Analytical Approaches for Optimal Placement of Distributed Generation Sources in Power Systems

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***Abstract*—Power system deregulation and the shortage of transmission capacities have led to increased interest in distributed generation (DG) sources. Proper location of DGs in power systems is important for obtaining their maximum potential benefits. This paper presents analytical methods to determine the optimal location to place a DG in radial as well as networked systems to minimize the power loss of the system. Simulation results are given to verify the proposed analytical approaches.**

***Index Terms*—Analytical approach, distributed generation, optimal placement, power loss.**

I. Introduction

THE EVER-increasing need for electrical power generation, steady progress in the power deregulation and utility restructuring, and tight constraints over the construction of new transmission lines for long distance power transmission have created increased interest in distributed power generation. Distributed generation (DG) devices can be strategically placed in power systems for grid reinforcement, reducing power losses and on-peak operating costs, improving voltage profiles and load factors, deferring or eliminating for system upgrades, and improving system integrity, reliability, and efficiency [1]–[5].

These DG sources are normally placed close to consumption centers and are added mostly at the distribution level. They are relatively small in size (relative to the power capacity of the system in which they are placed) and modular in structure. A common strategy to find the site of DG is to minimize the power loss of the system [2]–[5]. Another method for placing DG is to apply rules that are often used in sitting shunt capacitors in distribution systems. A “2/3 rule” is presented in [6] to place DG on a radial feeder with uniformly distributed load, where it is suggested to install DG of approximately 2/3 capacity of the incoming generation at approximately 2/3 of the length of line. This rule is simple and easy to use, but it cannot be applied directly to a feeder with other types of load distribution, or to a networked system. References [1] and [7] present power flow algorithms to find the optimal size of DG at each load bus in a networked system assuming that every load bus can have a DG source.

This paper presents analytical approaches for optimal placement of DG with unity power factor in power systems. First, placement of DG in a radial feeder is analyzed and the theoretical optimal site (bus) for adding DG is obtained for different types of loads and DG sources. Then, a method is presented to find the optimal bus for placing DG in a networked system based on bus admittance matrix, generation information and load distribution of the system. The proposed methods are tested by a series of simulations on radial feeders, an IEEE 6-bus test system [1], an IEEE 30-bus test system [11], and a subset of it, to show the effectiveness of the proposed methods in determining the optimal bus for placing DG.

In practice, there are more constraints on the availability of DG sources, and we may only have one or a few DGs with limited output available to add. Therefore, in this study the DG size is not considered to be optimized. The procedure to determine the optimal bus for placing DG may also need to take into account other factors, such as economic and geographic considerations. These factors are not discussed in this paper.

Manuscript received May 11, 2004. This work was supported by the National Science Foundation under Grant ECS-0135229 and by Montana State University. Paper no. TPWRS-00472-2003.

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Digital Object Identifier 10.1109/TPWRS.2004.836189

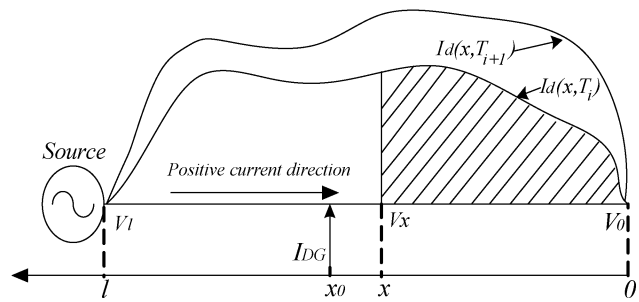


Fig. 1. A feeder with distributed loads along the line.

II. Optimal Placement of DG on a Radial Feeder

To simplify the analysis, only overhead transmission lines with uniformly distributed parameters are considered, i.e., R and L per unit length are the same along the feeder while C and G per unit length are neglected. The loads along the feeder are assumed to vary in discrete time duration; for example, the feeder load distributions along the line for time durations T_i and T_{i+1} are shown in Fig. 1.

A. Theoretical Analysis

First consider a radial feeder without DG. During the time duration T_i, the loads are distributed along the line with the phasor current density $I_d(x,T_i)$ as shown in Fig. 1.

The phasor feeder current at point x is

$$I(x,T_i) = \int_0^x I_d(x,T_i)dx. \tag{1}$$

Assuming the impedance per unit length of the line is $Z = R + jX$ (Ω/km), then the incremental power loss and phasor voltage drop at point x are

$$dP(x,T_i) = \left(\left|\int_0^x I_d(x,T_i)dx\right|\right)^2 \cdot Rdx \tag{2}$$

$$dV(x,T_i) = \left(\int_0^x I_d(x,T_i)dx\right) \cdot Zdx. \tag{3}$$

The total power loss along the feeder within the time duration T_i is

$$P_{loss}(T_i) = \int_0^l dP(x,T_i) = \int_0^l \left(\left|\int_0^x I_d(x,T_i)dx\right|\right)^2 \cdot Rdx. \tag{4}$$

The voltage drop between point x and the receiving end is

$$\begin{aligned} V_{drop}(x,T_i) &= V_x(T_i) - V_0(T_i) \\ &= \int_0^x dV(x,T_i) \\ &= \int_0^x \int_0^x I_d(x,T_i)dx \cdot Zdx \end{aligned} \tag{5}$$

and the voltage at point x is

$$\begin{aligned} V_x(T_i) &= V_0(T_i) + V_{drop}(x,T_i) \\ &= V_l(T_i) - V_{drop}(l,T_i) + V_{drop}(x,T_i). \end{aligned} \tag{6}$$

The total voltage drop across the feeder is

$$\begin{aligned} V_{drop}(l,T_i) &= V_l(T_i) - V_0(T_i) \\ &= \int_0^l dV(x,T_i) \\ &= \int_0^l \int_0^x I_d(x,T_i)dx \cdot Zdx. \end{aligned} \tag{7}$$

