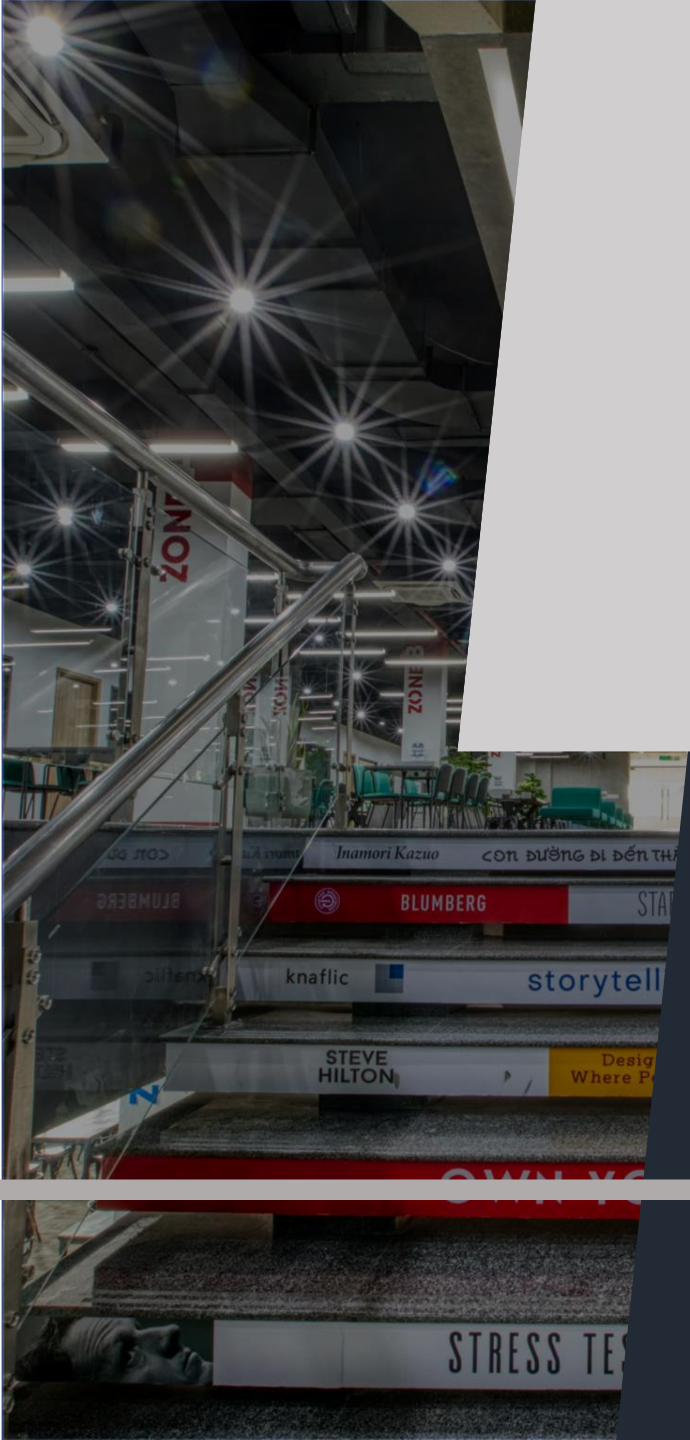

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CSE301 – DATABASE

Functional Dependencies

Functional Dependencies

- A functional dependency (FD) is a relationship between two attributes (typically between the Primary Key and other non-key attributes within a table).
- For any relation R, attribute Y is functionally dependent on attribute X (usually the Primary Key), if for every valid instance of X, that value of X uniquely determines the value of Y.
- This relationship is indicated by $X \rightarrow Y$
- The left side of the above FD diagram is called the determinant, and the right side is the dependent.

Examples of Functional Dependencies

R

A	B
1	6
2	7
3	8
4	9

- $A \rightarrow B$, if for every valid instance of A, that value of A uniquely determines the value of B.
- $\{1 \rightarrow 6\}, \{2 \rightarrow 7\}, \{3 \rightarrow 8\}, \{4 \rightarrow 9\}$
- *B has the same value for the same value as A.*

R

A	B
1	6
2	7
3	8
2	9

- $A \not\rightarrow B$, if for every valid instance of A, that value of A not uniquely determines the value of B.
- $\{1 \rightarrow 6\}, \{2 \rightarrow 7\}, \{3 \rightarrow 8\}, \{2 \rightarrow 9\}$
- *B has not the same value for the same value as A.*

Functional Dependencies

- We can mathematically represent $X \rightarrow Y$,
  - When $X \subseteq R$ and $Y \subseteq R$
  - If $T_1[X] = T_2[X]$, then $T_1[Y] = T_2[Y]$

