



# Data Structures

## Basic Abstract Data Types - List

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# ◆ Content

- Definition
- Operations of Lists
- Implementation of Lists
  - Array Implementation of Lists
  - Linked List

- A **list** is a sequence of zero or more elements of a given type (which we generally call the *elementtype*).
- We often represent such a list by a comma-separated sequence of elements:  $a_1, a_2, a_3, \dots, a_n$  where  $n \geq 0$ , and each  $a_i$  is of type *elementtype*.
- The number  $n$  of elements is said to be the **length** of the list.
- If
  - $n = 0$ , we have an **empty list**, one which has no elements.
  - $n \geq 1$ , we say that  $a_1$  is the **first** element and  $a_n$  is the **last** element.
- Elements of list can be linearly ordered according to their position on the list. We say the element:
  - $a_i$  **precedes**  $a_{i+1}$  for  $i = 1, 2, \dots, n-1$ ,
  - $a_i$  **follows**  $a_{i-1}$  for  $i = 2, 3, \dots, n$ .
  - $a_i$  is at **position**  $i$ .

## ◆ Operations of Lists

Operation	Use
<b>makenullList(L)</b>	Causes L to become an empty list.
<b>emptyList(L)</b>	Check if list L is empty or not.
<b>fullList(L)</b>	Check if list L is full or not.
<b>first(L)</b>	Returns the first position on list L. If L is empty, the position returned is endList(L).
<b>endList(L)</b>	Return the position following position n in an n-element list L.
<b>insertList(x,P,L)</b>	Insert x at position P in list L. If list L has no position p, the result is undefined.
<b>deleteList(P,L)</b>	Delete the element at position P of list L. The result is undefined if L has no position P or if $P = \text{endList}(L)$ .

Operation	Use
<b>retrieve(P,L)</b>	Returns the element at position P on list L. The result is undefined if $P = \text{endList}(L)$ or if L has no position P.
<b>locate(x,L)</b>	Returns the position of x on list L. If x appears more than once, then the position of the first occurrence is returned. If x does not appear at all, then $\text{endList}(L)$ is returned.
<b>next(P,L)</b>	Return the positions following position P on list L. If P is the last position on L, then $\text{next}(P, L) = \text{endList}(L)$ . next is undefined if P is $\text{endList}(L)$ . The result is undefined if L has no position P.
<b>previous(P,L)</b>	Return the positions preceding position P on list L. previous is undefined if P is 1. The result is undefined if L has no position P.
<b>printList(L)</b>	Print the elements of L in the order of occurrence.

## ◆ Operations of Lists

Example: What operations do we call if we want to add an element  $x$  to the beginning or the end of the list?

- Add an element  $x$  to the **beginning** of the list

```
insertList(x, first(L), L)
```

- Add an element  $x$  to the **end** of the list

```
insertList(x, endList(L), L)
```

Example: Let the function  $swap(p,q)$  swap the contents of two elements at positions  $p$  and  $q$  in the list. Use operations on lists, write a function  $sort$  (where input parameter is a list  $L$ ) to sort the list in ascending order?

```
void sort(List L){
    Position p,q;
    p= first(L);
    while (p!=endList(L)){
        q=next(p,L);
        while (q!=endList(L)){
            if (retrieve(p,L) > retrieve(q,L))
                swap(p,q);
            q=next(q,L);
        }
        p=next(p,L);
    }
}
```

# ◆ Array Implementation of Lists (Array List)

- Declaration of data
- Declaration of operations



# ◆ Array Implementation of Lists (Array List)

Index	Elements	Position
0	First Element	1
1	Second Element	2
2	...	3
Last-1 → 3	Last Element	4 ← Last
4		5
...		...
MaxLength-1		MaxLength

Array

- Use an array (**Elements**) to store contiguous elements, starting from the first position.
- Must estimate the maximum number of elements in the list (*MaxLength*).
- Must store the current length of the list (**Last**).

# ◆ Delaration of Data

## A structure in C

- Declaring (creating, defining) a structure:

```
typedef struct
```

```
{  
    <datatype>    <elementname1>;
```

```
    ...
```

```
    <datatype>    <elementnamen>;
```

```
} <structurename>;
```

- Declaring a structure variable:

