EXERCISE 9 REPORT

Course: **Discrete Mathematics and Declarative programming** - Class code: **CSBU110.Q11.KHBC**
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References and related links *(if any)*	
Feedback *(optional)*: *+ Comments* *+ Problems, bugs* *+ Suggestions*	

[Report in detail, using your own format, with screenshots and explanation, describing what you have done and what you have observed. Regarding programming tasks, you also need to list the important code snippets, screenshots when running your program followed by explanation]

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1. 9.1

1. Exercise 1:

Week 9

9.1

1. List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

a) $a = b$

$R = \{(a, b) \mid a = b\}$

$\Rightarrow R = \{(0,0), (1,1), (2,2), (3,3)\}$

b) $a + b = 4$

$R = \{(a, b) \mid a + b = 4\}$

$\Rightarrow R = \{(4,0), (3,1), (2,2), (1,3)\}$

c) $a > b$

$R = \{(a, b) \mid a > b\}$

$\Rightarrow R = \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2)$
$(4,0), (4,1), (4,2), (4,3)\}$

d) $a \mid b \Rightarrow b = ka$

$R = \{(a, b) \mid a \mid b\}$

$\Rightarrow R = \{(1,0), (2,0), (3,0), (4,0), (1,1), (1,2)$
$(2,2), (1,3), (1,4), (2,4), (4,4), (3,3)\}$

e) $\gcd(a, b) = 1$

$R = \{(a, b) \mid \gcd(a, b) = 1\}$

$\Rightarrow R = \{(0,1), (1,0), (1,2), (2,1), (1,3), (3,1),$
$(4,1), (2,3), (3,2), (4,3)\}$

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f) $\operatorname{lcm}(a, b) = 2$

$R = \{(a, b) \mid \operatorname{lcm}(a, b) = 2\}$

$\Rightarrow R = \{(1,2), (2,1), (2,2)\}$

2. Exercise 3:

3 For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive

a) $R_1 = \{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

R_1 is not reflexive since $\exists 1, 4 \in A$ and $(1,1), (4,4) \notin R$

R_1 is not symmetric since $\exists\ 2, 4 \in A$, $2 R 4$ and $4 \not R 2$

R_1 is ~~antisymmetric~~ transitive $\forall a, b \in A$ $\begin{cases} aRb \\ bRc \end{cases}$ then aRc

R_1 is not antisymmetric $\exists\ 2, 3 \in A$, $\begin{cases} 2R3 \\ 3R2 \\ 2 \neq 3 \end{cases}$

b) $R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

R_2 is reflexive since $\forall a \in A$, aRa

R_2 is symmetric since $\forall a, b \in A$, $[aRb] \Rightarrow [bRa]$

R_2 is antisymmetric since $\exists\ 1, 2 \in A$, $\begin{cases} 1R2 \\ 2R1 \\ 2 \neq 1 \end{cases}$

R_2 is transitive since $\forall a, b \in A$, $\begin{cases} aRb \\ bRc \end{cases}$, aRc

c) $R_3 = \{(2,4),(4,2)\}$

R_3 is not reflexive since $\exists 1,2,3,4 \in A$, $(1,1),(2,2),(3,3),(4,4) \notin R$

R_3 is symmetric since $\forall a,b \in A$, $[aRb] \Rightarrow [bRa]$

R_3 is not antisymmetric since $\exists 2,4 \in A$ $\begin{cases} 2R4 \\ 4R2 \\ 2 \neq 4 \end{cases}$

R_3 is not transitive since $\exists 2,4 \in A$ $\begin{cases} 2R4 \\ 4R2 \\ (2,2) \notin R \end{cases}$

d) $R_4 = \{(1,2),(2,3),(3,4)\}$

R_4 is not reflexive since $\exists 1 \in A$, $(1,1) \notin R$

R_4 is not symmetric since $\exists 1,2 \in A$, $1R2$ and $2 \not R 1$

R_4 is anti symmetric since $\forall a,b \in A$, $\begin{cases} aRb \\ b \not R a \end{cases}$