Now, consider a DG is added into the feeder at the location x_0, shown in Fig. 1. In general, the load current density $I_d(x,T_i)$ will change (normally decrease) as a result of adding DG due to improvements in the voltage profile along the line. This change in the load current density will cause the feeder current to decrease. The feeder current between the source (at $x = l$) and the location of DG (at $x = x_0$) will also change as a result of the injected current source $I_{\text{DG}}(T_{\text{i}})$. However, the change in feeder current due to the change in the load current density is generally much smaller than the change in the feeder current due to the injected current $I_{\text{DG}}(T_{\text{i}})$. For the purpose of analysis, the change in the load current density, resulted from the addition of DG, is neglected in the paper. Therefore, the load current density $I_d(x,T_i)$, used in (1), is also used for obtaining the feeder current after adding DG. In this case, the feeder current can be written as follows:

$$I(x,T_i) = \begin{cases} \int_0^x I_d(x,T_i)dx & 0 \le x \le x_0 \\ \int_0^x I_d(x,T_i)dx - I_{DG}(T_i) & x_0 \le x \le l. \end{cases} \tag{8}$$

The corresponding power loss and voltage drop in the feeder are

$$\begin{aligned} &P_{loss}(x_0,T_i) \\ &= \int_0^{x0} \left(\left|\int_0^x I_d(x,T_i)dx\right|\right)^2 \cdot Rdx \\ &\quad + \int_{x0}^l \left(\left|\int_0^x I_d(x,T_i)dx - I_{DG}(T_i)\right|\right)^2 \cdot Rdx \end{aligned} \tag{9}$$

$$\begin{aligned} &V_{drop}(x,T_i) \\ &= \begin{cases} \int_0^x \int_0^x I_d(x,T_i)dx \cdot Zdx & 0 \le x \le x_0 \\ \int_0^{x0} \int_0^x I_d(x,T_i)dx \cdot Zdx & \\ \quad + \int_{x0}^x \left(\int_0^x I_d(x,T_i)dx - I_{DG}(Ti)\right) \cdot Zdx & x_0 \le x \le l. \end{cases} \end{aligned} \tag{10}$$

The average power loss in a given time period T is

$$\overline{P_{loss}}(x_0) = \frac{1}{T} \sum_{i=1}^{N_t} P_{loss}(x_0,T_i)T_i \tag{11}$$

where N_t is the number of time durations in the time period T

$$T = \sum_{i=1}^{N_t} T_i. \tag{12}$$

Equation (6) can still be used under this situation to calculate the voltage at point x by using $V_{drop}(x,T_i)$ obtained from (10).

B. Procedure to Find the Optimal Location of DG on a Radial Feeder

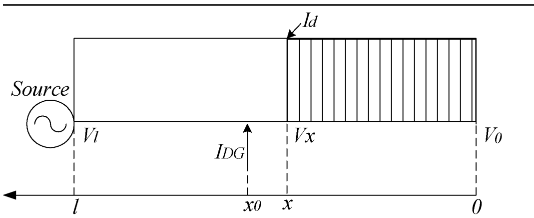
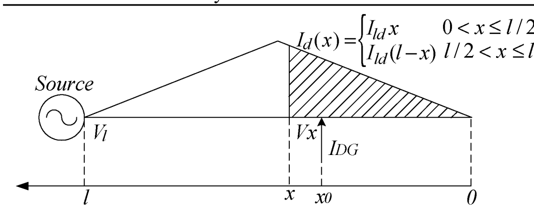
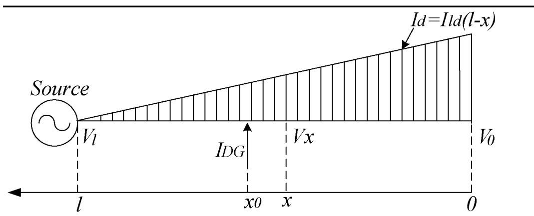
The goal is to add DG in a location to minimize the total average power loss and assure that the voltages V_x along the feeder are in the acceptable range, 1 ± 0.05 p.u., i.e.,

$$\frac{d\overline{P_{loss}}(x_0)}{dx_0} = 0. \tag{13}$$

The solution x_0 of the above equation will give the optimal site for minimizing the power loss, but it cannot guarantee that all the voltages along the feeder are in the acceptable range. If the voltage regulation cannot be satisfied at the same time, the DG can be placed around x_0 to satisfy the voltage regulation rule while decreasing the power loss as much as possible, or the DG size can be increased. The analytical procedure to determine the optimal point to place DG on a radial feeder is given as follows.

1) Find the distributed load $I_d(x,T_i)$ along the feeder.
2) Get the output current of DG, $I_{DG}(T_i)$.

TABLE I
THEORETICAL ANALYSIS RESULTS OF CASE STUDIES WITH TIME INVARIANT LOADS AND DGS

Cases (Assuming that DG supplies all the loads in each case)	$P_{loss}^{\ 0}$ (Power loss before adding DG)	P_{loss} (Power loss after adding DG)	Percent of power loss reduction (%)	Optimal place x_0
 Uniformly distributed load	$I_d^2 Rl^3/3$	$I_d^2 Rl^3/12$	75%	$\frac{l}{2}$
 Centrally distributed load	$\frac{23}{960} I_{ld}^2 Rl$	$\frac{1}{320} I_{ld} Rl^5$	87%	$\frac{l}{2}$
 Increasingly distributed load	$0.1333 I_{ld}^2 Rl^5$	$0.01555 I_{ld} Rl^5$	88%	$(1-\sqrt{2}/2)l$

3) Use (9) and (11) to calculate $\overline{P_{loss}}(x_0)$ and find the solution x_0 of (13).
4) Use (6) and (10) to check whether the voltage regulation is satisfied.
5) If all the voltages are in the acceptable range, then the calculated x_0 is the optimal spot (x_{op}) to add DG.
6) If x_0 doesn't meet the voltage regulation rule, then move the DG to see whether there is a point around point x_0, where all bus voltages are in the acceptable range.
7) If no point on the feeder can satisfy the voltage regulation rule, then increase the size of DG and repeat steps 2) to 7).
8) Sometimes more than one DG may be needed. Under this situation, the feeder can be divided into several segments and steps 1) to 7) can be applied to each segment.