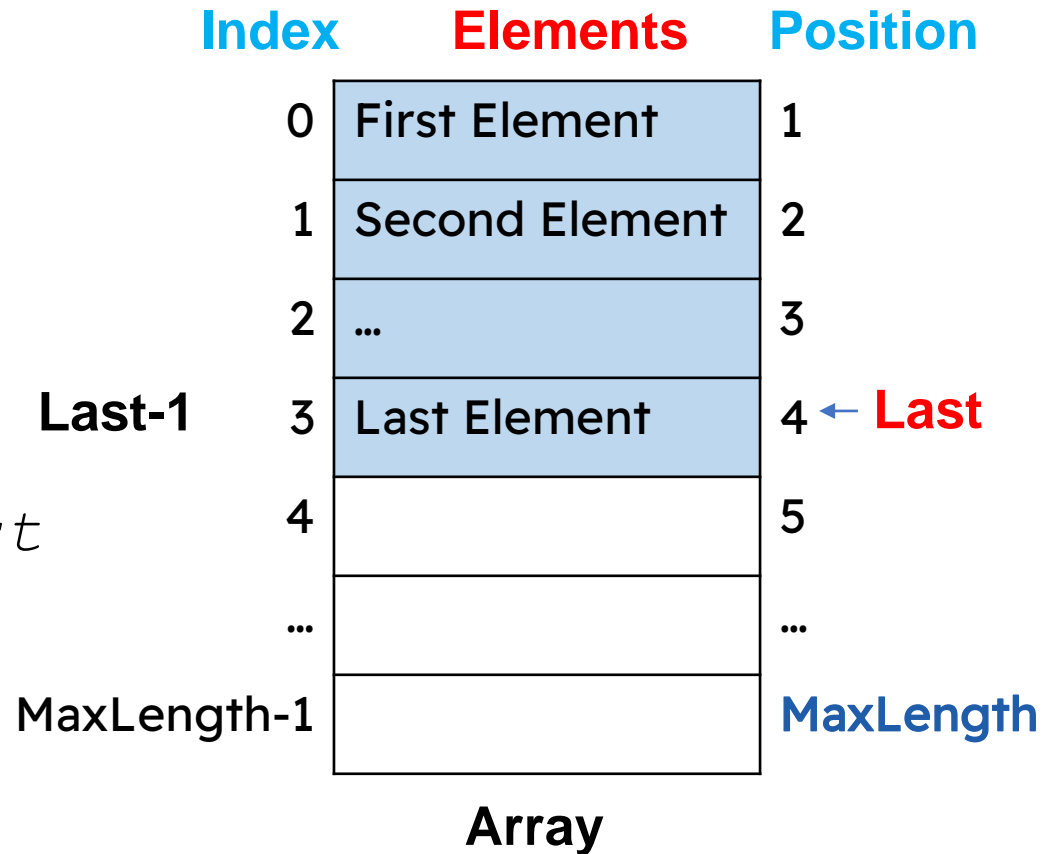
```
<structurename> <variablename>;
```

Index	Elements	Position
0	First Element	1
1	Second Element	2
2	...	3
3	Last Element	4 ← Last
4		5
...		...
MaxLength-1		MaxLength

Array

```
// the maximum size of list
#define MaxLength n
// the element type of list
typedef datatype ElementType;
// the position type of elements
typedef int Position;
typedef struct {
    // array consisting elements of list
    ElementType Elements[MaxLength];
    // the current length of list
    Position Last;
} List;

List L;
```



## Declaration of Data

Example: Declare a list that can contain up to 300 integers?

```
// the maximum size of list
```

```
#define MaxLength 300
```

```
// the element type of list
```

```
typedef int ElementType;
```

```
// the position type of elements
```

```
typedef int Position;
```

```
typedef struct {
```

```
    // array consisting elements of list
```

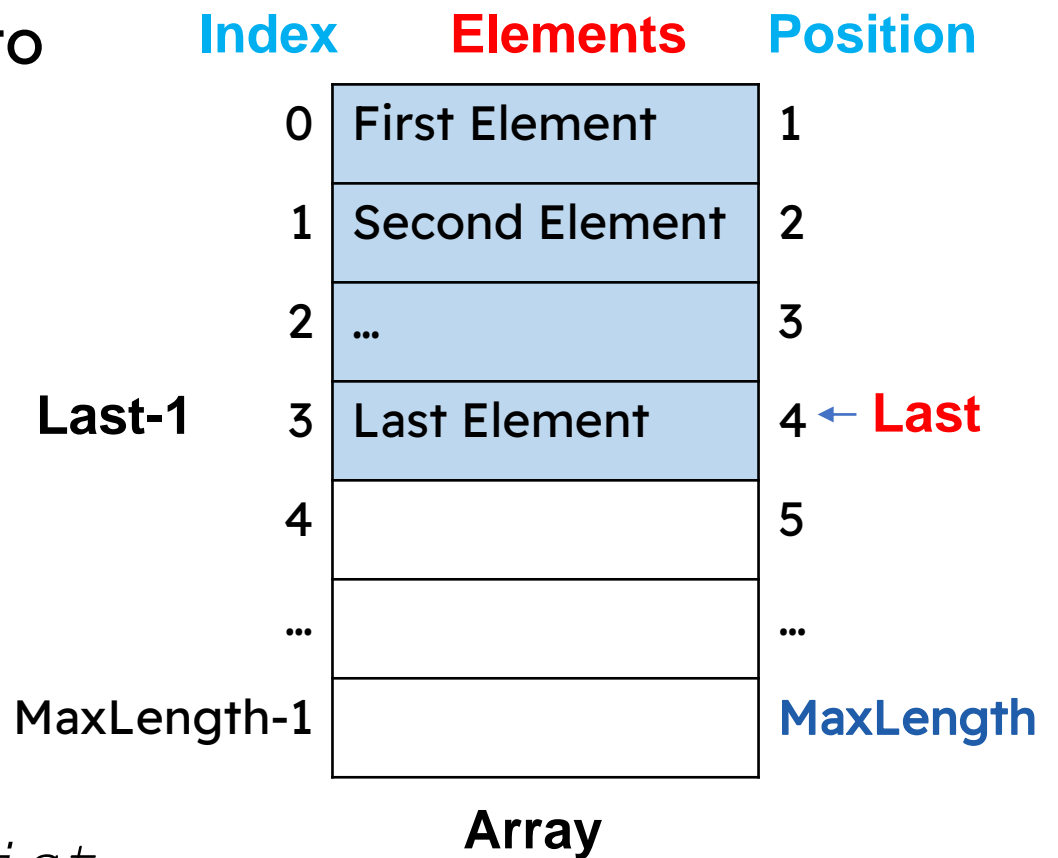
```
    ElementType Elements[MaxLength];
```

```
    // the current length of list
```

```
    Position      Last;
```

```
} List;
```

```
List L;
```



## Declaration of Data

# Exercise

- A line of text on the screen has a maximum of 80 characters. In fact, at a particular time the number of characters in the line is less than 80 so an integer variable is maintained to hold the actual number of characters in the line.
- To represent a line of text on the screen, we use an array list where each element of the line is a character.  
For example: 1 line of text

*Toi di hoc. Ban o nha.*

is recorded as a list with the following form.

DÒNG VĂN BẢN

Chỉ số mảng	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	..		(Số ký tự tối đa) -1
Phần tử	T	o	i		d	i		h	o	c	.		B	a	n		o		n	h	a	.				
Vị trí	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	...		(Số ký tự tối đa)

↑  
Số ký tự

- Let's write a declaration for the text line data type Line (where the number of maximum characters is 80) with the elements described as shown above by using the array list.

```
// the maximum size of list
#define MaxLength n
// the element type of list
typedef datatype ElementType;
// the position type of elements
typedef int Position;
typedef struct {
    // array consisting elements of list
    ElementType Elements[MaxLength];
    // the current length of list
    Position Last;
} List;
List L;
```

DÒNG VĂN BẢN

Chỉ số mảng	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	..		(Số ký tự tối đa) -1
Phần tử	T	o	i		d	i		h	o	c	.		B	a	n		o		n	h	a	.				
Vị trí	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	...		(Số ký tự tối đa)