R_4 is not transitive since $\exists 1,2,3 \in A$, $1R2$ and $2R3$ and $1 \not R 3$

e) $R_5 = \{(1,1),(2,2),(3,3),(4,4)\}$

R_5 is reflexive since $\forall a \in A$, aRa

R_5 is anti symmetric since $\forall a,b \in A$, $\begin{cases} aRb \\ bRa \end{cases}$, then $a = b$

R_5 is antisymmetric since $\forall a,b \in A$, $[aRb] \Rightarrow [bRa]$

R_5 is transitive since $\forall a,b,c \in A$ $\begin{cases} aRb \\ bRc \end{cases}$, then aRc

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f) $R_6 = \{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

R_6 is not reflexive since $\forall a \in A$, $a \not R a$

R_6 is not symmetric since $\exists 1,4 \in A$, $1R4$ and $4 \not R 1$

R_6 is not antisymmetric since $\exists 1,3 \in A$, $\begin{cases} 1R3 \\ 3R1 \\ 1 \neq 3 \end{cases}$

R_6 is not transitive since $\exists 1,3 \in A$, $\begin{cases} 1R3 \\ 3R1 \\ 1 \not R 1 \end{cases}$

3. Exercise 7:

7. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$

a) $x \neq y$

$R_1 = \{(x, y) \mid x \neq y\}$

R_1 is not reflexive since $\forall a \in \mathbb{Z}$, $(a, a) \notin R$ $(x \neq y)$

R_1 is symmetric since $\forall x, y \in \mathbb{Z}$ $[x R y] \Rightarrow [y R x]$

R_1 is not antisymmetric since $\forall x, y \in \mathbb{Z}$, $\begin{cases} x R y \\ y R x \\ x \neq y \end{cases}$

R_1 is not transition since $\forall x, y \in \mathbb{Z}$, $\begin{cases} x R y \\ y R x \\ x \not R x \end{cases}$

b) $x y \geq 1$

$R_2 = \{(x, y) \mid x.y \geq 1\}$

R_2 is not reflexive since $(a, a) \in R$

$a.a \geq 1 \Rightarrow a \geq 1$ or $a \leq -1$

$\Rightarrow \exists 0 \in \mathbb{Z}$, $(0, 0) \notin R$

R_2 is symmetric since $\forall x, y \in \mathbb{Z}$

$(x, y) \in R \Rightarrow x.y \geq 1 \Rightarrow (y, x) \in R$

R_2 is not anti symmetric since $\exists 2, 3 \in \mathbb{Z}$, $\begin{cases} (2,3) \in R \\ (3,2) \in R \\ 3 \neq 2 \end{cases}$

R_2 is transitive since $\forall x, y \in \mathbb{Z}$

~~Case 1 $x, y, z \in \mathbb{Z}^+$~~

$\begin{cases} x R y \\ y R z \end{cases} \Rightarrow x R z$

c) $x = y + 1$ or $x = y - 1$

$R_3 = \{(x,y) \mid x = y+1 \text{ or } x = y-1\}$

R_3 is not reflexive since $\forall a \in \mathbb{Z}$, ~~(a~~

$$\begin{cases} a = a + 1 \\ a = a - 1 \end{cases} \Rightarrow \left[\begin{array}{l} 0 = 1 \ (F) \\ 0 = -1 \ (F) \end{array}\right. \Rightarrow (a,a) \notin R$$

R_3 is symmetric since $\forall x, y \in \mathbb{Z}$, ~~(x,y)~~

$$(x,y) \in R \Rightarrow \left[\begin{array}{l} x = y - 1 \\ x = y + 1 \end{array}\right. \Rightarrow \left[\begin{array}{l} y = x + 1 \\ y = x - 1 \end{array}\right. \Rightarrow (y,x) \in R$$

