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in the IEEE Transactions on Power Systems, Vol. 19 No. 4, November 2004, 2068 – 2076.

Optimal Placement of Distributed Generation on a Radial Feeder

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***Abstract*—Distributed generation (DG) can be used to generate a customer's entire electricity supply; for peak shaving; for standby or emergency generation; as a green power source; or for increased reliability. In this paper, analytical approaches for optimal placement of DG with unity power factor in power systems are presented. Placement of DG in a radial feeder is analyzed and the theoretical optimal site (bus) for adding DG is obtained for different types of loads and DG sources. The proposed methods are tested by a series of simulations on radial feeders to show the effectiveness of the proposed methods in determining the optimal bus for placing DG.**

Keywords-Distributed generation; radial feeder; load

I. INTRODUCTION

Distributed generation generally refers to small-scale electric power generators that produce electricity at a site close to customers or that are tied to an electric distribution system. Distributed generators include, but are not limited to synchronous generators, induction generators, reciprocating engines, microturbines, combustion gas turbines, fuel cells, solar photovoltaics, and wind turbines. Distributed generation devices can be strategically placed in power systems for grid reinforcement, reducing power losses and on-peak operating costs, improving voltage profiles and load factors, deferring or eliminating system upgrades, and improving system integrity, reliability and efficiency [1]-[5].

A common strategy to find the site of DG is to minimize the power loss of the system [2]-[5]. The voltage at each bus is in the acceptable range and the line flows are within the limits. Another method for placing DG is to apply rules that are often used in sitting shunt capacitors in distribution systems. A "2/3 rule" is presented in [6] to place DG on a radial feeder with uniformly distributed load, where it is suggested to install DG of approximately 2/3 capacity of the incoming generation at approximately 2/3 of the length of line.

In this paper, analytical approaches for optimal placement of DG with unity power factor in power systems are presented. Placement of DG in a radial feeder is analyzed and the theoretical optimal site (bus) for adding DG is obtained for different types of loads and DG sources. The proposed methods are tested by a series of simulations on radial feeders to show the effectiveness of the proposed methods in determining the optimal bus for placing DG.

II. THEORETICAL ANALYSIS

To simplify the analysis, only overhead transmission lines with uniformly distributed parameters are considered, i.e., R and L (series resistance and inductance) per unit length are the same along the feeder while the shunt capacitance and susceptance of lines are neglected. The loads along the feeder are assumed to vary in discrete time duration; for example, the feeder load distributions along the line for time durations T_i and T_{i+1} are shown in Figure 1.

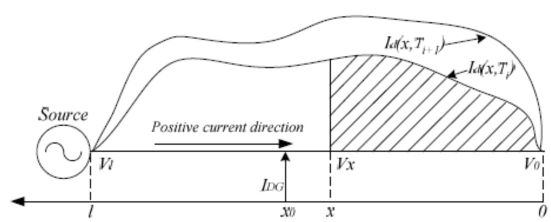

Figure 1. A feeder with distributed loads along the line.

First consider a radial feeder without DG. During the time duration T_i, the loads are distributed along the line with the phasor current density $I_d(x,T_i)$ as shown in Figure 1.

The phasor feeder current at point x is:

$$I(x,T_i) = \int_0^x I_d(x,T_i)dx \tag{1}$$

Assuming the impedance per unit length of the line is $Z = R + jX(\Omega / km)$, then the incremental power loss and phasor voltage drop at point x are:

$$dP(x,T_i) = \left[\left|\int_0^x I_d(x,T_i)dx\right|\right]^2 \cdot Rdx \tag{2}$$

$$dV(x,T_i)=\left[\left|\int_0^x I_d(x,T_i)dx\right|\right]\cdot Zdx \tag{3}$$

The total power loss along the feeder within the time duration T_i is:

$$\begin{aligned} P_{loss}(T_i) &= \int_0^l dP(x,T_i) \\ &= \int_0^l \left[\left|\int_0^x I_d(x,T_i)dx\right|\right]^2 \cdot Rdx \end{aligned} \tag{4}$$