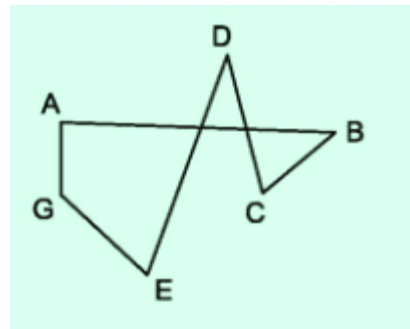
Số ký tự

Index	Elements	Position
0	First Element	1
1	Second Element	2
2	...	3
<b>Last-1</b>	Last Element	4 ← <b>Last</b>
4		5
...		...
MaxLength-1		MaxLength

Array

## Exercise

- A point in 2-dimensional space is described as a structure consisting of two  $x$  and  $y$  fields of integer type. A polygon is made up of many points, for example the polygon below is made up of 6 points:  $A(x_A, y_A)$ ,  $B(x_B, y_B)$ ,  $C(x_C, y_C)$ ,  $D(x_D, y_D)$ ,  $E(x_E, y_E)$ ,  $G(x_G, y_G)$ .



- Suppose we represent a polygon by an array list with a maximum of 100 points. Write a declaration for the **Polygon** type representing polygons as described above.

# ◆ Declaration of Operations

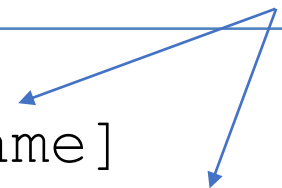
## ■ Defining a function

*Call by value*

*Call by reference*

*Formal parameters (arguments)*

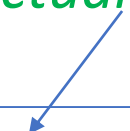
```
result_type  function_name (  
    [parameter_type  parameter_name]  
    [, [parameter_type  parameter_name] [...]] )  
{  
    [declaration of local variables]  
    [statements defining what the function does]  
    [return [expression];]  
}
```



## ■ Calling a function in C

*Actual parameters*

```
function_name ([a list of parameters])
```





# ◆ Declaration of Operations

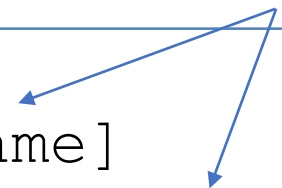
## ■ Defining a function

*Call by value*

*Call by reference*

*Formal parameters (arguments)*

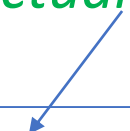
```
result_type function_name (  
    void      [parameter_type    parameter_name]  
              [, [parameter_type    parameter_name] [...]] )  
{  
    [declaration of local variables]  
    [statements defining what the function does]  
    [return [expression]]  
}
```



## ■ Calling a function

*Actual parameters*

```
function_name ([a list of parameters])
```



# Declaration of Operations

Index	Elements	Position
0	First Element	1
1	Second Element	2
2	...	3
3	Last Element	4 ← Last
4		5
...		...
MaxLength-1		MaxLength

Array

Operator name
makenullList(L)
emptyList(L)
fullList(L)
first(L)
endList(L)
insertList(x,P,L)
deleteList(P,L)
retrieve(P,L)
locate(x,L)
next(P,L)
previous(P,L)
printList(L)

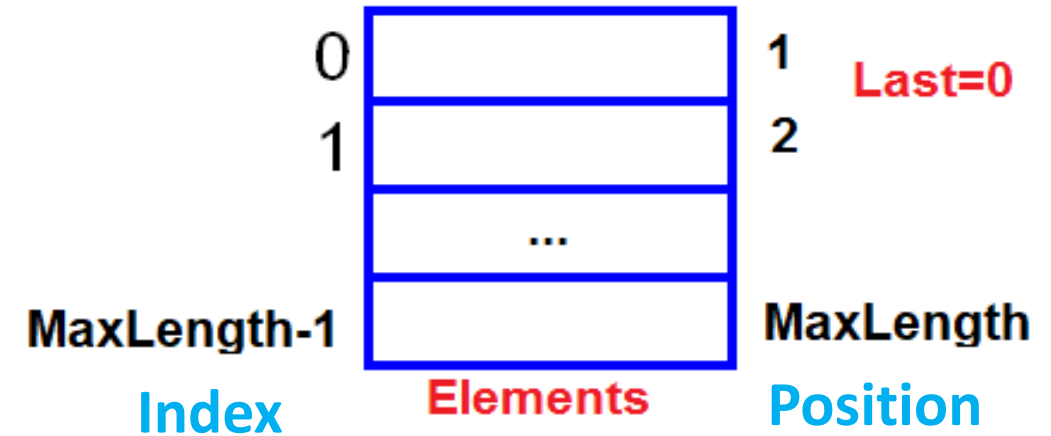
```

result_type  function_name (
    [parameter_type  parameter_name]
    [, [parameter_type  parameter_name] [...]] )
{
    [declaration of local variables]
    [statements defining what the function does]
    [return [expression] ;]
}

