Design of Microwave Filters With Arbitrary Frequency Response Based on Digital Methods

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***Abstract*—This letter presents a novel general technique for the design of microwave filters with arbitrary frequency response. It is based on the translation of the microwave specifications to the digital domain, where the well known and readily available digital filter design techniques are applied. By means of these digital techniques, the method provides a straightforward procedure to calculate the poles and zeros corresponding to the analog frequency response that satisfies the target specifications. From the poles and zeros, the microwave filter can be readily obtained using conventional techniques. As an example to demonstrate the proposed technique, a filter with user-defined specifications over two independent passbands has been implemented and successfully tested in microstrip technology.**

***Index Terms*—Arbitrary frequency response, digital filter design, microwave filter.**

I. Introduction

Up TO now, the systematic design of microwave filters has been mainly limited to the use of the well-known classical functions, i.e., Butterworth, *et al.* [1], [2]. However, more sophisticated and general functions like Zolotarev have been also explored [3] and specifically, for the case of multipassband filters, some limited analytical techniques have been proposed [4], although usually direct optimization methods have been applied for these cases, with the involved difficulties [5], [6].

The aim of this letter is to demonstrate that microwave filters with especially demanding magnitude responses can be synthesized by using the well established and readily available digital filter design techniques [7]. The proposed methodology rests on the translation of the target specifications from the analog to the digital domain, where the microwave designer can take immediate advantage of these sophisticated and continuously developing digital techniques, without requiring a deep understanding of the complex mathematics involved.

The details of the novel filter design technique will be explained in Section II and, afterward, an example of a two-passband microstrip filter with arbitrary specifications will be presented showing the excellent performance of the technique.

II. Microwave Filter Design

The proposed design technique comprises several steps sketched in Fig. 1 and detailed below.

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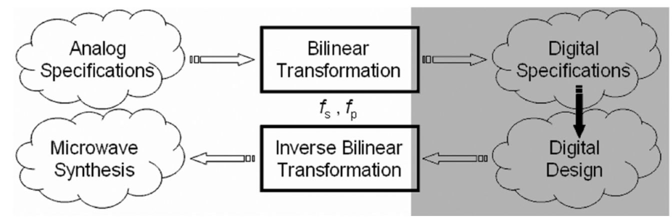

Fig. 1. Proposed general method of design.

A. From Analog Specifications to Digital Specifications

The desired microwave filter is characterized by a set of analog specifications. From them, the digital design specifications are obtained by means of a bilinear transformation [7]

$$z = \frac{a+s}{a-s}; \quad a = \frac{2\pi \cdot f_p}{\tan\left(\pi \cdot \frac{f_p}{f_s}\right)} \tag{1}$$

which is an algebraic transformation between the analog Laplace-transform s-plane, and the digital z-transform plane. In order to relate the analog frequency, f [Hz], with the digital frequency, ω [rad/sample], (1) has to be valued at $s = \mathrm{j} \cdot \Omega$, where $\Omega = 2\pi \cdot f$, and $z = e^{\mathrm{j}\cdot\omega}$ [7]. As seen in (1), the bilinear transformation employs two parameters: 1) the sampling frequency, f_s, which must satisfy the Nyquist criterion ($f_s \geq 2 \cdot f_{\max}$, being $f_{\max}$ the maximum frequency in the specifications with significant spectral content) and 2) the pre-warping frequency, f_p, which can be fixed to make the digital requirements more relaxed, as it will be shown later.

B. Digital Design

There are many efficient techniques to design a digital filter satisfying the (digital) specifications obtained in the above subsection. Furthermore, they are readily accessible and subject to a continuous improvement [7].

In this letter, we have used an iterative quasi-Newton type algorithm, available in MATLAB, which minimizes the error function defined as the p-norm of the weighted difference between the target filter frequency response and the designed one. The value of p is increased step-by-step so that the resulting design tends to minimize the maximum modulus of the error function. The algorithm provides some parameters that allow us to control the design process with high flexibility. The most relevant ones are the filter order, the weighting vector and the range of values for p. As a result, the poles and zeros of the digital filter are obtained.

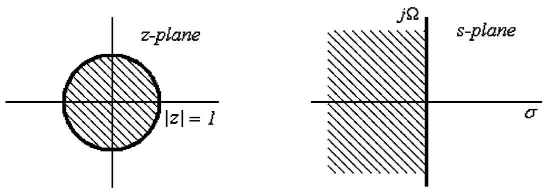

Fig. 2. Mapping between z-plane and s-plane.