C. Case Studies With Time Invariant Loads and DGs

Table I shows the results of analyses, using the foregoing procedure, to find the optimal location for placing DG on radial feeders with three different load distributions: uniformly distributed load, centrally distributed load and uniformly increasing distributed load. In the results given in Table I, it is assumed that the DG supplies all the loads on the feeder in each case, and the distribution system supplies the system losses.

It is noted from this table that the DG reduces the system power losses significantly when it is located properly.

The 2/3 rule presented in [6] works well when the load is uniformly distributed along the feeder, but it gives inaccurate results if the load configuration is different. For a uniformly distributed load, if the DG supplies 2/3 of the total load ($I_{DG} = I_{ld}l^2/6$), the optimal site is at $x_0 = l/3$ according to (9). This result is exactly the same as that given in [6]. However, when the loads are centrally and increasingly distributed and the DG provides 2/3 of the total load, the optimal location for placing DG turns out to be $l/\sqrt{6}$ and $(1-\sqrt{2/3})l$, respectively, which differ from what the "2/3 rule" suggests in [6].

D. Case Study With Time Varying Load and DG

The same feeders studied in part C, but with time varying load and DG, are analyzed in this case. The analysis is given for uniformly distributed load only. The analyzes for other types of loads follow similarly.

Assuming that DG is located at point x_0, then according to (9), the effective power loss is

$$P_{loss}(x_0,T_i) = RI_d(T_i)I_{DG}(T_i)\left(x_0^2 - l^2\right) - RI_{DG}^2(T_i)(x_0 - l) + \frac{RI_d^2(T_i)l^3}{3} \quad (14)$$

where $I_d(T_i) = I_{load}(T_i)/l$ and $I_{load}(T_i)$ is the load current at the very sending end of the feeder. The average power loss in a given time period T is

$$\overline{P_{loss}}(x_0) = C_1 + \frac{Rx_0^2}{T}\sum_{i=1}^{N_t} I_d(T_i)I_{DG}(T_i)T_i - \frac{Rx_0}{T}\sum_{i=1}^{N_t} I_{DG}^2(T_i)T_i \quad (15)$$

where $C_1 = (1/T)\sum_{i=1}^{N_t}[(RI_d^2(T_i)l^3)/(3) - RI_d(T_i)I_{DG}(T_i)l^2 + RI_{DG}^2(T_i)l]T_i$.

Setting $(d\overline{P_{loss}}(x_0))/(dx_0) = 0$, x_0 is obtained to be

$$x_0 = \frac{\sum_{i=1}^{N_t} I_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} I_d(T_i)I_{DG}(T_i)T_i} = \frac{l\cdot\sum_{i=1}^{N_t} I_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} I_{load}(T_i)I_{DG}(T_i)T_i}. \quad (16)$$

Assuming that all bus voltages along the feeder are in the acceptable range, (16) can be approximated as

$$x_0 \approx \frac{\sum_{i=1}^{N_t} P_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} P_d(T_i)P_{DG}(T_i)T_i} = \frac{l\cdot\sum_{i=1}^{N_t} P_{DG}^2(T_i)T_i}{2\sum_{i=1}^{N_t} P_{load}(T_i)P_{DG}(T_i)T_i} \quad (17)$$

where $P_d(T_i) = P_{load}(T_i)/l$ and $P_{load}(T_i)$ is the total load along the feeder in the time duration T_i.

III. Optimal Placement of DG in Networked Systems

The theoretical analysis for placing a DG in networked systems is different and more complicated than in a radial feeder. To simplify the analysis, only one DG is considered to be added to the system.

Consider the system shown in Fig. 2 with a DG added to the system to reinforce it. The system has N buses and loads, and the DG is located at a bus, say bus j. The main external power is injected into bus 1, which is taken as slack bus. The objective is to find the bus to install the DG so that the total system power loss is minimized and the voltage level at each bus is held in the acceptable range, 1 ± 0.05 p.u.

Before the DG is added to the system, the bus admittance matrix is

$$Y_{bus}^0 = \begin{bmatrix} Y_{11}^0 & Y_{12}^0 & \dots & Y_{1j}^0 & \dots & Y_{1N}^0 \\ & & \dots & & \dots & \\ & & \dots & & \dots & \\ Y_{N1}^0 & Y_{N2}^0 & \dots & Y_{Nj}^0 & \dots & Y_{NN}^0 \end{bmatrix} \quad (18)$$

where superscript ^{0} denotes the original system.