The voltage drop between point x and the receiving end is:

$$\begin{aligned} V_{drop}(x,T_i) &= V_x(T_i)-V_0(T_i) \\ &= \int_0^x dV(x,T_i)=\int_0^x\int_0^x I_d(x,T_i)dx\cdot Zdx \end{aligned} \tag{5}$$

And the voltage at point x is:

$$\begin{aligned} V_x(T_i) &= V_0(T_i)+V_{drop}(x,T_i) \\ &= V_l(T_i)-V_{drop}(l,T_i)+V_{drop}(x,T_i) \end{aligned} \tag{6}$$

The total voltage drop across the feeder is:

$$\begin{aligned} V_{drop}(l,T_i) &= V_l(T_i)-V_0(T_i) \\ &= \int_0^l dV(x,T_i)=\int_0^l\int_0^x I_d(x,T_i)dx\cdot Zdx \end{aligned} \tag{7}$$

Now, consider a DG is added into the feeder at the location x_0, shown in Figure 1. In general, the load current density $I_d(x,T_i)$ will change (normally decrease) as a result of adding DG due to improvements in the voltage profile along the line. This change in the load current density will cause the feeder current to decrease. The feeder current between the source (at $x=l$) and the location of DG (at $x=x_0$) will also change as a result of the injected current source $I_{DG}(T_i)$. However, the change in feeder current due to the change in the load current density is generally much smaller than the change in the feeder current due to the injected current $I_{DG}(T_i)$. For the purpose of analysis, the change in the load current density, resulted from the addition of DG, is neglected in this study. Therefore, the load current density $I_d(x,T_i)$, used in (1), is also used for obtaining the feeder current after adding DG. In this case, the feeder current can be written as follows:

$$I(x,T_i)=\begin{cases}\int_0^x I_d(x,T_i)dx & 0\le x\le x_0 \\ \int_0^x I_d(x,T_i)dx-I_{DG}(T_i), x_0\le x\le l\end{cases} \tag{8}$$

The corresponding power loss and voltage drop in the feeder are:

$$\begin{aligned} P_{loss}(x_0,T_i) &= \int_0^{x_0}\left[\left|\int_0^x I_d(x,T_i)dx\right|\right]^2\cdot Rdx + \\ &\int_{x_0}^l\left[\left|\int_0^x I_d(x,T_i)dx-I_{DG}(T_i)\right|\right]^2\cdot Rdx \end{aligned} \tag{9}$$

$$V_{drop}(x,T_i)=\begin{cases}\int_0^x\int_0^x I_d(x,T_i)dx\cdot Zdx, 0\le x\le x_0 \\ \int_{x_0}^x\left[\int_0^x I_d(x,T_i)dx-I_{DG}(T_i)\right]\cdot Zdx+ \\ \int_0^{x_0}\int_0^x I_d(x,T_i)dx\cdot Zdx, x_0\le x\le l\end{cases} \tag{10}$$

The average power loss in a given time period T is:

$$\overline{P_{loss}}(x_0)=\frac{1}{T}\sum_{i=1}^{N_t}P_{loss}(x_0,T_i)T_i \tag{11}$$

where N_t is the number of time durations in the time period T.

$$T=\sum_{i=1}^{N_t}T_i \tag{12}$$

Equation (6) can still be used under this situation to calculate the voltage at point x by using $V_{drop}(x,T_i)$ obtained from equation (10).

The goal is to add DG in a location to minimize the total average power loss and assure that the voltages V_x along the feeder are in the acceptable range, 1±0.05 p.u., i.e.,

$$\frac{d\overline{P_{loss}}(x_0)}{dx_0}=0 \tag{13}$$

III. Procedure to Find the Optimal Location of DG on a Radial Feeder

The solution x_0 of the above equation will give the optimal site for minimizing the power loss, but it cannot guarantee that all the voltages along the feeder are in the acceptable range. If the voltage regulation cannot be satisfied at the same time, the DG can be placed around x_0 to satisfy the voltage regulation rule while decreasing the power loss as much as possible or the DG size can be increased. The analytical procedure to determine the optimal point to place DG on a radial feeder is given as follows.

Step 1:Find the distributed load $I_d(x,T_i)$ along the feeder.

Step 2:Get the output current of DG, $I_{DG}(T_i)$.