R_3 is not antisymmetric since $\exists\, 0, 1 \in \mathbb{Z}$, $\begin{cases} (1,0) \in R \\ (0,1) \in R \\ 1 \neq 0 \end{cases}$

R_3 is not transitive since $\exists\, 0,1,2 \in \mathbb{Z}$, $\begin{cases} (0,1) \in R \\ (1,2) \in R \\ (0,2) \notin R \end{cases}$

d) $x \equiv y \pmod 7$

$R_4 = \{(x,y) \mid x \equiv y \pmod 7\}$

R_4 is reflexive since $\forall a \in \mathbb{Z}$, $(a,a) \in R$

$a \equiv a \pmod 7 \Rightarrow 7 \mid (a - a) \Rightarrow 7 \mid 0$ (true)

R_4 is symmetric since $\forall x, y \in \mathbb{Z}$

$$\begin{aligned} (x,y) \in R &\Rightarrow x \equiv y \pmod 7 \\ &\Rightarrow (x - y) = k7 \\ &\Rightarrow (y - x) = -k7 \Rightarrow y \equiv x \pmod 7 \\ &\Rightarrow (y,x) \in R \end{aligned}$$

R_4 is no antisymmetric since $\exists\, 14, 7 \in \mathbb{Z}$ $\begin{cases} (14,7) \in R \\ (7,14) \in R \\ 14 \neq 7 \end{cases}$

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R_4 is transitive since $\forall x, y \in \mathbb{Z}$

$$\begin{cases}(x,y) \in R \\ (y,z) \in R\end{cases} \Rightarrow \begin{cases} x - y = k7 \\ y - z = l7 \end{cases}$$

$$x - z = (k + l)7 \Rightarrow x \equiv z \pmod 7$$

e) x is a multiple of y

$R_5 = \{(x,y) \mid x \text{ is a multiple of } y\}$

R_5 is reflexive since, $\forall a \in \mathbb{Z}$,

$(a,a) \in R \Rightarrow a = n \cdot a \Rightarrow n = 1$

R_5 is not symmetric since $\exists\, 2, 4 \in \mathbb{Z}$, $\begin{cases}(4,2) \in R \\ (2,4) \notin R\end{cases}$

R_5 is not antisymmetric since $\exists\, 2, -2 \in \mathbb{Z}$ $\begin{cases}(2,-2) \in R \\ (-2,2) \in R \\ 2 \neq -2\end{cases}$

R_5 is transition since $\forall x, y \in \mathbb{Z}$

$$\begin{cases}(x,y) \in R \\ (y,z) \in R\end{cases} \Rightarrow \begin{cases} x = ky \\ y = mz \end{cases} \Rightarrow x = k\,m\,z$$

$$\Rightarrow (x,z) \in R$$

f) x and y are both negative or both nonnegative

$R_6 = \{(x,y) \mid x, y \geq 0 \text{ or } x, y < 0\}$

R_6 is reflexive since $\forall a \in \mathbb{Z}$. $(a,a) \in R$

R_6 is symmetric since $\forall x, y \in \mathbb{Z}$, $[xRy] \Rightarrow [yRx]$

R_6 is not antisymmetric since $\exists\, 0, 2 \in \mathbb{Z}$, $\begin{cases} 0R2 \\ 2R0 \\ 0 \neq 2 \end{cases}$

R_6 is transitive since $\forall x, y \in \mathbb{Z}$

$\begin{cases} (x,y) \in R \\ (y,z) \in R \end{cases} \Rightarrow \begin{cases} x, y \geq 0 \text{ or } x, y < 0 \\ y, z \geq 0 \text{ or } y, z < 0 \end{cases}$

