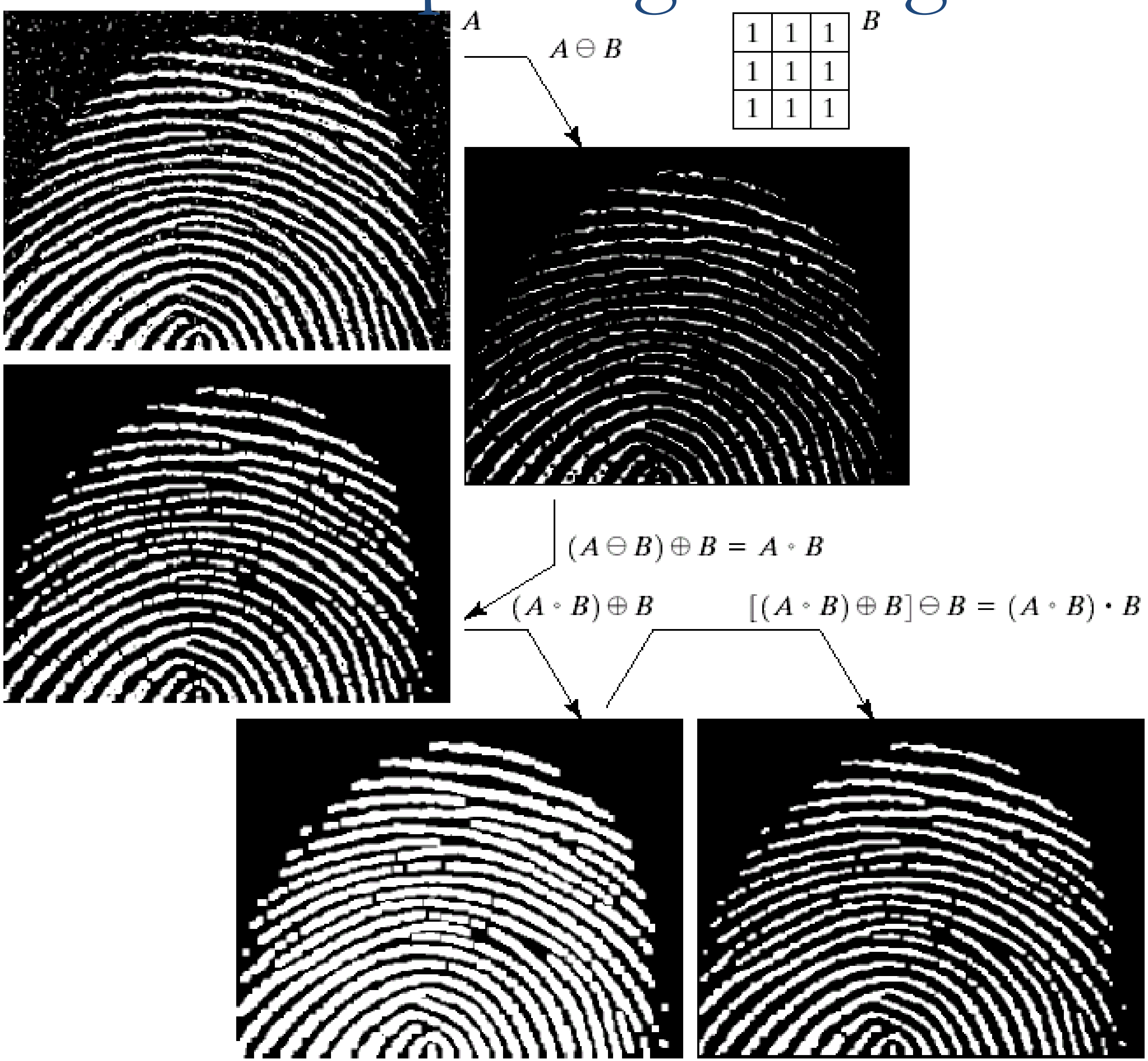


FIGURE 25-10
Morphological operations. Four basic morphological operations are used in the processing of binary images: *erosion*, *dilation*, *opening*, and *closing*. Figure (a) shows an example binary image. Figures (b) to (e) show the result of applying these operations to the image in (a).