R

	X	Y
T1	1	6
	2	7
T2	1	6
	4	9

Examples of Functional Dependencies

- Which of the following FD is not valid?

  a) $A \rightarrow B$

  b) $B \rightarrow C$

  c) $BC \rightarrow A$

  d) $AC \rightarrow B$

R

A	B	C
1	2	3
4	2	3
5	3	3

- $A \rightarrow B$, $\{1 \rightarrow 2\}, \{4 \rightarrow 2\}, \{5 \rightarrow 3\}$ ***Valid***
- $B \rightarrow C$, $\{2 \rightarrow 3\}, \{2 \rightarrow 3\}, \{3 \rightarrow 3\}$ ***Valid***
- $BC \rightarrow A$, $\{2,3 \rightarrow 1\}, \{2,3 \rightarrow 4\}, \{3,3 \rightarrow 5\}$ ***Not Valid***
  - *Right hand side has not the same value for the same value as left hand side.*
- $AC \rightarrow B$, $\{1,3 \rightarrow 2\}, \{4,3 \rightarrow 2\}, \{5,3 \rightarrow 3\}$ ***Valid***

Examples of Functional Dependencies

- Which of the following FD is valid?

  a) $XY \rightarrow Z,\ Z \rightarrow Y$

  b) $XZ \rightarrow X,\ Y \rightarrow Z$

  c) $YZ \rightarrow X,\ Z \rightarrow X$

  d) $XZ \rightarrow Y,\ Y \rightarrow Z$

R

X	Y	Z
1	4	3
1	5	3
4	6	3
3	2	2

c) $YZ \rightarrow X,\ Z \rightarrow X$

- $YZ \rightarrow X$
  - $\{4,3 \rightarrow 1\}, \{5,3 \rightarrow 1\}, \{6,3 \rightarrow 4\}, \{2,2 \rightarrow 3\}$
- $Z \rightarrow X$
  - $\{3 \rightarrow 1\}, \{3 \rightarrow 1\}, \{3 \rightarrow 4\}, \{2 \rightarrow 3\}$

Option c is not valid

a) $XY \rightarrow Z,\ Z \rightarrow Y$

- $XY \rightarrow Z$
  - $\{1,4 \rightarrow 3\}, \{1,5 \rightarrow 3\}, \{4,6 \rightarrow 3\}, \{3,2 \rightarrow 2\}$
- $Z \rightarrow Y$
  - $\{3 \rightarrow 4\}, \{3 \rightarrow 5\}, \{3 \rightarrow 6\}, \{2 \rightarrow 2\}$

Option a is not valid

b) $XZ \rightarrow X,\ Y \rightarrow Z$

- $XZ \rightarrow X$
  - $\{1,3 \rightarrow 1\}, \{1,3 \rightarrow 1\}, \{4,3 \rightarrow 4\}, \{3,2 \rightarrow 3\}$
- $Y \rightarrow Z$
  - $\{4 \rightarrow 3\}, \{5 \rightarrow 3\}, \{6 \rightarrow 3\}, \{2 \rightarrow 2\}$

Option b is valid

d) $XZ \rightarrow Y,\ Y \rightarrow Z$

- $XZ \rightarrow X$
  - $\{1,3 \rightarrow 4\}, \{1,3 \rightarrow 5\}, \{4,3 \rightarrow 6\}, \{3,2 \rightarrow 2\}$
- $Y \rightarrow Z$
  - $\{4 \rightarrow 3\}, \{5 \rightarrow 3\}, \{6 \rightarrow 3\}, \{2 \rightarrow 2\}$

Option d is not valid

Exercise on Functional Dependencies

- Which of the following FD is correct?

  a) $A \rightarrow BC$

  b) $DE \rightarrow C$

  c) $C \rightarrow DE$

  d) $BC \rightarrow A$

R

A	B	C	D	E
A	2	3	4	5
2	A	3	4	5
A	2	3	6	5
a	2	3	6	6

- Which of the following FD is not correct?

  a) $XY \rightarrow Z,\ Z \rightarrow Y$

  b) $YZ \rightarrow X,\ Y \rightarrow Z$

  c) $XZ \rightarrow X,\ Z \rightarrow X$

  d) $XZ \rightarrow Y,\ Y \rightarrow Z$

R

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

Classification of Functional Dependencies

- Trivial functional dependency
  - ✓ $A \rightarrow B$ has trivial functional dependency if B is a subset of A ($B \subseteq A$).
  - ✓ Examples: ***A → A, AB → B, {Employee_id, Employee_Name} → Employee_Id***
- Non-trivial functional dependency
  - ✓ $A \rightarrow B$ has a non-trivial functional dependency if B is not a subset of A.
  - ✓ If there is at least one attribute in right hand side that is not present in the left-hand side.
  - ✓ Examples: ***AB → BC, {ID} → {ID, DOB}, {Roll, Name} → {Roll, Name, Phone}***
  - ✓ When A intersection B is NULL, then $A \rightarrow B$ is called as complete non-trivial.
  - ✓ *Example: A → B, B → C, {ID} → {Name}, {Name} → {DOB}*