```

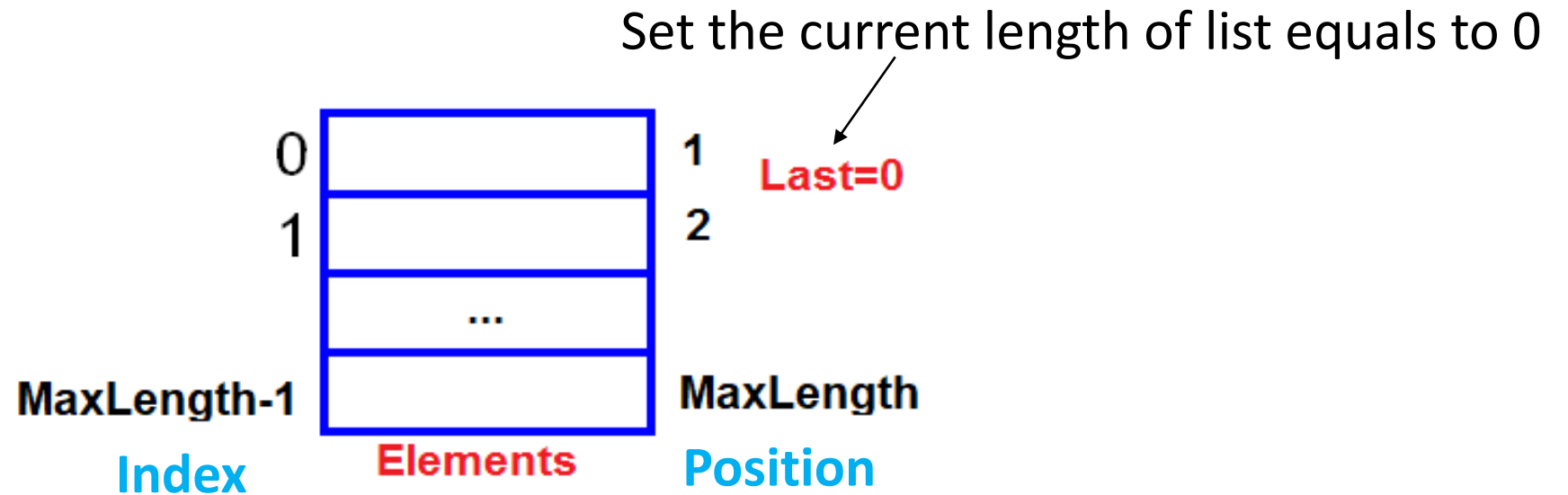
# Creating an Empty List

```
// the maximum size of list
#define MaxLength n
// the element type of list
typedef datatype ElementType;
// the position type of elements
typedef int Position;
typedef struct {
    // array consisting elements of list
    ElementType Elements[MaxLength];
    // the current length of list
    Position Last;
} List;
```



```
result_type function_name (
    [parameter_type parameter_name]
    [, [parameter_type parameter_name] [...]] )
{
    [declaration of local variables]
    [statements defining what the function does]
    [return [expression];]
}
```

## ◆ Creating an Empty List

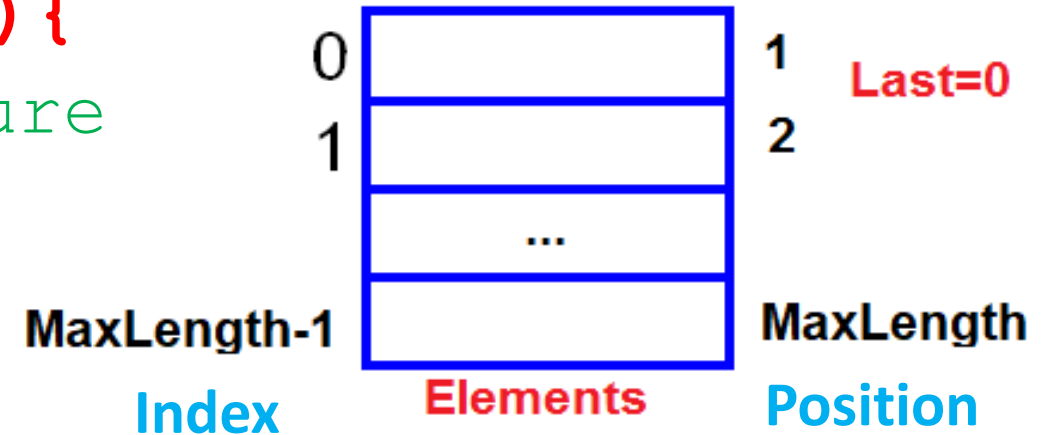


```
void makenullList (List *pL) {  
    // Pointer to structure  
    pL->Last=0;  
}
```

(\*pL).Last=0

## ◆ Creating an Empty List

```
void makenullList(List *pL) {  
    // Pointer to structure  
    pL->Last=0;  
}
```



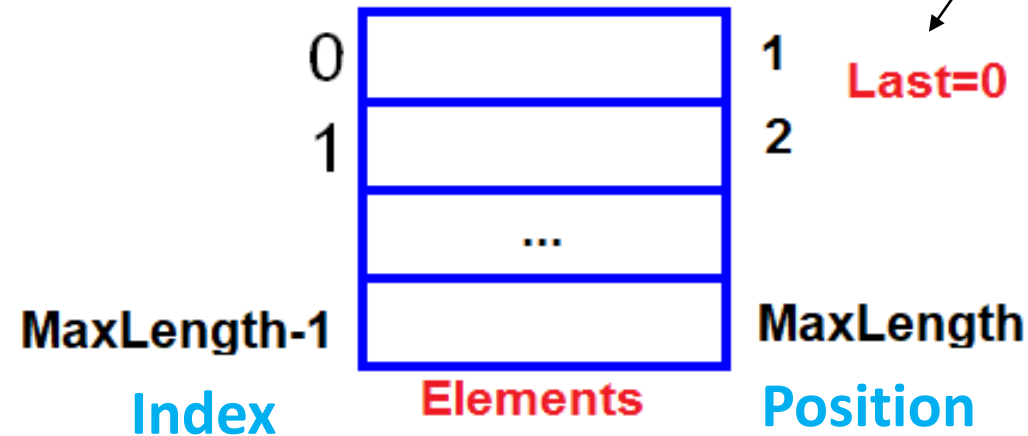
Suppose that we have a variable declaration:

```
List L;
```

Write a call to the `makenullList()` function to initialize an empty list `L`?

## ◆ Creating an Empty List

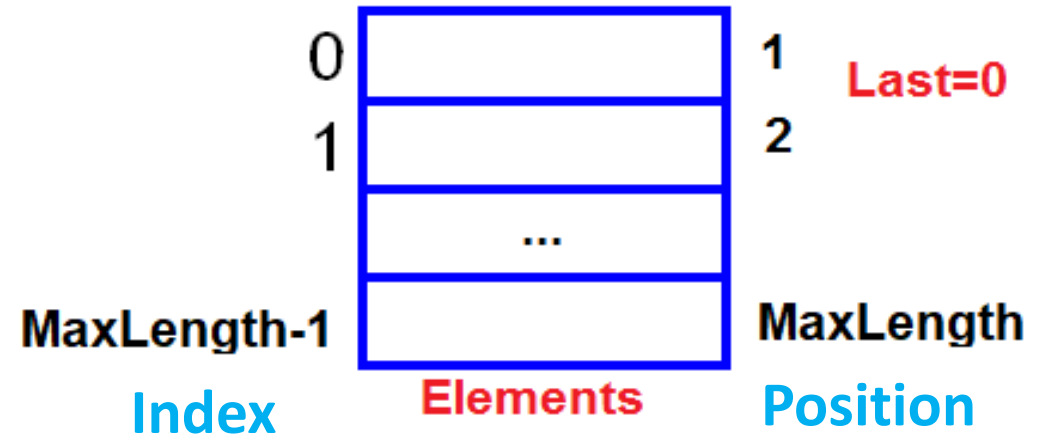
Set the current length of list equals to 0