Assuming that the DG is located at bus j, the system admittance matrix is changed from Y_{bus}^0 to Y_{bus} by considering that bus 1 and bus j are connected together. Actually there is no line to connect those buses together, but the imaginary line will help in finding the optimal location to add DG. Y_{bus} is one dimension less than Y_{bus}^0 except when the DG is located at bus 1. If the DG is at bus 1, Y_{bus} matrix will be the same as Y_{bus}^0. To obtain the new matrix Y_{bus} when the DG source is connected, we treat the system as connecting bus 1 and j by eliminating bus j in Y_{bus}^0 [10]. The new matrix is

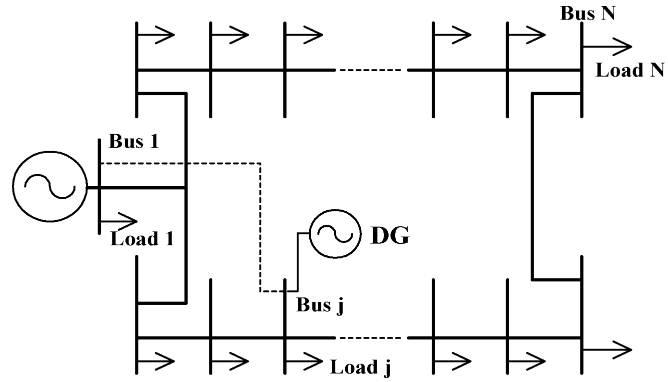


Fig. 2. A networked power system.

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1k} & \dots & Y_{1(N-1)} \\ & & \dots & & \dots & \\ & & \dots & & \dots & \\ Y_{(N-1)1} & Y_{(N-1)2} & \dots & Y_{(N-1)k} & \dots & Y_{(N-1)(N-1)} \end{bmatrix} \quad (19)$$

where

$$\begin{aligned} Y_{11} &= Y_{11}^0 + Y_{jj}^0 + 2Y_{1j}^0 \\ Y_{1k} &= Y_{1k}^0 + Y_{jk}^0, k = 2,\dots,j-1 \\ Y_{1k} &= Y_{1(k+1)}^0 + Y_{j(k+1)}^0, k = j,\dots,N-1 \\ Y_{k1} &= Y_{1k}, k = 2,\dots,N-1 \\ Y_{ik} &= Y_{ik}^0, 2 \le (i,k) \le j-1 \\ Y_{ik} &= Y_{i(k+1)}^0, 2 \le i \le j-1, j \le k \le N-1 \\ Y_{ik} &= Y_{(i+1)k}^0, j \le i \le N-1, 2 \le k \le j-1 \\ Y_{ik} &= Y_{(i+1)(k+1)}^0, j \le (i,k) \le N-1. \end{aligned}$$

The new bus impedance matrix Z_{bus} is as shown in (20), at the bottom of the page.

Suppose the complex load and generated power of the original system are

$$\begin{aligned} S_L^0 &= \left[S_{L1}^0, S_{L2}^0, \dots, S_{Li}^0, \dots . S_{LN}^0\right], \text{ and} \\ S_G^0 &= \left[S_{G1}^0, S_{G2}^0, \dots, S_{Gi}^0, \dots . S_{GN}^0\right], \ i = 1,2,\dots,N \end{aligned} \quad (21)$$

where $S_{Li}^0 = P_{Li}^0 + jQ_{Li}^0$ and $S_{Gi}^0 = P_{Gi}^0 + jQ_{Gi}^0$.

A new load vector S_L is set up as follows:

$$S_L = [S_{L1}, S_{L2}, \dots, S_{Li}, \dots . S_{LN}] \quad (22)$$

$$Z_{bus} = Y_{bus}^{-1} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1k} & \dots & Z_{1(N-1)} \\ & & \dots & & \dots & \\ & & \dots & & \dots & \\ Z_{(N-1)1} & Z_{(N-1)2} & \dots & Z_{(N-1)k} & \dots & Z_{(N-1)(N-1)} \end{bmatrix} \quad (20)$$

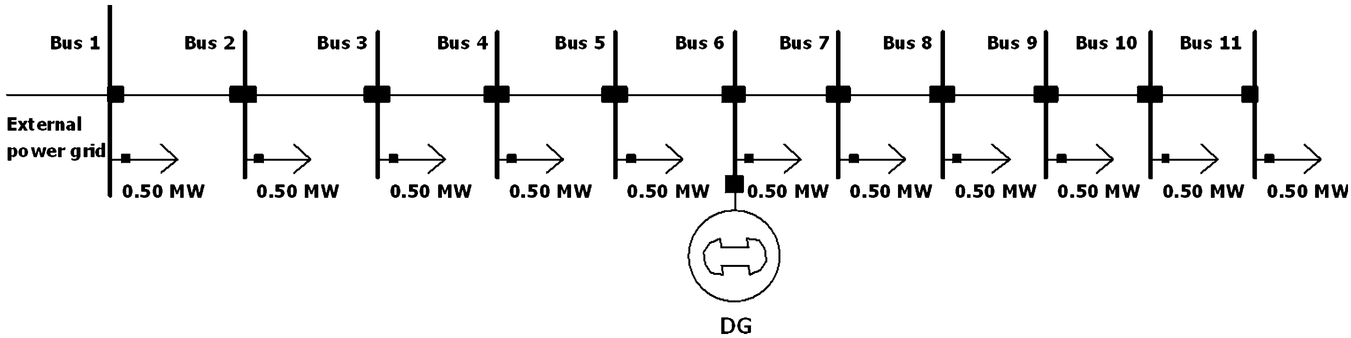


Fig. 3. A radial feeder with uniformly distributed loads.

where

$$S_{Li} = P_{Li} + jQ_{Li}$$
$$S_{Li} = 0, \quad \text{for } i = 1 \text{ (slack bus)}$$
$$S_{Li} = S_{Li}^0, \quad \text{for } i = \text{load buses}$$
$$S_{Li} = \begin{cases} P_{Li}^0 - P_{Gi}^0 + j0, & P_{Li} > P_{Gi} \\ 0, & P_{Li} \le P_{Gi} \end{cases}, \quad \text{for } i = \text{P} - \text{V buses.}$$

Note that at the slack bus (bus 1) $S_{L1} = 0$; it is assumed that the real and reactive power consumed by the load are supplied directly by the external generation at that bus. Also, at a voltage controlled (P-V) bus, $Q_{Li} = 0$; it is assumed that the load reactive power can be supplied by the external power source at the P-V bus.