$\Rightarrow x, z \geq 0$ or $x, z < 0$

g) $x = y^2$

$R_7 = \{(x, y) \mid x = y^2\}$

R_7 is not Reflexive since $\forall a \in \mathbb{Z}$,

$a = a^2 \Leftrightarrow a^2 - a = 0 \Leftrightarrow a = 0$

R_7 is not symmetric since $\exists 4, 2 \in \mathbb{Z}$, $\begin{cases} 4 R 2 \\ 2 \not{R} 4 \end{cases}$

R_7 is antisymmetric since $\forall x, y \in \mathbb{Z}$

$\begin{cases} x R y \\ y R x \end{cases} \Rightarrow \begin{cases} x = y^2 \\ y = x^2 \end{cases} \Rightarrow \begin{cases} x = x^4 \\ y = x^2 \end{cases} \Rightarrow \begin{cases} x = 0 & y = 0 \\ x = 1 & y = 1 \end{cases}$

R_7 is not transitive since $\exists 4, 2, 16$ $\begin{cases} 4 R 2 \\ 16 R 4 \\ 16 \not{R} 2 \end{cases}$

h) $x \geq y^2$

$R_8 = \{(x, y) \mid x \geq y^2\}$

R_8 is not reflexive since $\forall a \in \mathbb{Z}$

$a \geq a^2 \Leftrightarrow 0 \geq a^2 - a \Leftrightarrow 1 \geq a \geq 0$

R_8 is not symmetric since $\exists 1, 2 \in \mathbb{Z}$, $\begin{cases} 2 R 1 \\ 1 \not{R} 2 \end{cases}$

R_8 is antisymmetric since $\forall x, y \in \mathbb{Z}$

$\begin{cases} x R y \\ y R x \end{cases} \Rightarrow \begin{cases} x \geq y^2 \\ y \geq x^2 \end{cases} \Rightarrow \begin{cases} x \geq x^4 \\ y \geq x^2 \end{cases} \Rightarrow \begin{cases} x = 1, y = 1 \\ x = 0, y = 0 \end{cases}$

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R_8 is not transitive since $\forall\, x, y \in \mathbb{Z}$

$$\begin{cases} x R y \\ y R z \end{cases} \Rightarrow \begin{cases} x \geq y^2 \\ y \geq z^2 \end{cases} \Rightarrow \begin{cases} x \geq z^4 \geq z^2 \\ y \geq z^2 \end{cases}$$

