Note Template

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Abstract

This is a note template, with all but minimal compilable files provided. Feel free to adjust for your usage.

Now let's start a simple demo for you to take fancy notes in LaTeX!

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3.1 Introduction

Sep. 4 2025

Chapter 4

Matching Under Transferable Utility: Theory

Lecture 4: Week 4

Section 4.1: Accounting for Unobservable Heterogeneity

Mar. 11 2025

1. Background and Problem

- **Core Problem**: Real-world matching (e.g., marriage) is influenced not only by observable traits like income or education but also by unobservable heterogeneity (e.g., personal preferences, attractiveness). Simple models based solely on income or education (see Subsection 3.4.2) predict strict assortative matching (highest income pairs with highest, second-highest with second-highest, and no singles), which does not align with reality:
  - *Model Prediction*: Strict positive assortative matching with no singles.
  - *Reality*: Positive assortative matching exists (e.g., high-income often pairs with high-income), but it is not strict, and the probability of being single increases as income decreases.
- **Goal**: Develop a more flexible model that accounts for unobservable differences to explain real-world matching patterns.

2. Model Setup: Introducing Random Shocks

- **Basic Model**:
  - Individuals are categorized by types I and J (e.g., high or low income, denoted as K or L).
  - Pairwise surplus:
  $$S(i,j) = z(I,J) + \varepsilon_{i,j},$$
  where $z(I,J)$ is the deterministic component, and $\varepsilon_{i,j}$ is a stochastic term capturing unobservable preferences.
  - Surplus for singles:
  $$S(i,\emptyset) = z(I,\emptyset) + \varepsilon_{i\emptyset}, \quad S(\emptyset,j) = z(\emptyset,J) + \varepsilon_{\emptyset j}.$$
- **Separability Assumption**:
  - The random term is separable:
  $$\varepsilon_{i,j} = \alpha_I^i + \beta_J^j,$$
  where α_I^i depends on i's category I and individual characteristics, and β_J^j depends on j's category J.
  - This assumes the random term depends only on categories, not on the specific identity of the partner.

3. Why the Separability Assumption?

- **Reality Consideration**: Matching in reality depends on unobservable traits (e.g., appearance, personality), which are modeled as random shocks $\varepsilon_{i,j}$.
- **Problem with Independence**: If $\varepsilon_{i,j}$ is assumed independent, it leads to unrealistic predictions (Chiappori et al., 2015):
  - With large populations, the maximum $\varepsilon_{i,j}$ either converges to a fixed value (if bounded) or to infinity (if unbounded), overshadowing economic factors and predicting near-zero singlehood rates.
- **Benefits of Separability**:
  - Simplifies the model: $\varepsilon_{i,j} = \alpha_I^i + \beta_J^j$ depends only on categories, reducing computational complexity.
  - More realistic: For example, women of a certain type may generally prefer men of a certain type (captured by β_J^j), independent of specific traits.

4. Main Result (Proposition 1)

- **Conditions**: The surplus satisfies the separability assumption $\varepsilon_{i,j} = \alpha_I^i + \beta_J^j$, with α and β independently distributed.
- **Result**:
  - Payoff for woman i:

$$u_i = U^{I,i} + \alpha_I^i, \quad \text{and for man } j: \quad v_j = V^{J,j} + \beta_J^j.$$

  - $U^{I,i}$ and $V^{J,j}$ are deterministic components depending only on categories I and J.
  - Woman i matches with a man of category J if:

$$U^{I,i} + \alpha_I^i \geq V^{L,i} + \alpha_L^i \quad \text{for all } L.$$

  - She remains single if:

$$\alpha_I^i \geq U^{L,i} + \alpha_L^i.$$

- **Intuition**: Women choose partners based on their category (e.g., high or low income), not their specific identity. Their payoff depends on their preference α_I^i and the category's value $U^{I,i}$. If the singlehood payoff is higher, they stay single.

5. Comparative Statics (Proposition 3)

- **Conditions**: Compare two models with the separability assumption:
  - Model 1: $S(i,j) = z(I,J) + \alpha_I^i + \beta_J^j$,
  - Model 2: $\tilde{S}(i,j) = \tilde{z}(I,J) + \tilde{\alpha}_I^i + \tilde{\beta}_J^j$,
  - Random terms have the same distribution; the difference lies in the deterministic parts z and $\tilde{z}$.
- **Result**:
  - If z is more supermodular than $\tilde{z}$, i.e.,

$$z(K,K) + z(L,L) - z(K,L) - z(L,K) \geq \tilde{z}(K,K) + \tilde{z}(L,L) - \tilde{z}(K,L) - \tilde{z}(L,K),$$

    then Model 1 has more assortative matching than Model 2:

$$\sigma_{KK} + \sigma_{LL} \geq \tilde{\sigma}_{KK} + \tilde{\sigma}_{LL}.$$

  - If $\tilde{z}$ is not supermodular, i.e.,

$$\tilde{z}(K,K) + \tilde{z}(L,L) = \tilde{z}(K,L) + \tilde{z}(L,K),$$

    then Model 2 results in random matching.

- **Intuition**: Stronger supermodularity (higher surplus for same-type pairs) leads to more assortative matching (e.g., high income with high income). If supermodularity vanishes, matching becomes random.

6. Real-World Implications and Limitations

- **Implications**:
  - Explains why assortative matching in reality is not strict (due to random shocks) and why singlehood rates increase as income decreases (low-income individuals may prefer singlehood).
  - Supermodularity drives matching patterns: stronger supermodularity leads to more assortative matching; weaker supermodularity results in more random matching.
- **Limitations**:
  - The separability assumption is restrictive: In reality, preferences may depend on specific traits of the partner (e.g., appearance), not just their category.
  - Handling random shocks remains complex: Independence leads to unrealistic results, and specifying a covariance structure is hard to estimate.

Summary

- **Key Contribution**: Introduces the separability assumption to incorporate unobservable heterogeneity, explaining non-strict assortative matching and singlehood patterns in reality.
- **Main Results**:
  - Women choose partners based on categories, with payoffs determined by deterministic components and random preferences.
  - Supermodularity determines the matching pattern: stronger supermodularity leads to more assortative matching.
- **Real-World Application**: Explains why high-income individuals often pair with high-income partners, but not always, and why low-income individuals are more likely to remain single.

Chapter 5

Stylized Facts on Marriage

Lecture 6: Week 6

Appendix

Appendix A

Additional Proofs

A.1 Proof of ??

We can now prove **??**.

Proof of ??. See here. ■