Some Morphological Algorithms



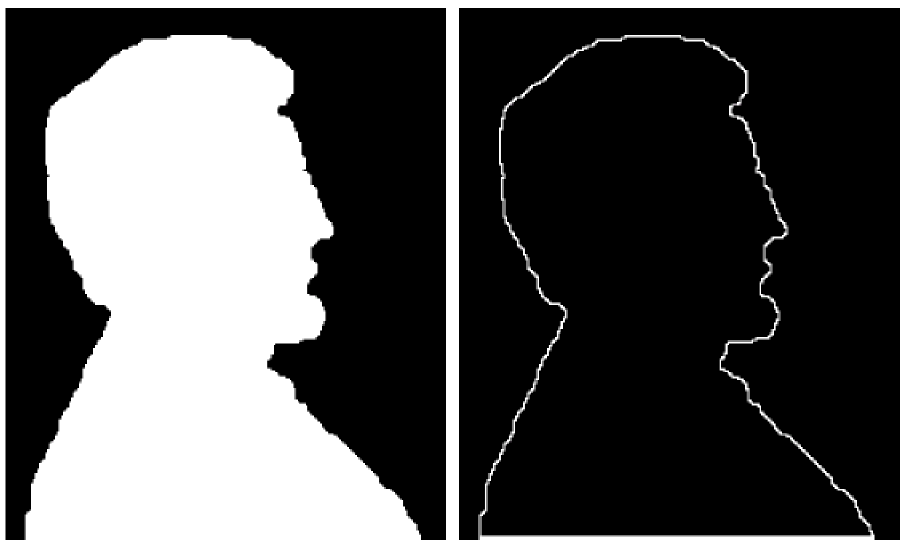
a b
c d
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Some Morphological Algorithms

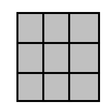
- Boundary of a set, A, can be found by

  $A - (A \ominus B)$



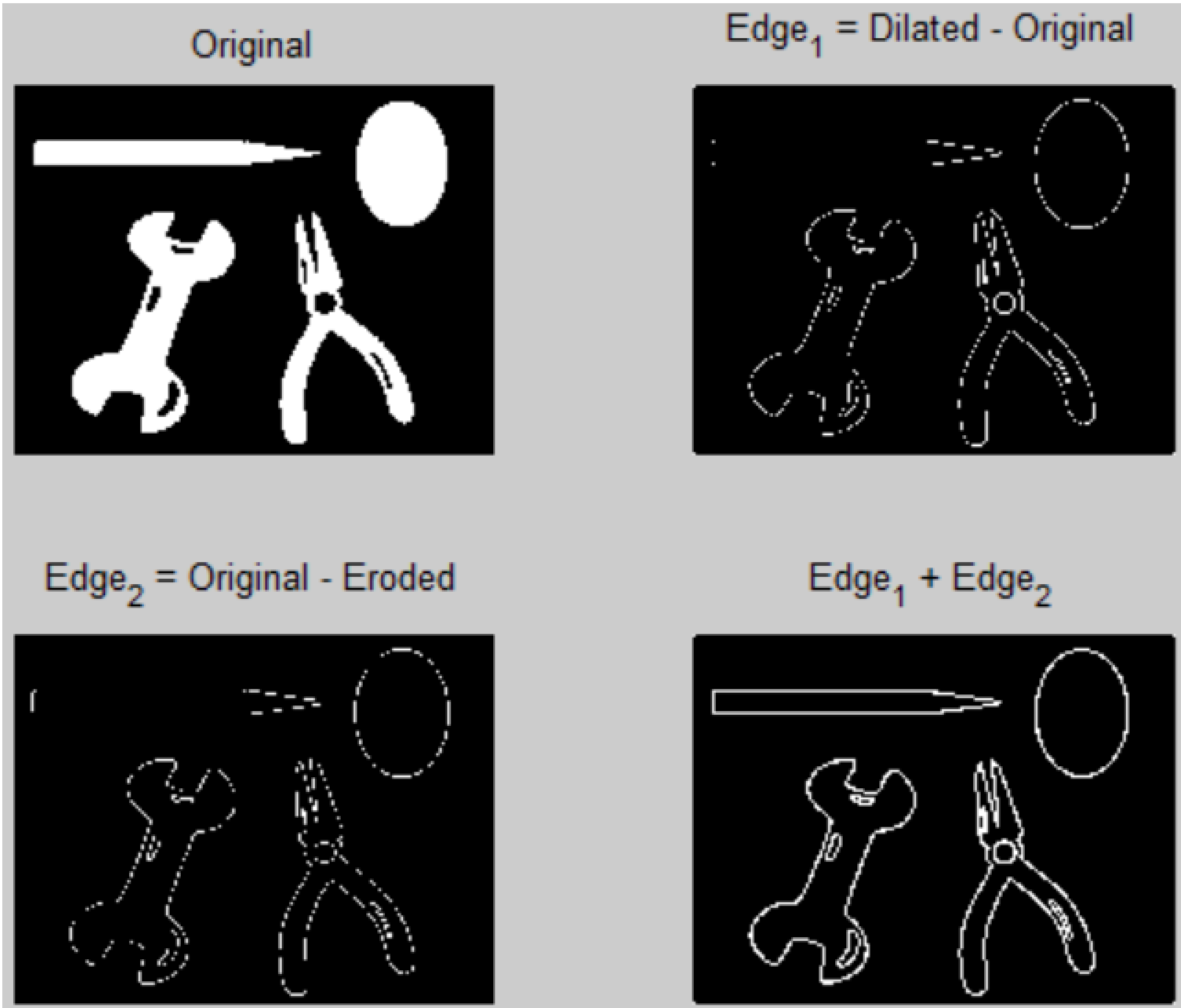
a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