C. Digital to Analog: Inverse Bilinear Transformation

The bilinear transformation already used in Section II-A has been also typically employed to transform classical analog filters, such as Butterworth or Chebyshev, into recursive digital filters [7]. In this letter, we will use this transformation in the opposite way, for the first time to our knowledge, in order to transform the digital filter into the pursued analog one. This gives rise to the inverse bilinear transformation, defined as in the following equation:

$$s = a \cdot \frac{z-1}{z+1}. \tag{2}$$

Using it, the analog poles and zeros are obtained from the digital poles and zeros of Section II-B.

The inverse bilinear transformation has the following advantageous features.

1) *Mapping Properties:* Inspecting (2), three important regions can be distinguished as depicted in Fig. 2: the shaded(white) region in the z-plane is mapped to the shaded(white) region in the s-plane, while the unit circle in the z-plane ($z = e^{j \cdot \omega}$) is mapped to the imaginary axis ($s = j \cdot \Omega$) in the s-plane. As a consequence, it can be shown that a causal and stable digital filter is transformed into a causal and stable analog filter as intended.
2) *Warping Effect:* As it can be easily inferred, the analytical relationship that holds between the digital (ω) and the analog (f) frequencies is not linear. This produces the so-called warping effect. However, since the whole design method includes also the bilinear transformation in subsection A, the total effect cancels out completely, providing a distortion-free procedure.

D. Synthesis of the Microwave Filter

The application of the inverse bilinear transformation results in the poles and zeros, and hence in the analytical frequency response, $S_{21}(s)$, of the microwave filter. From these data, different classical implementation procedures may be followed [2]. In this letter, the microwave filter will be implemented as a stepped-impedance filter in microstrip technology after deriving the associated ladder network of lumped elements. However, it is important to stress that other technologies (even nonplanar) and/or implementation schemes could be used [2].

The ladder network of lumped elements that implements the frequency response, $S_{21}(s)$, is shown in Fig. 3. The values of the elements L_i, C_j can be easily obtained using the classical continuous fraction expansion method as explained in [8].

The stepped-impedance technique allows us to implement the lumped-element ladder network in microstrip as a transmission line alternating high impedance (Z_{high}) and low impedance

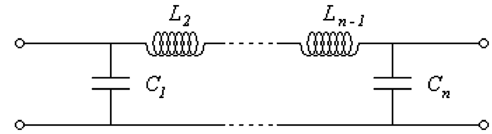

Fig. 3. Ladder network of lumped-element.

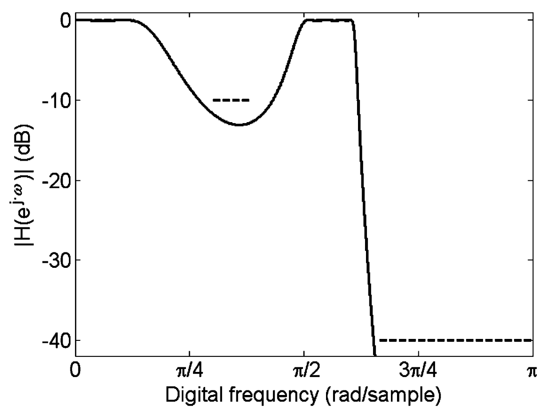

Fig. 4. Digital specifications (horizontal dashed lines) together with the frequency response of the obtained digital filter.

(Z_{low}) sections. The following design equations provide the required section lengths, l, in meters [2]

$$l_{i,\mathrm{high}} = \frac{v_p \cdot L_i}{Z_{\mathrm{high}}}; \quad l_{j,\mathrm{low}} = v_p \cdot C_j \cdot Z_{\mathrm{low}} \tag{3}$$

where v_p is the phase velocity in the microstrip line. The discontinuities between the high- and low-impedance sections produce some fringing capacitances [2], whose parasitic effect can be compensated by means of a simple final optimization process in the section lengths.

III. Example

To demonstrate the proposed design methodology, in this section we are going to design an 11th order filter implemented in microstrip technology. Once the analog specifications are chosen (passbands from dc to 1 GHz and from 4 to 5.5 GHz, with return losses around 15 dB; and stopbands from 2 to 3 GHz with rejection level around 10 dB, and from 7 to 9 GHz with rejection level better than 40 dB), the digital specifications are obtained using the bilinear transformation, (1), as explained in Section II-A, with $f_s = 2 \cdot f_{\max} = 14$ GHz and f_p fixed at 2.4 GHz to make the digital specifications more relaxed at the first stopband.

The digital filter design has been carried out as explained in Section II-B, and its frequency response is depicted in Fig. 4 together with the digital specifications. Applying the inverse bilinear transformation as explained in Section II-C, (2), the digital poles and zeros obtained are mapped into the analog domain. The resulting analytical frequency response of the analog filter is depicted in Fig. 5 together with the target analog specifications which are properly satisfied.