$$\Rightarrow x R z$$

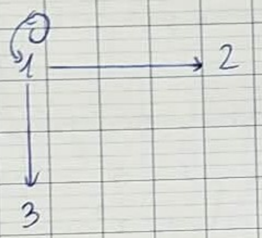
2. 5.3

1. Exercise 18:

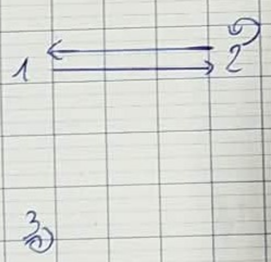
9.3

18 Draw the directed graphs representing each of the relations from Exercise 1

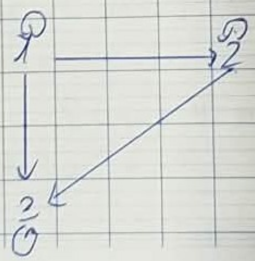
a) {(1,1), (1,2), (1,3)}



b) {(1,2), (2,1), (2,2), (3,3)}



c) {(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)}



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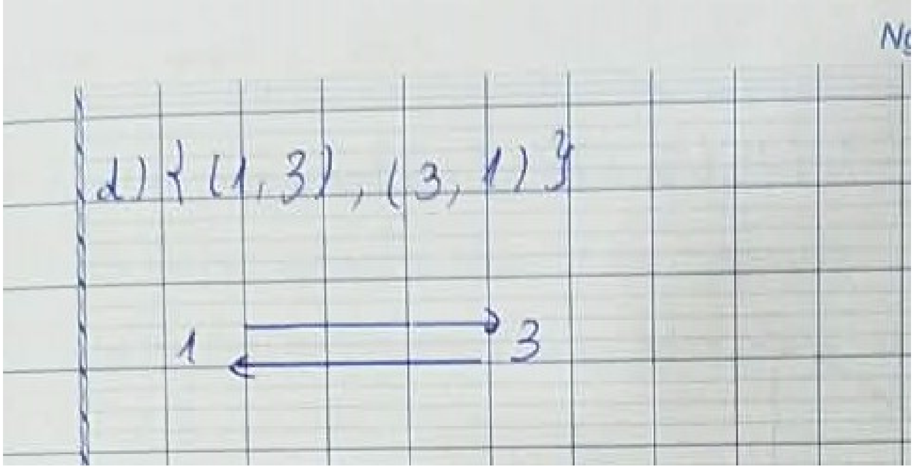
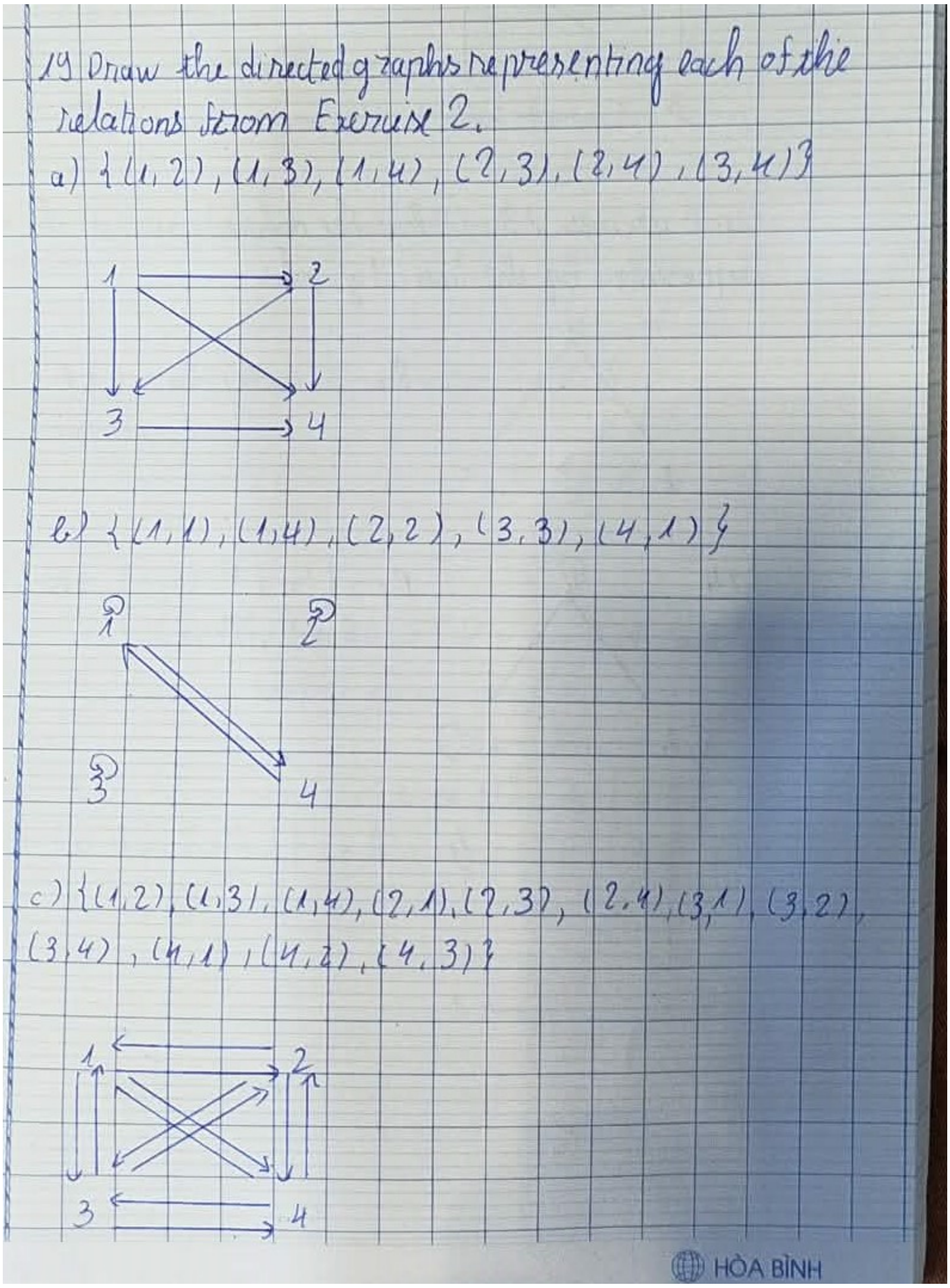