- Write the function `makenullList` that takes no parameters and returns an empty list?

```
List makenullList () {  
    List L; // L is a structure variable  
    L.Last=0;  
    return L; }
```

## ◆ Creating an Empty List

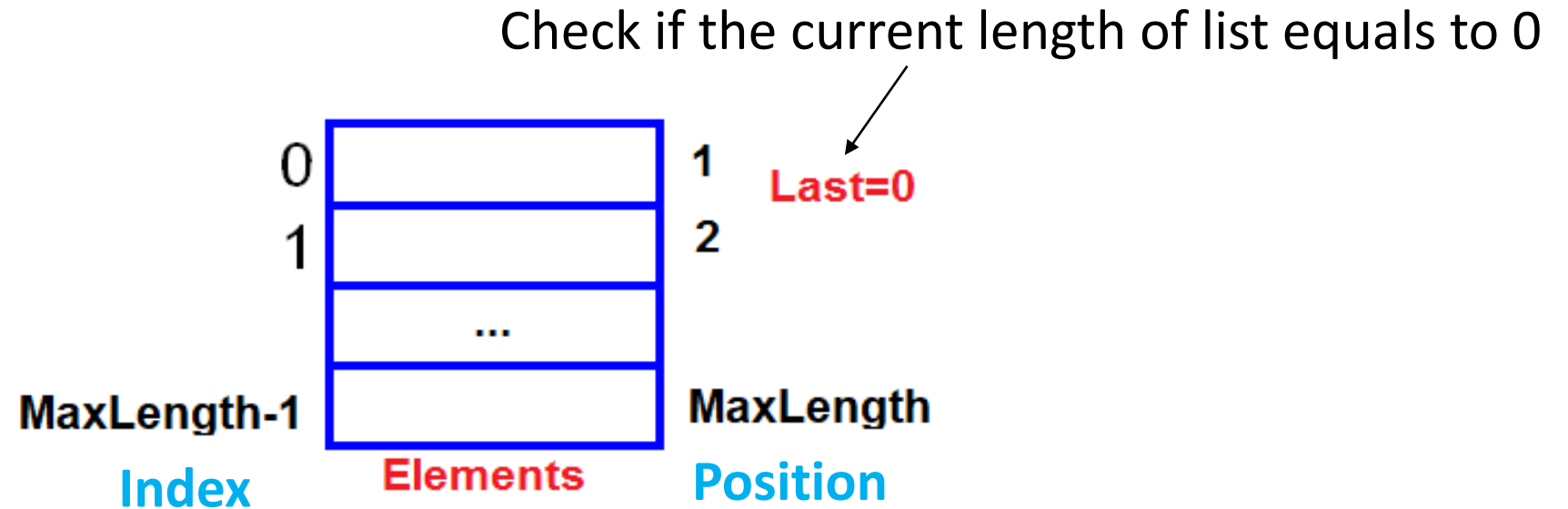


- Given the function to initialize an empty list.

```
List makenullList () {  
    List L;  
    L.Last=0;  
    return L;    }
```

- Suppose that we have a variable declaration: `List L;`
- Write a call to the `makenullList()` function to initialize an empty list `L`?

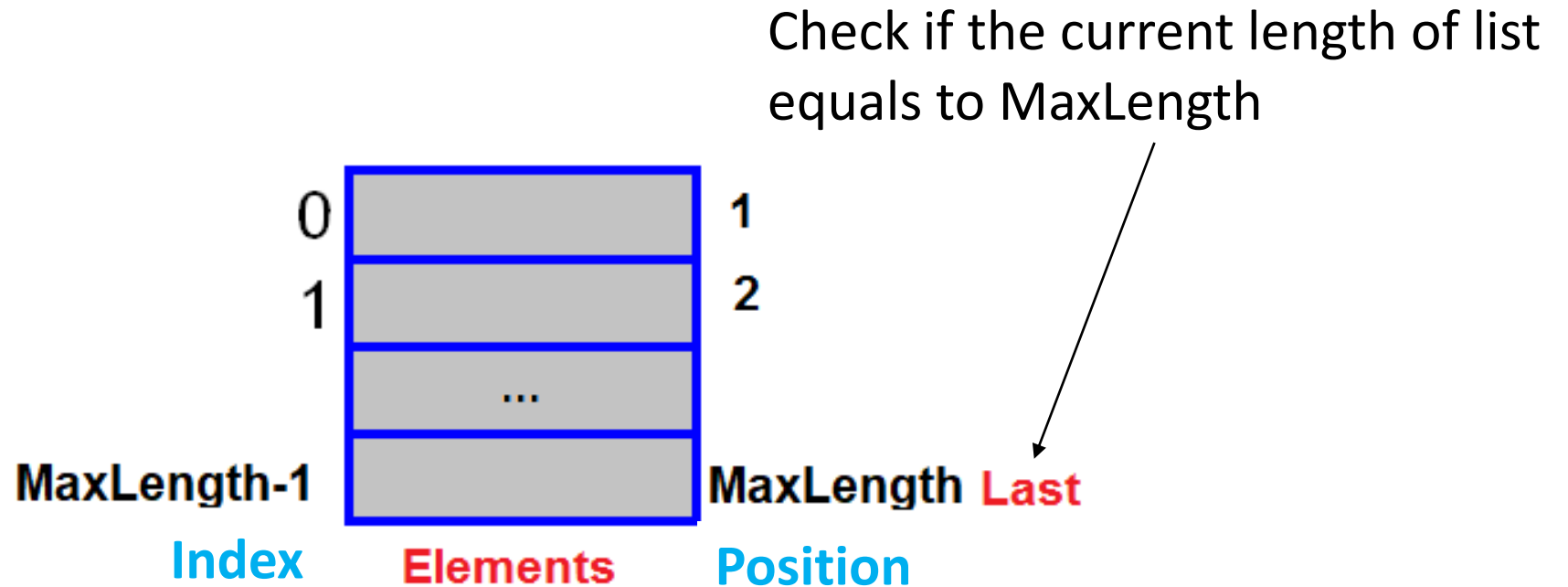
## ◆ Checking an Empty List



```
int emptyList(List L) {  
    return L.Last==0;  
}
```