Classification of Functional Dependencies

- Fully functional dependency
  - Given R and $A \rightarrow B$, then B is fully functional dependent on A if there is no Z where Z is a proper subset of A ($Z \subset A$) such that $Z \rightarrow B$.
  - Examples: {*AB* → *C*, *A* → *D*} ***is a fully FD***, {*AB* → *C*, *A* → *C*} ***is not a fully FD***
  - MaSV, MaMH → TenSV, TenMH,Diem and MaSV ≠> TenSV, TenMH,Diem
- Partial functional dependency
  - Given a relation R with FD F defined on the attributes of R and K as a candidate key, if X is a proper subset of K ($X \subset K$) and if and only if $X \rightarrow A$, then A said to be partially dependent on K.
  - Examples: *R(ABCD), Key(AB)*, ***FD: {A → C}.*** *C is partially dependent on A.*
  - MaSV, MaMH → TenSV, TenMH, Diem and MASV –>TenSV, MaMH -> TenMH

Classification of Functional Dependencies

- Transitive functional dependency
  - ✓ If $A \rightarrow B$ and $B \rightarrow C$, then C is transitively functional dependent on A such that $A \rightarrow C$.
  - ✓ Examples: $\{AB \rightarrow C, C \rightarrow D\}$, **So, $\{AB \rightarrow D\}$**

Armstrong's axioms / Inference Rules

Reflexive Rule (IR1)

- ✓ If Y is a subset of X ($X \supseteq Y$), then X determines Y ($X \rightarrow Y$).
- ✓ Examples: {$AB \rightarrow A$}, {$Employee\_id, Employee\_Name$} $\rightarrow$ {$Employee\_Name$}

Augmentation Rule (IR2)

- ✓ If X determines Y ($X \rightarrow Y$), then XZ determines YZ ($XZ \rightarrow YZ$). for any Z.
- ✓ Examples: *R(ABCD), if A $\rightarrow$ B then AC $\rightarrow$ BC*

Transitive Rule (IR3)

- ✓ If X determines Y ($X \rightarrow Y$) and Y determine Z ($Y \rightarrow Z$), then X must also determine Z ($X \rightarrow Z$).

Armstrong's axioms / Inference Rules

- Union Rule (IR4)
  - ✓ If X determines Y ($X \rightarrow Y$) and X determines Z ($X \rightarrow Z$), then X must also determine Y and Z ($X \rightarrow YZ$).
  - ✓ Examples: *R(ABCD), if $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow BC$.*
- Decomposition Rule (IR5)
  - ✓ If X determines Y and Z ($X \rightarrow YZ$), then X determines Y ($X \rightarrow Y$) and X determines Z ($Y \rightarrow Z$) separately.
  - ✓ Examples: *R(ABCD), if $A \rightarrow BC$, then $A \rightarrow B$ and $A \rightarrow C$*
- Pseudo Transitive Rule (IR6)
  - ✓ If X determines Y ($X \rightarrow Y$) and YZ determines W ($YZ \rightarrow W$) , then XZ determines W ($XZ \rightarrow W$).

Closure Set of Attributes

- X^+ is the set of all attributes that can be determine using the given set X(attributes).

- *If "F" is a functional dependency then closure of functional dependency can be denoted using "{F}+".*

Closure Set of Attributes

- There are three steps to calculate closure of functional dependency. These are:
  - ✓ Step-1 : Add the attributes which are present on Left Hand Side in the original functional dependency.
  - ✓ Step-2 : Now, add the attributes present on the Right Hand Side of the functional dependency.
  - ✓ Step-3 :
    - With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies.
    - Repeat this process until all the possible attributes which can be derived are added in the closure.

Closure Set of Attributes

Input: Functional dependencies (FDs), Target set of attributes (X)

Output: Closure of X, denoted as X+

Step 1: Initialize X+ with attributes in X

Step 2: Repeat until no new attributes can be added to X+

  For each functional dependency ($Y \rightarrow Z$) in FDs:

    If Y is a subset of X+:

      Add all attributes of Z to X+

Return X+ as the closure of X

Example of Closure Set of Attributes

- Consider the table Student_details having (Roll_no, Name, Marks, Location) as the attributes and having two functional dependencies.