d) {(1,3), (3,1)}

2. **Exercise 19:**



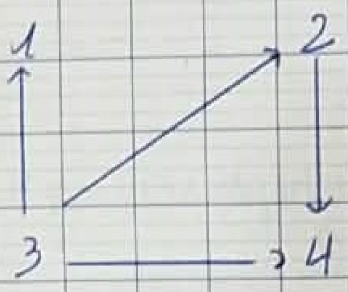
19 Draw the directed graphs representing each of the relations from Exercise 2.

a) {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}

b) {(1,1), (1,4), (2,2), (3,3), (4,1)}

c) {(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

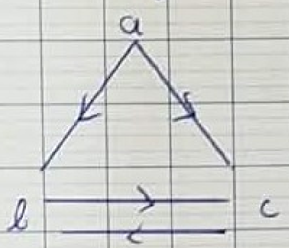
d) $\{(2,4),(3,1),(3,2),(3,4)\}$



3. **Exercise 23, 24, 25:**

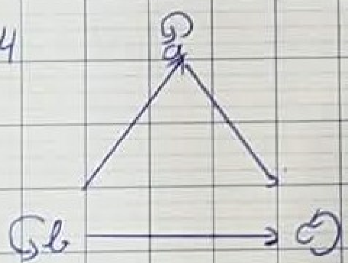
In Exercises 23 - 28 list the ordered pairs in the relations represented by the directed graphs

23



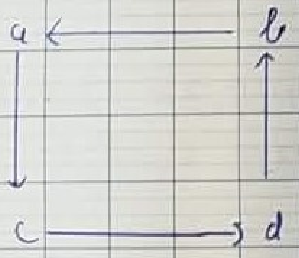
$R_1 = \{(a,b),(a,c),(b,c),(c,b)\}$

24



$R_2 = \{(a,a),(a,c),(b,b),(b,a),(b,c),(c,c)\}$

25



$R_3 = \{(a,c),(b,a),(c,d),(d,b)\}$

4. **Exercise 31:**

31. Determine whether the relations represented by the directed graphs shown in Exercises 23 - 25 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive

23

R_1 is not reflexive since $a, b, c \in A$, $(a,a), (b,b), (c,c) \notin R$

R_1 is not symmetric since $\begin{cases} a R b \\ b \not R a \end{cases}$

R_1 is not antisymmetric since $\begin{cases} b R c \\ c R b \\ c \neq b \end{cases}$

R_1 is not transitive since $\begin{cases} b R c \\ c R b \\ b \not R b \end{cases}$

24

R_2 is reflexive since $\forall a, b, c \in A$, $(a,a), (b,b), (c,c) \in R$

R_2 is not symmetric since $\begin{cases} a \not R c \\ c \not R a \end{cases}$

R_2 is antisymmetric since $\forall a, b \in A$, $\begin{cases} a R b \\ b \not R a \end{cases}$

R_2 is not transitive since $\forall a, c \in A$ $\begin{cases} a R a \\ a R c \\ a \not R c \end{cases}$