## ◆ Checking a Full List



```
int fullList(List L)    {  
    return L.Last==MaxLength;  
}
```

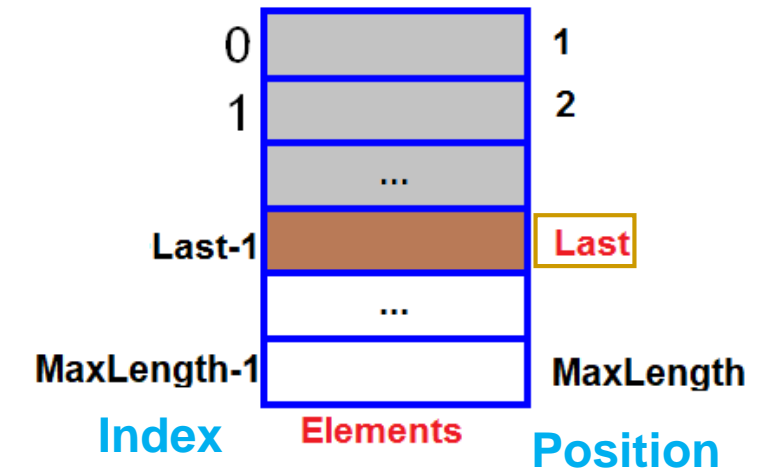
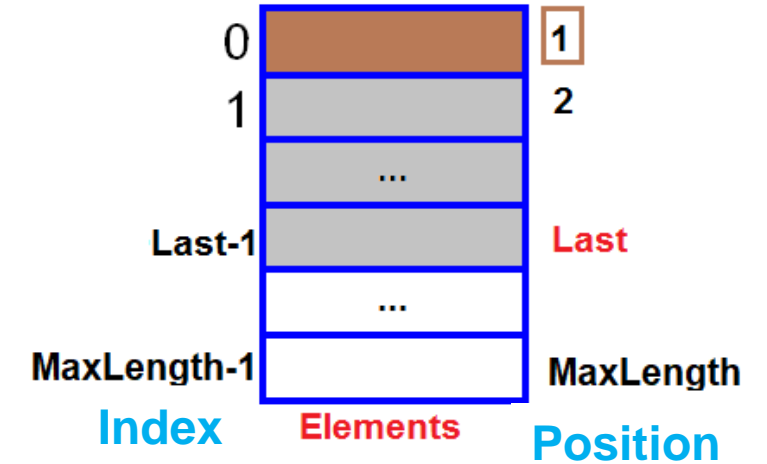
## ◆ Determining Positions (first, endList)

- Determining the first position of list.

```
Position first(List L) {  
    return 1;  
}
```

- Determining the position after the last element of list.

```
Position endList(List L) {  
    return L.Last+1;  
}
```



## ◆ Inserting an Element

- Example: insert the element  $x='k'$  into position  $P=3$  in the list  $L$ ?
  - Case 1: If  $L$  is the full list  
=> show the error message

Index	Elements	Position
0		1
1		2
2		3
3		4
4		5
...		...
MaxLength-1		MaxLength ← Last

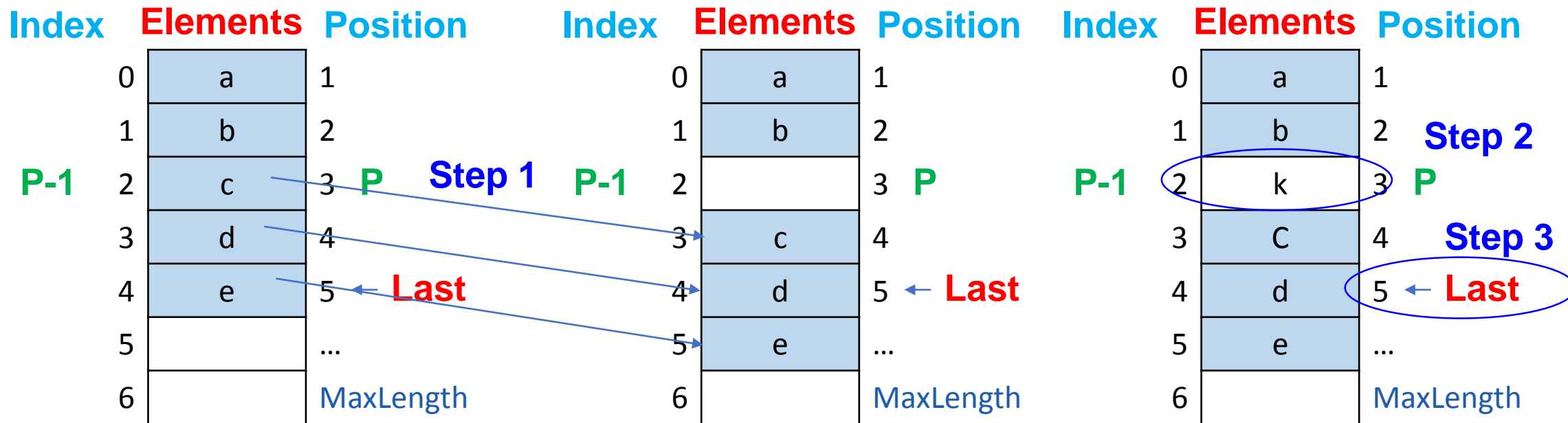
## ◆ Inserting an Element

- Example: insert the element  $x='k'$  into position  $P=3$  in the list  $L$ ?
  - Case 2: If position  $P$  is invalid (i.e.  $P \notin [1..Last+1]$ )  
 $\Rightarrow$  show the error message

Index	Elements	Position
0		1
1		2
2		3
3		4 ← Last
4		5
...		...
MaxLength-1		MaxLength

## ◆ Inserting an Element

- Example: insert the element  $x='k'$  into position  $P=3$  in the list  $L$ ?
  - Case 3:



- Step 1: Moving elements at  $p$  and following positions (from position  $P$  to position Last) to the next higher position.
- Step 2: Inserting  $x$  at position  $P$
- Step 3: Increasing the current length of list by 1.