To find the optimal point to place the DG, we set up an objective function for DG at each bus j as follows:

$$f_j = \sum_{i=1}^{j-1} R_{1i}(j)|S_{Li}|^2 + \sum_{i=j+1}^{N} R_{1i}(j)|S_{Li}|^2, \quad j = 2, \ldots N \tag{23}$$

where $R_{1i}(j)$ is the equivalent resistance between bus 1 and bus i when DG is located at bus j, $j \neq 1$

$$R_{1i}(j) = \begin{cases} Real\,(Z_{11} + Z_{ii} - 2Z_{1i}) & i < j \\ Real\,\left(Z_{11} + Z_{(i-1)(i-1)} - 2Z_{1(i-1)}\right) & i > j. \end{cases} \tag{24}$$

When the DG is located at bus 1 ($j = 1$), the objective function will be

$$f_1 = \sum_{i=1}^{N} R_{1i}|S_{Li}|^2. \tag{25}$$

Note that in this case, Y_{bus} (Z_{bus}) will be the same as Y_{bus}^0 (Z_{bus}^0) and $R_{11} = 0$.

The goal is to find the optimal bus m where the objective function reaches its minimum value

$$f_m = Min(f_j), \quad j = 1, 2, \ldots N. \tag{26}$$

The theoretical procedure to find the optimal bus to place DG in a networked system can be summarized as follows.

1) Find the matrix Y_{bus}^0 and set up the load vector S_L.
2) Compute Y_{bus} and the corresponding Z_{bus} for different DG locations.
3) Calculate the equivalent resistances according to (24).
4) Use (23) and (25) to calculate objective function values for DG at different buses and find the optimal bus m.
5) If all the voltages are in the acceptable range when the DG is located at bus m, then bus m is the optimal site.
6) If some bus voltages do not meet the voltage rule, then move the DG around bus m to satisfy the voltage rule.
7) If there is no bus that can satisfy the voltage regulation rule, try a different size DG and repeat steps 5) and 6).

Though the discussion here is under the assumption that only one DG source is added to the system, it can be easily extended to the systems with multiple DG sources. By connecting all the DG buses and slack bus together through imaginary lines, the new Y_{bus} matrix and the corresponding objective function can be established by the method presented above.

TABLE II
SIMULATION RESULTS OF CASE STUDIES WITH TIME INVARIANT LOADS AND DG

Line Loading	Optimal Bus No. (Simulation)	Optimal Place (Theoretical)	Total Power Losses (kW)	
			Without DG	With DG
Uniform	6	6	1.7785	0.2106
Central	6	6	0.2589	0.0275
Increasing	8	8	0.8099	0.0606

IV. Simulation Results

Several simulation studies were carried out to verify the results obtained analytically for both radial and network-connected systems.

A. Radial Feeder With Time Invariant Loads and DG

A radial feeder with a time invariant DG was simulated under uniformly distributed, centrally distributed and increasingly distributed loads. The simulated system for uniformly distributed loads is shown in Fig. 3. The system architecture is the same when the loads are centrally distributed or increasingly distributed. The line parameters, DG and load sizes are listed in Appendix A.

In Table II, the optimal bus for placing DG to minimize the total system power loss is given for each load distribution. The total system power losses are given both with and without DG. It is noted that the simulation results agree well with theoretical values. While some bus voltages fall far out of the acceptable range when there is no DG in the system, all the bus voltages are within 1 ± 0.05 p.u. with the DG added.

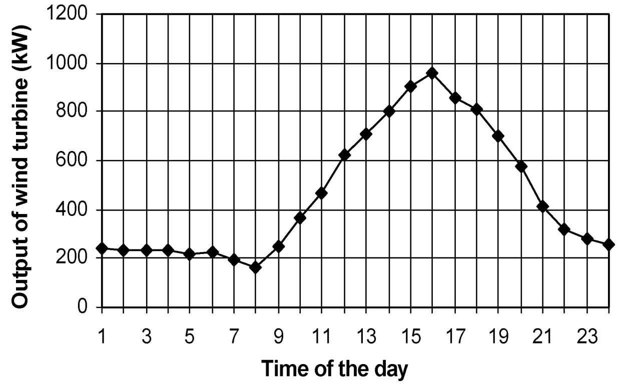


Fig. 4. Annual daily average output power profile of a 1-MW wind-DG.

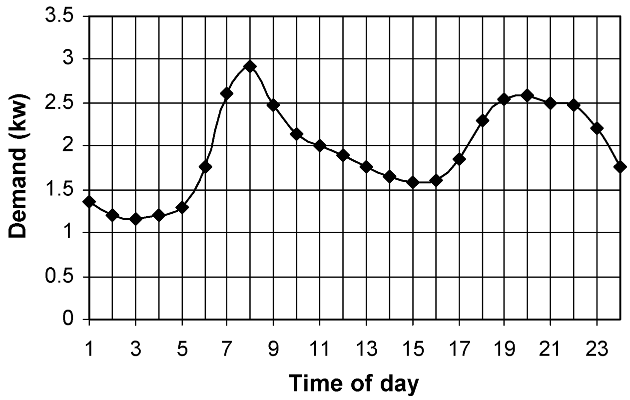


Fig. 5. Daily average demand of a typical house.

B. Radial Feeder With Time Varying Loads and DG

This part of the study is helpful in understanding the effect of variable power DG [such as wind and photovoltaic (PV)] sources on distribution systems with time varying loads. In practice, the site of such DG sources may be mainly determined by meteorological and geographic factors. However, DG sources with predictable output power (such as fuel cells and microturbines) can be placed at any bus in the distribution system to achieve optimal result.

