CSE301 – DATABASE

Functional Dependencies

Functional Dependencies

❑ A functional dependency (FD) is a relationship between two attributes (typically between the Primary Key and other non-key attributes within a table).

❑ For any relation R, attribute Y is functionally dependent on attribute X (usually the Primary Key), if for every valid instance of X, that value of X uniquely determines the value of Y.

❑ This relationship is indicated by $X \rightarrow Y$

❑ The left side of the above FD diagram is called the determinant, and the right side is the dependent.

Examples of Functional Dependencies

R

A	B
1	6
2	7
3	8
4	9

❑ A → B, if for every valid instance of A, that value of A uniquely determines the value of B.

❑ *{1 → 6}, {2 → 7}, {3 → 8}, {4 → 9}*

❑ *B has the same value for the same value as A.*

R

A	B
1	6
2	7
3	8
2	9

❑ A ↛ B, if for every valid instance of A, that value of A not uniquely determines the value of B.

❑ *{1 → 6}, {2 → 7}, {3 → 8}, {2 → 9}*

❑ *B has not the same value for the same value as A.*

Functional Dependencies

❑ We can mathematically represent $X \rightarrow Y$,

➢ When $X \subseteq R$ and $Y \subseteq R$

➢ If $T_1[X] = T_2[X]$, then $T_1[Y] = T_2[Y]$

R

X	Y
1	6
2	7
1	6
4	9

T1 →

T2 →

Examples of Functional Dependencies

❑ Which of the following FD is not valid?

    a) $A \rightarrow B$

    b) $B \rightarrow C$

    c) $BC \rightarrow A$

    d) $AC \rightarrow B$

R

A	B	C
1	2	3
4	2	3
5	3	3

❑ $A \rightarrow B$, $\{1 \rightarrow 2\}$, $\{4 \rightarrow 2\}$, $\{5 \rightarrow 3\}$ *Valid*

❑ $B \rightarrow C$, $\{2 \rightarrow 3\}$, $\{2 \rightarrow 3\}$, $\{3 \rightarrow 3\}$ *Valid*

❑ $BC \rightarrow A$, $\{2,3 \rightarrow 1\}$, $\{2,3 \rightarrow 4\}$, $\{3,3 \rightarrow 5\}$ *Not Valid*

    ➢ *Right hand side has not the same value for the same value as left hand side.*

❑ $AC \rightarrow B$, $\{1,3 \rightarrow 2\}$, $\{4,3 \rightarrow 2\}$, $\{5,3 \rightarrow 3\}$ *Valid*

Examples of Functional Dependencies

❑ Which of the following FD is valid?

    a) $XY \rightarrow Z, Z \rightarrow Y$

    b) $XZ \rightarrow X, Y \rightarrow Z$

    c) $YZ \rightarrow X, Z \rightarrow X$

    d) $XZ \rightarrow Y, Y \rightarrow Z$

R

X	Y	Z
1	4	3
1	5	3
4	6	3
3	2	2

c) $YZ \rightarrow X, Z \rightarrow X$

❑ $YZ \rightarrow X$
  ➢ $\{4,3 \rightarrow 1\}, \{5,3 \rightarrow 1\},$
     $\{6,3 \rightarrow 4\}, \{2,2 \rightarrow 3\}$
❑ $Z \rightarrow X$
  ➢ $\{3 \rightarrow 1\}, \{3 \rightarrow 1\},$
     $\{3 \rightarrow 4\}, \{2 \rightarrow 3\}$

Option c is not valid

a) $XY \rightarrow Z, Z \rightarrow Y$

❑ $XY \rightarrow Z$
  ➢ $\{1,4 \rightarrow 3\}, \{1,5 \rightarrow 3\},$
     $\{4,6 \rightarrow 3\}, \{3,2 \rightarrow 2\}$
❑ $Z \rightarrow Y$
  ➢ $\{3 \rightarrow 4\}, \{3 \rightarrow 5\},$
     $\{3 \rightarrow 6\}, \{2 \rightarrow 2\}$

Option a is not valid

b) $XZ \rightarrow X, Y \rightarrow Z$

❑ $XZ \rightarrow X$
  ➢ $\{1,3 \rightarrow 1\}, \{1,3 \rightarrow 1\},$
     $\{4,3 \rightarrow 4\}, \{3,2 \rightarrow 3\}$
❑ $Y \rightarrow Z$
  ➢ $\{4 \rightarrow 3\}, \{5 \rightarrow 3\},$
     $\{6 \rightarrow 3\}, \{2 \rightarrow 2\}$