Step 3:Use equations (9) and (11) to calculate $\overline{P_{loss}}(x_0)$ and find the solution x_0 of equation (13).

Step 4:Use equations (6) and (10) to check whether the voltage regulation is satisfied.

Step 5:If all the voltages are in the acceptable range, then the calculated x_0 is the optimal spot (x_{op}) to add DG.

Step 6:If x_0 doesn't meet the voltage regulation rule, then move the DG to see whether there is a point around point x_0, where all bus voltages are in the acceptable range.

Step 7:If no point on the feeder can satisfy the voltage regulation rule, then increase the size of DG and repeat steps 2 to 7.

Step 8:Sometimes more than one DG may be needed. Under this situation, the feeder can be divided into several segments and steps 1 to 7 can be applied to each segment.

The flow chart of the procedure is shown in Figure 2.

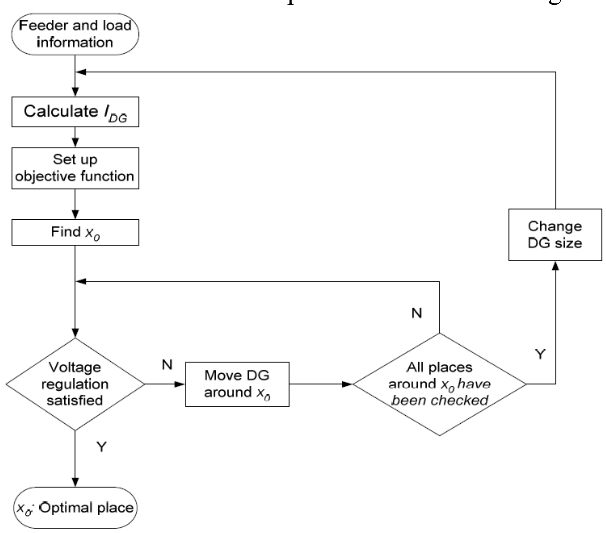

Figure 2. Procedure for finding optimal place for a DG source in a radial feeder.

IV. CASE STUDIES WITH TIME INVARIANT LOADS AND DGS

Table I shows the results of analyses, using the foregoing procedure, to find the optimal location for placing DG on radial feeders with three different load distributions: uniformly distributed load, centrally distributed load and uniformly increasing distributed load. In the results given in Table I, it is assumed that the DG supplies all the loads on the feeder in each case, and the distribution system supplies the system losses. It is noted from the table that the DG reduces the system power losses significantly when it is located properly.

TABLE I. THEORETICAL ANALYSIS RESULTS OF CASE STUDIES WITH TIME INVARIANT LOADS AND DGS

Cases (Assuming that DG supplies all the loads in each case)	P^0_{loss} **(Power loss before adding DG)**	P_{loss} **(Power loss after adding DG)**	**Percent of power loss reduction**	**Optimal place** x_0
Uniformly distributed load	$I_d^2 R l^3 / 3$	$I_d^2 R l^3 / 12$	75%	$\frac{l}{2}$
Centrally distributed load	$\frac{23}{960} I_{ld}^2 R l^5$	$\frac{1}{320} I_{ld}^2 R l^5$	87%	$\frac{l}{2}$
Increasingly distributed load	$0.13 I_{ld}^2 R l^5$	$\frac{2}{125} I_{ld}^2 R l^5$	88%	$\frac{(1-\sqrt{2})l}{2}$

The 2/3 rule presented in [6] works well when the load is uniformly distributed along the feeder, but it gives inaccurate results if the load configuration is different. For a uniformly distributed load, if the DG supplies 2/3 of the total load ($I_{DG} = I_{ld} l^2 / 6$), the optimal site is at $x_0 = l/3$ according to equation (9). This result is exactly the same as that given in [6]. However, when the loads are centrally and increasingly distributed and the DG provides 2/3 of the total load, the optimal location for placing DG turns out to be $l/\sqrt{6}$ and $(1-\sqrt{2/3})l$ respectively, which differ from what the "2/3 rule" suggests in [6].

V. CASE STUDY WITH TIME VARYING LOAD AND DG

The same feeders as in the previous section (shown in Figure 1), but with time varying load and DG, are analyzed in this case. The analysis is given for uniformly distributed load only. The analyses for other types of loads follow similarly.