The feeder shown in Fig. 3 is also used to simulate the situation with time varying uniformly distributed loads and DG. A wind-turbine generator is considered as the time varying DG source. Actual wind data taken in a rural area in south central Montana were used to obtain the output power of a simulated (1-MW) wind turbine, as shown in Fig. 4 [5], [8]. It shows the annual daily average output power of the turbine, which can be viewed as the filtered version of the turbine's output power. The daily average demand of a typical house in the northwestern United States, shown in Fig. 5 [9], is used here as one unit of the time varying loads. The loads are assumed to be uniformly distributed along the feeder with 100 houses at each bus.

The simulated wind-DG was installed at different buses and the total system power loss was obtained in each case, shown in Fig. 6. It is noticed from this figure that total feeder power loss reaches a minimum value when the wind-DG is placed at bus 10 in Fig. 3. The theoretical optimal position to place the wind-DG is obtained (using (17) and the generation and demand data shown in Figs. 4 and 5) to be $x_0 \approx 0.14l$. The position with this distance from the end of feeder in Fig. 3 is between bus 9 and 10, but closer to bus 10, which is the same bus obtained from the simulation results shown in Fig. 6.

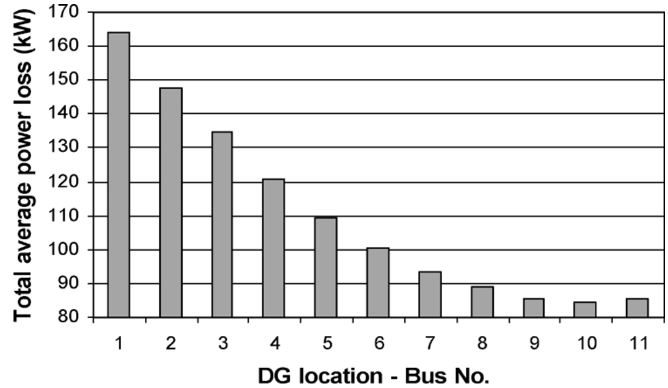


Fig. 6. Power losses of the radial feeder with the wind-DG at different buses.

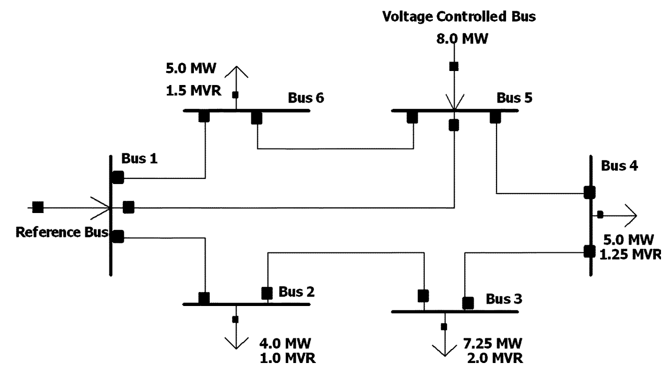


Fig. 7. 6-bus networked power system studied.

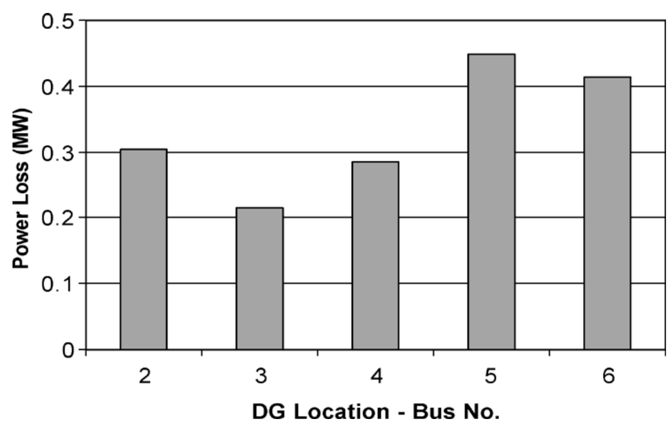


Fig. 8. Power losses of the system in Fig. 7 with a 5-MW DG.

C. Networked Systems

The 25-kV IEEE 6-bus system shown in Fig. 7 [1], which can be considered as a subtransmission/distribution system, was applied to verify the method presented in Section III. The parameters of this system are given in Appendix B. A 5-MW DG was added to reinforce the system. Total system power loss was obtained from the results of power flow studies when DG was placed at different buses (Fig. 8). It is noted from this figure that minimum power loss is achieved when DG is placed at bus 3. The values of the objective function for the system were obtained by applying the proposed analytical approach when the DG was placed at different buses. These values are shown in the bar chart of Fig. 9. It is noted from this figure that the objective function is also at its minimum when the DG is placed at bus 3, indicating that the result obtained from the proposed analytical method is the same as the simulation result.

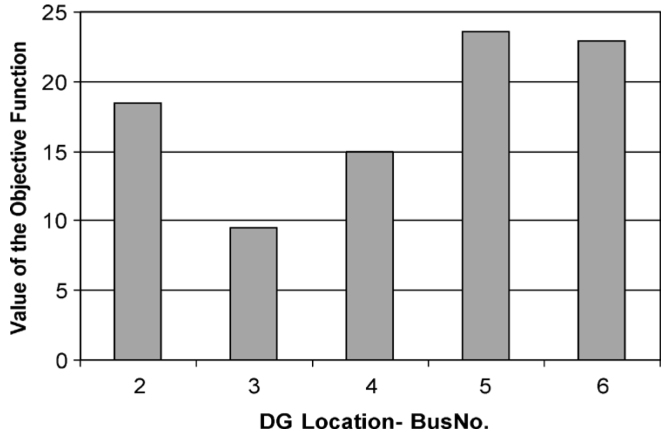


Fig. 9. Values of the objective function of the system in Fig. 7.