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25

R_3 is not reflexive since $\forall a \in A$, $a \not R a$

R_3 is not symmetric since $\forall a, c \in A$ $\begin{cases} c \not R a \\ a R c \end{cases}$

R_3 is antisymmetric since $\forall a, b \in A$ $\begin{cases} a \not R b \\ b R a \end{cases}$

R_3 is not transitive since $\forall a, c, d \in A$ $\begin{cases} a R c \\ c R d \\ a \not R d \end{cases}$

3. 9.5

1. Exercise 1:

9.5

1 Which of these relations on $\{0,1,2,3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack

a) $R_1 = \{(0,0), (1,1), (2,2), (3,3)\}$

R_1 is reflexive since $\forall a \in A$, aRa

R_1 is symmetric since $\forall a, b \in A$ $\begin{cases} aRb \\ bRa \end{cases}$

R_1 is transitive since $\forall a, b \in A$ $\begin{cases} aRb \\ b\cancel{R}c \end{cases}$

$\Rightarrow R_1$ is equivalence

b) $R_2 = \{(0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3)\}$

R_2 is not reflexive since $1 \in A$, $1 \cancel{R} 1$

R_2 is symmetric since, $\forall a, b \in A$, $[aRb] \Rightarrow [bRa]$

R_2 is not transitive since $0, 2, 3 \in A$ $\begin{cases} 0R2 \\ 2R3 \\ 0\cancel{R}3 \end{cases}$

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$\nexists (0,0) \Rightarrow R_2$ is not equivalence

c) $R_3 = \{(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)\}$

R_3 is reflexive since $\forall a \in A$, aRa

R_3 is symmetric since $\forall a,b \in A$, $[aRb] \Rightarrow [bRa]$

R_3 is transitive since $\forall a,b \in A$, $\begin{cases} aRb \\ bRc \end{cases} \Rightarrow aRc$

$\Rightarrow R_3$ is equivalence

d) $R_4 = \{(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

R_4 is reflexive since $\forall a \in A$, aRa

R_4 is symmetric since $\forall a,b$, $[aRb] \Rightarrow [bRa]$

R_4 is not transitive since $1,3,2 \in A$ $\begin{cases} 1R3 \\ 3R2 \\ 1 \not R 2 \end{cases}$

$\Rightarrow R_4$ is not equivalence

e) $R_5 = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)\}$

R_5 is reflexive since $\forall a \in A$, aRa

R_5 is not symmetric since $1,2 \in A$, $\begin{cases} 1R2 \\ 2 \not R 1 \end{cases}$

R_5 is transitive since $\forall a,b \in A$ $\begin{cases} aRb \\ bRc \end{cases} \Rightarrow aRc$

2. Exercise 2:

2 Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lacke

a) $R_1 = \{(a, b) \mid$ a and b are the same age $\}$

R_1 is reflexive since $\forall a \in A$, ~~a have same~~ aRa

R_1 is symmetric since $\forall a, b \in A$, $\begin{cases} aRb \Rightarrow bRa \\ \cancel{\&} \end{cases}$

R_1 is transitive since $\forall a, b \in A$ $\begin{cases} aRb \\ bRc \end{cases} \Rightarrow aRc$

$\Rightarrow$ equivalence

b) $R_2 = \{(a, b) \mid$ a and b have same parents $\}$

R_2 is reflexive since $\forall a \in A$, aRa

R_2 is symmetric since $\forall a, b \in A$, $aRb \Rightarrow bRa$

R_2 is transitive since $\forall a, b \in A$ $\begin{cases} aRb \\ bRc \end{cases} \Rightarrow aRc$

$\Rightarrow$ equivalence

c) $R_3 = \{(a, b) \mid$ a and b share a common request $\}$

R_3 is reflexive since $\forall a \in A$, aRa

R_3 is symmetric since $\forall a, b \in A$, $aRb \Rightarrow bRa$

R_3 is not transitive since $\exists a, b \in A$

$\begin{cases} aRb \\ bRc \end{cases} \Rightarrow \begin{cases} \text{a and b share mother} \\ \text{b and c share } \cancel{\text{mo}}\text{father} \end{cases} \Rightarrow a \not R c$