Assuming that DG is located at point x_0, then according to equation (9), the effective power loss is:

$$P_{loss}(x_0, T_i) = R I_d(T_i) I_{DG}(T_i)(x_0{}^2 - l^2) - R I_{DG}^2(T_i)(x_0 - l) + \frac{R I_d^2(T_i) l^3}{3} \qquad (14)$$

where $I_d(T_i) = I_{load}(T_i)/l$ and $I_{load}(T_i)$ is the load current at the very sending end of the feeder. The average power loss in a given time period T is:

$$\overline{P_{loss}}(x_0) = C_1 + \frac{R x_0^2}{T} \sum_{i=1}^{N_t} I_d(T_i) I_{DG}(T_i) T_i - \frac{R x_0}{T} \sum_{i=1}^{N_t} I_{DG}^2(T_i) T_i \qquad (15)$$

where

$$C_1 = \frac{1}{T}\sum_{i=1}^{N_t}\Big[RI_d^2(T_i)l^3/3 - RI_d(T_i)I_{DG}(T_i)l^2 + RI_{DG}^2(T_i)l\Big]T_i$$

Setting $\dfrac{d\overline{P_{loss}}(x_0)}{dx_0} = 0$, x_0 is obtained to be:

$$x_0 = \frac{\sum_{i=1}^{N_t} I_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} I_d(T_i)I_{DG}(T_i)T_i} = \frac{l\cdot\sum_{i=1}^{N_t} I_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} I_{load}(T_i)I_{DG}(T_i)T_i} \tag{16}$$

Assuming that all bus voltages along the feeder are in the acceptable range, equation (16) can be approximated as

$$x_0 \approx \frac{\sum_{i=1}^{N_t} P_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} P_d(T_i)P_{DG}(T_i)T_i} = \frac{l\cdot\sum_{i=1}^{N_t} P_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} P_{load}(T_i)P_{DG}(T_i)T_i} \tag{17}$$

where $P_d(T_i) = P_{load}(T_i)/l$ and $P_{load}(T_i)$ is the total load along the feeder in the time duration T_i.

VI. Simulation Results

A. Radial Feeder with Time Invariant Loads and DG

A radial feeder with a time invariant DG was simulated under uniformly distributed, centrally distributed and increasingly distributed loads. The simulated system for uniformly distributed loads is shown in Figure 3. The system architecture is the same when the loads are centrally distributed or increasingly distributed.

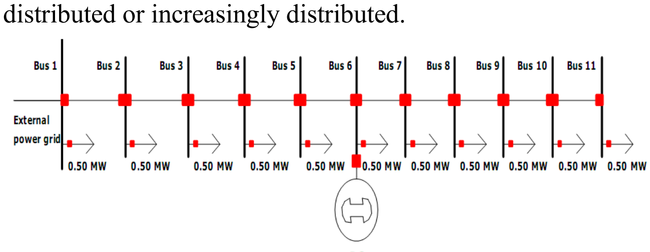

Figure 3. A radial feeder with uniformly distributed loads.

TABLE II. Simulation Results of Case Studies with Time Invariant Loads and DG

Line Loading	Optimal Bus No. (Simulation)	Optimal Place (Theoretical)	Total Power Losses (kW)	
			Without DG	***With DG***
Uniform	6	6	1.7785	0.2106
Central	6	6	0.2589	0.0275
Increasing	8	8	0.8099	0.0606

In Table Ⅱ, the optimal bus for placing DG to minimize the total system power loss is given for each load distribution. The total system power losses are given both with and without DG. It is noted that the simulation results agree well with theoretical values. While some bus voltages fall far out of the acceptable range when there is no DG in the system, all the bus voltages are within 1±0.05 p.u. with the DG added.

B. Radial Feeder with Time Varying Loads and DG

This part of the study is helpful in understanding the effect of variable power DG (such as wind and photovoltaic (PV)) sources on distribution systems with time varying loads. In practice, the site of such DG sources may be mainly determined by meteorological and geographic factors. However, DG sources with predictable output power (such as fuel cells and microturbines) can be placed at any bus in the distribution system to achieve optimal result.