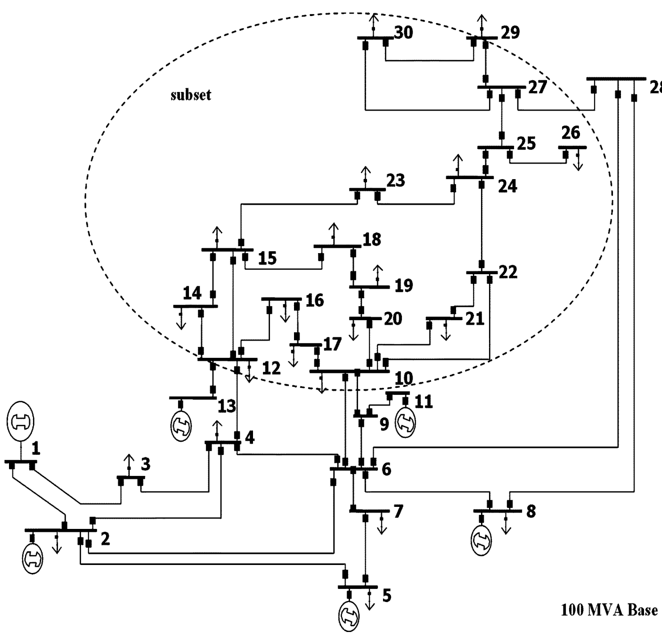


Fig. 10. IEEE 30-bus test system.

The proposed method was also tested on the IEEE 30-bus test system shown in Fig. 10, which can be considered as a meshed transmission/subtransmission system [11]. The system has 30 buses (mainly 132- and 33-kV buses) and 41 lines. The system bus data is given in Appendix C. A 15-MW DG (about 5% of the total system load of $283 + j126.2$ MVA) is considered to be added to reinforce the system. The total power loss of the system reaches a minimum value when DG is located at bus 5, as shown in Fig. 11. The optimal bus determined by the method proposed in this paper is also bus 5, as given in Fig. 12.

A DG source may not be connected directly to a 132-kV bus (bus 5 in Fig. 10) as determined by the proposed method. For this reason, a subset of the 30-bus test system with lower voltage level (33 kV), as indicated in Fig. 10, was also chosen to test the proposed method. The new system has 18 buses, 22 lines, and a total load of $104.7 + j50.8$ MVA. A 5-MW DG (about 5% of the total load of the subset) was added to the system. Simulation results for this system are given in Fig. 13. It is noted that the total system loss reaches a minimum value when the DG is located at bus 30. The optimal place suggested by the proposed method is also bus 30, as shown in Fig. 14.

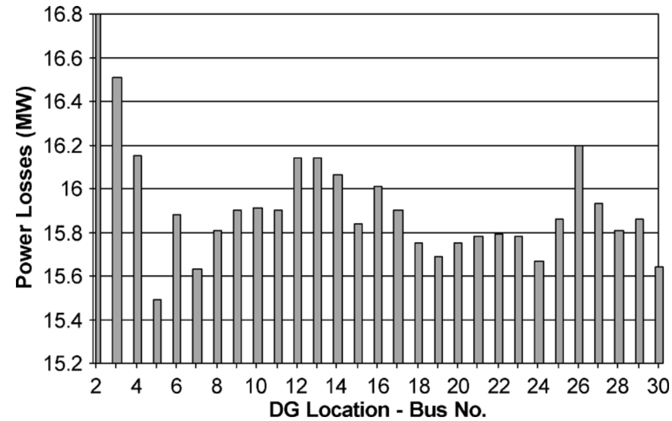


Fig. 11. Power losses of the IEEE 30-bus test system with a 15-MW DG.

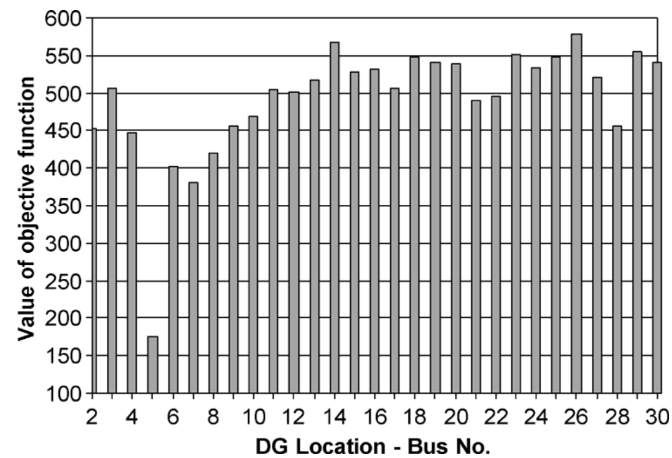


Fig. 12. Values of the objective function of the IEEE 30-bus system.

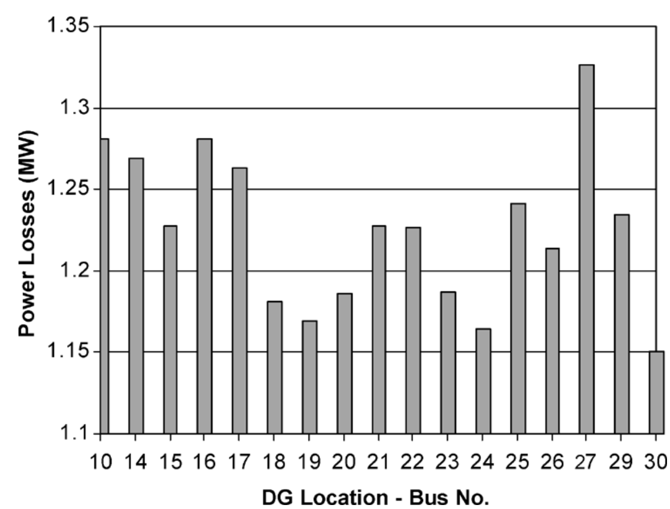


Fig. 13. Power losses of the subset system in Fig. 10 with a 5-MW DG.