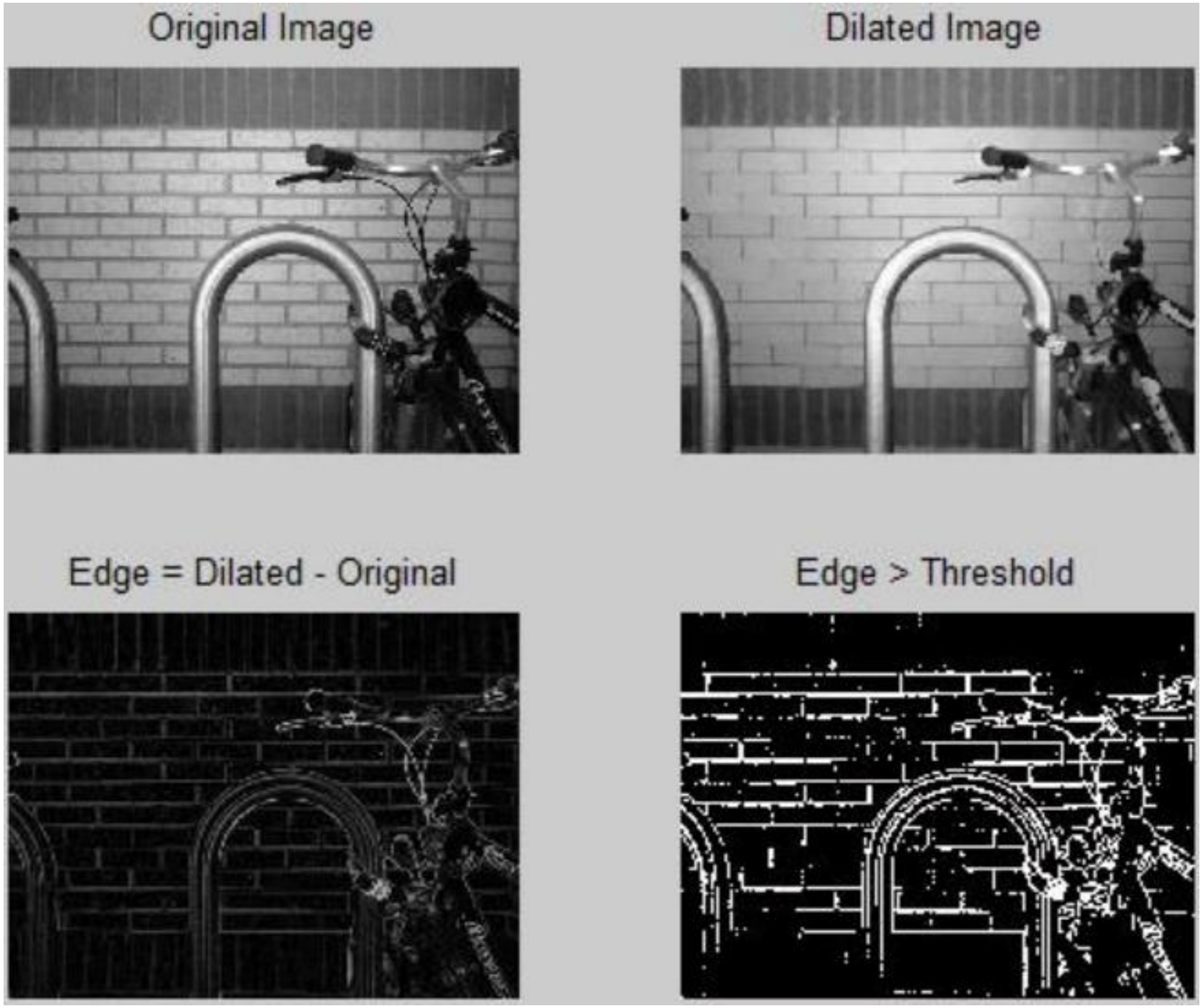


B

Some Morphological Algorithms



Some Morphological Algorithms



Summary

TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)