$\Rightarrow$ not equivalence

d) $R_4 = \{(a,b) \mid$ a and b have met$\}$

R_4 is not reflexive since $\forall a \in A$, a can't meet themself

R_4 is symmetric since $\forall a,b \in A$, $aRb \Rightarrow bRa$

R_4 is transitive since $\forall a,b \in A$, $\begin{cases} \text{a and b have met} \\ \text{b and c have meet} \\ \text{a and c can or can't meet} \end{cases}$

$\Rightarrow$ not equivalence

e) $R_5 = \{(a,b) \mid$ a and b speak a common language$\}$

R_4 is reflexive since $\forall a \in A$, aRa

R_4 is symmetric since $\forall a,b \in A$, $aRb \Rightarrow bRa$

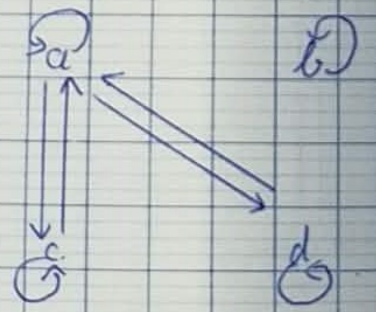
R_4 is transitive since $\forall a,b \in A$, $\begin{cases} aRb \\ bRc \end{cases} \Rightarrow aRc$

$\Rightarrow$ equivalence

3. **Exercise 21, 22, 23:**

In Exercises 21 - 23 determine whether the relation with the directed graph shown is an equivalence relation

21.



$R_1 = \{(a,a),(a,c),(a,d),(b,b),(c,a),(c,c),(d,a),(d,d)\}$

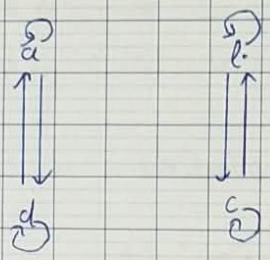
R_1 is reflexive since $\forall x \in A$, xRx

R_1 is symmetric since $\forall x, y \in A$, $[xRy] \Rightarrow [yRx]$

R_1 is transitive since $\forall x, y \in A$, $\begin{cases} xRy \\ yRz \end{cases} \rightarrow xRz$

$\Rightarrow$ equivalence

22.



$R_2 = \{(a,a), (a,d), (b,b), (b,c), (c,b), (c,c), (d,a), (d,d)\}$

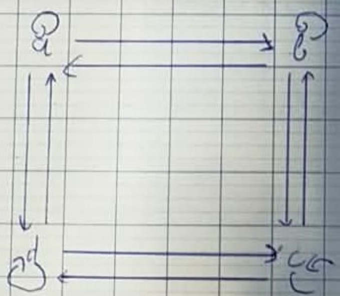
R_2 is reflexive since $\forall x \in A$, xRx

R_2 is symmetric since $\forall x, y \in A$, $[xRy] \Rightarrow [yRx]$

R_2 is transitive since $\forall x, y \in A$, $\begin{cases} xRy \\ yRz \end{cases} \Rightarrow xRz$

$\Rightarrow$ equivalence

23.



$R_3 = \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,c), (c,b), (c,c), (c,d), (d,a), (d,c), (d,d)\}$

R_3 is reflexive since $\forall x \in A$, xRx

R_3 is symmetric since $\forall x, y \in A$, $[xRy] \Rightarrow [yRx]$

R_3 is not transitive since $\forall b, y \in A$, $\begin{cases} bRy \\ yRd \\ b \not R d \end{cases}$ $\cancel{\Rightarrow xRz}$

$\Rightarrow$ equi

$\Rightarrow$ not equivalent