  FD1 : Roll_no → Name, Marks

  FD2 : Name → Marks, Location

- Now, we will calculate the closure of all the attributes present in the relation using the three steps mentioned below. Find closure set of attributes of{Roll_no}+
- Step-1 : add attributes present on the LHS of the first functional dependency to the closure.

  {Roll_no}+ = {Roll_no}

- Step-2 : add attributes present on the RHS of the original functional dependency to the closure.

  {Roll_no}+ = {Roll_no, Name, Marks}

Example of Closure Set of Attributes

- Step-3 : Add the other possible attributes which can be derived using attributes present on the RHS of the closure.
  - So, Roll_No attribute cannot functionally determine any attribute, but Name attribute can determine other attributes such as Marks and Location using 2nd Functional Dependency.
  - Therefore, complete closure of Roll_No will be :

    {Roll_no}+ = {Roll_No, Marks, Name, Location}

Example of Closure Set of Attributes

- Example 1: R (ABCDEFG)
  - {A $\rightarrow$ B, BC $\rightarrow$ DE, AEG $\rightarrow$ G}
  - Find $(AC)^+$ = ?

- $(AC)^+$ = AC

= ABC (A $\rightarrow$ B)

= ABCDE (BC $\rightarrow$ DE)

= ABCDE

- Example 2: R (ABCDE)
  - {A $\rightarrow$ BC, CD $\rightarrow$ E, B $\rightarrow$ D, E $\rightarrow$ A}
  - Find $(B)^+$ = ?

- $(B)^+$ = B

= BD (B $\rightarrow$ D)

= BD

- Example 3: R (ABCDEF)
  - {AB $\rightarrow$ C, CD $\rightarrow$ E, DE $\rightarrow$ B}
  - Find $(AB)^+$ = ?

- $(AB)^+$ = AB

= ABC (AB $\rightarrow$ C)

Example of Closure Set of Attributes

- Example 1: R (ABCDEFGH)
  - {A $\rightarrow$ BC, CD $\rightarrow$ E, E $\rightarrow$ C, D $\rightarrow$ AEH, ABH $\rightarrow$ BD, DH $\rightarrow$ BC}
  - Find $(BCD)^+ \rightarrow$ H ?
  - $(BCD)^+$ = BCD

  = BCDE (CD $\longrightarrow$ E)

  = ABCDEH (D $\rightarrow$ AEH)

  So, $(BCD)^+ \rightarrow$ H is valid

- Exercise 1: R (ABCDEF)
  - {A $\rightarrow$ BC, CD $\rightarrow$ EF, B $\rightarrow$ D, E $\rightarrow$ A}
  - Find $(AE)^+$ = ?

- Exercise 2: R (ABCDEF)
  - {AB $\rightarrow$ C, CD $\rightarrow$ E, DE $\rightarrow$ B}
  - Find $(AB)^+$ = ?

- Exercise 3: Student (Roll, Name, DoB, Phone, Course)
  - {Roll, Name} $\rightarrow$ Phone, {Course, DoB} $\rightarrow$ Roll, Course $\rightarrow$ Name, Roll $\rightarrow$ Name}
  - Find $(Course)^+$ = ?

Equivalence Of Functional Dependencies

- R (ACDEH)

- F: {$A \rightarrow C$, $AC \rightarrow D$, $E \rightarrow AD$, $E \rightarrow H$}
- G: {$A \rightarrow CD$, $E \rightarrow AH$}

- $(A)^+ = ACD$
- $(AC)^+ = ACD$
- $(E)^+ = EAH = EAHCD$

- So, $F \subseteq G$

- $(A)^+ = ACD$
- $(E)^+ = EADCH$

So, $G \subseteq F$

- Find the correct option

a) $F \subseteq G$

b) $G \subseteq F$

c) $F = G$

d) $F \neq G$

- $F \subseteq G$ and $G \subseteq F$, so, $F = G$

Equivalence Of Functional Dependencies

- R (ACDEH)
- F: $\{A \rightarrow C,\ AC \rightarrow D,\ E \rightarrow AD,\ E \rightarrow H\}$
- G: $\{A \rightarrow CD,\ E \rightarrow AH\}$
- Check both FD's are equivalent or not.

- R (ABC)
- F: $\{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$
- G: $\{A \rightarrow BC,\ B \rightarrow A,\ C \rightarrow A\}$
- Find the correct option

  a) $F \subseteq G$

  b) $G \subseteq F$

  c) $F = G$

  d) $F \neq G$

- R (VWXYZ)
- F: $\{W \rightarrow X,\ WX \rightarrow Y,\ Z \rightarrow WY,\ Z \rightarrow V\}$
- G: $\{W \rightarrow XY,\ Z \rightarrow WX\}$
- Find the correct option

  a) $F \subseteq G$

  b) $G \subseteq F$

  c) $F = G$

  d) $F \neq G$