## Algorithm

# Inserting an Element

To insert  $x$  into position  $P$  of the list  $L$ , we do the following:

- If the list is full, an error message is reported.
- Else,
  - If position  $P$  is invalid, an error message is reported.
  - Else:
    - Moving elements (from position  $P$  to position Last) to the next higher position.
    - Inserting new element  $x$  at position  $P$ .
    - Increasing the current length of list by 1.

```

void insertList(ElementType x, Position P, List *pL) {
    if (pL->Last==MaxLength)
        printf("List is full");
    else if ((P<1) || (P>(pL->Last+1)))
        printf("Position is invalid");
    else {
        Position Q;
        for (Q=pL->Last; Q>=P; Q--)
            pL->Elements[Q]=pL->Elements[Q-1];
        pL->Elements[P-1]=x;
        pL->Last++;
    }
}

```

## Inserting an Element

```

void insertList(ElementType x, Position P, List *pL) {
    if (pL->Last==MaxLength) → fullList(*pL)
        printf("List is full");
    else if ((P<1) || (P>(pL->Last+1)))
        printf("Position is invalid");
    else {
        Position Q;
        (P<first(*pL) || (P>endList(*pL)))
        for (Q=pL->Last; Q>=P; Q--)
            pL->Elements[Q]=pL->Elements[Q-1];
        pL->Elements[P-1]=x;
        pL->Last++;
    }
}

```

## Inserting an Element

- Determining the complexity of insertList()?



## ◆ Inserting an Element

- Using operations on array lists, write the function `readList()` to input a list of  $n$  integers?

## ◆ Deleting an Element

- Example: delete the element at position P=4 of the list L?

- Case 1: If L is the empty list  
=> show the error message

Index	Elements	Position
0		1
1		2
2		3
3		4
4		5
...		...
MaxLength-1		MaxLength

**Last=0**

## ◆ Deleting an Element

- Example: delete the element at position P=4 of the list L?

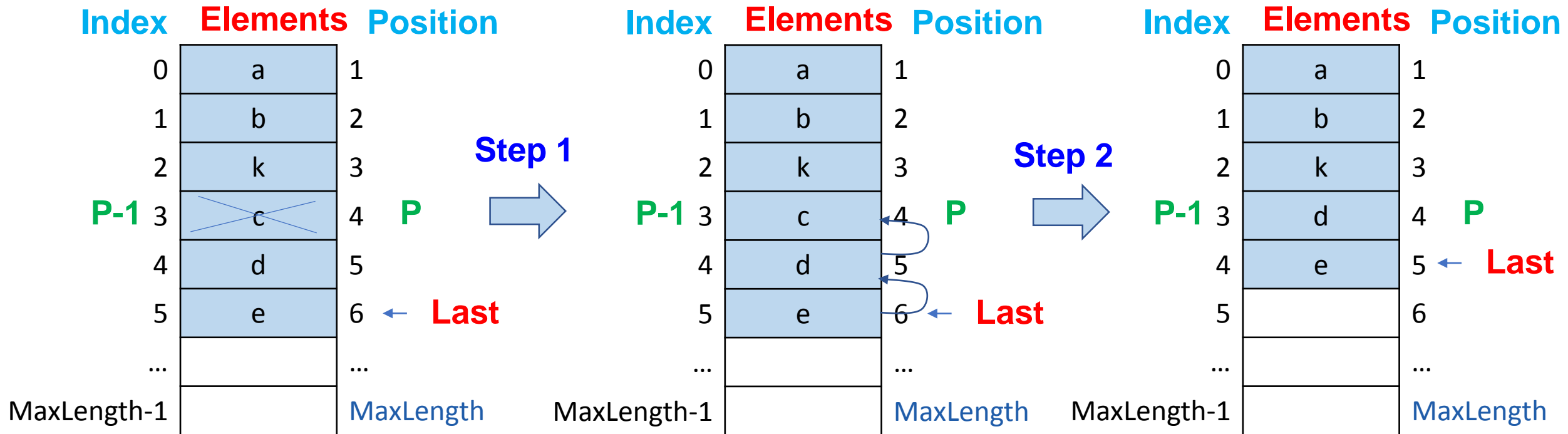
- Case 2: If position P is invalid (i.e.  $P \notin [1..Last]$ )  
=> show the error message

Index	Elements	Position
0		1
1		2
2		3
3		4 ← Last
4		5
...		...
MaxLength-1		MaxLength

- Example: delete the element at position  $P=4$  of the list  $L$ ?



- Case 3:



- Step 1: Moving elements from position  $P+1$  to position Last to the previous lower position.
- Step 2: Reducing the current length of list by 1.



## Algorithm

# Deleting an Element

To delete the element at position  $P$  of the list  $L$ , we do the following:

- If position  $P$  is invalid, an error message is reported.
- Else,
  - If the list is empty, an error message is reported.
  - Else:
    - Moving elements from position  $P+1$  to position Last to the previous lower position.
    - Reducing the current length of list by 1.

```

void deleteList(Position P,List *pL){
    if ((P<1) || (P>pL->Last))
        printf("Position is invalid");
    else if (emptyList(*pL))
        printf("List is empty");
    else{
        Position Q;
        for (Q=P;Q<pL->Last;Q++)
            pL->Elements[Q-1]=pL->Elements[Q];
        pL->Last--;
    }
}

```

## Deleting an Element

```

void deleteList(Position P,List *pL){
    if ((P<1) || (P>pL->Last))
        printf("Position is invalid");
    else if (emptyList(*pL))
        printf("List is empty");
    else{
        Position Q;
        for (Q=P;Q<pL->Last;Q++)
            pL->Elements[Q-1]=pL->Elements[Q];
        pL->Last--;
        for (Q=P-1;Q<pL->Last-1;Q++)
            pL->Elements[Q]=pL->Elements[Q+1];
    }
}