From the analog poles and zeros obtained in the mapping process, a lumped-element ladder network is obtained as explained in Section II-D. The capacitance values in pF are: $C_1 = 1.45$, $C_3 = 2.39$, $C_5 = 0.69$, $C_7 = 0.71$, $C_9 = 2.28$, and $C_{11} =$

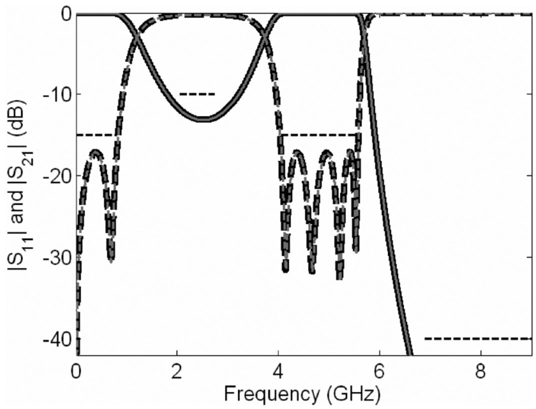

Fig. 5. Analog specification (horizontal dashed lines) together with the simulated $|S_{21}|$ (continuous line) and $|S_{11}|$ (dashed line) parameters of the analytical analog filter (thick line) and the lumped-element ladder network (thin grey line).

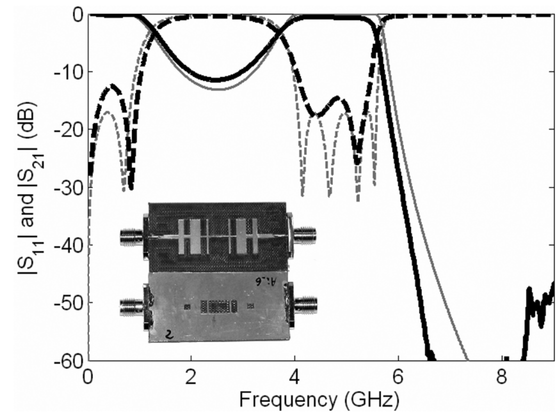

Fig. 6. $|S_{21}|$ (continuous line) and $|S_{11}|$ (dashed line) for the target analog response (thin grey line) and for the measured microstrip filter (thick line).

1.3. The inductance values in nH are: $L_2 = 1.08$, $L_4 = 2.43$, $L_6 = 6.42$, $L_8 = 2.38$, and $L_{10} = 1.15$. The frequency response of this lumped-element network has been simulated using Agilent ADS and it is shown in Fig. 5. As it can be seen, it matches perfectly with the desired analytical analog response.

The microwave filter has been finally implemented as a stepped-impedance filter in microstrip technology using a CUCLAD-250 LX substrate ($\varepsilon_r = 2.43$, dielectric thickness $h = 0.49$ mm) with 50 Ω input and output ports. The low impedance sections are realized using a strip-width $W_{\max} = 12$ mm ($Z_{\text{low}} = 8.92\ \Omega, \varepsilon_{\text{eff}} = 2.32$). In order to ease the manufacturing process, the high impedance sections are implemented using two different width values and slots etched in the ground plane [9]: strip-width $W_{\min} = 1$ mm and slot width $S = 2$ mm ($Z_{\text{high},1} = 97.02\ \Omega, \varepsilon_{\text{eff}} = 1.58$) for the first and the last section, and $W_{\min} = 0.2$ mm and $S = 4.5$ mm ($Z_{\text{high},2} = 208.71\ \Omega, \varepsilon_{\text{eff}} = 1.43$) for the remaining sections. The required section lengths are calculated using (3) and optimized as explained in Section II-D. The final lengths in millimeters are: $l_1 = 1.884$, $l_2 = 2.026$, $l_3 = 3.591$, $l_4 = 2.040$, $l_5 = 0.628$, $l_6 = 6.835$, $l_7 = 0.640$, $l_8 = 1.975$, $l_9 = 3.376$, $l_{10} = 2.201$, and $l_{11} = 1.626$. The microstrip filter has been fabricated using a numerical milling machine. Its S_{11} and S_{21} parameters have been measured by means of an Agilent 8722ES Vector Network Analyzer and are depicted in Fig. 6 together with the target analog response. A very good agreement is obtained between the target response and the measurements, confirming the very good performance of the proposed technique.

IV. Conclusion

We have proposed a novel technique to design microwave filters with arbitrary frequency response based on digital methods. The novel technique allows the microwave designer to take immediate advantage of the efficient and sophisticated digital filter design methods that are widely available in commercial software packages. The technique has been successfully validated using a stepped-impedance implementation in microstrip technology, but it can be also similarly applied with no restrictions to other technologies or implementation schemes.

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