Option b is valid

d) $XZ \rightarrow Y, Y \rightarrow Z$

❑ $XZ \rightarrow X$
  ➢ $\{1,3 \rightarrow 4\}, \{1,3 \rightarrow 5\},$
     $\{4,3 \rightarrow 6\}, \{3,2 \rightarrow 2\}$
❑ $Y \rightarrow Z$
  ➢ $\{4 \rightarrow 3\}, \{5 \rightarrow 3\},$
     $\{6 \rightarrow 3\}, \{2 \rightarrow 2\}$

Option d is not valid

Exercise on Functional Dependencies

❑ Which of the following FD is correct?

    a) A → BC

    b) DE → C

    c) C → DE

    d) BC → A

R

A	B	C	D	E
A	2	3	4	5
2	A	3	4	5
A	2	3	6	5
a	2	3	6	6

❑ Which of the following FD is not correct?

    a) XY → Z, Z → Y

    b) YZ → X, Y → Z

    c) XZ → X, Z → X

    d) XZ → Y, Y → Z

R

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

Classification of Functional Dependencies

❑ Trivial functional dependency

✓ A → B has trivial functional dependency if B is a subset of A (B ⊆ A).

✓ Examples: *A → A, AB → B, {Employee_id, Employee_Name} → Employee_Id*

❑ Non-trivial functional dependency

✓ A → B has a non-trivial functional dependency if B is not a subset of A.

✓ If there is at least one attribute in right hand side that is not present in the left-hand side.

✓ Examples: *AB → BC, {ID} → {ID, DOB}, {Roll, Name} → {Roll, Name, Phone}*

✓ When A intersection B is NULL, then A → B is called as complete non-trivial.

✓ *Example: A → B, B → C, {ID} → {Name}, {Name} → {DOB}*

Classification of Functional Dependencies

❑ Fully functional dependency

   ✓ Given R and A → B, then B is fully functional dependent on A if there is no Z where Z is a proper subset of A (Z ⊂ A) such that Z → B.

   ✓ Examples: *{AB → C, A → D} is a fully FD,* *{AB → C, A → C} is not a fully FD*

   ✓ MaSV, MaMH → TenSV, TenMH,Diem and MaSV ≠> TenSV, TenMH,Diem

❑ Partial functional dependency

   ✓ Given a relation R with FD F defined on the attributes of R and K as a candidate key, if X is a proper subset of K (X⊂ K) and if and only if X → A, then A said to be partially dependent on K.

   ✓ Examples: *R(ABCD), Key(AB), FD: {A → C}. C is partially dependent on A.*

   ✓ MaSV, MaMH → TenSV, TenMH, Diem and MASV –>TenSV, MaMH -> TenMH

Classification of Functional Dependencies

❑ Transitive functional dependency

  ✓ If A → B and B → C, then C is transitively functional dependent on A such that A → C.

  ✓ Examples: {AB → C, C → D}, So, {AB → D}

Armstrong's axioms / Inference Rules

❑ Reflexive Rule (IR1)

  ✓ If Y is a subset of X (X ⊇ Y), then X determines Y (X → Y).

  ✓ Examples: *{AB → A}, {Employee_id, Employee_Name} → {Employee_Name}*

❑ Augmentation Rule (IR2)

  ✓ If X determines Y (X → Y), then XZ determines YZ (XZ → YZ). for any Z.

  ✓ Examples: *R(ABCD), if A → B then AC → BC*

❑ Transitive Rule (IR3)

  ✓ If X determines Y (X → Y) and Y determine Z (Y → Z), then X must also determine Z (X → Z).

Armstrong's axioms / Inference Rules

❑ Union Rule (IR4)

  ✓ If X determines Y (X → Y) and X determines Z (X → Z), then X must also determine Y and Z (X → YZ).

  ✓ Examples: *R(ABCD), if A → B and A → C then A →   BC.*

❑ Decomposition Rule (IR5)

  ✓ If X determines Y and Z (X → YZ), then X determines Y (X → Y) and X determines Z (Y → Z) separately.

  ✓ Examples: *R(ABCD), if A →   BC, then A → B and A → C*

❑ Pseudo Transitive Rule (IR6)

  ✓ If X determines Y (X → Y) and YZ determines W (YZ → W) , then XZ determines W (XZ → W).

Closure Set of Attributes

❏ X^+ is the set of all attributes that can be determine using the given set X(attributes).

❏ *If "F" is a functional dependency then closure of functional dependency can be denoted using "{F}+".*

Closure Set of Attributes

❑ There are three steps to calculate closure of functional dependency. These are:

✓ Step-1 : Add the attributes which are present on Left Hand Side in the original functional dependency.