V. Conclusion

This paper presents analytical approaches to determine the optimal location for placing DG in both radial and networked systems to minimize power losses. The proposed approaches are not iterative algorithms, like power flow programs. Therefore, there is no convergence problems involved, and results could be obtained very quickly. A series of simulation studies have been

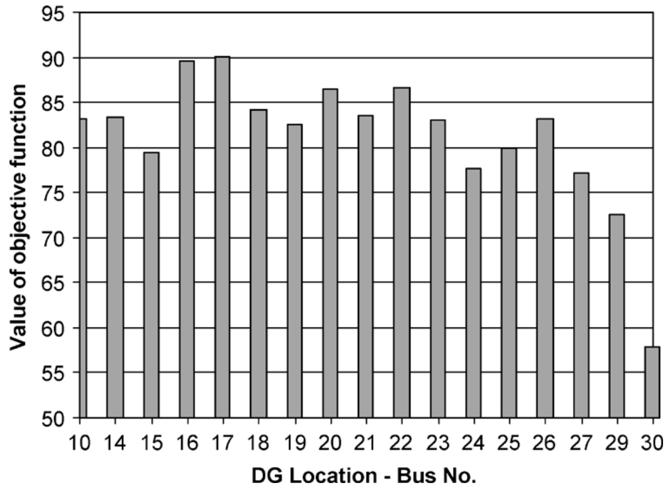


Fig. 14. Values of the objective function of the subset system in Fig. 10.

conducted to verify the validity of the proposed approaches, and results show that the proposed methods work well.

In practice, there are other constraints which may affect the DG placement. Nevertheless, methodologies presented in this paper can be effective, instructive, and helpful to system designers in selecting proper sites to place DGs.

APPENDIX A
PARAMETERS OF THE SYSTEM IN FIG. 3

TABLE A-1

Line parameters (AWG ACSR 1/0)	Line spacing = 1.32m (equal spacing assumed) R = 0. 538Ω, X_L = 0.4626 Ω Bus Voltage: 12.5kV Line length between two neighboring buses: 2.5km										
Load type	Load at each bus (MW)										
	1	2	3	4	5	6	7	8	9	10	11
Uniformly distributed	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Centrally distributed	0.05	0.1	0.2	0.3	0.4	0.5	0.4	0.3	0.2	0.1	0.05
Increasingly distributed	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55
DG size (MW)	Uniformly			Centrally				Increasingly			
	5.5			2.6				3.3			

APPENDIX B
PARAMETERS OF THE SYSTEM IN FIG. 7 [1]

TABLE B-1
BUS DATA

Bus No.	Voltage (p.u.)	Bus Power (MVA)
1	1.0+j0.0	Slack bus
2	—	-4.0 -j1.00
3	—	-7.25-j2.00
4	—	-5.00-j1.25
5	$\lvert V_5 \rvert = 1.0$	8.00
6	—	-5.00-j1.50

TABLE B-2
LINE DATA

From	To	Z_{serial} (p.u.)	Y_{shunt}(p.u.)
1	2	0.2238+j0.5090	j0.0012
2	3	0.2238+j0.5090	j0.0012
3	4	0.2238+j0.5090	j0.0012
4	5	0.2238+j0.5090	j0.0012
5	6	0.2238+j0.5090	j0.0012
6	1	0.2276+j0.2961	j0.0025
1	5	0.2603+j0.7382	j0.0008

APPENDIX C
BUS DATA OF THE IEEE 30-BUS TEST SYSTEM [11]

TABLE C-1

Bus No.	Type	Load (p.u.)	Rated Bus Voltage (kV)	Bus Voltage (p.u.)
1	Swing	0.0	132	1.060
2	P-V	0.217+j0.127	132	1.043
3	P-Q	0.024+j0.012	132	—
4	P-Q	0.076+j0.016	132	—
5	P-V	0.942+j0.19	132	1.010
6	P-Q	0.0	132	—
7	P-Q	0.228+j0.109	132	—
8	P-V	0.3+j0.3	132	1.010
9	P-Q	0.0	69.0	—
10	P-Q	0.058+j0.02	33.0	—
11	P-V	0.0	11.0	1.082
12	P-Q	0.112+j0.075	33.0	—
13	P-V	0.0	11.0	1.071
14	P-Q	0.062+j0.016	33.0	—
15	P-Q	0.082+j0.025	33.0	—
16	P-Q	0.035+j0.018	33.0	—
17	P-Q	0.09+j0.058	33.0	—
18	P-Q	0.032+j0.009	33.0	—
19	P-Q	0.095+j0.034	33.0	—
20	P-Q	0.022+j0.007	33.0	—
21	P-Q	0.175+j0.112	33.0	—
22	P-Q	0.0	33.0	—
23	P-Q	0.032+j0.016	33.0	—
24	P-Q	0.087+j0.067	33.0	—
25	P-Q	0.0	33.0	—
26	P-Q	0.035+j0.023	33.0	—
27	P-Q	0.0	33.0	—
28	P-Q	0.0	33.0	—
29	P-Q	0.024+j0.009	33.0	—
30	P-Q	0.106+j0.019	33.0	—

(100 MVA Base)

ACKNOWLEDGMENT

Power flow studies in this paper were carried out using PowerWorld Simulator.

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