```

## Deleting an Element

pL->Last==0

**for** (Q=P;Q<pL->Last;Q++)

pL->Elements[Q-1]=pL->Elements[Q];

pL->Last--;

**for** (Q=P-1;Q<pL->Last-1;Q++)

pL->Elements[Q]=pL->Elements[Q+1];

## ◆ Determining Positions (next, previous)

- Determining the previous position of P

```
Position next (Position P, List L) {  
    return P+1;  
}
```

- Determining the next position of P

```
Position previous (Position P, List L) {  
    return P-1;  
}
```

Index	Elements	Position
0		1
...		...
P-1	Element at P	P
...		...
Last-1		Last
...		...
...		...
MaxLength-1		MaxLength



## ◆ Returning the Element

- Returning the element at position P of list L

Index	Elements	Position
0		1
...		...
P-1	Element at P	P
...		...
Last-1		Last
...		...
...		...
MaxLength-1		MaxLength

```
ElementType retrieve(Position P, List L) {  
    return L.Elements[P-1];  
}
```



## Algorithm

# Searching the Element $x$

To find the element  $x$  of the list  $L$ , we do the following:

- Starting from the first element of list, we search from the beginning of list until we find  $x$  or the end of list.
  - If the element at position  $P$  equals to  $x$   $\text{retrieve}(P, L) == x$  then stop searching.
  - Else (if the element at position  $P$  differs from  $x$ ), then go to the next position  $P = \text{next}(P, L)$
- Returns the position of the first occurrence of  $x$  or  $\text{endList}(L)$  if  $x$  does not appear at all.

## ◆ Searching the Element x

```
Position locate (ElementType x, List L) {  
    Position P;  
    int Found = 0;  
    P = first(L);  
    while ((P != endList(L)) && (Found == 0))  
        if (retrieve(P, L) == x) Found = 1;    !Found  
        else P = next(P, L);  
    return P;  
}
```

- Determining the complexity of locate ()?

## ◆ Searching the Element x

```
Position locate (ElementType x, List L) {  
    Position P;  
    P = 1;  
    while (P != L.Last+1)  
        if ( L.Elements[P-1] == x) return P;  
        else P = P+1;  
    return P;  
}
```

- Determining the complexity of locate ()?

## ◆ Printing the Elements

- Determining the complexity of printList ()?

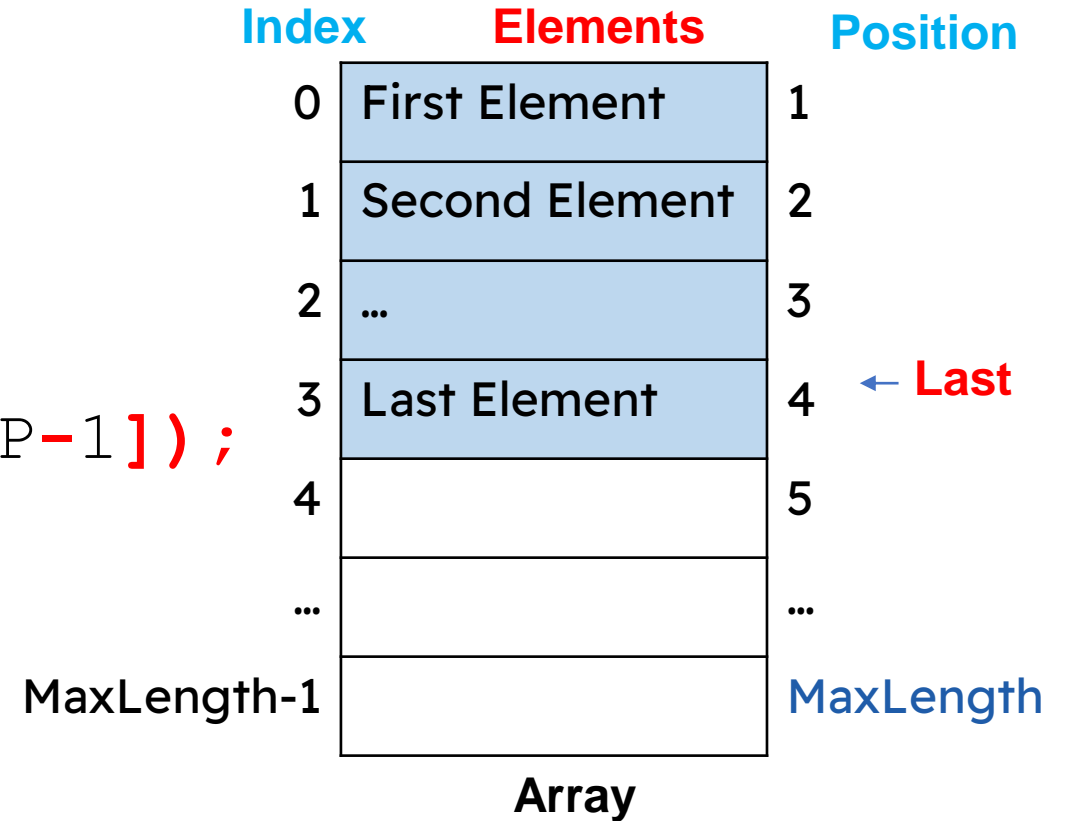
```
void printList (List L) {  
    Position P = first (L);  
    while (P != endList (L)) {  
        printf ("%d ", retrieve (P, L));  
        P = next (P, L);  
    }  
    printf ("\n");  
}
```

Index	Elements	Position
0	First Element	1
1	Second Element	2
2	...	3
3	Last Element	4 ← Last
4		5
...		...
MaxLength-1		MaxLength

Array

## ◆ Printing the Elements

```
void printList(List L) {  
    Position P = 1;  
    while (P != L.Last+1) {  
        printf("%d ", L.Elements[P-1]);  
        P = P+1;  
    }  
    printf("\n");  
}
```



## Exercise

Using operations of array lists to write programs:

- Inputting a list of  $n$  integers and displaying that list on the screen.
- Inserting the element  $x$  at position  $P$  of the list.  $x$  and  $P$  are inputted from the keyboard.
- Deleting the first occurrence of  $x$  from the list.  $x$  is inputted from the keyboard.
- Writing the function `delete_duplicate(LIST &L)` that removes duplicate values in the list.



# Q&A

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