✓ Step-2 : Now, add the attributes present on the Right Hand Side of the functional dependency.

✓ Step-3 :

➢ With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies.

➢ Repeat this process until all the possible attributes which can be derived are added in the closure.

Closure Set of Attributes

Input: Functional dependencies (FDs), Target set of attributes (X)

Output: Closure of X, denoted as X+

Step 1: Initialize X+ with attributes in X

Step 2: Repeat until no new attributes can be added to X+

    For each functional dependency (Y → Z) in FDs:

        If Y is a subset of X+:

            Add all attributes of Z to X+

Return X+ as the closure of X

Example of Closure Set of Attributes

- Consider the table Student_details having (Roll_no, Name, Marks, Location) as the attributes and having two functional dependencies.

    FD1 : Roll_no → Name, Marks

    FD2 : Name → Marks, Location

- Now, we will calculate the closure of all the attributes present in the relation using the three steps mentioned below. Find closure set of attributes of {Roll_no}+

- Step-1 : add attributes present on the LHS of the first functional dependency to the closure.

    {Roll_no}+ = {Roll_no}

- Step-2 : add attributes present on the RHS of the original functional dependency to the closure.

    {Roll_no}+ = {Roll_no, Name, Marks}

Example of Closure Set of Attributes

▪ Step-3 : Add the other possible attributes which can be derived using attributes present on the RHS of the closure.

- So, Roll_No attribute cannot functionally determine any attribute, but Name attribute can determine other attributes such as Marks and Location using 2nd Functional Dependency.

- Therefore, complete closure of Roll_No will be :

    {Roll_no}+ = {Roll_No, Marks, Name, Location}

Example of Closure Set of Attributes

- Example 1: R (ABCDEFG)
  - $\{A \rightarrow B, BC \rightarrow DE, AEG \rightarrow G\}$
  - Find $(AC)^+ = ?$

- Example 2: R (ABCDE)
  - $\{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$
  - Find $(B)^+ = ?$

- Example 3: R (ABCDEF)
  - $\{AB \rightarrow C, CD \rightarrow E, DE \rightarrow B\}$
  - Find $(AB)^+ = ?$

- $(AC)^+ = AC$

$= ABC \quad (A \rightarrow B)$

$= ABCDE \quad (BC \rightarrow DE)$

$= ABCDE$

- $(B)^+ = B$

$= BD \quad (B \rightarrow D)$

$= BD$

- $(AB)^+ = AB$

$= ABC \quad (AB \rightarrow C)$

Example of Closure Set of Attributes

- Example 1: R (ABCDEFGH)
  - {A → BC, CD → E, E → C, D → AEH, ABH → BD, DH → BC}
  - Find $(BCD)^+ → H$?

  - $(BCD)^+ = BCD$

  = BCDE (CD → E)

  = ABCDEH (D → AEH)

  So, $(BCD)^+ → H$ is valid

- Exercise 1: R (ABCDEF)
  - {A → BC, CD → EF, B → D, E → A}
  - Find $(AE)^+ = ?$

- Exercise 2: R (ABCDEF)
  - {AB → C, CD → E, DE → B}
  - Find $(AB)^+ = ?$

- Exercise 3: Student (Roll, Name, DoB, Phone, Course)
  - {Roll, Name} → Phone, {Course, DoB} → Roll, Course → Name, Roll → Name}
  - Find $(Course)^+ = ?$

Equivalence Of Functional Dependencies

- R (ACDEH)

- F: {A → C, AC → D,
    E → AD, E → H}

- G: {A → CD, E → AH}

- (A)⁺ = ACD

- (AC)⁺ = ACD

- (E)⁺ = EAH = EAHCD

- So, F ⊆ G

- (A)⁺ = ACD

- (E)⁺ = EADCH

So, G ⊆ F

- Find the correct option

  a) F ⊆ G

  b) G ⊆ F

  c) F = G

  d) F ≠ G

- F ⊆ G and G ⊆ F, so, F = G

Equivalence Of Functional Dependencies

- R (ACDEH)

- F: {A → C, AC → D, E → AD, E → H}

- G: {A → CD, E → AH}

- Check both FD's are equivalent or not.

- R (ABC)

- F: {A → B, B → C, C → A}

- G: {A → BC, B → A, C → A}

- Find the correct option

  a) F ⊆ G

  b) G ⊆ F

  c) F = G

  d) F ≠ G

- R (VWXYZ)

- F: {W → X, WX → Y, Z → WY, Z → V}

- G: {W → XY, Z → WX}

- Find the correct option

  a) F ⊆ G

  b) G ⊆ F

  c) F = G

  d) F ≠ G