The feeder shown in Figure 3 is also used to simulate the situation with time varying uniformly distributed loads and DG. A wind-turbine generator is considered as the time varying DG source. Actual wind data taken in a rural area were used to obtain the output power of a simulated (1-MW) wind turbine, as shown in Figure 4 [5],[7]. It shows the annual daily average output power of the turbine, which can be viewed as the filtered version of the turbine's output power. The daily average demand of a typical house, shown in Figure 5 [8],[9], is used here as one unit of the time varying loads. The loads are assumed to be uniformly distributed along the feeder with 100 houses at each bus.

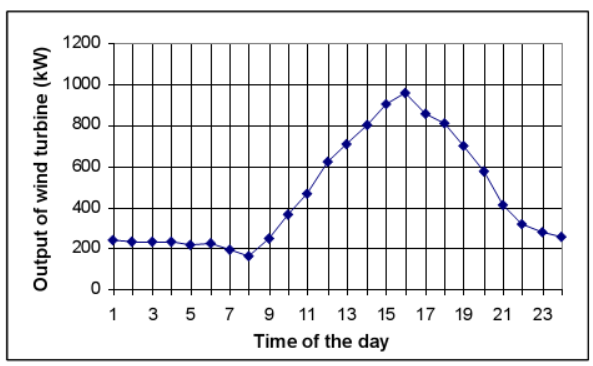

Figure 4. Annual daily average output power profile of a 1-MW wind-DG.

The simulated wind-DG was installed at different buses and the total system power loss was obtained in each case, shown in Figure 6. It is noticed from this figure that total feeder power loss reaches a minimum value when the wind-

DG is placed at bus 10 in Figure 3. The theoretical optimal position to place the wind-DG is obtained (using equation (17) and the generation and demand data shown in Figures 4 and 5) to be $x_0 \approx 0.14l$. The position with this distance from the end of feeder in Figure 3 is between bus 9 and 10, but closer to bus 10, which is the same bus obtained from the simulation results shown in Figure 6.

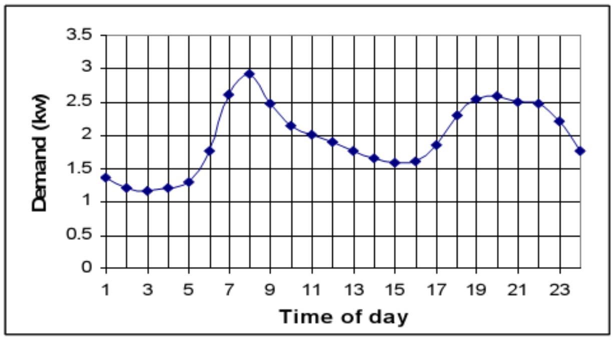

Figure 5. Daily average demand of a typical house.

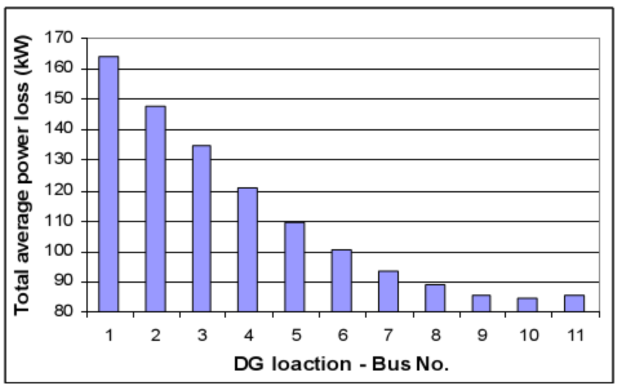

Figure 6. Power losses of the radial feeder with the wind-DG at different buses.

VII. Conclusion

Analytical approaches are presented in this paper to determine the optimal location for placing DG in radial system to minimize power losses. The proposed approaches are not iterative algorithms, like power flow programs. Therefore, there is no convergence problems involved, and results could be obtained very quickly. A series of simulation studies were also conducted to verify the validity of the proposed approaches, and results show that the proposed methods work well.

In practice, there are other constraints which may affect the DG placement. Nevertheless, methodologies presented in this paper can be effective, instructive and helpful to system designers in selecting proper sites to place DGs.

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